Exploring Countries

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Collected data on over 100 countries, including:

- GDP per capita
- Population
- Regime type
- Military expenditures
- ...

What can we do to understand our data better?

Dimensionality Reduction

Problem: Suppose that we wish to visualize *n* observations with measurements on a set of p features: $X_1, X_2, ..., X_p$ as part of exploratory data analysis. How do we do it?

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Solution: We want to find a low-dimensional representation of the data that captures as much information as possible. This is called dimensionality reduction.

Why Dimensionality Reduction?

Can be used for:

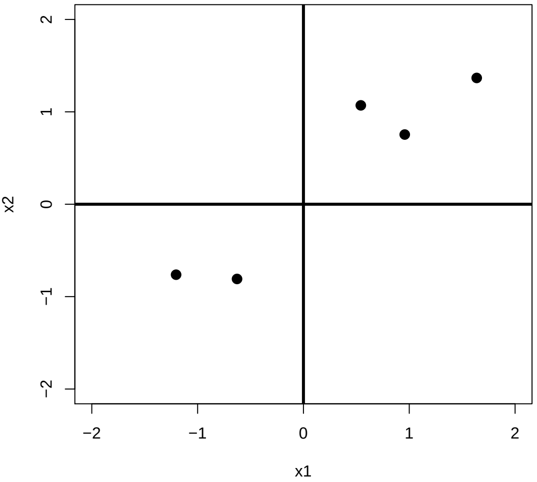
- data summary / exploration
- data visualization (of observations, or variables)
- producing features for use in supervised learning problems.
- data compression (use less memory)

Goal: summarize data in a lower-dimensional space
Method: Principal-Component Analysis
Key Terms:

- Dimensionality Reduction
- Projection
- Principle Component Analysis (PCA)
- Centered
- Scaled
- PC loadings
- PC scores
- Biplot
- Scree Plot
- Proportion of variance explained (PVE)
- Elbow

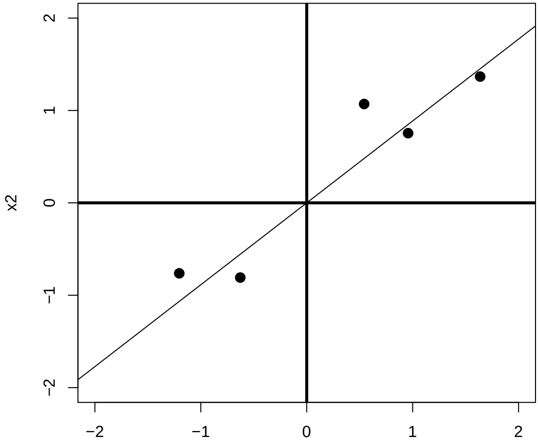
What does it mean to reduce data?

How do we reduce this data from 2 dimensions to 1 dimension?



What does it mean to reduce data?

Identify a new dimension on which to project (aka "map" or "embed") the data.



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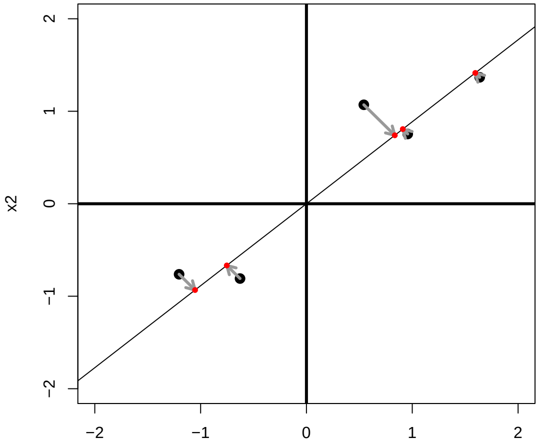
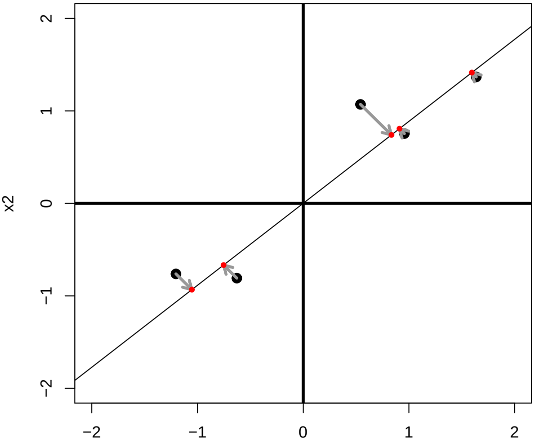
Projecting a point simply means finding the location on the line which is closest to the point.

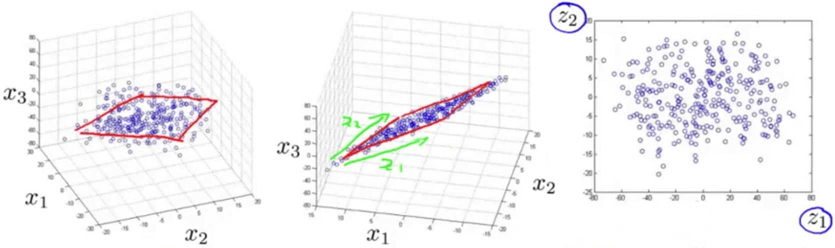


What does it mean to reduce data?

We can now represent each point with this new feature (z_1).
2 dimensions (on x_1 and x_2) $\rightarrow$ 1 dimension (on z_1)



From 3D to 2D



Principle Component Analysis

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- PCA seeks a small number of dimensions that are as interesting as possible.
- By "interesting," we mean the amount that the observations vary along each dimension.

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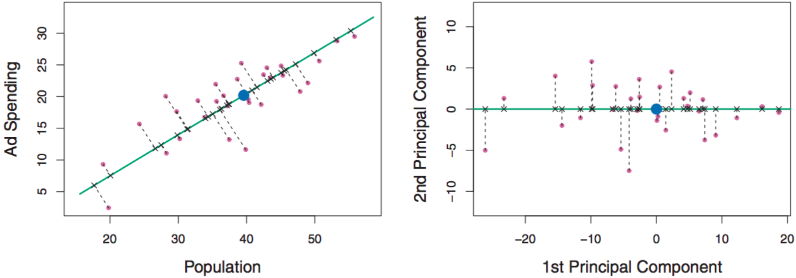
- **Each succeeding component** in turn has the highest variance possible under the constraint that it is orthogonal to the preceding components.
- Finds up to p unique components.

Another interpretation of principle components

The first principal component defines the line that is as close as possible to the data (using average squared Euclidean distance).



PCA Inputs

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1. Centered to have mean zero (i.e. de-meaning).
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   - `colMeans(feature) = 0`

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   - Compute mean of each feature.
   - Subtract each value of a feature by the mean of that feature.
   - `colMeans(feature) = 0`
2. Scaled for comparability.
   - Different features on different scales or units of measurement (number of bedrooms versus price of house)
   - Rescale variables to have comparable ranges of values
   - `standard deviation(feature) = 1`

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3. Principal component eigenvalues: defines "interestness" of each component.

An Example: USArrests

	Murder	Assault	UrbanPop	Rape
Alabama	13.20	236	58	21.20
Alaska	10.00	263	48	44.50
Arizona	8.10	294	80	31.00
Arkansas	8.80	190	50	19.50
California	9.00	276	91	40.60
Colorado	7.90	204	78	38.70

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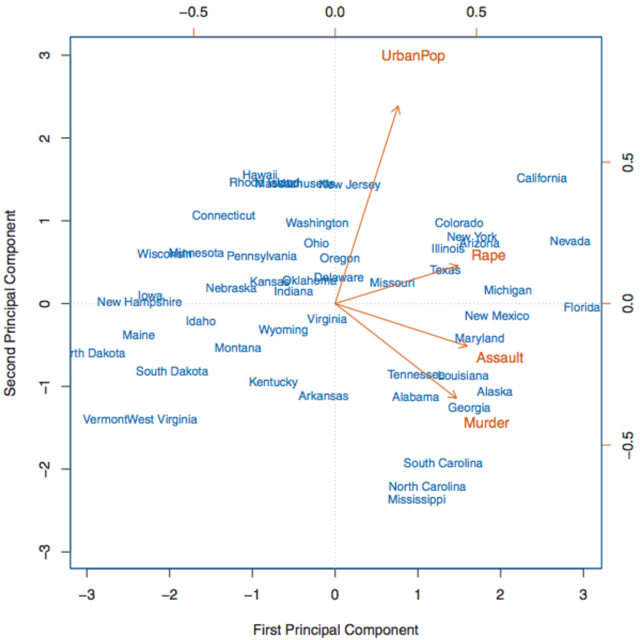
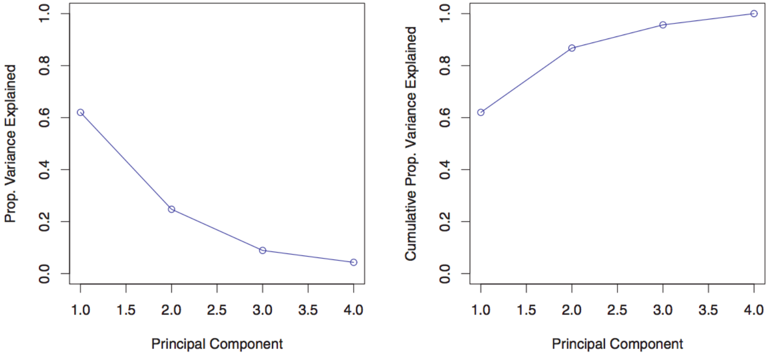
   - Loading vector: length p = 4.
   - Score vector: length n = 50.
3. Plot PC1 against PC2.


-0.5
0.0
0.5
UrbanPop
Rape
Assault
Murder
Hawaii
New Jersey
California
Connecticut
Washington
Colorado
New York
Arizona
Nevada
Ohio
Illinois
Pennsylvania
Oregon
Texas
Nebraska
Indiana
Delaware
Missouri
Michigan
New Hampshire
Florida
Idaho
Virginia
New Mexico
Maine
Wyoming
Maryland
rth Dakota
Montana
South Dakota
Kentucky
Tennessee
Louisiana
Arkansas
Alabama
Alaska
Georgia
VermontWest Virginia
South Carolina
North Carolina
Mississippi
Second Principal Component
First Principal Component
-3
-2
-1
0
1
2
3

How much variation is explained by each principal component?

Scree Plot: Visualize proportion of variance explained (PVE) by each component



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We choose the smallest number of principal components required to explain a sizable amount of variation in the data.

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Eyeball the scree plot, and look for the elbow (the point at which the proportion of variance explained by each subsequent principal component drops off.)

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To the R Code!