Introduction to Machine Learning for Social Science

Class 2: Supervised Learning and Regression

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Fill out this survey:
https://tinyurl.com/ps150b

Put up your yellow sticky note when finished.

Homework 1 Due Jan 23

At which you point, you get another one.

Predicting Election Results

Goal: forecast election winner

Potential predictors

1) GDP Growth
2) Polling data (Incumbent Presidential Popularity)

Conjecture: use relationship in prior elections to predict future election

GDP Growth, popularity $\rightsquigarrow$ input variables.

- *predictors, independent variables, features, attributes, variables*
- X, X_1 (GDP growth), X_2 (popularity)

Election winner $\rightsquigarrow$ output variables.

- *response, dependent variable, outcome, target, labels*
- Y
  - Quantitative (e.g, $15, 3.14, -82000$) $\rightsquigarrow$ Regression
  - Categorical (e.g, Republican/Democrat, 0/1, High/Medium/Low) $\rightsquigarrow$ Classification

We assume some relationship between Y and $X = (X_1, X_2, ..., X_p)$, such that:

$$Y = f(X) + \epsilon$$

where f is fixed but unknown function of $X_1, ..., X_p$, and ϵ is a random error term that is is independent of X and has mean zero. .

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Machine learning: estimating f with $\hat{f}$.

Why Estimate f?

Two main reasons that we may wish to estimate f:

1. *prediction*
   - $\hat{Y} = \hat{f}(X)$
   - $\hat{f}$ is treated as a *black box*.
   - Better model = more accurate predictions of $\hat{Y} \approx Y$

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We'll focus mostly on prediction in this class.

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2012	-.08	51.1
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5. Compare predicted response value ($\hat{Y}$) with true response value (Y) for observations in test / validation data to evaluate performance (e.g., mean squared error).

There is no free lunch in statistics: no one method dominates all others over all possible data sets. Selecting the best method is hard.

Test your knowledge

Explain whether each scenario is a classification or regression problem, and indicate whether we are most interested in inference or prediction. Finally, provide n and p.

We collect a set of data on the top 500 firms in the US. For each firm we record profit, number of employees, industry and the CEO salary. We are interested in understanding which factors affect CEO salary.

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Answer: Regression, inference, $n = 500, p = 3$.

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We are considering launching a new product and wish to know whether it will be a success or a failure. We collect data on 20 similar products that were previously launched. For each product we have recorded whether it was a success or failure, price charged for the product, marketing budget, competition price, and ten other variables.

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Answer: Classification, prediction, $n = 20, p = 13$.

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Answer: f is the true function that maps X onto Y. $\hat{f}(X)$ is the estimated / prediction function trained on sample data, mapping observed X onto observed Y.

Linear Regression

Predicting Election Results

Goal: Predict Incumbent Vote Share (create prediction function)

- Use relationship in prior elections to predict future election
- Training data (In sample) $\rightsquigarrow$ Testing data (Out of sample)

Method: Linear Regression: Simple (today) and Multiple (next week)

Evaluation (Focus of next lectures):

1) In sample fit (training data)
2) Out of sample fit (test data)

Key Terms:

- Linear Regression, Simple Regression, Multiple Regression
- Cost function
- Sum of Squared Residuals
- In sample, Out of Sample

Key Techniques and `R` Functions

- Linear algebra operations and terms
  - Inner product
  - Matrix
- `lm`, `plot` , `%*%`

Time for Change Model (Abramowitz, Linzer)

Predict Incumbent Vote Share with political and economic fundamentals

1) GDP Growth
2) Incumbent Presidential Popularity
3) Incumbent Party

To the R Code!

What is Linear Regression?

Linear regression is a simple approach for supervised learning.

- Around since 1800s.
- Still a widely used tool for predicting quantitative response.
- Good jumping-off point for newer approaches.
- We need to understand it before moving on!

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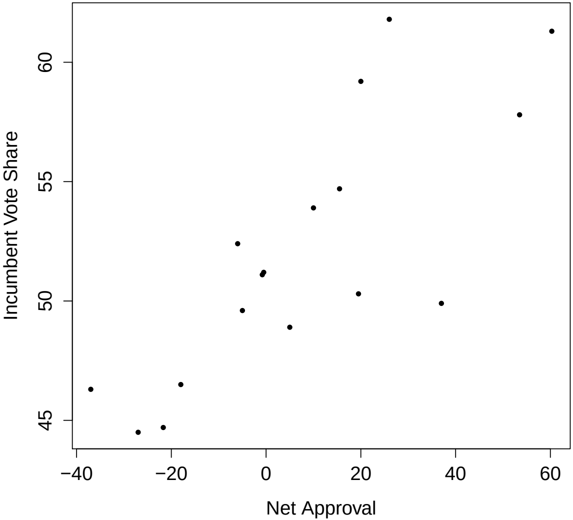
Simple linear regression:

- Assumes a linear relationship between quantitative response Y and a **single** variable X.
- Also called bivariate regression because there are two variables (X and Y)

Bivariate Regression: Geometric Perspective



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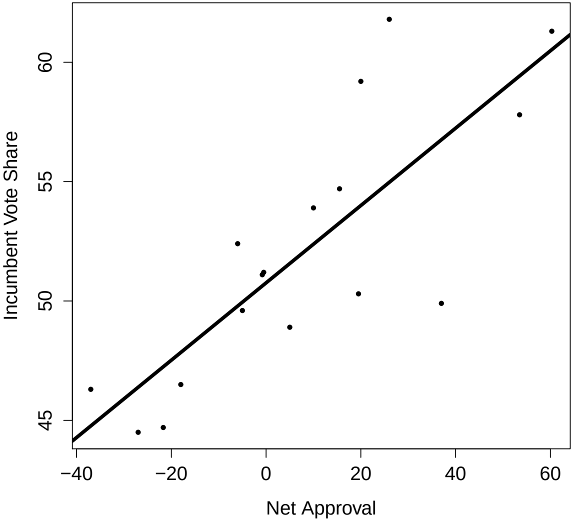

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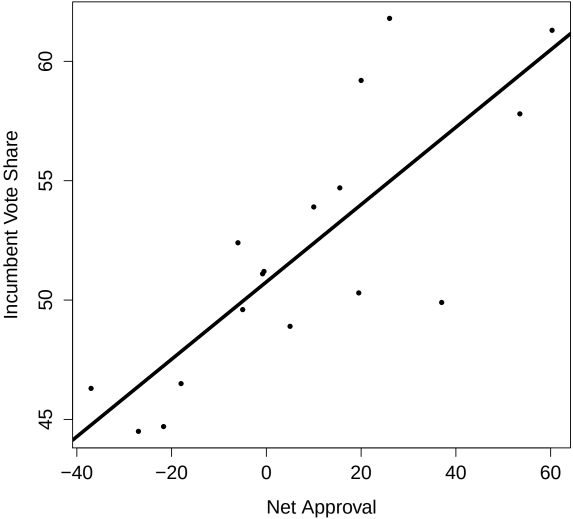
We use our training data to produce estimates:

$$\widehat{\text{Vote}}_i = \widehat{\beta}_0 + \widehat{\beta}_1 \text{Approval}_i + \epsilon_i$$

Geometric and Function Perspective Combined



1) What corresponds to β_0? (intercept)
2) What corresponds to β_1? (slope)
3) What corresponds to ϵ_i ? (residual)
4) What is an in-sample estimate and what is an out-of-sample estimates?

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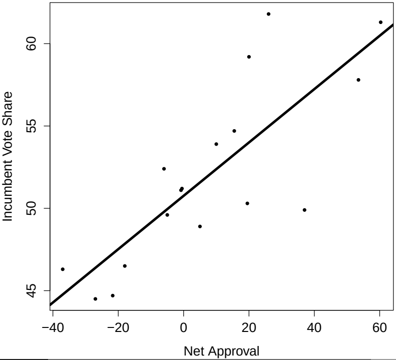

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Residual / Error: 51.1 - 50.12 = 0.98

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- irreducible error ϵ (catch-all for what we miss with this simple model)

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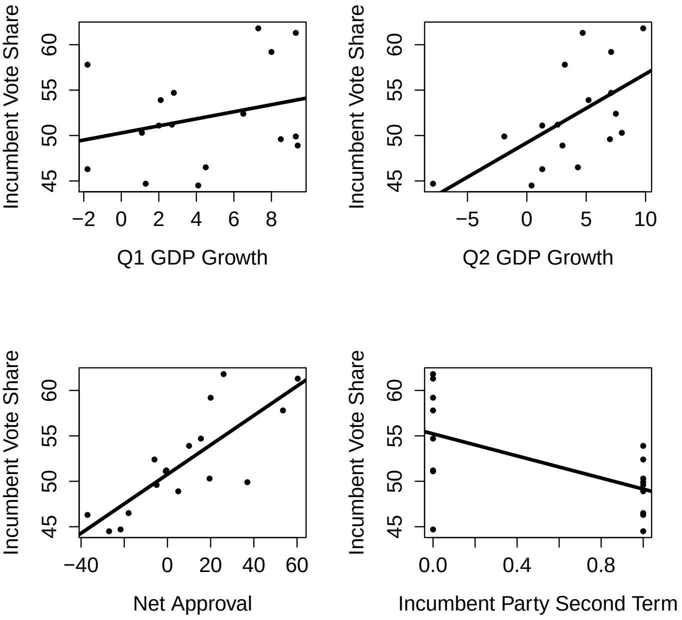
Incumbent Vote Share
45
50
55
60
−2
0
2
4
6
8
Q1 GDP Growth
Incumbent Vote Share
45
50
55
60
−5
0
5
10
Q2 GDP Growth
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50
55
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20
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60
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50
55
60
0.0
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Incumbent Party Second Term

Up next:
Multivariate Regression

Bonus: Gradient Descent