Introduction to Machine Learning for Social Science

Class 7: LASSO Regression & Cross Validation

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Questions?

Supervised Learning $\rightsquigarrow$ Text analysis

Goal: predict article from National desk (1) or other desk (0)
Method: LASSO Regression
Evaluation:

1) In Sample Fit
2) Out of Sample Fit

Key Terms:

- Overfitting
- Regularization
- LASSO
- Mean Squared Error (MSE)
- Bias-Variance Trade-off
- Cross validation

Key R Functions and Terms

- `glmnet`, `cv.glmnet`

LASSO Regression

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Linear regression: Choose $\beta's$ to minimize sum of squared residuals

$$\beta_{\text{OLS}} \quad = \quad \text{argmin}_{\boldsymbol{\beta}} \sum_{i=1}^{N} (Y_i - \boldsymbol{\beta} \cdot \boldsymbol{x}_i)^2$$

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$$\boldsymbol{\beta}_{\text{LASSO}} = \text{argmin}_{\boldsymbol{\beta}} \sum_{i=1}^{N} (Y_i - \boldsymbol{\beta} \cdot \boldsymbol{x}_i)^2 + \lambda \underbrace{\sum_{p=1}^{P} |\beta_p|}_{\text{penalty}}$$

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What does λ do?

How Do We Choose λ?

Best $\rightsquigarrow$ best performing model

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Define:

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\begin{aligned}
\widehat{\boldsymbol{\beta}}^{\lambda} &= \text{Coefficients at } \lambda \\
\widehat{p}_{i,\lambda} &= \Pr(Y_i = 1 | \boldsymbol{X}_i, \widehat{\boldsymbol{\beta}}^{\lambda})
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Best $\rightsquigarrow$ best performing model $\rightsquigarrow$ lowest mean squared error (MSE).

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\end{aligned}$$

Loss Function

Goal: Find λ that minimizes MSE (loss function).

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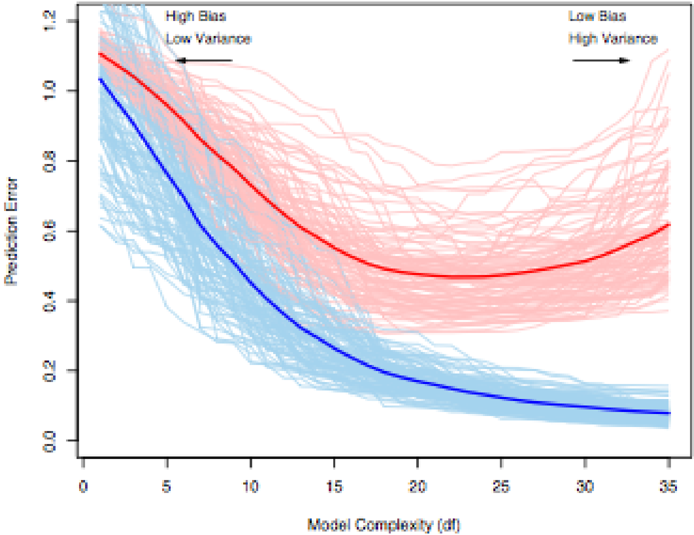
- Optimize in-sample fit?
- Optimize out-of-sample fit?

In-Sample Fit

In sample fit is greedy: adding more variables will always improve fit

Bad way to choose: in sample fit!



Bias-Variance Tradeoff

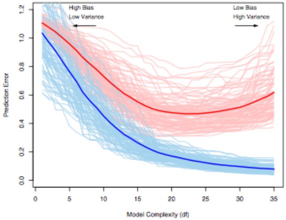


FIGURE 7.1. *Behavior of test sample and training sample error as the model complexity is varied. The light blue curves show the training error* $\overline{\mathrm{err}}$, *while the light red curves show the conditional test error* $\mathrm{Err}_{\mathcal{T}}$ *for 100 training sets of size 50 each, as the model complexity is increased. The solid curves show the expected test error* Err *and the expected training error* $\mathrm{E}[\overline{\mathrm{err}}]$.

Model overfit$\rightsquigarrow$ in sample error is optimistic:

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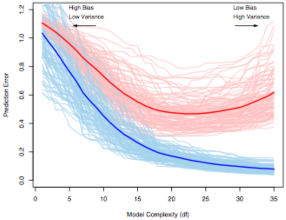

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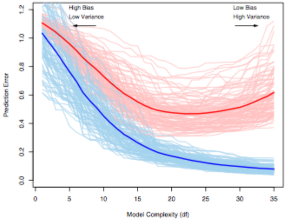

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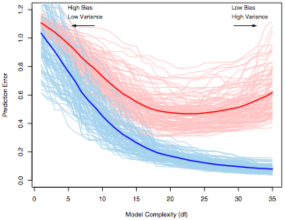

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- Additional model complexity: idiosyncratic features of the training set.

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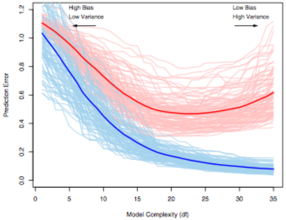

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- Reduces error in both training and test set.
- Additional model complexity: idiosyncratic features of the training set.
- Reduces error in training set, increases error in test set.

Bias-Variance Decomposition

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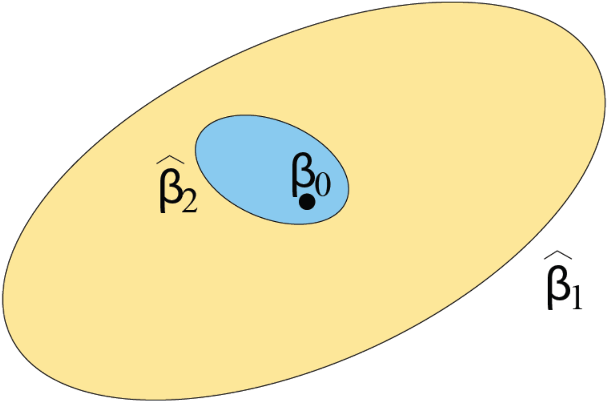
- Bias: error from erroneous assumptions in the learning algorithm. High bias can cause an algorithm to miss the relevant relations between features and target outputs (underfitting).

- Variance: error from sensitivity to small fluctuations in the training set. High variance can cause an algorithm to model the random noise in the training data, rather than the intended outputs (overfitting).

Bias-Variance Decomposition

Bias-Variance Tradeoff

Introducing bias can help minimize variance, leading to better performance.



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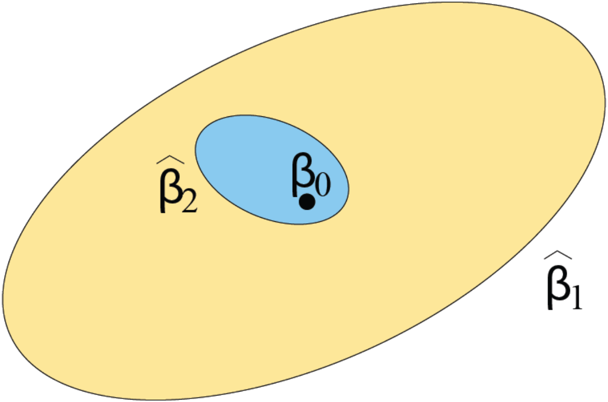
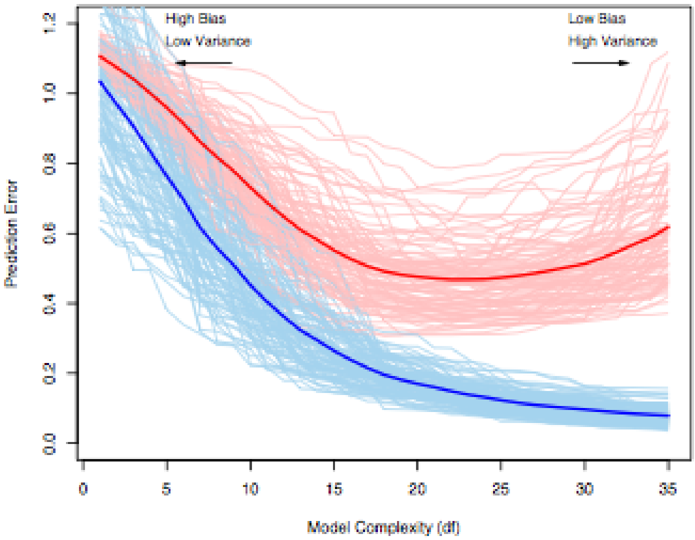


This is what lasso does!

The Goal:

Find the sweet spot between bias and variance using out-of-sample data.



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Strategy:

- Divide data into K groups
- Train data on $K - 1$ groups.

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Strategy:

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- Predict values for K^{th}
- Summarize performance with loss function:
    - Mean square error, Absolute error, Prediction error, ...
- Final choice: model with optimal CV score

How Do We Select K? (HTF, Section 7.10)

Common values of K

- $K = 5$: Five fold cross validation
- $K = 10$: Ten fold cross validation
- $K = N$: Leave one out cross validation (LOOCV)

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Considerations:

- How sensitive are inferences to number of coded documents? (ISL, pg 181-184)
    - 50 labeled documents
    - $K = N \rightarrow 49$ documents to train,
    - $K = 10 \rightarrow 45$ documents to train
    - $K = 5 \rightarrow 40$ documents to train

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- How long will it take to run models?
  - $K-$fold cross validation requires $K \times$ One model run
- Bias-Variance in Cross Validation:
  - LOOCV: Less bias
  - K = 5 or 10: Less Variance
  - In brief: k = 5 or 10 shown to be "sweet spot", yielding low test error estimates

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