Introduction to Machine Learning for Social Science

Class 6: LASSO Regression

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Some Review

Evaluating Model Fit

- Evaluate fit with gold standard data
- In sample: dependent variable of model
- Out of sample: held out data

Assessing Classification Performance

Measures of classification performance

	Actual Label	
Guess	Yea	Nay
Yea	True Yea	False Yea
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$$\text{Recall} = \frac{\text{True Yea}}{\text{True Yea} + \text{False Nay}}$$

$$F = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

Supervised Learning $\rightsquigarrow$ Text analysis

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4) Method to extrapolate from hand coding to unlabeled documents

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We've preprocessed the data $\rightsquigarrow$ Create a Document-Term Matrix

Goal: predict article from National desk (1) or other desk (0)
Method: LASSO Regression
Evaluation:

1) In Sample Accuracy
2) Out of Sample Accuracy

Key Terms:

- Overfitting
- Regularization
- LASSO
- Mean Squared Error (MSE)
- Bias-Variance Trade-off
- Cross validation

Key R Functions and Terms

- `glmnet`, `cv.glmnet`

Document-Term Matrices

$$X = \begin{array}{llllll} & \text{Word1} & \text{Word2} & \text{Word3} & \ldots & \text{WordP} \\ \text{Doc1} & 1 & 0 & 0 & \ldots & 3 \\ \text{Doc2} & 0 & 2 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \\ \text{DocN} & 0 & 0 & 0 & \ldots & 5 \end{array}$$

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Let $p = (\Pr(\text{Desk}_i = 1))$

$$\text{logit}(p) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_P X_P$$

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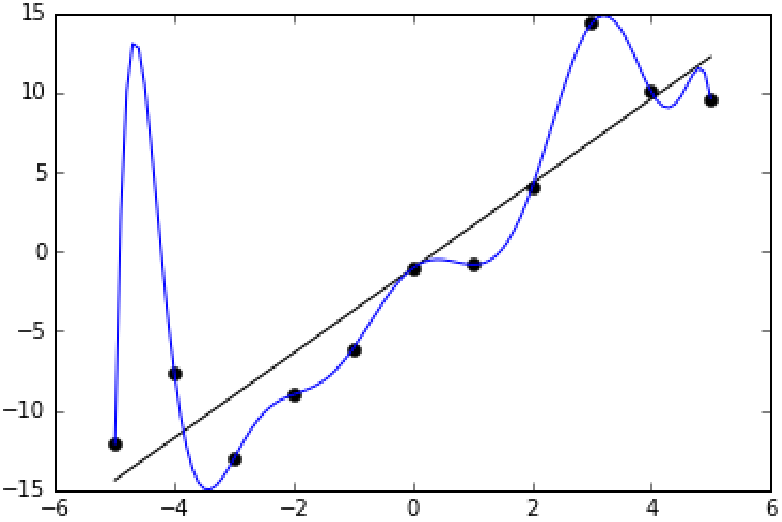
$\rightsquigarrow$ overfitting.

Overfitting



R Code

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2.) Regularization
   - Keep all features, but shrink magnitude / value of parameters close to zero (ridge).
   - Keep all features, but shrink magnitude / value of (some) parameters to zero (lasso).

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Linear regression: Choose $\beta's$ to minimize sum of squared residuals

$$\boldsymbol{\beta}_{\text{OLS}} = \text{argmin}_{\boldsymbol{\beta}} \sum_{i=1}^{N} (Y_i - \boldsymbol{\beta} \cdot \boldsymbol{x}_i)^2$$

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What does λ do?

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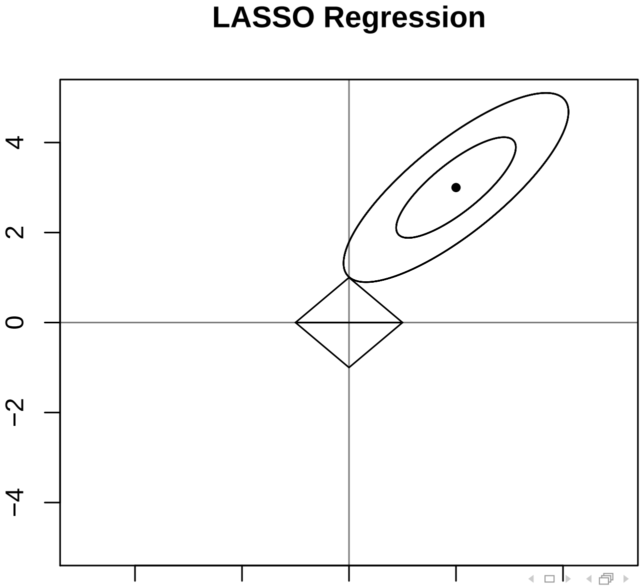
$$\sum_{j=1}^{2} |\tilde{\beta}_j| = 1 + 0 = 1$$

LASSO Penalty: Geometry



R Code!

Methods/Metrics for:

1) Choosing λ
2) Assessing model performance