# Introduction to Machine Learning for Social Science

## Class 12: Text Similarity & Distance

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# Announcements

## Group projects

- Instructions on canvas
- Fill out preference form by March 1.

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Extra Credit (+10 points)

1. Find distinctive words using ‘text.no.noun‘ column.
2. Provide a brief qualitative analysis of the distinctive terms. What do these suggest about the ways American media portray women in the West vs. Middle East?
3. Classify documents using these terms. Identify whether the results are better or worse than the previous analysis, and briefly explain why you think this happened.

# Plan for the day

1. Loose ends: Evaluating dictionary methods and distinctive methods.
2. Introducing unsupervised learning.
3. Text similarity and distance.

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- Face validity (do these results make sense?)
- Convergence (do different metrics lead to the same result?)
- "Gold Standard" (do our results align with human coding?)

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Supervised learning and Unsupervised learning are not competitors!

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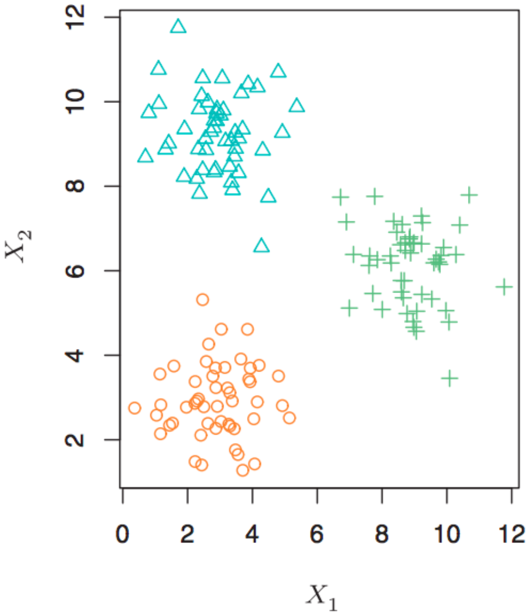
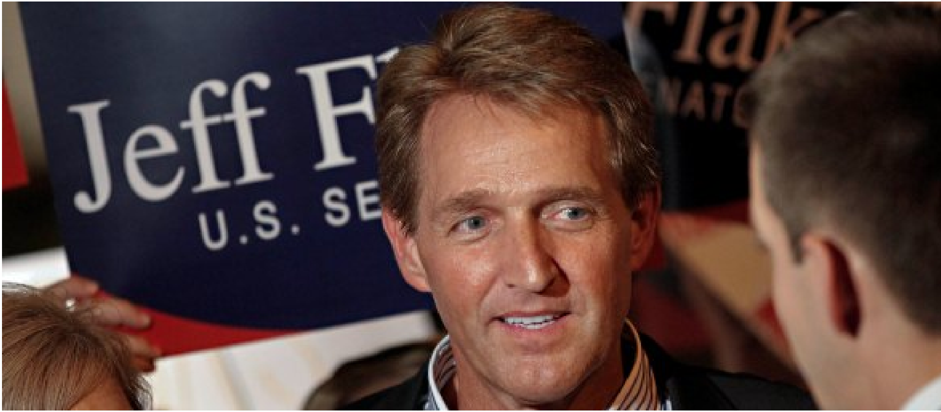
## $\rightsquigarrow$Cluster analysis / Clustering

- Goal is to ascertain, on the basis of $x_1, x_2, ..., x_n$, whether the observations fall into relatively distinct groups.
- These groups are interesting because the may correspond to some category or quantity of interest.



12
10
8
6
4
2
$X_2$
0
2
4
6
8
10
12
$X_1$


Jeff F
U.S. SE
NAT

Today (and Tuesday): Cluster press releases

Goal: partition documents such that:

- similar documents are together
- dissimilar documents are apart

Method: Clustering methods

Game Plan:

1) What makes two data points (i.e. documents) similar?
2) How do we find a good partition?
3) How do we interpret the clusters?

Key Terms:

- (Multidimensional) Space
- Distance
- Euclidean Distance
- Cosine Distance
- Cluster Analysis / Clustering
- K-means
- Centroid

What makes two documents similar?

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Similar $=$ Geometrically Close

Dissimilar $=$ Geometrically Distant

# Texts and Geometry

Consider a document-term matrix

$$\boldsymbol{X} = \begin{pmatrix} 1 & 2 & 0 & \ldots & 0 \\ 0 & 0 & 3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \ldots & 3 \end{pmatrix}$$

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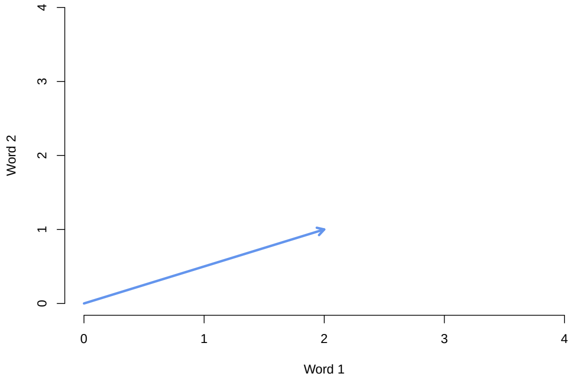
By transforming our text into a word count vector, we are representing it as a point in a multidimensional space

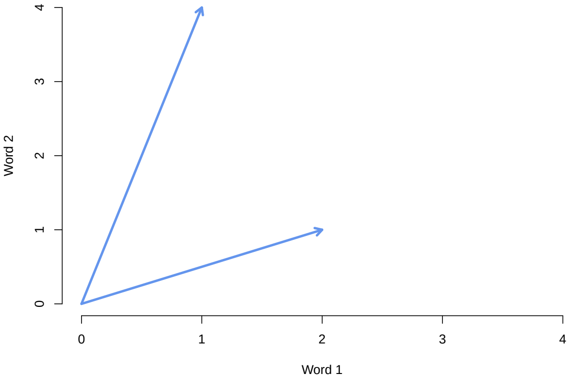
- Provides a geometry
- Natural notions of distance and similarity
- Tools from linear algebra to calculate distances mathematically.

# Texts in Space




Doc1 = "Wait? No wait." $\rightsquigarrow (2, 1)$

# Texts in Space

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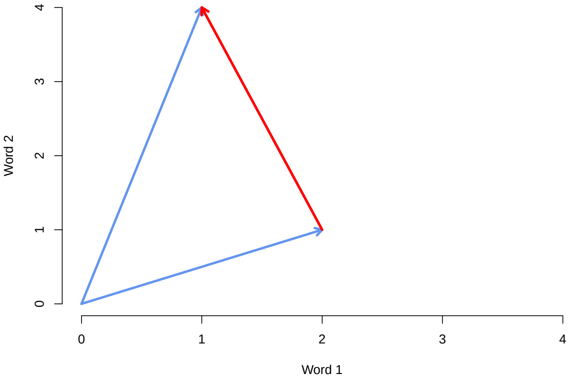


Doc1 = "Wait? No wait." $\rightsquigarrow (2, 1)$
Doc2 = "No, wait! No, no, no!" $\rightsquigarrow (1, 4)$

Suppose $\boldsymbol{X}_1 = (1, 4)$ and $\boldsymbol{X}_2 = (2, 1)$.

The Euclidean distance (aka norm) between $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ (or from $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$) is the length of the line segment connecting them.

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This generalizes beyond 2 dimensions!

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# Test your knowledge

The Euclidean distance between any documents $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ is:

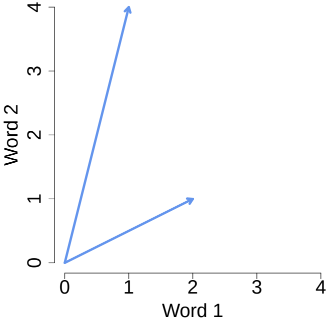
$$d(\boldsymbol{X}_1, \boldsymbol{X}_2) \;=\; \sqrt{\sum_{p=1}^{P} (x_{1p} - x_{2p})^2}$$

Suppose

- $\boldsymbol{X}_1 =$ Oh na na na.
- $\boldsymbol{X}_2 =$ Oh, me? Na.

Calculate the euclidean distance between these two documents.

# Problem(?) with Euclidean Distance

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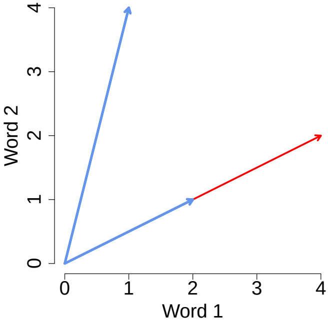


$$\begin{aligned} \boldsymbol{X}_1 &= (2, 1) \\ \boldsymbol{X}_2 &= (1, 4) \\ \boldsymbol{X}_3 &= 2\boldsymbol{X}_1 = (4, 2) \\ d(\boldsymbol{X}_3, \boldsymbol{X}_2) &= \sqrt{(4-1)^2 + (2-4)^2} \\ &= \sqrt{13} \end{aligned}$$

Euclidean distance depends on document-length.

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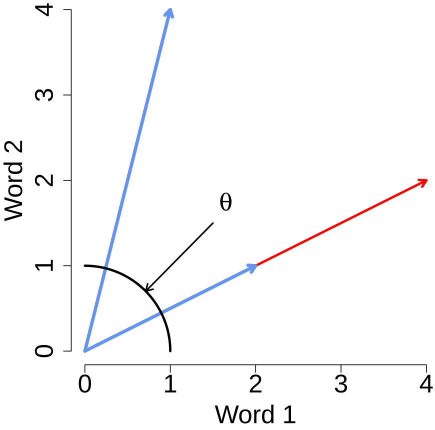


Cosine Similarity

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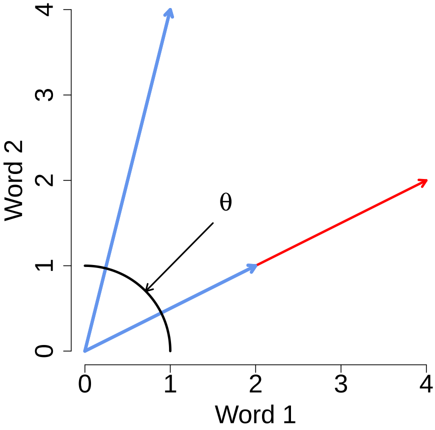


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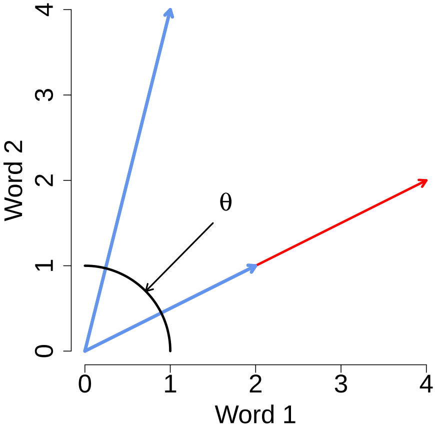


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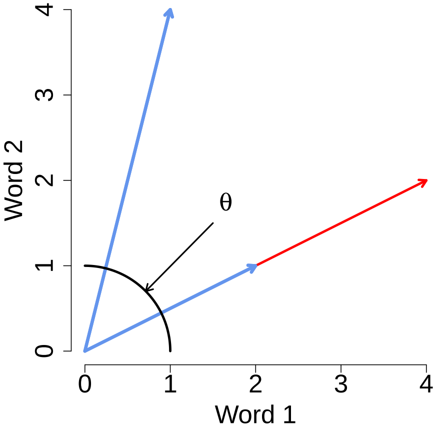


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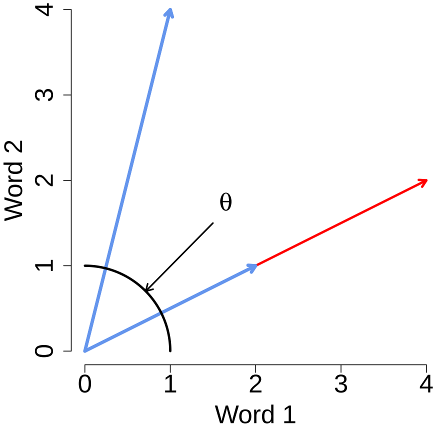


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- Other applications
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## Tomorrow

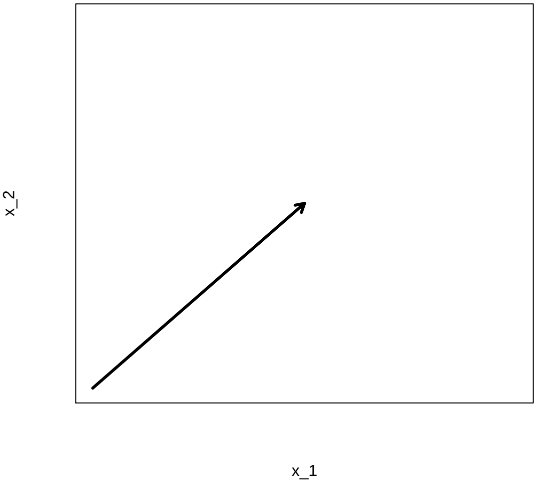
- How do we find a good partition?
- How do we interpret the clusters?

To the `R` code!

# Bonus Slides

For those who heart math.

# Vector Length



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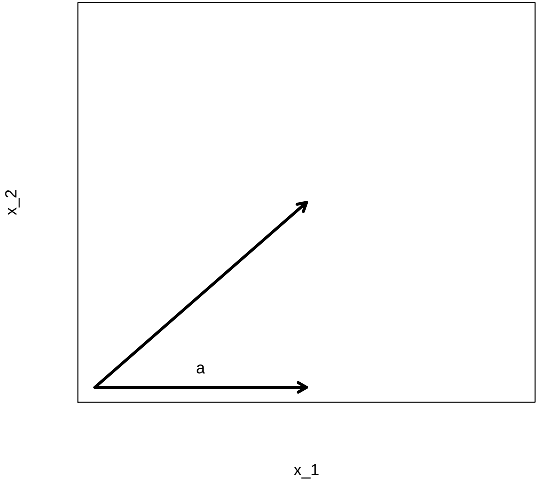


- Pythogorean Theorem: Side with length $a$

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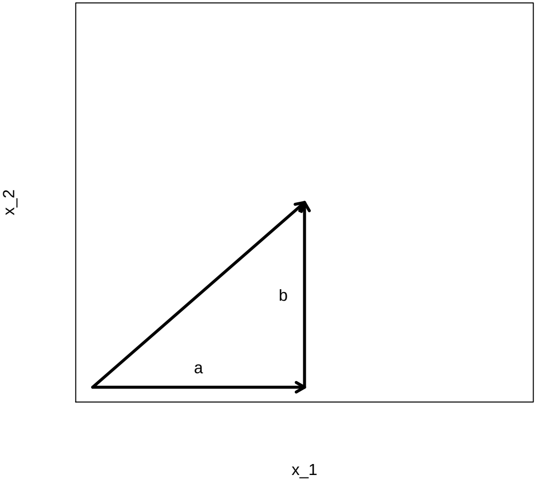


- Pythogorean Theorem: Side with length *a*
- Side with length *b* and right triangle

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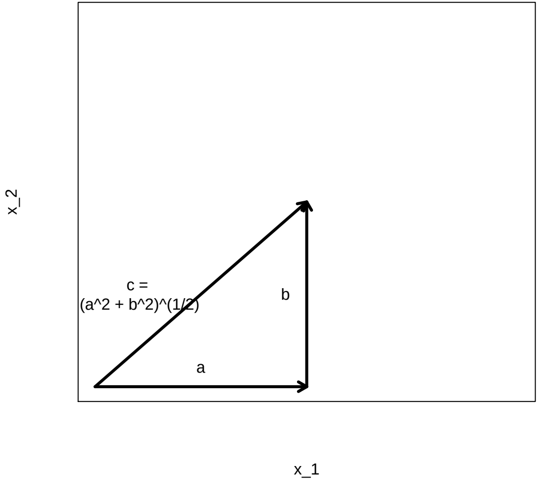


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- $c = \sqrt{a^2 + b^2}$

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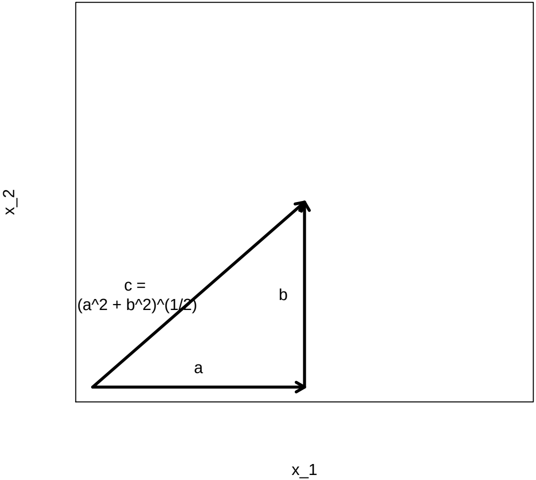


- Pythogorean Theorem: Side with length $a$
- Side with length $b$ and right triangle
- $c = \sqrt{a^2 + b^2}$
- Extends beyond 2 dimensions

# Vector (Euclidean) Length

Suppose $\boldsymbol{X}_i$ is a document (row from an $N \times K$ document-term matrix).

Then, we will define its length as

$$
\begin{aligned}
||\boldsymbol{X}_i|| &= \sqrt{(\boldsymbol{X}_i \cdot \boldsymbol{X}_i)} \\
&= \sqrt{(X_{i1}^2 + X_{i2}^2 + X_{i3}^2 + \ldots + X_{iK}^2)} \\
&= \sqrt{\sum_{k=1}^{K} X_{ik}^2}
\end{aligned}
$$

# Cosine Similarity

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$$\cos\theta \quad = \quad \left(\frac{X_1}{||X_1||}\right) \cdot \left(\frac{X_2}{||X_2||}\right)$$

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\begin{aligned}
\cos\theta &= \left(\frac{X_1}{||X_1||}\right) \cdot \left(\frac{X_2}{||X_2||}\right) \\
\frac{(4,2)}{||(4,2)||} &= (0.89, 0.45)
\end{aligned}
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\frac{(2,1)}{||(2,1)||} &= (0.89, 0.45)
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\frac{(2,1)}{||(2,1)||} &= (0.89, 0.45)\\
\frac{(1,4)}{||(1,4)||} &= (0.24, 0.97)
\end{aligned}
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\frac{(2,1)}{||(2,1)||} &= (0.89, 0.45)\\
\frac{(1,4)}{||(1,4)||} &= (0.24, 0.97)\\
(0.89, 0.45)\cdot(0.24, 0.97) &= 0.65
\end{aligned}
$$

# Cosine Similarity

$$\begin{aligned}
\cos\theta &= \left(\frac{X_1}{||X_1||}\right)\cdot\left(\frac{X_2}{||X_2||}\right)\\
\frac{(4,2)}{||(4,2)||} &= (0.89, 0.45)\\
\frac{(2,1)}{||(2,1)||} &= (0.89, 0.45)\\
\frac{(1,4)}{||(1,4)||} &= (0.24, 0.97)\\
(0.89, 0.45)\cdot(0.24, 0.97) &= 0.65\\
\text{cos dissimilarity} &= 1-\cos\theta
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