Design and Analysis of Algorithms
Lecture 1: Introduction

Ke Xu and Yongxin Tong

（许可 与 童咏昕）

School of CSE, Beihang University

Outline

- About Us
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

Instructors

Ke Xu
Professor

State Key Lab. of Software Development Environment

Research Interests: Algorithms, Phase Transitions in NPC, Logic and Complexity, Data Mining

Yongxin Tong
Professor

State Key Lab. of Software Development Environment

Research Interests: Big Data, Crowd Intelligence, Data Mining, Federated/Reinforcement Learning

Contact and TAs

- Contact
  - Ke Xu
    - Email: kexu@nlsde.buaa.edu.cn
    - Homepage: http://sites.nlsde.buaa.edu.cn/~kexu/
  - Yongxin Tong
    - Email: yxtong@buaa.edu.cn
    - Homepage: http://sites.nlsde.buaa.edu.cn/~yxtong/
- TAs
  - Yexuan Shi (Ph.D. Student)
    - Email: skyxuan@buaa.edu.cn
  - Qiaoyang Liu(Master Student)
    - Email: qiaoyangliu@buaa.edu.cn
  - Ruisheng Zhang (Master Student)
    - Email: rszhang@buaa.edu.cn
  - Xiaomin Pengmao (Senior Undergraduate Student)
    - Email: prime21@buaa.edu.cn

Instructor: Yongxin Tong

- Beihang University (2015.4 - Current)
  - “Zhuoyue Program” Professor
  - State Key Lab. of Software Development Environment
  - Research Interests: **Big Data** and **Crowd Intelligence**

- HKUST (2010.8 – 2015.3)
  - Research Assistant Professor (2014.2 – 2015.3)
    - CSE Department, focused on big data and crowdsourcing
  - Ph.D. Student and Candidate (2010.8 – 2014.1)
    - CSE Department, focused on data mining and crowdsourcing

Instructors: Yongxin Tong

Yongxin Tong 童 咏 昕

Professor
Big Data Analysis Group
State Key Laboratory of Software Development Environment
School of Computer Science and Engineering
Beihang University (BUAA)

Office: Room 1811, Baiyan Building

E-mail: yxtong AT buaa.edu.cn or yongxintong AT gmail.com

[**Short Bio**] [**Research**] [**Publications**] [**Awards**] [**Experiences**] [**Professional Services**] [**Misc.**]

Short Biography

Yongxin Tong is a Professor in the State Key Laboratory of Software Development Environment (SKLSDE) of the School of Computer Science and Engineering at Beihang University (BUAA). He received a Ph.D. degree in Computing Science and Engineering from the Department of Computer Science and Engineering, The Hong Kong University of Science and Technology (HKUST), under Prof. Lei Chen's supervision. He also received a Master degree in Software Engineering at Beihang University and a Double Bachelor degree in Economics from China Centre for Economic Research (CCER) at Peking University.

Research Interests

- Crowdsourcing
- Spatio-temporal Big Data
- Federated Learning and Data Federation Services
- Differential Privacy and Secure Multi-Party Computation
- Uncertain Data Mining and Management
- Social Network Analysis

Our Recent Tutorials and Surveys

- **Yongxin Tong**, Zimu Zhou, Yuxiang Zeng, Lei Chen, Cyrus Shahabi. "Spatial Crowdsourcing: A Survey", *The VLDB Journal* (**VLDBJ**), 29(1): 217–250, January 2020. [VLDB 2017 Tutorial Slides]
- Qiang Yang, Yang Liu, Tianjian Chen, **Yongxin Tong**. "Federated Machine Learning: Concept and Applications", *ACM Transactions on Intelligent Systems and Technology* (**TIST**), 10(2): No.12, February 2019.

Homepage：http://sites.nlsde.buaa.edu.cn/~yxtong/

Faculty Members in SKLSDE


李未教授


马殿富教授


吕卫锋教授


尹宝林教授


蔡维德教授


马世龙教授


张玉平教授

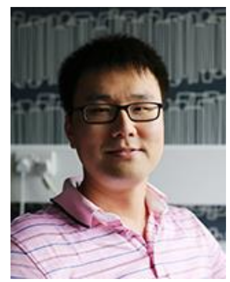

许可教授


张辉教授


郎波教授


杨钦教授


吴文峻教授


朱皞罡教授


童咏昕教授


杜博文教授


刘祥龙副教授


刘瑞副教授


丁嵘副教授


吕江花副教授


诸彤宇副教授


罗杰副教授


王德庆副教授


孟宪海博士


李吉刚博士

Outline

- About Us
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

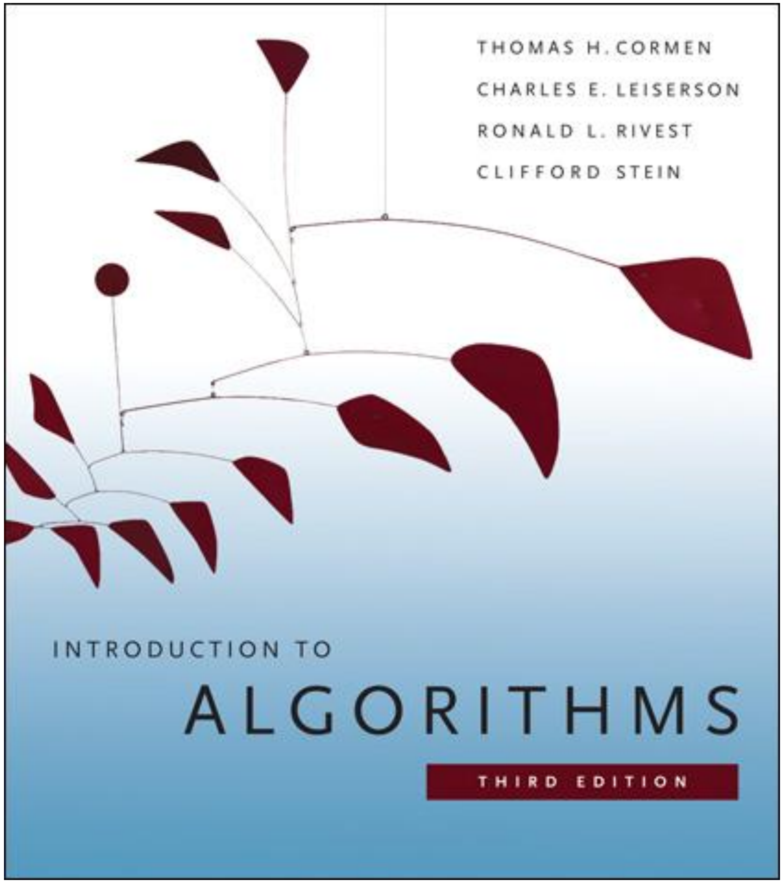
MOOC

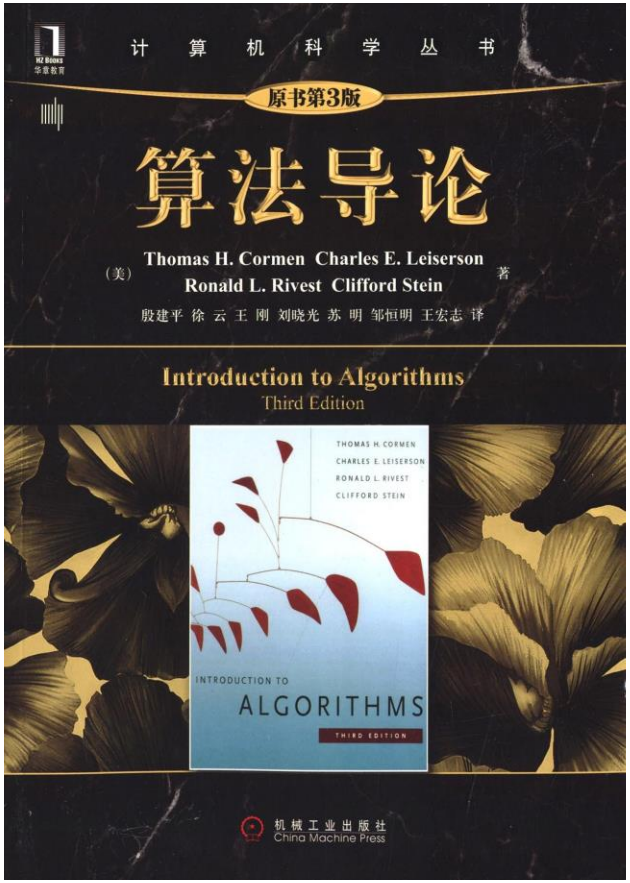
- 辅助线上MOOC课程：算法设计与分析（中文）

中国大学MOOC

Textbook

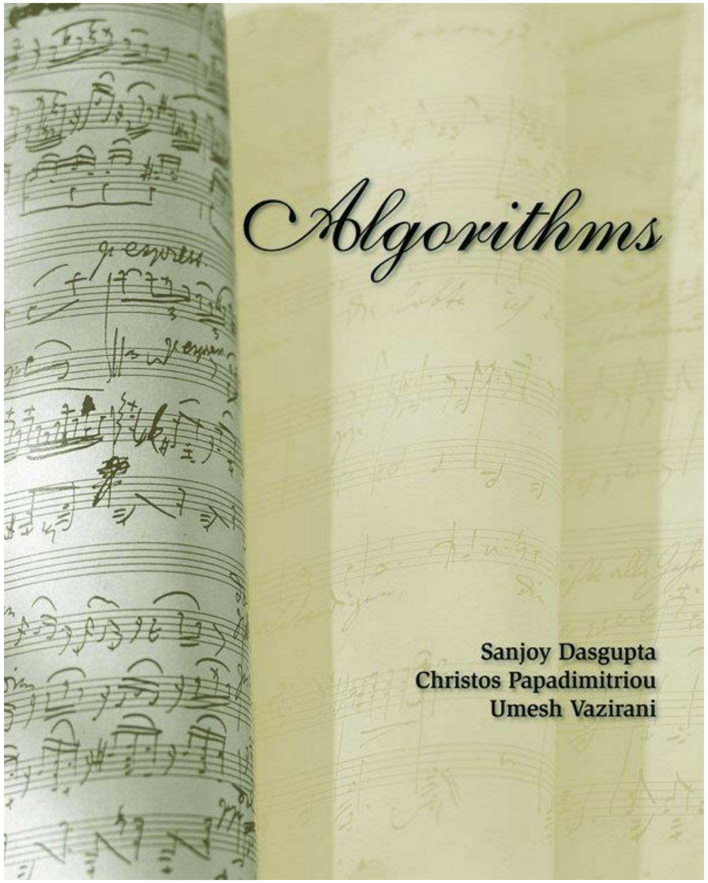
- Textbook: *Introduction to Algorithms* (3rd ed.)
  - by Cormen, Leiserson, Rivest and Stein (CLRS)
  - Prepublication version available online

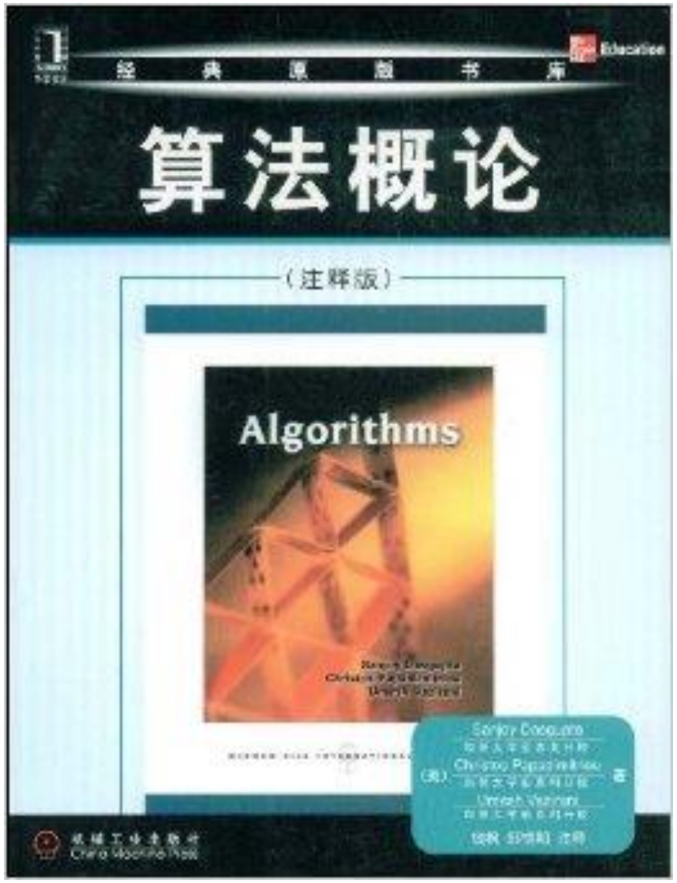




References (1)

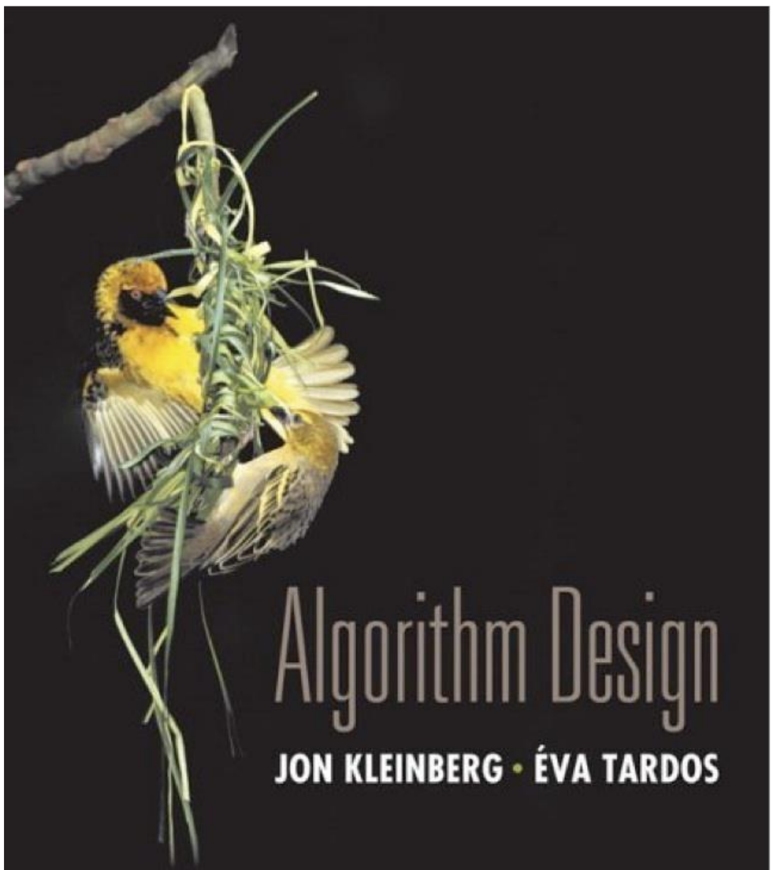
- Reference: *Algorithms*
  - by Dasgupta, Papadimitriou, and Vazirani (DPV)
  - Prepublication version available online

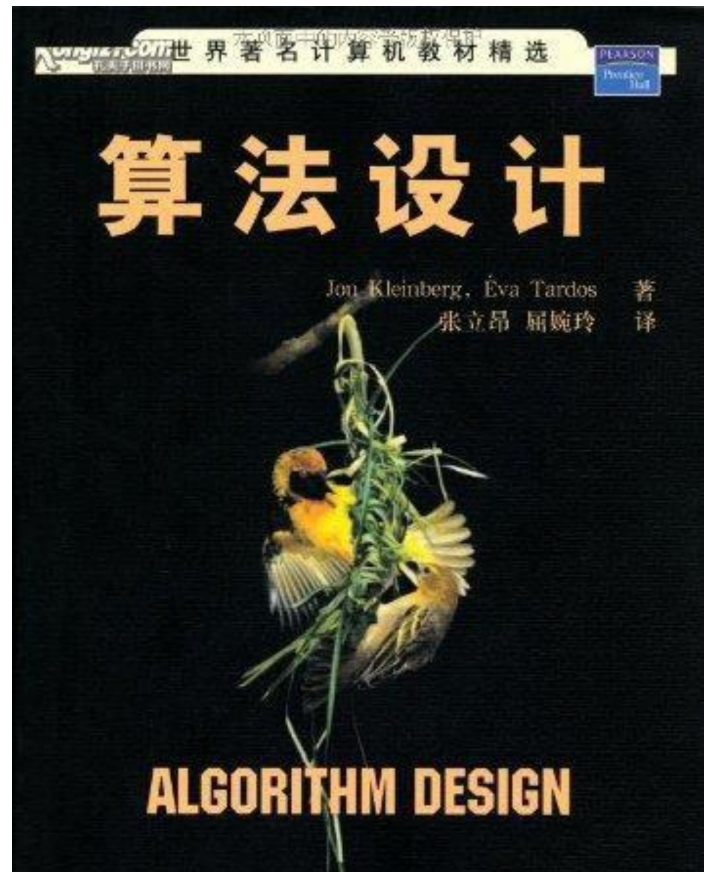




References (2)

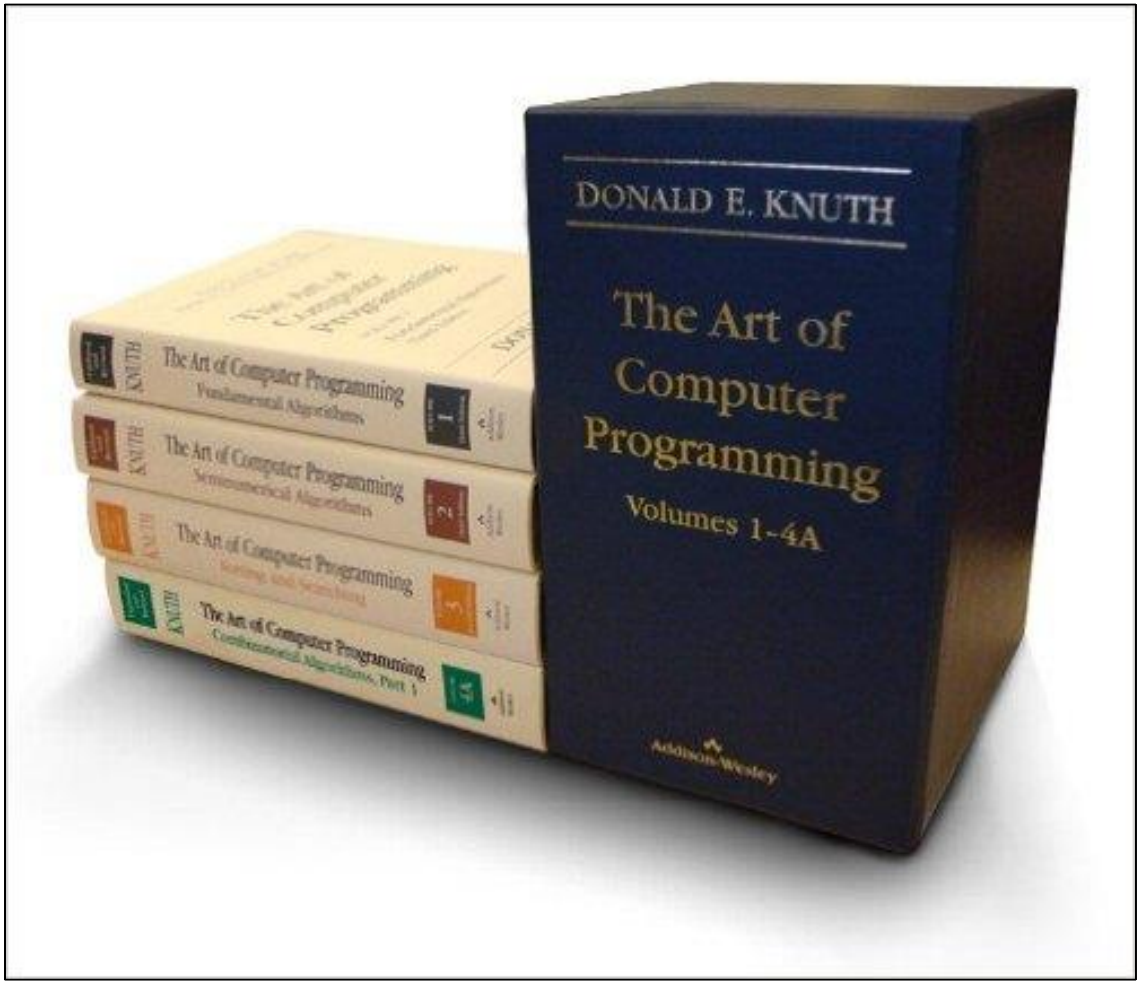
- Reference: *Algorithm Design*
  - by Kleinberg and Tardos (KT)





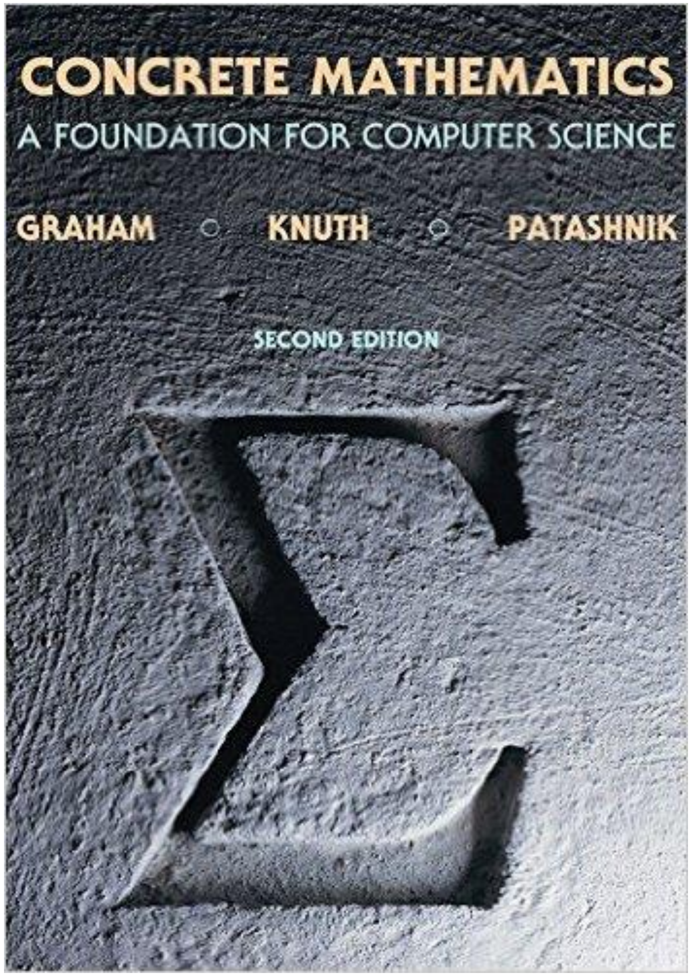
References (3)

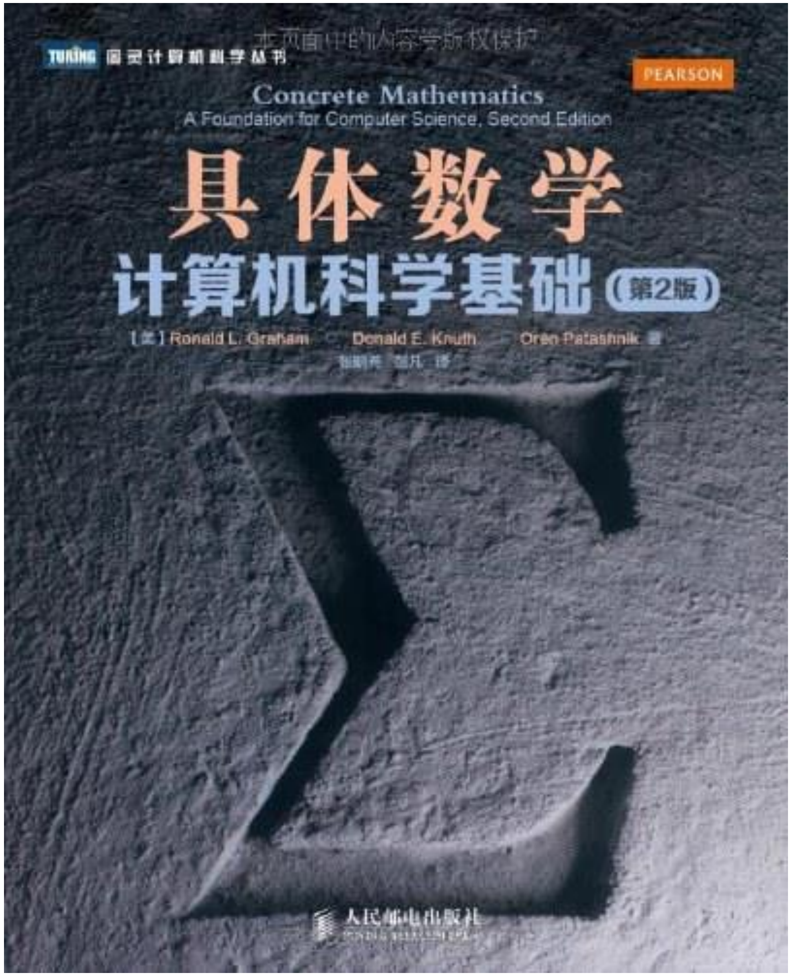
- Reference: *The Art of Computer Programming*
  - by Donald E. Knuth



References (4)

- Reference: *Concrete Mathematics* (2nd ed.)
  - by Graham, Knuth, Patashnik (GKP)





Prerequisites

- We assume you know:
  - Linked Lists, Stacks, Queues
  - Binary Search Trees
    - Traversals
    - Searching (but not analysis)
- What have you learnt previously?
  - Graph algorithms
    - Breadth-first search (BFS)
    - Depth-first search (DFS)
    - Topological sort (TS)
    - Minimum Spanning Trees (MST)
    - Dijkstra's shortest path algorithm (SP)

Tentative Syllabus

- Basics
  - Asymptotic Notations and Recurrences
- Divide and Conquer Algorithms
  - MCS Problem, PM Problem, and Quicksort
- Dynamic Programming Algorithms
  - 0-1 Knapsack, Rod-Cutting, CMM, LCS, and MDE
- Greedy Algorithms
  - Huffman Coding and Fractional Knapsack
- Graph Algorithms
  - BFS, DFS, SP, MST, Max Flow and Matching
- Dealing with Hard Problems
  - Problem Classes (P, NP, NPC) and Approximation Alg.

Lectures and Tutorials

- Lectures
  - Slides will be available on course web page.
- Tutorials (补充练习)
  - There will be 12 tutorials in this semester.
  - The tutorials will provide more examples to illustrate the material you learnt in class.
  - The first tutorial will be released on next week.

Grading Scheme

- (40%) Four Assignments
  - Each assignment has 10% scores.
  - Each takes 14 days. The first one will be released on Sep. 21.
  - After each submission due, we will post the solution and **WON'T** accept any assignment.
  - Failing to do any of these will be considered **PLAGIARISM**, and may result in a failing grade in the course.
- (60%) Final Exam
  - It covers the materials of the entire semester.

Coding and Online Judge

- Practice makes prefect
  - We strongly recommend you to practice on OJ, to better understand the algorithms learned in the lectures.
- Famous OJ
  - For beginers:
    - 洛谷: https://www.luogu.com.cn/training/list
    - Leetcode: https://leetcode.com/
    - Atcoder: https://atcoder.jp/
  - Advanced OJ:
    - 杭电oj: http://acm.hdu.edu.cn/typeclass.php
    - Codeforces: https://codeforces.com/problemset

Classroom Etiquette

- **No roll-call in our class !**
- Turn off cell phone ringers.
  - No phone conversations in room.
- Latecomers should enter quietly.
- No LOUD talking among selves during lectures.

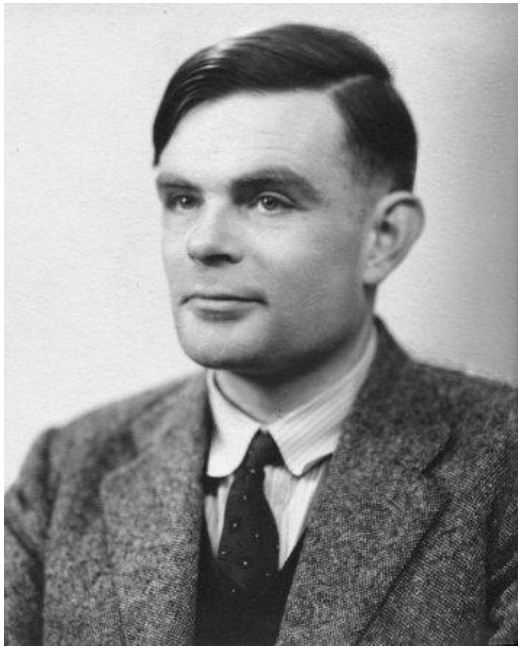
WeChat Group

Outline

- About Us
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

A.M. Turing Award

From 2007 to 2013, the award was accompanied by a prize of US $250,000 by Intel and Google. Since 2014, the award has been accompanied by a prize of US $1 million by Google.



Alan M. Turing

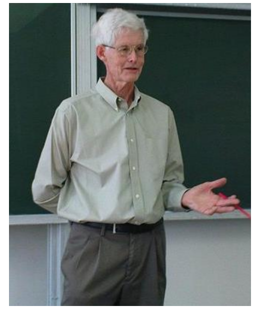
Nobel Prize of Computing

Since 1966，there have been 72 recipients of A.M. Turing Award!
This year is the 53rd anniversary of A.M. Turing Award!

A.M. Turing Award Winners for Algorithms

Donald E. Knuth
1974, USA

Robert W. Floyd
1978, USA

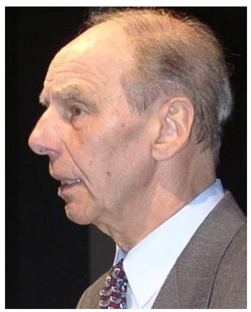


Stephen A. Cook
1982, USA

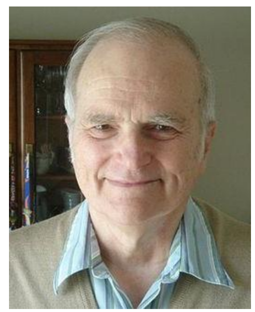
Richard M. Karp
1985, USA

John Hopcroft
1986, USA

Robert Tarjan
1986, USA



Juris Hartmanis
1993, Latvia



Richard E. Stearns
1993, USA

Manuel Blum
1995, Venezuela

Andrew Yao
2000, China

Leslie G. Valiant
2010, Hungarian

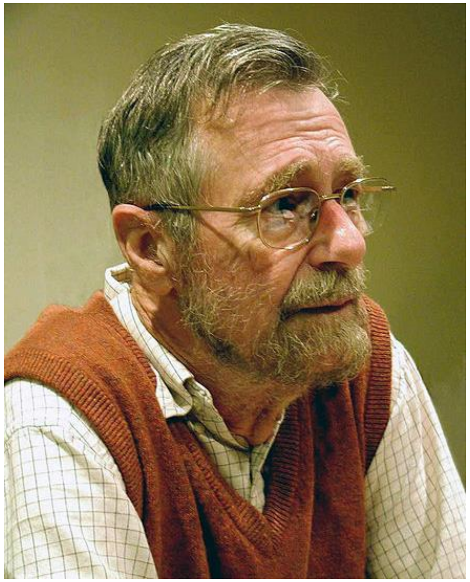
Silvio Micali
2012, Italy

Shafi Goldwasser
2012, USA

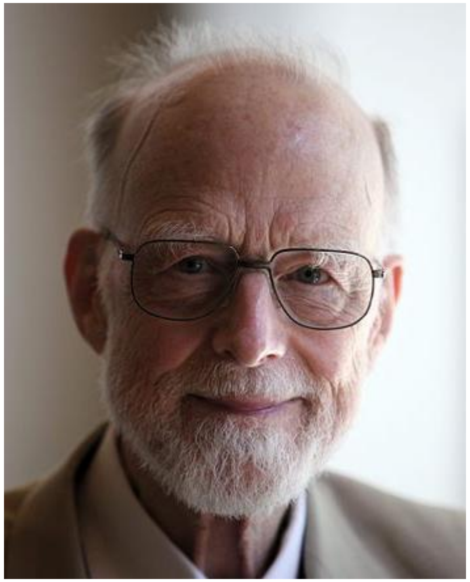
Martin Hellman
2015, USA

Whitfield Diffie
2015, USA

Other Related A.M. Turing Award Winners

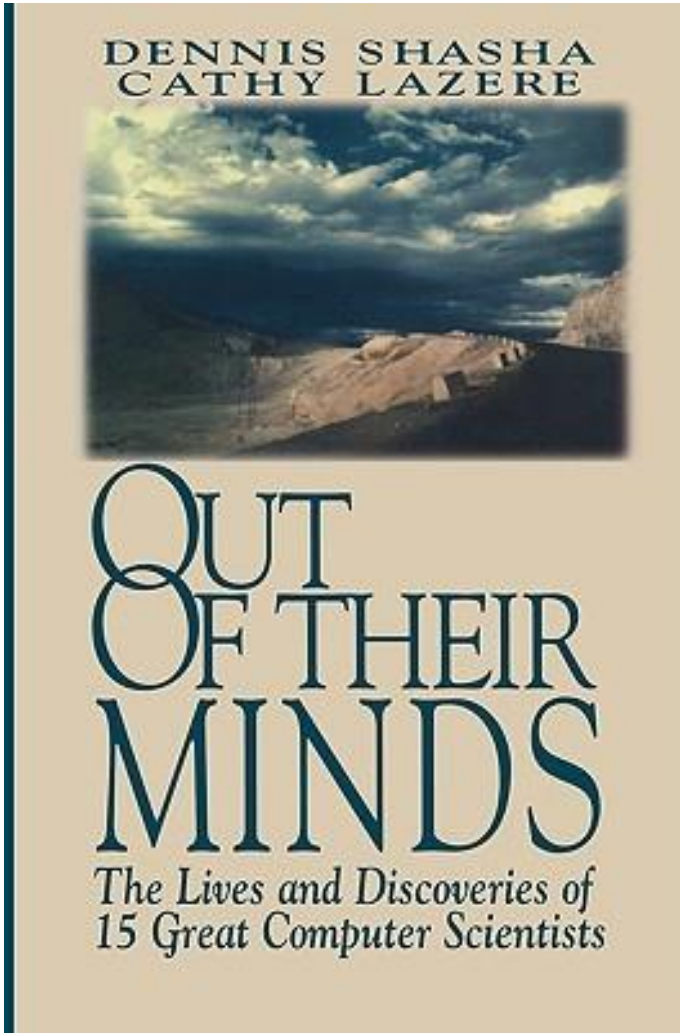


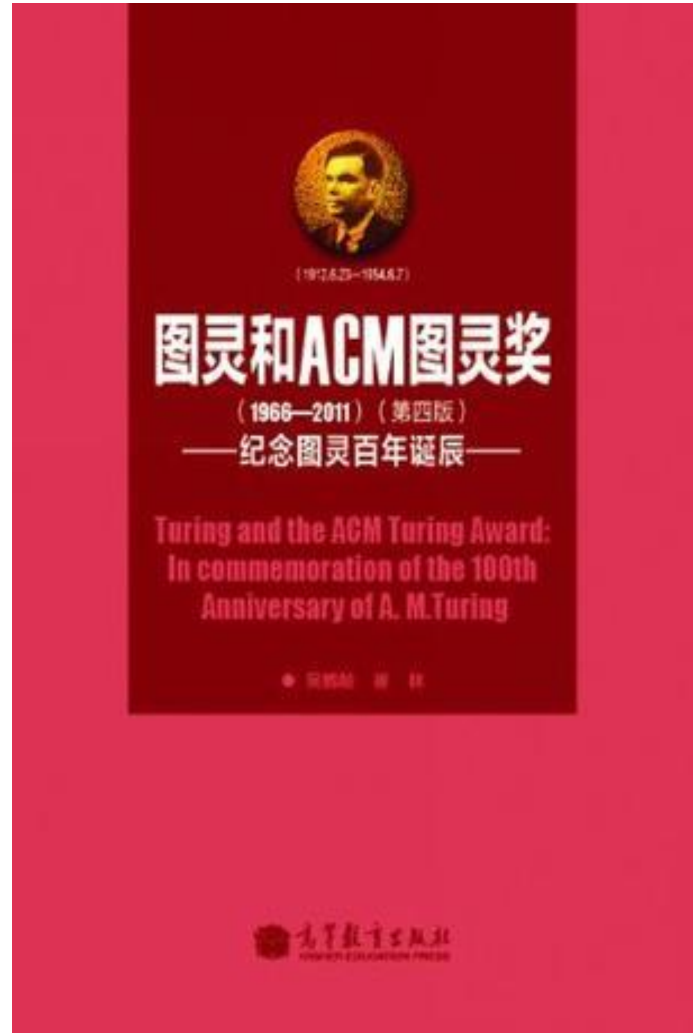
Edsger W. Dijkstra
The Recipient in 1972,
Netherlands,
Contributions: ALGOL Father,
Related Work: Dijkstra Algorithm



Tony Hoare
The Recipient in 1980,
UK,
Contributions: Hoare logic,
Related Work: QuickSort

Books of A.M. Turing Award Winners





Outline

- About Us
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

What is this course about?

Example (Chain Matrix Multiplication)

$$A = C = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}.$$

$$B = D = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

Want: $ABCD = ?$

- Method 1: $(AB)(CD)$
- Method 2: $A((BC)D)$

Method 1 is much more efficient than Method 2.
(Expand the expression on board)

What is this course about?

- There is usually more than one algorithm for solving a problem.
- Some algorithms are more efficient than others.
- We want the most efficient algorithm.

What is this course about?

- If we have a number of alternative algorithms for solving a problem, how do we know which is the most efficient?
- To do so, we need to analyze each of them to determine its efficiency.
- Of course, we must also make sure the algorithm is correct.

What is this course about?

- In this course, we will discuss fundamental techniques for:
  - Designing efficient algorithms,
  - Proving the correctness of algorithms,
  - Analyzing the running times of algorithms

- Note:
  - Analysis and design go hand-in-hand: *By analyzing the running times of algorithms, we will know how to design fast algorithms*

Outline

- About Us
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

Computational Problem

Definition

A computational problem is a specification of the desired input-output relationship

Example (Computational Problem)

Sorting

- Input: Sequence of n numbers $\langle a_1, \cdots, a_n \rangle$
- Output: Permutation (reordering)

$$\langle a'_1, a'_2, \cdots, a'_n \rangle$$

such that $a'_1 \leq a'_2 \leq \cdots \leq a'_n$

Instance

Definition

A problem instance is any valid input to the problem.

Example (Instance of the Sorting Problem)

$$\langle 8, 3, 6, 7, 1, 2, 9 \rangle$$

Algorithm

Definition

An algorithm is a well defined computational procedure that transforms inputs into outputs, achieving the desired input-output relationship

Definition

A correct algorithm halts with the correct output for every input instance. We can then say that the algorithm solves the problem

Example: Insertion Sort

- An incremental approach: To sort a given array of length n,
  at the *i*th step it sorts the array of the first *i* items by making use of the sorted array of the first *i* - 1 items

Example

Sort $A = \langle 6, 3, 2, 4, 5 \rangle$ with insertion sort

Step 1: $\langle 6, 3, 2, 4, 5 \rangle$

Step 2: $\langle 3, 6, 2, 4, 5 \rangle$

Step 3: $\langle 2, 3, 6, 4, 5 \rangle$

Step 4: $\langle 2, 3, 4, 6, 5 \rangle$

Step 5: $\langle 2, 3, 4, 5, 6 \rangle$

Example: Insertion Sort

Pseudocode:

Input: $A[1 \ldots n]$ is an array of numbers
for $j \leftarrow 2$ **to** n **do**
    key $\leftarrow A[j]$;
    $i \leftarrow j - 1$;
    while $i \geq 1$ *and* $A[i] > key$ **do**
        $A[i+1] \leftarrow A[i]$;
        $i \leftarrow i - 1$;
    end
    $A[i+1] \leftarrow$ key;
end

	key	

Sorted   Unsorted

Where in the sorted part to put "key"?

Outline

- About Us
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

Analyzing Algorithms

- Predict resource utilization
  - Memory (space complexity)
  - Running time (time complexity) -- focus of this course
    - depends on the speed of the computer
    - depends on the implementation details
    - depends on the input, especially on the size of the input
- In light of the above factors, how can we compare different algorithms in terms of their running times?
- We want to find a way of measuring running times that is mathematically elegant and machine-independent.

Machine-independent running time

- We will measure the running time as the number of primitive operations (e.g., addition, multiplication, comparisons) used by the algorithm
- We will measure the running time as a function of the input size. Let n denote the input size and let T(n) denote the running time for input of size *n*.
- Input size ***n***: rigorous definition given later
  - Sorting: number of items to be sorted
  - Graphs: number of vertices and edges

Three Kinds of Analysis: I

Best Case: An instance for a given size n that results in the fastest possible running time.

Example (Insertion sort)

$$A[1] \leq A[2] \leq A[3] \leq \cdots \leq A[n]$$

The number of comparisons needed is equal to

$$\underbrace{1 + 1 + 1 + \cdots + 1}_{n-1} = n - 1 = \Theta(n)$$

	key	
Sorted		Unsorted

"key" is compared to only the element right before it.

Three Kinds of Analysis: II

Worst Case: An instance for a given size n that results in the slowest possible running time.

Example (Insertion sort)

$$A[1] \geq A[2] \geq A[3] \geq \cdots \geq A[n]$$

The number of comparisons needed is equal to

$$1 + 2 + \cdots + (n-1) = \frac{n(n-1)}{2} = \Theta(n^2)$$

	key	

Sorted Unsorted

"key" is compared to everything element before it.

Three Kinds of Analysis: III

Average Case: Running time averaged over all possible instances for the given size, assuming some probability distribution on the instances.

Example (Insertion sort)

$\Theta(n^2)$, assuming that each of the $n!$ instances is equally likely (uniform distribution).

	key	

Sorted Unsorted

On average, "key" is compared to half of the elements before it.

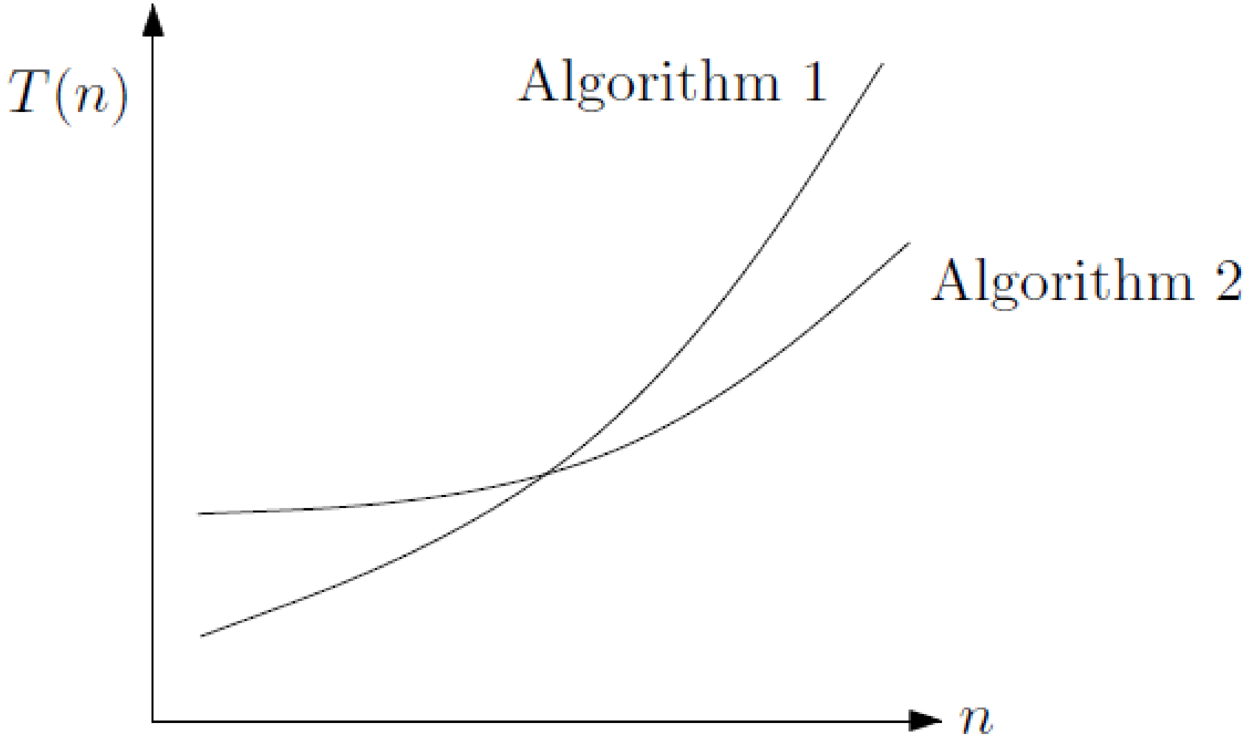
Three Kinds of Analysis

- Best case: Clearly useless
- Worst case: Commonly used, will also be used in this course
  - Gives a running time guarantee no matter what the input is
  - Fair comparison among different algorithms
- Average case: Used sometimes
  - Need to assume some distribution: real-world inputs are seldom uniformly random!
  - Analysis is complicated
  - Will not be used in this course

Outline

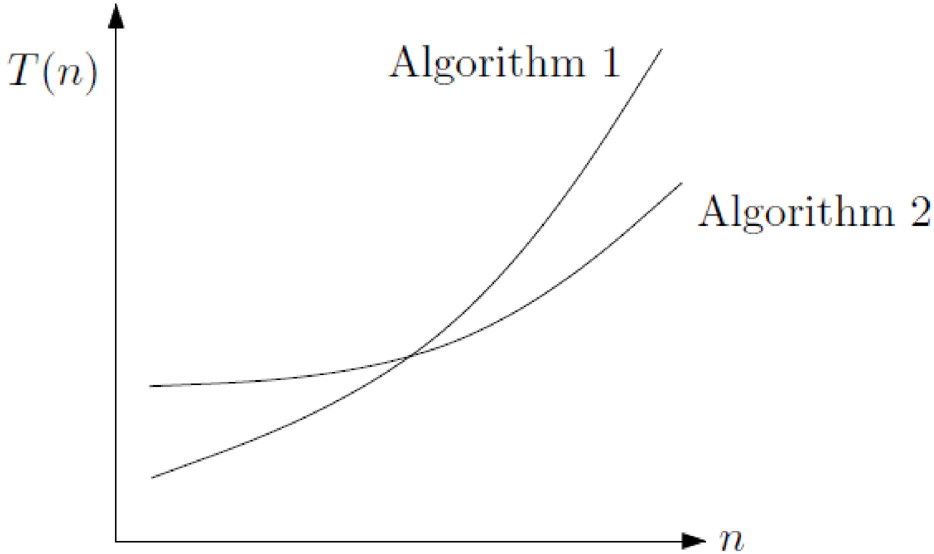
- About Us
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

Comparing Time Complexity



- Which algorithm is superior for large n?
  - T(n) for Algorithm 1 is $3n^3 + 6n^2 - 4n + 17$
  - T(n) for Algorithm 2 is $7n^2 - 8n + 20$
- Clearly, Algorithm 2 is superior.

Asymptotic Analysis



- T(n) for Algorithm 1 is $3n^3 + 6n^2 - 4n + 17 = \Theta(n^3)$
- T(n) for Algorithm 2 is $7n^2 - 8n + 20 = \Theta(n^2)$

Θ-notation

- Drop low-order terms; ignore leading constants
- Look at growth of T(n) as $n \rightarrow \infty$
- When n is large enough, a $\Theta(n^2)$ algorithm always beats a $\Theta(n^3)$ algorithm

谢谢

BDA
Beihang University