Gmacs

A generalized size-structured stock assessment model

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Outline

Indices

Symbol	Description
g	group
h	sex
i	year
j	time step (years)
k	gear or fleet
ℓ	index for size class
m	index for maturity state
o	index for shell condition

Estimated parameters

Symbol	Description
M_0	Instantaneous natural mortality rate
$\bar{R}$	Average recruitment
$\ddot{R}$	Initial recruitment
α_r	Mode of size-at-recruitment
β_r	Shape parameter for size-at-recruitment
R_0	Unfished average recruitment
κ	Recruitment compensation ratio

$$M_{0,h} > 0, \bar{R} > 0, \ddot{R} > 0, \alpha_r > 0, \beta_r > 0, R_0 > 0, \kappa > 1.0$$

$$\Theta = \{M_0, \bar{R}, \ddot{R}, \alpha_r, \beta_r, R_0, \kappa, \alpha_h, \beta_h, \varphi_h, \boldsymbol{\nu}, \boldsymbol{\xi}\}.$$

Growth parameters

Symbol	Description
α_h	Mode of size-at-recruitment
β_h	Shape parameter for size-at-recruitment
φ_h	Instantaneous natural mortality rate

$$\Phi = \{\alpha_h, \beta_h, \varphi_h\}.$$

Stuff

Symbol	Description
$w_{h,\ell}$	Mean weight at length
$m_{h,\ell}$	Average proportion mature at length

Growth

The average molt increment from size class ℓ to ℓ' is assumed to be sex-specific and is defined by the linear function

$$a_{h,\ell} = \frac{\alpha_h + \beta_h \ell}{\varphi_h}.$$

The probability of transitioning from size class ℓ to ℓ' assumes that variation in molt increments follows a gamma distribution

$$p(\ell, \ell')_h = \boldsymbol{G}_h = \int_{\ell}^{\ell+\Delta\ell} \frac{\ell^{a_{h,\ell-1}} \exp\left(\frac{\ell}{\varphi_h}\right)}{\Gamma(a_{h,\ell})\ell^{a_{h,\ell}}} \quad \text{where} \quad \Delta\ell = \ell' - \ell.$$

Specifically

$$\boldsymbol{G} = G_{\ell,\ell'} = \begin{pmatrix} G_{1,1} & G_{1,2} & \dots & G_{1,n} \\ G_{2,1} & G_{2,2} & \dots & G_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n,1} & G_{n,2} & \dots & G_{n,n} \end{pmatrix} \quad \text{where} \quad \sum_{\ell'} G_{\ell,\ell'} = 1 \quad \forall \ell.$$

Growth

Growth increment

../../examples/bbrkc/OneSex/figure/gi.png

Growth

Growth transition

../../examples/bbrkc/OneSex/figure/growth_transition.p

Natural mortality

Natural mortality (M) is assumed to be sex-specific (h), size-independent (ℓ), and may or may not be constant over time (i). If time-varying natural mortality is used, the model constrains $M_{h,i}$ to be a random-walk process with variance σ_M^2

$$M_{h,i+1} = \begin{cases} \bar{M}_h \\ M_{h,i} e^{\delta_i} \end{cases} ,$$

where

$$\delta_i \sim \mathcal{N}\left(0, \sigma_M^2\right).$$

A time-varying natural mortality can be estimated for all years (i), or for specified blocks of years ($\iota \in i$).

Natural mortality

Assuming natural mortality is time-varying constrained by variance ($\sigma_M^2 = 0.04$) at four specific knots (1976, 1980, 1985, 1994)

../../examples/bbrkc/OneSex/figure/M_t.png

Selectivity and retention

The probability of catching an animal of sex h, in year i, in fishery k, of length ℓ (i.e. selectivity) is

$$s_{h,i,k,\ell} = (1 + \exp(-(\ell - a_{h,i,k})/\sigma^s_{h,i,k}))^{-1}.$$

The probability of an animal of sex h, in year i, in fishery k, of length ℓ being retained is

$$y_{h,i,k,\ell} = (1 + \exp(-(r_{h,i,k} - \ell)/\sigma^y_{h,i,k}))^{-1}.$$

The joint probability of vulnerability due to fishing and discard mortality is

$$\nu_{h,i,k,\ell} = s_{h,i,k,\ell}\left[y_{h,i,k,\ell} + (1 - y_{h,i,k,\ell})\xi_{h,k}\right],$$

where $\xi_{h,k}$ is the discard mortality rate for sex h in fishery k.

Selectivity and retention

Assuming that selectivity for the NMFS trawl fishery is split into two blocks (1975-1981 and 1982-2014) and that retention is constant with time $y_{h,i,k,\ell} = y_{h,k,\ell}$

../../examples/bbrkc/OneSex/figure/selectivity.pn

Fishing mortality

$$\boldsymbol{F}_{k,i} = \exp\left(\bar{\boldsymbol{f}}_k + \Psi_{k,i}\right),$$
$$\boldsymbol{f}_{h,i} = \sum_k \boldsymbol{F}_{k,i}\nu_{h,i,k,\ell},$$

Growth and survival

Growth and survival

$$\boldsymbol{A}_h = \boldsymbol{G}_h \left[\exp(-\bar{M}_h)\boldsymbol{I} \right].$$

Growth and survival including fishing mortality

$$\boldsymbol{B}_{h,i} = \boldsymbol{G}_h \left[\exp(-M_{h,i} - \boldsymbol{f}_{h,i})\boldsymbol{I} \right].$$

Survivorship to length

Survivorship to length

$$\boldsymbol{u}_h = -(\boldsymbol{A}_h - \boldsymbol{I})^{-1}(p(\boldsymbol{r})),$$
$$\boldsymbol{v}_h = -(\boldsymbol{B}_h - \boldsymbol{I})^{-1}(p(\boldsymbol{r})).$$

Steady-state conditions

$$B_0 = R_0 \sum_h \lambda_h \sum_\ell \boldsymbol{u}_h w_{h,\ell} m_{h,\ell},$$
$$\tilde{B} = \tilde{R} \sum_h \lambda_h \sum_\ell \boldsymbol{v}_h w_{h,\ell} m_{h,\ell}.$$

Recruitment

Recruitment size-distribution

$$\alpha = \frac{\alpha_r}{\beta_r},$$

$$p(\boldsymbol{r}_i) = \int_{x_\ell - 0.5\Delta x}^{x_\ell + 0.5\Delta x} \frac{x^{\alpha-1} \exp\left(\frac{x}{\beta_r}\right)}{\Gamma(\alpha) x^{\alpha}} dx,$$

$$\boldsymbol{r}_{h,i} = 0.5 p(\boldsymbol{r}_i) \ddot{R}.$$

Population dynamics

Initial numbers at length

$$\boldsymbol{n}_{h,i} = \left[-\left(\boldsymbol{A}_h - \boldsymbol{I}\right)^{-1} \boldsymbol{r}_{h,i} \right] e^{\boldsymbol{\nu}} \quad \text{where} \quad i = 1.$$

The numbers in each size-class in the following time-step ($\boldsymbol{n}_{h,i+1}$) is the product of the numbers in each size-class in the previous time-step ($\boldsymbol{n}_{h,i}$), size-specific growth and survival ($\boldsymbol{A}_{h,i}$), plus new recuits ($\boldsymbol{r}_{h,i}$)

$$\boldsymbol{n}_{h,i+1} = \boldsymbol{n}_{h,i} \boldsymbol{A}_{h,i} + \boldsymbol{r}_{h,i} \quad \text{where} \quad i > 1.$$

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1. Notation and symbols
   - Indices
   - Key parameters
   - Growth parameters
   - Stuff
2. Growth and survival
   - Growth
   - Natural mortality
   - Growth and survival
3. Steady-state conditions
   - Survivorship to length

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Data weighting

Log-likelihood, likelihood, distribution

$$\ell(\mu, \sigma^2, \lambda; x) = \lambda \left(-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma^2) - \frac{1}{2\sigma^2}(x-\mu)^2 \right)$$

$$p(x|\mu, \sigma^2, \lambda) = \exp(\lambda) \left(2\pi\sigma^2\right)^{-\frac{1}{2}} \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2 \right]$$

$$x|\mu, \sigma^2, \lambda \sim \mathcal{N}\left(\mu, \lambda\sigma^2\right)$$