EE412 Foundation of Big Data Analytics, Fall 2023
HW4

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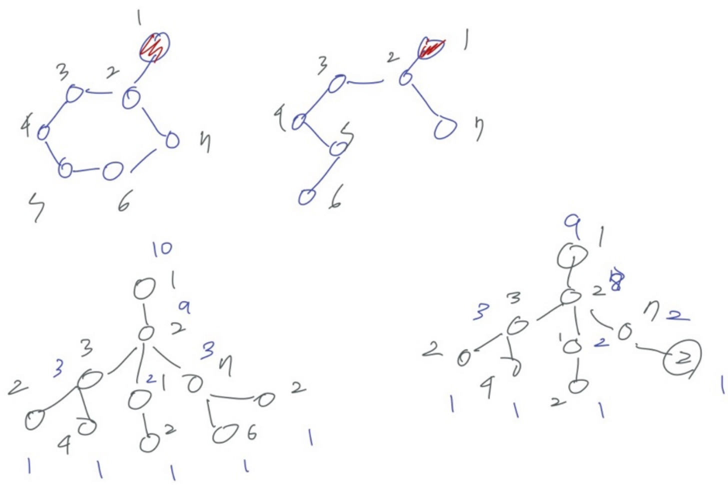
Discussion Group (People with whom you discussed ideas used in your answers):
- 김기영

On-line or hardcopy documents used as part of your answers:

1 - a)



=) 3 layer 이면 node 1에 대해서 10과 9로 다르게 embedding 됨

1 - b)

$$M(h_v^k) = \begin{cases} 1 & , \text{if } h_v^k = 1 \\ 0 & , \text{otherwise} \end{cases}$$

$$h_{N(v)}^{k+1} = \max_{u \in N(v)} \left(M(h_k^u), 0 \right)$$

$$h_v^{k+1} = \max \left(M(h_v^k), h_{N(v)}^{k+1} \right)$$

2-a)

i) $1-(p^2+p^2-2p+1) = -2p^2+2p = \text{GINI}$

$$\frac{d^2\,\text{GINI}}{dp^2} = \frac{d}{dp}(-4p+2) = -4 < 0 \Rightarrow \text{concave}$$

ii) Entropy $\quad p\log_2\left(\frac{1}{p}\right) + (1-p)\log_2\left(\frac{1}{1-p}\right)$

$$= -p\log_2 p - (1-p)\log(1-p)$$

$$\frac{d^2\,\text{Entropy}}{dp^2} = \frac{d}{dp}\left(\frac{d\,\text{Entropy}}{dp}\right) = \frac{d}{dp}\left(-\log_2 p - \frac{1}{\ln 2} + \log_2(1-p) - (1-p)\cdot\frac{1}{\ln 2}\cdot\frac{-1}{1-p}\right)$$

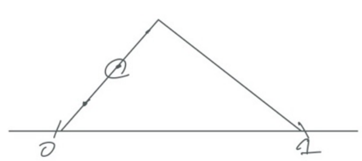
$$= \frac{d}{dp}\left(-\log_2 p - \frac{1}{\ln 2} + \log_2(1-p) + \frac{1}{\ln 2}\right)$$

$$= \frac{d}{dp}\left(\log_2(1-p) - \log_2 p\right)$$

$$= \frac{1}{\ln 2}\cdot\frac{-1}{1-p} - \frac{1}{\ln 2}\cdot\frac{1}{p} < 0$$

iii) $\underline{p < 0.5}$ $\quad 1-\max(p, 1-p) = p$

$p \geq 0.5$ $\quad 1-\max(p, (1-p)) = 1-p$



$x = 0.1 \quad y = 0.5, \quad z = 0.3$

$z = 0.3 \quad f(z) = 0.3$

$x = 0.1 \quad f(x) = 0.1$

$y = 0.5 \quad f(y) = 0.5$

$f(z) = 0.3$

same

$$\frac{y-z}{y-x}f(x) + \frac{z-x}{y-x}f(y) = \frac{0.2}{0.4}\,0.1 + \frac{0.2}{0.4}\,0.5 = \frac{1}{2}\cdot(0.6) = 0.3$$

2-b)

① - GINI Index

$P_{soccer} = \frac{5}{12}$, $P_{cricket} = \frac{3}{12}$, $P_{Hockey} = \frac{2}{12}$, $P_{baseball} = \frac{2}{12}$

$$1 - \left(\frac{25}{144} + \frac{9}{144} + \frac{4}{144} + \frac{4}{144} \right) = 1 - \frac{42}{144} = \frac{102}{144}$$

① - accuracy

$$1 - \max\left(\frac{5}{12}, \frac{3}{12}, \frac{2}{12}, \frac{2}{12} \right) = \frac{7}{12}$$

② - GINI Index

$P_{soccer} = \frac{5}{6}$, $P_{cricket} = \frac{1}{6}$

$$1 - \left(\frac{25}{36} + \frac{1}{36} \right) = 1 - \frac{26}{36} = \frac{10}{36} = \frac{5}{18}$$

② - accuracy

$$1 - \max\left(\frac{5}{6}, \frac{1}{6} \right) = \frac{1}{6}$$

③ - GINI Index

$P_{hockey} = \frac{2}{6}$, $P_{baseball} = \frac{2}{6}$, $P_{cricket} = \frac{2}{6}$

$$1 - \left(\frac{4}{36} + \frac{4}{36} + \frac{4}{36} \right) = 1 - \frac{12}{36} = \frac{24}{36} = \frac{2}{3}$$

③ - accuracy

$$1 - \frac{2}{6} = \frac{2}{3}$$

④ - GINI Index

$P_{baseball} = \frac{2}{4}$ $\quad$ $P_{cricket} = \frac{2}{4}$

$$1 - \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2}$$

④ - accuracy

$$1 - \frac{1}{2} = \frac{1}{2}$$

3 -a)

i) $\left(1 - e^{-km/n}\right)^k$

$k=3$, $n=$ 8 billion. $m=$ 1 billion. $\left(1 - e^{-3 \cdot \frac{1}{8}}\right)^3 = 0.030$

$k=4$, $n=$ 8 billion, $m=$ 1 billion. $\left(1 - e^{-4 \cdot \frac{1}{8}}\right)^4 = 0.024$

ii) $1 - k/n$

$\frac{k}{n} = \frac{1}{x}$

$x = \frac{n}{k}$

$\left(1 - \frac{k}{n}\right)^m$ $\quad 1 - \left(1 - \frac{k}{n}\right)^m = 1 - \left(1 - \frac{k}{n}\right)^{m \cdot \frac{n}{k} \cdot \frac{k}{n}}$

$= 1 - \left(1 - \frac{1}{x}\right)^{m \cdot x \cdot \frac{k}{n}}$

$= 1 - e^{-m \cdot \frac{k}{n}}$ for each array

for every k array

$\frac{d}{dx}\left(f(x)\right)^x$

$\left(1 - e^{-m \cdot \frac{k}{n}}\right)^k$

iii) optimal value of $k = \frac{n}{m} \ln 2$ $\quad \left(1 - e^{-m \cdot \frac{k}{n}}\right)^k \cdot \ln.$

3-b)

i)

a) $h(x) = 2x+1 \mod 32$

elem	hash	bit	tail length
3	7	00111	0
1	3	00011	0
4	9	01001	0
1	3	00011	0
5	11	01011	0
9	19	10011	0
2	5	00101	0
6	13	01101	0
5	11	01011	0

$\Rightarrow$ estimated distinct elem

$\Rightarrow 2^0 = 1$

b) $h(x) = 3x+7 \bmod 32$

elem	hash	bit	tail Length
3	16	10000	4
1	10	01010	1
4	19	10011	0
1	10	01010	1
5	22	10110	1
9	2	00010	1
2	13	01101	0
6	25	11001	0
5	22	10110	1

⇒ number of distinct elements ⇒ $2^4 = 16$

c) $h(x) = 4x \bmod 32$

elem	hash	bit	tail length
3	12	01100	2
1	4	00100	2
4	16	10000	4
1	4	00100	2
5	20	10100	2
9	4	00100	2
2	8	01000	3
6	24	11000	3
5	20	10100	2

⇒ number of distinct elements ⇒ $2^4 = 16$

ii)

hash function이 $ax+b \bmod 2^k$ 일때, 이 숫자는 k-bit length로 나타내진다. 전체 element를 담아낼수 있는 정도의 k bit가 있어야 한다. 위 예시를 보았을때 $k=5$는 충분하다.

하지만 hash function에 의해 2^R로 측정하는게 너무 fluctuation이 심하므로 일단 적절한 hash function을 사용해야한다.

또한 각 hash function의 결과를 평균을 내서 distinct eleme