

1.

单带图灵机, 初始带内容为[>, 1, 1, ..., 1, □, 1, ..., 1, □, □, ...]

输出从初始带上第三个□位置开始写

$$\begin{aligned}
 & \langle q_{start}, \triangleright \rangle \rightarrow \langle q_{find}, \triangleright, R \rangle \\
 & \langle q_{find}, 1 \rangle \rightarrow \langle q_{find}, 1, R \rangle, \langle q_{find}, \square \rangle \rightarrow \langle q_{mult}, R, \square \rangle \\
 & \langle q_{mult}, 0 \rangle \rightarrow \langle q_{mult}, 0, R \rangle, \langle q_{mult}, 1 \rangle \rightarrow \langle q_{copy1}, 0, L \rangle, \langle q_{mult}, \square \rangle \rightarrow \langle q_{halt}, 0, S \rangle \\
 & \langle q_{copy1}, 0 \rangle \rightarrow \langle q_{copy1}, 0, L \rangle, \langle q_{copy1}, \square \rangle \rightarrow \langle q_{copy2}, \square, L \rangle \\
 & \langle q_{copy2}, 0 \rangle \rightarrow \langle q_{copy2}, 0, L \rangle, \langle q_{copy2}, 1 \rangle \rightarrow \langle q_{copy3}, 0, R \rangle, \langle q_{copy2}, \triangleright \rangle \rightarrow \langle q_{revive}, \triangleright, R \rangle \\
 & \langle q_{copy3}, 0 \rangle \rightarrow \langle q_{copy3}, 0, R \rangle, \langle q_{copy3}, \square \rangle \rightarrow \langle q_{copy4}, \square, R \rangle \\
 & \langle q_{copy4}, 0 \rangle \rightarrow \langle q_{copy4}, 0, R \rangle, \langle q_{copy4}, 1 \rangle \rightarrow \langle q_{copy4}, 1, R \rangle, \langle q_{copy4}, \square \rangle \rightarrow \langle q_{copy5}, \square, R \rangle \\
 & \langle q_{copy5}, 1 \rangle \rightarrow \langle q_{copy5}, 1, R \rangle, \langle q_{copy5}, \square \rangle \rightarrow \langle q_{back1}, 1, L \rangle \\
 & \langle q_{back1}, 1 \rangle \rightarrow \langle q_{back1}, 1, L \rangle, \langle q_{back1}, \square \rangle \rightarrow \langle q_{back1}, \square, L \rangle, \langle q_{back1}, 0 \rangle \rightarrow \langle q_{back2}, 0, L \rangle \\
 & \langle q_{back2}, 0 \rangle \rightarrow \langle q_{back2}, 0, L \rangle, \langle q_{back2}, \square \rangle \rightarrow \langle q_{copy2}, \square, L \rangle \\
 & \langle q_{revive}, 0 \rangle \rightarrow \langle q_{revive}, 1, R \rangle, \langle q_{revive}, \square \rangle \rightarrow \langle q_{mult}, \square, R \rangle
 \end{aligned}$$

2.

1. 把字母表{0, 1}看作编码基础。所有图灵机的描述都可以写成有限长的字符串 (用某种编码)。因此图灵机的集合是可数的。记为{M₀, M₁, M₂, ...}。
2. 对于每台图灵机M_i, 它计算一个函数f_i: {0, 1}^{*} → {0, 1}^{*}。于是所有可计算的总函数构成的集合至多可数。
3. 现在看所有从{0, 1}^{*}到{0, 1}^{*}的函数的总集合F = {f: {0, 1}^{*} → {0, 1}^{*}}的势。注意{0, 1}^{*}可数, 势是N₀。但是从可数集合到一个至少含两个元素的集合的函数集的势是不可数 (确切地说, 势是2^{N₀}, 等价于R的势, 不过这还是悖论)。
4. 因此|F| = 2^{N₀}, 但可计算函数集合是可数的⇒几乎所有函数不在可计算集合中。

3.

$$\begin{aligned}
 & \langle q_{start}, \triangleright, \triangleright \rangle \rightarrow \langle q_{init}, \triangleright, R, R \rangle \\
 & \langle q_{init}, 1, \square \rangle \rightarrow \langle q_{add}, 0, S, S \rangle \\
 & \langle q_{add}, 1, 0 \rangle \rightarrow \langle q_{add}, 1, R, S \rangle, \langle q_{add}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{add}, \square, 0 \rangle \rightarrow \langle q_{halt}, 0, S, S \rangle, \langle q_{add}, \square, 1 \rangle \rightarrow \langle q_{halt}, 1, S, S \rangle \\
 & \langle q_{sp}, 1, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 1, 0 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle \\
 & \langle q_{back}, 1, 0 \rangle \rightarrow \langle q_{back}, 0, S, L \rangle, \langle q_{back}, 1, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{back}, 1, \triangleright \rangle \rightarrow \langle q_{add}, \triangleright, R, R \rangle \\
 & T(n) = O(n)
 \end{aligned}$$

$$\begin{aligned}
 & \langle q_{start}, \triangleright, \triangleright \rangle \rightarrow \langle q_{init}, \triangleright, R, R \rangle \\
 & \langle q_{init}, 0, \square \rangle \rightarrow \langle q_{init}, 0, R, R \rangle, \langle q_{init}, 1, \square \rangle \rightarrow \langle q_{init}, 0, R, R \rangle, \langle q_{init}, \square, \square \rangle \rightarrow \langle q_{init}, \square, L, L \rangle, \langle q_{init}, 0, 0 \rangle \rightarrow \langle q_{init}, 0, L, L \rangle, \\
 & \langle q_{init}, 1, 0 \rangle \rightarrow \langle q_{init}, 0, L, L \rangle, \langle q_{init}, \triangleright, \triangleright \rangle \rightarrow \langle q_{sub}, \triangleright, S, R \rangle \\
 & \langle q_{sub}, \triangleright, 0 \rangle \rightarrow \langle q_{judge}, 1, R, S \rangle, \langle q_{sub}, \triangleright, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle \\
 & \langle q_{judge}, 0, 0 \rangle \rightarrow \langle q_{judge}, 0, R, R \rangle, \langle q_{judge}, 1, 1 \rangle \rightarrow \langle q_{judge}, 1, R, R \rangle, \langle q_{judge}, 0, 1 \rangle \rightarrow \langle q_{adder}, 1, R, R \rangle, \\
 & \langle q_{judge}, 1, 0 \rangle \rightarrow \langle q_{adder}, 0, R, R \rangle, \langle q_{judge}, \square, \square \rangle \rightarrow \langle q_{halt}, \square, S, S \rangle \\
 & \langle q_{adder}, 0, 0 \rangle \rightarrow \langle q_{adder}, 0, R, R \rangle, \langle q_{adder}, 0, 1 \rangle \rightarrow \langle q_{adder}, 1, R, R \rangle, \langle q_{adder}, 1, 0 \rangle \rightarrow \langle q_{adder}, 0, R, R \rangle, \\
 & \langle q_{adder}, 1, 1 \rangle \rightarrow \langle q_{adder}, 1, R, R \rangle, \langle q_{adder}, \square, \square \rangle \rightarrow \langle q_{realadder}, \square, S, R \rangle \\
 & \langle q_{realadder}, \square, 1 \rangle \rightarrow \langle q_{realadder}, 1, S, R \rangle, \langle q_{realadder}, \square, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle \\
 & \langle q_{back}, \square, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{back}, \square, \square \rangle \rightarrow \langle q_{back}, \square, L, L \rangle, \langle q_{back}, 0, 0 \rangle \rightarrow \langle q_{back}, 0, L, L \rangle, \langle q_{back}, 0, 1 \rangle \rightarrow \langle q_{back}, 1, L, L \rangle, \\
 & \langle q_{back}, 1, 0 \rangle \rightarrow \langle q_{back}, 0, L, L \rangle, \langle q_{back}, 1, 1 \rangle \rightarrow \langle q_{back}, 1, L, L \rangle, \langle q_{back}, \triangleright, \triangleright \rangle \rightarrow \langle q_{judge}, \triangleright, R, R \rangle, \langle q_{back}, \triangleright, 0 \rangle \rightarrow \langle q_{back}, 0, S, L \rangle, \\
 & \langle q_{back}, \triangleright, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle \\
 & \langle q_{sp}, \triangleright, 0 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, \triangleright, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle \\
 & T(n) = O(2^n), 这是由于输出的1进制数至少有2ⁿ⁻¹位
 \end{aligned}$$

4.

$\langle q_{start}, \triangleright, \triangleright \rangle \rightarrow \langle q_{init}, \triangleright, S, R \rangle$
 $\langle q_{init}, \triangleright, \square \rangle \rightarrow \langle q_{alg}, 0, R, S \rangle$
 $\langle q_{alg}, 1, 0 \rangle \rightarrow \langle q_{alg}, 1, R, S \rangle, \langle q_{alg}, 0, 0 \rangle \rightarrow \langle q_{alg}, 1, R, S \rangle,$
 $\langle q_{alg}, 1, \square \rangle \rightarrow \langle q_{alg}, 1, R, S \rangle, \langle q_{alg}, 0, \square \rangle \rightarrow \langle q_{alg}, 1, R, S \rangle$
 $\langle q_{alg}, 0, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{alg}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle$
 $\langle q_{alg}, \square, 0 \rangle \rightarrow \langle q_{halt}, 0, S, S \rangle, \langle q_{alg}, \square, 1 \rangle \rightarrow \langle q_{halt}, 1, S, S \rangle$
 $\langle q_{sp}, 0, 0 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 0, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{sp}, 0, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle$
 $\langle q_{sp}, 1, 0 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{sp}, 1, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle$
 $\langle q_{back}, 0, 0 \rangle \rightarrow \langle q_{back}, 0, S, L \rangle, \langle q_{back}, 0, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{back}, 1, 0 \rangle \rightarrow \langle q_{back}, 0, S, L \rangle, \langle q_{back}, 1, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle$
 $\langle q_{back}, 0, \triangleright \rangle \rightarrow \langle q_{alg}, \triangleright, R, R \rangle, \langle q_{back}, 1, \triangleright \rangle \rightarrow \langle q_{alg}, \triangleright, R, R \rangle$

$$T(n) = O(n)$$