

# 1.

单带图灵机, 初始带内容为 $[\triangleright, 1, 1, \dots, 1, \square, 1, \dots, 1, \square, \square, \dots]$

输出从初始带上第三个 $\square$ 位置开始写

$\langle q_{start}, \triangleright \rangle \rightarrow \langle q_{find}, \triangleright, R \rangle$   
 $\langle q_{find}, 1 \rangle \rightarrow \langle q_{find}, 1, R \rangle, \langle q_{find}, \square \rangle \rightarrow \langle q_{mult}, R, \square \rangle$   
 $\langle q_{mult}, 0 \rangle \rightarrow \langle q_{mult}, 0, R \rangle, \langle q_{mult}, 1 \rangle \rightarrow \langle q_{copy1}, 0, L \rangle, \langle q_{mult}, \square \rangle \rightarrow \langle q_{halt}, 0, S \rangle$   
 $\langle q_{copy1}, 0 \rangle \rightarrow \langle q_{copy1}, 0, L \rangle, \langle q_{copy1}, \square \rangle \rightarrow \langle q_{copy2}, \square, L \rangle$   
 $\langle q_{copy2}, 0 \rangle \rightarrow \langle q_{copy2}, 0, L \rangle, \langle q_{copy2}, 1 \rangle \rightarrow \langle q_{copy3}, 0, R \rangle, \langle q_{copy2}, \triangleright \rangle \rightarrow \langle q_{revive}, \triangleright, R \rangle$   
 $\langle q_{copy3}, 0 \rangle \rightarrow \langle q_{copy3}, 0, R \rangle, \langle q_{copy3}, \square \rangle \rightarrow \langle q_{copy4}, \square, R \rangle$   
 $\langle q_{copy4}, 0 \rangle \rightarrow \langle q_{copy4}, 0, R \rangle, \langle q_{copy4}, 1 \rangle \rightarrow \langle q_{copy4}, 1, R \rangle, \langle q_{copy4}, \square \rangle \rightarrow \langle q_{copy5}, \square, R \rangle$   
 $\langle q_{copy5}, 1 \rangle \rightarrow \langle q_{copy5}, 1, R \rangle, \langle q_{copy5}, \square \rangle \rightarrow \langle q_{back1}, 1, L \rangle$   
 $\langle q_{back1}, 1 \rangle \rightarrow \langle q_{back1}, 1, L \rangle, \langle q_{back1}, \square \rangle \rightarrow \langle q_{back1}, \square, L \rangle, \langle q_{back1}, 0 \rangle \rightarrow \langle q_{back2}, 0, L \rangle$   
 $\langle q_{back2}, 0 \rangle \rightarrow \langle q_{back2}, 0, L \rangle, \langle q_{back2}, \square \rangle \rightarrow \langle q_{copy2}, \square, L \rangle$   
 $\langle q_{revive}, 0 \rangle \rightarrow \langle q_{revive}, 1, R \rangle, \langle q_{revive}, \square \rangle \rightarrow \langle q_{mult}, \square, R \rangle$

# 2.

1. 把字母表 $\{0, 1\}$ 看作编码基础。所有图灵机的描述都可以写成有限长的字符串（用某种编码）。因此图灵机的集合是可数的。记为 $\{M_0, M_1, M_2, \dots\}$ 。
2. 对于每台图灵机 $M_i$ , 它计算一个函数 $f_i: \{0, 1\}^* \rightarrow \{0, 1\}^*$ 。于是所有可计算的总函数构成的集合至多可数。
3. 现在看所有从 $\{0, 1\}^*$ 到 $\{0, 1\}^*$ 的函数的总集合 $\mathcal{F} = \{f: \{0, 1\}^* \rightarrow \{0, 1\}^*\}$ 的势。注意 $\{0, 1\}^*$ 可数, 势是 $\aleph_0$ 。但是从可数集到一个至少含两个元素的集合的函数集的势是不可数（确切地说, 势是 $2^{\aleph_0}$ , 等价于 $\mathbb{R}$ 的势, 不过这还是悖论）。
4. 因此 $|\mathcal{F}| = 2^{\aleph_0}$ , 但可计算函数集合是可数的 $\Rightarrow$ 几乎所有函数不在可计算集合中。

# 3.

$\langle q_{start}, \triangleright, \triangleright \rangle \rightarrow \langle q_{init}, \triangleright, R, R \rangle$   
 $\langle q_{init}, 1, \square \rangle \rightarrow \langle q_{add}, 0, S, S \rangle$   
 $\langle q_{add}, 1, 0 \rangle \rightarrow \langle q_{add}, 1, R, S \rangle, \langle q_{add}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{add}, \square, 0 \rangle \rightarrow \langle q_{halt}, 0, S, S \rangle, \langle q_{add}, \square, 1 \rangle \rightarrow \langle q_{halt}, 1, S, S \rangle$   
 $\langle q_{sp}, 1, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 1, 0 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle$   
 $\langle q_{back}, 1, 0 \rangle \rightarrow \langle q_{back}, 0, S, L \rangle, \langle q_{back}, 1, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{back}, 1, \triangleright \rangle \rightarrow \langle q_{add}, \triangleright, R, R \rangle$   
 $T(n) = O(n)$

$\langle q_{start}, \triangleright, \triangleright \rangle \rightarrow \langle q_{init}, \triangleright, R, R \rangle$   
 $\langle q_{init}, 0, \square \rangle \rightarrow \langle q_{init}, 0, R, R \rangle, \langle q_{init}, 1, \square \rangle \rightarrow \langle q_{init}, 0, R, R \rangle, \langle q_{init}, \square, \square \rangle \rightarrow \langle q_{init}, \square, L, L \rangle, \langle q_{init}, 0, 0 \rangle \rightarrow \langle q_{init}, 0, L, L \rangle,$   
 $\langle q_{init}, 1, 0 \rangle \rightarrow \langle q_{init}, 0, L, L \rangle, \langle q_{init}, \triangleright, \triangleright \rangle \rightarrow \langle q_{sub}, \triangleright, S, R \rangle$   
 $\langle q_{sub}, \triangleright, 0 \rangle \rightarrow \langle q_{judge}, 1, R, S \rangle, \langle q_{sub}, \triangleright, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle$   
 $\langle q_{judge}, 0, 0 \rangle \rightarrow \langle q_{judge}, 0, R, R \rangle, \langle q_{judge}, 1, 1 \rangle \rightarrow \langle q_{judge}, 1, R, R \rangle, \langle q_{judge}, 0, 1 \rangle \rightarrow \langle q_{adder}, 1, R, R \rangle,$   
 $\langle q_{judge}, 1, 0 \rangle \rightarrow \langle q_{adder}, 0, R, R \rangle, \langle q_{judge}, \square, \square \rangle \rightarrow \langle q_{halt}, \square, S, S \rangle$   
 $\langle q_{adder}, 0, 0 \rangle \rightarrow \langle q_{adder}, 0, R, R \rangle, \langle q_{adder}, 0, 1 \rangle \rightarrow \langle q_{adder}, 1, R, R \rangle, \langle q_{adder}, 1, 0 \rangle \rightarrow \langle q_{adder}, 0, R, R \rangle,$   
 $\langle q_{adder}, 1, 1 \rangle \rightarrow \langle q_{adder}, 1, R, R \rangle, \langle q_{adder}, \square, \square \rangle \rightarrow \langle q_{realadder}, \square, S, R \rangle$   
 $\langle q_{realadder}, \square, 1 \rangle \rightarrow \langle q_{realadder}, 1, S, R \rangle, \langle q_{realadder}, \square, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle$   
 $\langle q_{back}, \square, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{back}, \square, \square \rangle \rightarrow \langle q_{back}, \square, L, L \rangle, \langle q_{back}, 0, 0 \rangle \rightarrow \langle q_{back}, 0, L, L \rangle, \langle q_{back}, 0, 1 \rangle \rightarrow \langle q_{back}, 1, L, L \rangle,$   
 $\langle q_{back}, 1, 0 \rangle \rightarrow \langle q_{back}, 0, L, L \rangle, \langle q_{back}, 1, 1 \rangle \rightarrow \langle q_{back}, 1, L, L \rangle, \langle q_{back}, \triangleright, \triangleright \rangle \rightarrow \langle q_{judge}, \triangleright, R, R \rangle, \langle q_{back}, \triangleright, 0 \rangle \rightarrow \langle q_{back}, 0, S, L \rangle,$   
 $\langle q_{back}, \triangleright, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle$   
 $\langle q_{sp}, \triangleright, 0 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, \triangleright, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle$   
 $T(n) = O(2^n)$ , 这是由于输出的1进制数至少有 $2^{n-1}$ 位

## 4.

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$$\begin{aligned} \langle q_{start}, \triangleright, \triangleright \rangle &\rightarrow \langle q_{init}, \triangleright, S, R \rangle \\ \langle q_{init}, \triangleright, \square \rangle &\rightarrow \langle q_{alg}, 0, R, S \rangle \\ \langle q_{alg}, 1, 0 \rangle &\rightarrow \langle q_{alg}, 1, R, S \rangle, \langle q_{alg}, 0, 0 \rangle \rightarrow \langle q_{alg}, 1, R, S \rangle, \\ \langle q_{alg}, 1, \square \rangle &\rightarrow \langle q_{alg}, 1, R, S \rangle, \langle q_{alg}, 0, \square \rangle \rightarrow \langle q_{alg}, 1, R, S \rangle \\ \langle q_{alg}, 0, 1 \rangle &\rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{alg}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle \\ \langle q_{alg}, \square, 0 \rangle &\rightarrow \langle q_{halt}, 0, S, S \rangle, \langle q_{alg}, \square, 1 \rangle \rightarrow \langle q_{halt}, 1, S, S \rangle \\ \langle q_{sp}, 0, 0 \rangle &\rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 0, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{sp}, 0, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle \\ \langle q_{sp}, 1, 0 \rangle &\rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{sp}, 1, 1 \rangle \rightarrow \langle q_{sp}, 0, S, R \rangle, \langle q_{sp}, 1, \square \rangle \rightarrow \langle q_{back}, 1, S, L \rangle \\ \langle q_{back}, 0, 0 \rangle &\rightarrow \langle q_{back}, 0, S, L \rangle, \langle q_{back}, 0, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle, \langle q_{back}, 1, 0 \rangle \rightarrow \langle q_{back}, 0, S, L \rangle, \langle q_{back}, 1, 1 \rangle \rightarrow \langle q_{back}, 1, S, L \rangle \\ \langle q_{back}, 0, \triangleright \rangle &\rightarrow \langle q_{alg}, \triangleright, R, R \rangle, \langle q_{back}, 1, \triangleright \rangle \rightarrow \langle q_{alg}, \triangleright, R, R \rangle \\ T(n) &= O(n) \end{aligned}$$