Structural VAR Analysis for Germany

Fiscal and Monetary Policy Effects

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Zusammenfassung

This report presents a quarterly Vector Autoregression (VAR) model for Germany. The model jointly examines government spending growth (gov_g), tax revenue growth (tax_g), real GDP growth (gdp_g), inflation ($infl$), and the short-term interest rate (i). The data spans 2000Q2–2025Q3 (102 quarters). We estimate a VAR with four lags (VAR(4)), selected based on standard information criteria. Results are reported using lag-selection metrics, stability checks, residual diagnostics, orthogonalised impulse-response functions with bootstrap confidence bands, and short-term forecasts with 95% prediction intervals.

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1 Introduction

To study how fiscal policy (government spending and taxation) interacts with real activity, inflation, and monetary policy over time, we require a framework that captures dynamic feedback among macroeconomic variables. A VAR model is suitable because all included variables are treated as endogenous and can respond to their own lags and the lags of other variables.

This report constructs a consistent quarterly dataset for Germany, applies stationarity-inducing transformations where appropriate, estimates a VAR model with a constant term, and uses impulse-response analysis to interpret the dynamic effects of structural shocks. Standard diagnostics are reported, and the fitted model is also used to generate short-term forecasts.

2 Data

2.1 Data construction

Constructing the dataset involved three steps: (i) collecting the underlying macroeconomic time series, (ii) ensuring consistency across quarterly observation windows, and (iii) transforming the series into growth rates suitable for VAR estimation.

Quarterly figures for real GDP and real government spending are used; both series are available at quarterly frequency and are already seasonally adjusted. Inflation is constructed from a price index by computing the within-quarter average of the monthly index (e.g., HICP/CPI) and converting it into quarterly inflation via log differences:

$$\pi_t = 100 \times \Delta \log(P_t),$$

where P_t denotes the quarterly average of the price index.

The short-term interest rate i_t is measured as the quarterly average of the 3-month money-market rate (Euribor 3M) in levels (not log-transformed).

Tax revenues are taken from the provided fiscal tables and transformed into quarterly growth rates using

$$g_t^{\text{tax}} = 100 \times \Delta \log(T_t),$$

where T_t denotes tax revenues. No additional seasonal adjustment is applied to the tax series; therefore, seasonal revenue patterns may be reflected in the estimated dynamics.

2.2 Summary statistics

The summary statistics highlight substantial differences in scale and volatility across the series used for estimation. Government spending growth (gov_g) averages around 0.8% per quarter, but exhibits considerable variability. Tax revenue growth (tax_g) is substantially more volatile, suggesting strong sensitivity to economic cycles, policy changes, and potentially seasonal or administrative effects.

Real GDP growth (gdp_g) has a small mean but a wide range, reflecting major macroeconomic shocks during the sample period. Inflation ($infl$) is comparatively less volatile but still displays meaningful variation. The short-term interest rate (i) ranges from negative values to above 5%, capturing episodes of accommodative and tightening monetary policy.

Tabelle 1: Descriptive statistics of endogenous variables (estimation sample)

Variable	Mean	SD	Min	Max
gov_g	0.787	2.039	-9.864	9.340
tax_g	0.745	8.130	-17.162	17.332
gdp_g	0.245	1.509	-9.293	8.321
$infl$	0.513	0.716	-0.864	3.310
i	1.604	1.782	-0.567	5.023

3 Methodology

3.1 Transformations

Let x_t denote a quarterly time series. All non-interest-rate variables are transformed into growth rates:

$$g_t = 100 \times (\log x_t - \log x_{t-1}),$$

applied to gov_g, tax_g, gdp_g, and $infl$ (constructed as above). This yields approximate quarterly percentage changes and supports stationarity.

3.2 VAR specification

The reduced-form VAR of order p is

$$y_t = c + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \tag{1}$$

where y_t is the vector of endogenous variables and u_t denotes reduced-form innovations.

The baseline model includes a constant term and four lags (VAR(4)). Lag length is selected using standard information criteria computed for candidate orders $p = 1, \ldots, 8$.

Tabelle 2: Lag-order selection criteria (candidate lags 1–8)

Lag	AIC	SC (BIC)	HQ	FPE
1	2.5576	3.3693	2.8855	12.9161
2	1.7202	3.2083	2.3213	5.6156
3	0.8483	3.0128	1.7226	2.3751
4	0.1623	3.0032	1.3098	1.2222
5	0.3431	3.8605	1.7639	1.5176
6	0.2835	4.4772	1.9774	1.5091
7	0.2621	5.1322	2.2293	1.5964
8	-0.0508	5.4958	2.1896	1.3001

Table 2 shows that AIC favors a larger lag length, whereas HQ, SC/BIC, and FPE suggest four lags. Choosing $p = 4$ balances parsimony and flexibility and corresponds to one year of dynamics in quarterly data.

3.3 Identification and impulse responses

Structural shocks are identified using a recursive (Cholesky) decomposition of the residual covariance matrix. The ordering is

$$gov_g \to tax_g \to gdp_g \to infl \to i,$$

implying contemporaneous effects from earlier to later variables within a quarter, but not vice versa.

Orthogonalised impulse response functions (IRFs) are computed over 0–20 quarters. Uncertainty is quantified using 95% bootstrap confidence intervals based on 2,000 replications.

Tabelle 3: Sample, specification, and IRF settings

Item	**Value**
Country	Germany
Frequency	Quarterly
Raw panel sample	2000Q2–2025Q3 (N = 102)
VAR effective sample	2001Q2–2025Q3 (N = 98), $p = 4$ lags
Endogenous variables	gov_g, tax_g, gdp_g, $infl$, i
Deterministic terms	Constant
Identification for IRFs	Cholesky (orthogonalised)
Ordering	$gov_g \rightarrow tax_g \rightarrow gdp_g \rightarrow infl \rightarrow i$
IRF horizon / bands	0–20 quarters; 95% bootstrap CI, 2,000 runs

4 Results

4.1 Stability and residual diagnostics

Model stability is evaluated via the moduli of the companion-matrix roots. The VAR is stable if all moduli are strictly less than one.

Tabelle 4: Moduli of characteristic roots (stability check)

Root #	Modulus
1	0.9733
2	0.9611
3	0.9611
4	0.8633
5	0.8633
6	0.7999
7	0.7999
8	0.7405
9	0.7405
10	0.6773
11	0.6773
12	0.6752
13	0.6667
14	0.6667
15	0.5218
16	0.5218
17	0.4731
18	0.4731
19	0.3855
20	0.0048

All moduli are below one, indicating that the VAR satisfies the stability condition. The largest root is close to unity, suggesting high persistence, which is typical for macroeconomic series but remains within the stable region.

Tabelle 5: Model diagnostics

Diagnostic	Result
Stability (all roots < 1)	TRUE
Portmanteau autocorrelation (lags $= 12$)	$\chi^2 = 245.29$, df $= 200$, $p = 0.0159$
Portmanteau autocorrelation (lags $= 16$)	$\chi^2 = 300.86$, df $= 300$, $p = 0.4751$
Normality (multivariate Jarque–Bera)	$\chi^2 = 513.08$, df $= 10$, $p < 2.2 \times 10^{-16}$
Skewness component	$\chi^2 = 66.08$, df $= 5$, $p = 6.7 \times 10^{-13}$
Kurtosis component	$\chi^2 = 446.99$, df $= 5$, $p < 2.2 \times 10^{-16}$
ARCH (multivariate, lags $= 12$)	$\chi^2 = 1290$, df $= 2700$, $p = 1.000$

Residual diagnostics show some autocorrelation at 12 lags but not at 16 lags. Multivariate normality is rejected (common in macro data), while no evidence of multivariate ARCH effects is detected. This supports the use of bootstrap confidence bands for IRFs.

4.2 Residual covariance and correlations

Tabelle 6: Residual covariance matrix

	gov_g	tax_g	gdp_g	$infl$	i
gov_g	3.7932	-1.5327	-0.8815	0.0316	0.0447
tax_g	-1.5327	7.5781	2.3161	0.0879	-0.0375
gdp_g	-0.8815	2.3161	2.1782	0.0711	0.0843
$infl$	0.0316	0.0879	0.0711	0.2720	0.0223
i	0.0447	-0.0375	0.0843	0.0223	0.0707

The diagonal elements indicate that shocks to tax revenue growth have the largest variance. Several off-diagonal elements are non-zero, highlighting contemporaneous correlation across reduced-form innovations.

Tabelle 7: Residual correlation matrix

	gov_g	tax_g	gdp_g	$infl$	i
gov_g	1.0000	-0.2859	-0.3067	0.0311	0.0864
tax_g	-0.2859	1.0000	0.5701	0.0613	-0.0512
gdp_g	-0.3067	0.5701	1.0000	0.0923	0.2150
$infl$	0.0311	0.0613	0.0923	1.0000	0.1611
i	0.0864	-0.0512	0.2150	0.1611	1.0000

The strongest contemporaneous correlation is between tax_g and gdp_g, suggesting a close within-quarter link between revenue surprises and output. This underscores the importance of orthogonalisation (Cholesky) for interpreting structural IRFs and implies sensitivity to the chosen ordering.

4.3 Selected coefficient patterns

Due to the large number of parameters in a VAR, we focus on selected statistically significant coefficients.

Tabelle 8: Significant coefficients — Real GDP growth (gdp_g)

Term	Estimate	CI 2.5%	CI 97.5%	p-value
$infl_{t-4}$	-0.8657	-1.4507	-0.2806	0.0043
Constant	1.0873	0.2140	1.9606	0.0154

In the GDP growth equation, lagged inflation at four quarters enters negatively, suggesting that higher inflation about one year earlier is associated with lower current GDP growth. The constant term captures the average growth component not explained by included lags.

Tabelle 9: Significant coefficients — Interest rate (i)

Term	Estimate	CI 2.5%	CI 97.5%	p-value
i_{t-2}	-0.8182	-1.2515	-0.3849	0.0003
$infl_{t-1}$	0.1091	0.0032	0.2149	0.0436
i_{t-1}	1.5764	1.3373	1.8155	$< 2.2 \times 10^{-16}$

The interest rate equation shows strong persistence (large coefficient on i_{t-1}). Lagged inflation is positive, consistent with a policy reaction to inflation.

Tabelle 10: Significant coefficients — Inflation ($infl$)

Term	Estimate	CI 2.5%	CI 97.5%	p-value
tax_{gt-2}	0.0555	0.0164	0.0945	0.0060
$infl_{t-1}$	0.2692	0.0616	0.4769	0.0117
gdp_{gt-2}	0.1155	0.0158	0.2153	0.0238
$infl_{t-3}$	0.2288	0.0230	0.4346	0.0298
gdp_{gt-1}	0.0900	0.0009	0.1789	0.0478
$infl_{t-4}$	0.4318	0.2250	0.6385	8.2×10^{-5}

Inflation is persistent (significant own lags) and responds positively to lagged GDP growth. Tax revenue growth at lag two is also positive, though interpretation requires caution given the aggregate nature of the tax measure.

Tabelle 11: Significant coefficients — Tax revenue growth (tax_g)

Term	Estimate	CI 2.5%	CI 97.5%	p-value
$infl_{t-4}$	-1.8420	-2.9333	-0.7507	0.0012
tax_{gt-3}	-0.3408	-0.5435	-0.1381	0.0013
tax_{gt-1}	-0.3193	-0.5187	-0.1200	0.0021
tax_{gt-2}	-0.2274	-0.4336	-0.0213	0.0311
gdp_{gt-3}	0.5586	0.0428	1.0744	0.0342
tax_{gt-4}	0.4847	0.2992	0.6703	1.6×10^{-6}

Tax revenue growth exhibits strong dependence on its own lags and delayed business cycle effects (positive coefficient on gdp_{gt-3}). The pattern of negative short lags and a positive fourth lag is consistent with mean reversion and an annual timing component.

4.4 Impulse response functions

Orthogonalised impulse response functions are computed under the ordering $gov_g \rightarrow tax_g \rightarrow gdp_g \rightarrow infl \rightarrow i$, with 95% bootstrap confidence intervals (2,000 runs).

Tabelle 12: IRF peaks and whether the 95% band excludes zero (selected responses)

Shock	Response	|Peak IRF|	Peak IRF	Peak horizon	Any horizon CI excludes 0?
gov_g	gdp_g	0.453	-0.453	0	No
gov_g	$infl$	0.068	-0.068	1	No
tax_g	gdp_g	0.743	0.743	0	Yes (first: 0, 4)
tax_g	$infl$	0.236	0.236	2	Yes (first: 1, 2, 6)
i	gdp_g	0.300	0.300	1	No
i	$infl$	0.069	-0.069	2	No
i	i	0.422	0.422	2	Yes (first: 0, 1, 2)

Output growth shows a short initial decline after a government-spending shock, followed by partial recovery and a return toward zero. However, the uncertainty band is wide and includes zero across horizons, so the average effect is not statistically distinguishable from zero in this setup.

Inflation also fluctuates around zero after a government-spending shock, with bands overlapping zero throughout. By contrast, a tax growth shock is associated with a positive inflation response at selected horizons, and the confidence band excludes zero early on.

irf_infl_tax.pdf

Abbildung 1: IRF of inflation to a tax growth shock (orthogonalised; 95% bootstrap CI, 2,000 runs)

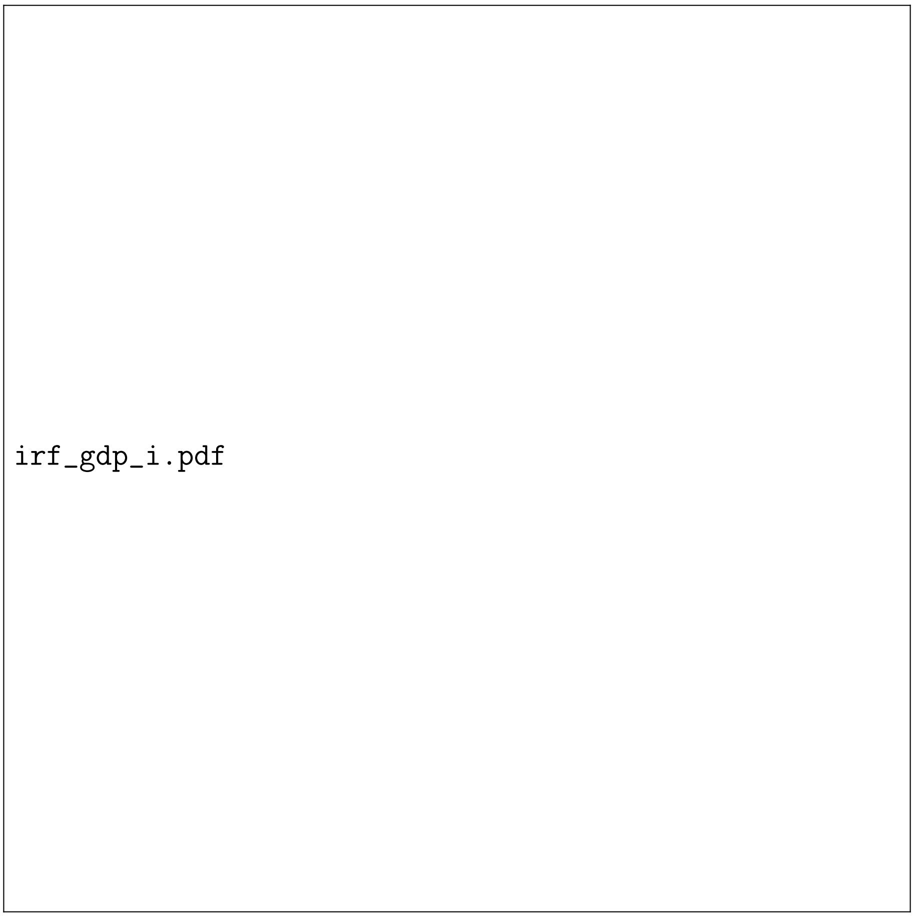

Abbildung 2: IRF of real GDP growth to an interest rate shock (orthogonalised; 95% bootstrap CI, 2,000 runs)

irf_infl_i.pdf

Abbildung 3: IRF of inflation to an interest rate shock (orthogonalised; 95% bootstrap CI, 2,000 runs) irf_i_i.pdf

Abbildung 4: IRF of the interest rate to its own shock (orthogonalised; 95% bootstrap CI, 2,000 runs)

4.5 Forecasts

The fitted VAR model is used to generate forecasts up to eight quarters ahead. Point forecasts are accompanied by 95% prediction intervals.

Tabelle 13: Forecasts for government spending growth (gov_g)

Horizon	Forecast	Lower	Upper
1	0.320	-3.497	4.137
2	0.812	-3.458	5.082
3	0.977	-3.320	5.273
4	1.067	-3.291	5.424
5	0.656	-3.733	5.045
6	0.938	-3.517	5.393
7	0.667	-3.799	5.132
8	0.935	-3.542	5.413

Tabelle 14: Forecasts for tax revenue growth (tax_g)

Horizon	Forecast	Lower	Upper
1	9.306	3.911	14.702
2	-4.329	-9.976	1.319
3	1.115	-4.892	7.122
4	-3.667	-9.776	2.442
5	8.357	1.243	15.471
6	-3.548	-10.952	3.855
7	1.231	-6.424	8.887
8	-3.204	-10.950	4.542

Tabelle 15: Forecasts for real GDP growth (gdp_g)

Horizon	Forecast	Lower	Upper
1	0.137	-2.756	3.029
2	-0.029	-3.097	3.039
3	0.010	-3.068	3.088
4	0.544	-2.568	3.655
5	0.123	-3.127	3.374
6	-0.062	-3.325	3.200
7	0.163	-3.105	3.432
8	0.551	-2.739	3.841

Tabelle 16: Forecasts for inflation ($infl$)

Horizon	Forecast	Lower	Upper
1	0.148	-0.874	1.170
2	0.601	-0.531	1.733
3	0.987	-0.276	2.251
4	0.565	-0.733	1.863
5	0.160	-1.223	1.543
6	0.587	-0.828	2.002
7	0.871	-0.578	2.319
8	0.531	-0.929	1.991

Tabelle 17: Forecasts for the interest rate (i)

Horizon	Forecast	Lower	Upper
1	1.904	1.383	2.425
2	1.746	0.729	2.764
3	1.698	0.254	3.143
4	1.668	-0.144	3.480
5	1.642	-0.478	3.762
6	1.575	-0.801	3.950
7	1.570	-1.015	4.156
8	1.567	-1.198	4.332

5 Discussion and limitations

The results should be interpreted with caution.

First, identification relies on a recursive ordering (Cholesky). This implies that variables ordered earlier can affect later variables contemporaneously within a quarter, whereas the interest rate is ordered last and therefore does not affect fiscal and real variables instantaneously. Impulse responses could differ under alternative plausible orderings or other identification approaches, such as sign restrictions or narrative instruments.

Second, fiscal and tax series often display strong seasonality and institutional timing. While log differences reduce trends, they do not guarantee removal of all seasonal components, especially for tax revenues. Future work could apply explicit seasonal adjustment (e.g., X-13ARIMA-SEATS) or include seasonal dummies.

Third, the sample spans major structural events (euro introduction, the global financial crisis, the sovereign debt crisis, COVID-19, and the 2022–2023 energy shock). Parameter stability may not hold across these regimes. Robustness checks could include sub-sample estimation, dummy variables for breaks, or time-varying parameter VARs.

6 Conclusion

The baseline quarterly VAR(4) specification for Germany is dynamically stable and yields interpretable dynamics over 2000Q2–2025Q3. The interest rate shows the clearest persistence, while estimated effects of fiscal and tax shocks on GDP growth and inflation are generally small and often statistically uncertain at the quarterly frequency. Despite limitations, the framework provides a transparent starting point for policy discussion and can be strengthened via robustness checks on identification, seasonality, and structural stability.