



Introduction Wilf-Zeilberger's method Gosper's algorithm Implementation Results Discussion and conclusions on the conclusions of the conclusions on the conclusions of the conclusions on the conclusions on the conclusions on the conclusions on the conclusions of the conclusions on the conclusions of the conclusions o

Polynomial Computer Algebra and implementation of Wilf-Zeilberger's method

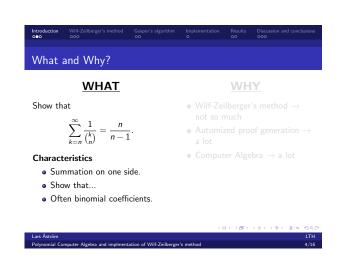
Polynomial Computer Algebra and implementation of

What is the thesis about?

Wilf-Zeilberger's method

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Introduction 000	Wilf-Zeilberger's method 000	Gosper's algorithm 00		Results 00	Discussion and conclusions			
What a	and Why?							
	WHAT			WH	<u>IY</u>			
Show th	$\sum_{k=n}^{\infty} \frac{1}{\binom{k}{n}} = \frac{n}{n-1}$		 Wilf-Zeilberger's method → not so much Automized proof generation - a lot 					
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WHAT

Show that

$$\sum_{k=n}^{\infty} \frac{1}{\binom{k}{n}} = \frac{n}{n-1}.$$

Characteristics

- Summation on one side.
- Show that...
- Often binomial coefficients.

WHY

- $\bullet \ \ \text{Wilf-Zeilberger's method} \ \to \ \$ not so much

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What and Why?

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Show that

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WHY

- $\bullet \ \ \text{Wilf-Zeilberger's method} \ \to$ not so much
- ullet Automized proof generation ightarrow
- $\bullet \ \, \mathsf{Computer} \ \, \mathsf{Algebra} \to \mathsf{a} \ \, \mathsf{lot} \\$

Historical background

Short version of the thesis

Important findings

- 1960s: Computer Algebra
- 1978: Gosper's Algorithm
- 1990: Wilf-Zeilberger's method
- 1994: WZ implemented in Mathematica

Short version of the thesis

- Polynomials

- \bullet Used for implementation of WZ
- Polynomial

$$p(k) = a_0 + a_1 k + \ldots + a_m k^m$$
 is stored as

$$[a_0, a_1, \ldots, a_m]$$

• Coefficients ai can be integers or polynomials

Short version of the thesis

- Wilf-Zeilberger's method

- \bullet Used to prove identities on the form

$$\sum_{k} F(n,k) = 1$$

• Does this by proving
$$\sum_{k}^{k} F(n+1,k) = \sum_{k} F(n,k)$$

• Which is done by "changing variables"

Short version of the thesis

- Gosper's algorithm

... I Computer Algebra and implmentation of Wilf-Zeilberger's method

- \bullet An algorithm to find a function ${\cal S}$ such

$$a_k = S_k - S_{k-1}$$

• Finds the change of variables needed in

Short version of the thesis

- Results
- The program writes formal proofs
- Proves 80% of the examples
- $\bullet\,$ The remaining seem impossible to prove by WZ method

Short version of the thesis

- Conclusions
- The program seems to work well, although cannot solve all examples
- Computer Algebra quickly gets complicated

What problems can be solved?

Problems on the form

can be solved. Problems on the form

What problems can be solved?

Problems on the form

$$\sum_k F(n,k) = 1$$

can be solved. Problems on the form

$$\sum A(n,k)=B(n)$$

can get converted to the right form.

The idea

Want to prove

$$\sum_k F(n,k) = 1$$

$$\sum_{k} F(n+1,k) - F(n,k) = \sum_{k} G(n,k+1) - G(n,k) = 0.$$

$$\sum_{i} F(n,k)$$

The idea

Want to prove

$$\sum_{k} F(n,k) = 1$$

Find G(n,k) such that F(n+1,k)-F(n,k)=G(n,k+1)-G(n,k)and $\lim_{k \to \pm \infty} \mathcal{G}(n,k) = 0.$ Now

$$\sum_{k} F(n+1,k) - F(n,k) = \sum_{k} G(n,k+1) - G(n,k) = 0.$$

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$$\sum_{k} F(n+1,k) - F(n,k) = \sum_{k} G(n,k+1) - G(n,k) = 0.$$

Therefore

$$\sum_{i} F(n,k)$$

is constant for all n, and if we can evaluate for one n then we are done.

Steps of the method

Start with

$$\sum_{k} A(n,k) = B(n)$$

- Find G(n, k) such that the
- \bigcirc Show that $\sum_k F(n',k) = 1$ for

$$\sum_{k} \binom{n}{k} = 2$$

- $(n,k) = \frac{\binom{n}{k}}{2^n}$
- **a** Let $G(n,k) = -\frac{\binom{n}{k-1}}{2^{n+1}}$
- For n = 0 we have $\sum_{k} F(n, k) = \frac{\binom{0}{0}}{2^{0}} = 1$, thus we have proved the identity.

Steps of the method

Start with $\sum A(n,k)=B(n)$

- ② Let $F(n,k) = \frac{A(n,k)}{B(n)}$
- \bigcirc Find G(n, k) such that the
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$$\sum_{k} \binom{n}{k} = 2^{n}$$

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Steps of the method

Start with $\sum_{k} A(n,k) = B(n)$

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Steps of the method

- Start with $\sum_{k} A(n, k) = B(n)$
- Find G(n, k) such that the conditions are satisfied
- Show that $\sum_{k} F(n', k) = 1$ for some n'
- Show that

$$\sum_{k} \binom{n}{k} = 2^{n}$$

- $F(n,k) = \frac{\binom{n}{k}}{2^n}$
- **a** Let $G(n,k) = -\frac{\binom{n}{k-1}}{2^{n+1}}$
- For n = 0 we have $\sum_{k} F(n, k) = \frac{\binom{n}{2}}{2^{0}} = 1$, thus we have proved the identity.

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Steps of the method

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Given an expression a_k , Gosper's algorithm finds an expression S_k such that

 $a_k = S_k - S_{k-1}.$

With $a_k=F(n+1,k)-F(n,k)$ we get that $G(n,k)=S_{k-1}$ makes the first condition in Wilf-Zeilberger's method fulfilled.

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Polynomial Computer Algebra and implmentation of Wilf-Zeilberger's method

What problems can be solved?

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Steps of the algorithm

Find polynomials p_k, q_k, r_k such that $gcd(q_k, r_{k+j}) = 1$ $\forall j \geq 0$ and $\frac{a_k}{a_{k-1}} = \frac{p_k}{p_{k-1}} \frac{q_k}{r_k}$ Sind polynomial f_k

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Find polynomials

Steps of the algorithm

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- Find polynomials p_k, q_k, r_k such that $gcd(q_k, r_{k+j}) = 1$ $\forall j \ge 0 \text{ and}$ $\frac{a_k}{a_{k-1}} = \frac{p_k}{p_{k-1}} \frac{q_k}{r_k}$
- \odot Find polynomial f_k such that $p_k = q_{k+1}f_k - r_kf_{k-1}$
- Now we see that $S_k S_{k-1} = \frac{q_{k+1}}{\rho_k} f_k a_k \frac{q_k}{\rho_{k-1}} f_{k-1} a_{k-1} =$ $\begin{aligned} & p_k & p_{k-1} \\ & = \frac{a_k}{p_k} \left(q_{k+1} f_k - \frac{q_k}{p_{k-1}} f_{k-1} p_k \frac{a_{k-1}}{a_k} \right) = \\ & = \frac{a_k}{p_k} \left(q_{k+1} f_k - \frac{q_k}{p_{k-1}} f_{k-1} p_k \frac{p_{k-1}}{p_k} \frac{f_k}{q_k} \right) = \\ & = \frac{a_k}{p_k} \left(q_{k+1} f_k - r_k f_{k-1} \right) = \frac{a_k}{p_k} p_k = a_k, \end{aligned}$ which means that this S_k indeed is a solution.

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- $p_k = q_{k+1}f_k r_kf_{k-1}$
- For $\sum_{k} \binom{n}{k} = 2^n$ we get $\frac{a_k}{a_{k-1}} = \frac{(2k - n - 1)(n + 2 - k)}{k(2k - n - 3)}$ which gives us $p_k = 2k - n - 1$, $q_k = n + 2 - k, \ r_k = k.$
- In
- Now we get $S_k = -\frac{n+1-k}{2k-n-1}a_k = -\frac{\binom{n}{k}}{2^{n+1}}$, which corresponds to the G(n,k) we got

Steps of the algorithm

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- $2k n 1 = (n + 1 k)f_k kf_{k-1}$ we see that $f_k = -1$ gives a solution.

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Steps of the algorithm

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- in the previous example.

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2300 lines of code

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- 50% methods for polynomials and Wilf-Zeilberger's method

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- 50% methods for polynomials and Wilf-Zeilberger's method
- 20% parsing

2300 lines of code

- 50% methods for polynomials and Wilf-Zeilberger's method
- 20% parsing
- 30% testing of the methods

Results as statistics

ullet 10 examples for training, 10 for validation

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- The automatic solver manages to prove 8 of each
- The remaining examples seem to be unsolvable using Wilf-Zeilberger's method

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Results as statistics

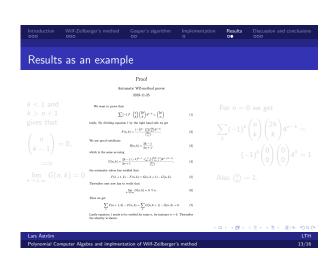
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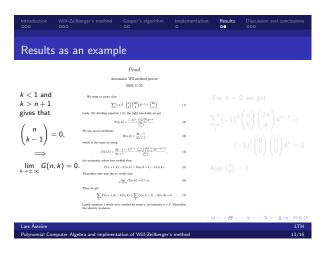
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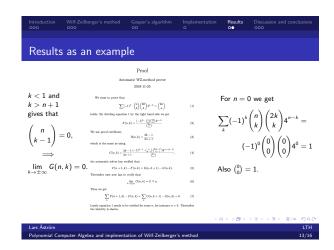


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Introduction Wilf-Zeilberger's method Gosper's algorithm Implementation Results 0 Discussion and conclusions

DOES NOT WORK

- Cannot come up with solution, only prove
- Some parts are left for the us
- Similar examples with different results

WORKS WELL

- Solves most examples
- Gives a solution quickly

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- Can Wilf-Zeilberger's method be used on other types of problems? (not binomial coefficients)
- Combine the program with guessing solution to identity
- Computer algebra in general



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Thank you for listening!

Polynomial $p(k)=a_0+a_1k+\ldots+a_mk^m$ is stored as $[a_0,a_1,\ldots,a_m].$

Polynomials – Representation

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Polynomials 1 – Example

The polynomial $p(k,m) = 1 + k^2 + km - m^2 + km^2 + k^2m^2$ is stored as [[1,0,1],[0,1,0],[-1,1,1]].

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ial Computer Algebra and implmentation of Wilf-Zeilberger's method

Polynomials – Addition

Assume we want to add $f = [f_0, \dots, f_{m_f}]$ and $g = [g_0, \dots, g_{m_g}]$. Then we

$$-f_{\cdot} \perp \sigma_{\cdot}$$

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Extra slides

Polynomials – Addition

Assume we want to add $f = [f_0, \dots, f_{m_f}]$ and $g = [g_0, \dots, g_{m_g}].$ Then we get $h = [h_0, \ldots, h_m]$ where $m = max(m_f, m_g)$. Then we have that

Extra slides

Polynomials – Addition

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 $h_i = f_i + g_i,$

if f_i and g_i are integers. Otherwise we get

Extra slides

Polynomials – Addition

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if f_i and g_i are integers. Otherwise we get

 $h_i = ADD(f_i, g_i).$

Extra slides

Polynomials - Multiplication

Assume we want to multiply $f = [f_0, \dots, f_{m_f}]$ and $g = [g_0, \dots, g_{m_g}]$. Then

$$h_i = \sum_{k=0}^{\infty} MULTIPLY(f_k, g_{i-k}).$$

Polynomials - Multiplication

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$$h_i = \sum_{i=1}^i f_k \cdot g_{i-k},$$

$$h_i = \sum_{k=0}^{n} MULTIPLY(f_k, g_{i-k}).$$

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Extra slides

Polynomials - Multiplication

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$$h_i = \sum_{k=0}^i f_k \cdot g_{i-k},$$

if f_i and g_i are of one and the same variable. Otherwise we get

$$h_i = \sum_{k=0}^{n} MULTIPLY(f_k, g_{i-k}).$$

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Extra slides

Polynomials - Multiplication

Assume we want to multiply $f = [f_0, \dots, f_{m_f}]$ and $g = [g_0, \dots, g_{m_g}]$. Then we get $h = [h_0, \dots, h_m]$ where $m = m_f + m_g$. Then we have that $h_i = \sum_{k=0}^i f_k \cdot g_{i-k},$ if f_i and g_i are of one and the same variable. Otherwise we get $h_i = \sum_{k=0}^i MULTIPLY(f_k, g_{i-k}).$

$$h_i = \sum_{i=1}^{i} f_k \cdot g_{i-k},$$

$$h_i = \sum_{i}^{i} MULTIPLY(f_k, g_{i-k}).$$

Extra slides

Polynomials – Division

Usually division (a divided by b) is done by finding polynomials q, r such

$$a = q \cdot b + r$$
,

and deg(r) < deg(b). This is not possible in integer coefficients. Therefore

$$f \cdot a = q \cdot b + r,$$

Extra slides

Polynomials – Division

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Extra slides

Polynomials – Division

Usually division (a divided by b) is done by finding polynomials q, r such

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and deg(r) < deg(b). This is not possible in integer coefficients. Therefore we use q, r, f such that

$$f \cdot a = q \cdot b + r$$
,

deg(r) < deg(b) and f has the same variable setup as the coefficients of a and b.

Extra slides

Polynomials – GCD

We get gcd by Euclid's algorithm. With division as

 $a = q \cdot D +$

gcd is usually done by

$$gcd(a,b) = a$$
 if $b = 0$ else $gcd(b,r)$

With division a

$$f \cdot a = g \cdot b + r$$

we get gcd by

$$gcd(a,b) = a \text{ if } b = 0 \text{ else } \frac{g}{\bar{g}}gcd(\bar{a},\bar{b}),$$

where \bar{x} denotes gcd of the coefficients in x and g = gcd(b, r)

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Extra slides 00000€00

Polynomials – GCD

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$$a = q \cdot b + r$$

 $\ensuremath{\mathit{gcd}}$ is usually done by

$$gcd(a,b) = a$$
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$$f \cdot g = g \cdot h + g$$

we get gcd hy

$$g(d(a,b) = a \text{ if } b = 0 \text{ else } \frac{g}{g}g(d(\bar{a},\bar{b}))$$

where \bar{y} denotes σcd of the coefficients in y and $\sigma = \sigma cd$

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Proof generation

Steps of proof generation

- Get input and parse
- Parse input \rightarrow get F(n,k) and $\frac{a_k}{a_{k-1}}$
- Get G(n, k) from Gosper's algorithm
- Write proof in IATEXformat
- Highlight parts that the user need to complete

40 × 40 × 42 × 42 × 21 ± 40 4

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10 + 1**5** + 12 + 12 + 2| = 10 0

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Extra slides

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Extra slides

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4 D > 4 Q > 4 S > 4 S > 3 L S | 2 L S | 2 L S |

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Proof generation

Steps of proof generation

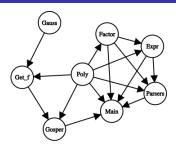
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Dependencies of the code



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