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The latest “ace up your sleeve” in the search for the optimal liquidity stress testing model: Benford’s law testing

“It is a part of probability that many improbable things will happen”

Aristotele

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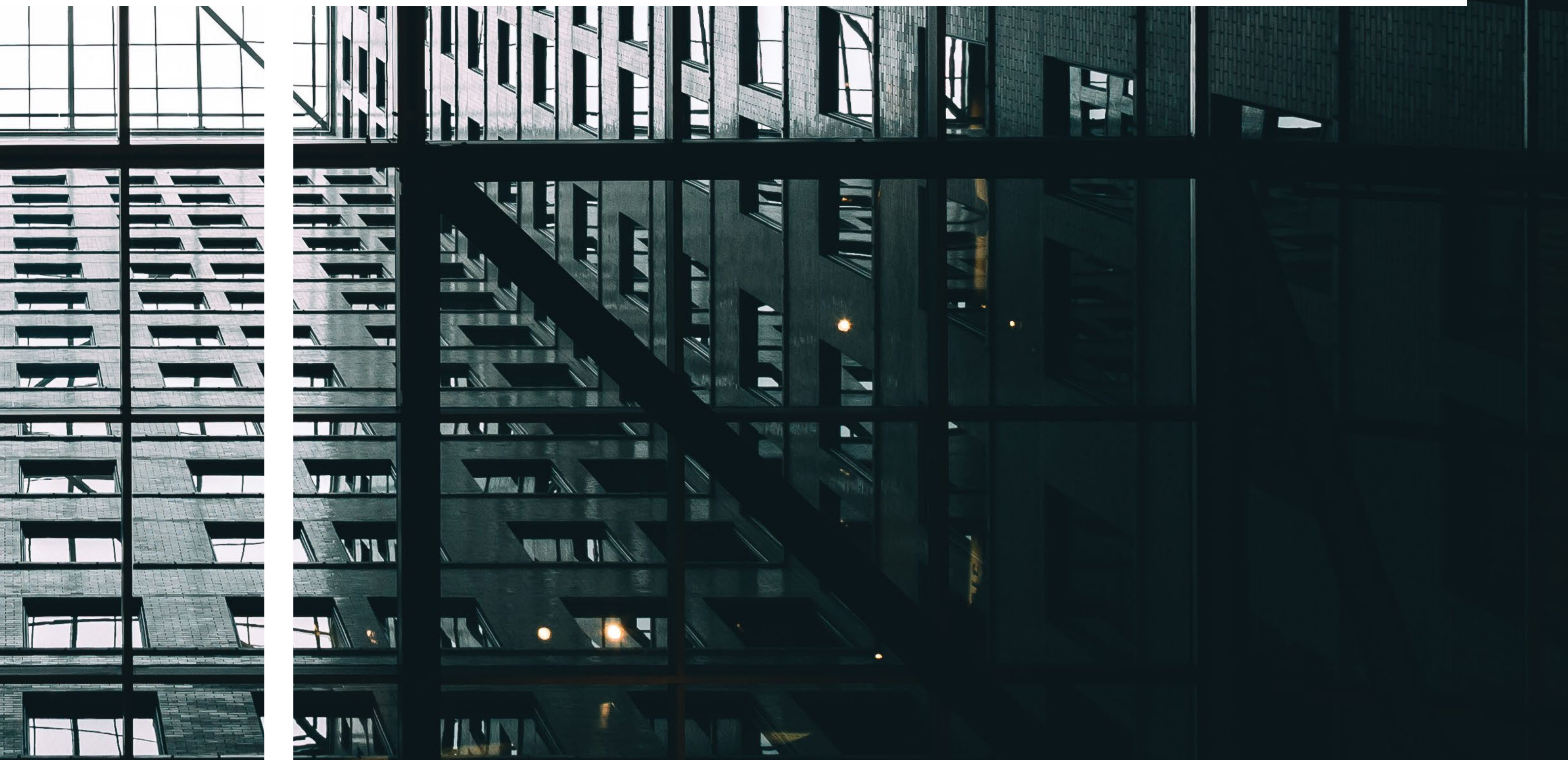


Why should Benford's law matter to you?

If your work has anything to do with **predictive models, stress tests** or similar things, this publication will provide you with some great insights into how **Benford's law** can be used to better evaluate the robustness of your models and to understand how realistic their output is.

We focus here in particular on **liquidity stress testing**, an area in which we have vast experience: anyone working in treasury and liquidity modelling could learn something new about how to optimise their models using the data they have.

That being said, our findings can be applied to any predictive model, as there are a multitude of possibilities. Anyone working with models will understand the extensive potential of this powerful tool.

The beauty of Benford's law is that it uses a relatively **small amount of data and computational power**, and yet it returns some **great insights into how good your predictive model is** – provided you set up Benford's testing correctly. At PwC, we have experience in this: this document will help you understand the basics of our approach, and we are happy to work with you to ensure you exploit our findings to the fullest.

Benford's law: giving new life to an old tool

This publication covers a topic that, unfortunately, is still not widely known about in the financial services community, outside the audit practice. Most readers (except those of you who are auditors) are probably not familiar with Benford's law, although some might have come across it thanks to a recent docu-series[1] uncovering the most unbelievable findings linked to this extraordinary law.

As experts in liquidity and funding modelling, we are constantly looking for better, more effective and predictive models, and in this quest we stumbled across Benford's law – already used as a detective tool in auditing, but not yet applied to financial modelling. By applying this law to liquidity model outputs, we are able to quickly identify potential red flags, thus making it easier to evaluate the quality of a model.

In this paper, you will read about the fundamentals of this law, the key goals of a liquidity stress test model and how Benford's law can be used to help achieve those goals, with the support of our expert team.

[1] "Connected", by Latif Nasser – Episode 4: "Digits".

What is Benford's law?

Benford's law[2] applies to many naturally occurring series of number, and it can predict the frequency distribution of the first digits in those numbers. Although one would naturally assume a uniform distribution of the first digits (as represented by the orange series in Figure 1), in many real-life series the distribution is skewed towards small digits, as shown by the series in grey, i.e. the expected distribution as per Benford's law.

In a nutshell, in many real-life sets of numbers, 1 is the first digit 30.1% of the time, 2 occurs 17.6% of the times, and so on.

Although this was originally just based on observations of real datasets (the discovery of Benford's law goes back to 1881), in recent years robust proof for this law was put forward. After the validity of the law was established, the mathematician Boyle was able to demonstrate that dataset elements that have been processed by mathematical operators (such as multiplication and division) also follow Benford's law. Thanks to this discovery, it was proven that accounting numbers generally appear to follow Benford's law distribution[3] – something that opened the door for a number of applications in this field.

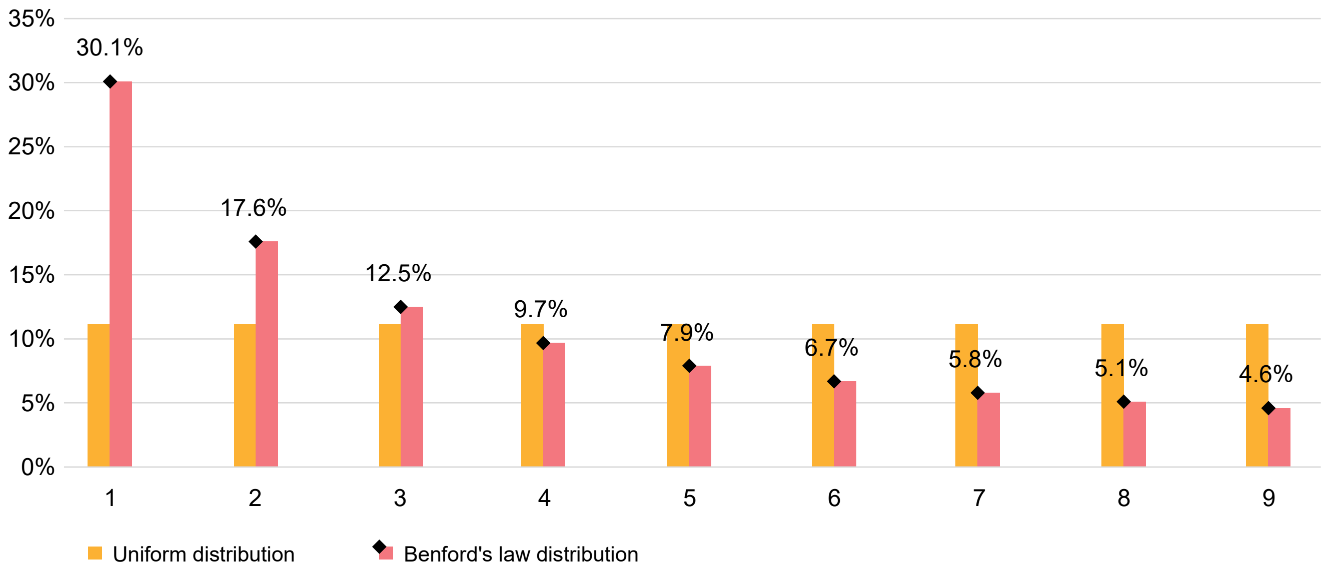


Figure 1: Leading digits and their expected occurrence according to Benford's law – in comparison with a uniform distribution

How does it work?

To understand how Benford's law works, imagine a dataset containing every single accounting record which – once aggregated – gives us the balance sheet data of a global bank. Imagine these records to be as granular as possible, i.e. every transaction from any entity within that bank is recorded on a separate line.

For each of these records, a balance sheet amount is indicated – now, those balance sheet amounts are essentially a series of real-life data. Imagine you record, in a separate column, the first (or 'leading') digit of each of those numbers. Considering our assumptions, one would expect this new column to be populated with a 'uniform' distribution of each digit, i.e. we would expect each of these digits to appear with the same frequency (series in orange in Figure 1). This is where Benford's law comes into play: those leading digits will be distributed according to Benford's law, i.e. the series in pink in Figure 1. Mind-blowing, right?

Note that Benford's law does not apply to just any data-set: for the law to hold, all numbers must be equally likely to appear and the numbers must span multiple orders of magnitude (e.g. range from 100 to 10,000,000).

[2] Also called the Newcomb–Benford law, the law of anomalous numbers, or the first-digit law.
[3] Nirosh Kuruppu, The application of Benford's law in fraud detection: a systematic methodology, September 2019.

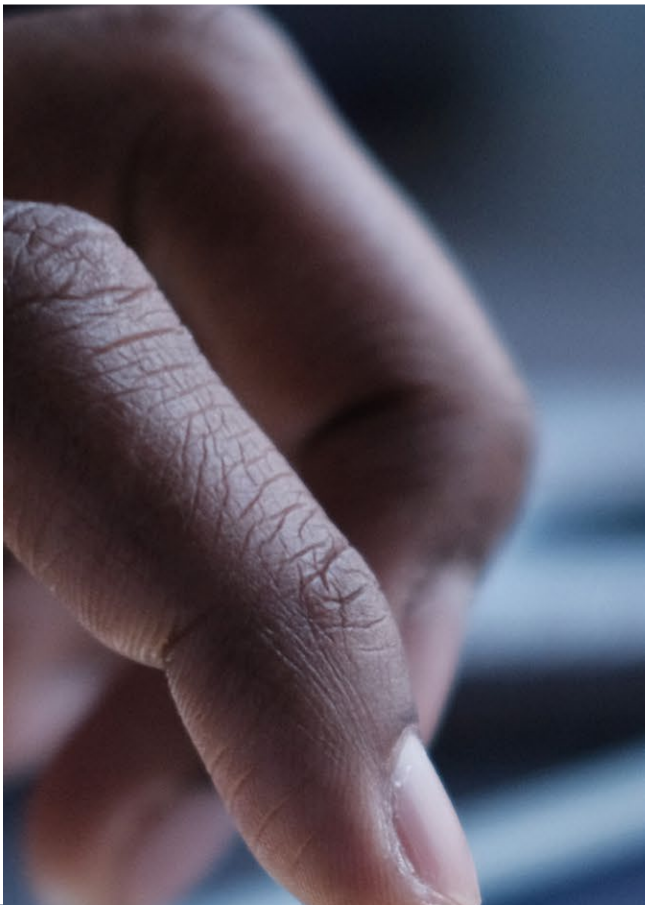
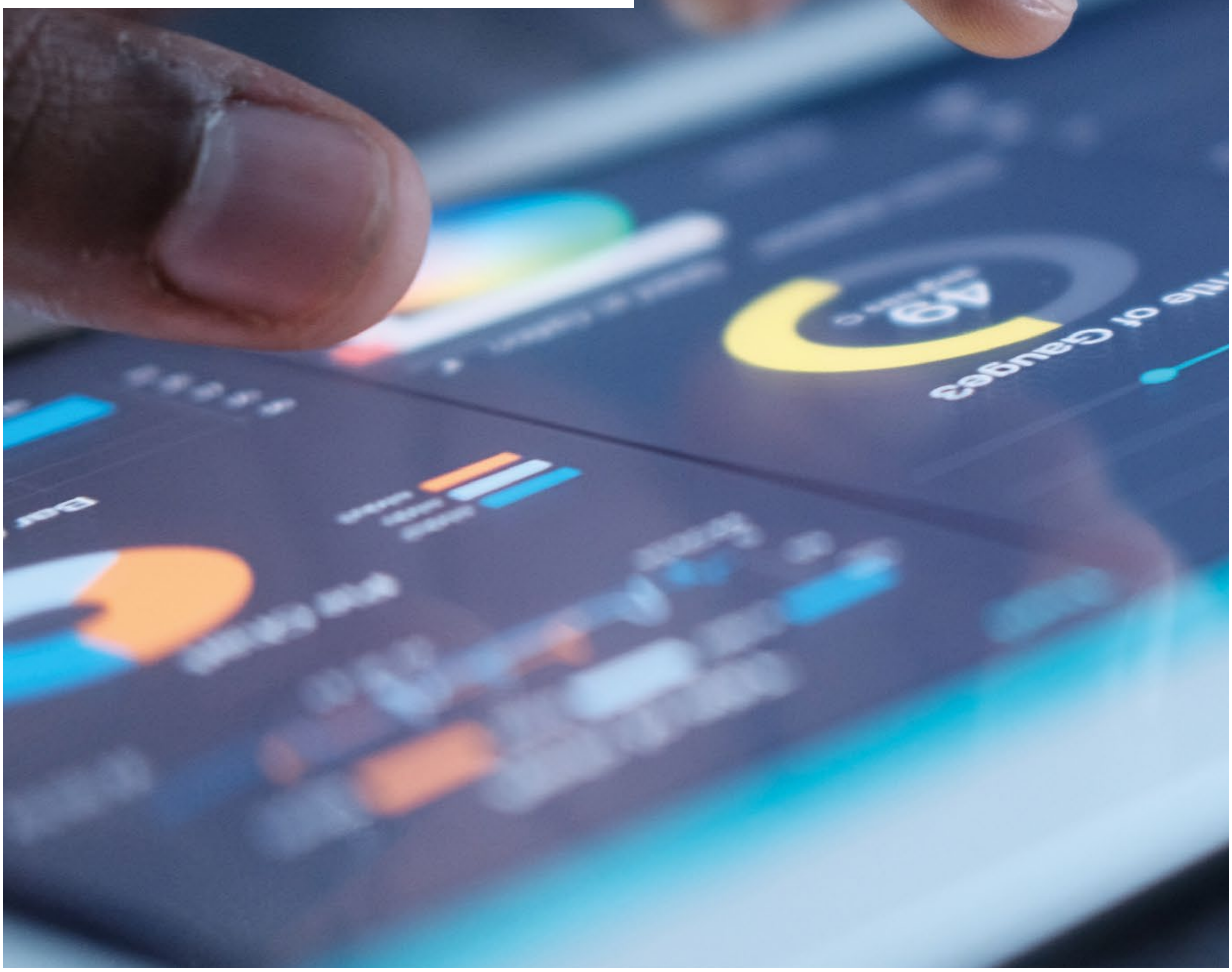
A cool test to detect fraud

One of the most common uses of Benford’s law nowadays is fraud detection.

Starting from the assumption that real accounting numbers should follow Benford’s law, auditors have been comparing the distribution of leading digits in accounting records with the Benford distribution. Of course, deviation from Benford’s law by itself does not provide any concrete evidence of fraud, but it does represent a red flag that a certain dataset might have been manipulated.

For example, let’s assume a company has a policy whereby only expenses above CHF 50 are thoroughly checked. Employees willing to defraud the company might charge a number of fictitious expenses within the limit (e.g. all at CHF 49). These records are not ‘naturally occurring’ and will skew the distribution towards the digit 4 – hence suggesting the auditor should check the accounts that are deviating from Benford’s law.

Nowadays, a range of auditing software exists that leverages this law to automatically check for possible instances of fraud, but the scope of application is still relatively limited.

Predictive liquidity modeling for bank's: liquidity stress testing

Our team focuses on liquidity and funding modelling, and we are constantly on the lookout for ways to improve the quality of our work. Benford's law is the latest addition to our tools.

Liquidity stress testing – introduction for beginners

Financial institutions need to build up a portfolio that is resilient to possible future stress. Normally, resilience is expected in two areas:

- **Capital:** financial institutions need to ensure that – even after stress events – they can maintain an adequate level of capital
- **Liquidity:** financial institutions record a number of inflows and outflows of cash every day. In a stress situation, they should still hold, or be able to collect, enough liquidity to cover cash outflows.

The amount of capital and liquidity is normally calculated daily. This, however, does not tell us how well a bank would fare in the event of a stress scenario. For this reason, financial institutions are required to model the evolution of their capital and liquidity in potential stress scenarios – the second is achieved via liquidity stress testing.

Liquidity stress test modelling can be schematised as follows:

1 Definition of a stress scenario (e.g. a market-wide financial crisis, similar to what we experienced in 2008, or an entity-specific event, e.g. a fraud affecting the entity is made public)

2 Definition of specific assumptions that define how the various components of the balance sheet would react to the stress scenario (e.g. how will the scenario affect the value of the trading assets? How will the depositors react to the news of a financial crisis?)

3 The current balance sheet amount is taken as a baseline, and assumptions are used to estimate future cash flows for each item in the balance sheet, at different future dates

4 For each future date, the estimated cash flows are aggregated to identify areas where the liquidity position might be inadequate (i.e. estimated inflows materially lower than outflows, thus anticipating an unsustainable financial position)

What makes for good liquidity modelling?

Liquidity stress testing is more complex than capital stress testing, as we do not simply estimate the future value of assets, but need to make extra assumptions about the liquidity flows linked to that asset.

For example, we can assume that certain financial assets will decrease in value during a crisis, but that change may not materialise in the form of a cash flow unless the financial institution decides to sell those assets at a loss. Assumptions specific to the cash flow component are hence needed when modelling liquidity.

Given this complexity, it is fundamental to identify clear KPIs / success criteria when it comes to liquidity stress test modelling. Some of the key criteria should be:

- **Predictive power:** a good model should be able to correctly predict the liquidity position of a financial institution during a crisis
- **Flexibility:** the model should allow for the use of different assumptions, so that different scenarios (and assumptions) can be tested
- **Consistency:** assumptions related to different areas of the balance sheet need to be consistent
- **Realistic:** assumptions and model output should be sensible – a very basic example is that we should not assume a deposit outflow volume that is larger than the initial balance sheet amount of the deposits.

Some of these criteria are easily verified when building the model, while it is harder for others. A tricky one is certainly the predictive power: as stress tests are meant to predict so-called 'tail events', we very rarely (if ever) observe in the market the exact same scenario as the ones modelled by financial institutions. Hence, as measuring predictive power is not easily done, model specialists try to increase predictive power by making use of 'proven assumptions' – i.e. looking at past tail events and trying to replicate how certain balance sheet components have reacted to those.

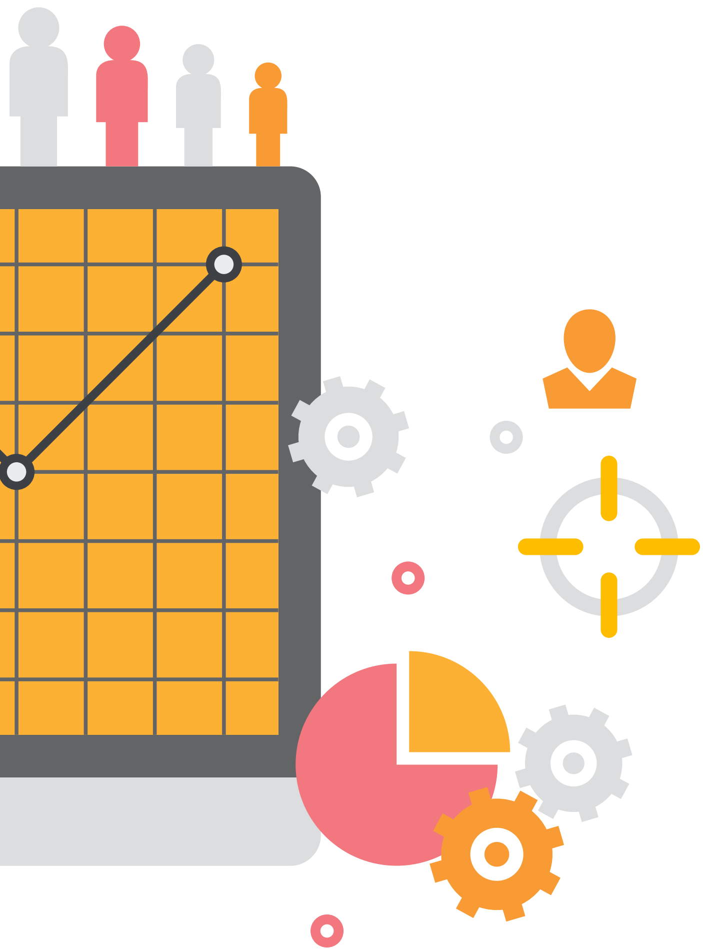
Another criterion that is complex to verify is 'realism': some assumptions can be easily excluded as unrealistic (e.g. all financial assets losing 95% of their value in the first day of a market stress seems an impossible case), while others might sound realistic but lead to an unrealistic output. For example, one can consider a set of assumptions that – taken singularly – seem plausible but, once applied at the same time, lead to an unrealistic outcome (e.g. a material switch in the asset/liability composition that the management knows to not be plausible). This is where we believe Benford's law offers a possible solution.

Benford's law can help you perfect your liquidity stress testing capabilities

Get rid of unrealistic model assumptions

Normally, any assumption identified for a liquidity stress test model is carefully evaluated by the senior management of the financial institution: among other criteria, they can confirm how plausible an assumption is, often based on historical observations.

However, this might not be enough: some assumptions are based on scenarios that have never happened or, as mentioned above, a mix of different realistic assumptions could result in an implausible scenario. To try and overcome these limitations, we have tested how Benford's law can be applied, and summarised the steps we took as well as our findings (from two specific use cases).



1 Check the baseline

As the baseline for any liquidity stress test is the balance sheet current amounts, the starting point is to verify whether this baseline follows the leading digit expected distribution, as per Benford's law.

For this test, one should extract the entire accounting record at different dates, to make sure that the distribution holds true also through time. For most financial institutions, this should be the case.

2 Run your model

This test can be performed on a specific area of the balance sheet (provided a material number of records are available), or on the entire model output. Also, it can be used to test how all the assumptions interact within the model, or to test one specific assumption.

In this step, one should simply run the model output (entirely or for specific areas/assumptions), i.e. calculate the expected cash flows for each record.

3 Estimate future balance sheet levels

Provided that balance sheet amounts have, historically, been following Benford's law, one could reasonably expect that any good predictive model for those balance sheet amounts would lead to results that are also in line with Benford's law.

Liquidity stress test models do not directly forecast futurebalance sheet amounts, but rather changes in liquidity for a certain time period. As such, future balance sheet levels should be calculated as part of this test. Although the change in liquidity is not always fully reflected as a change in the balance sheet, for the sake of this test one could estimate future balance sheet amounts by adding any liquidity inflow/outflow estimated in the forecasted period to the original balance.[4]

[4] We highlight that this value will not provide the exact estimation of the future balance based on the assumptions, and it is only used as proxy for the purposes of this test.

4 Calculate leading digit distribution for the expected balance sheet amounts

Once you have the leading digit distribution for the expected balance sheet amount, you can compare it to Benford's expected distribution. This can also be done for specific areas of the balance sheet.

Any material deviation from Benford's distribution should lead to further investigation, as it tells us we have modelled a set of data that is not 'realistic' under this law. For example, we might have capped cash flows to a certain level, or decreased the value of certain assets to the extent where they no longer show the required variability in scale for Benford's law to hold.

In this case, Benford's law would still only act as a red flag: after investigation, the financial institution might decide to keep the assumptions unchanged. However, one may find – during the investigation – that some critical assumptions need to change.

Note that we can detect deviations from Benford's law visually in a distribution chart only when those deviations are large. To detect smaller deviations, we need to employ a more sophisticated statistical test. While one can employ statistical tests such as Anderson-Darling or Cramer-von-Mises distribution tests, here a simple squares deviation measure (SSD) would likely suffice. This is calculated as follows:

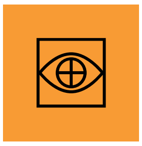
$$SSD = \sum_{i=1}^{9} \left[Observed\ \%\ of\ Digit_i - 100 * Log_{10}\left(1 + \frac{1}{Digit_i}\right) \right]^2$$

An SSD value below 2 is considered to be ideal, while values below 25 are still within a good range to conclude that the data follows Benford's law to a reasonable degree. Note: the SSD is calculated against a logarithmic scale as that is used to calculate the distribution of leading digits for Benford's law.

The future of treasury is in the cloud

At PwC, we strongly believe in innovation and that digitalisation will benefit all companies that are able to correctly capitalise on it. When it comes to the treasury function, we have a clear view of its future: it is in the cloud. By migrating cross-functional data into an integrated database in the cloud, the treasury function of the future will be able to generate better and more reliable analytics, which will then lead to better and more informed decision-making.

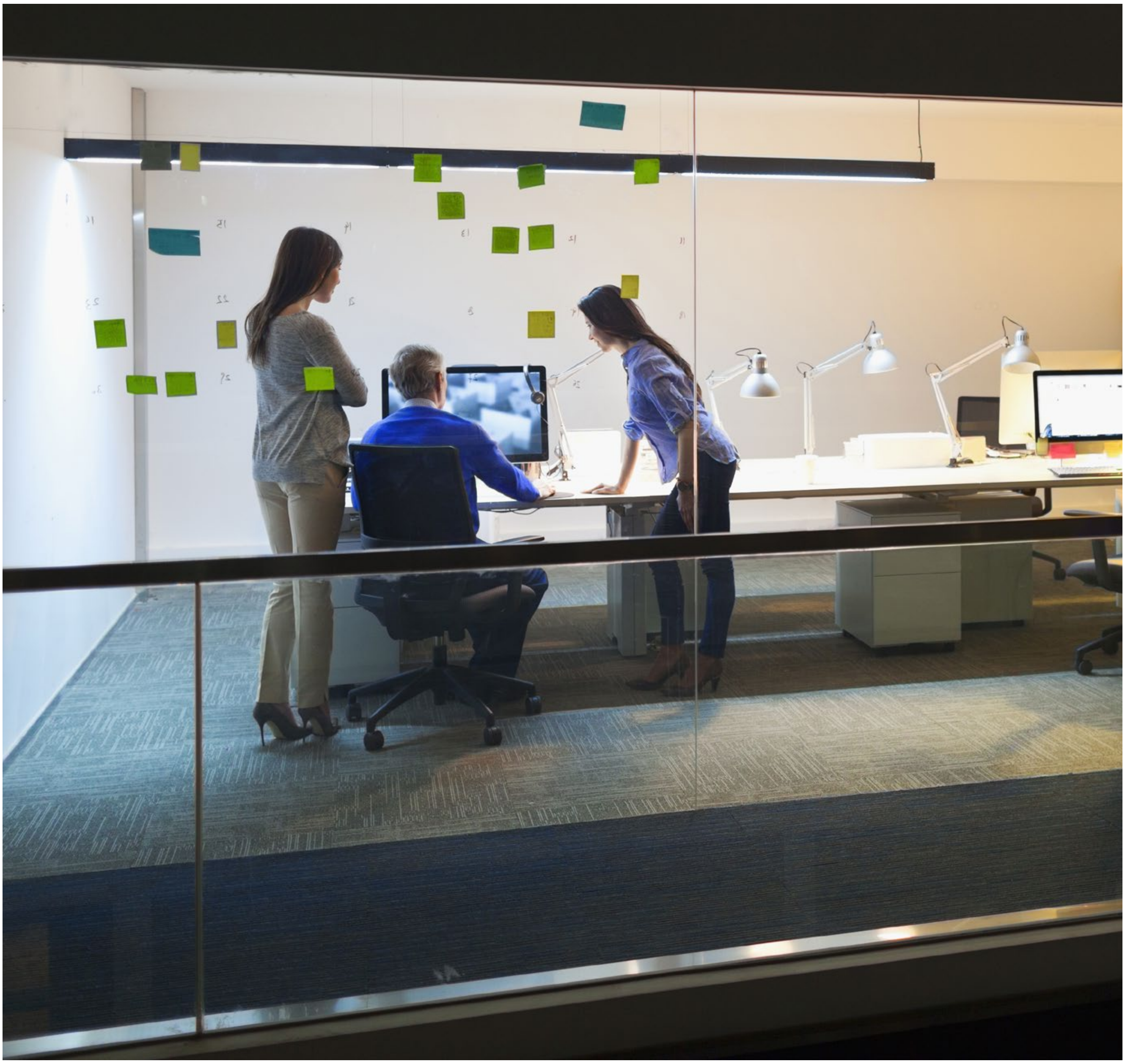
Benford's law could easily be used in this context as well: simple Benford's testing could be done on data and analytics in the cloud, thus providing greater assurance regarding the quality of that data. If you want to know more about PwC's vision of the future of treasury, get in touch!



Case study: capping your cash flows may lead to unrealistic model outputs

A common assumption in liquidity stress testing is that the bank may only sell a certain quantity of a particular type of asset within a given period. For example, the bank may assume it cannot sell more than $5,000,000 worth of US equities per day. These assumptions, known as capacity constraints, are included to model the fact that there is a limit to the amount of a particular asset that can be bought or sold without affecting its price. To see this, imagine a case where you have five apples to sell; one person is willing to buy two apples for $10 per apple, while the second is willing to buy three apples for $5 per apple. Clearly, if you sell two apples or less you will sell them at $10 per apple, while if you sell three apples you will need to accept $5 for the third apple. If this were a liquidity stress testing model, we could set a capacity constraint of two apples a day and assume a price of $10 per apple.

While these assumptions may be realistic, if those limits are reached too often (i.e. are too often binding), the result will likely violate Benford's law. Coming back to the apple example, if the capacity constraint were set at two apples a day, the distribution of apple sales would be 2, 2, 1; which may cause the resulting model output to violate Benford's law.



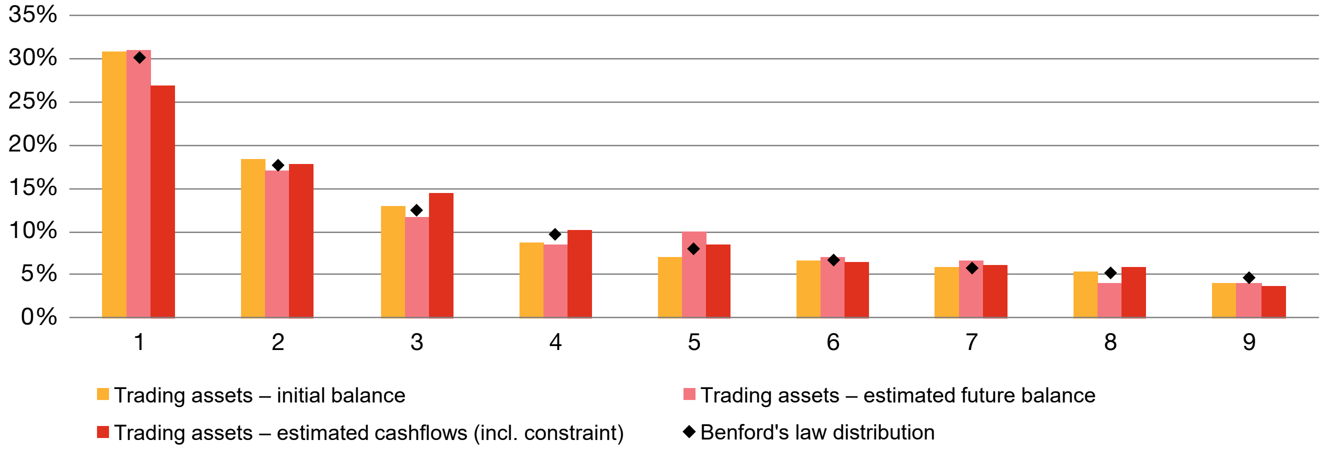


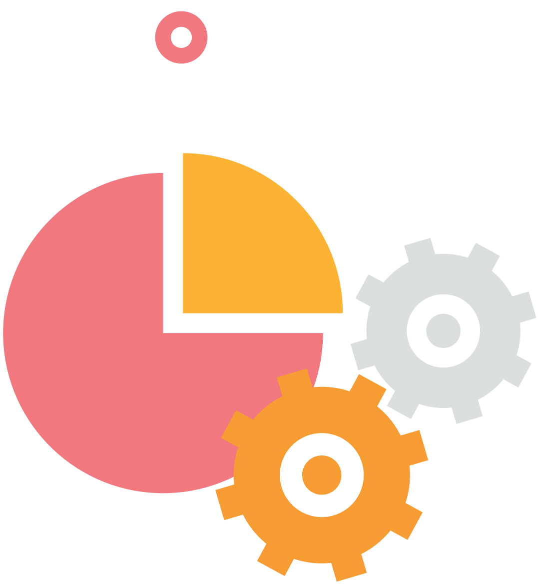
Figure 2: First digit distribution Securities Financing Liquidity Modelling

As a real-life example for this case study, we report here the case of a large European universal bank, where we tested the robustness of their capacity constraints applied to the sale of trading assets within their liquidity stress testing. For this test, we created a distribution of the first digits of the bank's current balance sheet figures, as well as the distribution of the estimated future balance sheet figures, under the assumption that in the liquidity stress scenario assets are sold within pre-defined constraints.

Figure 2 shows the results of our test:

The left bars indicate the distribution of the first digits for the balance sheet before the stress event (i.e. current figures). This follows Benford's law well with an SSD metric of 3.24.	The middle bars are related to the estimated cash flow figures, to which a constraint is applied. One can see this is already deviating from Benford's law, with an SSD metric of 9.99.	The right bars indicate the estimated future balance sheet, including 12 months of modelled stress cash flow. Due to the application of constraints to cash flows, the final SSD metric has deteriorated and is now 15.56.

Note that this alone may not be a cause for concern, but it suggests that further investigation is needed in order to confirm that the constraints are constructed in a 'natural' way (e.g. constructed from data in a stressed period) and are sufficiently granular.



Identify errors that affect your model output

Wrong implementation of assumptions
Liquidity stress testing models are complex – especially when it comes to modelling the trading assets area of the balance sheet, where records are interconnected as they are part of a 'chain' of transactions. Due to this, even good assumptions might not be correctly reflected in the model, thus leading to wrong outcomes.

Mistakes in the extraction code
Most banks run stress testing models on specific engines that write the model output on dedicated databases. These databases contain a massive amount of information, so it is normal that any analysis of model output normally starts with some form of coding to extract a portion of the data from the dataset. Usually, a model generates the expected output, but a mistake in the extraction code may lead to unexpected results.

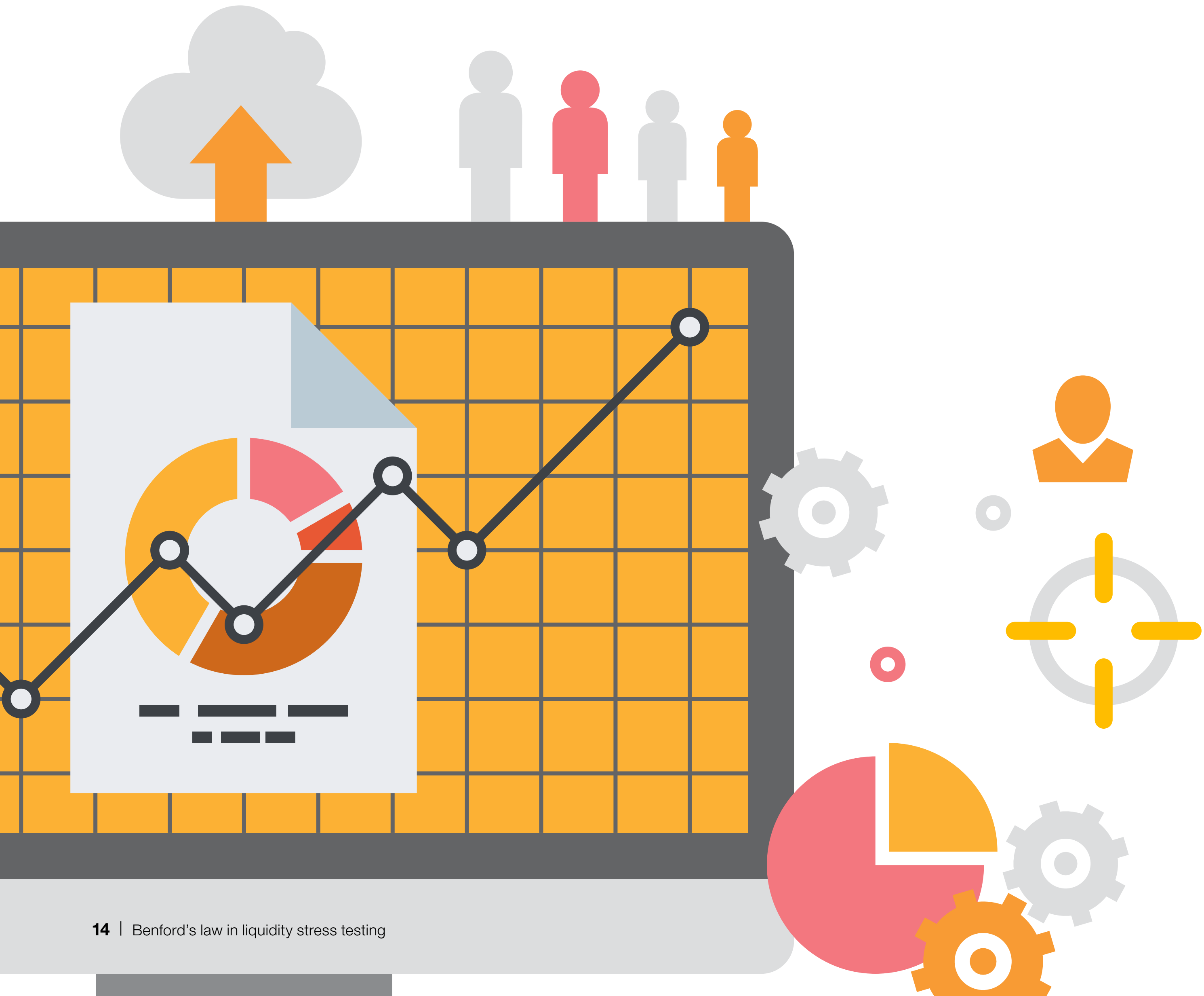
Most of the time, the two types of errors described above are easily identified, however it may be the case that they remain unnoticed for a while, for different reasons.

What we suggest is to pre-emptively run Benford's law tests to quickly identify and investigate any potential red flags.

The test itself is performed following the same steps described in the previous chapter. Here we suggest including 'Benford testing' as part of your model output – anytime a new test is run in your model, this check should be performed, in order to flag any potential causes of concern to the model team.

Similarly, **'Benford testing'** could be included in any piece of code used to extract data. This would help make sure that no small errors in your coding lead to an incorrect extraction.

Again, these are to be used as 'red flag' checks, in addition to any existing model validation check a financial institution already has in place.



Case study: saving time by checking your extraction first

To make the error checking process more efficient, one could first test the model output for violations using Benford's testing before continuing with any other tests one would have done otherwise.

Figure 3 shows the initial balances of three sections of a liquidity stress testing model at a large European bank: deposits, loans and trading assets. Visually, no major deviations from Benford's law can be detected. Similarly, the SSD metrics are 2.11, 1.31 and 5.5 for the deposits, loans and trading assets parts, respectively. The metrics themselves confirm adherence to Benford's law, but they can also be compared over time to check for changing trends in the data (e.g. by comparing the SSDs from this month with last month's output).

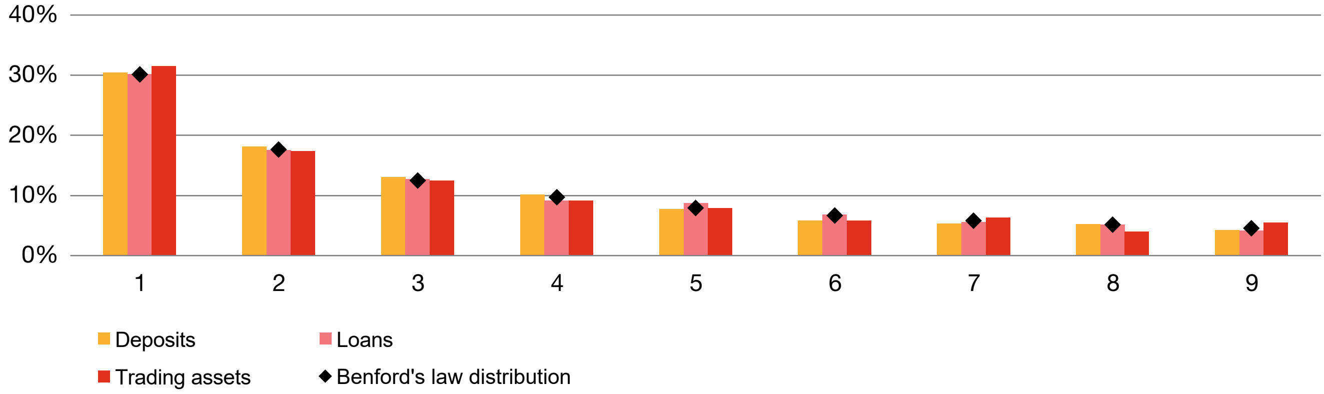


Figure 3: Testing of Benford's law on the balance sheet of a universal bank

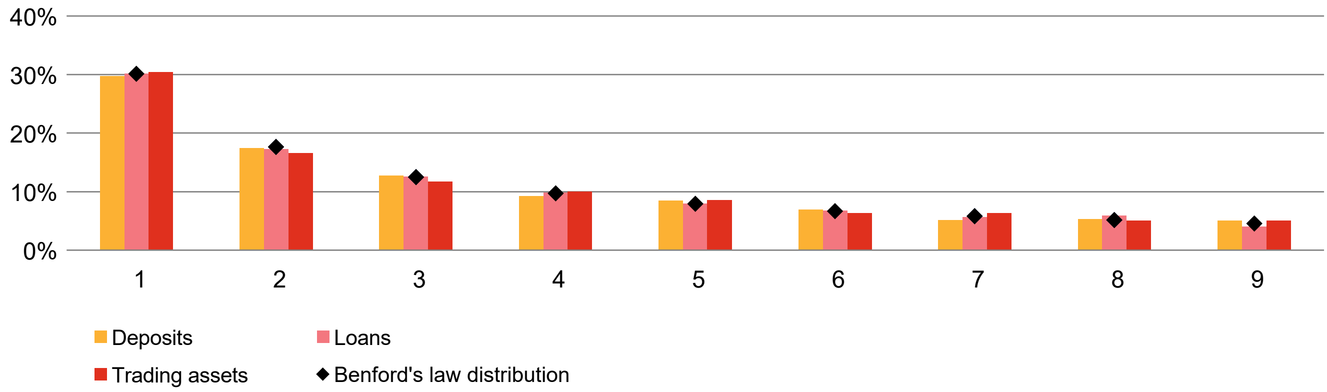


Figure 4: Testing of Benford's law on the projected balance sheet of the same bank as in Figure 3

Figure 4 shows the distributions of projected balance sheets for the same three sections after 12 months of modelled stress cash flows have been taken into account. Again, visually, no major deviations from Benford's law can be detected and the SSD metrics are 1.45, 1.09 and 2.92 for the deposits, loans and trading assets parts, respectively. These results indicate the model is not generating 'unnatural' output and can be compared to previous results to confirm the model has not changed unexpectedly. An 'incorrect' extraction might instead lead to deviations from Benford's law in the distribution of first digits, thus suggesting that the way the data was extracted should be checked.

How can PwC help you?

We are a global team of professionals, with deep expertise in liquidity and funding transformation programmes. We have supported our clients in building reliable liquidity and funding models, and we can help you achieve the same results, following our standard methodology.

When it comes to applying Benford's law to check your model outputs, we offer a number of solutions. Thanks to our team, we can deliver any of the following, starting from initial design to final implementation:

1. Ad-hoc checks of model output:

A one-off exercise to check the output of your existing models. At the end of the exercise, you will receive a report showing which areas of your model may need further investigation, as suggested by major deviations from Benford's law.

Our experience: PwC has successfully supported a number of financial institutions in evaluating the robustness of their model outputs, identifying areas of strength and weakness.

2. Complete review of your model:

As liquidity and funding experts, we can review the entire current model within your bank. Benford's law checks would be one of the many tools we can use to help you identify improvement areas and we can support you with any required transformation process.

Our experience: on multiple occasions, PwC has successfully supported its clients in reviewing their full existing models, as well as building new models for their stress testing capabilities.

3. Built-in checks within your model:

We can include built-in Benford's law checks in your model that can automatically flag when a new assumption may affect the robustness of your model output. Similarly, we can help you code built-in checks to apply to any data extrapolated from your database.

Our experience: PwC has a large team of experts, with extensive experience in writing codes using multiple languages and including built-in checks.

For additional information, please contact our experts.

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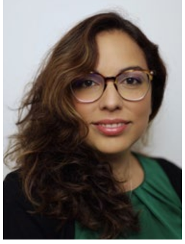

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