Homework 5

Daniel Kreuzberger, Lars Kuehmichel, David Weinand

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1 Expression for Gaussian Elimination

We have $Ax = b$ with a tridiagonal matrix A of dimension $N \times N$, so we can start directly without having to reorder the rows. The first row of A will stay the same, so we do an iteration over $N-1$ other rows:

```
for i := 1 to N-1:
    factor = A_{i+1,i} / A_{i,i}
    # calculate the factor with which we have to
       multiply the previous row and add it to the
       next row to eleminate the left off-diagonal
       element
    b_{i+1} = b_{i+1} - factor * b_i
    # update the vector b so the solution stays the
       same
    for k := 1 to N:
        A_{i+1,k} = A_{i+1,k} - factor * A_{i,k}
        # update the row i+1
```

Wit this algorithm, we get an upper right triangular matrix A and an updated vector b. By doing backward substitution, we can get the solution of the linear equation system.

2 Backward Substitution

We have $Ax = b$ with an upper right triangular matrix A of dimension $N \times N$. We have to start with:

```
x_N = b_N / A_{N,N}

for i := 1 to N-1:
    for k := N-(i-1) to N:
        n_{N-i} = n_{N-i} - A_{N-i,k} * x_k
        # since the off-diagonal elements are not
           zero, we have to subtract them times the
           solution x
        x_{N-i} = b_{N-1} / A_{N-i,N-i}
```

with this algorithm, we get the solution vector x.