Solution of exercise 2.1

We have the following equation:

$\frac{dN}{dt} = rN(1 - \frac{N}{K}) - \frac{BN^2}{A^2+N^2}$

We do the dimension analysis:

r is of dimension $[\frac{1}{t}]$

A has the same dimension as N and K

Therefore $\frac{BN^2}{A^2+N^2}$ has to be in dimensions of $[\frac{N}{t}]$:

B has dimension $[\frac{N}{t}]$

We use $n = \frac{N}{A}$:

$\frac{dN}{dt} = rnA(1 - \frac{An}{K}) - \frac{B(An)^2}{A^2+(An)^2} = rnA(1 - \frac{An}{K}) - \frac{Bn^2}{1+n^2}$

since $dN = Adn$, we get:

$\frac{dn}{dt} = rn(1 - \frac{An}{K}) - \frac{Bn^2}{A(1+n^2)}$
For the dimensionless time we use $\tau = \frac{A}{B}t$, and $dt = \frac{B}{A}d\tau$:

$\frac{dn}{d\tau} = \frac{rA}{B}n(1 - \frac{An}{K}) - \frac{n^2}{1+n^2}$

In the last step we can define $\alpha = \frac{rA}{B}$ and $\beta = \frac{A}{K}$, which are both dimensionless and get the fully dimensionless form:

$\frac{dn}{d\tau} = \alpha n(1 - \beta n) - \frac{n^2}{1+n^2}$

Solution of exercise 2.2

We analytically determined the cubic function and got:

$f = \alpha * n * (1 - \beta * n) - \frac{n^2}{(1+n^2)}$

To be a fix point, this have to be zero. By varying and plotting the functions, we determined some areas: (We number the points with growing n)

Two real solutions for: α in $[-\infty, -0.45]$, while the first fix point is stable and the second one is unstable

Four real solutions for α in $[-0.5, 0)$, while the first and the third fix points are stable and the second and fourth are unstable

One real solution for $\alpha = 0$, which is neither stable nor unstable because $f'(n) = 0$

Two real solutions for α in $(0, \inf]$, while the first fix point is unstable and the second one $(n > 0)$ is stable

There is one exception which you can see when you decrease the step size: At around $\alpha = 0.55$ there is a small area where there are four real solutions of the function