Solution of exercise c

We start with:
$Q = \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{a})$

Then we calculate Q^4
$\langle\Psi_n|Q^4|\Psi_m\rangle = (\frac{1}{\sqrt{2}})^4\langle\Psi_n|(\hat{a}^\dagger + \hat{a})^4|\Psi_m\rangle =$

since we only have to consider the diagonal elements (Here is some laborious calculation needed, which I have done on an extra sheet of paper):

$= 0.25 * \langle\Psi_n|m\sqrt{m^2-1} + (m+1)(m+2) + 2m(m+1) + m^2|\Psi_m\rangle =$
$(n\sqrt{n^2-1} + (n+1)(n+2) + 2n(n+1) + n^2)\delta_{nm} = Q^4_{diag}$

So we have for h:
$h_{nm} = (n + \frac{1}{2} + \lambda \times 0.25(n\sqrt{n^2-1} + (n+1)(n+2) + 2n(n+1) + n^2))\delta_{nm}$

As h is a diagonal matrix, the values on the diagonal are the eigenvalues:
$eigenvalue_n = (n + \frac{1}{2} + \lambda \times 0.25(n\sqrt{n^2-1} + (n+1)(n+2) + 2n(n+1) + n^2))$
$eigenvalue_1 = 1.5 + 2.75\lambda$
$eigenvalue_2 = 2.5 + 7.866\lambda$
and so on...