Ex07

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(1)

- Lemma: $(X^T X + \tau \mathbb{1}_D)^{-1} X^T = X^T (X X^T + \tau \mathbb{1}_N)^{-1}$

  Proof:
  - $X \in \mathbb{R}^{N \times D}$
  - $(X^T X + \tau \mathbb{1}_D) X^T = X^T X X^T + \tau X^T = X^T (X X^T + \tau \mathbb{1}_N)$
  - multiply by $(X^T X + \tau \mathbb{1}_D)^{-1}$ from left
    " " $(X X^T + \tau \mathbb{1}_N)^{-1}$ from right

  $\Rightarrow X^T (X X^T + \tau \mathbb{1}_N)^{-1} = (X^T X + \tau \mathbb{1}_D)^{-1} X^T$ //

- $\hat{\alpha} = (X X^T + \tau \mathbb{1}_N)^{-1} Y$, show $\hat{\beta} = X^T \hat{\alpha}$ where $\hat{\beta} = (X^T X + \tau \mathbb{1}_D)^{-1} X^T Y$

  $X^T \hat{\alpha} = X^T (X X^T + \tau \mathbb{1}_N)^{-1} Y \overset{\text{Lemma}}{=} (X^T X + \tau \mathbb{1}_D)^{-1} X^T Y = \hat{\beta}$ //