Probability for Data Science

Random Variables

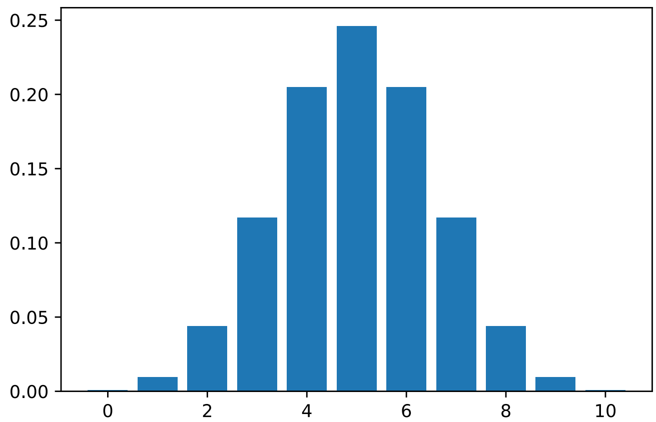
Random variables are functions with numerical outcomes that occur with some level of uncertainty. For example, rolling a 6-sided die could be considered a random variable with possible outcomes {1,2,3,4,5,6}.

Discrete and Continuous Random Variables

Discrete random variables have countable values, such as the outcome of a 6-sided die roll.
Continuous random variables have an uncountable amount of possible values and are typically measurements, such as the height of a randomly chosen person or the temperature on a randomly chosen day.

Probability Mass Functions

A probability mass function (PMF) defines the probability that a discrete random variable is equal to an exact value.
In the provided graph, the height of each bar represents the probability of observing a particular number of heads (the numbers on the x-axis) in 10 fair coin flips.



Probability Mass Functions in Python

The `binom.pmf()` method from the `scipy.stats` module can be used to calculate the probability of observing a specific value in a random experiment.
For example, the provided code calculates the probability of observing exactly 4 heads from 10 fair coin flips.

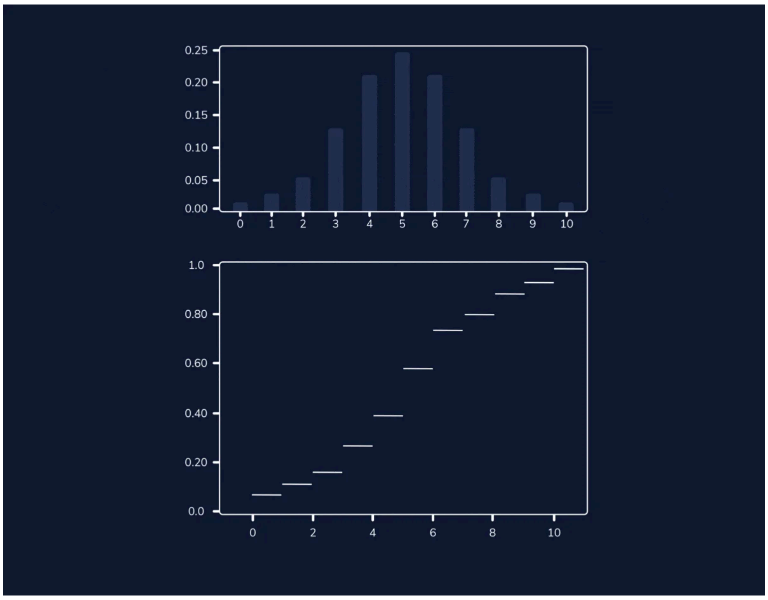
```
import scipy.stats as stats

print(stats.binom.pmf(4, 10, 0.5))

# Output:
# 0.2050781250000022
```

Cumulative Distribution Function

A cumulative distribution function (CDF) for a random variable is defined as the probability that the random variable is less than or equal to a specific value.
In the provided GIF, we can see that as x increases, the height of the CDF is equal to the total height of equal or smaller values from the PMF.



Calculating Probability Using the CDF

The `binom.cdf()` method from the `scipy.stats` module can be used to calculate the probability of observing a specific value or less using the cumulative density function.
The given code calculates the probability of observing 4 or fewer heads from 10 fair coin flips.

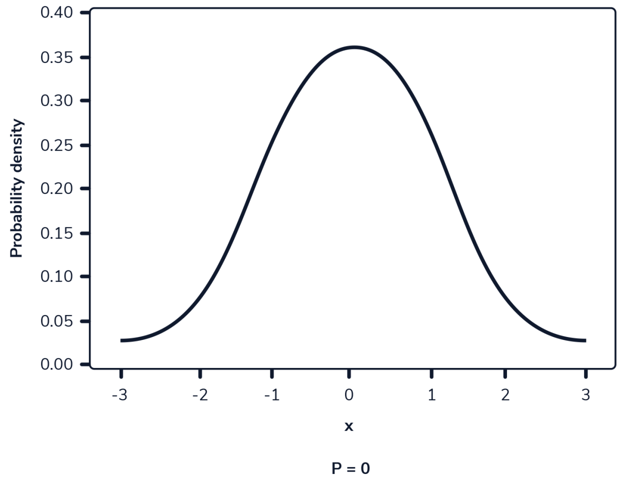
```
import scipy.stats as stats

print(stats.binom.cdf(4, 10, 0.5))

# Output:
# 0.3769531250000001
```

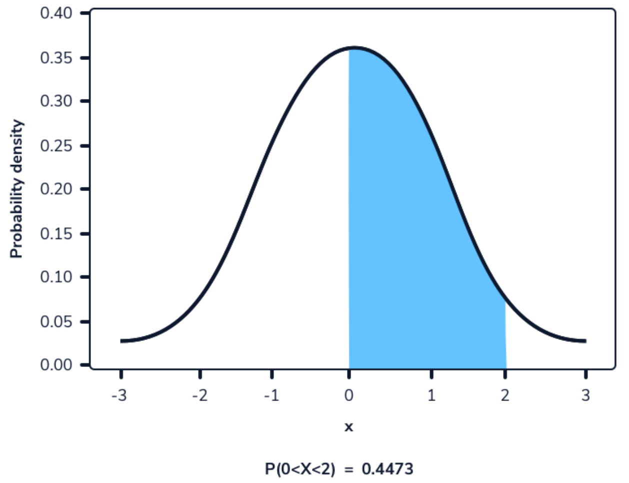
Probability Density Functions

For a continuous random variable, the probability density function (PDF) is defined such that the area underneath the PDF curve in a given range is equal to the probability of the random variable equalling a value in that range.
The provided gif shows how we can visualize the area under the curve between two values.



Probability Density Function at a Single Point

The probability that a continuous random variable equals any exact value is zero. This is because the area underneath the PDF for a single point is zero.
In the provided gif, as the endpoints on the x-axis get closer together, the area under the curve decreases. When we try to take the area of a single point, we get 0.



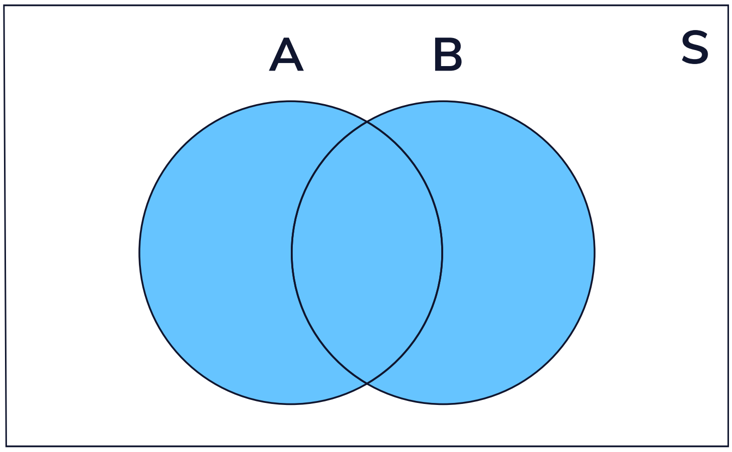
Parameters of Probability Distributions

Probability distributions have parameters that control the exact shape of the distribution.
For example, the binomial probability distribution describes a random variable that represents the number of sucesses in a number of trials (n) with some fixed probability of success in each trial (p). The parameters of the binomial distribution are therefore *n* and *p*. For example, the number of heads observed in 10 flips of a fair coin follows a binomial distribution with n=10 and p=0.5.

Union

The *union* of two sets encompasses any element that exists in either one or both of them. We can represent this visually as a *venn diagram* as shown. Union is often represented as:

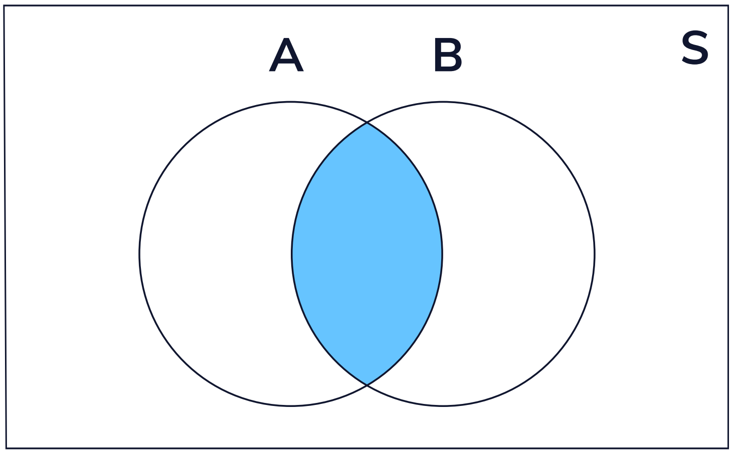
$$(A \; or \; B)$$



Intersection

The intersection between two sets encompasses any element that exists in BOTH sets and is often written out as:

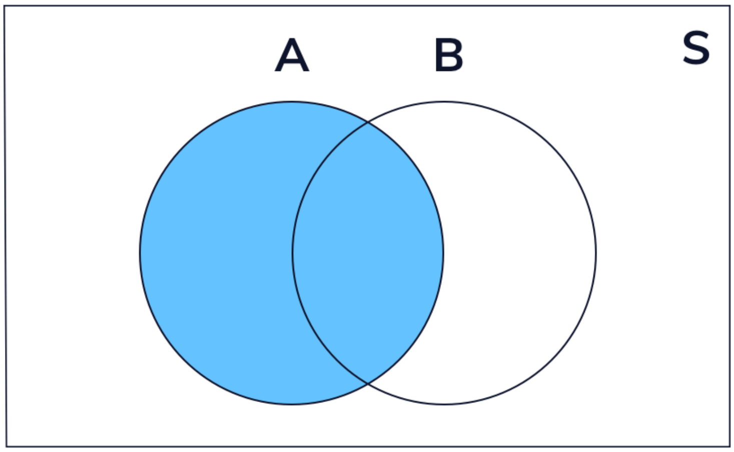
$$(A \; and \; B)$$



Addition Rule

If there are two events, A and B, the addition rule states that the probability of event A or B occurring is the sum of the probability of each event minus the probability of the intersection:

$$P(A \; or \; B) = P(A) + P(B) - P(A \; and \; \ldots$$



P(A or B) = P(A)

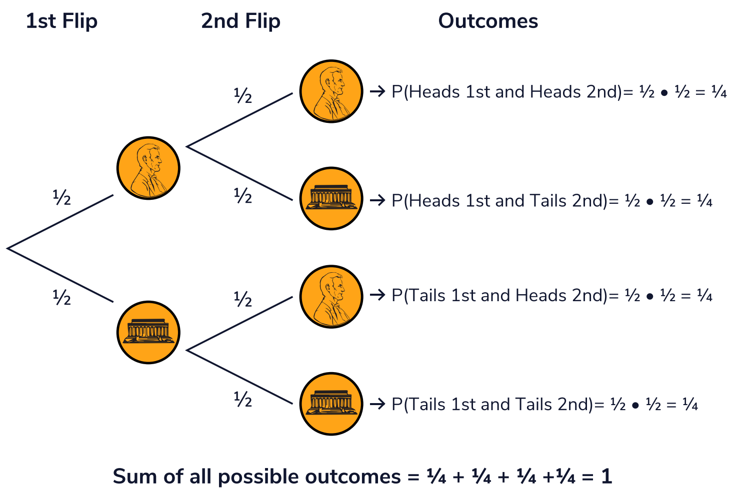
If the events are mutually exclusive, this formula simplifies to:

$$P(A \; or \; B) = P(A) + P(B)$$

Multiplication Rule

The multiplication rule is used to find the probability of two events, *A* and *B*, happening simultaneously. The general formula is:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$



For independent events, this formula simplifies to:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

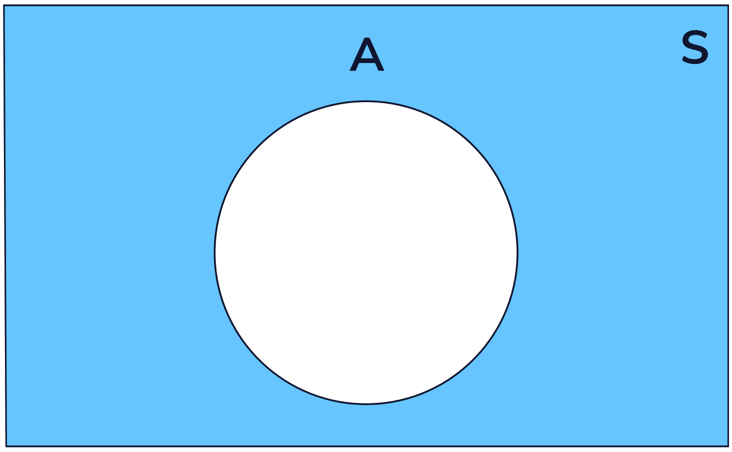
This is because the following is true for independent events:

$$P(B \mid A) = P(B)$$

The tree diagram shown displays an example of the multiplication rule for independent events.

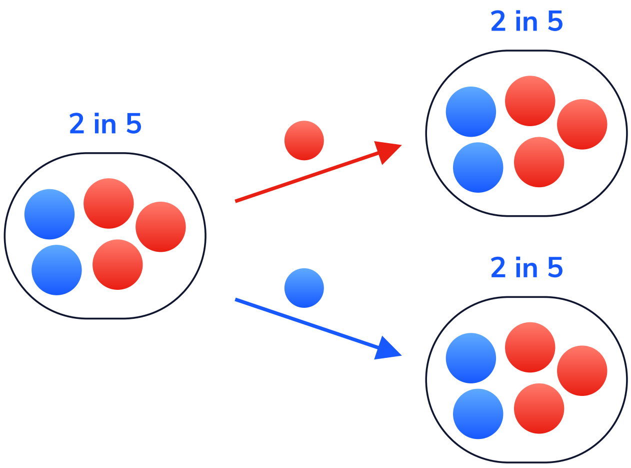
Complement

The complement of a set consists of all possible outcomes outside of the set.
Let's say set *A* is rolling an odd number with a 6-sided die: *{1, 3, 5}*. The complement of this set would be rolling an even number: *{2, 4, 6}*.
We can write the complement of set *A* as A^C. One key feature of complements is that a set and its complement cover the entire sample space. In this die roll example, the set of even numbers and odd numbers would cover all possible rolls: *{1, 2, 3, 4, 5, 6}*.



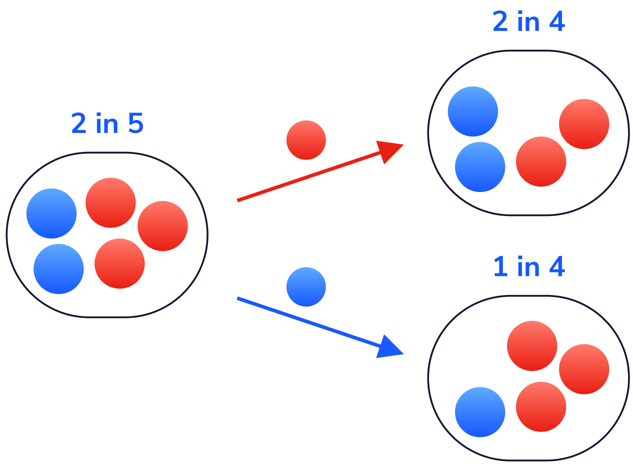
Independent Events

Two events are *independent* if the occurrence of one event does not affect the probability of the other one occurring.
Let's say we have a bag of five marbles: three are red and two are blue. If we select two marbles out of the bag WITH replacement, the probability of selecting a blue marble second is independent of the outcome of the first event.
The diagram below outlines the independent nature of these events. Whether a red marble or a blue marble is chosen randomly first, the chance of selecting a blue marble second is always 2 in 5.



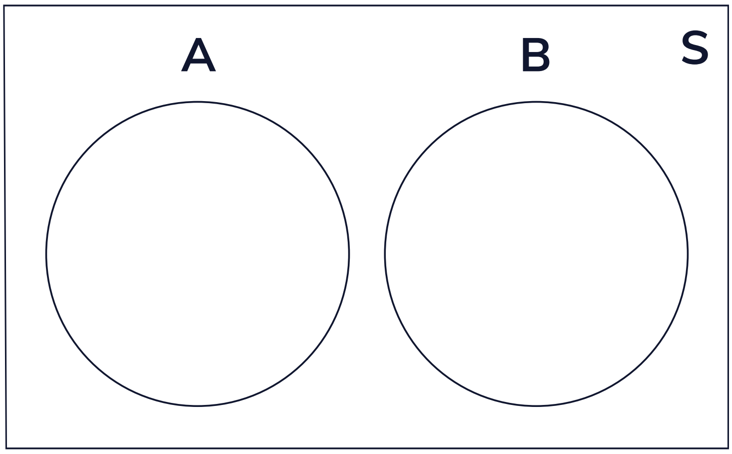
Dependent Events

Two events are *dependent* if the occurrence of one event does affect the probability of the other one occurring.
Let's say we have a bag of five marbles: three are red and two are blue. If we select two marbles out of the bag WITHOUT replacement, the probability of selecting a blue marble second depends on the outcome of the first event.
The diagram below outlines this dependency. If a red marble is randomly selected first, the chance of selecting a blue marble second is 2 in 4. Meanwhile, if a blue marble is randomly selected first, the chance of selecting a blue marble second is 1 in 4.



Mutually Exclusive Events

Two events are considered *mutually exclusive* if they cannot occur at the same time. For example, consider a single coin flip: the events "tails" and "heads" are mutually exclusive because we cannot get both tails and heads on a single flip.
We can visualize two mutually exclusive events as a pair of non-overlapping circles. They do not overlap because there is no outcome for one event that is also in the sample space for the other.



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Conditional Probability

Conditional probability is the probability of one event occurring, given that another one has already occurred. We can represent this with the following notation:

$$\text{Probability of event A occurring given e}$$
$$P(A \mid B)$$

For independent events, the following is true for events *A* and *B*:

$$P(A \mid B) = P(A)$$
$$\text{and}$$
$$P(B \mid A) = P(B)$$

Bayes' Theorem

Bayes' theorem is a useful tool to find the probability of an event based on prior knowledge. The formula for Bayes' theorem is:

$$P(B \mid A) = \frac{P(A \mid B) \cdot P(B)}{P(A)}$$

The Poisson Distribution

The Poisson distribution is a probability distribution that represents the number of times an event occurs in a fixed time and/or space interval and is defined by parameter λ (lambda).
Examples of events that can be described by the Poisson distribution include the number of bikes crossing an intersection in a specific hour and the number of meteors seen in a minute of a meteor shower.

Expected Value

The *expected value* of a probability distribution is the weighted (by probability) average of all possible outcomes. For different random variables, we can generally derive a formula for the expected value based on the parameters. For example, the expected value of the binomial distribution is n*p.
The expected value of the Poisson distribution is the parameter λ (lambda).
Mathematically:

$$X \sim Binomial(n, p),\ E(X) = n \times p$$

$$Y \sim Poisson(\lambda),\ E(Y) = \lambda$$

Variance of a Probability Distribution

The *variance* of a probability distribution measures the spread of possible values. Similarly to expected value, we can generally write an equation for the variance of a particular distribution as a function of the parameters.
For example:

$$X \sim Binomial(n, p),\ Var(X) = n \times p \times$$

$$Y \sim Poisson(\lambda),\ Var(Y) = \lambda$$

Sum of Expected Values

For two random variables, *X* and *Y*, the expected value of the sum of *X* and *Y* is equal to the sum of the expected values. Mathematically:

$$E(X + Y) = E(X) + E(Y)$$

Adding a Constant to an Expected Value

If we add a constant *c* to a random variable *X*, the expected value of *X + c* is equal to the original expected value of *X* plus *c*.
Mathematically:

$$E(X + c) = E(X) + c$$

Multiplying an Expectation by a Constant

If we multiply a random variable *X* by a constant *c*, the expected value of *c*X* equals the original expected value of *X* times *c*.
Mathematically:

$$E(c \times X) = c \times E(X)$$

Adding a Constant to Variance

If we add a constant *c* to a random variable *X*, the variance of the random variable will not change.
Mathematically:

$$Var(X + c) = Var(X)$$

Multiplying Variance by a Constant

If we multiply a random variable *X* by a constant *c*, the variance of *c**X* equals the original expected value of *X* times *c* squared.
Mathematically:

$$Var(c \times X) = c^2 \times Var(X)$$

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