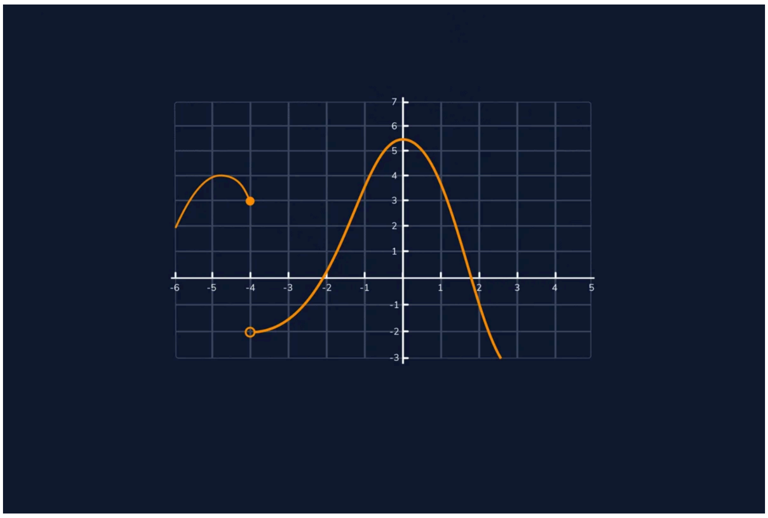
Differential Calculus

Limits

Limits quantify what happens to the values of a function as we approach a given point. This can be defined notationally as:

$$\lim_{x \to 6} f(x) = L$$

We can read this in simple terms as “the limit as *x* goes to 6 of f(x) approaches some value L”.

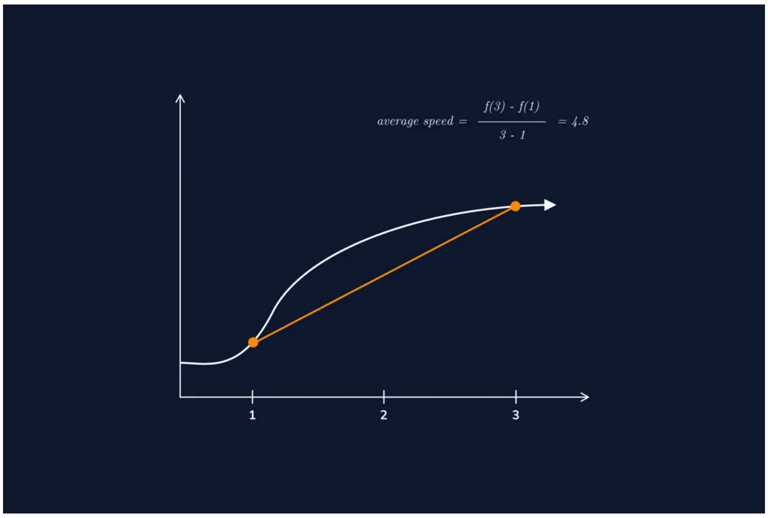


Limit Definition of the Derivative

The *limit definition of the derivative* proves how to measure the instantaneous rate of change of a function at a specific point by looking at an infinitesimally small range of *x* values.

$$instantaneous\ rate\ of\ change = \lim_{h \to 0} \frac{f(}{\;}$$

The animation provided shows that as we look at a smaller range of *x* values, we approach the instantaneous range of a point.

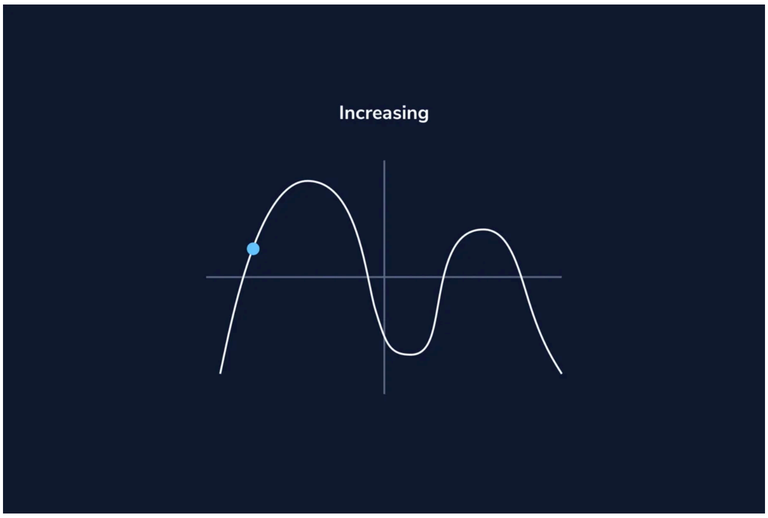


Derivative Properties

The *derivative* is the slope of a tangent line at a specific point, and the derivative of a function *f(x)* is denoted as *f′(x)*. We can use the derivative of a function to determine where the function is increasing, decreasing, at a minimum or maximum value, or at an inflection point.
If *f′(x)* = 0, then the function is not changing. This can mean one of a few things.

- It may mean that the function has reached a *local maximum* (or minimum). A local maximum is a value of x where *f′(x)* changes from positive to negative and thus hits 0 along the way. In *f(x)*, the local maximum is lower than all the points around it.
- It may also mean that the function has reached what is called a *local maximum*. Our local maximum is higher than the points around it. When *f′(x)* goes from negative values to 0 to positive values, a local maximum forms.
- It may be an *inflection point*. This is a point where a function has a change in the direction of curvature. For example, the curve of the function goes from "facing down" to "facing up." Finding inflection points involves a second derivative test, which we will not get to in this lesson.

If *f′(x) > 0*, the function is increasing, and if *f′(x) < 0*, the function is decreasing.



Derivatives in Python

We can use the `np.gradient()` function from the NumPy library to calculate derivatives of functions represented by arrays. The code block shown shows how to calculate the derivative of the function $f(x) = x^3 + 2$ using the `gradient()` function.

```
from math import pow

# dx is the "step" between each x value
dx = 0.05
def f(x):
  # to calculate the y values of the function
  return pow(x, 3) + 2
# x values
f_array_x = [x for x in np.arange(0,4,dx)]
# y values
f_array_y = [f(x) for x in np.arange(0,4,dx)]

# derivative calculation
f_array_deriv = np.gradient(f_array_y, dx)
```

Calculating Derivatives

To take the derivative of polynomial functions, we use the *power rule*. This states the following:

$$\frac{d}{dx}x^n = nx^{n-1}$$

There are rules even beyond the power rule. Many common functions have defined derivatives. Here are some common ones:

$$\frac{d}{dx}ln(x) = \frac{1}{x}$$
$$\frac{d}{dx}e^x = e^x$$
$$\frac{d}{dx}sin(x) = cos(x)$$
$$\frac{d}{dx}cos(x) = -sin(x)$$

Derivative Rules

There are general rules we can use to calculate derivatives.
The derivative of a constant is equal to zero:

$$\frac{d}{dx}c = 0$$

Derivatives are *linear operators*, meaning that we can pull constants out of derivative calculations:

$$\frac{d}{dx}cf(x) = cf'(x)$$

The derivative of a sum is the sum of the derivatives, meaning we can say the following:

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

We define the derivative of two products as the following:

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g$$
$$f(x) = u(x)v(x) \rightarrow f'(x) = u(x)v'(x) +$$