

# Functional programming

# What is it?

# Functions?

# Functions?

- Mapping
- Takes multiple arguments
- Produces one result
- Can be *named*
- Have a *body*

# Function example

- Let's define a function that doubles its argument

*double x = x + x*

- Named *double*
- Argument *x*
- Body *x + x*

# Function application

*double 3*

= { apply double }

*3 + 3*

= { apply + }

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# Nested application

*double (double 2)*

= { apply inner double }

*double (2 + 2)*

= { apply + }

*double 4*

= { apply double }

*4 + 4*

= { apply + }

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# Does order matter?

Yes!



# What is functional programming?

- A *style* of programming
- The basic *method of computation* is the *application of functions to arguments*
- Functions are *first-class citizens*

# Contrast with OO

- Object-oriented programming is also a *style* of programming
- *Combine data and operations* on data in *objects*
- Data contained in object is called *state*
- The state of methods is *mutated* by calling methods

# What is a functional programming language?

# A functional programming language...

supports  
and encourages  
a functional style

# Declarative programming

- Functional programming tends to be more *declarative*
- Say what you want
- Not how you want it

# Imperative example

- Sum of numbers 1 to n

*count := 0*

*total := 0*

***repeat***

*count := count + 1*

*total := total + count*

***until***

*count = n*

# Declarative example

- Sum of numbers 1 to n

*sum [1..n]*

# Haskell



# Haskell is...

- ... a *pure* functional programming language
- ... a general-purpose language
- ... a static typed language
- ... widely used in academia
- ... used in industry

# The Haskell Tool Stack

# Glasgow Haskell Compiler

- The main implementation of Haskell
- Both an interpreter and compiler
- Multi-platform
- Noted for rich feature set and high performance

# Where to download?

<https://docs.haskellstack.org/en/stable/README/>

# Conventions for this course

- We use GHCi (the GHC interpreter)
- Files have the .hs-extension
- We provide type descriptions as much as possible (more on that later)

# Demonstration

# Basics

- Comments start with --
- Functions in scripts can span multiple lines
- Whitespace matters in multi-line functions!

# Types and classes



# Types in Haskell (1)

- In Haskell everything has a type
- Types can be *inferred*
- Types can (but don't have to be) made explicit

# Types in Haskell (2)

- We denote the type of something with the notation  $v :: T$
- This means  $v$  has type  $T$
- Functions have types too!
- $f :: T \rightarrow U$
- This means function  $f$  maps from something of type  $T$  to type  $U$

# Types in Haskell (3) – basic types

- Bool – logical values
- Char – single characters
- String – string of characters
- Int – fixed-precision integers
- Integer – arbitrary-precision integers
- Float – single-precision floating point numbers
- Double – double-precision floating point numbers

# Types in Haskell (4) - Lists

- Sequence of elements of the same type
- A list is denoted by elements enclosed in [ ]
- The type of a list is denoted by another type in [ ]
- For example:  
*[False, True, False] :: [Bool]*  
*[ 'a', 'b', 'c', 'd' ] :: [Char]*  
*[ "Michiel", "Joris", "Liesbeth", "Ron" ] :: [String]*
- Lists can be infinite! (we get to that later)

# Convenient prelude list functions (1)

- *head xs* – returns head of list *xs*
- *tail xs* – returns tail of list *xs*
- *[1, 2, 3] !! 1* - *!!* returns *n*th element of list (zero-based)
- *take n xs* – returns first *n* elements of list
- *drop n xs* – remove first *n* elements of list
- *length xs* – calculate length of list *xs*
- *sum xs* – calculate the sum of elements of list *xs*

# Convenient prelude list functions (2)

- *product xs* – calculate the product of elements of list xs
- *[1, 2, 3] ++ [4, 5]* - ++ concatenates lists
- *reverse xs* – reverses list xs

# Types in Haskell (5) - Tuples

- Finite sequence of components of possibly different types
- Components are comma-separated, enclosed by ( ), as are the types
- For example:

*(False, True) :: (Bool, Bool)*

*(False, 'a', True) :: (Bool, Char, Bool)*

*("Michiel", False) :: (String, Bool)*

# Types in Haskell (6) - Functions

- Functions (obviously) have types as well
- We can provide a type for a function as follows:  
$$\text{add} :: (\text{Int}, \text{Int}) \rightarrow \text{Int}$$
$$\text{add } (x, y) = x + y$$
- Another example  
$$\text{zeroto} :: \text{Int} \rightarrow [\text{Int}]$$
$$\text{zeroto } n = [0..n]$$
- Note that functions don't have to be *total* functions for their provided types



# Currying and partial application

- Functions in Haskell (and many other functional languages) can return functions
- Consider the following function:  
$$add :: Int \rightarrow (Int \rightarrow Int)$$
$$add\ x\ y = x + y$$
- This is called a *curried* function
- Why is this useful?
- Partial application!
- The arrow is right-associative, e.g.  $Int \rightarrow Int \rightarrow Int == Int \rightarrow (Int \rightarrow Int)$

# Polymorphic types

- The Prelude function *length* calculates the length of any type of list
- *length* is polymorphic!
- This is done through *type variables*
- Type variables are denoted by lower-case letters in a type definition
- *length* :: *[a]* -> *Int*

# Overloading and type classes

- Let's redefine add (again)
- Add should obviously be able to add any number, but not other types
- We use a type class and overloading to redefine add:  
$$\text{add} :: \text{Num } a \Rightarrow a \rightarrow a \rightarrow a$$
$$\text{add } x \ y = x + y$$
- *Num a* is called a class constraint, and constrains the type variable *a* to type class Num

# Basic type classes

- Eq – equality types (can be compared using `==` and `/=`)
- Ord – ordered types (can be compared using `<` `<=` `>=` `>`, *min* and *max*)
- Show – types convertible to String
- Read – types convertible from String
- Num – numeric types
- Integral – integral types
- Fractional – fractional types

# Defining functions

- There are various tools for defining functions:
  - Combine existing functions
  - Using conditional expressions
  - Using guarded equations
  - Using pattern matching

# Combine existing functions

- Very simple: use existing functions in the body of your new function. For example:

*even :: Integral a => a -> Bool*

*even n = n `mod` 2 == 0*

*splitAt :: Int -> [a] -> ([a], [a])*

*splitAt n xs = (take n xs, drop n xs)*

*recip :: Fractional a => a -> a*

*recip n = 1 / n*

# Conditional expressions

- *If*-statements. They work similar to Java. For example:

*abs :: Int -> Int*

*abs n = if n >= 0 then n else -n*

*signum :: Int -> Int*

*signum n = if n < 0 then -1 else  
          if n == 0 then 0 else 1*

- Haskell if-statements *always* require an *else*-branch!

# Guarded equations

- Guarded equations are often a more elegant way to express conditions than if-statements. Examples:

$$\begin{array}{ll} \text{abs } n & | \ n \geq 0 \quad = \ n \\ & | \ \text{otherwise} \quad = \ -n \end{array}$$
$$\begin{array}{ll} \text{signum } n & | \ n < 0 \quad = \ -1 \\ & | \ n == 0 \quad = \ 0 \\ & | \ \text{otherwise} \quad = \ 1 \end{array}$$

- Otherwise is similar to *default* in switch-statements in Java



# Pattern matching (1)

- Many functions can be expressed particularly intuitive using *pattern matching*
- Pattern matching defines a function as a sequence of patterns, which are evaluated in order. For example:

```
and' :: Bool -> Bool -> Bool
and' True True   = True
and' True False  = False
and' False True  = False
and' False False = False
```

# Pattern matching (2)

- Patterns can be simplified using *wildcards* and the evaluation order.
- A simpler version of *and'*:

$$\begin{aligned} \text{and}' &:: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool} \\ \text{and}' \text{ True True} &= \text{True} \\ \text{and}' \_ \_ &= \text{False} \end{aligned}$$

# Pattern matching (3)

- Patterns can be used with tuples as well

```
fst      :: (a, b) -> a  
fst (x, _) = x
```

```
snd      :: (a, b) -> b  
snd (_, y) = y
```

# Pattern matching (5)

- Patterns are even more powerful with lists.
- A function that tests if a list consists of 3 characters starting with a:

```
test :: [Char] -> Bool
test ['a', _, _] = True
test _           = False
```

# Interlude – the cons-operator

- Lists are actually not primitive, but constructed using the *cons-operator*:

$$\begin{aligned} & [1, 2, 3] \\ = & 1 : [2, 3] \\ = & 1 : (2 : [3]) \\ = & 1 : (2 : (3 : [])) \end{aligned}$$

# Pattern matching (6)

- Patterns can use the cons-operator
- Cons-patterns need to be parenthesised because of function precedence
- A more general test function:

```
test      :: [Char] -> Bool
test ('a' : _) = True
test _       = False
```

# Pattern matching (6)

- Head and tail use pattern matching!

```
head      :: [a] -> a
```

```
head (x:_) = x
```

```
tail      :: [a] -> [a]
```

```
tail (_:xs) = xs
```

# Local definitions

- The *where*-keyword can be used in functions to define a local function or value.

- A function that returns the first  $n$  odd integers can be defined as:

```
odds      :: Int -> [Int]
odds n    = map f [0.. n - 1]
           where f x = x * 2 + 1
```



# Lambda expressions

- Alternative to regular functions
- Nameless
- Can be used to formalise currying
- Can be used when returning functions from a function, for example defining a constant function:  
$$\text{const} \quad :: a \rightarrow (b \rightarrow a)$$
$$\text{const } x = \_ \rightarrow x$$
- Can be used to avoid naming a function that is used only once, for example with *map*

# List comprehensions

- Allow functions on lists to be defined in a simple manner
- Allow for the construction of new lists from existing lists
- Example:  
 $[x^2 \mid x \leftarrow [1..5]]$
- $\mid$  is read as 'such that'
- $\leftarrow$  is read as 'drawn from'

# Generators (1)

- `x <- [1..5]` is called a *generator*
- List comprehensions can have multiple generators, for example:  
`[(x, y) | x <- [1,2,3], y <- [4,5]]`
- The order of generators matters, this produces a different result:  
`[(x,y) | y <- [4,5], x <- [1,2,3]]`
- Generators can be dependant (right can depend on left):  
`[(x,y) | x <- [1..3], y <- [x..3]]`
- Another example:  

```
concat      :: [[a]] -> [a]
concat xss = [x | xs <- xss, x <- xs]
```

# Generators (2)

- Wildcards can be used as well
- For example, to get the first elements of each pair in a list:  

```
firsts      :: [(a, b)] -> [a]  
firsts ps = [x | (x,_) <- ps]
```

# Guards (1)

- Used to filter the products of generators
- For example finding all positive factors of an integer:  

```
factors    :: Int -> [Int]
factors n = [x | x <- [1..n], n `mod` x == 0]
```
- Finding prime if a number is prime (using factors):  

```
prime      :: Int -> Bool
prime n = factors n == [1, n]
```
- Finding primes up to n:  

```
primes     :: Int -> [Int]
primes n = [x | x <- [2..n], prime x]
```

## Guards (2)

- Finding values by key in a lookup table
- Represent lookup table as a list of key-value pairs
- Implementation of 'find':

```
find      :: Eq a => a -> [(a, b)] -> [b]  
find k t = [v | (k', v) <- t, k == k']
```

# The *zip* function

- The library function *zip* creates a list by pairing successive elements of two existing lists, until one or both are exhausted
- For example, creating a list of all adjacent pairs of a list:  
pairs :: [a] -> [(a, a)]  
pairs xs = zip xs (tail xs)
- This can be used to see if a list is sorted. How?

# String comprehensions

- Strings are actually lists of characters, so everything that works on lists, also works on Strings!