Inhomogeneous Material Properties in Solid Mechanical Systems

Thore Bergmann
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Advisor: Dr. Stefan Sandfeld

Continuum Mechanical Foundations and Notations

The von Mises Stress or "Equivalent Tensile Stress"

How can one compare two stresses?

- Cauchy stress tensor with 9 compoenents

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

- The von Mises stress or equivalent tensile stress

$$\sigma_v^2 = \frac{1}{2}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{22})^2 + 6(\sigma_{23}^2 + \sigma_{31}^2 + \sigma_{12}^2)\right]$$

What are the properties of the von Mises stress?

- scalar stress measure
- two stress states with equal distortion energy have equal von Mises stresses
- used to formulate the von Mises yield criterion

What is the von Mises yield criterion?

- criterion, which is used to predict yielding of materials
- begins when the second deviatoric stress inavriant J_2 rechaes a critical value

$$\sigma_v = \sigma_y = \sqrt{3J_2}$$

- stress values larger than the yield stress cannot be reached

In which space does the von Mises yield surface live in and how does the – theoretical – surface look like?

- in the principal stress space
- like a cylinder (with the rotational axis: $\sigma_x = \sigma_y = \sigma_z$)

What are principal stresses?

- the stresses on the diagonal line of the Cauchy stress tensor

What is the work conjugate counterpart of the equivalent stress and how is it defined?

- equivalent strain or von Mises strain ε_{eq}

$$\varepsilon_{eq} = \sqrt{\frac{2}{3}\boldsymbol{\varepsilon}^{\text{dev}} : \boldsymbol{\varepsilon}^{\text{dev}}}$$

with

$$\boldsymbol{\varepsilon}^{\text{dev}} = \boldsymbol{\varepsilon} - \frac{1}{3}\text{tr}(\boldsymbol{\varepsilon})\mathbf{1}$$

What are the first and the second invariant of the strain tensor?

$$J_1 = \mathrm{tr}(\boldsymbol{\varepsilon}), \qquad\qquad J_2 = \frac{1}{2}\sigma^{\mathrm{dev}} : \sigma^{\mathrm{dev}}$$

- the second invariant can also be defined in several other ways (e.g. in dependence of the strain tensor itself)

What is the expression for the equivalent stress in terms of the deviatoric parts of the stress tensor?

$$\sigma_{\mathrm{eq}} = \sqrt{\frac{3}{2}\boldsymbol{\sigma}^{\mathrm{dev}} : \boldsymbol{\sigma}^{\mathrm{dev}}}$$

What are the plain strain conditions and why are they how they are?

- the length of a structure in one direction is much larger than in the other two directions
- strains associated with the larger direction are constrained by the nearby material and are small compared to the cross-sectional strains

How do the strain and stress tensor lokk like in the plane strain approximation?

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ \varepsilon_{21} & \varepsilon_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad\qquad \boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

What is elastic material behaviour?

- the path during loading is also followed during unloading
- reversible behaviour

What is plastic material behaviour?

- residual irreversible deformation for the loading - unloading experiment

What is ideal plastic behaviour?

- irreversible plastic deformation occurs when a threshold is reached (yield stress)
- no hardening (stress stays constant for increasing strain)
- except for this the behaviour is ideally elastic

What is the yield condition for ideally plastic behaviour?

$$f(\sigma, \sigma_y) \leq 0$$

Of which term is the total strain made of?

- an elastic part and a plastic part

$$\varepsilon = \varepsilon^e \; + \; \varepsilon^p$$

How is hardening defined in the elastic-plastic framework?

- threshold evolves with loading history

What is the difference between isotropic and kinematic hardening?
(Bilder)

What is the relationship between effective plastic strain $\bar{\varepsilon}^p$ and effective stress $\bar{\sigma}$ under linear hardening behaviour?

$$\Delta\bar{\varepsilon}^p = \frac{\bar{\sigma}(t) - \sigma_y}{E^p}, \qquad E^p = \frac{E\ E_T}{E\ -\ E_T}$$

- E^T: hardening modulus **(Bilder)**

What is necessary to describe the effective stress of isotropic linear hardening?

- the hardening modulus E^T **(Bilder)**
- $\bar{\sigma}$ can be expressed as a function of E^T only

What is the difference between yield stress and flow stress?

Averaging Elastic Material Properties with FEM

Axel fragen ob das wichtig ist...

Modelling of Shear Band Formation

...see other summaries (e.g. Statistical Analysis in Materials Simulation)