Exercise Sheet #7

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Problem 1 (*Truncation errors*) 4 Pts

(a) Write a loop to sum over $\sum_i^n 1/i$ with $n = 16, 100, 333$. Repeat that using the data types `float`, `double` and `long double`. Print the output in a formatted way, considering the number of digits that are relevant for variables of the respective data type.

(b) Show that the following expressions are equivalent

- $\left((a+b)(a-b)\right)^2$
- $\left(a^2+b^2\right)^2 - 4(ab)^2$
- $\left(a^2-b^2\right)^2$

We choose $a = 10^8 + 2$ and $b = 10^8 - 1$. Write a code to calculate the above expressions, using data type `double`.Why does one of the results differ strongly?

Problem 2 (*Recursive functions*) 7 Pts

(a) Find a reduction formula for the following integral.

$$I_n = \int_0^1 x^n \mathrm{e}^{ax} \mathrm{d}x \tag{1}$$

(Hint: Integration by parts)

(b) Implement forward and backward recursion, setting the parameter $a = 1$. (Hint: For the backward recursion you can assume the Integral to be ≈ 0 for $n = 50$.)

(c) By successively reinserting the reduction formula into itself, i.e.

$$I_n = \frac{1}{a}\mathrm{e}^a - \frac{n}{a}\left(\frac{1}{a}\mathrm{e}^a - \frac{n-1}{a}\left(\frac{1}{a}\mathrm{e}^a - \frac{n-2}{a} I_{n-3}\right)\right), \tag{2}$$

find a series representation for the integral and write a program to implement this series!

(d) Print the forward, backward recursion and the series representation for a range of n for $a < 1$ and $a > 1$. Where do the single methods brake down? What is the reason? How is the stability of the formulas related to the parameter a?

Problem 3 (*Triangle map*) 9 Pts

Equivalently to the example of the logistic map discussed in the lecture we can examine the triangle map:

$$y_{n+1} = \begin{cases} ry_n & \text{if } 0 \leq y_n < 1/2 \\ r - ry_n & \text{if } 1/2 \leq y_n \leq 1 \end{cases},$$

For $0 \leq r \leq 1$ the function converges for any initial point $y_0 \in [0, 1]$ to a fixpoint. Chaos, meaning you are not able to predict y_{n+1} for a given y_n, appears for values $1 < r \leq 2$.

(a) Write a function to compute the iterations of the triangle map using forward recursion and starting from a random point $y_0 \in [0, 1]$ for a maximum number of iterations $N = 500$ and for 500 different r: $r \in [0, 2]$. Neglect the first 300 iterations for each r value, and print out y_n and r for the rest.

(b) Rewrite your code such that the results were streamed into a file called `results_triangle_map.dat` and write a simple gnuplot script which plots y_n against r.