2.a

The integral reads

$$\int_0^1 dx x^n e^{ax}$$

using integration by parts we can rewrite it as

$$\int_0^1 dx x^n e^{ax} = x^n \frac{1}{a} e^{ax}\Big|_0^1 - \int_0^1 dx n x^{n-1} \frac{1}{a} e^{ax}$$

$$= \frac{e^a}{a} - \frac{n}{a} \int_0^1 dx x^{n-1} e^{ax}$$

therefore our reduction formula reads

$$I_n = \frac{e^a}{a} - \frac{n}{a} I_{n-1}$$

For n = 0 we have

$$I_0 = \int_0^1 dx e^{ax} = \frac{1}{a}(e^a - 1)$$

2.c

Using the formula given on the sheet we can write the integral as the following series

$$I_n = \frac{e^a}{a} - \frac{n e^a}{a^2} + \frac{n(n-1)e^a}{a^3} - \frac{n(n-1)(n-2)e^a}{a^4} + ...$$

which is equivalent to

$$\sum_{i=0}^{n} (-1)^i \frac{n!}{(n-i)!} \frac{e^a}{a^{i+1}} + (-1)^{n+1} \frac{n!}{a^{n+1}}$$

where the last term is taking the '-1' in the definition of I_0 into account.

2.d

Looking at a small error for I_{n-1} of ϵ we get an error for I_n of $-\frac{n}{a}\epsilon$. This error will increase exponentially with n. Therefore if a is small, the error will be much bigger. That explains why all of the three methods break down for small a at $n < 30$.
Looking at the series we can see that here the terms are proportional to $\frac{1}{a^i}$ which goes to infinity for small a. So we also expect larger errors for small a.