# Optimization of Business Processes - Assignment 1

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## 1 Introduction

All results in this report were done with the following parameters as described in the part $d$ of the assignment: $\mu_1 = 1.0, \mu_2 = 1.1, \mu_3 = 0.9, B_1 = B_2 = 5$. When a machine was made deterministic its service time was simply the expected service time, therefore for a deterministic machine $i$ holds: $S_i = \mathbb{E}S_i = \frac{1}{\mu_i}$. For the simulations $n = 1000$ iterations were done, each with a warm-up period of 1000 time units and a run of 10000 time units after that.

## 2 Question 1

After applying the iterative algorithm as described in equation 4.1 of the lecture notes, the stationary distribution as in Figure 1 was obtained. From this figure it becomes clear that the most occupied states are indeed the states where the second buffer is (almost) completely full. This indicates that machine 3 is the bottleneck of the system, which is expected since it has the lowest service rate. Interestingly, the state $(0, 6)$ is occupied much longer than the states surrounding it. This is most likely, because the fastest machine, machine 2, is not active in this state, which causes the system to stay longer in that state.

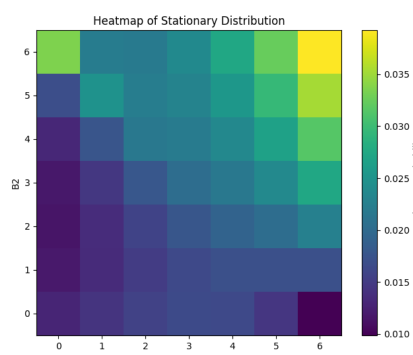


Figure 1: Stationary Distribution of the System

The throughput of the system was then calculated based on the stationary distribution of the system with the following equation:

$$\text{throughput} = \mu_1 \cdot \left[1 - \sum_{i=0}^{B_2+1} \pi(B_1 + 1, i)\right] \tag{1}$$

This resulted in a throughput of approximately 0.7995. Since the limit for the throughput is $\min_i \mu_i = 0.9$, it shows that the obtained throughput could be realistic. It also shows that there is room for improvement in the throughput. Therefore, if a higher throughput is desired both the buffer sizes and the service rates (in particular $\mu_3$) can be increased.

# 3 Question 2

Another way of estimating throughput is using a discrete event simulation. This was also applied and a confidence interval was constructed based on several iterations. The confidence interval used was a two sided $95\% - \text{CI}$ constructed in the following way:

$$95\% - \text{CI} = \mu_T \pm t_{999;0.975} \cdot \frac{\sigma_T}{\sqrt{n}} \approx \mu_T \pm 1.962 \cdot \frac{\sigma_T}{\sqrt{n}} \tag{2}$$

Here $\mu_T$ and $\sigma_T$ are the sample mean and sample standard deviation of the throughputs obtained during the iterations. This resulted in the following statistics: $\mu_T \approx 0.7999, \sigma_T \approx 0.0066, 95\% - \text{CI} \approx [0.7994, 0.8003]$. The sample mean is close to the obtained value of question 1. Additionally, the obtained value from question 1 also lies within the confidence interval, indicating that we cannot reject that the throughput is indeed the answer of question 1.

# 4 Question 3

The influence of a single deterministic machine was also tested. To this end, a confidence interval was constructed when each machine separately was deterministic. The confidence interval used was the same as in question 2. The results are shown in Table 1. This table shows that throughput increases if any machine becomes deterministic. Additionally, it shows that if this machine is closer towards the last machine, this increase is larger. Interestingly, the standard deviation of the throughput does decrease if the last machine becomes deterministic, however, it does not do so for the other machines. This might be because the last machine is the bottleneck of the system, and that a more predictable in- and output of the machine is preferable because other machines will have less idle time, therefore increasing throughput.

| Deterministic Machine | Sample Mean | Standard Deviation | CI |
|---|---|---|---|
| 1 | 0.8294 | 0.0071 | [0.8290,0.8299] |
| 2 | 0.8419 | 0.0067 | [0.8415, 0.8423] |
| 3 | 0.8499 | 0.0043 | [0.8495, 0.8502] |

Table 1: Results Deterministic Machine