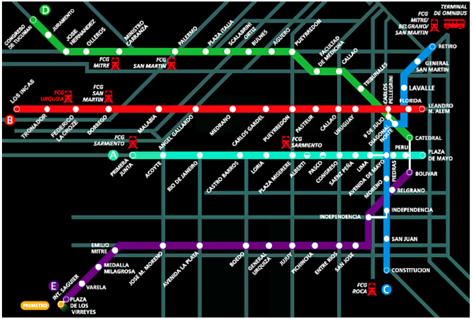
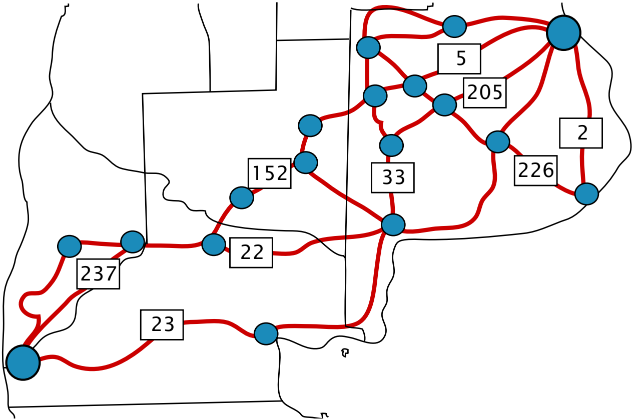
Teoría de grafos: Reseña histórica
Algoritmos y Estructuras de Datos III

Grafos en la vida real



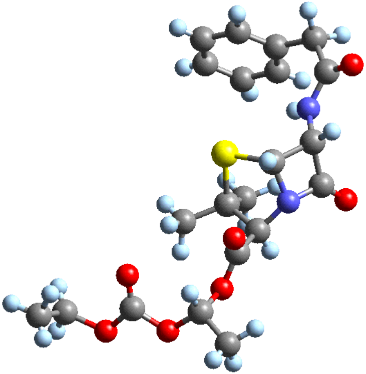
Red de subtes o metro

Grafos en la vida real



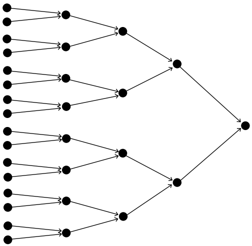
Rutas entre ciudades

Grafos en la vida real



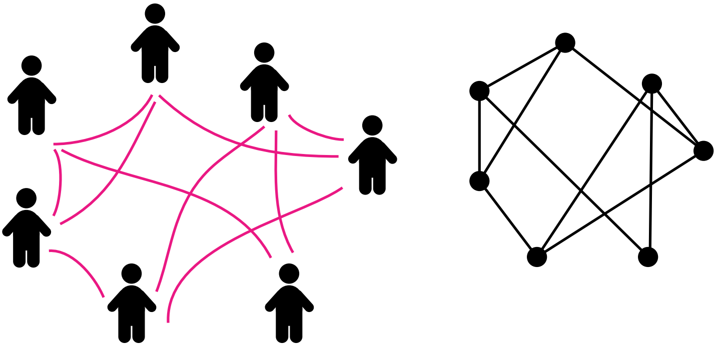
Molécula de penicilina

Grafos en la vida real



Draw de un torneo de tenis

Grafos en la vida real



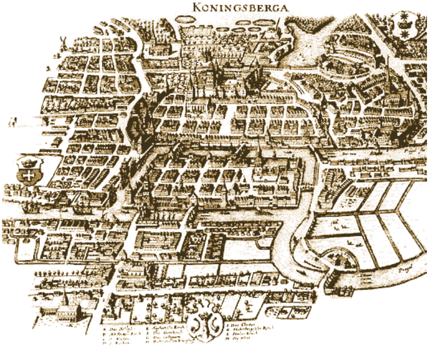
Relaciones sociales

El origen: Los puentes de Königsberg

Leonhard Euler (1707–1783)

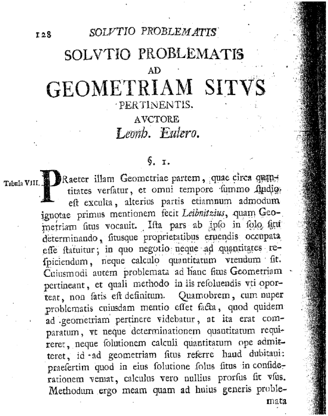
El origen: Los puentes de Königsberg

- La ciudad de Königsberg (hoy Kaliningrado) tenía en el siglo XVIII siete puentes.
- Euler (1735) planteó (y resolvió) el problema de cruzar por todos ellos exactamente una vez y volver al punto de partida.



El origen: Los puentes de Königsberg

- L. Euler, *Solutio problematis ad geometriam situs pertinentis* (26 de Agosto de 1735) [E53].

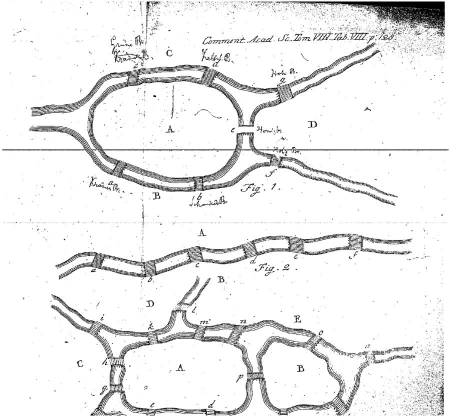


128 SOLVTIO PROBLEMATIS

SOLVTIO PROBLEMATIS
AD
GEOMETRIAM SITVS
PERTINENTIS.
AVCTORE
Leonh. Eulero.

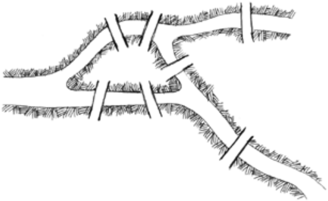
§. 1.

Tabula VIII. Praeter illam Geometriae partem, quae circa quantitates verſatur, et omni tempore ſummo ſtudio eſt exculta, alterius partis etiamnum admodum ignotae primus mentionem fecit *Leibnitzius*, quam Geometriam ſitus vocauit. Iſta pars ab ipſo in ſolo ſitu determinando, ſitusque proprietatibus eruendis occupata eſſe ſtatuitur; in quo negotio neque ad quantitates reſpiciendum, neque calculo quantitatum vtendum ſit. Cuiusmodi autem problemata ad hanc ſitus Geometriam pertineant, et quali methodo in iis reſoluendis vti oporteat, non ſatis eſt definitum. Quamobrem, cum nuper problematis cuiusdam mentio eſſet facta, quod quidem ad geometriam pertinere videbatur, at ita erat comparatum, vt neque determinationem quantitatum requireret, neque ſolutionem calculi quantitatum ope admitteret, id ad geometriam ſitus referre haud dubitaui: praeſertim quod in eius ſolutione ſolus ſitus in conſiderationem veniat, calculus vero nullius prorſus ſit vſus. Methodum ergo meam quam ad huius generis problemata



El origen: Los puentes de Königsberg

- Euler mostró que el problema no tiene solución y dio una condición necesaria para el caso general.
- Carl Hierholzer (1840-1871) mostró en 1871 que esta condición es también suficiente, y formalizó la demostración.



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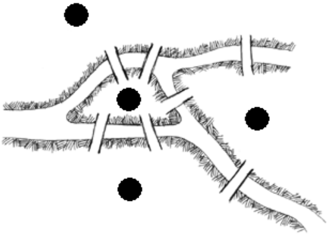

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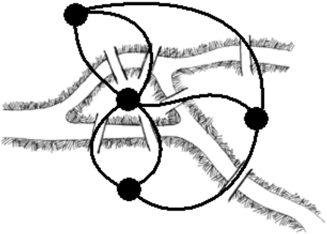
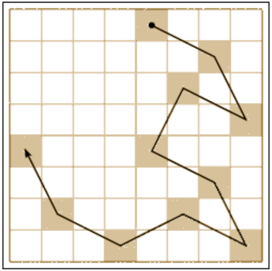


El origen: Los puentes de Königsberg

Segundo acto: El problema del caballo

Definición. Un caballo de ajedrez debe visitar todas las casillas pasando exactamente una vez por cada una.

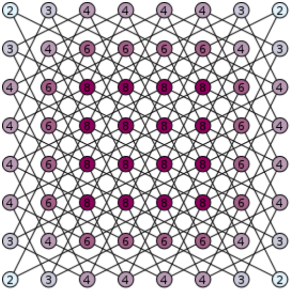


Segundo acto: El problema del caballo

- La referencia más temprana a este problema es del siglo IX.
- Alexandre-Theophile Vandermonde (1735–1796) estudió este problema, pero no encontró una solución.
  - A. Vandermonde, *Remarques sur des problèmes de situation*. Académie des Sciences (1771).
- El primer algoritmo (heurístico!) fue presentado en 1823. En términos modernos, es una heurística golosa que en cada paso se mueve al vecino de menor grado.
  - H. C. Warnsdorff, *Des Rösselsprungs einfachste und allgemeinste Lösung* (1823).

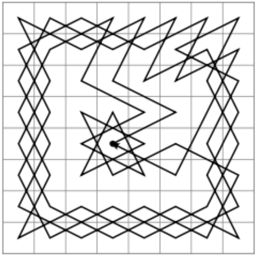
Segundo acto: El problema del caballo

Este problema corresponde a encontrar un circuito Hamiltoniano en el siguiente grafo:



Segundo acto: El problema del caballo

Una solución para el caso de 8×8:

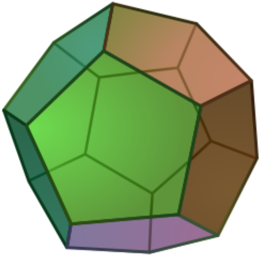


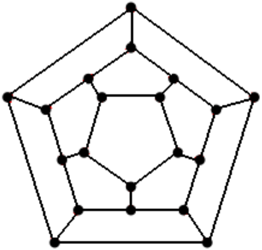
Tercer acto: Más sobre grafos Hamiltonianos

Sir William Hamilton (1805–1865)

Tercer acto: Más sobre grafos Hamiltonianos

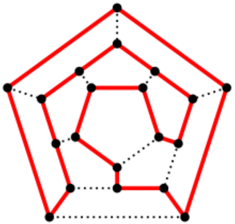
- Hamilton (1857) inventó el juego icosiano, que consiste en encontrar un camino que pase por todos los vértices de un dodecaedro y que retorne al punto de partida.



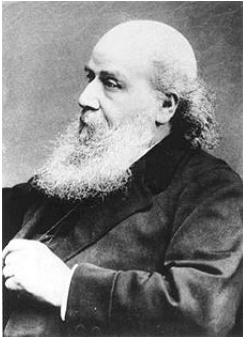


Tercer acto: Más sobre grafos Hamiltonianos

- Un recorrido con estas propiedades se llama actualmente circuito hamiltoniano, y para este caso particular se puede encontrar una solución.



La partida de nacimiento: Sylvester



James Sylvester (1814–1897)

El término grafo (graph) fue introducido en 1887 por Sylvester, en el contexto de análisis algebraico de estructuras moleculares.

La partida de nacimiento: Sylvester

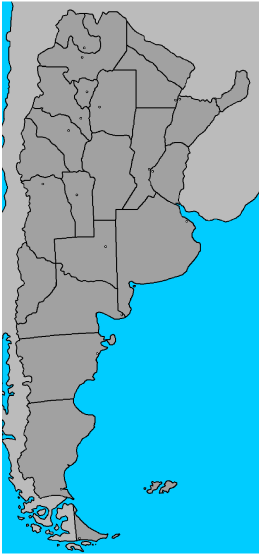
"The theory of ramification is one of pure colligation, for it takes no account of magnitude or position; geometrical lines are used, but have no more real bearing than those employed in genealogical tables have in explaining the laws of procreation."

El desafío: La conjetura de los cuatro colores

Conjetura. Todo mapa se puede colorear usando 4 colores, de modo tal que regiones adyacentes usen colores distintos.

- Las regiones deben ser contiguas.
- Dos regiones no se consideran adyacentes si sólo se intersecan en un punto.

El desafío: La conjetura de los cuatro colores



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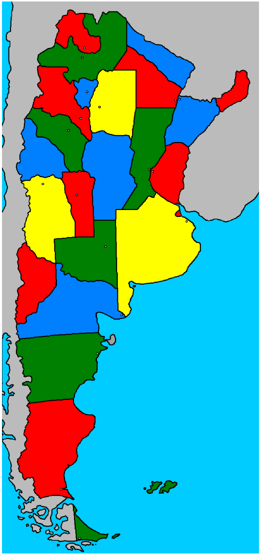

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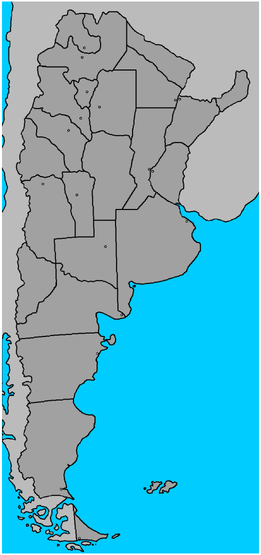

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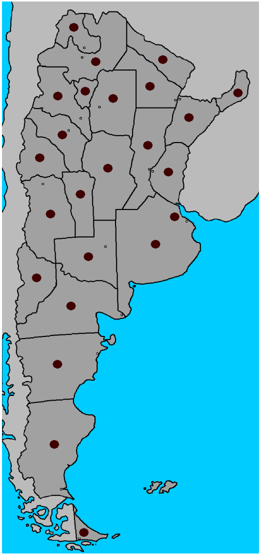

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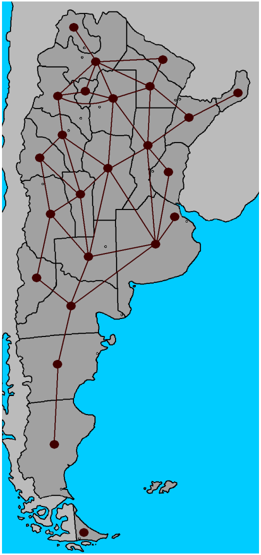

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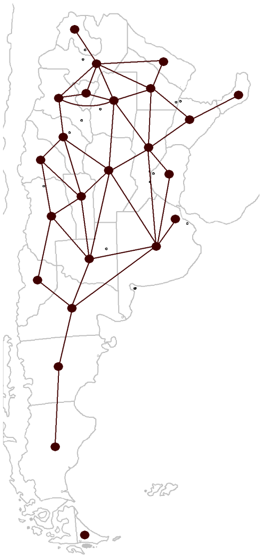


El desafío: La conjetura de los cuatro colores

- August Möbius (1790–1868) conocía este problema, aunque es posible que no sea él mismo quien lo haya propuesto por primera vez.
- Francis Guthrie (1831–1899) redescubrió la conjetura mientras coloreaba un mapa de Inglaterra, y su hermano la comunicó a Augustus De Morgan (1806–1871).
  - "A student of mine asked me to day to give him a reason for a fact which I did not know was a fact –and do not yet. He says that if a figure be any how divided and the compartments differently coloured so that figures with any portion of common boundary line are differently coloured –four colours may be wanted but not more– the following is his case in which four colours are wanted. Query cannot a necessity for five or more be invented."

El desafío: La conjetura de los cuatro colores

- Alfred Kempe (1849–1922) dio una demostración en 1879, pero Percy Heawood (1861–1955) encontró en 1890 un error. Al mismo tiempo, demostró el teorema de los cinco colores.

- Oystein Ore (1899–1968) y Joel Stemple mostraron en 1969 que la conjetura es cierta para todos los mapas de hasta 40 regiones.

- Kenneth May, *The origin of the four-color conjecture*. Isis 56 (1965) 346–348 ...

El desafío: La conjetura de los cuatro colores

THE ORIGIN OF THE FOUR–COLOR CONJECTURE

By Kenneth O. May *

Considering the fame and tender age of the four-color conjecture,[1] our knowledge of its origins is surprisingly vague. The well-known tradition appears to stem from W. W. R. Ball's *Mathematical Recreations and Essays,* whose first edition appeared in 1892. There it is said that "the problem was mentioned by A. F. Möbius in his lectures in 1840 . . . but it was not until Francis Guthrie communicated it to De Morgan about 1850 that attention was generally called to it . . . it is said that the fact had been familiar to practical mapmakers for a long time previously." [2] In spite of repetition by later writers, this tradition does not correspond to the facts.

In the first place there is no evidence that mapmakers were or are aware of the sufficiency of four colors. A sampling of atlases in the large collection of the Library of Congress indicates no tendency to minimize the number of colors used. Maps utilizing only four colors are rare, and those that do usually require only three. Books on cartography and the history of mapmaking do not mention the four-color property, though they often discuss various other problems relating to the coloring of maps.

If cartographers are aware of the four-color conjecture, they have certainly kept the secret well. But their lack of interest is quite understandable. Before the invention of printing it was as easy to use many as few colors. With the development of printing, the possibility of printing one color over another and of using such devices as hatching and shading provided the mapmaker with an unlimited variety of colors. Moreover, the coloring of a geographical map is quite different from the formal problem posed by mathematicians because of such desiderata as coloring colonies the same as the mother country and the reservation of certain colors for terrain features, e.g. blue for water. The four-color conjecture cannot claim either origin or application in cartography.

To support his statement about Möbius, Ball refers to an article by Baltzer, a former student of Möbius and the editor of his collected works.[3] However, as has been pointed out by H. S. M. Coxeter,[4] this article shows merely that Weiske communicated to Möbius a puzzle whose solution amounted to the claim that it is impossible to have five regions each having a common boundary with every

* Carleton College and University of California, Berkeley. This article is based on work done during the tenure of a Science Faculty Fellowship from the National Science Foundation. It was presented to the Minnesota section of the Mathematical Association of America on 3 November 1962 and to the Midwest Junto of the History of Science Society on 5 April 1963. I am indebted to S. Schuster, G. A. Dirac, J. Dyer-Bennet, Oystein Ore, and H. S. M. Coxeter for discussion and suggestions.

[1] In nontechnical terms the four-color conjecture is usually stated as follows: Any map on a plane or the surface of a sphere can be colored with only four colors so that no two adjacent countries have the same color. Each country must consist of a single connected region, and adjacent countries are those having a boundary line (not merely a single point) in common. The conjecture has acted as a catalyst in the branch of mathematics known as combinatorial topology and is closely related to the currently fashionable field of graph theory. More than half a century of work by many (some say all) mathematicians has yielded proofs only for special cases (up to 35 countries by 1940). The consensus is that the conjecture is correct but unlikely to be proved in general. It seems destined to retain for some time the distinction of being both the simplest and most fascinating unsolved problem of mathematics.

[2] W. W. Rouse Ball, *Mathematical Recreations and Essays,* rev. H. S. M. Coxeter (London: Macmillan, 1959), p. 223.

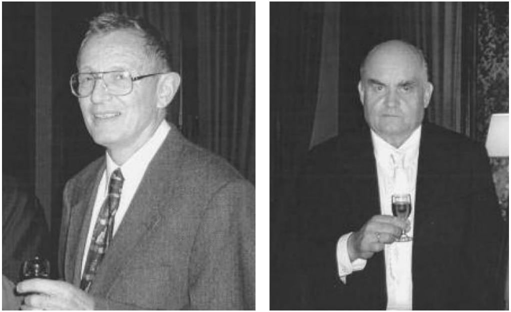
[3] R. Baltzer, "Eine Erinnerung an Möbius und seinen Freund Weiske," *Bericht über die Verhandlungen der Sächsischen Akademie der Wissenschaften zu Leipzig. Math.-Nat. Kl.,* 1885, *37*:1–6.

[4] H. S. M. Coxeter, "The Four-Color Map Problem," *Mathematics Teacher,* 1959, *52*:283–289.

El desafío: La conjetura de los cuatro colores

- “The conjecture has acted as a catalyst in the branch of mathematics known as combinatorial topology and is closely related to the currently fashionable field of graph theory. More than half a century of work by many (some say all) mathematicians has yielded proofs for special cases.
- “The consensus is that the conjecture is correct but unlikely to be proved in general. It seems destined to retain for some time the distinction of being both the simplest and most fascinating unsolved problem of mathematics.”

El desafío: La conjetura de los cuatro colores



Appel y Haken en 2002

- La primera demostración fue dada en 1976 por Kenneth Appel (1932–) y Wolfgang Haken (1928–), coronando una “carrera por la demostración”.

El desafío: La conjetura de los cuatro colores

- Appel y Haken redujeron todos los contraejemplos posibles a 1936 contrajemplos minimales.
- Utilizando un programa de computadora, verificaron que todos esos posibles contraejemplos se pueden colorear con cuatro colores (¿dijo "un ... programa de computadora"?).
- **Estado actual:** Algoritmo $O(n^2)$ para colorear un mapa con 4 colores (N. Robertson, D. Sanders, P. Seymour y R. Thomas, 1996).
- Demostración simplificada con 633 configuraciones minimales.