

```
> #ex 1:
> f :=x->2*x*(1+y^2)
```

$$f := x \mapsto 2 \cdot x \cdot (y^2 + 1)$$

```
> ecdif1:=diff(y(x),x)=2*x*(1+y(x)^2)
```

$$ecdif1 := \frac{d}{dx} y(x) = 2 x (1 + y(x)^2)$$

```
> dsolve(ecdif1,y(x))
```

$$y(x) = \tan(x^2 + 2 c_1)$$

```
> expand(y(x) = tan(x^2+2*c__1));
```

$$y(x) = \frac{\frac{2 \tan(c_1)}{1 - \tan(c_1)^2} + \tan(x^2)}{1 - \frac{2 \tan(c_1) \tan(x^2)}{1 - \tan(c_1)^2}}$$

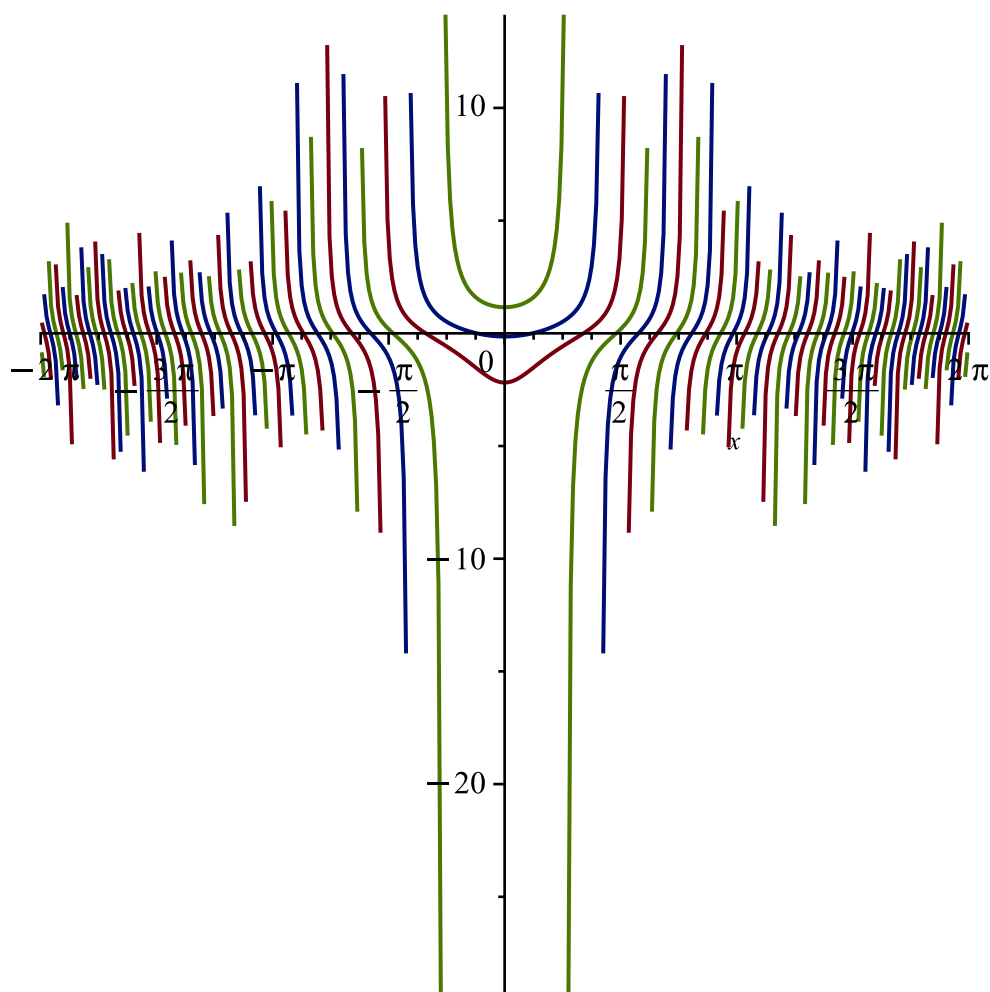
```
> with(plots):
> sol1:=dsolve(ecdif1,y(x))
```

$$sol1 := y(x) = \tan(x^2 + 2 c_1)$$

```
> y1:=(x,c)->tan(x^2+c)
```

$$y1 := (x, c) \mapsto \tan(x^2 + c)$$

```
> plot([y1(x,2),y1(x,3),y1(x,4)],x=-2*Pi..2*Pi)
```



```
> ecdif2:=diff(y(x),x)=-2*x/(x^2-1)*(y(x))^2
```

$$ecdif2 := \frac{d}{dx} y(x) = -\frac{2xy(x)^2}{x^2-1} \quad (7)$$

```
> dsolve(ecdif2,y(x))
```

$$y(x) = \frac{1}{\ln(x-1) + \ln(x+1) + c_1} \quad (8)$$

```
> sol2:=dsolve(ecdif2,y(x))
```

$$sol2 := y(x) = \frac{1}{\ln(x-1) + \ln(x+1) + c_1} \quad (9)$$

```
> lhs((9)) - rhs((9)) = 0;
```

$$y(x) - \frac{1}{\ln(x-1) + \ln(x+1) + c_1} = 0 \quad (10)$$

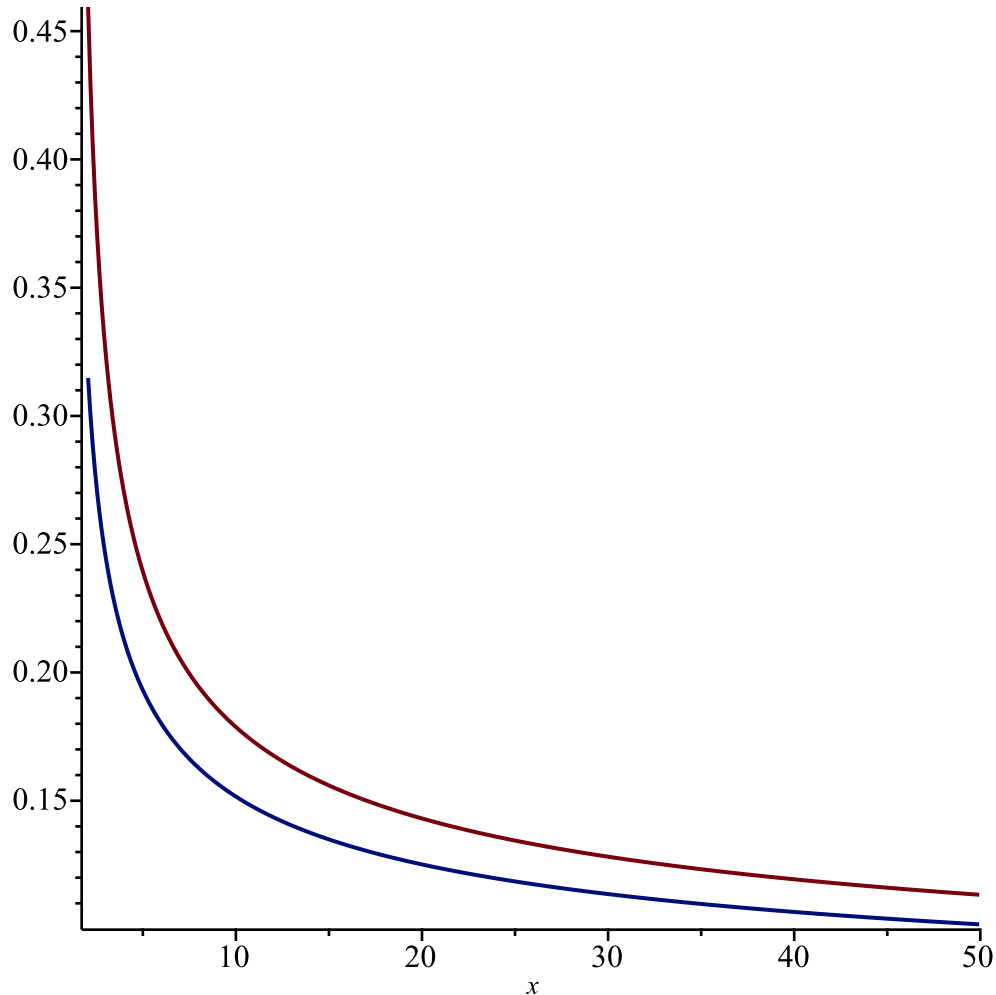
```
> right_hand_expr:= rhs(sol2)
```

$$right_hand_expr := \frac{1}{\ln(x-1) + \ln(x+1) + c_1} \quad (11)$$

```
> y2:=unapply(right_hand_expr,x,c__1)
```

$$y2 := (x, c_1) \mapsto right_hand_expr \quad (12)$$

```
> plot([y2(x,1),y2(x,2)],x=2..50)
```



```
> ecdif3:=diff(y(x),x)=1/2+1/2*(y(x)/x)^2
```

$$ecdif3 := \frac{d}{dx} y(x) = \frac{1}{2} + \frac{y(x)^2}{2x^2} \quad (13)$$

```
> restart
```

```
> ecdif4:=2*x*diff(y(x),x)=x^2+(y(x))^2
```

$$ecdif4 := 2x \left(\frac{d}{dx} y(x) \right) = x^2 + y(x)^2 \quad (14)$$

```
> dsolve(ecdif4,y(x))
```

$$y(x) = \frac{c_1 x \text{BesselY}\left(1, \frac{x}{2}\right)}{c_1 \text{BesselY}\left(0, \frac{x}{2}\right) + \text{BesselJ}\left(0, \frac{x}{2}\right)} + \frac{\text{BesselJ}\left(1, \frac{x}{2}\right) x}{c_1 \text{BesselY}\left(0, \frac{x}{2}\right) + \text{BesselJ}\left(0, \frac{x}{2}\right)} \quad (15)$$

```
> with(plots):
```

```
> restart
```

```
> ecdif4:=diff(y(x),x)=-x/y(x)
```

$$ecdif4 := \frac{d}{dx} y(x) = -\frac{x}{y(x)} \quad (16)$$

```
> dsolve(ecdif4,y(x))
```

$$y(x) = \sqrt{-x^2 + c_1}, y(x) = -\sqrt{-x^2 + c_1} \quad (17)$$

```
> sol4:=dsolve(ecdif4,y(x))
```

$$sol4 := y(x) = \sqrt{-x^2 + c_1}, y(x) = -\sqrt{-x^2 + c_1} \quad (18)$$

```
> y4:=unapply(rhs(sol4[1]),x,c__1)
```

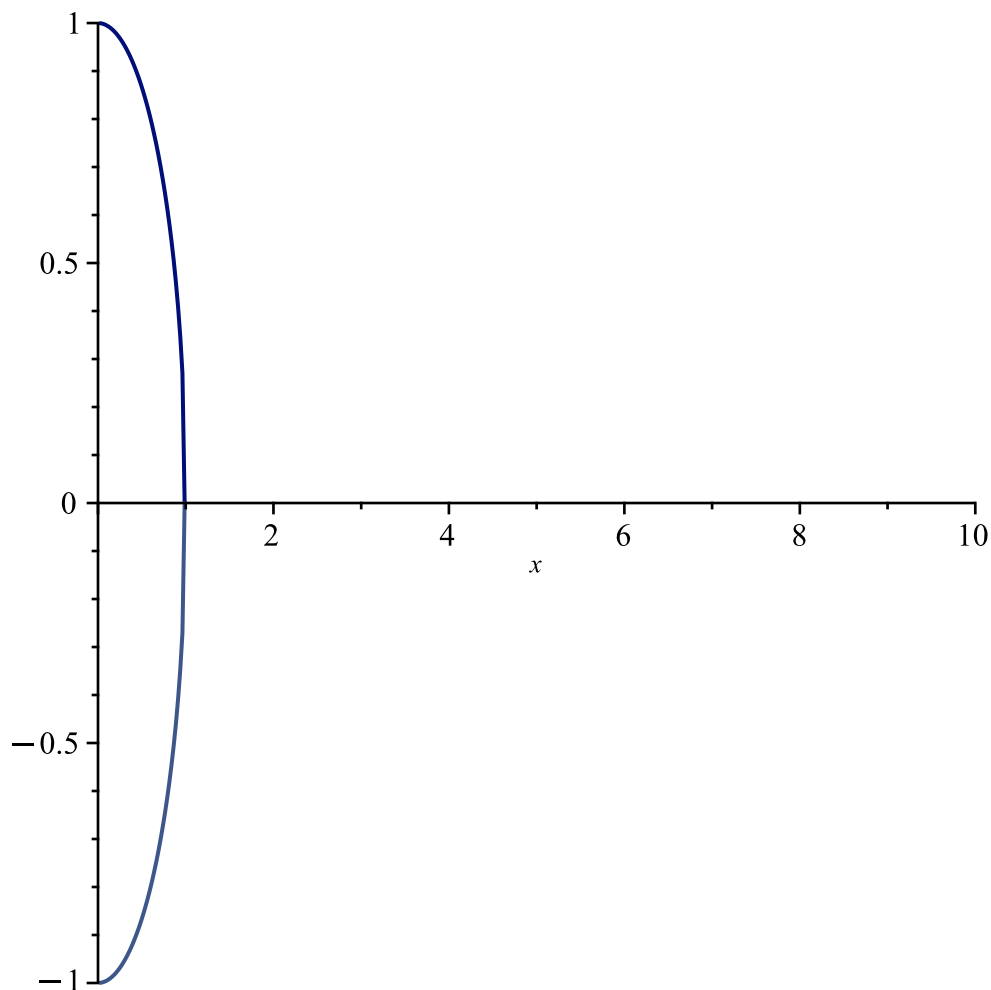
$$y4 := (x, c_1) \mapsto \sqrt{-x^2 + c_1} \quad (19)$$

```
> y5:=unapply(rhs(sol4[2]),x,c__1)
```

$$y5 := (x, c_1) \mapsto -\sqrt{-x^2 + c_1} \quad (20)$$

```
> with(plots):
```

```
> plot([y4(x,0),y4(x,1),y5(x,0),y5(x,1)],x=0..10)
```



```
> restart
```

```
> ecdif5:=diff(y(x),x)=-x/(y(x))^3
```

$$ecdif5 := \frac{d}{dx} y(x) = -\frac{x}{y(x)^3} \quad (21)$$

```
> dsolve(ecdif5,y(x))
```

$$y(x) = (-2x^2 + c_1)^{1/4}, y(x) = -(-2x^2 + c_1)^{1/4}, y(x) = -I(-2x^2 + c_1)^{1/4}, y(x) = I(-2x^2 + c_1)^{1/4} \quad (22)$$

$$-2x^2 + c_1)^{1/4}$$

```
> sol5:=dsolve(ecdif5,y(x))
```

$$\text{sol5} := y(x) = (-2x^2 + c_1)^{1/4}, y(x) = -(-2x^2 + c_1)^{1/4}, y(x) = -I(-2x^2 + c_1)^{1/4}, y(x) = I(-2x^2 + c_1)^{1/4} \quad (23)$$

```
> y1:=unapply(rhs(sol5[1]),x,c__1)
```

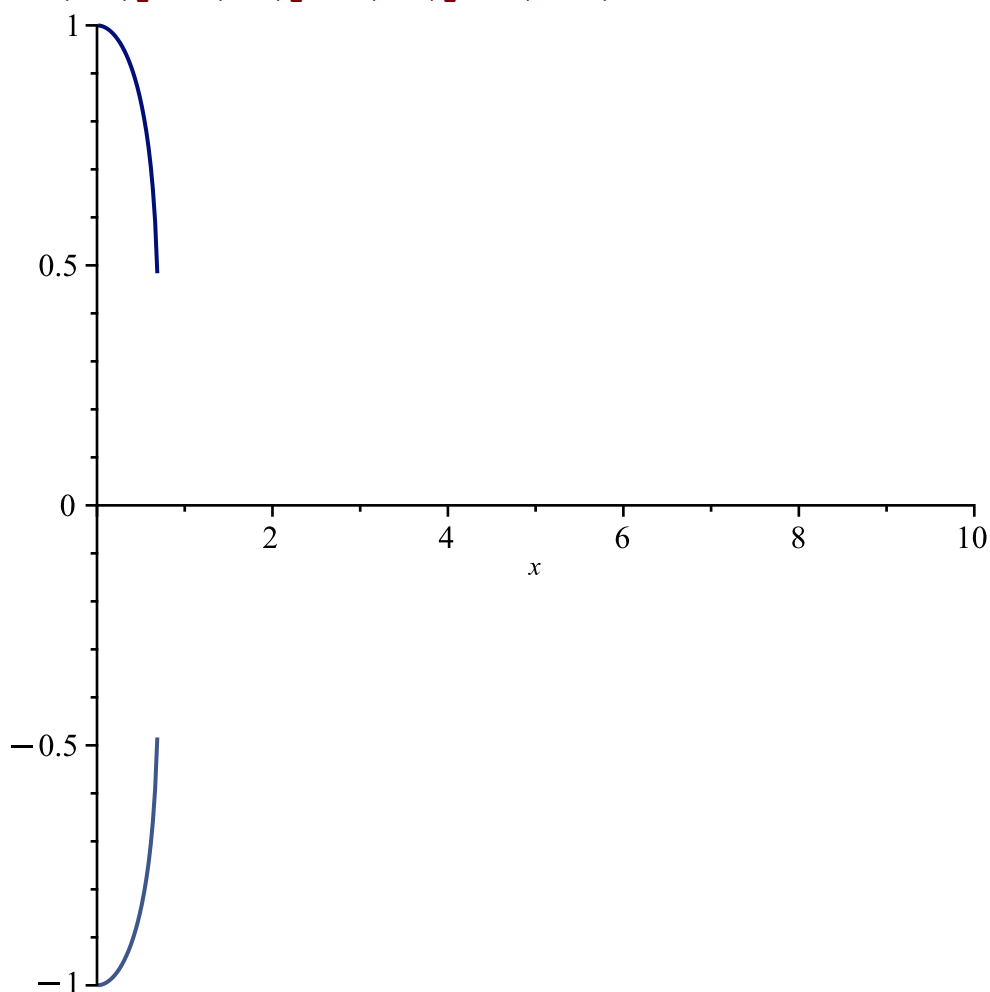
$$y1 := (x, c_1) \mapsto (-2 \cdot x^2 + c_1)^{1/4} \quad (24)$$

```
> y2:=unapply(rhs(sol5[2]),x,c__1)
```

$$y2 := (x, c_1) \mapsto -(-2 \cdot x^2 + c_1)^{1/4} \quad (25)$$

```
> with(plots):
```

```
> plot([y1(x,0),y1(x,1),y2(x,0),y2(x,1)],x=0..10)
```



```
> restart
```

```
> ecdif6:=diff(y(x),x)=-(x+y(x))/y(x)
```

$$\text{ecdif6} := \frac{d}{dx} y(x) = -\frac{x + y(x)}{y(x)} \quad (26)$$

```
> dsolve(ecdif6,y(x))
```

(27)

$$y(x) = \frac{\sqrt{3} x \tan\left(\text{RootOf}\left(\sqrt{3} \ln\left(\frac{3x^2}{4} + \frac{3x^2 \tan(_Z)^2}{4}\right) + 2\sqrt{3} c_1 - 2_Z\right)\right)}{2} - \frac{x}{2} \quad (27)$$

```
> sol6:=dsolve(ecdif6,y(x))
```

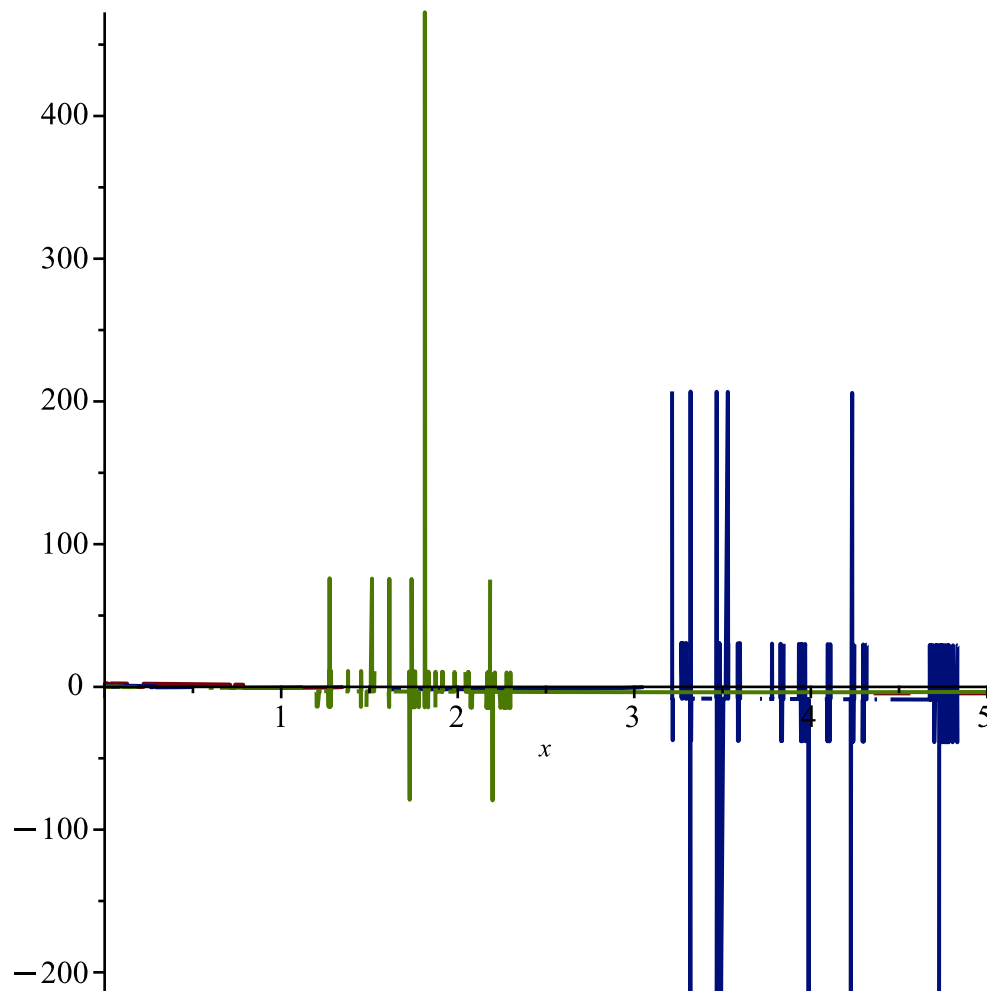
$$\text{sol6} := y(x) = \frac{\sqrt{3} x \tan\left(\text{RootOf}\left(\sqrt{3} \ln\left(\frac{3x^2}{4} + \frac{3x^2 \tan(_Z)^2}{4}\right) + 2\sqrt{3} c_1 - 2_Z\right)\right)}{2} - \frac{x}{2} \quad (28)$$

```
> y6:=unapply(rhs(sol6),x,c__1)
```

$$y6 := (x, c_1) \mapsto \frac{\sqrt{3} \cdot x \cdot \tan\left(\text{RootOf}\left(\sqrt{3} \cdot \ln\left(\frac{3 \cdot x^2}{4} + \frac{3 \cdot x^2 \cdot \tan(_Z)^2}{4}\right) + 2 \cdot \sqrt{3} \cdot c_1 - 2 \cdot _Z\right)\right)}{2} - \frac{x}{2} \quad (29)$$

```
> with(plots):
```

```
> plot([y6(x,0),y6(x,1),y6(x,2)],x=0..5)
```



```
> restart
```

```
> ecdif7:=diff(y(x),x)+y(x)*tan(x)=1/cos(x)
```

$$ecdif7 := \frac{d}{dx} y(x) + y(x) \tan(x) = \frac{1}{\cos(x)} \quad (30)$$

```
> dsolve(ecdif7, y(x))
```

$$y(x) = (\tan(x) + c_1) \cos(x) \quad (31)$$

```
> sol7:=dsolve(ecdif7, y(x))
```

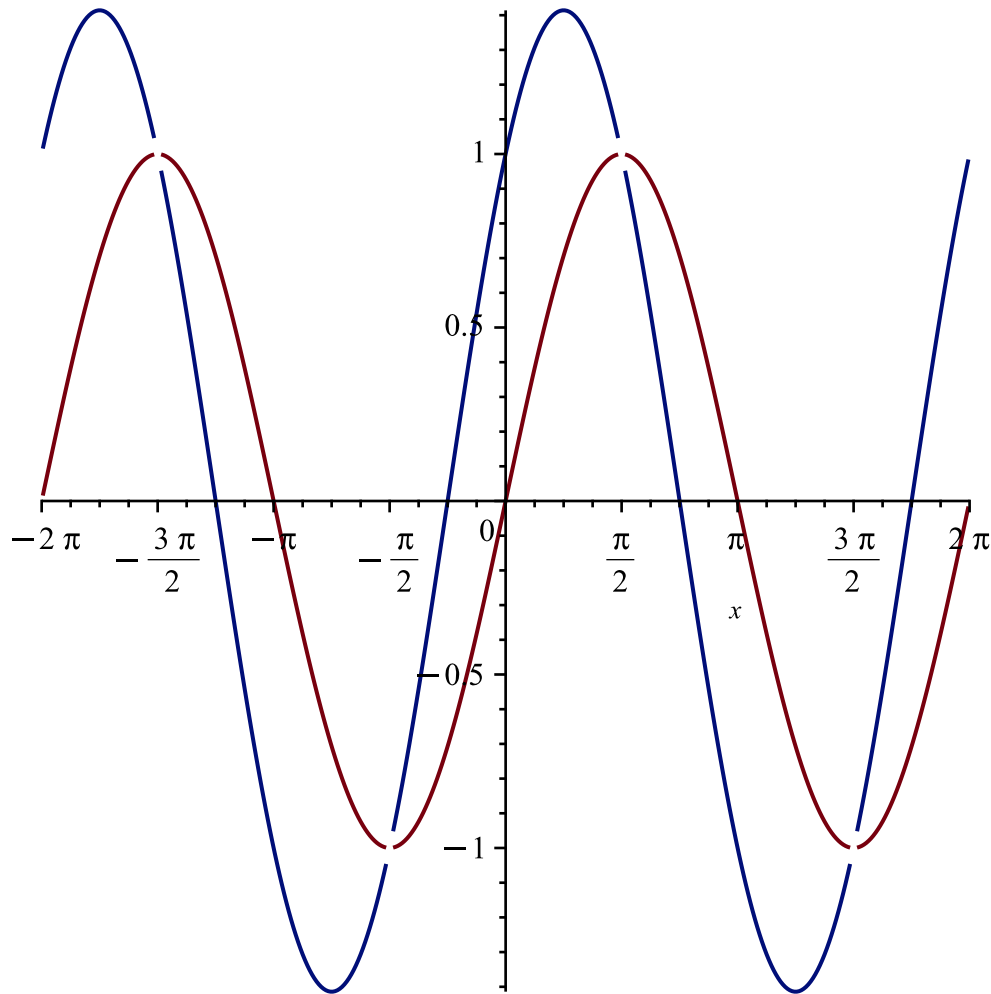
$$sol7 := y(x) = (\tan(x) + c_1) \cos(x) \quad (32)$$

```
> y7:=unapply(rhs(sol7), x, c__1)
```

$$y7 := (x, c_1) \mapsto (\tan(x) + c_1) \cdot \cos(x) \quad (33)$$

```
> with(plots):
```

```
> plot([y7(x,0), y7(x,1)], x=-2*Pi..2*Pi)
```



```
> restart
```

```
> ecdif8:=diff(y(x), x)+2/x*y(x)=x^3
```

$$ecdif8 := \frac{d}{dx} y(x) + \frac{2y(x)}{x} = x^3 \quad (34)$$

```
> dsolve(ecdif8, y(x))
```

$$y(x) = \frac{\frac{x^6}{6} + c_1}{x^2} \quad (35)$$

```
> sol8:=dsolve(ecdif8,y(x))
```

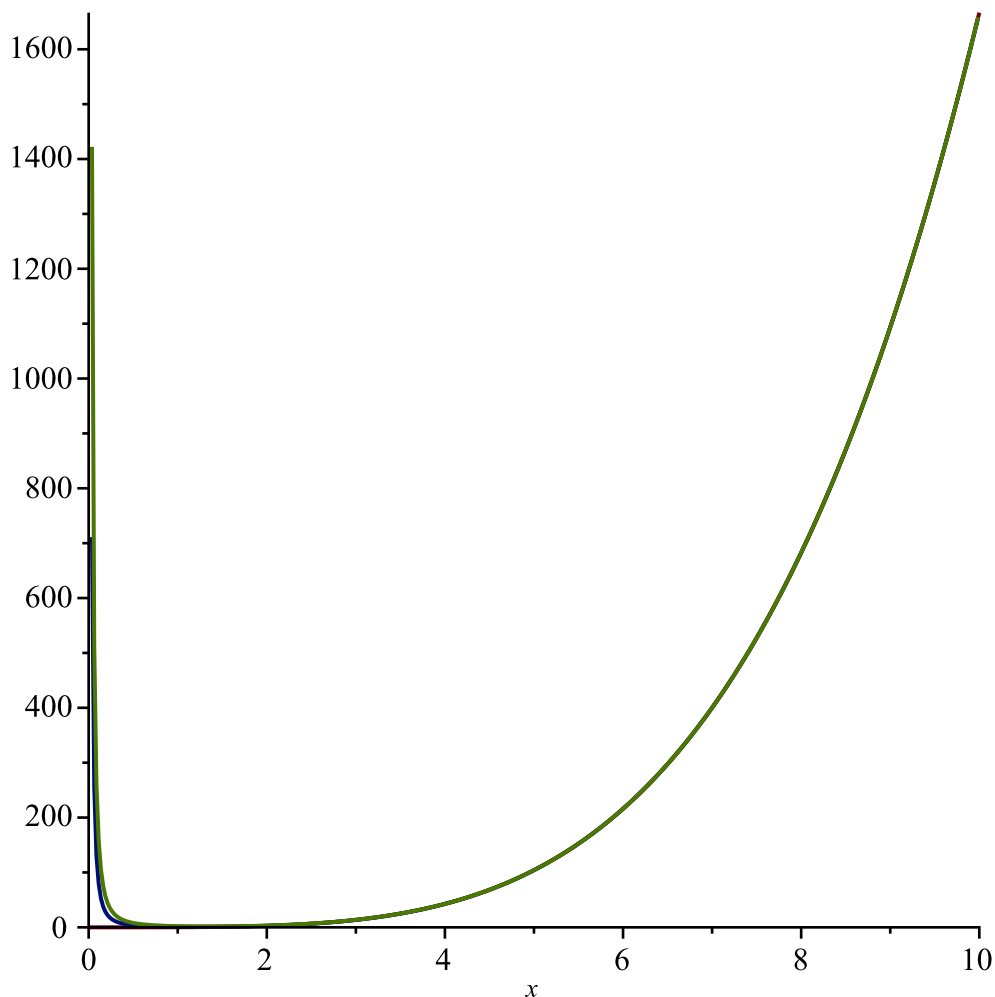
$$sol8 := y(x) = \frac{\frac{x^6}{6} + c_1}{x^2} \quad (36)$$

```
> y8:=unapply(rhs(sol8),x,c__1)
```

$$y8 := (x, c_1) \mapsto \frac{\frac{x^6}{6} + c_1}{x^2} \quad (37)$$

```
> with(plots):
```

```
> plot([y8(x,0),y8(x,1),y8(x,2)],x=0..10)
```



```
> restart
```

```
> ecdif9:=diff(y(x),x$2)+diff(y(x),x)=sin(x)+cos(x)
```

$$ecdif9 := \frac{d^2}{dx^2} y(x) + \frac{d}{dx} y(x) = \sin(x) + \cos(x) \quad (38)$$

```
> dsolve(ecdif9,y(x))
```

$$y(x) = -e^{-x} c_1 - \cos(x) + c_2 \quad (39)$$

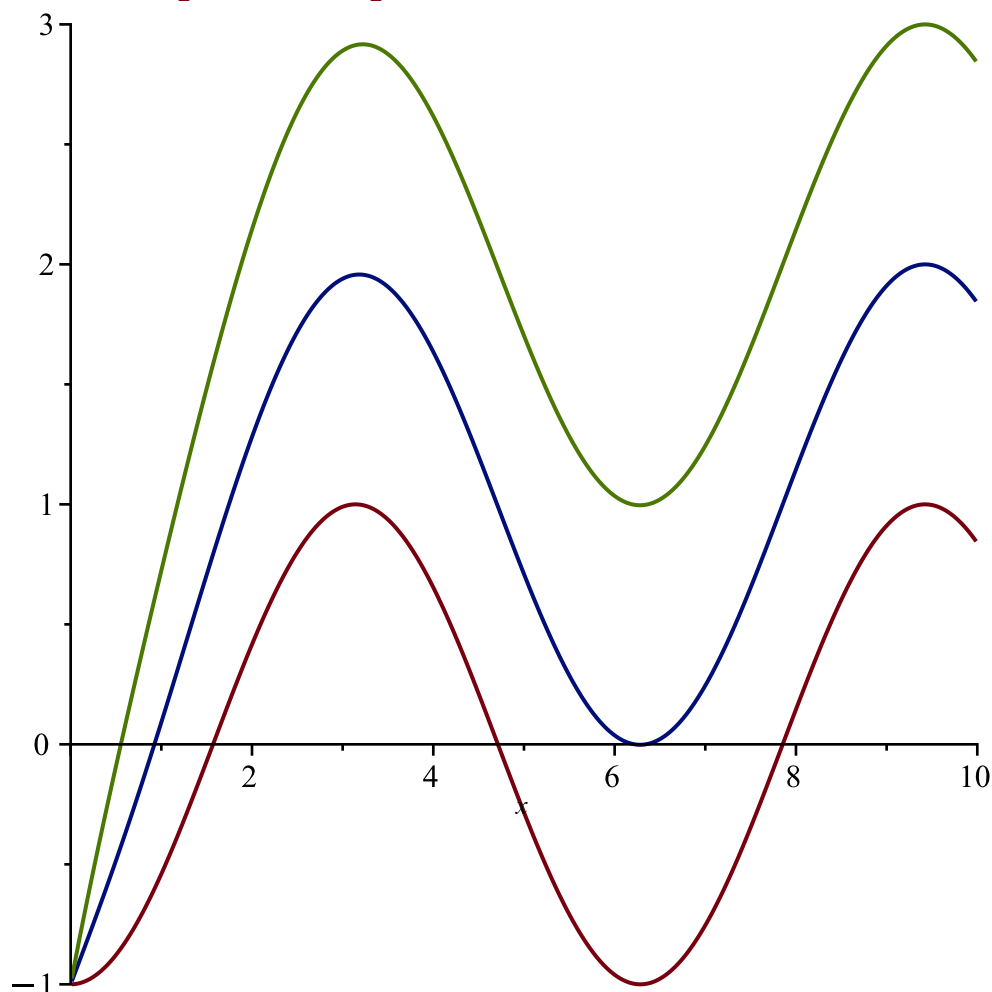
```
> sol9:=dsolve(ecdif9,y(x))
```

$$sol9 := y(x) = -e^{-x} c_1 - \cos(x) + c_2 \quad (40)$$


```
> y9:=unapply(rhs(sol9),x,c__1,c__2)
```

$$y9 := (x, c_1, c_2) \mapsto -e^{-x} \cdot c_1 - \cos(x) + c_2 \quad (41)$$

```
> with(plots):
> plot([y9(x,0,0),y9(x,1,1),y9(x,2,2)],x=0..10)
```



```
> restart
> ecdif10:=diff(y(x),x$2)-y(x)=exp(2*x)
```

$$ecdif10 := \frac{d^2}{dx^2} y(x) - y(x) = e^{2x} \quad (42)$$

```
> dsolve(ecdif10,y(x))
```

$$y(x) = e^x c_2 + e^{-x} c_1 + \frac{e^{2x}}{3} \quad (43)$$

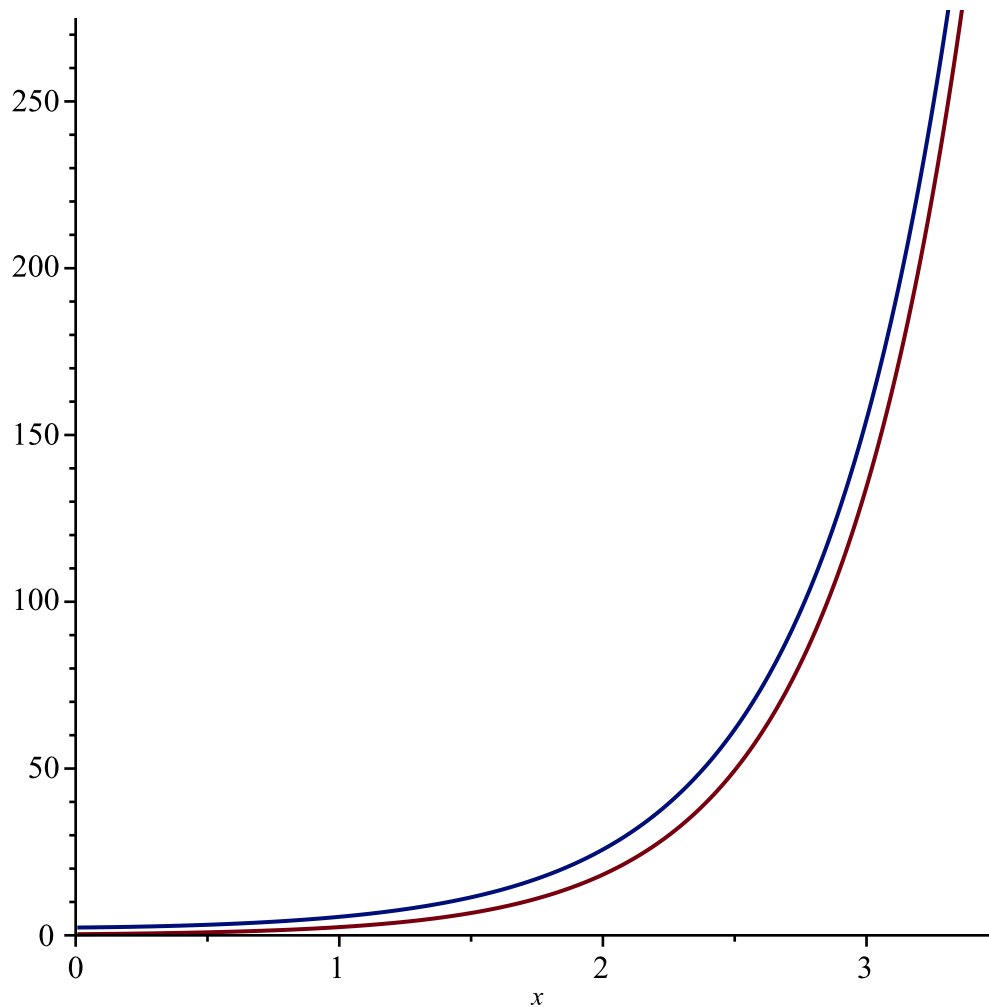
```
> sol10:=dsolve(ecdif10,y(x))
```

$$sol10 := y(x) = e^x c_2 + e^{-x} c_1 + \frac{e^{2x}}{3} \quad (44)$$

```
> y10:=unapply(rhs(sol10),x,c__1,c__2)
```

$$y10 := (x, c_1, c_2) \mapsto e^x \cdot c_2 + e^{-x} \cdot c_1 + \frac{e^{2 \cdot x}}{3} \quad (45)$$

```
> with(plots):
> plot([y10(x,0,0),y10(x,1,1)],x=0..5)
```



```
> #ex 2:
> restart
```

```
> with(DEtools):
```

```
> ecdif1:=diff(y(x),x)=1+y(x)^2
```

$$ecdif1 := \frac{d}{dx} y(x) = 1 + y(x)^2 \quad (46)$$

```
> cond_in:=y(0)=1
```

$$cond_in := y(0) = 1 \quad (47)$$

```
> sol1:=dsolve({ecdif1,cond_in},y(x))
```

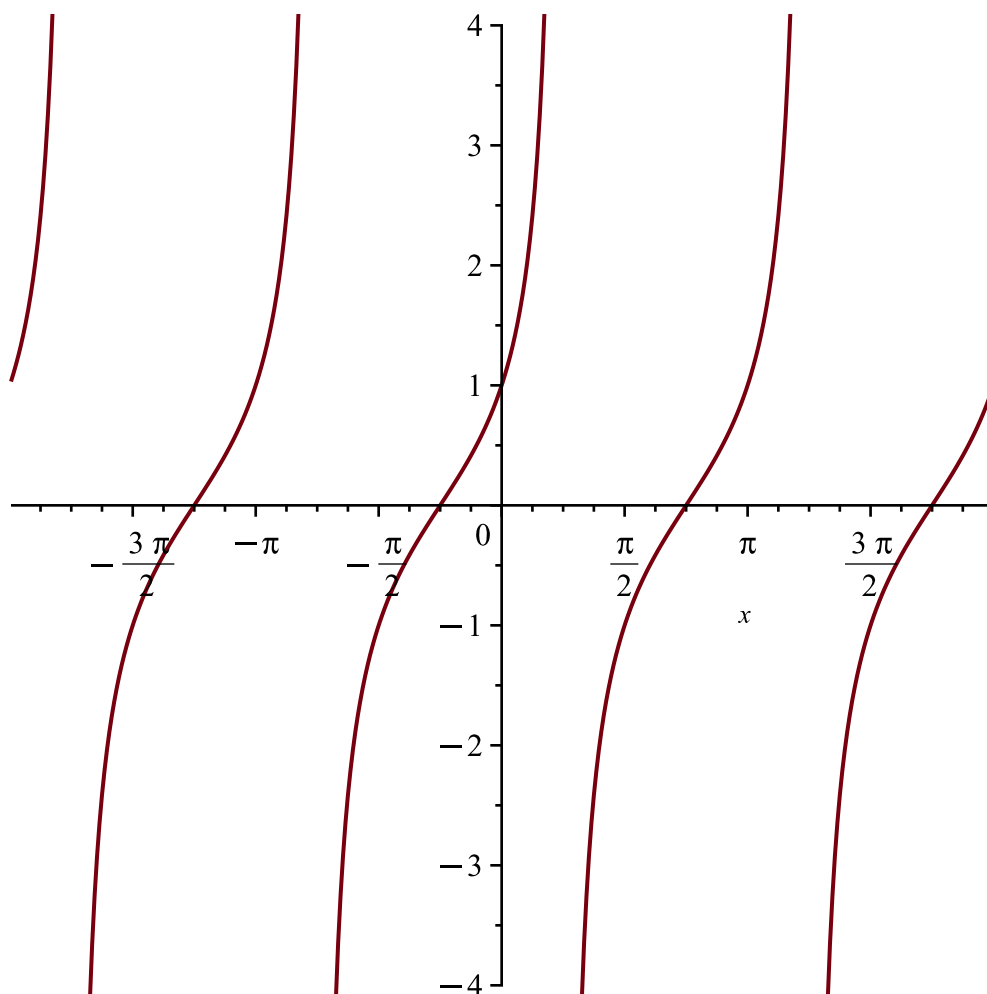
$$sol1 := y(x) = \tan\left(x + \frac{\pi}{4}\right) \quad (48)$$

```
> y1:=unapply(rhs(sol1),x)
```

$$y1 := x \mapsto \tan\left(x + \frac{\pi}{4}\right) \quad (49)$$

```
> with(plots):
```

```
> plot(y1(x),x=-2*Pi..2*Pi)
```



```
> ecdif2:=diff(y(x),x)=1/(1-x^2)*y(x)+1+x
```

$$ecdif2 := \frac{d}{dx} y(x) = \frac{y(x)}{-x^2 + 1} + 1 + x \quad (50)$$

```
> cond_in2:=y(0)=0
```

$$cond_in2 := y(0) = 0 \quad (51)$$

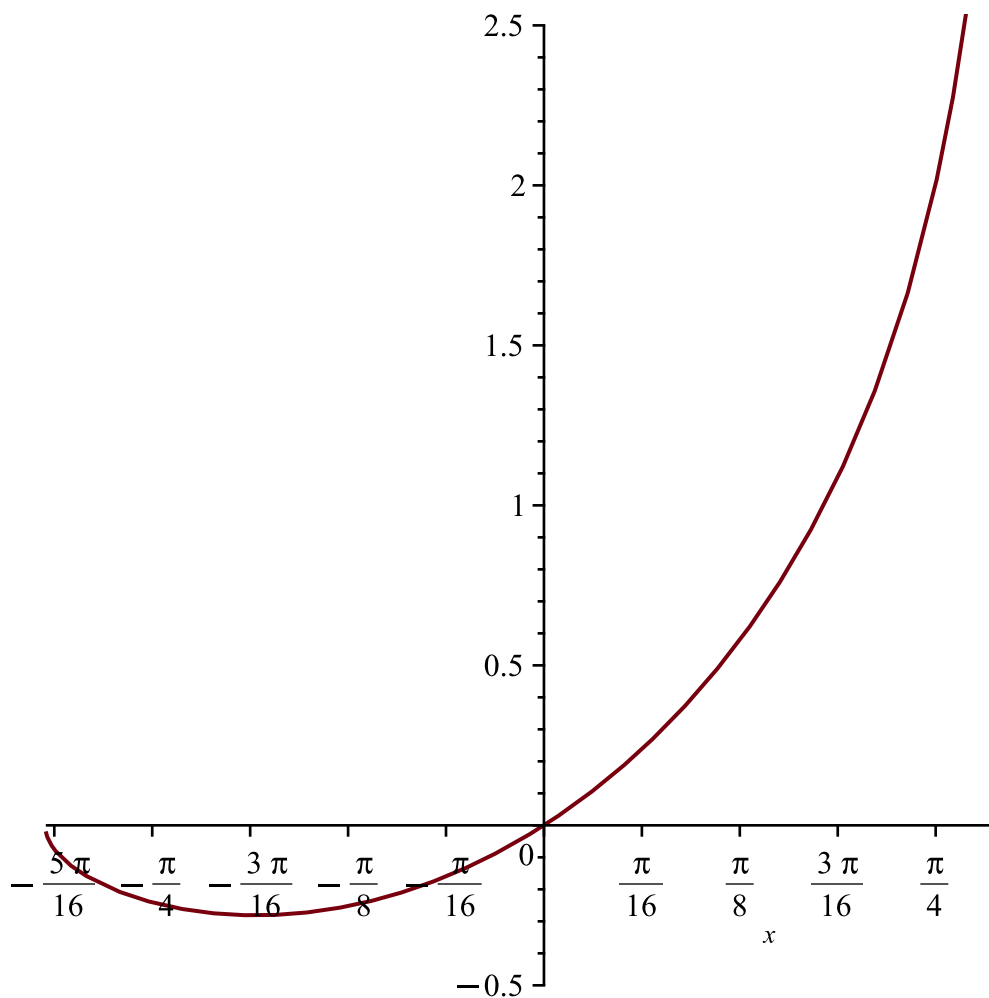
```
> sol2:=dsolve({ecdif2,cond_in2},y(x))
```

$$sol2 := y(x) = \frac{(x \sqrt{-x^2 + 1} + \arcsin(x)) (x + 1)}{2 \sqrt{-x^2 + 1}} \quad (52)$$

```
> y2:=unapply(rhs(sol2),x)
```

$$y2 := x \mapsto \frac{(x \cdot \sqrt{-x^2 + 1} + \arcsin(x)) \cdot (x + 1)}{2 \cdot \sqrt{-x^2 + 1}} \quad (53)$$

```
> plot(y2(x),x=-2*Pi..2*Pi)
```



```
> ecdif3:=diff(y(x),x$2)-5*diff(y(x),x)+4*y(x)=0
```

$$ecdif3 := \frac{d^2}{dx^2} y(x) - 5 \frac{d}{dx} y(x) + 4 y(x) = 0 \quad (54)$$

```
> cond_in3:=y(0)=5
```

$$cond_in3 := y(0) = 5 \quad (55)$$

```
> cond_in31:=D(y)(0)=8
```

$$cond_in31 := D(y)(0) = 8 \quad (56)$$

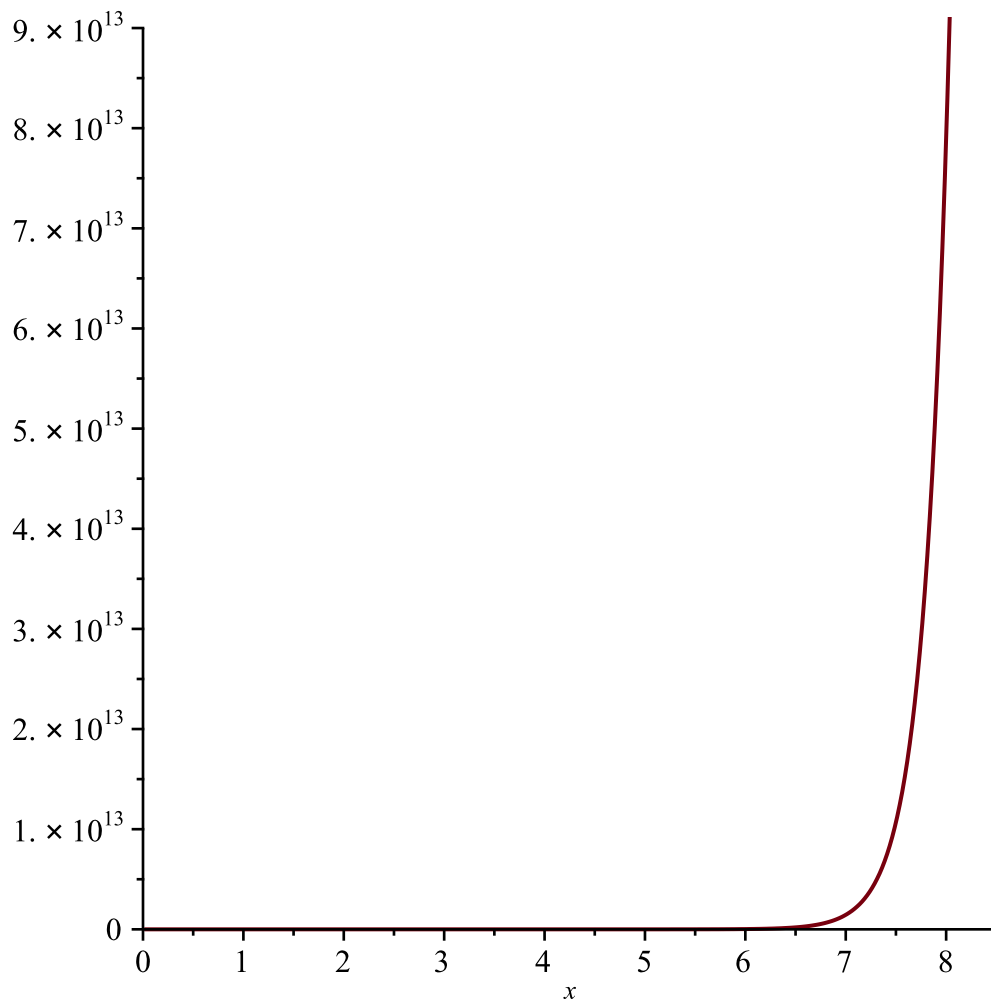
```
> sol3:=dsolve({ecdif3,cond_in3,cond_in31},y(x))
```

$$sol3 := y(x) = 4 e^x + e^{4x} \quad (57)$$

```
> y3:=unapply(rhs(sol3),x)
```

$$y3 := x \mapsto 4 \cdot e^x + e^{4 \cdot x} \quad (58)$$

```
> plot(y3(x),x=0..10)
```



```
> ecdif4:=diff(y(x),x$2)-4*diff(y(x),x)+5*y(x)=2*x^2*exp(x)
```

$$ecdif4 := \frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 5 y(x) = 2 x^2 e^x \quad (59)$$

```
> cond_in4:=y(0)=2
```

$$cond_in4 := y(0) = 2 \quad (60)$$

```
> cond_in41:=D(y)(0)=3
```

$$cond_in41 := D(y)(0) = 3 \quad (61)$$

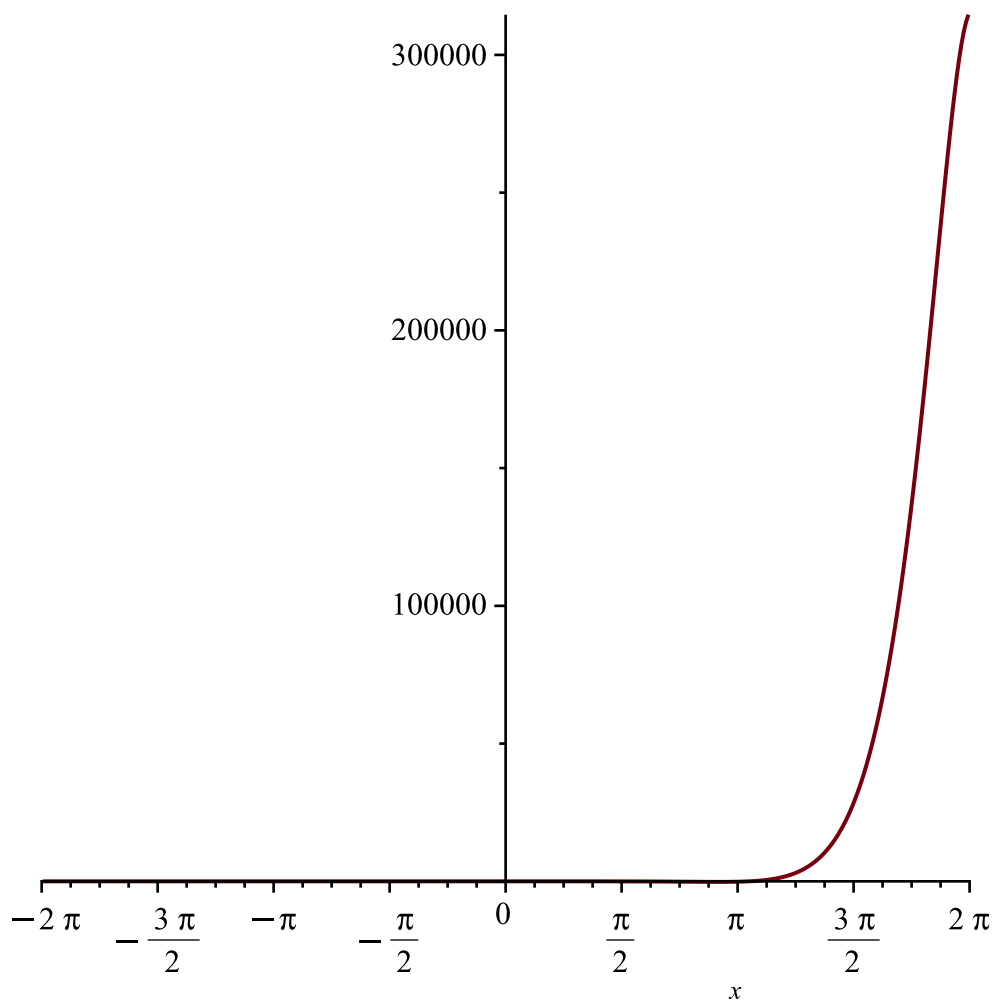
```
> sol4:=dsolve({ecdif4,cond_in4,cond_in41},y(x))
```

$$sol4 := y(x) = (\cos(x) - 2 \sin(x)) e^{2x} + (x + 1)^2 e^x \quad (62)$$

```
> y4:=unapply(rhs(sol4),x)
```

$$y4 := x \mapsto (\cos(x) - 2 \sin(x)) \cdot e^{2x} + (x + 1)^2 \cdot e^x \quad (63)$$

```
> plot(y4(x),x=-2*Pi..2*Pi)
```



```
> restart
```

```
> #ex 3:
```

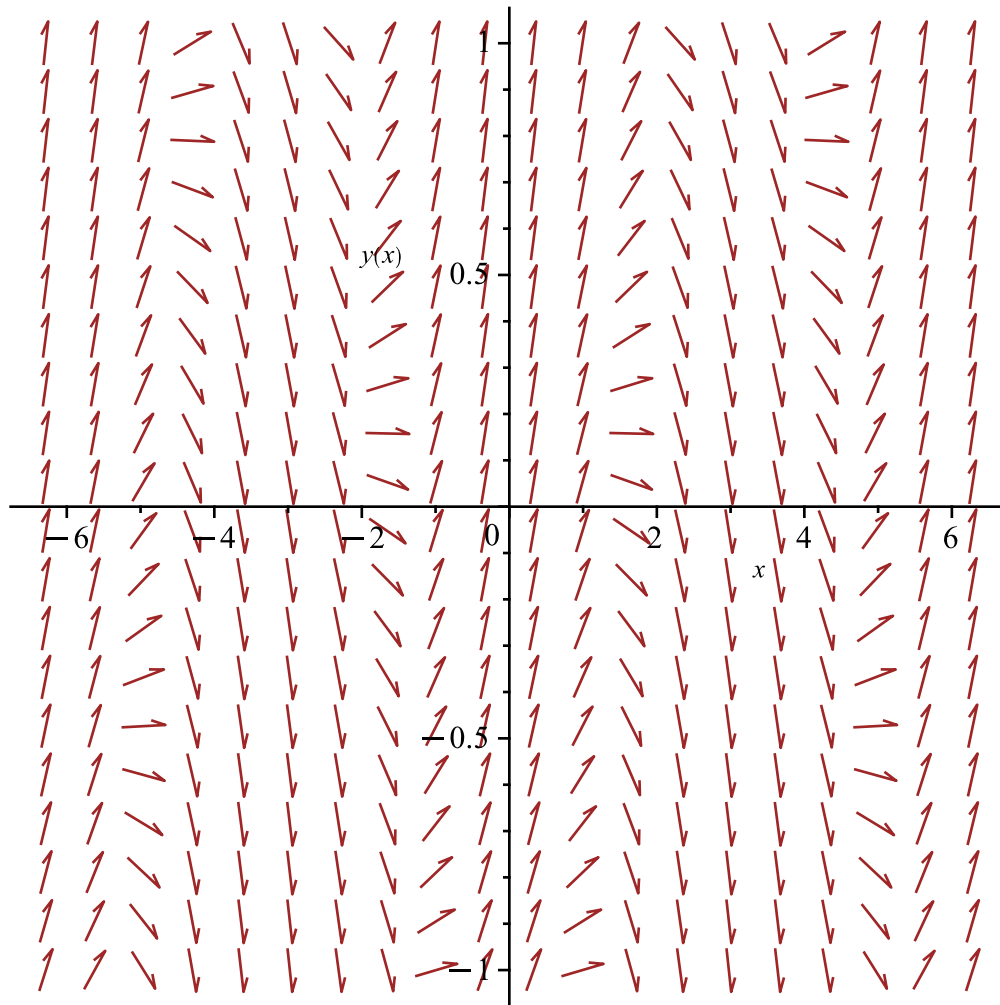
```
> with(DEtools): with(plots):
```

```
> ecdif:=diff(y(x),x)-1/2*y(x)=cos(x)
```

$$ecdif := \frac{d}{dx} y(x) - \frac{y(x)}{2} = \cos(x)$$

(64)

```
> DEplot(ecdif,y(x),x=-2*Pi..2*Pi,y=-1..1)
```



```
> cond_in:=y(0)=a
```

$$cond_in := y(0) = a$$

(65)

```
> sol1:=dsolve({ecdif,cond_in},y(x))
```

$$sol1 := y(x) = -\frac{2 \cos(x)}{5} + \frac{4 \sin(x)}{5} + e^{\frac{x}{2}} \left(a + \frac{2}{5} \right)$$

(66)

```
> a:=-2/5
```

$$a := -\frac{2}{5}$$

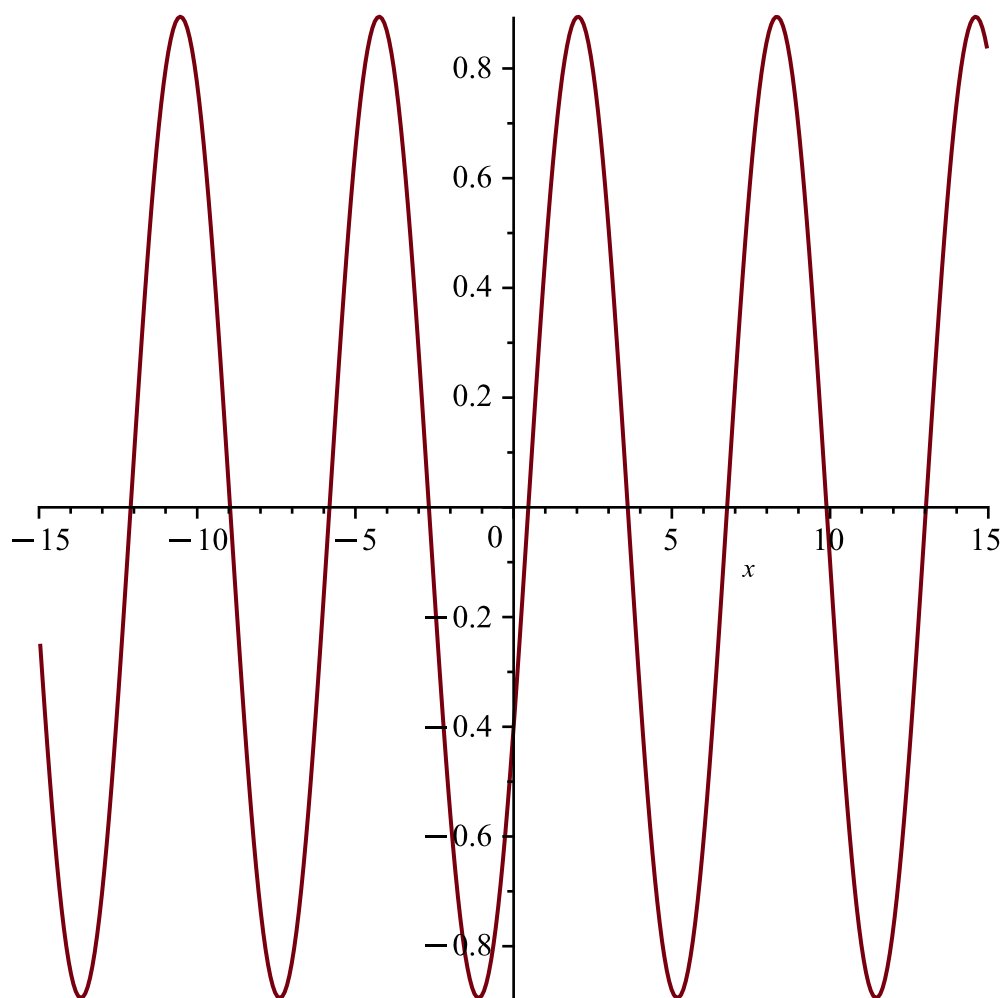
(67)

```
> y1:=unapply(rhs(sol1),x)
```

$$y1 := x \mapsto -\frac{2 \cdot \cos(x)}{5} + \frac{4 \cdot \sin(x)}{5}$$

(68)

```
> plot(y1(x),x=-15..15)
```



```
> restart: #ex 4:
```

```
> ecdif1:=diff(y(x),x)=aa*y(x)+b
```

$$ecdif1 := \frac{d}{dx} y(x) = aa y(x) + b \quad (69)$$

```
> sol2:=dsolve(ecdif1,y(x))
```

$$sol2 := y(x) = -\frac{b}{aa} + e^{aa x} c_1 \quad (70)$$

```
> y4:=unapply(rhs(sol2),x,c__1)
```

$$y4 := (x, c_1) \mapsto -\frac{b}{aa} + e^{aa \cdot x} \cdot c_1 \quad (71)$$

```
> cond_in:=y(0)=m
```

$$cond_in := y(0) = m \quad (72)$$

```
> sol21:=dsolve({ecdif1,cond_in},y(x))
```

$$sol21 := y(x) = \frac{(m aa + b) e^{aa x} - b}{aa} \quad (73)$$

```
> m:=5*b/2
```

$$m := \frac{5 b}{2} \quad (74)$$

```
> cond_in1:=y(0)=1
```

(75)

$$\text{cond_in1} := y(0) = 1 \quad (75)$$

> sol22:=dsolve({ecdif1,cond_in1},y(x))

$$\text{sol22} := y(x) = \frac{e^{aa \cdot x} (aa + b) - b}{aa} \quad (76)$$

> y42:=unapply(rhs(sol22),x)

$$y42 := x \mapsto \frac{e^{aa \cdot x} \cdot (aa + b) - b}{aa} \quad (77)$$

> e1:=y42(2)=2*exp(2)-1

$$e1 := \frac{e^{2aa} (aa + b) - b}{aa} = 2e^2 - 1 \quad (78)$$

> e2:=y42(3)=2*exp(3)-1

$$e2 := \frac{e^{3aa} (aa + b) - b}{aa} = 2e^3 - 1 \quad (79)$$

> sist:=e1,e2

$$\text{sist} := e1, e2 \quad (80)$$

> #de terminat

> restart: #ex 5:

> ecdif1:=diff(y(x),x\$2)-diff(y(x),x)-2*y(x)=0

$$\text{ecdif1} := \frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) - 2y(x) = 0 \quad (81)$$

> cond_in1:=y(0)=a

$$\text{cond_in1} := y(0) = a \quad (82)$$

> cond_in2:=D(y)(0)=2

$$\text{cond_in2} := D(y)(0) = 2 \quad (83)$$

> sol:=dsolve({ecdif1,cond_in1,cond_in2},y(x))

$$\text{sol} := y(x) = \frac{(-2 + 2a)e^{-x}}{3} + \frac{e^{2x}(a + 2)}{3} \quad (84)$$

> limit(sol,x=infinity)

$$\lim_{x \rightarrow \infty} y(x) = \text{signum}(a + 2) \infty \quad (85)$$

> a:=-2

$$a := -2 \quad (86)$$

> limit(sol,x=infinity)

$$\lim_{x \rightarrow \infty} y(x) = 0 \quad (87)$$

> restart:

> #ex 1:

> ec11:=diff(y(x),x\$2)+4*y(x)=1/cos(2*x)

$$\text{ec11} := \frac{d^2}{dx^2} y(x) + 4y(x) = \frac{1}{\cos(2x)} \quad (88)$$

> sol11:=dsolve(ec11,y(x))

$$(89)$$

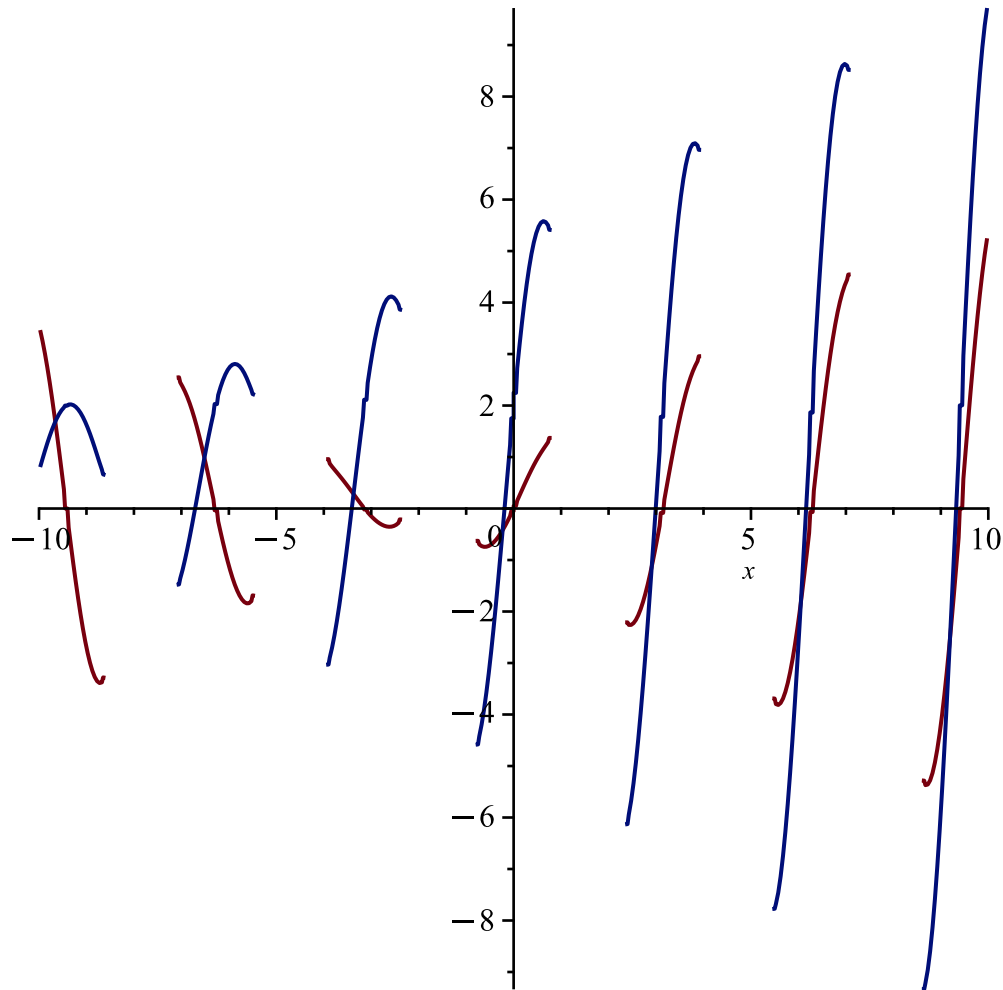
$$sol11 := y(x) = \sin(2x) c_2 + \cos(2x) c_1 + \frac{x \sin(2x)}{2} + \frac{\ln(\cos(2x)) \cos(2x)}{4} \quad (89)$$

> with (plots) :

> y11:=unapply(rhs(sol11),x,c__1,c__2)

$$y11 := (x, c_1, c_2) \mapsto \sin(2 \cdot x) \cdot c_2 + \cos(2 \cdot x) \cdot c_1 + \frac{x \cdot \sin(2 \cdot x)}{2} + \frac{\ln(\cos(2 \cdot x)) \cdot \cos(2 \cdot x)}{4} \quad (90)$$

> plot([y11(x,0,1),y11(x,2,5)],x=-10..10)



> ec12:=diff(y(x),x\$2)-diff(y(x),x)=1/(1+exp(x))

$$ec12 := \frac{d^2}{dx^2} y(x) - \frac{d}{dx} y(x) = \frac{1}{1 + e^x} \quad (91)$$

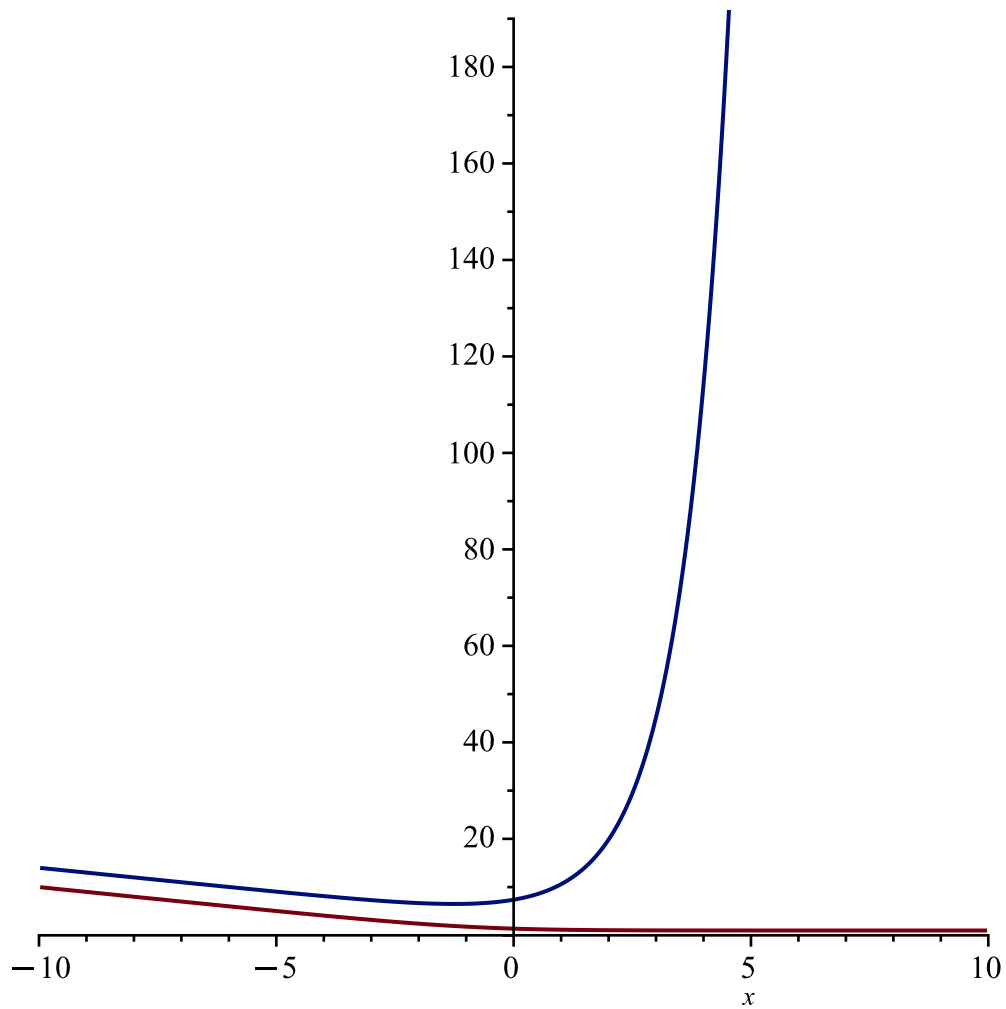
> sol12:=dsolve(ec12,y(x))

$$sol12 := y(x) = -x + e^x c_1 + \ln(1 + e^x) (1 + e^x) - 1 - e^x \ln(e^x) + c_2 \quad (92)$$

> y12:=unapply(rhs(sol12),x,c__1,c__2)

$$y12 := (x, c_1, c_2) \mapsto -x + e^x \cdot c_1 + \ln(1 + e^x) \cdot (1 + e^x) - 1 - e^x \cdot \ln(e^x) + c_2 \quad (93)$$

> plot([y12(x,0,1),y12(x,2,5)],x=-10..10)



```

> #ex 2:
> restart:
> with(DEtools): with(plots):
> ecdif3:=diff(y(x),x)-2*y(x)=-x^2

```

$$ecdif3 := \frac{d}{dx} y(x) - 2 y(x) = -x^2 \quad (94)$$

```

> cond_in3:=y(0)=1/4

```

$$cond_in3 := y(0) = \frac{1}{4} \quad (95)$$

```

> sol3:=dsolve({ecdif3,cond_in3},y(x))

```

$$sol3 := y(x) = \frac{1}{2} x^2 + \frac{1}{2} x + \frac{1}{4} \quad (96)$$

```

> y3:=unapply(rhs(sol3),x)

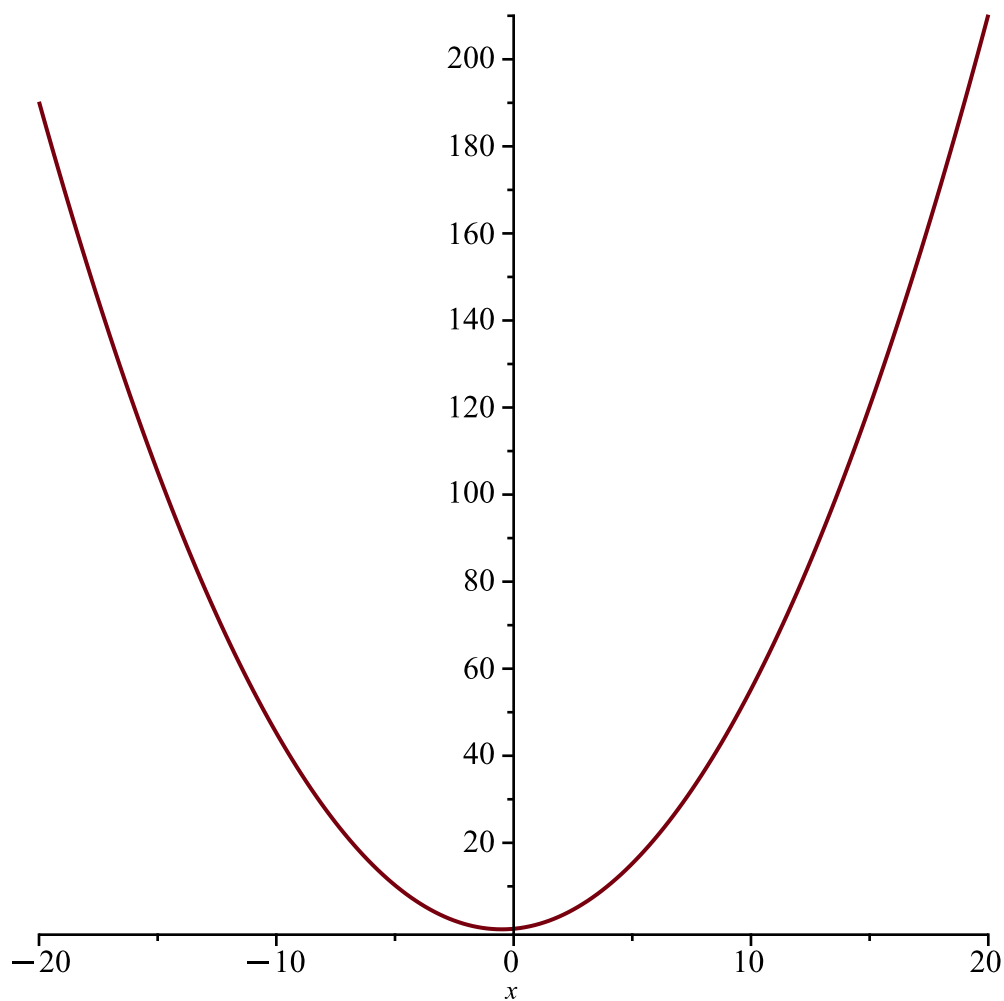
```

$$y3 := x \mapsto \frac{1}{2} \cdot x^2 + \frac{1}{2} \cdot x + \frac{1}{4} \quad (97)$$

```

> plot(y3(x),x=-20..20)

```



```
> ecdif6:=diff(y(x),x$2)+4*y(x)=4*(sin(2*x)+cos(2*x))
```

$$ecdif6 := \frac{d^2}{dx^2} y(x) + 4 y(x) = 4 \sin(2x) + 4 \cos(2x) \quad (98)$$

```
> cond_in1:=y(Pi)=2*Pi
```

$$cond_in1 := y(\pi) = 2\pi \quad (99)$$

```
> cond_in2:=D(y)(Pi)=2*Pi
```

$$cond_in2 := D(y)(\pi) = 2\pi \quad (100)$$

```
> sol6:=dsolve({ecdif6,cond_in1,cond_in2},y(x))
```

$$sol6 := y(x) = (-x + 3\pi) \cos(2x) + \frac{\sin(2x)(2x + 1)}{2} \quad (101)$$

```
> y6:=unapply(rhs(sol6),x)
```

$$y6 := x \mapsto (-x + 3\pi) \cdot \cos(2x) + \frac{\sin(2x) \cdot (2x + 1)}{2} \quad (102)$$

```
> plot(y6(x),x=-25..25)
```

