Laborator 3

1. Considerăm curba Bézier cubică dată de punctele de control $\mathbf{b}_0(-3,1)$, $\mathbf{b}_1(-4,4)$, $\mathbf{b}_2(4,4)$, $\mathbf{b}_3(3,1)$.

(i) Determinaţi punctele de control ale curbei, privită ca o curbă de gradul 4.

(ii) Reprezentaţi grafic (cu gnuplot, geogebra, maxima, matplotlib, ...) pe acelaşi sistem de axe: curba, punctele de control şi poligonul de control ale curbei privite ca o curbă de gradul 3, precum şi punctele de control şi poligonul de control ale curbei, privite ca o curbă de gradul 4.

(i) $c_0 = b_0$

$$c_k = \frac{m+1-k}{m+1} b_k + \frac{k}{m+1} b_{k-1}, \quad k = 1 \dots m$$

$$c_{m+1} = b_m$$

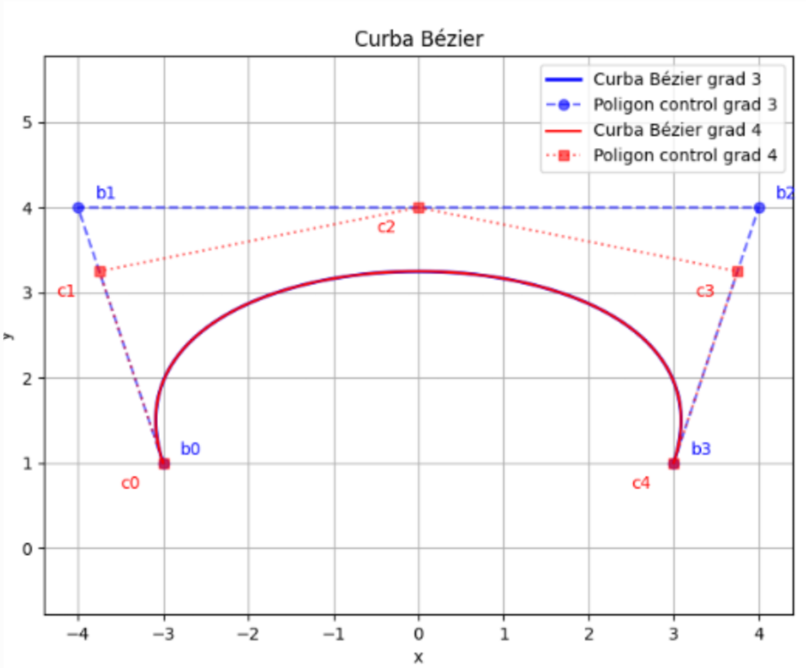
Pt. $m = 3 \Rightarrow c_0 = b_0 \Rightarrow c_0(-3,1)$

$$c_1 = \frac{3}{4} b_1 + \frac{1}{4} b_0 \Rightarrow c_1\left(-\frac{15}{4}, \frac{13}{4}\right)$$

$$c_2 = \frac{2}{4} b_2 + \frac{2}{4} b_1 \Rightarrow c_2(0,4)$$

$$c_3 = \frac{1}{4} b_3 + \frac{3}{4} b_2 \Rightarrow c_3\left(\frac{15}{4}, \frac{13}{4}\right)$$

$$c_4 = b_3 \Rightarrow c_4(3,1)$$



2. Considerăm curba Bézier de gradul 3 de la problema precedentă.

(i) Divizaţi curba la valoarea $1/3$ a parametrului, determinând punctele de control şi parametrizările pe intervalul $[0, 1]$ pentru fiecare dintre cele două segmente.

(ii) Reprezentaţi grafic (cu gnuplot, geogebra, maxima, matplotlib, …), pe acelaşi sistem de axe: cele două segmente de curbă (cu culori diferite), punctele de control şi poligonul de control pentru curba originală, precum şi punctele de control şi poligoanele de control pentru cele două segmente obţinute prin divizare.

(i) Calc. pct. intermediare. $t = \frac{1}{3}$, $1-t = \frac{2}{3}$.

Nivel 1
$$b_0^1 = (1-t)\,b_0^0 + t\,b_1^0 = \frac{2}{3} b_0^0 + \frac{1}{3} b_1^0 = \left(-\frac{10}{3}, 2\right)$$
$$b_1^1 = \frac{2}{3} b_1^0 + \frac{1}{3} b_2^0 = \left(-\frac{4}{3}, 4\right)$$
$$b_2^1 = \frac{2}{3} b_2^0 + \frac{1}{3} b_3^0 = \left(\frac{11}{3}, 3\right)$$

Nivel 2:
$$b_0^2 = (1-t)\,b_0^1 + t\,b_1^1 = \frac{2}{3} b_0^1 + \frac{1}{3} b_1^0 = \left(-\frac{8}{3}, \frac{8}{3}\right)$$
$$b_1^2 = \frac{2}{3} b_1^1 + \frac{1}{3} b_2^1 = \left(\frac{1}{3}, \frac{11}{3}\right)$$

Nivel 3:
$$b_0^3 = (1-t)\,b_0^2 + t\,b_1^2 = \frac{2}{3} b_0^2 + \frac{1}{3} b_1^2 = \left(-\frac{5}{3}, 3\right)$$

$\Rightarrow$ Pt. $[a,b] = [0, \frac{1}{3}]$

$c_0 = b_0 \Rightarrow c_0(-3, 1)$

$c_1 = b_0^1 \Rightarrow c_1\left(-\frac{10}{3}, 2\right)$

$c_2 = b_0^2 \Rightarrow c_2\left(-\frac{8}{3}, \frac{8}{3}\right)$

$c_3 = b_0^3 \Rightarrow c_3\left(-\frac{5}{3}, 3\right)$

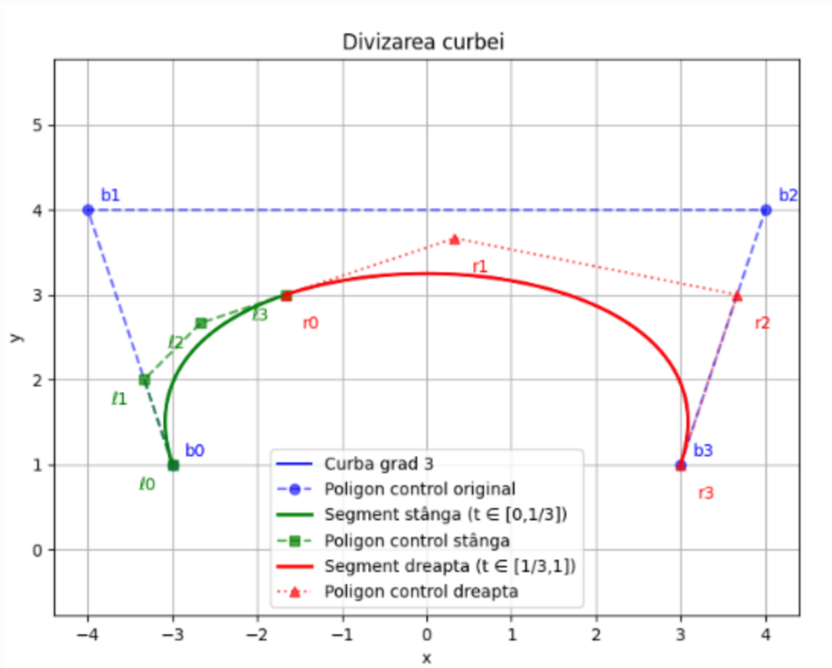
Pt. $[a,b] = [\frac{1}{3}, 1]$

$c_0 = b_0^3 \Rightarrow c_0 = \left(-\frac{5}{3}, 3\right)$

$c_1 = b_1^2 \Rightarrow c_1 = \left(\frac{1}{3}, \frac{11}{3}\right)$

$c_2 = b_2^1 \Rightarrow c_2\left(\frac{11}{3}, 3\right)$

$c_3 = b_3^0 \Rightarrow c_3(3, 1)$



V2)

$$c_k = C(a)^{n-k}\, C(b)^k \begin{pmatrix} b_0 \\ \vdots \\ b_n \end{pmatrix}$$

Particularizând formula pe datele problemei:

$n=3$, $b_0(-3,1)$, $b_1(-4,4)$, $b_2(4,4)$, $b_3(3,1)$, $[a,b]=[0,\frac{1}{3}]$, resp. $[a,b]=[\frac{1}{3},1]$

$$c_0 = C(a)^3 C(b)^0 \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(a)C_2(a)C_3(a)\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$c_1 = C(a)^2 C(b)^1 \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(a)C_2(a)C_3(b)\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$$c_2 = C(a)^1 C(b)^2 \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(a)C_2(b)C_3(b)\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$c_3 = C(a)^0 C(b)^3 \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = C_1(b)C_2(b)C_3(b)\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$C_1(a) = \begin{bmatrix} 1-a & a \end{bmatrix}$$

$$C_2(a) = \begin{bmatrix} 1-a & a & 0 \\ 0 & 1-a & a \end{bmatrix}$$

$$C_3(a) = \begin{bmatrix} 1-a & a & 0 & 0 \\ 0 & 1-a & a & 0 \\ 0 & 0 & 1-a & a \end{bmatrix}$$

$$C_3(b) = \begin{bmatrix} 1-b & b & 0 & 0 \\ 0 & 1-b & b & 0 \\ 0 & 0 & 1-b & b \end{bmatrix}$$

$$C_2(b) = \begin{bmatrix} 1-b & b & 0 \\ 0 & 1-b & b \end{bmatrix}$$

$$C_1(b) = \begin{bmatrix} 1-b & b \end{bmatrix}$$

Efectuând calculele se obține:

pt. $[a,b]=[0,\frac{1}{3}]$: $c_0(-3,1)$, $c_1(-\frac{10}{3},2)$, $c_2(-\frac{8}{3},\frac{8}{3})$, $c_3(-\frac{5}{3},3)$

pt. $[a,b]=[\frac{1}{3},1]$: $c_0(-\frac{5}{3},3)$, $c_1(\frac{1}{3},\frac{11}{3})$, $c_2(\frac{11}{3},3)$, $c_3(3,1)$