```
> #model1,ex 1:
> ec1:=diff(y(x),x)-y(x)/x=m*x
```

$$ec1 := \frac{d}{dx}\, y(x) - \frac{y(x)}{x} = m\,x \quad (1)$$

```
> dsolve(ec1,y(x))
```

$$y(x) = \left(m\,x + c_1\right) x \quad (2)$$

```
> cond_in1:=y(1)=1
```

$$cond\_in1 := y(1) = 1 \quad (3)$$

```
> sist:=ec1,cond_in1
```

$$sist := \frac{d}{dx}\, y(x) - \frac{y(x)}{x} = m\,x,\ y(1) = 1 \quad (4)$$

```
> sol2:=dsolve({sist},y(x))
```

$$sol2 := y(x) = x\,(1 + (x-1)\,m) \quad (5)$$

```
> # 0 = 2*(1+ (2-1)*m) => 2+2*m=0 => m = -1
> m:=-1
```

$$m := -1 \quad (6)$$

```
> sol2:=dsolve({sist},y(x))
```

$$sol2 := y(x) = -x\,(-2 + x) \quad (7)$$

```
> expand(sol2)
```

$$y(x) = -x^2 + 2\,x \quad (8)$$

```
> #ex2:
> restart
> ecdif:=x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0
```

$$ecdif := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) + 3\,x \left(\frac{d}{dx}\, y(x)\right) + y(x) = 0 \quad (9)$$

```
> sol:=dsolve(ecdif)
```

$$sol := y(x) = \frac{c_1}{x} + \frac{c_2 \ln(x)}{x} \quad (10)$$

```
> with(plots):
> cond_in:=y(1)=1,D(y)(1)=1
```

$$cond\_in := y(1) = 1,\ \mathrm{D}(y)(1) = 1 \quad (11)$$

```
> sist:=ecdif,cond_in
```

$$sist := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) + 3\,x \left(\frac{d}{dx}\, y(x)\right) + y(x) = 0,\ y(1) = 1,\ \mathrm{D}(y)(1) = 1 \quad (12)$$

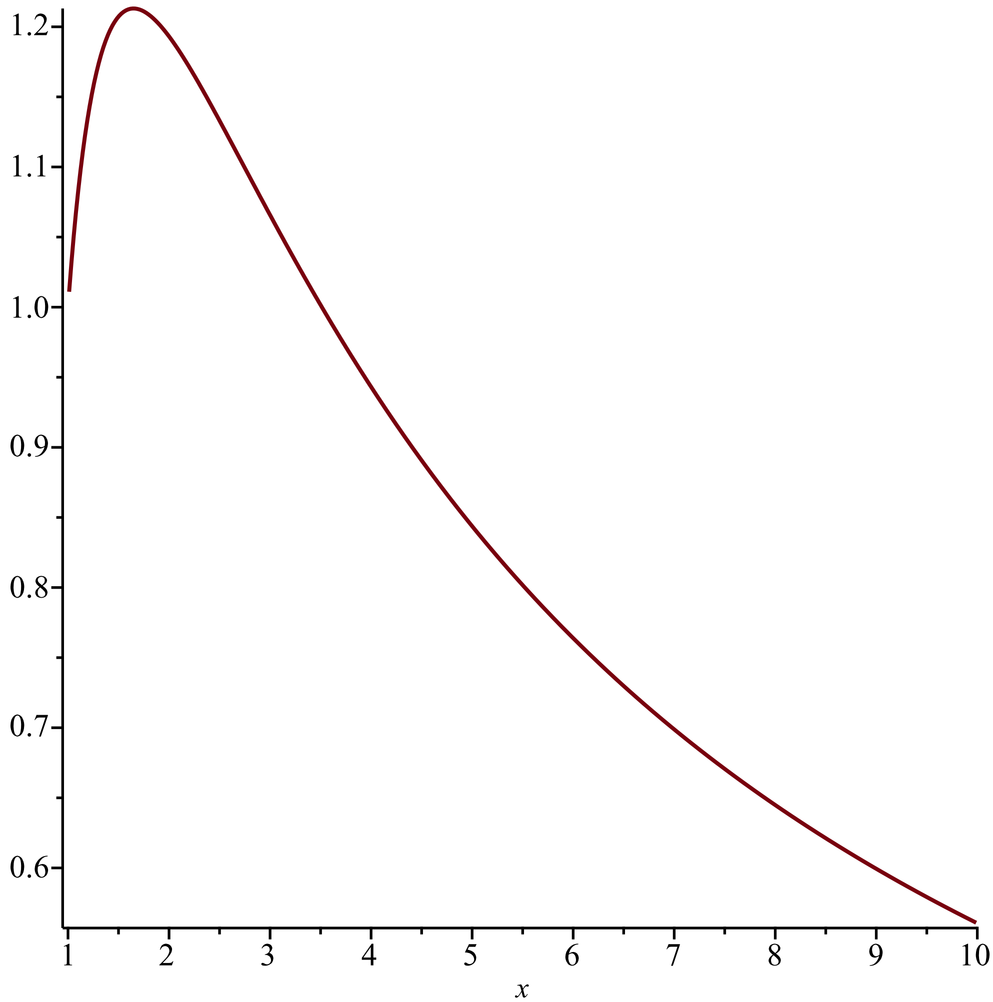
```
> sol2:=dsolve({sist},y(x))
```

$$sol2 := y(x) = \frac{1 + 2\ln(x)}{x} \quad (13)$$

```
> y2:=unapply(rhs(sol2),x)
```

$$y2 := x \mapsto \frac{1 + 2\cdot\ln(x)}{x} \quad (14)$$

```
> plot(y2(x),x=1..10)
```




```
> #ex3:
> restart: with(plots): with(DEtools):
> f__1:=x->x*(x+1)*(2-x)
```

$$f_1 := x \mapsto x \cdot (x+1) \cdot (2-x) \quad \textbf{(15)}$$

```
> ecdif:=diff(x(t),t)=f__1(x(t))
```

$$ecdif := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = x(t)\ (x(t)+1)\ (2-x(t)) \quad \textbf{(16)}$$

```
> pct_ech:=solve(f__1(x)=0,x)
```

$$pct\_ech := -1, 0, 2 \quad \textbf{(17)}$$

```
> D(f__1)(pct_ech[1])
```

$$-3 \quad \textbf{(18)}$$

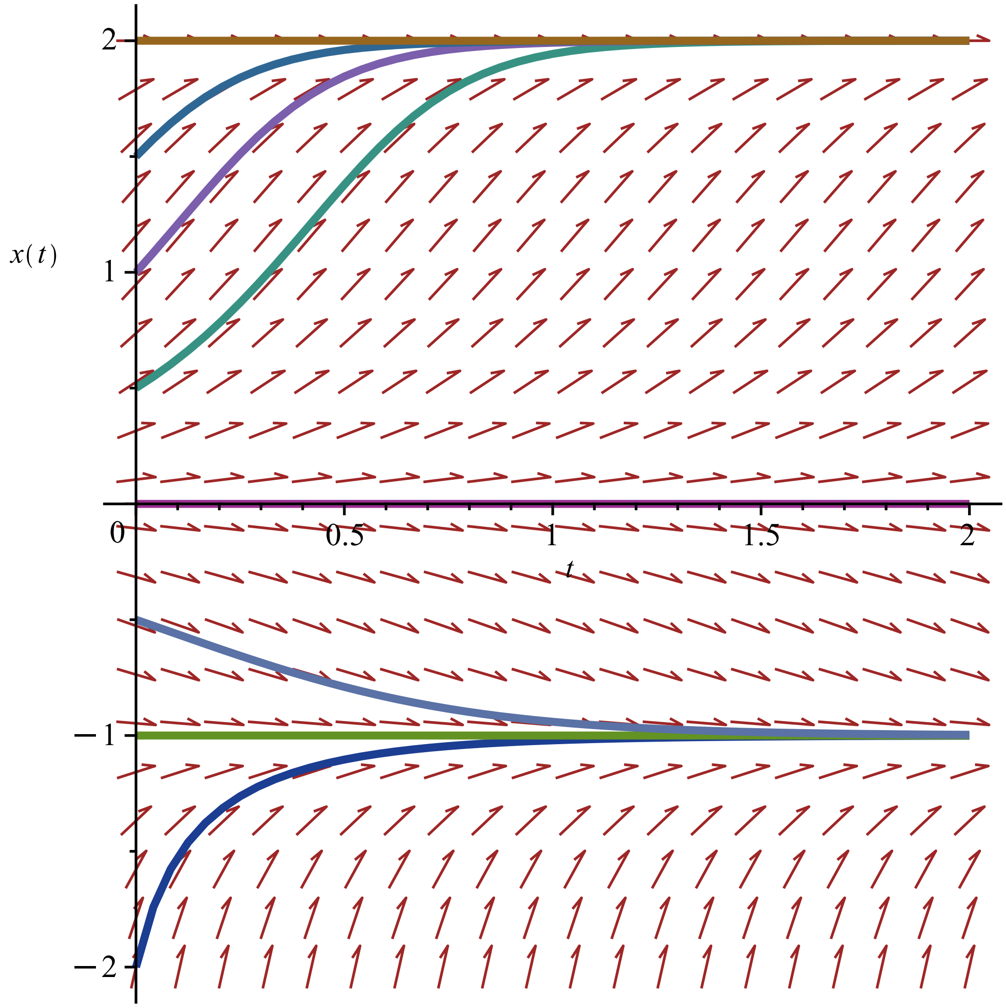
```
> D(f__1)(pct_ech[2])
```

$$2 \quad \textbf{(19)}$$

```
> D(f__1)(pct_ech[3])
```

$$-6 \quad \textbf{(20)}$$

```
> DEplot(ecdif,x(t), t=0..2, [[x(0)=-2],[x(0)=-1],[x(0)=-0.5],[x(0)
  =0],[x(0)=0.5],[x(0)=1],[x(0)=3/2],[x(0)=2]])
```




```
> restart: with(plots): with(DEtools):
> #ex4:
> ec1:=diff(y__1(t),t)=-7*y__1(t)-6*y__2(t)
```

$$ec1 := \frac{d}{dt}\, y_1(t) = -7\, y_1(t) - 6\, y_2(t) \quad \textbf{(21)}$$

```
> ec2:=diff(y__2(t),t)=12*y__1(t)+10*y__2(t)
```

$$ec2 := \frac{d}{dt}\, y_2(t) = 12\, y_1(t) + 10\, y_2(t) \quad \textbf{(22)}$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt}\, y_1(t) = -7\, y_1(t) - 6\, y_2(t),\ \frac{d}{dt}\, y_2(t) = 12\, y_1(t) + 10\, y_2(t) \quad \textbf{(23)}$$

```
> sol:=dsolve({sist},{y__1(t),y__2(t)})
```

$$sol := \left\{ y_1(t) = c_1\, e^{2t} + c_2\, e^{t},\ y_2(t) = -\frac{3\, c_1\, e^{2t}}{2} - \frac{4\, c_2\, e^{t}}{3} \right\} \quad \textbf{(24)}$$

```
> cond_in:=y__1(0)=2,y__2(0)=4
```

$$cond\_in := y_1(0) = 2,\ y_2(0) = 4 \quad \textbf{(25)}$$

```
> sist2:={ec1,ec2,cond_in}
```

$$sist2 := \left\{ \frac{d}{dt}\, y_1(t) = -7\, y_1(t) - 6\, y_2(t),\ \frac{d}{dt}\, y_2(t) = 12\, y_1(t) + 10\, y_2(t),\ y_1(0) = 2,\ y_2(0) = 4 \right\} \quad \textbf{(26)}$$

```
> sol2:=dsolve(sist2,{y__1(t),y__2(t)})
```
$$sol2 := \{y_1(t) = -40\,e^{2t} + 42\,e^{t}, y_2(t) = 60\,e^{2t} - 56\,e^{t}\} \quad (27)$$

```
> restart
> #model2,ex1:
> ec:=x*diff(y(x),x)=m*x^2+y(x)
```
$$ec := x\left(\frac{d}{dx}\,y(x)\right) = m\,x^2 + y(x) \quad (28)$$

```
> sol1:=dsolve(ec,y(x))
```
$$sol1 := y(x) = (m\,x + c_1)\,x \quad (29)$$

```
> cond_in:=y(1)=2
```
$$cond\_in := y(1) = 2 \quad (30)$$

```
> sol2:=dsolve({ec,cond_in},y(x))
```
$$sol2 := y(x) = (2 + (x-1)\,m)\,x \quad (31)$$

```
> #pt A(3,1) => 1=(2+2m)*3 => 1=6+6m =>m=-5/6
> m:=-5/6
```
$$m := -\frac{5}{6} \quad (32)$$

```
> f:=x->x*(17-5*x)/6
```
$$f := x \mapsto \frac{x\cdot(17 - 5\cdot x)}{6} \quad (33)$$

```
> sol3:=solve(f(x)=0,x)
```
$$sol3 := 0, \frac{17}{5} \quad (34)$$

```
> #ex2:
> restart: with(plots):
> ec:=x^2*diff(y(x),x$2)-2*x*diff(y(x),x)+2*y(x)=0
```
$$ec := x^2\left(\frac{d^2}{dx^2}\,y(x)\right) - 2\,x\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0 \quad (35)$$

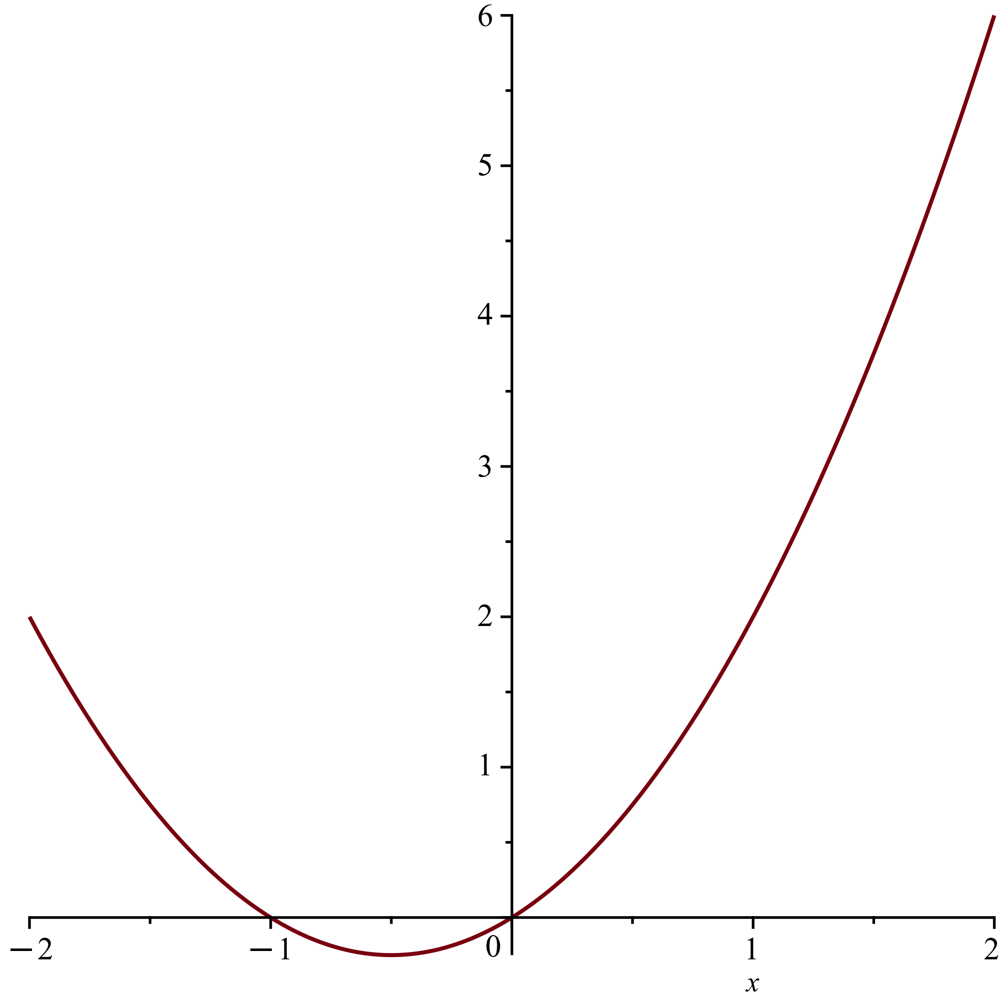
```
> sol:=dsolve(ec,y(x))
```
$$sol := y(x) = c_2\,x^2 + c_1\,x \quad (36)$$

```
> cond_in:=y(1)=2,D(y)(1)=3
```
$$cond\_in := y(1) = 2, D(y)(1) = 3 \quad (37)$$

```
> sol2:=dsolve({ec,cond_in},y(x))
```
$$sol2 := y(x) = x^2 + x \quad (38)$$

```
> y2:=unapply(rhs(sol2),x)
```
$$y2 := x \mapsto x^2 + x \quad (39)$$

```
> plot(y2(x),x=-2..2)
```




```
> #ex3:
> restart: with(DEtools):
> f__1:=x->-x^3+x^2+2*x
```

$$f_1 := x \mapsto -x^3 + x^2 + 2 \cdot x \quad \textbf{(40)}$$

```
> ec:=diff(x(t),t)=f__1(x(t))
```

$$ec := \frac{d}{dt}\, x(t) = -x(t)^3 + x(t)^2 + 2\, x(t) \quad \textbf{(41)}$$

```
> pct_ech:=solve(f__1(x),x)
```

$$pct\_ech := 0, 2, -1 \quad \textbf{(42)}$$

```
> D(f__1)(pct_ech[1])
```

$$2 \quad \textbf{(43)}$$

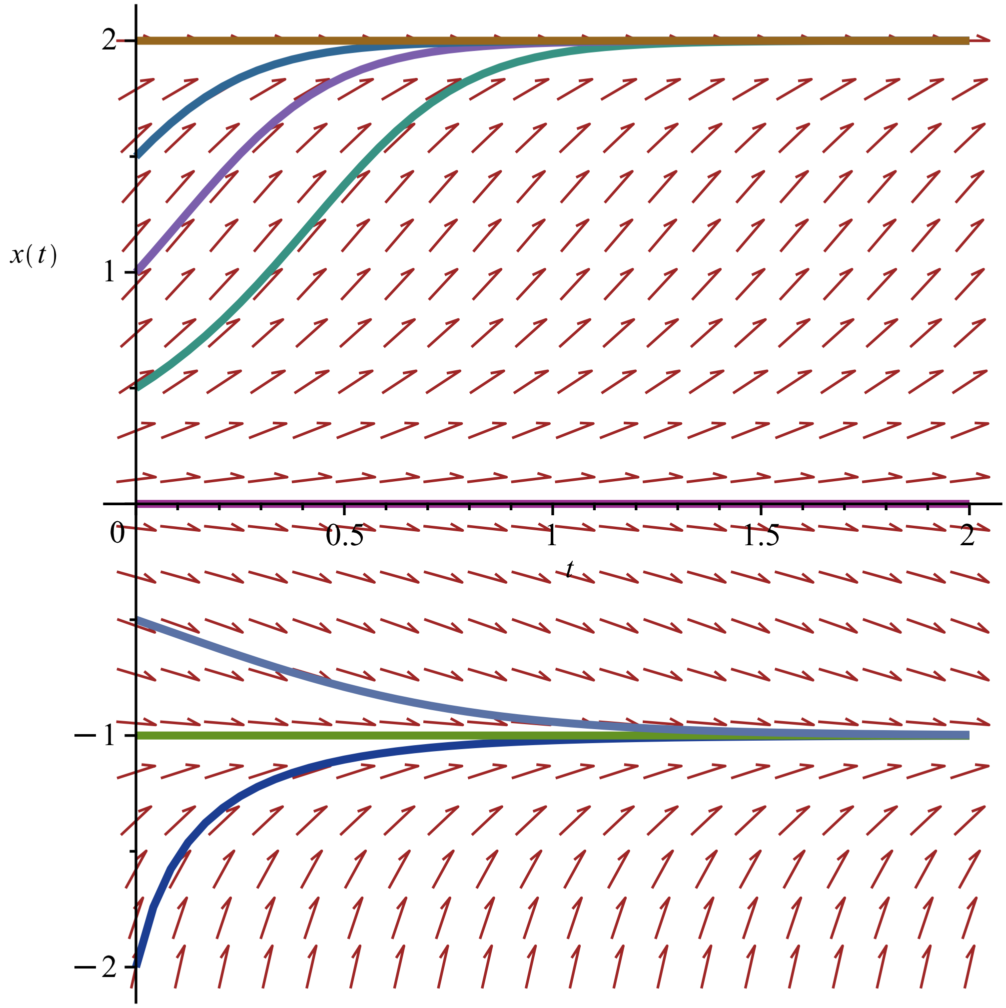
```
> D(f__1)(pct_ech[2])
```

$$-6 \quad \textbf{(44)}$$

```
> D(f__1)(pct_ech[3])
```

$$-3 \quad \textbf{(45)}$$

```
> # pct 0 e instabil, restul local as stabile
> DEplot(ec,x(t),t=0..2, [[x(0)=-2],[x(0)=-1],[x(0)=-0.5],[x(0)=0],
  [x(0)=0.5],[x(0)=1],[x(0)=3/2],[x(0)=2]])
```




```
> #ex4:
> restart
> ec1:=diff(y__1(x),x)=9*y__1(x)+21*y__2(x)
```

$$ec1 := \frac{d}{dx}\, y_1(x) = 9\, y_1(x) + 21\, y_2(x) \quad (46)$$

```
> ec2:=diff(y__2(x),x)=-2*y__1(x)-4*y__2(x)
```

$$ec2 := \frac{d}{dx}\, y_2(x) = -2\, y_1(x) - 4\, y_2(x) \quad (47)$$

```
> sol:=dsolve({ec1,ec2},{y__1(x),y__2(x)})
```

$$sol := \left\{ y_1(x) = c_1\, e^{3x} + c_2\, e^{2x},\, y_2(x) = -\frac{2\, c_1\, e^{3x}}{7} - \frac{c_2\, e^{2x}}{3} \right\} \quad (48)$$

```
> cond_in:=y__1(0)=2,y__2(0)=5
```

$$cond\_in := y_1(0) = 2,\, y_2(0) = 5 \quad (49)$$

```
> sol2:=dsolve({ec1,ec2,cond_in},{y__1(x),y__2(x)})
```

$$sol2 := \left\{ y_1(x) = 119\, e^{3x} - 117\, e^{2x},\, y_2(x) = -34\, e^{3x} + 39\, e^{2x} \right\} \quad (50)$$

```
> #restart
> #model3,ex1:
> ecdif:=diff(T(t),t)=-k*(T(t)-T__m)
```

$$ecdif := \frac{d}{dt}\, T(t) = -k\left(T(t) - T_m\right) \tag{51}$$

```
> cond_in:=T(0)=T__0
```

$$cond\_in := T(0) = T_0 \tag{52}$$

```
> sol:=dsolve({ecdif,cond_in},T(t))
```

$$sol := T(t) = T_m + e^{-k t}\left(T_0 - T_m\right) \tag{53}$$

```
> cond_in2:=T__m=5,T__0=40,T__1=10
```

$$cond\_in2 := T_m = 5, T_0 = 40, T_1 = 10 \tag{54}$$

```
> TT:=unapply(rhs(sol),t,T__m,T__0,k)
```

$$TT := \left(t, T_m, T_0, k\right) \mapsto T_m + e^{-k\cdot t}\cdot\left(T_0 - T_m\right) \tag{55}$$

```
> ec1:=TT(t,5,40,k)=10
```

$$ec1 := 5 + 35\, e^{-k t} = 10 \tag{56}$$

```
> k:=solve(ec1,k)
```

$$k := \frac{\ln(7)}{t} \tag{57}$$

```
> # ???
> #ex2:
> restart
> ec1:=diff(x(t),t)=4*x(t)+6*y(t)
```

$$ec1 := \frac{d}{dt}\, x(t) = 4\, x(t) + 6\, y(t) \tag{58}$$

```
> ec2:=diff(y(t),t)=2*x(t)+3*y(t)
```

$$ec2 := \frac{d}{dt}\, y(t) = 2\, x(t) + 3\, y(t) \tag{59}$$

```
> sol:=dsolve({ec1,ec2},{x(t),y(t)})
```

$$sol := \left\{x(t) = c_1 + c_2\, e^{7 t}, y(t) = \frac{c_2\, e^{7 t}}{2} - \frac{2\, c_1}{3}\right\} \tag{60}$$

```
> cond_in:=x(0)=2,y(0)=2
```

$$cond\_in := x(0) = 2, y(0) = 2 \tag{61}$$

```
> sol2:=dsolve({ec1,ec2,cond_in},{x(t),y(t)})
```

$$sol2 := \left\{x(t) = -\frac{6}{7} + \frac{20\, e^{7 t}}{7}, y(t) = \frac{10\, e^{7 t}}{7} + \frac{4}{7}\right\} \tag{62}$$

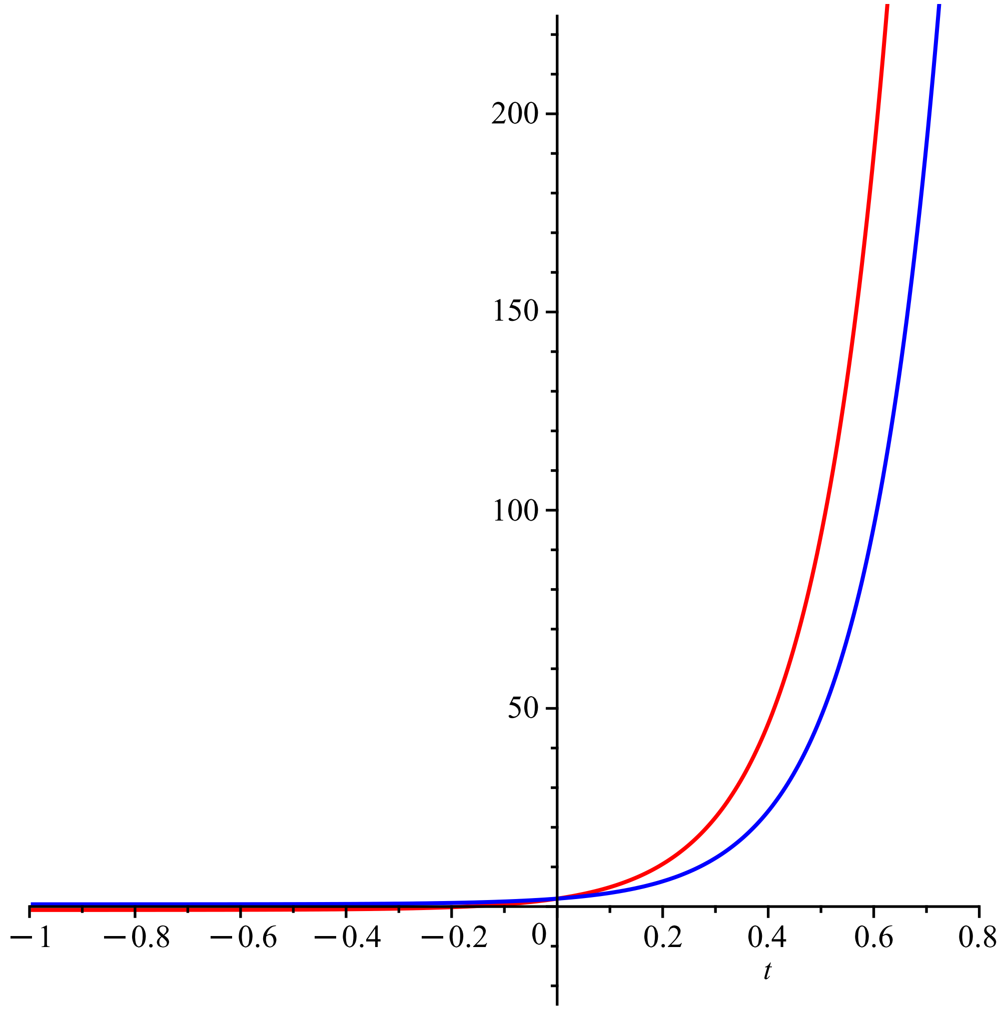
```
> with(plots):
> xx:=unapply(rhs(sol2[1]),x)
```

$$xx := x \mapsto -\frac{6}{7} + \frac{20\cdot e^{7\cdot t}}{7} \tag{63}$$

```
> yy:=unapply(rhs(sol2[2]),y)
```

$$yy := y \mapsto \frac{10\cdot e^{7\cdot t}}{7} + \frac{4}{7} \tag{64}$$

```
> plot([xx(t),yy(t)],t=-1..1,color=[red,blue])
```




```
> #ex3:
> restart
> f__1:=x->x*(2-x)*(x-4)
```

$$f_1 := x \mapsto x \cdot (2 - x) \cdot (x - 4) \tag{65}$$

```
> ec:=diff(x(t),t)=f__1(x(t))
```

$$ec := \frac{d}{dt}\, x(t) = x(t)\, (2 - x(t))\, (x(t) - 4) \tag{66}$$

```
> pct_ech:=solve(f__1(x),x)
```

$$pct\_ech := 0, 2, 4 \tag{67}$$

```
> D(f__1)(pct_ech[1])
```

$$-8 \tag{68}$$

```
> #local stabil as
> D(f__1)(pct_ech[2])
```

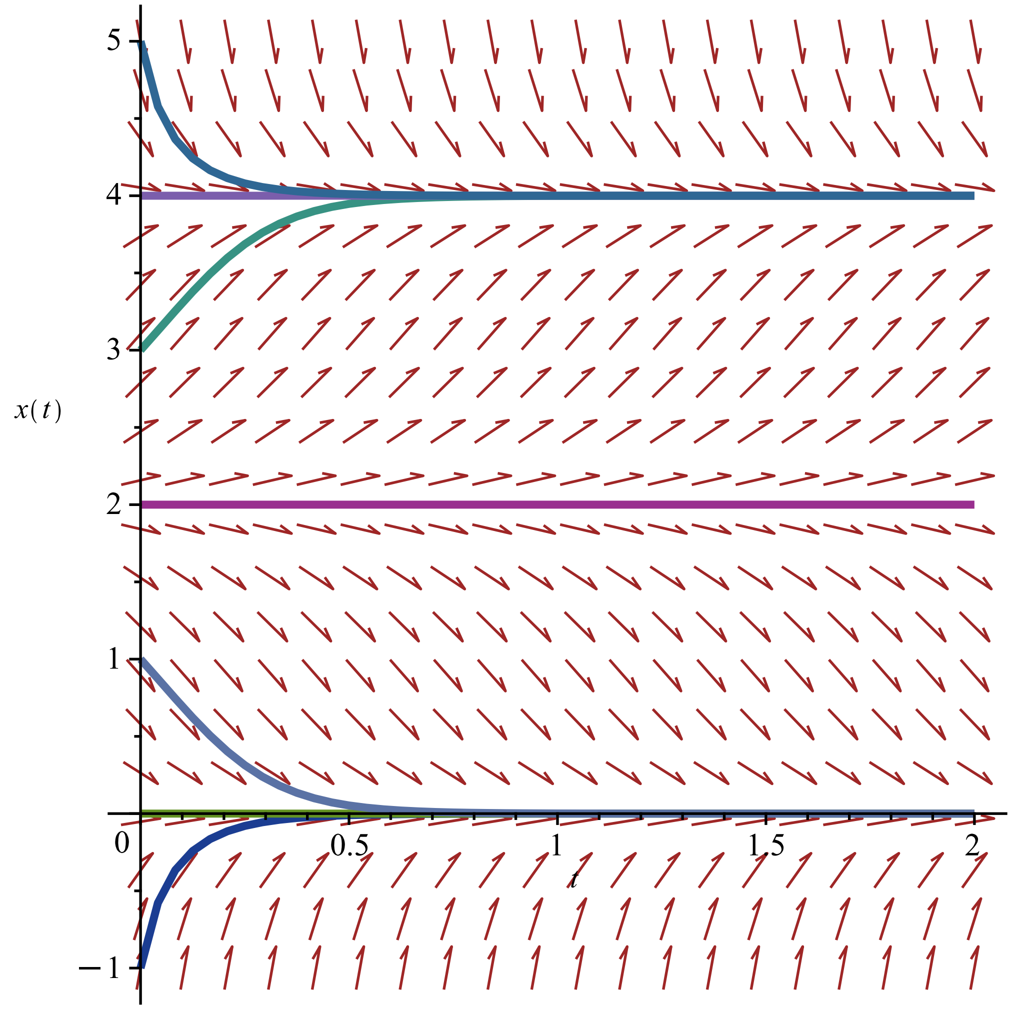
$$4 \tag{69}$$

```
> #instabil
> D(f__1)(pct_ech[3])
```

$$-8 \tag{70}$$

```
> #local stabil as
> with(DEtools):
```

```
> DEplot(ec,x(t),t=0..2,[[x(0)=-1],[x(0)=0],[x(0)=1],[x(0)=2],[x(0)=3],[x(0)=4],[x(0)=5]])
```




```
> #ex4:
> restart: with(linalg):
> f__1:=(x,y)->x*y-1
```

$$f_1 := (x, y) \mapsto x \cdot y - 1 \tag{71}$$

```
> f__2:=(x,y)->x^2-16*y^2
```

$$f_2 := (x, y) \mapsto x^2 - 16 \cdot y^2 \tag{72}$$

```
> ec1:=diff(x(t),t)=f__1(x(t),y(t))
```

$$ec1 := \frac{d}{dt} x(t) = x(t)\, y(t) - 1 \tag{73}$$

```
> ec2:=diff(y(t),t)=f__2(x(t),y(t))
```

$$ec2 := \frac{d}{dt} y(t) = x(t)^2 - 16\, y(t)^2 \tag{74}$$

```
> sist:={ec1,ec2}
```

$$sist := \left\{ \frac{d}{dt} x(t) = x(t)\, y(t) - 1, \frac{d}{dt} y(t) = x(t)^2 - 16\, y(t)^2 \right\} \tag{75}$$

```
> pct_ech:=solve({f__1(x,y)=0,f__2(x,y)=0},{x,y})
```

(76)

$$pct\_ech := \left\{x = -2\, RootOf(\_Z^2+1),\, y = \frac{RootOf(\_Z^2+1)}{2}\right\}, \left\{x = 2, y = \frac{1}{2}\right\}, \left\{x = -2, y = -\frac{1}{2}\right\} \quad (76)$$

```
> J:=jacobian([f__1(x,y),f__2(x,y)],[x,y])
```

$$J := \begin{bmatrix} y & x \\ 2\,x & -32\,y \end{bmatrix} \quad (77)$$

```
> A:=subs(pct_ech[1,1],pct_ech[1,2],eval(J))
```

$$A := \begin{bmatrix} \frac{RootOf(\_Z^2+1)}{2} & -2\, RootOf(\_Z^2+1) \\ -4\, RootOf(\_Z^2+1) & -16\, RootOf(\_Z^2+1) \end{bmatrix} \quad (78)$$

```
> eigenvals(A)
```

$$RootOf(31\, RootOf(\_Z^2+1)\,\_Z + 2\,\_Z^2 + 32) \quad (79)$$

```
> B:=subs(pct_ech[2,1],pct_ech[2,2],eval(J))
```

$$B := \begin{bmatrix} \frac{1}{2} & 2 \\ 4 & -16 \end{bmatrix} \quad (80)$$

```
> eigenvals(B)
```

$$-\frac{31}{4} + \frac{\sqrt{1217}}{4}, -\frac{31}{4} - \frac{\sqrt{1217}}{4} \quad (81)$$

```
> C:=subs(pct_ech[3,1],pct_ech[3,2],eval(J))
```

$$C := \begin{bmatrix} -\frac{1}{2} & -2 \\ -4 & 16 \end{bmatrix} \quad (82)$$

```
> eigenvals(C)
```

$$\frac{31}{4} + \frac{\sqrt{1217}}{4}, \frac{31}{4} - \frac{\sqrt{1217}}{4} \quad (83)$$

```
> # tip puncte
  rootof ???
```

```
> restart
> #model4,ex1:
> ec:=diff(y(x),x)-y(x)/x=m*x
```

$$ec := \frac{d}{dx}\, y(x) - \frac{y(x)}{x} = m\,x \quad (84)$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = (m\,x + c_1)\,x \quad (85)$$

```
> cond_in:=y(1)=1
```

$$cond\_in := y(1) = 1 \quad (86)$$

```
> sol2:=dsolve({ec,cond_in},y(x))
```

$$sol2 := y(x) = (1 + (x - 1)\, m)\, x \quad (87)$$

```
> #A(2,0)
> f:=(x,m)->x*(1+m*(x-1))
```

$$f := (x, m) \mapsto x \cdot (1 + m \cdot (x - 1)) \quad (88)$$

```
> m__rez:=solve(f(2,m)=0,m)
```

$$m_{rez} := -1 \quad (89)$$

```
> m:=-1
```

$$m := -1 \quad (90)$$

```
> f__2:=x->(2-x)*x
```

$$f_2 := x \mapsto (2 - x) \cdot x \quad (91)$$

```
> sol__f:=solve(f__2(x)=0,x)
```

$$sol_f := 0, 2 \quad (92)$$

```
> #ex2:
> restart
> ec:=x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0
```

$$ec := x^2 \left(\frac{d^2}{dx^2}\, y(x) \right) + 3\, x \left(\frac{d}{dx}\, y(x) \right) + y(x) = 0 \quad (93)$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = \frac{c_1}{x} + \frac{c_2 \ln(x)}{x} \quad (94)$$

```
> cond_in:=y(1)=1,D(y)(1)=1
```

$$cond\_in := y(1) = 1, \mathrm{D}(y)(1) = 1 \quad (95)$$

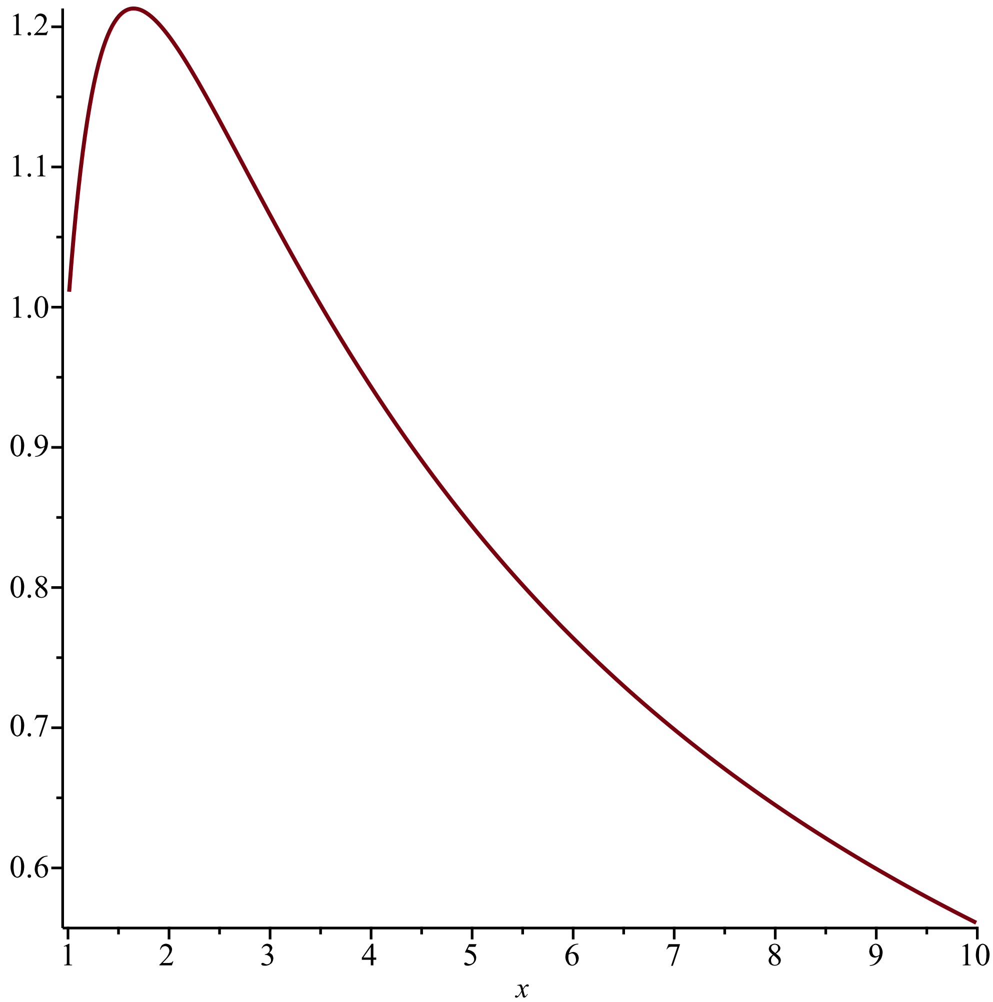
```
> sol2:=dsolve({ec,cond_in},y(x))
```

$$sol2 := y(x) = \frac{1 + 2 \ln(x)}{x} \quad (96)$$

```
> y__sol:=unapply(rhs(sol2),x)
```

$$y_{sol} := x \mapsto \frac{1 + 2 \cdot \ln(x)}{x} \quad (97)$$

```
> with(plots):
> plot(y__sol(x),x=1..10)
```




```
> #ex3:
> restart
> f:=x->x*(x+1)*(2-x)
```

$$f := x \mapsto x \cdot (x+1) \cdot (2-x) \quad \textbf{(98)}$$

```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{d}{dt} x(t) = x(t) \ (x(t)+1) \ (2-x(t)) \quad \textbf{(99)}$$

```
> pct_ech:=solve(f(x)=0,x)
```

$$pct\_ech := -1, 0, 2 \quad \textbf{(100)}$$

```
> D(f)(pct_ech[1])
```

$$-3 \quad \textbf{(101)}$$

```
> #local stabil as
> D(f)(pct_ech[2])
```

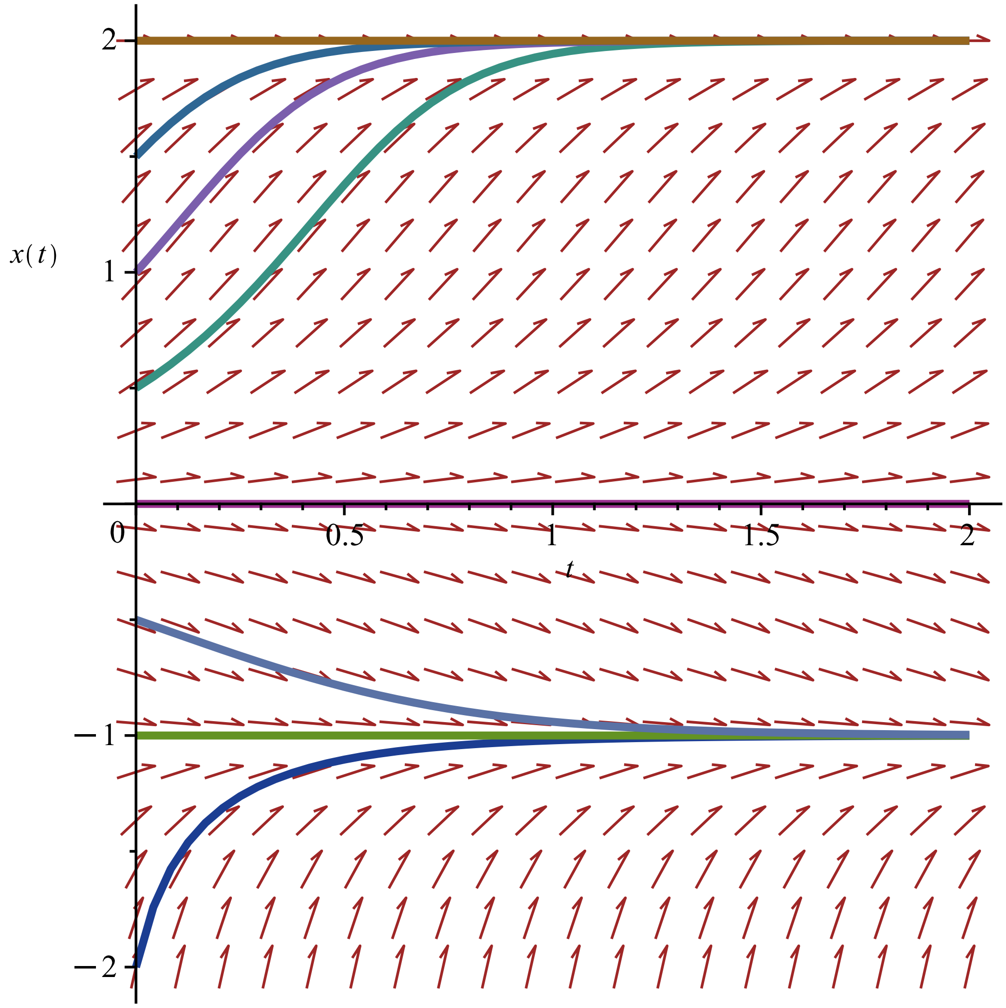
$$2 \quad \textbf{(102)}$$

```
> #instabil
> D(f)(pct_ech[3])
```

$$-6 \quad \textbf{(103)}$$

```
> #local stabil as
> with(DEtools):
> DEplot(ec,x(t),t=0..2,[[x(0)=-2],[x(0)=-1],[x(0)=-1/2],[x(0)=0],
```

```
[x(0)=1/2],[x(0)=1],[x(0)=3/2],[x(0)=2]])
```




```
> #ex4:
> restart
> f1:=(x,y)->y^2-8*x
```

$$f1 := (x, y) \mapsto y^2 - 8 \cdot x \tag{104}$$

```
> f2:=(x,y)->x^2-y
```

$$f2 := (x, y) \mapsto x^2 - y \tag{105}$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = y(t)^2 - 8\, x(t) \tag{106}$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = x(t)^2 - y(t) \tag{107}$$

```
> pct_ech:=solve({f1(x,y)=0,f2(x,y)=0},{x,y})
```

$$pct\_ech := \{x = 0, y = 0\}, \{x = 2, y = 4\}, \{x = -2\, RootOf(\_Z^2 + \_Z + 1) - 2, y = 4\, RootOf(\_Z^2 + \_Z + 1)\} \tag{108}$$

```
> with(linalg):
> J:=jacobian([f1(x,y),f2(x,y)],[x,y])
```

(109)

$$J := \begin{bmatrix} -8 & 2y \\ 2x & -1 \end{bmatrix} \quad (109)$$

```
> A:=subs(pct_ech[1,1],pct_ech[1,2],eval(J))
```

$$A := \begin{bmatrix} -8 & 0 \\ 0 & -1 \end{bmatrix} \quad (110)$$

```
> eigenvals(A)
```

$$-8, -1 \quad (111)$$

```
> B:=subs(pct_ech[2,1],pct_ech[2,2],eval(J))
```

$$B := \begin{bmatrix} -8 & 8 \\ 4 & -1 \end{bmatrix} \quad (112)$$

```
> eigenvals(B)
```

$$-\frac{9}{2} + \frac{\sqrt{177}}{2}, -\frac{9}{2} - \frac{\sqrt{177}}{2} \quad (113)$$

```
> C:=subs(pct_ech[3,1],pct_ech[3,2],eval(J))
```

$$C := \begin{bmatrix} -8 & 8\, RootOf(\_Z^2 + \_Z + 1) \\ -4\, RootOf(\_Z^2 + \_Z + 1) - 4 & -1 \end{bmatrix} \quad (114)$$

```
> eigenvals(C)
```

$$RootOf(\_Z^2 + 9\_Z - 24) \quad (115)$$

```
> #A,B - pct de tip nod
```

```
> simplify(pct_ech[3,1])
```

$$x = -2\, RootOf(\_Z^2 + \_Z + 1) - 2 \quad (116)$$

```
> restart
> #model5,ex1:
> ec:=x*diff(y(x),x)+k*y(x)=x^4
```

$$ec := x \left(\frac{d}{dx}\, y(x) \right) + k\, y(x) = x^4 \quad (117)$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = \frac{x^4}{4+k} + x^{-k}\, c_1 \quad (118)$$

```
> cond_in:=y(1)=1/(k+4)
```

$$cond\_in := y(1) = \frac{1}{4+k} \quad (119)$$

```
> sol2:=dsolve({ec,cond_in},y(x))
```

$$sol2 := y(x) = \frac{x^4}{4+k} \quad (120)$$

```
> f:=(x,k)->x^4/(4+k)
```

(121)

$$f := (x, k) \mapsto \frac{x^4}{4 + k} \tag{121}$$

```
> k__sol:=solve(f(2,k)=16,k)
```

$$k_{sol} := -3 \tag{122}$$

```
> k:=-3
```

$$k := -3 \tag{123}$$

```
> f__sol:=x->x^4
```

$$f_{sol} := x \mapsto x^4 \tag{124}$$

```
> rez:=solve(f__sol(x)=0,x)
```

$$rez := 0, 0, 0, 0 \tag{125}$$

```
> #ex2:
> restart: with(DEtools): with(linalg):
> f:=x->-x^3+2*x^2+3*x
```

$$f := x \mapsto -x^3 + 2 \cdot x^2 + 3 \cdot x \tag{126}$$

```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = -x(t)^3 + 2\, x(t)^2 + 3\, x(t) \tag{127}$$

```
> pct_ech:=solve(f(x)=0,x)
```

$$pct\_ech := 0, 3, -1 \tag{128}$$

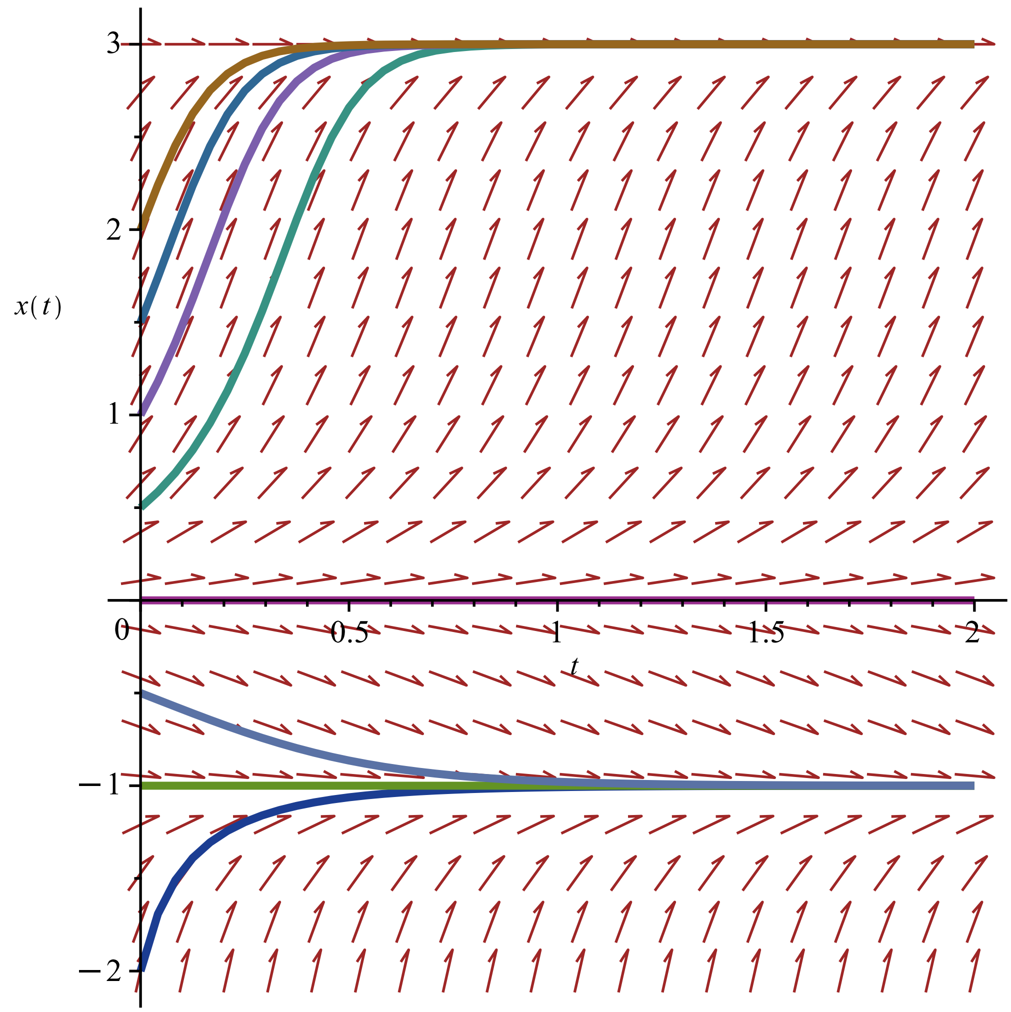
```
> D(f)(pct_ech[1])
```

$$3 \tag{129}$$

```
> #instabil
> D(f)(pct_ech[2])
```

$$-12 \tag{130}$$

```
> #local stabil as
> DEplot(ec,x(t),t=0..2, [[x(0)=-2],[x(0)=-1],[x(0)=-0.5],[x(0)=0],
  [x(0)=0.5],[x(0)=1],[x(0)=3/2],[x(0)=2]])
```




```
> #ex3:
> restart: with(plots):
> ec:=x^2*diff(y(x),x$2)-4*diff(y(x),x)+4*y(x)=0
```

$$ec := x^2 \left(\frac{d^2}{dx^2}\, y(x)\right) - 4\, \frac{d}{dx}\, y(x) + 4\, y(x) = 0 \tag{131}$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = \frac{c_1\, e^{-\frac{2}{x}} \left(\left(\mathrm{I}\sqrt{15}\, x + x + 4\right) \mathrm{BesselI}\left(\frac{\mathrm{I}}{2}\sqrt{15}, \frac{2}{x}\right) + 4\, \mathrm{BesselI}\left(\frac{\mathrm{I}\sqrt{15}}{2} + 1, \frac{2}{x}\right)\right)}{\sqrt{x}} + \frac{1}{\sqrt{x}} \left(c_2\, e^{-\frac{2}{x}} \left(\left(\mathrm{I}\sqrt{15}\, x + x + 4\right) \mathrm{BesselK}\left(\frac{\mathrm{I}}{2}\sqrt{15}, \frac{2}{x}\right) - 4\, \mathrm{BesselK}\left(\frac{\mathrm{I}\sqrt{15}}{2} + 1, \frac{2}{x}\right)\right)\right) \tag{132}$$

```
> cond_in:=y(1)=2,D(y)(1)=1
```

$$cond\_in := y(1) = 2, \mathrm{D}(y)(1) = 1 \tag{133}$$

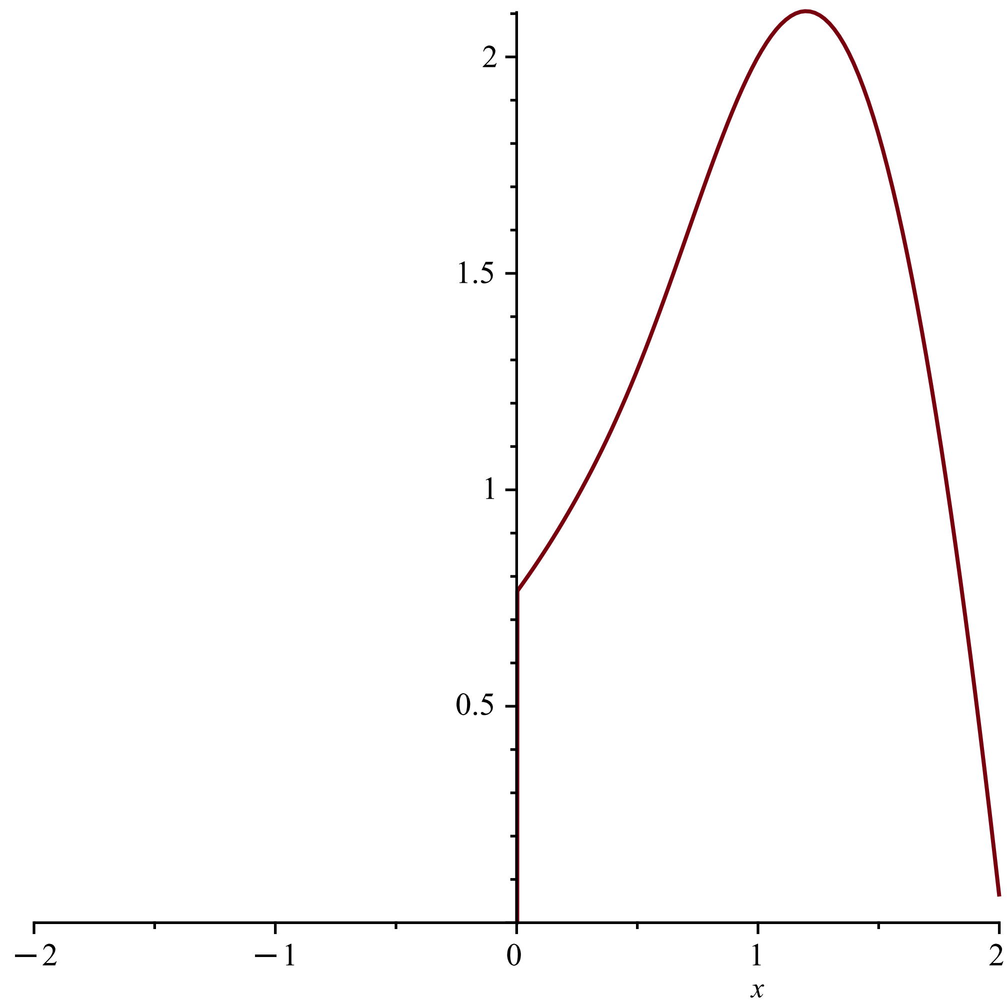
```
> sol2:=dsolve({ec,cond_in},y(x))
```

$$
\mathit{sol2} := y(x) = -\frac{1}{16\sqrt{x}}\left(\left(\mathrm{I}\sqrt{15}\,\mathrm{BesselK}\left(\frac{\mathrm{I}}{2}\sqrt{15},2\right) - 11\,\mathrm{BesselK}\left(\frac{\mathrm{I}}{2}\sqrt{15},2\right) - 4\,\mathrm{BesselK}\left(\frac{\mathrm{I}\sqrt{15}}{2}+1,2\right)\right)\mathrm{e}^{2}\,\mathrm{e}^{-\frac{2}{x}}\left(\left(\mathrm{I}\sqrt{15}\,x + x + 4\right)\mathrm{BesselI}\left(\frac{\mathrm{I}}{2}\sqrt{15},\frac{2}{x}\right) + 4\,\mathrm{BesselI}\left(\frac{\mathrm{I}\sqrt{15}}{2}+1,\frac{2}{x}\right)\right)\right) + \frac{1}{16\sqrt{x}}\left(\mathrm{e}^{2}\left(\mathrm{I}\sqrt{15}\,\mathrm{BesselI}\left(\frac{\mathrm{I}}{2}\sqrt{15},2\right) - 11\,\mathrm{BesselI}\left(\frac{\mathrm{I}}{2}\sqrt{15},2\right) + 4\,\mathrm{BesselI}\left(\frac{\mathrm{I}\sqrt{15}}{2}+1,2\right)\right)\mathrm{e}^{-\frac{2}{x}}\left(\left(\mathrm{I}\sqrt{15}\,x + x + 4\right)\mathrm{BesselK}\left(\frac{\mathrm{I}}{2}\sqrt{15},\frac{2}{x}\right) - 4\,\mathrm{BesselK}\left(\frac{\mathrm{I}\sqrt{15}}{2}+1,\frac{2}{x}\right)\right)\right) \tag{134}
$$

```
> yy:=unapply(rhs(sol2),x)
```

$$
yy := x \mapsto -\frac{1}{16\cdot\sqrt{x}}\left(\left(\mathrm{I}\cdot\sqrt{15}\cdot\mathrm{BesselK}\left(\frac{\mathrm{I}}{2}\cdot\sqrt{15},2\right) - 11\cdot\mathrm{BesselK}\left(\frac{\mathrm{I}}{2}\cdot\sqrt{15},2\right) - 4\cdot\mathrm{BesselK}\left(\frac{\mathrm{I}\cdot\sqrt{15}}{2}+1,2\right)\right)\cdot\mathrm{e}^{2}\cdot\mathrm{e}^{-\frac{2}{x}}\cdot\left(\left(\mathrm{I}\cdot\sqrt{15}\cdot x + x + 4\right)\cdot\mathrm{BesselI}\left(\frac{\mathrm{I}}{2}\cdot\sqrt{15},\frac{2}{x}\right) + 4\cdot\mathrm{BesselI}\left(\frac{\mathrm{I}\cdot\sqrt{15}}{2}+1,\frac{2}{x}\right)\right)\right) + \frac{1}{16\cdot\sqrt{x}}\left(\mathrm{e}^{2}\cdot\left(\mathrm{I}\cdot\sqrt{15}\cdot\mathrm{BesselI}\left(\frac{\mathrm{I}}{2}\cdot\sqrt{15},2\right) - 11\cdot\mathrm{BesselI}\left(\frac{\mathrm{I}}{2}\cdot\sqrt{15},2\right) + 4\cdot\mathrm{BesselI}\left(\frac{\mathrm{I}\cdot\sqrt{15}}{2}+1,2\right)\right)\cdot\mathrm{e}^{-\frac{2}{x}}\cdot\left(\left(\mathrm{I}\cdot\sqrt{15}\cdot x + x + 4\right)\cdot\mathrm{BesselK}\left(\frac{\mathrm{I}}{2}\cdot\sqrt{15},\frac{2}{x}\right) - 4\cdot\mathrm{BesselK}\left(\frac{\mathrm{I}\cdot\sqrt{15}}{2}+1,\frac{2}{x}\right)\right)\right) \tag{135}
$$

```
> plot(yy(x),x=-2..2)
```




```
> #ex4:
> restart:
> ec1:=diff(y__1(t),t)=-5*y__1(t)+9*y__2(t)
```

$$ec1 := \frac{d}{dt}\, y_1(t) = -5\, y_1(t) + 9\, y_2(t) \tag{136}$$

```
> ec2:=diff(y__2(t),t)=-6*y__1(t)+10*y__2(t)
```

$$ec2 := \frac{d}{dt}\, y_2(t) = -6\, y_1(t) + 10\, y_2(t) \tag{137}$$

```
> sol:=dsolve({ec1,ec2},{y__1,y__2})
```

$$sol := \left\{ y_1(t) = c_1\, e^{t} + c_2\, e^{4\,t},\, y_2(t) = \frac{2\, c_1\, e^{t}}{3} + c_2\, e^{4\,t} \right\} \tag{138}$$

```
> cond_in:=y__1(0)=2,y__2(0)=3
```

$$cond\_in := y_1(0) = 2,\, y_2(0) = 3 \tag{139}$$

```
> sol2:=dsolve({ec1,ec2,cond_in},{y__1(t),y__2(t)})
```

$$sol2 := \left\{ y_1(t) = -3\, e^{t} + 5\, e^{4\,t},\, y_2(t) = -2\, e^{t} + 5\, e^{4\,t} \right\} \tag{140}$$

```
> restart
> #model6,ex1:
> ec:=x*diff(y(x),x)+k*y(x)=x^4+y(x)
```

$$ec := x\left(\frac{d}{dx}\,y(x)\right) + k\,y(x) = x^4 + y(x) \tag{141}$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = \frac{x^4}{3+k} + x^{-k+1}\,c_1 \tag{142}$$

```
> cond_in:=y(1)=1/(k+3)
```

$$cond\_in := y(1) = \frac{1}{3+k} \tag{143}$$

```
> sol2:=dsolve({ec,cond_in},y(x))
```

$$sol2 := y(x) = \frac{x^4}{3+k} \tag{144}$$

```
> f:=(x,k)->x^4/(3+k)
```

$$f := (x, k) \mapsto \frac{x^4}{k+3} \tag{145}$$

```
> k__rez:=solve(f(3,k)=81,k)
```

$$k_{rez} := -2 \tag{146}$$

```
> rez:=solve(f(x,-2)=0,x)
```

$$rez := 0, 0, 0, 0 \tag{147}$$

```
> #ex2:
> restart
> ec:=x^2*diff(y(x),x$2)+2*x*diff(y(x),x)+4*y(x)=0
```

$$ec := x^2\left(\frac{d^2}{dx^2}\,y(x)\right) + 2\,x\left(\frac{d}{dx}\,y(x)\right) + 4\,y(x) = 0 \tag{148}$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = \frac{c_1 \sin\left(\frac{\sqrt{15}\,\ln(x)}{2}\right)}{\sqrt{x}} + \frac{c_2 \cos\left(\frac{\sqrt{15}\,\ln(x)}{2}\right)}{\sqrt{x}} \tag{149}$$

```
> cond_in:=y(1)=0,D(y)(1)=1
```

$$cond\_in := y(1) = 0, D(y)(1) = 1 \tag{150}$$

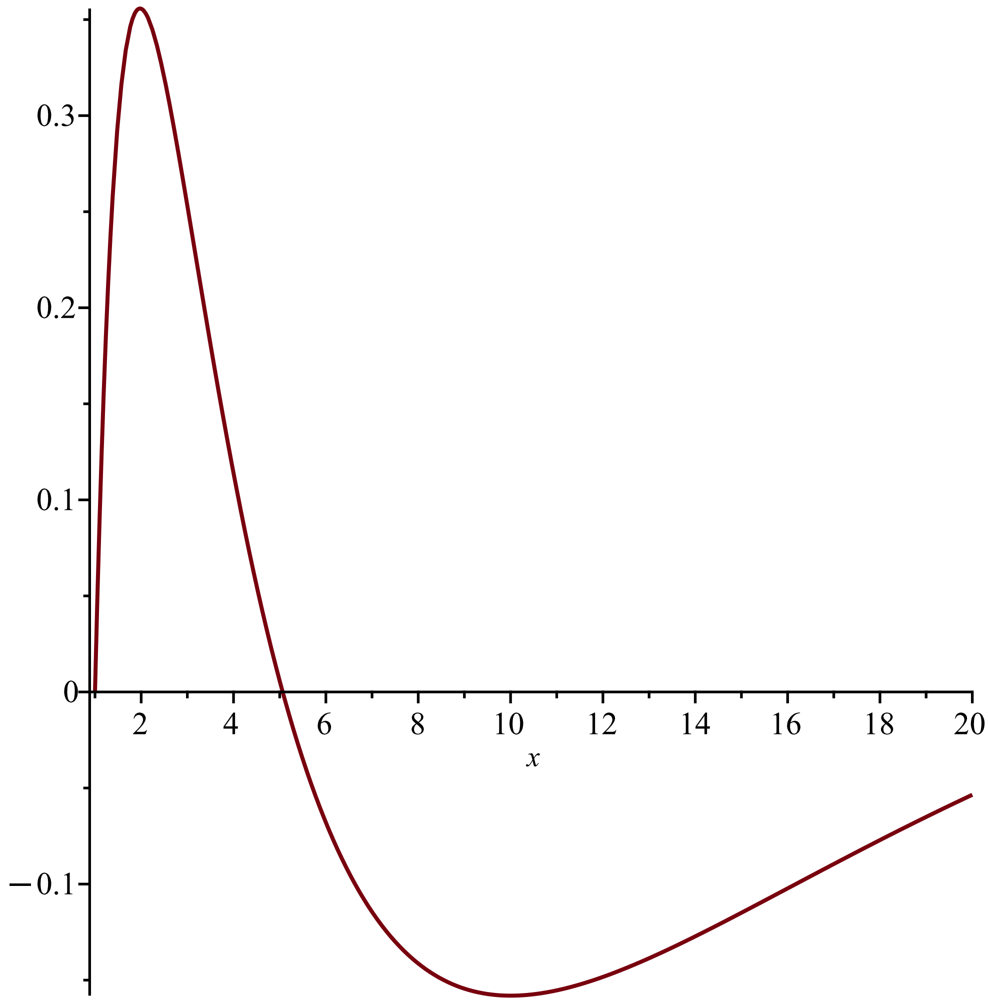
```
> sol2:=dsolve({ec,cond_in},y(x))
```

$$sol2 := y(x) = \frac{2\sqrt{15}\,\sin\left(\frac{\sqrt{15}\,\ln(x)}{2}\right)}{15\sqrt{x}} \tag{151}$$

```
> yy:=unapply(rhs(sol2),x)
```

$$yy := x \mapsto \frac{2\cdot\sqrt{15}\cdot\sin\left(\frac{\sqrt{15}\cdot\ln(x)}{2}\right)}{15\cdot\sqrt{x}} \tag{152}$$

```
> with(plots):
> plot(yy(x),x=1..20)
```




```
> #ex3:
> restart
> f:=x->(3-x)*(x+1)*x
```

$$f := x \mapsto (3 - x)\cdot(x + 1)\cdot x \quad (153)$$

```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{d}{dt}\, x(t) = (3 - x(t))\ (x(t) + 1)\ x(t) \quad (154)$$

```
> pct_ech:=solve(f(x)=0,x)
```

$$pct\_ech := -1, 0, 3 \quad (155)$$

```
> D(f)(pct_ech[1])
```

$$-4 \quad (156)$$

```
> #local as stabil
> D(f)(pct_ech[2])
```

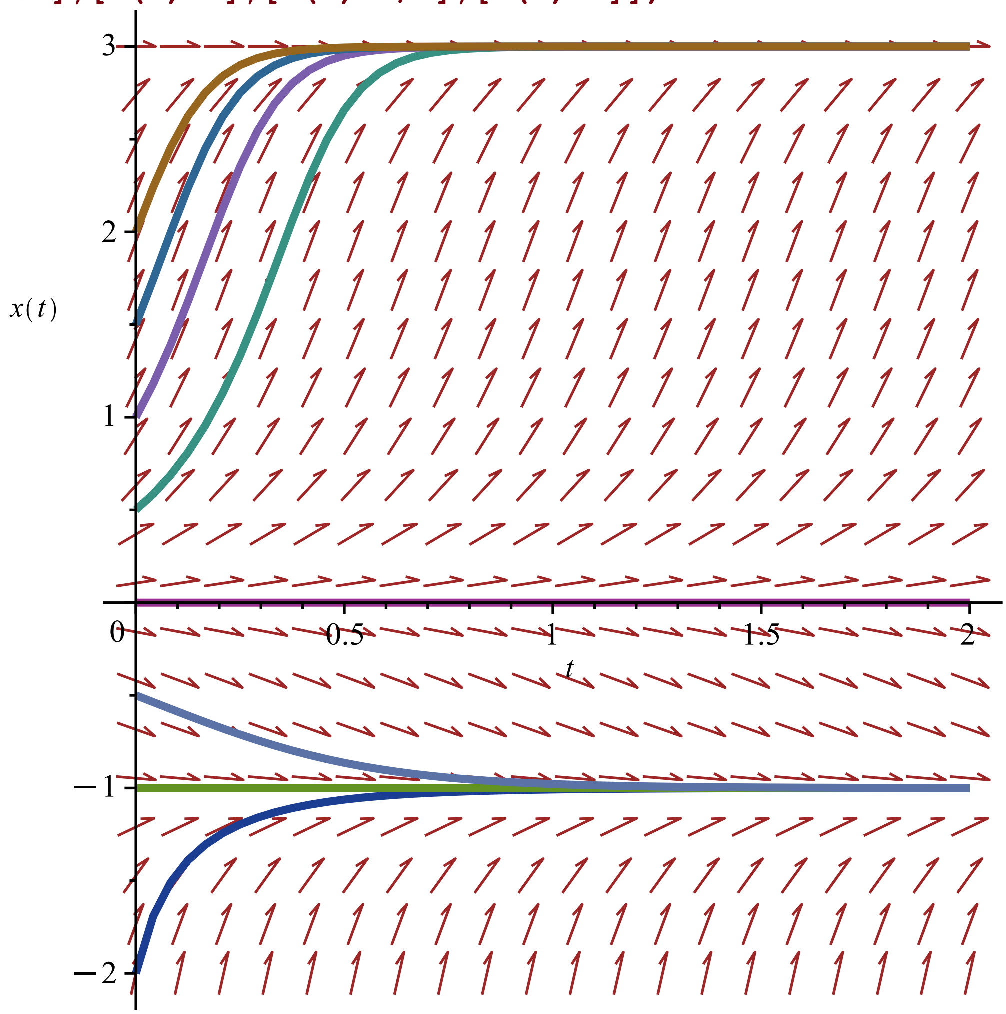
$$3 \quad (157)$$

```
> #instabil
> D(f)(pct_ech[3])
```

$$-12 \quad (158)$$

```
> #local as stabil
> with(DEtools):
> DEplot(ec,x(t),t=0..2, [[x(0)=-2],[x(0)=-1],[x(0)=-0.5],[x(0)=0],
```

```
[x(0)=0.5],[x(0)=1],[x(0)=3/2],[x(0)=2]])
```




```
> #ex4:
> restart
> ec1:=diff(y__1(x),x)=-y__1(x)-2*y__2(x)
```

$$ec1 := \frac{d}{dx}\, y_1(x) = -y_1(x) - 2\, y_2(x) \qquad \textbf{(159)}$$

```
> ec2:=diff(y__2(x),x)=6*y__1(x)+6*y__2(x)
```

$$ec2 := \frac{d}{dx}\, y_2(x) = 6\, y_1(x) + 6\, y_2(x) \qquad \textbf{(160)}$$

```
> sol:=dsolve({ec1,ec2},{y__1(x),y__2(x)})
```

$$sol := \left\{ y_1(x) = c_1\, e^{3x} + c_2\, e^{2x}, y_2(x) = -2\, c_1\, e^{3x} - \frac{3\, c_2\, e^{2x}}{2} \right\} \qquad \textbf{(161)}$$

```
> cond_in:=y__1(0)=2,y__2(0)=5
```

$$cond\_in := y_1(0) = 2, y_2(0) = 5 \qquad \textbf{(162)}$$

```
> sol2:=dsolve({ec1,ec2,cond_in},{y__1(x),y__2(x)})
```

$$sol2 := \left\{ y_1(x) = -16\, e^{3x} + 18\, e^{2x}, y_2(x) = 32\, e^{3x} - 27\, e^{2x} \right\} \qquad \textbf{(163)}$$

```
> #subiect 2022, ex1:
> restart
```

```
> ecdif:=diff(x(t),t)=r__0*x(t)*(1-x(t)/k)
```
$$ecdif := \frac{d}{dt}\, x(t) = r_0\, x(t)\left(1 - \frac{x(t)}{k}\right) \tag{164}$$

```
> cond_in:=x(0)=x__0
```
$$cond\_in := x(0) = x_0 \tag{165}$$

```
> sol:=dsolve({ecdif,cond_in},x(t))
```
$$sol := x(t) = \frac{k\, x_0}{\left(k - x_0\right) e^{-r_0 t} + x_0} \tag{166}$$

```
> xV:=unapply(rhs(sol),t,x__0,r__0,k)
```
$$xV := \left(t, x_0, r_0, k\right) \mapsto \frac{k \cdot x_0}{\left(k - x_0\right) \cdot e^{-r_0 \cdot t} + x_0} \tag{167}$$

```
> ec1:=xV(10,40,r__0,k)=400
```
$$ec1 := \frac{40\, k}{(k - 40)\, e^{-10\, r_0} + 40} = 400 \tag{168}$$

```
> ec2:=xV(10,400,r__0,k)=800
```
$$ec2 := \frac{400\, k}{(k - 400)\, e^{-10\, r_0} + 400} = 800 \tag{169}$$

```
> sol__f:=dsolve({ec1,ec2},{r__0,k})
```
$$sol_f := \left\{k = 850, r_0 = \frac{\ln(18)}{10}\right\} \tag{170}$$

```
> #model 8,ex1:
> restart
> ec:=diff(y(x),x)-m*y(x)=x
```
$$ec := \frac{d}{dx}\, y(x) - m\, y(x) = x \tag{171}$$

```
> sol:=dsolve(ec,y(x))
```
$$sol := y(x) = -\frac{x}{m} - \frac{1}{m^2} + e^{m x}\, c_1 \tag{172}$$

```
> cond_in:=y(0)=-1
```
$$cond\_in := y(0) = -1 \tag{173}$$

```
> sol2:=dsolve({ec,cond_in},y(x))
```
$$sol2 := y(x) = \frac{-e^{m x}\, m^2 - m\, x + e^{m x} - 1}{m^2} \tag{174}$$

```
> f:=(x,m)->(-exp(m*x)*m^2-m*x+exp(m*x)-1)/m^2
```
$$f := (x, m) \mapsto \frac{-e^{m \cdot x} \cdot m^2 - m \cdot x + e^{m \cdot x} - 1}{m^2} \tag{175}$$

```
> m__rez:=solve(f(1,m)=0,m)
```
$$m_{rez} := -1 \tag{176}$$

```
> rez:=solve(f(x,-1)=0,x)
```

$$rez := 1 \tag{177}$$

```
> #ex2:
> restart
> ec:=x^2*diff(y(x),x$2)+3*x*diff(y(x),x)+y(x)=0
```

$$ec := x^2 \left(\frac{d^2}{dx^2} y(x) \right) + 3\,x \left(\frac{d}{dx} y(x) \right) + y(x) = 0 \tag{178}$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = \frac{c_1}{x} + \frac{c_2 \ln(x)}{x} \tag{179}$$

```
> cond_in:=y(1)=1,D(y)(1)=1
```

$$cond\_in := y(1) = 1, \mathrm{D}(y)(1) = 1 \tag{180}$$

```
> sol2:=dsolve({ec,cond_in},y(x))
```

$$sol2 := y(x) = \frac{1 + 2\ln(x)}{x} \tag{181}$$

```
> #ex3:
> restart
> f:=x->x*(x+1)*(2-x)
```

$$f := x \mapsto x \cdot (x+1) \cdot (2-x) \tag{182}$$

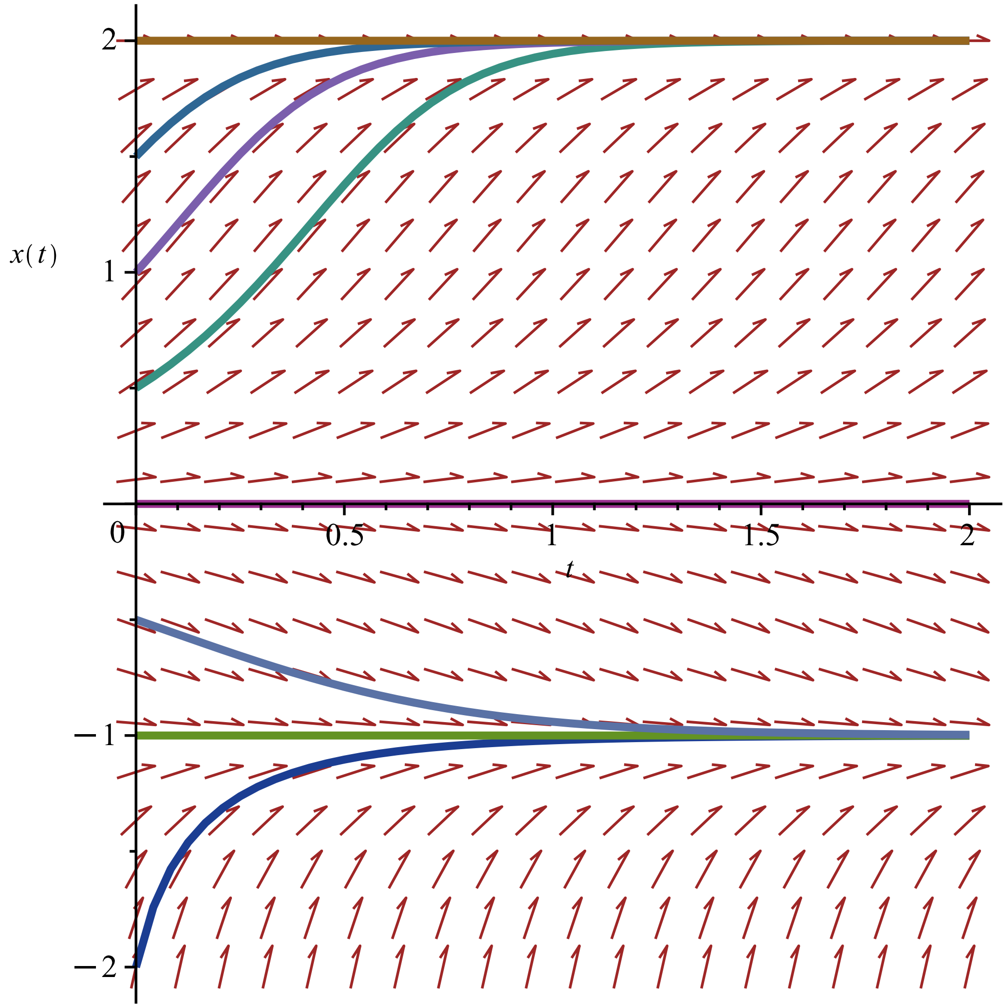
```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{d}{dt} x(t) = x(t)\,(x(t)+1)\,(2-x(t)) \tag{183}$$

```
> pct_ech:=solve(f(x)=0,x)
```

$$pct\_ech := -1, 0, 2 \tag{184}$$

```
> with(DEtools):
> DEplot(ec,x(t),t=0..2,[[x(0)=-2],[x(0)=-1],[x(0)=-0.5],[x(0)=0],
  [x(0)=0.5],[x(0)=1],[x(0)=3/2],[x(0)=2]])
```




```
> #model9,ex1 - la model3:
> restart
> #ex2,3 la fel ca restu:
> #ex4:
> f__1:=(x,y)->4*x-x*y^2
```

$$f_1 := (x, y) \mapsto 4\cdot x - x\cdot y^2 \tag{185}$$

```
> f__2:=(x,y)->x-y
```

$$f_2 := (x, y) \mapsto x - y \tag{186}$$

```
> ec1:=diff(x(t),t)=f__1(x(t),y(t))
```

$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = 4\, x(t) - x(t)\, y(t)^2 \tag{187}$$

```
> ec2:=diff(y(t),t)=f__2(x(t),y(t))
```

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = x(t) - y(t) \tag{188}$$

```
> pct_ech:=solve({f__1(x,y)=0,f__2(x,y)=0},{x,y})
```

$$pct\_ech := \{x = 0, y = 0\}, \{x = 2, y = 2\}, \{x = -2, y = -2\} \tag{189}$$

```
> with(DEtools): with(linalg):
> J:=jacobian([f__1(x,y),f__2(x,y)],[x,y])
```

(190)

$$J := \begin{bmatrix} -y^2+4 & -2\,x\,y \\ 1 & -1 \end{bmatrix} \tag{190}$$

```
> A:=subs(pct_ech[1,1],pct_ech[1,2],eval(J))
```

$$A := \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix} \tag{191}$$

```
> eigenvals(A)
```

$$4, -1 \tag{192}$$

```
> #de tip sa
> B:=subs(pct_ech[2,1],pct_ech[2,2],eval(J))
```

$$B := \begin{bmatrix} 0 & -8 \\ 1 & -1 \end{bmatrix} \tag{193}$$

```
> eigenvals(B)
```

$$-\frac{1}{2}+\frac{\mathrm{I}\sqrt{31}}{2}, -\frac{1}{2}-\frac{\mathrm{I}\sqrt{31}}{2} \tag{194}$$

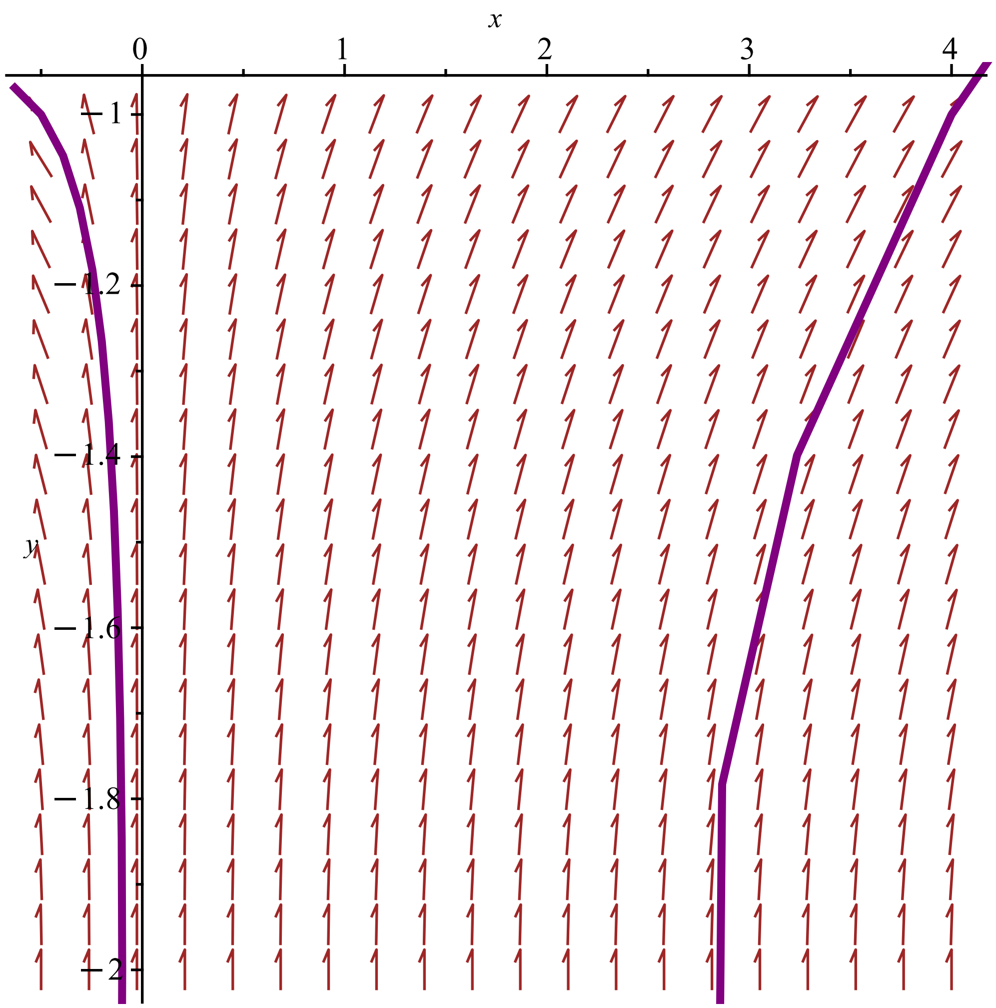
```
> #de tip focus
> C:=subs(pct_ech[3,1],pct_ech[3,2],eval(J))
```

$$C := \begin{bmatrix} 0 & -8 \\ 1 & -1 \end{bmatrix} \tag{195}$$

```
> eigenvals(C)
```

$$-\frac{1}{2}+\frac{\mathrm{I}\sqrt{31}}{2}, -\frac{1}{2}-\frac{\mathrm{I}\sqrt{31}}{2} \tag{196}$$

```
> #de tip focus
> DEplot([ec1,ec2],[x(t),y(t)],t=-2..2,x=-1/2..4,y=-2..-1,[[x(0)=4,
  y(0)=-1],[x(0)=-0.5,y(0)=-1],[x(0)=2,y(0)=-1/2]],linecolor=
  purple,stepsize=0.1)
```




```
> #model x:
> restart
> ecdif:=diff(x(t),t)=r*x(t)
```

$$ecdif := \frac{d}{dt}\, x(t) = r\, x(t) \tag{197}$$

```
> cond_in:=x(0)=x__0
```

$$cond\_in := x(0) = x_0 \tag{198}$$

```
> sol:=dsolve({ecdif,cond_in},x(t))
```

$$sol := x(t) = x_0\, e^{r\, t} \tag{199}$$

```
> xM:=unapply(rhs(sol),t,x__0,r)
```

$$xM := (t, x_0, r) \mapsto x_0 \cdot e^{r \cdot t} \tag{200}$$

```
> r:=solve(xM(10,1000,r)=30000,r)
```

$$r := \frac{\ln(30)}{10} \tag{201}$$

```
> #restu se repeta ;)

> restart
> #2024.1, ex1:
```

```
> ec:=diff(y(x),x)+y(x)=x*exp(x)*y(x)^3
```

$$ec := \frac{d}{dx}\, y(x) + y(x) = x\, e^{x}\, y(x)^{3} \tag{202}$$

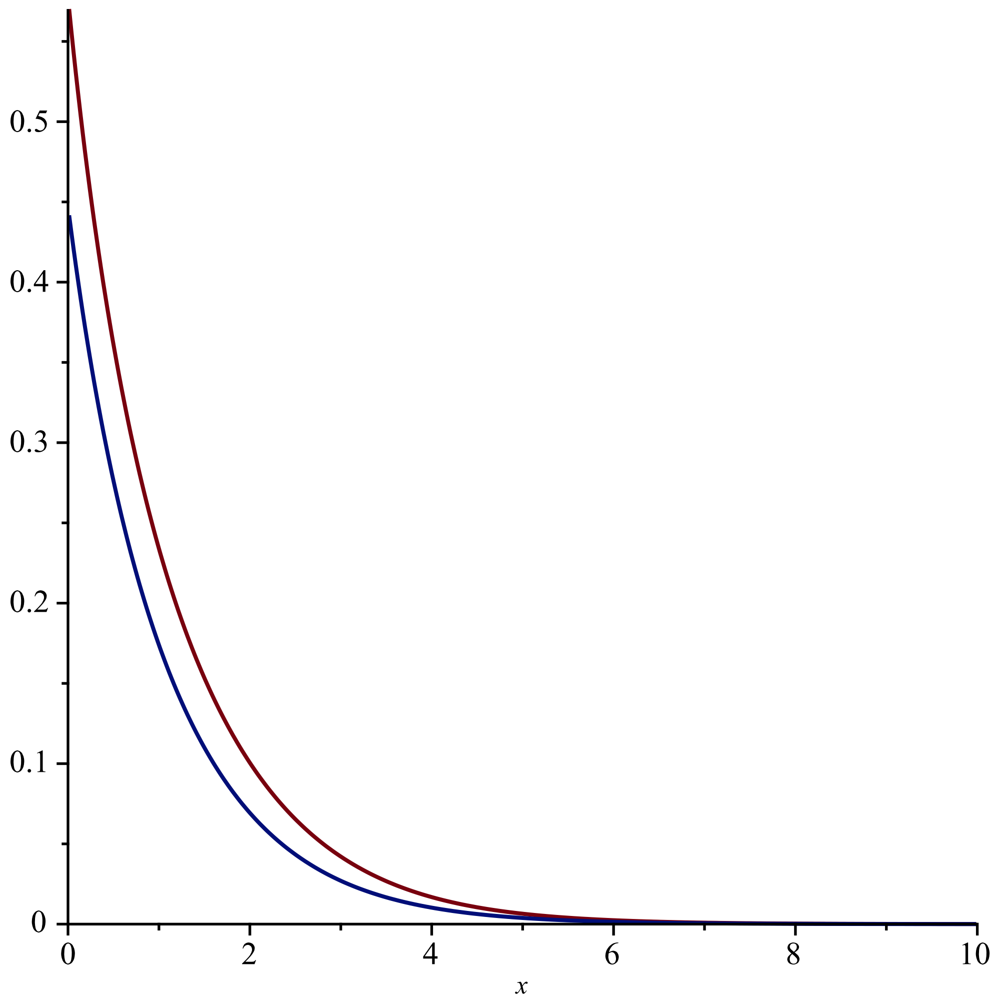
```
> with(plots):
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = \frac{1}{\sqrt{e^{2\,x}\, c_1 + 2\,x\, e^{x} + 2\, e^{x}}},\; y(x) = -\frac{1}{\sqrt{e^{2\,x}\, c_1 + 2\,x\, e^{x} + 2\, e^{x}}} \tag{203}$$

```
> yy:=unapply(rhs(sol[1]),x,c__1)
```

$$yy := (x, c_1) \mapsto \frac{1}{\sqrt{e^{2\cdot x}\cdot c_1 + 2\cdot x\cdot e^{x} + 2\cdot e^{x}}} \tag{204}$$

```
> plot([yy(x,1),yy(x,3)],x=0..10)
```




```
> cond_in:=y(0)=1
```

$$cond\_in := y(0) = 1 \tag{205}$$

```
> sol__C:=dsolve({ec,cond_in},y(x))
```

$$sol_C := y(x) = \frac{1}{\sqrt{-e^{x}\,(e^{x} - 2\,x - 2)}} \tag{206}$$

```
> #ex2:
> restart
```

```
> ec:=(1+x^2)*diff(y(x),x$2)+4*x*diff(y(x),x)-10*y(x)=0
```

$$ec := (x^2+1)\left(\frac{d^2}{dx^2}\,y(x)\right)+4\,x\left(\frac{d}{dx}\,y(x)\right)-10\,y(x)=0 \tag{207}$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x)=c_1\,(5\,x^2+1)+\frac{c_2\,((15\,x^4+18\,x^2+3)\arctan(x)+15\,x^3+13\,x)}{x^2+1} \tag{208}$$

```
> cond_in:=y(0)=a,D(y)(0)=1
```

$$cond\_in := y(0)=a,\ D(y)(0)=1 \tag{209}$$

```
> sol__1:=dsolve({ec,cond_in},y(x))
```

$$sol_1 := y(x)=a\,(5\,x^2+1)+\frac{(15\,x^4+18\,x^2+3)\arctan(x)+15\,x^3+13\,x}{16\,(x^2+1)} \tag{210}$$

```
> limit(sol__1,x=infinity)
```

$$\lim_{x\to\infty} y(x)=\mathrm{signum}\left(\frac{3\,\pi}{32}+a\right)\infty \tag{211}$$

```
> a:=-3*Pi/32
```

$$a := -\frac{3\,\pi}{32} \tag{212}$$

```
> limit(sol__1,x=infinity)
```

$$\lim_{x\to\infty} y(x)=0 \tag{213}$$

```
> #ex3:
> restart: with(DEtools): with(linalg):
> ec__1:=diff(x(t),t)=x(t)-3*y(t)
```

$$ec_1 := \frac{d}{dt}\,x(t)=x(t)-3\,y(t) \tag{214}$$

```
> ec__2:=diff(y(t),t)=3*x(t)-y(t)
```

$$ec_2 := \frac{d}{dt}\,y(t)=3\,x(t)-y(t) \tag{215}$$

```
> sol__g:=dsolve({ec__1,ec__2},{x(t),y(t)})
```

$$sol_g := \left\{x(t)=c_1\sin(2\sqrt{2}\,t)+c_2\cos(2\sqrt{2}\,t),\ y(t)=-\frac{2\,c_1\sqrt{2}\cos(2\sqrt{2}\,t)}{3}+\frac{2\,c_2\sqrt{2}\sin(2\sqrt{2}\,t)}{3}+\frac{c_1\sin(2\sqrt{2}\,t)}{3}+\frac{c_2\cos(2\sqrt{2}\,t)}{3}\right\} \tag{216}$$

```
> cond_in:=x(0)=0,y(0)=1
```

$$cond\_in := x(0)=0,\ y(0)=1 \tag{217}$$

```
> sol__c:=dsolve({ec__1,ec__2,cond_in},{x(t),y(t)})
```

$$sol_c := \left\{x(t)=-\frac{3\sqrt{2}\sin(2\sqrt{2}\,t)}{4},\ y(t)=\cos(2\sqrt{2}\,t)-\frac{\sqrt{2}\sin(2\sqrt{2}\,t)}{4}\right\} \tag{218}$$

```
> A:=matrix([[1,-3],[3,-1]])
```

$$A := \begin{bmatrix} 1 & -3 \\ 3 & -1 \end{bmatrix} \tag{219}$$

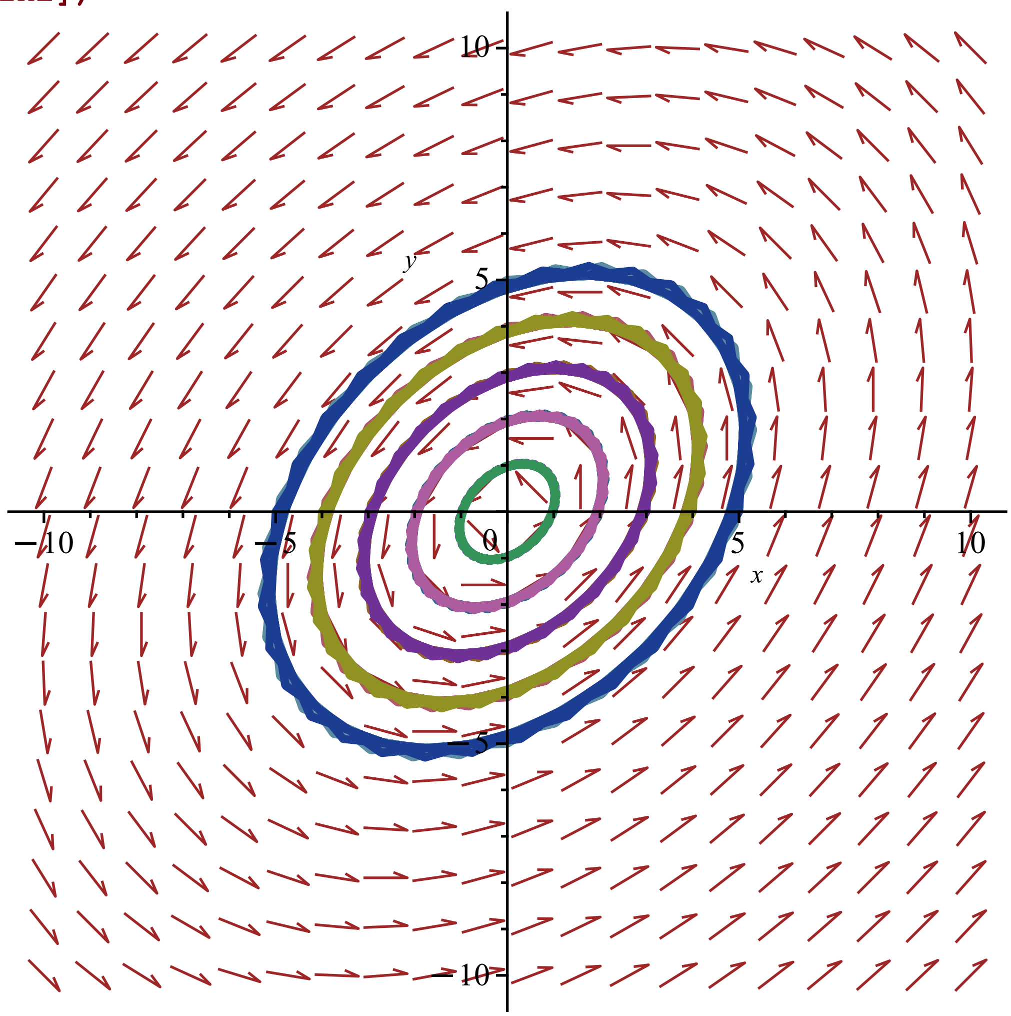
```
> eigenvals(A)
```

$$2\,\mathrm{I}\sqrt{2},\ -2\,\mathrm{I}\sqrt{2} \tag{220}$$

```
> cond_in2:=[x(0)=0,y(0)=i]$i=0..5,[x(0)=-i,y(0)=0]$i=0..5,[x(0)=0,
  y(0)=-i]$i=0..5,[x(0)=-i,y(0)=0]$i=0..5
```

$$cond\_in2 := [x(0)=0, y(0)=0], [x(0)=0, y(0)=1], [x(0)=0, y(0)=2], [x(0)=0, y(0)=3], [x(0)=0, y(0)=4], [x(0)=0, y(0)=5], [x(0)=0, y(0)=0], [x(0)=-1, y(0)=0], [x(0)=-2, y(0)=0], [x(0)=-3, y(0)=0], [x(0)=-4, y(0)=0], [x(0)=-5, y(0)=0], [x(0)=0, y(0)=0], [x(0)=0, y(0)=-1], [x(0)=0, y(0)=-2], [x(0)=0, y(0)=-3], [x(0)=0, y(0)=-4], [x(0)=0, y(0)=-5], [x(0)=0, y(0)=0], [x(0)=-1, y(0)=0], [x(0)=-2, y(0)=0], [x(0)=-3, y(0)=0], [x(0)=-4, y(0)=0], [x(0)=-5, y(0)=0] \tag{221}$$

```
> DEplot({ec__1,ec__2},[x(t),y(t)],t=-5..5,x=-10..10,y=-10..10,
  [cond_in2])
```




```
> limit(sol__c[1],t=infinity)
```

$$\lim_{t\to\infty} x(t) = -\frac{3\sqrt{2}}{4} .. \frac{3\sqrt{2}}{4} \tag{222}$$

```
> limit(sol__c[2],t=infinity)
```

(223)

$$\lim_{t \to \infty} y(t) = -1 - \frac{\sqrt{2}}{4} \,..1 + \frac{\sqrt{2}}{4} \quad (223)$$

```
> #cond nu e indeplinita
> #ex4:
> restart
> ec:=diff(x(t),t)=-k*x(t)
```

$$ec := \frac{d}{dt}\, x(t) = -0.5\, x(t) \quad (224)$$

```
> cond_in:=x(0)=x__0
```

$$cond\_in := x(0) = x_0 \quad (225)$$

```
> sist:=ec,cond_in
```

$$sist := \frac{d}{dt}\, x(t) = -k\, x(t), x(0) = x_0 \quad (226)$$

```
> k:=0.5;x__0:=15
```

$$k := 0.5$$

$$x_0 := 15 \quad (227)$$

```
> sol:=dsolve({sist},x(t))
```

$$sol := x(t) = 15\, e^{-\frac{t}{2}} \quad (228)$$

```
> restart: ec:=diff(x(t),t)=-k*x(t)
```

$$ec := \frac{d}{dt}\, x(t) = -k\, x(t) \quad (229)$$

```
> cond_in:=x(0)=x__0
```

$$cond\_in := x(0) = x_0 \quad (230)$$

```
> sist:=ec,cond_in
```

$$sist := \frac{d}{dt}\, x(t) = -k\, x(t), x(0) = x_0 \quad (231)$$

```
> sol:=dsolve({sist},x(t))
```

$$sol := x(t) = x_0\, e^{-k t} \quad (232)$$

```
> xx:=unapply(rhs(sol),t,x__0,k)
```

$$xx := (t, x_0, k) \mapsto x_0 \cdot e^{-k \cdot t} \quad (233)$$

```
> ec1:=xx(5,8,k)=2
```

$$ec1 := 8\, e^{-5 k} = 2 \quad (234)$$

```
> k:=solve(ec1)
```

$$k := \frac{2 \ln(2)}{5} \quad (235)$$

```
> #ex5:
> restart: with(DEtools): with(linalg):
> f1:=(x,y)->2*y
```

$$f1 := (x, y) \mapsto 2 \cdot y \quad (236)$$

```
> f2:=(x,y)->x*(1-x^2)+y
```

(237)

$$f2 := (x, y) \mapsto x \cdot \left(1 - x^2\right) + y \tag{237}$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{d}{dt}\, x(t) = 2\, y(t) \tag{238}$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{d}{dt}\, y(t) = x(t)\, \left(1 - x(t)^2\right) + y(t) \tag{239}$$

```
> pct_ech:=solve({f1(x,y)=0,f2(x,y)=0},{x,y})
```

$$pct\_ech := \{x = 0, y = 0\}, \{x = 1, y = 0\}, \{x = -1, y = 0\} \tag{240}$$

```
> J:=jacobian([f1(x,y),f2(x,y)],[x,y])
```

$$J := \begin{bmatrix} 0 & 2 \\ -3\, x^2 + 1 & 1 \end{bmatrix} \tag{241}$$

```
> A:=subs(pct_ech[1,1],pct_ech[1,2],eval(J))
```

$$A := \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \tag{242}$$

```
> eigenvals(A)
```

$$2, -1 \tag{243}$$

```
> #instabil.de tip sa
> B:=subs(pct_ech[2,1],pct_ech[2,2],eval(J))
```

$$B := \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} \tag{244}$$

```
> eigenvals(B)
```

$$\frac{1}{2} + \frac{I\sqrt{15}}{2}, \frac{1}{2} - \frac{I\sqrt{15}}{2} \tag{245}$$

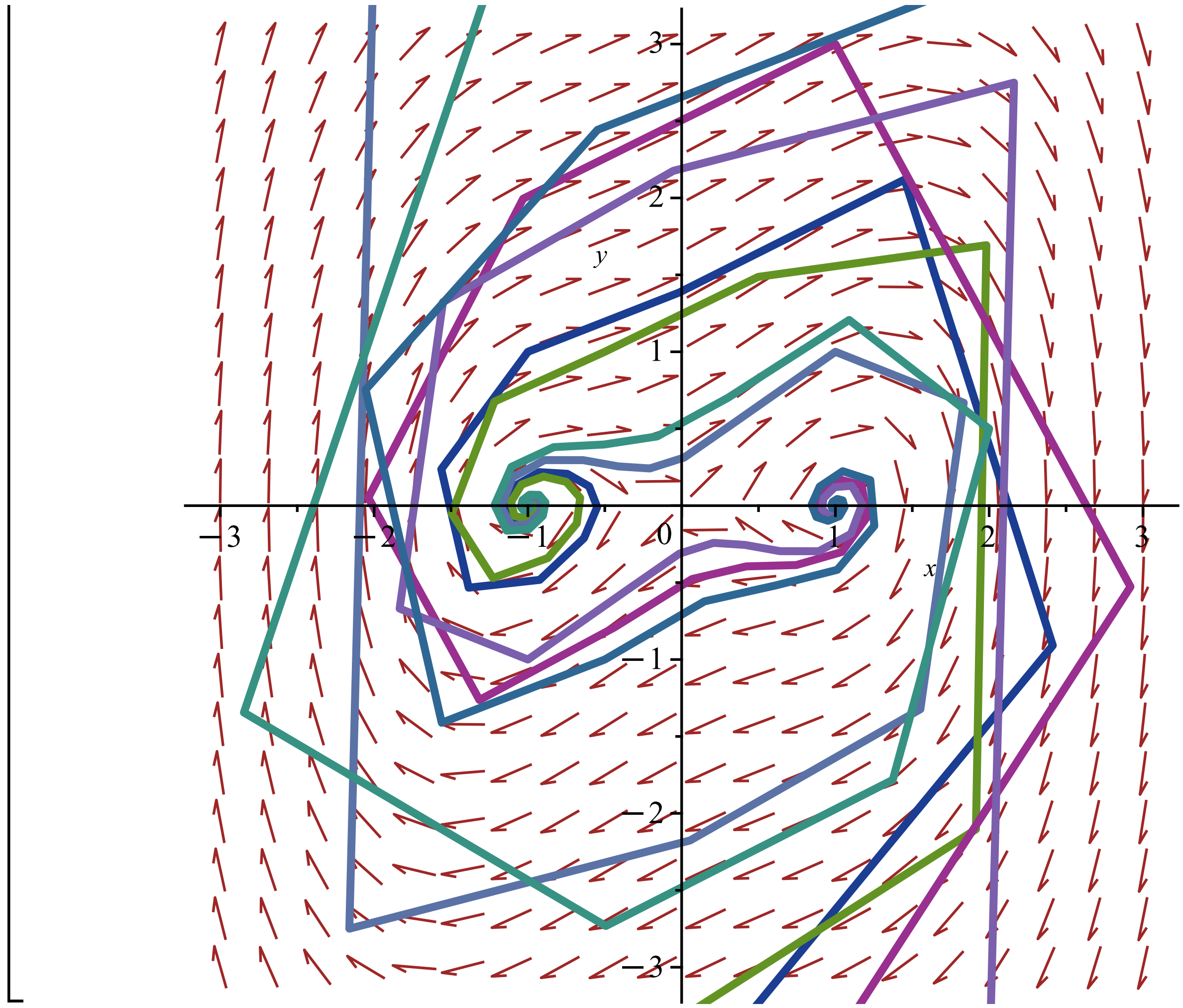
```
> #instabil de tip focus
> C:=subs(pct_ech[3,1],pct_ech[3,2],eval(J))
```

$$C := \begin{bmatrix} 0 & 2 \\ -2 & 1 \end{bmatrix} \tag{246}$$

```
> eigenvals(C)
```

$$\frac{1}{2} + \frac{I\sqrt{15}}{2}, \frac{1}{2} - \frac{I\sqrt{15}}{2} \tag{247}$$

```
> #instabil de tip focus
> DEplot([ec1,ec2],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[[x(0)=-1,
  y(0)=1],[x(0)=-1/2,y(0)=1],[x(0)=1,y(0)=1],[x(0)=1,y(0)=3],[x(0)=
  2,y(0)=1/2],[x(0)=-1,y(0)=-1],[x(0)=-1/2,y(0)=-1]])
```



y
x
3
2
1
0
−1
−2
−3