Laborator 1

· triunghi ABC cu vârfurile $A(1,1), B(4,1), C(2,3)$.

1. Determinați imaginea ΔABC printr-o scalare simplă neuniformă, de factori de scală $(2,1)$, relativ la punctul $Q(2,2)$, urmată de o rotație de unghi $90°$ față de origine.

· scalare

$$Scale(Q,2,1) = Trans(2,2) \cdot Scale(2,1) \cdot Trans(-2,-2) =$$

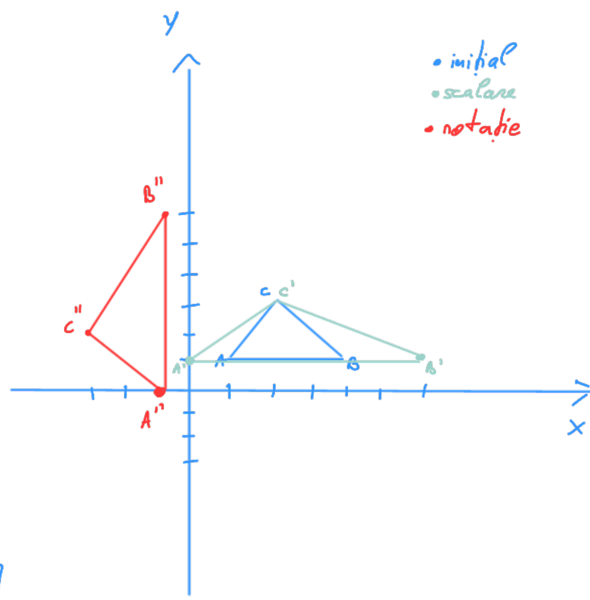
$$Trans(2,2) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad Trans(-2,-2) = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Scale(2,1) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow Scale(Q,2,1) = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A' \;\; B' \;\; C'] = Scale(Q,2,1) \cdot [A \;\; B \;\; C] = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{array}{c} \begin{matrix} A' & B' & C' \end{matrix} \\ \begin{bmatrix} 0 & 6 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \Rightarrow \begin{matrix} A'(0,1) \\ B'(6,1) \\ C'(2,3) \end{matrix}$$



· rotație

$$T = Rot(90°) \cdot S(Q,2,1)$$

$$Rot(90°) = \begin{bmatrix} \cos 90 & -\sin 90 & 1 \\ \sin 90 & \cos 90 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A'' \;\; B'' \;\; C''] = T \cdot [A \;\; B \;\; C] = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{array}{c} \begin{matrix} A'' & B'' & C'' \end{matrix} \\ \begin{bmatrix} -1 & -1 & -3 \\ 0 & 6 & 2 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

2. Det. imag. ΔABC printr-o forfecare de unghi $45°$, relativ la pct. $Q(2,2)$ în direcția vect $v(2,1)$.

$$Shear(Q, v, tg\,\theta) = \begin{pmatrix} 1+tg\,\theta\, v_1 v_2 & -tg\,\theta\, v_1^2 & tg\,\theta \cdot v_1(v_1 q_2 - v_2 q_1) \\ tg\,\theta\, v_2^2 & 1-tg\,\theta\, v_1 v_2 & tg\,\theta \cdot v_2(v_1 q_2 - v_2 q_1) \\ 0 & 0 & 1 \end{pmatrix}$$

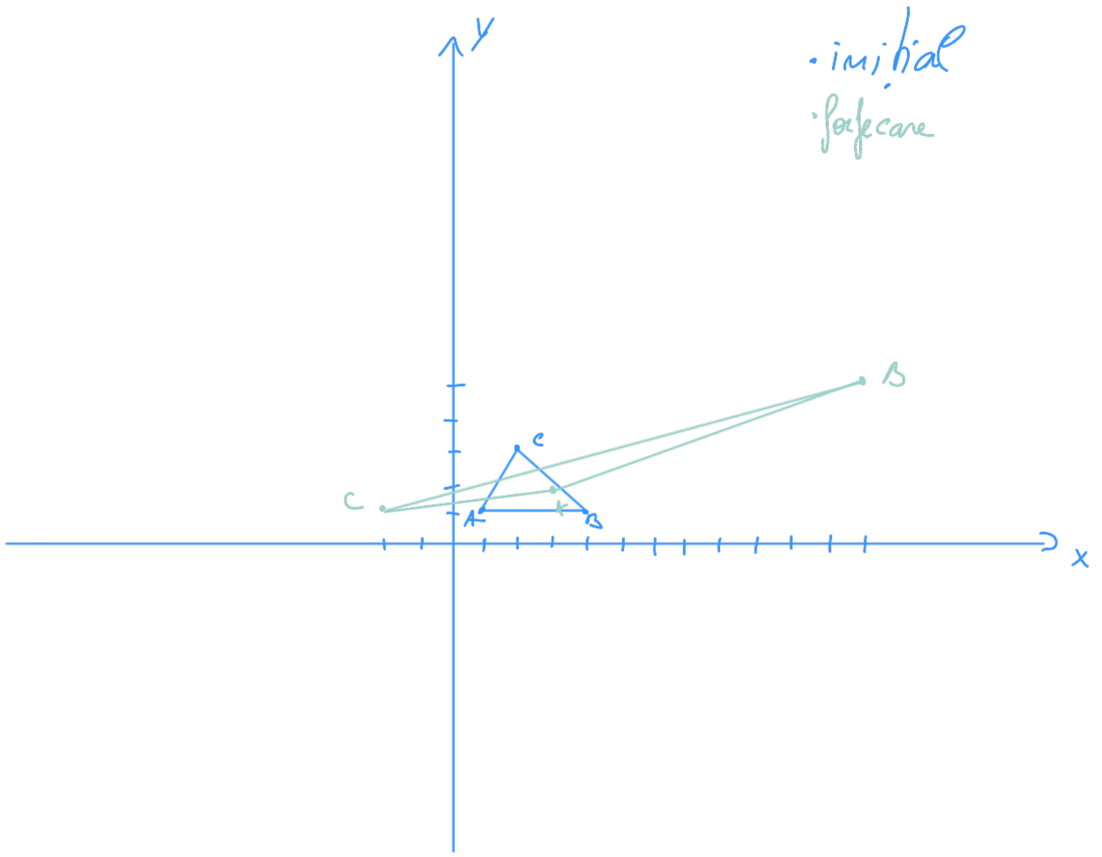
$tg\ 45 = 1$

$$Shear(Q, v, tg\,\theta) = \begin{pmatrix} 3 & -4 & 4 \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[A' \ B' \ C'] = Shear(Q, v, tg\,\theta) \cdot [A \ B \ C] = Shear(Q, v, tg\,\theta) \cdot \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 & -2 \\ 2 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

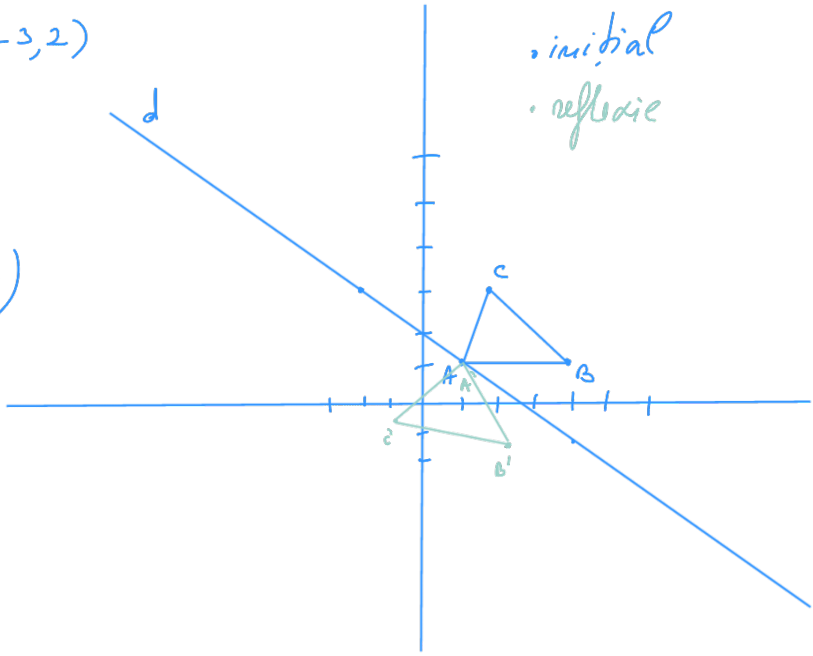
$\Rightarrow$ $A'(3,2)$
$B'(12,5)$
$C'(-2,1)$



3. Det. imag. ΔABC prin reflexia relativ la dreapta $2x+3y-5=0$.

$d: 2x+3y-5=0$, $\vec{n}(2,3)$ $\Rightarrow A(1,1)\in d$, $\vec{v}(-3,2)$

$tg\,\theta = -\frac{2}{3} \Rightarrow \theta = \text{arctg}\left(-\frac{2}{3}\right)$



1) $Trans(-1,-1) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ → translație de vector $(0, \frac{c}{5})$

2) rotire cu $-\theta$

$$R(-\theta) = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) & 0 \\ \sin(-\theta) & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) reflexie relativ la Ox

$$R_{Ox} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4) rotire cu θ

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cos\theta = -\frac{3}{\sqrt{3^2+2^2}} = -\frac{3}{\sqrt{13}}$$

$$\sin\theta = \frac{2}{\sqrt{13}}$$

5) $Trans(1,1) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$R_f = Trans(-1,-1)\cdot R(-\theta)\cdot R_{Ox}\cdot R(\theta)\cdot Trans(1,1) = \begin{bmatrix} \frac{5}{13} & -\frac{12}{13} & \frac{20}{13} \\ -\frac{12}{13} & -\frac{5}{13} & \frac{30}{13} \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A'\ B'\ C'] = R_f\cdot[A\ B\ C] = R_f\cdot\begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 1 & \frac{28}{13} & -\frac{6}{13} \\ 1 & -\frac{23}{13} & -\frac{9}{13} \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow$

$A'(1,1)$

$B'\left(\frac{28}{13}, -\frac{23}{13}\right)$

$C'\left(-\frac{6}{13}, -\frac{9}{13}\right)$

4. Det. imag. ΔABC prin rotația cu $90°$ în jurul punctului C, urmată de reflexia relativ la dreapta AB.

• rotație $90°$

$$Rot(90°) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Trans(2,3) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Trans(-2,-3) = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_1 = Trans(2,3) \cdot Rot(90) \cdot Trans(-2,-3) = \begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A' \ B' \ C'] = T_1 \cdot [A \ B \ C] = T_1 \cdot \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{array}{c} \begin{matrix} A' & B' & C' \end{matrix} \\ \begin{bmatrix} 4 & 4 & 2 \\ 2 & 5 & 3 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow$ $A'(4,2)$, $B'(4,5)$, $C'(2,3)$

• reflexie față de $AB: y=1$

$$T_{\downarrow} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \qquad T_{\uparrow} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad R_{Ox} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = T_{\downarrow} \cdot R_{Ox} \cdot T_{\uparrow} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T = T_2 \cdot T_1 = \begin{bmatrix} 0 & -1 & 5 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A'' \ B'' \ C''] = T \cdot [A \ B \ C] = T \begin{bmatrix} 1 & 4 & 2 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{array}{c} \begin{matrix} A'' & B'' & C'' \end{matrix} \\ \begin{bmatrix} 4 & 4 & 2 \\ 0 & -3 & -1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$\Rightarrow$ $A''(4,0)$

$B''(4,-3)$

$C''(2,-1)$