

```
> #ex 1:
> ecdif1:=diff(x(t),t)=-k*x(t)
```

$$ecdif1 := \frac{d}{dt} x(t) = -k x(t) \quad (1)$$

```
> cond_in:=x(0)=x__0
```

$$cond_in := x(0) = x_0 \quad (2)$$

```
> sol:=dsolve({ecdif1,cond_in},x(t))
```

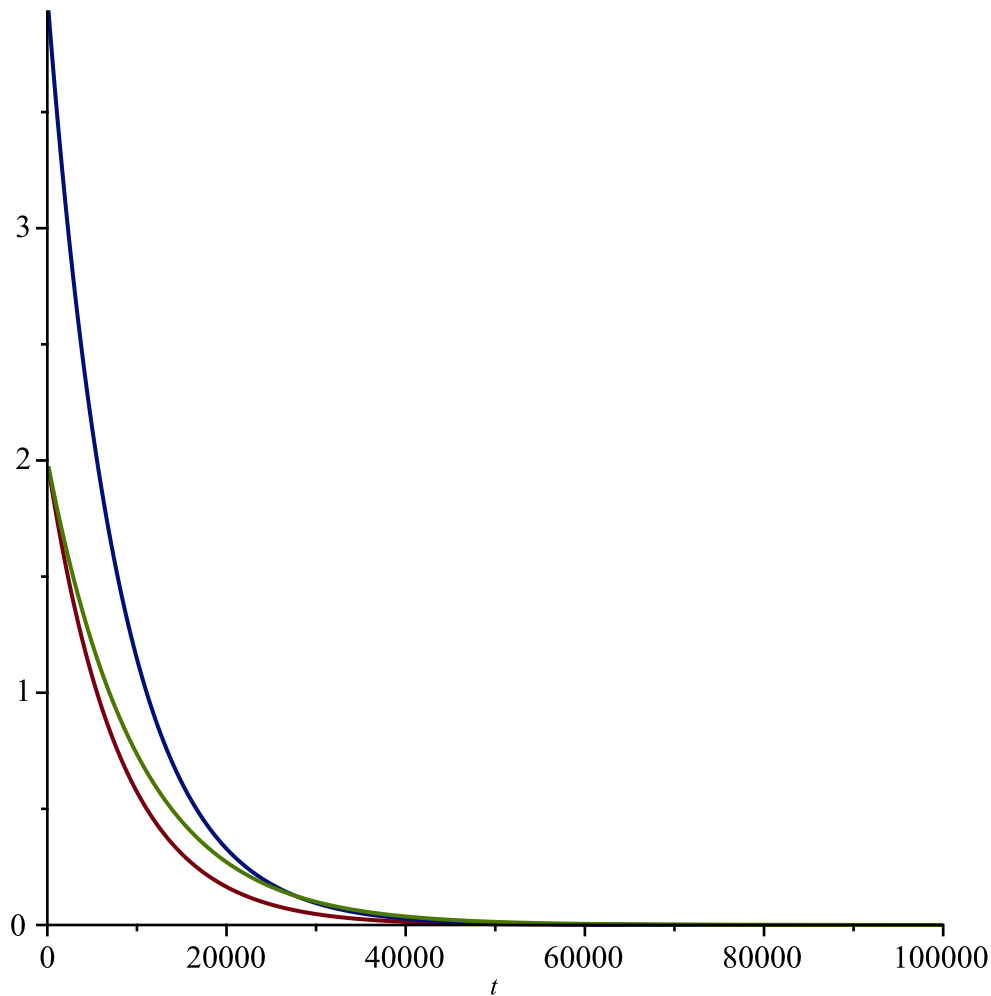
$$sol := x(t) = x_0 e^{-kt} \quad (3)$$

```
> xx:=unapply(rhs(sol),t,x__0,k)
```

$$xx := (t, x_0, k) \mapsto x_0 \cdot e^{-k \cdot t} \quad (4)$$

```
> with(plots):
```

```
> plot([xx(t,2,1/8000),xx(t,4,1/8000),xx(t,2,1/10000)],t=0..100000)
```



```
> #xx(t,x__0,k)
```

```
> ec1:=xx(5730,x__0,k)=x__0/2
```

$$ec1 := x_0 e^{-5730 k} = \frac{x_0}{2} \quad (5)$$

```
> k__c14:=solve(ec1,k)
```

$$k_{c14} := \frac{\ln(2)}{5730} \quad (6)$$

$$\begin{aligned} &> \text{ec2} := \text{xx}(t, x_0, k_c14) = 0.2 * x_0 \\ & \qquad \qquad \qquad ec2 := x_0 e^{-k_{c14} t} = 0.2 x_0 \end{aligned} \tag{7}$$

$$\begin{aligned} &> \text{timp} := \text{solve}(\text{ec2}, t) \\ & \qquad \qquad \qquad timp := 13304.64798 \end{aligned} \tag{8}$$

$$\begin{aligned} &> \text{ec3} := \text{xx}(t, x_0, k_c14) = 0.9157 * x_0 \\ & \qquad \qquad \qquad ec3 := x_0 e^{-\frac{\ln(2) t}{5730}} = 0.9157 x_0 \end{aligned} \tag{9}$$

$$\begin{aligned} &> t1 := \text{solve}(\text{ec3}, t) \\ & \qquad \qquad \qquad t1 := 728.0141045 \end{aligned} \tag{10}$$

$$\begin{aligned} &> \text{ec4} := \text{xx}(t, x_0, k_c14) = 0.93021 * x_0 \\ & \qquad \qquad \qquad ec4 := x_0 e^{-\frac{\ln(2) t}{5730}} = 0.93021 x_0 \end{aligned} \tag{11}$$

$$\begin{aligned} &> t2 := \text{solve}(\text{ec4}, t) \\ & \qquad \qquad \qquad t2 := 598.0495293 \end{aligned} \tag{12}$$

$$\begin{aligned} &> T1 := 1988 - t1 \\ & \qquad \qquad \qquad T1 := 1259.985896 \end{aligned} \tag{13}$$

$$\begin{aligned} &> T2 := 1988 - t2 \\ & \qquad \qquad \qquad T2 := 1389.950471 \end{aligned} \tag{14}$$

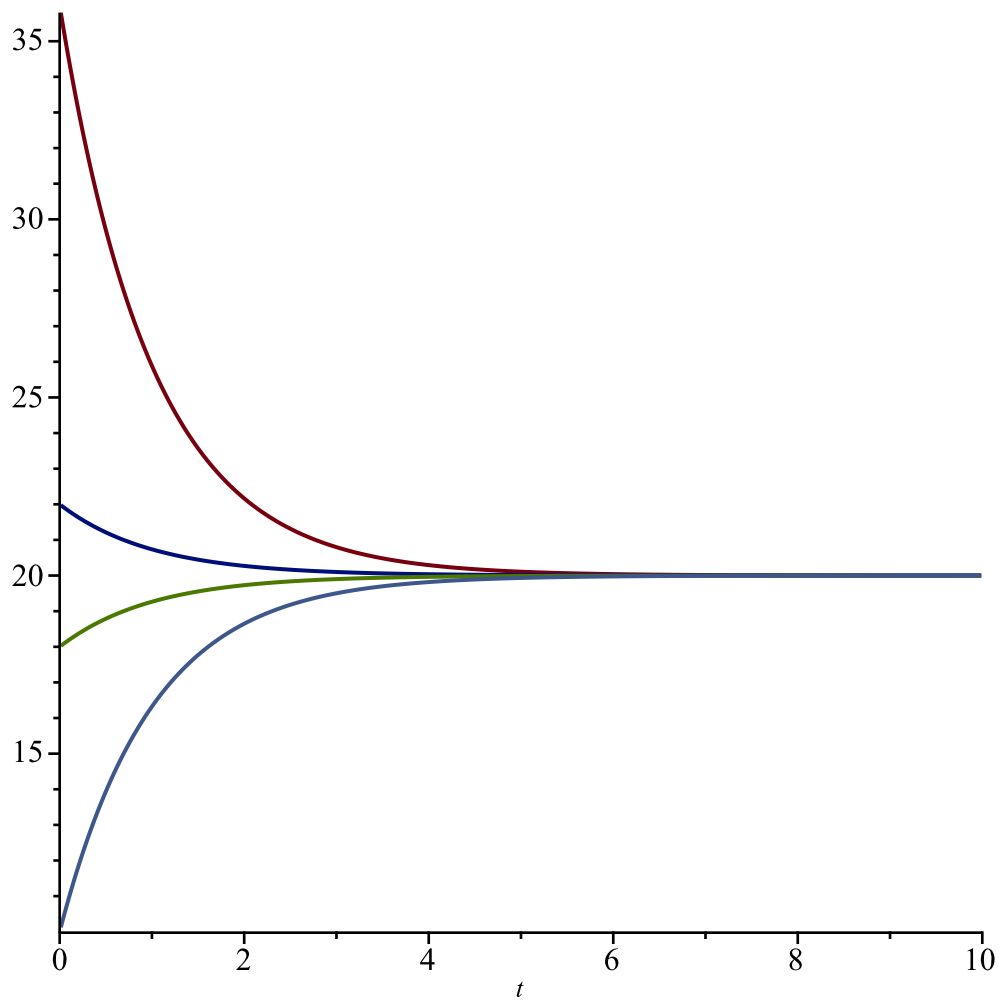
$$\begin{aligned} &> \# \text{ex2:} \\ &> \text{restart} \\ &> \text{ecdif} := \text{diff}(T(t), t) = -k * (T(t) - T_m) \\ & \qquad \qquad \qquad ecdif := \frac{d}{dt} T(t) = -k (T(t) - T_m) \end{aligned} \tag{15}$$

$$\begin{aligned} &> \text{cond_in} := T(0) = T_0 \\ & \qquad \qquad \qquad cond_in := T(0) = T_0 \end{aligned} \tag{16}$$

$$\begin{aligned} &> \text{sol} := \text{dsolve}(\{\text{ecdif}, \text{cond_in}\}, T(t)) \\ & \qquad \qquad \qquad sol := T(t) = T_m + e^{-kt} (T_0 - T_m) \end{aligned} \tag{17}$$

$$\begin{aligned} &> \text{with}(\text{plots}): \\ &> \text{TT} := \text{unapply}(\text{rhs}(\text{sol}), t, k, T_0, T_m) \\ & \qquad \qquad \qquad TT := (t, k, T_0, T_m) \mapsto T_m + e^{-k \cdot t} \cdot (T_0 - T_m) \end{aligned} \tag{18}$$

$$> \text{plot}([TT(t, 1, 36, 20), TT(t, 1, 22, 20), TT(t, 1, 18, 20), TT(t, 1, 10, 20)], t = 0..10)$$



```
> T__0:=36;T__m:=21;T__1:=34.22;T__2:=34.11
```

$$T_0 := 36$$

$$T_m := 21$$

$$T_1 := 34.22$$

$$T_2 := 34.11$$

(19)

```
> ec1:=TT(t,k,T__0,T__m)=T__1
```

$$ec1 := 21 + 15 e^{-kt} = 34.22$$

(20)

```
> ec2:=TT(t+1,k,T__0,T__m)=T__2
```

$$ec2 := 21 + 15 e^{-k(t+1)} = 34.11$$

(21)

```
> sist:=ec1,ec2
```

$$sist := 21 + 15 e^{-kt} = 34.22, 21 + 15 e^{-k(t+1)} = 34.11$$

(22)

```
> sols:=solve({sist},{t,k})
```

$$sols := \{k = 0.008355536648, t = 15.11804352\}$$

(23)

```
> timp:=sols[2]
```

$$timp := t = 15.11804352$$

(24)

```
> ora:=11.30-timp+24
```

$$ora := -t + 35.30 = 20.18195648$$

(25)

```

> #ex3
> restart
> ecdif1:=diff(x(t),t)=r*x(t)
                                     
$$ecdif1 := \frac{d}{dt} x(t) = r x(t) \quad (26)$$

> cond_in1:=x(0)=x__0
                                     
$$cond\_in1 := x(0) = x_0 \quad (27)$$

> ecdif2:=diff(x(t),t)=r__0*x(t)*(1-x(t)/K)
                                     
$$ecdif2 := \frac{d}{dt} x(t) = r_0 x(t) \left(1 - \frac{x(t)}{K}\right) \quad (28)$$

> cond_in2:=x(0)=x__0
                                     
$$cond\_in2 := x(0) = x_0 \quad (29)$$

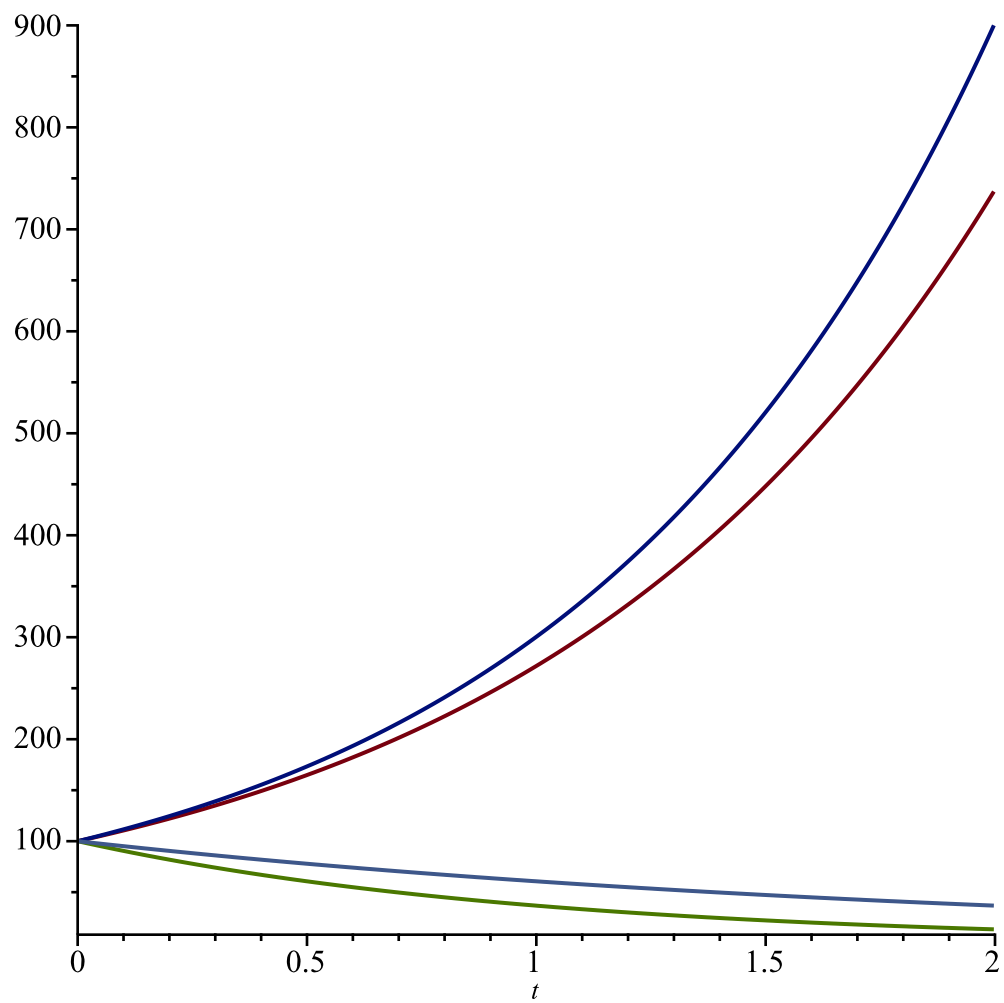
> sol1:=dsolve({ecdif1,cond_in1},x(t))
                                     
$$sol1 := x(t) = x_0 e^{rt} \quad (30)$$

> sol2:=dsolve({ecdif2,cond_in2},x(t))
                                     
$$sol2 := x(t) = \frac{K x_0}{(K - x_0) e^{-r_0 t} + x_0} \quad (31)$$

> xM:=unapply(rhs(sol1),t,x__0,r)
                                     
$$xM := (t, x_0, r) \mapsto x_0 \cdot e^{r \cdot t} \quad (32)$$

> with(plots):
> plot([xM(t,100,1),xM(t,100,1.1),xM(t,100,-1),xM(t,100,-0.5)],t=0.
.2)

```

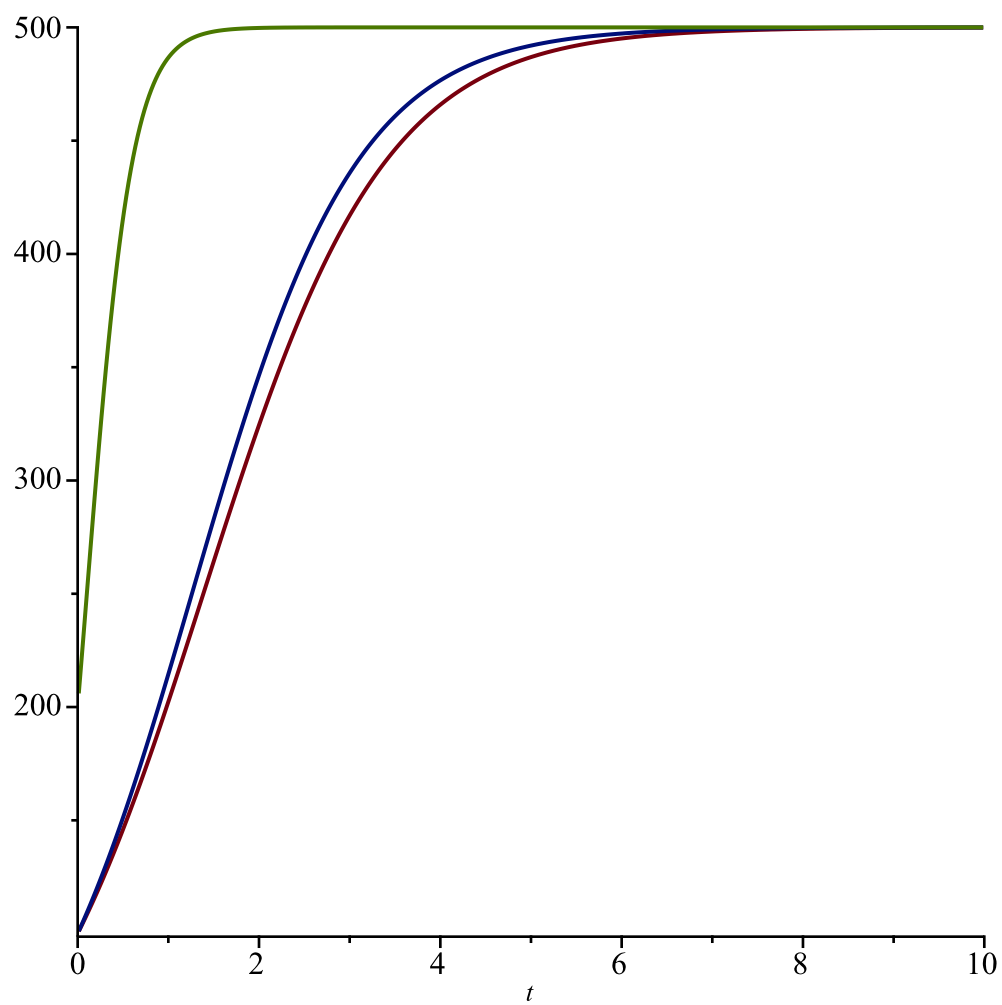


```
> xV:=unapply(rhs(sol2),t,x__0,r__0,K)
```

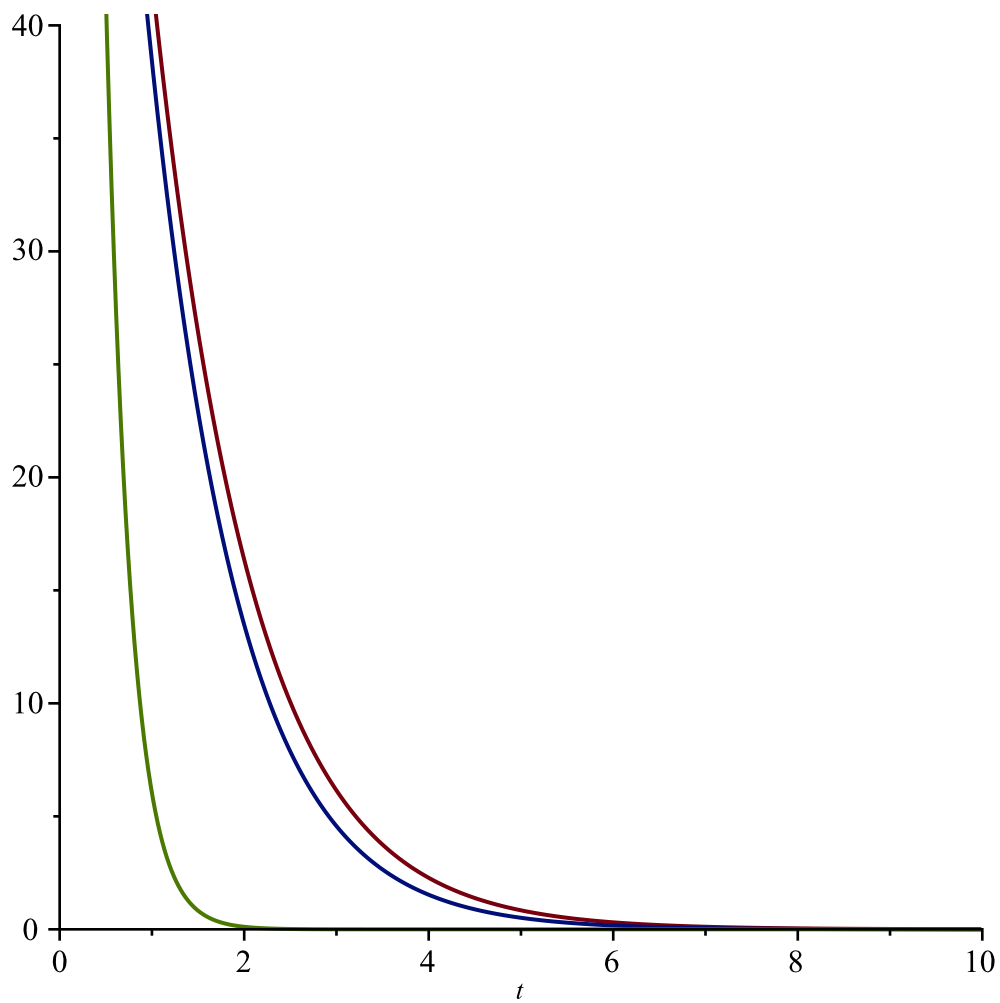
$$xV := (t, x_0, r_0, K) \mapsto \frac{K \cdot x_0}{(K - x_0) \cdot e^{-r_0 \cdot t} + x_0}$$

(33)

```
> plot([xV(t,100,1,500),xV(t,100,1.1,500),xV(t,200,4,500)],t=0..10)
```



```
> plot([xV(t,100,-1,500),xV(t,100,-1.1,500),xV(t,200,-4,500)],t=0.  
.10)
```



```
> x__0:=25*10^3;final:=30*10^3;timp:=2
```

$$x_0 := 25000$$

$$final := 30000$$

$$timp := 2$$

(34)

```
> ec:=xM(timp,x__0,r)=final
```

$$ec := 25000 e^{2r} = 30000$$

(35)

```
> rM:=solve(ec,r)
```

$$rM := \frac{\ln\left(\frac{6}{5}\right)}{2}$$

(36)

```
> est:=xM(5,x__0,rM);evalf(%)
```

$$est := 7200 \sqrt{30}$$

$$39436.02414$$

(37)

```
> x__0:=20*10^3;f1:=40*10^3;t1:=2;f2:=50*10^3;t2:=3
```

$$x_0 := 20000$$

$$f1 := 40000$$

$$t1 := 2$$

$$f2 := 50000$$

$$t2 := 3 \quad (38)$$

> ec1:=xV(t1,x__0,r__0,K)=f1

$$ec1 := \frac{20000 K}{(K - 20000) e^{-2 r_0} + 20000} = 40000 \quad (39)$$

> ec2:=xV(t2,x__0,r__0,K)=f2

$$ec2 := \frac{20000 K}{(K - 20000) e^{-3 r_0} + 20000} = 50000 \quad (40)$$

> sist:=ec1,ec2

$$sist := \frac{20000 K}{(K - 20000) e^{-2 r_0} + 20000} = 40000, \frac{20000 K}{(K - 20000) e^{-3 r_0} + 20000} = 50000 \quad (41)$$

> s:=solve({sist},{r__0,K})

$$s := \left\{ K = \frac{200000 \operatorname{RootOf}(5 _Z^2 - _Z - 1)}{7} + \frac{400000}{7}, r_0 = -\ln(\operatorname{RootOf}(5 _Z^2 - _Z - 1)) \right\} \quad (42)$$

> r00:=evalf(rhs(s[2]))

$$r00 := 0.5829348290 \quad (43)$$

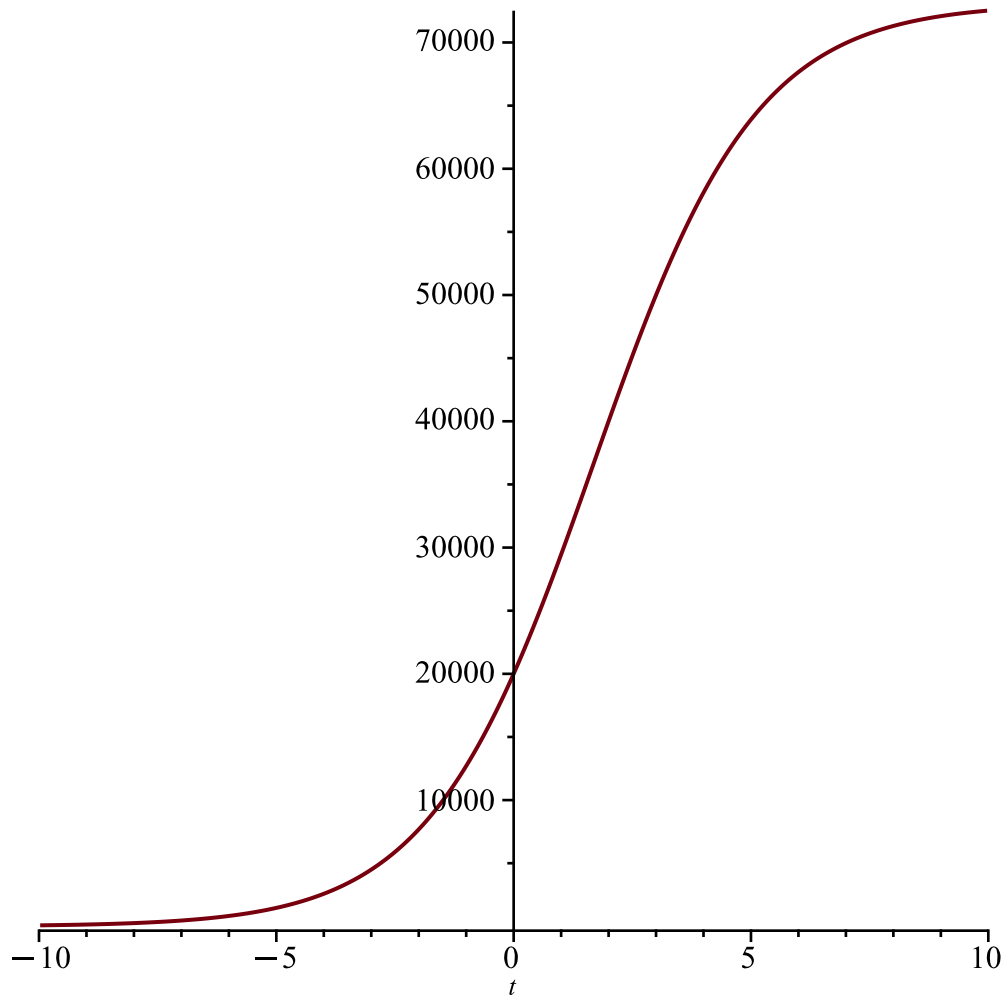
> K1:=evalf(rhs(s[1]))

$$K1 := 73093.07341 \quad (44)$$

> est1:=xV(7,x__0,r00,K1)

$$est1 := 69954.95937 \quad (45)$$

> plot(xV(t,x__0,r00,K1,t=0..30))



```
> #ex:
```

```
> restart
```

```
> ecdif:=diff(v(x),x)*v(x)=-g*R^2/(x+R)^2
```

$$ecdif := \left(\frac{d}{dx} v(x) \right) v(x) = - \frac{g R^2}{(x + R)^2} \quad (46)$$

```
> cond_in:=v(0)=v__0
```

$$cond_in := v(0) = v_0 \quad (47)$$

```
> sol:=dsolve({ecdif,cond_in},v(x),implicit)
```

$$sol := v(x)^2 - \frac{2 g R^2}{x + R} + 2 g R - v_0^2 = 0 \quad (48)$$

```
> ec:=lhs(sol)=0
```

$$ec := v(x)^2 - \frac{2 g R^2}{x + R} + 2 g R - v_0^2 = 0 \quad (49)$$

```
> vs:=solve(ec,v(x))
```

$$vs := \frac{\sqrt{-(x + R) (2 g R x - v_0^2 R - v_0^2 x)}}{x + R}, - \frac{\sqrt{-(x + R) (2 g R x - v_0^2 R - v_0^2 x)}}{x + R} \quad (50)$$

```
> vv:=unapply(vs[1],x,v__0,g,R)
```

(51)

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> #de terminat b,c,d,e :))
```