

$$\begin{aligned}
&> \text{ec1}:=\mathbf{x}*\text{diff}(\mathbf{y}(\mathbf{x}),\mathbf{x})+\mathbf{y}(\mathbf{x})=\exp(\mathbf{x}) \\
&\qquad\qquad\qquad ec1 := x \left(\frac{d}{dx} y(x) \right) + y(x) = e^x \tag{1} \\
&= \\
&> \text{cond}:=\mathbf{y}(\mathbf{a})=\mathbf{b} \\
&\qquad\qquad\qquad cond := y(a) = b \tag{2} \\
&= \\
&> \text{sist}:=\{\text{ec1},\text{cond}\} \\
&\qquad\qquad\qquad sist := \left\{ x \left(\frac{d}{dx} y(x) \right) + y(x) = e^x, y(a) = b \right\} \tag{3} \\
&= \\
&> \text{sol1}:=\text{dsolve}(\text{ec1},\mathbf{y}(\mathbf{x})) \\
&\qquad\qquad\qquad sol1 := y(x) = \frac{e^x + c_1}{x} \tag{4} \\
&= \\
&> \text{sol2}:=\text{dsolve}(\text{sist},\{\mathbf{y}(\mathbf{x})\}) \\
&\qquad\qquad\qquad sol2 := y(x) = \frac{e^x + b a - e^a}{x} \tag{5} \\
&= \\
&> \text{restart} \\
&> \text{ec1}:=\text{diff}(\mathbf{y1}(\mathbf{x}),\mathbf{x})=\mathbf{y1}(\mathbf{x})+\mathbf{y2}(\mathbf{x}) \\
&\qquad\qquad\qquad ec1 := \frac{d}{dx} y1(x) = y1(x) + y2(x) \tag{6} \\
&= \\
&> \text{ec2}:=\text{diff}(\mathbf{y2}(\mathbf{x}),\mathbf{x})=-2*\mathbf{y1}(\mathbf{x})+4*\mathbf{y2}(\mathbf{x}) \\
&\qquad\qquad\qquad ec2 := \frac{d}{dx} y2(x) = -2 y1(x) + 4 y2(x) \tag{7} \\
&= \\
&> \text{sol}:=\text{dsolve}(\{\text{ec1},\text{ec2}\},\{\mathbf{y1}(\mathbf{x}),\mathbf{y2}(\mathbf{x})\}) \\
&\qquad\qquad\qquad sol := \{y1(x) = c_1 e^{2x} + c_2 e^{3x}, y2(x) = c_1 e^{2x} + 2 c_2 e^{3x}\} \tag{8} \\
&= \\
&> \text{cond}:=\mathbf{y1}(0)=0,\mathbf{y2}(0)=-1 \\
&\qquad\qquad\qquad cond := y1(0) = 0, y2(0) = -1 \tag{9} \\
&= \\
&> \text{sol2}:=\text{dsolve}(\{\text{ec1},\text{ec2},\text{cond}\},\{\mathbf{y1}(\mathbf{x}),\mathbf{y2}(\mathbf{x})\}) \\
&\qquad\qquad\qquad sol2 := \{y1(x) = e^{2x} - e^{3x}, y2(x) = e^{2x} - 2 e^{3x}\} \tag{10} \\
&= \\
&> \text{restart} \\
&> \text{ec1}:=\text{diff}(\mathbf{y}(\mathbf{x}),\mathbf{x}\$2)-4*\text{diff}(\mathbf{y}(\mathbf{x}),\mathbf{x})+5*\mathbf{y}(\mathbf{x})=2*\mathbf{x}^2*\exp(\mathbf{x}) \\
&\qquad\qquad\qquad ec1 := \frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 5 y(x) = 2 x^2 e^x \tag{11} \\
&= \\
&> \text{sol}:=\text{dsolve}(\text{ec1},\mathbf{y}(\mathbf{x})) \\
&\qquad\qquad\qquad sol := y(x) = c_2 e^{2x} \sin(x) + c_1 e^{2x} \cos(x) + (x+1)^2 e^x \tag{12} \\
&= \\
&> \text{cond}:=\mathbf{y}(0)=2,\mathbf{D}(\mathbf{y})(0)=3 \\
&\qquad\qquad\qquad cond := y(0) = 2, D(y)(0) = 3 \tag{13} \\
&= \\
&> \text{solv}:=\text{dsolve}(\{\text{ec1},\text{cond}\},\{\mathbf{y}(\mathbf{x})\}) \\
&\qquad\qquad\qquad solv := y(x) = (\cos(x) - 2 \sin(x)) e^{2x} + (x+1)^2 e^x \tag{14} \\
&= \\
&> \text{restart} \\
&> \text{ec}:=\text{diff}(\mathbf{y}(\mathbf{x}),\mathbf{x}\$2)+4*\mathbf{y}(\mathbf{x})=4*\mathbf{x}
\end{aligned}$$

$$ec := \frac{d^2}{dx^2} y(x) + 4 y(x) = 4 x \quad (15)$$

> cond:=y(Pi)=0,D(y)(Pi)=1

$$cond := y(\pi) = 0, D(y)(\pi) = 1 \quad (16)$$

> sol:=dsolve({ec,cond},{y(x)})

$$sol := y(x) = -\cos(2 x) \pi + x \quad (17)$$

> ec1:=diff(y(x),x\$2)+4*y(x)=0

$$ec1 := \frac{d^2}{dx^2} y(x) + 4 y(x) = 0 \quad (18)$$

> solv:=dsolve(ec1,y(x))

$$solv := y(x) = c_1 \sin(2 x) + c_2 \cos(2 x) \quad (19)$$

> restart

> ec:=diff(y(x),x\$2)+6*diff(y(x),x)+9*y(x)=6*cos(x)+8*sin(x)

$$ec := \frac{d^2}{dx^2} y(x) + 6 \frac{d}{dx} y(x) + 9 y(x) = 6 \cos(x) + 8 \sin(x) \quad (20)$$

> cond:=y(0)=1,D(y)(0)=1

$$cond := y(0) = 1, D(y)(0) = 1 \quad (21)$$

> sol:=dsolve(ec,y(x))

$$sol := y(x) = e^{-3 x} c_2 + e^{-3 x} x c_1 + \sin(x) \quad (22)$$

> sol2:=dsolve({ec,cond},{y(x)})

$$sol2 := y(x) = e^{-3 x} + 3 e^{-3 x} x + \sin(x) \quad (23)$$

> restart

> ec:=diff(y(x),x\$2)+lambda*y(x)=0

$$ec := \frac{d^2}{dx^2} y(x) + \lambda y(x) = 0 \quad (24)$$

> cond:=y(0)=0,y(1)=0

$$cond := y(0) = 0, y(1) = 0 \quad (25)$$

> sol:=dsolve(ec,y(x))

$$sol := y(x) = c_1 \sin(\sqrt{\lambda} x) + c_2 \cos(\sqrt{\lambda} x) \quad (26)$$

> sol2:=dsolve({ec,cond},{y(x),lambda})

$$sol2 := \left\{ \lambda = c_1, y(x) = 0 \right\}, \left\{ \lambda = \pi^2 _Z1 \sim^2, y(x) = c_2 \sin\left(\sqrt{\pi^2 _Z1 \sim^2} x\right) \right\} \quad (27)$$

> restart

> ec:=x^2*diff(y(x),x)*cos(1/x)-y(x)*sin(1/x)=-1

$$ec := x^2 \left(\frac{d}{dx} y(x) \right) \cos\left(\frac{1}{x}\right) - y(x) \sin\left(\frac{1}{x}\right) = -1 \quad (28)$$

> sol:=dsolve(ec,y(x))

$$sol := y(x) = \left(\tan\left(\frac{1}{x}\right) + c_1 \right) \cos\left(\frac{1}{x}\right) \quad (29)$$

> cond:=limit(y(x),x=infinity)=0

$$(30)$$

$$cond := \lim_{x \rightarrow \infty} y(x) = 0 \quad (30)$$

```
> solv:=dsolve({ec,cond},{y(x)})
```

$$solv := \left[\left\{ \lim_{x \rightarrow \infty} y(x) \right\} \right] \quad (31)$$

```
> restart; with(linalg): with(DEtools):
```

```
> ec1:=diff(x(t),t)=x(t)
```

$$ec1 := \frac{d}{dt} x(t) = x(t) \quad (32)$$

```
> ec2:=diff(y(t),t)=x(t)+2*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2y(t) \quad (33)$$

```
> sist1:=ec1,ec2
```

$$sist1 := \frac{d}{dt} x(t) = x(t), \frac{d}{dt} y(t) = x(t) + 2y(t) \quad (34)$$

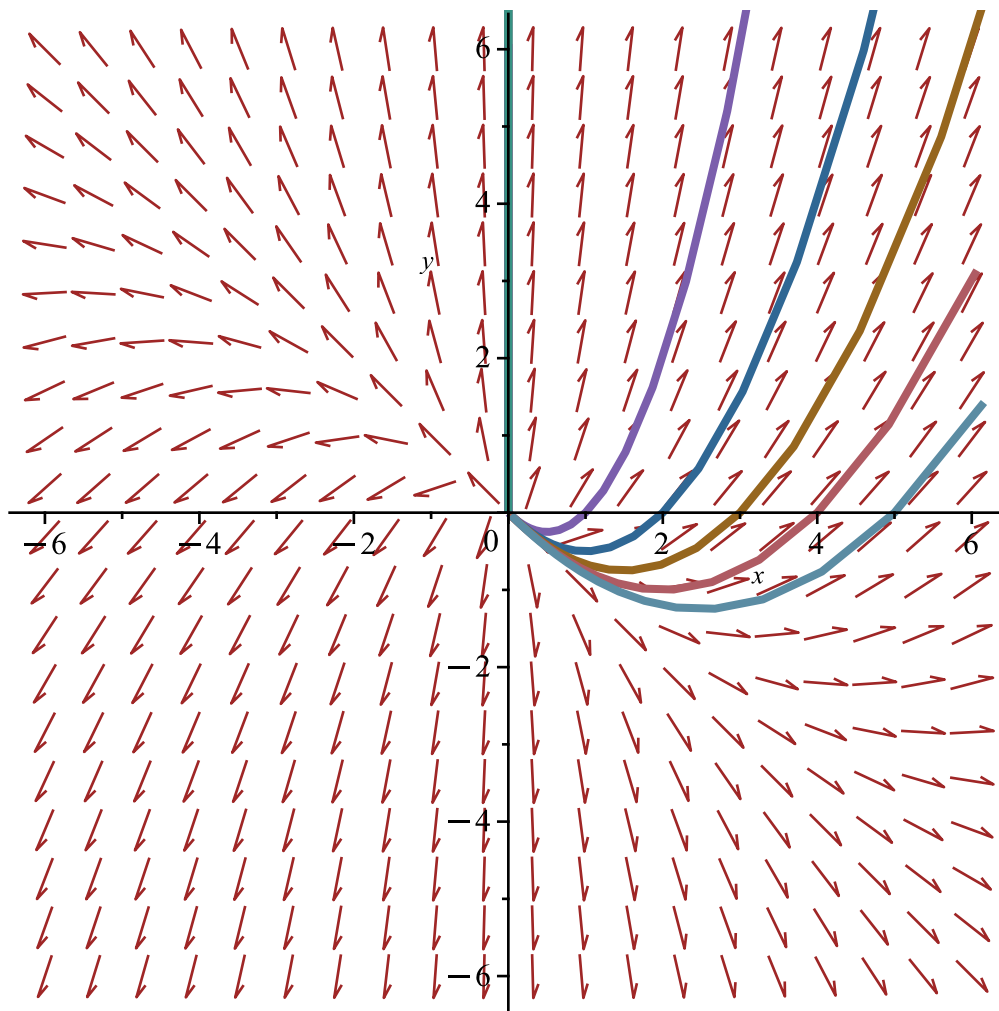
```
> A:=matrix([[1,0],[1,2]])
```

$$A := \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \quad (35)$$

```
> eigenvals(A)
```

$$1, 2 \quad (36)$$

```
> DEplot([sist1],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[[x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0]])
```



```
> sol:=dsolve({sist},{x(t),y(t)})
Error, (in dsolve) not a system with respect to the unknowns {x(t), y(t)}
```

```
> restart
> ec1:=x*diff(y(x),x$2)+diff(y(x),x)=4*x
      ec1 := x \left( \frac{d^2}{dx^2} y(x) \right) + \frac{d}{dx} y(x) = 4x \quad (37)
```

```
> cond:=y(1)=1,D(y)(1)=4
      cond := y(1) = 1, D(y)(1) = 4 \quad (38)
```

```
> sol:=dsolve(ec1,y(x))
      sol := y(x) = x^2 + c_1 \ln(x) + c_2 \quad (39)
```

```
> sol2:=dsolve({ec1,cond},{y(x)})
      sol2 := y(x) = x^2 + 2 \ln(x) \quad (40)
```

```
> restart
```

```
> ec:=x^2*diff(y(x),x)=x*y(x)+y(x)^2
      ec := x^2 \left( \frac{d}{dx} y(x) \right) = x y(x) + y(x)^2 \quad (41)
```

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = -\frac{x}{\ln(x) - c_1} \quad (42)$$

```
> ec2:=diff(y(x),x$2)-4*diff(y(x),x)+8*y(x)=(25*x-5)*exp(x)
```

$$ec2 := \frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 8 y(x) = (25x - 5) e^x \quad (43)$$

```
> sol2:=dsolve(ec2,y(x))
```

$$sol2 := y(x) = e^{2x} \sin(2x) c_2 + e^{2x} \cos(2x) c_1 + (5x + 1) e^x \quad (44)$$

```
> restart
```

```
> ec1:=diff(y(t),t)=-4*x(t)
```

$$ec1 := \frac{d}{dt} y(t) = -4x(t) \quad (45)$$

```
> ec2:=diff(x(t),t)=y(t)
```

$$ec2 := \frac{d}{dt} x(t) = y(t) \quad (46)$$

```
> sol:=dsolve({ec1,ec2},{x(t),y(t)})
```

$$sol := \{x(t) = c_1 \sin(2t) + c_2 \cos(2t), y(t) = 2c_1 \cos(2t) - 2c_2 \sin(2t)\} \quad (47)$$

```
> cond:=x(0)=eta__1,y(0)=eta__2
```

$$cond := x(0) = \eta_1, y(0) = \eta_2 \quad (48)$$

```
> sol2:=dsolve({ec1,ec2,cond},{x(t),y(t)})
```

$$sol2 := \left\{ x(t) = \frac{\eta_2 \sin(2t)}{2} + \eta_1 \cos(2t), y(t) = \eta_2 \cos(2t) - 2\eta_1 \sin(2t) \right\} \quad (49)$$

```
> restart
```

```
> ec:=diff(y(x),x$2)+6*diff(y(x),x)+9*y(x)=27*x^2
```

$$ec := \frac{d^2}{dx^2} y(x) + 6 \frac{d}{dx} y(x) + 9 y(x) = 27x^2 \quad (50)$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = e^{-3x} c_2 + e^{-3x} x c_1 + 3x^2 - 4x + 2 \quad (51)$$

```
> restart
```

```
> ec:=diff(y(x),x$2)-1/x*diff(y(x),x)=2
```

$$ec := \frac{d^2}{dx^2} y(x) - \frac{\frac{d}{dx} y(x)}{x} = 2 \quad (52)$$

```
> sol:=dsolve(ec,y(x))
```

$$sol := y(x) = x^2 \ln(x) - \frac{x^2}{2} + \frac{c_1 x^2}{2} + c_2 \quad (53)$$

```
> cond:=y(1)=0,y(2)=4*ln(2)
```

$$cond := y(1) = 0, y(2) = 4 \ln(2) \quad (54)$$

```
> sol2:=dsolve({ec,cond},{y(x)})
```

(55)

$$sol2 := y(x) = x^2 \ln(x) \quad (55)$$

> restart

> ec:=(1+x^3)*diff(y(x),x\$2)-3*x^2*diff(y(x),x)=0

$$ec := (x^3 + 1) \left(\frac{d^2}{dx^2} y(x) \right) - 3x^2 \left(\frac{d}{dx} y(x) \right) = 0 \quad (56)$$

> cond:=y(0)=0,y(2)=6

$$cond := y(0) = 0, y(2) = 6 \quad (57)$$

> sol:=dsolve(ec,y(x))

$$sol := y(x) = c_1 + \left(\frac{1}{4} x^4 + x \right) c_2 \quad (58)$$

> sol2:=dsolve({ec,cond},{y(x)})

$$sol2 := y(x) = \frac{1}{4} x^4 + x \quad (59)$$

> restart

> ec:=2*(x+y(x))*diff(y(x),x)=y(x)

$$ec := 2(x + y(x)) \left(\frac{d}{dx} y(x) \right) = y(x) \quad (60)$$

> sol:=dsolve(ec,y(x))

$$sol := y(x) = \frac{1 + \sqrt{c_1 x + 1}}{c_1}, y(x) = -\frac{-1 + \sqrt{c_1 x + 1}}{c_1} \quad (61)$$

> ec2:=(x*y(x)-x^2)*diff(y(x),x)=x^2+y(x)^2

$$ec2 := (x y(x) - x^2) \left(\frac{d}{dx} y(x) \right) = x^2 + y(x)^2 \quad (62)$$

> dsolve(ec2,y(x))

$$y(x) = x e^{-\text{LambertW}\left(-\frac{c_1}{2} \frac{e^{-\frac{1}{2}}}{2\sqrt{x}}\right) - \frac{c_1}{2} - \frac{1}{2} - \frac{\ln(x)}{2}} - x \quad (63)$$

> ec3:=diff(y(x),x\$2)+4*diff(y(x),x)+5*y(x)=(5*x+3)*exp(x)

$$ec3 := \frac{d^2}{dx^2} y(x) + 4 \frac{d}{dx} y(x) + 5 y(x) = (5x + 3) e^x \quad (64)$$

> dsolve(ec3,y(x))

$$y(x) = e^{-2x} \sin(x) c_2 + e^{-2x} \cos(x) c_1 + \frac{e^x x}{2} \quad (65)$$

> ec100:=(2*x+1)*diff(z(x),x\$2)-2*diff(z(x),x)=4

$$ec100 := (2x + 1) \left(\frac{d^2}{dx^2} z(x) \right) - 2 \frac{d}{dx} z(x) = 4 \quad (66)$$

> dsolve(ec100,z(x))

$$z(x) = c_1 (x^2 + x) - 2x + c_2 \quad (67)$$

> cond:=z(0)=2,z(1)=2

$$cond := z(0) = 2, z(1) = 2 \quad (68)$$

$$\begin{aligned} &> \text{dsolve}(\{\text{ec100}, \text{cond}\}, \{z(x)\}) \\ & z(x) = x^2 - x + 2 \end{aligned} \quad (69)$$

$$\begin{aligned} &> \text{ec} := \text{diff}(y(x), x\$2) + 4 * \text{diff}(y(x), x) + 5 * y(x) = 2 * \exp(-x) \\ & ec := \frac{d^2}{dx^2} y(x) + 4 \frac{d}{dx} y(x) + 5 y(x) = 2 e^{-x} \end{aligned} \quad (70)$$

$$\begin{aligned} &> \text{dsolve}(ec, y(x)) \\ & y(x) = e^{-2x} \sin(x) c_2 + e^{-2x} \cos(x) c_1 + e^{-x} \end{aligned} \quad (71)$$

$$\begin{aligned} &> \text{restart} \\ &> \text{ec} := x * \text{diff}(y(x), x) = y(x) * (\ln(y(x)) - 2 * \ln(x)) \\ & ec := x \left(\frac{d}{dx} y(x) \right) = y(x) (\ln(y(x)) - 2 \ln(x)) \end{aligned} \quad (72)$$

$$\begin{aligned} &> \text{dsolve}(ec, y(x)) \\ & y(x) = e^{c_1 x} e^2 x^2 \end{aligned} \quad (73)$$

$$\begin{aligned} &> \text{infolevel}[\text{dsolve}] := 0 \\ & \text{infolevel}_{\text{dsolve}} := 0 \end{aligned} \quad (74)$$

$$\begin{aligned} &> ?\text{dsolve} \\ &> \text{dsolve}(ec, y(x)) \\ & y(x) = e^{c_1 x} e^2 x^2 \end{aligned} \quad (75)$$

$$\begin{aligned} &> \text{restart} \\ &> \text{ec} := \text{diff}(y(x), x\$2) - \exp(x) / (\exp(x) + 1) * \text{diff}(y(x), x) = \exp(-x) + 1 \\ & ec := \frac{d^2}{dx^2} y(x) - \frac{e^x \left(\frac{d}{dx} y(x) \right)}{e^x + 1} = e^{-x} + 1 \end{aligned} \quad (76)$$

$$\begin{aligned} &> \text{dsolve}(ec, y(x)) \\ & y(x) = x c_1 - x + c_1 e^x + e^{-x} + c_2 \end{aligned} \quad (77)$$

$$\begin{aligned} &> \text{restart} \\ &> \text{ec} := \text{diff}(y(x), x\$2) - 4 * \text{diff}(y(x), x) + 6 * y(x) = 12 * x + 16 \\ & ec := \frac{d^2}{dx^2} y(x) - 4 \frac{d}{dx} y(x) + 6 y(x) = 12 x + 16 \end{aligned} \quad (78)$$

$$\begin{aligned} &> \text{dsolve}(ec, y(x)) \\ & y(x) = e^{2x} \sin(\sqrt{2} x) c_2 + e^{2x} \cos(\sqrt{2} x) c_1 + 2 x + 4 \end{aligned} \quad (79)$$

$$\begin{aligned} &> \text{restart} \\ &> \text{ec} := \text{diff}(y(x), x\$2) - 1 / (x * \ln(x)) * \text{diff}(y(x), x) = 12 * x^2 * \ln(x) \\ & ec := \frac{d^2}{dx^2} y(x) - \frac{\frac{d}{dx} y(x)}{x \ln(x)} = 12 x^2 \ln(x) \end{aligned} \quad (80)$$

$$\begin{aligned} &> \text{dsolve}(ec, y(x)) \\ & y(x) = x^4 \ln(x) - \frac{x^4}{4} + c_1 (x \ln(x) - x) + c_2 \end{aligned} \quad (81)$$

$$> \text{cond_in} := y(1) = -1/4, y(2) = \exp(4)$$

$$cond_in := y(1) = -\frac{1}{4}, y(2) = e^4 \tag{82}$$

> dsolve({ec,cond_in},{y(x)})

$$y(x) = x^4 \ln(x) - \frac{x^4}{4} + \frac{(-16 \ln(2) + e^4 + 4)(x \ln(x) - x)}{2 \ln(2) - 1} + \frac{-16 \ln(2) + e^4 + 4}{2 \ln(2) - 1} \tag{83}$$