**Using uniformly distributed random numbers to produce random numbers that follow a distribution**

Given a probability of finding a number between x' and (x'+dx') of p(x') dx' where x' can go from a to x, we can define a cumulative distribution function such that



Here, F(x) goes from 0 to 1. We associate F(x) with a uniform random number between 0 and 1. F(x) goes from 0 to 1 and the random number goes from 0 to 1. Let’s call our random number ‘’. Then,



We need to invert this equation and solve for :



So, we choose a random number between 0 and 1, plug it into our function g, and produce a random number that is distributed according to the distribution .

Example 1: Spherically symmetric distribution of polar angles

How do we choose the random θ’s of spherical coordinates. We know that the probability of choosing a certain θ is proportional to the solid angle covered by θ from θ to (θ+dθ). A range of solid angle is given by



If we integrate over the angle φ, we gain a factor of 2π.



So, we know that the probability of obtaining a given value of θ is given by:



The fact that the cumulative distribution runs from 0 to 1 allows us to solve for the constant:



If we carry out this integral from 0 to π, then F(θ) is, by definition, equal to 1. It turns out, then, that the constant is equal to ½. Then,



Solving for θ,



The azimuthal angle is easier:



Example 2: Impact parameters

Let’s say that we are performing classical scattering on a hard sphere of radius . We wish to distribute the impact parameters for the scattering correctly. That is, we need the probability distribution of getting an impact parameter “” or in our case “” to be consistent with the notation above. What is the probability of getting an impact parameter between a given value  and ? Well, the number that go into a given cross-sectional area will be proportional to the area. So, the fraction of particles that go into the range  to  is



So,



The rest is easy



We associate our random variable  with , so that



Inverting, to see how to obtain our impact parameters from our random variable, we get

