Advanced Lab II: Muon Lifetime

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In this paper, we will describe our measurement of the average lifetime of a cosmic-ray muon. Measurements were for this study were taken via a plastic scintillation bar coupled with a photo-multiplier tube. We found that the muon's average lifetime is $\tau = 2152 \pm 68$ ns 95% CI, which agrees with the accepted value of 2197 ns.

I. INTRODUCTION

Cosmic ray muons created in the upper atmosphere rain down on the Earth at a rate of approximately 1 muon per minute per cm^2 at energies of 4 GeV at sea level. In order to detect these subatomic particles, this experiment was conducted primarily using a plastic scintillation bar coupled with a photo-multiplier. This piece of equipment operates such that when a cosmic ray muon strikes a nucleus and stops in the scintillation bar's frame, a photon is emitted, which is then detected by the photo-multiplier tube. Shortly after, (typically a few microseconds) the muon will decay, emitting another photon that will again be collected by the photo-multiplier tube. Given a sufficiently large collection of decay times, a fairly accurate average decay time can be calculated given the muon's exponential probability of decay with time.

II. THEORETICAL MODEL

A muon's probability of decaying increasing exponentially with time, such that

$$N(t) = N_0 e^{-t/\tau}, \tag{1}$$

where $N(t)$ is the number of undecayed muons remaining in a sample, N_0 is the initial number of muons at time $t = 0$ in said sample, and τ is the muon's average lifetime. Taking the derivative of Eq. 1 with respect to time, we get

$$\frac{d}{dt}N(t) = -\frac{N_0}{\tau}e^{-t/\tau}.$$

Thus, the number of muons to decay, $N_{\mathrm{decayed}}(t)$, in some small time interval Δt is

$$N_{\mathrm{decayed}}(t) \approx |\dot{N}(t)|\Delta t = \frac{N_0 \Delta t}{\tau}e^{-t/\tau}$$

$$N_{\mathrm{decayed}}(t) \approx N_{\mathrm{decayed}}(0)e^{-t/\tau} + |C|, \tag{2}$$

where $N_{\mathrm{decayed}}(0)$ is the number of muons to decay during the first time interval from $t = 0$ to $t = \Delta t$, and C is a constant included to account for uniform noise due to the cosmic background. Applying a curve fit of the form of Eq. 2 should then yield a value for τ.

III. EXPERIMENT

A. Procedure

Our study was carried out via the following procedure:

1. Connect scintillation bar and photo-multiplier together to oscilloscope, which in turn is connected to a computer running custom the custom program described below.

2. Begin data collection, recording "double peaks" that signify the closely-spaced collision and decay of a muon hitting a nucleus in the scintillation plastic.

3. Collect as many collision-decay pair data points as is reasonably possible. In the case of this paper, our setup ran for about a week, and yielded 11700 points.

B. Data

Following the data collection phase, our amassed 11700 "double peak" events were stored and exported to a csv file. This data was collected using a Pico Technologies 2206B MSO desktop oscilloscope. The collection process was automated via a `custom program` written using Pico Technology's C++ `SDK` (and the extremely helpful examples on the company's `github page`). A sample two-peak waveform collected by the program is displayed below.

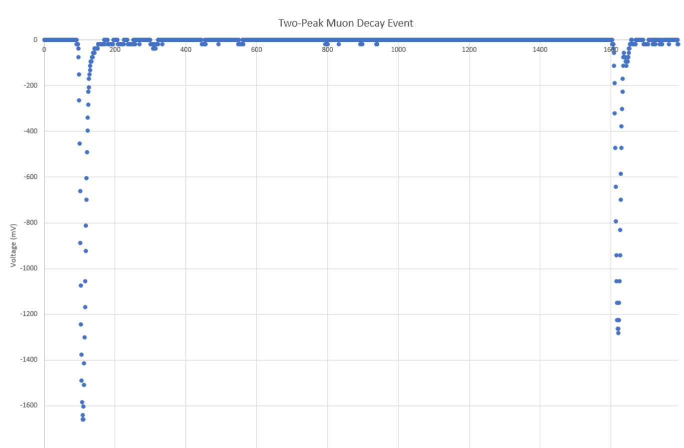

Fig. 1. A sample waveform captured by the program's peak finding algorithm. The algorithm detected peaks of −1660 mV and −1283 mV, with a peak to peak Δt (and thus decay time) of 1511 ns.

The collected data was then plotted using Mathematica in order to show the number of decay events registered in each Δt "bin," which in this particular case $\Delta t = 100$ ns. During this plotting and fitting, the first 10 bins of data points of the collection were excluded, as the very first proved to be an outlier. This can reasonably be explained by the 2206B MSO's 1 ns sampling interval. Such a sampling rate makes double peak detection process intractable for some events, as two closely spaced peaks will appear as one. Thus, decay events with shorter lifespans were underrepresented in the data collected.

A fit in the form of Eq. 2 was then applied to the data set, with a weight of

$$\sqrt{N_{\text{decayed}}(t)}$$

applied to each bin, where $N_{\text{decayed}}(t)$ is the number of decay events in a given time bin beginning at time t. Said plot is as appears below.

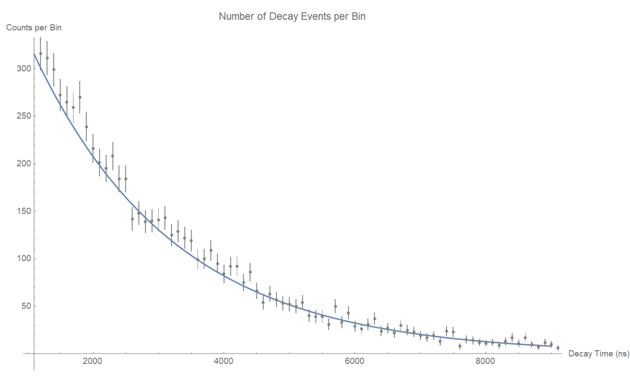

Fig. 2. A plot of the number of decay events in each Δt bin, with a curve fit applied of the form of Eq. 2.

Said curve fit gives us the particular form of Eq. 2,

$$N_{\text{decayed}}(t) = 525.637 e^{t/-2152.38} + \left(3.23 \cdot 10^{-9}\right).$$

IV. ANALYSIS

The error bars as shown in Fig. 1 reflect the uncertainty of the count of decay events in each bin. The 68% CI was calculated using the poisson statistics, such that

$$u(N) = \sqrt{N_{\text{decayed}}(t)}.$$

Each of these uncertainties were then multiplied by 2 to yield each bin's 95% CI, as shown on the plot.

Directly from Mathematica, the 95% uncertainty for the τ component of our curve fit is 68 ns. We thus find our muon average lifetime to be

$$\tau = 2152 \pm 68 \text{ ns } 95\% \text{ CI}.$$

This matches very closely with the accepted value of $\tau = 2197$ ns.

While our measurement closely matched with the expected result, possible sources of unaccounted error are still abound. As discussed above, the sampling rate of our oscilloscope severely limits our ability to detect rapid decay events, throwing off the data analysis. The software was also written to only collect data in 50,100 ns blocks (100 ns before the trigger, 50,000 following the trigger), thus eliminating some longer decay events from the measurement pool. This cutoff point was chosen somewhat arbitrarily, and could be easily extended up to 33,554,342 ns (the maximum number of samples allowed under the constraints of the oscilloscope's internal memory). This extension could collect longer decay events, but would also slow the data collection process, as well introduce additional complications (for peak detection and/or data analysis) with the raised probability of collecting multiple decay events in the same sample. More sophisticated peak detection algorithms could be explored as a means to improve collected results, or to potentially pick up a higher quantity of rapid decays in which there is significant overlap between peaks, causing an incorrect single peak classification. Furthermore, as the scintillation tube is a nuclear environment, μ^- can stimulate inverse beta decay, which provides an additional mode of decay not accounted for in this experiment's model.

V. CONCLUSION

In our study, we found the average lifetime of a cosmic ray muon is $\tau = 2152 \pm 68$ ns 95% CI, which agrees with the accepted value of 2197 ns. Some potential changes that could be implemented to improve this measurement are accounting for the potential sources of error outlined in the Analysis section (issues with software, inverse beta decay), running the apparatus for a longer period of time to gather more data points, or using more than one scintillation bar to mitigate against material defects.

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