MAC 2311 Review for Final is extra Credit **CALCULUS I** **Fall 2011**

	Bradenton campus	**Venice campus**
Instructor: Davidson Pierre	**Math Office: 27-108**	**613**
Instructor's Office: 27 -114	**Math Office phone: 752-5224**	**408-1476**
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Instructor's e-mail: pierred@scf.edu	**Math Lab Hrs:**	**http://www.scf.edu/pages/144.asp**

Office Hours: MW 1 – 2:30pm; F 1 – 2pm; TR 12:30 – 3:30pm.

COURSE DESCRIPTION: This course is the first of a three-course sequence consisting of MAC 2311, MAC 2312, and MAC 2313. This course includes the study of limits, differentiation and integration of algebraic, trigonometric and transcendental functions, and L'Hôpital's rule. Course performance standards are available at www.scf.edu/pages/1160.asp and in the Math Labs.

PREREQUISITES: MAC 1140 Precalculus and MAC 1114 Trigonometry or MAC 2147 Precalculus Algebra/Trigonometry with a grade of "C" or better or equivalent. Student Enrollment in any mathematics course is contingent upon approval of the mathematics department. This means that students who have been misplaced may have their schedule changed. Students already with credit for MAC 2311 cannot subsequently get credit for MAC 2233.

TEXT: **<u>Calculus – Early Transcendental Functions</u> – 4th edition; Larson, Hostetler, Edwards**

MATERIALS: A ***graphing*** calculator is required; **a TI-83 Plus, TI-84 Plus, or TI-86 is strongly recommended**. Calculators can be used during exams with the exception of those calculators with symbolic manipulation capabilities (e.g. TI-89, TI-92)

ADDITIONAL MATERIALS: Students will be informed by the instructor regarding the use of WebAssign (WA) (optional). Student Solutions manual (optional) is available in the bookstore and videotapes and tutoring are available in the Academic Resource Center (ARC).

EXAMINATIONS: There will be four exams and a required comprehensive final examination.
NO MAKE-UP EXAMS WILL BE GIVEN.

GRADING: Your grade in the course is determined by the percentage of points earned during the semester.
A grade of 60% or better must be earned on the final exam in order to pass the course.

	POINTS	**SCALE**
4 Exams	400	90 – 100% = A
4 Study Guides	100	
Homework	100	80 – 89% = B
Final Exam (cumulative)	200	70 – 79% = C
		60 – 69% = D
		0 – 59% = F

if % for final is higher than any of the exams, he will replace the lowest test score

*Instructor will choose composition of these points.

GORDON RULE: This course meets the Florida State Board of Education Rule Number 6A-10.30. For the purpose of this rule, a grade of "C" or better shall be considered successful completion.

ATTENDANCE: All late arrivals, early departures and absences must be discussed and cleared with the instructor. More than 3 hours of unexcused absences or excessive tardiness may result in your withdrawal from the course.

WITHDRAWAL: In accordance with the State College of Florida policy, as stated in the college catalog, students may withdraw from any course, or all courses, without academic penalty, by the withdrawal deadline listed in the State College of Florida academic calendar. This semester, the withdrawal date is **October 28, 2011.** Students should take responsibility to initiate the withdrawal procedure but are strongly encouraged to talk with their instructors before taking any withdrawal action. In addition, students should note that faculty may also withdraw students for violating policies, procedures or conditions of the class, as outlined in individual class syllabi, and such action could affect financial aid eligibility.

CELL PHONE POLICY: Cell phone etiquette must be observed: In-class usage is restricted to emergency situations. Cell phones are not allowed to be used during tests, not even as a calculator. Inform the instructor before class of any extenuating circumstances. Electronic devices as iPods, Blackberries, etc are not permitted to be used or worn in class.

MISCONDUCT: Students are required to adhere to statements regarding student misconduct outlined in official State College of Florida publications including the Catalog and the Student Handbook and www.scf.edu/pages/306.asp. The minimal consequence of failure to adhere to these statements is withdrawal from the course.

Tentative Academic Calendar Fall 2011

MAC 2311

Week	Sections Covered	Topics Covered	Suggested Homework Assignments
1 M 8/22 – F 8/26	2.1 2.2	A Preview of Calculus Finding Limits Graphically and Numerically	~~(2.1) p. 67: 1-11 odd~~ ~~(2.2) p. 74: 1-27 odd, 31-49 odd, 65, 67, 69~~
2 M 8/29 – F 9/2	2.3 2.4 2.5	Evaluating Limits Analytically Continuity and One-Sided Limits Infinite Limits	~~(2.3) p. 87: 1-123 eoo~~ ~~(2.4) p. 98: 1-69 eoo, 91, 93, 95~~ ~~(2.5) p. 108: 1-51 odd, 65, 69~~
3 *M 9/5 – F 9/9 **Labor Day Holiday – College Closed*	**Test 1** 3.1 3.2	The Derivative and Tangent Line Problem Basic Differentiation Rules and Rates of Change	~~(3.1) p. 123: 1-23 odd, 33-41 odd, 57, 59, 81, 83, 85~~ ~~(3.2) p. 136: 1-65 odd, 73, 75, 113~~
4 M 9/12 – F 9/16	3.3 3.4	Product and Quotient Rules/Higher Order Derivatives;The Chain Rule	~~(3.3) p. 147: 1-57 odd, 77, 79, 81, 97-107 odd~~ ~~(3.4) p. 161: 1-35 odd, 47-111 eoo, 123-127~~
5 M 9/19 – F 9/23	3.5 3.6 3.7	Implicit Differentiation Derivatives of Inverse Functions Related Rates	~~(3.5) p. 171: 1-45 odd, 65-73 odd~~ ~~(3.6) p. 179: 19-51 odd~~ ~~(3.7) p. 187: 1-39 odd, 43~~
6 M 9/26 – F 9/30	3.8 **Test 2**	Newton's Method	~~(3.8) p. 195: 1-15 odd~~
7 M 10/3 – **F 10/7 ***Faculty/Staff Development Day*	4.1 4.2 4.3	Extrema on an Interval Rolle's Theorem and the Mean Value Theorem Inc/Dec and the 1st Derivative Test	~~(4.1) p. 209: 1-39 odd~~ ~~(4.2) p. 216: 1-23 odd, 33, 37, 43-51 odd, 63~~ ~~(4.3) p. 226 1-53 eoo, 63-71 odd~~
8 M 10/10 - F 10/14	4.4 4.5 4.6	Concavity and the 2nd Derivative Test Limits at Infinity Summary of Curve Sketching	~~(4.4) p. 235: 1-53 eoo, 67, 69, 73~~ ~~(4.5) p. 245: 1-37 odd, 47, 51, 63, 57, 73~~ ~~(4.6) p. 255: 1-57 eoo~~
9 M 10/17 – F 10/21	4.7 4.8	Optimization Problems Differentials	~~(4.7) p. 265: 1-29 odd, 33, 35, 39, 49~~ ~~(4.8) p. 276: 1-31 odd~~
10 M 10/24– *F 10/28 10-27 *~~*F 10/28*~~ *– last day to withdraw without academic penalty (no refund)*	**Test 3** 5.1	Antiderivatives and Indefinite Integration Make up Exam on this week	(5.1) p. 291: 1-45 eoo, 63, 67, 77-93 odd
11 M 10/31– F 11/4	5.2 5.3 5.4	Area Riemann Sums and Definite Integrals The Fundamental Th of Calculus	(5.2) p. 303: 1-43 odd, 47, 51 (5.3) p. 314: 1-43 odd (5.4) p. 327: 5-61 odd, 75, 87-105 odd
12 M 11/7 – F 11/11 ** 11/11 - Veteran's Day Holiday – College Closed*	5.5 5.6	Integration by Substitution Numerical Integration	(5.5) p. 340: 1-33 odd, 47-83 odd, 87, 95-109 odd (5.6) p. 350: 1-37 odd
13 M 11/14– F 11/18	5.7 5.8	The ln Function: Integration Inverse Trig Functions: Integration	(5.7) p. 358: 1-37 odd, 49-55 odd (5.8) p. 366: 1-45 odd
14 M 11/21– *F 11/25 *W, R, F 11/23-11/25 **Thanksgiving Holidays- College Closed*	5.9 Review	Hyperbolic Functions	(5.9) p. 377: 1-33 odd, 37-79 odd
15 M 11/28 – F 12/2	**Test 4** Review for Final	Review for Final	Review for Final
16 M 12/5 – *F 12/9 **F 12/9 – Classes End.*	Review for Final	Review for Final	Review for Final
M 12/12 – R 12/15	**Final Exams** *Note: The Final Exam Schedule can be viewed at: www.scf.edu/pages/153.asp*		
F 12/16	**Final Grades Due 2:00 p.m.**		

****Friday, 10/7, Faculty/Staff Development Day – no day classes or academic support labs. Classes scheduled 4:00 pm and later will meet as usual.**

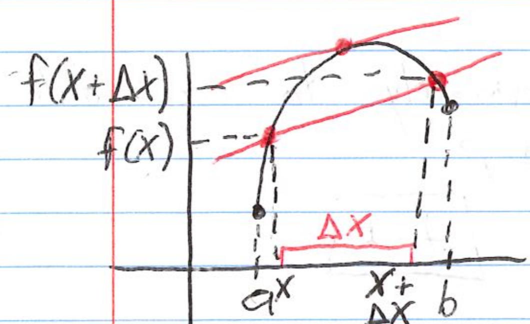
***Friday, 10/28, last day to withdraw without academic penalty (no refund).**

Section 2.1
A Preview of Calculus

Extra Credit

1) What is Calculus? 1 paragraph.

2) Why is Calculus important for your major? 1 paragraph



Touches 1 point the line is Tangent

Touches 2 points the line is Secant

$(\overset{x_1}{X}, \overset{y_1}{f(X)})$

Slope of Secant Line $(\underset{x_2}{X+\Delta X}, \underset{y_2}{f(X+\Delta X)})$

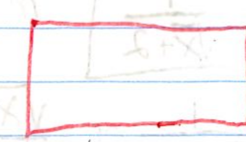
$$m = \frac{y_2 - y_1}{X_2 - X_1} = \frac{f(X+\Delta X) - f(X)}{(\cancel{X}+\Delta X) - \cancel{X}}$$

$$m = \frac{f(X+\Delta X) - f(X)}{\Delta X}$$

$$\boxed{\Delta X = h}$$

$$m = \frac{f(X+\Delta X) - f(X)}{h}$$

$A = \frac{1}{2} b \cdot h$



$A = b \cdot h$

Pg 67

2.1 Examples

4) $f(x) = 0.08x$

Rate of change = Slope

Precalculus

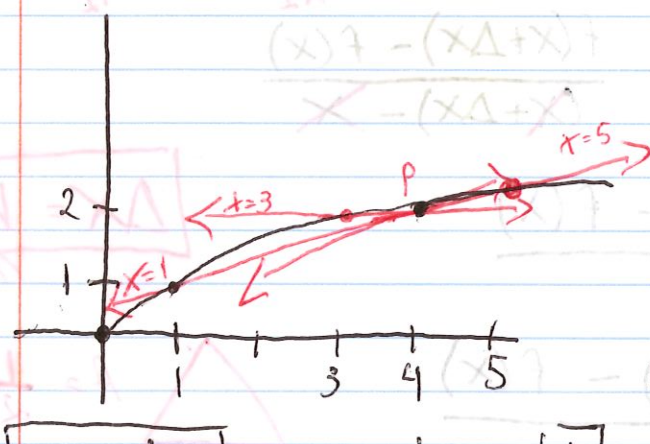
$\boxed{m = 0.08}$

6) Need Calculus

width = 2 units
height = 2.5 units approtimately to accomodate for the gap

$A = w \cdot h = 2 \cdot (2.5)$ $\boxed{A = 5}$

8) $f(x) = \sqrt{x}$
P(4, 2) when $x = 1, 3, 5$



P(4,2)

b) $(\overset{x_1}{4}, \overset{y_1}{2}), (\overset{x_2}{x}, \overset{y_2}{\sqrt{x}})$

$m = \dfrac{\sqrt{x} - 2}{x - 4} =$

$m = \dfrac{\cancel{(\sqrt{x}-2)}}{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}$

$\boxed{m = \dfrac{1}{\sqrt{x}+2}}$ $m = \dfrac{1}{\sqrt{1}+2} = \boxed{\dfrac{1}{3}}$

$m = \dfrac{1}{\sqrt{3}+2}$ or $\dfrac{\sqrt{x}-2}{x-4} = \dfrac{\sqrt{3}-2}{3-4} = \boxed{2-\sqrt{3}}$

$\boxed{m = \sqrt{5} - 2}$

c) Slope @ (4, 2)

$$m = \frac{1}{\sqrt{x}+2} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

10) a) $f(x) = \sin x$

$$A = A_1 + A_2 + A_3 + A_4$$
$$= wh_1 + wh_2 + wh_3 + wh_4$$
$$= w(h_1 + h_2 + h_3 + h_4)$$
$$= \frac{\pi}{4}\left(\sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4} + \sin\pi\right)$$
$$= \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0\right)$$
$$A = \frac{\pi}{4}(\sqrt{2}+1) = \boxed{A = 1.896}$$

b) width $= \frac{\pi}{6}$

$$A = \frac{\pi}{6}\left(\sin\frac{\pi}{6} + \sin\frac{\pi}{3} + \sin\frac{\pi}{2} + \sin\frac{2\pi}{3} + \sin\frac{\pi}{6} + \sin\pi\right)$$

$$A = \frac{\pi}{6}\left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} + \frac{1}{2} + 0\right)$$

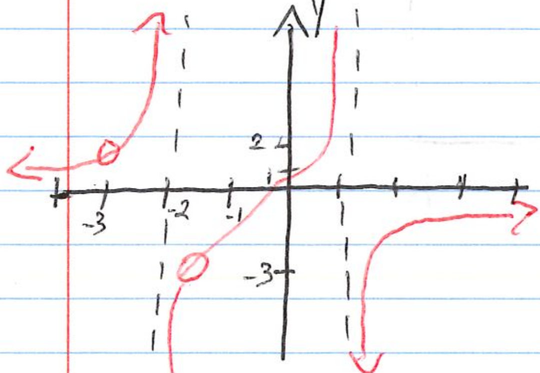
$$A = \frac{\pi}{6}(2 + \sqrt{3})$$

$$\boxed{A = 1.954}$$

2.2 Finding Limits

$\lim_{x \to c} f(x) = L$

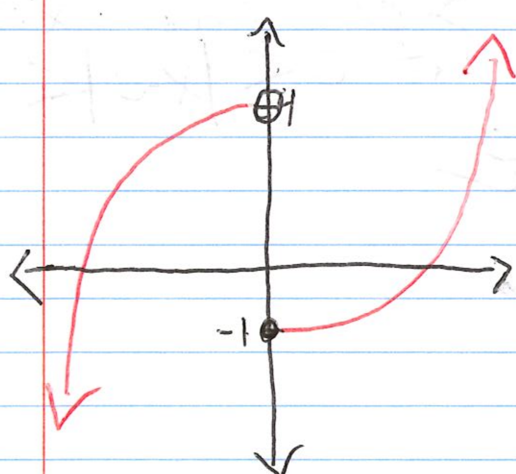
"Limit of f(x) as x approaches c equal to L"



1) $\lim_{x \to 4} f(x) = \boxed{-1}$ Value on y axis

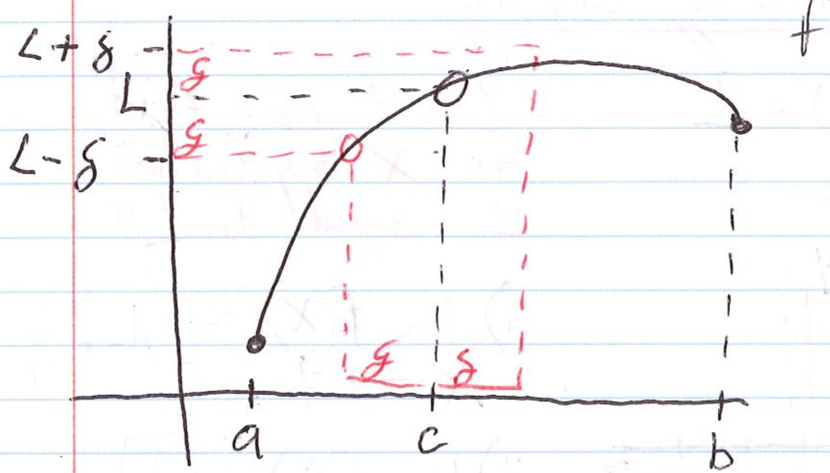
2) $\lim_{x \to -3} f(x) = \boxed{2}$

3) $\lim_{x \to -1.8} f(x) = \boxed{-3}$



1) $\lim_{x \to 0} f(x) =$ if lim from the left & right do not approach same value it $\boxed{\text{does not exist}}$

$\lim_{x \to 0^+} f(x)$



$$\lim_{x \to c} f(x) = L$$

means for all $\varepsilon > 0$ there exists a $\delta > 0$ Such that $0 < |x-c| < \delta$, then $|f(x) - L| < \varepsilon$

Examples 2.2

4) $$\lim_{x \to -3} \frac{\sqrt{1-x} - 2}{x+3} = \boxed{-0.25}$$

x	-3.1	-3.01	-3.001	-2.999	-2.99
$f(x)$	-.2485	-.2498	-.25	-.25	-.2502

6) $$\lim_{x \to 0} \frac{\cos x - 1}{x} =$$ does not exist since values to left & right are not same

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	.04996	.005	5×10^{-4}	-5×10^{-4}	-.005	-.05

14) $$\lim_{x \to 1} f(x) = \boxed{4}$$ Since the limit is a $y=4$

16) $$\lim_{x \to 0} \frac{4}{2+e^{1/x}} =$$ does not exist Since limit of one is 2 & other is 0

20) $\lim_{x \to 2} \frac{1}{x-2}$ = does not exist since from left it is $-\infty$ & from right its ∞

22) a) $f(-2)$ = does not exist since its an V-assymptote

b) $\lim_{x \to -2} f(x)$ = does not exist. Going to ∞

c) $f(0) = \boxed{4}$

d) $\lim_{x \to 0} f(x)$ = does not exist

e) $f(2)$ = does not exist since its a hole

f) $\lim_{x \to 2} f(x) = 0.5$

g) $f(4) = \boxed{2}$

h) $\lim_{x \to 4} f(x)$ = does not exist

$c = 4$

$f(x) = \left(4 - \frac{x}{2}\right)$

$\varepsilon = .01$

36) $\lim_{x \to 4} \left(4 - \frac{x}{2}\right)$ find L replace x by $x \to 4$

$4 - \frac{4}{2} = \boxed{2}$ $\boxed{L = 2}$

Start at $|f(x) - L| < \varepsilon$

$|x - c| < \delta$

So $\lim_{x \to 4} \left(4 - \frac{x}{2}\right) = 2$ means there is an $\varepsilon > 0$ such that $\delta > 0$, $|f(x) - L| < \varepsilon$

$\left|\left(4 - \frac{x}{2}\right) - 2\right| < 0.01$

$\left|4 - \frac{x}{2} - 2\right| < 0.01$

$\left|2 - \frac{x}{2}\right| < 0.01$

$\left|-\frac{1}{2}(x - 4)\right| < 0.01$

$\frac{1}{2}|x - 4| < 0.01$

$2 \cdot \frac{1}{2}|x - 4| < 0.01 \cdot 2$

$|x - 4| < 0.02$ $\boxed{\text{so } \delta = .02}$

$\boxed{\delta = 0.02}$

$c = -3$
$L = -1$
$f(x) = (2x+5)$

So $|f(x) - L| < \varepsilon$
$\downarrow$
$|x - c| < \delta$

40) $\lim_{x \to -3} (2x+5)$

find L
$2(-3)+5 = -6+5$
$\boxed{L = -1}$

$\lim_{x \to -3} (2x+5) = -1$

there is an $\varepsilon > 0$ for each $\delta > 0$ such that $|f(x) - L| < \varepsilon$

$$|(2x+5) - (-1)| < \varepsilon$$

$$|2x+6| < \varepsilon$$

$$2|x+3| < \varepsilon$$

$$|x+3| < \frac{\varepsilon}{2} \quad \text{so} \quad \boxed{\delta = \frac{\varepsilon}{2}}$$

42) $L = \frac{2}{3}(1) + 9 = \boxed{L = \frac{29}{3}}$

$c = 1$
$L = \frac{29}{3}$
$f(x) = \frac{2}{3}x + 9$

there is an $\varepsilon > 0$ for $\delta > 0$ such that $|f(x) - L| < \varepsilon$

$$\left|\left(\frac{2}{3}x + 9\right) - \frac{29}{3}\right| < \varepsilon$$

$$\left|\frac{2}{3}x + 9 - \frac{29}{3}\right|$$

$$\left|\frac{2}{3}x - \frac{2}{3}\right| < \varepsilon$$

$$\frac{2}{3}|x-1| < \varepsilon$$

$$|x-1| < \frac{3}{2}\varepsilon$$

$$\boxed{\text{Let } \delta = \frac{3}{2}\varepsilon}$$

$c = 3$
$L = 0$
$f(x) = |x-3|$

48) $\lim_{x \to 3} |x-3|$ $\qquad L = |3-3| = 0$

$\lim_{x \to 3} |x-3| = 0$ means there is a $\varepsilon > 0$ for a $\delta > 0$ such that

$$|f(x) - L| < \varepsilon$$

$$\big||x-3| - 0\big| < \varepsilon$$

$$\big||x-3|\big| < \varepsilon$$

$$|x-3| < \varepsilon \qquad \boxed{\text{Let } \delta = \varepsilon}$$

2.3 Evaluating Limits Analytically

Let $f(x)$ & $G(x)$ be functions and a, b, c, n, L, M be real numbers. Also let $\lim_{x \to c} f(x) = L$

and $\lim_{x \to c} G(x) = M$

1) $\lim_{x \to c} af(x) = a \lim_{x \to c} f(x) = aL$

2) $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)} = \sqrt[n]{L}$

3) $\lim_{x \to c} \left[f(x) \pm g(x)\right] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L \pm M$

4) $\lim_{x \to c} \left[g(x)\right]^b = \left[\lim_{x \to c} g(x)\right]^b = M^b$

5) $\lim_{x \to c} \sin x = \sin c$

6) $\lim_{x \to c} \cos x = \cos c$

7) $\lim_{x \to c} \tan x = \tan c$

8) $\lim_{x \to c} b = b$

9) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

10) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

Squeeze Theorem

if $h(x) \le g(x) \le f(x)$ and

$\lim_{x \to c} h(x) = L = \lim_{x \to c} f(x)$, then

$\lim_{x \to c} g(x) = L$

11) $\lim_{x \to c} \ln(f(x)) = \ln\left[\lim_{x \to c} f(x)\right] = \ln(L)$

12) $\lim_{x \to c} f[g(x)] = f\left[\lim_{x \to c} g(x)\right]$

2.3 Examples

10) $\lim_{x \to 1} (3x^3 - 4x^2 + 3) = 3(1)^3 - 4(1)^2 + 3$

$3 - 4 + 3$

$= \boxed{2}$

22) $\lim_{x \to} \sin\left(\frac{\pi x}{2}\right) = \sin\left(\frac{\pi}{2}\right) = \boxed{1}$

32) $\lim_{x \to 1} \ln\left(\frac{x}{e^x}\right) = \ln\left[\lim_{x \to 1} \frac{x}{e^x}\right]$

$= \ln\left(\frac{1}{e^1}\right) = \ln e^{-1} = \boxed{-1}$

$\ln e^M = M$ Since $\ln_e e^M = M$

36) $f(x) = 2x^2 - 3x + 1$,

$g(x) = \sqrt[3]{x+6}$

a) $\lim_{x \to 4} f(x) = 2(4)^2 - 3(4) + 1$

$= \boxed{21}$

b) $\lim_{x \to 21} g(x) = \sqrt[3]{21+6} = \sqrt[3]{27}$

$\boxed{= 3}$

c) $\lim\limits_{x \to 4} g\left[f(x)\right] = g\left[\lim\limits_{x \to 4} f(x)\right]$

$= g(21) = \sqrt[3]{21+6} = \boxed{3}$

40) $\lim\limits_{x \to c} f(x) = 27$

a) $\lim\limits_{x \to c} \sqrt[3]{f(x)} = \sqrt{\lim\limits_{x \to c} f(x)} = \sqrt[3]{27} = \boxed{3}$

b) $\lim\limits_{x \to c} \frac{f(x)}{18} = \frac{\lim\limits_{x \to c} f(x)}{\lim\limits_{x \to c} 18} = \frac{27}{18} = \boxed{\frac{3}{2}}$

c) $\lim\limits_{x \to c} \left[f(x)\right]^2 = \left[\lim\limits_{x \to c} f(x)\right]^2 = 27^2 = \boxed{729}$

d) $\lim\limits_{x \to c} \left[f(x)\right]^{\frac{2}{3}} = \left[\lim\limits_{x \to c} f(x)\right]^{\frac{2}{3}} = 27^{\frac{2}{3}}$

$= \left(3^3\right)^{\frac{2}{3}} = 3^2 = \boxed{9}$

42) $h(x) = \frac{x^2 - 3x}{x}$

a) $\lim_{x \to -2} f(x) = \boxed{-5}$

$\lim_{x \to -2} h(x) = \frac{x^2 - 3x}{x} = \frac{\cancel{x}(x-3)}{\cancel{x}} = -2-3 = \boxed{-5}$

b) $\lim_{x \to 0} = -3$

$\lim_{x \to 0} h(x) = x - 3 = 0 - 3 = \boxed{-3}$

50) $\lim_{x \to 0} \frac{e^{2x} - 1}{e^{x} - 1} = \frac{e^0 - 1}{e^0 - 1} = \frac{1-1}{1-1} = \boxed{\frac{0}{0}}$

* means you have to factor first!
so start over

$\lim_{x \to 0} \frac{e^{2x} - 1}{e^x - 1} = \frac{(e^x)^2 - (1)^2}{e^x - 1} =$

$= \frac{\cancel{(e^x - 1)}(e^x + 1)}{\cancel{e^x - 1}} = \lim_{x \to 0} (e^x + 1) = e^0 + 1$

$= 1 + 1 = \boxed{2}$

56) $\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$ factor!

$$\lim_{x \to 0} \frac{\sqrt{3+x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3+x} + \sqrt{3}}{\sqrt{3+x} + \sqrt{3}}$$

$$\lim_{x \to 0} \frac{(3+\cancel{x}) - 3}{\cancel{x}(\sqrt{3+x} + \sqrt{3})} = \frac{1}{\sqrt{3+x} + \sqrt{3}}$$

$$\lim_{x \to 0} \frac{1}{\sqrt{3+x} + \sqrt{3}} = \frac{1}{\sqrt{3+0} + \sqrt{3}}$$

$$= \boxed{\frac{1}{2\sqrt{3}} \text{ or } \frac{\sqrt{3}}{6}}$$

72) $\lim_{\theta \to 0} \frac{\cos\theta \operatorname{Tan}\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{\cancel{\cos\theta} \cdot \left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)}{\theta}$

$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta}$ #9 $= \boxed{1}$

78) $\lim_{x \to \frac{\pi}{4}} \frac{1 - \operatorname{Tan} x}{\sin x - \cos x} = \frac{1 - \frac{\sin x}{\cos x}}{\sin x - \cos x}$

$= \lim_{x \to \frac{\pi}{4}} \frac{\frac{\cos x - \sin x}{\cos x}}{\sin x - \cos x} = \frac{\cos x - \sin x}{\cos x} \cdot \frac{1}{\sin x - \cos x}$

$= \lim_{x \to \frac{\pi}{4}} \frac{-\cancel{(\sin x - \cos x)}}{\cos x \cancel{(\sin x - \cos x)}} =$

$= \lim_{x \to \frac{\pi}{4}} -\frac{1}{\cos x} = -\frac{1}{\cos\frac{\pi}{4}} = -\frac{1}{\frac{\sqrt{2}}{2}} =$

$\boxed{= -\frac{2}{\sqrt{2}} \text{ or } -\sqrt{2}}$

$(x+\Delta x)(x+\Delta x)$

$x^2 + x\Delta x + x\Delta x + (\Delta x)^2 = x^2 + 2x\Delta x + \Delta x^2$

62) $\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x} = \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 \cancel{- x^2}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}}$

$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x + 0 = \boxed{2x}$

92) $f(x) = x^2 - 4x$

$\lim_{\Delta x \to 0} \frac{\boxed{f(x+\Delta x)} - f(x)}{\Delta x}$

1) $f(x+\Delta x) = (x+\Delta x)^2 - 4(x+\Delta x)$

$= (x + 2x\Delta x + \Delta x^2) - 4x - 4\Delta x$

2) $f(x+\Delta x) - f(x) = (x^2 + 2x\Delta x + (\Delta x)^2 - 4x - 4\Delta x$

$\to \quad - (x^2 - 4x)$

$= \cancel{x^2} + 2x\Delta x + \Delta x^2 \cancel{- 4x} - 4\Delta x \cancel{- x^2} + \cancel{4x}$

$= 2x\Delta x + \Delta x^2 - 4\Delta x$

3) $\frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{2x\cancel{\Delta x}}{\cancel{\Delta x}} + \frac{(\Delta x)^{\cancel{2}}}{\cancel{\Delta x}} - \frac{4\cancel{(\Delta x)}}{\cancel{\Delta x}}$

$= \lim_{\Delta x \to 0} 2x + \Delta x - 4 = \boxed{2x - 4}$

4) $\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} 2x + \Delta x - 4$

$= 2x + (0) - 4 = \boxed{2x-4}$

94) $\lim_{x \to c} f(x) = ?$

$b - |x-a| \leq f(x) \leq b + |x-a|$

$c = a$

$\lim_{x \to c} b - |x-a| = \lim_{x \to a} b - |x-a|$

$b - |a-a| = b - 0 = \boxed{b}$

and

$\lim_{x \to a} b + |x-a| = \lim_{x \to a} b \, |a-a|$

$= b + 0 = \boxed{b}$ $\boxed{\lim_{x \to c} f(x) = b}$

Squeeze Theo!

2.4 Continuity & One Sided Limits

Let f be a fxn
$f(x)$ is continuous at c if the folowing is true

1) $f(c)$ exists

2) $\lim_{x \to c} f(x)$ exists

3) $\lim_{x \to c} f(x) = f(c)$

If $\lim_{x \to c^-} f(x) = L$ and the

$\lim_{x \to c^+} f(x) = L$, then $\lim_{x \to c} f(x) = L$

Intermediate Value Theorem

Let f be a continuos fxn on $[a, b]$.
If there is a value K between $f(a)$ & $f(b)$, then there exists at least 1 number c such that
$f(c) = K$

2.4 Examples

4) 1) $f(-2) = 1$

2) $\lim_{x \to -2} f(x) = 2$

Since $\lim_{x \to -2} f(x) \neq f(-2)$

f is not continuos at $x = -2$

a) $\lim_{x \to -2^-} f(x) = 2$ b) $\lim_{x \to -2^+} f(x) = 2$

c) $\lim_{x \to -2} f(x) = \boxed{2}$

6) a) $\lim_{x \to -1^-} f(x) = 2$

b) $\lim_{x \to -1^+} f(x) = 0$

c) Since the $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$

then the limit does not exist

10) $\lim_{x \to 4^-} \dfrac{\sqrt{x}-2}{x-4} = \dfrac{\cancel{\sqrt{x}-2}}{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}$

$= \lim_{x \to 4^-} \dfrac{1}{\sqrt{x}+2} = \dfrac{1}{\sqrt{4}+2} = \boxed{\dfrac{1}{4}}$

16) $f(x) = \begin{cases} x^2-4x+6 \,; & x < 2 \\ -x^2+4x-2 \,; & x \geq 2 \end{cases}$

find Lim from left & right

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^2-4x+6)$

$= 2^2-4(2)+6 = \boxed{2}$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (-x^2+4x-2)$

$= -(2)^2+4(2)-2 = \boxed{2}$

Since $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$, then the $\lim_{x \to 2} f(x) = 2$

28) $\lim_{x \to 5^+} \ln \frac{x}{\sqrt{x-4}}$

$\ln \left[\lim_{x \to 5^+} \frac{x}{\sqrt{x-4}} \right]$

$= \ln \left[\frac{5}{\sqrt{5-4}} \right] = \boxed{\ln 5}$

30) f is continuos everywhere except at $x = -1$ because $f(-1)$ does not exist

36) $g(t) = \frac{1}{x^2 - 4}$ $\quad [-1, 2]$

g is continuos at $[-1, 2)$

because $g(2)$ does not exist

42) $f(x) = \dfrac{x}{x^2-1}$

f is continuos everywhere except $x=1$ and $x=-1$, these discontinuities are non-removable

cant factor out

46) $f(x) = \dfrac{(x-1)}{x^2+x-2} = \dfrac{\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}$

$= \dfrac{1}{(x+2)}$

f is continuos everywhere except $x=-2$ and $x=1$. At $x=-2$ the disontinuity is non-removable

50) $f(x) \begin{cases} -2x+3; & x<1 \\ x^2; & x \geq 1 \end{cases}$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (-2x+3) = -2(1)+3 = \boxed{1}$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} x^2 = (1)^2 = \boxed{1}$

Since The $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x)$

then $\lim\limits_{x\to 1} f(x) = 1$

2) $f(1) = (1)^2 = 1$

3) $\lim\limits_{x \to 1} f(x) = f(1) = 1$

therefore, f is continuous everywhere

52) Piecewise are not continuos at the boundries!

$f(x) = \begin{cases} -2x & x \leq 2 \end{cases} = \lim\limits_{x \to 2^-} = -2(2) = \boxed{-4}$

$\lim\limits_{x \to 2^+} (x^2 - 4x + 1) = (2)^2 - 4(2) + 1 = \boxed{-3}$

Since $\lim\limits_{x \to 2^-} f(x) \neq \lim\limits_{x \to 2^+} f(x)$

then $\lim\limits_{x \to 2} f(x)$ does not exist.

$\boxed{\text{f is continuous everywhere except at } x=2 \text{ because } \lim\limits_{x \to 2} f(x) \text{ does not exist}}$

ON Exam

64) $g(x) = \begin{cases} \frac{4\sin x}{x}, & x < 0 \\ a - 2x, & x \geq 0 \end{cases}$

$\lim_{x \to 0^-} G(x) = \lim_{x \to 0^-} \left(\frac{4\sin x}{x}\right) = 4\left(\lim_{x \to 0^-} \frac{\sin x}{x}\right)$

$= 4(1) = \boxed{4}$

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (a - 2x) = a - 2(0) = a$

$\boxed{\text{Since } \lim_{x \to 0^-} = \lim_{x \to 0^+} \text{ then } a = 4}$

68) $f(x) = \frac{1}{\sqrt{x}}$ $\quad G(x) = x - 1$

$h(x) = f[g(x)]$

$f[x-1] = \frac{1}{\sqrt{x-1}} = \quad h(x) = \frac{1}{\sqrt{x-1}}$

den cant be zero so $x - 1 > 0$

$\boxed{x > 1}$

$\boxed{h(x) \text{ is continuous at } x > 1}$

94) $f(x) = \frac{x^2+x}{x-1}$, $\left[\frac{5}{2}, 4\right]$, $f(c)=6$

find f of boundries

$$\frac{\left(\frac{5}{2}\right)^2+\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)-1} = \text{calculator} = \boxed{5.83}$$

$$\frac{(4)^2+(4)}{4-1} = \boxed{6.6\overline{6}}$$

f of a must be less than K
& f of b must be greater than K

$f(a) = 5.83 < K = C = 6$
$f(b) = 6.6\overline{6} > K = C = 6$

$$f(c) = 6 = \frac{c^2+c}{c-1} = \frac{6}{1}$$

$(c^2+c) = 6c-6$

$= c^2 - 5c + 6 = 0$
$(c-3)(c-2) = 0$
$\boxed{c=3}$ $\quad c=2$

We pick 3 since its between the boundries $\left[\frac{5}{2}, 4\right]$

2.5 Infinite Limits

$\frac{K}{0} \longrightarrow \infty$ K approaches infinity

$\frac{0}{K} = 0$ $\frac{0}{0}$ = Factor!

Vertical Assymptotes (VA)

If $\lim\limits_{x \to c} f(x) = \pm\infty$, then

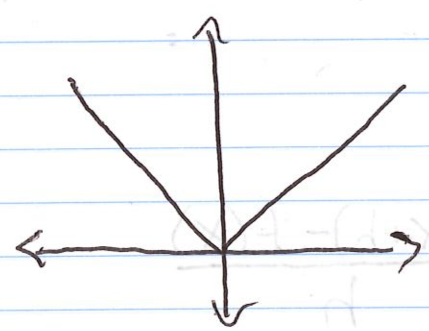
$x = c$ is V.A.

If $f(x)$ is ~~continuous~~ differentiable at $x=c$, then f is continuous at $x=c$

* So if you prove something is differentiable, then it is also continuous

These fxns have no derivative

absolute value fxn



Tangent line will be vertical so it will be undefined

2.1 Examples

8) $g(x) = 5 - x^2$ at $(2, 1)$

$m = g'(2)$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

first find $g(x+h)$

1) $g(x+h) = 5 - (x+h)^2$

$= 5 - x^2 - 2xh - h^2$

2) $g(x+h) - g(x) = (5 - x^2 - 2xh - h^2) - (5 - x^2)$

$= 5 - x^2 - 2xh - h^2 - 5 + x^2$

$= -2xh - h^2$

3) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2xh}{h} - \dfrac{h^2}{h} = \boxed{2x - h}$

4) $g'(x) = \lim_{h \to 0} \dfrac{g(x+h) - g(x)}{h}$

$= \lim_{h \to 0} (-2x - h) = \quad g'(x) = -2x$

$m = g'(2) = -2(2)$

$\boxed{m = -4}$

20) $f(x) = x^3 + x^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = (x+h)^3 + (x+h)^2$

$f(x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x^2 + 2xh + h^2$

2) $f(x+h) - f(x) = x^3 + 3x^2h + 3xh^2 + h^3 + x^2 + 2xh + h^2$
$- (x^3 + x^2)$

$= \cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{x^2} + 2xh + h^2 \cancel{- x^3} \cancel{- x^2}$

$= 3x^2h + 3xh^2 + h^3 + 2xh + h^2$

3) $$\frac{f(x+h) - f(x)}{h} = \frac{3x^2h}{h} + \frac{3xh^2}{h} + \frac{h^3}{h} + \frac{2xh}{h} + \frac{h^2}{h}$$

$= 3x^2 + 3xh + h^2 + 2x + h$

4) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} (3x^2 + 3xh + h^2 + 2x + h)$$

$= 3x^2 + 3x(0) + (0)^2 + 2x + 0$

$= 3x^2 + 2x$

$$\boxed{f'(x) = 3x^2 + 2x}$$

24) $f(x) = \dfrac{4}{\sqrt{x}}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = \dfrac{4}{\sqrt{x+h}}$

2) $f(x+h) - f(x) = \dfrac{4}{\sqrt{x+h}} - \dfrac{4}{\sqrt{x}}$

$$= \frac{4(\sqrt{x}) - 4(\sqrt{x+h})}{(\sqrt{x+h})(\sqrt{x})}$$

$$= \frac{4\sqrt{x} - 4\sqrt{x+h}}{(\sqrt{x+h})(\sqrt{x})}$$

3) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{1}{h} \cdot \dfrac{4\sqrt{x} - 4\sqrt{x+h}}{(\sqrt{x+h})(\sqrt{x})}$

$$= \frac{4\sqrt{x} - 4\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

4) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$(4\sqrt{x})^2 - (4\sqrt{x+h})^2 = 16x - 16(x+h)$$

$$= \cancel{16x} - \cancel{16x} - 16h = \boxed{-16h}$$

$$f'(x) = \lim_{h\to 0} \frac{4\sqrt{x} - 4\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$$

factor or Rationalize

$$= \frac{4\sqrt{x} - 4\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \frac{4\sqrt{x}+4\sqrt{x+h}}{4\sqrt{x}+4\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{-16\cancel{h}}{\cancel{h}\sqrt{x}\sqrt{x+h}(4\sqrt{x}+4\sqrt{x+h})}$$

$$\lim_{h\to 0} \frac{-16}{\sqrt{x}\sqrt{x+h}(4\sqrt{x}+4\sqrt{x+h})}$$

$$= \frac{-16}{\sqrt{x}\sqrt{x+0}(4\sqrt{x}+4\sqrt{x+0})}$$

$$= \frac{-\cancel{16}}{x(\cancel{8}\sqrt{x})} = -\frac{2}{x\sqrt{x}}$$

$$\boxed{f'(x) = -\frac{2}{x\sqrt{x}}}$$

Parallel = Same Slope

36) $f(x) = \dfrac{1}{\sqrt{x-1}}$ Parallel to $x + 2y + 7 = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

find Slope of ∥ line

$y = -\frac{1}{2}x - \frac{7}{2}$ $\boxed{m = -\frac{1}{2}}$

1) $f(x+h) = \dfrac{1}{\sqrt{(x+h)-1}}$

2) $f(x+h) - f(x) = \dfrac{1}{\sqrt{(x+h)-1}} - \dfrac{1}{\sqrt{x-1}}$

$$= \frac{1\sqrt{x-1} - 1\sqrt{(x+h)-1}}{\sqrt{x+h-1}\ \sqrt{x-1}}$$

3) $= \dfrac{f(x+h) - f(x)}{h} = \dfrac{1}{h} \cdot \dfrac{\sqrt{x-1} - \sqrt{x+h-1}}{\sqrt{x+h-1}\ \sqrt{x-1}}$

$$= \frac{\sqrt{x-1} - \sqrt{x+h-1}}{h\sqrt{x+h-1}\ \sqrt{x-1}}$$

$\hookrightarrow$

4) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\lim_{h \to 0} \frac{\sqrt{x-1} - \sqrt{x+h-1}}{h\sqrt{x+h-1}\sqrt{x-1}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x-1} - \sqrt{x+h-1}}{h\sqrt{x+h-1}\sqrt{x-1}} \cdot \frac{\sqrt{x-1} + \sqrt{x+h-1}}{\sqrt{x-1} + \sqrt{x+h-1}}$$

$$\lim_{h \to 0} \frac{-h}{h\sqrt{x-1}\sqrt{x+h-1}(\sqrt{x-1} + \sqrt{x+h-1})}$$

$$= \lim_{h \to 0} \frac{-1}{\sqrt{x-1}\sqrt{x+h-1}(\sqrt{x-1} + \sqrt{x+h-1}}$$

$$= \frac{-1}{\sqrt{x-1}\sqrt{x+0-1}\,(\sqrt{x-1} + \sqrt{x+0-1})}$$

$$= \frac{-1}{x-1\,(2\sqrt{x-1})} = f'(x) = \frac{-1}{2(x-1)\sqrt{x-1}}$$

Since $2a\sqrt{a} = a \cdot a^{\frac{1}{2}} = 1 + \frac{1}{2} = a^{\frac{3}{2}}$

So $f'(x) = -\dfrac{1}{2}(x-1)^{-\frac{3}{2}}$

Since $a^{-2} = 1$

$\frac{1}{a^2} = \frac{1}{1}$

$a^2 = 1$

1) $X = \pm K^{\frac{m}{n}}$ if m is even

2) $X = K^{\frac{m}{n}}$ if m is odd

$X^{\frac{m}{n}} = K$

Slope of || lines $= -\frac{1}{2}(X-1)^{-\frac{3}{2}} = -\frac{1}{2}$

$(X-1)^{-\frac{3}{2}} = 1$

$(X-1)^{\frac{3}{2}} = 1$

$X-1 = 1^{\frac{2}{3}}$

$X-1 = 1$

$X = 2$

$Y = f(x) = \frac{1}{\sqrt{x-1}}$

$X = 2$

$\begin{array}{cc} x & y \\ (2, & 1) \end{array}$

$Y = \frac{1}{\sqrt{2-1}} = \boxed{1}$

$m = -\frac{1}{2}$

$Y - Y_1 = -\frac{1}{2}(X - X_1)$

$Y - 1 = -\frac{1}{2}(X-2)$

$Y - 1 = -\frac{1}{2}x + 1$

$\boxed{Y = -\frac{1}{2}X + 2}$

58) $f(x) = x^2$ find equations of the 2 crossing lines

Use (x_0, y_0) for unknown point

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - y_0}{1 - x_0}$$

Since $f(x) = x^2$ so $\frac{-3 - x_0^2}{1 - x_0}$

$y_0 = x_0^2$

$m = f'(x_0)$

1) $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

2) $f(x+h) - f(x) = \cancel{x^2} + 2xh + h^2 - \cancel{x^2}$

$= 2xh + h^2$

3) $\frac{2xh}{h} + \frac{h^2}{h} = \boxed{2x+h}$

4) $\lim_{h \to 0} 2x + h = 2x + 0 = \boxed{2x}$

$$\boxed{f'(x) = 2x}$$

So $m = f'(x_0) = 2x_0$

$\frac{-3 - x_0^2}{1 - x_0} = 2x_0$ cross mult!

$\longrightarrow$

$$1(-3-x_0^2) = 2x_0(1-x_0)$$
$$-3-x_0^2 = 2x_0 - 2x_0^2$$
$$2x_0^2 - x_0^2 - 2x_0 - 3 = 0$$
$$x_0^2 - 2x_0 - 3 = 0$$
$$(x_0-3)(x_0+1) = 0$$
$$\boxed{x_0 = 3} \quad \boxed{x_0 = -1}$$

$y_0 = x_0^2$
$y_0 = 3^2$
$\boxed{y_0 = 9}$
$(3, 9)$

$y_0 = x_0^2$
$y_0 = -1^2$
$\boxed{y_0 = 1}$
$(-1, 1)$

$x_0 = 3$
$m = f'(x_0) = 2x_0$
$m = 2(3) = \boxed{6}$
$(3, 9);\ m = 6$

$y - y_1 = m(x - x_1)$
$y - 9 = 6(x-3)$
$y - 9 = 6x - 3$
$\boxed{y = 6x - 9}$

$x_0 = -1$
$m = f'(x_0) = 2x_0$
$m = 2(-1) = -2$
$(-1, 1);\ m = -2$

$y - y_1 = m(x - x_1)$
$y - 1 = -2(x+1)$
$y - 1 = -2x - 2$
$\boxed{y = -2x - 1}$

3.2 Differentiation Rules

1) $f(x) = c$ then $f'(x) = 0$

2) $f(x) = x$ " $f'(x) = 1$

3) $f(x) = x^n$ " $f'(x) = nx^{(n-1)}$

4) $f(x) = e^x$ " $f'(x) = e^x$

5) $f(x) = \sin x$ " $f'(x) = \cos x$

6) $f(x) = \cos x$ " $f'(x) = -\sin x$

7) $f(x) = \tan x$ " $f'(x) = \sec^2 x$

8) $f(x) = \sec x$ " $f'(x) = \sec x \tan x$

9) $f(x) = \cot x$ " $f'(x) = -\csc^2 x$

10) $f(x) = \csc x$ " $f'(x) = -\csc x \cot x$

11) $[c\,f(x)]' = c\,f'(x)$

12) $[f(x) \pm G(x)]' = f'(x) \pm G'(x)$

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3.2 Examples

4) $f(x) = -6$

#1 $f(x) = c$ so $f'(x) = 0$

8) $g(x) = \sqrt[6]{x}$ $\sqrt[n]{x^p} = n^{\frac{p}{n}}$

$g(x) = x^{\frac{1}{6}}$ = #6 $\frac{1}{6}x^{(\frac{1}{6}-1)} = \frac{1}{6}x^{(-\frac{5}{6})}$

$$\boxed{g'(x) = \frac{1}{6x^{\frac{5}{6}}}}$$

16) $f(x) = 2x^3 - 4x^2 + 3x$

$f(x) = 2(3)x^{(3-1)} - 4(2)x^{(2-1)} + 3(1)$

$$\boxed{f'(x) = 6x^2 - 8x + 3}$$

24) $y = \frac{3}{4}e^x + 2\cos x$

#6 $$\boxed{y' = \frac{3}{4}e^x - 2\sin x}$$

28) $y = \frac{\pi}{(5x)^2} = y = \frac{\pi}{25x^2}$

$= \frac{\pi}{25} x^{-2}$

Differentiate $= \frac{\pi}{25}(-2)x^{(-2-1)}$

Simp: $\boxed{-\frac{2\pi}{25} x^{-3} \text{ or } -\frac{2\pi}{25x^3}}$

36) $g(t) = 2 + 3\cos t \quad ; \quad (\pi, -1)$

Slope = derivative at x value (π)

$g(t) = 0 - 3\sin t$

Slope $= g'(\pi) = -3\sin\pi$

$g'(\pi) = 0$

$\boxed{\text{Slope} = 0}$

40) $f(t) = x + \frac{1}{x^2}$

$f'(x) = x + x^{-2}$

$f(x) = 1 + (-2)x^{(-2-1)}$

$\boxed{f(x) = 1 - 2x^{-3}}$ or $\boxed{f'(x) = 1 - \frac{2}{x^3}}$

48) $f(t) = t^{\frac{2}{3}} - t^{\frac{1}{3}} + 4$

$f'(t) = \frac{2}{3}t^{(\frac{2}{3}-1)} - \frac{1}{3}t^{(\frac{1}{3}-1)} + 0$

$\boxed{f'(t) = \frac{2}{3}t^{-\frac{1}{3}} - \frac{1}{3}^{-\frac{2}{3}}}$

or

$\boxed{f'(t) = \frac{2}{3t^{\frac{1}{3}}} - \frac{1}{3t^{\frac{2}{3}}}}$

60) $y = \sqrt{3}x + 2\cos x, \quad 0 \leq x < 2\pi$

where slope is zero (hor Tan. line)

$y' = 0$

#2 + 6

$y' = \sqrt{3} - 2\sin x$

$y' = \sqrt{3} - 2\sin x = 0$

$\sin x = \frac{\sqrt{3}}{2}$

$\to x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

$x = \frac{\pi}{3}, \; y = \frac{\sqrt{3}\pi}{3} + 2\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}\pi}{3} + 1$

$\boxed{y = \frac{\sqrt{3}\pi + 3}{3}}$

$\left(\frac{\pi}{3}, \frac{\sqrt{3}\pi + 3}{3}\right)$

$x = \frac{2\pi}{3}$, $y = \frac{2\sqrt{3}\,\pi}{3} + 2\cos\left(\frac{2\pi}{3}\right)$

$= \frac{2\sqrt{3}\,\pi}{3} - 1 = \boxed{y = \frac{2\sqrt{3}\pi - 3}{3} \quad \left(\frac{2\pi}{3}, \frac{2\sqrt{3}\pi - 3}{3}\right)}$

64) $f(x) = K - x^2$; $y = -4x + 7$

$(K - x^2)' = (-4x + 7)'$

$0 - 2x = -4 + 0 = -2x = -4$

$\boxed{x = 2}$

Since $K - x^2 = -4x + 7$

$K - (2)^2 = -4(2) + 7$

$\boxed{K = 3}$

Two lines intersecting each other means they equal each other

74) $y = x$; $y = \frac{1}{x}$

$\frac{x}{1} = \frac{1}{x}$ $x(x) = 1(1)$

$x^2 = 1$

$x = \pm\sqrt{1} = \boxed{x = \pm 1}$

$y = x$

$\boxed{y' = 1}$

$y = \frac{1}{x}$

$y' = 1x^{-1} = (-1)x^{(-1-1)} = \boxed{y' = -x^{-2}}$

Slope = when $x = 1$

$\boxed{y' = 1}$

$y' = -\frac{1}{x^2} = \boxed{y' = -1}$

When $x = -1$

$y' = 1$

$\boxed{y(-1) = 1}$

$y' = -\frac{1}{x^2}$

$y'(-1) = -\frac{1}{(-1)^2} = \boxed{y(-1) = -1}$

114) $f(x) \begin{cases} \cos x, & x < 0 \\ ax+b, & x \geq 0 \end{cases}$

$\lim_{x \to 0^-} \cos x = \cos(0) = \boxed{1}$

$\lim_{x \to 0^+} ax+b = a(0)+b = \boxed{b}$

b must be 1 to be continuous

$f(x) = \begin{cases} \cos x, & x < 0 \\ ax+1, & x \geq 0 \end{cases}$

$f'(x) \begin{cases} -\sin x & x < 0 \\ a & x \geq 0 \end{cases}$ Rules

to be differentiable they must be equal to each other

$-\sin(0) = 0$ so $\boxed{a = 0}$

3.3 Product & Quotient Rule

13) $\left[f(x) \cdot G(x)\right]' = f'(x) \cdot G(x) + G'(x) \cdot F(x)$

14) $[U \cdot V]' = U' \cdot V + V' \cdot U$

15) $\left[\frac{f(t)}{g(t)}\right]' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{[g(x)]^2}$

16) $\left[\frac{U}{V}\right]' = \frac{U' \cdot V - V' \cdot U}{V^2}$

17) $S(t)$
$V(t) = S'(t)$

18) $S(t)$
$a(t) = V'(t)$ or $a(t) = S''(t)$

19) $f(x) = 2x^7 - 8x^6 + 3x^2 - 10$
$f^{V}(\cancel{x}) = ?$

$f'(x) = 14x^6 - 48x^5 + 6x$
$f''(x) = 84x^5 - 240x^4 + 6$
$f'''(x) = 420x^4 - 960x^3$
$f^{IV}(x) = 1680x^3 - 2880x^2$
$f^{V}(x) =$

3.3 Examples

6) $g(x) = \sqrt{x}\ \sin x$

$g(x) = x^{\frac{1}{2}} \cdot \sin x$ $\qquad U = x^{\frac{1}{2}}$ so $U' = \frac{1}{2}x^{-\frac{1}{2}}$

$\qquad V = \sin x \qquad V' = \cos x$

$g(x) = \frac{1}{2x^{\frac{1}{2}}} \sin x + (\cos x) \cdot x^{\frac{1}{2}}$

$g(x) = \frac{1}{2x^{\frac{1}{2}}} \left[\sin x + 2x \cos x\right]$

12) $f(x) = \frac{\cos t}{t^3}$ $\qquad U = \cos t \qquad U' = -\sin t$

$\qquad V = t^3$ so $V' = 3t^2$

$$f'(x) = \frac{(-\sin t) \cdot t^3 - 3t^2 (\cos t)}{(t^3)^2}$$

$$f'(x) = \frac{-t^2 \left[t \sin t + 3 \cos t\right]}{t^6}$$

$$\boxed{f'(x) - \frac{\left[t \sin t + 3 \cos t\right]}{t^4}}$$

$(x^3-1) = a^3-b^3 = (x-1)(x^2+x+1)$

$f'(c) = ?$

14) $f(x) = (x^2-2x+1)(x^3-1) \qquad c = 1$

$U = (x^2-2x+1)$, $U' = 2x-1$ or $2(x-1)$

$V = (x^3-1) \qquad V' = 3x^2$

$f'(x) = 2(x-1)(x^3-1) + 3x^2(x^2-2x+1)$

$f'(1) = 2(1-1)(1^3-1) + 3(1)^2(1^2-2(1)+1)$

$f'(1) = 2(0)(0) + 3(1)(0)$

$\boxed{f'(1) = 0}$

18) $f(x) = \dfrac{\sin x}{x} \qquad c = \dfrac{\pi}{6}$

$U = \sin x$, $U' = \cos x$

$V = x$, $V' = 1$

$f'(x) = \dfrac{(\cos x)x - 1(\sin x)}{x^2}$

$\boxed{f'(x) = \dfrac{x\cos x - \sin x}{x^2}}$

$f'(c) = \dfrac{\frac{\pi}{6}\left(\cos\frac{\pi}{6}\right) - \sin\frac{\pi}{6}}{\left(\frac{\pi}{6}\right)^2}$

$$= \frac{\left(\frac{\pi}{6}\right)\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}\frac{6}{6}}{\frac{\pi^2}{36}} = \frac{\frac{6\sqrt{3}\pi}{12}}{\frac{\pi^2}{36}}$$

$$= \frac{6\sqrt{3}\pi}{12} \cdot \frac{36}{\pi^2} = \boxed{\frac{18\sqrt{3}}{\pi}}$$

24) $y = \frac{5}{4x^2}$

$y = \frac{5}{4}x^{-2}$

Diff $= \frac{5}{4}(-2)x^{(-2-1)}$

Simp $= -\frac{5}{2}x^{-3}$

26) $y = \frac{3x^2 - 5}{7}$

$y = \frac{3}{7}x^2 - \frac{5}{7}$

Diff $\frac{3}{7}(2)x^{(2-1)} - 0$

Simp $\frac{6}{7}x$

34) $h(x) = (x^2+1)^2$

$h(x) = x^4 + 2x^2 + 1$

$\boxed{h'(x)\ 4x^3 + 4x + 0}$

40) $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$ c is a constant

$u = c^2 - x^2$, $u' = -2x$

$v = c^2 + x^2$, $v' = 2x$

$f'(x) = \dfrac{-2x(c^2+x^2) - 2x(c^2-x^2)}{(c^2+x^2)^2}$

$= \dfrac{-2c^2x - 2x^3 - 2c^2x + 2x^3}{(c^2+x^2)^2}$

$f'(x)\ \boxed{= \dfrac{-4c^2x}{(c^2+x^2)^2}}$

3.5 Implicit Differentiation

Explicit → $y = 2x^2 - 7x^3 + 5$

Implicit → $x^3y^2 + \cos xy - \ln xy = 20$

$(u \cdot v)' = (x^3y^2)' = 3x^2y^2 + 2yy' \cdot x^3$

$U = x^3$ $\quad V = y^2$

$U' = 3x^2$ $\quad V' = 2y \cdot y'$

* 3.4 The Chain Rule

Ex: $(2x^3 - 7x^2 + 5)^{50}$ Chain Rule!

let $U = 2x^3 - 7x^2 + 5$

$[U^{50}]' = 50U^{49} \cdot U'$

$= 50(2x^3 - 7x^2 + 5)^{49} \cdot (6x^2 - 14x)$

$U = x$

$f(U) = U^n$

$f'(U) = nU^{(n-1)} \cdot U'$

- Rule #20) $[\ln U]' = \frac{U'}{U}$
- 21) $[a^U]' = U'(\ln a)a^U$

Ex: $[3^{(x^2-5x+3)}]' = (2x-5)(\ln 3)3^{(x^2-5x+3)}$

- 22) $[\log_a U] = \frac{U'}{(\ln a)U}$
- 23) $f(U) = cU^n$

$f'(U) = cnU^{(n-1)} \cdot U'$

24) $[ce^u]' = ce^u \cdot u'$

25) $(\sin u)' = u' \cdot \cos u$

26) $(\cos u)' = -u' \cdot \sin u$

To Simplify:

a) $\dfrac{a^n}{a^m} = a^{(n-m)}$ if $n > m$

b) $\dfrac{a^n}{a^m} = \dfrac{1}{a^{m-n}}$ if $m > n$

c) $\dfrac{a^n}{a^m} = 1$ if $m = n$

3.5 examples

8) $e^{xy} + x^2 + y^2 = 10$

$(y + xy')e^{xy} + 2x - 2yy' = 0$

$ye^{xy} + xy'e^{xy} + 2x - 2yy' = 0$

$ye^{xy} + 2x = 2yy' - xy'e^{xy}$

$ye^{xy} + 2x = y'(2y - xe^{xy})$

$$y' = \frac{ye^{xy} + 2x}{2y - xe^{xy}}$$

14) $(\sin \pi x + \cos \pi y)^2 = 2$

$2(\sin \pi x + \cos \pi y) \cdot$

→ $(\pi \cos \pi x - \pi y' \sin \pi y) = 0$

Set factors to zero

$\pi \cos \pi x - \pi y' \sin \pi y = 0$

$$y' = \frac{\pi \cos \pi x}{\pi \sin \pi y} = \frac{\cos \pi x}{\sin \pi y}$$

$\pi \cos \pi x = \pi y' \sin \pi y$ → then divide the sin part

$(\sin \pi x)' = \cos \pi x \cdot \pi$

$= \cancel{\pi \cos \pi x}$

$(\cos \pi y)' = (-\sin \pi y)\pi y'$

$= -\pi y' \sin \pi y$

$\frac{a}{b} + \frac{c}{b} = \frac{d}{b}$

$= \boxed{a+c=d}$

$(\ln x)' = \frac{(x)'}{x} = \frac{1}{x}$

$(\ln y)' = \frac{y'}{y}$ because y' is the fxn we need!

20) $\ln xy + 5x = 30$

$\ln x + \ln y + 5x = 30$

* $\boxed{(\ln x)' = \frac{x'}{x}}$ $\quad \frac{1}{x}\frac{y}{y} + \frac{y'}{y}\frac{x}{x} + \frac{5}{1}\frac{xy}{xy} = 0$ $\quad$ LCD $= xy$

$$\frac{y + xy' + 5xy}{xy} = 0$$

$$y' = \frac{-5xy - y}{x}$$

22) $x^2 + y^2 - 4x + 6y + 9 = 0$ (This eq is a circle)

Solve for y so complete the square

$(x^2 - 4x + (\frac{4}{2})^2) + (y^2 + 6y + (\frac{6}{2})^2) = -9 + (\frac{4}{2})^2 + (\frac{6}{2})^2$

$(x-2)^2 + (y+3)^2 = -9 + 4 + 9$

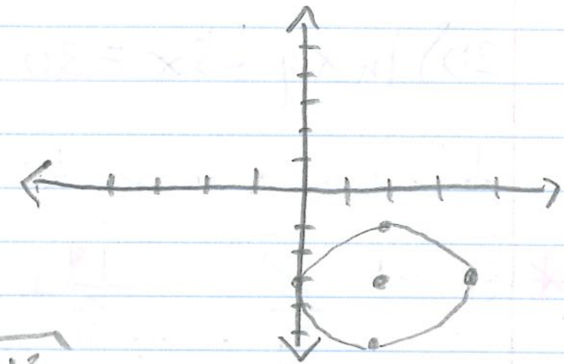
$(x-2)^2 + (y+3)^2 = 4$

$(y+3)^2 = 4 - (x-2)^2$

$y + 3 = \pm\sqrt{4-(x-2)^2}$

$\boxed{y = -3 \pm \sqrt{4-(x-2)^2}}$ → addition & Sub cannot be simplified

b) center $(2,-3)$
radius $= 2$



c) $y = -3 \pm \sqrt{4-(x-2)^2}$

$$y = -3 \pm \left[4-(x-2)^2\right]^{\frac{1}{2}}$$

$$y' = 0 \pm \frac{1}{2}\left[4-(x-2)^2\right]^{-\frac{1}{2}} \cdot -2(x-2)$$

$$y' = \frac{-(x-2)}{\pm\sqrt{4-(x-2)^2}}$$

$$y' = \frac{-(x-2)}{-3 \pm\sqrt{4-(x-2)^2}+3}$$

$$y' = \frac{-(x-2)}{y+3}$$

d) derivative of original eq.

$(3y^2)' = 6yy'$

$$(x^2+y^2-4\cancel{x}+6y+9=0)'$$

$$2x+2yy'-4+6y'+0=0$$

$$2yy'+6y' = -2x+4$$

$$y'(2y+6) = -2x+4$$

$$y' = \frac{-2x+4}{(2y+6)} = \frac{-2(x-2)}{2(y+3)} = \frac{-(x-2)}{(y+3)}$$

$(3x^2y)' = 6xy + y' \cdot 3x^2 = \underline{6xy + 3x^2y'}$

$(3xy^2)' = 3y^2 + 2yy' \cdot 3x = \underline{3y^2 + 6xyy'}$

28) $(x+y)^3 = x^3 + y^3$, $(-1, 1)$

$x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3$

$3x^2y + 3xy^2 = 0$

$6xy + 3x^2y' + 3y^2 + 6xyy' = 0$

$3x^2y' + 6xyy' = -6xy - 3y^2$

$$y' = \frac{-6xy - 3y^2}{3x^2 + 6xy}$$

b) $\dfrac{-6(-1)(1) - 3(1)^2}{3(-1)^2 + 6(-1)(1)} = \dfrac{6-3}{3-6} = \dfrac{3}{-3} = \boxed{-1}$

38) find slope = find derivative at a given point

$x^3 + y^3 - 6xy = 0$, $\left(\frac{4}{3}, \frac{8}{3}\right)$

$(-6xy)'$

$-6y - 6xy'$

$3x^2 + 3y^2y' - 6y - 6xy' = 0$

$3y^2y' - 6xy' = 6y - 3x^2$ → factor out y' then divide

$$\boxed{y' = \frac{6y - 3x^2}{3y^2 - 6x}}$$

$m = y'$ at the point

$$= \frac{6\left(\frac{8}{3}\right) - 3\left(\frac{4}{3}\right)^2}{3\left(\frac{8}{3}\right)^2 - 6\left(\frac{4}{3}\right)} = \frac{\frac{16(3)}{1(3)} - \frac{16}{3}}{\frac{64}{3} - \frac{8(3)}{1(3)}} = \frac{\frac{48-16}{3}}{\frac{64-24}{3}}$$

$$= \frac{32}{40} = \boxed{\frac{4}{5}}$$

$(x^2y^2)' = 2xy^2 + 2yy' \cdot x^2$

46) Find equation $y - y_1 = m(x - x_1)$

$y^2(x^2+y^2) = 2x^2$, $(1,1)$

$m = y'(x_1, y_1)$

$x^2y^2 + y^4 = 2x^2$

$2xy^2 + 2x^2yy' + 4y^3 \cdot y' = 4x$

$2x^2yy' + 4y^3y' = 4x - 2xy^2$

$y'(2x^2y + 4y^3) = 4x - 2xy^2$

$y' = \dfrac{4x - 2xy^2}{2x^2y + 4y^3}$

$m = y' = \dfrac{4(1) - 2(1)(1)^2}{2(1)^2(1) + 4(1)^3} = \boxed{\dfrac{1}{3}}$

$y - y_1 = m(x - x_1) = \quad y - 1 = \frac{1}{3}(x - 1)$

$y - 1 = \frac{1}{3}x - \frac{1}{3}$

$\boxed{y = \frac{1}{3}x + \frac{2}{3}}$

properties needed = 1) $\ln M^p = p \ln M$
2) $\ln MN = \ln M + \ln N$
3) $\ln\left(\frac{M}{N}\right) = \ln M - \ln N$

70) $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

$\ln y = \ln\left(\dfrac{(x+1)(x+2)}{(x-1)(x-2)}\right)$

$\ln y = \ln(x+1) + \ln(x+2) - [\ln(x-1) - \ln(x-2)]$

$\ln y = \ln(x+1) + \ln(x+2) - \ln(x-1) - \ln(x-2)$

$\dfrac{y'}{y} = \dfrac{1(x-1)(x+2)(x-2)}{(x+1)(x-1)(x+2)(x+2)} + \dfrac{1(x+1)(x-2)(x-1)}{(x+2)(x+1)(x-1)(x-2)} - \dfrac{1(x+1)(x+2)(x-2)}{(x-1)(x+1)(x+2)(x-2)} - \dfrac{1(x+1)(x+2)(x-1)}{(x-2)}$

$(x-1)(x^2-4) = x^3 - 4x - x^2 + 4$
$(x-2)(x^2-1) = x^3 - x - 2x^2 + 2$
$(x+1)(x^2-4) = x^3 - 4x + x^2 - 4$
$(x+2)(x^2-1) = x^3 - x + 2x^2 - 2$

$\dfrac{y'}{y} = \dfrac{x^3 - x^2 - 4x + 4 + x^3 - 2x^2 - x + 2 - x^3 - x^2 + 4x + 4 - x^3 + x - 2x^2 + 2}{(x+1)(x-2)(x+2)(x-1)}$

cubes cancel
x cancel out as well

$\dfrac{y'}{y} = \dfrac{-6x^2 + 12}{(x+1)(x-1)(x+2)(x-2)}$

$y' = \dfrac{-6x^2 + 12}{(x+1)(x-1)(x+2)(x-2)} \cdot (y)$ * y was given at the beggining

$y' = \left[\dfrac{\cancel{(x+1)}\cancel{(x+2)}}{(x-1)(x-2)}\right] \cdot \left[\dfrac{-6x^2 + 12}{\cancel{LCD}}\right]$

$\boxed{y' = \dfrac{-6(x^2-2)}{(x-1)^2(x-2)^2}}$

$$\left(\frac{1}{x}\right)' = (x^{-1})' = -1x^{-2} = -\frac{1}{x^2}$$

$$-\frac{1}{\cancel{x^2}} \cdot \frac{\cancel{x}}{1+x} = \frac{1}{1+x} \cdot \frac{1}{x}$$

74) $y = (1+x)^{\frac{1}{x}}$

$$\ln y = \ln (1+x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x} \ln (1+x)$$

$$\frac{y'}{y} = -\frac{1}{x^2}\left[\ln(1+x)\right] + \frac{1}{1+x} \cdot \frac{1}{x}$$

$$\frac{y'}{y} = \left[-\frac{1}{x^2}\left[\ln(1+x) - \frac{x}{1+x}\right]\right] \cdot y$$

$$y' = (1+x)^{\frac{1}{x}} \cdot \left[-\frac{1}{x^2}\left[\ln(1+x) - \frac{x}{1+x}\right]\right]$$

$$y' = -\frac{1}{x^2}(1+x)^{\frac{1}{x}}\left[\ln(1+x) - \frac{x}{1+x}\right]$$

38) $f(x) = \sec^{-1} x + \csc^{-1} x$

$$= \frac{1}{|x|\sqrt{x^2-1}} + \frac{-1}{|x|\sqrt{}}$$

example #1 on page 175 may be on exam

3.6 Derivative of inverse fxns

Remember formulas on page 179

3.6 Examples

20) $f(t) = \sin^{-1}(t)^2$ #19 formula

$$f'(t) = \frac{2t}{\sqrt{1-(t^2)^2}}$$

$$f'(t) = \frac{2t}{\sqrt{1-t^4}}$$

28) $g(x) = e^x \sin^{-1}(x)$

$$g'(x) = e^x \sin^{-1}(x) + \frac{1}{\sqrt{1-x^2}} \cdot e^x$$

$$\boxed{g'(x) = e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right)}$$

40) $y = x\tan^{-1}(2x) - \frac{1}{4}\ln(1+4x^2)$

$y' = 1\cdot\tan^{-1}(2x) + \frac{2}{1+(2x)^2}\cdot x - \frac{1}{4}\cdot\frac{8x}{1+4x^2}$

$y' = \tan^{-1}(2x) + \cancel{\frac{2}{1+4x^2}} - \cancel{\frac{2x}{1+4x^2}}$

$\boxed{y' = \tan^{-1}(2x)}$

92) $y = \ln\sqrt[3]{\frac{x-2}{x+2}}$

$y = \ln\left(\frac{x-2}{x+2}\right)^{1/3}$

$y = \frac{1}{3}\ln\left(\frac{x-2}{x+2}\right)$

$y = \frac{1}{3}\left[\ln(x-2) - \ln(x+2)\right]$

$y' = \frac{1}{3}\left[\frac{1(x+2)}{x-2(x+2)} - \frac{1(x-2)}{x+2(x-2)}\right]$

$y' = \frac{1}{3}\left[\frac{x+2-x+2}{(x-2)(x+2)}\right]$

$\boxed{y' = \frac{4}{3(x^2-4)}}$

a) $\log m^p = P\log m$

b) $\ln\frac{M}{N} = \ln M - \ln N$

c) $\log MN = \log M + \log N$

d) $\log_a U = \frac{\ln U}{\ln a}$

$= \frac{1}{\ln a}\cdot\ln U$

Section 3.4 examples

Section 3.4 examples

106) $g(x) = \sqrt{x} + e^x \ln x$

$U = e^x$ | $V = \ln x$

$U' = e^x$ | $V' = \frac{1}{x}$

$g(x) = x^{1/2} + e^x \ln x$

$g'(x) = \frac{1}{2}x^{-1/2} + e^x \ln x + \frac{1}{x} \cdot e^x$

$g'(x) = \frac{1}{2}x^{-1/2} + e^x \ln x + x^{-1} e^x$

$g''(x) = \frac{1}{2}\left(-\frac{1}{2}\right)x^{-3/2} + e^x \ln x + \frac{1}{x}e^x + (-x^{-2})e^x + e^x \cdot x^{-1}$

$U = x^{-1}$ | $V = e^x$

$U' = -x^{-2}$ | $V' = e^x$

$$\boxed{g''(x) = -\frac{1}{4}x^{-3/2} + e^x \ln x + \frac{2}{x}e^x - \frac{e^x}{x^2}}$$

122) $y = 2e^{1-x^2}$; $(\overset{x_1}{1}, \overset{y_1}{2})$

$m = y'(1)$

$= -4(1)e^{(1-1^2)}$

$= -4(1)(1)$

$= -4$

$y - y_1 = m(x - x_1)$

$y - 2 = -4(x - 1)$

$y - 2 = -4x + 4$

$y = -4x + 4 + 2$

$$\boxed{y = -4x + 6}$$

32) $y = \dfrac{x}{\sqrt{x^4+2}}$

$U = x$

$U' = 1$

$V = (x^4+2)^{\frac{1}{2}}$

$V' = \frac{1}{2}(x^4+2)^{-\frac{1}{2}} \cdot (u^4)'$

$V' = 2x^3(x^4+2)$

$y = \dfrac{x}{(x^4+2)^{\frac{1}{2}}}$

$y'(x) = \dfrac{1 \cdot (x^4+2)^{\frac{1}{2}} - 2x^3 \cdot (x^4+2)^{-\frac{1}{2}} \cdot x}{\left[(x^4+2)^{\frac{1}{2}}\right]^2}$

$\dfrac{a^{-\frac{1}{2}}}{a} = \dfrac{1}{a^{3/2}}$

$1 + \frac{1}{2} = \frac{3}{2}$

$y'(x) = \dfrac{(x^4+2)^{-1/2}\left[\overset{x^4+2}{(x^4+2)'} - 2x^4\right]}{(x^4+2)}$

$$\boxed{y'(x) = \dfrac{(2-x^4)}{(x^4+2)^{\frac{1}{2}}}}$$

36) $g(x) = \left(\dfrac{3x^2-1}{2x+5}\right)^3$

$U = 3x^2-1$ $\quad U' = 6x$

$V = 2x+5$ $\quad V' = 2$

$12x^2+30x - 6x^2+2$

$\dfrac{6x(2x+5) - 2(3x^2-1)}{(2x+5)^2}$

$\dfrac{6x^2+30x+2}{(2x+5)^2}$

$2(3x^2+15x+1)$

$g'(x) = 3\left(\dfrac{3x^2-1}{2x+5}\right)^2 \cdot \left[\dfrac{2(3x^2+15+1)}{(2x+5)^2}\right]$

$$g'(x) = \frac{3(3x^2-1)^2}{(2x+5)^2} \cdot \frac{2(3x^2+15x+1)}{(2x+5)^2}$$

$$\boxed{g'(x) = \frac{6(3x^2-1)\cdot(3x^2+15x+1)}{(2x+5)^4}}$$

50) a) $y = e^{3x}$, $(0,1)$

$m = y'(0)$

$y' = 1e^{2x} \cdot 2$

$y' = 2e^{2x}$

$m = y'(0)$

$= 2e^{2(0)}$

$m = 2(1)$

$\boxed{m=2}$

b) $y = e^{-2x}$; $(0,1)$

$y' = e^{-2x} \cdot (-2)$

$y' = -2e^{-2x}$

$m = y'(0)$

$= 2e^{-2(0)}$

$= -2(1)$

$\boxed{m=-2}$

48) $y = \operatorname{Sec}^{-1} 4x$ $\quad\left(\frac{\sqrt{2}}{4}, \frac{\pi}{4}\right)$

$$y' = \frac{4}{|4x| - \sqrt{(4x)^2 - 1}} = \frac{1}{x\sqrt{16x^2 - 1}}$$

$$m = y'\left(\frac{\sqrt{2}}{4}\right) = \frac{1}{\left(\frac{\sqrt{2}}{4}\right)\sqrt{16\left(\sqrt{2}/4\right)^2 - 1}}$$

$m = 2\sqrt{2} \longrightarrow y - y_1 = m(x - x_1)$

$$\boxed{y - \frac{\pi}{4} = 2\sqrt{2}\left(x - \frac{\sqrt{2}}{4}\right)}$$

52) $g(x) = \operatorname{Tan}^{-1} x \quad ; \; x = 1$

$$g'(x) = \frac{u'}{1 + u^2} = \frac{1}{1 + x^2}$$

$$g'(x) = \frac{1}{1 + 1^2} = \frac{1}{2}$$

$\operatorname{Tan}^{-1}(1) = \theta$

$\operatorname{Tan}\theta = 1$

$\theta = \frac{\pi}{4}$

$y = \frac{\pi}{4}$ so $y - y_1 = m(x - x_1)$

$$\boxed{y - \frac{\pi}{4} = \frac{1}{2}(x - 1)}$$

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3.7 Related Rates

$$(x)' = \frac{dx}{dt} \quad \text{or} \quad x'$$

$$(y)' = \frac{dy}{dt} \quad \text{or} \quad y'$$

Ex: $2x^4y^3 = 6$

$$8x^3\frac{dx}{dt} \cdot y^3 + 6y^2\frac{dy}{dt} \cdot x^4 = 0$$

3.7 Examples

4) $x^2 + y^2 = 25$

a) $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

a) $2(3) \cdot (8) + 2(4)\frac{dy}{dt} = 0$

$$48 + 8\frac{dy}{dt} = 0$$

$$\boxed{\frac{dy}{dt} = -6}$$

b) $2(4)\cdot\frac{dx}{dt}+2(3)\cdot(-2)=0$

$8\frac{dx}{dt}-12=0$

$\frac{dx}{dt}=\frac{12}{8}$ $\boxed{\frac{dx}{dt}=\frac{3}{2}}$

8) $y=\sin x$ $\frac{dx}{dt}=2\text{cm}$ $\frac{dy}{dt}=?$

$y'=\cos x$

$\frac{dy}{dt}=(\cos x)\frac{dx}{dt}$

$\frac{dy}{dt}=2\cos x$

b) $x=\frac{\pi}{4}$

$\frac{dy}{dt}=2\cos\frac{\pi}{4}$

$\boxed{\frac{dy}{dt}=\sqrt{2}}$

a) $\frac{dy}{dt}=2\cos\frac{\pi}{6}$

$\frac{dy}{dt}=2\left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\frac{dy}{dt}=\sqrt{3}}$

c) $x=\frac{\pi}{6}$

$\frac{dy}{dt}=2\left(\frac{1}{2}\right)$

$\boxed{\frac{dy}{dt}=1}$

18) Volume

$V_{sphere} = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 3\left(\frac{4}{3}\pi r\right)^2 \cdot \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

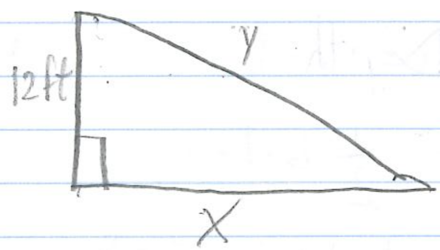
$$\frac{dv}{dt} = 4\pi r^2 \cdot (2)$$

a) $\frac{dv}{dt} = 8\pi(6)^2 = 288\pi$

$\frac{dv}{dt} = 8\pi(24)^2 = 4{,}608\pi$

b) rate of change of the volume is proportional to the square of the radius

30) Boating:



$$y^2 = x^2 + (12)^2$$

$$y^2 = x^2 + 144$$

$$2y \cdot \frac{dy}{dt} = 2x \cdot \left(\frac{dx}{dt}\right)$$

a) pulling rope = decreasing

so rate of change $= 4$ is $\frac{dy}{dt}$

$\frac{dy}{dt} = -4$ $\quad y = 13\text{ ft}$

So $y^2 = x^2 + 144$ is $(13)^2 = x^2 + 144$

$169 - 144 = x^2$ $\quad \boxed{x = 5}$

$$2y \cdot \frac{dy}{dt} = 2x \cdot \frac{dx}{dt}$$

$$2(13) \cdot (-4) = 2(5) \cdot \frac{dx}{dt}$$

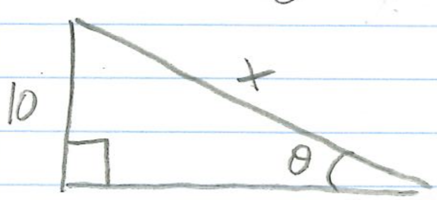
$10 \frac{dx}{dt} = -104$ $\quad \boxed{\frac{dx}{dt} = -10.4}$

b) $\frac{dx}{dt} = 4.$ $\quad y = 13 \;;\; x = 5$

$2(13) \cdot \frac{dy}{dt} = 2(5) \cdot (4)$

$26 \frac{dy}{dt} = 40$ $\quad \boxed{\frac{dy}{dt} = 1.54}$

44) Angle of elevation



$\frac{dx}{dt} = -1$

$X = 25$

$\sin\theta = \frac{10}{X} = \sin\theta = \frac{10}{25} = \theta = \sin^{-1}\left(\frac{10}{25}\right)$

$\boxed{\theta = .4115}$

$\sin\theta = 10x^{-1}$

$(\cos\theta) \frac{d\theta}{dt} = -10x^{-2} \frac{dx}{dt}$

So: $[\cos(.4115)] \cdot \frac{d\theta}{dt} = \frac{-10}{25^2} \cdot (-1)$

$.9165 \cdot \left(\frac{d\theta}{dt}\right) = 0.016$

$\frac{d\theta}{dt} = \frac{0.016}{.9165}$ $\quad \boxed{\frac{d\theta}{dt} = 0.017}$

3.8 Newtons Method

how to find the zero of a complicated fxn like:

Ex: $\cos x^2 - 25x^2 + 3 = 0$

Newtons Method: $X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$

3.8 Examples

2) $2x^2 - 3$ $\qquad x^2 = 1$

$f'(x) = 4x$

$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$

$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$

$X_2 = 1 - \frac{f(1)}{f'(1)}$ → show this much work and we're good

* On Calculator:

$\boxed{Y_1} = X - ((2x^2 - 3)/(4x))$

$\boxed{2nd}$ $\boxed{Graph}$ Xvalue = 1 Plug in your x values

$x - ((f(x)) / (f'(x)))$

$X_2 = 1.25$ ✓

now do a second iteration

$$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)}$$

$$X_3 = 1.25 - \frac{f(1.25)}{f'(1.25)}$$

8) $f(x) = x - 2(x+1)^{\frac{1}{2}}$
$f'(x) = 1 - (x+1)^{-\frac{1}{2}}$

$$X_{n-1} = X_n - \frac{f(X_1)}{f'(X_1)}$$

$X_1 = 4$

$$X_2 = 4 - \frac{f(4)}{f'(4)}$$

$X_2 = 4.854101$

$X_3 = \ldots$

$X_3 = 4.828446$

$X_4 = \ldots$ $X_4 = 4.828427$

The zero is 4.828

14) $f(x) = \frac{1}{2}x^4 - 3x - 3$

$f'(x) = 2x^3 - 3$

→ $x_1 = -1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = -1 - \frac{f(-1)}{f'(-1)}$

$x_2 = -.9$

$x_3 = -.9 - \frac{f(-.9)}{f'(-.9)}$

$x_3 = -.\underbrace{89}3707$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

You can stop once the first 3 decimall places are the same

$x_4 = -.\underbrace{89}3686$

→ $x_1 = 2$

$x_2 = 2 - \frac{f(2)}{f'(2)}$

$x_2 = 2.076923$

$$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)}$$

$X_3 = 2.\underbrace{072}040$

$$X_4 = X_3 - \frac{f(X_3)}{f'(X_3)}$$

$X_4 = 2.\underbrace{072}019$

The zeros are $-.893$ & 2.072

16) $f(x) = x^3 - \cos x$

$f'(x) = 3x^2 + \sin x$

$X_1 = 1$

$$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

$X_2 = .880332$

$$X_3 = X_2 - \frac{f(X_2)}{f'(X_2)}$$

$X_3 = .865684$

$$X_4 = X_3 - \frac{f(X_3)}{f'(X_3)}$$

$X_4 = \underbrace{.865}474$

The zero is .865

If you have 2 fxns like f(x) & G(x), then graph them, see who is higher, subtract higher fxn from lower fxn by setting them equal to each other, then bring the lower fxn over to set everything to zero, simplify & call the new fxn h(x) & proceed with finding the derivative from there.

Ex: $f(x) = 2x+4$

$G(x) = x+2$

$h(x) = 2x+4 = x+2$

$h(x) = 2x+4-x-2=0$

$\rightarrow x+2=0$

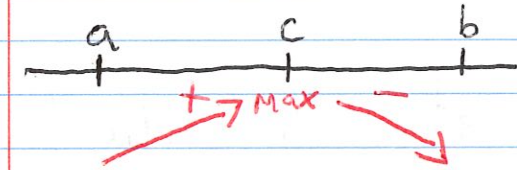
$h'(x) = 1$

...

4.1 Extrema on Interval

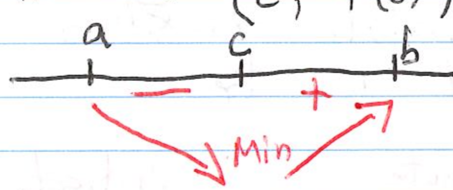
Let $f(x)$ be a continuous fxn on $[a, b]$, and have "C" be a number between "a" and "b"

1) if $f'(x) > 0$ on (a, c) and $f'(x) < 0$ on (c, b) then "f" has a maximum on (a, b). The maximum is at "c" and $f(c)$. $(c, f(c))$



Sign of $f'(x)$

2) if $f'(x) < 0$ on (a, c) and $f'(x) > 0$ on (c, b) then "f" has a minimum on (a, b). The minimum is at $(c, f(c))$



3) $f(x)$ has a critical value at "C" if $f'(c) = 0$ or $f'(c)$ is undefined or does not exist. The point $(c, f(c))$ is called a critical point

4) if $f'(x)$ is > 0 on (a,b) then "f" is increasing on (a,b)

5) if $f'(x)$ is < 0 on (a,b) then "f" is decreasing on (a,b)

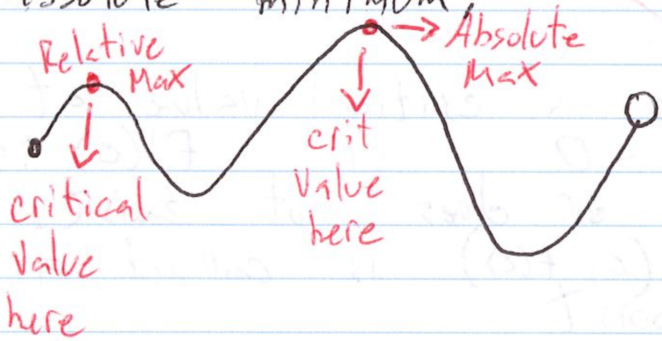
Absolute Extreme

a) find all critical numbers of "F" on (a,b)

b) Evaluate "F" at each critical numbers from (a)

c) Evaluate $F(a)$ & $f(b)$

d) The largest number from (b) and (c) is the absolute maxima & the smallest is the absolute minimum.



critical values are at every turn

Pg 209

Absolute Min/Max is highest/lowest point in the graph

$2a^{-3}b^3 + 6a^{-3}b^7$

$2a^{-3}b^3(1 + 3a^2b^4)$

4.1 Examples

2) A = absolute Min
B = Relative Max because G has higher pts
C = None no change in direction
D = Rel Min
E = Rel Max
F = Rel Min
G = None since its open

6) $f(x) = -3x\sqrt{x+1}$

$f'(x) = -3x(x+1)^{\frac{1}{2}}$

$f'(x) = -3(x+1)^{\frac{1}{2}} + \frac{1}{2}(x+1)^{-\frac{1}{2}}(-3x)$

$f'(x) = -\frac{3}{2}(x+1)^{-\frac{1}{2}}\left[2(x+1) + x\right]$

$f'(x) - \frac{3}{2}(x+1)^{-\frac{1}{2}}\left[3x+2\right]$

$x = -\frac{2}{3}$

$f'(-\frac{2}{3}) = -\frac{3}{2}(-\frac{2}{3}+1)^{-\frac{1}{2}}(3(-\frac{3}{2})+2)$

$\boxed{f'(-\frac{2}{3}) = 0}$

$(a^u)' = u'(\ln a)\,a^u$

20) $G(x) = 4x^2(3^x)$

$G'(x) = 4(2x(3^x) + (1)(\ln 3)(3^x) \cdot (x^2))$

$G'(x) = 4x(3^x)[2 + (\ln 3)(x)]$

$G'(x) = 0$

$4x = 0$ | $2 + x\ln 3 = 0$

$x = 0$ | $x = -\dfrac{2}{\ln 3}$

dont bother with $3^x = 0$ Since anything to power & can never be zero

24) $f(x) = x^2 + 2x - 4$; $[-1, 1]$

a) $f'(x) = 2x + 2$

$2x + 2 = 0$

$\boxed{x = -1} \longrightarrow$ crit #

b) $f(-1) = (-1)^2 + 2(-1) - 4$

$\boxed{= -5}$

c) $f(1) = (1)^2 + 2(1) - 4$

$\boxed{= -1}$

d) absolute max at $(1, -1)$
and absolute min at $(-1, -5)$

26) $f(x) = x^3 - 12x$; $[0,4]$

a) $f'(x) = 3x^2 - 12$

$\frac{3x^2}{3} - \frac{12}{3} = \frac{0}{3}$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$\boxed{x = -2}$ $x = 2$

not in domain

b) $f(2) = (2)^3 - 12(2) = \boxed{-16}$

c) $f(0) = 0^3 - 12(0) = \boxed{0}$

$f(4) = 4^3 - 12(4) = \boxed{16}$

d) absolute max @ $(4, 16)$

absolute min @ $(2, -16)$

38) $y = x^2 - 2 - \cos x$; $[-1, 3]$

a) $y' = 2x + \sin x$

$2x + \sin x = 0$

$x = 0$ $\rightarrow$ you can use Newtons Method

b) $y(0) = 0^2 - 2 - \cos(0) = \boxed{-3}$

c) $y(-1) = -1^2 - 2 - \cos(-1)$

$= -1 - \pi$

$\boxed{= -4.14...}$

$\cos(-1) = \theta$

$\cos\theta = -1$

$\theta = \pi$

$y(3) = 3^2 - 2 - \cos(3)$

$= 7 - \cos(3)$

$\boxed{= 7.9..}$

d) Absolute Max @ $(3, 7.9..)$

Ab bsolute Min @ $(0, -3)$

4.2 Rolles Theorem & Mean Value Theorem

Let $f(x)$ be a continuous fxn on $[a,b]$ and differentiable (a,b).
If $f(a) = f(b)$ then there exists a number "C" between (a,b) such that $f'(c) = 0$

or more #'s

Mean Value Theorem

Let $f(x)$ be a continuous fxn on $[a,b]$ and differentiable on (a,b).

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

There is at least a number "C" between (a,b) such that $f'(c) =$

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4.2 Examples

4) $f(x) = \sqrt{(2 - x^{2/3})^3}$; $[-1, 1]$

$f(-1) = f(1) \rightarrow$ need this

$f(-1) = \sqrt{(2-(-1)^{\frac{2}{3}})^3} = \boxed{1}$ ✓

$f(1) = \sqrt{(2-(1)^{2/3})^3} = \boxed{1}$ ✓

$f'(x) = (2-x^{\frac{2}{3}})^{\frac{3}{2}}$

$f'(x) = \frac{3}{2}(2-x^{\frac{2}{3}})^{\frac{3}{2}} + \left(-\frac{2}{3}x^{-\frac{1}{3}}\right)$

$f'(x) = -\frac{\sqrt{2-x^{\frac{2}{3}}}}{x^{1/3}}$ $\rightarrow$ not diff at 0 since undefined

f` is not differentiable at $x=0$
Rolle's theorem does not apply

8) $f(x) = -3x(x+1)^{\frac{1}{2}}$

$-3x(x+1)^{\frac{1}{2}} = 0$

$-3x = 0$ / $x+1=0$

$x=0$ / $x=-1$

$\boxed{[-1, 0] \text{ domain}}$

$f'(x) = -3\left(x(x+1)^{\frac{1}{2}}\right)$

$f'(x) = -3\left[1(x+1)^{\frac{1}{2}} + \frac{1}{2}(x+1)^{-\frac{1}{2}} \cdot x\right]$

$f'(x) = -\frac{3}{2}(x+1)^{-\frac{1}{2}}\left[2(x+1) + x\right]$

$f'(x) = -\frac{3}{2}(x+1)^{-\frac{1}{2}}\left[3x+2\right]$

$f'(x) = \frac{-3(3x+2)}{2(x+1)^{\frac{1}{2}}}$

$f'(x) = 0 = \quad 3x+2=0$

$\boxed{x = -\frac{2}{3}}$

14) $f(x) = (x-3)(x+1)^2$; $[-1, 3]$

$f(a) = f(b)$?

$f(-1) = (-1-3)(-1+1)^2 = \boxed{0}$ ✓

$f(3) = (3-3)(3+1)^2 = \boxed{0}$ ✓

Since $f(-1) = f(-3)$ we can apply Rolle's theo

$f'(x) = 1(x+1)^2 + (2x+1) \cdot (x-3)$

$f'(x) = (x+1)\left[(x+1) + 2(x-3)\right]$

$f'(x) = (x+1)\left[3x-5\right]$

c needs to be between a, b

$f'(x) = 0 =$

$x+1=0$ / $3x-5=0$

$\boxed{x=-1}$ ✗ / $\boxed{x=\frac{5}{3}}$ ✓

on boundry so not valid

24) $f(x) = \cos 2x \quad \left[-\frac{\pi}{12}, \frac{\pi}{6}\right]$

$f\left(-\frac{\pi}{12}\right) = \cos\left(2\left(-\frac{\pi}{12}\right)\right)$

$= \boxed{\cos\left(-\frac{\pi}{6}\right)} = \boxed{\frac{\sqrt{3}}{2}}$ X

$f\left(\frac{\pi}{6}\right) = \cos 2\left(\frac{\pi}{6}\right) = \boxed{\cos\left(\frac{\pi}{3}\right)} = \boxed{\frac{1}{2}}$ X

Since $f\left(-\frac{\pi}{12}\right) \neq f\left(\frac{\pi}{6}\right)$ then we cannot apply Rolle's theo

34) $c(x) = 10\left(\frac{1}{x} + \frac{x}{x+3}\right)$

a) $c(3) = 10\left(\frac{1}{3} + \frac{3}{6}\right) = \boxed{\frac{25}{3}}$

$c(6) = 10\left(\frac{1}{6} + \frac{6}{9}\right) = \boxed{\frac{25}{3}}$

$c(3) = c(6)$

b) $c'(x)\ 10\left(x^{-1} + \frac{x}{x+3}\right)$

$c'(x) = 10\left(-1x^{-2} + \frac{1(x+3) - (x)1}{(x+3)^2}\right)$

$c'(x) = 10\left(-\frac{1}{x^2} + \frac{3}{(x+3)^2}\right)$

$\rightarrow$

$$c'(x) = 0$$

$$\frac{10}{10}\left(-\frac{1}{x^2} + \frac{3}{x^2+6x+9}\right) = \frac{0}{10}$$

$$-\frac{1}{x^2} + \frac{3}{x^2+6x+9} = 0$$

$$\frac{3}{x^2+6x+9} = \frac{1}{x^2}$$

$$3x^2 = x^2+6x+9$$
$$2x^2-6x-9=0$$

$$a=2,\ b=-6\quad c=-9$$

$$x = \frac{6 \pm \sqrt{(-6)^2-4(2)(-9)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{36+72}}{4} \longrightarrow \frac{6 \pm \sqrt{108}}{4} \longrightarrow \frac{6 \pm \sqrt{36 \cdot 3}}{4}$$

$$= \frac{6 \pm 6\sqrt{3}}{4} = \frac{2(3 \pm 3\sqrt{3})}{2(2)} = \frac{3 \pm 3\sqrt{3}}{2}$$

$$x = \frac{3+3\sqrt{3}}{2} \quad \Big| \quad \frac{3-3\sqrt{3}}{2}$$

$$\boxed{x = 4.09} \quad \Big| \quad \boxed{x = -1.098} \longrightarrow \text{Not in domain}$$

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

They equal eachother

46) $f(x) = \frac{x+1}{x}$; $\left[\frac{1}{2}, 2\right]$

$f(x) = \cancel{\frac{x}{x}} + \frac{1}{x} = 1 + \frac{1}{x}$

$f\left(\frac{1}{2}\right) = 1 + \frac{1}{\frac{1}{2}} = \boxed{3}$

$f(2) = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$

$$\frac{f(2) - f\left(\frac{1}{2}\right)}{2 - \frac{1}{2}} = \frac{\frac{3}{2} - 3}{\frac{3}{2}} = \boxed{-1}$$

$f(x)' = -\frac{1}{x^2}$

$$f'(x) = \frac{f(2) - f\left(\frac{1}{2}\right)}{2 - \frac{1}{2}}$$

$\frac{1}{x^2} = -1$

$x^2 = 1$

$x = \pm 1$

$\boxed{x = -1}$ So let $\boxed{c = 1}$

Not in domain

No denominator to make undef
so OK to use mean value

50) $f(x) = 2\sin x + \sin 2x \quad [0, \pi]$

$f(0) \; 2(\sin 0) + \sin 2(0) = \boxed{0}$

$f(\pi) = 2\sin\pi + \sin 2\pi = \boxed{0}$

$f'(x) = 2\cos x + 2\cos 2x$

$f'(x) = \dfrac{f(\pi) - f(0)}{\pi - 0}$

$\dfrac{2}{2}\cos x + \dfrac{2}{2}\cos 2x = \dfrac{0}{2}$

$\cos x + \cos 2x = 0$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

1) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
2) $\cos 2\theta = 2\cos^2\theta - 1$
3) $\cos 2\theta = 1 - 2\sin^2\theta$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \boxed{\frac{\pi}{3}}, \frac{5\pi}{3}$ → not in domain

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$ → boundry

4.3 Increasing & Decreasing

Let "f" be a continuous fxn on $[a, b]$ and differentiable on (a, b)

1) if $f'(x) > 0$ for all x in (a, b), then "f" is increasing on $[a, b]$

2) if $f'(x) < 0$ for all x in (a, b) then "f" is decreasing on $[a, b]$

First Derivative Test

Let "f" be a continuous fxn on $[a, b]$ and differentiable on (a, b) and "c" be a critical number such that $f'(c) = 0$

a) if the signs of $f'(c)$ change from positive to negative, then "f" has a ~~minimum~~ Maximum at $(c, f(c))$