

A) Oscillatory Motion Homework

Using Hooke's Law

- 1) A 4N block rests on a vertical mass-less spring. The spring is compressed by 8 m. What is the spring constant of the spring? (0.5N/m)

Diagram showing a 0.4 kg block resting on a vertical spring. The spring is compressed by 8 m. The forces acting on the block are the normal force $F_R = 4\text{N}$ (up), the weight $F_w = -4\text{N}$ (down), and the spring force $F_s = -4\text{N}$ (down). The displacement is $\Delta X = -8\text{m}$.

$$F_R = -kx$$

$$4\text{N} = -k(-8)$$

$$-k = \frac{4\text{N}}{8\text{m}}$$

$$-k = -\frac{1}{2}$$

$$\boxed{k = \frac{1}{2}\text{N/m}}$$

- 2) A 12kg block rests on a vertical mass-less spring. The spring is compressed by 3 m. What is the spring constant of the spring? (40N/m)

Diagram showing a 12 kg block resting on a vertical spring. The spring is compressed by 3 m. The forces acting on the block are the normal force $F_R = 120\text{N}$ (up), the weight $F_w = -120\text{N}$ (down), and the spring force $F_s = -120\text{N}$ (down). The displacement is $\Delta X = -3\text{m}$.

$$F_R = -kx$$

$$-k = \frac{F_R}{x}$$

$$-k = \frac{120}{-3}$$

$$-k = -40\text{N/m}$$

$$\boxed{k = 40\text{N/m}}$$

- 3) A 6N block rests on a vertical mass-less spring. The spring's spring constant is 4 N/m. By how much is the spring compressed? (1.5m)

Diagram showing a 0.6 kg block resting on a vertical spring. The spring's spring constant is 4 N/m. The forces acting on the block are the normal force $F_R = 6\text{N}$ (up), the weight $F_w = -6\text{N}$ (down), and the spring force $F_s = -6\text{N}$ (down). The displacement is $\Delta X = ?$.

$$F_R = -kx$$

$$x = \frac{F_R}{-k}$$

$$x = \frac{6\text{N}}{-4\text{N/m}}$$

$$\boxed{x = \frac{3}{2}\text{m}}$$

- 4) A 4 kg block rests on a vertical mass-less spring. The spring is 12m long when uncompressed. The spring is compressed by 8m. What is the spring constant of the spring? (5N/m)

Diagram showing a 4 kg block resting on a spring. The spring is compressed by 8m from its original length of 12m. The forces acting on the block are the weight $F_w = -40\text{N}$ pointing down, and the spring force $F_R = 40\text{N}$ pointing up.

$F_R = 40\text{N}$	$F_R = -kx$
$F_w = -40\text{N}$	$-k = \frac{F_R}{x}$
$\Delta x = -8\text{m}$	$-k = \frac{40\text{N}}{-8\text{m}}$
$k = ?$	$-k = -5\text{N/m}$

$\boxed{k = 5\text{N/m}}$

- 5) A block that weighs X Newton rests on a vertical mass-less spring. The spring is Y meters long when uncompressed. The spring is Z meters long when compressed. What is the spring constant of the spring? ($\frac{X}{Y-Z}\text{ N/m}$)

Diagram showing a block of weight X resting on a spring. The spring is compressed by $(Y-Z)$ m from its original length of Y m.

$F_R = X\text{N}$	$F_R = -kx$
$F_w = -X\text{N}$	$-k = \frac{F_R}{x}$
$\Delta x = Y-Z\text{m}$	$k = -\frac{F_R}{x}$

$\boxed{k = \frac{-X}{Y-Z}}$

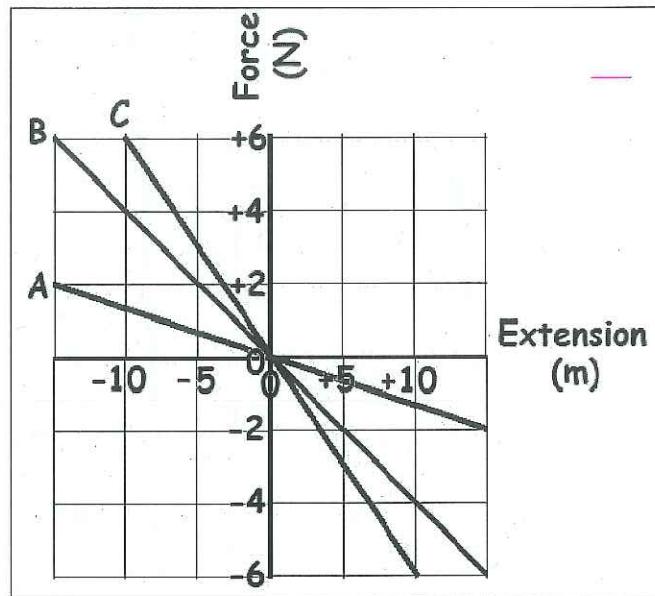
why?
 $\frac{+X}{Y-Z}$

Because $\Delta x = -(Y-Z)$

$\therefore k = \frac{-X}{-Y+Z}$

$\boxed{k = \frac{X}{Y-Z}}$

Consider this graph:



- 6) What is the spring constant associated with trace A on the graph? (0.133 N/m)

$$F_R = -kx$$
$$y = mx + b$$
$$-k \approx mx$$
$$\frac{\text{rise}}{\text{run}} = \text{Slope} = -k$$
$$\text{Slope} = \frac{-2}{15} = -k$$
$$-k = -0.133$$
$$k = 0.133 \text{ N/m}$$

- 7) What is the spring constant associated with trace B on the graph? (0.4 N/m)

$$-k = \text{Slope}$$

$$-k = \frac{-4}{10}$$

$$k = 0.4 \text{ N/m}$$

- 8) What is the spring constant associated with trace C on the graph? (0.6 N/m)

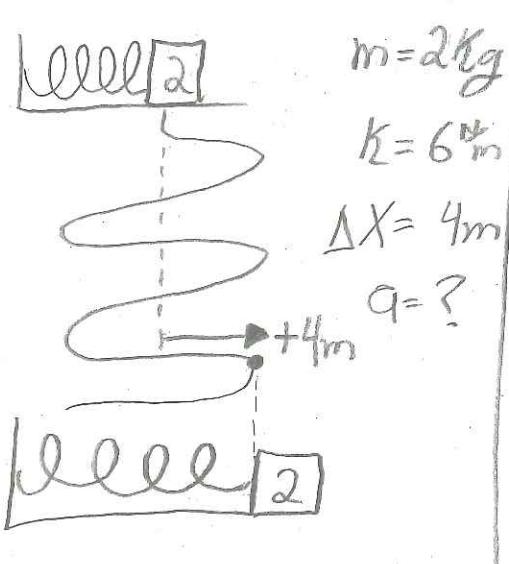
$$-k = \text{Slope}$$

$$-k = \frac{-6}{10}$$

$$k = 0.6 \text{ N/m}$$

Acceleration of a load at a specific displacement during Simple Harmonic Motion

- 9) A 2 kg mass is attached to the end of a horizontal mass-less spring that has a spring constant of 6 N/m. The mass displays simple harmonic oscillation while sliding over a frictionless horizontal surface. What is the acceleration of the mass when it has a displacement of +4m with respect to its equilibrium position? (-12 m/s²)



$$m = 2\text{kg}$$

$$K = 6\text{N/m}$$

$$\Delta X = 4\text{m}$$

$$a = ?$$

$$\text{eqn } F_R = -kx \quad : \quad -kx = ma$$

Sum of the forces horizontally in this system is the restoring force of the spring. $F_R = \sum F_h$

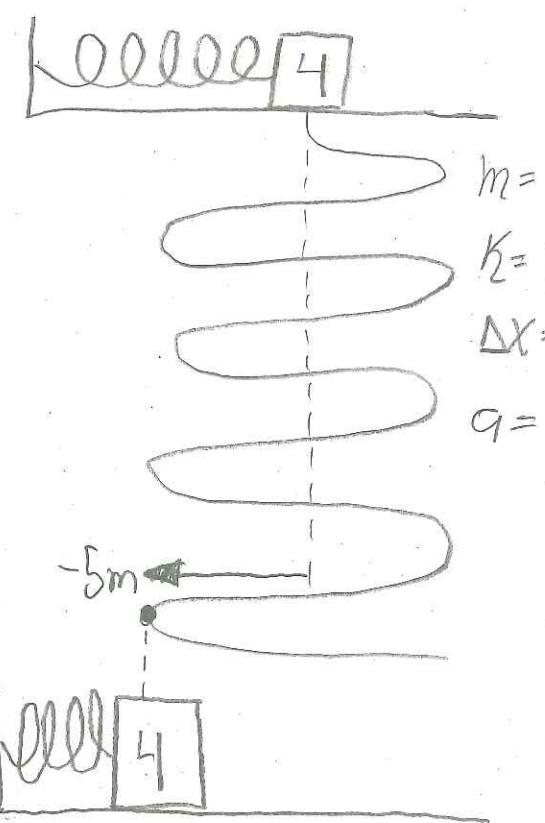
$$a = \frac{-kx}{m}$$

$$a = \frac{-6(4)}{2}$$

$$a = -12\text{m/s}^2$$

negative acceleration means it's being pulled back towards middle

- 10) A 4 kg mass is attached to the end of a horizontal mass-less spring that has a spring constant of 8 N/m. The mass displays simple harmonic oscillation while sliding over a frictionless horizontal surface. What is the acceleration of the mass when it has a displacement of -5m with respect to its equilibrium position? (+10 m/s²)



$$m = 4\text{kg}$$

$$K = 8\text{N/m}$$

$$\Delta X = -5\text{m}$$

$$a = ?$$

$$F_R = -kx \quad F = ma$$

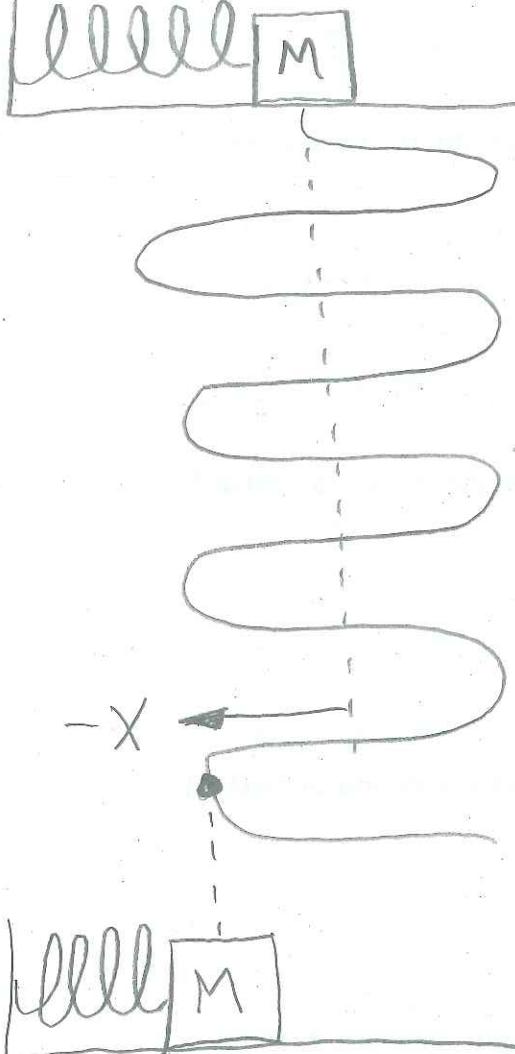
$$-kx = ma$$

$$a = \frac{-kx}{m}$$

$$a = \frac{-8(-5)}{4}$$

$$a = 10\text{m/s}^2$$

- 11) An M kg mass is attached to the end of a horizontal mass-less spring that has a spring constant of K N/m. The mass displays simple harmonic oscillation while sliding over a frictionless horizontal surface. What is the acceleration of the mass when it has a displacement of $-X$ meters with respect to its equilibrium position? ($a = \left(\frac{KX}{M}\right) m/s^2$)



$$K = K \text{ N/m}$$

$$m = M \text{ kg}$$

$$\Delta X = -X \text{ m}$$

$$a = ?$$

$$F_R = -KX$$

$$F = ma$$

$$-KX = ma$$

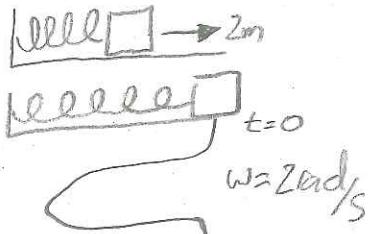
$$a = -\frac{KX}{m}$$

$$a = -\frac{K(-X)}{M}$$

$$a = \frac{KX}{M} \text{ m/s}^2$$

SHM Motion at specific times using A and ω

- 12) A block is attached to the end of a horizontal mass-less spring. The block oscillates with simple harmonic motion. The block is moved 2m to the right of its equilibrium position. The block is released at time $t = 0s$. The block oscillates with a frequency of 2rad/s .



$$A = 2\text{m}$$

$$\omega = 2 \text{ rad/s}$$

$$t = 0$$

- i) What is the amplitude of the block's oscillation? (2m)

The block can never travel more than the initial stretched 2m

$$A = 2\text{m}$$

- ii) What is the displacement of the block at $t=0.3\text{s}$? (+1.65 m)

$$X = A \cos(\omega t) \rightarrow X = 2 \cos(0.6)$$

$$X = 2 \cos(2(0.3))$$

$$X = 1.65\text{m}$$

- iii) What is the instantaneous velocity of the block at $t=0.3\text{s}$? (-2.26 m/s)

$$V = -A\omega \sin(\omega t)$$

$$V = -2(2) \sin(2(0.3))$$

$$V = -4 \sin(0.6)$$

$$V = -2.26\text{m/s}$$

- iv) What is the instantaneous acceleration of the block at $t=0.3\text{s}$? (-6.60 m/s 2)

$$a = -A\omega^2 \cos(\omega t)$$

$$a = -2(2)^2 \cos(2(0.3))$$

$$a = -8 \cos(0.6)$$

$$a = -6.60\text{m/s}^2$$

A block is attached to the end of a horizontal mass-less spring. The block oscillates with simple harmonic motion. The block is moved to 4m to the right of its equilibrium position. The block is released at time $t = 0$ s. The block oscillates with a frequency of 8rad/s.



$$A = 4\text{m}$$



$$\omega \text{ or } f = 8\text{ rad/s}$$

v) What is the amplitude of the block's oscillation? (4m)

$$A = 4\text{m}$$

vi) What is the displacement of the block at $t=0.12\text{s}$? (+2.29 m)

$$X = A \cos(\omega t)$$

$$X = 4 \cos(0.96)$$

$$X = 4 \cos(8(0.12))$$

$$X = 2.29\text{m}$$

vii) What is the instantaneous velocity of the block at $t=0.12\text{s}$? (-26.21 m/s)

$$V = -A\omega \sin(\omega t)$$

$$V = -32 \sin(0.96)$$

$$V = -4(8) \sin(8(0.12))$$

$$V = -26.21\text{m/s}$$

viii) What is the instantaneous acceleration of the block at $t=0.12\text{s}$? (-146.82 m/s²)

$$a = -A\omega^2 \cos(\omega t)$$

$$a = -4(8)^2 \cos(8(0.12))$$

$$a = -146.82\text{m/s}^2$$

$$a = -256 \cos(0.96)$$

- 13) A block is attached to the end of a horizontal mass-less spring. The block oscillates with simple harmonic motion. The block is moved to 8m to the right of its equilibrium position. The block is released at time $t = 0$ s. The block oscillates with a frequency of 1.4rad/s.



$$A = 8\text{m}$$

$$\omega = f = 1.4 \frac{\text{rad}}{\text{s}}$$

i) What is the amplitude of the block's oscillation? (8m)

$$A = 4\text{m}$$

ii) What is the displacement of the block at $t=0.2\text{s}$? (+7.67 m)

$$x = A \cos(\omega t)$$

$$x = 8 \cos(1.4(0.2))$$

$$x = 8 \cos(1.4(0.2))$$

$$x = 7.68\text{m}$$

iii) What is the instantaneous velocity of the block at $t=0.2\text{s}$? (-3.09 m/s)

$$v = -A\omega \sin(\omega t)$$

$$v = -11.2 \sin(0.28)$$

$$v = -8(1.4) \sin(1.4(0.2))$$

$$v = -3.10\text{m/s}$$

iv) What is the instantaneous acceleration of the block at $t=0.2\text{s}$? (-15.07 m/s²)

$$a = -A\omega^2 \cos(\omega t)$$

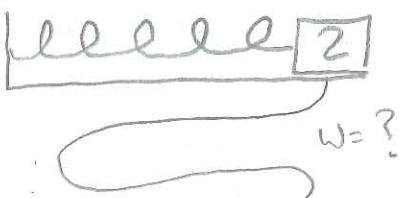
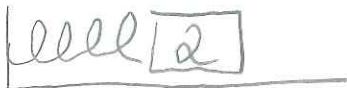
$$a = -15.68 \cos(0.28)$$

$$a = -8(1.4)^2 \cos(1.4(0.2))$$

$$a = -15.07\text{m/s}^2$$

Angular Frequency of Vibration based on k and m

- 14) A 2kg block is attached to the end of a mass-less spring that has a spring constant of 4N/m. The mass is displaced from equilibrium and released. With what angular frequency does the mass oscillate? (1.4 rad/s)



$$m = 2 \text{ kg}$$

$$k = 4 \text{ N/m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{4}{2}}$$

$$\boxed{\omega = 1.4 \text{ rad/s}}$$

- 15) A block is attached to the end of a mass-less spring that has a spring constant of 5 N/m. The mass is displaced from equilibrium and released. The mass oscillates with an angular frequency of 15 radians/s. What is the mass of the block? (0.022 kg)



$$\omega = 15 \text{ rad/s}$$

$$k = 5 \text{ N/m}$$

$$\omega = 15 \text{ rad/s}$$

$$m = ?$$

$$\omega = \sqrt{\frac{k}{m}}$$

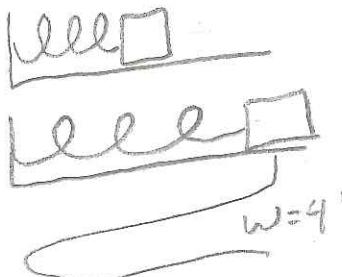
$$\omega^2 = \frac{k}{m}$$

$$m = \frac{k}{\omega^2}$$

$$m = \frac{5}{15^2}$$

$$\boxed{m = 0.022 \text{ kg}}$$

- 16) A block is attached to the end of a mass-less spring. The mass is displaced from equilibrium and released. The mass oscillates at 4 radians/s. What would be the new frequency of oscillation if the mass of the block was doubled and the spring constant were quartered? (1.41 rad/s)



$$\omega = 4 \frac{\text{rad}}{\text{s}}$$

$$\frac{\text{New } \omega_2 = \sqrt{\frac{K_2}{m_2}}}{\text{Old } \omega_1 = \sqrt{\frac{K_1}{m_1}}} = \frac{\omega_2 = \sqrt{\frac{.25}{2}}}{4 = \sqrt{\frac{1}{1}}} = \frac{\omega_2}{4} = \frac{\sqrt{\frac{1}{8}}}{1}$$

$\omega_1 = 4 \frac{\text{rad}}{\text{s}}$	$\omega_2 = ?$
$m_1 = 1$	$m_2 = 2$
$K_1 = 1$	$K_2 = .25$

$$\omega_2 = 4 \left(\sqrt{\frac{1}{8}} \right) \rightarrow \boxed{\omega_2 = 1.41 \frac{\text{rad}}{\text{s}}}$$

- 17) A block of mass X kg is attached to the end of a mass-less spring that has a spring constant of Y N/m. The mass is displaced from equilibrium and released. With what angular frequency does the mass oscillate? ($\left(\sqrt{\frac{Y}{X}} \right) \text{rad/s}$)



$$K = Y \frac{\text{N}}{\text{m}}$$

$$m = X$$

$$\omega = ?$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$\boxed{\omega = \sqrt{\frac{Y}{X}} \frac{\text{rad}}{\text{s}}}$$

Converting between Angular frequency and Frequency in Hertz

- 18) Express an angular frequency of 1.4 rad/s in hertz. (0.22 Hz)

$$\begin{array}{l|l} \omega = 1.4 \frac{\text{rad}}{\text{s}} & \omega = 2\pi f \\ f = ? & f = \frac{\omega}{2\pi} \end{array} \rightarrow f = \frac{1.4}{2\pi}$$

$f = 0.22 \text{ Hz}$

- 19) Express a frequency of 34Hz as an angular frequency. (213.6 rad/s)

$$\begin{array}{l} \omega = 2\pi f \\ \omega = 2\pi(34) \end{array} \rightarrow \boxed{\omega = 213.6 \frac{\text{rad}}{\text{s}}}$$

- 20) Express an angular frequency of 4.2×10^{-5} rad/s in hertz. (6.7×10^{-6} Hz)

$$\begin{array}{l} \omega = 2\pi f \\ f = \frac{\omega}{2\pi} \end{array} \rightarrow f = \frac{4.2 \times 10^{-5}}{2\pi}$$

$f = 6.7 \times 10^{-6} \text{ Hz}$

- 21) Express a frequency of 5MHz as an angular frequency. (3.14×10^7 rad/s)

$$\begin{array}{l} f = 5 \text{ MHz} \\ f = 5 \times 10^6 \text{ Hz} \\ \omega = 2\pi f \end{array} \rightarrow \boxed{\omega = 3.14 \times 10^7 \frac{\text{rad}}{\text{s}}}$$

Converting between Angular frequency and Period of Oscillation

22) What is the period of an oscillation that has an angular frequency of 1.7 rad/s?

(3.70s)

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$T = 2\pi \left(\frac{1}{\omega}\right)$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{1.7}$$

$$T = 3.70 \text{ s}$$

23) What is the angular frequency of an oscillator that has a period of 0.4s? (15.7 rad/s)

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{0.4}$$

$$\omega = 15.7 \frac{\text{rad}}{\text{s}}$$

24) What is the period of an oscillation that has an angular frequency of 2.6×10^{-4} rad/s?

(24,166 s)

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{2.6 \times 10^{-4}}$$

$$T = 24,166 \text{ s}$$

25) What is the angular frequency of an oscillator that has a period of $6\mu\text{s}$? (1.05×10^6 rad/s)

$$\omega = \frac{2\pi}{T} \rightarrow \omega = \frac{2\pi}{6 \times 10^{-6}}$$

$$T = 6 \times 10^{-6}$$

$$\omega = 1.05 \times 10^6 \frac{\text{rad}}{\text{s}}$$

Spring Potential Energy

- 26) A spring has a spring constant of 4N/m. The spring is compressed by 3m. How much elastic potential energy is stored in the spring? (18J)

$K = 4 \text{ N/m}$	$PE_s = \frac{1}{2} Kx^2$
$\Delta x = -3 \text{ m}$	$PE_s = \frac{1}{2}(4)(-3)^2$
$PE_s = ?$	$PE_s = 2(9)$
	$\boxed{PE_s = 18 \text{ J}}$

- 27) A spring is compressed to such a degree that it stores 20J of spring potential energy. How much energy would be stored in a spring that had twice the spring constant and three times the compression? (360 J)

$PE_{s1} = 20 \text{ J}$	$PE_s = ?$
$K_1 = 1$	$K_2 = 2$
$\Delta x_1 = 1$	$\Delta x_2 = 3$
New $\frac{PE_{s2}}{20} = \frac{\frac{1}{2}K_2(\Delta x_2)^2}{\frac{1}{2}K_1(\Delta x_1)^2}$	$\rightarrow \frac{PE_{s2}}{20} = \frac{\frac{1}{2}(2)(3)^2}{\frac{1}{2}(1)(1)}$
OLD $PE_{s1} = \frac{1}{2}K_1(\Delta x_1)^2$	
	$\frac{PE_{s2}}{20} = 18 \rightarrow PE_{s2} = 18(20)$

$$\boxed{PE_{s2} = 360 \text{ J}}$$

- 28) A spring of spring constant 6×10^8 N/m is compressed by 3mm. How much elastic potential energy is stored in the spring? (2700 J)

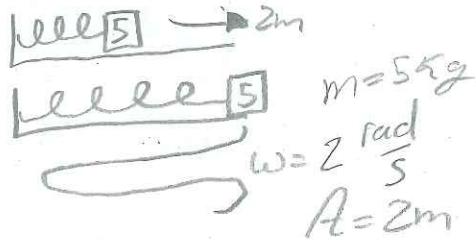
|||||
||||| - 3mm - |
 $k = 6 \times 10^8 \text{ N/m}$
 $x = 3 \times 10^{-3} \text{ m}$

$$PE_s = \frac{1}{2} k(x)^2$$
$$PE_s = \frac{1}{2} (6 \times 10^8) (3 \times 10^{-3})^2$$
$$PE_s = \frac{1}{2} (5400)$$

$PE_s = 2700 \text{ J}$

Instantaneous KE, PE_s, and ME

- 29) A 5kg block is attached to the end of a horizontal mass-less spring. The block oscillates with simple harmonic motion. The block is moved 2m to the right of its equilibrium position. The block is released at time $t = 0s$. The block oscillates with a frequency of 2rad/s.



- i) What is the KE of the system at $t=2.1\text{s}$? (30.39 J)

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ v &= -A\omega \sin(\omega t) \\ v^2 &= A^2\omega^2 \sin^2(\omega t) \\ KE &= \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t) \end{aligned}$$

$$\rightarrow \frac{1}{2}(5)(2)^2(2)^2 \sin^2(2(2.1))$$

$$KE = \frac{1}{2}(5)(16) \sin^2(4.2)$$

$$KE = 40 \sin^2(4.2)$$

$$\rightarrow KE = 30.39 \text{ J}$$

- ii) What is the PE of the system at $t=2.1\text{s}$? (9.61 J)

$$\begin{aligned} PE_{\text{spring}} &= \frac{1}{2}KX^2 & PE_s &= \frac{1}{2}(mw^2)(A^2 \cos^2(\omega t)) \\ X &= A \cos(\omega t) & PE_s &= \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t) \\ X^2 &= A^2 \cos^2(\omega t) & PE_s &= \frac{1}{2}(5)(2)^2(2)^2 \cos^2(2(2.1)) \\ K &= mw^2 & PE_s &= 40 \cos^2(4.2) \end{aligned}$$

$$\rightarrow PE_s = 9.61 \text{ J}$$

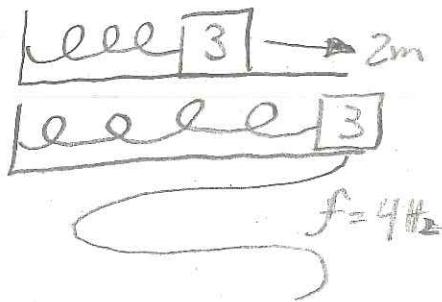
- iii) What is the ME of the system at $t=2.1\text{s}$? (40 J)

$$ME = KE + PE_s$$

$$ME = 30.39 + 9.61$$

$$\boxed{ME = 40 \text{ J}}$$

- 30) A 3kg block is attached to the end of a horizontal mass-less spring. The block oscillates with simple harmonic motion. The block is moved 2m to the right of its equilibrium position. The block is released at time $t = 0$ s. The block oscillates with a frequency of 4Hz.



$$\omega = 2\pi f$$

$$\omega = 25.13 \frac{\text{rad}}{\text{s}}$$

$$m = 3 \text{ kg}$$

$$A = X = 2 \text{ m}$$

- i) What is the KE of the system at $t=0.6$ s? (1309.4 J)

$$KE = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t)$$

$$KE = \frac{1}{2} (3)(25.13)^2 (2)^2 \sin^2(25.13 \cdot 0.6)$$

$$KE = 3789.10 \sin^2(15.078)$$

$$\rightarrow 1309.4 \text{ J}$$

- ii) What is the PE of the system at $t=0.6$ s? (2480.5 J)

$$PE = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t)$$

$$PE = \frac{1}{2} (3)(25.13)^2 (2)^2 \cos^2(25.13 \cdot 0.6)$$

$$PE = 2480.5 \text{ J}$$

- iii) What is the ME of the system at $t=0.6$ s? (3789.9 J)

$$ME = KE + PE$$

$$ME = 1309.4 + 2480.5$$

$$ME = 3789.9 \text{ J}$$

Chapter 16 - Wave Motion - Questions

Oscillation - Cycles/second and Hertz

- 1) An oscillation exhibits 5 cycles a second. What is its frequency? (5Hz)

2

$$f = \frac{5 \text{ cycles}}{1 \text{ sec}} = 5 \text{ Hz}$$

1 cycle per sec = 1 Hz

- 2) An oscillation exhibits 9 cycles a second. What is its frequency? (9Hz)

$$f = 9 \text{ Hz}$$

- 3) An oscillation exhibits 12 cycles a second. What is its frequency? (12Hz)

$$f = 12 \text{ Hz}$$

Oscillation - frequency and period

- 4) An oscillation has a frequency of 5 Hz. What is its period? (0.2s)

$$f = 5 \text{ Hz}$$
$$T = \frac{1}{f} = \boxed{T = \frac{1}{5}}$$

- 5) An oscillation has a frequency of 2 Hz. What is its period? (0.5s)

$$T = \frac{1}{f} = \frac{1}{2} = \boxed{T = 0.5 \text{ s}}$$

- 6) An oscillation has a frequency of 0.4 Hz. What is its period? (2.5s)

$$T = \frac{1}{f} = T = \frac{1}{0.4} = \boxed{T = 2.5 \text{ s}}$$

Oscillation - period and angular frequency

- 7) An oscillator has a period of 0.2 s. What is its angular frequency? (31.4 rad/s)

$$T = 0.2 \quad | \quad T = \frac{1}{f} \quad | \quad T = \frac{2\pi}{\omega} \quad | \quad \omega = \frac{2\pi}{0.2}$$

$$\omega = ? \quad | \quad f = \frac{\omega}{2\pi} \quad | \quad \omega = \frac{2\pi}{T} \quad | \quad \boxed{\omega = 31.4 \text{ rad/s}}$$

- 8) An oscillator has a period of 0.5 s. What is its angular frequency? (12.6 rad/s)

$$T = 0.5 \quad | \quad T = \frac{1}{f} \quad | \quad T = \frac{2\pi}{\omega} \quad | \quad \omega = \frac{2\pi}{0.5}$$

$$\omega = ? \quad | \quad f = \frac{\omega}{2\pi} \quad | \quad \omega = \frac{2\pi}{T} \quad | \quad \boxed{\omega = 12.6 \text{ rad/s}}$$

- 9) An oscillator has a period of 2.5 s. What is its angular frequency? (2.5 rad/s)

$$T = 2.5 \quad | \quad \omega = \frac{2\pi}{T} \quad | \quad \boxed{\omega = 2.5 \text{ rad/s}}$$

$$\omega = ? \quad | \quad \omega = \frac{2\pi}{2.5}$$

Waves - wavelength and wave number

- 10) A wave has a wavelength of 5 m. What is its wave number? (0.2 waves/m)



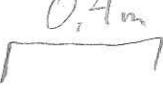
$$\lambda = 5 \text{ m} \quad | \quad \tilde{V} = \frac{1}{\lambda} \quad | \quad \boxed{\tilde{V} = 0.2 \text{ wave/m}}$$

- 11) A wave has a wavelength of 2 m. What is its wave number? (0.5 waves/m)



$$\lambda = 2 \text{ m} \quad | \quad \tilde{V} = \frac{1}{\lambda} = \frac{1}{2} \quad | \quad \boxed{\tilde{V} = 0.5 \text{ wave/m}}$$

- 12) A wave has a wavelength of 0.4 m. What is its wave number? (2.5 waves/m)



$$\lambda = 0.4 \text{ m} \quad | \quad \tilde{V} = \frac{1}{\lambda} = \frac{1}{0.4} \quad | \quad \boxed{\tilde{V} = 2.5 \text{ wave/m}}$$

1 Dim Traveling Waves

- 13) At time $t=0$ s a transverse pulse wave is described by the function $y = \frac{6}{x^2 + 8}$. What is

the function that describes the pulse if it is travelling at 3 m/s in a positive x direction? $\square y = \frac{6}{(x - 3t)^2 + 8} \square$

at $t=0$ $y = \frac{6}{x^2 + 8}$

Like cones.

$$x^2 + y^2 = \text{at origin}$$

$(x - 3t)^2$ shifted to
the right
(displaced)

$$y = f(x - vt)$$

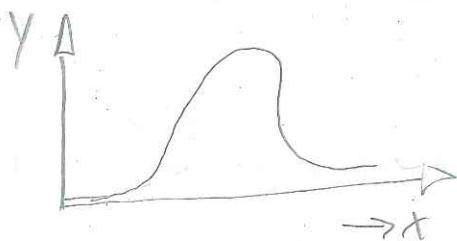
$$y = \frac{6}{(x - 3t)^2 + 8}$$

$$\boxed{y = \frac{6}{(x - 3t)^2 + 8}}$$

$$V = 3 \text{ m/s}$$

- 14) At time $t=0$ s a transverse pulse wave is described by the function $y = \frac{3}{x^2 + 2}$. What is

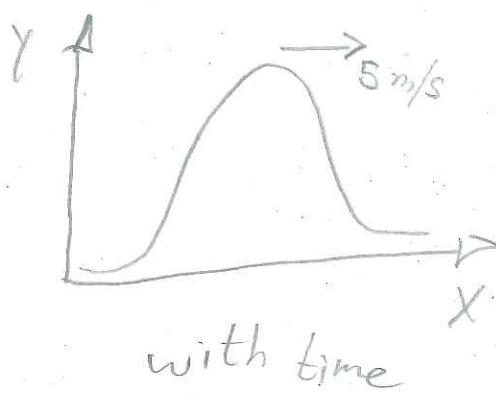
the function that describes the pulse if it is travelling at 5 m/s in a positive x direction? $\square y = \frac{3}{(x - 5t)^2 + 2} \square$



$$y = \frac{3}{x^2 + 2} \quad @ t=0$$

$$V = 5 \text{ m/s}$$

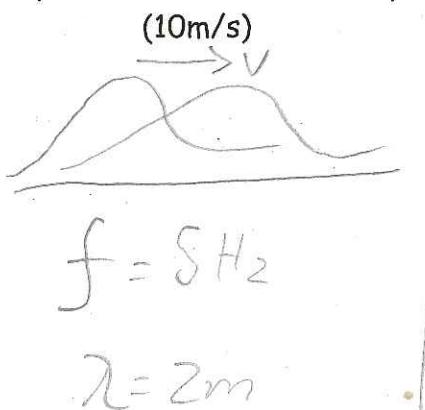
$$y = f(x - vt) \quad \text{to right}$$



$$\boxed{y = \frac{3}{(x - 5t)^2 + 2}}$$

Wave Speed

- 15) A wave has a frequency of 5 Hz and a wavelength of 2m. What is its wave speed?

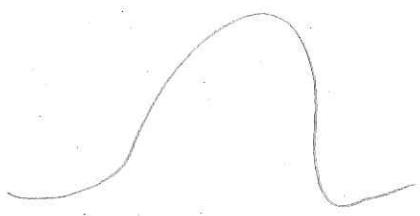


$$V = f\lambda$$

$$V = 5(2)$$

$$V = 10 \text{ m/s}$$

- 16) A wave has a frequency of 9Hz and a wavelength of 3m. What is its wave speed?
(27m/s)

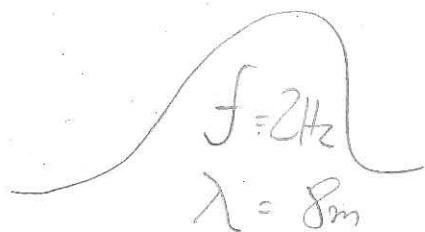


$$f = 9 \text{ Hz} \rightarrow V = 9(3)$$

$$\lambda = 3 \text{ m}$$

$$\boxed{V = 27 \text{ m/s}}$$

- 17) A wave has a frequency of 2Hz and a wavelength of 8m. What is its wave speed?
(16m/s)



$$V = f\lambda$$

$$V = 2(8) \rightarrow$$

$$\boxed{V = 16 \frac{\text{m}}{\text{s}}}$$

Sinusoidal Wave Function from A, ω , x and t

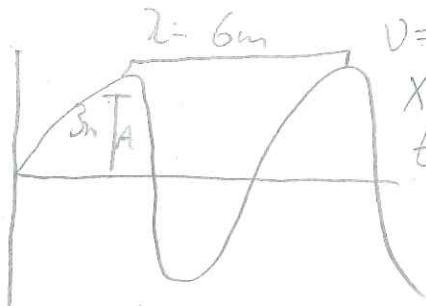
(hint: Use $y = A \sin \left[\frac{2\pi}{\lambda} (x - vt) \right]$. (Work in radians.)

- 18) A sinusoidal wave has amplitude of 8m. It has a wavelength of 3m. It has a velocity of 5m/s to the right. What is the y-axis displacement of a point at a location 2m to the right of the origin at time $t = 1.6$ s. (0.0 m)

$$y = A \sin \left(2\pi \frac{(x-vt)}{\lambda} \right)$$

$A = 8\text{m}$ $\lambda = 3\text{m}$ $v = 5\text{m/s}$ $x = 2\text{m}$ $t = 1.6$	$y = 8 \sin \left(2\pi \frac{(2 - 5(1.6))}{3} \right)$ $y = 8 \sin \left(2\pi \frac{(2 - 8)}{3} \right)$ $y = 8 \sin \left(2\pi \left(-\frac{6}{3} \right) \right)$ $y = 8 \sin (2\pi(-2))$	$\Rightarrow y = 8 \sin (-4\pi)$ $y = 8(0)$ <div style="border: 1px solid black; padding: 5px; display: inline-block;">$y = 0\text{m}$</div>
---	--	---

- 19) A sinusoidal wave has amplitude of 3m. It has a wavelength of 6m. It has a velocity of 2m/s to the right. What is the y axis displacement of a point at a location 1.6m to the right of the origin at time $t = 0.3$ s? (+2.60 m)



$$y = A \sin \left[2\pi \frac{(x-vt)}{\lambda} \right]$$

$$y = 3 \sin \left[2\pi \frac{(1.6 - 2(0.3))}{6} \right]$$

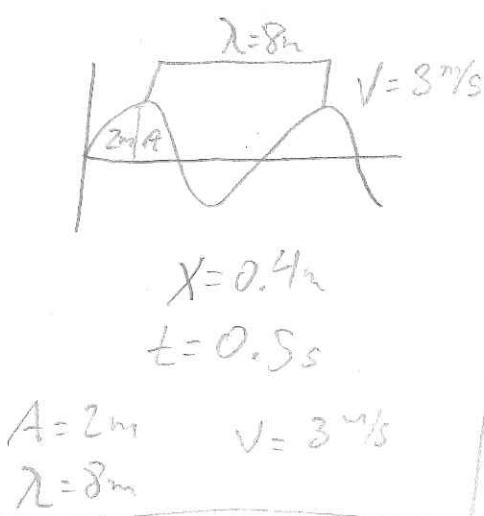
$$y = 3 \sin \left[2\pi \frac{(1.6 - 0.6)}{6} \right]$$

$$y = 3 \sin \left[2\pi \cdot \left(\frac{1}{3} \right) \right]$$

$$y = 3 \sin \left(\frac{\pi}{3} \right)$$

$y = \frac{3\sqrt{3}}{2} = 2.6\text{m}$

- 20) A sinusoidal wave has an amplitude of 2m. It has a wavelength of 8m. It has a velocity of 3m/s to the right. What is the y axis displacement of a point at a location 0.4m to the right of the origin at time $t = 0.5s$? (-1.5 m)



$$y = A \sin \left[2\pi \frac{(x - vt)}{\lambda} \right]$$

$$y = 2 \sin \left[2\pi \frac{(0.4 - (3 \cdot 0.5))}{8} \right]$$

$$y = 2 \sin \left[2\pi \frac{(0.4 - 1.5)}{8} \right]$$

$$y = 2 \sin \left[2\pi \left(-\frac{1.1}{8} \right) \right]$$

$$y = 2 \sin \left[2\pi \left(-\frac{11}{80} \right) \right]$$

$$y = 2 \sin \left[-\frac{11}{40}\pi \right]$$

$$\boxed{y = -1.5\text{m}}$$

Sinusoidal Wave Function from A k, ω , x and t

y or k ?

(hint: Use $y = A \sin(kx - \omega t)$. Work in radians.)

- 21) A sinusoidal wave has amplitude of 4 m. It has an angular wave number of 2.3 rad/m. It has an angular frequency of 1.7 rad/s. What is the y axis displacement of a point at a location 2 m to the right of the origin at time $t = 3.5$ s? (-3.90 m)



$$A = 4 \text{ m}$$

$$K = 2.3 \frac{\text{rad}}{\text{m}}$$

$$\omega = 1.7 \frac{\text{rad}}{\text{s}}$$

$$X = 2 \text{ m}$$

$$t = 3.5 \text{ s}$$

$$y = A \sin [Kx - \omega t]$$

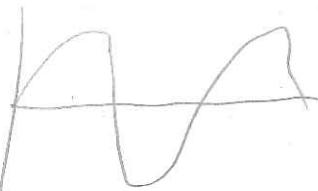
$$y = 4 \sin [2.3(2) - 1.7(3.5)]$$

$$y = 4 \sin [4.6 - 5.9]$$

$$y = 4 \sin [-1.35]$$

$$y = -3.90 \text{ m}$$

- 22) A sinusoidal wave has an amplitude of 9 m. It has an angular wave number of 0.5 rad/m. It has an angular frequency of 0.3 rad/s. What is the y axis displacement of a point at a location 1.2 m to the right of the origin at time $t = 0.7$ s? (+1.52 m)



$$A = 9 \text{ m}$$

$$K = 0.5 \frac{\text{rad}}{\text{m}}$$

$$\omega = 0.3 \frac{\text{rad}}{\text{s}}$$

$$t = 0.7 \text{ s}$$

$$X = 1.2 \text{ m}$$

$$y = A \sin [Kx - \omega t]$$

$$y = 9 \sin [0.5(1.2) - 0.3(0.7)]$$

$$y = 9 \sin [0.6 - 0.21]$$

$$y = 9 \sin [0.39]$$

$$y = 3.42 \text{ m}$$

Sinusoidal Wave Velocity

- 23) What is the velocity of a sinusoidal wave that has a frequency of 34Hz and a wavelength of 2.3m? (78.2 m/s)

$$\lambda = 2.3 \text{ m}$$

$$f = 34 \text{ Hz}$$

$$V = \lambda f$$

$$V = 2.3(34)$$

$$\rightarrow V = 78.2 \text{ m/s}$$

- 24) What is the velocity of a sinusoidal wave that has a frequency of 17Hz and a wavelength of 9.3m? (158.1 m/s)

$$\lambda = 9.3$$

$$f = 17 \text{ Hz}$$

$$V = \lambda f$$

$$V = 9.3(17)$$

$$\rightarrow V = 158.1 \text{ m/s}$$

- 25) What is the velocity of a sinusoidal wave that has a frequency of 56Hz and a wavelength of 0.2m? (11.2 m/s)

$$V = 56(0.2) \rightarrow V = 11.2 \text{ m/s}$$

- 26) What is the velocity of a sinusoidal wave that has a period of 0.4s and a wavelength of 2.3m? (5.75m/s)

$$V = \lambda f \quad V = 2.3(2.3)$$

$$T = \frac{1}{f} \quad f = \frac{1}{T} = \frac{1}{0.4} = \frac{5}{2}$$

$$\rightarrow V = 5.75 \text{ m/s}$$

- 27) What is the velocity of a sinusoidal wave that has a period of 1.2s and a wavelength of 0.7m? (0.58m/s)

$$T = 1.2 \text{ s} \quad V = \lambda f \rightarrow V = 0.7 \left(\frac{5}{6} \right)$$

$$\lambda = 0.7 \text{ m} \quad f = \frac{5}{6} \quad \boxed{V = 0.58 \text{ m/s}}$$

- 28) What is the velocity of a sinusoidal wave that has a period of 0.2s and a wavelength of 1.9m? (9.5m/s)

$$T = 0.2 \text{ s} \quad V = \lambda f \rightarrow V = 1.9(5)$$

$$\lambda = 1.9 \text{ m} \quad f = 5 \text{ Hz} \quad \boxed{V = 9.5 \text{ m/s}}$$

- 29) What is the velocity of a sinusoidal wave that has an angular frequency of 1.2 rad/s and an angular wave number of 0.4m⁻¹? (3.0 m/s)

$$\omega = 1.2 \frac{\text{rad}}{\text{s}} \quad \begin{cases} \omega = 2\pi f \\ f = \frac{\omega}{2\pi} \end{cases} \quad \begin{cases} \omega = 2\pi f \\ f = \frac{1.2}{2\pi} \end{cases} \quad \begin{cases} \lambda = \frac{2\pi}{K} \\ K = 2\pi \lambda \end{cases} \quad \begin{cases} V = \frac{\omega}{2\pi} \frac{2\pi}{K} = \frac{\omega}{K} = \frac{1.2}{0.4} \\ V = 3.0 \text{ m/s} \end{cases}$$

- 30) What is the velocity of a sinusoidal wave that has an angular frequency of 0.34 rad/s and an angular wave number of 0.21m⁻¹? (1.62 m/s)

$$\begin{cases} V = \lambda f \\ K = \frac{2\pi}{\lambda} \end{cases} \quad \begin{cases} \lambda = \frac{2\pi}{K} \\ f = \frac{\omega}{2\pi} \end{cases} \quad \begin{cases} V = \frac{2\pi}{K} \cdot \frac{\omega}{2\pi} \\ V = \frac{\omega}{K} \end{cases} \quad \begin{cases} V = \frac{0.34}{0.21} \\ V = 1.62 \text{ m/s} \end{cases}$$

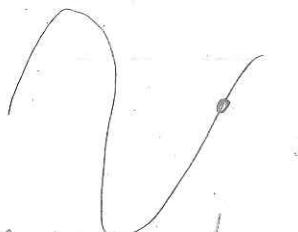
- 31) What is the velocity of a sinusoidal wave that has an angular frequency of 0.23 rad/s and an angular wave number of 0.83m⁻¹? (0.28 m/s)

$$\begin{cases} \omega = 0.23 \\ K = 0.83 \end{cases} \quad \begin{cases} V = \frac{\omega}{K} \\ V = \frac{0.23}{0.83} \end{cases} \quad \boxed{V = 0.28 \text{ m/s}}$$

Sinusoidal Waves on a String

A sinusoidal wave on a horizontal string has amplitude of 0.3m an angular wave number of 0.4m^{-1} , and an angular frequency of 0.6rad/s .

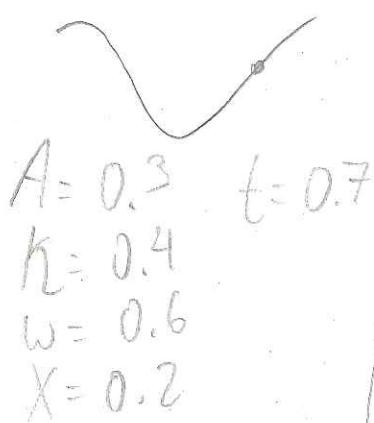
- 32) What is its y-axis deflection at a point 0.2m from the origin at $t=0.7\text{s}$? (-0.10m)



$$\begin{aligned} A &= 0.3 \text{ m} \\ k &= 0.4 \\ \omega &= 0.6 \\ x &= 0.2 \end{aligned}$$

$$\begin{aligned} Y &= A \sin(kx - \omega t) \\ Y &= 0.3 \sin(0.4(0.2) - (0.6)(0.7)) \\ Y &= 0.3 \sin(0.08 - 0.42) \\ Y &= 0.3 \sin(-0.34) \rightarrow \boxed{Y = -0.10 \text{ m}} \end{aligned}$$

- 33) What is its y-axis velocity at a point 0.2m from the origin at $t=0.7\text{s}$? (-0.17m/s)



$$\begin{aligned} A &= 0.3 \\ t &= 0.7 \\ k &= 0.4 \\ \omega &= 0.6 \\ x &= 0.2 \end{aligned}$$

$$\begin{aligned} Y &= A \sin(kx - \omega t) \\ V &= \frac{dY}{dt} = -A\omega \cos(kx - \omega t) \\ V &= -(0.3)(0.6) \cos(0.4(0.2) - (0.6)(0.7)) \\ V &= -0.18 \cos(-0.34) \rightarrow \boxed{V = -0.17 \text{ m/s}} \end{aligned}$$

- 34) What is its y-axis acceleration at a point 0.2m from the origin at $t=0.7\text{s}$? (+0.036m/s²)

$$\begin{aligned} Y &= A \sin(kx - \omega t) \\ V &= \frac{dY}{dt} = -A\omega \cos(kx - \omega t) \\ a &= \frac{d^2Y}{dt^2} = -A\omega^2 \sin(kx - \omega t) \end{aligned}$$

$$\begin{aligned} a &= -0.18 \sin(-0.34) \\ a &= 0.036 \text{ m/s}^2 \end{aligned}$$

Velocity of a Wave on a String

- 35) What is the velocity of a wave on a horizontal string whose tension is 4N and whose mass/unit length is 3kg/m? (1.15m/s)

$$V = \sqrt{\frac{T}{\mu}}$$

T = tension

$\mu = \frac{\text{mass}}{\text{unit length}}$

$$V = ?$$

$$T = 4N$$

$$\mu = 3 \text{ kg/m}$$

$$V = \sqrt{\frac{T}{\mu}}$$

$$V = \sqrt{\frac{4}{3}} =$$

$$V = 1.154 \text{ m/s}$$

- 36) What is the velocity of a wave on a horizontal string whose tension is 8N and whose mass/unit length is 0.6kg/m? (3.65m/s)

$$T = 8N$$

$$\mu = 0.6 \text{ kg/m}$$

$$V = \sqrt{\frac{T}{\mu}}$$

$$V = \sqrt{\frac{8}{0.6}}$$

$$V = 3.65 \text{ m/s}$$

- 37) What is the velocity of a wave on a horizontal 3m length of string whose tension is 4N and whose mass is 6kg? (1.41m/s)

$$V = \sqrt{\frac{T}{\mu}}$$

$$\mu = \frac{m}{L} = \frac{6 \text{ kg}}{3 \text{ m}} = 2 \text{ kg/m}$$

$$m = 6 \text{ kg}$$

$$L = 3 \text{ m}$$

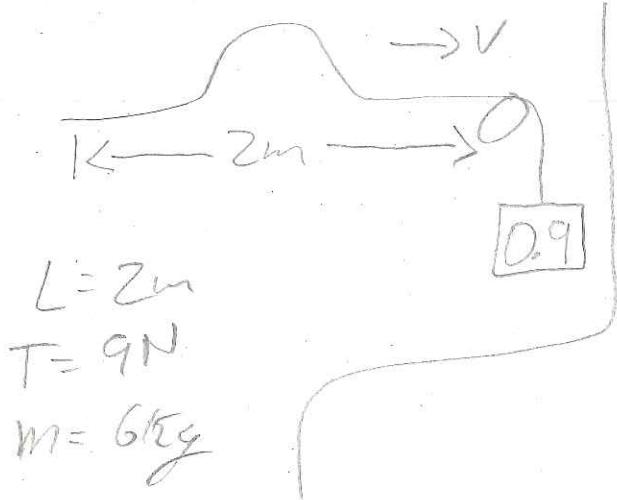
$$T = 4N$$

$$V = \sqrt{\frac{4}{2}}$$

$$V = \sqrt{2}$$

$$V = 1.41 \text{ m/s}$$

- 38) What is the velocity of a wave on a horizontal 2m length of string whose tension is 9N and whose mass is 6kg? (1.73m/s)

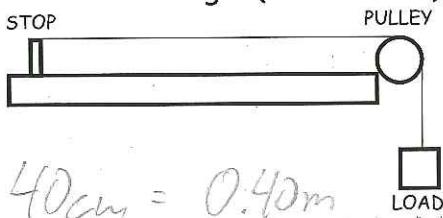


$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}} = \sqrt{\frac{9}{6/2}}$$

$$\mu = \frac{m}{L} = \frac{6}{2} = 3$$

$$V = \sqrt{\frac{9}{3}} = \sqrt{3} = 1.73 \text{ m/s}$$

- 39) A 40cm long string is stretched from a stop, over a pulley and down to a 9kg load. The string has a mass of 2mg. What velocity would a travelling wave have along this string? (4242.6 m/s)



$$L = 40\text{cm} = 0.40\text{m}$$

$$M = 2\text{mg} = 2 \times 10^{-6} \text{kg}$$

$$\mu = \frac{2 \times 10^{-6}}{0.40} = 5 \times 10^{-6}$$

$$T = 90\text{N}$$

$$V = \sqrt{\frac{T}{\mu}}$$

$$V = \sqrt{\frac{90}{5 \times 10^{-6}}} = 4242.6 \text{ m/s}$$

$$V = 4242.6 \text{ m/s}$$

Energy in one Wavelength and Power transmitted by a string

A string has a mass/unit length is 3kg/m. It carries a wave that has an angular frequency of 4rad/s. The amplitude of the wave is 2m. The wavelength of the wave is 5m.

- 40) What is the total kinetic energy in one wavelength of the wave? (240J)

$$K_{\lambda} = \frac{1}{4} \mu w^2 A^2 \lambda$$

$$K_{\lambda} = \frac{1}{4} (3)(4)^2 (2)^2 (5)$$

$$K_{\lambda} = 3(16)(5)$$

$$K_{\lambda} = 15(16)$$

$$K_{\lambda} = 240J$$

- 41) What is the total potential energy in one wavelength on the wave? (240J)

$$U_{\lambda} = K_{\lambda} = 240J$$

- 42) What is the total mechanical energy in one wavelength of the wave? (480J)

$$U_{\lambda} + K_{\lambda} = 480J$$

- 43) What power is transmitted by the wave? (305.5W)

$$P = \frac{E_{\lambda}}{T} = \frac{U_{\lambda} + K_{\lambda}}{T} = \frac{480J}{2\pi/4} = \frac{480J}{\pi/2} = 480 \cdot \frac{2}{\pi}$$

$$T = \frac{1}{f}$$

$$\omega = 2\pi f \rightarrow \omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$\frac{960}{\pi} = P = 305.5W$$

C) Sound Waves Homework

Speed of Sound through a Medium

- 1) Sound propagates through a medium that has a bulk modulus of 8 N/m^2 . The density of the medium is 2 kg/m^3 . What is the speed of sound in this medium? (2.0 m/s)

$$V_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{8}{2}} = \sqrt{4} = \boxed{V_s = 2 \text{ m/s}}$$

- 2) Sound propagates through a medium that has a bulk modulus of 64 N/m^2 . The density of the medium is 4 kg/m^3 . What is the speed of sound in this medium? (4m/s)

$$V_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{64}{4}} = \sqrt{16} = \boxed{V_s = 4 \text{ m/s}}$$

- 3) Sound propagates through a medium that has a bulk modulus of 81 N/m^2 . The density of the medium is 1 kg/m^3 . What is the speed of sound in this medium? (9m/s)

$$V_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{81}{1}} = \sqrt{81} = \boxed{9 \text{ m/s}}$$

Speed of sound at a specific temperature of air

- 4) Sound travels through air that has a temperature of 60°C . What is the speed of sound in such air? (365.6 m/s)

$$\sqrt{V_s} = 331 \sqrt{1 + \frac{T_c}{273}}$$
$$\sqrt{V_s} = 331 \sqrt{1 + \frac{60}{273}}$$
$$\sqrt{V_s} = 331 \sqrt{\frac{111}{91}}$$
$$\boxed{\sqrt{V_s} = 365.6 \text{ m/s}}$$

- 5) Sound travels through air that has a temperature of 0°C . What is the speed of sound in such air? (331 m/s)

$$\sqrt{V_s} = 331 \sqrt{1 + \frac{0}{273}}$$
$$\boxed{\sqrt{V_s} = 331 \text{ m/s}}$$

- 6) Sound travels through air that has a temperature of -30°C . What is the speed of sound in such air? (312.3 m/s)

$$\sqrt{V_s} = 331 \sqrt{1 + \frac{-30}{273}}$$
$$\sqrt{V_s} = 331 \sqrt{0.8901}$$
$$\boxed{\sqrt{V_s} = 312.3 \text{ m/s}}$$

$$\Delta P_{\max} = \rho V \omega X_{\max}$$

Pressure Amplitude because of a sound wave

- 7) A sound passes through a medium. The density of the medium is 4kg/m^3 . The velocity of the sound in the medium is 10m/s . The angular frequency of the sound is 6rad/s . The maximum displacement of the gas particles is 0.2m . What is the maximum Pressure Amplitude caused by the sound? (48 Pa)

$$\rho = 4$$

$$V = 10$$

$$\omega = 6$$

$$X_{\max} = 0.2$$

$$\Delta P_{\max} = \rho V \omega X_{\max}$$

$$\Delta P_{\max} = 4(10)(6)(0.2)$$

$$\boxed{\Delta P_{\max} = 48 \text{ Pa}}$$

- 8) A sound passes through a medium. The density of the medium is 0.1kg/m^3 . The velocity of the sound in the medium is 300m/s . The angular frequency of the sound is 200rad/s . The maximum displacement of the gas particles is 0.01m . What is the maximum Pressure Amplitude caused by the sound? (60 Pa)

$$\rho = 0.1 \frac{\text{kg}}{\text{m}^3}$$

$$V = 300 \text{ m/s}$$

$$\omega = 200 \frac{\text{rad}}{\text{s}}$$

$$X_{\max} = 0.01\text{m}$$

$$\Delta P = \rho V \omega X_{\max}$$

$$\Delta P = (0.1)(300)(200)(0.01)$$

$$\boxed{\Delta P = 60 \text{ Pa}}$$

- 9) A sound passes through a medium. The density of the medium is 0.5kg/m^3 . The velocity of the sound in the medium is 350m/s . The angular frequency of the sound is 4000rad/s . The maximum displacement of the gas particles is 0.02m . What is the maximum Pressure Amplitude caused by the sound? (14,000 Pa)

$$\rho = 0.5$$

$$V = 350$$

$$\omega = 4000$$

$$X_{\max} = 0.02$$

$$\Delta P_{\max} = \rho V \omega X_{\max}$$

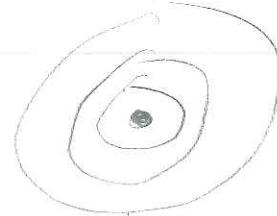
$$\Delta P_{\max} (0.5)(350)(4000)$$

$$\boxed{\Delta P_{\max} = 14,000 \text{ Pa}}$$

Intensity of Periodic Waves

- 10) A source of sound radiates energy in all directions. The source radiates 40 J of energy each second. What is the intensity of sound at any point that is 2m from the source?
(0.796 W/m²)

$$\begin{aligned} I &= \frac{P}{A} \\ P &= 40 \text{ Js} \\ \text{Area of Sphere} &= 4\pi r^2 \\ r &= 2 \text{ m} \end{aligned}$$
$$\boxed{I = \frac{40}{4\pi(2)^2}}$$
$$\boxed{I = 0.796 \text{ W/m}^2}$$



- 11) A source of sound radiates energy in all directions. The source radiates 40 J of energy each second. What is the intensity of sound at any point that is 4m from the source?
(0.199 W/m²)

$$\begin{aligned} I &= \frac{P}{A} \\ \text{Area} &= 4\pi r^2 \\ r &= 4 \text{ m} \end{aligned}$$
$$\boxed{I = \frac{40}{4\pi(4)^2}}$$
$$\boxed{I = 0.199 \text{ W/m}^2}$$



- 12) A source of sound radiates energy in all directions. The source radiates 40 J of energy each second. What is the intensity of sound at any point that is 8m from the source?
(0.050 W/m²)

$$\begin{aligned} I &= \frac{P}{A} \\ \text{Area} &= 4\pi r^2 \\ r &= 8 \text{ m} \end{aligned}$$
$$\boxed{I = \frac{40}{4\pi(8)^2}}$$
$$\boxed{I = 0.050 \text{ W/m}^2}$$



Decibel Scale

13) What is the sound level of a sound that has an intensity of $5 \times 10^{-6} \text{ W/m}^2$? (66.99 dB)

$$B = 10 \log \left(\frac{I}{I_{E-12}} \right)$$
$$I = 5 \times 10^{-6}$$
$$B = 10 \log \left(\frac{5 \times 10^{-6}}{10^{-12}} \right)$$
$$B = 66.99 \text{ dB}$$

14) What is the sound level of a sound that has an intensity of $10 \times 10^{-6} \text{ W/m}^2$? (70.00 dB)

$$B = 10 \log \left(\frac{I}{I_{E-12}} \right)$$
$$B = 10 \log \left(\frac{10 \times 10^{-6}}{10^{-12}} \right)$$

$$B = 70.00 \text{ dB}$$

15) What is the sound level of a sound that has an intensity of $15 \times 10^{-6} \text{ W/m}^2$? (71.76 dB)

$$B = 10 \log \left(\frac{I}{I_{E-12}} \right)$$
$$I = 15 \times 10^{-6}$$
$$B = 10 \log \left(\frac{15 \times 10^{-6}}{10^{-12}} \right)$$
$$B = 71.76 \text{ dB}$$

16) What is the sound level of a sound that has an intensity of $50 \times 10^{-6} \text{ W/m}^2$? (76.99 dB)

$$B = 10 \log \left(\frac{I}{I_{E-12}} \right)$$
$$I = 50 \times 10^{-6}$$
$$B = 10 \log \left(\frac{50 \times 10^{-6}}{10^{-12}} \right)$$

$$B = 76.99 \text{ dB}$$

Doppler Effect

- 17) A source of sound has a frequency of 100 Hz. It travels towards a stationary observer at 30 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source of sound approaches? (111.11 Hz)

$$\bullet)) \rightarrow v_s = 30 \text{ m/s}$$

$$f_s = 100 \text{ Hz}$$

$$V = 300 \text{ m/s}$$

$$f_o = f_s \left(\frac{V + V_o}{V - V_s} \right)$$

$$f_o = 100 \left(\frac{300 + 0}{300 - (-30)} \right)$$

$$f_o = 100 \left(\frac{300}{270} \right)$$

$$f_o = 100 \left(\frac{10}{9} \right)$$

$$f_o = 111.11 \text{ Hz}$$

- 18) A source of sound has a frequency of 100 Hz. It travels away from a stationary observer at 30 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source of sound moves away? (90.91 Hz)

$$\bullet)) \leftarrow v_s = -30 \text{ m/s}$$

$$f_s = 100 \text{ Hz}$$

$$V = 300 \text{ m/s}$$

$$f_o = f_s \left(\frac{V + V_o}{V - V_s} \right)$$

$$f_o = f_s \left(\frac{300 + 0}{300 - (-30)} \right)$$

$$f_o = f_s \left(\frac{300}{330} \right)$$

$$f_o = 100 \left(\frac{10}{11} \right)$$

$$f_o = 90.91 \text{ Hz}$$

- 19) A source of sound has a frequency of 200 Hz. It travels towards a stationary observer at 50 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source of sound approaches? (240.00 Hz)

$$\bullet)) \rightarrow v_s = 50 \text{ m/s}$$

$$f_s = 200 \text{ Hz}$$

$$V = 300 \text{ m/s}$$

$$f_o = f_s \left(\frac{V + V_o}{V - V_s} \right)$$

$$f_o = 200 \left(\frac{300 + 0}{300 - (50)} \right)$$

$$f_o = 200 \left(\frac{300}{250} \right)$$

$$f_o = 200 \left(\frac{6}{5} \right)$$

$$f_o = 240 \text{ Hz}$$

- 20) A source of sound has a frequency of 200 Hz. It travels away from a stationary observer at 50 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source of sound moves away? (171.43 Hz)

$$\bullet)) \left(\begin{array}{l} \text{← } -50 \text{ m/s} = V_s \\ f_s = 200 \text{ Hz} \\ V = 300 \text{ m/s} \end{array} \right) \left\{ \begin{array}{l} f_o = f_s \left(\frac{V + V_o}{V - V_s} \right) \\ f_o = 200 \left(\frac{300 + 0}{300 - (-50)} \right) \\ f_o = 200 \left(\frac{300}{350} \right) \end{array} \right. \rightarrow f_o = 200 \left(\frac{6}{7} \right) \boxed{f_o = 171.43 \text{ Hz}}$$

- 21) A source of sound has a frequency of 100 Hz. The observer travels towards it at 30 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source approaches? (110.00 Hz)

$$\bullet)) \left(\begin{array}{l} \text{← } V_o = 30 \text{ m/s} \\ f_s = 100 \text{ Hz} \\ V = 300 \text{ m/s} \end{array} \right) \left\{ \begin{array}{l} f_o = f_s \left(\frac{V + V_o}{V - V_s} \right) \\ f_o = 100 \left(\frac{300 + 30}{300 - 0} \right) \\ f_o = 100 \left(\frac{330}{300} \right) \end{array} \right. \rightarrow f_o = 100 (1.1) \boxed{f_o = 110.0 \text{ Hz}}$$

- 22) A source of sound has a frequency of 100 Hz. The observer travels away from it at 30 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source recedes? (90.00 Hz)

$$\bullet)) \left(\begin{array}{l} \text{→ } V_o = -30 \text{ m/s} \\ f_s = 100 \text{ Hz} \\ V = 300 \text{ m/s} \end{array} \right) \left\{ \begin{array}{l} f_o = f_s \left(\frac{V + V_o}{V - V_s} \right) \\ f_o = 100 \left(\frac{300 - 30}{300 - 0} \right) \\ f_o = 100 \left(\frac{270}{300} \right) \end{array} \right. \rightarrow f_o = 100 (.9) \boxed{f_o = 90 \text{ Hz}}$$

- 23) A source of sound has a frequency of 200 Hz. The observer travels towards it at 50 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source approaches? (233.33 Hz)

•))  $v_o = 50 \text{ m/s}$

$$f_s = 200 \text{ Hz}$$

$$V = 300 \text{ m/s}$$

$$f_o = f_s \left(\frac{V + v_o}{V - v_s} \right)$$

$$f_o = 200 \left(\frac{300 + 50}{300 - 0} \right)$$

$$f_o = 200 \left(\frac{350}{300} \right)$$

$$\rightarrow f_o = 200 (1.17)$$

$$\boxed{f_o = 233.3 \text{ Hz}}$$

- 24) A source of sound has a frequency of 200 Hz. The observer travels away from it at 50 m/s. The speed of sound is 300 m/s. What frequency will the observer hear as the source recedes? (166.67 Hz)

•)))  $v_o = -50 \text{ m/s}$

$$f_s = 200 \text{ Hz}$$

$$V = 300 \text{ m/s}$$

$$f_o = f_s \left(\frac{V + v_o}{V - v_s} \right)$$

$$f_o = 200 \left(\frac{300 - 50}{300 - 0} \right)$$

$$f_o = 200 \left(\frac{250}{300} \right)$$

$$f_o = 200 \left(\frac{5}{6} \right)$$

$$\boxed{f_o = 166.67 \text{ Hz}}$$

Mach Number

- 25) A source of sound travels through air at 450 m/s. The speed of sound in air on at that time is 300m/s. What is the Mack Number for this speed? (1.5)

$$\begin{aligned} \sqrt{s} &= 300 \text{ m/s} \\ v_s &= 450 \text{ m/s} \end{aligned}$$
$$MN = \frac{\sqrt{s}}{v_s}$$
$$MN = \frac{300}{450}$$
$$MN = 1.50$$

- 26) A source of sound travels through air at 350m/s. The speed of sound in air on at that time is 310m/s. What is the Mack Number for this speed? (1.13)

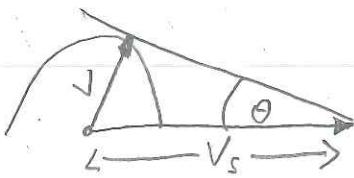
$$\begin{aligned} \sqrt{s} &= 350 \text{ m/s} \\ v_s &= 310 \text{ m/s} \end{aligned}$$
$$MN = \frac{\sqrt{s}}{v_s}$$
$$MN = \frac{350}{310}$$
$$MN = 1.13$$

- 27) A source of sound travels through air at 670m/s. The speed of sound in air on at that time is 330m/s. What is the Mack Number for this speed? (2.03)

$$\begin{aligned} \sqrt{s} &= 670 \text{ m/s} \\ v_s &= 330 \text{ m/s} \end{aligned}$$
$$MN = \frac{\sqrt{s}}{v_s}$$
$$MN = \frac{670}{330}$$
$$MN = 2.03$$

Shock Wave Mach Angle

- 28) A source of sound travels through air at 450 m/s. The speed of sound in air on at that time is 300m/s. What is the shock wave Mach Angle for this speed? (41.81*)



$$V_s = 450 \text{ m/s}$$

$$V = 300 \text{ m/s}$$

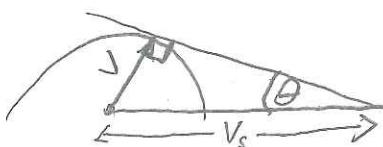
$$\sin \theta = \frac{V}{V_s}$$

$$\sin \theta = \frac{300}{450}$$

$$\theta = \sin^{-1} \left(\frac{300}{450} \right)$$

$$\theta = 41.81^\circ$$

- 29) A source of sound travels through air at 350m/s. The speed of sound in air on at that time is 310m/s. What is the shock wave Mach Angle for this speed? (62.34*)



$$V_s = 350 \text{ m/s}$$

$$V = 310 \text{ m/s}$$

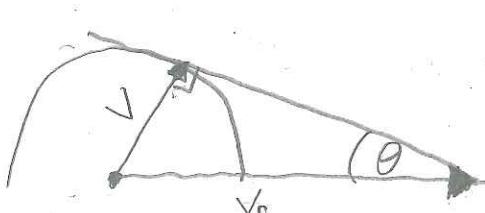
$$\sin \theta = \frac{V}{V_s}$$

$$\sin \theta = \frac{310}{350}$$

$$\theta = \sin^{-1} \left(\frac{310}{350} \right)$$

$$\theta = 62.34^\circ$$

- 30) A source of sound travels through air at 670m/s. The speed of sound in air on at that time is 330m/s. What is the shock wave Mach Angle for this speed? (29.51*)



$$V_s = 670 \text{ m/s}$$

$$V = 330 \text{ m/s}$$

$$\sin \theta = \frac{V}{V_s}$$

$$\sin \theta = \frac{330}{670}$$

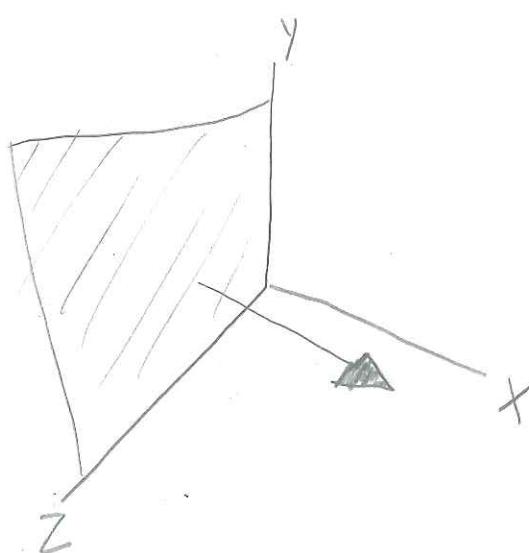
$$\theta = \sin^{-1} \left(\frac{330}{670} \right)$$

$$\theta = 29.51^\circ$$

Gauss Law Homework

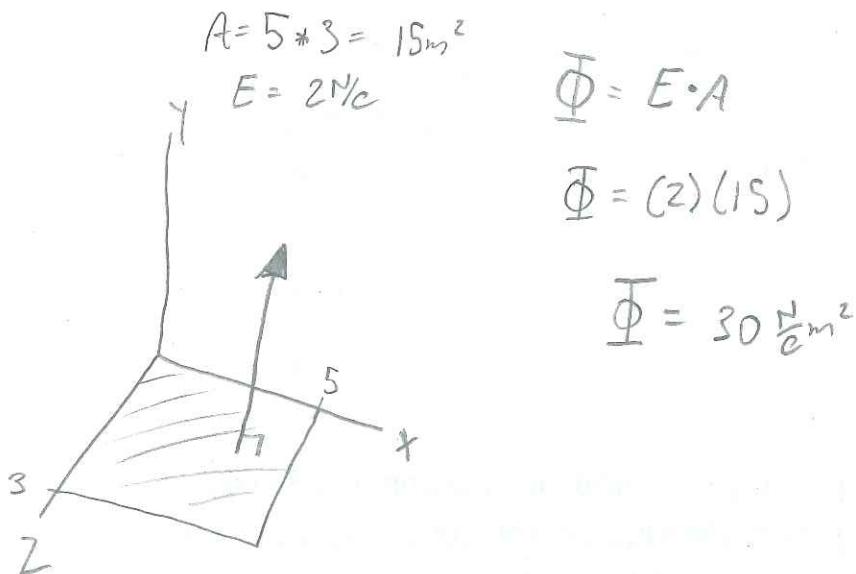
Electric Flux through a flat perpendicular surface

- 1) An electric field of 6 N/C points in the positive X direction. What is the electric flux that passes through a 4 m^2 surface that is in the YZ plane? ($24 \text{ Nm}^2/\text{C}$) (Draw a diagram showing the electric field vector and the plane.) (Is the electric flux a vector or a scalar?) (a scalar)



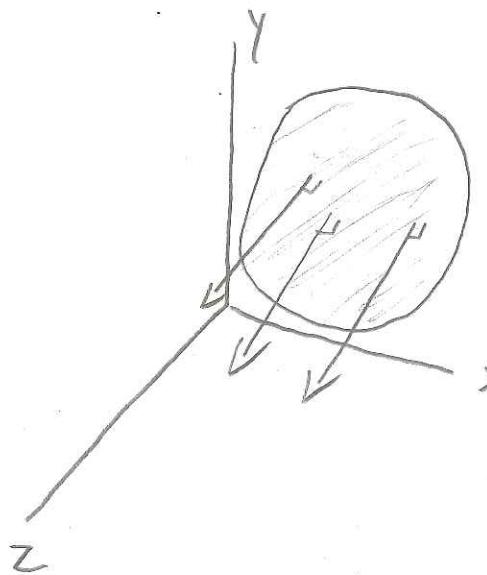
$$\begin{aligned} \vec{E} &= 6 \frac{\text{N}}{\text{C}} \\ A &= 4 \text{ m}^2 \\ \Phi &= E \cdot A \\ \Phi &= (6)(4) \\ \Phi &= 24 \frac{\text{Nm}^2}{\text{C}} \end{aligned}$$

- 2) An electric field of 2 N/C points in the positive Y direction. What is the electric flux that passes through a surface that is in the XZ plane if the area is rectangular measuring 3 m by 5 m ? ($30 \text{ Nm}^2/\text{C}$)



$$\begin{aligned} A &= 5 * 3 = 15 \text{ m}^2 \\ E &= 2 \frac{\text{N}}{\text{C}} \\ \Phi &= E \cdot A \\ \Phi &= (2)(15) \\ \Phi &= 30 \frac{\text{Nm}^2}{\text{C}} \end{aligned}$$

- 3) An electric field of 6 N/C points in the positive Z direction. What is the electric flux that passes through a surface that is in the XY plane if the area is circular with a radius of 3 m? (170 Nm²/C)



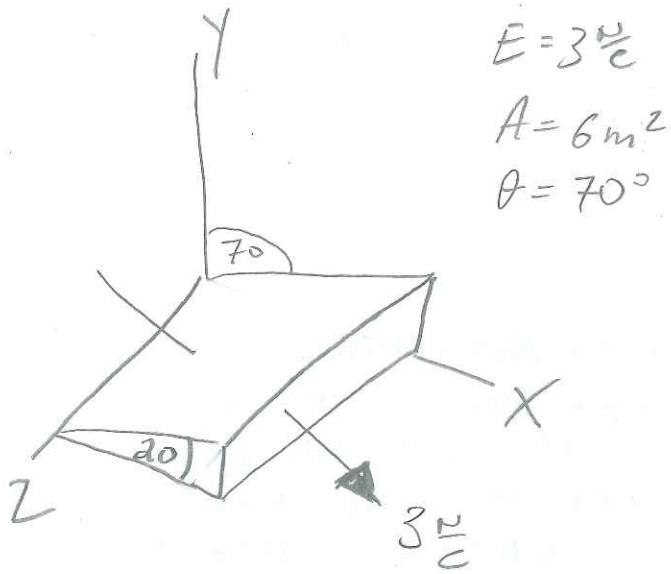
$$A = \pi r^2 = \pi(3)^2 = 9\pi$$
$$E = 6 \text{ N/C}$$

$$\Phi = E \cdot A$$

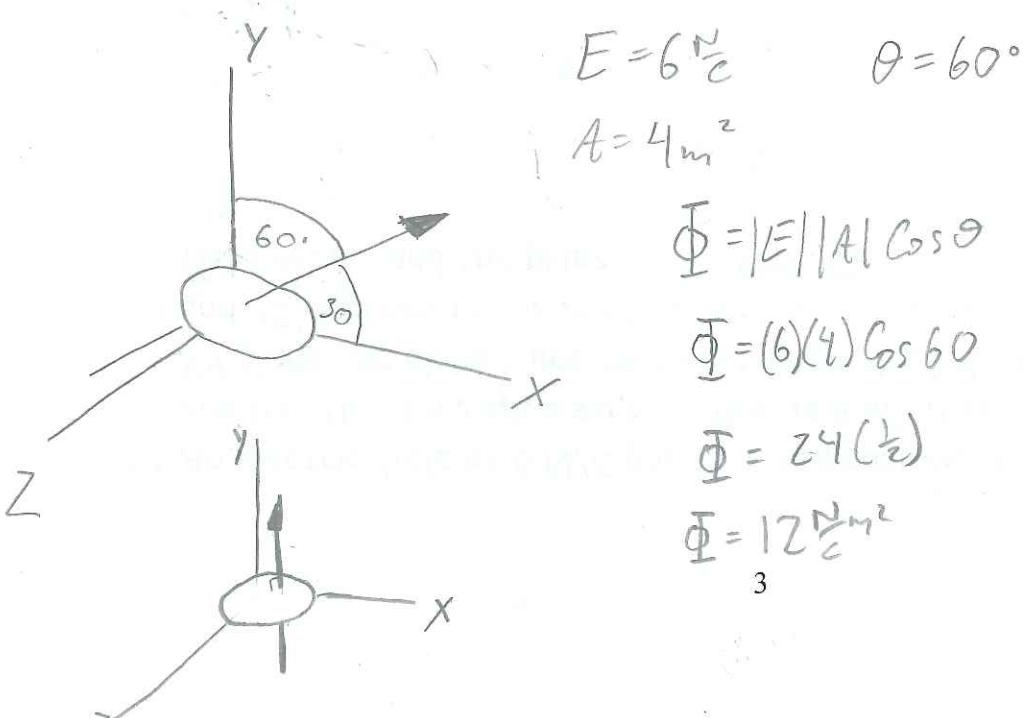
$$\Phi = (6)(9\pi) = 169.6 \frac{\text{N}}{\text{C}} \cdot \text{m}^2$$

Electric Flux through a flat inclined Surface

- 4) An electric field of 3 N/C points in the positive X direction. What is the electric flux through a surface that is 6 m² if the plane of the surface is inclined at 20 degrees to the positive X axis? (Draw a diagram showing the electric field vector and the plane). (6.16 Nm²/C)



- 5) An electric field of 6 N/C points in the positive Y direction. What is the electric flux through a surface that is 4 m² if its surface normal is in the XY plane and is also inclined along a line that is 60 degrees to the positive Y axis and 30 degrees to the positive X axis? (Draw a diagram showing the electric field vector and the plane). (12 Nm²/C)



- 6) An electric field of 6 N/C points in the positive X direction. What is the electric flux through a surface that is 4 m^2 , if its surface normal is in the XY plane and along a line that is inclined at 60 degrees to the positive Y axis and 30 degrees to the positive X axis? (Draw a diagram showing the electric field vector and the plane). (21 Nm²/C)

$E = 6 \frac{\text{N}}{\text{C}}$

$A = 4 \text{ m}^2$

$\Phi = EA \cos \theta$

$\theta = 30^\circ$

$\Phi = (6)(4) \cos 30$

$\Phi = 24 \left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\Phi = 21 \frac{\text{N}}{\text{C}} \text{ m}^2}$

- 7) An electric field of 6 N/C points in the positive X direction. What is the electric flux through a surface that is 4 m^2 if its surface normal is in the XZ plane and is along a line that is inclined at 60 degrees to the positive Z axis and 30 degrees to the positive X axis? (Draw a diagram showing the electric field vector and the plane). (21 Nm²/C)

$E = 6 \frac{\text{N}}{\text{C}}$

$A = 4 \text{ m}^2$

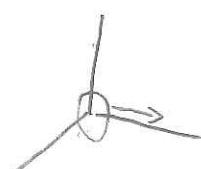
$\Phi = EA \cos \theta$

$\theta = 30^\circ$

$\Phi = (6)(4) \cos 30$

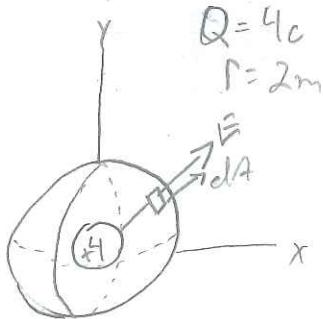
$\Phi = 24 \left(\frac{\sqrt{3}}{2}\right)$

$\boxed{\Phi = 21 \frac{\text{N}}{\text{C}} \text{ m}^2}$



Gauss' Law

- 8) A 4C positive charge sits at the center of a spherical surface of radius 2m. What is the electric flux through the surface? (Draw a diagram showing the charge, electric field and surface). (Why is the electric flux positive?)
 $(+(4/\epsilon_0) \text{ Nm}^2/\text{C})$



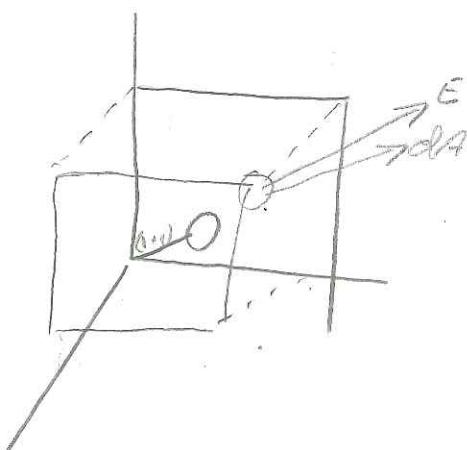
$$\underline{\Phi} = \frac{Q_{in}}{\epsilon_0}$$

$$\underline{\Phi} = \frac{4}{\epsilon_0} \frac{\text{N m}^2}{\text{C}}$$

- 9) A 4C positive charge sits at the center of a spherical surface of radius 10 m. What is the electric flux through the surface? (Draw a diagram showing the charge, electric field and surface). (Why is the electric flux positive?)
 $(+(4/\epsilon_0) \text{ Nm}^2/\text{C})$

- 10) A $4C$ negative charge sits at the center of a spherical surface of radius 6 m . What is the electric flux through the surface? (Draw a diagram showing the charge, electric field and surface). (Why is the electric flux negative?) $(-(4/\epsilon_0)\text{ Nm}^2/\text{C})$

- 11) A $4C$ positive charge sits at the point $(1i+1j+1k)\text{m}$. A cubic gaussian surface is 2m on edge with one corner at the origin and its opposite corner at $(2i+2j+2k)\text{m}$. What is the electric flux through the surface? (Draw a diagram showing the charge, electric field and surface.) (Why is the electric flux positive?) $(+(4/\epsilon_0)\text{ Nm}^2/\text{C})$



$$\Phi = \frac{Q_{in}}{\epsilon_0}$$
$$\boxed{\Phi = \frac{4}{\epsilon_0} \frac{\text{Nm}^2}{\text{C}}}$$

- 12) A $4C$ positive charge sits at the point $(1i+1j+1k)m$. A cubic gaussian surface is 2 m on edge with one corner at the origin and its opposite corner at $(-2i-2j-2k)m$. What is the electric flux through the surface? (Draw a diagram showing the charge, electric field and surface.) (Why is the electric flux positive?) $(0 \text{ Nm}^2/\text{C})$

Using Gauss' Law to Find an Electric Field near a point charge

- 13) A 5C positive charge is located at the origin. Consider a spherical Gaussian surface of 2m radius centered on the origin. Use this geometry to determine the electric field at any point that is 2m from the point charge. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($1.125 \times 10^{10} \text{ N/C}$)

$$\Phi = \int E \cdot dA = \frac{Q_{in}}{\epsilon_0} \quad | \quad E \parallel dA \quad E \text{ is const}$$

$$dA_{\text{Sphere}} = 4\pi r^2 \quad | \quad E = \frac{Q_{in}}{4\pi r^2 \epsilon_0} \rightarrow E = K_e \frac{Q_{in}}{r^2} \rightarrow E = K_e \left(\frac{5}{2^2} \right)$$

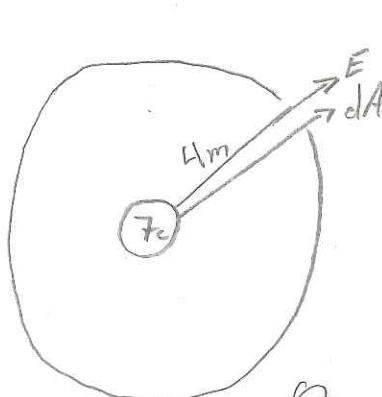
$$K_e = \frac{1}{4\pi \epsilon_0} \quad | \quad r = 2\text{m}$$

$$Q_{in} = 5\text{C}$$

$$E = K_e \frac{5}{4} = E = \frac{(9 \times 10^9)(5)}{4}$$

$$E = 1.125 \times 10^{10} \frac{\text{N}}{\text{C}}$$

- 14) A 7C positive charge is located at the origin. Consider a spherical Gaussian surface of 4 m radius centered on the origin. Use this geometry to determine the electric field at any point that is 4m from the point charge. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($3.94 \times 10^9 \text{ N/C}$)



$$\Phi = \int E \cdot dA = \frac{Q_{in}}{\epsilon_0} \quad | \quad E \parallel dA \quad \& \quad E \text{ is constant}$$

$$E \cdot \int dA = \frac{Q_{in}}{\epsilon_0} \rightarrow E (4\pi r^2) = \frac{Q_{in}}{\epsilon_0}$$

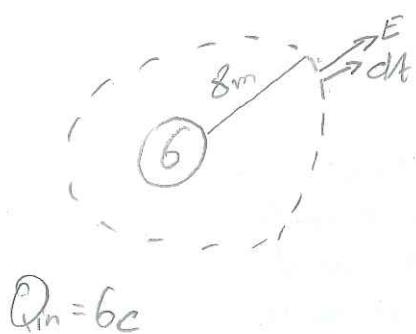
$$E = \frac{Q_{in}}{4\pi r^2 \epsilon_0} \rightarrow E = K_e \frac{Q_{in}}{r^2} \rightarrow E = K_e \frac{7}{4^2}$$

$$Q_{in} = 7\text{C}$$

$$r = 4\text{m}$$

$$E = \frac{(9 \times 10^9)(7)}{16} \rightarrow E = 3.94 \times 10^9 \frac{\text{N}}{\text{C}}$$

- 15) A 6 C positive charge is located at the origin. Determine the electric field at any point that is 8 m from the point charge. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($8.44 \times 10^8 \text{ N/C}$)



$$\bar{\Phi} = \int E \cdot dA = \frac{Q_{in}}{\epsilon_0} \quad E \parallel dA \text{ & } E \text{ is constant}$$

$$\bar{\Phi} = E \int dA = \frac{Q_{in}}{\epsilon_0} \rightarrow E(4\pi r^2) = \frac{Q_{in}}{\epsilon_0}$$

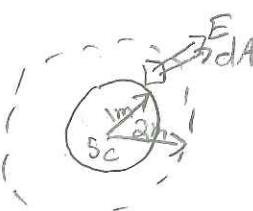
$$E = \frac{Q_{in}}{4\pi r^2 \epsilon_0} \rightarrow K_e = \frac{1}{4\pi \epsilon_0} \rightarrow E = K_e \frac{Q_{in}}{r^2}$$

$$E = \frac{(9 \times 10^9)(6)}{8^2} =$$

$$\boxed{E = 8.44 \times 10^8 \text{ N/C}}$$

Using Gauss' Law to Find an Electric Field near spherical charge distribution

- 16) A 5C positive spherical charge distribution is located at the origin. It has a 1m radius. Consider a spherical Gaussian surface of 2 m radius centered on the origin. Use this geometry to determine the electric field at any point that is 2m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($1.125 \times 10^{10} \text{ N/C}$)



charge radius = 1m
GS radius = 2m
 $a = 1\text{m}$
 $r = 2\text{m}$

Proof

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{in}}{\epsilon_0}$$

E is constant $\oint dA = 4\pi r^2$

$$E \cdot 4\pi r^2 = \frac{Q_{in}}{\epsilon_0}$$

$$E = \frac{Q_{in}}{4\pi r^2 \epsilon_0} \rightarrow k_e = \frac{1}{4\pi \epsilon_0} \therefore E = k_e \frac{Q_{in}}{r^2}$$

$$E = k_e \frac{(S)}{r^2} \Rightarrow \boxed{E = 1.125 \times 10^{10} \frac{\text{N}}{\text{C}}}$$

- 17) A 3C positive spherical charge distribution is located at the origin. It has a 4m radius. Consider a spherical Gaussian surface of 7m radius centered on the origin. Use this geometry to determine the electric field at any point that is 7m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($5.51 \times 10^8 \text{ N/C}$)

- 18) A 4C positive spherical charge distribution is located at the origin. It has a 2m radius. Determine the electric field at any point that is 7m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($7.35 \times 10^8 \text{ N/C}$)

Using Gauss' Law to Find an Electric Field inside spherical charge distribution

- 19) A 5C positive spherical charge distribution is located at the origin. It has a 4m radius. Consider a spherical Gaussian surface of 2m radius centered on the origin. Use this geometry to determine the electric field at any point that is 2m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($1.41 \times 10^9 \text{ N/C}$)

Diagram showing a spherical charge distribution of radius $R = 4\text{m}$. A Gaussian surface of radius $r = 2\text{m}$ is drawn around the origin. A small differential area element dA is shown on the Gaussian surface, with a normal vector \hat{n} pointing outwards. A vector E represents the electric field at this point.

Given:

- Total Charge $Q = 5\text{C}$
- Radius of charge distribution $R = 4\text{m}$
- Radius of Gaussian surface $r = 2\text{m}$
- Electric field E

Proof:

$$E = k_e \frac{Qr}{R^3}$$

$$\Phi = \oint E \cdot dA = \frac{Q_{in}}{\epsilon_0} \quad E \parallel dA$$

$$E \oint dA = \frac{Q_{in}}{\epsilon_0} \quad \oint dA = 4\pi r^2$$

$$E(4\pi r^2) = \frac{Q_{in}}{\epsilon_0} \rightarrow E = \frac{Q_{in}}{4\pi r^2 \epsilon_0} \quad k_e = \frac{1}{4\pi \epsilon_0}$$

$$E = k_e \frac{Q_{in}}{r^2} \rightarrow Q_{in} = \frac{\text{Total Charge}}{\text{Charge Vol}} * (\text{GS Vol}) = \frac{5}{4\pi R^3} * (4\pi r^3)$$

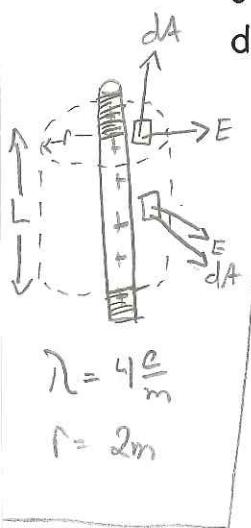
$$E = k_e \frac{5r^3}{r^2 R^3} \rightarrow E = k_e \frac{5r}{R^3} \rightarrow E = \frac{10}{64} (9 \times 10^9) \quad \boxed{E = 1.41 \times 10^9 \text{ N/C}}$$

- 20) A 3C positive spherical charge distribution is located at the origin. It has a 9m radius. Consider a spherical Gaussian surface of 5m radius centered on the origin. Use this geometry to determine the electric field at any point that is 5m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($1.85 \times 10^8 \text{ N/C}$)

- 21) A $7C$ positive spherical charge distribution is located at the origin. It has a $8m$ radius. Determine the electric field at any point that is $2m$ from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($2.46 \times 10^8 \text{ N/C}$)

Using Gauss' Law to Find an Electric Field near an infinitely long linear charge distribution

- 22) An infinitely long cylinder of positive charge has a charge density of 4C/m . Determine the electric field at any point that is 2m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($3.60 \times 10^{10} \text{ N/C}$)



Proof E is const., $E \parallel dA$

$$\Phi = SE \cdot dA = \frac{Q_{in}}{\epsilon_0} \rightarrow E \cdot 2\pi r L = \frac{Q_{in}}{\epsilon_0} \rightarrow \int dA = 2\pi r L \therefore E(2\pi r L) = \frac{Q_{in}}{\epsilon_0}$$

$$E = \frac{Q_{in}}{2\pi r L \epsilon_0} \rightarrow Q_{in} = \lambda L \therefore E = \frac{\lambda L}{2\pi r L \epsilon_0} \rightarrow E = \frac{\lambda}{2\pi r \epsilon_0}$$

$$k_e = \frac{1}{4\pi r \epsilon_0} \rightarrow \frac{4\pi}{2} = \left(\frac{1}{k_e \epsilon_0}\right) \div 2 \rightarrow (2\pi) = \frac{1}{2k_e \epsilon_0} \therefore E = \frac{\lambda}{\left(\frac{1}{2k_e \epsilon_0}\right) r \epsilon_0}$$

$$E = \frac{\lambda}{\frac{r}{2k_e}} = E = \lambda \cdot \frac{2k_e}{r} \rightarrow E = (4) \frac{2k_e}{r}$$

$$E = 4k_e \rightarrow E = 3.60 \times 10^{10} \frac{\text{N}}{\text{C}}$$

- 23) An infinitely long cylinder of positive charge has a charge density of 6C/m . Determine the electric field at any point that is 5m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($2.16 \times 10^{10} \text{ N/C}$)

- 24) An infinitely long cylinder of positive charge has a charge density of 2C/m . Determine the electric field at any point that is 9m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($4.00 \times 10^9 \text{ N/C}$)

Using Gauss' Law to Find an Electric Field near an infinitely large planar charge distribution

- 25) An infinitely large plane of positive charge has a charge density of 4C/m^2 . Determine the electric field at any point that is 2m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($2.22 \times 10^{11} \text{ N/C}$)
- 26) An infinitely large plane of positive charge has a charge density of 3C/m^2 . Determine the electric field at any point that is 2m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($1.67 \times 10^{11} \text{ N/C}$)

- 27) An infinitely large plane of positive charge has a charge density of 6C/m^2 . Determine the electric field at any point that is 2m from the center of the charge distribution. (Draw a diagram showing the charge distribution, Gaussian surface and electric field lines.) ($3.33 \times 10^{11} \text{ N/C}$)

Chapter 25 Skills - Current and Resistance

Current

- 1) 6 Coulomb of charge passes a cross-sectional area of a copper wire in 3 seconds. What is the current flowing through the wire? (2A)

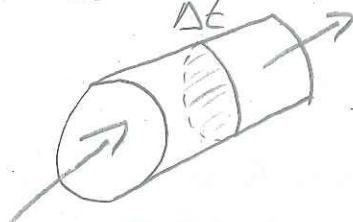
$$Q = 6C$$

$$I = \frac{6}{3}$$

$$\Delta t = 3s$$

$$I = \frac{Q}{\Delta t}$$

$$I = 2 \frac{C}{s}$$



$$\boxed{I = 2A}$$

- 2) 18 Coulomb of charge passes a cross-sectional area of a copper wire in 6 seconds. What is the current flowing through the wire? (3A)

$$I = \frac{Q}{\Delta t}$$

$$I = \frac{18}{6}$$

$$\boxed{I = 3 \text{ Amps}}$$

- 3) 24 C of charge passes a cross-sectional area of a copper wire in 4 seconds. What is the current flowing through the wire? (6A)

$$I = \frac{Q}{\Delta t}$$

$$I = \frac{24}{4}$$

$$\boxed{I = 6 \text{ Amps}}$$

- 4) 6nC of charge passes a cross-sectional area of a copper wire in 2ms . What is the current flowing through the wire? ($3\mu\text{A}$)

$$\frac{I = 6 \times 10^{-9}}{2 \times 10^{-3}}$$

$$I = 3 \times 10^{-6} \text{ A}$$

- 5) $6\mu\text{C}$ of charge passes a cross-sectional area of a copper wire in 2ns . What is the current flowing through the wire? (3kA)

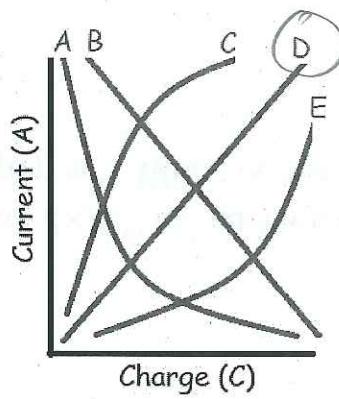
- 6) 6A flows through a wire. What would the new current be if twice as much charge passed a cross-sectional area in half the time? (24A)

$$\frac{I_N}{I_0} = \frac{\frac{Q}{At}}{\frac{Q}{At}} \rightarrow I_N = (6)(4)$$
$$\frac{I_N}{6\text{A}} = \frac{\frac{Q}{At/2}}{1} \rightarrow I_N = 24\text{A}$$

7) 9A flows through a wire. What would the new current be if half as much charge passed a cross-sectional area in quadruple the time? (1.12 A)

8) An EMF is applied across a length of wire. Which trace best represents the relationship between current and the amount of charge passing a cross-sectional area in a wire? Everything else is kept constant. (D)

$$I = \frac{Q}{\Delta t}$$



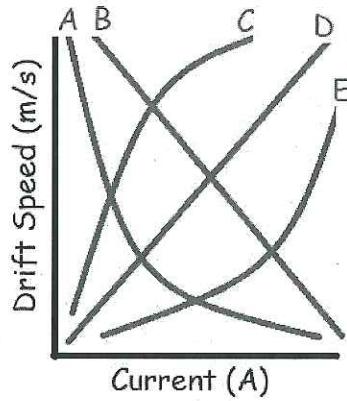
Charge Carrier Drift Speed

9) A wire that has a cross-sectional area of 2m^2 carries a 5A current. The wire contains 300 charge carrying electrons per m^3 . What is the drift speed of the charge carriers? ($5.2 \times 10^{16} \text{ m/s}$)

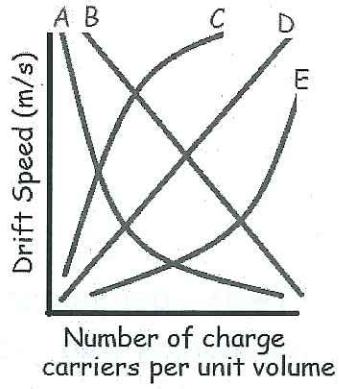
10) A wire of copper that has a cross-sectional area of $2 \times 10^{-6} \text{ m}^2$ carries a 6A current. The wire contains 9×10^{28} charge carriers per m^3 . What is the drift speed of the charge carriers? ($2 \times 10^{-4} \text{ m/s}$)

- 11) A wire of Aluminum that has a cross-sectional area of $2 \times 10^{-6} \text{ m}^2$ carries a 6A current. The wire contains 6×10^{28} charge carriers per m^3 . What is the drift speed of the charge carriers? ($3.1 \times 10^{-4} \text{ m/s}$)

- 12) An EMF is applied across a conductor. Which trace best represents the relationship between the average drift speed of charge carriers in a conductor and the current flowing through the conductor? Everything else is kept constant. (D)



- 13) An EMF is applied across a conductor. Which trace best represents the relationship between the average drift speed of charge carriers in a conductor and the number of charge carriers per unit volume within the conductor? Everything else is kept constant. (A)



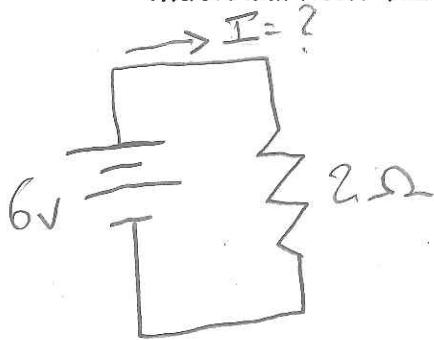
- 14) 12A of current flows through a wire. What would be the new current if the drift speed of charge carriers within the wire was doubled, the area of the wire was halved, the number of charge carriers per unit volume was quartered, and the charge per unit charge carrier was doubled? (6A)

Current Density

- 15) What is the current density in a wire that has a cross-sectional area of 2 m^2 and carries a current of 5A ? (2.5 A/m^2)
- 16) What is the current density in a wire that has a cross-sectional area of 4m^2 and carries a current of 8A ? (2.0 A/m^2)
- 17) What is the current density in a cylindrical wire that has a radius of 1m and carries a current of 8A ? ($8/\pi \text{ A/m}^2$)
- 18) What is the current density in a cylindrical wire that has a radius of 0.5m and carries a current of 12A ? ($48/\pi \text{ A/m}^2$)

Ohm's Law

- 19) A 6 volt potential difference is applied across a 2 ohm resistor. How much current flows through the resistor? (3A)



$$\Delta V = IR$$

$$I = \frac{\Delta V}{R}$$

$$I = \frac{6}{2}$$

$$I = 3A$$

- 20) A 4 Amp current flows through a 12 ohm resistor. What is the potential difference across the resistor? (48V)

$$I = 4A$$

$$\Delta V = IR$$

$$R = 12\Omega$$

$$\Delta V = 4(12)$$

$$\Delta V = ?$$

$$\Delta V = 48V$$

- 21) A 9 volt potential difference causes 2 Amp to flow through a resistor. What is the value of the resistance? (4.5 Ω)

$$\Delta V = 9V$$

$$\Delta V = IR$$

$$I = 2A$$

$$R = \frac{\Delta V}{I}$$

$$R =$$

$$R = \frac{9}{2}$$

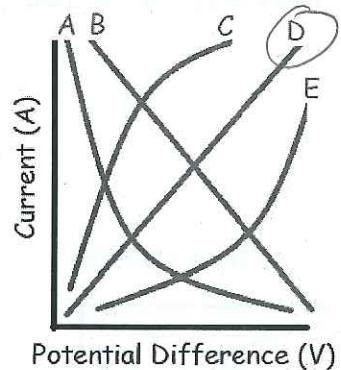
$$R = 4.5\Omega$$

- 22) An 8 mV potential difference causes 2 μ A to flow through a resistor. What is the value of the resistance? (4k Ω)

- 23) An EMF is applied to a resistor. Which trace best represents the relationship between the Current flow through the resistor, and the Potential Difference that exists across it? Resistance is kept constant. (D)

$$\Delta V = I R$$

$$\boxed{\Delta V = I}$$

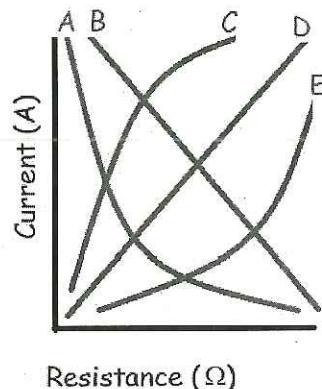


- 24) An EMF is applied to a resistor. Which trace best represents the relationship between the Current flow through the resistor, and the Resistance of the resistor? The EMF is kept constant. (A)

$$\Delta V = IR$$

$$I = \frac{\Delta V}{R}$$

Inverse



- 25) A current of 12 amps flows through a resistor. What would the new current be if the potential difference across the resistor was halved and the resistance of the resistor was tripled? Everything else is kept constant. (2 A)

Conductivity and Resistivity

- 26) A material has a resistivity of 4 ohm-meter. What is its conductivity?
(0.25 siemens/meter)

$$\rho = 4 \Omega m \rightarrow \sigma = \frac{1}{4}$$
$$\rho\sigma = 1$$
$$\sigma = \frac{1}{\rho}$$
$$\sigma = 0.25 \frac{\text{Siemens}}{\text{m}}$$

- 27) A material has a resistivity of 2 ohms-meter. What is its conductivity?
(0.5 siemens/meter)

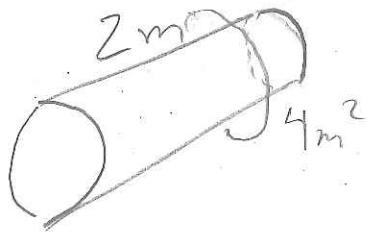
$$\rho = 2 \Omega m \rightarrow \sigma = \frac{1}{2}$$
$$\rho\sigma = 1$$
$$\sigma = \frac{1}{\rho}$$
$$\sigma = \frac{1}{2} \frac{\text{Siemens}}{\text{m}}$$

- 28) A material has a resistivity of 0.2 ohms-meter. What is its conductivity? (5 siemens/meter)

$$\rho = \frac{2}{10} \rightarrow \sigma = \frac{10}{2}$$
$$\sigma = \frac{1}{\frac{2}{10}}$$
$$\sigma = 5 \frac{\text{Siemens}}{\text{m}}$$

Resistivity and resistance

- 29) A cylindrical resistor of length 2 m and cross-sectional area of 4 m^2 is made from a material that has a resistivity of $5 \Omega\text{m}$. What is the resistance of the component? (2.5 Ω)



$$R = \frac{\rho L}{A}$$

$$R = \frac{10}{4}$$

$$L = 2 \text{ m}$$

$$A = 4 \text{ m}^2$$

$$\rho = 5 \Omega\text{m}$$

$$R = 2.5 \Omega$$

- 30) A cylindrical resistor of length 6 m and cross-sectional area of 2 m^2 is made from a material that has a resistivity of $8 \Omega\text{m}$. What is the resistance of the component? (24 Ω)

- 31) A cylindrical resistor of length 2 m and cross-sectional area of 0.5 m^2 is made from a material that has a resistivity of $4 \Omega\text{m}$. What is the resistance of the component? (16 Ω)

Resistivity and temperature

?

- 32) A material has a resistivity of 2 ohms/m at 20 degrees C. The same material has a resistivity of 7 ohms/m at 25 degrees C. What is the coefficient of resistivity of the material? ($0.5 \text{ }^{\circ}\text{C}^{-1}$)

$$\rho_0 = 2 \text{ }\Omega/\text{m}$$

$$T_0 = 20 \text{ }^{\circ}\text{C}$$

$$\rho = 7 \text{ }\Omega/\text{m}$$

$$T = 25 \text{ }^{\circ}\text{C}$$

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

$$\frac{\rho - \rho_0}{\rho_0} = \alpha$$

$$\frac{7 - 2}{2} = \alpha$$

$$\alpha = \frac{2.5}{5}$$

$$\alpha = \frac{5/2}{5}$$

$$\boxed{\alpha = \frac{1}{2} \text{ }^{\circ}\text{C}^{-1}}$$

- 33) A material has a resistivity of 8 ohms/m at 20 degrees C. The same material has a resistivity of 20 ohms/m at 22 degrees C. What is the coefficient of resistivity of the material? ($0.75 \text{ }^{\circ}\text{C}^{-1}$)

Power

- 34) How much power is dissipated in a resistor if a potential difference of 5 V causes 3 A to flow through it? (15W)

$$P = \frac{J}{S}$$

$$I = \text{Amps} = \frac{Q}{S}$$

$$V = \frac{J}{Q}$$

$$P = IV$$

$$P = 3 \cdot 5$$

$$P = 15 \text{ J/S}$$

$$\boxed{P = 15 \text{ W}}$$

- 35) How much power is dissipated in a resistor if a potential difference of 8 V causes 2 A to flow through it? (16W)

$$I = 2A$$

$$\Delta V = 8V$$

$$P = IV$$

$$\boxed{P = 16 \text{ W}}$$

36) How much power is dissipated in a resistor if a potential difference of 4 V causes 5 A to flow through it? (20W)

37) How much power is dissipated in a 9 ohm resistor if a potential difference of 3 V is applied across it? (1W)

$$P = ? \quad I = \frac{\Delta V}{R} = I = \frac{3}{9} = I = \frac{1}{3}$$

$$R = 9 \Omega$$

$$\Delta V = 3 V$$

$$\Delta V = I R$$

$$P = I V$$

$$\boxed{P = 1 W}$$

38) How much power is dissipated in a 12 ohm resistor if a potential difference of 6 V is applied across it? (3W)

39) How much power is dissipated in a 20 ohm resistor if a potential difference of 5 V is applied across it? (1.25W)

40) How much power is dissipated in a 10 ohm resistor if a 4 A current flows through it? (160W)

$$P = ?$$

$$P = IV$$

$$I = 4A$$

$$R = 10 \Omega$$

$$\Delta V = IR$$

$$\Delta V = 40V$$

41) How much power is dissipated in a 5 ohm resistor if a 6 A current flows through it? (180W)

- 42) How much power is dissipated in a 2 ohm resistor if a 3 A current flows through it? (18W)

$$P = IV \rightarrow P = (3)(6)$$

$$R = 2\Omega$$

$$I = 3A$$

$$\Delta V = 6V$$

$P = 18W$

- 43) A resistor dissipates 12 W. How much power would another resistor dissipate if it had twice as much current flowing through it and three times the resistance? (144W)

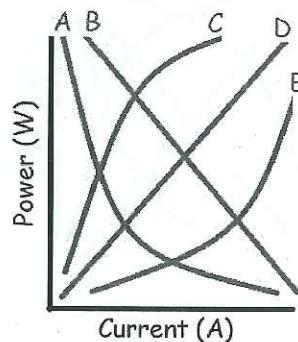
$$\frac{P}{12} = \frac{(2)(2 \cdot 3)}{1} \rightarrow P = (12)(12)$$

$$\Delta V = IR$$

$P = 144W$

- 44) An EMF is applied to a resistor. Which trace best represents the relationship between the Power that is dissipated in a resistor, and the current that flows through it? The EMF is kept constant. (E)

$P = IV$ why E?

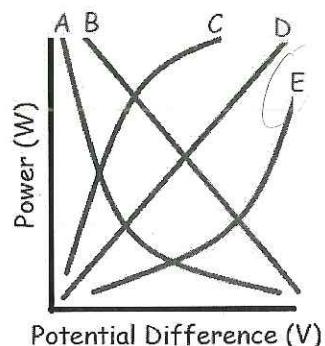


- 45) An EMF is applied to a resistor. Which trace best represents the relationship between the Power that is dissipated in a resistor, and the potential difference that is applied across it? The resistance of the resistor is kept constant. (E)

$$P = IV$$

$$P = \frac{V^2}{R}$$

$$P = \frac{V}{R}$$



- 46) An EMF is applied to a resistor. Which trace best represents the relationship between the Power that is dissipated in a resistor, and the value of the resistance? The EMF is kept constant. (A)

$$P = IV$$

$$P = \frac{V^2}{R}$$

$$P = \frac{I}{R}$$

