1, 3, 5, 9, 21

Steven Romeiro

1 - 14 - 15

$\frac{53}{53}$ 100%

Homework 2.1

(1) $f(x) = x^3 + 1$

a) $[2, 3]$ $f(2) = (2)^3 + 1 = \boxed{9}$

$f(3) = (3)^3 + 1 = \boxed{27}$

$$m = \frac{f(3) - f(2)}{3 - 2} = \frac{27 - 9}{3 - 2} = \boxed{18}$$

b) $[-1, 1]$ $f(-1) = (-1)^3 + 1 = \boxed{0}$

$f(1) = (1)^3 + 1 = \boxed{2}$

$$m = \frac{f(1) - f(-1)}{1 - (-1)} = \frac{2 - 0}{1 + 1} = \boxed{1}$$

(3) $h(t) = \cot t$

a) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ $\cot = \frac{\cos t}{\sin t} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

$$\frac{\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} - \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}}{\frac{3\pi}{4} - \frac{\pi}{4}} = \frac{-2}{\frac{2\pi}{4}}$$

$$= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$\boxed{= -\frac{4}{\pi}}$$

(5) $R(\theta) = \sqrt{4\theta+1}$ $[0, 2]$

$R(0) = \sqrt{4(0)+1} = \sqrt{1} = 1$

$R(2) = \sqrt{4(2)+1} = \sqrt{9} = 3$

$$m = \frac{R(2) - R(0)}{2 - 0} = \frac{3-1}{2-0} = \boxed{1}$$

(9) $$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) - 3 - (x^2-2x-3)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{3} - \cancel{x^2} + \cancel{2x} + \cancel{3}}{h}$$

$$\frac{2xh + h^2 - 2h}{h} = \boxed{2x + h - 2}$$

(21) a) $$\frac{15 - 0}{1 - 0} = \boxed{15 mph/hr}$$

$$\frac{20 - 15}{2.5 - 1} = \frac{5}{1.5} = \frac{5}{\frac{3}{2}} = 5 \cdot \frac{2}{3} = \boxed{3.3\overline{3}}$$

$$\frac{30 - 20}{3.5 - 2.5} = \frac{10}{1} \quad \boxed{10 mph}$$

b) $t = \frac{1}{2}$ $(0,0)(1, 15)$ $(.25, 5)$ $(.75, 12.5)$

$$\frac{15 - 0}{1 - 0} = 1.5 mph$$

$t = 2$ drawing a tangent line through $t = 2$ reveals a straight horizontal line. Speed = 0 mph

$t = 3 \quad (2.5, 20)\ (3.5, 30) \ \vert \ (2.75, 21)\ (3.25, 24)$

$\frac{30-20}{3.5-2.5} = 10\text{mph} \ \vert \ \frac{24-21}{3.25-2.75} = \frac{3}{.5} = 6\text{mph}$

c) at $t = 3.5$ hrs where the line is the steepest.

$(3.25, 24)\ (3.75, 34) \ \vert \ (3.40, 29)\ (3.60, 32.5)$

$\frac{34-24}{3.75-3.25} = \frac{10}{.5} = \boxed{20\text{mph}} \ \vert \ \frac{32.5-29}{3.6-3.4} = \boxed{17.5}$

Steven
Romeiro
1, 3, 13, 15, 16, 19
25, 29, 35, 43

Homework 2.2

(1) a) DNE b) 1 c) 0 d) .5
JMP

(3) a) T b) T c) F d) F e) F f) F e) T

(13) $\lim_{x \to 6} 8(x-5)(x-7) = 8(x^2-12x+35)$
$= 8(6^2-12(6)+35) = \boxed{-8}$

(15) $\lim_{x \to 2} \frac{2x+5}{11-x^3} = \frac{2(2)+5}{11-(2)^3} = \boxed{3}$

(16) $\lim_{x \to 2/3} (8-3x)(2x-1) = -6x^2+3x+16x-8$
$= -6\left(\frac{2}{3}\right)^2 + 19\left(\frac{2}{3}\right) - 8 = \boxed{8}$

(19) $\lim_{x \to -3} (5-x)^{4/3} = (5+3)^{4/3} = \boxed{16}$

(25) $\lim_{x \to -5} \frac{x^2+3x-10}{x+5} = \frac{\cancel{(x+5)}(x-2)}{\cancel{x+5}} = x-2$

$\lim_{x \to -5} (x-2) = \boxed{-7}$

(29) $\lim_{x \to -2} \frac{-2x-4}{x^3+2x^2} = \frac{4-4}{-8-8} = \boxed{0}$? $-\frac{1}{2}$

(35) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \frac{x^{\frac{1}{2}}-3}{(x^{\frac{1}{2}}-3)(x^{\frac{1}{2}}+3)}$

$\frac{1}{x^{\frac{1}{2}}+3} = \lim_{x \to 9} \frac{1}{\sqrt{9}+3} = \frac{1}{3+3}$

$= \boxed{\frac{1}{6}}$

(43) $\lim_{x \to 0} (2\sin x - 1)$

$= 2(0) - 1 = \boxed{-1}$

#15, 23, 31, 33
37, 39
Steven Romeiro

Homework 2.3

(15) $f(x) = x+1 \quad L=5 \quad c=4 \quad \varepsilon = .01$

$0 < |x - x_0| < \delta \longrightarrow |f(x) - L| < \varepsilon$

$0 < |x-4| < \delta \longrightarrow |(x+1)-5| < .01$

$0 < |x-4| < \delta \longrightarrow |x-4| < .01$

$0 < |x-4| < \delta \longrightarrow |x-4| < .01$

$\therefore \boxed{\delta = .01}$ so $(x-\delta, x+\delta)$

(23) $f(x) = x^2 \quad L=4 \quad c=-2 \quad \varepsilon = .5$

$0 < |x-x_0| < \delta \longrightarrow |f(x) - L| < \varepsilon$

$0 < |x+2| < \delta \longrightarrow |x^2-4| < .5$

$0 < |x+2| < \delta \longrightarrow |(x+2)(x-2)| < .5$

$0 < |x-2| < \delta \longrightarrow |(x-2)| < \dfrac{.5}{(x+2)}$

$\therefore \boxed{\delta = \dfrac{.5}{x+2}}$

(31) $f(x) = 3-2x \quad c=3 \quad \varepsilon = .02$

$0 < |x-x_0| < \delta \longrightarrow |f(x) - L| < \varepsilon$

$0 < |x-3| < \delta \longrightarrow |(3-2x) - L| < .02$

$\lim_{x \to 3} 3 - 2(3) = -3 = L$

$0 < |x-3| < \delta \longrightarrow |(3-2x)+3| < .02$

$0 < |x-3|\ \delta \longrightarrow |-2x+6| < .02$

$0 < |x-3|\ \delta \longrightarrow |-2||(x-3)| < .02$

$0 < |x-3|\ \delta \longrightarrow |x-3| < \dfrac{.02}{+2}$

$\therefore \boxed{\delta = .01}$

33 $f(x) = \frac{x^2-4}{x-2}$ $c=2$ $\varepsilon = .05$

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = x+2 = \boxed{L=4}$$

$0 < |x-2| < \delta \rightarrow \left|\left(\frac{x^2-4}{x-2}\right) - 4\right| < .05$

$0 < |x-2| < \delta \rightarrow |x+2-4| < .05$

$0 < |x-2| < \delta \rightarrow |x-2| < .05$

$\therefore$ $\boxed{\delta = .05}$

37 $\lim_{x\to 4} (9-x) = 5$

$0 < |x-x_0| < \delta \rightarrow |f(x)-L| < \varepsilon$

$0 < |x-4| < \delta \rightarrow |9-x-5| < \varepsilon$

$0 < |x-4| < \delta \rightarrow |-x+4| < \varepsilon$

$0 < |x-4| < \delta \rightarrow |x-4| < \varepsilon$

$\therefore$ $\boxed{\delta = \varepsilon}$

39 $\lim_{x\to 9} \sqrt{x-5} = 2$

$0 < |x-x_0| < \delta \rightarrow |f(x)-L| < \varepsilon$

$0 < |x-9| < \delta \rightarrow |\sqrt{x-5}-2| < \varepsilon$

$0 < |x-9| < \delta \rightarrow |$

So $\underset{+2}{-\varepsilon} < \underset{+2}{\sqrt{x-5}-2} < \underset{+2}{\varepsilon}$

$2-\varepsilon < \sqrt{x-5} < \varepsilon+2$

$(2-\varepsilon)^2 < x-5 < (\varepsilon+2)^2$

$(2-\varepsilon)^2+5 < x < (\varepsilon+2)^2+5$

So now solve for $|x-9| < \delta$

$-\delta < x-9 < \delta$

$-\delta+9 < x < \delta+9$

If $\delta = \varepsilon$ then

$-\delta+9 = (2-\varepsilon)^2+5$ or $\delta+9 = (2+\varepsilon)^2+5$

$-\delta+9 = 4-4\varepsilon+\varepsilon^2+5$ or $\delta+9 = 4+4\varepsilon+\varepsilon^2+5$

$-\delta+9 = \varepsilon^2-4\varepsilon+9$ or $\delta = 4\varepsilon+\varepsilon^2$

$-\delta = \varepsilon^2-4\varepsilon$ or $\delta = 4\varepsilon+\varepsilon^2$

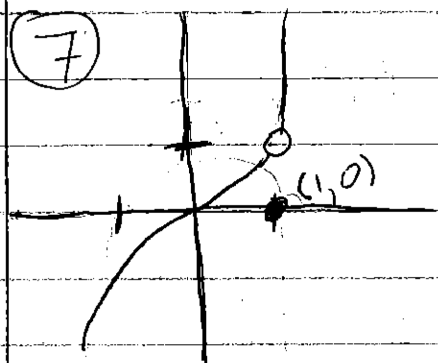
$\boxed{\delta = -\varepsilon^2+4\varepsilon}$

#1, 7, 11, 13, 21
25, 29

Homework 2.4

(1) a) T b) T c) F d) T e) T f) T
g) F h) F i) F J) F K) F l) F

(7)



b) $\lim_{x \to 1^-} x^3 = \boxed{1}$ $\lim_{x \to 1^+} x^3 = \boxed{1}$

c) Yes. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = L$

(11) $\lim_{x \to -0.5^-} \sqrt{\frac{x+2}{x+1}} = \sqrt{\frac{-.5+2}{-.5+1}} = \sqrt{\frac{1.5}{.5}} = \boxed{\sqrt{3}}$

(13) $\lim_{x \to -2^+} \left(\frac{x}{x+1}\right)\left(\frac{2x+5}{x^2+x}\right) = \left(\frac{-2}{-2+1}\right)\left(\frac{2(-2)+5}{-2^2-2}\right)$

$= (2)\left(\frac{1}{2}\right) = \boxed{1}$

(21) $\lim_{\theta \to 0} \frac{\sin\sqrt{2\theta}}{\sqrt{2\theta}}$ = Let $\sqrt{2\theta} = \theta$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \boxed{1}$

Let $t = 2x \rightarrow x = \frac{t}{2}$

(25) $\lim_{x\to 0} \frac{\tan 2x}{x} = \lim_{x\to 0} \frac{\frac{\sin 2x}{\cos 2x}}{x}$

$= \frac{\frac{\sin 2x}{1} \cdot \frac{1}{\cos 2x}}{x} = \frac{\sin 2\left(\frac{t}{2}\right) \cdot \frac{1}{\cos 2\left(\frac{t}{2}\right)}}{\frac{t}{2}}$

$\frac{\sin t \cdot \frac{1}{\cos t}}{t\left(\frac{1}{2}\right)} = \frac{\sin t \cdot \frac{1}{\cos t}}{t} \cdot \left(\frac{2}{1}\right)$

$\frac{\sin t}{t} \cdot \frac{\frac{1}{\cos t}}{1} \cdot 2 = 1 \cdot \frac{\frac{1}{\cos t}}{1} \cdot 2$

$= \frac{2}{\cos t} \quad \lim_{t\to 0} \frac{2}{\cos t} = \boxed{2}$

(29) $\lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x} = \frac{x + x(\cos x)}{\sin x \cos x}$

$= \frac{x(1+\cos x)}{\frac{1}{2}\sin 2x}$? or $\frac{x(1+\cos x)}{\sin x \cos x}$

$\rightarrow$ No

$= \frac{x}{\sin x} \cdot \frac{1+\cos x}{\cos x}$

$= \frac{x}{\sin x} \cdot 2$

$= 1 \cdot 2 = \boxed{2}$

(29) $$\lim_{x\to 0} \frac{x + x\cos x}{\sin x \cos x} = \frac{x(1+\cos x)}{\sin x \cos x}$$

$$\frac{x}{\sin x} \cdot \frac{1+\cos x}{\cos x} = \frac{1}{\frac{\sin x}{x}} \cdot \frac{1+\cos x}{\cos x}$$

$$\frac{\lim_{x\to 0} 1}{\lim_{x\to 0} \frac{\sin x}{x}} \cdot \frac{\lim_{x\to 0} 1 + \lim_{x\to 0} \cos x}{\lim_{x\to 0} \cos x} =$$

$$\frac{1}{1} \cdot \frac{1+1}{1} = 1 \cdot 2 = \boxed{2}$$

13, 15, 19, 21, 25
29, 53, 59

2.5 Homework

(13) $y = \frac{1}{x-2} - 3x$ $\quad x-2=0$
$x = 2$ = discont.

function continuous at all points except @ $x=2$ $\quad (-\infty,2)\cup(2,\infty)$

(15) $y = \frac{x+1}{x^2-4x+3} = \frac{x+1}{(x-3)(x-1)}$

$x-3=0 \qquad x-1=0$
$x=3 \qquad x=1$

function continuous on all points of x except at $x=3$ & $x=1$ $\quad (-\infty,1)\cup(1,3)\cup(3,\infty)$

(19) $y = \frac{\cos x}{x} \qquad x \neq 0$

function continuous on all x points except at $x=0$ $\quad (-\infty,0)\cup(0,\infty)$

(21) $y = \csc 2x = \frac{1}{\sin 2x} \qquad \sin 2x \neq 0$

$2x = \theta \qquad \sin\theta = 0$ @ $\pi, 2\pi, 3\pi \ldots K\pi$

$\theta = 2x = \pi, 2\pi, 3\pi \ldots K\pi$
$x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \ldots \frac{K\pi}{2}$

Cannot have anything negative under the radical so ≥ 0

? (25) $y = \sqrt{2x+3}$ $\quad 2x+3 \geq 0$

$2x \geq -3$

$x \geq -\frac{3}{2}$

function continuous on it's domain $x \geq -\frac{3}{2}$

? (29) $g(x) = \dfrac{x^2 - x - 6}{x - 3}$ $\quad x \neq 3$

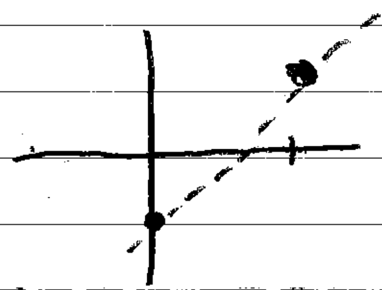
why find limit

$\lim_{x \to 3} \dfrac{x^2 - x - 6}{x - 3} = \dfrac{\cancel{(x-3)}(x+2)}{\cancel{x-3}} = \lim_{x \to 3} x+2$

$\lim_{x \to 3} = 5$ $\quad \lim_{x \to 3} g(x) \neq g(c)$

Function continuous on all points of x except $x = 3$ $\quad (-\infty, 3) \cup (3, \infty)$

(53)



Continuous so crosses $x = 0$ at some point

(59) Ex: $\quad \dfrac{x^2 - 4}{x - 2} = \dfrac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$

$x \neq 2$

Pg 115

#13, 15, 17, 21, 23, 25, 31, 37
39, 43, 53, 55, 63, 65, 67
81, 101

2.6 Homework

(13) $$\lim_{x\to\pm\infty} \frac{2x+3}{5x+7} = \frac{\frac{2x}{x}+\frac{3}{x}}{\frac{5x}{x}+\frac{7}{x}} = \frac{2+\frac{3}{x}}{5+\frac{7}{x}}$$

$$\lim_{x\to\pm\infty} \frac{2+\frac{3}{x}}{5+\frac{7}{x}} = \frac{2+0}{5+0} = \boxed{\frac{2}{5}}$$

(15) $$\lim_{x\to\pm\infty} \frac{x+1}{x^2+3} = \frac{\frac{x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{3}{x^2}} = \frac{\frac{1}{x}+\frac{1}{x^2}}{1+\frac{3}{x^2}}$$

$$\lim_{x\to\pm\infty} \frac{0+0}{1+0} = \boxed{0}$$

(17) $$\lim_{x\to\pm\infty} \frac{7x^3}{x^3-3x^2+6x} = \frac{\frac{7x^3}{x^3}}{\frac{x^3}{x^3}-\frac{3x^2}{x^3}+\frac{6x}{x^3}}$$

$$\lim_{x\to\pm\infty} \frac{7}{1-\frac{3}{x}+\frac{6}{x^2}} = \boxed{7}$$

(21) $$\lim_{x\to\pm\infty} \frac{3x^7+5x^2-1}{6x^3-7x+3} = \frac{3+\frac{5}{x^5}-\frac{1}{x^7}}{\frac{6}{x^4}-\frac{7}{x^6}+\frac{3}{x^7}}$$

$$\lim_{x\to\pm\infty} \frac{3+0-0}{0+0+0} = \frac{3}{\infty^{\pm}} = \boxed{\infty^{\pm}}$$

(23) $\lim_{x\to\infty} \sqrt{\dfrac{8x^2-3}{2x^2+x}} = \left(\dfrac{\frac{8x^2}{x^2} - \frac{3}{x^2}}{\frac{2x^2}{x^2} + \frac{x}{x^2}}\right)^{\frac{1}{2}}$

$= \left(\dfrac{8 - \frac{3}{x^2}}{2 + \frac{1}{x}}\right)^{1/2} = \lim_{x\to\infty} \sqrt{\dfrac{8 - 3/x^2}{2 + \frac{1}{x}}}$

$= \lim_{x\to\infty} \sqrt{\dfrac{8 - 0^+}{2 + 0^+}} = \lim_{x\to\infty} \dfrac{2\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}} = \boxed{2}$

(25) $\lim_{x\to -\infty} \left(\dfrac{1-x^3}{x^2+7x}\right)^5 = \lim_{x\to -\infty} \left(\dfrac{1-0^-}{0^- + 0^-}\right)^5$

$\dfrac{1+0^+}{0^- + 0^-} = \boxed{\infty}$

(31) $\lim_{x\to\infty} \dfrac{2x^{5/3} - x^{1/3} + 7}{x^{8/5} + 3x + x^{\frac{1}{2}}}$

$= \lim_{x\to\infty} \dfrac{0^+ - 0^+ + 0^+}{0^+ + 3 + 0^+} = \dfrac{0^+}{3} = \boxed{0}$

(37) $\lim_{x\to 0^+} \frac{1}{3x} = \frac{1}{3(.001)} = \frac{1}{3(.00001)} = \boxed{\infty}$

(39) $\lim_{x\to 2^-} \frac{3}{x-2} = \frac{3}{(1.99)-2} = \frac{3}{-.01} = \frac{3}{0^-} = \boxed{\infty}$

(43) $\lim_{x\to 7} \frac{4}{(x-7)^2}$

$\lim_{x\to 7^+} \frac{4}{(7.001-7)^2} \quad \frac{4}{0^+} = \boxed{\infty}$

$\lim_{x\to 7^-} \frac{4}{(6.99-7)^2} = \frac{4}{0^+} = \boxed{+\infty}$

$\lim_{x\to 7^-} = \lim_{x\to 7^+} \therefore \lim_{x\to 7} = \boxed{\infty}$

(53) $\lim \frac{1}{x^2-4}$ as

a) $x\to 2^+ = \frac{1}{0^+} = \boxed{\infty}$

b) $x\to 2^- = \frac{1}{0^-} = \boxed{-\infty}$

c) $x\to -2^+ = \frac{1}{0^-} \quad \boxed{-\infty}$

d) $x\to -2^- = \frac{1}{0^+} \quad \boxed{\infty}$

55 $\lim \left(\frac{x^2}{2} - \frac{1}{x}\right)$ as

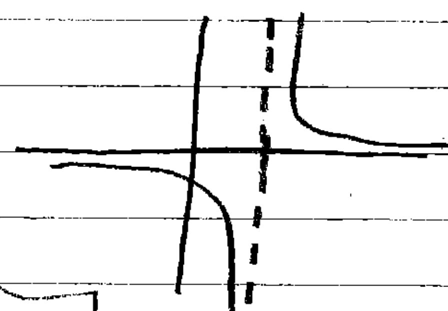
a) $x \to 0^+ = \frac{(.0001)^2}{2} - \frac{1}{.0001} = -\infty$

b) $x \to 0^- = \frac{0^-}{2} - \frac{1}{(-0^-)} = \boxed{\infty}$

c) $x \to 2^{\frac{1}{3}} = \frac{(2^{\frac{1}{3}})^2}{2} - \frac{1}{2^{\frac{1}{3}}} = \frac{2^{\frac{2}{3}}}{2} - \frac{1}{2^{1/3}} = \boxed{0}$

d) $x \to -1 = \frac{(-1)^2}{2} - \frac{1}{-1} = \frac{1}{2} + 1 = \boxed{\frac{3}{2}}$

63 $y = \frac{1}{x-1}$



V.A.: $x - 1 = 0$

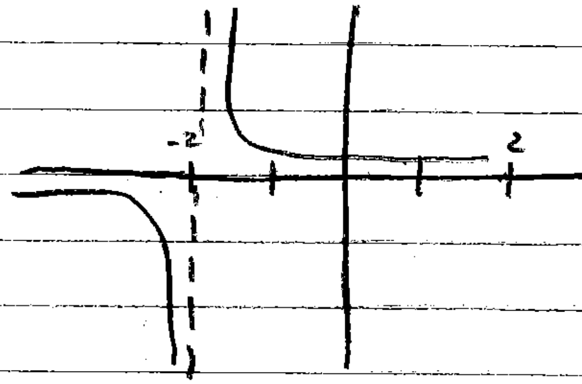
$\boxed{x = 1}$

H.A.: $\lim\limits_{x \to 1^+} \frac{1}{x-1} = \boxed{\infty}$

$(-\infty, 1) \cup (1, \infty)$

$\lim\limits_{x \to 1^-} \frac{1}{x-1} = \boxed{-\infty}$

65 $y = \frac{1}{2x+4}$



VA: $2x+4=0$

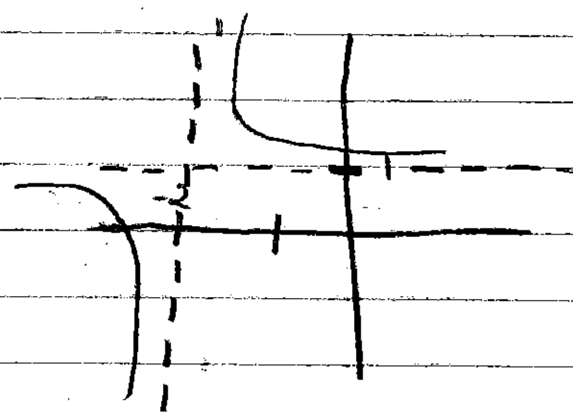
$2x=-4$

$x=-2$

$\lim\limits_{x \to -2^+} \frac{1}{0^+} = \infty$

$\lim\limits_{x \to 2^-} \frac{1}{0^-} = \boxed{-\infty}$

67 $y = \frac{x+3}{x+2}$



VA: $x+2=0$

$\boxed{x=-2}$

HA: $\lim\limits_{x \to -2} \frac{\overset{0}{x}+3}{\overset{0}{x}+2} = \boxed{y=1}$

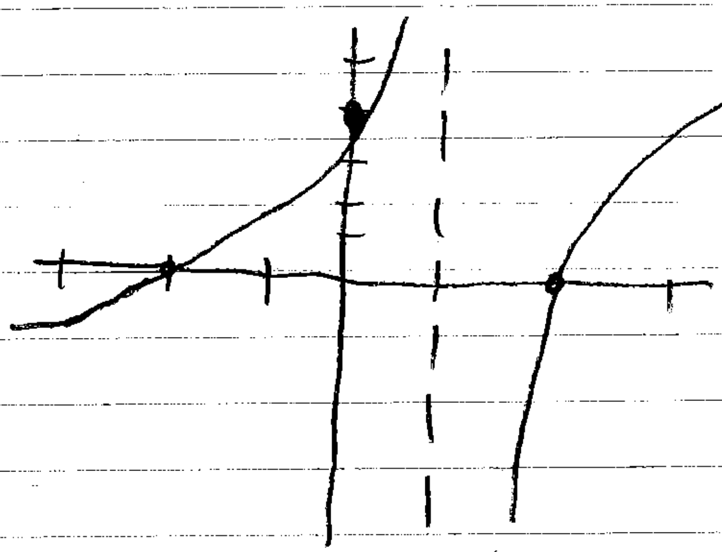
$\boxed{\text{O.A: none}}$

81) $\lim\limits_{x \to \infty} \left(\sqrt{x^2+25} - \sqrt{x^3-1}\right)$

$\left(\sqrt{0^2-25} - \sqrt{0^3-1}\right)$

$\lim\limits_{x \to \infty} \left(\sqrt{25} - \sqrt{1}\right)$

101) $y = \dfrac{x^2-4}{x-1}$



Asymptote @ $x = 1$

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Homework 3.1

$\frac{100}{103}$ 97%

Steven Romeiro
#5, 9, 11, 13, 19, 21, 27, 29, 31

(5) $y = 4 - x^2 \quad , (-1, 3)$

$$y' = \frac{f(x+h) - f(x)}{h}$$

$$y' = \frac{4 - (x+h)^2 - (4 - x^2)}{h}$$

$$y' = \frac{\cancel{4} - \cancel{x^2} + 2xh - h^2 - \cancel{4} + \cancel{x^2}}{h}$$

$$y' = \frac{-2xh - h^2}{h}$$

$$y' = -2x - h$$

$$y' = \lim_{h \to 0} -2x - 0$$

$$y' = -2(-1)$$

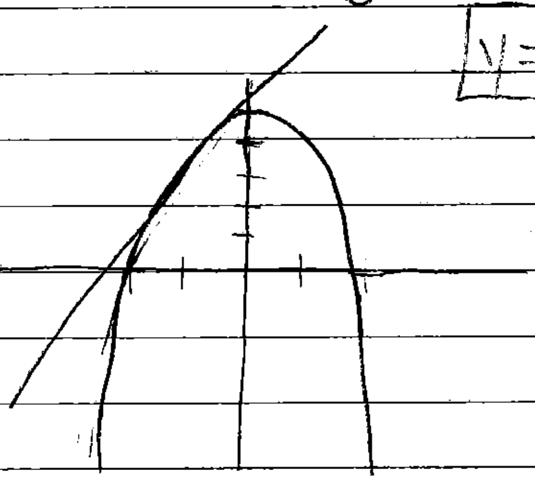
$$\boxed{y' = 2}$$

$$y = 2x + b$$

$$3 = 2(-1) + b$$

$$b = 5$$

$$\boxed{y = 2x + 5}$$



(9) $y = x^3 \quad (-2, -8)$

$$y' = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$y' = \lim_{h \to 0} 3x^2 + 3xh + h^2$$

$y' = 3x^2$ evaluated @ $x = -2$

$y' = 3(-2)^2$

$\boxed{y' = 12}$

$y = 12x + b$

$b = y - 12x$

$b = -8 - 12(-2)$

$b = 16$

$\boxed{y = 12x + 16}$

(11) $f(x) = x^2 + 1 \quad (2,5)$

$$y' = \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h}$$

$$y' = \frac{\cancel{x^2} + 2xh + h^2 \cancel{+1} - \cancel{x^2} \cancel{-1}}{h}$$

$$y' = \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h}$$

$$y' = \lim_{h \to 0} 2x + h$$

$y' = 2x$ evaluated @ $x = 2$

$$y' = 2(2)$$

$$\boxed{y' = 4}$$

$$y = 4x + b$$

$$b = y - 4x$$

$$b = 5 - 4(2)$$

$$b = -3$$

$$\boxed{y = 4x - 3}$$

(13) $g(x) = \dfrac{x}{x-2}$ $(3, 3)$

$$g'(x) = \lim_{h \to 0} \frac{\dfrac{x+h}{x+h-2} - \dfrac{x}{x-2}}{h}$$

$$g'(x) = \frac{\dfrac{(x-2)(x+h)}{(x-2)(x+h-2)} - \dfrac{x^2+xh-2x}{(x-2)(x+h-2)}}{h}$$

$$g'(x) \frac{\dfrac{x^2+xh-2x-2h-(x^2+xh-2x)}{(x-2)(x+h-2)}}{h}$$

$$g'(x) \frac{\cancel{x^2}+\cancel{xh}-\cancel{2x}-2h-\cancel{x^2}-\cancel{xh}+\cancel{2x}}{(x-2)(x+h-2)}$$

$$g'(x) = \frac{-2h}{(x-2)(x+h-2)} \cdot \frac{1}{h}$$

$$g'(x) = \frac{-2}{(x-2)(x+h-2)}$$

$$g'(x) \lim_{h \to 0} \frac{-2}{(x-2)(x+\cancel{h}^{0}-2)} = g'(x) = \frac{-2}{(x-2)^2}$$

@ $x = 3$ $g'(3) = \dfrac{-2}{(3-2)^2} = \boxed{g'(3) = -2}$ $\Rightarrow$

$y = -2x + b$

$b = y + 2x$

$b = 3 + 2(3)$

$b = 9$

$\boxed{y = -2x + 9}$

(19) $y = 5x - 3x^2 \qquad x = 1$

$$y' = \lim_{h \to 0} \frac{5(x+h) - 3(x+h)^2 - (5x - 3x^2)}{h}$$

$$y' = \frac{5x + 5h - 3(x^2 + 2xh + h^2) - 5x + 3x^2}{h}$$

$$y' = \frac{\cancel{5x} + 5h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{5x} + \cancel{3x^2}}{h}$$

$$y' = \frac{5h - 6xh - 3h^2}{h} = \frac{h(5 - 6x - 3h)}{h}$$

$$y' = \lim_{h \to 0} 5 - 6x - \cancel{3h}^{\,0} = \quad y' = 5 - 6x$$

@ $x = 1 \quad y' = 5 - 6(1)$

$\boxed{y' = -1}$

(21) $y = \dfrac{1}{x-1}$ $\quad \cancel{x} = 3$

$$y' = \lim_{h \to 0} \frac{\dfrac{1}{x+h-1} - \dfrac{1}{x-1}}{h}$$

$$y' = \frac{\dfrac{1(x-1)}{(x+h-1)(x-1)} - \dfrac{1(x+h-1)}{(x+h-1)(x-1)}}{h}$$

$$y' = \frac{\dfrac{\cancel{x} \cancel{-1} - \cancel{x} - h \cancel{+1}}{(x-1)(x+h-1)}}{h}$$

$$y' = \frac{-\cancel{h}}{(x-1)(x+h-1)} \cdot \frac{1}{\cancel{h}}$$

$$y' = \lim_{h \to 0} \frac{-1}{(x-1)(x+\cancel{h}^{0}-1)} = \frac{-1}{(x-1)^2}$$

@ $x = 3$ $\quad y' = \dfrac{-1}{(3-1)^2} = \boxed{-\dfrac{1}{4}}$

(27) Slope $= -1$ $\quad y = \frac{1}{x-1}$

$y' = \frac{-1}{(x-1)^2}$ from previous problem

$y' = \text{Slope} = \frac{-1}{(x-1)^2}$

$-1 = \frac{-1}{(x-1)^2} \rightarrow -1(x-1)^2 = -1$

$= (x-1)^2 = \frac{-1}{-1} \rightarrow (x-1)^2 = 1$

$x-1 = \sqrt{1}$ $\quad y = \frac{1}{2-1}$

$x-1 = 1$

$\boxed{x=2}$ $\quad \boxed{y=1}$

$y = -x + b$

$b = y + x$

$b = 1 + 2$

$\boxed{b = 3}$

$\boxed{y = -x + 3 \quad (2,1)}$

(29) $f(t) = 100 - 4.9t^2$

how fast is it falling after 2 seconds = Rate of change

$$f'(t) = \frac{100 - 4.9(t+h)^2 - 100 + 4.9t^2}{h}$$

$$f'(t) = \frac{\cancel{100} - \cancel{4.9t^2} + 9.8th + 4.9h^2 - \cancel{100} + \cancel{4.9t^2}}{h}$$

$$f'(t) = \frac{9.8th + 4.9h^2}{h} = \frac{h(9.8t + 4.9h)}{h}$$

$$f'(t) = \lim_{h \to 0} 9.8t + 4.9h = \boxed{f'(t) = 9.8t}$$

$$f'(2) = 9.8(2) = \boxed{f'(2) = 19.6 \text{ falling}}$$

(31) $A = \pi r^2$ @ $r = 2$

$$A'(r) = \frac{\pi(r+h)^2 - \pi r^2}{h}$$

$$A'(r) = \frac{\cancel{\pi r^2} + 2\pi r h + \pi h^2 - \cancel{\pi r^2}}{h}$$

$$A'(r) = \frac{2\pi r h + \pi h^2}{h}$$

$$A'(r) = \lim_{h \to 0} 2\pi r + \pi \cancel{h}^{\to 0}$$

$$A'(r) = 2\pi r$$

$$A'(2) = 2\pi(2)$$

$$\boxed{A'(2) = 4\pi}$$

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#1, 3, 7, 9, 13

3.2 Homework

1) $f(x) = 4 - x^2 \quad f'(-3),\ f'(0),\ f'(1)$

$$f'(x) = \lim_{h \to 0} \frac{4 - (x+h)^2 - 4 + x^2}{h}$$

$$f'(x) = \frac{\cancel{4} - \cancel{x^2} - 2xh - h^2 - \cancel{4} + \cancel{x^2}}{h}$$

$$f'(x) = \frac{-2xh - h^2}{h} = \frac{h(-2x - h)}{h}$$

$$f'(x) = \lim_{h \to 0} -2x - \cancel{h}^{\to 0}$$

$$\boxed{f'(x) = -2x}$$

$$\boxed{f'(-3) = 6}$$

$$\boxed{f'(0) = 0}$$

$$\boxed{f(1) = -2}$$

$t^{-2} = -2t^{-3}$

③ $g(t) = \frac{1}{t^2}$ $g'(-1), g'(2), g'(\sqrt{3})$

$$g'(t) = \frac{\frac{1}{(t+h)^2} - \frac{1}{t^2}}{h}$$

$$g'(t) = \frac{\frac{t^2}{t^2(t+h)^2} - \frac{(t+h)^2}{t^2(t+h)^2}}{h}$$

$$g'(t) = \frac{\frac{\cancel{t^2} - \cancel{t^2} - 2th - h^2}{t^2(t+h)^2}}{h}$$

$$g'(t) = \frac{-2th - h^2}{t^2(t+h)^2} \cdot \frac{1}{h}$$

$$g'(t) = \frac{-2\cancel{t} - h}{\cancel{t^2}(t+h)^2} = \frac{-2-h}{t(t+h)^2}$$

$$g'(t) = \lim_{h \to 0} \frac{-2-h}{t(t+h)^2} = \boxed{\frac{-2}{t^3}}$$

$\boxed{g'(-1) = 1}$ $\boxed{g'(2) = -\frac{1}{4}}$ $\boxed{g'(\sqrt{3}) = \frac{-2}{3}}$

7 $\frac{dy}{dx}$ if $y = 2x^3$

$$y' = \frac{2(x+h)^3 - 2x^3}{h}$$

$$y' = \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + 2h^3 - \cancel{2x^3}}{h}$$

$$y' = \frac{6x^2h + 6xh^2 + 2h^3}{h}$$

$$y' = 6x^2 + 6xh + 2h^2$$

$$y' = \lim_{h \to 0} 6x^2 + 6x\cancel{h}^{\to 0} + 2\cancel{h^2}^{\to 0}$$

$$\boxed{y' = 6x^2}$$

9) $\frac{dy}{dx}$ if $y = \frac{x}{2x+1}$

$$y' = \lim_{h \to 0} \frac{\frac{x+h}{2(x+h)+1} - \frac{x}{2x+1}}{h}$$

$$y' = \frac{\frac{(2x+1)(x+h) - x(2(x+h)+1)}{(2(x+h)+1)(2x+1)}}{h}$$

$$y' = \frac{\frac{\cancel{2x^2} + \cancel{2xh} + \cancel{x} + h - \cancel{2x^2} - \cancel{2xh} - \cancel{x}}{(2(x+h)+1)(2x+1)}}{h}$$

$$y' = \frac{h}{(2(x+h)+1)(2x+1)} \cdot \frac{1}{h}$$

$$y' = \frac{1}{(2(x+h)+1)(2x+1)}$$

$$y' = \lim_{h \to 0} \frac{1}{(2(x+\cancel{h}^{\to 0})+1)(2x+1)}$$

$$\boxed{y' = \frac{1}{(2x+1)^2}}$$

(13) $f(x) = x + \frac{9}{x} \qquad x = -3$

$$f'(x) = \lim_{h \to 0} \frac{x+h+\frac{9}{x+h} - x - \frac{9}{x}}{h}$$

$$f'(x) = \frac{\frac{\cancel{x^3}+\cancel{hx^2}+x^2h+h^2x+\cancel{9x^2}+\cancel{9xh}-\cancel{x^3}-\cancel{hx^2}-9x^2-9}{(x+h)(x)}}{h}$$

$$f'(x) = \frac{x^2h + h^2x}{(x+h)(x)} \cdot \frac{1}{h}$$

$$f'(x) = \frac{x^2+hx}{(x+h)(x)} = f'(x) = \lim_{h \to 0} \frac{x^2+\cancel{h}^{\,0}x}{(x+\cancel{h}^{\,0})(x)}$$

$$f'(x) = \frac{x^2}{x^2} \qquad \boxed{f'(x) = 1}$$

#1, 5, 7, 11, 13, 17, 23, 25
29, 33, 45, 51, 55, 57

3.3 Homework

(1) $y = -x^2 + 3$

$y' = -2x$

$\boxed{y'' = -2}$

(5) $y = \frac{4x^3}{3} - x + 2e^x$

$y' = 4x^2 - 1 + 2e^x$

$\boxed{y'' = 8x + 2e^x}$

(7) $w = 3z^{-2} - \frac{1}{z}$

$w' = -6z^{-3} + \frac{1}{z^2}$

$\boxed{w'' = 18z^{-4} - \frac{2}{z^3}}$

(11) $r = \frac{1}{3s^2} - \frac{5}{2s} = \frac{1}{3}s^{-2} - \frac{5}{2}s^{-1}$

$r' = -\frac{2}{3}s^{-3} + \frac{5}{2}s^{-2}$

$\boxed{r'' = 2s^{-4} - 5s^{-3}}$

(13) $y = (3-x^2)(x^3-x+1)$

$y' = (3-x^2)(3x^2-1) + (x^3-x+1)(-2x)$

$y' = 9x^2 - 3 - 3x^4 - 2x^4 + 2x^2 - 2x$

$\boxed{y' = -5x^4 + 11x^2 - 2x - 3}$ //

(17) $y = \dfrac{2x+5}{3x-2} =$

$y' = \dfrac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2}$

$y' = \dfrac{6x-4-6x-15}{(3x-2)^2}$

$y' = \dfrac{-19}{(3x+2)^2}$

(23) $f(x) = \dfrac{\sqrt{s}-1}{\sqrt{s}+1}$

$$f'(x) = \frac{(s^{\frac{1}{2}}+1)(\frac{1}{2}s^{-\frac{1}{2}}) + (s^{\frac{1}{2}}-1)(\frac{1}{2}s^{-\frac{1}{2}})}{(\sqrt{s}+1)^2}$$

$$f'(x) = \frac{s^0 + \frac{1}{2}s^{-\frac{1}{2}} + s^0 - \frac{1}{2}s^{-\frac{1}{2}}}{(\sqrt{s}+1)^2}$$

$$f'(x) = \frac{1}{(\sqrt{s}+1)^2}$$

(25) $V = \dfrac{1+x-4\sqrt{x}}{x}$

$$V' = \frac{(x)(1-2x^{-\frac{1}{2}}) + (1+x-4x^{\frac{1}{2}})(1)}{(x)^2}$$

$$V' = \frac{x - \frac{2x}{x^{\frac{1}{2}}} + 1 + x - 4x^{\frac{1}{2}}}{x^2}$$

$$V' = \frac{x^{3/2} - 2x + x^{\frac{1}{2}} + x^{\frac{3}{2}} - 4x}{x^{\frac{1}{2}}} \cdot \frac{1}{x^2}$$

$\hookrightarrow$

$$v' = \frac{x^{3/2} - 2x + x^{\frac{1}{2}} + x^{3/2} - 4x}{x^{\frac{1}{2}}} \cdot \frac{1}{x^2}$$

$$v' = \frac{x^{3/2} - 2x + x^{\frac{1}{2}} + x^{3/2} - 4x}{x}$$

$$v' = \frac{2x^{3/2} - 6x + x^{\frac{1}{2}}}{x}$$

(29) $y = 2e^{-x} + e^{3x}$

$$y' = \frac{2}{e^{x}} + 3e^{2x} \cdot e^{x}$$

$$y' = -2e^{-2x} \cdot e^{x} + 3e^{2x} \cdot e^{x}$$

$$y' = -2e^{-x} + 3e^{3x}$$

(33) $y = x^{9/4} + e^{-2x}$

$y = x^{9/4} + e^{-2x}$

$y' = \frac{9}{4}x^{\frac{5}{4}} + -2e^{-3x} \cdot e^{x}$

$y' = \frac{9}{4}x^{\frac{5}{4}} - 2e^{-2x}$

(45) $y = \frac{x^3+7}{x}$

$$y' = \frac{x(3x^2) - (x^3+7)(1)}{x^2}$$

$$y' = \frac{3x^3 - x^3 - 7}{x^2} = \frac{2x^3-7}{x^2}$$

$$y'' = \frac{(x^2)(6x^2) - (2x^3-7)(2x)}{x^4}$$

$$y'' = \frac{6x^4 - 4x^4 + 14x}{x^4}$$

$$y'' = \frac{x(2x^3+14)}{x^4}$$

$$\boxed{y'' = \frac{2x^3+14}{x^3}}$$

51) $w = 3z^2(e^{2z})$

$w' = 3z^2 \cdot (2e^{2z}) + 6z \cdot (e^{2z})$

$w' = 3z^2 2e^{2z} + 6ze^{2e}$

55) $y = x^3 - 4x + 1$ $\quad (2, 1)$

$y' = 3x^2 - 4$

$y'(2) = 3(2)^2 - 4$

$y'(2) = 12 - 4$

$y'(2) = 8$

$y = 8x + b$

$b = y - 8x$

$b = 1 - 8(2)$

$b = -15$

$y = 8x - 15$

equation of line perpendicular to $y = 8x - 15$ will have a slope that is its negative reciprocal

$m = -\frac{1}{m} \therefore \quad 8 = \frac{-1}{8}$

perpendicular equation = $\boxed{y = -\frac{1}{8}x - 15}$

or $y - 1 = -\frac{1}{8}(x - 2)$

$y - 1 = -\frac{1}{8}x + \frac{1}{4}$

$\boxed{y = -\frac{1}{8}x + \frac{5}{4}}$

57) $y = \dfrac{4x}{x^2+1}$ $(1, 2)$

$$y' = \frac{(x^2+1)(4) - (4x)(2x)}{(x^2+1)^2}$$

$$y' = \frac{4x^2+4-8x^2}{(x^2+1)^2} = \frac{-4x^2+4}{(x^2+1)^2}$$

$$y' = \frac{-4(x^2-1)}{(x^2+1)^2} = \frac{-4(x+1)(x-1)}{(x^2+1)^2}$$

$$y' = \frac{-4(x-1)}{x^2+1}$$

$$y'(1) = \frac{-4(1-1)}{1^2+1} = \frac{0}{1} = 0$$

$y = (0)x + b$

$b = y$

$b = 2$

$y = (0)x + 2$

$\boxed{y = 2}$

pg 153 #1, 3, 7, 9, 23, 25

3.4 Homework

(1) $S = t^2 - 3t + 2 \qquad 0 \le t \le 2$

a) $S' = 2t - 3 \qquad S(0) = 2,\ S(2) = 0$

$\boxed{S'(2) = 2(2) - 3 = 1}$

b) $S'' = 2$

(3) $S = -t^3 + 3t^2 - 3t \qquad 0 \le t \le 3$

a) $S(0) = 0,\ S(3) = -9,$

$S' = -3t^2 + 6t - 3,\ S'(3) = -27 + 18 - 3 = -17$

b) $S'' = -6t + 6,\ S''(0) = 6,\ S''(3) = -12$

(7) $S = t^3 - 6t^2 + 9t$

? a) $S' = 3t^2 - 12t + 9,\ S'(1) = 3(1) - 12(1) + 9 = 0\ \frac{m}{sec}$

$S'' = 3t - 12 \qquad S''(1) = 3(1) - 12 = -9\ \frac{m}{sec^2}$

$3t - 12 = 0$

$t = 4 \qquad S'(4) = 3(16) - 12(4) + 9 = 9\ \frac{m}{sec}$

c) $S(1) - S(0)$ & $S(2) - S(1)$

(9) $S = 1.86t^2$ mars & $S = 11.44t^2$ Jupiter

rock from Velocity $0 \rightarrow 27.8\ \frac{m}{sec}$

mars $= S' = 3.72t$ Jupiter $= S' = 22.88t$

$27.8 = 3.72t$ $27.8 = 22.88t$

$\boxed{t = 7.47 \text{ sec}}$ $\boxed{t = 1.22 \text{ sec}}$

(23) $C(x) = 2000 + 100x - 0.1x^2$

a) Average cost $= \dfrac{C(x)}{x}$ for 100

$$= \frac{2000 + 100(100) - 0.1(100)^2}{100}$$

$$= \frac{11000}{100} = \boxed{\$110}$$

b) Marginal Cost

$C'(x) = 100 - 0.2x$

$C'(100) = 100 - 20 = \boxed{\$80}$

c) $C'(99) = 100 - 19.8 = \boxed{\$80.2}$

$C(100) = 2000 + 100(100) - 0.1(100)^2$

$C(100) = \boxed{\$11000}$

$C(101) = 11679.9$

$$\$11079.9 - \$11,000 = \boxed{\$79.9}$$

25) $b = 10^6 + 10^4 t - 10^3 t^2$

$b' = 10^4 - 20^3 t$

a) $b'(0) = 10^4 - 20^3(0) = \boxed{10^4 \text{ bac}}$

b) $b'(5) = 10^4 - 20^3(5) = \boxed{-30^4 \text{ bac}}$

c) $b'(10) = 10^4 - 20^3(10) = \boxed{-70^4 \text{ bac}}$

1, 3, 7, 11, 16, 19, 23, 27, 31
35, 37, 47, 49

3.5 Homework

(1) $y = -10x + 3\cos x$

$\boxed{y' = -10 - 3\sin x}$

(3) $y = x^2 \cdot \cos x$

$y' = x^2(-\sin x) + 2x(\cos x)$

$\boxed{y' = -x^2 \sin x + 2x \cos x}$

(7) $f(x) = \sin x \tan x$

$f'(x) = \cos x(\tan x) + (\sin x)(\sec^2 x)$

$f'(x) = \sin x + \sin x \sec^2 x$

(11) $y = \dfrac{\cot x}{1 + \cot x}$

$$y' = \frac{(1 + \cot x)(-\csc^2 x) - (-\csc^2 x)(\cot x)}{(1 + \cot x)^2}$$

$$y' = \frac{-\csc^2 x - \csc^2 x \cot x + \csc^2 x \cot x}{(1 + \cot x)^2}$$

$$\boxed{y' = \frac{-\csc^2 x}{(1 + \cot x)^2}}$$

(16) $y = x^3\cos x - 2x\sin x - 2\cos x$

$y' = 3x^2\cos x - x^3\sin x - \cancel{2\sin x} + 2x\cos x + \cancel{2\sin x}$

$$\boxed{y' = 3x^2\cos x - x^3\sin x + 2x\cos x}$$

(19) $S = \tan t - e^{-t}$

$$\boxed{\frac{ds}{dt} = \sec^2 t + e^{-t}}$$

(23) $r = 4 - \theta^2\sin\theta$

$$\frac{dr}{d\theta} = -\left(2\theta\sin\theta + \theta^2\cos\theta\right)$$

$$\frac{dr}{d\theta} = -2\theta\sin\theta - \theta^2\cos\theta$$

$$\frac{dr}{d\theta} = -\theta\left(2\sin\theta + \theta\cos\theta\right)$$

(27) $p = 5 + \frac{1}{\cot q}$

$$\frac{dP}{dq} = \frac{(\cot q)(0) - (1)(-\csc^2 q)}{\cot^2 q}$$

$$\frac{dP}{dq} = \frac{\csc^2 q}{\cot^2 q} = \frac{\frac{1}{\sin^2 q}}{\frac{\cos^2 q}{\sin^2 q}} = \frac{1}{\cancel{\sin^2 q}} \cdot \frac{\cancel{\sin^2 q}}{\cos^2 q}$$

$$\boxed{\frac{dP}{dq} = \sec^2 q}$$

(31) $p = \frac{q \sin q}{q^2 - 1} = \frac{q \sin q}{(q-1)^2}$

$$p' = \frac{\cancel{(q-1)^2} (\sin q + q \cos q) - (2)(q-1)(1) + q \sin q}{\cancel{(q-1)}^4}$$

$$p' = \frac{\sin q + q \cos q - (2q - 2 + q \sin q)}{(q-1)^2}$$

35 $y = \sin x \qquad -3\pi/2 \leq v \leq 2\pi$

$x = -\pi, 0, 3\pi/2$

(37) $y = \sec x \qquad -\pi/2 \leq x \leq \pi/2$

$x = -\pi/3, \frac{\pi}{4}$

$$\frac{dy}{dx} = \sec x \tan x = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \frac{\sin x}{\cos^2 x}$$

$x = -\frac{\pi}{3}$

$$y = \sec\left(\frac{-\pi}{3}\right) = \frac{1}{\cos\left(\frac{-\pi}{3}\right)} = \frac{1}{\frac{1}{2}} = \boxed{2} \qquad \left(-\frac{\pi}{3}, 2\right)$$

$$y' = \frac{\sin x}{\cos^2 x} = \frac{\sin -\frac{\pi}{3}}{\cos^2\left(-\frac{\pi}{3}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\left(\frac{1}{2}\right)^2} = -\frac{4\sqrt{3}}{2}$$

$$= \boxed{-2\sqrt{3}}$$

$x = \frac{\pi}{4}$

$$y = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$$

(47) $\lim_{x \to 2} \sin\left(\frac{1}{x} - \frac{1}{2}\right)$

$f(x) = \sin\left(\frac{1}{x} - \frac{1}{2}\right)$ @ $x = 2$

$f(x) = \sin\frac{1}{x} \cdot \cos\frac{1}{2} - \cos\frac{1}{x} \sin\frac{1}{2}$

$$f'(x) = \left[\sin\frac{1}{x}\left(-\sin\frac{1}{2}\right) + \cos\frac{1}{x}\left(\cos\frac{1}{2}\right)\right] - \left[\cos\frac{1}{x}\left(\cos\frac{1}{2}\right) - \sin\frac{1}{x}\sin\frac{1}{2}\right]$$

$$f'(x) = -\sin\frac{1}{x}\sin\frac{1}{2} + \cancel{\cos\frac{1}{x}\cos\frac{1}{2}} - \cancel{\cos\frac{1}{x}\cos\frac{1}{2}} + \sin\frac{1}{x}\sin\frac{1}{2}$$

$$f'(x) = \cancel{-\sin\frac{1}{x}\sin\frac{1}{2}} + \cancel{\sin\frac{1}{x}\sin\frac{1}{2}}$$

$$\boxed{f'(x) = 0}$$

(49) $\lim_{\theta \to \frac{\pi}{6}^-} \dfrac{\sin .52 - \frac{1}{2}}{.52 - \frac{\pi}{6}} = .86692$

$\lim_{\theta \to \frac{\pi}{6}^+} \dfrac{\sin .53 - \frac{1}{2}}{.53 - \frac{\pi}{6}} = .8644$

$\boxed{\dfrac{\sqrt{3}}{2}}$

#9, 13, 15, 21, 23, 27, 29
35, 43, 51, 61, 71, 73

3.6 Homework

(1) $y = 6u - 9 \qquad u = \frac{1}{2}x^4$

$\frac{dy}{du} = 6 \qquad \frac{du}{dx} = 2x^3$

$\frac{dy}{dx} = 6 \cdot 2x^3 = \boxed{12x^3}$

(9) $y = (2x + 1)^5$

$y' = 5(2x+1)^4 \cdot 2$

$y' = 10(2x+1)^4$

(13) $y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4$

$y' = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3 \cdot \left(\frac{1}{4}x + 1 + \frac{1}{x^2}\right)$

$y' = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3 \cdot \left(x + 4 + \frac{4}{x^2}\right)$

(15) $y = (\sec x + \tan x)(\sec x - \tan x)$

$$y' = (\sec x + \tan x)(\tan x \sec x - \sec^2 x) + (\sec x - \tan x)(\tan x \sec x + \sec^2 x)$$

$$y' = \sec^2 x \tan x - \cancel{\sec^3 x} + \cancel{\tan^2 x \sec x} - \sec^2 x \tan x + \sec^2 x \tan x + \cancel{\sec^3 x} - \cancel{\tan^2 x \sec x} - \sec^2 x \tan x$$

$$y' = 2 \sec^2 x \tan x - 2 \sec^2 x \tan x$$

$$\boxed{y' = 0}$$

(21) $s = \dfrac{1 + \csc t}{1 - \csc t}$

$$s' = \frac{(1 - \csc t)(-\cot t \csc t) - (1 - \csc t)(\cot t \csc t)}{(1 - \csc t)^2}$$

$$s' = \frac{-\cot \csc t + \cot t \csc^2 t - \cot t \csc t + \cot t \csc^2 t}{(1 - \csc t)^2}$$

$$s' = \frac{-2 \cot t \csc t + 2 \cot t \csc^2 t}{(1 - \csc t)^2}$$

23 $p = (3-t)^{\frac{1}{2}}$

$$p' = \frac{1}{2}(3-t)^{-\frac{1}{2}}(-1)$$

$$\boxed{p' = -\frac{1}{2(3-t)^{\frac{1}{2}}}}$$

27 $r = (\csc\theta + \cot\theta)^{-1}$

?

$$r' = -1(\csc\theta + \cot\theta)^{-2} \cdot (-\cot\csc\theta - \csc^2\theta)$$

$$r' = \frac{\cot\theta\csc\theta + \csc^2\theta}{(\csc\theta + \cot\theta)^2}$$

$$r' = \frac{\frac{\cos\theta}{\sin^2\theta} + \frac{1}{\sin^2\theta}}{(\csc\theta + \cot\theta)^2} = \frac{\frac{\cos\theta + 1}{\sin^2\theta}}{(\csc\theta + \cot\theta)^2}$$

(29) $y = \underbrace{x^2 \sin^4 x}_{\text{Product Rule}} + x\cos^{-2} x$

$y' = (2x)(\sin^4 x) + (x^2)(4\sin^3 x)(\cos x)$
$+ (1)(\cos^{-2} x) + (x)(-2\cos^{-3} x)(-\sin x)$

$$\boxed{y' = 2x\sin^4 x + 4x^2\sin^3 x\cos x + \cos^{-2} x + 2x\cos^{-3} x\sin x}$$

(35) $y = xe^{-x} + e^{x^3}$

$y' = (1)(e^{-x}) + x(-e^{-x}) + e^{x^3} \cdot \ln e \cdot 3x^2$

$y' = e^{-x} - xe^{-x} + 3x^2 e^{x^3}$

$$\boxed{y' = e^{-x}(1-x) + 3x^2 e^{x^3}}$$

43 $f(\theta) = \left(\dfrac{\sin\theta}{1+\cos\theta}\right)^2$

$$f'(\theta) = 2\left(\frac{\sin\theta}{1+\cos\theta}\right)\cdot\left(\frac{(1+\cos\theta)(\cos\theta)-(-\sin\theta)(\sin\theta)}{(1+\cos\theta)^2}\right)$$

$\rightarrow \cos\theta$

$$f'(\theta) = 2\left(\frac{\sin\theta}{1+\cos\theta}\right)\left(\frac{1+\cos^2\theta+\sin^2\theta}{(1+\cos\theta)^2}\right)$$

$$f'(\theta) = 2\left(\frac{\sin\theta}{1+\cos\theta}\right)\left(\frac{1+1}{(1+\cos\theta)^2}\right)$$

$$f'(\theta) = 2\left(\frac{2\sin\theta}{(1+\cos\theta)^3}\right)$$

$$f'(\theta) = 4\left(\frac{\sin\theta}{(1+\cos\theta)^3}\right)$$

(51) $y = \sin^2(\pi t - 2)$

$y' = 2\sin(\pi t-2)\cdot\cos(\pi t-2)\cdot(\pi)$

$\boxed{y' = 2\pi\sin(\pi t-2)\cos(\pi t-2)}$

(61) $y = \sin\left(\cos(2t-5)\right)$

$y' = \cos\left(\cos(2t-5)\right)\cdot\left(-\sin(2t-5)\right)(2)$

$\boxed{y' = -2\cos\left(\cos(2t-5)\right)\sin(2t-5)}$

(71) $y = \left(1+\frac{1}{x}\right)^3$

$y' = 3\left(1+\frac{1}{x}\right)^2\left(-\frac{1}{x^2}\right) = \left(1+\frac{1}{x}\right)^2\left(\frac{-3}{x^2}\right)$

? $y'' = 2\left(1+\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\left(\frac{-3}{x^2}\right) + \left(1+\frac{1}{x}\right)^2\left(\frac{6}{x^3}\right)$

$y'' = 2\left(\frac{3}{x^4}+\frac{3}{x^5}\right) +$

73 $$y = \frac{1}{9}\cot(3x-1)$$

$$y' = \frac{1}{9} - \csc^2(3x-1)(3)$$

$$y' = -\frac{3}{9}\csc^2(3x-1)$$

$$y' = -\frac{1}{3}\csc^2(3x-1)$$

$$y'' = -\frac{1}{3}(2)(\csc(3x-1))(-\cot(3x-1)\csc(3x-1))(3)$$

$$y'' = -\frac{6}{3}\left(\csc(3x-1)\right)\left(-\cot(3x-1)\csc(3x-1)\right)$$

$$y'' = 2\csc(3x-1)\cot(3x-1)\csc(3x-1)$$

$$\boxed{y'' = 2\csc^2(3x-1)\cot(3x-1)}$$

#1, 7, 11, 15, 21, 23, 31, 33, 37

3.7 Homework

(1) $x^2y + xy^2 = 6$

$$x^2\left(\frac{dy}{dx}\right) + 2x(y) + x\left(2y\frac{dy}{dx}\right) + y^2 = 0$$

$$x^2\frac{dy}{dx} + x2y\frac{dy}{dx} = -2xy - y^2$$

$$\frac{dy}{dx}\left(x^2 + 2xy\right) = -2xy - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}}$$

(7) $y^2 = \dfrac{x-1}{x+1}$

$$2y\frac{dy}{dx} = \frac{(x+1)(1) - (1)(x-1)}{(x+1)^2}$$

$$2y\frac{dy}{dx} = \frac{x+1 - x+1}{(x+1)^2}$$

$$2y\frac{dy}{dx} = \frac{2}{(x+1)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{y(x+1)^2}}$$

(11) $x + \tan(xy) = 0$

$$1 + \sec^2(xy) \cdot \left[(x)\frac{dy}{dx} + y\right] = 0$$

$$\sec^2(xy) \cdot \left(x\frac{dy}{dx}\right) = -1 - y\sec^2(xy)$$

$$\frac{dy}{dx}\left(x\sec^2(xy)\right) = -1 - y$$

? $$\boxed{\frac{dy}{dx} = \frac{-1-y}{x\sec^2(xy)}}$$

(15) $e^{2x} = \sin(x+3y)$

? $\rightarrow$ $$2e^{2x} = \cos(x+3y)\left[(1) + 3\frac{dy}{dx}\right] \leftarrow ?$$

$$-3\frac{dy}{dx} = \cos(x+3y) - 2e^{2x}$$

$$\frac{dy}{dx} = \frac{-\cos(x+3y) + 2e^{2x}}{3}$$

$$\boxed{\frac{dy}{dx} = \frac{2e^{2x} - \cos(x+3y)}{3}}$$?

(21) $x^2 + y^2 = 1$

$$2x + 2y\frac{dy}{dx} \rightarrow 2y\frac{dy}{dx} = -2x$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$$\frac{d^2y}{dx^2} = \frac{(y)(-1) - (-x)(1 \cdot y')}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y - (-xy')}{y^2} = \frac{-y - \left(-x \cdot \left(\frac{-x}{y}\right)\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y - \left(\frac{x^2}{y}\right)}{y^2} = \frac{\frac{-y^2}{y} - \frac{x^2}{y}}{y^2} = \frac{\frac{-y - x^2}{y}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{-y - x^2}{y} \cdot \frac{1}{y^2} = \boxed{\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y^3}}$$

23 $y^2 = e^{x^2} + 2x$

$$2yy' = e^{x^2} \cdot \ln e \cdot 2x + 2$$

$$2yy' = 2xe^{x^2} + 2$$

$$y' = \frac{2xe^{x^2}+2}{2y} = \boxed{y' = \frac{xe^{x^2}+1}{y}}$$

$$y'' = \frac{(y)\left(e^{x^2} + x(2xe^{x^2})\right) - (y')(xe^{x^2}+1)}{y^2}$$

missing +1

$$y'' = \frac{(y)(e^{x^2}+2x^2e^{x^2}) - \left(\frac{xe^{x^2}+1}{y}\right)(xe^{x^2}+1)}{y^2}$$

$$y'' = \frac{(y)(e^{x^2}+2x^2e^{x^2}) - \frac{(xe^{x^2}+1)^2}{y}}{y^2}$$

$$y'' = \frac{\frac{y^2(e^{x^2}+2x^2e^{x^2}) - (xe^{x^2}+1)^2}{y}}{y^2}$$

$$y'' = \frac{y^2(e^{x^2}+2x^2e^{x^2}) - (xe^{x^2}+1)^2}{y^3}$$

$$y'' = \frac{e^{x^2}(1+2x^2) - (xe^{x^2}+1)^2}{y}$$

31) $x^2 + xy - y^2 = 1$ $\quad (2,3)$

$(2)^2 + (2)(3) - (3)^2 = 1 \rightarrow \boxed{1 = 1}$ ✓

$2x + y + xy' - 2yy' = 0$

$xy' - 2yy' = -2x - y$

$y'(x - 2y) = -2x - y$

$y' = \dfrac{-2x - y}{x - 2y} = \dfrac{-2(2) - 3}{2 - 2(3)} = \boxed{y' = \dfrac{7}{4}}$

$$\boxed{y - 3 = \frac{7}{4}(x - 2)}$$

$y'_{normal} = -\dfrac{4}{7}$

$$\boxed{y - 3 = -\frac{4}{7}(x - 2)}$$

(35) $6x^2+3xy+2y^2+17y-6=0$ $(-1,0)$

$6(-1)^2+3(-1)(0)+2(0)^2+17(0)-6=0$ $\boxed{0=0}$ ✓

$12x+3y+3xy'+4yy'+17y'=0$

$3xy'+4yy'+17y'=-12x-3y$

$y'(3x+4y+17)=-12x-3y$

$y'=\dfrac{-12x-3y}{3x+4y+17}=\dfrac{-12(-1)-3(0)}{3(-1)+4(0)+17}=\boxed{y'=\dfrac{6}{7}}$

$\boxed{y-0=\dfrac{6}{7}(x+1)}$ //

$y'_\perp=-\dfrac{7}{6}$ $\therefore$ $\boxed{y-0=-\dfrac{7}{6}(x+1)}$ //

(37) $2xy + \pi \sin y = 2\pi \quad (1, \frac{\pi}{2})$

$2(1)(\frac{\pi}{2}) + \pi \sin(\frac{\pi}{2}) = 2\pi \quad \boxed{2\pi = 2\pi}$ ✓

$2y + 2xy' + \pi \cos y \cdot y' = 0$

$2xy' + \pi \cos y \cdot y' = -2y$

$y'(2x + \pi \cos y) = -2y$

$y' = \dfrac{-2y}{2x + \pi \cos y} = \dfrac{-2(\frac{\pi}{2})}{2(1) + \pi \cos \frac{\pi}{2}} = \boxed{y' = -\dfrac{\pi}{2}}$

$\boxed{y - \dfrac{\pi}{2} = -\dfrac{\pi}{2}(x-1)}$

$y'_\perp = \dfrac{2}{\pi} \quad \therefore \quad \boxed{y - \dfrac{\pi}{2} = \dfrac{2}{\pi}(x-1)}$

11, 17, 21, 27, 37, 41, 45, 49
67, 71, 83, 89, 93

3.8 Homework

(11) $y = \ln 3x + x$ $\qquad \ln u = \frac{1}{u} \cdot \frac{du}{dx}$

$y' = \frac{1}{3x} \cdot 3 + 1$

$\boxed{y' = \frac{1}{x} + 1}$

(17) $y = \ln(\theta+1) - e^{\theta}$ $\qquad \ln u = \frac{1}{u} \cdot \frac{du}{dx}$

$\boxed{y' = \frac{1}{\theta+1} - e^{\theta}}$

(21) $y = x(\ln x)^2$

$y' = (\ln x)^2 + x\left(2\ln x \cdot \frac{1}{x}\right)$

$y' = 2\ln x + 2x \ln x$

?

$\boxed{y' = \ln x(\cancel{2} + 2x)}$

(37) $y = \ln(\sec(\ln\theta))$

$y' = \frac{1}{\sec(\ln\theta)} \cdot (\tan(\ln\theta)\sec(\ln\theta)) \cdot \frac{1}{\theta}$

$y' = \frac{(\tan(\ln\theta)\sec(\ln\theta))}{\sec(\ln\theta)\ \theta}$ $\qquad$ Let $\ln\theta = x$

$y' = \frac{\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}}{\frac{1}{\cos x} \cdot \theta} = \frac{\sin x}{\cos^2 x} \cdot \frac{\cos x}{\theta}$

$a^u \cdot \ln a \cdot \frac{du}{dx}$

$\frac{\sin X}{\cos^2 X} \cdot \frac{\cos X}{\theta} = \boxed{y' = \frac{\tan(\ln\theta)}{\theta}}$

(41) $y = \sqrt{X(x+1)} \longrightarrow y = (X(X+1))^{\frac{1}{2}}$

$y' = \frac{1}{2}(X(X+1))^{-\frac{1}{2}} \cdot [1 \cdot (X+1) + X \cdot (1+0)]$

$y' = \frac{1}{2}(X(X+1))^{-\frac{1}{2}} \cdot (X+1+X)$

$\boxed{y' = \frac{2x+1}{2\sqrt{x(x+1)}}}$

(45) $y = (\sin\theta)\sqrt{\theta+3}$

$y = \sin\theta\,(\theta+3)^{\frac{1}{2}}$

$y' = \cos\theta\,(\theta+3)^{\frac{1}{2}} + \sin\theta\left[\frac{1}{2}(\theta+3)^{-\frac{1}{2}} \cdot 1\right]$

$y' = \cos\theta\,(\theta+3)^{\frac{1}{2}} + \frac{1}{2}\sin\theta\,(\theta+3)^{-\frac{1}{2}}$

?

$y' = \cos\theta\,(\theta+3)^{\frac{1}{2}} + \frac{\sin\theta}{2(\theta+3)^{\frac{1}{2}}}$

$\boxed{y' = \frac{\cos\theta\,(\theta+3) + \sin\theta}{\sqrt{\theta+3}}}$

OR

$\ln y = \ln(\sin\theta) + \frac{1}{2}\ln(\theta+3)$

$= \frac{1}{y} \cdot y' = \frac{1}{\sin\theta} \cdot \cos\theta + \frac{1}{2(\theta+3)}$

$\boxed{y' = \sin\theta\,(\theta+3)^{\frac{1}{2}}\left(\cot\theta + \frac{1}{2(\theta+3)}\right)}$

(49) $y = \dfrac{\theta + 5}{\theta \cos\theta}$

$$\ln y = \ln \frac{\theta + 5}{\theta\cos\theta}$$

$$\ln y = \ln(\theta+5) - \ln(\theta\cos\theta)$$

$$\ln y = \ln(\theta+5) - [\ln\theta + \ln\cos\theta]$$

$$\ln y = \ln(\theta+5) - \ln\theta - \ln\cos\theta$$

$$\frac{1}{y}\cdot y' = \frac{1}{\theta+5} - \frac{1}{\theta} - \frac{1}{\cos\theta}(-\sin\theta)$$

$$y' = y\left(\frac{1}{\theta+5} - \frac{1}{\theta} - \frac{1}{\cos\theta}(\sin\theta)\right)$$

$$\boxed{y' = \frac{\theta+5}{\theta\cos\theta}\left(\frac{1}{\theta+5} - \frac{1}{\theta} + \tan\theta\right)}$$

67) $y = 2^x$ $\qquad a^u \cdot \ln a \cdot \frac{du}{dx}$

$\frac{dy}{dx} = 2^x \ln 2 \, (1)$

71) $y = x^{\pi}$

$\frac{dy}{dx} = \pi x^{\pi - 1}$

83) $y = \log_5 e^x \longrightarrow y = \frac{\ln e^x}{\ln 5}$

$y' = \frac{1}{\ln 5} \cdot \frac{1}{e^x} \cdot e^x$

$\boxed{y' = \frac{1}{\ln 5}}$

89) $y = (x+1)^x$ $\qquad \ln u = \frac{1}{u} \cdot \frac{du}{dx}$

$\ln y = \ln (x+1)^x$

$\ln y = x \ln(x+1)$

$\frac{1}{y} \cdot y' = 1 \cdot \ln(x+1) + x\left(\frac{1}{x+1} \cdot 1\right)$

$\frac{1}{y} \cdot y' = \ln(x+1) + \frac{x}{x+1}$

$\boxed{y' = (x+1)^x \left(\ln(x+1) + \frac{x}{x+1}\right)}$

93) $y = (\sin x)^x$

$\ln y = x \ln(\sin x)$

$\frac{1}{y} \cdot y' = \ln \sin x + x\left(\frac{1}{\sin x} \cdot \cos x\right)$

$\frac{1}{y} y' = \ln(\sin x) + x \cot x$

$$\boxed{y' = (\sin x)^x \left[\ln(\sin x) + x \cot x\right]}$$

#21, 23, 25, 27, 35
39, 49

3.9 Homework

21) $y = \cos^{-1}(x^2)$ $\qquad U = x^2$

$D_x \cos^{-1}(U) = -\frac{1}{\sqrt{1-U^2}} \cdot \frac{dU}{dx}$

$y' = -\frac{1}{\sqrt{1-x^4}} \cdot 2x = \boxed{y' = \frac{-2x}{\sqrt{1-x^4}}}$

23) $y = \sin^{-1}(\sqrt{2t})$ $\qquad U = \sqrt{2t}$

$D_x \sin^{-1}(U) = \frac{1}{\sqrt{1-U^2}} \cdot \frac{dU}{dx}$

$y' = \frac{1}{\sqrt{1-2t}} \cdot \frac{1}{2} \cdot (2t)^{-\frac{1}{2}} \cdot 2 = \frac{1}{\sqrt{1-2t}} \cdot \frac{1}{2} \cdot \frac{2}{\sqrt{2t}}$

$y' = \frac{1}{\sqrt{1-2t}} \cdot \frac{1}{\sqrt{2t}}$ or $y' = \frac{\frac{1}{\sqrt{2t}}}{\sqrt{1-2t}}$?

? $y' = \frac{1}{\sqrt{2t} \cdot \sqrt{1-2t}}$ OR $\boxed{y' = \frac{\sqrt{2t}}{\sqrt{1-2t}}}$?

$$D_x \sec^{-1}(u) = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

25 $y = \sec^{-1}(2s+1)$ $\qquad u = (2s+1)$

$$y' = \frac{1}{(2s+1)\sqrt{(2s+1)^2-1}} \cdot 2$$

$$\boxed{y' = \frac{2}{(2s+1)\sqrt{(2s+1)^2-1}}}$$ Simplify

$$\downarrow$$

$$y' = \frac{2}{(2s+1)\sqrt{4s^2+4s+1-1}}$$

$$y' = \frac{2}{(2s+1)\sqrt{4(s^2+s)}}$$

$$y' = \frac{2}{(2s+1)\cdot\sqrt{4}\cdot\sqrt{s^2+s}}$$

$$\boxed{y' = \frac{1}{(2s+1)\sqrt{s^2+s}}}$$ //

$$D_x \csc^{-1}(u) = \frac{-1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

27 $y = \csc^{-1}(x^2+1)$ $\quad u = (x^2+1)$

$$y' = \frac{-1}{(x^2+1)\sqrt{(x^2+1)^2-1}} \cdot 2x$$

$$y' = \frac{-2x}{(x^2+1)\sqrt{(x^2+1)^2-1}}$$

$$y' = \frac{-2x}{(x^2+1)\sqrt{x^4+2x^2+1-1}}$$

$$\boxed{y' = \frac{-2x}{(x^2+1)\sqrt{x^2(x^2+2)}}}$$

$$y' = \frac{-2x}{(x^2+1)\sqrt{x^2}\sqrt{x^2+2}}$$

Simplify ?

$$\boxed{y' = \frac{-2}{(x^2+1)\sqrt{x^2+2}}}$$

$$D_x \csc^{-1}(u) = \frac{-1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx}$$

35) $y = \csc^{-1}(e^x)$ $\qquad u = e^x$

$$y' = \frac{-1}{(e^x)\sqrt{(e^x)^2-1}} \cdot e^x$$

$$y' = \frac{-e^x}{e^x\sqrt{e^{2x}-1}} = \boxed{y' = \frac{-1}{\sqrt{e^{2x}-1}}}$$

39) $y = \tan^{-1}(\sqrt{x^2-1}) + \csc^{-1} x$

$$y' = \frac{1}{1+(\sqrt{x^2-1})^2} \cdot \frac{1}{\cancel{2}}(x^2-1)^{-\frac{1}{2}} \cdot \cancel{2}x + \left[\frac{-1}{|x|\sqrt{x^2-1}}\right]$$

$$y' = \frac{1}{1+x^2-1} \cdot \frac{x}{\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}}$$

$$y' = \frac{1}{x^2} \cdot \frac{\cancel{x}}{\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}}$$

$$y' = \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}}$$

$$\boxed{y' = 0}$$

49 a) $\sec^{-1} 0$ b) $\sin^{-1}\sqrt{2}$

Not defined @ a & b

1,3,5,7,19,25,39,43

3.11 Homework -

1) $f(x) = x^3 - 2x + 3 \qquad a = 2$

$L(x) = f(a) + f'(a)(x-a)$

$f'(x) = 3x^2 - 2$

$L(2) = 2^3 - 2(2) + 3 + [(3(2^2) - 2)(x-2)]$

$L(2) = 7 + (10)(x-2)$

$L(2) = 7 + 10x - 20$

$\boxed{L(2) = 10x - 13}$

3) $f(x) = x + \frac{1}{x}, \qquad a = 1$

$L(x) = f(a) + f'(a)(x-a)$

$f'(x) = 1 - \frac{1}{x^2}$

$L(1) = 1 + 1 + [(1 - \frac{1}{1^2})(x-1)]$

$L(1) = 2 + [0(x-1)]$

$L(1) = 2 + 0$

$\boxed{L(1) = 2}$

$f(x) = \tan x \qquad a = \pi$

$L(x) = f(a) + f'(a)(x-a)$

$f'(a) = \sec^2 x$

$L(\pi) = \tan(\pi) + (\sec^2(\pi) \cdot (x-\pi))$

$L(\pi) = 0 + ((-1)^2(x-\pi))$

$\boxed{L(\pi) = x - \pi}$

7 $f(x) = x^2 + 2x \qquad a = 0.1$

$L(x) = f(a) + f'(a)(x-a)$

$f'(x) = 2x + 2$

ⓐ $a = 0$

$L(0) = 0^2 + 2(0) + [2(0) + 2(x-0)$

$\boxed{L(0) = 2x}$

19 $y = x^3 - 3(x)^{\frac{1}{2}}$

$dy = 3x^2 dx - \frac{3}{2}(x)^{-\frac{1}{2}} dx$

$dy = 3x^2 dx - \frac{3}{2\sqrt{x}} dx$

$$\boxed{dy = dx\left(3x^2 - \frac{3}{2\sqrt{x}}\right)}$$

(25) $y = \sin(5\sqrt{x})$

$dy = \cos(5\sqrt{x}) \cdot \frac{5}{2}(x)^{-\frac{1}{2}}\, dx$

$dy = \frac{5}{2\sqrt{x}} \cos(5\sqrt{x})\, dx$

(39) $f(x) = x^2 + 2x \quad x_0 = 1 \quad dx = 0.1$

$\Delta f = f(1 + 0.1) - f(1)$

$\Delta f = (1.1)^2 + 2(1.1) - (1^2 + 2(1))$

$\Delta f = 1.21 + 2.2 - 3$

$\boxed{\Delta f = 0.41}$

$f'(x) = 2x + 2(dx) \rightarrow 4(0.1)$

$\boxed{f'(x) = .4}$

$|\Delta f - df| = |0.41 - .4| = \boxed{0.01}$

(43) $f(x) = x^{-1} = \frac{1}{x}$ $\quad X_0 = 0.5,\ dx = 0.1$

$\Delta f = \left(\frac{1}{.5} + .1\right) - \frac{1}{.5}$

$\boxed{\Delta f = 0.1}$

$f'(x) = -\frac{1}{x^2}(dx)$ $\quad @\ x = 0.5$

$\boxed{f'(x) = 0.4}$

$|\Delta f - df|$

$|0.1 - 0.4| \quad = \boxed{0.3}$

#5, 7, 13, 17, 21, 23

Steven Romeiro

3.10 Homework

(5) $y = x^2 \quad \frac{dx}{dt} = 3 \quad \frac{dy}{dt} = ? \quad x = -1$

$$\frac{dy}{dt} = 2x\frac{dx}{dt} \longrightarrow \frac{dy}{dt} = 2(-1)(3) = \boxed{\frac{dy}{dt} = -6}$$

(7) $x^2 + y^2 = 25 \quad \frac{dx}{dt} = -2 \quad x = 3,\ y = 4 \quad \frac{dy}{dt} = ?$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{2x\frac{dx}{dt}}{2y} \longrightarrow \frac{dy}{dt} = \frac{2(-2)(3)}{2(4)}$$

$$\boxed{\frac{dy}{dt} = -\frac{3}{2}}$$

(15) $V = IR \quad dV = 1\ \frac{v}{sec} \quad dI = \frac{1}{3}\ \frac{amp}{sec}$

$\frac{dR}{dt} = \frac{V}{I} \quad \boxed{\frac{dR}{dt} = 3} \qquad$ c) ? $\longleftarrow$?

d) $V = 12,\ I = 2 \quad R = ?$

$$R = \frac{V}{I} \longrightarrow \frac{dr}{dt} = \frac{I\left(\frac{dV}{dt}\right) - \left(\frac{dI}{dt}\right)(V)}{I^2}$$

$$\frac{dr}{dt} = \frac{2(1) - \frac{1}{3}(12)}{2^2} = \boxed{-\frac{1}{2}\ \frac{ohms}{sec}} \quad ? \quad \frac{3}{2}$$

17 $S = \sqrt{x^2 + y^2}$ $(x, 0)$ & $(0, y)$

a) $S^2 = x^2 + y^2$

$$2S\frac{ds}{dt} = 2x\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{2x\frac{dx}{dt}}{2S}$$

$$\boxed{\frac{ds}{dt} = \frac{x}{\sqrt{x^2+y^2}} \cdot \frac{dx}{dt}}$$

b) $S^2 = x^2 + y^2$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\boxed{\frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{\sqrt{x^2+y^2}}}$$

c) $S^2 = x^2 + y^2$

$x^2 = S^2 - y^2$

$$\frac{dx}{dt} = \frac{0 - 2y\frac{dy}{dt}}{2x}$$

$$\boxed{\frac{dx}{dt} = \frac{y}{\sqrt{S^2-y^2}} \cdot \frac{dy}{dt}}$$

(21) $\frac{dL}{dt} = \frac{-2\,cm}{sec}$ $\quad \frac{dw}{dt} = 2\,\frac{cm}{sec}$ $\quad L = 12\,cm$

$W = 5\,cm$

a) $A = L \cdot W$ $\quad \frac{dA}{dt} = \frac{dL}{dt} \cdot W + L \cdot \frac{dw}{dt}$

$\frac{dA}{dt} = -2(5) + 12(2)$ $\quad \boxed{\frac{dA}{dt} = 14\,\frac{cm^2}{sec}}$

b) $P = 2L + 2W$ $\quad \frac{dP}{dt} = 2\frac{dL}{dt} + 2\frac{dw}{dt}$

$\frac{dP}{dt} = 2(-2) + 2(2) = \boxed{\frac{dP}{dt} = 0\,\frac{cm}{sec}}$

c) Diagonal $= \sqrt{12^2 + 5^2}$

Diagonal $= \underline{13\,cm}$

Let Diagonal $= D$

$\therefore\ D = \sqrt{L^2 + W^2} = D^2 = L^2 + W^2$

$2D\,\frac{dD}{dt} = 2L\,\frac{dl}{dt} + 2w\,\frac{dw}{dt}$ $\quad \boxed{\frac{dD}{dt} = -\frac{14}{13}\,\frac{cm}{sec}}$

$\frac{dD}{dt} = \frac{L\,\frac{dl}{dt} + W\,\frac{dw}{dt}}{D}$

(23) $X = 12\,ft \quad \frac{dx}{dt} = 5\frac{ft}{sec}$

a) $\frac{dy}{dt} = ?$ $\quad x^2 + y^2 = 13^2$

$y = \sqrt{13^2 - 12^2}$

$2y\frac{dy}{dt} = -\cancel{2}x\frac{dx}{dt}$ $\quad \underline{y = 5}$

$\frac{dy}{dt} = -\frac{12}{5} \cdot 5$ $\quad \boxed{\frac{dy}{dt} = -12\frac{ft}{sec}}$

b) $A = \frac{1}{2} b \cdot h = \frac{1}{2} x y$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}(y) + \frac{1}{2}x\left(\frac{dy}{dt}\right)$

$\boxed{\frac{dA}{dt} = -59.5\frac{ft^2}{sec}}$

c) $\tan\theta = \frac{y}{x}$ $\quad \theta = \arctan\left(\frac{5}{12}\right)$

$\theta = .395$

$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - \frac{dx}{dt}(y)}{x^2}$

$\frac{1}{\cos^2(.395)} \cdot \frac{d\theta}{dt} = \frac{-169}{144}$ $\quad \boxed{\frac{d\theta}{dt} = -.981\frac{rad}{sec}}$

$\frac{d\theta}{dt} = -1.1736\left(\cos^2(.395)\right)$

Bonus Question ✓ 10 Steven Romeiro

Ex:

$f(x) = \frac{-2}{2x-1}$

find eq. of line @ $x=0$

$= \dfrac{\frac{-2}{2(x+h)-1} - \frac{-2}{2x-1}}{h}$

$\dfrac{\frac{-2(2x-1)}{(2x-1)(2(x+h)-1)} - \frac{-2(2(x+h)-1)}{(2x-1)(2(x+h)-1)}}{h}$

$\dfrac{\frac{-4x+2}{(2x-1)(2(x+h)-1)} - \frac{-4(x+h)+2}{(2x-1)(2(x+h)-1)}}{h}$

$\dfrac{\frac{-4x+2+4(x+h)-2}{(2x-1)(2(x+h)-1)}}{h} = \dfrac{\frac{-\cancel{4x}+\cancel{2}+\cancel{4x}+4h-\cancel{2}}{(2x-1)(2(x+h)-1)}}{h}$

$= \frac{4h}{(2x-1)(2(x+h)-1)} \cdot \frac{1}{h} = \frac{4}{(2x-1)(2(x+h)-1)}$

$\lim_{h\to 0} \frac{4}{(2x-1)(2(x+h)+1)} = \frac{4}{(2x-1)(2x-1)}$ ↳

$f'(x) = \dfrac{4}{(2x-1)(2x+1)}$ $f'(x) = \dfrac{4}{(2x-1)^2}$

$\boxed{f'(x) = \dfrac{4}{4x^2-1}}$ $f'(0) = 4$

$f'(0) = \dfrac{4}{4(0)^2-1} = \boxed{f'(0) = -4}$

$f(0) = \dfrac{-2}{2x-1} = \dfrac{-2}{2(0)-1}$

$\boxed{f(0) = 2}$

$y = -4x + b \quad (0, 2)$

$\therefore\ 2 = -4x + b$

$b = 2 + 4x \quad @\ x = 0$

$b = 2 + 4(0)$

$b = 2$

equation of the tangent line

$\boxed{y = -4x + 2 \quad (0, 2)}$

Steven Romeiro

\# 1, 3, 7, 11, 17, 19, 21, 25, 31, 39, 45, 49, 53, 61

$\frac{72}{78}$ 92%

4.1 Homework

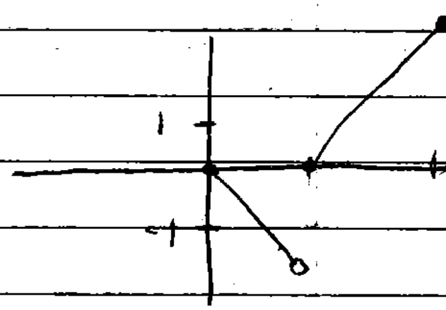
(1) absolute min = C_2 Since f is a continuous function
Absolute max = b in interval [a, b]

(3) Absolute min = None
Absolute max = C

(7) No absolute Max, no absolute min

(11) $f'(x)$ = Graph C

(17) $g(x) \begin{cases} -x & 0 \le x < 1 \\ x-1 & 1 \le x \le 2 \end{cases}$

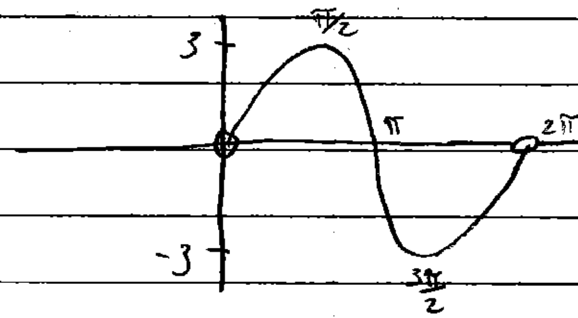


Abs Max = 2 Abs Min = None

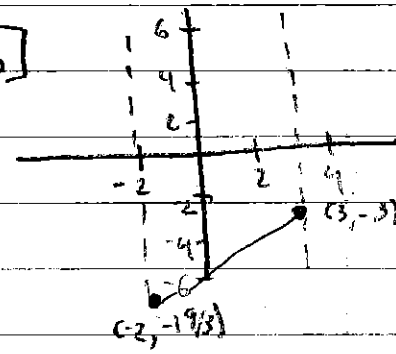
(19) $y = 3\sin x, \quad 0 < x < 2\pi$

Abs Max @ $x = \frac{\pi}{2}$

Abs Min @ $x = \frac{3\pi}{2}$



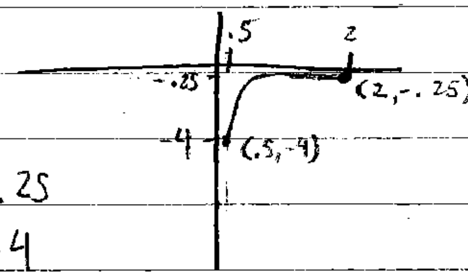
(21) $f(x) = \frac{2}{3}x - 5$ $[-2, 3]$



$f'(x) = \frac{2}{3}$

Abs Min $= -19/3$ Abs Max $= -3$

(25) $f(x) = -\frac{1}{x^2}$ $[0.5, 2]$



$f'(x) = +\frac{2}{x^3} \neq 0$ Abs Max $= -.25$

$x \neq 0$ Abs Min $= -4$

(31) $f(\theta) = \sin\theta$ $[-\frac{\pi}{2}, \frac{5\pi}{6}]$

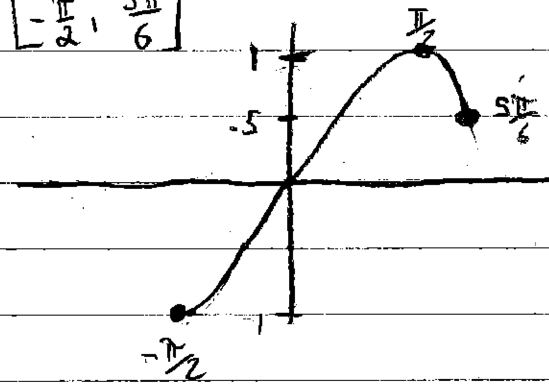
$f'(\theta) = \cos\theta$

$\theta = \cos^{-1} 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \ldots$

Abs Max $= 1$

Abs Min $= -1$



How w/o graphing Calc? Need graphing Calc

39 $f(x) = \frac{1}{x} + \ln x$ $[0.5, 4]$

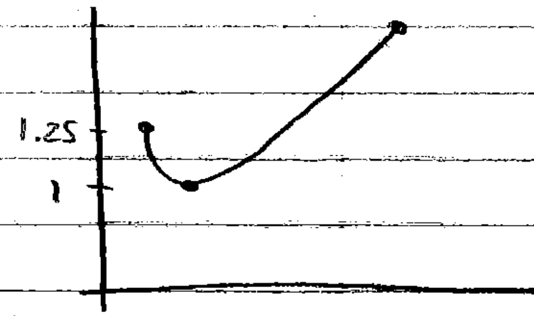
$f'(x) = -\frac{1}{x^2} + \frac{1}{x}$

$-\frac{1}{x^2} + \frac{1}{x} = 0$

$\frac{-1+x}{x^2} = 0$

$x \neq 0$ @ $x = 1$ $\frac{-1+1}{1^2} = \frac{0}{1} = 0$

$f(0.5) = 1.307$
$f(1) = 1$
$f(4) = 1.636$



Absolute max @ $x = 4$
Absolute min @ $x = 1$

(45) $y = x^2 - 6x + 7$

$y' = 2x - 6$

$0 = 2x - 6$

$\boxed{x = 3}$ critical point

(49) $y = x^2 + \frac{2}{x}$

$y' = 2x - \frac{2}{x^2}$

$0 = 2x - \frac{2}{x^2}$

$2x = \frac{2}{x^2}$

$2x^3 = 2$

$x^3 = 1$ $\boxed{x = 1}$ critical point

(53) $y = 2x^2 - 8x + 9$

$y' = 4x - 8$

$0 = 4x - 8$

$4x = 8$

$x = 2 \rightarrow y = 2(2)^2 - 8(2) + 9$ $y = 1$

$\boxed{\text{Min @ } x = 2 \text{ is } 1}$

(61) $y = \frac{x}{x^2+1}$

$y' = \frac{(x^2+1)(1) - 2x(x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$: $x = 1$ & $x = -1$

$y = \frac{1}{1^2+1} = \frac{1}{2}$ Max @ $x = 1$ $(1, \frac{1}{2})$

$y = \frac{-1}{(-1)^2+1} = -\frac{1}{2}$ Min @ $x = -1$ $(-1, -\frac{1}{2})$

#1, 3, 10, 13, 21, 25, 33, 37, 39, 43
45, 47

4.2 Homework —

$$\frac{f(b) - f(a)}{b - a} = f'(x)$$

(1) $f(x) = x^2 + 2x - 1 \quad [0, 1]$

$$\frac{f(1) - f(0)}{1 - 0} = \frac{2 - 1}{1} = 1$$

$f'(x) = 2x + 2 \rightarrow \quad 2x + 2 = 1$

$$2x = -1$$

$$\boxed{x = -\frac{1}{2}}$$

(3) $f(x) = x + \frac{1}{x} \quad [\frac{1}{2}, 2]$

$$\frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = \frac{1.5 - (-1.5)}{2 - .5} = \frac{3}{1.5} = \frac{3}{\frac{3}{2}} = \boxed{2}$$

$f(x) = x + x^{-1} = f'(x) = 1 - \frac{1}{x^2}$

$1 - \frac{1}{x^2} = 2 = \frac{-1}{x^2} = 1 = \quad x^2 = -1$

$$\boxed{x = \pm\sqrt{-1}}$$

?

(10) $f(x) = x^{4/5} \quad [0, 1]$

$$\frac{f(1) - f(0)}{1 - 0} = 1 \qquad f'(x) = \frac{4}{5}x^{-\frac{1}{5}} = \frac{4}{5x^{\frac{1}{5}}}$$

$$\frac{4}{5x^{\frac{1}{5}}} = 1 \rightarrow 5x^{\frac{1}{5}} = 4 \rightarrow x^{\frac{1}{5}} = \frac{4}{5}$$

$$\boxed{x = \sqrt[5]{4/5}}$$

?

Now what?

13 $f(x) = \begin{cases} x^2 - x, & -2 \le x \le -1 \\ 2x^2 - 3x - 3, & -1 < x \le 0 \end{cases}$

21 $f(x) = x^4 + 3x + 1 \qquad [-2, -1]$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(-1) - f(-2)}{-1 + 2} = \frac{-1 - 11}{1} \qquad ?$$

$$= -12$$

$f'(x) = 4x^3 + 3 \rightarrow 4x^3 + 3 = -12$

$4x^3 = -15$

$x^3 = -\frac{15}{4}$ $\quad \boxed{x = -1.554}$

(25) $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8 \quad (-\infty, \infty)$?

(33) a. $y' = x$ $\boxed{y = \frac{x^2}{2} + c}$ b. $y' = x^2$ $\boxed{y = \frac{x^3}{3} + c}$ c. $y' = x^3$ $\boxed{y = \frac{x^4}{4} + c}$

(37) a. $y' = \sin 2t$ $\boxed{y = \frac{-\cos 2t}{2} + c}$ b. $y' = \cos\frac{t}{2}$ $\boxed{y = 2\sin\frac{t}{2} + c}$

c. $y' = \sqrt{\theta} - \sec^2\theta$

$\boxed{y = \frac{2}{3}\theta^{\frac{3}{2}} - \tan\theta} + c$

(39) $f'(x) = 2x - 1$

$\boxed{f(x) = x^2 - x}$

(43) $V = 9.8t + 5$, $S(0) = 10$

$\boxed{S(t) = 4.9t^2 + 5t + 10}$

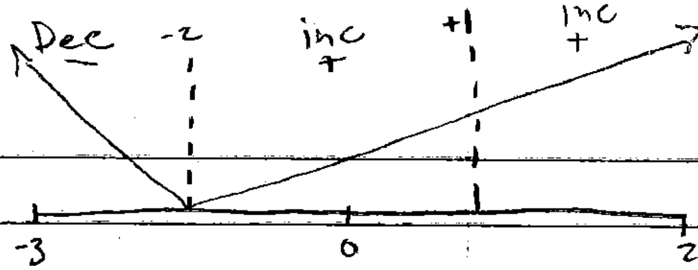
(45) $V = \sin \pi t$ $S(0) = 0$

$$\boxed{S(t) = -\frac{\cos(\pi t)}{\pi} - \frac{1}{\pi}}$$

(47) $a = e^x$, $V(0) = 20$, $S(0) = 5$

$$\boxed{\begin{array}{l} V(t) = e^x + 19 \\ S(t) = e^x + 19t + 4 \end{array}}$$

#3, 7, 19, 25, 35, 45, 49, 53, 57, 61



4.3 Homework

(3) $f'(x) = (x-1)^2(x+2)$

a) $(x-1)^2(x+2) = 0$

$(x-1)^2 = 0$ $\quad$ $(x+2) = 0$

$x - 1 = 0$ $\quad$ $x + 2 = 0$

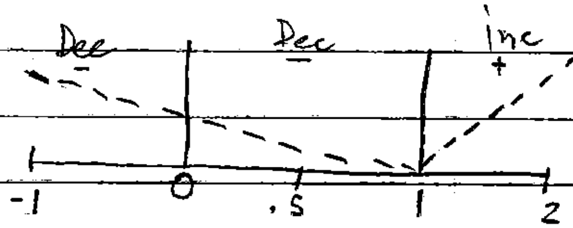
$\boxed{x = 1}$ $\quad$ $\boxed{x = -2}$

b) Decreasing: $(-\infty, -2)$

increasing: $(-2, 1) \cup (1, \infty)$

c) local in $x = -2$ & no local Max

(7) $f'(x) = \dfrac{x^2(x-1)}{x+2}$, $x \neq -2$



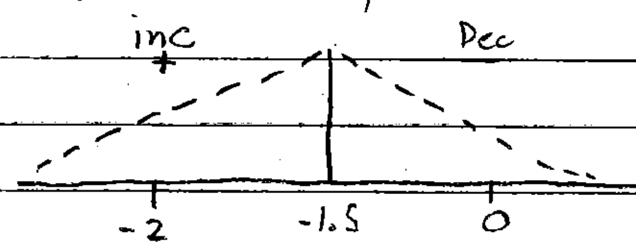
a) $x^2(x-1) = 0$

$\boxed{x = 0}$ & $\boxed{x = 1}$

b) Decreasing: $(-2, 0) \cup (0, 1)$

increasing: $(-\infty, -2) \cup (1, \infty)$

c) local min $x = 1$, no local Max



19) $f(x) = -x^2 - 3x + 3$

$f'(x) = -2x - 3$

$-2x - 3 = 0$

$\boxed{x = -\frac{3}{2}}$

$f(-1.5) = -7.125$ Absolute Max

$f(x)$ increasing: $(-\infty, -1.5)$

$f(x)$ Decreasing: $(-1.5, \infty)$

25) $f(r) = 3r^3 + 16r$

$f'(r) = 9r^2 + 16$

? $\left\{ \begin{array}{l} 9r^2+16=0 \\ \cancel{\sqrt{9r^2+16} = \sqrt{0}} \\ \cancel{3r+4=0} \\ \cancel{r = -\frac{4}{3}} \end{array} \right.$ or $\dfrac{-0 \pm \sqrt{0^2-4(9)(16)}}{2(9)}$

$\dfrac{\sqrt{-576}}{18}$ nonreal

cant square an addition

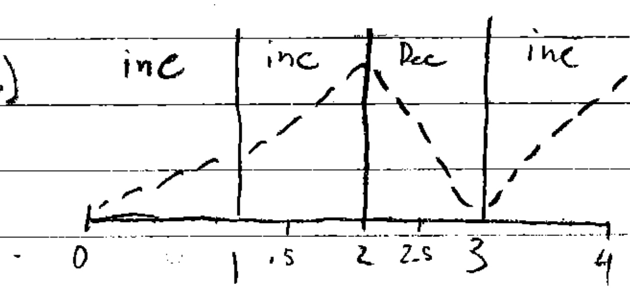
35) $f(x) = \dfrac{x^2-3}{x-2}$ $\quad x \neq -2$

$f'(x) = \dfrac{(x-2)(2x)-(1)(x^2-3)}{(x-2)^2}$

$f'(x) = \dfrac{2x^2-4x-x^2+3}{(x-2)^2}$

$f'(x) = \dfrac{x^2-4x+3}{(x-2)^2}$

$f'(x) = \dfrac{(x-3)(x-1)}{(x-2)^2}$



$f(1) = 2$

$f(2) = $ UND

$f(3) = 6$

$x-3=0$ $\quad \boxed{x=3}$

$x-1=0$ $\quad \boxed{x=1}$

increasing: $(-\infty, 2) \cup (3, \infty)$

Decreasing: $(2, 3)$

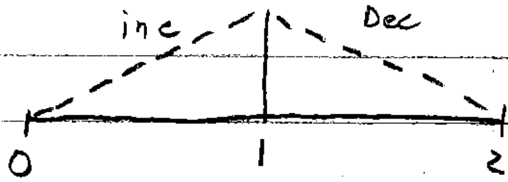
local min @ $x=3$, $f(3)=6$

(45) $f(x) = 2x - x^2, \quad -\infty < x \leq 2$

a) $f'(x) = 2 - 2x$

$2x - 2 = 0$

$x = 1$



local & absolute Max $f(1) = 1$

@ $x = 1$, $(1,1)$ local min @ $x = 2$, $(2,0)$

(49) $f(t) = 12t - t^3, \quad -3 \leq t < \infty$

$f'(t) = 12 - 3t^2$

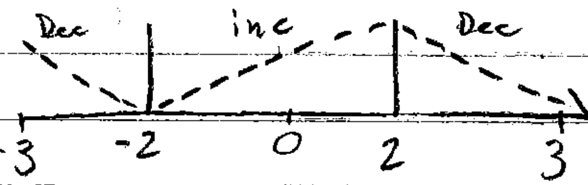
$3t^2 - 12 = 0$

$3t^2 = 12$

$t^2 = \frac{12}{3}$

$t = \sqrt{4}$ $\boxed{t \pm 2}$

No Absolute Min



$f(-3) = -9$

$f(-2) = -16$ local Min $(-2, -16)$

$f(2) = 16$ Absolute Max $(2, 16)$

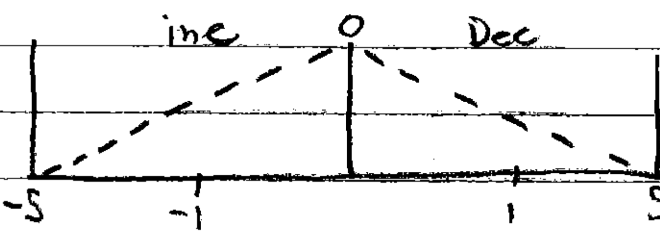
(53) $f(x) = \sqrt{25 - x^2}, \quad -5 \leq x \leq 5$

$f'(x) = (x^2 - 25)^{\frac{1}{2}}$

$f'(x) = \dfrac{(x^2-25)^{-\frac{1}{2}}(2x)}{2}$

$f'(x) = \dfrac{2x}{\sqrt{x^2-25}\,(2)}$

$f'(x) = \dfrac{x}{(x^2-25)^{\frac{1}{2}}}$



$f(-5) = 0$ Absolute Min $(-5, 0)$

$f(0) = 5$ Absolute Max $(0, 5)$

$f(5) = 0$ Absolute Min $(5, 0)$

increasing: $f(x) > 0$ $(-5, 0)$

Decreasing: $f(x) < 0$ $(0, 5)$

(57) $f(x) = \sin 2x$, $0 \leq x \leq \pi$

$f'(x) = 2\cos 2x$

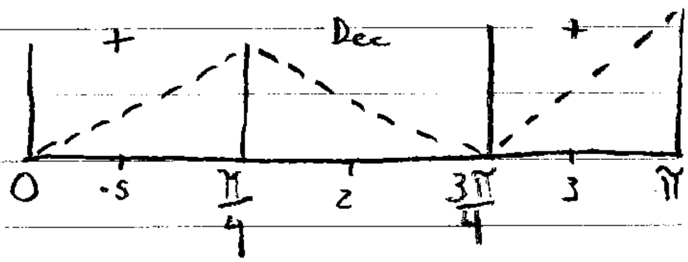
$2\cos 2x = 0$

$\cos 2x = 0$

$2x = \cos^{-1}(0)$

$2x = 1.571$

$\boxed{x = .785} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$



$f(0) = 0$ local Min $(0, 0)$

$f(\frac{\pi}{4}) = 1$ Absolute Max $(\frac{\pi}{4}, 1)$

$f(\frac{3\pi}{4}) = -1$ Absolute Min $(\frac{3\pi}{4}, -1)$

$f(\pi) = 0$ local Max $(\pi, 0)$

(61) $f(x) = \frac{x}{2} - 2\sin\frac{x}{2}$ $0 \leq x \leq 2\pi$

$f'(x) = \frac{1}{2} - \cos\frac{x}{2}$

$\frac{1}{2} - \cos x/2 = 0$

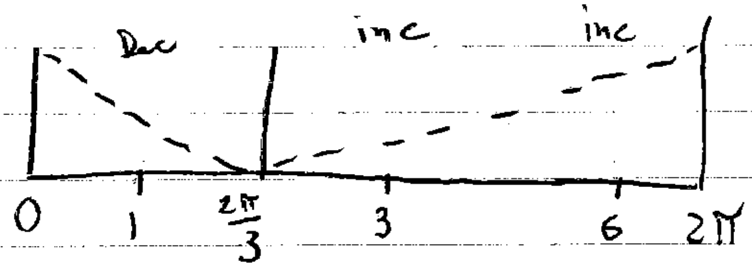
let $x = \frac{x}{2}$ $\cos\frac{x}{2} = \frac{1}{2}$

$\therefore$ $\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\frac{x}{2} = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{2\pi}{3}, \frac{10\pi}{3}$



$f(0) = 0$ local Max

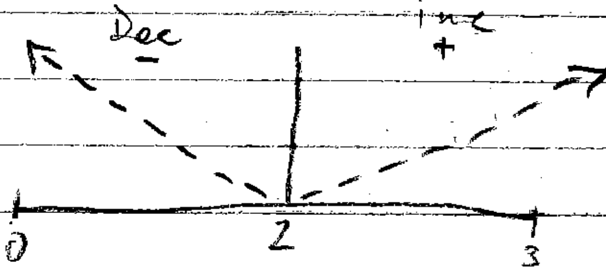
$f(\frac{2\pi}{3}) = -.685$ local Min

$f(2\pi) = \pi$ local Max

#9, 11, 17, 23, 43, 53, 59, 67, 85, 89, 97

4.4 Homework

(9) $y = x^2 - 4x + 3$



$y' = 2x - 4$

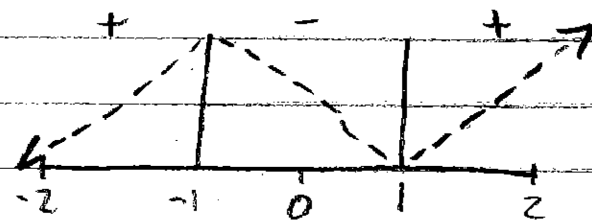
$2x - 4 = 0$

$\boxed{x = 2}$ extrema

$f(2) = 2^2 - 4(2) + 3$

$f(2) = -1$ Abs min $(2, -1)$

(11) $y = x^3 - 3x + 3$



$y' = 3x^2 - 3$

$3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = \pm 1$

$f(-1) = 5$ local max $(-1, 5)$

$f(1) = 1$ local min $(1, 1)$

$y'' = 6x$

$6x = 0$ $\boxed{x = 0}$ possible poi

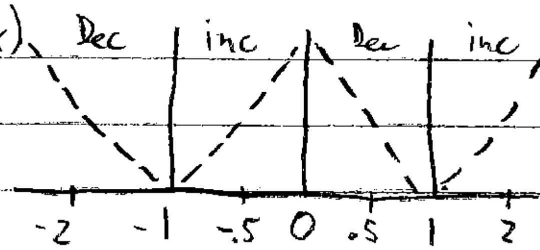
$f''(-1) = 6(-1) = \text{neg}$

$f(0) = 3$ $(0, 3)$ poi

$f''(1) = 6(1) = \text{pos}$

(17) $y = x^4 - 2x^2 = x^2(x^2 - 2)$

$y' = 4x^3 - 4x = 2x(x^2 - 2) + (x^2)(2x)$



$y' = 4x^3 - 4x = 2x^3 - 4x + 2x^3$

$y' = 4x^3 - 4x = 4x^3 - 4x$

$4x^3 - 4x = 0$

$y'' = 12x^2 - 4$

$4x(x^2 - 1) = 0$

$12x^2 - 4 = 0$

$f(-1) = -1$ Absolute Min

$x = \pm 1$

$x^2 = \frac{1}{3} = \pm \frac{1}{\sqrt{3}}$

$f(0) = 0$ local Max

$4x = 0$

Poi: $\left(\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$ or $\left(-\frac{1}{\sqrt{3}}, -\frac{5}{9}\right)$

$f(1) = 1$ Absolute Min

$x = 0$

$f(x) > 0$: $(-1, 0) \cup (1, \infty)$

$f(x) < 0$: $(-\infty, -1) \cup (0, 1)$

(23) $y = x + \sin x$ $\qquad 0 \leq x \leq 2\pi$

$y' = 1 + \cos x$

$\cos x = -1$

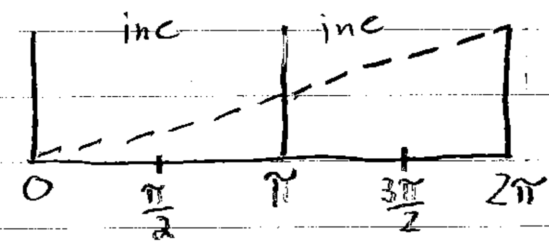
$x = \pi, 3\pi, 5\pi \ldots$

$y'' = -\sin x$

$-\sin x = 0$

$\sin^{-1}(0) = 0$ $\quad x = \pi, 2\pi$

? What about 2π? $\quad$ Poi: (π, π)



$f(0) = 0$ Absolute Min

$f(\pi) = \pi$

$f(2\pi) = 2\pi$ Absolute Max

(43) $y = \dfrac{8x}{x^2+4}$

$$y' = \frac{(x^2+4)(8) - 2x(8x)}{(x^2+4)^2} = \frac{8x^2 + 32 - 16x^2}{(x^2+4)^2}$$

$$y' = \frac{-8x^2+32}{(x^2+4)^2} = \frac{-8(x^2+4)}{(x^2+4)^2}$$

$$y' = \frac{-8}{(x^2+4)} = 0$$

(53) $y = e^x - 2e^{-x} - 3x$

$y' = e^x - 2e^{-x}(-1) - 3$

$y' = e^x + 2e^{-x} - 3$

$0 = e^x \ \frac{2}{e^x} - 3$

$0 = \frac{e^{2x} + 2 - 3e^x}{e^x}$

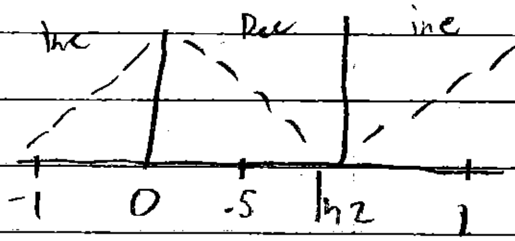
$0 = \frac{(e^x - 1)(e^x - 2)}{e^x}$

$e^x - 1 = 0$ $e^x - 2 = 0$

$e^x = 1$ $e^x = 2$

$x = 0$ $\ln e^x = \ln 2$

$x = \ln 2$



$f(0) = -1$ local Max

$f(\ln 2) = 1 - 3\ln 2$ local Min

$y'' = e^x + 2e^{-x}(-1)$

$y'' = e^x - 2e^{-x}$

$y'' = \frac{e^{2x} - 2}{e^x}$

$e^{2x} - 2 = 0$

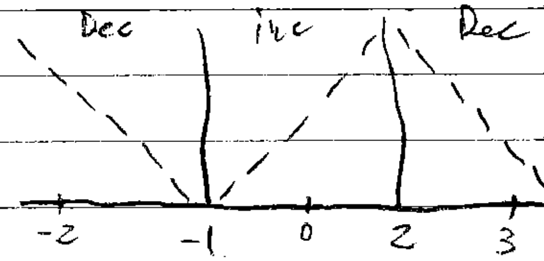
$2x = \ln 2$

Poi: $x = \frac{\ln 2}{2}$

$y = 2x + \frac{1}{2}x^2 - \frac{1}{3}x^3$

(59) $y' = 2 + x - x^2$

$y'' = 1 - 2x$



$2 + x - x^2 = 0$

$(x+1)(x+2)$

$\boxed{x = -1 \text{ \& } x = 2}$

$2x - 1 = 0$

$x = \frac{1}{2}$

$\boxed{\text{Poi: } f(\tfrac{1}{2}) = \tfrac{13}{12}}$

(67) $y' = \sec^2 x$ $\qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$y' = (\sec x)^2$

$y'' = 2\sec x \cdot \sec x \tan x$ $\quad \to \sec^2 x = 0$

$y'' = 2\sec^2 x \tan x$ $\qquad \frac{1}{\cos^2 x} \neq 0$

$y'' = 2 \, \frac{1}{\cos^2 x} \cdot \frac{\sin x}{\cos x}$

$y'' = 2 \cdot \frac{\sin x}{\cos^3 x} \longrightarrow y'' = \frac{2\sin x}{\cos^3 x}$

$x \neq -\frac{\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad 2\sin x = 0$

$\sin x = 0$

Poi: $x = -\pi, 0, \pi$

No critical Points

(85) $y = \dfrac{2x^2 + x - 1}{x^2 - 1}$

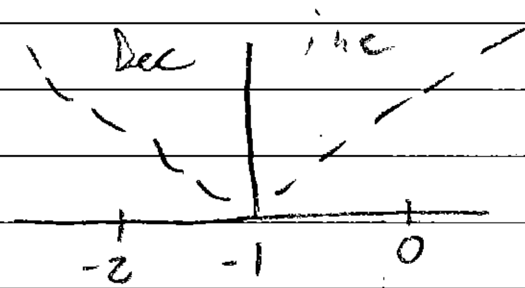
$$y' = \frac{(x^2-1)(4x+1) - (2x)(2x^2+x-1)}{(x^2-1)^2}$$

$$y' = \frac{4x^3 + x^2 - 4x - 1 - (4x^3 + 2x^2 - 2x)}{(x^2-1)^2}$$

$$y' = \frac{\cancel{4x^3} + x^2 - 4x - 1 - \cancel{4x^3} - 2x^2 + 2x}{(x^2-1)^2}$$

$$y' = \frac{-x^2 - 2x - 1}{(x^2-1)^2}$$

$$y' = \frac{-(x+1)(x+1)}{[(x+1)(x-1)]^2}$$



$$y' = \frac{-(x+1)}{(x-1)^2}$$

$-(x+1) = 0 \quad x \neq 1$

$-x - 1 = 0$

$\boxed{x = -1}$

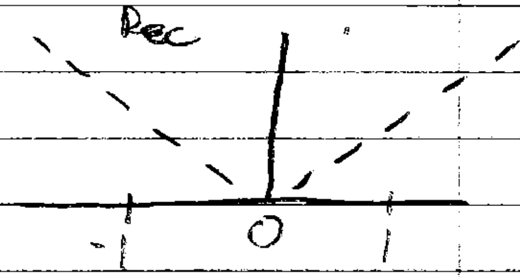
(89) $\frac{1}{x^2-1}$

$$y' = \frac{(x^2-1)(0)-(1)(2x)}{(x^2-1)^2}$$

$$y' = \frac{-2x}{(x^2-1)^2}$$

$-2x=0$

$\boxed{x=0}$



$$y'' = \frac{(x^4-2x^2+1)(-2)-(-2x)((2x^2-2)(2x))}{(x^2-1)^4}$$

$$y'' = \frac{-2x^4+4x^2-2-(-8x^4+8x^2)}{(x^2-1)^4}$$

$$y'' = \frac{-2x^4+4x^2-2+8x^4-8x^2}{(x^2-1)^4}$$

$\boxed{f(0)=-1}$ Abs Min

$$y'' = \frac{6x^4-4x^2-2}{(x^2-1)^4}$$

$$y'' = \frac{2(3x^4-2x^2-1)}{(x^2-1)^4}$$

Poi: $x=.805$

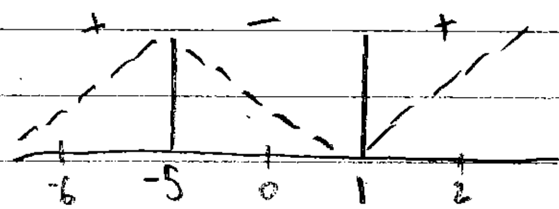
$x=-.138$

(97) $y = \dfrac{x^3 - 3x^2 + 3x - 1}{x^2 + x - 2} = \dfrac{(x^3 - 1) + (-3x^2 + 3x)}{(x+2)(x-1)}$

$y = \dfrac{(x-1)(x^2 + x + 1) + (-3x)(x-1)}{(x+2)(x-1)}$

$y' = \dfrac{(x^2 + x - 2)(3x^2 - 6x + 3) - (x^3 - 3x^2 + 3x - 1)(2x + 1)}{(x^2 + x - 2)^2}$

Zero's @ $x = 1$ & $x = -5$



#7, 11, 13, 15, 27, 33, 46, 51, 63

4.5 Homework

7. $\lim_{x\to 2} \dfrac{x-2}{x^2-4} = \dfrac{0}{0}$

$\lim_{x\to 2} \dfrac{1}{2x} = \dfrac{1}{2(2)} = \boxed{\dfrac{1}{4}}$

11. $\lim_{x\to \infty} \dfrac{5x^3-2x}{7x^3+3} = \dfrac{\infty}{\infty}$

$\lim_{x\to \infty} \dfrac{15x^2-2}{21x^2}$

$\lim_{x\to \infty} \dfrac{30x}{42x}$

$\lim_{x\to \infty} \dfrac{30}{42} = \boxed{\dfrac{5}{7}}$

13. $\lim_{t\to 0} \dfrac{\sin t^2}{t} = \dfrac{0}{0}$

$\lim_{t\to 0} \dfrac{2t\cos t^2}{1}$

$\lim_{t\to 0} 2t \cdot \cos t^2 = 2(0)$

$\boxed{= 0}$

15 $\lim_{x \to 0} \dfrac{8x^2}{\cos x - 1} = \dfrac{0}{0}$

$\lim_{x \to 0} \dfrac{16x}{-\sin x} = \dfrac{0}{0}$

$\lim_{x \to 0} \dfrac{16}{-\cos x} = \dfrac{16}{-1} = \boxed{-16}$

27 $\lim_{\theta \to 0} \dfrac{3^{\sin\theta} - 1}{\theta} = \dfrac{0}{0}$

$\lim_{\theta \to 0} \dfrac{3^{\sin\theta}(\ln 3)(\cos\theta)}{1}$

$3^0 (\ln 3)(\cos 0)$

$1 (\ln 3)(1)$

$\boxed{\ln 3}$

4.5 (33) $\lim_{x\to 0^+} \dfrac{\ln(x^2+2x)}{\ln x}$ $\quad \dfrac{-\infty}{-\infty}$

$\lim_{x\to 0^+} \dfrac{\dfrac{1}{x^2+2x} \cdot 2x+2}{\dfrac{1}{x}}$

$\lim_{x\to 0^+} \dfrac{x(2x+2)}{x^2+2x}$

$\lim_{x\to 0^+} \dfrac{2x^2+2x}{x^2+2x} = \dfrac{0}{0}$

$\lim_{x\to 0^+} \dfrac{4x+2}{2x+2} = \boxed{1}$

(46) $\lim_{x\to\infty} x^2 e^{-x}$ $\quad \infty \cdot 0$

$\lim_{x\to\infty} \dfrac{x^2}{e^x}$ $\quad \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{e^x(2x) - e^x(x^2)}{e^{2x}}$

$\lim_{x\to\infty} \dfrac{2xe^x - x^2e^x}{e^{2x}}$ $\quad \hookrightarrow$

$$\lim_{x\to\infty} \frac{e^x(2x-x^2)}{e^{2x}} = \lim_{x\to\infty} \frac{2x-x^2}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{2e^x - 2xe^x - 2xe^x + x^2e^x}{e^{2x}} = \lim_{x\to\infty} \frac{x^2e^x - 4xe^x + 2e^x}{e^{2x}}$$

$$\lim_{x\to\infty} \frac{e^x(x^2-4x+2)}{e^{2x}} = \lim_{x\to\infty} \frac{x^2-4x+2}{e^x}$$

? $$\frac{(\infty)^2 - 4(\infty) + 2}{e^\infty}$$

51) $$\lim_{x\to 1^+} x^{\frac{1}{1-x}}$$

$$\ln y = \lim_{x\to 1^+} \frac{1}{1-x}(\ln x)$$

$$\ln y = \lim_{x\to 1^+} \frac{\ln x}{1-x} = \frac{0}{0}$$

$$\ln y = \lim_{x\to 1^+} \frac{(1-x)(\frac{1}{x}) - (-1)(\ln x)}{(1-x)^2}$$

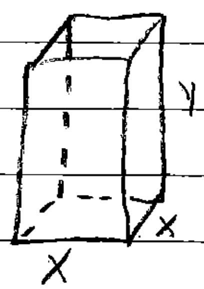
$$\ln y = \lim_{x\to 1^+} \frac{\frac{1-x}{x} + \ln x}{(1-x)^2} = \frac{0}{0} = \ln y = \lim_{x\to 1^+} (x^2-2x+1)\Big(\frac{x(-1)-(1)(1-x)}{x^2}$$

#5, 9, 19, 23, 27, 37

4.6 Homework

5

9

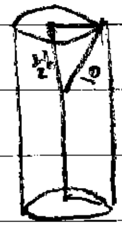


$V = x^2 y$

$A = x^2 + 4xy$

$500 = x^2 y$

(19)



(23)

$\frac{1}{2}$ hemisphere Area $= \frac{1}{2}(4\pi r^2) = 2\pi r^2$

cyl wall Area $= 2\pi r h$

$C = 2\pi r h + 2(2\pi r^2)$ Hemi: $2x$

$C = 2\pi r h + 4\pi r^2$

Vol is constant

$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \pi r^2 h + \frac{2}{3}\pi r^3$

$V - \frac{2}{3}\pi r^3 = \pi r^2 h$

$h = \dfrac{V - \frac{2}{3}\pi r^3}{\pi r^2} = \dfrac{V}{\pi r^2} - \dfrac{2r}{3}$

now replace h in cost f(x)

#1, 3, 5, 13

4.7 Homework

(1) $x^2+x-1=0 \qquad x_0=-1 \;\; \& \;\; x_0=1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = x_1 = -1 - \frac{f(-1)}{f'(-1)} = \boxed{x_1=-2}$ L

$x_1 = 1 - \frac{f(1)}{f'(1)} = \boxed{x_1 = \frac{2}{3}}$ R

$x_2 = -2 - \frac{f(-2)}{f'(-2)} = \boxed{x_2 = -\frac{5}{3}}$ L

$x_2 = \frac{2}{3} - \frac{f(2/3)}{f'(2/3)} = \boxed{x_2 = \frac{13}{21}}$ R

(3) $f(x) = x^4+x-3 \qquad x_{0L}=-1 \quad x_{0R}=1$

$x_{1L} = -1 - \frac{f(-1)}{f'(-1)} = x_{1L} = -1.999999$

$x_{2L} = -1.999999 - \frac{f(-1.999999)}{f'(-1.999999)} = \boxed{x_{2L} = -1.645}$

$x_{1R} = 1 - \frac{f(1)}{f'(1)} = x_{1R} = 1.1999999$

$x_{2R} = 1.1999999 - \frac{f(1.1999999)}{f'(1.1999999)} = \boxed{x_{2R} = 1.165}$

(5) $f(x) = x^4 - 2$ $\quad X_0 = 1$

$X_1 = 1 - \frac{f(1)}{f'(1)} = X_1 = 1.25$

$X_2 = 1.25 - \frac{f(1.25)}{f'(1.25)} = \boxed{X_2 = 1.1935}$

(13) $Y = \tan x$ $\quad X_0 = \frac{\pi}{4}$

$X_1 = \frac{\pi}{4} - \frac{f(\frac{\pi}{4})}{f'(\frac{\pi}{4})} = X_1 = .2853988$

$X_2 = .2853988 - \frac{f(.2853988)}{f'(.2853988)} = X_2 = .015247$

$X_3 = .015247 - \frac{f(.015247)}{f'(.015247)} = X_3 = .000002$

Answer $\boxed{X_4 = .000002} - \frac{f(.000002)}{f'(.000002)} = X_4 = .000000$

$\boxed{X_5 = .000000} - \frac{f(.000000)}{f'(.000000)} = \boxed{X_5 = .000000}$

equal

#3, 7, 11, 15, 19, 25, 29, 35, 41, 45, 51, 55

4.8 Homework

(3) a. $-3x^{-4}$ $\boxed{x^{-3}}$ b. x^{-4} $\boxed{-\frac{1}{3}x^{-3}}$ c. $x^{-4}+2x+3$ $\boxed{-\frac{1}{3}x^{-3}+x^{2}+3x+C}$

(7) a. $\frac{3}{2}x^{\frac{1}{2}} = \boxed{x^{\frac{3}{2}}+C}$ b. $\frac{1}{2}x^{-\frac{1}{2}}$ $\boxed{x^{\frac{1}{2}}+C}$ c. $x^{\frac{1}{2}}+x^{-\frac{1}{2}}$ $\boxed{\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C}$

(11) a. $\frac{1}{x} = \boxed{\ln x + C}$ b. $\frac{7}{x} = \boxed{\ln 7x + C}$ c. $1-5/x$ $\boxed{x - \ln 5x + C}$

(15) a. $\sec^2 x$ $\boxed{\tan x + C}$ b. $\frac{2}{3}\sec^2\frac{x}{3}$ $\boxed{2\tan\frac{x}{3}}$ c. $-\sec^2\frac{3}{2}x$ $\boxed{-\frac{2}{3}\tan\frac{3}{2}x+C}$

(19) a. $e^{3x} =$ $\boxed{\frac{1}{3}e^{3x}+C}$ b. e^{-x} $\boxed{-e^{-x}+C}$ c. $e^{x/2}$ $\boxed{2e^{x/2}+C}$

(25) $\int (x+1)\,dx = \boxed{\frac{1}{2}x^2+x+C}$

(29) $\int (2x^3-5x+7)\,dx = \boxed{\frac{1}{2}x^4-\frac{5}{2}x^2+7x+C}$

35) $\int (x^{\frac{1}{2}} + x^{\frac{1}{3}})\,dx = \boxed{\frac{2}{3}x^{\frac{3}{2}} + \frac{3}{4}x^{\frac{4}{3}} + c}$

41) $\int \left(\frac{t \cdot t^{\frac{1}{2}} + t^{\frac{1}{2}}}{t^2}\right) dx = \int \left((t^{\frac{3}{2}} + t^{\frac{1}{2}})(t^{-2})\right) dx$

$\int \left(t^{-\frac{1}{2}} + t^{-\frac{3}{2}}\right) dx = -2t^{\frac{1}{2}} - -2t^{-\frac{1}{2}} + C$

$= \boxed{-2\sqrt{t} - \frac{2}{\sqrt{t}} + C}$

45) $\int 7 \sin\frac{\theta}{3}\, d\theta = \boxed{-21 \cos\frac{\theta}{3} + C}$

51) $\int (e^{3x} + 4^x)\,dx = \frac{1}{3}e^{3x} + \left(\frac{1}{1 \cdot \ln 4}\right)4^x + C$

$= \boxed{\frac{1}{3}e^{3x} + \frac{4^x}{\ln 4} + C}$

55) $\int (4 \sec x \tan x - 2\sec^2 x)\,dx$

$\boxed{= 4\sec x - 2\tan x + C}$

Steven Romeiro

1,4,5,7,13,17

40/64 60%

Homework 5.1

1) $f(x) = x^2 \qquad x=0 \to x=1$

a) $\Delta x = \frac{1-0}{2} = \boxed{\frac{1}{2}}$

$A = f(0)\cdot\frac{1}{2} + f(\frac{1}{2})\cdot\frac{1}{2} \to A = 0 + .125 \to \boxed{A = \frac{1}{8}}$

b) $\Delta x = \frac{1-0}{4} = \boxed{\frac{1}{4}}$

$A = [f(0) + f(.25) + f(.5) + f(.75)]\cdot\frac{1}{4} \to \boxed{A = .21875}$

c) $\Delta x = \frac{1}{2}$

$A = f(.5)\cdot\frac{1}{2} + f(1)\cdot\frac{1}{2} \to A = .25\cdot\frac{1}{2} + 1\cdot\frac{1}{2} = \boxed{A = .625}$

d) $\Delta x = \frac{1}{4}$

$[f(.25) + f(.5) + f(.75) + f(1)]\cdot\frac{1}{4}$

$\boxed{A = .46875}$

4) $f(x) = x^2 - 4 \qquad x = -2 \rightarrow x = 2$

a) $\Delta x = \frac{2+2}{2} = 2$

$f(-2) \cdot 2 + f(0) \cdot 2 \rightarrow A = 0$

b) $\Delta x = \frac{2+2}{4} = 1$

$[f(-2) + f(-1) + f(0) + f(1)] \cdot 1 \rightarrow \boxed{A = -10}$ //

c) $\Delta x = 2$

$f(0) \cdot 2 + f(2) \cdot 2 = \boxed{A = -8}$

d) $\Delta x = 1$

$[f(-1) + f(0) + f(1) + f(2)] \cdot 1 = \boxed{A = -10}$ //

$A = L \cdot W$

$L = \frac{A}{W}$

(5) $f(x) = x^2 \qquad x=0 \rightarrow x=1$

$\Delta x = \frac{1-0}{2} = \frac{1}{2}$

$f(.25) \cdot \frac{1}{2} + f(.75) \cdot \frac{1}{2} \rightarrow \boxed{A = .3125}$

$\Delta x \ \frac{1-0}{4} = \frac{1}{4}$

$[f(.125) + f(.375) + f(.625) + f(.875)] \cdot \frac{1}{4} = \boxed{A = .328125}$

(7) $f(x) = \frac{1}{x} \qquad x=1 \rightarrow x=5$

$\Delta x = \frac{5-1}{2} = \boxed{2}$

$f(2) \cdot 2 + f(4) \cdot 2 \rightarrow A = 1 + \frac{1}{2} = \boxed{A = 1.5}$

$\Delta x = \frac{5-1}{4} = \boxed{1}$

$[f(.5) + f(1.5) + f(2.5) + f(3.5)] \cdot 1 = \boxed{A = 1.5746}$

13 $\Delta X = 1 = \frac{5-0}{5} = 1$

$A = 19.41(1) + 11.77(1) + 7.14(1) + 4.33(1) + 2.53(1)$

why lower → $=$ Lower limit $= 45.28$

$A = 32.00(1) + 19.41(1) + 11.77(1) + 7.14(1) + 4.33(1)$

why upper → Upper limit $= 74.65$

? $\Delta x = \frac{5-0}{3} = \frac{5}{3}$

hour → upper $= 74.65\left(\frac{5}{3}\right)$

17 $f(t) = (1/2) + \sin^2 \pi t \qquad [0, 2]$

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$\left[f(.25) + f(.75) + f(1.25) + f(1.75)\right] \cdot \frac{1}{2}$

why 1?? $\boxed{= 2}$

1, 3, 7, 9, 14, 15, 21, 25, 29, 33, 35

5.2 Homework —

1) $\sum_{k=1}^{2} \frac{6k}{k+1} = \frac{6(1)}{1+1} = \boxed{3}$

$+ \quad 7$

$= \frac{6(2)}{2+1} = \boxed{\frac{12}{3}}$

3) $\sum_{k=1}^{4} \cos k\pi = -1 + 1 + (-1) + 1 = \boxed{0}$

7) b) $\sum_{k=0}^{5} 2^k$

9) b) $\sum_{k=0}^{2} \frac{(-1)^k}{k+1}$

14) $\sum_{n=1}^{5} n(2)$

15) $\sum_{n=1}^{5} (-1)^{n+1} \frac{1}{n}$

(21) $\displaystyle\sum_{k=1}^{7} (-2k) = -2+(-4)+(-6)+(-8)+(-10)+(-12)+(-14)$

$\boxed{= -56}$

(25) $\displaystyle\sum_{n=1}^{5} n(3n+5) = 8+22+42+68+100+138+182$

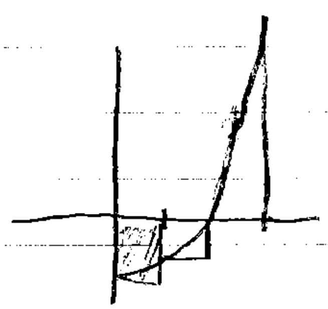
$= \boxed{560}$

(29) $\displaystyle\sum_{k=1}^{7} 3 = 7(3) = \boxed{21}$

(33) $f(x) = x^2 - 1 \quad [0, 2]$

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

a) $[f(0) + f(.5) + f(1) + f(1.5)] \cdot \frac{1}{2} = -\frac{1}{4}$



#9, 41-55 odd, 59

Homework 5.3

9) a) $\int_2^2 g(x)dx = \boxed{0}$ b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = \boxed{-8}$

c) $\int_1^2 3f(x)dx = 3(-4) = \boxed{-12}$

d) $\int_2^5 f(x)\,dx = \int_1^5 f(x) - \int_1^2 f(x) =$

e) $\int_1^5 [f(x) - g(x)]\,dx = 6 - 8 = \boxed{-2}$

f) $\int_1^5 [4f(x) - g(x)]dx = 4(6) - 8 = \boxed{16}$

41) $\int_3^1 7\,dx = -\int_1^3 7\,dx = 7x\Big|_1^3$

$= -[7(3) - 7(1)] = \boxed{-14}$

(43) $\displaystyle\int_0^2 (2t-3)\,dt = t^2-3t \Big|_0^2$

$(2^2-3(2)) - (0^2-3(0))$

$2^2-3(2) = \boxed{-2}$

(45) $\displaystyle\int_2^1 \left(1+\frac{z}{2}\right)dz = -\int_1^2 \left(1+\frac{z}{2}\right)dz$

$= x + \frac{1}{2}\cdot\frac{z^2}{2} = -\left[x+\frac{z^2}{4}\right]\Big|_1^2$

$-\left[\left(2+\frac{2^2}{4}\right)-\left(1+\frac{1^2}{4}\right)\right] = -\left[(2+1)-\left(1+\frac{1}{4}\right)\right]$

$-\left[3-1-\frac{1}{4}\right] = -\left(\frac{12-4-1}{4}\right) = \boxed{-\frac{7}{4}}$

(47) $\displaystyle\int_1^2 3u^2\,du = u^3 \Big|_1^2$

$= (2^3)-(1^3) = 8-1 = \boxed{7}$

(49) $\displaystyle\int_0^2 (3x^2+x-5)\,dx = x^3+\frac{x^2}{2}-5x \Big|_0^2$

$= \left(2^3+\frac{2^2}{2}-5(2)\right) - 0$

$= 8+2-10$

$= \boxed{0}$

(51) $y = 3x^2 = \int_0^b 3x^2\,dx = x^3 \Big|_0^b = \boxed{b^3}$

(53) $y = 2x = \int_0^b 2x\,dx = x^2 \Big|_0^b = \boxed{b^2}$

(55) $f(x) = x^2 - 1 \quad [0, \sqrt{3}]$

$$\int_0^{\sqrt{3}} (x^2 - 1)\,dx = \frac{x^3}{3} - x \Big|_0^{\sqrt{3}}$$

$$AV = \frac{1}{b-a} \int_0^{\sqrt{3}} (x^2 - 1)\,dx = \frac{1}{\sqrt{3}} \left(\frac{x^3}{3} - x \right) \Big|_0^{\sqrt{3}}$$

$$= \left[\frac{(\sqrt{3})^3}{3} - \sqrt{3} - (0) \right] \frac{1}{\sqrt{3}}$$

$$= \left[\frac{3}{3} - \sqrt{3} \right] \frac{1}{\sqrt{3}}$$

$$= \frac{1 - \sqrt{3}}{\sqrt{3}} = \frac{1}{\sqrt{3}} - 1$$

$$= \boxed{-.4226}$$

59) $f(t) = (t-1)^2 \quad [0, 3]$

$$AV = \frac{1}{3-0}\int_0^3 (t-1)^2\,dt$$

$U = t-1$

$dU = dt$

$$\frac{1}{3}\int_0^3 U^2\,du = \frac{1}{3}\left(\frac{U^3}{3}\right)\Big]_0^3$$

?

$$\frac{1}{3}\left[\frac{(t-1)^3}{3} - 0\right] = \frac{1}{3}\left(\frac{(3-1)^3}{3}\right)$$

$$= \frac{1}{3}\left(\frac{8}{3}\right) = \boxed{\frac{8}{9}}$$

1-57 every other odd

5.4 Homework

(1) $\int_0^2 x(x-3)\,dx = \int_0^2 (x^2-3x)\,dx = \frac{x^3}{3} - \frac{3}{2}x^2 \Big|_0^2$

$= \boxed{-\frac{10}{3}}$

(5) $\int_1^4 \left(3x^2 - \frac{x^3}{4}\right) dx = x^3 - \frac{1}{16}x^4 \Big|_1^4 = \boxed{\frac{753}{16}}$

(9) $\int_0^{\pi/3} 2\sec^2 x\,dx = 2\tan x \Big|_0^{\pi/3} = \boxed{2\sqrt{3}}$

(13) $-\int_0^{\pi/2} \frac{1+\cos 2t}{2}\,dt = -\int_0^{\pi/2} \left(\frac{1}{2} + \frac{\cos 2t}{2}\right) dt$

$\int_0^{\pi/2} \frac{1}{2}\,dt + \int_0^{\pi/2} \frac{\cos 2t}{2}\,dt$

$= \frac{1}{2}t + \frac{1}{2}\cdot\frac{1}{2}\sin 2t = \frac{1}{2}t + \frac{1}{4}\sin 2t \Big|_0^{\pi/2}$

$= -\boxed{\frac{\pi}{4}}$

(17) $\int_0^{\pi/8} \sin 2x \, dx = -\frac{1}{2}\cos 2x \Big|_0^{\pi/8} = \boxed{-.177}$

(21) $-\int_1^{\sqrt{2}} \left(\frac{u^7}{2} - u^{-5}\right) du = -\int_1^{\sqrt{2}} \left(\frac{1}{2}u^7 - u^{-5}\right) du$

$= -\left(\frac{1}{16}u^8 + \frac{1}{4}u^{-4}\right)\Big|_1^{\sqrt{2}} = \boxed{-\frac{3}{4}}$

(25) $\int_{\frac{\pi}{2}}^{\pi} \left(\frac{\sin 2x}{2\sin x}\right) dx = \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\left(\frac{\sin 2x}{\sin x}\right) dx$

$= \frac{1}{2}\left[\frac{1}{2}\left(\frac{-\cos 2x}{-\cos x}\right)\right]\Big|_{\frac{\pi}{2}}^{\pi} = \boxed{-1}$

(29) $\int_0^{\ln 2} e^{3x} \, dx = \frac{1}{3}e^{3x}\Big|_0^{\ln 2} = \boxed{\frac{7}{3}}$

(33) $\int_2^4 x^{\pi-1} \, dx = \frac{x^{\pi}}{\pi}\Big|_2^4 = \boxed{\frac{1}{\pi}\left(4^{\pi} - 2^{\pi}\right)}$

(37) $\displaystyle\int_2^5 \frac{1}{\sqrt{1+x^2}}\cdot x\,dx \qquad U=(1+x^2)$
$\qquad\qquad\qquad\qquad\qquad du = 2x\,dx$

$$\frac{1}{2}\int_2^5 \frac{1}{(1+x^2)^{\frac{1}{2}}}\cdot 2x\,dx = \frac{1}{2}\int_2^5 u^{-\frac{1}{2}}\,du$$

$$= \frac{1}{2}\left[2u^{\frac{1}{2}}\right]\Big|_2^5 = u^{\frac{1}{2}}\Big|_2^5 = (1+x^2)^{\frac{1}{2}}\Big|_2^5 = \boxed{\sqrt{26}-\sqrt{5}}$$

(41) $\displaystyle\frac{d}{dt}\int_0^{t^4}\sqrt{u}\,du = \left(\sqrt{t^4}\right)\cdot(4t^3)$

$$= t^2\cdot 4t^3 = \boxed{4t^5}$$

(45) $\displaystyle y = \int_0^x (1+t^2)^{\frac{1}{2}}\,dt = \boxed{(1+x^2)^{\frac{1}{2}}\cdot 1}$

(49) $\displaystyle\int_{-1}^x \frac{t^2}{t^2+4}\,dt - \int_3^x \frac{t^2}{t^2+4}\,dt = \frac{x^2}{x^2+4} - \left(\frac{x^2}{x^2+4}\right) = \boxed{0}$

(53) $$\int_0^{e^{x^2}} \frac{1}{(t)^{\frac{1}{2}}}\,dt = \int_0^{e^{x^2}} t^{-\frac{1}{2}}\,dt$$

$$= \left(e^{x^2}\right)^{-\frac{1}{2}} \cdot 2x = \boxed{2x e^{-\frac{1}{2}x^2}}$$