MAC 2311 – Calculus w/Analytic Geometry I
Worksheet – Section 6.5 – Work
Lesson Problems

$F = kx$ $\quad x = 1.8 - 1.0 = 0.8$

1) A spring has a natural length of 1 meter. A force of 24N holds the spring stretched to a total length of 1.8 m.
(a) Find the force constant k.
(b) How much work will it take to stretch the spring 2 m beyond its natural length? $\quad F = 30(x)$
(c) How far will a 45 N force stretch the spring?

a) $24N = k(0.8m) = \quad K = \frac{24N}{.8m} = \boxed{K = 30\frac{N}{m}}$

b) $W = \int_0^2 30x\,dx = \frac{30}{2}x^2 \Big|_0^2 = 15x^2\Big|_0^2 = \boxed{60\frac{N}{m}}$

c) $F = Kx \rightarrow 45N = 30x \rightarrow \boxed{X = 1.5m}$

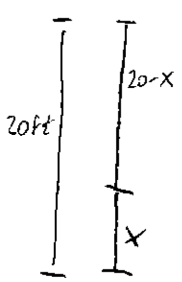
2) When a spring is expanded 1 foot from its natural position and held fixed, the force necessary to hold it is 30 pounds. Find the work required to stretch the spring from 3 to 6 feet. $\xrightarrow{1ft}$ $F = 30lbs$

$W = Fd \qquad d = 6 - 3 = 3ft \qquad F = Kx \qquad \frac{30}{1} = K \quad \boxed{K = 30}$

$W = \int_3^6 30x\,dx = 15x^2 \Big|_3^6 = \boxed{405\ ft\cdot lbs}$

$F = Kx$
$F = 30x$

3) A bucket weighing 1000 pounds is to be lifted from the bottom of a shaft that is 20 feet deep. The weight of the cable used to hoist it is 10 pounds per foot. How much work is done lifting the bucket to the top of the shaft?



$K = 10\frac{lbs}{ft} \qquad W_{total} = W_{bucket} + W_{chain}$

$F = 10(20-x)$

$W_{bucket} = \overset{F}{1000}(\overset{d}{20})$

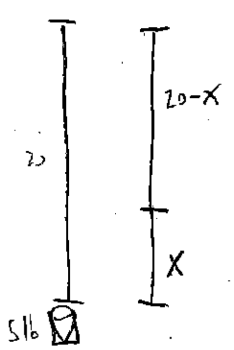
$W_{bucket} = 20,000\ ft\cdot lbs$

$W_{chain} = \int_0^{20} 10(20-x)\,dx$

$= 10\left(20x - \frac{1}{2}x^2\right)\Big|_0^{20} = 2000\ ft\cdot lbs$

$W_t = 20,000 + 2000 \qquad \boxed{W_t = 22,000\ ft\cdot lbs}$

4) A 5-lb bucket is lifted from the ground into the air by pulling in 20ft of rope at a constant speed. The rope weighs 0.08 pounds per foot. How much work was spent lifting the bucket and rope?



$K = \frac{.08\ lbs}{1\ ft}$

$F = 0.08(20-x) =$

$W_{bucket} = \overset{F}{5}(\overset{d}{20})$

$W_{bucket} = 100\ lbs\cdot ft$

$W_{rope} = \int_0^{20} .08(20-x)\,dx$

$= .08\left(20x - \frac{1}{2}x^2\right)\Big|_0^{20}$

$W_{rope} = 16\ lbs\cdot ft$

$\boxed{W_t = 116\ lbs\cdot ft}$

MAC 2311 – Calculus w/Analytic Geometry I
Worksheet – Section 6.5 – Work
Practice Problems

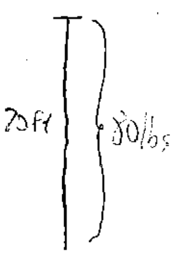
$F = kx$

1) If a spring's natural length is 1 foot and it requires 10 lbs. of force to hold this spring stretched to a length 1.5 feet, how much work is done stretching this spring from 2 feet to 3?

$10 \text{ lbs} = K(.5\text{ft}) \rightarrow \boxed{K = 20 \frac{\text{lbs}}{\text{ft}}}$

2) A spring has a natural length of 20 cm. A 40 N force is required to stretch (and hold the spring) to a length of 30 cm. How much work is done in stretching the spring from 35 cm to 38 cm?

3) A 20 ft. cable weighs 80 lbs. and hangs from the ceiling of a building without touching the floor. Determine the work that must be done to lift the bottom end of the chain all the way up until it touches the ceiling.



$F = Kx \qquad K = 4 \frac{\text{lbs}}{\text{ft}}$

$\frac{80 \text{ lbs}}{20 \text{ ft}} \; K$

$F = 4(20 - x)$

$\int_0^{20} 4(20-x)\,dx = 4\left(20x - \frac{1}{2}x^2\right)\Big|_0^{20}$

$\boxed{W = 800 \text{ ft} \cdot \text{lbs}}$

4) If 6 foot-pounds of work are required to compress a spring 1 foot from its natural length, find the work necessary to compress the spring 1 extra foot.

$W = F \cdot d$

$6 = K(1)$

$K = 6$

$6 = \int_0^1 Kx\,dx = 6 = K \cdot \frac{1}{2}x^2 \Big|_0^1 : \quad 6 = \frac{K}{2} \quad K = 12$

$W = \int_1^2 12x\,dx = 6x^2 \Big|_1^2 = \boxed{18 \text{ ft} \cdot \text{lbs}}$

5) If 6 foot-pounds of work are required to compress a spring from its natural length of 10 feet to a length of 9 feet, find the work necessary to stretch the spring from its natural length to a length of 12 feet.

$w = 6 \text{ ft} \cdot \text{lbs}$

$$6 = \int_0^1 Kx\,dx = \frac{K}{2}x^2 \Big|_0^1 = 6 = \frac{K}{2} = K = 12$$

$$W = \int_0^2 12x\,dx = 6x^2 \Big|_0^2 = \boxed{24 \text{ ft} \cdot \text{lbs}}$$

6) A bucket of cement weighing 200 pounds is hoisted by means of a windlass from the ground to the tenth story of an office building, 80 feet above the ground. Assume that a chain weighing 1 pound per foot is used in lifting the bucket. Find the work required to make the lift.

$w = 200 \text{ lbs} \cdot 8 \text{ ft}$

$W_{bucket} = 16{,}000 \text{ ft} \cdot \text{lbs}$

$$W_{chain} \int_0^{80} 1(80 - x)\,dx = 80x - \frac{1}{2}x^2 \Big|_0^{80}$$

$$= 3{,}200 \text{ lbs} \cdot \text{ft}$$

7) Find the work required to compress a spring from its natural length of 1ft to a length of 0.75 ft. if the force constant is 16 foot-lbs.

8) A certain amount of work is done to stretch a spring 3 in. The force to keep it stretched is 16 lb. How much additional work is done to stretch the spring an additional 4 in.

$$F = Kx \rightarrow 16 = K(3) = K = \frac{16}{3}$$

$$\int_3^7 \frac{16}{3}x\,dx = \frac{16}{6}x^2 \Big|_3^7 = \boxed{106.667 \text{ in} \cdot \text{lbs}}$$

MAC 2311 - Calculus w/Analytic Geometry I — Name S. R.

Replacement Test - Comprehensive — Date

Find the limit, if it exists.

1) $\lim\limits_{x\to -6} \dfrac{x^2+3x-18}{x^2+2x-24}$ $\dfrac{36-18-18}{36-36}$ $\dfrac{0}{0} = \dfrac{2x+3}{2x+2} = \dfrac{2(-6)+3}{2(-6)+2} = \dfrac{-12+3}{-12+2} = \dfrac{9}{10}$

A) Does not exist B) $\frac{3}{10}$ C) $-\frac{9}{10}$ D) $\frac{9}{10}$

Use l'Hopital's Rule to evaluate the limit.

2) $\lim\limits_{x\to 0} \dfrac{\cos 9x - 1}{x^2} = \dfrac{-9\sin 9x}{2x} = \dfrac{9\cdot -\cos 9x\cdot 9}{2} = \dfrac{81\cdot -1}{2} = -\dfrac{81}{2}$

A) $-\frac{81}{2}$ B) $\frac{81}{2}$ C) 0 D) $\frac{9}{2}$

Find the derivative of the function.

3) $y = \dfrac{x^3}{x-1} = \dfrac{(x-1)(3x^2)-(1)(x^3)}{(x-1)^2} = \dfrac{3x^3-3x^2-x^3}{(x-1)^2} = \dfrac{2x^3-3x^2}{(x-1)^2}$

A) $y' = \dfrac{2x^3-3x^2}{(x-1)^2}$ B) $y' = \dfrac{-2x^3-3x^2}{(x-1)^2}$ C) $y' = \dfrac{2x^3+3x^2}{(x-1)^2}$ D) $y' = \dfrac{-2x^3+3x^2}{(x-1)^2}$

4) $r = (\sec\theta + \tan\theta)^{-5}$ $-5(\sec\theta+\tan\theta)^{-6}\cdot(\tan\theta\sec\theta + \sec^2\theta)$

A) $-5(\sec\theta+\tan\theta)^{-6}$ B) $-5(\sec\theta\tan\theta + \sec^2\theta)^{-6}$

C) $-5(\sec\theta+\tan\theta)^{-6}(\tan^2\theta + \sec\theta\tan\theta)$ D) $\dfrac{-5\sec\theta}{(\sec\theta+\tan\theta)^5}$

$\dfrac{-5(\tan\theta\sec\theta+\sec^2\theta)}{(\sec\theta+\tan\theta)^6} = \dfrac{-5\sec\theta(\tan\theta+\sec\theta)}{(\sec\theta+\tan\theta)^6}$ $-5\sec\theta$

Find the derivative of y with respect to x, t, or θ, as appropriate.

5) $y = e^{8-8x} = e^{8-8x}(-8) = -8e^{8-8x}$

A) $-8e^{8-8x}$ B) e^{-8} C) $8e^{8-8x}$ D) $-8\ln(8-8x)$

Find the derivative of y with respect to x.

6) $y = \tan^{-1}\sqrt{11x}$ $\dfrac{1}{1+11x}\cdot\dfrac{1}{2}(11x)^{-\frac{1}{2}}\cdot 11 = \dfrac{1}{1+11x}\cdot\dfrac{11}{2\sqrt{11x}} = \dfrac{11}{2\sqrt{11x}(1+11x)}$

A) $\dfrac{1}{22\sqrt{11x}(1+11x)}$ B) $\dfrac{11}{2(1+11x)\sqrt{11x}}$ C) $\dfrac{1}{\sqrt{1-11x}}$ D) $\dfrac{1}{1+11x}$

Use implicit differentiation to find dy/dx.

7) $2xy - y^2 = 1$

A) $\dfrac{x}{y-x}$ B) $\dfrac{x}{x-y}$ C) $\dfrac{y}{x-y}$ D) $\dfrac{y}{y-x}$

$2x\left(\dfrac{dy}{dx}\right) + 2y - 2y\left(\dfrac{dy}{dx}\right) = 0$

$2x\cdot y' - 2y(y') = -2y$ $y' = \dfrac{-2y}{2x-2y} = \dfrac{-y}{x-y}$

$y'(2x-2y) = -2y$

$U = 5t^2 - 1$
$du = 10t$

$\int \frac{ds}{dt} = \int 20t(5t^2-1)^3 > S(t) = \left(\frac{1}{2}\int 2 \cdot 10t(5t^2-1)^3\right.$ need 10t so $\frac{1}{2} \cdot 20t$

~~Already~~ Already 20t

$= \frac{1}{2}\int U^3 du = \frac{1}{2} \cdot \frac{U^4}{4} + C = \frac{U^4}{8} + C$

Solve the initial value problem.

8) $\frac{ds}{dt} = 20t(5t^2 - 1)^3, \quad s(1) = 4$

$\frac{1}{2}\int U^3$

$\frac{U^4}{8} + C$

$4 = \frac{U^4}{8} + C$

A) $s = \frac{1}{2}(5t^2 - 1)^4$

B) $s = \frac{1}{2}(5t^2 - 1)^4 + 4$

C) $s = \frac{1}{2}(5t^2 - 1)^4 - 124$

D) $s = (5t^2 - 1)^4 - 252$

$C = -\frac{(5(1)^2-1)^4}{8} + 4$

$-\frac{256+32}{8}$

$4 = \frac{5(1^2)-1}{8} + C = 4 - \frac{1}{2} = C$

$S = \frac{5t^2-1}{8} + \frac{7}{2}$

Evaluate the integral.

9) $\int 2x^2 \sqrt[4]{5 + 4x^3}\, dx$

$\int 2x^2 (5+4x^3)^{\frac{1}{4}} dx > \frac{1}{6}\int U^{\frac{1}{4}} du = \frac{1}{6} \cdot \frac{4}{5} U^{5/4} + C = \frac{2}{15}(4x^3+5)^{5/4} + C$

$U = 4x^3 + 5$
$du = 12x^2 dx$

A) $\frac{8}{5}\left(5 + 4x^3\right)^{5/4} + C$

B) $\frac{2}{15}\left(5 + 4x^3\right)^{5/4} + C$

C) $-\frac{4}{3}\left(5 + 4x^3\right)^{-3/4} + C$

D) $2\left(5 + 4x^3\right)^{5/4} + C$

$U = 2\theta + 9$
$du = 2\, d\theta$

10) $\int \csc^2 (2\theta + 9)\, d\theta$

$\frac{1}{2}\int \csc^2(u) du = -\frac{1}{2}\cot(2\theta+9) + C$

A) $-\cot (2\theta + 9) + C$

B) $2 \cot (2\theta + 9) + C$

C) $-\frac{1}{2} \cot (2\theta + 9) + C$

D) $4 \csc (2\theta + 9) \cot (2\theta + 9) + C$

$U = \sin^{-1} x$
$du = \frac{1}{\sqrt{1-x^2}} dx$

11) $\int \frac{(\sin^{-1} x)^3}{\sqrt{1 - x^2}}\, dx = \int (\sin^{-1} x)^3 \cdot \frac{1}{\sqrt{1-x^2}} dx > \int U^3 du = \frac{U^4}{4} + C = \frac{(\sin^{-1} x)^4}{4} + C$

A) $3(\sin^{-1} x)^2 + C$

B) $\frac{(\sin^{-1} x)^4}{4} + C$

C) $\frac{\ln (\sin x)}{\sqrt{1 - x^2}} + C$

D) $(\cos^{-1} x)^4 + C$

Find the area enclosed by the given curves.

12) $y = 2x - x^2, \; y = 2x - 4$

A) $\frac{32}{3}$

B) $\frac{37}{3}$

C) $\frac{31}{3}$

D) $\frac{34}{3}$

Use the washer method to find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.

13) $y = x^2 + 3, \; y = 3x + 3$

A) $\frac{297}{5}\pi$

B) 27π

C) $\frac{297}{10}\pi$

D) $\frac{513}{5}\pi$

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves and lines about the y-axis. Use the calculator to calculate the volume.

14) $y = 3x^2, \; y = 3\sqrt{x}$

A) $\frac{9}{4}\pi$

B) $\frac{9}{5}\pi$

C) $\frac{9}{20}\pi$

D) $\frac{9}{10}\pi$

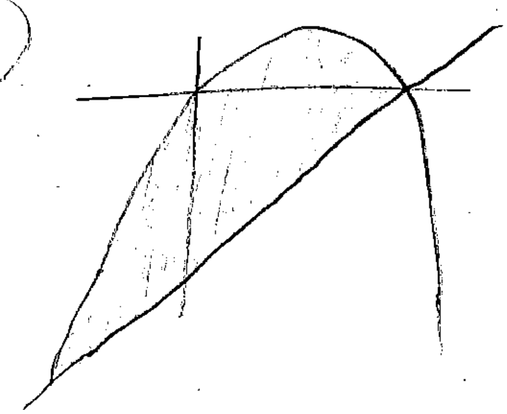
-1

MAC 2311 – Calculus w/Analytic Geometry I 93% Name Shawn Romeiro

Replacement Test – Answer Sheet Date

1) D
2) A
3) A
4) D
5) A
6) B
7) D
8) ~~D~~ C
9) B
10) C
11) B
12) A
13) A
14) D

12



Poi

$2x - x^2 = 2x - 4$

$2x - x^2 - 2x + 4 = 0$

$-x^2 + 4 = 0$

$x^2 = 4$

$x = \pm 2$

f(x) = top − bottom

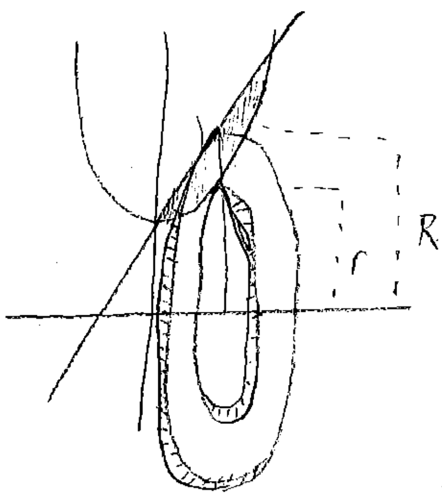
$2x - x^2 - [2x - 4]$

$2x - x^2 - 2x + 4$

$$\int_{-2}^{2} -x^2 + 4 \, dx = -\frac{x^3}{3} + 4x \Big|_{-2}^{2}$$

$$= \boxed{\frac{32}{3}}$$

13



$V = \pi R^2$

$$V = \int_0^3 \pi (R^2 - r^2)\, dx$$

$$V = \int_0^3 \pi \left([3x+3]^2 - [x^2+3]^2\right) dx$$

$$= \boxed{\frac{297}{5}\pi}$$

Poi

$x^2 + 3 = 3x + 3$

$x^2 - 3x + 3 - 3$

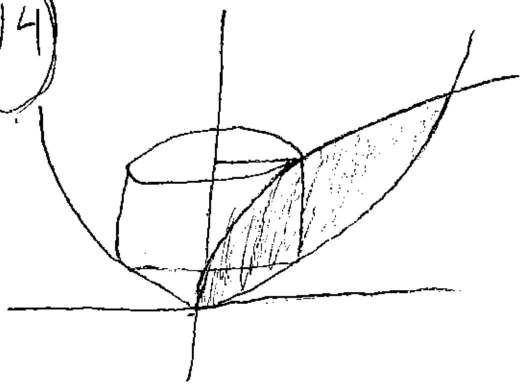
$x^2 - 3x = 0$

$x = 0, x = 3$

$R = 3x + 3 - 0$

$r = x^2 + 3 - 0$

(14)



Volume $= \int 2\pi r \cdot (f(x) - g(x)) \cdot dx$
top - bottom

$r = x$

$y = 3x^2$

$y = 3\sqrt{x}$

$Ht = 3\sqrt{x} - 3x^2$

Poi

$x = 0, x = 1$

$\int_0^1 2\pi x \left(3\sqrt{x} - 3x^2\right)$

$V = \int_0^1 2\pi \left(3x^{3/2} - 3x^3\right)dx = 2\pi\left(\frac{2}{5} \cdot 3x^{5/2} - \frac{3}{4}x^4\right)$

$= 2\pi \left(\frac{6}{5}x^{5/2} - \frac{3}{4}x^4\right)\Big|_0^1$

$2\pi \frac{9}{20} = \boxed{\frac{9}{10}\pi}$

$\frac{1}{3}$

MAC 2311 - Calculus w/Analytic Geometry I

Volume of Solids of Rotation – Method of Shells

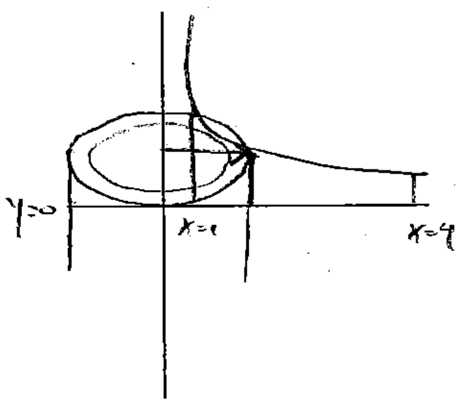
Lesson Problems

About the y-axis: $Volume = \int_a^b 2\pi x \cdot f(x) \cdot dx$

About the x-axis: $Volume = \int_c^d 2\pi y \cdot f(y) \cdot dy$

Sketch the graphs, shade the bounded region, draw a typical shell, set up the integral to find the volume, and calculate the volume.

1) $y = \frac{1}{\sqrt{x}}$ $y=0$ $x=1$ *and* $x=4$ - about the y-axis



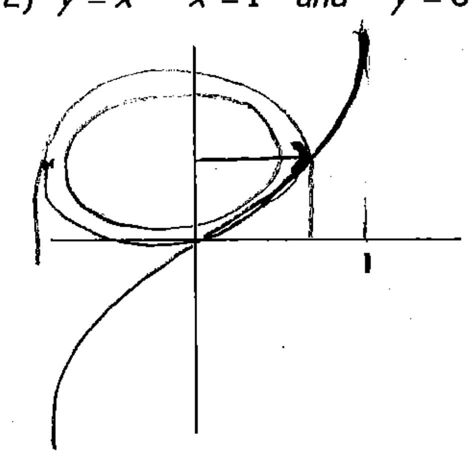
radius = x (it's always some x value)

$$\int_a^b 2\pi r \cdot Ht \cdot th$$

$$\int_1^4 2\pi x\left(\frac{1}{x}\right)dx = \int_1^4 2\pi x^{\frac{1}{2}}dx = \boxed{29.322}$$

$$2\pi \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_1^4 = \frac{4}{3}\pi x^{\frac{3}{2}}\Big|_1^4$$

2) $y = x^3$ $x=1$ *and* $y=0$ - about the y-axis



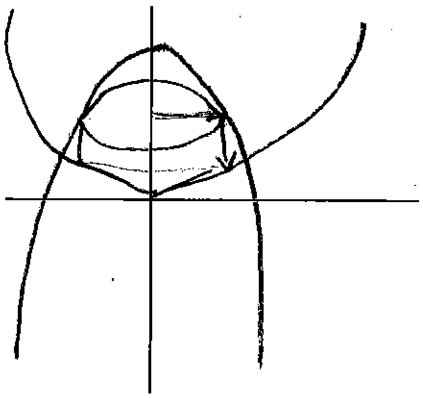
Ht = x^3 poi = $x^3 = 0 = x$ $x=0$ & $x=1$

r = x

$$V = \int_0^1 2\pi x(x^3)dx = \int_0^1 2\pi(x^4)dx$$

$$2\pi \cdot \frac{1}{5}x^5\Big|_0^1 = 1.257$$

3) $y = x^2 \quad y = 2 - x^2 \quad x = 0$ (Region in the 1st quadrant) rotated about the y-axis



$r = x$

top btm

$Ht = 2 - x^2 - x^2 = 2 - 2x^2$

poi: $x^2 = 2 - x^2$

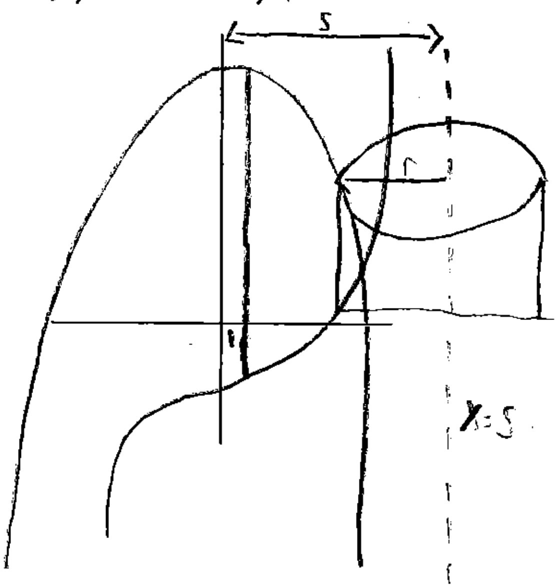
$x^2 = 1$

$x = \pm 1$ only interested in $+1$

$$\int_0^1 2\pi x\left(2 - 2x^2\right)dx$$

$$\int_0^1 2\pi\left(2x - 2x^3\right)dx = \boxed{3.142}$$

4) $y = x^3 - 2 \quad y = -x^2 + 8$ *and* $x = 1$ - about the line $x = 5$



$r = 5 - x$

$Ht = -x^2 + 8 - (x^3 - 2)$

$Ht = -x^2 + 8 - x^3 + 2$

$Ht = -x^3 - x^2 + 10$

poi

$x^3 - 2 = -x^2 + 8$

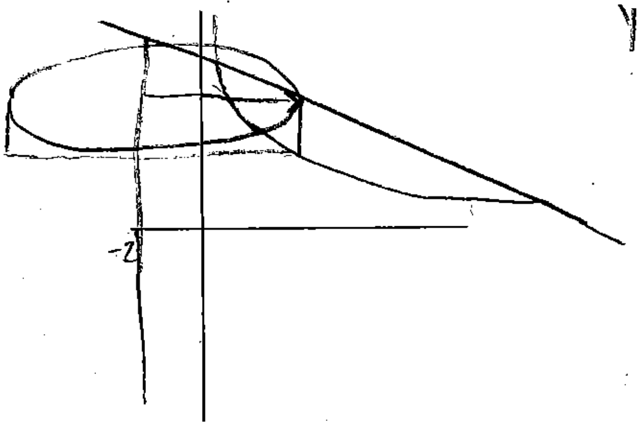
$x = 1.867$

$$V = \int_1^{1.867} 2\pi(5 - x)(-x^3 - x^2 + 10)dx$$

don't even foil

$V = 93.800$

5) $xy = 4$ and $x + y = 5$ - about the line at $x = -2$



$y = \frac{4}{x}$, $y = 5 - x$

$r = x - (-2) = x + 2$

$Ht = 5 - x - \left(\frac{4}{x}\right) = 5 - x - \frac{4}{x}$

poi

$\frac{4}{x} = -x + 5$

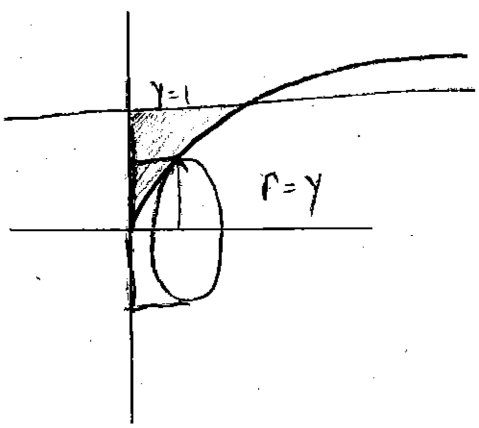
$4 = -x^2 + 5x = x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$$\int_1^4 2\pi(x+2)\left(-x+5-\frac{4}{x}\right)dx$$

$V = 52.339$

6) $y^2 = x$ $y = 1$ and $x = 0$ - about the x-axis



$x = y^2$ $x = 0$

$Ht = x_2 - x_1$

$Ht = y^2 - 0 = y^2$

poi

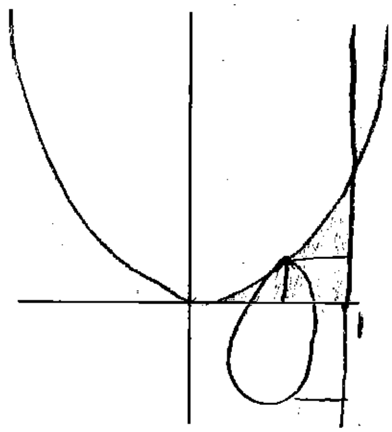
$y^2 = 0$

$y = 0$

$$V = \int_0^1 2\pi(y)(y^2)dy = \int_0^1 2\pi y^3 dy = 2\pi \frac{y^4}{4}\Bigg|_0^1 = \frac{\pi}{2}y^4\Bigg|_0^1$$

$$\boxed{V = 1.571 = \frac{\pi}{2}}$$

7) $y = x^2 \quad y = 0 \quad and \quad x = 1$ - about the x-axis



$r = y$

$x = 1 \quad + \quad x = \pm\sqrt{y}$

$Ht = 1 - \sqrt{y}$

poi

$1 = \sqrt{y}$

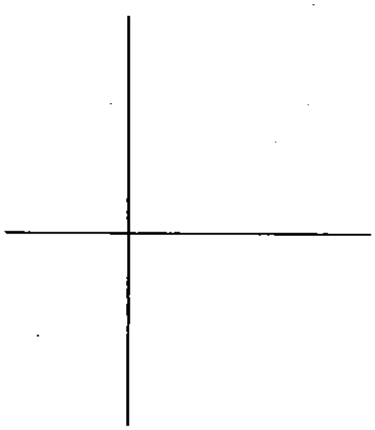
$y = 1, \; y = 0$

$$\int_0^1 2\pi(y)(1-\sqrt{y})\,dy = \int_0^1 2\pi(y - y^{1.5})\,dy$$

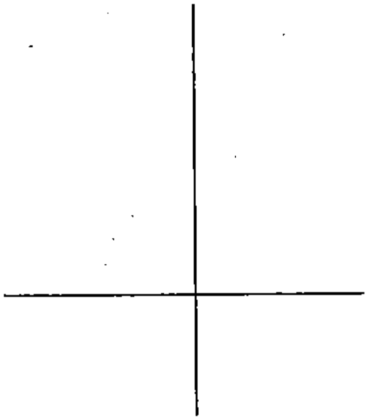
$$= 2\pi \; \frac{y^2}{2} - \frac{y^{5/2}}{\frac{5}{2}} \Big|_0^1 = 2\pi\left(\frac{y^2}{2} - \frac{2y^{5/2}}{5}\right)\Big|_0^1$$

$$\boxed{V = 0.628}$$

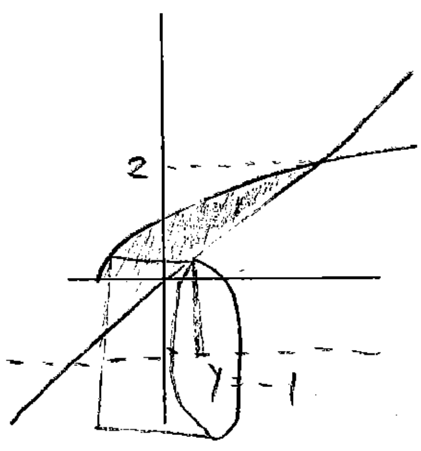
8) $y = x^2 \quad y = 4 \quad and \quad x = 0$ (Region in the 1st quadrant) rotated about the x-axis



9) $x = (y-1)^2$ *and* $x = y+1$ - about the x-axis



10) $y = \sqrt{x+2}$ $y = x$ *and* $y = 0$ - about the line $y = -1$ $\int_a^b 2\pi r \cdot Ht \cdot th$



$r = y - (-1)$ $y = x$

$r = y + 1$ $y = \sqrt{x+2} : y^2 - 2 = x$

$Ht = y - y^2 - 2 = -y^2 + y - 2$

poi

$y^2 - 2 = y$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$y = -1$ No vlc $y = +2$

$V = \int_0^2 2\pi (y+1)(-y^2+y-2)\,dy$

$\boxed{V = 37.699}$

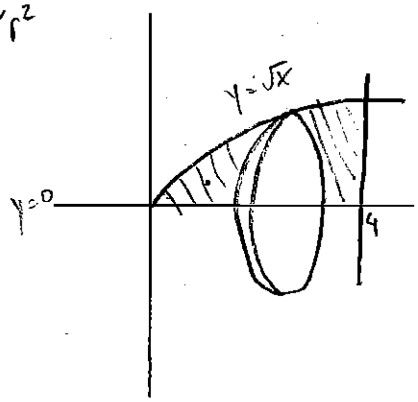
MAC 2311 - Calculus w/Analytic Geometry I

Volume of Solids of Rotation – Disk and Washer Methods

Sketch the graphs, shade the bounded region, and draw a typical disk or washer. Set up the integral to find the volume, integrate, and calculate the volume.

1) $y=\sqrt{x}$, $x=4$, and $y=0$ about the x-axis

$A=\pi r^2$



Radius $=\sqrt{x}-0$

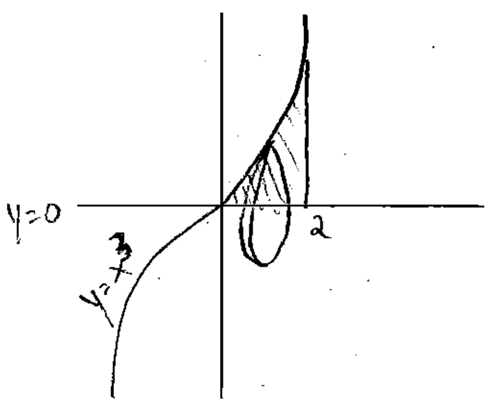
$$\int_0^4 \pi(\sqrt{x})^2 dx$$

$$\int_0^4 \pi x\,dx = \frac{\pi}{2}x^2\Big|_0^4$$

$$\frac{\pi}{2}(4)^2 - 0$$

$$8\pi = \boxed{25.133}$$

2) $y=x^3$, $y=0$, and $x=2$ about the x-axis



$$\int_0^2 \pi(x^3)^2 dx$$

$$\int_0^2 \pi x^6 dx = \frac{\pi}{7}x^7\Big|_0^2$$

$$\frac{\pi}{7}(2)^7 - 0$$

$$\frac{\pi}{7}\cdot 128 = \boxed{57.446}$$

Poi =

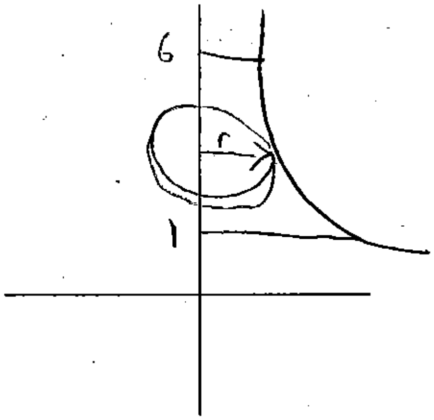
$0=x^3$

$0=0$

$r=x^3-0$

$r=x^3$

3) $x = \frac{2}{y}$, and $y = 1$, $y = 6$, and $x = 0$ about the y-axis



$$V = \int_1^6 \pi \left(\frac{2}{y}\right)^2 dy$$

$$\int_1^6 \pi \frac{4}{y^2}\, dy =$$

$$\int_1^6 \pi 4 y^{-2} dy = \pi 4\left(-\frac{1}{y}\right) = \pi \frac{-4}{y} \Big|_1^6$$

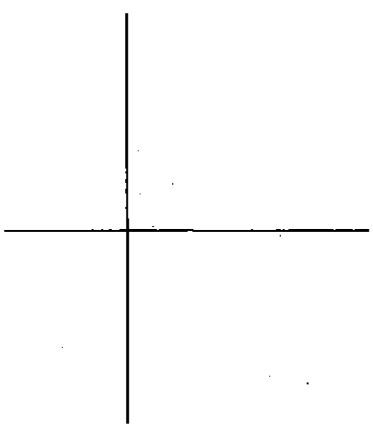
$$= \boxed{10.472}$$

right - Left

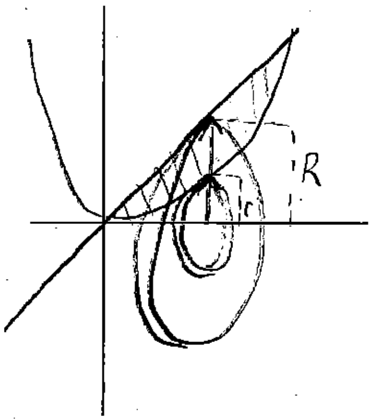
$r = \frac{2}{y} - 0$

$r = \frac{2}{y}$

4) $x = y^{2/3}$, $y = 8$, and $x = 0$ about the y-axis



5) $y = 3x$, *and* $y = 2x^2$ about the x-axis



Poi

$3x = 2x^2$

$0 = 2x^2 - 3x$

$0 = x(2x-3)$

$x = 0$ & $x = \frac{3}{2}$

$$\int_0^{3/2} \pi (R^2 - r^2)\,dx$$

$$\int_0^{3/2} \pi\, (3x)^2 - (2x^2)^2\,dx$$

$R = 3x - 0$

$R = 3x$

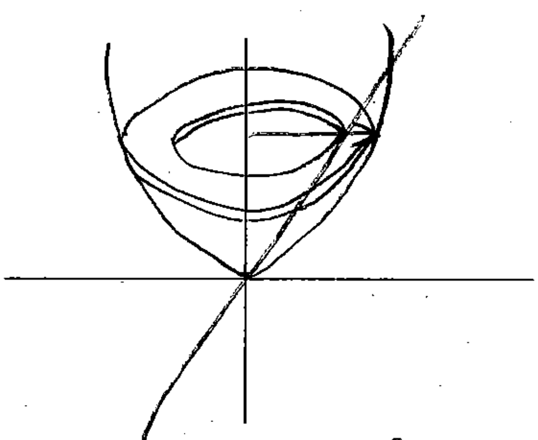
$r = 2x^2 - 0$

$r = 2x^2$

$$\int_0^{3/2} \pi \left(9x^2 - 4x^4\right) dx = \pi \left[\frac{9}{3}x^3 - \frac{4}{5}x^5\right]\Big|_0^{3/2}$$

$$= \boxed{12.723}$$

6) $y = 3x$, $y = 2x^2$ about the y-axis



$x^2 = \frac{y}{2}$

$x = \frac{\sqrt{y}}{\sqrt{2}}$

$x = \frac{y}{3}$

Poi =

$\sqrt{\frac{y}{2}} = \frac{y}{3}$

$\frac{y}{2} = \frac{y^2}{9}$

$\frac{y^2}{9} - \frac{y}{2} = 0$

$y\left(\frac{y}{9} - \frac{1}{2}\right) = 0$

$y = 0$, $y = \frac{9}{2}$

$R = \sqrt{\frac{y}{2}} - 0$

$R = \sqrt{\frac{y}{2}}$

$r = \frac{y}{3} - 0$

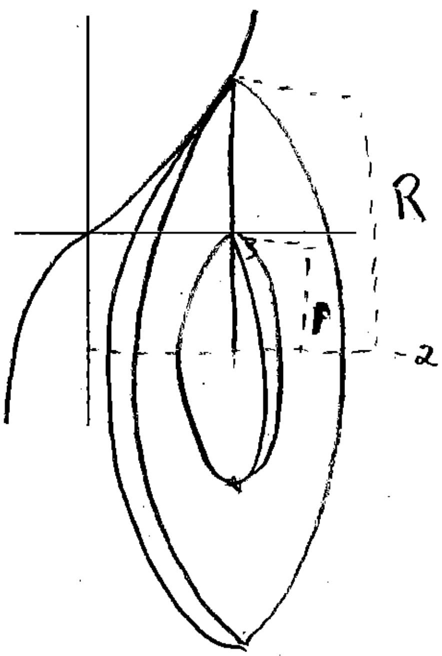
$r = \frac{y}{3}$

$$\int_0^{9/2} \pi \left(\sqrt{\frac{y}{2}}\right)^2 - \left(\frac{y}{3}\right)^2 dy$$

$$\int_0^{9/2} \pi \left(\frac{y}{2} - \frac{y^2}{9}\right) dy$$

$$= \left(\frac{1}{4}y^2 - \frac{1}{27}y^3\right)\Big|_0^{9/2} = \boxed{5.301}$$

7) $y = x^3$, $y = 0$, *and* $x = 3$ about the line $y = -2$



$$V = \int_0^3 \pi\left((x^3+2)^2 - (2)^2\right)dx$$

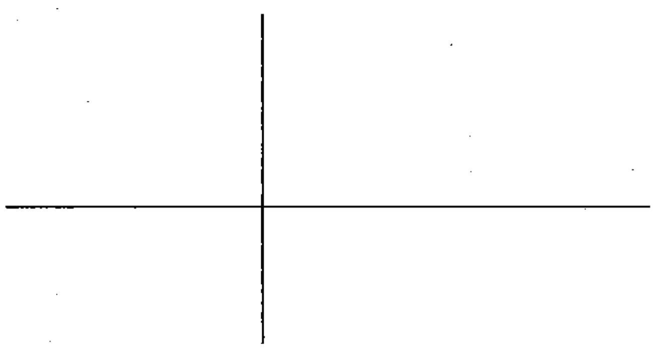
$$\int_0^3 \pi\left(x^6 + 4x^3 + 4 - 4\right)dx$$

$$\int_0^3 \pi\left(x^6 + 4x^3\right)dx = \pi\left(\frac{x^7}{7} + \frac{4}{4}x^4\right)$$

$$= \pi\left(\frac{1}{7}x^7 + x^4\right)\Big|_0^3 = 1235.992$$

$R = x^3 - (-2)$
$R = x^3 + 2$
$r = 0 - (-2)$
$r = 2$

8) $y = x^3$, $y = 0$, *and* $x = 3$ about the line $x = -1$

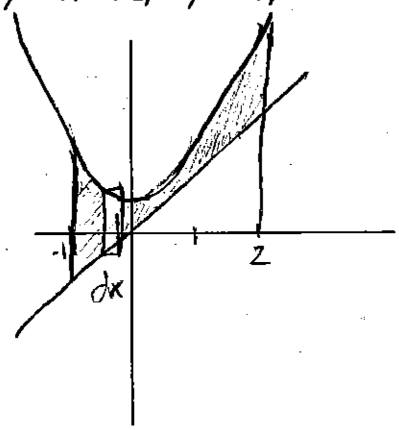


MAC 2311 - Calculus w/Analytic Geometry I

Area Between Two Curves (A)

Sketch the graphs, shade the bounded region and find the area bounded by the given expressions. Use vertical boxes (dx) for problems 1, 2, and 3. Use horizontal boxes (dy) for problems 4, 5, and 6.

1) $y = x^2 + 1, \quad y = x, \quad x = -1, \quad and \quad x = 2$



$$A = \int_{-1}^{2} l \cdot w = \int_{-1}^{2} (f(x) - g(x))\,dx$$

$$\int_{-1}^{2} (x^2 + 1 - x)\,dx = \left. \frac{x^3}{3} + x - \frac{x^2}{2} \right|_{-1}^{2}$$

$$\frac{2^3}{3} + 2 - \frac{2^2}{2} - \left[\frac{(-1)^3}{3} + (-1) - \frac{(-1)^2}{2} \right]$$

$$\boxed{A = \frac{9}{2}}$$

2) $y = \sqrt{x} \quad and \quad y = \frac{x}{4}$

Points of intersection is where these equations equal each other

$$\sqrt{x} = \frac{x}{4}$$

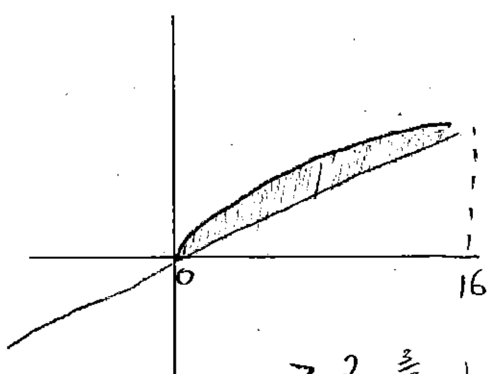
$$4\sqrt{x} = x$$

$$16x = x^2$$

$$x^2 - 16x = 0$$

$$x(x - 16) = 0$$

$$x = 0 \quad \& \quad x = 16$$



$$\int_{0}^{16} (\sqrt{x} - x/4)\,dx$$

$$\int_{0}^{16} \left(x^{\frac{1}{2}} - \frac{1}{4}x\right)dx \rightarrow \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{4}\frac{x^2}{2}$$

$$\left. \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{8}x^2 \right|_{0}^{16}$$

$$= \boxed{10.667}$$

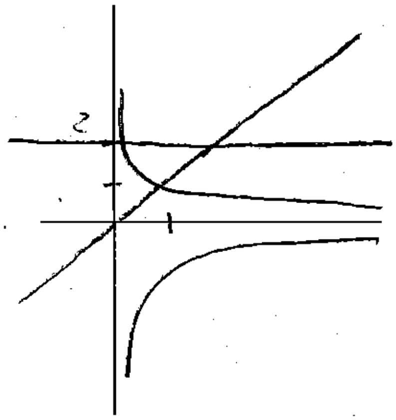
$y^2 = \frac{1}{x} \rightarrow y = \pm\sqrt{\frac{1}{x}} \rightarrow y = \pm \frac{1}{\sqrt{x}}$

3) $x = \frac{1}{y^2}$, $y = x$, and $y = 2$

Poi = $\frac{1}{y^2} = y$ or $\boxed{\text{Calc} > .5}$

$1 = y^3$

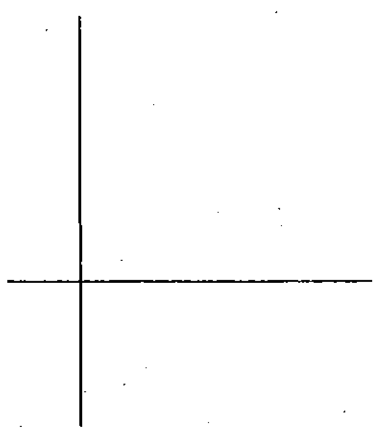
$\boxed{y = 1}$



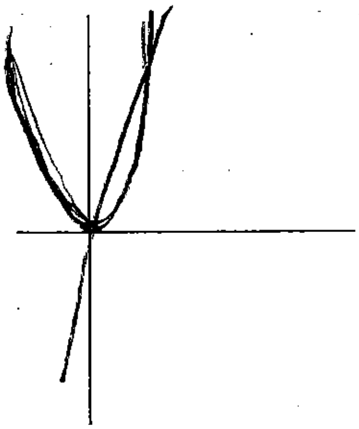
$$\int_1^2 \left(y - \frac{1}{y^2}\right) dy = \int_1^2 \left(y - y^{-2}\right) dy$$

$$\left. \frac{y^2}{2} + \frac{1}{y} \right|_1^2 = \boxed{1}$$

4) $x = 2 - y^2$, $y = -x$.



5) $y = x^2$ and $y = 4x$ use vertical boxes (dx)



Poi $=$ $4x = x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

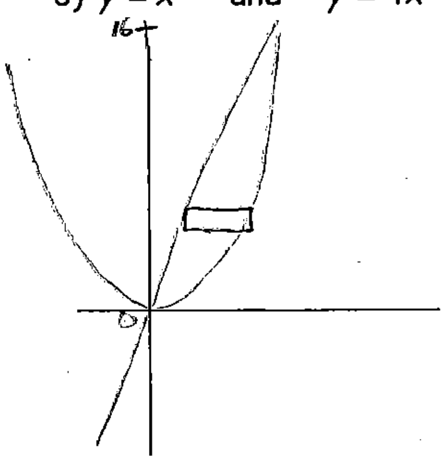
$x = 0,\ x = 4$

$$\int_0^4 (4x - x^2)\,dx$$

$$= 2x^2 - \frac{x^3}{3}\Big|_0^4$$

$$\boxed{A = 10.667}$$

6) $y = x^2$ and $y = 4x$ use horizontal boxes (dy)



$x = \frac{y}{4}$ & $x = \pm\sqrt{y}$

$\frac{y}{4} = \sqrt{y}$

$y = 4\sqrt{y}$

$y^2 = 16y$

$y^2 - 16y = 0$

$y(y-16) = 0$

$y = 0,\ y = 16$

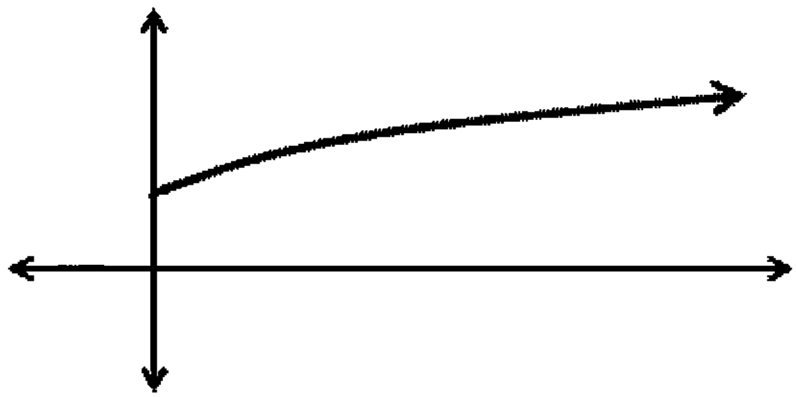
$$\int_0^{16} \left(\sqrt{y} - \frac{y}{4}\right) dy$$

MAC 2311 - Calculus w/Analytic Geometry I Name________________________________

Review for Test #4- Chapter 5 **SHOW ALL WORK** Date________________________________

Use a Riemann sum to estimate the area under the curve of the function on the given interval. Partition the interval into rectangles of equal width and evaluate the area using the Left-hand endpoint, Right-hand endpoint and Midpoint methods.

1) $f(x) = \sqrt{x} + 2$ on [2, 5] divided into 6 subintervals



MAC 2311 - Calculus w/Analytic Geometry I   Name____________________

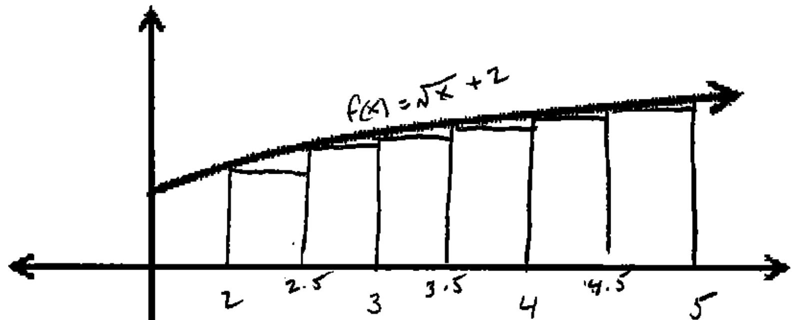
Review for Test #3 - Chapter 5   **SHOW ALL WORK**   Date____________________

Use a Riemann sum to estimate the area under the curve of the function on the given interval. Partition the interval into rectangles of equal width and evaluate the area using the Left-hand endpoint, Right-hand endpoint and Midpoint methods.

1) $f(x) = \sqrt{x} + 2$ on [2, 5] divided into 6 subintervals

$\Delta x = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2}$

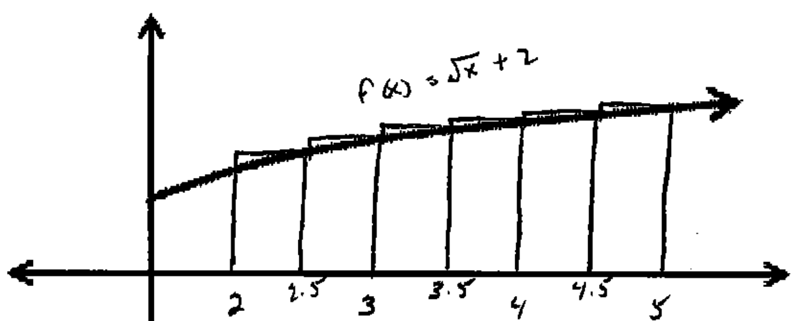
LH



$A = f(2)\frac{1}{2} + f(2.5)\frac{1}{2} + f(3)\frac{1}{2} + f(3.5)\frac{1}{2} + f(4)\frac{1}{2} + f(4.5)\frac{1}{2}$

$A = 11.360$

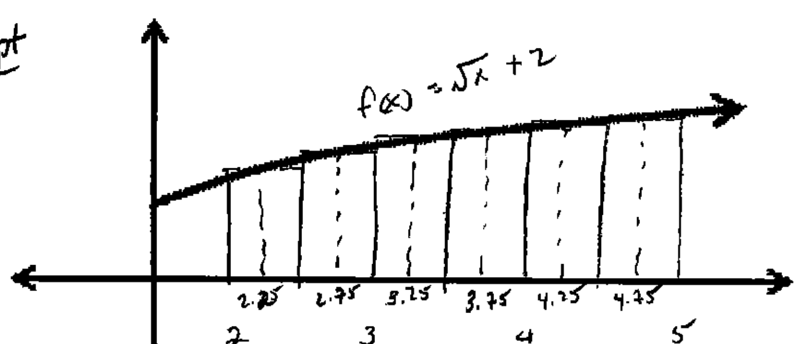
RH



$A = f(2.5)\frac{1}{2} + f(3)\frac{1}{2} + f(3.5)\frac{1}{2} + f(4)\frac{1}{2} + f(4.5)\frac{1}{2} + f(5)\frac{1}{2}$

$A = 11.771$

mid pt



$A = f(2.25)\frac{1}{2} + f(2.75)\frac{1}{2} + f(3.25)\frac{1}{2} + f(3.75)\frac{1}{2} + f(4.25)\frac{1}{2} + f(4.75)\frac{1}{2}$

$A = 11.569$

Write the sum without sigma notation and evaluate it.

2) $\sum_{k=1}^{2} \frac{18k}{k+37}$

A) $\frac{18}{1+37} + \frac{36}{2+37} = \frac{54}{77}$

B) $\frac{18}{1+37} + \frac{36}{2+37} = \frac{108}{247}$

C) $\frac{18}{1+37} + \frac{36}{2+37} = \frac{345}{247}$

D) $\frac{18}{1+37} + \frac{18}{2+37} = \frac{231}{247}$

3) $\sum_{k=1}^{3} (-1)^k (k-2)^2$

A) $-(1-2)^2 - 2(2-2)^2 - 3(3-2)^2 = -4$

B) $(1-2)^2 - (3-2)^2 = -2$

C) $-(1-2)^2 + (2-2)^2 - (3-2)^2 = 2$

D) $-(1-2)^2 + (2-2)^2 - (3-2)^2 = -2$

Express the sum in sigma notation.

4) $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$

A) $\sum_{k=1}^{5} \left(\frac{1}{3}\right)^{k-1}$

B) $\sum_{k=1}^{4} \left(\frac{1}{3}\right)^{k-1}$

C) $\sum_{k=1}^{4} \left(\frac{1}{3}\right)^{k}$

D) $\sum_{k=0}^{4} \left(\frac{1}{3}\right)^{k+1}$

5) $3 + 6 + 9 + 12 + 15$

A) $\sum_{k=2}^{5} 3(k-1)$

B) $\sum_{k=1}^{6} 3k$

C) $\sum_{k=1}^{5} 3(k+1)$

D) $\sum_{k=0}^{4} 3(k+1)$

Solve the problem.

6) Suppose that f and g are continuous and that $\int_5^9 f(x)\,dx = -6$ and $\int_5^9 g(x)\,dx = 8$. Find $\int_5^9 [5f(x) + g(x)]\,dx$.

A) -22

B) 10

C) 34

D) 13

7) Suppose that f and g are continuous and that $\int_6^{10} f(x)\,dx = -6$ and $\int_6^{10} g(x)\,dx = 10$. Find $\int_{10}^{6} [g(x) - f(x)]\,dx$.

A) 16

B) 4

C) -16

D) -4

Find the average value of the function over the given interval.

8) $f(x) = -2x + 4$ on $[-4, 2]$

A) 12

B) 2

C) 36

D) 6

9) $y = 6 - x^2$; $[-5, 4]$

A) - 5

B) $\frac{223}{27}$

C) $\frac{43}{27}$

D) - 1

Find the derivative.

10) $\frac{d}{dt}\int_0^{\sin t} \frac{1}{9-u^2}\,du$ Plug in upper limit function & multiply by its derivative

A) $\frac{\cos t}{9-\sin^2 t}$ B) $\frac{-\cos t}{9-\sin^2 t}$ C) $\frac{1}{\cos t\,(9-\sin^2 t)}$ D) $\frac{1}{9-\sin^2 t}$

11) $y=\int_0^{x^8} \cos\sqrt{t}\,dt$ $\cos\sqrt{x^8}\cdot 8x^7 = \boxed{\cos x^4 \cdot 8x^7}$

A) $8x^7\cos(x^4)$ B) $\cos(x^4)-1$ C) $\sin(x^4)$ D) $\cos(x^4)$

Evaluate the integral.

12) $\int 9(3x-1)^{-6}\,dx$ find U & du $U=3x-1$ $du=3$ →

A) $-\frac{6}{5}(3x-1)^{-5}+C$ B) $-\frac{3}{7}(3x-1)^{-7}+C$ C) $(3x-1)^{-5}+C$ D) $-\frac{3}{5}(3x-1)^{-5}+C$

13) $\int \frac{32s^3\,ds}{\sqrt{10-s^4}}$

A) $\frac{16s^4}{\sqrt{10-s^4}}$ B) $\frac{-8}{2\sqrt{10-s^4}}+C$ C) $-16\sqrt{10-s^4}+C$ D) $-16s^3\sqrt{10-s^4}+C$

14) $\int \csc^2 6\theta \cot 6\theta\;d\theta$

A) $\frac{1}{6}\csc^3 6\theta\cot^2 6\theta+C$

B) $-\frac{1}{12}\cot^2 6\theta+C$

C) $\frac{1}{12}\cot^2\theta+C$

D) $-\frac{1}{12}\tan^2 6\theta+C$

15) $\int \frac{x\,dx}{(7x^2+3)^5}$

A) $-\frac{1}{56}(7x^2+3)^{-4}+C$

B) $-\frac{1}{14}(7x^2+3)^{-6}+C$

C) $-\frac{7}{3}(7x^2+3)^{-6}+C$

D) $-\frac{7}{3}(7x^2+3)^{-4}+C$

16) $\int \csc^2(2\theta+9)\,d\theta$

$U=(2\theta+9)$

$du=2\,d\theta$

A) $2\cot(2\theta+9)+C$

B) $-\cot(2\theta+9)+C$

C) $4\csc(2\theta+9)\cot(2\theta+9)+C$

D) $-\frac{1}{2}\cot(2\theta+9)+C$

$\frac{1}{2}\int \csc^2(2\theta+9)\cdot 2\,d\theta = \frac{1}{2}\int \csc^2 u\,du = \frac{1}{2}\cdot(-\cot u)+C$

$= \boxed{-\frac{1}{2}\cot(2\theta+9)+C}$

(12) $3\int \frac{1}{3} 9 (3x-1)^{-6} dx \rightarrow \boxed{\frac{-3}{5}(3x-1)^{-5} + C}$

$3\int u^{-6} du$

$3\frac{u^{-5}}{-5} + C$

(13) $\int \frac{32 s^3 \, ds}{\sqrt{10-s^4}} = \int 32 s^3 (10-s^4)^{-\frac{1}{2}} du$ $\quad u = 10 - s^4$

$du = -4s^3 ds$

$-8\int -\frac{1}{8} 32 s^3 (10-s^4)^{-\frac{1}{2}} ds = -8\int u^{-\frac{1}{2}} du = -8 \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= -16 u^{\frac{1}{2}} + C = \boxed{-16(10-s^4)^{\frac{1}{2}} + C}$

(14) $\int \csc^2 6\theta \cot 6\theta \, d\theta$ $\quad u = \cot 6\theta$

$du = -\csc^2 6\theta \cdot (6) = -6\csc^2 6\theta \, d\theta$

$-\frac{1}{6}\int \cot 6\theta \cdot -6\csc^2 6\theta \, d\theta = -\frac{1}{6}\int u \, du = -\frac{1}{6} \cdot \frac{u^2}{2} + C$

$= -\frac{1}{12} u^2 + C = \boxed{-\frac{1}{12}(\cot 6\theta)^2 + C}$

(15) $\int x(7x^2+3)^{-5} dx = \frac{1}{14}\int (7x^2+3)^{-5} 14x \, dx$ $\quad u = 7x^2 + 3$

$du = 14x \, dx$

$= \frac{1}{14}\int u^{-5} du = \frac{1}{14} \cdot \frac{u^{-4}}{-4} + C = -\frac{1}{56} u^{-4} + C = \boxed{-\frac{1}{56}(7x^2+3)^{-4} + C}$

17) $\int \frac{1}{t^2} \sin\left(\frac{3}{t} + 6\right) dt$

A) $-\cos\left(\frac{3}{t} + 6\right) + C$

B) $\frac{1}{3} \cos\left(\frac{3}{t} + 6\right) + C$

C) $3 \cos\left(\frac{3}{t} + 6\right) + C$

D) $-\frac{1}{3} \cos\left(\frac{3}{t} + 6\right) + C$

18) $\int \frac{e^x\, dx}{\sqrt{1 - e^{2x}}}$

A) $e^x \sin^{-1}(e^x) + C$

B) $\sin^{-1}(e^x) + C$

C) $\sec^{-1}(e^x) + C$

D) $-2\sqrt{1 - e^{2x}} + C$

Solve the initial value problem.

19) $\frac{dy}{dx} = x \cos(9x^2)$, $y(0) = 11$

A) $y = \frac{1}{u} \sin(u)$

B) $y = \sin(9x^2) + 11$

C) $y = \frac{1}{18} \sin(9x^2) + 11$

D) $y = \frac{x^2}{2} \sin(9x^2) + 11$

20) $\frac{ds}{dt} = (4t + 3) \sin(2t^2 + 3t)$, $s(0) = -5$

A) $s = \cos(2t^2 + 3t) - 6$

B) $s = \sin(2t^2 + 3t) - 5$

C) $s = -\cos(2t^2 + 3t) - 4$

D) $s = -\cos(4t + 3) - \frac{5}{2}$

Evaluate the integral.

21) $\int_{3\pi/2}^{2\pi} \theta\, d\theta$

A) $\frac{7\pi^2}{8}$

B) $\frac{9\pi^2}{8}$

C) $\frac{\pi^2}{2}$

D) $\frac{\pi^2}{8}$

22) $\int_0^{1/5} t^2\, dt$

A) $\frac{1}{375}$

B) $-\frac{1}{5}$

C) 375

D) $-\frac{1}{375}$

23) $\int_2^{\sqrt{6}} (z - \sqrt{6})\, dz$

A) $-\sqrt{6}$

B) $-5\sqrt{6}$

C) $-3\sqrt{6}$

D) $-5 + 2\sqrt{6}$

(17) $\int t^{-2} \cdot \sin\left(\frac{3}{t}+6\right) dt = \int t^{-2} \sin(3t^{-1}+6)\, dt$

$u = 3t^{-1}+6$

$du = -3t^{-2}\, dt$

$-\frac{1}{3}\int \sin(3t^{-1}+6) \cdot -3t^{-2}\, dt = -\frac{1}{3}\int \sin u\, du = -\frac{1}{3}(-\cos u)+C$

$= \frac{1}{3}\cos u + C = \boxed{\frac{1}{3}\cos(3t^{-1}+6)+C}$

(18) $\int \frac{e^x\, dx}{\sqrt{1-(e^x)^2}} = \int \frac{1}{\sqrt{1-u^2}} \cdot du = \sin^{-1} u + C$

$u = e^x$

$du = e^x\, dx$

$\to \boxed{\sin^{-1}(e^x)+C}$

(19) $\frac{dy}{dx} = x\cos(9x^2)$, $y(0) = 11$

$f(0) = 11$

$u = 9x^2$

$du = 18x\, dx$

$dy = x\cos(9x^2)\, dx$

$\int dy = \int x\cos(9x^2)\, dx = y = \frac{1}{18}\int \cos(9x^2)\, 18x\, dx$

$y = \frac{1}{18}\int \cos u\, du = y = \frac{1}{18}\sin u + C$

$\boxed{f(x) = \frac{1}{18}\sin(9x^2)+11}$

$y = \frac{1}{18}\sin(9x^2)+C \to 11 = \frac{1}{18}\sin(9(0)^2)+C = \boxed{11 = C}$

(21) $\left.\dfrac{\theta^2}{2}\right|_{\frac{3\pi}{2}}^{2\pi}$

$= \dfrac{(2\pi)^2}{2} - \dfrac{\left(\frac{3\pi}{2}\right)^2}{2} = \dfrac{4\pi^2}{2} - \dfrac{\frac{9\pi^2}{4}}{2} = \dfrac{4\pi^2}{2} \quad \dfrac{9\pi^2}{8}$

$= \dfrac{16\pi^2}{8} - \dfrac{9\pi^2}{8} = \boxed{\dfrac{7\pi^2}{8}}$

(22) $\displaystyle\int_0^{1/5} t^2\,dt = \left.\dfrac{t^3}{3}\right|_0^{1/5} = \boxed{.0027}$

(23) $\displaystyle\int_2^{\sqrt{6}} (z-\sqrt{6})\,dz = \left.\dfrac{z^2}{2} - \sqrt{6}\,z\right|_2^{\sqrt{6}} = \boxed{-.1010}$

$U = 2t^2 + 3t$

$du = 4t+3 \ dt$

20 $\int ds = \int (4t+3) \sin (2t^2+3t) \, dt$

$\int ds = \int \sin u \, du$

$S = -\cos u + C$

$S = -\cos(2t^2+3t) + C$

$-5 = -\cos(0) + C$

$-5 = -1 + C$

$C = -4$

$S = \boxed{-\cos(2t^2+3t) - 4}$

24) $\int_0^7 (3x^2 + x + 8)\, dx$ $= X^3 + \frac{X^2}{2} + 8x \Big|_0^7 = \boxed{423.5}$

A) 64 B) $\frac{2485}{3}$ C) $\frac{847}{2}$ D) 189

25) $\int_1^4 \left(t + \frac{1}{t}\right)^2 dt$ $\int_1^4 (t+t^{-1})^2 dt = \int_1^4 (t^2+2+\frac{1}{t^2})dt = \left(\frac{t^3}{3} + 2t - \frac{1}{t}\right)\Big|_1^4 = \boxed{27.75}$

A) $\frac{79}{4}$ B) $\frac{349}{12}$ C) $\frac{111}{4}$ D) $\frac{365}{12}$

26) $\int_{\pi/2}^{3\pi/2} 10 \cos x\, dx$ $= 10 \sin x \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = 10\left(\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}\right) = -10 - 10 = \boxed{-20}$

A) 10 B) -20 C) -10 D) 20

27) $\int_0^1 \frac{10\, r\, dr}{\sqrt{9 + 5r^2}}$ $= 1.4833 = \int_0^1 10r\,(9+5r^2)^{-\frac{1}{2}} dr =$ $u = 9+5r^2$, $du = 10r\,dr$ $\int_0^1 u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} \Big|_0^1$

A) $2\sqrt{14} - 6$ B) $\frac{\sqrt{14}}{2} - \frac{3}{2}$ C) $\sqrt{14} - 3$ D) $-2\sqrt{14} + 6$

28) $\int_{\pi/3}^{2\pi} 4 \cos^3 x \sin x\, dx$

A) $-\frac{32769}{524288}$ B) $\frac{15}{16}$ C) $\frac{15}{16}$ D) $-\frac{15}{4}$

29) $\int_{\pi/8}^{\pi/4} 2 \cot(2\theta)\, d\theta$

A) $-\frac{\ln 2}{2}$ B) $\frac{\ln 4}{2}$ C) $\ln 2$ D) $\frac{\ln 2}{2}$

Sketch the graphs, shade the bounded region, draw a typical horizontal or vertical box, set up the integral, integrate and state the area.

30) $y = 2x - x^2$, $y = 2x - 4$ use "dx"

A) $\frac{32}{3}$ B) $\frac{37}{3}$ C) $\frac{31}{3}$ D) $\frac{34}{3}$

31) $y = x^3$, $y = 4x$ area in the first quadrant only and use "dy"

A) 16 B) 8 C) 4 D) 2

(28) $\displaystyle\int_{\frac{\pi}{3}}^{2\pi} 4\cos^3 x \sin x\, dx$

$U = \cos x$
$dU = -\sin x\, dx$

$\displaystyle -4\int_{\frac{\pi}{3}}^{2\pi} -\frac{1}{4}\cdot 4\cos^3 x \sin x\, dx = -4\int_{\frac{\pi}{3}}^{2\pi} U^3\, dU = -4\,\frac{U^4}{4}\Big|_{\frac{\pi}{3}}^{2\pi}$

$\displaystyle = -U^4\Big|_{\frac{\pi}{3}}^{2\pi} = -\cos^4 x\Big|_{\frac{\pi}{3}}^{2\pi} \rightarrow -\cos^4(2\pi) + \cos^4\left(\frac{\pi}{3}\right)$

$= -(1)^4 + \left(\frac{1}{2}\right)^4 = \boxed{-\frac{15}{16}}$

---

(29) $\displaystyle\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} 2\cot 2\theta\, d\theta = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} 2\,\frac{\cos 2\theta}{\sin 2\theta}\, d\theta$

$U = \sin 2\theta$
$dU = \cos 2\theta \cdot 2\, d\theta$

$\displaystyle \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1}{U}\, dU = \ln U\Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}} = \ln(\sin(2\theta))\Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}} =$

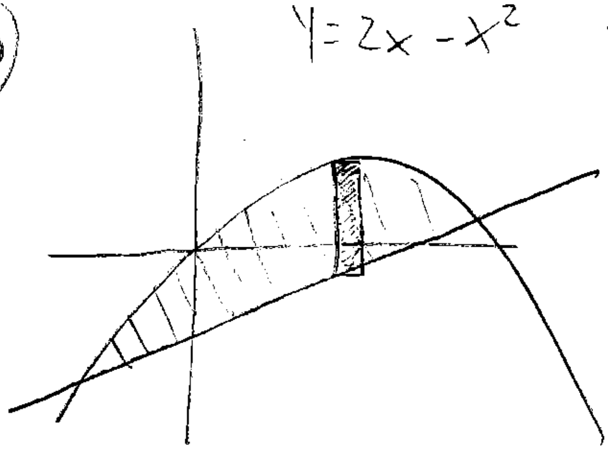
$\ln \sin 2\left(\frac{\pi}{4}\right) - \ln \sin 2\left(\frac{\pi}{8}\right) = \ln \sin\left(\frac{\pi}{2}\right) - \ln \sin\left(\frac{\pi}{4}\right)$

$= \ln 1 - \ln\frac{\sqrt{2}}{2} = 0 - \ln\frac{\sqrt{2}}{2} = \boxed{-\ln\frac{\sqrt{2}}{2}}$

Answer Key
Testname: MAC 2311 - REVIEW FOR TEST #4 - CHAPTER 5

1)
2) C
3) D
4) C
5) D
6) A
7) C
8) D
9) D
10) A
11) A
12) D
13) C
14) B
15) A
16) D
17) B
18) B
19) C
20) C
21) A
22) A
23) D
24) C
25) C
26) B
27) A
28) C
29) D
30) A
31) ~~B~~ C

(30) $Y = 2x - x^2 \quad y = 2x - 4 \quad$ POI $= 2x - x^2 = 2x - 4$



$2x - x^2 - 2x + 4 = 0$

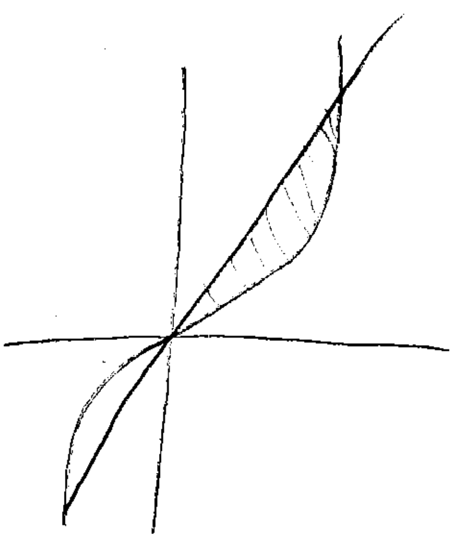
$x^2 - 4 = 0$

$x = -2, x = 2$

$$\int_{-2}^{2} [2x - x^2 - (2x - 4)]\,dx$$

$$\int_{-2}^{2} (2x - x^2 - 2x + 4)\,dx = \int_{-2}^{2} (-x^2 + 4)\,dx$$

$$= -\frac{x^3}{3} + 4x \Big|_{-2}^{2} = \boxed{10.67}$$



$Y = x^3, \; Y = 4x \qquad$ POI $= y^{\frac{1}{3}} = \frac{1}{4}y$

$y^{\frac{1}{3}} = x, \; \frac{1}{4}y = x$

$Y = \frac{1}{64}y^3$

$64y = y^3$

$y^3 - 64y = 0$

$y(y^2 - 64) = 0$

$Y = 0, \; Y = 8, \; Y = -8$

$$\int_0^8 y^{\frac{1}{3}} - \frac{1}{4}y\,dy$$

$$= \frac{3}{4}y^{4/3} - \frac{1}{8}y^2 \Big|_0^8$$

$$= \boxed{4}$$

MAC 2311 - Calculus w/Analytic Geometry I

Volumes of Solids - Cross Sections

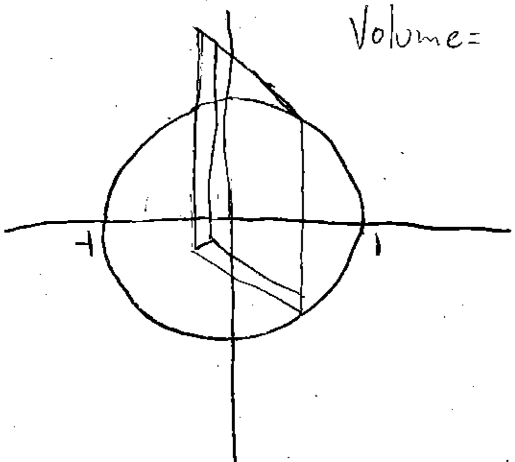
Lesson Problems

1. The base of the solid lies in the area bounded by $y=\sqrt{1-x^2}$, $y=-\sqrt{1-x^2}$, $x=-1$, and $x=1$. Cross-sections perpendicular to the x-axis are squares with a side of the square lying on the coordinate plane.

thickness = dx

Volume = l·w·ht

$$\text{Volume} = \int_{-1}^{1} \underbrace{(2\sqrt{1-x^2})}_{l}\underbrace{(2\sqrt{1-x^2})}_{ht}\,\underbrace{dx}_{w}$$



$$\int_{-1}^{1} 4(1-x^2)\,dx$$

$$\int_{-1}^{1} (4-4x^2)\,dx = 4x - \frac{4}{3}x^3 \Big|_{-1}^{1}$$

$$= \left[4(1) - \frac{4}{3}(1)^3\right] - \left[4(-1) - \frac{4}{3}(-1)^3\right]$$

$$\boxed{= 5.333}$$

Side = top f(x) - bottom f(x)

Side = $\sqrt{1-x^2} - (-\sqrt{1-x^2})$

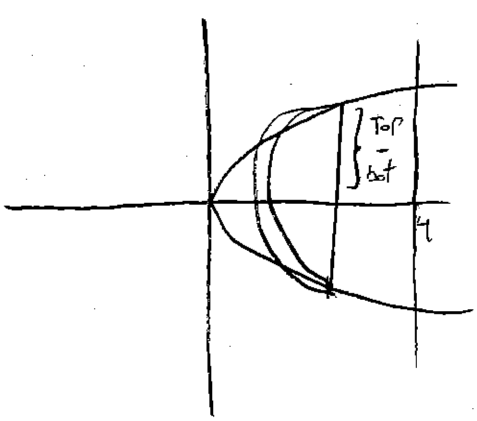
Side = $2\sqrt{1-x^2}$

2. The base of the solid lies in the area bounded by $y=\sqrt{x}$, $y=-\sqrt{x}$, $x=0$, and $x=4$. Cross-sections perpendicular to the x-axis are semicircles with the diameter running between the curves.

$A_{cir} = \pi r^2$

thickness = height

$$\int_0^4 \frac{1}{2}\pi r^2\,dx$$



$$\int_0^4 \frac{1}{2}\pi(\sqrt{x})^2\,dx$$

$$\int_0^4 \frac{1}{2}\pi x\,dx = \frac{1}{4}\pi x^2 \Big|_0^4$$

$$= \frac{\pi}{4}x^2 \Big|_0^4 =$$

Volume$_{\text{semi cir}}$ $= \frac{1}{2}\pi r^2 \cdot dx$

$$\left(\frac{\pi}{4}(4)\right) - \left(\frac{\pi}{4}(0)\right) = \boxed{12.566}$$

radius = $\sqrt{x} - 0$

3. The base of the solid lies in the area bounded by $y = \sqrt{1-x^2}$, $y = -\sqrt{1-x^2}$, $x = -1$, and $x = 1$. Cross-sections perpendicular to the x-axis are semicircles with the diameter running between the curves.

~~4. A solid lies in the area bounded by $y = \sqrt{1-x^2}$, $y = -\sqrt{1-x^2}$, $x = -1$, and $x = 1$. Cross-sections perpendicular to the x-axis are squares with the diagonal of the square running between the two semicircles.~~

MAC 2311 – Calculus w/Analytic Geometry I

Integration

"u du" Substitution

1) $\int 3(3x-1)^5\,dx$

$u = 3x-1$

$du = 3dx$

$\int (3x-1)^5 3dx$

$\int u^5 du$

$= \frac{u^6}{6} + C$

$= \frac{1}{6}(3x-1)^6 + C$

2) $\int (6x+7)^3\,dx$

$u = 6x+7$

$du = 6dx$

$\frac{1}{6}\int (6x+7)^3 6dx$

$\frac{1}{6}\int u^3 du$

$= \frac{1}{6}\left(\frac{u^4}{4}+c\right)$

$= \frac{1}{6}\left(\frac{(6x+7)^4}{4}+c\right)$

$= \boxed{\frac{(6x+7)^4}{24} + C}$

3) $\int \sqrt{(1-4x)}\,dx$

$u = 1-4x$

$du = -4dx$

$-\frac{1}{4}\int (1-4x)^{\frac{1}{2}} -4dx$

$-\frac{1}{4}\int u^{\frac{1}{2}} du$

$-\frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}}+c\right)$

$-\frac{1}{4}\left(\frac{2}{3}(1-4x)^{\frac{3}{2}}+c\right)$

$\boxed{-\frac{1}{6}(1-4x)^{\frac{3}{2}} + C}$

4) $\int x(2x^2+1)^6\,dx$

$u =$

$du =$

5) $\int (2x^2+1)^2\,dx$

$u =$

$du =$

6) $\int x^2(x^3-1)^{10}\,dx$

$u = x^3-1$

$du = 3x^2dx$

$\frac{1}{3}\int 3x^2(x^3-1)^{10}dx$

$\frac{1}{3}\int u^{10} du$

$\frac{1}{3}\cdot\left(\frac{u^{11}}{11}+c\right)$

$\frac{1}{3}\left(\frac{(x^3-1)^{11}}{11}+c\right)$

$\boxed{\frac{(x^3-1)^{11}}{33} + C}$

7) $\int x^4\sqrt{3x^5-5}\,dx$

$u =$

$du =$

8) $\int \sin^{10}(x)\cos(x)\,dx$ $\int u^{10}\,du$

$u = \sin x$

$du = \cos x\,dx$ $\frac{u^{11}}{11} + C$

$\boxed{\frac{(\sin x)^{11}}{11} + C}$

9) $\int 6x^2\sin\left(x^3\right)dx$

$u =$

$du =$

10) $\int \sin^2(2x)\cos(2x)\,dx$

$u =$

$du =$

11) $\int \frac{4y}{\sqrt{2y^2+1}}\,dy$ $\int (2y^2+1)^{-\frac{1}{2}} \cdot 4y\,dy$

$u = 2y^2+1$ $\int u^{-\frac{1}{2}}\,du$

$du = 4y\,dy$ $2u^{\frac{1}{2}} + C$

$\boxed{2(2y^2+1)^{\frac{1}{2}} + C}$

12) $\int \frac{5x}{(4+x^2)^2}\,dx$ $\int \frac{1}{(4+x^2)^2} \cdot 5x\,dx$

$u = 4+x^2$ $\int (4+x^2)^{-2} \cdot 5x\,dx$

$du = 2x\,dx$ $5 \cdot \frac{1}{2}\int u^{-2}\,dx$

$\frac{5}{2}\int u^{-2}\,dx$

$\frac{5}{2}\left(\frac{u^{-1}}{-1} + C\right)$

$\frac{5}{2}\left(-\frac{1}{u} + C\right)$

$\boxed{-\frac{5}{2(4+x^2)} + C}$

Review

1. ~~[illegible]~~ $f(x) = 8\sqrt{x} - x + 5 \quad (64, 5)$

$f(x) = 8x^{\frac{1}{2}} - x + 5$

$f'(x) = \frac{4}{x^{\frac{1}{2}}} - 1 \quad @ \; x = 64$

$f'(x) = \frac{4}{\sqrt{64}} - 1 \qquad \boxed{f'(x) = -\frac{1}{2}}$

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x - 64)$

$y - 5 = -\frac{1}{2} + 32$

$\boxed{y = -\frac{1}{2}x + 37}$

$\frac{dy}{dx}$ if $y = \frac{x}{8x - 2}$

$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{x+h}{8(x+h) - 2} - \frac{x}{8x - 2}}{h}$

$= \frac{\frac{(x+h)(8x-2) - [(x)(8(x+h) - 2)]}{(8x-2)(8 \cdot (x+h) - 2)}}{h}$ ↳

$$\frac{8x^2 - 2x + 8xh - 2h - 8x^2 - 8xh + 2x}{h(8x+8h-2)(8x-2)}$$

$$= \lim_{h \to 0} \frac{-2h}{h(8x+8h-2)(8x-2)}$$

$$= \boxed{\lim_{h \to} \frac{-2}{(8x-2)^2}}$$

3 $y = \dfrac{27}{x^2+2}$ $(1, 9)$

$$y' = \frac{(x^2+2)(0) - 27(2x)}{(x^2+2)^2}$$

$$y' = \frac{-54x}{(x^2+2)^2} \quad @\ x = 1$$

$$y' = \frac{-54}{9} = \boxed{-6}$$

$$\boxed{y - 9 = -6(x-1)}$$

(4) $f(x) = 2x^2 - 3x$ tangent line $\parallel$ to $y = 13x + 5$

$y' = 4x - 3$ $y' = 13$

$13 = 4x - 3$ $y = 2(4)^2 - 3(4)$

$\boxed{x = 4}$ $(4, 20)$ $\boxed{y = 20}$

(5) $s(t) = t^3 - 9t^2 + 24t$

$v(t) = 3t^2 - 18t + 24$ $v(t) = 0$

$0 = 3(t^2 - 6t + 8)$

$0 = 3((t-2)(t-4))$

$v(t) = 0 = t = 2$ & $t = 4$

$$a(t) = 6t - 18 \begin{cases} a(2) = 6(2) - 18 = \boxed{-6} \\ a(4) = 6(4) - 18 = \boxed{6} \end{cases}$$

(6) $s(t) = t^3 - 9t^2 + 24t$ $t = 0$ $t = 5$

$s(0) = 0$ > 20

$s(2) = 20$ > 4

$s(4) = 16$ > 4 +

$s(5) = 20$

$\boxed{28\ m}$

(7) $S(t) = t^3 - 9t^2 + 24t$ Displacement $t=0$ $t=5$

Displacement = End − begining

$S(5) - S(0) = 20 - 0 = 20m$ (right)

(8) $S(t) = 120t - 4t^2$

$V(t) = 120 - 8t$ $V(t) = 0 \rightarrow$ max height

$0 = 120 - 8t$

$\boxed{t = 15 \text{ sec}}$

how high does it go?

$S(15) = 120(15) - 4(15)^2$

$\boxed{S(15) = 900m}$

(9) $S(t) = 120t - 4t^2$ when it hits the ground?

$V(t) = 120 - 8t$ $S(t) = 0$ ground

$S(t) = 0$

$0 = 120t - 4t^2$ $\boxed{V(30) = -120 \frac{m}{sec}}$

$0 = 4t(30 - t)$

$4t = 0$ $30 - t = 0$

$\boxed{t = 0}$ $\boxed{t = 30}$

(10) $s(t) = 120t - 4t^2$

@ 100 ft above ground
$s(t) = 100$

$100 = 120t - 4t^2$

$4t^2 - 120t + 100 = 0$

$4(t^2 - 30t + 25) = 0$

$$t = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(25)}}{2(1)}$$

$t = 0.858$ sec , 29.142 sec

(11) $y = \frac{8}{x} + 3\sec x$

$$\frac{dy}{dx} = -8x^{-2} + 3\sec x(\tan x)$$

$$\boxed{\frac{dy}{dx} = \frac{-8}{x^2} + 3\sec x(\tan x)}$$

(12) $y = \frac{11}{\sin x} + \frac{1}{\cot x}$

$y = 11\csc x + \tan x$

$\boxed{y' = -11\csc x \cot x + \sec^2 x}$

(13) $s = t^5 - \csc t + 11$

$\frac{ds}{dt} = 5t^4 - (-\csc t \cot t)$

(14) $y = \frac{1}{6}(8x+7)^3 + (1 - x^{-3})^{-1}$

$\frac{dy}{dx} = \frac{1}{6} \cdot 3(8x+7)^2(8) - (1-x^{-3})^{-2}(3x^{-4})$

$= \frac{1}{2}(8x-7)^2 \cdot 8 - (1 - \frac{1}{x^3})^{-2} \cdot \frac{3}{x^4}$

15) $h(x) = \left(\dfrac{\cos x}{1+\sin x}\right)^5$

$h'(x) = 5\left(\dfrac{\cos x}{1+\sin x}\right)^4 \left(\dfrac{(1+\sin x)(-\sin x) - \cos x\,(\cos x)}{(1+\sin x)^2}\right)$

16) $y = \cos^5(\pi t - 19)$

$$\frac{dy}{dt} = 5(\cos^4(\pi t - 19))(-\sin(\pi t - 19))(\pi)$$

$$\boxed{\frac{dy}{dt} = -5\pi(\cos^4(\pi t - 19))(\sin(\pi t - 19))}$$

17) $y = t^5(t^4 - 6)^4$

$$\frac{dy}{dt} = 5t^4(t^4-6)^4 + (t^5)(4(t^4-6)^3(4t^3))$$

$$\frac{dy}{dt} = 16t^8(t^4-6)^3 + 5t^4(t^4-6)^4$$

$$\frac{dy}{dt} = t^4(t^4-6)^3(16t^4 + 5(t^4-6))$$

$$= \boxed{t^4(t^4-6)^3(21t^4 - 30)}$$

$$\frac{dy}{dt} = 4\left(e^{\cos(t/3)}\right)^3 .$$

(18) $y = \left(e^{\cos(t/3)}\right)^4$

$$\frac{dy}{dt} = 4\left(e^{\cos\frac{t}{3}}\right)^3 \left(-\sin\frac{t}{3}\right)\left(\frac{1}{3}\right)$$

$$\frac{dy}{dt} - \frac{4}{3} t \sin\frac{t}{3} \left(e^{\cos\frac{t}{3}}\right)^3$$

(19) $\frac{x+y}{x-y} = x^2 + y^2$

Don't Try !

$(x-y)$

(20) $\cos xy + x^5 = y^5$

$$(-\sin xy)\left(x \cdot \frac{dy}{dx} + 1y\right) + 5x^4 = 5y^4 \frac{dy}{dx}$$

$$-x\sin xy \frac{dy}{dx} - y\sin xy + 5x^4 - 5y^4\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(-x\sin xy - 5y^4\right) = -5x^4 + y\sin xy$$

$$\boxed{\frac{dy}{dx} = \frac{-5x^4 + y\sin xy}{-x\sin xy - 5y^4}}$$

(21) $y^5 + x^3 = y^2 + 12x$ tangent @ $(0, 1)$

$$5y^4 \frac{dy}{dx} + 3x^2 = 2y\frac{dy}{dx} + 12$$

$$5y^4 \frac{dy}{dx} - 2y\frac{dy}{dx} = 12 - 3x^2$$

$$\frac{dy}{dx} = \frac{12 - 3x^2}{5y^4 - 2y} = \frac{12}{3} = \boxed{4}$$

$$y - 1 = 4(x - 0)$$

$$\boxed{y = 4x + 1}$$

(22) $3x^2y - \pi\cos y = 4\pi$ normal @ $(1, \pi)$

$$3x^2\frac{dy}{dx} + 6xy + \pi\sin y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}\left(3x^2 + \pi\sin y\right) = -6xy$$

$$\frac{dy}{dx} = \frac{-6xy}{3x^2 + \pi\sin y} = -2\pi$$

normal $= +\frac{1}{2\pi}$

$$\boxed{y - \pi = \frac{1}{2\pi}(x - 1)}$$

(23) $y = 10e^{\theta}(\sin\theta - \cos\theta)$

$$\frac{dy}{d\theta} = 10e^{\theta}(\cos\theta + \sin\theta) + 10e^{\theta}(\sin\theta - \cos\theta)$$

$$= \cancel{10e^{\theta}\cos\theta} + 10e^{\theta}\sin\theta + 10e^{\theta}\sin\theta - \cancel{10e^{\theta}\cos\theta}$$

$$\boxed{\frac{dy}{d\theta} = 20e^{\theta}\sin\theta}$$

(24) $y = \ln\left(\frac{e^{\theta}}{9+e^{\theta}}\right)$

$y = \ln e^{\theta} - \ln(9+e^{\theta})$

$y = \theta \ln e - \ln(9+e^{\theta})$

$y = \theta \cdot 1 - \ln(9+e^{\theta})$

$$\frac{dy}{d\theta} = 1 - \frac{1}{9+e^{\theta}} \cdot e^{\theta}$$

$$\frac{dy}{d\theta} = 1 - \frac{e^{\theta}}{9+e^{\theta}} = \frac{9+e^{\theta}-e^{\theta}}{9+e^{\theta}}$$

$$\boxed{\frac{dy}{d\theta} = \frac{9}{9+e^{\theta}}}$$

25 $y = e^{\sin t}(\ln t^3 + 3)$

$$\frac{dy}{dt} = e^{\sin t}\left(\frac{1}{t^3} \cdot 3t^2 + 0\right) + e^{\sin t}\cos t(\ln t^3 + 3)$$

$$\frac{e^{\sin t}\, 3t^2}{t} + e^{\sin t}\cos t(\ln t^3 + 3)$$

$$\boxed{= e^{\sin t}\left(\frac{3}{t} + \cos t(\ln t^3 + 3)\right)}$$

26 $\tan y = e^x + \ln 7x$

$$\sec^2 y \frac{dy}{dt} = e^x + \frac{1}{7x} \cdot 7$$

$$\sec^2 y \frac{dy}{dx} = e^x + \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{e^x + \frac{1}{x}}{\sec^2 y}}$$

(27) $y = x^{5\sin x}$

$\ln y = \ln x^{5\sin x} = \ln y = 5\sin x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = 5\sin x \cdot \frac{1}{x} + 5\cos x \cdot \ln x$

$$\boxed{\frac{dy}{dx} = x^{5\sin x}\left(\frac{5}{x}\sin x + 5\cos x \ln x\right)}$$

2

(29) $y = 2\sin^{-1}(4x^4)$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{\sqrt{1-(4x^4)^2}} \cdot 16x^3$$

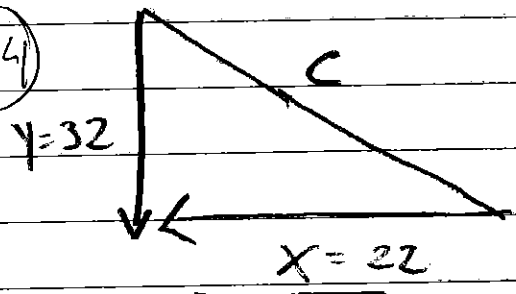
(32) $f(x) = \sqrt[3]{x}$ $\quad a=27$

$f(x) = x^{\frac{1}{3}}$ $\quad f'(x) = \frac{1}{3} \cdot x^{-\frac{2}{3}}$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = \sqrt[3]{27} + \frac{1}{3(\sqrt[3]{27})^2}(x-27)$

34



$\frac{dy}{dt} = -192$

$\frac{dx}{dt} = -298$

$c = \sqrt{x^2+y^2} \longrightarrow c = 38.883$

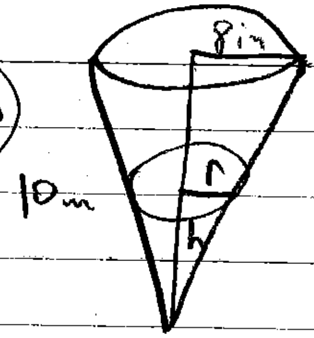
$x^2 + y^2 = c^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$

$x\frac{dx}{dt} + y\frac{dy}{dt} = c\frac{dc}{dt}$

$\frac{dc}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{c}$

35



find $\frac{dV}{dt}$ when $h = 8$ inch

$\frac{dh}{dt} = -1$

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{9}{10}$

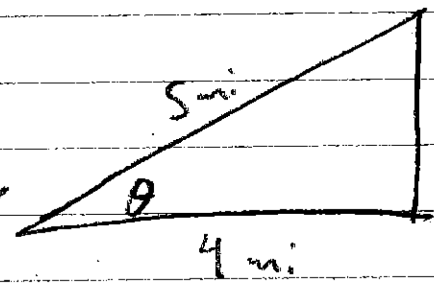
$V = \frac{1}{3}\pi\left(\frac{9}{10}h\right)^2 h$

$r = \frac{9}{10}h$

$V = \frac{\pi}{3} \cdot \frac{81}{100} h^3$

$V = \frac{27\pi}{100}$

(36)



$y = 3$ mi $\qquad \frac{dy}{dt} = 6 \frac{mi}{sec}$

$4^2 + 3^2 = c^2$

$\boxed{c = 5}$

$\tan\theta = \frac{y}{x} \rightarrow$ constant

$\tan\theta = y \cdot \frac{1}{4}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{4}\frac{dy}{dt}$

$\cos\theta = \frac{4}{5}$

$\cos^2\theta = \left(\frac{4}{5}\right)^2$

$\cos^2\theta = \frac{16}{25}$

$\frac{1}{\cos^2\theta} \cdot \frac{d\theta}{dt} = \frac{1}{4}\frac{dy}{dt}$

$\frac{1}{\frac{16}{25}} \cdot \frac{d\theta}{dt} = \frac{1}{4}\frac{dy}{dt}$

$\frac{d\theta}{dt} = \frac{1}{4}\frac{dy}{dt} \cdot \frac{16}{25}$

$\frac{d\theta}{dt} = \frac{1}{4}(6) \cdot \frac{16}{25}$

5) $S = t^3 - 9t^2 + 24t$ a(t) when V(t) = 0

$V(t) = 3t^2 - 18t + 24$

$0 = 3t^2 - 18t + 24$ $(t-2)(t-4)$

$0 = 3(t^2 - 6t + 8)$ $t = 2$ $t = 4$

$a(t) = 6t - 18$, $a(2) = -6$, $a(4) = 6$

6)
$$\left.\begin{array}{l} S(0) = 0 \\ S(2) = 20 \\ S(4) = 16 \\ S(5) = 20 \end{array}\begin{array}{l} 20 \\ 4 \\ 4 \end{array}\right\}$$
added $= 28\,m$

7) $S(5) - S(0) = 20\,m$

8) $S(t) = 120t - 4t^2$ How high, how long?

highest when Velocity $= 0$

$S'(t) = V(t) = 120 - 8t$

$0 = 120 - 8t \rightarrow t = 15$ sec

position of rock at 15 sec is highest point

$S(15) = 120(15) - 4(15)^2$

$S(15) = 900\,m$

9) $S(t) = 120t - 4t^2$ ground $S(t) = 0$

$0 = 120t - 4t^2$

$0 = 4(t^2 - 30t)$ $V(30) = 120 - 8(30)$

$0 = 4t(t - 30)$ $V(30) = -120 \frac{m}{sec}$

$t = 0$ & $t = 30$

10) $100 = 120t - 4t^2$

$\cancel{100 = 4t(t+30)}$ $-4t^2 + 120t - 100$

$-4(t^2 - 30t + 25)$

$t = \dfrac{-(-30) \pm \sqrt{(-30)^2 - 4(1)(25)}}{2(1)} =$ $t = 29.142$ sec

$t = .8579$ sec

17) $y = (e^{\cos(t/3)})^4$

$\dfrac{dy}{dt} = 4(e^{\cos(t/3)})^3 \cdot \cancel{\cos(\frac{t}{3})}\ -\sin(\frac{t}{3}) \cdot \frac{1}{3}$

33) $\dfrac{dA}{dt} = \dfrac{2\,mm^2}{sec}$ $r = 176mm$ $\dfrac{dr}{dt} = ?$

$A = \pi r^2$

MAC 2311 - Calculus w/Analytic Geometry I

Spring 2015

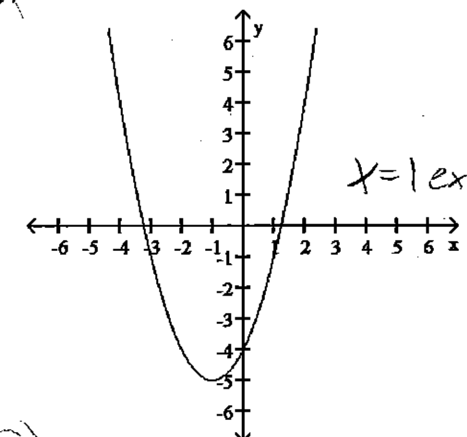
Review for Test #3 - Chapter 4

Solve the problem.

1) Find the graph that matches the given table.

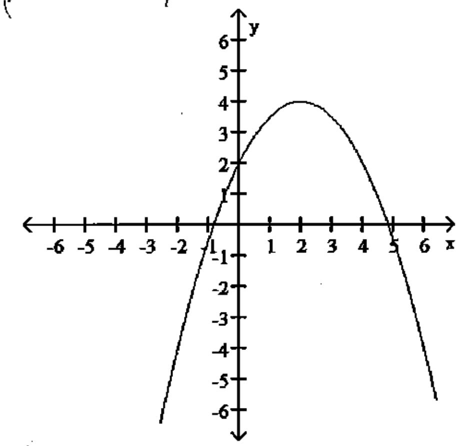
x	f'(x)
-1	0
1	does not exist
3	0

~~A)~~

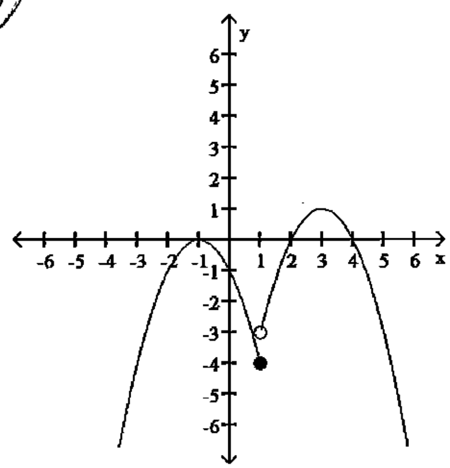


$x=1$ exist

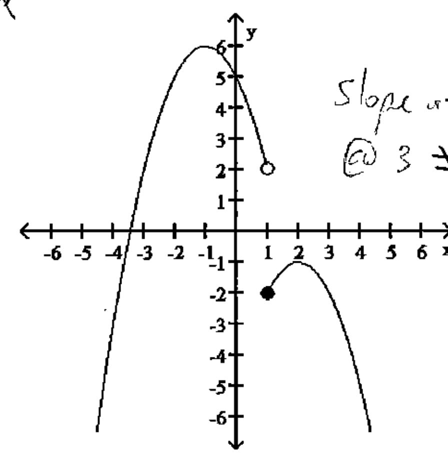
~~B)~~ Slope of Tan $-1 \neq 0$



C)



~~D)~~



Slope of Tan
@ 3 $\neq 0$

Determine whether the function satisfies the hypotheses of the Mean Value Theorem for the given interval.

2) $g(x) = x^{3/4}$, [0,5]

A) No

B) Yes

$g'(x) = \frac{3}{4}x^{-1/4}$

$= \frac{3}{4x^{\frac{1}{4}}}$

#1 $g(x)$ is continuous on $[0,5]$

#2 $g'(x)$ is differentiable on $(0,5)$
does not include endpoint

#9 $\frac{f\left(-\frac{\sqrt{3}}{3}\right) - f\left(\frac{\sqrt{3}}{3}\right)}{-\frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3}} = .907$

$f'(x) = \frac{1}{1+x^2} = .907$

$1+x^2 = \frac{1}{.907}$

$x^2 = 0.1025$

$\boxed{x = \pm .320}$

#5 $f(x) = x^{4/3}$

$f'(x) = \frac{4}{3}x^{\frac{1}{3}}$

$f'(x) = 0$ @ $x = 0$

$f(-1) = 1$

$f(0) = 0$ Abs Min

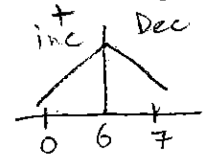
$f(8) = 16$ Abs Max

$g'(x) = -2x + 12$

$0 = -2x + 12$

$x = 6$

inc Dec



Find the absolute extreme values of the function on the interval.

3) $g(x) = -x^2 + 12x - 32, \ 4 \le x \le 8$

A) absolute maximum is 68 at x = 6; absolute minimum is 0 at 8 and 0 at x = 4

B) absolute maximum is 4 at x = 6; absolute minimum is 0 at 8 and 0 at x = 4

C) absolute maximum is 5 at x = 7; absolute minimum is 0 at 8 and 0 at x = 4

D) absolute maximum is 4 at x = 7; absolute minimum is 0 at 8 and 0 at x = 4

$f(4) = 0$

$f(6) = 4$ Abs Max

$f(8) = 0$

4) $f(x) = \csc x, \ -\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$

A) absolute maximum is -1 at x = π; absolute minimum is 1 at x = 0

B) absolute maximum is 1 at x = π; absolute minimum is -1 at x = π

C) absolute maximum does not exist; absolute minimum does not exist

D) absolute maximum is 0 at x = $-\pi$; absolute minimum is -1 at x = π

$f(x) = \csc x = \frac{1}{\sin x}$

$f(x) = \text{UND} = x = 0, \pi, 3\pi, 3\pi$

$f'(x) = -\csc x \cot x = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$

$f'(x) = 0 \ @ \ x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

$f'(x)$ UND $= x = 0, \pi$

No Abs extreme value

Find the absolute extreme values of the function on the interval.

5) $f(x) = x^{4/3}, \ -1 \le x \le 8$

A) absolute maximum is 64 at x = 8; absolute minimum is 0 at x = 0

B) absolute maximum is 16 at x = 8; absolute minimum does not exist

C) absolute maximum is 16 at x = 8; absolute minimum is 1 at x = -1

D) absolute maximum is 16 at x = 8; absolute minimum is 0 at x = 0

6) $f(x) = -6e^{-x^2}, -\infty < x < \infty$

A) Minimum value is -6 at x = 0; maximum value is $-\frac{6}{e}$ at x = 1

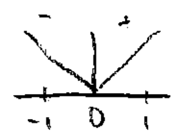
B) No minimum value and no maximum value

C) Minimum value is - 6 at x = 0; no maximum value

D) Maximum value is - 6 at x = 0; minimum value

$f'(x) = -6e^{-x^2}(-2x)$

$f'(x) = \frac{12x}{e^{x^2}}$



$f'(x) = 0 \ @ \ x = 0$

$f(0) = -\frac{6}{1} = -6$ min val Abs Min

7) $f(x) = \ln(-x), -7 \le x \le -1$

A) Minimum value is 0 at x = -1; no maximum value

B) Maximum value is 0 at x = -1; minimum value is -ln 7 at x = -7

C) No minimum value; no maximum value

D) Minimum value is 0 at x = -1; maximum value is ln 7 at x = -7

Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the function and interval.

8) $f(x) = x^2 + 5x + 2, \ [1, 2]$

$f'(x) = 2x + 5$ $\quad \frac{f(2) - f(1)}{2 - 1} = \frac{16 - 8}{1} = 8 \ \therefore \ f'(c) = 2x + 5$, $8 = 2x + 5$ $\quad \boxed{x = \frac{3}{2}}$

A) $\frac{3}{2}$ $\quad$ B) 1, 2 $\quad$ C) 0, $\frac{3}{2}$ $\quad$ D) $-\frac{3}{2}, \frac{3}{2}$

9) $f(x) = \tan^{-1} x, \left[-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right]$ $\quad$ Round to the nearest thousandth.

A) 0.320 $\quad$ B) 0, 0.320 $\quad$ C) ±0.320 $\quad$ D) -0.320, 0, 0.320

Answer the question.

10) A trucker handed in a ticket at a toll booth showing that in 3 hours he had covered 222 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

$\frac{222 \text{ mi}}{3 \text{ hr}} = 74 \text{ mph} \rightarrow$ AVG.

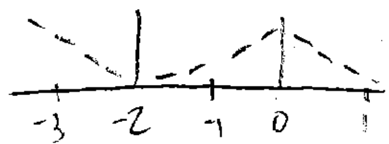
15) $y' = \frac{(x^2+2x+2)(1) - (x+1)(2x+2)}{(x^2+2x+2)^2} = \frac{x^2+2x+2-2x^2-4x-2}{(x^2+2x+2)^2}$

$f'(x) = \frac{-x^2-2x}{(x^2+2x+2)^2} \longrightarrow -x^2-2x = 0$

$x(-x-2) = 0$

No values will make this UND

$x = 0$ & $x = -2$



$f(-2) = -.5$ local min

$f(0) = .5$ local max

Determine all critical points for the function.

11) $f(x) = x^3 - 12x + 5$

A) $x = 2$
B) $x = -2$
C) $x = -2$ and $x = 2$
D) $x = -2$, $x = 0$, and $x = 2$

12) $f(x) = 20x^3 - 3x^5$

A) $x = 2$
B) $x = -2$
C) $x = -2$ and $x = 2$
D) $x = 0$, $x = -2$, and $x = 2$

13) $y = 4x^2 - 128\sqrt{x}$ $\quad f'(x) = 8x - 128 \cdot \frac{1}{2} \cdot x^{-\frac{1}{2}}$

$f'(x) = 8x - \frac{64}{x^{\frac{1}{2}}} = 0$

$8x^{3/2} - 64 = 0 \longrightarrow x = 8^{\frac{2}{3}} = \boxed{4}$

$x = 0$ UND

A) $x = 0$
B) $x = 0$, $x = 4$, and $x = -4$
C) $x = 4$
D) $x = 0$ and $x = 4$

Find the extreme values of the function and where they occur.

14) $y = x^3 - 12x + 2$

A) Local maximum at (0, 0).
B) Local maximum at (2, -14), local minimum at (-2, 18).
C) Local maximum at (-2, 18), local minimum at (2, -14).
D) None

15) $y = \dfrac{x + 1}{x^2 + 2x + 2}$

A) None
B) The maximum is $-\frac{1}{2}$ at $x = 0$; the minimum is $\frac{1}{2}$ at $x = -2$.
C) The maximum is $\frac{1}{2}$ at $x = 0$; the minimum is $-\frac{1}{2}$ at $x = -2$.
D) The maximum is 2 at $x = 0$; the minimum is $\frac{1}{2}$ at $x = -2$.

16) $y = \dfrac{\ln x}{x^2}$ $\quad \ln x > 0$

A) Maximum value is $\frac{1}{2e}$ at $x = e^{1/2}$; minimum value is 0 at $x = 1$.
B) Maximum value is $\frac{1}{2e}$ at $x = e^{1/2}$; no minimum value.
C) Minimum value is $\frac{1}{2e}$ at $x = e^{1/2}$; no maximum value.
D) None

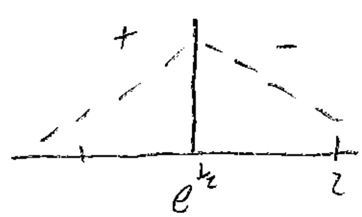
$f'(x) = \dfrac{x^2 \cdot \frac{1}{x} - \ln x \cdot 2x}{x^4} = \dfrac{x - 2x\ln x}{x^4} = \dfrac{x(1 - 2\ln x)}{x^4} = \dfrac{1 - 2\ln x}{x^3}$

$1 - 2\ln x = 0$

$2\ln x = 1$

$\ln x = \frac{1}{2}$

$\boxed{x = e^{\frac{1}{2}}}$



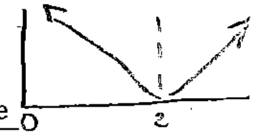
Absolute Max @ $x = e^{\frac{1}{2}}$

$Y = x^{8/3} - 16x^{2/3}$

$Y' = \frac{8}{3}x^{5/3} - \frac{32}{3}x^{-\frac{1}{3}} = \frac{8}{3}x^{-\frac{1}{3}}\left[x^{6/3} - 4\right] = \frac{8}{3x^{1/3}}\left[x^2 - 4\right] = 0$

Find the derivative at each critical point and determine the local extreme values.

$x = \cancel{-2}, 2$



17) $y = x^{2/3}(x^2 - 16);\ x \geq 0$

~~A)~~

Critical Pt.	derivative	Extremum	Value
x = 0	~~0~~	maximum	0
x = 2	0	minimum	-19.049

~~B)~~

Critical Pt.	derivative	Extremum	Value
x = 0	Undefined	local max	0
x = 2	0	minimum	31.748

C)

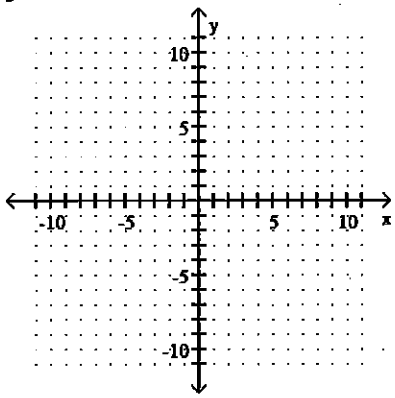
Critical Pt.	derivative	Extremum	Value
x = 0	Undefined	local max	0
x = 2	0	minimum	-19.049

~~D)~~

Critical Pt.	derivative	Extremum	Value
x = 0	Undefined	local max	~~4~~
x = 2	0	minimum	-19.049

Identify all local extrema and inflection points.

18) $y = -x^4 + 2x^2 - 9$



Use l'Hopital's Rule to evaluate the limit.

19) $\lim\limits_{x \to 9} \frac{x^2 - 81}{x - 9} = \frac{0}{0}$ $\quad \lim\limits_{x \to 9} \frac{2x}{1} = 2(9) = \boxed{18}$

A) -18 B) -9 C) 18 D) 9

20) $\lim\limits_{x \to 0} \frac{\cos 9x - 1}{x^2} = \frac{0}{0}$ $\quad \lim\limits_{x \to 0} \frac{(-\sin 9x)9}{2x} = \frac{0}{0} = \lim\limits_{x \to 0} \frac{(-\cos 9x)81}{2} = \boxed{-\frac{81}{2}}$

A) $\frac{9}{2}$ B) $\frac{81}{2}$ C) 0 D) $-\frac{81}{2}$

21) $\lim\limits_{x \to 0} \frac{x}{\sin x} = \frac{0}{0}$ $\quad \lim\limits_{x \to 0} \frac{1}{\cos x} = \boxed{1}$

A) -1 B) 0 C) 1 D) $\frac{1}{2}$

22) $\lim\limits_{x \to \infty} \frac{16x^2 + 7x - 5}{13x^2 - 5x + 18} = \frac{16}{13}$

A) 1 B) $\frac{16}{13}$ C) $-\frac{16}{13}$ D) $\frac{13}{16}$

#27 $V = l \cdot w \cdot h$

$V(x) = (40-2x)(40-2x)(x)$

$V(x) = (40-2x)(40x-2x^2)$

$V(x) = 1600x - 160x^2 + 4x^3$

$V'(x) = 1600 - 320x + 12x^2$

$V'(x) = 0 \qquad x = 6.67 \ \& \ x = 20$

20 doesn't work as its min

$x = 6.67$ in

$40 - 2x = 26.67$ in

28) $V = x \cdot x \cdot h$

$60 = x^2 \cdot h$

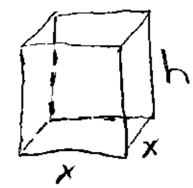
$\frac{60}{x^2} = h$

$SA = 4(xh) + x^2$

$SA = 4xh + x^2$

$SA = 4x \cdot \frac{60}{x^2} + x^2$

$SA = \frac{240}{x} + x^2$



$SA = 240x^{-1} + x^2$

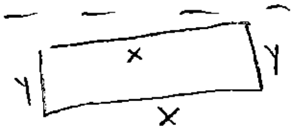
$SA' = -240x^{-2} + 2x = 0$

$0 = \frac{-240}{x^2} + 2x \qquad LCD = x^2$

$0 = -240 + 2x^3$

$x^3 = 120 = \boxed{x = 4.9\text{ft}} \quad \boxed{h = \frac{60}{4.9^2} = 2.5}$

min cost



29) $Cost = 2x + 2y$

$Cost = \$7(2x) + \$5(2y)$

$Cost = 14x + 10y$

$Cost = 14x + 10\left(\frac{840}{x}\right)$

$C(x) = 14x + 8400x^{-1}$

$C'(x) = 14 - 8400x^{-2}$

$0 = 14 - \frac{8400}{x^2}$

$LCD = x^2$

$0 = 14x^2 - 8400$

$14x^2 = 8400$

$\boxed{x = \pm 24.5}$

can't be minus

$Area = 840 ft^2 = l \cdot w$

$840 = l \cdot w$

$840 = x \cdot y$

$y = 840/x$

$y = \frac{840}{24.5}$

$\boxed{y = 34.3\text{ft}}$

Use l'Hôpital's rule to find the limit.

23) $\lim\limits_{x \to 0} \dfrac{\sin 6x}{\tan 4x}$ $\frac{0}{0}$ $\lim\limits_{x\to 0} \dfrac{(\cos 6x)6}{(\sec^2 4x)4} = \dfrac{6}{4}\cos(6x) \cdot \cos^2(4x) \to \dfrac{3}{2}\cos(6x)\cdot\cos^2(4x) = \boxed{\dfrac{3}{2}}$

$\boxed{\text{A) } \frac{3}{2}}$ B) $\frac{2}{3}$ C) 0 D) $-\frac{3}{2}$

24) $\lim\limits_{x \to \infty} x \sin \dfrac{20}{x}$ $\infty \cdot 0$ $\dfrac{\sin(20x^{-1})}{\frac{1}{x}} = \dfrac{0}{0} = \lim\limits_{x\to\infty} \dfrac{\cos(20x^{-1})(-20x^{-2})}{-x^{-2}} = \lim\limits_{x\to\infty} 20\cos\left(\frac{20}{x}\right) \to 20 \cdot 1 = 20$

A) 0 B) $\frac{1}{20}$ C) 1 D) 20

25) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 6x} - x\right)$

A) 6 B) 3 C) 0 D) - 3

Find the limit.

26) $\lim\limits_{x \to 0^+} x^{-2/\ln x}$

A) e^2 B) $\frac{1}{e^2}$ C) $\frac{1}{e}$ D) -2

Solve the problem.

27) From a thin piece of cardboard 40 in. by 40 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

A) 20 in. × 20 in. × 10 in.; 4000 in^3
B) 26.7 in. × 26.7 in. × 13.3 in.; 9481.5 in^3
C) 26.7 in. × 26.7 in. × 6.7 in.; 4740.7 in^3
D) 13.3 in. × 13.3 in. × 13.3 in.; 2370.4 in^3

28) A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 60 ft^3. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

A) 3.9 ft × 3.9 ft × 3.9 ft
B) 5.6 ft × 5.6 ft × 1.9 ft
C) 4.9 ft × 4.9 ft × 2.5 ft = 60.025
D) 11 ft × 11 ft × 0.5 ft

29) A rectangular field is to be enclosed on four sides with a fence. Fencing costs \$7 per foot for two opposite sides, and \$5 per foot for the other two sides. Find the dimensions of the field of area 840 ft^2 that would be the cheapest to enclose. = min

A) 24.5 ft @ \$7 by 34.3 ft @ \$5
B) 40.6 ft @ \$7 by 20.7 ft @ \$5
C) 34.3 ft @ \$7 by 24.5 ft @ \$5
D) 20.7 ft @ \$7 by 40.6 ft @ \$5

Find the most general antiderivative.

30) $\int (5x^3 + 8x + 3)\,dx$

A) $5x^4 + 8x^2 + 3x + C$
B) $15x^4 + 16x^2 + 3x + C$
C) $\frac{5}{4}x^4 + 4x^2 + 3x + C$
D) $15x^2 + 8 + C$

(31) $\int\left(\frac{\sqrt{y}}{6}+\frac{3}{\sqrt{y}}\right)dy = \int\left(\frac{1}{6}y^{\frac{1}{2}}+3y^{-\frac{1}{2}}\right)dy$

$= \frac{1}{6}\cdot\frac{y^{\frac{3}{2}}}{\frac{3}{2}} + 3\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + C = \boxed{\frac{1}{9}y^{\frac{3}{2}} + 6y^{\frac{1}{2}} + C}$

(33) $\int\left(\frac{\frac{1}{\cos\theta}}{\frac{1}{\cos\theta}-\cos\theta}\right)d\theta = \frac{1}{\cos\theta}\cdot\frac{1}{\frac{1}{\cos\theta}-\cos\theta} = \frac{1}{\cos\theta\left(\frac{1}{\cos\theta}-\cos\theta\right)}$

$= \frac{1}{1-\cos^2\theta}$ → identity $= \frac{1}{\sin^2\theta} = \csc^2\theta\, d\theta$

$\int(\csc^2\theta)\,d\theta = \boxed{-\cot\theta + C}$

$-\cot\theta + C$

---

(26) $y = x^{-\frac{2}{\ln x}}$

$\ln y = \lim_{x\to 0}\ln x^{-\frac{2}{\ln x}} \rightarrow \ln y = -\frac{2}{\ln x}\cdot\lim_{x\to 0^+}\ln x$

$\ln y = \lim_{x\to 0^+} -\frac{2}{\cancel{\ln x}}\cdot \cancel{\ln x}$

$\ln y = -2$

$e^{\ln y} = e^{-2}$

$y = e^{-2} \rightarrow \boxed{y = \frac{1}{e^2}}$

31) $\int \left(\frac{\sqrt{y}}{6} + \frac{3}{\sqrt{y}}\right) dy$

A) $\frac{1}{4}y^{3/2} + \frac{1}{6}\sqrt{y} + C$ B) $\frac{1}{9}y^{3/2} - 6\sqrt{y} + C$ C) $\frac{1}{12}\sqrt{y} - \frac{1}{6\sqrt{y}} + C$ D) $\frac{1}{9}y^{3/2} + 6\sqrt{y} + C$

32) $\int (-9 \sec^2 x)\, dx$

A) $-9 \tan x + C$ B) $9 \cot x + C$ C) $\frac{\tan x}{9} + C$ D) $-9 \cot x + C$

33) $\int \frac{\sec \theta}{\sec \theta - \cos \theta} d\theta$

A) $\cos^2 \theta + C$ B) $-\cot \theta + C$ C) $\theta + \tan \theta + C$ D) $\cot \theta + C$

Solve the initial value problem. = Find C

34) $\frac{dy}{dx} = 3x^{-3/4}$, $y(1) = 1$

A) $y = 12x^{1/4} - 11$ B) $y = 12x^{1/4} + 12$ C) $y = -\frac{3}{4}x^{-7/4} - \frac{11}{4}$ D) $y = 3x^{1/4} - 2$

35) $\frac{dr}{dt} = 4t + \sec^2 t$, $r(-\pi) = 4$ $dr = \int (4t + \sec^2 t)\, dt$

A) $r = 2t^2 + \cot t + 4 - 2\pi^2$ B) $r = 4t^2 + \tan t + 4 - 4\pi^2$

C) $r = 4 + \tan t + 0$ D) $r = 2t^2 + \tan t + 4 - 2\pi^2$

$r(t) = \frac{4t^2}{2} + \tan t + C$

$r(t) = 2t^2 + \tan t + C$

$4 = 2t^2 + \tan t + C$

$4 = 2(-\pi)^2 + \tan(-\pi) + C$

$4 = 2\pi^2 + C$

$4 - 2\pi^2 = C$ $\therefore$ $\boxed{r = 2t^2 + \tan t + (4 - 2\pi^2)}$

36) Plan and plot a graph the meets the following criteria. Each block represents one unit.

$f(-4) = 6,\ f(5) = 2$ $(-4,6)\ (5,2)$

$f'(-4) = 0$

$f'(x) > 0$ for $(-\infty, -4) \cup (2, \infty)$

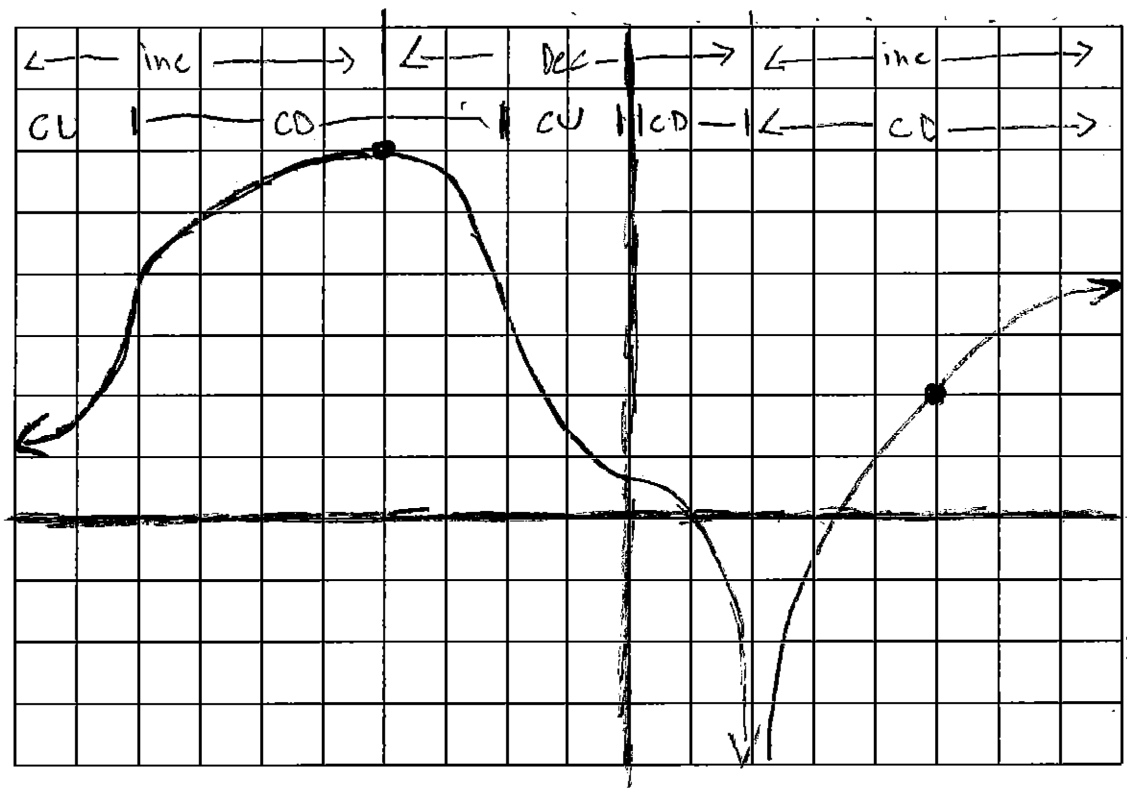
$f'(x) < 0$ for $(-4, 2)$

$f''(-7) = f''(-2) = 0$

$f''(x) > 0$ for $(-\infty, -7) \cup (-2, 0)$

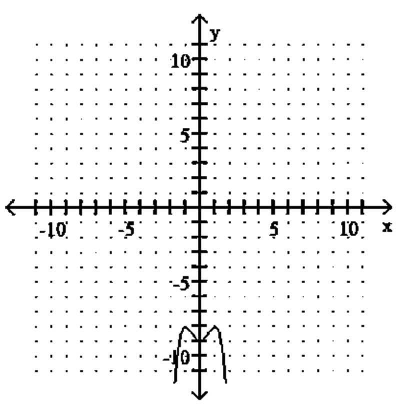
$f''(x) < 0$ for $(-7, -2) \cup (0, 2) \cup (2, \infty)$

$\lim_{x \to -\infty} f(x) = 1$, $\lim_{x \to \infty} f(x) =$ [redacted] 4 $\lim_{x \to 2^-} f(x) = -\infty$, $\lim_{x \to 2^+} f(x) = -\infty$



Answer Key
Testname: MAC 2311 - REV T3 W ANS - CHAPTER 4

1) C
2) B
3) B
4) C
5) D
6) C
7) D
8) A
9) C
10) As the trucker's average speed was 74 mph, the Mean Value Theorem implies that the trucker must have been going that speed at least once during the trip.
11) C
12) D
13) D
14) C
15) C
16) B
17) C
18) Absolute maxima: (-1, -8), (1, -8)
Local minimum: (0, -9)
Inflection points: $\left(-\sqrt{\frac{1}{3}}, \frac{8}{9}\right), \left(\sqrt{\frac{1}{3}}, \frac{8}{9}\right)$

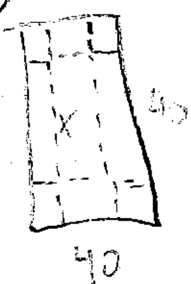


19) C
20) D
21) C
22) B
23) A
24) D
25) B
26) B
27) C
28) C
29) A
30) C
31) D
32) A

33) B
34) A
35) D
36)

(17) $f(x) = x^{2/3}$

(27)



$V = l \cdot w \cdot h$

$V = (40-2x)(40-2x) \cdot x$

$V = 1600x - 160x^2 + 4x^3$

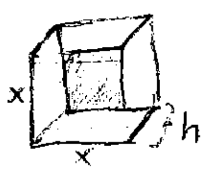
$V' = 1600 - 320x + 12x^2$

$0 = 1600 - 320x + 12x^2$

$x = 6.67 \qquad 40 - 2(6.67) = 26.67$

$26.67 \times 26.67 \times 6.67$

(28)



$V = 60ft^3$

$V = x \cdot x \cdot h$

$V = x^2 h$

$60 = x^2 h$

$h = \frac{60}{x^2}$

$SA = 4(xh) + x^2$

$SA = 4x\left(\frac{60}{x^2}\right) + x^2$

$SA = \frac{240}{x} + x^2$

(34) $\frac{dy}{dx} = 3x^{-3/4} \qquad f(x) = 1$

$f(1) = 1 \qquad (1,1)$

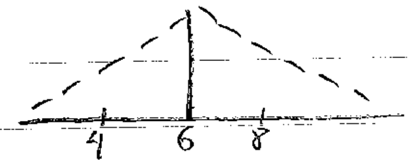
$\frac{1}{\frac{1}{4}} \cdot 3x^{\frac{1}{4}} = 12x^{\frac{1}{4}} + c$

$1 = 12(1)^{\frac{1}{4}} + c$

$1 = 12 + c$

$c = -11$

③ $g(x) = -x^2 + 12x - 32 \quad [4, 8]$



$g'(x) = -2x + 12 = 0$

$-2x = -12 \Rightarrow x = \frac{12}{2} \Rightarrow x = 6$

$f(4) = 0, \quad f(6) = 4, \quad f(8) = 0$

④ $\csc x, \quad [-\frac{\pi}{2}, \frac{3\pi}{2}]$ $\quad \boxed{\text{No Abs extreme values}}$

?

$f'(x) = -\cot x \csc x$

$f'(x) = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\frac{\cos x}{\sin^2 x} \quad x \neq 0, \pi, 2\pi$ UND

$\cos x = 0$ @ $x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

⑥ $f(x) = -6e^{-x^2} \quad (-\infty, \infty)$

$f'(x) \; -6e^{-x^2} \cdot (-2x) = 12xe^{-x^2} = \frac{12x}{e^{x^2}}$

$f'(x) = \frac{12x}{e^{x^2}}$ @ $x = 0 \quad f(0) = -6$ Abs Min

⑧ $f(x) = x^2 + 5x + 2 \quad [1, 2]$

$\frac{f(2) - f(1)}{2 - 1} = \frac{16 - 8}{1} = 8$

$f'(x) = 2x + 5$

$2x + 5 = 8$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

(11) $f(x) = x^3 - 12x + 5$

$f'(x) = 3x^2 - 12$

$0 = 3x^2 - 12$

$3x^2 = 12 \rightarrow x^2 = \frac{12}{3} = x^2 = 4 \Rightarrow x = \pm 2$

(12) $f(x) = 20x^3 - 3x^5$

$f'(x) = 60x^2 - 15x^4$

$0 = 15x^2(4 - x^2)$

$x = 0$ & $x = \pm 2$

(13) $f(x) = \frac{\ln x}{x^2}$

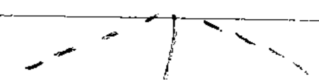
$f'(x) = \ln x \cdot x^{-2}$

$f'(x) = \frac{1}{x} \cdot x^{-2} + \ln x(-2x^{-3})$

$f'(x) = \frac{1}{x^3} + \frac{\ln x(-2)}{x^{-3}}$

$f'(x) = \frac{1 - 2\ln x}{x^3}$

$x \neq 0$



$e^{\frac{1}{2}}$

$0 = 1 - 2\ln x$

$2\ln x = 1$ $\qquad f(e^{\frac{1}{2}}) = \frac{1}{2e}$ abs Max

$\ln x = \frac{1}{2}$

$x = e^{\frac{1}{2}}$

MAC 2311 - Calculus w/Analytic Geometry I

REVIEW for Test #2 - Chapter 3- Differentiation

Find an equation for the tangent to the curve at the given point.

1) $f(x) = 8\sqrt{x} - x + 5,\ (64, 5)$

A) $y = 5$ B) $y = -\frac{1}{2}x + 5$ C) $y = \frac{1}{2}x - 37$ D) $y = -\frac{1}{2}x + 37$

Find the indicated derivative using the formal definition of the derivative.

2) $\frac{dy}{dx}$ if $y = \frac{x}{8x - 2}$

A) $-\frac{2x}{(8x - 2)^2}$ B) $-\frac{2}{(8x - 2)^2}$ C) $-\frac{2}{8x - 2}$ D) $\frac{16x - 2}{(8x - 2)^2}$

Provide an appropriate response.

3) Find an equation for the tangent to the curve $y = \frac{27}{x^2 + 2}$ at the point (1, 9).

A) $y = -6x + 15$ B) $y = -3x + 12$ C) $y = -6$ D) $y = 6x + 3$

4) Find all points (x, y) on the graph of $f(x) = 2x^2 - 3x$ with tangent lines parallel to the line $y = 13x + 5$.

A) (8, 20) B) (4, 32) C) (4, 20) D) (0, 0), (4, 20)

Solve the problem.

5) At time t (sec), the position of a body moving along the s-axis is $s = t^3 - 9t^2 + 24t$ m. Find the body's acceleration each time the velocity is zero.

A) $a(2) = 6\ m/sec^2,\ a(4) = -6\ m/sec^2$ B) $a(2) = 0\ m/sec^2,\ a(4) = 0\ m/sec^2$

C) $a(4) = 24\ m/sec^2,\ a(8) = 4\ m/sec^2$ D) $a(2) = -6\ m/sec^2,\ a(4) = 6\ m/sec^2$

6) At time t, the position of a body moving along the s-axis is $s = t^3 - 9t^2 + 24t$ m. Find the total distance traveled by the body from t = 0 to t = 5.

A) 63 m B) 28 m C) 31 m D) 27 m

7) At time t, the position of a body moving along the s-axis is $s = t^3 - 9t^2 + 24t$ m. Find the displacement of the body from t = 0 to t = 5.

A) 20 m B) 45 m C) 49 m D) 105 m

8) A rock is thrown vertically upward from the surface of an airless planet. It reaches a height of $s = 120t - 4t^2$ meters in t seconds. How high does the rock go? How long does it take the rock to reach its highest point?

A) 900 m, 15 sec B) 1800 m, 30 sec C) 3480 m, 30 sec D) 1785 m, 15 sec

9) A rock is thrown vertically upward from the surface of an airless planet. It reaches a height of $s = 120t - 4t^2$ meters in t seconds. When will the rock hit the ground? What is the velocity at impact?

A) 5 sec, 150 mps B) 15 sec, -130 mps C) 26 sec, 110 mps D) 30 sec, -120 mps

10) A rock is thrown vertically upward from the surface of an airless planet. It reaches a height of $s = 120t - 4t^2$ meters in t seconds. When will the rock be at 100 feet above the ground?

A) 15 sec, 25 sec
B) 4.5 sec
C) 29.142 sec, 0.858 sec
D) 26 sec

Find the derivative.

11) $y = \frac{8}{x} + 3 \sec x$

A) $y' = -\frac{8}{x^2} + 3 \tan^2 x$
B) $y' = -\frac{8}{x^2} - 3 \csc x$
C) $y' = \frac{8}{x^2} - 3 \sec x \tan x$
D) $y' = -\frac{8}{x^2} + 3 \sec x \tan x$

12) $y = \frac{11}{\sin x} + \frac{1}{\cot x}$

A) $y' = 11 \cos x - \csc^2 x$
B) $y' = -11 \csc x \cot x + \sec^2 x$
C) $y' = 11 \csc x \cot x - \csc^2 x$
D) $y' = 11 \csc x \cot x - \sec^2 x$

13) $s = t^5 - \csc t + 11$

A) $\frac{ds}{dt} = 5t^4 + \cot^2 t$
B) $\frac{ds}{dt} = 5t^4 + \csc t \cot t$
C) $\frac{ds}{dt} = t^4 - \cot^2 t + 11$
D) $\frac{ds}{dt} = 5t^4 - \csc t \cot t$

Find the derivative of the function.

14) $y = \frac{1}{6}(8x + 7)^3 + \left(1 - \frac{1}{x^3}\right)^{-1}$

A) $\frac{4}{3}(8x + 7)^2 + \frac{3}{x^4}\left(1 - \frac{1}{x^3}\right)^{-2}$
B) $\frac{1}{2}(8x)^2 - \left(\frac{3}{x^4}\right)^{-2}$
C) $4(8x + 7)^2 - \frac{3}{x^4}\left(1 - \frac{1}{x^3}\right)^{-2}$
D) $\frac{1}{2}(8x + 7)^2 - \left(1 - \frac{1}{x^3}\right)^{-2}$

15) $h(x) = \left(\frac{\cos x}{1 + \sin x}\right)^5$

A) $\frac{-5 \cos^4 x}{(1 + \sin x)^5}$
B) $\left(-\frac{4 \sin x}{\cos x}\right)\left(\frac{\cos x}{1 + \sin x}\right)^4$
C) $-5\left(\frac{\sin x}{\cos x}\right)^4$
D) $5\left(\frac{\cos x}{1 + \sin x}\right)^4$

Find dy/dt.

16) $y = \cos^5(\pi t - 19)$

A) $-5\pi \cos^4(\pi t - 19) \sin(\pi t - 19)$
B) $-5 \cos^4(\pi t - 19) \sin(\pi t - 19)$
C) $-5\pi \sin^4(\pi t - 19)$
D) $5 \cos^4(\pi t - 19)$

17) $y = t^5(t^4 - 6)^4$

A) $t^4(t^4 - 6)^3(21t^4 - 30)$

B) $5t^4(t^4 - 6)^3(16t^4 - 6)$

C) $80t^{18}(t^4 - 6)^3$

D) $t^5(t^4 - 6)^3(21t^3 - 30)$

18) $y = \left(e^{\cos(t/3)}\right)^4$

A) $\frac{4}{3}\cos\left(\frac{t}{3}\right)e^{4\cos(t/3)}$

B) $-\frac{4}{3}\sin\left(\frac{t}{3}\right)e^{4\cos(t/3)}$

C) $\frac{4}{3}\sin\left(\frac{t}{3}\right)e^{3\cos(t/3)}$

D) $-\frac{4}{3}\left(e^{\sin(t/3)}\right)^3$

Use implicit differentiation to find dy/dx.

19) $\frac{x + y}{x - y} = x^2 + y^2$

A) $\frac{x(x - y)^2 - y}{x + y(x - y)^2}$

B) $\frac{x(x - y)^2 - y}{x - y(x - y)^2}$

C) $\frac{x(x - y)^2 + y}{x - y(x - y)^2}$

D) $\frac{x(x - y)^2 + y}{x + y(x - y)^2}$

20) $\cos xy + x^5 = y^5$

A) $\frac{5x^4 - x\sin xy}{5y^4}$

B) $\frac{5x^4 + y\sin xy}{5y^4 - x\sin xy}$

C) $\frac{5x^4 + x\sin xy}{5y^4}$

D) $\frac{5x^4 - y\sin xy}{5y^4 + x\sin xy}$

At the given point, find the slope of the curve, the line that is tangent to the curve, or the line that is normal to the curve, as requested.

21) $y^5 + x^3 = y^2 + 12x$, tangent at $(0, 1)$

A) $y = -\frac{12}{7}x$

B) $y = 4x + 1$

C) $y = \frac{12}{5}x + 1$

D) $y = -\frac{12}{5}x - 1$

22) $3x^2y - \pi\cos y = 4\pi$, normal at $(1, \pi)$

A) $y = \frac{1}{2\pi}x - \frac{1}{2\pi} + \pi$

B) $y = -\frac{1}{\pi}x + \frac{1}{\pi} + \pi$

C) $y = \frac{1}{\pi}x - \frac{1}{\pi} + \pi$

D) $y = -2\pi x + 3\pi$

Find the derivative of y with respect to x, t, or θ, as appropriate.

23) $y = 10e^{\theta}(\sin\theta - \cos\theta)$

A) $10e^{\theta}(\sin\theta - \cos\theta) + 10e^{\theta}$

B) $20e^{\theta}\sin\theta$

C) $20e^{\theta}(\sin\theta - \cos\theta)$

D) 0

24) $y = \ln\left(\frac{e^{\theta}}{9 + e^{\theta}}\right)$

A) $\frac{9 + e^{\theta}}{e^{\theta}}$

B) $\ln\left(\frac{9}{9 + e^{\theta}}\right)$

C) $\frac{9}{9 + e^{\theta}}$

D) $\frac{9 + 2e^{\theta}}{9 + e^{\theta}}$

25) $y = e^{\sin t}(\ln t^3 + 3)$

A) $e^{\cos t}(\cos t)(\ln t^3 + 3) + \dfrac{3e^{\sin t}}{t}$

B) $e^{\sin t}\left((\cos t)(\ln t^3 + 3) + \dfrac{3}{t}\right)$

C) $\dfrac{3e^{\sin t}\cos t}{t}$

D) $e^{\sin t}\left(\ln t^3 + 3 + \dfrac{3}{t}\right)$

Find $\dfrac{dy}{dx}$.

26) $\tan y = e^x + \ln 7x$

A) $\dfrac{xe^x + 1}{x\sec^2 y}$

B) $e^x + \dfrac{7}{x} - \sec^2 y$

C) $\dfrac{xe^x + 7}{x\sec^2 y}$

D) $\dfrac{e^x + 7}{\sin^2 y}$

Use logarithmic differentiation to find the derivative of y with respect to the independent variable.

27) $y = x^{5\sin x}$

A) $x^{\sin x}\left(\cos x \ln x + \dfrac{\sin x}{x}\right)$

B) $5x^{5\sin x}\left(\cos x \ln x + \dfrac{\sin x}{x}\right)$

C) $5 \sin x \ln x$

D) $5\cos x \ln x + \dfrac{\sin x}{x}$

28) $y = (\sin x)^{\cos x}$

A) $\cos x \ln(\sin x)$

B) $\cos x \cot x - \ln(\sin x)$

C) $(\sin x)^{\cos x}(\cos x \cot x - \sin x \ln(\sin x))$

D) $\cos x \cot x - \sin x \ln(\sin x)$

Find the derivative of y with respect to x.

29) $y = 2\sin^{-1}(4x^4)$

A) $\dfrac{32x^3}{\sqrt{1 - 16x^8}}$

B) $\dfrac{32x^3}{\sqrt{1 - 16x^4}}$

C) $\dfrac{2}{\sqrt{1 - 16x^8}}$

D) $\dfrac{32x^3}{1 - 16x^8}$

30) $y = \tan^{-1}(\ln 3x)$

A) $\dfrac{1}{1 + (\ln 3x)^2}$

B) $\dfrac{3}{x(1 + (\ln 3x)^2)}$

C) $\dfrac{1}{x(1 + (\ln 3x)^2)}$

D) $\dfrac{1}{x\sqrt{1 + (\ln 3x)^2}}$

Find the linearization L(x) of f(x) at x = a.

31) $f(x) = \sqrt{2x + 36}$, $a = 0$

A) $L(x) = \frac{1}{3}x + 6$

B) $L(x) = \frac{1}{3}x - 6$

C) $L(x) = \frac{1}{6}x - 6$

D) $L(x) = \frac{1}{6}x + 6$

32) $f(x) = \sqrt[3]{x}$, $a = 27$

A) $L(x) = \frac{1}{27}x + 1$

B) $L(x) = \frac{1}{9}x + 6$

C) $L(x) = \frac{1}{27}x + 2$

D) $L(x) = \frac{1}{9}x + 1$

Solve the problem.

33) Water is falling on a surface, wetting a circular area that is expanding at a rate of 2 mm^2/s. How fast is the radius of the wetted area expanding when the radius is 176 mm? (Round your answer to four decimal places.)

A) 0.0018 mm/s B) 0.0114 mm/s C) 552.9198 mm/s D) 0.0036 mm/s

Solve the problem. Round your answer, if appropriate. Round all answers to three decimal places.

34) One airplane flying south is approaching an airport at a rate of 192 km/hr. A second airplane flying west approaches the airport at a rate of 298 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 32 km away from the airport and the westbound plane is 22 km from the airport.

A) -654.123 km/hr B) -163.527 km/hr C) -327.041 km/hr D) -490.131 km/hr

35) Water is being drained from a container which has the shape of an inverted right circular cone. The container has a radius of 9.00 inches at the top and a height of 10.0 inches. At the instant when the water in the container is 8.00 inches deep, the surface level is falling at a rate of 1 in./sec. Find the rate at which water is being drained from the container.

A) 128.774 $in.^3s$ B) 175.681 $in.^3/s$ C) 156.229 $in.^3/s$ D) 2.860 $in.^3/s$

36) A rocket is fired vertically into the sky at a rate of 6 miles per second. An observer is located 4 miles away from the launching pad (horizontal distance). How fast is the observer's angle of elevation changing when the rocket is at an altitude of 3 miles?

A) 0.96 rad/sec B) 0.362 rad/sec C) 3.344 rad/sec D) 0.844 rad/sec

Answer Key
Testname: MAC 2311 - REVIEW FOR TEST 2 - CH 3

1) D
2) B
3) A
4) C
5) D
6) B
7) A
8) A
9) D
10) C
11) D
12) B
13) B
14) C
15) A
16) A
17) A
18) B
19) C
20) D
21) B
22) A
23) B
24) C
25) B
26) A
27) B
28) C
29) A
30) C
31) D
32) C
33) A
34) C
35) D
36) A

MAC 2311 – Calculus w/Analytical Geometry I
Optimization Problems

1) The position function of a projectile launched vertically upward from an elevated position is $s(t) = -16t^2 + 87t + 129$. Position (s) is measured in feet and time (t) is measured in seconds.

a) When will the projectile hit the ground? $= t \to s(t) = 0$

b) What is the impact velocity?

c) When will the projectile reach its maximum height?

d) What is its maximum height?

a) $0 = -16t^2 + 87t + 129 = \dfrac{-(87) \pm \sqrt{(87)^2 - 4(-16)(129)}}{2(-16)} = -1.212$ sec & 6.650 sec

$V(t) = -32t + 87$

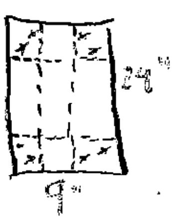
b) $V(t) = -32(6.65) + 87 = -125.8 \frac{ft}{sec}$

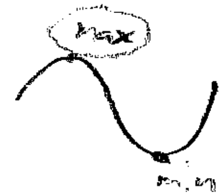
c) $0 = -32t + 87 = 32t = 87 \rightarrow t = \frac{87}{32} \rightarrow \boxed{t = 2.719 \text{ sec}}$

d) $S(2.719) = 16(2.719)^2 + 87(2.719) + 129$

$S(2.719) = \boxed{247.266 ft}$

2) A cardboard box manufacturing company has to maximize the volume of a box made from a 24-inch by 9-inch sheet of cardboard. What are the dimensions of the box?





$V = l \cdot w \cdot h$

$l = 24 - 2x$

$w = 9 - 2x$

$h = x$

$V = (24-2x)(9-2x)(x)$

$V = 216x - 18x^2 - 48x^2 + 4x^3$

$V = 4x^3 - 66x^2 + 216x$

$V' = 12x^2 - 132x + 216$

$0 = 12x^2 - 132x + 216$

$0 = 12(x-2)(x-9)$

$x = 2$ in & $\cancel{x = 9}$ too big

$x =$ height

$l = 24 - 2(2) = 20$ in

$w = 9 - 2(2) = 5$ in

$\boxed{20 \text{ in} \times 5 \text{ in} \times 2 \text{ in}}$

3) A box is to be constructed where the base length is 3 times the base width. The material used to build the top and bottom cost \$10 per square foot and the material used to build the sides cost \$6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box.

B+ T

$C = 10 \cdot 2(3x \cdot x) + 6 \cdot 2 \cdot (3x \cdot h) + 6 \cdot 2\,(x \cdot h)$

$C = 60x^2 + 36xh + 12xh$

$C = 60x^2 + 48xh$

$C = 60x^2 + 48x\left(\frac{50}{3x^2}\right)$

$C = 60x^2 + \frac{2400}{3x}$

$C = 60x^2 + 800x^{-1}$

$C' = 120x - 800x^{-2}$

$0 = 120x - \frac{800}{x^2}$

$V = (3x)x \cdot h$

$V = 3x^2h$

$50 = 3x^2h$

$h = \frac{50}{3x^2}$

$0 = 120x^3 - 800$

$x^3 = \frac{800}{120}$

$x = \sqrt[3]{\frac{20}{3}} = \boxed{x = 1.882\,ft}$

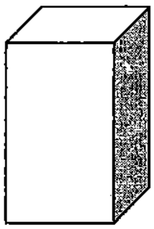
$\boxed{3x = 5.646\,ft}$

$h = \frac{50}{3(1.882)^2} = \boxed{4.706\,ft}$

$\boxed{5.646\,ft \times 1.882\,ft \times 4.706\,ft}$

4) A printer need to make a poster that will have a total area of 200 in² and will have 1 inch margins on the sides, a 2 inch margin on the top and a 1.5 inch margin on the bottom. What dimensions will give the largest printed area?

5) An open rectangular box with a square base is to be made from 48 square feet of material. What dimensions will result in a box with the largest possible volume?



MAC 2311 - Calculus w/Analytical Geometry 1

Worksheet - Curve Sketching

Sketch a graph of the function from the given information. Plan ahead prior to plotting the x and y axis. Each box is 1 unit in length and 1 unit in height. The graph must be neat, legible, and fit within the provided grid system.

1)

f is continuous.

$f(2)=4,\ f(4)=3$ (2,4), (4,3)

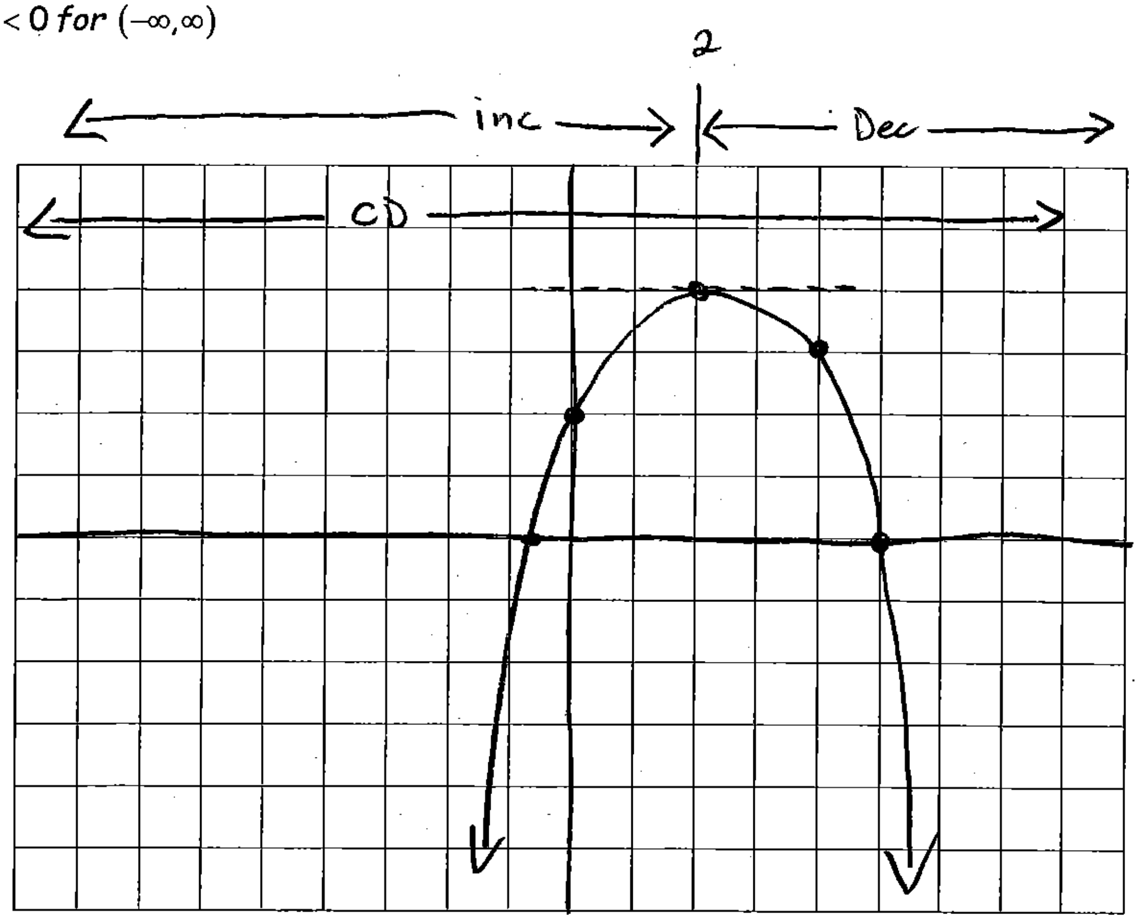
x-intercepts $=-\frac{1}{2}, 5$ y-intercept $=2$

$f'(2)=0$

$f'(x)>0$ for $(-\infty,2)$

$f'(x)<0$ for $(2,\infty)$

$f''(x)<0$ for $(-\infty,\infty)$



State the extreme values and their classification.

Sketch a graph of the function from the given information. Plan ahead prior to plotting the x and y axis. Each box is 1 unit in length and 1 unit in height. The graph must be neat, legible, and fit within the provided grid system.

2)

f is continuous.

$f(0)=5, \quad f(3)=2, \quad f(5)=-1, \quad f(8)=1$

$f'(0)=f'(5)=0$

$f'(x)>0$ *for* $(-\infty,0)\cup(5,\infty)$

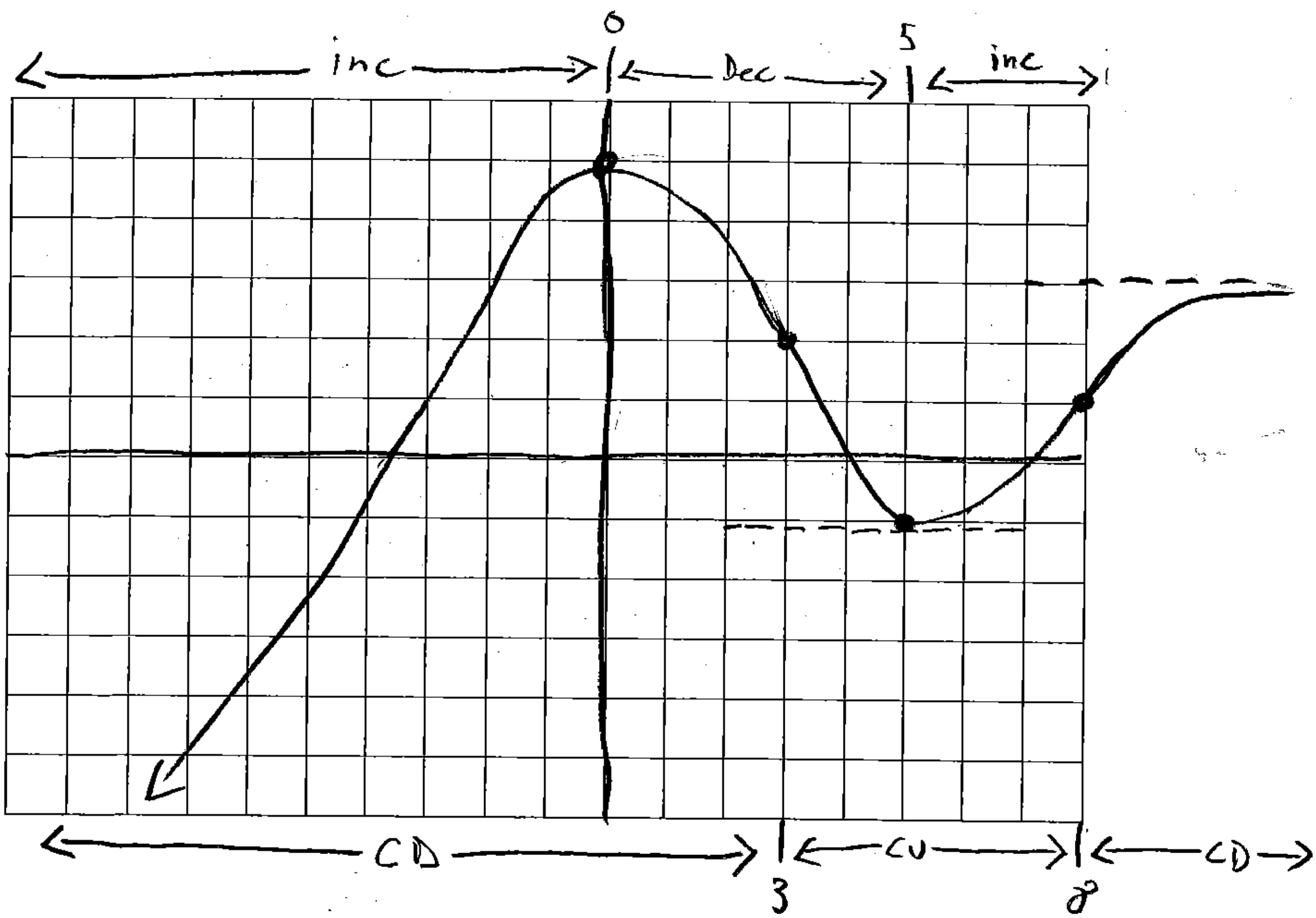
$f'(x)<0$ *for* $(0,5)$

$f''(3)=f''(8)=0$

$f''(x)>0$ *for* $(3,8)$

$f''(x)<0$ *for* $(-\infty,3)\cup(8,\infty)$

$\lim_{x\to\infty} f(x)=3$



State the extreme values and their type.

Sketch a graph of the function from the given information. Plan ahead prior to plotting the x and y axis. Each box is 1 unit in length and 1 unit in height. The graph must be neat, legible, and fit within the provided grid system.

3)

f is continuous.

$f(-9)=4, \quad f(-4)=5, \quad f(-2)=2, \quad f(0)=4, \quad f(-6)=7$

$f'(-6)=f'(-2)=0$

$f'(x)>0$ *for* $(-\infty, -6) \cup (-2, \infty)$

$f'(x)<0$ *for* $(-6, -2)$

$f''(-9)=f''(-4)=f''(0)=0$

$f''(x)>0$ *for* $(-\infty,-9) \cup (-4,0)$

$f''(x)<0$ *for* $(-9,-4) \cup (0,\infty)$

$\lim_{x\to-\infty} f(x)=-1, \qquad \lim_{x\to\infty} f(x)=6$

State the extreme values and their classification.

MAC 2311 – Calculus with Analytical Geometry I

Related Rates

1. Read the problem, pull out essential information and identify a formula to be used.

2. Sketch a diagram if possible.

3. Write down any known rate of change & the rate of change you are looking for.

4. Be careful with signs...if the amount is decreasing, the rate of change is negative.

5. Pay attention to whether quantities are constant or varying.

6. Set up an equation involving the appropriate quantities.

7. Differentiate with respect to t using implicit differentiation.

8. Plug in known items (you may need to find some quantities using geometry).

9. Solve for the item you are looking for, most often this will be a rate of change.

10. State your final answer with the appropriate units.

1. $If x = y^3 - y \, and \frac{dy}{dt} = 5, \, then \, what \, is \, the \, value \, of \frac{dx}{dt} when \, y = 2?$

$$X = y^3 - y$$

$$\frac{dx}{dt} = 3y^2 \frac{dy}{dt} - \frac{dy}{dt}$$

$$\frac{dx}{dt} = 3(2^2)(5) - 5$$

$$\boxed{\frac{dx}{dt} = 55}$$

2. $If r + s^2 + v^3 = 12, \frac{dr}{dt} = 4, \, and \frac{ds}{dt} = -3, \, then \, what \, is \, the \, value \, of \frac{dv}{dt} when \; r = 3, \, s = 1, \, and \, v = 4?$

$$r + s^2 + v^3 = 12$$

$$\frac{dr}{dt} + 2s\frac{ds}{dt} + 3v^2\frac{dv}{dt} = 0$$

$$4 + 2(1)(-3) + 3(4^2)\frac{dv}{dt}$$

$$48\frac{dv}{dt} = 2$$

$$\frac{dv}{dt} = \frac{2}{48}$$

$$= \boxed{\frac{dv}{dt} = 0.0417}$$

MAC 2311 – Calculus with Analytic Geometry I
The Derivative as a Rate of Change
Rectilinear and Projectile Motion

Position: $s(t)$ Velocity: $v(t)=\frac{ds}{dt}$ Acceleration: $a(t)=\frac{dv}{dt}=\frac{d^2s}{dt^2}$

1) The position of a skateboarder at any time t (in seconds) is given by the function $s(t)=t^3-8t^2+8t$ measured in feet.

a) What are the velocity and acceleration functions in terms of t?

$S'(t) = v(t) = 3t^2 - 16t + 8$

$S''(t) = a(t) = 6t - 16$

b) When is the skateboarder at rest?

$0 = 3t^2 - 16t + 8$

$0 = (3t$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \frac{16 \pm \sqrt{256 - 4(3)(8)}}{6}$

$t = \frac{16 \pm \sqrt{160}}{6} = \frac{16 \pm 4\sqrt{10}}{6}$

$t = 0.558$
&
$t = 4.775$

c) What is the position(s) of the skateboarder when at rest?

$S(.558) = (.558)^3 - 8(.558)^2 + 8(.558) = 2.147$ ft

$s(4.775) = (4.775)^3 - 8(4.775)^2 + 8(4.775) = -35.332$ ft

d) What are the position, velocity, and acceleration of the skateboarder at three seconds and five seconds?

$S(3) = (3)^3 - 8(3)^2 + 8(3) = \boxed{-21}$, $S(5) = \boxed{-35}$

$v(3) = 3(3)^2 - 16(3) + 8 = \boxed{-12.99}$, $v(5) = \boxed{3.000001}$

$a(3) = 6(3) - 16 = \boxed{2}$, $a(5) = \boxed{14}$

~~e) Sketch a motion schematic of the skateboarder. Be sure to label (1) the initial position, (2) the initial velocity, (3) the position and velocity at any critical time, and (4) the times at which these occur.~~

f) What is the total distance traveled by the skateboarder in the first five seconds?

$|S(5) - S(0)| = |-35 - 0| = |-35| = 35 \text{ ft}$

$S(0) = 0 \text{ ft}$

$S(.558) = 2.147$

$S(4.775) = -35.332$

$S(5) = -35$

$2.147 - 0 = 2.147$

$-35.332 - 2.147 = -37.479$

$-35 + (-35.332) = 0.332$

ADD $= 2.147 + 37.479 + 0.332$

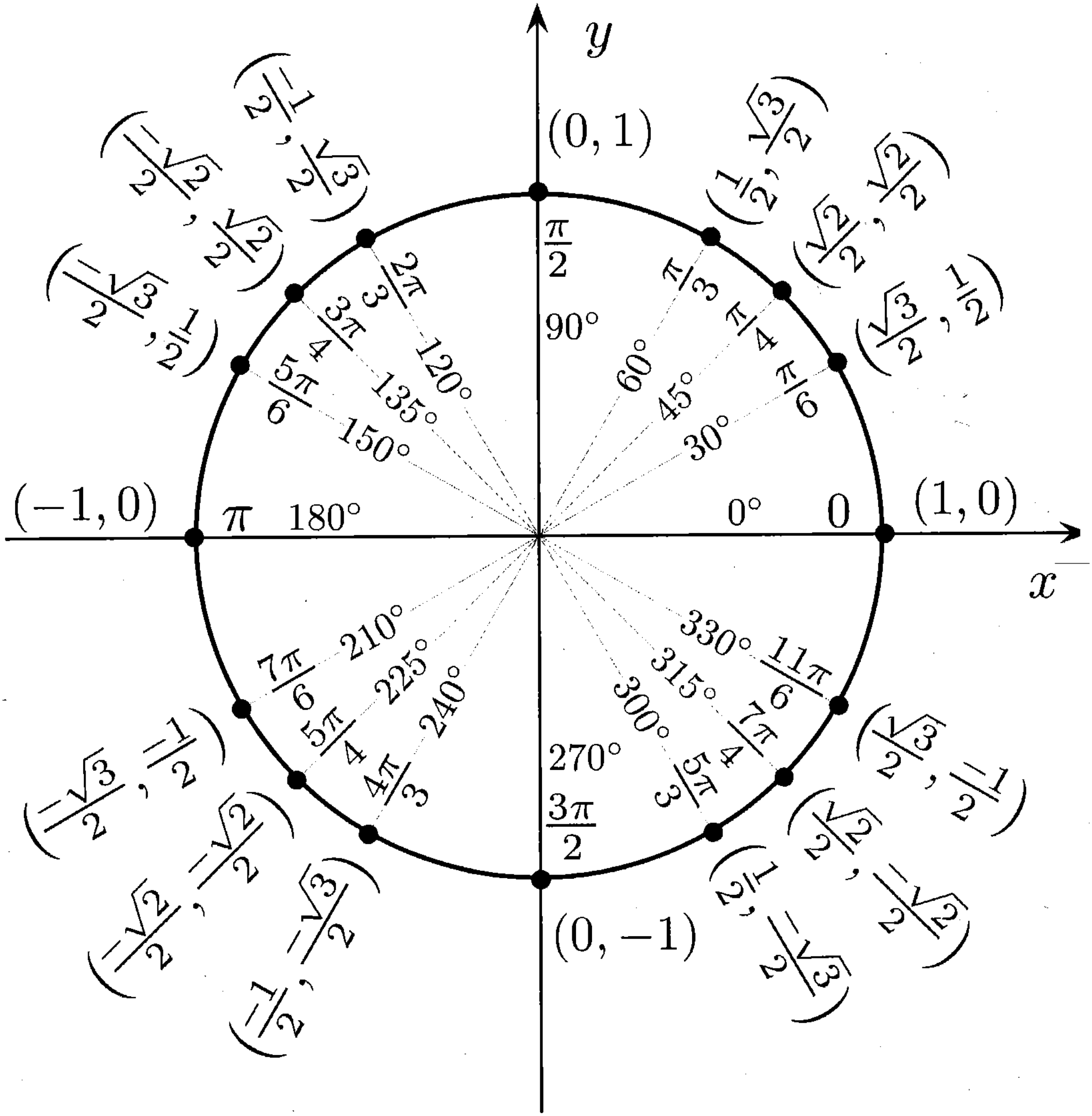
$\boxed{= 39.958}$

g) What is the displacement of the skateboarder after the first five seconds?

$S(5) - S(0) = -35 \text{ ft}$

h) When is the skateboarder moving to the right and to the left? Use interval notation for your answers.

	Sin	Cos	Tan	csc	Sec	cot
$0°$ 2π	0	1	0	—	1	—
$30°$ $\pi/6$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45°$ $\pi/4$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$ $\pi/3$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$90°$ $\pi/2$	1	0	—	1	—	0
$120°$ $2\pi/3$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\frac{\sqrt{3}}{3}$
$135°$ $3\pi/4$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
$150°$ $5\pi/6$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
$180°$ π	0	-1	0	—	-1	—
$225°$ $5\pi/4$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
$270°$ $3\pi/2$	-1	0	—	-1	—	0
$315°$ $7\pi/4$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1



y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
π/2
90°
π/3
60°
π/4
45°
π/6
30°
0
0°
2π/3
120°
3π/4
135°
5π/6
150°
π
180°
7π/6
210°
5π/4
225°
4π/3
240°
3π/2
270°
5π/3
300°
7π/4
315°
11π/6
330°
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(√2/2, −√2/2)
(√3/2, −1/2)

Related Rates

1. Read problem, pull out essential information & identify formula to use
2. Sketch a diagram
3. Write down any Known rate of change & rate you need
4. Signs... decreasing rates are negative
5. Pay attention to whether quantities are constant or varying
6. Set up equation with appropriate quantities
7. Differentiate with respect to "t" using implicit Differentiation
8. Plug in Known items. Might need geometry
9. Solve for items. Most likely rate of change
10. Answer with appropriate units.

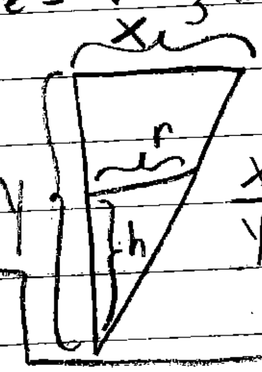
Circle Area = $A = \pi r^2$

Volume Cone = $V = \frac{1}{3}\pi r^2 h$

Triangle Area = $\frac{1}{2}bh$

Volume Sphere = $V = \frac{4}{3}\pi r^3$

Similar Triangles:



$$\frac{x}{y} = \frac{r}{h}$$

Quadratic Formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Derivative of a function

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

if $m_{tan} = x$

then $m_{normal} = -\frac{1}{x}$

Differentiation Rules

Constant = $f'(C) = 0$

Constant Multiple = $f'(C \cdot U) = C \cdot \frac{du}{dx}$

Product = $f'(U \cdot V) = U \cdot \frac{dv}{dx} + V \cdot \frac{du}{dx}$

Quotient = $f'\left(\frac{U}{V}\right) = \frac{V \cdot (U') - U \cdot (V')}{V^2}$

Sin = $f'(\sin x) = \cos x$

Cos = $f'(\cos x) = -\sin x$

tan = $f'(\tan x) = \sec^2 x$

Sec = $f'(\sec x) = \sec x \tan x$

Csc = $f'(\csc x) = -\csc x \cot x$

cot = $f'(\cot x) = -\csc^2 x$

$\sin^{-1} = f'(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \cdot \frac{du}{dx}$

Power = $f'(x^n) = nx^{n-1}$

Chain = $f'(g(x)) = f'(g(x)) \cdot g'(x)$

Logs = $f'(e^u) = e^u \cdot \frac{du}{dx}$

Logs = $f'(a^u) = a^x \cdot \ln a \cdot \frac{du}{dx}$

Logs = $f'(\ln U) = \frac{1}{x} \cdot \frac{du}{dx}$

Logs = $f'(\log_a U) = \frac{1}{U \cdot \ln a} \cdot \frac{du}{dx}$

$\tan^{-1} = f'(\tan^{-1} x) = \frac{1}{1+x^2} \cdot \frac{du}{dx}$

$\cot^{-1} = f'(\cot^{-1} x) = \frac{-1}{1+x^2} \cdot \frac{du}{dx}$

$\sec^{-1} = f'(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}} \cdot \frac{du}{dx}$

$\csc^{-1} = f'(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}} \cdot \frac{du}{dx}$

$\cos^{-1} = f'(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}} \cdot \frac{du}{dx}$

$v(t) = s'(t) = \frac{ds}{dt}$

$a(t) = v'(t) = \frac{dv}{dt}$

Speed = the absolute value of Velocity

Speed = $|v(t)| = |s'(t)|$

Cost of Production = $C(x)$ Average Cost = $\frac{C(x)}{x}$

Marginal Cost = $C'(x)$

finding Marginal cost of Producing $x+1$ Units = $C'(x)$

finding Actual Cost of Producing $x+1$ Unit = $C(x+1) - C(x)$

Linearization = $L(x) = f(a) + f'(a)(x-a)$ near $x = a$

Using Linearization to approximate the actual value of x just pick a whole number value near x

Approximation Error = $|\Delta y - dy|$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

Row 0
Row 1
Row 2
Row 3
Row 4

Ex: $(x+h)^3 =$
$(x^3 + 3x^2h + 3xh^2 + h^3)$

for any slope of a Tangent
$m_{tan} = -4-2h$

Precise Def of a Limit
$0 < |x - x_0| \delta \rightarrow |f(x) - L| \varepsilon$
$\lim_{x \to x_0} f(x) = L$

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

Rules for Continuity $\lim_{x \to c} f(x) = f(c)$
1) Function value must exist
2) Limit value must exist
3) Function value & Limit value must be equal

Removable Discontinuity
create a similar function
① $\lim_{x \to 2} g(x)$ exists $= 1$
③ $\lim_{x \to 2} g(x) = g(2)$ $1 = 1$
② $g(2)$ exists $= 1$

$g(x) = \begin{cases} f(x) & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$

$g(x)$ cont at $x = 2$

Limits involving infinity : Assymtotes
Horizontal Asymptote
$\lim_{x \to \infty} f(x) = b$ $\lim_{x \to -\infty} f(x) = b$

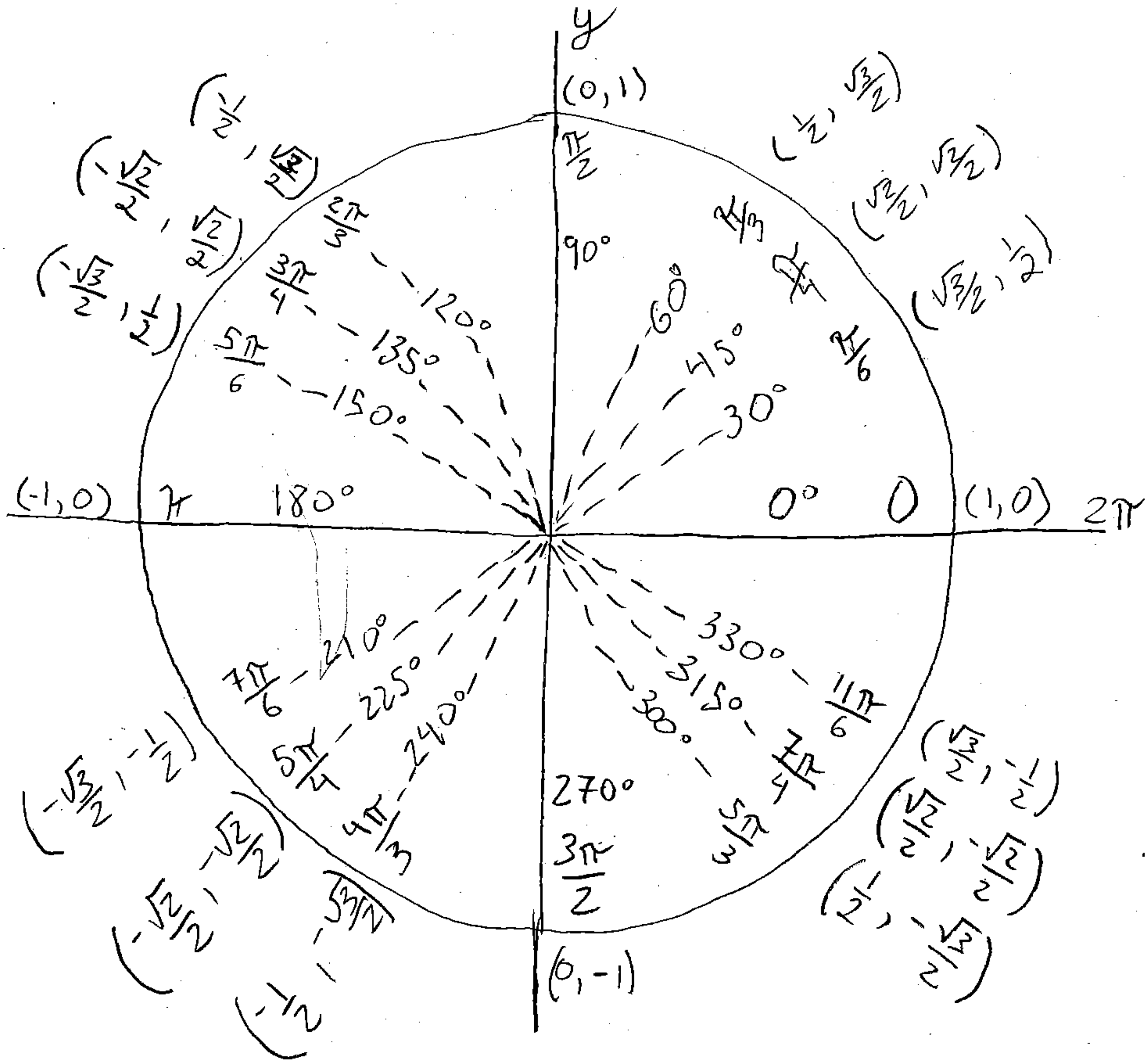
Vertical Asymptote
$\lim_{x \to b^+} f(x) = \pm \infty$ $\lim_{x \to b^-} f(x) = \pm \infty$

Rules approaching infinity
① Largest exponent in numerator $\frac{x^2}{x}$ no HA.
Since $\frac{1}{0^+} = \infty$
② Largest exponent in numerator & denominator equal take Coeficients
③ Largest exponent is in denominator we have H.A. @ $y = 0$

Long Division
Denominator $\overline{)\text{Nemerator}}$
draw the line
& change the sign

point slope formula
$y - y_1 = m(x - x_1)$

Slope of a curve
@ point p & equation of tangent line @ P $= \frac{f(x+h) - f(x)}{h}$



	0	30°	45°	60°	90°	120°	135°	150°	180°	225°	270°	315°
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	1	—	-1
CSC	—	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	—	$-\sqrt{2}$	-1	$-\sqrt{2}$
Sec	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	—	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\sqrt{2}$	—	$\sqrt{2}$
cot	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	—	1	0	-1

3. The radius r and the height h of a right circular cone are related to the cone's volume V by the equation $V = \frac{1}{3}\pi r^2 h$.

r constant?

$V = \frac{1}{3}\pi r^2 h$

$$\boxed{\frac{dV}{dt} = \frac{\pi}{3} r^2 \frac{dh}{dt}}$$

h constant?

$$\frac{dV}{dt} = \frac{2}{3}\pi r \frac{dr}{dt} h$$

$$\boxed{\frac{dV}{dt} = \frac{2}{3}\pi r h \frac{dr}{dt}}$$

$$\frac{dV}{dt} = \frac{2\pi}{3} r \frac{dr}{dt}(h) + \frac{\pi}{3} r^2 \left(\frac{dh}{dt}\right)$$

$$\boxed{\frac{dV}{dt} = \frac{\pi}{3} r^2 \frac{dh}{dt} + \frac{2\pi}{3} r h \frac{dr}{dt}}$$

4. An oil tanker strikes an iceberg and a hole is ripped open on its side. Oil is leaking out in a near circular shape. The radius of the oil spill is changing at a rate of 1.5 miles per hour. How fast is the area of the oil spill changing when the radius is 0.6 mile?

$\frac{dr}{dt} = 1.5 \frac{mi}{hr}$ $\quad r = 0.6$ mile $\quad \frac{dA}{dt} = ?$

Area of circle = πr^2

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi (0.6\, m) \cdot 1.5 \frac{mi}{hr}$$

$$\boxed{\frac{dA}{dt} = 5.655 \frac{mi^2}{hr}}$$

5. A balloon is being inflated at a rate of 10 cubic centimeters per second. How fast is the radius of a spherical balloon changing at the instant the radius is 5 centimeters?

$$\frac{dV}{dt} = 10 \frac{cm^3}{sec} \qquad r = 5 \text{ cm} \qquad V = \frac{4}{3}\pi r^3$$

$$\frac{dr}{dt} = ?$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$10 \frac{cm^3}{sec} = 4\pi (5)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{10}{4\pi \cdot 25}$$

$$\boxed{\frac{dr}{dt} = .0318 \frac{cm}{sec}}$$

6. A water authority is filling an inverted conical water storage tower at a rate of 9 cubic feet per minute. The height of the tank is 80 feet and the radius at the top is 40 feet. How fast is the water level inside the tank changing when the water level is 60 feet deep?

$$\frac{dv}{dt} = 9 \frac{ft^3}{mi}$$

$$h = 80 ft \text{ top} \quad r = 40 ft \text{ top} \qquad \frac{dh}{dt} = ?$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dh}{dt} @ h = 60 ft$$

$$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

$$\boxed{\frac{dh}{dt} = .00318 \frac{ft}{min}}$$

$$V = \frac{1}{12}\pi h^3$$

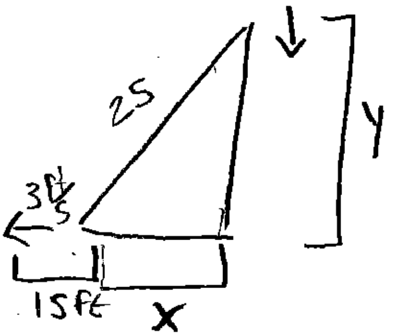
$$\frac{r}{h} = \frac{80}{40}$$

$$r = \frac{1}{2}h$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{9}{2027.4}$$

7. A 25-foot ladder is leaning against a wall. The bottom of the ladder is being pulled away from the wall at a rate of 3 feet per second. How fast is the top of the ladder moving at the instant the bottom of the ladder is 15 feet away from the wall?



$\frac{dx}{dt} = \frac{3\text{ ft}}{\text{sec}}$ $\quad x = 15\text{ ft}$

$x^2 + y^2 = 25^2\text{ft} \rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$y = \sqrt{400}$

$\underline{y = 20\text{ft}}$

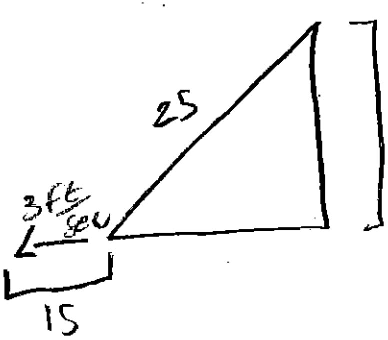
$2(15)(3) + 2(20)\frac{dy}{dt} = 0$

$90 + 40\frac{dy}{dt} = 0$

$40\frac{dy}{dt} = -90$

$\boxed{\frac{dy}{dt} = -2.25\,\frac{\text{ft}}{\text{sec}}}$

8. A 25-foot ladder is leaning against a wall. The bottom of the ladder is being pulled away from the wall at a rate of 3 feet per second. How fast is the top of the ladder moving at the instant the bottom of the ladder is 15 feet away from the wall? At the same instant, what is the rate of change of the area between the ground, the wall, and the ladder?



$A = \frac{1}{2}bh = \frac{1}{2}xy$ $\quad x = 15\text{ft},\ y = 15\text{ft}$

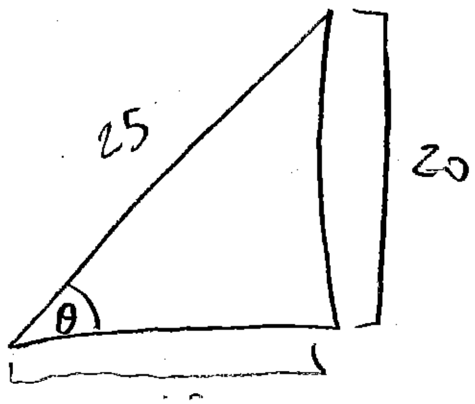
$\frac{dx}{dt} = 3\frac{\text{ft}}{\text{sec}}$ $\quad \frac{dy}{dt} = -2.25$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}(y) + \frac{1}{2}x\left(\frac{dy}{dt}\right)$

$\frac{dA}{dt} = 30 + (-16.875)$

$\boxed{\frac{dA}{dt} = 13.125\,\frac{\text{ft}^2}{\text{sec}}}$

9. A 25-foot ladder is leaning against a wall. The bottom of the ladder is being pulled away from the wall at a rate of 3 feet per second. How fast is the top of the ladder moving at the instant the bottom of the ladder is 15 feet away from the wall? At the same instant, what is the rate of change of the angle (θ) between the ground and the ladder?



$\frac{dx}{dt} = 3 \quad x = 15\text{ft} \quad y = 20\text{ft} \quad \frac{dy}{dt} = -2.25$

$\tan\theta = \frac{y}{x} \qquad \frac{d\theta}{dt} = ?$

$\tan\theta = \frac{20}{15} \qquad \theta = 0.9273$

$\tan\theta = \frac{y}{x}$

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x\left(\frac{dy}{dt}\right) - \left(\frac{dx}{dt}\right)(y)}{x^2}$$

$$\sec^2(.9273) \cdot \frac{d\theta}{dt} = \frac{15(-2.25) - 3(20)}{15^2}$$

$$\frac{1}{\cos^2(.9273)} \frac{d\theta}{dt} = -.4167$$

$$\boxed{\frac{d\theta}{dt} = -.150 \frac{\text{rad}}{\text{sec}}}$$

MAC 2311 - Calculus w/Analytic Geometry I Name________________

REVIEW for Test #1 - Chapter 2 - Limits and Continuity Date________________

Find the average rate of change of the function over the given interval.

1) $y = 5x^3 - 3x^2 - 8$, $[-8, -5]$

A) -236 B) 684 C) $-\frac{2052}{5}$ D) $\frac{708}{5}$

2) $g(t) = 4 + \tan t$, $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

A) $-\frac{3}{2}$ B) $\frac{4}{\pi}$ C) $-\frac{4}{\pi}$ D) 0

Find the slope of the curve at the given point P and an equation of the tangent line at P.

3) $y = x^2 + 5x$, $P(4, 36)$

A) slope is $\frac{1}{20}$; $y = \frac{x}{20} + \frac{1}{5}$ B) slope is $-\frac{4}{25}$; $y = -\frac{4x}{25} + \frac{8}{5}$

C) slope is 13; $y = 13x - 16$ D) slope is -39; $y = -39x - 80$

Find the limit.

4) $\lim_{x \to 2} (x^3 + 5x^2 - 7x + 1)$

A) 29 B) 0 C) does not exist D) 15

5) $\lim_{x \to 5} \sqrt{x^2 + 10x + 25}$

A) 100 B) 10 C) ± 10 D) does not exist

Find the limit if it exists.

6) $\lim_{x \to \frac{1}{6}} 6x\left(x - \frac{2}{5}\right)$

A) $\frac{17}{30}$ B) $-\frac{7}{5}$ C) $-\frac{7}{30}$ D) $-\frac{7}{180}$

7) $\lim_{x \to -3} (x + 3128)^{3/5}$

A) 25 B) 125 C) 625 D) -125

Find the limit, if it exists.

8) $\lim_{x \to 0} \frac{x^3 + 12x^2 - 5x}{5x}$

A) -1 B) 0 C) Does not exist D) 5

9) $\lim_{x \to 7} \frac{x^2 - 49}{x - 7}$

A) Does not exist B) 1 C) 7 D) 14

10) $\lim_{x \to 8} \frac{x^2 + 2x - 80}{x^2 - 64}$

A) Does not exist B) 0 C) $\frac{9}{8}$ D) $-\frac{1}{8}$

11) $\lim_{h \to 0} \frac{(x + h)^3 - x^3}{h}$

A) 0 B) $3x^2$ C) Does not exist D) $3x^2 + 3xh + h^2$

Find the limit.

12) $\lim_{x \to -\pi} \sqrt{x + 9} \cos(x + \pi)$

A) $\sqrt{9 - \pi}$ B) $-\sqrt{9 - \pi}$ C) 0 D) 1

Evaluate the limit using the difference quotient, $\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$, for the given x_0 and function f.

13) $f(x) = 3x^2 + 4$ for $x_0 = 4$

A) 24 B) 28 C) 48 D) Does not exist

14) $f(x) = 4\sqrt{x}$ for $x_0 = 4$

A) 4 B) 1 C) 8 D) Does not exist

A function f(x), a point x_0, the limit of f(x) as x approaches x_0, and a positive number ε is given. Find a number $\delta > 0$ such that for all x, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

15) $f(x) = 3x - 2$, $L = 1$, $x_0 = 1$, and $\varepsilon = 0.01$

A) 0.003333 B) 0.001667 C) 0.006667 D) 0.01

16) $f(x) = -10x + 5$, $L = -15$, $x_0 = 2$, and $\varepsilon = 0.01$

A) 0.004 B) -0.005 C) 0.002 D) 0.001

Find the relationship between epsilon and delta. Explain the relationship with an example.

17) $\lim_{x \to 5} (5x - 3) = 22$ $L = 22,\ c = 5$

Find the limit using $\lim_{x = 0} \frac{\sin x}{x} = 1$.

18) $\lim_{x \to 0} \frac{\tan 4x}{x}$

A) $\frac{1}{4}$ B) does not exist C) 4 D) 1

19) $\lim\limits_{x \to 0} \dfrac{\sin x \cos 4x}{x + x \cos 5x}$

A) $\frac{1}{2}$ B) 0 C) $\frac{4}{5}$ D) does not exist

Find the limit and determine if the function is continuous at the point being approached.

20) $\lim\limits_{x \to -\pi/2} \cos(5x - \cos 5x)$

A) 0; no B) does not exist; no C) 0; yes D) does not exist; yes

21) $\lim\limits_{x \to -\pi/2} \cos\left(\frac{3\pi}{2} \cos(\tan x)\right)$

A) 1; no B) 1; yes C) does not exist; yes D) does not exist; no

22) $\lim\limits_{x \to 1} \sin(x \sin^2 x + x \cos^2 x + 2)$

A) sin 3; yes B) does not exist; no C) sin 1; yes D) sin 3; no

Find the limit.

23) $\lim\limits_{x \to \infty} \dfrac{x^2 + 8x + 6}{x^3 - 9x^2 + 5}$

A) ∞ B) $\frac{6}{5}$ C) 1 D) 0

24) $\lim\limits_{x \to -\infty} \dfrac{-9x^2 - 8x + 2}{-3x^2 + 3x + 16} = \dfrac{-9}{-3} = \boxed{3}$

A) 3 B) 1 C) ∞ D) $\frac{1}{8}$

25) $\lim\limits_{x \to -\infty} \dfrac{\cos 2x}{x}$

A) 2 B) 1 C) $-\infty$ D) 0

Divide numerator and denominator by the highest power of x in the denominator to find the limit.

26) $\lim\limits_{t \to \infty} \dfrac{\sqrt{25t^2 - 125}}{t - 5}$

A) does not exist B) 5 C) 125 D) 25

Find the limit.

27) $\lim\limits_{x \to -2} \dfrac{1}{x + 2}$

A) $-\infty$ B) ∞ C) 1/2 D) Does not exist

28) $\lim\limits_{x \to 7^+} \frac{1}{x - 7}$

A) 0 B) -1 C) ∞ D) $-\infty$

29) $\lim\limits_{x \to -3^-} \frac{4}{x^2 - 9}$

A) 0 B) $-\infty$ C) -1 D) ∞

30) $\lim\limits_{x \to (\pi/2)^+} \tan x$

A) 0 B) ∞ C) $-\infty$ D) 1

Indentify the types of asymptotes and their equations that exist in each function.

31) $f(x) = \frac{2x^2}{4 - x^2}$

A) asymptotes: V.A.: $x = -2, x = 2$, O. A. : $y = 0$

B) asymptotes: V.A.: $x = -2, x = 2$, H.A.: $y = -2$

C) asymptotes: O.A.: $x = -2, x = 2$, H. A.: $y = 0$

D) asymptotes: H.A.: $x = -2, x = 2$, O.A.: $y = 2$

32) $f(x) = \frac{2 - x^2}{2x + 4}$

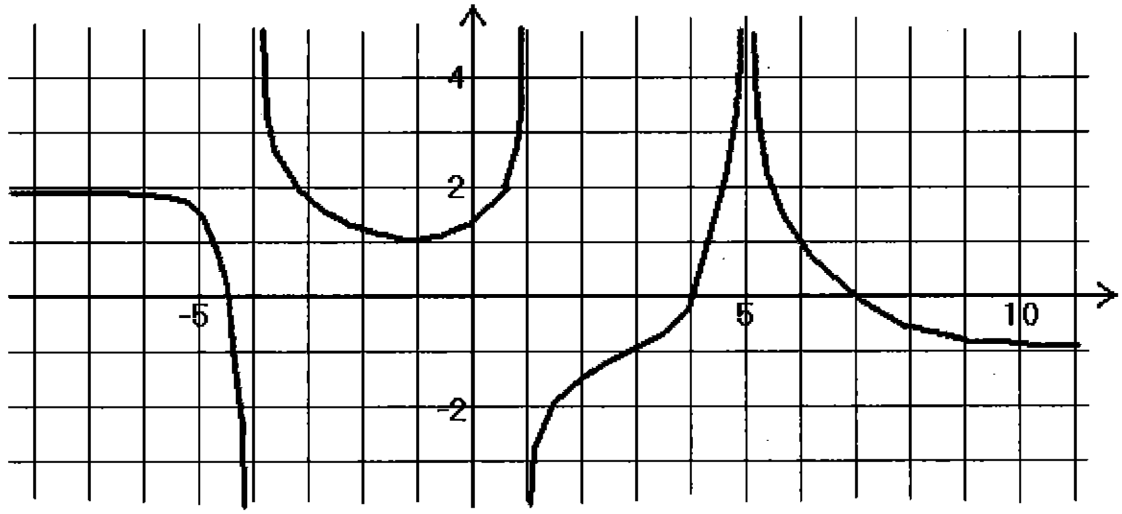
A) asymptotes: H.A.: $x = -2$, O.A.: $y = -\frac{1}{4}x + \frac{1}{2}$

B) asymptotes: V.A.: $x = -2$, O.A.: $y = -x + 2$

C) asymptotes: V.A.: $x = -4$, O.A. $y = -\frac{1}{8}x + \frac{1}{2}$

D) asymptotes: V.A.: $x = -2$, O.A>: $y = -\frac{1}{2}x + 1$

33) Use the following graph of the function f(x) to find the limits and function values



a. $\lim_{x \to 1^-} f(x) =$ ∞

b. $\lim_{x \to -4^+} f(x) =$ ∞

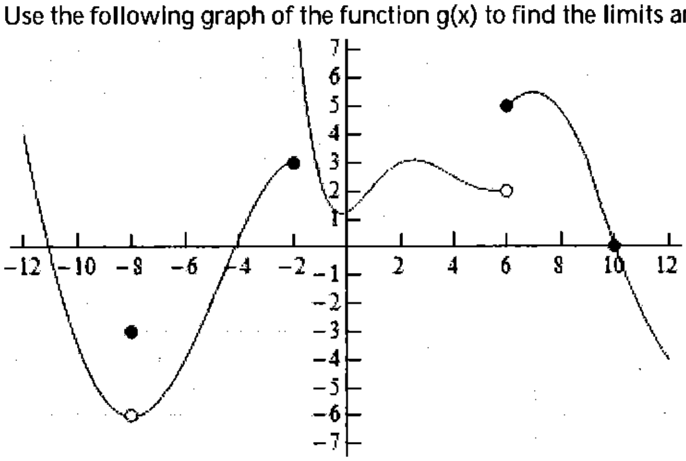
c. $\lim_{x \to \infty} f(x) =$ -4

d. f(-4)= DNE

e. f(7)= 0

f. $\lim_{x \to 7} f(x) =$ 0

34) Use the following graph of the function g(x) to find the limits and function values



a. $\lim_{x \to -8^+} g(x) =$ -6

b. $\lim_{x \to -6^-} g(x) =$ -4

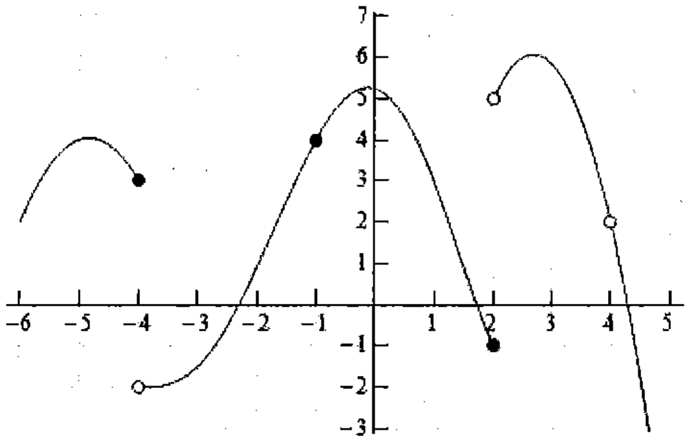
c. $\lim_{x \to 6} g(x) =$ DNE

d. g(-8)= -3

e. $\lim_{x \to 9^+} g(x) =$ 3

f. $\lim_{x \to -2} g(x) =$ DNE

35) Is the function h(x) continuous at the requested values of x? Use the theorem of Continuity at a Point to prove your answers.



a. x = -1

$f(-1) = 4$ exists

$\lim_{x \to -1} f(x) = 4$ exists

$f(-1) = \lim_{x \to -1} f(x)$

Continuous @ $x = -1$

b. x = 2

$f(2) = -1$

$\lim_{x \to 2} f(x) =$ DNE

disc @ $x = 2$

MAC 2311 – Calculus with Analytic Geometry I

Answers for Test #1 Review

1. B
2. B
3. B
4. D
5. B
6. C
7. B
8. A
9. D
10. C
11. B
12. A
13. A
14. B
15. A
16. D
17. $\delta = \frac{\epsilon}{5}$
18. C
19. A
20. C
21. D
22. A
23. D
24. A
25. D
26. B
27. D
28. C
29. D
30. C
31. B
32. D
33. a. ∞ b. $-\infty$ c. -1 d. DNE e. 0 f. 0
34. a. -6 b. -4 c. DNE d. -3 e. 3 f. DNE

35a. $f(c) exists$: $f(-1) = 4$

$\lim_{x\to c} f(x) exists$: $\lim_{x\to -1} f(x) = 4$

$\lim_{x\to c} f(x) = f(c)$: $4 = 4$

$\therefore f(x)\ is\ continuous\ at\ x = -1$

35b. $f(c) exists$: $f(2) = -1$

$\lim_{x\to c} f(x) exists$: $\lim_{x\to -1} f(x) = DNE$

$\therefore f(x)\ is\ not\ continuous\ at\ x = 2$

Review Test #1

① $y = 5x^3 - 3x^2 - 8 \quad [-8, -5]$

$$\frac{f(-5) - f(-8)}{-5 - (-8)} = \boxed{684}$$

② $g(t) = 4 + \tan t \quad \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$$\frac{g(\frac{\pi}{4}) - g(-\frac{\pi}{4})}{\frac{\pi}{4} - (-\frac{\pi}{4})} = 1.273 = \boxed{\frac{4}{\pi}}$$

$$\frac{4 + \tan(\frac{\pi}{4}) - (4 + \tan(-\frac{\pi}{4}))}{\frac{\pi}{4} + \frac{\pi}{4}} = \frac{4 + 1 - (4 - 1)}{\frac{\pi}{4} + \frac{\pi}{4}}$$

$$\frac{5 - 3}{\frac{2\pi}{4}} = \frac{2}{\frac{\pi}{2}} = 2 \cdot \frac{2}{\pi} = \boxed{\frac{4}{\pi}}$$

③ $y = x^2 + 5x \quad (4, 36)$

$$m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 5(x+h) - (x^2 + 5x)}{h}$$

$$\frac{\cancel{x^2} + 2xh + h^2 + \cancel{5x} + 5h - \cancel{x^2} - \cancel{5x}}{h}$$

$$= \frac{2xh + h^2 + 5h}{h} = \frac{h(2x + h + 5)}{h}$$

$$\lim_{h \to 0} 2x + h + 5 = m_{tan} = 2x + 5$$

$$y - y_1 = m_{tan}(x - x_1)$$

$$m_{tan} = 2(4) + 5 = m_{tan} = 13$$

$$y - 36 = 13(x - 4)$$

$$y - 36 = 13x - 52$$

$$\boxed{y = 13x - 16}$$

$$\begin{array}{ccccccccc} & & & & 1 & & & & - 0 \\ & & & 1 & & 1 & & & - 1 \\ & & 1 & & 2 & & 1 & & - 2 \\ & 1 & & 3 & & 3 & & 1 & - 3 \\ 1 & & 4 & & 6 & & 4 & & 1 - 4 \end{array}$$

(11) $\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$

$$\frac{x^3h^0 + 3x^2h + 3xh^2 + x^0h^3 - x^3}{h}$$

$$\frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2 + 3xh + h^2$$

$$\lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2 + 0 + 0 = \boxed{3x^2}$$

(13) $f(x) = 3x^2 + 4$ for $x_0 = 4$

$$\lim_{h\to} \frac{3(x+h)^2 + 4 - (3x^2+4)}{h}$$

$$= \frac{\cancel{3x^2} + 6xh + 3h^2 + \cancel{4} - \cancel{3x^2} - \cancel{4}}{h}$$

$$= 6x + 3h = \lim_{h\to 0} 6x + 0 = \boxed{6x}$$

$$x_0 = 4 \text{ so } 6(4) = \boxed{24}$$

14 $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$= \frac{4(\sqrt{x+h}) - 4\sqrt{x}}{h} \cdot \frac{4\sqrt{x+h} + 4\sqrt{x}}{4\sqrt{x+h} + 4\sqrt{x}}$$

$$= \frac{16(x+h) - 16x}{h(4\sqrt{x+h} + 4\sqrt{x})} = \frac{16x + 16h - 16x}{h(4\sqrt{x+h} + 4\sqrt{x})}$$

$$\lim_{h \to 0} \frac{16\cancel{h}}{\cancel{h}(4\sqrt{x+h} + 4\sqrt{x})}$$

$$\lim_{h \to 0} \frac{16}{4\sqrt{x+0} + 4\sqrt{x}} = \frac{16}{8\sqrt{x}} = \frac{2}{\sqrt{x}}$$

$X_0 = 4$ so $\dfrac{2}{\sqrt{4}} = \boxed{1}$

(16) $f(x) -10x+5 \quad L=-15 \quad X_0=2 \quad \varepsilon=.01$

$0 < |x-x_0| \, \delta \longrightarrow |f(x)-L| < \varepsilon$

$0 < |x-2| \, \delta \longrightarrow |(10x+5)-(-15)| < \varepsilon$

$0 < |x-2| \, \delta \longrightarrow |-10x+20| < .01$

$0 < |x-2| \, \delta \longrightarrow |-10(x-2)| < .01$

$0 < |x-2| \, \delta \longrightarrow 10\,|x-2| < .01$

$0 < |x-2| \, \delta \longrightarrow |x-2| < .001$

$$\boxed{\delta = \varepsilon = .001}$$

(18) $\lim\limits_{x\to 0} \dfrac{\tan 4x}{x} = \dfrac{0}{0}$

$$= \frac{\frac{\sin 4x}{\cos 4x}}{\frac{x}{1}} = \frac{\sin 4x}{\cos 4x} \cdot \frac{1}{x} = \frac{\sin 4x}{x \cos 4x}$$

Let $\theta = 4x$

$x = \dfrac{\theta}{4}$

$X \to 0 \therefore \theta \to 0$

$$= \frac{\sin 4 \cdot \frac{\theta}{4}}{\frac{\theta}{4} \cos 4 \cdot \frac{\theta}{4}} = \frac{\sin\theta}{\frac{\theta}{4}\cos\theta}$$

$$= \lim_{\theta\to 0} \frac{\sin\theta}{\frac{\theta}{4}\cos\theta} = \lim_{\theta\to 0} \frac{\sin\theta}{\frac{\theta\cos\theta}{4}}$$

$$= \frac{4\sin\theta}{\theta\cos\theta} = \frac{\sin\theta}{\theta} \cdot \frac{4}{\cos\theta} = 1 \cdot \frac{4}{1} = \boxed{4}$$

(25) $\lim\limits_{x \to -\infty} \dfrac{\cos 2x}{x} = \dfrac{\text{Some \#}}{-\infty} = \boxed{0}$

(26) $\lim\limits_{t \to \infty} \dfrac{\sqrt{25t^2 - 125}}{t-5} = \dfrac{\sqrt{25(t^2-5)}}{t-5}$

$\dfrac{\sqrt{25} \cdot \sqrt{t^2-5}}{t-5} = \dfrac{5 \cdot \sqrt{t^2-5}}{t-5}$

$= \dfrac{5 \cdot \sqrt{t^2+0}}{t-0} = \dfrac{5t}{t} = \boxed{5}$

(27) $\lim\limits_{x \to -2} \dfrac{1}{x+2}$

$\lim\limits_{x \to -2^-} \dfrac{1}{x+2} = \dfrac{1}{-0.01} = \dfrac{1}{-0.000001} = \dfrac{1}{0^-} = -\infty$

$\lim\limits_{x \to -2^+} \dfrac{1}{x+2} = \dfrac{1}{0^+} = \infty$

Right & Left limit do not equal each other $-\infty \neq \infty$
so Limit DNE

(17) $\lim_{x \to 5} (5x-3) = 22$

$0 < |x - x_0| < \delta \rightarrow |f(x) - L| < \varepsilon$

$0 < |x-5| < \delta \rightarrow |(5x-3)-22| < \varepsilon$

$\quad " \quad " \rightarrow |5x-25| < \varepsilon$

$\quad " \quad " \rightarrow |x-5| \ \varepsilon/5$

$0 < |x-5| < \delta \rightarrow |x-5| < \frac{\varepsilon}{5}$

$\boxed{\delta = \frac{\varepsilon}{5}}$

(20) $\lim_{x \to \left(\frac{-\pi}{2}\right)} \cos(5x - \cos 5x)$

$= \cos\left(\frac{-5\pi}{2} - \cos \frac{-5\pi}{2}\right)$

$= \cos\left(\frac{-5\pi}{2}\right) = \boxed{0}$

$f\left(\frac{-\pi}{2}\right) = \cos\left(\frac{-5\pi}{2} - \cos \frac{-5\pi}{2}\right)$

$= \cos\left(\frac{-5\pi}{2} - 0\right)$

$= \boxed{0}$

(6) $\lim_{x \to \frac{1}{6}} 6x\left(x - \frac{2}{5}\right) = 6\frac{1}{6}\left(\frac{1}{6} - \frac{2}{5}\right)$

$\left(\frac{1}{6} - \frac{2}{5}\right) \Rightarrow \left(\frac{5}{30} - \frac{12}{30}\right) = -\frac{7}{30} = \boxed{\frac{-7}{30}}$

(8) $\lim_{x \to 0} \frac{x^3 + 12x^2 - 5x}{5x} = \frac{\cancel{x}(x^2 + 12x - 5)}{5\cancel{x}}$

$\lim_{x \to 0} \frac{0 + 12(0) - 5}{5} = -\frac{5}{5} = \boxed{-1}$

(15) $f(x) = 3x - 2$, $L = 1$, $x_0 = 1$ & $\varepsilon = .01$

$0 < |x - x_0| < \delta \longrightarrow |f(x) - L| < \varepsilon$

$0 < |x - 1| < \delta \longrightarrow |(3x - 2) - 1| < \varepsilon$

$\quad '' \quad '' \longrightarrow |3x - 3| < \varepsilon$

$\quad '' \quad '' \longrightarrow |3(x - 1)| < \varepsilon$

$\quad '' \quad '' \longrightarrow |x - 1| < \frac{.01}{3}$

$\delta = \frac{\varepsilon}{3} \therefore \boxed{\delta = .00333}$

(28) $\lim_{x \to 7^+} \frac{1}{x-7} = \frac{1}{7.01-7} = \frac{1}{.01} = \frac{1}{0^+} = \boxed{\infty}$

(30) $\lim_{x \to \frac{\pi}{2}^+} \tan x = \lim_{x \to \frac{\pi}{2}^+} \frac{\sin x}{\cos x} = \frac{1}{0^+} = \boxed{\infty}$

(31) $f(x)\ \frac{2x^2}{4-x^2}$

H.A.: $\lim_{x \to \infty} \frac{2x^{2}}{-x^{2}+4} = \frac{2}{-1} = \boxed{-2} = \boxed{y=-2}$

V.A.: $4-x^2=0$

$-x^2=-4$

$x=\pm 2$

$\boxed{x=-2 \quad x=2}$

$\boxed{\text{O.A.: None}}$

(32) $f(x) = \frac{2-x^2}{2x+4}$

H.A.: $\lim_{x \to \infty} \frac{2-x^{2}}{2x^{0}+4} = -\infty$ $\boxed{\text{HA = None}}$

V.A: $2x+4=0$

$\boxed{x=-2}$

$$\begin{array}{r|l} & -\frac{1}{2}x + 1 \\ \hline \text{O.A.} \quad 2x+4 & -x^2 + 0x + 2 \\ & \underline{\pm \cancel{x^2} \pm 2x} \\ & 0 + \cancel{2x} + 2 \\ & \underline{\mp \cancel{2x} \mp 4} \\ & 6 \end{array}$$

O.A. = $\boxed{y = -\frac{1}{2} + 1}$

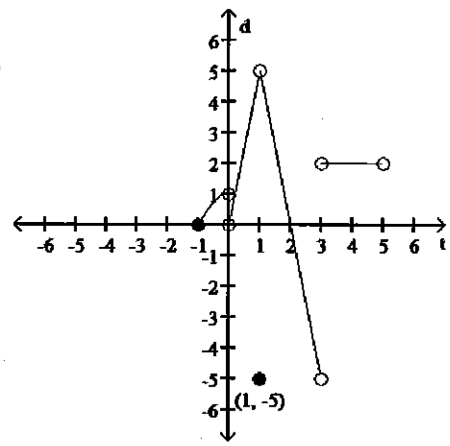
MAC 2311 - Calculus w/Analytic Geometry I   Name Steven Romero

5 Quizzes   **DUE DATE: Tuesday, May 5th, 6:00 PM**   Date

Answer the question.

1) Does $\lim_{x \to 1} f(x)$ exist?

$$f(x) = \begin{cases} -x^2 + 1, & -1 \le x < 0 \\ 5x, & 0 < x < 1 \\ -5, & x = 1 \\ -5x + 10 & 1 < x < 3 \\ 2, & 3 < x < 5 \end{cases}$$



A) Yes   B) No

Find the limit.

2) $\lim_{x \to -\infty} \dfrac{-10x^2 - 5x + 4}{-18x^2 - 5x + 7}$

A) 1   B) ∞ C) $\frac{4}{7}$ D) $\frac{5}{9}$

Find the derivative.

3) $y = 11x^{-2} + 5x^3 - 4x$

A) $-22x^{-1} + 15x^2$ B) $-22x^{-3} + 15x^2$ C) $-22x^{-1} + 15x^2 - 4$ D) $-22x^{-3} + 15x^2 - 4$

$-22x^{-3} + 15x^2 - 4$

Find y '.

4) $y = (5x - 2)(5x + 1)$

A) $25x - 5$ B) $50x - 15$ C) $50x - 5$ D) $50x - 2.5$

$y' = 5(5x+1) + (5x-2) \cdot 5$

$y' = 25x + 5 + 25x - 10$

$y' = 50x - 5$

Find the derivative of the function.

5) $y = \frac{x^3}{x - 1}$

A) $y' = \frac{2x^3 - 3x^2}{(x - 1)^2}$ B) $y' = \frac{2x^3 + 3x^2}{(x - 1)^2}$ C) $y' = \frac{-2x^3 + 3x^2}{(x - 1)^2}$ D) $y' = \frac{-2x^3 - 3x^2}{(x - 1)^2}$

$\frac{(x-1)3x^2 - 1(x^3)}{(x-1)^2} = \frac{3x^3 - 3x^2 - x^3}{(x-1)^2} = \frac{2x^3 - 3x^2}{(x-1)^2}$

Solve the problem.

6) The position of a body moving on a coordinate line is given by $s = t^2 - 5t + 6$, with s in meters and t in seconds. When, if ever, during the interval $0 \le t \le 5$ does the body change direction?

A) t = 2.5 sec
B) no change in direction
C) t = 10 sec
D) t = 5 sec

$v(t) = 2t - 5 \rightarrow 2t - 5 = 0$

$2t = 5$

$t = \frac{5}{2}$

Find the derivative.

7) $s = t^6 - \csc t + 6$

A) $\frac{ds}{dt} = 6t^5 + \csc t \cot t$
B) $\frac{ds}{dt} = 6t^5 + \cot^2 t$
C) $\frac{ds}{dt} = 6t^5 - \csc t \cot t$
D) $\frac{ds}{dt} = t^5 - \cot^2 t + 6$

$\frac{ds}{dt} = 6t^5 + \cot t \csc t$

Find the derivative of the function.

8) $q = \sqrt{19r - r^3}$

A) $\frac{19 - 3r^2}{2\sqrt{19r - r^3}}$ B) $\frac{1}{2\sqrt{19 - 3r^2}}$ C) $\frac{-3r^2}{\sqrt{19r - r^3}}$ D) $\frac{1}{2\sqrt{19r - r^3}}$

$\frac{dq}{dr} = (19r - r^3)^{\frac{1}{2}} = \frac{1}{2}(19r - r^3)^{-\frac{1}{2}} \cdot (19 - 3r^2)$

$= \frac{19 - 3r^2}{2\sqrt{19r - r^3}}$

Use implicit differentiation to find dy/dx.

9) $2xy - y^2 = 1$

A) $\frac{y}{y - x}$ B) $\frac{x}{y - x}$ C) $\frac{x}{x - y}$ D) $\frac{y}{x - y}$

$2(x \cdot y) - y^2 = 1$

$2(1y + x\frac{dy}{dx}) - 2y \cdot \frac{dy}{dx} = 0$

$2y + 2x\frac{dy}{dx} = 2y\frac{dy}{dx} \Rightarrow 2y = 2y\frac{dy}{dx} - 2x\frac{dy}{dx} > 2y = (2y - 2x)y'$

$y' = \frac{y}{y - x}$

$$157^2 + 248^2 = c^2$$

$$c^2 = 86153$$

$$c = 293.518$$

$\frac{1}{3}\theta^{-2/3}\frac{dr}{d\theta} - \frac{1}{3}r^{-2/3} = 0$

Find dr/dθ.

10) $\theta^{1/3} - r^{1/3} = 1$

A) $-\left(\frac{\theta}{r}\right)^{2/3}$ B) $-\left(\frac{r}{\theta}\right)^{2/3}$ C) $\left(\frac{r}{\theta}\right)^{2/3}$ D) $\left(\frac{\theta}{r}\right)^{2/3}$

$\sec^2 y \cdot \frac{dy}{dx} = e^x + \frac{1}{7x} \cdot 7 > \quad e^x + \frac{1}{x} >$

Find $\frac{dy}{dx}$.

11) $\tan y = e^x + \ln 7x$

A) $\frac{e^x + 7}{\sin^2 y}$ B) $e^x + \frac{7}{x} - \sec^2 y$ C) $\frac{xe^x + 1}{x \sec^2 y}$ D) $\frac{xe^x + 7}{x \sec^2 y}$

Find the derivative of y with respect to the independent variable.

12) $y = 6^{\ln 7t}$ $= 6^{(\ln(7t))} \cdot \ln 6 \cdot \frac{1}{7t} \cdot 7 = 6^{(\ln 7t)} \cdot \ln 6 \cdot \frac{1}{t}$

A) $\frac{\ln 6}{t} 6^{\ln 7t}$ B) $6^{\ln 7t}$ C) $\frac{7 \ln 6}{t} 6^{\ln 7t}$ D) $\frac{7 \ln 6}{t}$

Find the derivative of y with respect to x.

13) $y = -\cot^{-1} \frac{6x}{5}$ $+ \frac{1}{1 + \left(\frac{6x}{5}\right)^2} \cdot \frac{6}{5} = \frac{\frac{6}{5}}{1 + \frac{36x^2}{25}} = \frac{6}{5} \cdot \frac{25}{25 + 36x^2} = \frac{30}{36x^2 + 25}$

A) $\frac{25}{36x^2 + 25}$ B) $\frac{30}{36x^2 + 25}$ C) $\frac{-30}{36x^2 + 25}$ D) $\frac{6}{\sqrt{25 - 36x^2}}$

Solve the problem. Round your answer, if appropriate.

14) One airplane is approaching an airport from the north at 157 km/hr. A second airplane approaches from the east at 284 km/hr. Find the rate at which the distance between the planes changes when the southbound plane is 31 km away from the airport and the westbound plane is 17 km from the airport.

A) -411 km/hr B) -274 km/hr C) -137 km/hr D) -548 km/hr

Use l'Hopital's Rule to evaluate the limit.

15) $\lim_{x \to 1} \frac{x^3 - 6x^2 + 5}{x - 1}$ $\frac{0}{0}$ $3x^2 - 12x$ $3(1)^2 - 12(1) =$

A) 12 B) 9 C) -9 D) 15

Find the average value of the function over the given interval.

16) $y = 5 - x^2$; [-4, 5]

A) $\frac{74}{27}$ B) - 2 C) $-\frac{16}{3}$ D) $-\frac{46}{27}$

AV = $\frac{\text{Area under Curve}}{\Delta x}$ = $AV = \frac{1}{5+4} \int_{-4}^{5} 5 - x^2\, dx$

$= \frac{1}{9}\left(5x - \frac{x^3}{3}\right)\Big|_{-4}^{5} = \boxed{-2}$

Find the derivative.

17) $y = \int_0^{\sqrt{x}} 9t \cos(t^6)\,dt$

$$\frac{dy}{dx}\int_0^{x^{1/2}} 9t\cos(t^6)dt = 9(x^{\frac{1}{2}})\cos(x^3)\cdot\frac{1}{2\sqrt{x}}$$
$$= \frac{9\sqrt{x}}{2\sqrt{x}}\cdot\cos(x^3) = \boxed{\frac{9}{2}\cos(x^3)}$$

A) $9\sqrt{x}\cos(x^3)$ B) $6\sqrt{x}\cos(x^3)$

C) $9\cos(x^3) - 9\sqrt{x}\sin(x^3)$ D) $\frac{9}{2}\cos(x^3)$

Solve the initial value problem.

18) $\frac{dy}{dx} = 6\sin^2 x \cos x,\ y(0) = 4$

$U = \sin x$

$y = \int 6\sin^2 x\cdot\cos x\,dx$ $du = \cos x\,dx$

$\boxed{y = 2\sin^3 x + 4}$

A) $y = 2\sin^3 x + 4$ B) $y = 3\cos^2 x + 4$

C) $y = -2\sin^3 x - 4$ D) $y = 12\sin x\cos x + 4$

$y = 6\cdot\int \sin^2 x\cdot\cos x$

$y = 6\cdot\int u^2 du = 6\left(\frac{u^3}{3}\right) + C = 2(\sin x)^3 + C = 4$

$C = 4 - 2\sin^3(0)$

Evaluate the integral.

19) $\int 9x^2 \sqrt[4]{11 + 3x^3}\,dx$

$\int 9x^2(11+3x^3)^{1/4}dx$

$U = 11 + 3x^3$

$du = 9x^2 dx$

$\int U^{1/4}du = \boxed{\frac{4}{5}U^{5/4} + C}$

A) $9(11 + 3x^3)^{5/4} + C$ B) $\frac{4}{5}(11 + 3x^3)^{5/4} + C$

C) $-6(11 + 3x^3)^{-3/4} + C$ D) $\frac{36}{5}(11 + 3x^3)^{5/4} + C$

$U = 2 + \cos t$

$du = -\sin t\,dt$

$$\int \frac{1}{(2+\cos t)^4}\cdot\sin t\,dt = \int U^{-4}du = -\frac{1}{3}U^{-3} + C = -\frac{1}{3U^3} + C = \frac{1}{3(2+\cos)^3} + C$$

20) $\int \frac{\sin t}{(2 + \cos t)^4}\,dt$

A) $\frac{1}{3(2 + \cos t)^3} + C$ B) $\frac{1}{5(2 + \cos t)^5} + C$ C) $\frac{3}{(2 + \cos t)^3} + C$ D) $\frac{1}{(2 + \cos t)^3} + C$

Find the area enclosed by the given curves.

21) $y = 2x - x^2,\ y = 2x - 4$

A) $\frac{31}{3}$ B) $\frac{37}{3}$ C) $\frac{34}{3}$ D) $\frac{32}{3}$

22) $y = -4\sin x,\ y = \sin 2x,\ 0 \le x \le \pi$

A) $\frac{1}{2}$ B) 8 C) 4 D) 16

$$\int_0^{\pi} \sin 2x + 4\sin x\,dx = -\frac{1}{2}\cos 2x - 4\cos x\Big|_0^{\pi}$$
$$= -\frac{1}{2}\cos 2(\pi) - 4\cos(\pi) - \left[-\frac{1}{2}\cos 2(0) - 4\cos(0)\right]$$
$$= -\frac{1}{2} - 4 - \left[-\frac{1}{2} - 4\right]$$
$$= +\frac{7}{2} + \frac{9}{2}$$

4

(21) $y = 2x - x^2$; $y = 2x - 4$

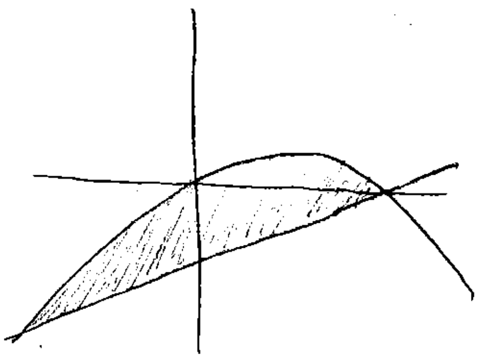
poi $= 2x - x^2 = 2x - 4$

$-x^2 + \cancel{2x} - \cancel{2x} + 4 = 0$

$-x^2 + 4 = 0$

$(x^2 - 4) = 0$

$x = \pm 2$

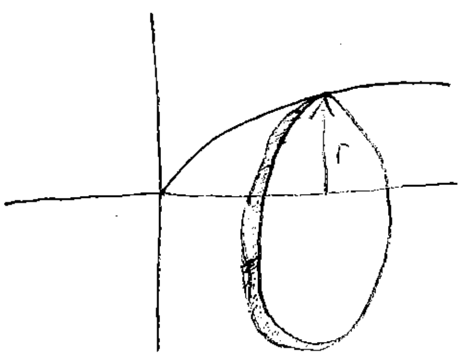


$$A = \int_{-2}^{2} 2x - x^2 - (2x - 4)\,dx$$

$$\int_{-2}^{2} 2x - x^2 - 2x + 4\,dx$$

$$\int_{-2}^{2} -x^2 + 4\,dx = -\frac{x^3}{3} + 4x \Big|_{-2}^{2} = \boxed{\frac{32}{3}}$$

(23) $y = \sqrt{x}$, $y = 0$, $x = 0$, $x = 9$



$r = \sqrt{x} - 0$

$r = \sqrt{x}$

$$\int_0^9 \pi (\sqrt{x})^2\,dx$$

$$\int_0^9 \pi x\,dx = \pi\left(\frac{x^2}{2}\right)\Big|_0^9$$

$$A = \boxed{\frac{81}{2}\pi}$$

Use the disk method to find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.

23) $y = \sqrt{x}$, $y = 0$, $x = 0$, $x = 9$

A) $\frac{81}{2}\pi$ B) 9π C) $\frac{9}{2}\pi$ D) 27π

Use the washer method to find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.

24) $y = \sqrt{3x}$, $y = 3$, $x = 0$

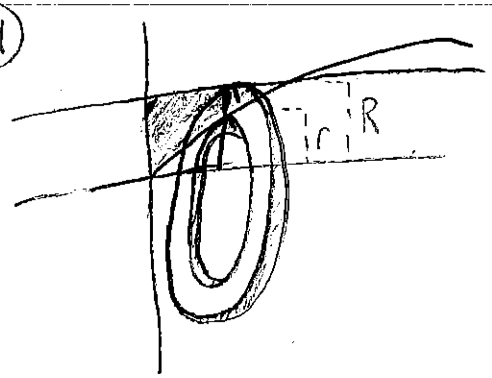
A) $\frac{27}{4}\pi$ B) 9π C) $\frac{27}{2}\pi$ D) 18π

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves and lines about the y-axis.

25) $y = 5x^2$, $y = 5\sqrt{x}$

A) $\frac{15}{4}\pi$ B) 3π C) $\frac{3}{4}\pi$ D) $\frac{3}{2}\pi$

24



$R = y = 3 - 0 = 3$

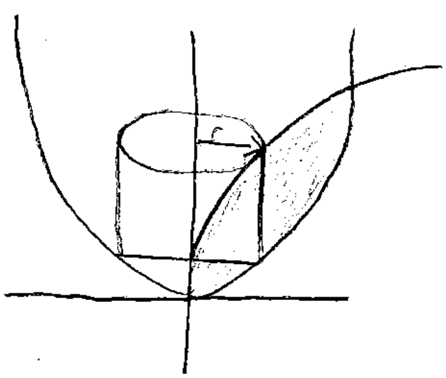
POI: $\sqrt{3}x = 3$, $3x = 9$, $X = \frac{9}{3}$, $\boxed{X = 3}$

$r = \sqrt{3x} - 0 = \sqrt{3x}$

$$V = \int_0^3 \pi(R^2 - r^2)\,dx = \int_0^3 \pi(9 - 3x)\,dx$$

$$V = \pi\left(9x - \frac{3}{2}x^2\right)\Big|_0^3 = \boxed{\frac{27\pi}{2}}$$

$$V = \int_a^b 2\pi r \cdot ht \cdot dx$$



$r = X$

$$\int_0^1 2\pi x\left(5x^{\frac{1}{2}} - 5x^2\right)dx = \int_0^1 2\pi\left(5x^{\frac{3}{2}} - 5x^3\right)dx$$

$$= 2\pi\left(2x^{5/2} - \frac{5}{4}x^4\right)\Big|_0^1 = \pi\left(4x^{5/2} - \frac{5}{2}x^4\right)\Big|_0^1$$

$$= \pi\left(4(1)^{5/2} - \frac{5}{2}(1)^4\right) = \left(4 - \frac{5}{2}\right)\pi$$

$$= \frac{8-5}{2} = \boxed{\frac{3}{2}\pi}$$

$ht = 5\sqrt{x} - 5x^2$

$ht = 5(x)^{\frac{1}{2}} - 5x^2$

POI

$5\sqrt{x} = 5x^2$

$25\cancel{x} = 2x^4$

$2x^4 - 25x = 0$

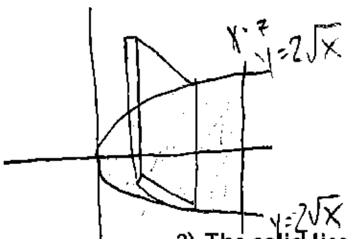
MAC 2311 - Calculus w/Analytic Geometry I   Name________________

Review for Test 5   Date________________

For all volume problems (a) plot the graphs, (b) shade the bounded region, (c) show a typical section, (d) set up the integral, (e) integrate, (f) and state the volume. If needed, set ups for limits, POIs, radii, height, etc. must be shown. Round all answers to three decimal places.

Find the volume of the described solid.

1) The solid lies between planes perpendicular to the x-axis at x = 0 and x = 7. The cross sections perpendicular to the x-axis between these planes are squares whose bases run from the parabola $y = -2\sqrt{x}$ to the parabola $y = 2\sqrt{x}$.



$S = 2\sqrt{x} - (-2\sqrt{x})$

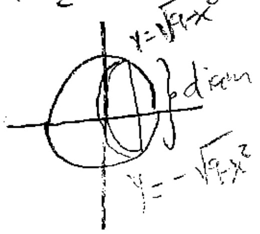
$S = 4x^{\frac{1}{2}}$

$\int_0^7 S^2\,dx = \int_0^7 \left(4x^{\frac{1}{2}}\right)^2 dx$

$\int_0^7 16x\,dx = \frac{16}{2}x^2 \Big|_0^7 = \boxed{392}$

2) The solid lies between planes perpendicular to the x-axis at x = - 3 and x = 3. The cross sections perpendicular to the x-axis are semicircles whose diameters run from $y = -\sqrt{9 - x^2}$ to $y = \sqrt{9 - x^2}$.

$A = \frac{1}{2}\pi r^2$



diameter $= \sqrt{9-x^2} - (-\sqrt{9-x^2})$

$d = 2\sqrt{9-x^2}$

r = half diameter

$r = \frac{1}{2}d = \sqrt{9-x^2}$

$\int_{-3}^{3} \frac{1}{2}\pi\left(\sqrt{9-x^2}\right)^2 dx$

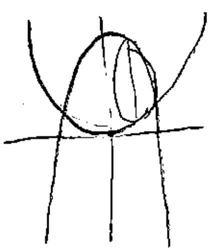
$\int_{-3}^{3} \frac{1}{2}\pi(9-x^2)\,dx$

$\frac{1}{2}\pi\int_{-3}^{3} 9-x^2\,dx$

$\frac{\pi}{2}\left(9x - \frac{x^3}{3}\right)\Big|_{-3}^{3}$

$= 56.549$

3) The solid lies between planes perpendicular to the x-axis at x = - 5 and x = 5. The cross sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 50 - x^2$.



diam $= 50 - x^2 - (x^2)$

$d = 50 - 2x^2$

poi

$x^2 = 50 - x^2$

$2x^2 = 50$

$x = \pm 5$

$r = \frac{1}{2}d = \frac{50-2x^2}{2}$

$r = 25 - x^2$

$V = \int_{-5}^{5} \pi\left(25 - x^2\right)^2 dx$

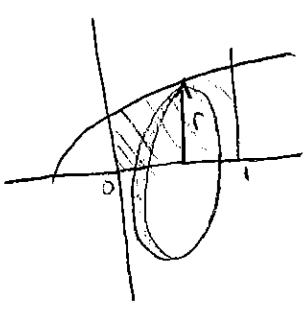
$\pi\int_{-5}^{5} (625 - 50x^2 + x^4)\,dx$

$\pi\left[625x - \frac{50}{3}x^3 + \frac{1}{5}x^5\right]\Big|_{-5}^{5}$

$V = 10{,}471.976$

Find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.

4) $y = \sqrt{2x + 3}$, y = 0, x = 0, x = 1



$r = \sqrt{2x+3}$

$V = \int_0^1 \pi r^2\,dx$

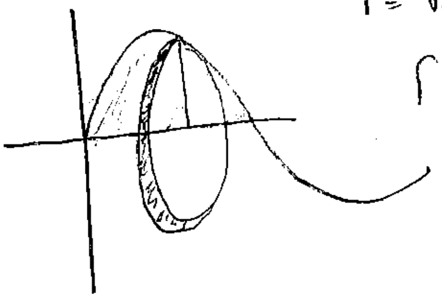
$\int_0^1 \pi\left(\sqrt{2x+3}\right)^2 dx = \boxed{12.566}$

$\pi\left(x^2 + 3x\right)\Big|_0^1$

$u = 3x$

$du = 3dx$ $\quad \pi \cdot \frac{1}{3}\int_0^{\pi/3} \sin u\, du$

5) $y = \sqrt{\sin 3x}$, $y = 0$, $0 \le x \le \frac{\pi}{3}$



$r = \sqrt{\sin 3x} - 0$

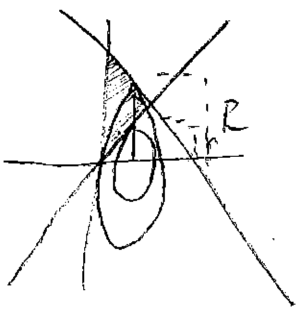
$r = \sqrt{\sin 3x}$

$$\int_0^{\frac{\pi}{3}} \pi\left(\sqrt{\sin 3x}\right)^2 dx = \boxed{2.094}$$

$\frac{\pi}{3}\left(-\cos u\right)\Big|_0^{\pi/3}$

$-\frac{\pi}{3}\cos(3x)\Big|_0^{\pi/3}$

6) $y = -4x + 8$, $y = 4x$, $x = 0$



$\int \pi(R^2 - r^2)\,dx$

$R = -4x + 8 - 0$

$r = 4x - 0$

$r = 4x$

Poi

$-4x + 8 = 4x$

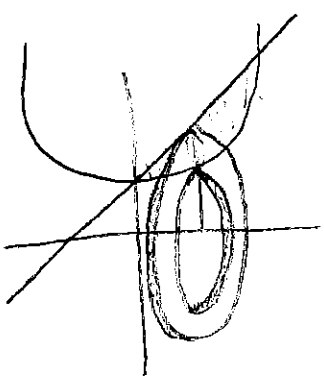
$8x = 8$

$x = 1$

$$\int_0^1 \pi\left[(-4x+8)^2 - (4x)^2\right] dx$$

$\boxed{V = 100.531}$

7) $y = x^2 + 3$, $y = 3x + 3$



$\int \pi(R^2 - r^2)\,dx$

$R = 3x + 3$

$r = x^2 + 3$

Poi

$3x + 3 = x^2$

$x^2 - 3x - 3 + 3$

$(x^2 - 3x) = 0$

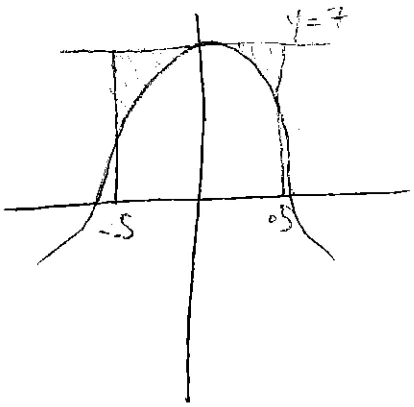
$x = 0, 3$

$$\int_0^3 \pi (3x+3)^2 - (x^2+3)^2 dx = \boxed{V = 186.611}$$

8) $y = 7\cos(\pi x)$, $y = 7$, $x = -0.5$, $x = 0.5$

$R = 7 - 0$

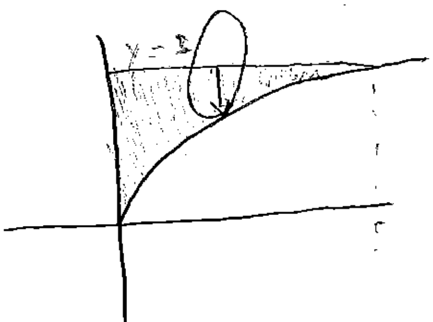
$r = 7\cos(\pi x) - 0$



$$V = \int_{-.5}^{.5} \pi\left((7)^2 - (7\cos(\pi x))^2\right) dx = \boxed{76.969}$$

Find the volume of the solid generated by revolving the region about the given line.

9) The region in the first quadrant bounded above by the line $y = 5$, below by the curve $y = \sqrt{5x}$, and on the left by the y-axis, about the line $y = 5$



Poi

$\sqrt{5x} = 5$

$5x = 25$

$x = 5$

$r = 5 - \sqrt{5x}$

$\int_0^5 \pi (5 - \sqrt{5x})^2$

$\pi \int_0^5 25 - 10\sqrt{5x} + 5x \, dx$

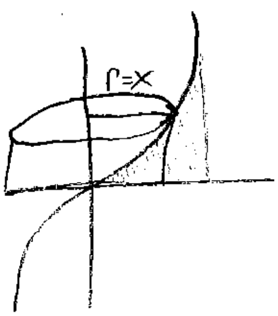
$\rightarrow \pi \left(25x - \frac{10\sqrt{5}\, x^{3/2}}{3/2} + \frac{5}{2}x^2\right)$

$\pi \left(25x - \frac{20\sqrt{5}\, x^{3/2}}{3} + \frac{5}{2}x^2\right)\Big|_0^5$

$= \boxed{65.450}$

Find the volume of the solid generated by revolving the region about the y-axis.

10) The region in the first quadrant bounded on the left by $y = x^3$, on the right by the line $x = 4$, and below by the x-axis



$r = x$

$ht = top - bottom$

$ht = x^3 - 0$

$ht = x^3$

$\int 2\pi r \cdot ht \cdot th$

$\int_0^4 2\pi x (x^3) dx$

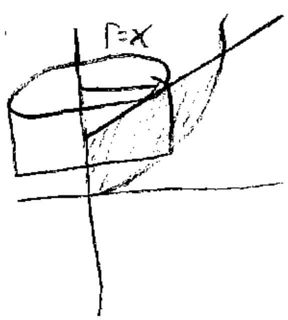
$\rightarrow 2\pi \left[\frac{x^5}{5}\right]\Big|_0^4$

$= \frac{2\pi x^5}{5}\Big|_0^4$

$\boxed{V = 1286.796}$

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves and lines about the y-axis.

11) $y = x^2$, $y = 3 + 2x$, for $x \geq 0$



$r = x$

$ht = 3 + 2x - x^2$

Poi

$3 + 2x - x^2 = 0$

$(x+1)(x-3)$

$x = -1, 3$

$\int_0^3 2\pi x \cdot (3 + 2x - x^2) dx$

$\int_0^3 2\pi (3x + 2x^2 - x^3) dx$

$2\pi \left(\frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{x^4}{4}\right)\Big|_0^3 = \boxed{70.686}$

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves and lines about the x-axis.

12) $x = 3y - y^2$, $x = 0$

$x = 3y - y^2$

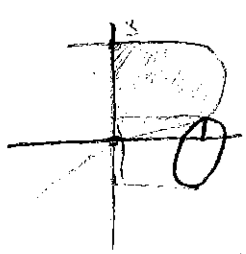
$x = 0$

Set =

$3y - y^2 = 0$

$y(y-3)$

$y = 0, 3$



$r = y$

$ht = 3y - y^2 - 0$

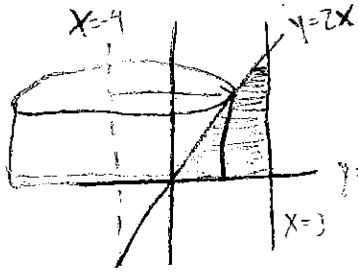
$V = \int_0^3 2\pi y (3y - y^2) dy$

$= \int_0^3 2\pi (3y^2 - y^3) dy$

$= 2\pi \left(y^3 - \frac{y^4}{4}\right)\Big|_0^3 = \boxed{42.412}$

Use the shell method to find the volume of the solid generated by revolving the region bounded by the given curves about the given lines.

13) y = 2x,   y = 0,   x = 3; revolve about the line x = -4



$r = X - (-4)$ Poi

$r = X + 4$ $2X = 0$ $X = 0$

$ht = 2X - 0$

$ht = 2X$

$\int^{3} 2\pi r \cdot ht \cdot th$

$\int_0^{3} 2\pi (X+4)(2X)\,dx$

$\int_0^{3} 2\pi (2X^2 + 8X)\,dx$

$2\pi\left(\frac{2}{3}X^3 + 4X^2\right)\Big|_0^3$

$= \boxed{339.292}$

Solve the problem.

14) The spring of a spring balance is 6.0 in. long when there is no weight on the balance, and it is 7.9 in. long with 8.0 lb hung from the balance. How much work is done in stretching it from 6.0 in. to a length of 13.5 in.?

$F = Kx$

$8 = K(7.9 - 6)$

$K = 4.211 \frac{lb}{in}$

$F = 4.211X$

$W = \int f(x)\,dx$

$13.5 - 6.0 = 7.5$ in

$\int_0^{7.5} 4.211x\,dx$

$\frac{4.211}{2}X^2 \Big|_0^{7.5} = \boxed{118.434 \text{ in-lbs}}$

15) It takes a force of 16,000 lb to compress a spring from its free height of 13 in. to its fully compressed height of 8 in. How much work does it take to compress the spring the first inch?

13 in

8 in

$16000 = k\,5$

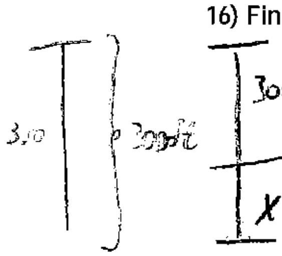
$3200 \frac{lb}{in} = k$

$F = 3200X$

$W = \int_0^1 3200X\,dx$

$= \frac{3200}{2}X^2 \Big|_0^1 = 1600$ in·lbs

16) Find the work done in winding up a 300-ft cable that weighs 3.00 lb/ft.



$K = 3 \frac{lb}{ft}$

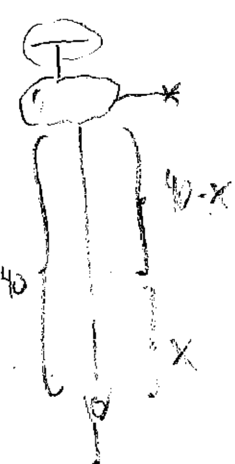
$F = 3(300 - X)$

$F = 900 - 3X$

$W = \int_0^{300} (900 - 3x)\,dx$

$900X - \frac{3}{2}X^2 \Big|_0^{300} = \boxed{135{,}000 \text{ ft}\cdot\text{lbs}}$

17) A rescue cable attached to a helicopter weighs 2 lb/ft. A 160-lb man grabs the end of the rope and is pulled from the ocean into the helicopter. How much work is done in lifting the man if the helicopter is 40 ft above the water?



$W_T = W_{man} + W_{cable}$

$W_M = 40 \cdot 160$ lb

$W_m = 6400$

$K_{cable} = 2 \frac{lbs}{ft}$

$F = 2(40 - X)$

$W_{cable} = \int_0^{40} 2(40 - X)\,dx$

$80X - X^2 \Big|_0^{40}$

$= 1600$

$W_T = 6400 + 1600 = 8000$ ft·lbs

Answer Key
Testname: MAC 2311 REV T5 - CH 6 NO MC

1) 392
2) 18π
3) $\frac{10000}{3}\pi$
4) 4π
5) $\frac{2}{3}\pi$
6) 32π
7) $\frac{297}{5}\pi$
8) $\frac{49}{2}\pi$
9) $\frac{125}{6}\pi$
10) $\frac{2048}{5}\pi$
11) $\frac{45}{2}\pi$
12) $\frac{27}{2}\pi$
13) 108π
14) 120 lb·in.
15) 1600 in. · lb
16) 135,000 ft·lb
17) 8000 ft · lb