Homework #1 (Covers Unit-1, Unit-2 and Unit-3)
CDA Computer Logic Design
Total Points: 100

Notes:
1. All homework should be done and submitted individually
2. Show all steps for each question to get full points (Use extra pages if required)
3. Submit electronically in canvas as a single pdf file
4. Follow instructions for each question
5. A' is the complement of A

Q1. Write True or False after each question. Subscript defines the base format. (8*3 = 24 points)

(a) Addition in binary: $(100010)_2 + (111000)_2 + (001101)_2 = (1100111)_2$
Answer: true

(b) Subtraction in octal: $(7305)_8 - (5274)_8 = (2011)_8$
Answer: false

(c) Division of $(110011)_2$ by $(1100)_2$ produces remainder $(10)_2$
Answer: false

(d) Multiplication in hexadecimal: $(5E2)_{16} \times (F04)_{16} = (585588)_{16}$
Answer: true

(e) Conversion: $(1011.101)_2 = (11.5)_8$
Answer: false

(f) Conversion: $(1011.1101)_2 = (C.B)_{16}$
Answer: false

(g) Conversion: $(5752.777)_8 = (BEA.1FF)_{16}$
Answer: false

(h) DeMorgan's law is limited to 2 variables.
Answer: false

Q1

a.
$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ \ \ \ \ \ \ \ } \\ 16\ 0010_2 \\ 11\ 1000_2 \\ +\ 00\ 1101_2 \\ \hline 1106111_2 \end{array}$$

b.
$$\begin{array}{r} 7305_8 \\ -\ 5274_8 \\ \hline 2031_8 \end{array}$$

c. $1100_2 \overline{)\ 110011_2}$ — 100 R 11$_2$

$$\begin{array}{r} 1100 \\ \hline 01 \\ 00 \\ \hline 011 \\ 000 \\ \hline 011 \end{array}$$

d.
$$\begin{array}{r} {\scriptstyle 5\ 13\ 3\ \ \ \ \ \ \ \ \ \ } \\ 5E2_{16} \\ \times\ F04_{16} \\ \hline 17888 \\ \hline 0 \\ 583E00 \\ \hline 5855888_{16} \end{array}$$

14 (E) $\overline{)\ 56}$ — 3 R 8

$$\begin{array}{r} 4 \\ \hline 56 \end{array} \qquad \begin{array}{r} 48 \\ \hline 8 \end{array}$$

$16\overline{)\ 23}$ — 1 R 7

$$\begin{array}{r} {\scriptstyle 2\ \ \ \ \ \ \ \ \ \ } \\ \times\ 15\ (F) \\ \hline 30 \end{array} \qquad 16\overline{)\ 30} \text{ — } 1 \text{ R } 14\ (E)$$

$$\begin{array}{r} {\scriptstyle 5\ \ \ \ \ \ \ \ \ \ } \\ \times\ 15\ (F) \\ \hline 75 \\ +13 \\ \hline 88 \end{array} \qquad \begin{array}{r} 5\text{ R }8 \\ 16\overline{)\ 88} \\ 80 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 14\ (E) \\ \times\ 15\ (F) \\ \hline 210 \\ +\ 1 \\ \hline 211 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ \ \ \ \ \ \ \ \ \ } \\ 16\overline{)\ 211} \\ 16 \\ \hline 51 \\ 48 \\ \hline 3 \end{array} \quad 13 \text{ R } 3$$

e. 1011.101_2

$2^3+2^1+2^0+2^{-1}+2^{-3}$

$8+2+1+\frac{1}{2}+\frac{1}{8}$

$11\frac{5}{8}$

11.5_8

$8^1+8^0+5(8^{-1})$

$9\frac{5}{8}$

f. 1011.1101_2

$2^3+2^1+2^0+2^{-1}+2^{-2}+2^{-4}$

$8+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{16}$

$11\frac{13}{16}$

$C.B_{16}$

$12(16^0)+11(16^{-1})$

$12\frac{11}{16}$

g. $5752.777_8 = BEA.FF8_{16}$

101 111 101 010 . 111 111 111

1011 | 1110 | 1010 . 111 | 111 | 111 | 000

B E A . F F 8

Q2. Conversion of numbers. (5*3 = 15 points)

(a) Hexadecimal 1FB to decimal

(b) Octal 374 to Binary

(c) Binary 101111110110 to Hexadecimal

(d) Decimal 627 to Trinary (Base 3)

(e) What is the base x in ~~$(2345)_x$~~ $(2400)_x = (1010)_7$

Answers:

(a) 1FB

$(1 \times 16^2) + (15 \times 16^1) + (11 \times 16^0)$

507

(b) 374

3 7 4

011 111 100

011111100

(c) 1011 1111 0110

B F 6

(d)

627	0
209	2
69	0
23	2
7	1
2	2

21 2020

(e) 1010_7

$(1 \times 7^3) + (0 \times 7^2) + (1 \times 7^1) + (0 \times 7^0)$

350

$2x^3 + 4x^2 = 350$

$x^3 + 2x^2 = 175$

$x = 5$

Q3. (a) A decimal integer is in the range of 102 to -102. How Many bits are required to represent any value in this range in 2's complement representation? (3 points)

(b) Assume the same number of bits as part (a), compute the following using 2's complement method and comment on the correctness of the result. (4*3 = 12 points)

(b1) 75 – 32
(b2) -57 + 99
(b3) -52 – 84
(b4) 37 + 93

Answers:

(a)

102 → 0110 0110
-102 → 1001 1001

8 bits

(b1)

75 → 0100 1011
-32 → 1110 0000

omit [1]
correct

[1] 0010 1011 = 43

(b2)

-57 → 1106 0111
99 → 0110 0011

omit [1]
correct

[1] 0010 1010 = 42

(b3)

-52 → 1100 1100
-84 → 1010 1100

omit [1]
overflow
incorrect

[1] 0111 1000 = 120

(b4)

37 → 0010 0101
93 → 0101 1101

overflow
incorrect

1000 0010 = -126

Q4. Simplify the following Boolean functions (3*5 = 15 points)

(a) F = XY + XY'

(b) F = (X + Y) (X + Y')

(c) F = Y'Z + X'YZ + XYZ

(d) F = (X + Y) (X' + Y + Z) (X' + Y + Z)

(e) F = X + XYZ + X'YZ + X'Y + WX + WX'

Answers:

(a) $XY + X\bar{Y}$

$X(Y + \bar{Y})$

$= X$

(b) $(X+Y)(X+\bar{Y})$

$XX + X\bar{Y} + XY + Y\bar{Y}$

$X + X(Y + \bar{Y})$

$X + X$

$= X$

(c) $\bar{Y}Z + \bar{X}YZ + XYZ$

$\bar{Y}Z + YZ(X + \bar{X})$

$Z(Y + \bar{Y})$

$= Z$

(d)

$XZ + Y$

(e) $X + XYZ + \bar{X}YZ + \bar{X}Y + WX + W\bar{X}$

$X + YZ(X + \bar{X}) + \bar{X}Y + W(X + \bar{X})$

$X + YZ + \bar{X}Y + W$

Q5. We can perform logical operations on strings of bits by considering each pair of bits separately (called bitwise operation). (3*3 = 9 points)

Given two strings A and B

A = 10110110

B = 11001000

Perform the bitwise operation using the following functions

(a) NAND

(b) NOR

(c) XOR

Answers:

(a)

NAND → 0111 1111

(b)

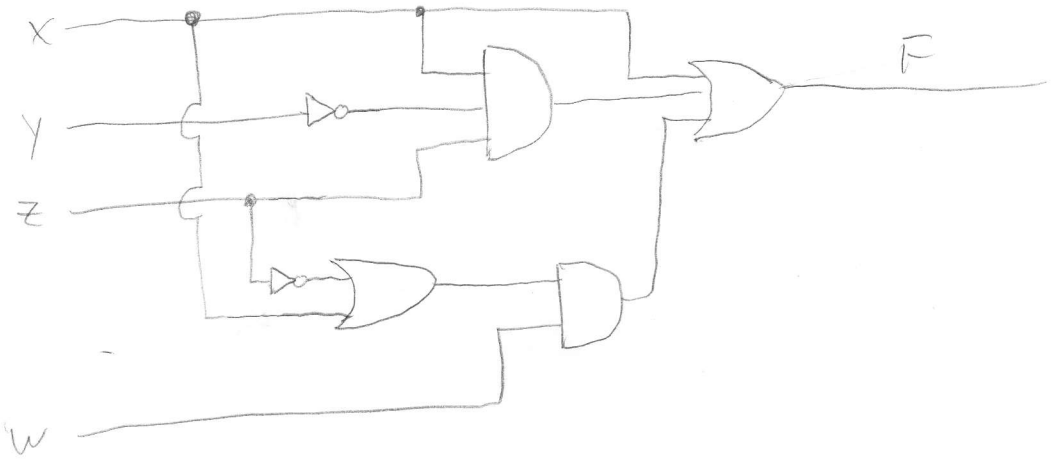
NOR → 0000 0001

(c)

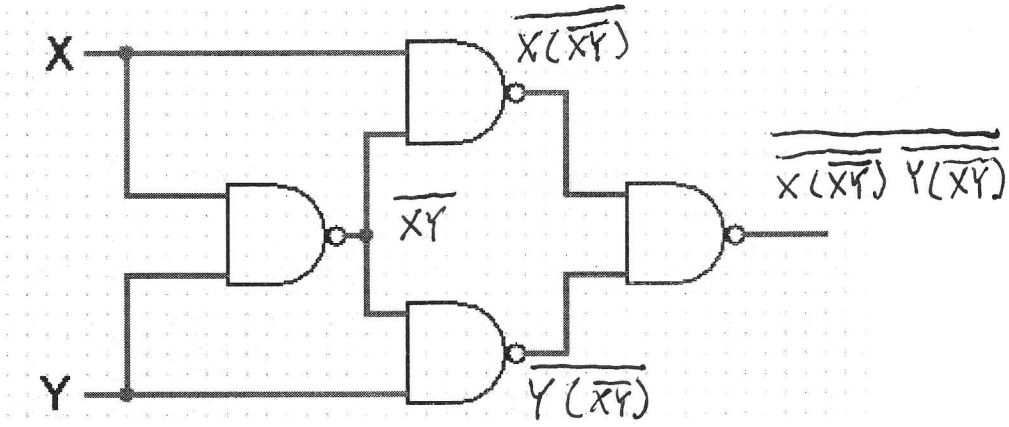
XOR → 0111 1110

Q6. (2*3 = 6 points)

(a) Draw the logic diagram for F = X + XY'Z + W(X+Z')



(b) Obtain the Reduced Boolean expression for the following circuit diagram (ignore coloring)



$\overline{X(\overline{XY})}\ \overline{Y(\overline{XY})}$

$\overline{X(\overline{XY})} + \overline{Y(\overline{XY})}$

$X(\overline{XY}) + Y(\overline{XY})$

$X(\overline{X}+\overline{Y}) + Y(\overline{X}+\overline{Y})$

$X\overline{X} + X\overline{Y} + \overline{X}Y + Y\overline{Y}$

$X\overline{Y} + \overline{X}Y$

Q7. Use DeMorgan's theorem to simplify the following expressions: (2*3 = 6 points)

(a) ((A' + B)' (C'+D')')'

(b) ((AB'C)' + (AB')')'

(a) $\overline{(\overline{(\overline{A}+B)}\ \overline{(\overline{C}+\overline{D})})}$

$(A\overline{B})(CD)$

$\overline{A}+B+\overline{C}+\overline{D}$

(b) $\overline{(\overline{(A\overline{B}C)}+\overline{(A\overline{B})})}$

$(\overline{A}+B+\overline{C})+(\overline{A}+B)$

$(A\overline{B}C)(A\overline{B})$

$A\overline{B}C$

Q8. Kyle, Patrick, Jorge and Steven are hungry college students. They want a quicker way to decide where to go for lunch, the Marshall Center or Juniper. The majority wins, except when Jorge and Steven both agree, then they win. Any other ties end with a trip to Juniper. What would be the design of the logic circuit that automatically selects the restaurant when everyone votes? Show truth table, minimized Boolean expression, and circuit diagram. (10 points)

Let 0 represent Juniper

A Kyle	B Patrick	C Steven	D Jorge	Place to eat
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$\bar{A}\bar{B}CD + \bar{A}B\bar{C}D + A\bar{B}CD + AB\bar{C}D + ABC\bar{D} + ABCD$

$CD(\bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB) + AB(\bar{C}D + C\bar{D})$

$CD(\bar{A}(B+\bar{B}) + A(B+\bar{B})) + AB(\bar{C}D + C\bar{D})$

$CD(A + \bar{A}) + AB(\bar{C}D + C\bar{D})$

$CD + AB(\bar{C}D + C\bar{D})$

$CD + AB(C \oplus D)$