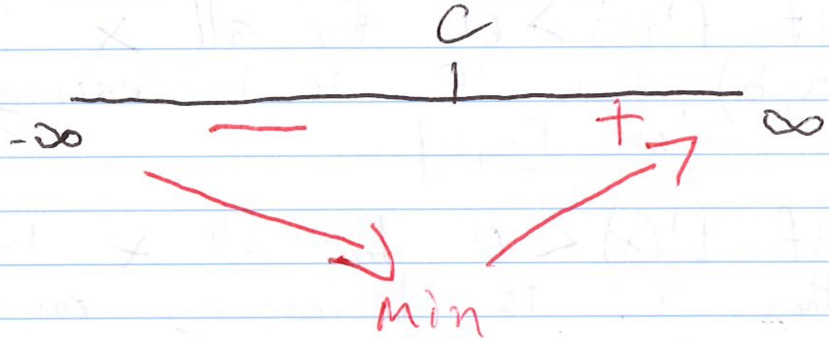
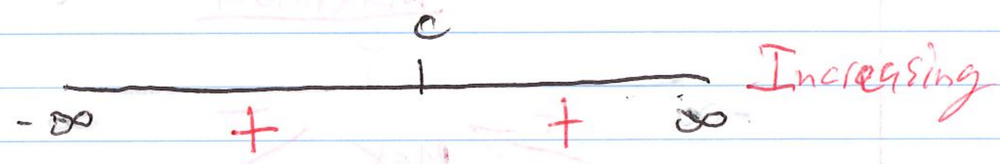
b) If the signs of $f'(c)$ from negative to positive then "f" has a minimum at $(c, f(c))$




c) If the signs of $f'(c)$ do not change from left to right then "f" is constant at "C"

# 4.3 Examples

6) $f(x) = x^4 - 2x^2$

find critical #
then you'll Know if
increase or decrease

$f'(x) = 4x^3 - 4x$

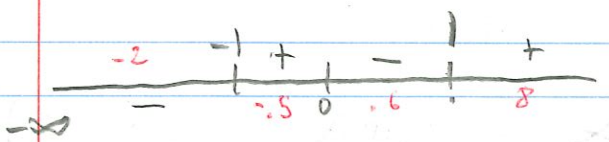
$\frac{4x^3}{4} - \frac{4x}{4} = \frac{0}{4}$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x-1)(x+1)$

$\boxed{x=0}$ $\boxed{x=1}$ $\boxed{x=-1}$




what are these
signs at those
points?

f is increasing on $(-1, 0)$ and $(1, \infty)$
and decreasing on $(-\infty, -1)$ and $(0, 1)$

12) $h(x) = 27x - x^3$

$f'(x)\ 27 - 3x^2$

$\frac{27}{-3} - \frac{3x^2}{-3} = \frac{0}{-3}$

$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$\boxed{x = -3}$ $\boxed{x = 3}$

$-4 \quad -3 \quad -2 \quad 3 \quad 4$

$-\infty \quad - \quad + \quad - \quad \infty$

f is increasing on $(-3, 3)$ and decreasing on $(-\infty, -3)$ and $(3, \infty)$

16) $f(x) = \cos^2 x - \cos x \qquad (0, 2\pi)$

$f'(x) = -2\cos x \sin x + \sin x$

$-2\cos x \sin x + \sin x = 0$

$\sin x(-2\cos x + 1) = 0$

$\sin x = 0$

$x = \pi$

$-2\cos x + 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

$\frac{\pi}{4} \quad \frac{\pi}{2} \quad \frac{3\pi}{2} \quad \frac{11\pi}{6}$

$0 \quad - \quad \pi/3 \quad + \quad \pi \quad - \quad \frac{5\pi}{3} \quad + \quad 2\pi$

f is increasing on $(\frac{\pi}{3}, \pi)$ and $(\frac{5\pi}{3}, 2\pi)$

decreasing on $(0, \frac{\pi}{3})$ and $(\pi, \frac{5\pi}{3})$

24) $f(x) = (x+2)^2(x-1)$

$f'(x) = 2(x+2)(x-1) + 1(x+2)^2$

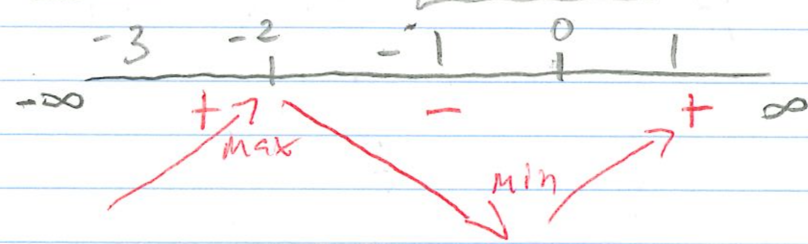
$f'(x) = (x+2)[2(x-1) + (x+2)]$

$f'(x) = 3x(x+2)$

$3x(x+2) = 0$

$3x = 0$ $\qquad$ $(x+2) = 0$

$\boxed{x = 0}$ $\qquad$ $\boxed{x = -2}$




$x = -2$

$f(-2) = (-2+2)^2(-2-1) = \boxed{0}$

$f(0) = (0+2)^2(0-1) = \boxed{-4}$

f is increasing on $(-\infty, -2)$ and $(0, \infty)$

f is decreasing on $(-2, 0)$

f has a Relative Max @ $(-2, 0)$

f has a Relative Min @ $(0, -4)$

38) $f(x) = \dfrac{x^2 - 3x - 4}{x-2}$

$$f'(x) = \frac{(2x-3)(x-2) - 1(x^2-3x-4)}{(x-2)^2}$$

$$f'(x) = \frac{x^2 - 4x + 10}{(x-2)^2}$$

$x^2 - 4x + 10 = 0$

Since $b^2 - 4ac = -24$ there are no real solutions

$x - 2 = 0$

$\boxed{x = 2}$

1 2 3

$\infty$ + + $\infty$ → constant increase

f is increasing on all real numbers and no Extrema.

because of $\ln x$

4b) $f(x) = \dfrac{x^3}{3} - \ln x \qquad (0, \infty)$

$f'(x) = x^2 - \dfrac{1}{x}$

$x^2 - \dfrac{1}{x} = 0$

$\dfrac{x^3 - 1}{x} = 0$

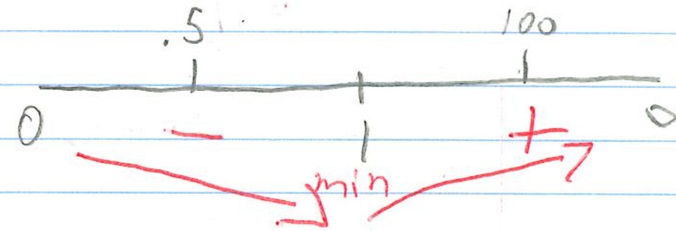
$x^3 - 1 = 0$

$(x-1)(x^2+x+1) = 0$

$x - 1 = 0$ | $x^2+x+1=0$

$x = 1$ | Since $b^2 - 4ac = -3$ No real sol

$x = 0$




$\rightarrow \ln(1) = 0$

$x = 1$

$f(1) = \frac{1}{3}(1)^3 - \ln(1) = \frac{1}{3}$

f is increasing on $(1, \infty)$ and decreasing on $(0, 1)$

f has a Rel Min at $(1, \frac{1}{3})$

54) $f(x) = \dfrac{\sin x}{1+\cos^2 x}$ $(0, 2\pi)$

$$f'(x) = \frac{\cos x\,(1+\cos^2 x) + 2\cos x \sin x(\sin x)}{(1+\cos^2 x)^2}$$

$$f'(x) = \frac{\cos x\left[(1+\cos^2 x) + 2\sin^2 x\right]}{(1+\cos^2 x)^2}$$

$$1+\underbrace{\cos^2 x + \sin^2 x}_{=1} + \sin^2 x$$

$$1+1+\sin^2 x$$

$$f'(x) = \frac{\cos x\,(2+\sin^2 x)}{(1+\cos^2 x)^2}$$

$f'(x) = 0$

$\cos x\,(2+\sin^2 x) = 0$

| | |
|---|---|
| $\cos x = 0$ | $2+\sin^2 x = 0$ |
| $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ | No real solution since sum of two squares $\neq 0$ |

$0 \quad + \quad \frac{\pi}{2} \quad - \quad \frac{3\pi}{2} \quad + \quad 2\pi$

$x = \frac{\pi}{2}$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin \frac{\pi}{2}}{1+\left(\cos\frac{\pi}{2}\right)^2} = 1$$

$x = \frac{3\pi}{2}$

$$F\left(\frac{3\pi}{2}\right) = \frac{\sin \frac{3\pi}{2}}{1+\left(\cos \frac{3\pi}{2}\right)^2} = -1$$

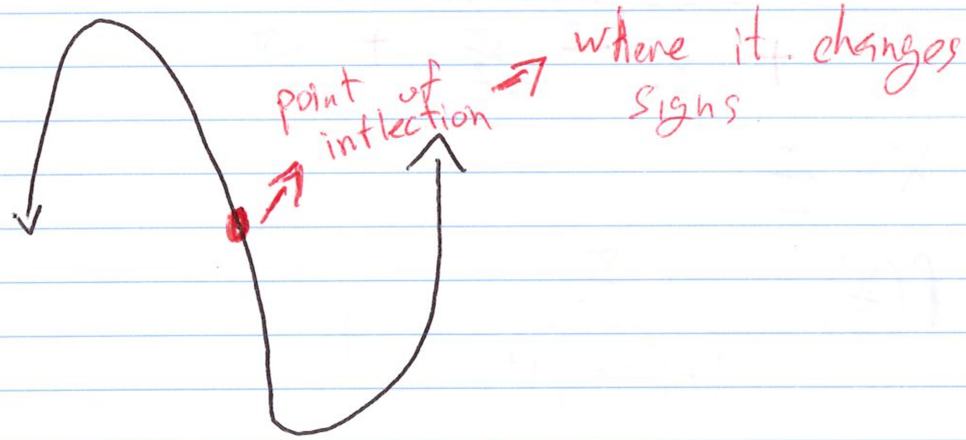
F is increasing on $\left(0, \frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2}, 2\pi\right)$

f is decreasing on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

f has a min @ $\left(\frac{3\pi}{2}, -1\right)$

F has a max @ $\left(\frac{\pi}{2}, 1\right)$

# 4.4 Concativity of a fxn




Let "f" be a continuous fxn on $[a, b]$ and differentiable on $(a, b)$ and "c" is a number in $(a, b)$

1) if $f''(x) > 0$ for all $x$ in $(a, b)$ then "f" concaves up


2) "$f''(x) < 0$ then "f" concaves down


3) The point $(c, f(c))$ is an inflection point if $f''(c) = 0$ or does not exist $\Rightarrow$ not crit #

# Second Derivative Test

Let "f" be a continuous fxn on $[a,b]$ and differentiable on $(a,b)$.
Let "c" be a critical value such that $f'(c)=0 \rightarrow$ crit #

1) If $f''(c) < 0$, "f" has a Max
2) If $f''(c) > 0$ then "f" has a Min
3) If $f''(c) = 0$, you cannot use the second derivative test

24) $f(x) = \sin x + \cos x \quad [0, 2\pi]$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

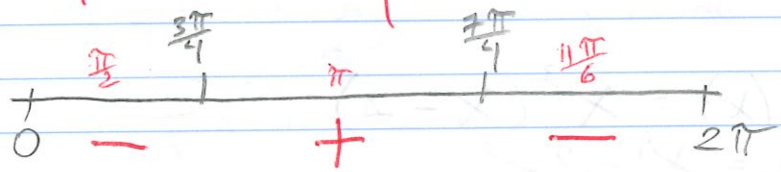
$-\sin x - \cos x = 0$

$-\sin x = \cos x$

They are only equal at angle 45°

SO...

$x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$




$f\left(\frac{3\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{4}\right)$

$f\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{2}}{2}\right) = \boxed{0}$

$f\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{0}$

Concaves up $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

Concaves down $\left(0, \frac{3\pi}{4}\right)$ & $\left(\frac{7\pi}{4}, 2\pi\right)$

inflection pts are $\left(\frac{3\pi}{4}, 0\right)$ & $\left(\frac{7\pi}{4}, 0\right)$

Since $g''(-2) < 0$, $g$ has Max @ $(-2, 0)$

Since $g''(4) < 0$, $g$ has Max @ $(4, 0)$

Since $g''(1) >$, $g$ has Min @ $(1, -\frac{81}{8})$

$$36)\quad g(x) = -\frac{1}{8}(x+2)^2(x-4)^2$$

$$f'(x) = -\frac{1}{4}(x+2)(x-4)^2 + 2(x-4)\left[-\frac{1}{8}(x+2)^2\right]$$

$$= -\frac{1}{4}(x+2)(x-4)\underbrace{\left[(x-4)+(x+2)\right]}_{=2x-2\;=2(x-1)}$$

$$f'(x) = -\frac{1}{2}(x+2)(x-4)(x-1)$$

$$f'(x) = 0 \quad \boxed{x=-2} \quad \boxed{x=+4} \quad \boxed{x=1}$$

$$(x+2)(x-4) = x^2-2x-8\,(x-1) =$$
$$= x^3 - x^2 - 2x^2 + 2x - 8x + 8 = x^3 - 3x^2 - 6x + 8$$

$$f'(x) = -\frac{1}{2}(x^3 - 3x^2 - 6x + 8)$$

$$f''(x) = -\frac{1}{2}(3x^2 - 6x - 6)$$

$$f''(-2) = -\frac{1}{2}(3(-2)^2 - 6(-2) - 6) = \boxed{-9}$$ Max @ X

$$f''(4) = -\frac{1}{2}(3(4)^2 - 6(4) - 6) = \boxed{-9}$$ Max @ X

$$f''(1) = -\frac{1}{2}(3(1)^2 - 6(1) - 6) = \boxed{\frac{9}{2}}$$ Min @ X

$$f(-2) = -\frac{1}{2}(-2+2)^2(-2-4)^2 = \boxed{0}$$

$$f(4) = -\frac{1}{8}(4+2)^2(4-4)^2 = \boxed{0}$$

$$f(1) = -\frac{1}{8}(1+2)^2(1-4)^2 = \boxed{-\frac{81}{8}}$$

$\frac{U'}{U} = \frac{\frac{1}{4}}{\frac{x}{4}} = \frac{1}{X}$

46) $y = x^2 \ln \frac{x}{4}$ $(0, \infty)$

$y' = 2x \left(\ln \frac{x}{4}\right) + x^2 \left(\frac{1}{x}\right)$

$y' = 2x \ln \frac{x}{4} + x$

$2x \ln \frac{x}{4} + x = 0$

$x\left(2 \ln \frac{x}{4} + 1\right) = 0$

$\boxed{x = 0}$ not in domain

$2 \ln \frac{x}{4} + 1 = 0$

$\ln \frac{x}{4} = -\frac{1}{2}$

$e^{-\frac{1}{2}} = \frac{x}{4}$

$\boxed{x = 4e^{-\frac{1}{2}}}$

Since $y = \ln x \rightarrow x = e^y$

so...

$e^{-\frac{1}{2}} = \frac{x}{4}$

$y'' = 2 \ln \frac{x}{4} + \frac{1}{x}(2x) + 1$

$y'' = 2 \ln \frac{x}{4} + 3$

$x = 4e^{-\frac{1}{2}}$

$\ln e^M = M$

$\ln \frac{\cancel{4}e^{-\frac{1}{2}}}{\cancel{4}} = -\frac{1}{2}$

$y''(4e^{-\frac{1}{2}}) = 2 \ln \frac{4e^{-\frac{1}{2}}}{4} + 3 = -1 + 3 = \boxed{2}$

$y(4e^{-\frac{1}{2}}) = (4e^{-\frac{1}{2}})^2 \ln \frac{4e^{-\frac{1}{2}}}{4}$

$y = 16e^{-1} \cdot \left(-\frac{1}{2}\right) = \boxed{-\frac{8}{e}}$

Since $y''(4e^{-\frac{1}{2}}) > 0$, $y$ has Min $\left(4e^{-\frac{1}{2}}, -\frac{8}{e}\right)$

$\log_3 x = \frac{\ln x}{\ln 3}$ = derivative = $\boxed{\frac{1}{\ln 3} \ln x}$

52) $y = x^2 \log_3 x = \frac{1}{\ln 3} x^2 \ln x$

$y' = \frac{1}{\ln 3}\left[2x \ln x + \frac{1}{x} \cdot x^2\right]$

$y' = \frac{1}{\ln 3}\left[2x \ln x + x\right]$

$y' = 0 \rightarrow 2x \ln x + x = 0$

$x(2\ln x + 1) = 0$

$\boxed{x = 0}$ $\quad 2\ln x + 1 = 0$

can't use $\quad 2\ln x = -1$

$\ln x = -\frac{1}{2}$

$\boxed{e^{-\frac{1}{2}} = x}$

$y'' = \frac{1}{\ln 3}\left[2\left(1 \cdot \ln x + \frac{1}{x} \cdot x\right) + 1\right]$

$y'' = \frac{1}{\ln 3}\left[2 \ln x + 2 + 1\right]$

$y'' = \frac{1}{\ln 3}\left[2 \ln x + 3\right]$

$y''(e^{-\frac{1}{2}}) = \frac{1}{\ln 3}\left[2 \ln e^{-\frac{1}{2}} + 3\right]$

$y''(e^{-\frac{1}{2}}) = \frac{2}{\ln 3} > 0$

$x = e^{-\frac{1}{2}}$

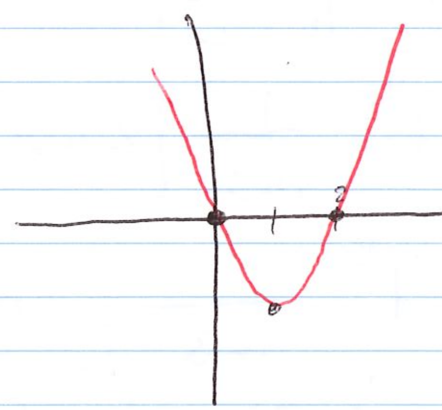
$y = \frac{1}{\ln 3}\left(e^{-\frac{1}{2}}\right)^2 \ln e^{-\frac{1}{2}}$

$y = \frac{1}{\ln 3} \cdot e^{-1} \cdot \left(-\frac{1}{2}\right) = \boxed{-\frac{1}{2e\ln 3}}$

Since the $y''(e^{-\frac{1}{2}}) >$ then $y$ has a Min @ $\left(e^{-\frac{1}{2}}, -\frac{1}{2e\ln 3}\right)$

70) $f(0) = f(2) = 0 \rightarrow$ cross $x$ axis @ 0, 2

$f'(x) < 0$ if $x < 1$



# 4.5 Limit at infinity

Let $f(x)$ be a Fxn
Let "a" and "b" be constants

1) if the $\lim_{x \to a} f(x)$ does not exist
then $x = a$ is a Vertical Asym

2) If the $\lim_{x \to \pm\infty} f(x) = b$ then
$y = b$ is a H.A.

3) If the $\lim_{x \to \pm\infty} f(x) = \pm\infty$ $y = ax + b$ is O.A.

---

a) $\lim_{x \to \pm\infty} \frac{\sin x}{x} = 0$

b) $\lim_{x \to +\infty} e^{x} = \infty$

c) $\lim_{x \to -\infty} e^{x} = 0$

# • 4.5 Examples

12) $f(x) = \dfrac{8}{\sqrt{x^2-3}}$

| $x$ | $10^0$ | $10^1$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ |
|---|---|---|---|---|---|---|---|
| $f(x)$ | ERR | 8.122 | 8.0012 | 8 | 8 | 8 | 8 |

16) $f(x) = 5x^2 - 3x + 7$

a) $h(x) = \dfrac{f(x)}{x}$ or $\lim\limits_{x\to\infty} \dfrac{f(x)}{x}$

$$\lim_{x\to\infty}\left(5x - 3 + \frac{7}{x}\right) = \boxed{\infty}$$

$5(\infty) - 3 + \frac{7}{\infty} =$

b) $\lim\limits_{x\to\infty} \dfrac{f(x)}{x^2} = \lim\limits_{x\to\infty}\left(5 - \dfrac{3}{x} + \dfrac{7}{x^2}\right)$

$(5 - 0 + 0) = \boxed{5}$

c) $\lim\limits_{x\to\infty} \dfrac{f(x)}{x^3} = \lim\limits_{x\to\infty}\left(\dfrac{5}{x} - \dfrac{3}{x^2} + \dfrac{7}{x^3}\right)$

$(0 - 0 + 0) = \boxed{0}$

divide all terms by largest degree of $X$

20) a) $\lim_{x \to \infty} \frac{5x^{3/2}}{4x^2+1}$

$$\lim_{x \to \infty} \frac{\frac{5x^{3/2}}{x^2}}{\frac{4x^2+1}{x^2}} = \lim_{x \to \infty} \frac{\frac{5}{x^{1/2}}}{4+\frac{1}{x^2}}$$

$$= \frac{0}{4+0} = \boxed{0}$$

b) $$\lim_{x \to \infty} \frac{5x^{3/2}}{4x^{3/2}+1} = \lim_{x \to \infty} \frac{\frac{5x^{3/2}}{x^{3/2}}}{\frac{4x^{3/2}+1}{x^{3/2}}}$$

$$= \frac{5}{4+\frac{1}{x^{3/2}}} = \frac{5}{4+0} = \boxed{\frac{5}{4}}$$

c) $$\lim_{x \to \infty} \frac{5x^{3/2}}{4x^{1/2}+1} = \lim_{x \to \infty} \frac{\frac{5x^{3/2}}{x^{3/2}}}{\frac{4x^{1/2}}{x^{3/2}}+\frac{1}{x^{3/2}}}$$

$$= \frac{5}{\frac{4}{x}+\frac{1}{x^{3/2}}} = \frac{5}{0+0} = \frac{5}{0} = \boxed{\infty}$$

$x \longrightarrow -\infty$ $(x < 0)$ so... $-\sqrt{x^2}$

$x \longrightarrow \infty$ $(x > 0)$ so... $\sqrt{x^2}$

28) $\lim\limits_{x \to \infty} \dfrac{x}{\sqrt{x^2+1}}$

$$\lim_{x \to -\infty} \frac{\frac{x}{-x}}{\sqrt{\frac{x^2}{x^2} + \frac{1}{x^2}}}$$

$\dfrac{\sqrt{x^2+1}}{-\sqrt{x^2}} = -\sqrt{\dfrac{x^2+1}{x^2}}$

$$\lim_{x \to -\infty} \frac{-1}{\sqrt{1+\frac{1}{x^2}}} = \frac{-1}{\sqrt{1+0}} = -\frac{1}{1} = \boxed{-1}$$

$$32)\ \lim_{x\to\infty} \frac{3(x-\cos x)}{x} \qquad \frac{x}{x} - \frac{\cos x}{x}$$

$$3\left(\lim_{x\to\infty}\left(1-\frac{\cos x}{x}\right)\right)$$

$$3(1-0) = \boxed{3}$$

* Since $\cos x$ is a constant from $-1$ to $1$ then its $\frac{\text{constant}}{\infty}$

$$36)\ \lim_{x\to-\infty} (2+5e^x)$$

$$2+5\lim_{x\to-\infty} e^x$$

* Since $e^{-\infty} = \frac{1}{e^{\infty}}$ constant over $\infty = 0$

$$2+5(0) = \boxed{2}$$

$$48)\ \lim_{x\to\infty} x\tan\frac{1}{x} \qquad x = \frac{1}{t}$$

$$\lim_{t\to0^+} \frac{1}{t}\tan t$$

$$\lim_{t\to0^+} \frac{\tan t}{t}$$

$$\frac{\frac{\sin t}{\cos t}}{\frac{t}{1}} = \frac{\sin t}{\cos t}\cdot\frac{1}{t} = \frac{\sin t}{t}\cdot\frac{1}{\cos t}$$

$$\lim_{t\to0^+}\left(\frac{\sin t}{t}\cdot\frac{1}{\cos t}\right)$$

$$(1)\cdot(1) = \boxed{1}$$

7 steps here it will be on test

64) $y = \dfrac{x-3}{x-2}$

① Domain $\{x \mid x \neq 2\}$

② Pull out all V.A. finding lim as x goes to that value

$$\lim_{x\to 2} \frac{x-3}{x-2} = \frac{2-3}{2-2} = \frac{-1}{0} = \text{undefined}$$

Since $\lim_{x\to} y$ does not exist

$\therefore x=2$, V.A.

③ Any HA? $\lim_{x\to\pm\infty}$

$$\lim_{x\to\pm\infty} \frac{x-3}{x-2} = \lim_{x\to\pm\infty} \frac{\frac{x}{x}-\frac{3}{x}}{\frac{x}{x}-\frac{2}{x}}$$

$$= \lim_{x\to\pm\infty} \frac{1-\frac{3}{x}}{1-\frac{2}{x}} = \frac{1-\frac{3}{\infty}}{1-\frac{2}{\infty}} = \frac{1-0}{1-0} = \boxed{1}$$

Since $\lim_{x\to\pm\infty} y = 1$ then $y=1$, HA

④ find X-int setting $f(x)=0$

fractions only set top $=0$

$x-3=0 \rightarrow x=3 \quad (3,0)$

y-int

$y(0)=\frac{0-3}{0-2}=\frac{3}{2}\ ; \ (0,\frac{3}{2})$

⑤ Analyzing sign of $f'(x)$

$$f'(x)=\frac{1(x-2)-1(x-3)}{(x-2)^2}=\frac{x-2-x+3}{(x-2)^2}$$

$$f'(x)=\frac{1}{(x-2)^2}$$

$-\infty$ — 1 — 2 — 3 — $\infty$

$+ \qquad +$

$x-2=0 \rightarrow x=2$

f has no Extrema, f is Increasing everywhere

⑥ Sign of $f''(x)$

$f''(x)=(x-2)^{-2}$

$f''(x)=-2(x-2)^{-3}=\frac{-2}{(x-2)^3}$

$x-2=0 \rightarrow x=2$

$-\infty$ — 1 — 2 — 3 — $\infty$

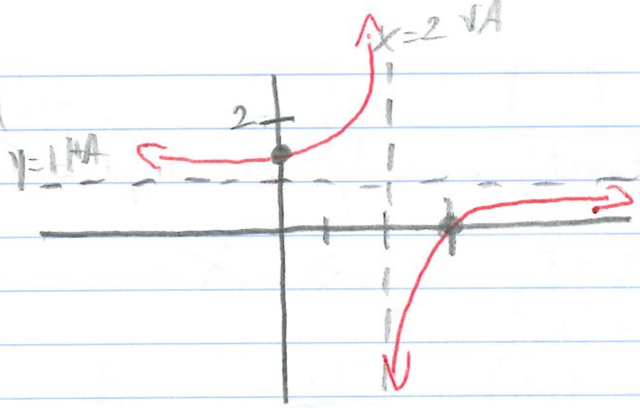
$+ \qquad -$

Concave UP $(-\infty,2)$

Concave Down $(2,\infty)$

f has no inflection point since $x=2$ is not in the domain

(7) Graph



# 4.6 Summary of Curve Sketch

See 7 Steps

## 4.6 Examples

10) $f(x) = \dfrac{x^2-1}{x^2-9}$

Always factor but top is sum of 2 squares *

$f(x) = \dfrac{x^2-1}{(x+3)(x-3)}$

1. Domain $\{x \mid x \neq -3, 3\}$

2. VA $\lim_{x \to -3} \dfrac{x^2+1}{(x+3)(x-3)} = \dfrac{-3^2+1}{(-3+3)(-3-3)} = \dfrac{10}{0}$

Since $\lim_{x \to -3} f(x)$ does not exist then $x=-3$ VA

$\lim_{x \to 3} \dfrac{x^2+1}{(x-3)(x+3)} = \dfrac{3^2+1}{(3-3)(3+3)} = \dfrac{10}{0}$

Since $\lim_{x \to 3} f(x)$ does not exist then $x = 3$, VA

③ HA

$$\lim_{x\to\pm\infty} \frac{x^2+1}{(x-3)(x+3)} = \frac{\frac{x^2}{x^2}+\frac{1}{x^2}}{\frac{(x+3)(x+3)}{x^2}} =$$

$$= \frac{1+\frac{1}{x^2}}{1-\frac{9}{x^2}} = \frac{1-\frac{1}{\infty}}{1-\frac{9}{\infty}} = \frac{1}{1} = \boxed{1}$$

Since $\lim\limits_{x\to\pm\infty} f(x) = 1$ $y=1$ HA

④ X-int Y-int

$x^2+1=0 \longrightarrow x=\sqrt{-1}$

$x^2=-1$ no real solution

* $f(0) = \frac{0^2+1}{(0-3)(0+3)} = \frac{1}{-9}$ $\left(0, -\frac{1}{9}\right)$

⑤ $f'(x)$

$$f'(x) = \frac{2x(x^2-9)-2x(x-1)}{(x^2-9)^2}$$

$$f'(x) = \frac{-20x}{(x^2-9)^2}$$

-3 0 3

$-\infty$ + + max − − $\infty$

$-20x=0$

$\boxed{x=0}$

Are they crit #'s? test values!

$x^2-9=0$

$\boxed{x=\pm 3}$

not crit #

not in Domain

$\boxed{f(0) = -\frac{1}{9}}$

f increases on $(-\infty, -3)$ & $(-3, 0)$
f decreases on $(0, 3)$ & $(3, \infty)$
f has a Max @ $(0, -\frac{1}{9})$

⑥ $f''(x)$

$$f''(x) = \frac{-20x}{(x^2-9)^2} =$$

$$= \frac{-20(x^2-9)^2 - (-20x)(2(x^2-9)^1 \cdot (2x))}{(x^2-9)^4}$$

$$f''(x) = \frac{-20(x^2-9)\left[(x^2-9) - 4x^2\right]}{(x^2-9)^4}$$

$$f''(x) = \frac{-20\left[-3x^2-9\right]}{(x^2-9)^3}$$

$$\frac{-20(-3x^2-9)}{-20} = \frac{0}{-20}$$

$$-3x^2 - 9 = 0$$

$x^2 + 3 = 0$ → Sum of 2 square
No Solution

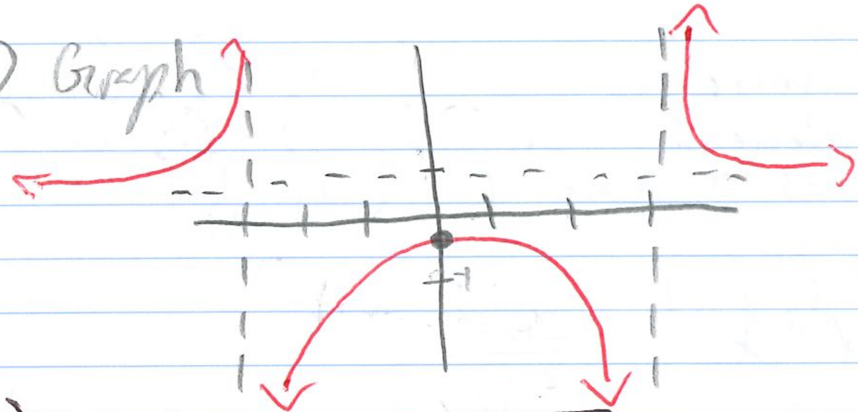
$$(x^2-9) = 0$$
$$x = \pm 3$$

$-\infty$ + $-3$ − $3$ + $\infty$

No inflection point since not Domain

Concaves up $(-\infty, -3)$, $(3, \infty)$
Concave Down $(-3, 3)$

7. Graph




22) $f(x) = x\sqrt{16-x^2}$

1. Domain $16-x^2 \geq 0$

$(x+4)(x-4) \geq 0$

$x \geq \pm 4$

-5 -4 0 4 5

— + —

Domain ; $[-4, 4]$

2. No VA since not rational fxn

3. No HA since not rational

4. x-int y-int

$x\sqrt{16-x^2} = 0$

$x = 0$ | $\sqrt{16-x^2} = \sqrt{0}$

* $(0,0)$ | $x^2 = 16$

$x = \pm 4$

$(4,0)\ (-4,0)$

y-int

$f(0) = 0\sqrt{16-0^2} = \boxed{0}$ $(0, 0)$

(5) $f'(t)$

$f'(x) = x(16-x^2)^{\frac{1}{2}}$

$f'(t) = 1(16-x^2)^{\frac{1}{2}} + \left(\frac{1}{2}(16-x^2)\cdot(-2x)\right)$

$f'(x) = (16-x^2)^{\frac{1}{2}} + \left[-(16-x^2)^{-\frac{1}{2}} \cdot x^2\right]$

$f'(t) = (16-x^2)^{-\frac{1}{2}}\left[(16-x^2) - x^2\right]$

$$f'(x) = \frac{-2x^2+16}{\sqrt{16-x^2}}$$

$-2x^2+16=0$

$x^2-8=0$

$\boxed{x = \pm 2\sqrt{2}}$ → Still in domain

$\sqrt{16-x^2}=0$

$\boxed{x = \pm 4}$

$-3 \quad -2\sqrt{2} \quad 0 \quad 2\sqrt{2} \quad 3$

$-4 \quad - \quad + \quad - \quad 4$

Min Max Min

$f(-2\sqrt{2}) = -2\sqrt{2}(\sqrt{16-(-2\sqrt{2})^2} = \boxed{-8}$

$f(2\sqrt{2}) = 2\sqrt{2}\sqrt{16-(2\sqrt{2})^2} = \boxed{8}$

f increases $(-2\sqrt{2}, 2\sqrt{2})$, decreasing $(-4, -2\sqrt{2})(2\sqrt{2}, 4)$

f has Max

Max @ $(2\sqrt{2}, 8)$

Min@ $(-2\sqrt{2}, -8)$

(6) $f''(x)$

$$f''(x) = \frac{-2x^2+16}{\sqrt{16-x^2}}$$

$$f''(x) = (-2x^2+16)(16-x^2)^{-\frac{1}{2}}$$

$$f''(x) = x(16-x^2)^{-\frac{3}{2}} \cdot (-2x^2+16) + (-4x)(16-x^2)^{-}$$

$$F''(x) = -2x(16-x^2)^{-\frac{3}{2}}\left[(x^2-8)+2(16-x^2)^{1}\right]$$

$$F''(x) = -2x(16-x^2)^{-\frac{3}{2}}\left[24-x^2\right]$$

$$f''(x) = \frac{-2x(24-x^2)}{(16-x^2)^{\frac{3}{2}}}$$

$-2x(24-x^2) = 0$

$x=0$ / $24-x^2=0$, $\boxed{x=\pm 2\sqrt{6}}$ Not in Dom / $16-x^2$, $x=\pm 4$

0

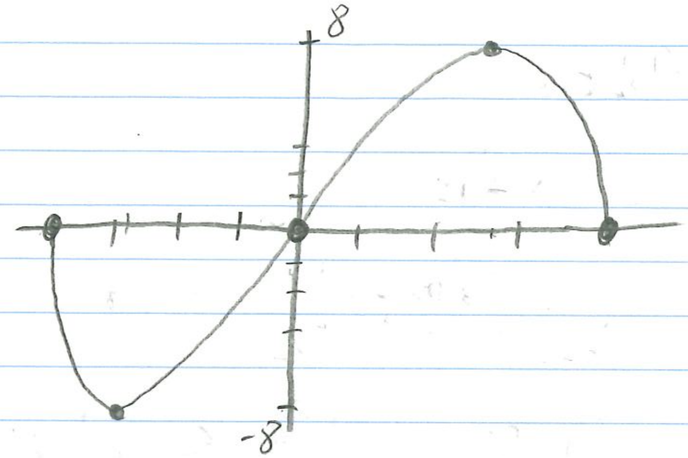
$-4$    $+$    $-$    $4$

* $F(0) = (0,0)$

f concaves up $(-4,0)$, Down $(0,4)$

f has an inflection pt on $(0,0)$

7)




28) $f(x) = \frac{1}{3}(x-1)^3 + 2$

1) Domain all real #'s $(-\infty, \infty)$

2) VA, No VA since not rational

3) HA, No HA or O.A

4) x-int

$^3\left(\frac{1}{3}(x-1)^3 + 2\right) = (0)^3$

Need Parenth

$(x-1)^3 + 6 = 0$

$(x-1)^3 = \sqrt[3]{-6}$

$x = 1 + \sqrt[3]{-6}$

$x = -.8171$

y-int

$f(0) = \frac{1}{3}(0-1)^3 + 2$

$= \frac{1}{3}(-1) + 2$

$-\frac{1}{3} + 2$

$f(0) = \frac{5}{3}$

Final

←8

5) $f'(x) = (x-1)^2$

$(x-1)^2 = 0$

$\boxed{x = 1}$

$-\infty$ + + $\infty$

f has no Extrema.

f increases everywhere

⑥ $f''(x) = 2(x-1)$

$2(x-1) = 0$

$x = 1$

$-\infty \quad - \quad 1 \quad + \quad \infty$

$f(1) = \frac{1}{3}(1-1)^3 + 2 = \boxed{2} \rightarrow$ inflection

f concaves up on $(1, \infty)$ & down on $(-\infty, 1)$

f has inflection point @ $(1, 2)$

⑦ Graph

34) $f(x) = x^4 - 8x^3 + 18x^2 - 16x + 5$

① Domain $(-\infty, \infty)$

② V.A. No VA since not a rational

③ H.A. No HA or O.A since no rational

④ X-int

$Q \rightarrow x^4 - 8x^3 + 18x^2 - 16x + 5 = 0 \rightarrow P$ Need Precalc

find $\frac{P}{Q}$ $P = \pm 5, \pm 1$ Put it in calc

$\boxed{\text{Zeroes} = 1, 5}$ $Q = \pm 1$ test potential rational zeros

$$\begin{array}{r|rrrrr} 1 & 1 & -8 & +18 & -16 & +5 \\ & & 1 & -7 & 11 & -5 \\ \hline & 1 & -7 & 11 & -5 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -7 & 11 & -5 \\ & & 5 & -10 & 5 \\ \hline & 1 & -2 & 1 & \boxed{0} \end{array}$$

$x^2 - 2x + 1 \longrightarrow (x-1)(x-1) = 0$

$\boxed{x = 5}$ & $\boxed{x = 1}$

$f(x) = (x-1)(x-5)(x-1)^2$

$f(x) = (x-5)(x-1)^3$

Y-int = $f(0) = (0-5)(0-1)^3$

$\boxed{f(0) = 5}$

⑤ $f'(x) = (x-5)(x-1)^3$

$f'(x) = (1)(x-1)^3 + (x-5)(3(x-1)^2)$

$f'(x) = (x-1)^2\left[(x-1) + 3(x-5)\right]$

$f'(x) = (x-1)^2\left[4x-16\right]$

$x-1=0$ $\quad$ $4x-16=0$

$\boxed{x=1}$ $\quad$ $\boxed{x=4}$

| $-\infty$ | | 1 | | 4 | | $\infty$ |
|---|---|---|---|---|---|---|
| | $-$ | | $-$ | | $+$ | |

$f(4) = (4-5)(4-1)^3 = \boxed{-27}$

f is decreasing on $(-\infty, 1)$ & $(1, 4)$

f increases on $(4, \infty)$, Rel Min on $(4, -27)$

⑥ $f''(x) = (x-1)^2(4x-16)$

$f''(x) = 2(x-1)(4x-16) + 4(x-1)^2$

$f''(x) = 2(x-1)\,4(x-4) + 4(x-1)^2$

$f''(x) = 4(x-1)\left[2(x-4) + (x-1)\right]$

$f''(x) = 4(x-1)(3x-9)$

$4(x-1)(3x-9) = 0$

$x-1=0$ $\quad$ $3x-9=0$

$\boxed{x=1}$ $\quad$ $\boxed{x=3}$

| $-\infty$ | | 1 | | 3 | | $\infty$ |
|---|---|---|---|---|---|---|
| | $+$ | | $-$ | | $+$ | |

$f(1) = (1-5)(1-1)^3 = \boxed{0}$

$f(3) = (3-5)(3-1)^3 = \boxed{-16}$

f concaves up on $(-\infty, 1)$ & $(3, \infty)$

f conceves down on $(1, 3)$

inflection point on $(1, 0)$ & $(3, -16)$

44) $y = \frac{1}{24}x^3 - \ln x$

1. Domain $x > 0$ $(0, \infty)$

2. V.A. $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1}{24x^3} - \ln x$

$\frac{1}{24}(0)^3 - \ln(0) = \boxed{\text{undefined}}$

Since the $\lim_{\cancel{x} \to 0} f(\cancel{x})$ does not exist $x = 0$ is VA

3. NO HA or O.A

4. X-int

$f(0) = \frac{1}{24}x^3 - \ln x = 0$

Zero is not part of the Domain so no y-int

~~X~~-int -

$\frac{1}{24}x^3 - \ln x = 0$ → Use calculator

use newtons method

⑤ $F'(x) = \frac{1}{8}x^2 - \frac{1}{x} = \frac{x^3-8}{8x}$

$\frac{x^3-8}{8x} = 0 \qquad x^3 - 8 = 0 \qquad \boxed{x = 2}$

$0 \quad - \quad 2 \quad + \quad \infty$

$f(2) = \frac{1}{24}(2)^3 - \ln 2 = $ calculator

$\boxed{F(2) = -.36}$

f is increasing on $(2, \infty)$ & decreasing on $(0,2)$ with a Rel Min @ $(2, -.36)$

⑥ $F''(x) = \frac{1}{8}x^2 - \frac{1}{x}$

$f''(x) = \frac{1}{4}x + \frac{1}{x^2} = \frac{x^3+4}{4x^2}$

$\frac{x^3+4}{4x^2} = 0 \qquad x^3 + 4 = 0$

$x = \sqrt[3]{-4} \rightarrow \boxed{x = -1.59}$

not in domain

Since $x = -1.59$ is not in domain then no inflection point.

Since $f''(x) > 0$ for all $x$ in the domain then $x$ concaves up from $(0, \infty)$

⑦ Graph

52) $F(x) = \cos x - \frac{1}{2}\cos 2x \quad [0, 2\pi]$

1) Domain $[0, 2\pi]$

2) No VA Trig Cos fxn

3) No HA, O.A Trig Cos fxn

4) X-int

$\cos x - \frac{1}{2}\cos 2x = 0$

$\cos x - \frac{1}{2}(2\cos^2 x - 1) = 0$

$-2\left(-\cos^2 x + \cos x + \frac{1}{2}\right) = (0)^{-2}$

$2\cos^2 x - 2\cos x - 1 = 0$

Quadratic formula $a=2$ $b=-2$ $c=-1$

$$\frac{2 \pm \sqrt{4-4(2)(-1)}}{4} = \frac{2 \pm \sqrt{4+8}}{4}$$

$$\cos x = \frac{2 \pm \sqrt{12}}{4}$$

$\boxed{\cos x = 1.366025}$ No Solution

$\cos x = -.366025$

$x = \cos^{-1}(-.366025)$

$\boxed{x = 1.94553}$ $(1.95, 0)$

y-Int

$$\cos 0 - \frac{1}{2}\cos 2(0) = \boxed{\frac{1}{2}} \quad \left(0, \frac{1}{2}\right)$$

$$f'(t) = -\sin x + \sin 2x$$

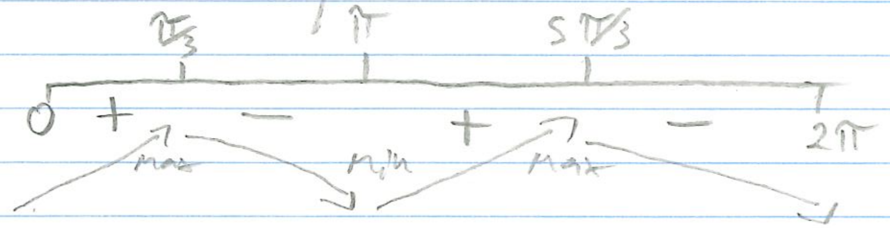
$$-\sin x + \sin 2x = 0$$

$$-\sin x + (2\sin x \cos x) = 0$$

$$-\sin x(1 - 2\cos x) = 0$$

$-\sin x = 0$ / $\cos x = \frac{1}{2}$

$x = 0, \pi, 2\pi$ / $x = \frac{\pi}{3}, \frac{5\pi}{3}$




$$f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} - \frac{1}{2}\cos 2\left(\frac{\pi}{3}\right) = \frac{3}{4}$$

$$f(\pi) = \cos\pi - \frac{1}{2}\cos 2(\pi) = -1.5$$

$$f\left(\frac{5\pi}{3}\right) = \cos\frac{5\pi}{3} - \frac{1}{2}\cos 2\left(\frac{5\pi}{3}\right) = \frac{3}{4}$$

f increases on $\left(0, \frac{\pi}{3}\right)$ & $\left(\pi, \frac{5\pi}{3}\right)$

f decreases on $\left(\frac{\pi}{3}, \pi\right)$ & $\left(\frac{5\pi}{3}, 2\pi\right)$

f has Rel Min @ $\left(\pi, -\frac{3}{2}\right)$

f has Rel Max @ $\left(\frac{\pi}{3}, \frac{3}{4}\right)$ & $\left(\frac{5\pi}{3}, \frac{3}{4}\right)$

$f''(x) = -\cos x + 2\cos(2x)$

$-\cos x + 2(2\cos^2 x - 1) = 0$

$4\cos^2 x - \cos x - 2$

$$\frac{1 \pm \sqrt{1 - 4(4)(-2)}}{8} = \frac{1 \pm \sqrt{33}}{8}$$

$$\cos x = \frac{1+\sqrt{33}}{8} = \boxed{\cos x = .843070}$$

or

$$\cos x = \frac{1-\sqrt{33}}{8} = \boxed{\cos x = -.593070}$$

# 4.7 Optimization

Word Problems !! yay . . .

## 4.7 Examples

4) Let $x$ be the first #
Let $y$ be the second #

$xy = 192$
$S = x + y$

$S = f(x)$

$y = \frac{192}{x}$

$f(x) = x + \frac{192}{x}$

$f'(x) = x + 192x^{-1}$
$f'(x)\ 1 - \frac{192}{x^2} = \frac{x^2 - 192}{x^2}$

$\frac{x^2 - 192}{x^2} = 0$

$x^2 - 192 = 0$
$\boxed{x = \sqrt{192}}$

instruction said 2 positive #'s

$y = \frac{192}{x} = \frac{192}{\sqrt{192}} \cdot \frac{\sqrt{192}}{\sqrt{192}} = \boxed{\sqrt{192}}$

The numbers are $\sqrt{192}$ & $\sqrt{192}$

Test these values at the second derivative to see if it concaves up or down for Max/Min

$$S''(x) = \frac{384}{x^3}$$

Since $S''(\sqrt{192}) > 0$, the sum is Min when $x = \sqrt{192}$ & $y = \sqrt{192}$

8) Let $x$ be first #
Let $y$ be Second #

$x^2 + y = 27 \rightarrow y = 27 - x^2$

$P = xy \rightarrow x(27 - x^2) = \boxed{27x - x^3}$

$f'(x) = 27 - 3x^2 = 0$

$3x^2 = 27$

$x = \pm 3 \rightarrow$ only use positive for this problem

$f''(x) = -6x$

$f''(3) = -6(3) = -18$

Since $f''(3) < 0$ then the product has a Max at $x = 3$

$y = 27 - (3)^2$

$\boxed{y = 18}$ Numbers are 3 & 18

The distance between these two points are closest when its at a minimum
Since $\sqrt{f(x)} < f(x)$ then just find $f'(x)$ then $f''(c)$

$x_1 \ y_1$
16) $f(x) = (x+1)^2$ $\qquad (5,3)$

$(x, (x+1)^2)$
$\ \ x_2 \qquad y_2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x-5)^2 + ((x+1)^2 - 3)^2}$

$(x+1)^2 = x^2 + 2x + 1$
$x^2 + 2x + 1 - 3$
$(x^2 + 2x - 2)^2$
$= x^4 + 2x^3 - 2x^2 + 2x^3 + 4x^2 - 4x$
$-2x^2 - 4x + 4$

$= \sqrt{x^2 - 10x + 25 + x^4 + 4x^3 - 8x + 4}$

$d = \sqrt{x^4 + 4x^3 + x^2 - 18x + 29}$

let $f(x)$ be the function under the radical

$f(x) = x^4 + 4x^3 + x^2 - 18x + 29$
$f'(x) = 4x^3 + 12x^2 + 2x - 18$

$4x^3 + 12x^2 + 2x - 18 = 0$ $\longrightarrow$ divide by 2
$2x^3 + 6x^2 + x - 9 = 0$ solve by finding zeros

| 1 | 2 | 6 | 1 | -9 |
|---|---|---|---|---|
| | | 2 | 8 | 9 |
| | 2 | 8 | 9 | 0 |

$2x^2 + 8x + 9 = 0$

Since $(b^2) - 4(a)(c) = \boxed{-8}$ no solution

$f(x) = (x-1)(2x^2 + 8x + 9) = 0$
$\boxed{x = 1}$

40) $V = \frac{4}{3}\pi r^3 + \pi r^2 h = 3000$

Surface Area $= 4\pi r^2 + 2\pi r h$

$C = 2K(4\pi r^2) + K(2\pi r h)$

$C = 8\pi K r^2 + 2\pi K r h$

$h = \frac{3000}{\pi r^2} - \frac{4\pi r^3}{3\pi r^2}$

$h = \frac{3000}{\pi r^2} - \frac{4}{3} r$

$8\pi K r^2 + 2\pi K r\left(\frac{3000}{\pi r^2} - \frac{4}{3} r\right)$

$C(r) = 8\pi K r^2 + \frac{6000K}{r} - \frac{8\pi K r^2}{3}$

$C(r) = 8K\left(\pi r^2 + 750 r^{-1} - \frac{\pi r^2}{3}\right)$

$C'(r) = 8K\left(2\pi r - \frac{750}{r^2} - \frac{2\pi r}{3}\right)$

$8K\left(\frac{6\pi r^3 - 2250 - 2\pi r^3}{3r^2}\right) = 0$

$8K\left(\frac{4\pi r^3 - 2250}{3r^2}\right)$

$$(4\pi r^3 - 2250) = 0$$

$$\boxed{r = \sqrt[3]{\frac{2250}{4\pi}}}$$

$$C''(r) = 8K\left(2\pi + \frac{1500}{r^3} - \frac{2}{3}\pi\right)$$

$$C''(r) = 8K\left(\frac{4\pi}{3} + \frac{1500}{r^3}\right)$$

$C''\left(\sqrt[3]{\frac{2250}{4\pi}}\right) > 0$, the cost is Min when $r = \sqrt[3]{\frac{2250}{4\pi}}$

$r = 5.64$

$$h = \frac{3000}{\pi r^2} - \frac{4}{3}r$$

$$\boxed{h = 22.5}$$

$\Delta x = dx = -0.1$

8) $y = 1 - 2x^2$ $\quad x = 0$

$\Delta y = f(x + \Delta x) - f(x)$

$\Delta y = f(0 + (-0.1)) - f(0)$

$\Delta y = f(-0.1) - f(0)$

$\Delta y = f(1 - 2(-0.1)^2) - (1 - 2(0)^2)$

$\boxed{\Delta y = -0.02}$ $\quad f'(x) = -4x$

$dy = -4x(dx)$

$dy = -4(0)(-0.1)$ $\quad \boxed{dy = 0}$

18) $y = e^{-0.5x} \cos 4x$

$dy = f'(x)\,dx$

$f'(x) = [-0.5e^{-0.5x}(\cos 4x) + (-4\sin 4x)e^{-0.5x}]\,dx$

$dy = -0.5e^{-0.5x}[\cos 4x + 8\sin 4x]\,dx$

32) $A = \frac{1}{2}bh$ $\quad b = 36 \quad h = 50$

$db = dh = \pm 0.25$

$dA = \frac{1}{2}[db(h) + dh(b)]$

$dA = \frac{1}{2}[\pm .25(50) + (\pm .25)(36)]$

$\boxed{dA = \pm 10.75}$

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$\sqrt[4]{(16)+(5)}$ $\quad x = 16$

$\Delta x = 5$

$f(x) = \sqrt[3]{x}$

$dx = -1$

$\approx f'(27)\,dx + f(27)$

$f' = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$

$f'(27) = \frac{1}{3(27)^{2/3}} = 0.037037$

$\sqrt[3]{26} \approx f'(27)\,dx + f(27)$

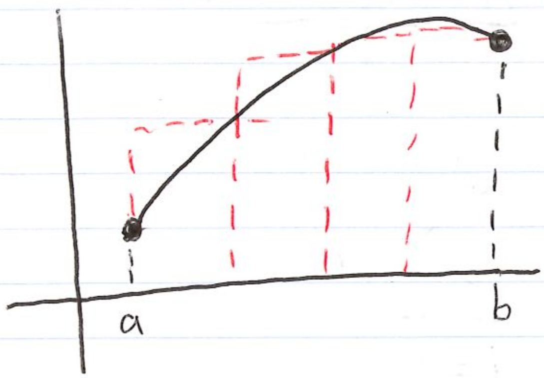
$\sqrt[3]{26} \approx 0.037037(-1) + 3$

$\boxed{\sqrt[3]{26} \approx 2.96296}$

# Great Math Websites

cramsters.com
calc101.com
Wolframalpha.com
BrightStorm.com
KhanAcademy.org

# 5.2 Area under the Curve




$$\Delta x = \frac{b-a}{n}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$$

$c_i = a + i\Delta x$

height $= f(c_i)\,\Delta x$

$$\underset{L}{f(c_i)}\ \underset{w}{\Delta x}$$

$$f(c_1)\Delta x + f(c_2)\Delta x + f(c_3)\Delta x + \dots + f(c_n)\Delta x$$

or

$$\sum_{i=1}^{n} f(c_i)\,\Delta x$$

1) $\displaystyle\sum_{i=1}^{n} c = nc$

2) $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

3) $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}$

4) $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4 + 2n^3 + n^2}{4}$

5) $\sum_{i=1}^{n} (2+3i) = \sum_{i=1}^{n} 2 + 3\sum_{i=1}^{n} i$

calculator = $\sum_{i=1}^{20} (i^2+3)$

= Sum(Seq($x^2$+3, x, 1, 20, 1))

variable

starting #

highest number
n

how many #'s to
jump between 1-20

# 5.2 Examples

6) $$\sum_{i=1}^{4}\left[(i-1)^2+(i+1)^3\right]$$

$$\left[(1-1)^2+(1+1)^3\right]+\left[(2-1)^2+(2+1)^3\right]+\left[(3-1)^2+(3+1)^3\right]$$
$$+\left[(4-1)^2+(4+1)^3\right]$$

$$=8+28+68+9+125=\boxed{238}$$

8) Write as a sum notation

$$\sum_{i=1}^{15}\frac{5}{1+i}$$

12) " " " " "

$$\sum_{i=1}^{n}\left[1-\left(\frac{2i}{n}-1\right)^2\right]\left(\frac{2}{i}\right)$$

20) $\sum_{i=1}^{10} i(i^2+1) = \sum_{i=1}^{10} (i^3+i)$

$= \sum_{i=1}^{10} i^3 + \sum_{i=1}^{10} i$

$\frac{10^4 + 2(10)^3 + 10^2}{4} + \frac{10(10+1)}{2} = \boxed{3,080}$

24) $w = 1$

S = upper Sum  s = lower Sum

$S = 1(5+5+4+2)$  $s = 1(4+4+2+0)$

$\boxed{S = 16}$  $\boxed{s = 10}$

28) $y = 4e^{-x}$

$S = 4(e^{-0} + e^{-.5} + e^{-1} + e^{-1.5}) = \boxed{S = 8.790161}$

$s = 4(e^{-.5} + e^{-1} + e^{-1.5} + e^{-2}) = \boxed{s = 5.3315}$

$*\ \frac{a}{b} \cdot \frac{d}{e} = \frac{a}{e} \cdot \frac{d}{b}$

32) $S(n)$ as $n \to \infty$

$$S(n) = \frac{64}{n^3}\left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= * \ \frac{64}{6}\left[\frac{2n^2+3n^2+n}{n^3}\right] = \frac{32}{3}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right]$$

$$\lim_{n\to\infty} S(n) = \lim_{n\to\infty} \frac{32}{3}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right]$$

$$\frac{32}{3}\left[\lim_{n\to\infty}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)\right]$$

$$= \frac{32}{3}\left[2 + 0 + 0\right] = \boxed{\frac{64}{3}}$$

36) $\displaystyle\sum_{j=1}^{n} \frac{4j+1}{n^2} = \frac{1}{n^2}\left(\sum_{j=1}^{n}(4j+1)\right)$

$$= \frac{1}{n^2}\left[\sum_{j=1}^{n} 4j + \sum_{j=1}^{n} 1\right]$$

$$= \frac{1}{n^2}\left[4\left(\frac{n^2+n}{2}\right) + n\right] = \frac{1}{n^2}\left[2n^2+3n\right]$$

$$= \boxed{2 + \frac{3}{n}}$$

$n=10 \quad 2 + \frac{3}{10} = \boxed{2.3}$

$n=100 \quad 2 + \frac{3}{100} = \boxed{2.03}$

$n=1000 \quad 2 + \frac{3}{1000} = \boxed{2.003}$

$$\left(1+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right) = \left(\frac{n+2i}{n}\right)^3\left(\frac{2}{n}\right) = \frac{(n+2i)^3}{n^3}\cdot\frac{2}{n}$$

$$= (n+2i)^3\cdot\frac{2}{n^4} = \left(n^3+6n^2i+12ni^2+8i^3\right)\left(\frac{2}{n^4}\right)$$

$$= \frac{2}{n}+\frac{12}{n^2}i+\frac{24}{n^3}i^2+\frac{16}{n^4}i^3$$

44) $$\lim_{n\to\infty}\sum_{i=1}^{n}\left(1+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}+\frac{12}{n^2}i+\frac{24}{n^3}i^2+\frac{16}{n^4}i^3$$

$$\lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{2}{n}+\frac{12}{n^2}\sum_{i=1}^{n}i+\frac{24}{n^3}\sum_{i=1}^{n}i^2+\frac{16}{n^3}\sum_{i=1}^{n}i^3\right]$$

$$\lim_{n\to\infty}\left[\frac{2}{n}(n)+\frac{12}{n^2}\left(\frac{n^2+n}{2}\right)+\frac{24}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right)+\frac{16}{n^4}\left(\frac{n^4+2n^3+n^2}{4}\right)\right]$$

$$\lim_{n\to\infty}\left[2+\left(6+\frac{6}{n}\right)+4\left(2+\frac{3}{n}+\frac{1}{n^2}\right)+4\left(1+\frac{2}{n}+\frac{1}{n^2}\right)\right]$$

$$= 2+(6+0)+4(2+0+0)+4(1+0+0)$$

$$= \boxed{20}$$

Use lim process = $\lim_{n\to\infty} f(c_i)\Delta x$

48) $y = 3x - 4$ $\qquad [\underset{a}{2}, \underset{b}{5}]$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

1) $\Delta x = \frac{b-a}{n} = \frac{5-2}{n} = \boxed{\frac{3}{n}}$

2) $c_i = a + \Delta x i = \boxed{2 + \frac{3}{n} i}$

3) $f(c_i) = 3\left(2 + \frac{3}{n} i\right) - 4 = 6 + \frac{9}{n} i - 4 = \boxed{2 + \frac{9}{n} i}$

4) $f(c_i)\Delta x = \left(2 + \frac{9}{n} i\right)\left(\frac{3}{n}\right) = \boxed{\frac{6}{n} + \frac{27}{n^2} i}$

5) $\sum_{i=1}^{n} f(c_i)\Delta x = \sum_{i=1}^{n} \frac{6}{n} + \frac{27}{n^2} \sum_{i=1}^{n} i$

$= \frac{6}{n}(n) + \frac{27}{n^2}\left(\frac{n^2+n}{2}\right) = 6 + \frac{27}{2}\left(1 + \frac{1}{n}\right)$

6) $\lim_{N\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x = \lim_{n\to\infty}\left[6 + \frac{27}{2}\left(1 + \frac{1}{n}\right)\right]$

$= 6 + \frac{27}{2}(1 + 0) = \boxed{\frac{39}{2}}$

52) $y = 1 - x^2$ $\qquad [-1, 1]$

$\qquad\qquad\qquad\qquad\qquad\qquad\quad a \quad b$

1) $\Delta x = \frac{b-a}{n} = \frac{1+1}{n} = \boxed{\frac{2}{n}}$

2) $c_i = a + \Delta x\, i = \boxed{-1 + \frac{2}{n} i}$

3) $f(c_i) = 1 - \left(-1 + \frac{2}{n} i\right)^2 = 1 - \frac{n^2 - 4ni + 4i^2}{n^2}$

$= \cancel{1} - \cancel{1} + \boxed{\frac{4}{n} i - \frac{4}{n^2} i^2}$

4) $f(c_i)\Delta x = \left(\frac{4}{n} i - \frac{4}{n^2} i^2\right)\left(\frac{2}{n}\right)$

$= \boxed{\frac{8}{n^2} i - \frac{8}{n^3} i^2}$

5) $\sum_{i=1}^{n} f(c_i)\Delta x = \frac{8}{n^2} \sum_{i=1}^{n} i - \frac{8}{n^3} \sum_{i=1}^{n} i^2$

$\frac{8}{n^2} \frac{(n^2+n)}{2} \quad \frac{8}{n^3} \left(\frac{2n^3 + 3n^2 + n}{6}\right)$

$= 4\left(1 + \frac{1}{n}\right) - \frac{4}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right)$

6) $\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$

$$\lim_{n\to\infty} \left[4\left(1+\frac{1}{n}\right) - \frac{4}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)\right]$$

$$A = 4(1+0) - \frac{4}{3}(2+0+0)$$

$$A = 4 - \frac{8}{3} \qquad \boxed{A = \frac{4}{3}}$$

# 5.3 Reiman Sums

$||\Delta||$ : norm of $\Delta$

$$A = \lim_{||\Delta|| \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

$$A = \int_a^b f(x)\,dx$$

5) $\int_a^a f(x)\,dx = 0$

6) $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

7) $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b G(x)\,dx$

$$c_i - c_{(i-1)} = \frac{i^3}{n^3} - \frac{(i-1)^3}{n^3} = \frac{i^3}{n^3} - \frac{i^3 - 3i^2 + 3i - 1}{n^3} = \frac{3i^2 - 3i + 1}{n^3}$$

# 5.3 Examples

2) $f(x) = 2\sqrt[3]{x}$ $\qquad [0, 1]$

$$c_i = \frac{i^3}{n^3}$$

1) $\Delta x_i = \frac{i^3}{n^3} - \frac{(i-1)^3}{n^3} = \boxed{\frac{1}{n^3}\left(3i^2 - 3i + 1\right)}$

3) $f(c_i) = 2\sqrt[3]{\frac{i^3}{n^3}} = \boxed{\left(\frac{2i}{n}\right)}\boxed{\left[\frac{1}{n^3}\left(3i^2 - 3i + 1\right)\right]} = \boxed{\frac{2}{n^4}\left(3i^3 - 3i^2 + i\right)}$

4) $f(c_i)\Delta x_i = \frac{2i}{n}\left(\frac{1}{n}\right) =$

5) $\sum_{i=1}^{n} f(c_i)\Delta x_i =$

$$\frac{2}{n^4}\left[3\sum_{i=1}^{n} i^3 - 3\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i\right]$$

$$= \frac{2}{n^4}\left[3\left(\frac{n^4 + 2n^3 + n^2}{4}\right) - 3\left(\frac{2n^3 + 3n^2 + n}{6}\right) + \left(\frac{n^2 + n}{2}\right)\right]$$

$$= \frac{3}{2}\left(1 + \frac{2}{n} + \frac{1}{n^2}\right) - \left(\frac{2}{n} + \frac{3}{n^2} + \frac{1}{n^3}\right) + \left(\frac{1}{n^2} + \frac{1}{n^3}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

$$\lim_{n\to\infty} \left[\frac{3}{2}\left(1+\frac{2}{n}+\frac{1}{n^2}\right)-\left(\frac{2}{n}+\frac{3}{n^2}-\frac{1}{n^3}\right)+\left(\frac{1}{n^2}+\frac{1}{n^3}\right)\right.$$

$$\frac{3}{2}(1+0+0)-(0+0+0)+(0+0)$$

$$\boxed{A=\frac{3}{2}}$$

8) $$\int_{-1}^{2} (3x^2+2)\,dx$$

$$A=\lim_{n\to\infty} \sum_{i=1}^{n} f(C_i)\,\Delta x$$

1) $\Delta x = \frac{b-a}{n} = \frac{2+1}{n} = \boxed{\frac{3}{n}}$

2) $C_i = a+\Delta x\, i = \boxed{1+\frac{3i}{n}}$

3) $f(C_i) = 3\left(1+\frac{3i}{n}\right)^2+2$

$= \left(\frac{-n+3i}{n}\right)^2 = \frac{(-n+3i)^2}{n^2} = \frac{n^2+6ni+9i^2}{n^2}$

$$3\left(1 - \frac{6i}{n} + \frac{9i^2}{n^2}\right) + 2$$

$$3 - \frac{18i}{n} + \frac{27i^2}{n^2} + 2$$

$$\boxed{5 - \frac{18i}{n} + \frac{27i^2}{n^2}}$$

4) $$f(c_i)\Delta x = \left(5 - \frac{18i}{n} + \frac{27i^2}{n^2}\right)\left(\frac{3}{n}\right)$$

$$= \frac{15}{n} - \frac{54i}{n^2} + \frac{81i^2}{n^3}$$

5) $$\sum_{i=1}^{n} f(c_i)\Delta x = \sum_{i=1}^{n}\frac{15}{n} - \frac{54}{n^2}\sum_{i=1}^{n} i + \frac{81}{n^3}\sum_{i=1}^{n} i^2$$

$$= \left(\frac{15}{n}\right)(n) - \frac{54}{n^2}\left(\frac{n^2+n}{2}\right) + \frac{81}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right)$$

$$= 15 - 27 - \frac{27}{n} + 27 + \frac{81}{2n} + \frac{27}{2n^2}$$

$$= 15 - \frac{27}{n} + \frac{81}{2n} + \frac{27}{2n^2}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$$= \lim_{n \to \infty} \left(15 - \frac{27}{n} + \frac{81}{2n} + \frac{27}{2n^2}\right)$$

$$A = 15 - 0 + 0 + 0$$

$$\boxed{A = 15}$$

12) $$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \left(\frac{3}{c_i^2}\right) \Delta x_i \quad [1, 3]$$

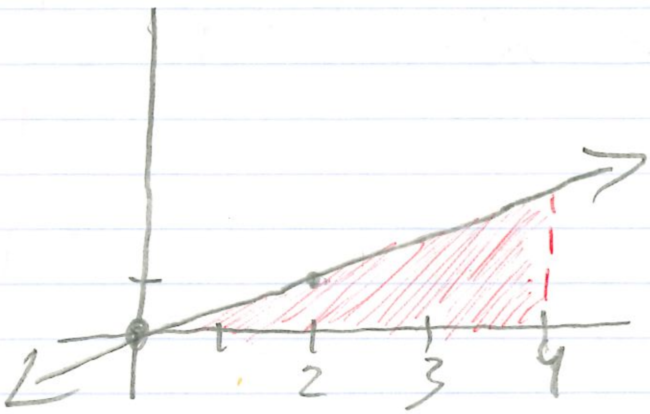
$$\int_1^3 \left(\frac{3}{x^2}\right) dx$$

18) $$\int_0^2 2e^{-x} \, dx$$

26) $f(x) = \frac{x}{2}$

$y = \frac{1}{2}x + 0$

$(0,0)$
$m = \frac{1}{2}$




$b = \boxed{4}$

$h = f(4) = \boxed{2}$

$$A = \int_0^4 \left(\frac{x}{2}\right)dx = \frac{1}{2}bh = \frac{1}{2}(4)(2) = \boxed{4}$$

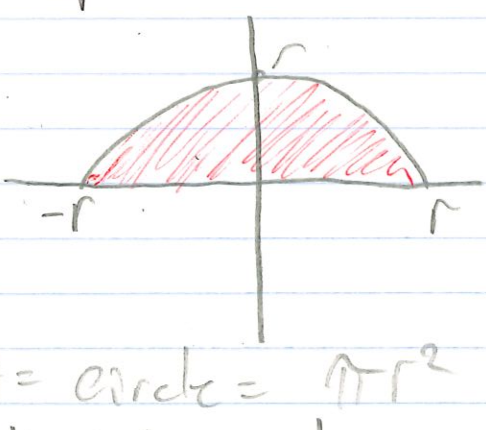
32) $\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx$

$f(x) = \sqrt{r^2 - x^2}$

$y = \sqrt{r^2 - x^2}$

$y^2 = r^2 - x^2$

$x^2 + y^2 = r^2 \rightarrow$ circle




$$\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = \boxed{\frac{1}{2}\pi r^2}$$

$A =$ circle $= \pi r^2$

$\frac{1}{2}$ circle $= \frac{1}{2}\pi r^2$

# 5.4 Examples

10) $\int_1^3 (3x^2+5x-4)\,dx$

$\left[x^3 + \frac{5}{2}x^2 - 4x\right]_1^3$

$\left[(3)^3 + \frac{5}{2}(3)^2 - 4(3)\right] - \left[(1)^3 + \frac{5}{2}(1)^2 - 4(1)\right]$

$37.5 - (-.5) = \boxed{38}$

22) $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}} = \int_{-8}^{-1} \frac{x - x^2}{2x^{\frac{1}{3}}}$

$\frac{x}{2x^{\frac{1}{3}}} - \frac{x^2}{2x^{\frac{1}{3}}} = \frac{1}{2}x^{-\frac{2}{3}} - \frac{1}{2}x^{\frac{5}{3}}$

$\frac{1}{2}\left[x^{-\frac{2}{3}} - x^{\frac{5}{3}}\right] = \int x^{-\frac{2}{3}}\,dx = \frac{1}{-\frac{2}{3}+1}x^{(-\frac{2}{3}+1)}$

$= \boxed{3x^{\frac{1}{3}}}$

$\int x^{\frac{5}{3}}\,dx = \frac{1}{\frac{5}{3}+1}x^{(\frac{5}{3}+1)} = \boxed{\frac{3}{8}x^{8/3}}$

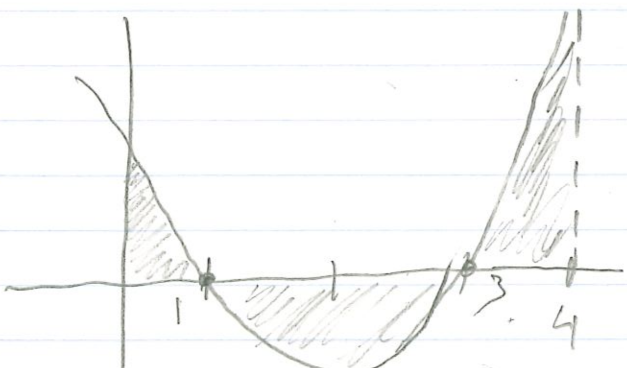
$$\frac{1}{2}\left(3x^{1/3} - \frac{3}{8}x^{(8/3)}\right)\Bigg|_{-8}^{-1}$$

$$= \left[\frac{1}{2}\left(3(-1)^{\frac{1}{3}} - \frac{3}{8}(-1)^{\frac{8}{3}}\right)\right] - \left[\frac{1}{2}(-8)^{\frac{1}{3}} - \frac{3}{8}(-8)^{\frac{8}{3}}\right]$$

$$= -1.688 - (-51) = \boxed{49.312}$$

26) $\displaystyle\int_0^4 |x^2 - 4x + 3|\, dx$ on Exam

go on calc & put this fxn in to see if wee cross x-axis




becouse under the curve

$$\int_0^1 (x^2 - 4x + 3)\, dx - \int_1^3 (x^2 - 4x + 3)\, dx + \int_3^4 (x^2 - 4x + 3)\, dx$$

$$\left(\frac{1}{3}x^3-2x^2+3x\right)\Big|_0^1 - \left(\frac{1}{3}x^3-2x^2+3x\right)\Big|_1^3$$

$$+\left(\frac{1}{3}x^3-2x^2+3x\right)\Big|_3^4$$

$$=\left[\left(\frac{1}{3}(0)^3-2(0)^2+3(0)\right)-\left(\frac{1}{3}(1)^3-2(1)^2+3(1)\right)\right]-\left[\frac{1}{3}(3)^3-2(3)^2 \dots\right]$$

$$=\left[1.3\bar{3}-0\right]-\left[0-1.3\bar{3}\right]+\left[1.3\bar{3}-0\right]$$

$$=\boxed{3.9\bar{9}}$$

32) $$\int_1^5 \frac{x+1}{x}\,dx = \int_1^5 \left(1+\frac{1}{x}\right)dx$$

$$= \int_1^5 (1)\,dx \quad \int_1^5 \frac{1}{x}\,dx$$

$$= \left(x+\ln x\right)\Big|_1^5 =$$

$$=(5+\ln 5)-(1+\ln 1) = 5+\ln 5-1+0$$

$$=\boxed{4+\ln 5}$$

$\ln 2e = \ln 2 + \ln e$

38) $\displaystyle\int_e^{2e} \left(\cos x - \frac{1}{x}\right) dx$

$\left(\sin x - \ln x\right)\Big|_e^{2e}$

$(\sin 2e - \ln 2e) - (\sin e - \ln e)$

$\boxed{\sin 2e - \ln 2 - \sin e}$

44) $y = x + \sin x$

$\displaystyle\int_0^{\pi} (x + \sin x)\, dx$

$\left(\frac{1}{2}x^2 - \cos x\right)\Big|_0^{\pi}$

$\left[\frac{1}{2}(\pi)^2 - \cos(\pi)\right] - \left[\frac{1}{2}(0)^2 - \cos(0)\right]$

$\frac{1}{2}\pi^2 + 1 - 0 + 1$

$= \boxed{\frac{1}{2}\pi^2 + 2}$

50) $y = e^x$

$$A = \int_0^2 e^x \, dx$$

$$= (e^x) \Big|_0^2$$

$$e^2 - e^0 = \boxed{e^2 - 1}$$

54) $f(x) = \cos x$ $\qquad [\overset{a}{-\frac{\pi}{3}}, \overset{b}{\frac{\pi}{3}}]$

Mean Value

find c value

$$\int_a^b f(x) \, dx = f(c)(b-a)$$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \cos x \, dx = \sin x \Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

$$(\sin \tfrac{\pi}{3}) - (\sin -\tfrac{\pi}{3}) = \frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right) = \boxed{\sqrt{3}}$$

$$(\cos c)\left(\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right)$$

$$= \frac{2\pi}{3} \cos c = \sqrt{3} \longrightarrow$$ set equal to each other

$$\cos c = \frac{3\sqrt{3}}{2\pi}$$

$$\boxed{c = \cos^{-1}\left(\frac{3\sqrt{3}}{2\pi}\right)}$$

60) $f(x) = \frac{1}{2x}$ $[1, 4]$

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{4-1}\int_a^b \frac{1}{2x}\,dx$$

$$\frac{1}{3}\int_a^b \frac{1}{2}\left(\frac{1}{x}\right)dx$$

$$\frac{1}{6}\int \frac{1}{x}\,dx = \frac{1}{6}(\ln x)\Big|_1^4$$

$$= \frac{1}{6}\left[\ln 4 - \ln 1\right] = \frac{1}{6}\left[\ln 4 - 0\right]$$

$$\boxed{\frac{1}{6}\ln 4}$$

74) $V = K(R^2 - r^2)$ $\quad [0, R]$

$$\frac{1}{b-a}\int_a^b V(R)\,dR$$

$$\frac{1}{R-0}\int_0^R K(R^2-r^2)\,dr$$

$$\frac{1}{R}(K)\int_0^R (R^2-r^2)\,dr$$

$$\frac{K}{R}\left[\frac{1}{3}R^3 - r^2R\right]\Big|_0^R$$

$$\frac{K}{R}\left[\frac{1}{3}R^3 - R^3\right] - \left[\frac{1}{3}(0)^3 - 0^3\right]$$

$$\boxed{-\frac{2}{3}R^2K}$$

88) $F(x) = \int_0^x t(t^2+1)\,dt$

$F(x) = \int_0^x (t^3+t)\,dt$

$\left(\frac{1}{4}t^4 + \frac{1}{2}t^2\right)\Big|_0^x$

$\left[\left(\frac{1}{4}x^4 + \frac{1}{2}x^2\right) - \frac{1}{4}0 + \frac{1}{2}(0)\right]$

$F(x) = \frac{1}{4}x^4 + \frac{1}{2}x^2$

$\boxed{F'(x) = x^3 + x}$

98) $F(x) = \int_1^x \sqrt[4]{t}\,dt$

$= \int_1^x t^{\frac{1}{4}}\,dt = \frac{1}{\frac{1}{4}+1}t = \frac{4t^{5/4}}{5}$

$\frac{4}{5}t^{5/4}\Big|_1^x = \frac{4}{5}(x)^{5/4} - \frac{4}{5}(1)^{5/4}$

$F'(x) = \frac{4}{5}x^{5/4} - \frac{4}{5} = \boxed{F'(x) = x^{\frac{1}{4}}}$

104) $F(x) = \int_2^{x^2} t^{-3}\,dt$

$\frac{1}{-3+1} t^{-3+1} = -\frac{t^{-2}}{2} \Big|_2^{x^2} = -\frac{1}{2t^2} \Big|_2^{x^2}$

$-\frac{1}{2(x^2)^2} + \frac{1}{8}$

$F(x) = -\frac{1}{2} x^{-4} + \frac{1}{8}$

$F'(x) = 2x^{-5}$

$F'(x) = \frac{2}{x^{-5}}$

# 5.5 Integration by Substitution

$y = (3x^2 + 2x)^4$

$\int y' = \int 4(3x^2+2x)^3 \cdot (6x+2)\,dx$

How do you get back from chain Rule?

if $u = 3x^2 + 2x$

then $\frac{du}{dx} = 6x+2 \longrightarrow du = 6x+2\,dx$

Ex: $4\int u^3 \cdot du \longrightarrow \frac{4u^4}{4} + C \longrightarrow u^4 + C$

now substitute...

$(3x^2+2x)^4 + C$

Tip: On Calc: $*$ Integrate on TI-83

$\int_0^2 (-\frac{1}{3}x)\,dx = \boxed{\text{Math}}\ \boxed{\#9 - \text{fnInt}} = \text{fnInt}((-\frac{1}{3})X, X, 0, 2)\ \boxed{\text{Enter}}$

OR

$\int_0^4 x^2\,dx = \boxed{Y=}\ x^2\ \boxed{\text{2nd}}\ \boxed{\text{calc}}\ \boxed{\#7\ \int f(x)dx}$ Lower = 0 Upper = 4

16) $\int t^3 \sqrt{t^4+5}\, dt$

$\int t^3 (t^4+5)^{\frac{1}{2}}\, dt$

$\boxed{\begin{aligned} u &= t^4+5 \\ du &= 4t^3\,dt \end{aligned}}$

$\underbrace{\frac{1}{4}\int (t^4+5)^{\frac{1}{2}}\, \underbrace{4t^3\,dt}}_{\text{add these constants}}$

$\frac{1}{4}\int u^{\frac{1}{2}}\, du$

$= \boxed{\frac{1}{4}\left(\frac{2}{3}u^{\frac{3}{2}}\right)+C} \longrightarrow \boxed{\frac{1}{6}(t^4+5)^{\frac{3}{2}}+C}$

25) $-1\int \underbrace{\left(1+\frac{1}{t}\right)^3}_{u} \underbrace{\left(\frac{-1}{t^2}\right)dt}_{du \text{ (needs a neg)}}$

$\boxed{u = 1+\frac{1}{t} = 1+t^{-1}}$

$\boxed{\begin{aligned} du &= -1t^{-2} \\ du &= \frac{-1}{t^2}\,dt \end{aligned}}$

$-\int u^3\, du$

$-\frac{u^4}{4}+C \longrightarrow \boxed{-\frac{1}{4}\left(1+\frac{1}{t}\right)^4+C}$

32) $\int \left( \frac{t^3}{3} + \frac{1}{4t^2} \right) dt$

$\int \left[ \frac{1}{3}(t^3) + \frac{1}{4}(t^{-2}) \right] dt$

$\frac{1}{3}\left(\frac{t^4}{4}\right) + \frac{1}{4}\left(\frac{t^{-1}}{-1}\right) + C$

$\boxed{\frac{1}{12}t^4 - \frac{1}{4t} + C}$

48) $\int 4x^3 \sin x^4 \, dx$

U is typically trig part

$\boxed{\begin{array}{l} U = x^4 \\ dU = 4x^3 dx \end{array}}$

$\int \sin(x)^4 \, 4x^3 \, dx$

$\int \sin U \, dU$

$-\cos U + C \longrightarrow \boxed{-\cos(x^4) + C}$

$$\frac{1}{4}\int\left(u^{3/2}-3u^{1/2}\right)du$$

$$\frac{1}{4}\left[\frac{1}{\frac{3}{2}+1}u^{\frac{3}{2}+1}-\frac{3}{\frac{1}{2}+1}u^{\frac{1}{2}+1}+C\right]$$

$$\frac{1}{4}\left[\frac{2}{5}u^{5/2}-2u^{3/2}+C\right]$$

$$\frac{1}{10}u^{5/2}-\frac{1}{2}u^{3/2}+C$$

$$\boxed{\frac{1}{10}(2x+3)^{5/2}-\frac{1}{2}(2x+3)^{3/2}+C}$$

100) $$\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx$$

$u = 1+2x^2$
$du = 4x\,dx$

$$\frac{1}{4}\int_0^2 (1+2x^2)^{-\frac{1}{2}} \cdot 4x\,dx$$

when $x=0$
$u = 1 + 2(0)^2$
$\boxed{u=1}$

$x=2$
$u = 1+2(2)^2$
$\boxed{u=9}$

$$\frac{1}{4}\int_1^9 u^{-\frac{1}{2}}\,du$$

$$\frac{1}{4}\left[\frac{1}{-\frac{1}{2}+1}u^{-\frac{1}{2}+1}\right]\Bigg|_1^9$$

$$= \frac{1}{4}\left[2u^{\frac{1}{2}}\right]\Bigg|_1^9$$

$$\frac{1}{4}\left[2\sqrt{9} - 2\sqrt{1}\right] \longrightarrow \frac{1}{4}\left[6-2\right] = 1$$

104) $\int_0^{\sqrt{2}} x e^{-(x^2/2)} dx$

$$\boxed{\begin{array}{l} u = -\frac{x^2}{2} \\ du = -x\,dx \end{array}}$$

$$-\int_0^{\sqrt{2}} e^{(-x^2/2)} (-x\,dx)$$

$x = 0$

$\boxed{u = 0}$

$x = \sqrt{2}$

$u = -\frac{\sqrt{2}^2}{2} = \boxed{-1}$

$$-\int_0^{-1} e^u \, du \longrightarrow -(-1)\int_{-1}^{0} e^u \, du$$

$$e^u \Big|_{-1}^{0}$$

$$e^0 - e^{-1} = \boxed{1 - \frac{1}{e}}$$

# 5.6 Numerical Integration

Let $f$ be a continuous fxn on $[a, b]$

## Trapezoidal Rule

1) $$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) \ldots f(x_n)\right]$$

2) Error: $$E \leq \frac{(b-a)^3}{12n^2}\left[\max|f''(x)|\right]$$

## Simpsons Rule

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4(fx_3) + \ldots + f(x_n)\right]$$

Error: $$E \leq \frac{(b-a)^5}{180n^4}\left[\max\left|f^{iv}(x)\right|\right]$$

## Left Endpoint

$X_0 = a$

$X_1 = X_0 + \Delta X$

$X_2 = X_1 + \Delta X$

$X_3 = X_2 + \Delta X$

$n$ is the number of subintervals

$\Delta X = \frac{b-a}{n}$

# 5.6 Examples

2) $\displaystyle\int_0^1 \left(\frac{x^2}{2}+1\right)dx \qquad n=4$

$f(x)=\frac{x^2}{2}+1$

$a=0,\ b=1$

$x_0=0$

$x_1=0+\frac{1}{4}=\boxed{\frac{1}{4}}$

$x_2=\frac{1}{4}+\frac{1}{4}=\boxed{\frac{1}{2}}$

$x_3=\frac{1}{2}+\frac{1}{4}=\boxed{\frac{3}{4}}$

$x_4=\frac{3}{4}+\frac{1}{4}=\boxed{1}$

$\Delta x=\frac{b-a}{n}=\frac{1-0}{4}=\boxed{\frac{1}{4}}$

$\displaystyle\int_0^1 \frac{x^2}{2}+1\,dx \approx \frac{1-0}{2(4)}\left[f(0)+2f(\tfrac{1}{4})+2f(\tfrac{1}{2})+2f(\tfrac{3}{4})+f(1)\right]$

use calc

$\approx \frac{1}{8}\left[1+2(1.03)+2(1.125)+2(1.281)+1.5\right]$

$\approx \boxed{1.1719}$

Simpsons Rule

$\frac{1-0}{3(4)}\left[f(0)+4f(\tfrac{1}{4})+2f(\tfrac{1}{2})+4f(\tfrac{3}{4})+f(1)\right]$

$\frac{1}{12}\left[1+4(1.03)+2(1.125)+4(1.281)+1.5\right]$

$\approx \boxed{1.1667}$

$$\int_0^1 \left(\frac{x^2}{2}+1\right)dx = \left(\frac{1}{2}\cdot\frac{1}{3}x^3 + x\right)\Big|_0^1$$

$$= \left(\frac{1}{6}x^3 + x\right)\Big|_0^1 \longrightarrow \left(\frac{1}{6}(1)^3+1\right) - \left(\frac{1}{6}(0)^3+0\right)$$

$= \boxed{1.1667} \longrightarrow$ Simpsons Rule closer

10) $\displaystyle\int_0^2 x\sqrt{x^2+1}\,dx$

$n = 4$
$a = 0, \quad b = 2$
$f(x) = x\sqrt{x^2+1}$

$x_0 = 0$
$x_1 = 0 + \frac{1}{2} = \frac{1}{2}$
$x_2 = \frac{1}{2} + \frac{1}{2} = 1$
$x_3 = 1 + \frac{1}{2} = \frac{3}{2}$
$x_4 = 2$

$\Delta x = \frac{2-0}{4} = \boxed{\frac{1}{2}}$

$$\int_0^2 x\sqrt{x^2+1}\,dx$$

$$\approx \frac{2-0}{2(4)}\left[f(0)+2f(\tfrac{1}{2})+2f(1)+2f(\tfrac{3}{2})+f(2)\right]$$

$$\approx \frac{1}{4}\left[0+2(.5590)+2(1.4142)+2(2.7042)+4.4721\right]$$

$$\approx \boxed{3.4567}$$

Simpsons $\approx \frac{2-0}{3(4)}\left[f(0)+4f(\tfrac{1}{2})+2f(1)+4f(\tfrac{3}{2})+2\right]$

$$\approx \frac{1}{6}\left[0+4(.5590)+2(1.4142)+4(2.7042)+4.4721\right]$$

$$\approx \boxed{3.3922}$$

$\hookrightarrow$

18) $$\int \frac{x^3-3x^2+4x-9}{x^2+3}\,dx$$ Long Division

$$\begin{array}{r} x-3 \\ x^2+3 \overline{\big)\, x^3-3x^2+4x-9} \\ \underline{x^3 \quad\quad +3x} \\ -3x^2+x \\ \underline{-3x^2 \quad\quad -9} \\ x \end{array}$$

$$\int \left( x-3+\frac{x}{x^2+3} \right) dx$$

$$\int \frac{x}{x^2+3}\,dx \qquad u=x^2+3 \qquad du=2x\,dx \qquad \frac{1}{2}\int \frac{1}{x^2+3}\,2x\,dx$$

$$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln u + C = \boxed{\frac{1}{2}\ln(x^2+3)+C}$$

$$\boxed{\frac{1}{2}x^2-3x+\frac{1}{2}\ln(x^2+3)+C}$$

24) $\int \frac{x(x-2)}{(x-1)^3}\,dx$ $\qquad \int \frac{x^2-2x+1-1}{(x-1)^3}\,dx$

$\int \frac{(x-1)^2-1}{(x-1)^3}\,dx \longrightarrow \int \frac{(x-1)^2}{(x-1)^3} - \int \frac{1}{(x-1)^3}$

$\int \frac{1}{x-1} - \int \frac{1}{(x-1)^3}$ $\qquad u = x-1$

$\qquad du = 1dx$

$\int \frac{1}{u}\,du - \int \frac{1}{u^3}\,du$

$\ln|u| - \left(\frac{-1}{2u^2}\right) + C$

$= \ln|x-1| + \frac{1}{2(x-1)^2} + C$

30) $\int \tan 5\theta \, d\theta \quad -\frac{1}{5}\int \frac{1}{\cos 5\theta} \cdot -5\sin 5\theta \, d\theta$

$$\boxed{\begin{aligned} u &= \cos 5\theta \\ du &= -5\sin 5\theta \, d\theta \end{aligned}}$$

$$-\frac{1}{5}\int \frac{1}{u}\, du = -\frac{1}{5}\ln|u| + C$$

$$\boxed{-\frac{1}{5}\ln\left|\cos 5\theta\right| + C}$$

54) $\int_0^1 \frac{x-1}{x+1}\, dx$ Long Division

$$\begin{array}{r} 1 \\ x+1 \enclose{longdiv}{x-1} \\ \underline{x+1} \\ \boxed{-2} \end{array} \qquad \int_0^1 1 - \frac{2}{x+1}\, dx$$

$$= x - 2\ln(x+1) \Big|_0^1$$

$$1 - 2\ln(1+1) - (0 - 2\ln(0+1))$$

$$= \boxed{1 - 2\ln(2)}$$

Since $\csc^2\theta = 1+\cot^2\theta$

then $\cot^2\theta = \underline{\csc^2\theta - 1}$

56) $$\int_{0.1}^{0.2} (\csc 2\theta - \cot 2\theta)^2 \, d\theta$$

$$\int_{.1}^{.2} (\underline{\csc^2 2\theta} - 2\csc 2\theta \cot 2\theta + \underline{\cot^2 2\theta}) \, d\theta$$

$$\int_{.1}^{.2} \underline{(2\csc^2 2\theta - 1) \, d\theta} - \int_{.1}^{.2} (\csc 2\theta \cot 2\theta) \cdot 2 \, d\theta$$

let $u = 2\theta$ when $\theta = .1$ when $\theta = .2$

$du = 2\,d\theta$ $\boxed{u = .2}$ $\boxed{u = .4}$

$$\int_{.1}^{.2} \csc^2 2\theta \cdot 2\,d\theta - \int_{.1}^{.2} 1\,d\theta - \int_{.1}^{.2} (\csc 2\theta \cot 2\theta) \cdot 2\,d\theta$$

$$\int_{.2}^{.4} \csc^2 u \, du - \int_{.1}^{.2} 1\,d\theta - \int_{.2}^{.4} (\csc u \cot u) \, du$$

$$-\cot u \Big|_{.2}^{.4} - \theta \Big|_{.1}^{.2} - (-\csc u) \Big|_{.2}^{.4}$$

$$[-\cot .4 + \cot .2] - (.2 - .1) + [\csc .4 - \csc .2]$$

$$= \boxed{.0023754}$$

# 5.8 Inverse Trig Fxn Integration

Pg 364 Know these formulas

- 5.8 Examples

6) $\int \frac{1}{4+(x-1)^2}\,dx$

Rule 19 $u = x-1$
$du = 1dx$

$\int \frac{1}{2^2+(x-1)^2}\,dx \longrightarrow \int \frac{1}{2^2+u^2}\,du$

$= \frac{1}{2}\tan^{-1}\frac{u}{2}+C$

$= \boxed{\frac{1}{2}\tan^{-1}\frac{(x-1)}{2}+C}$

8) $\int \frac{x^4 - 1}{x^2+1}\,dx \longrightarrow \int \frac{(x^2+1)(x^2-1)}{x^2+1}\,dx$

$\int x^2 - 1\,dx = \boxed{\frac{1}{3}x^3 - x + C}$

12) $\int \frac{1}{x\,(x^4-4)^{\frac{1}{2}}}\,dx$

$u = x^2$
$du = 2x\,dx$

Rule 20

$$\int \frac{1}{x\sqrt{(x^2)^2 - 2^2}}\,dx$$

$\frac{1}{2}\int \frac{1}{x \cdot x\sqrt{(x^2)^2 - 2^2}}\,2x\,dx = \frac{1}{2}\int \frac{1}{u\sqrt{u^2 - 2^2}}\,du$

$\frac{1}{2}\left(\frac{1}{2}\sec^{-1}\frac{u}{2} + C\right)$

$$= \boxed{\frac{1}{4}\sec^{-1}\frac{x^2}{2} + C}$$

18) $$\int \frac{4x+3}{\sqrt{1-x^2}}\,dx$$

$$\int \frac{4x}{\sqrt{1-x^2}}\,dx + \int \frac{3}{\sqrt{1-x^2}}\,dx$$

$$\int \frac{4x}{\sqrt{1-x^2}}\,dx \qquad \begin{aligned} u &= 1-x^2 \\ du &= -2x\,dx \end{aligned} \qquad \int \frac{1}{\sqrt{1-x^2}} \cdot 4x\,dx$$

$$-2\int \frac{1}{\sqrt{1-x^2}}(-2x)\,dx \longrightarrow -2\int u^{-\frac{1}{2}}\,du$$

$$= -2\left(2u^{\frac{1}{2}}\right)+C \longrightarrow \boxed{-4\sqrt{1-x^2}+C}$$

$$3\int \frac{1}{\sqrt{1^2-x^2}}\,dx \longrightarrow \boxed{3\sin^{-1}\frac{x}{1}+C}$$

$$\boxed{-4\sqrt{1-x^2}+3\sin^{-1}x+C}$$

28) $\displaystyle\int_{-\sqrt{3}}^{0} \frac{x}{1+x^2}\,dx$

$u = 1+x^2$
$du = 2x\,dx$

$\displaystyle\frac{1}{2}\int_{-\sqrt{3}}^{0} \frac{1}{1+x^2}\cdot 2x\,dx$

$x = -\sqrt{3}$
$\boxed{u = 4}$
$x = 0$
$\boxed{u = 1}$

$\displaystyle\frac{1}{2}\int_{4}^{1} \frac{1}{u}\,du \longrightarrow -\frac{1}{2}\int_{1}^{4} \frac{1}{u}\,du$

$\displaystyle\left(-\frac{1}{2}\ln u\right)\Big|_{1}^{4} = -\frac{1}{2}\left[\ln 4 - \ln 1\right]$

$\boxed{-\frac{1}{2}\ln 4}$ or $\boxed{-\ln 2}$ or $\boxed{\ln \frac{1}{2}}$

34) $\int \frac{2x-5}{x^2+2x+2}\,dx$

$\int \frac{2x+2-7}{x^2+2x+2}\,dx = \int \frac{7}{(x+1)^2+1}\,dx$

$\int \frac{2x+2}{x^2+2x+2} \longrightarrow \int \frac{1}{x^2+2x+2} \cdot (2x+2)\,dx$ $\quad u = x^2+2x+2$, $du = 2x+2$

$\int \frac{1}{u}\,du = \boxed{\ln u + C}$

Rule 19

$7\int \frac{1}{(x+1)^2+1^2}\,dx \qquad 7\left[\frac{1}{1}\,\text{Tan}^{-1}\,\frac{(x+1)}{1}\right] + C$

$= 7\,\text{Tan}^{-1}(x+1) + C$

$$\boxed{\ln(x^2+2x+2) - 7\,\text{Tan}^{-1}(x+1) + C}$$

40) $\displaystyle\int \frac{1}{(x-1)\sqrt{x^2-2x+1-1}}\,dx$

$\displaystyle\int \frac{1}{(x-1)\sqrt{(x-1)^2-1^2}}\,dx$

Rule # 20

$U = X-1$
$dU = 1\,dx$

$\displaystyle\int \frac{1}{U\sqrt{U^2-1^2}}\,dU = \frac{1}{1}\sec^{-1}\frac{U}{1}+C$

$\boxed{\sec^{-1}(x-1)+C}$

46) $\displaystyle\int_0^1 \frac{dx}{2\sqrt{3-x}\sqrt{x+1}}$

$U = \sqrt{x+1}$
$dU = \frac{1}{2\sqrt{x+1}}\,dx$

$\boxed{U^2-1 = X}$

$\sqrt{3-X} = \sqrt{3+1-U^2} = \sqrt{4-U^2}$ ⟶ Rule 18

$x=0$
$U=1$
$x=1$
$U=\sqrt{2}$

$\displaystyle\int_0^1 \frac{1}{\sqrt{3-X}}\cdot\frac{1}{2\sqrt{X+1}}\,dX$

$\displaystyle\int_1^{\sqrt{2}} \frac{1}{\sqrt{4-U^2}}\,du \rightarrow \left(\sin^{-1}\frac{U}{2}\right)\Big|_1^{\sqrt{2}}$

$$\sin^{-1}\frac{\sqrt{2}}{2} - \sin^{-1}\frac{1}{2}$$

$$\frac{\pi}{4} - \frac{\pi}{6} = \boxed{\frac{\pi}{12}}$$

# 5.9 Hyperbolic Functions

Cosh $x \rightarrow$ Hyperbolic cosine of $x$

They can be compared to exponential fxns
see page 369

See page 371 for hyperbolic identities

Ex: $(\cosh u) = u'(\sinh u)$ no neg sign for this

Know inverse hyperbolic functions
on page 373

$$\text{Slope Intercept Form} = \quad y = \underbrace{m}_{\text{Slope}}x + \underbrace{b}_{\text{y-intercept}} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

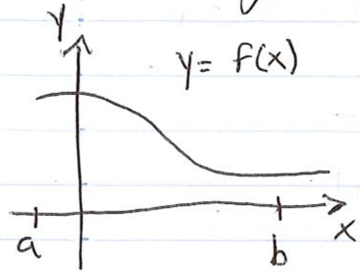
$$\text{Point Slope Form} = \quad y - y_1 = m(x - x_1)$$

$$\text{Distance Formula is } d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

5-19-14
Greg Henderson
Mac 2312

# Calculus II

## Arc Length




Recall that distance b/w 2 points on a plane is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

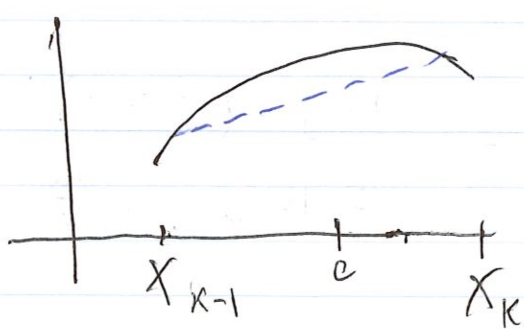
For a curve however we need to break up the curve into small partitions by drawing a straight line through it. We add all of these up to arrive at

$$\sum_{K-1}^{n} \sqrt{(X_{K-1} - X_K)^2 + (f(X_{K-1}) - f(X_K))^2}$$

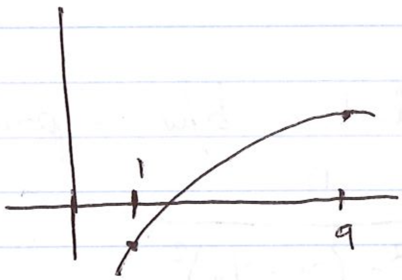
## Mean Value Theorem

Says

$$f'(c) = \frac{f(X_K) - f(X_{K-1})}{X_K - X_{K-1}}$$



Ex: $y = \frac{1}{3}x^{\frac{3}{2}} - x^{\frac{1}{2}}$




Length of this curve

$y = f(x) \quad a \le x \le b$

$$\int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Recall $(a-b)^2 = a^2 - 2ab + b^2$

$$y = \frac{1}{3}x^{\frac{3}{2}} - x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}} \qquad \text{so } \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}x - 2\left(\frac{1}{4}\right) + \frac{1}{4}x^{-1}$$

$$= \frac{1}{4}x - \frac{1}{2} + \frac{1}{4}x^{-1}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1} = \left(\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)^2$$

So $$\int_1^9 \sqrt{\left(\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)^2}\, dx$$

$$= \int_1^9 \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\, dx = \frac{1}{2} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \Bigg|_1^9$$

$$= \frac{1}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} \Bigg|_1^9 = 9 + 3 - \left(\frac{1}{3} + 1\right)$$

$$= 10\frac{2}{3} = \frac{32}{3}$$