Problem 1 (15 points): Multiple Choice.

1) Decide whether or not a statement: $512 = 2^8$
   A) Statement B) Not a statement
   1) A

2) Decide whether or not a statement: what time is it?
   A) Statement B) Not a statement
   2) B

3) Decide whether the statement is true or false: For all real number r, -r is a negative real number.
   A) True B) False
   3) B

4) p represents the statement "It is below freezing." q represents the statement "It is snowing."
   Translate the following compound statement into words: $\sim p \wedge \sim q$
   A) It's not below freezing or it's not snowing.
   B) It's not the case that it's below freezing and snowing.
   C) It's not below freezing and it's not raining today.
   D) It's below freezing or it's snowing.
   4) C

5) p represents the statement "It's below freezing." q represents the statement "It's snowing."
   Translate the following compound statement into words: $\sim p \vee \sim q$
   A) It's below freezing or it's snowing.
   B) It's below freezing and it's snowing.
   C) It's not below freezing or it's not snowing.
   D) It's not below freezing and it's not snowing.
   5) C

6) Given that $p \wedge q$ is true, what can you conclude about the truth values of p and q?
   A) Both p and q are false
   B) Exactly one of p and q is true
   C) At least one of p and q is false
   D) Both p and q are true
   6) D

7) Given that $\sim(p \vee q)$ is true, what can you conclude about the truth values of p and q?
   A) p and q have the same truth value
   B) Exactly one of p and q is false
   C) Both p and q are false
   D) At least one of p and q is false
   7) C

8) Given p is true, q is true, and r is false, find the truth value of the statement $\sim[(\sim q \to r) \to (q \vee r)]$
   A) False B) True
   8) A

9) True or False? The statement $\sim(q \to p)$ is equivalent to $\sim q \vee p$.
   A) False B) True $q \wedge \sim p$
   9) A

10) Determine whether the statement is true or false. If q is true then the statement $(p \wedge \sim q) \to p$ must be true.
    A) False B) True
    10) B

11) Determine whether the statement is true or false. $\forall$ real numbers m and n, $\sqrt{m+n} = \sqrt{m} + \sqrt{n}$.
    A) False B) True
    11) A

12) Determine whether the statement is true or false. $\exists x \in Z$ such that $\forall y \in Z, x = y + 1$.
A) False B) True
12) A

13) Determine whether the statement is true or false. $\forall x \in R, \exists y \in R$ such that $xy = 1$.
A) False B) True
13) ~~B~~ A

14) Let $D = E = \{1,2,3,4,5\}$. Use T (true) or F (false) to show the truth value of each of the following.
A) False B) True
a) $\forall x \in D, \exists y \in E$ such that x = y+1. 14a) A
b) $\exists x \in D$ such that $\forall y \in E, x < y$ 14b) A

Problem 2 [10 points] Write a negation for the statement.

1) Denim is out and linen is in.

Denim is in or linen is out.

2) If P is square, then p is a rectangle.

P is square and p is not a rectangle.

3) Every bird can fly.

Some birds canot fly.

4) Some old dogs can learn new tricks.

No old dogs can learn new tricks.

5) $\forall x \in D(\forall y \in E(P(x, y)))$

$\sim\forall x \in D(\forall y \in E(P(x,y)))$

$= \exists x \in D(\sim\forall y \in E(P(x,y)))$

$= \exists x \in D(\exists y \in E(\sim P(x,y)))$

Problem 3 [8 points] Rewrite it formally using quantifiers ∀ and ∃, symbols ∧, ∨, ~, and →, and variables. Then write the negation for each statement expressed in quantifiers ∀ and ∃, symbols ∧, ∨, ~, and→, and variables.

1) For any integer n, if n^2 is even then n is even.

let. even(x): x is even, $x \in \mathbb{Z}$

$\forall n \in \mathbb{Z}$, even(n^2) $\rightarrow$ even(n)

Negation: $\exists n \in \mathbb{Z}$ such that even(n^2) $\wedge$ ~even(n)

2) There is a triangle x such that for all squares y, x is above y.

let above(x, y) = x is above y

$\exists$ triangle x such that $\forall$ square y above(x, y)

Negation: $\forall$ triangle x $\exists$ square y ~above(x, y)

Problem 4 [4 points] Rewrite the statements without using the word necessary or sufficient.

1) Doing homework regularly is a necessary condition for Jim to pass the course.

If Jim doesn't doing homework regularly, he will not pass the course.

2) A sufficient condition for Jon's team to win the championship is that it win the rest of its games.

If Jon's team wins the rest of its games, then it will win the champion.

Problem 5 [6 points] Write the converse, inverse, and contrapositive for the following two statements.

1) If P is square, then P is a rectangle.

2) $\forall x \in R$, if $x > 2$, then $x^2 > 4$.

Problem 6 (6 Points)

Use a truth table to determine whether the argument is valid.

$$\begin{aligned} &\mathbf{p \vee q} \\ &\mathbf{p \rightarrow \sim q} \\ &\mathbf{p \rightarrow r} \\ \therefore\ &\mathbf{r} \end{aligned}$$

p	q	r	~q	p∨q	p→~q	p→r	r
T	T	T	F	T	F	T	T
T	T	F	F	T	F	F	F
T	F	T	T	T	T	T	T ✓
T	F	F	T	T	T	F	F
F	T	T	F	T	T	T	T ✓
F	T	F	F	T	T	T	F ✗
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	F

∴ Invalid

Problem 7 (10 Points)
Prove the following logical equivalence exists.

$$\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$$

a) Use true table to prove the above logical equivalence exists.
b) Use the logical equivalences in Theorem 2.1.1 to show the above logical equivalence exists. Supply a reason for each step.

$\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$

$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ De Morgan's laws

$\equiv \sim p \wedge (q \vee \sim q)$ Distributive laws

$\equiv \sim p \wedge t$ Negation laws

$\equiv \sim p$ Identity laws

Problem 8 [10 points] Let $P(x)$ be the predicate $x > 1/x$

a) [4 points] Write $P(2)$, $P(1/2)$, $P(-1)$, and $P(-8)$, and indicate which of these statements are true and which are false.

b) [3 points] Find the truth set of $P(x)$ if the domain of x is $\mathbb{Z}$ the set of all integers.

c) [3 points] If the domain is the set R^{+} of all positive real numbers, what is the truth set of $P(x)$?

a) $P(2)$: true $\quad 2 > 1/2$

$P(1/2)$: false $\quad 1/2 \not> 1/(1/2) = 2$

$P(-1)$: false $\quad -1 \not> 1/-1$

$P(-8)$: false $\quad -8 \not> 1/-8$

b). $x > 1$ or $-1 < x < 0$

c) $x > 1$

Problem 9 (6 Points)

Is the following argument valid or invalid? Justify your answer.

1) If a number is even, then twice that number is even.
The number 2n is even, for a particular number n.
Therefore, the particular number n is even.

Invalid, converse error

2) All healthy people eat an apple a day.
Herbert is not a healthy person.
Therefore, Herbert does not eat an apple a day.

Invalid, Inverse error

3) All freshmen must take writing.
Caroline is a freshman.
Therefore, Caroline must take writing.

Valid, Universal Modus Ponens

Problem 10 (10 Points)

A set of premises and a conclusion are given. Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, showing the argument form for each step. Assume all variables are statement variables.

a. $p \to q$
b. $r \vee s$
c. $\sim s \to \sim t$
d. $\sim q \vee s$
e. $\sim s$
f. $\sim p \wedge r \to u$
g. $w \vee t$
h. $\therefore u \wedge w$

(1). $\sim s \to \sim t$	by premise (c)
$\sim s$	by premise (e)
$\therefore \sim t$	by Modus Ponens
(2) $w \vee t$	by premise (g)
$\sim t$	by (1)
$\therefore w$	by Elimination
(3) $\sim q \vee s$	by premise (d)
$\sim s$	by premise (e)
$\therefore \sim q$	by Elimination
(4) $p \to q$	by premise (a)
$\sim q$	by (3)
$\therefore \sim p$	by Modus Tollens
(5) $r \vee s$	by premise (b)
$\sim s$	by premise (e)
$\therefore r$	by Elimination
(6) $\sim p$	by (4)
r	by (5)
$\therefore \sim p \wedge r$	by Conjunction
(7) $\sim p \wedge r \to u$	by premise (f)
$\sim p \wedge r$	by (6)
$\therefore u$	by Modus Ponens
(8) u	by (7)
w	by (2)
$\therefore u \wedge w$	by Conjunction