Calc III - Test 3 - Spring 2016

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Name Steven Romeiro

Instructor: Wendy Pogoda

Directions: This test covers chapter 15 (15.1 - 15.6). Show work for partial credit (partial credit will even be awarded on MC questions if a graph or work is shown). Each question is worth 15 points. Good luck!

Evaluate the integral.

1) $\int_{-8}^{2} \int_{-3}^{-1} dy\, dx$

$\int_{-8}^{2} [y]_{-3}^{-1} dx \rightarrow \int_{-8}^{2} [-1+3] dx \rightarrow \int_{-8}^{2} 2 dx \rightarrow [2x]_{-8}^{2}$

$= 2(2) - 2(-8) \rightarrow 4 - (-16) \rightarrow \boxed{20}$

Write an equivalent double integral with the order of integration reversed.

2) $\int_{1}^{6} \int_{0}^{\ln x} 2x\, dy\, dx$

$0 \leq y \leq \ln x$

$1 \leq x \leq 6$

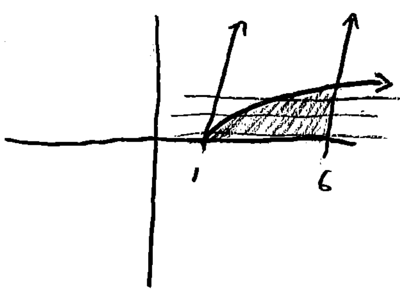
$y = \ln x \rightarrow x = e^y$

A) $\int_{0}^{\ln 6} \int_{e^y}^{6} 2x\, dx\, dy$

B) $\int_{0}^{\ln 6} \int_{1}^{6} 2x\, dx\, dy$

C) $\int_{0}^{\ln 6} \int_{1}^{6} 12x\, dx\, dy$

D) $\int_{0}^{\ln 6} \int_{e^y}^{6} 12x\, dx\, dy$



$e^y \leq x \leq 6$

$0 \leq y \leq \ln 6$

$$\boxed{\int_{0}^{\ln 6} \int_{e^y}^{6} 2x\, dx\, dy}$$

-0

$x^2 = -6x + 40$

$x^2 + 6x - 40 = 0$

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

3) The parabola $y = x^2$ and the line $y = -6x + 40$

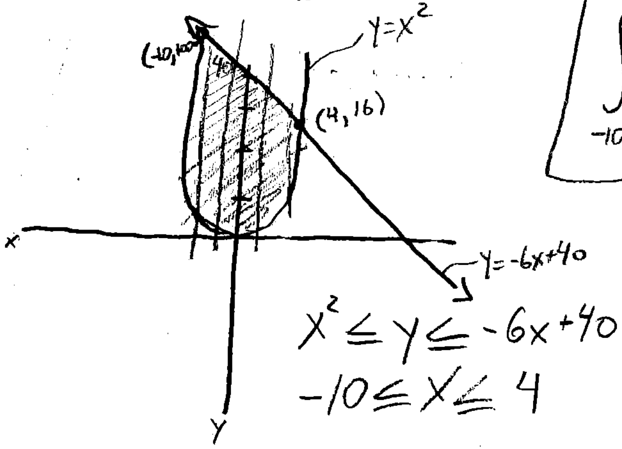
A) $\displaystyle\int_0^4 \int_{x^2}^{-6x+40} dy\,dx$

B) $\displaystyle\int_0^4 \int_{40}^{-6x+x^2} dy\,dx$

C) $\displaystyle\int_{-10}^4 \int_{x^2}^{-6x+40} dy\,dx$

D) $\displaystyle\int_{-10}^4 \int_0^{-6x+40-x^2} dx\,dy$

$$\boxed{\int_{-10}^{4}\int_{x^2}^{-6x+40} dy\,dx}$$

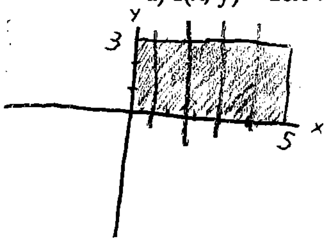


$x^2 \le y \le -6x + 40$

$-10 \le x \le 4$

Find the average value of the function f over the given region.

4) $f(x, y) = 10x + 5y$ over the rectangle $0 \le x \le 5,\ 0 \le y \le 3$.



Area $= 5(3) = 15$

$Av = \frac{1}{15}\int_0^5\int_0^3 [10x + 5y]\,dy\,dx \rightarrow \frac{1}{15}\int_0^5 \left[10xy + \frac{5}{2}y^2\right]_0^3 dx$

$Av = \frac{1}{15}\int_0^5 \left[30x + \frac{45}{2}\right] dx \rightarrow \frac{1}{15}\left[15x^2 + \frac{45}{2}x\right]_0^5$

$Av = \frac{1}{15}\left[375 + \frac{225}{2}\right] \rightarrow \frac{1}{15}\left[\frac{975}{2}\right]$

$$\boxed{Av = \frac{65}{2}}$$

$\sqrt{0}$

$x^2+y^2=169$

$r=13$

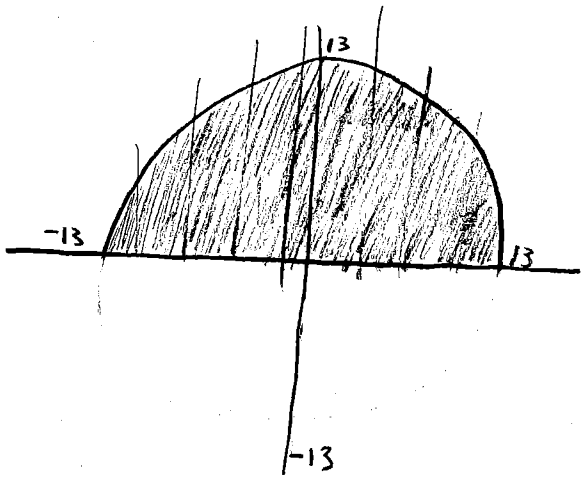
Change the Cartesian integral to an equivalent polar integral, and then evaluate.

5) $\displaystyle\int_{-13}^{13}\int_{0}^{\sqrt{169-x^2}} dy\,dx$

$$\int_0^{\pi}\int_0^{13} r\,dr\,d\theta \longrightarrow \int_0^{\pi}\left[\frac{1}{2}r^2\right]_0^{13}d\theta$$

$$\int_0^{\pi}\frac{169}{2}\,d\theta \longrightarrow \left[\frac{169}{2}\theta\right]_0^{\pi}$$

$$\boxed{\frac{169\pi}{2}}$$



$0\le r\le 13$

$0\le \theta\le \pi$

Find the volume of the indicated region.

6) the tetrahedron cut off from the first octant by the plane $\frac{x}{6}+\frac{y}{2}+\frac{z}{5}=1$

$0\le z\le 5(1-\frac{x}{6}-\frac{y}{2})$

$0\le y\le 2(1-\frac{x}{6})$

$0\le x\le 6$

$$V=\int_0^6\int_0^{2(1-x/6)}\int_0^{5(1-\frac{x}{6}-\frac{y}{2})} dz\,dy\,dx \longrightarrow \int_0^6\int_0^{2(1-\frac{x}{6})}\left[z\right]_0^{5(1-\frac{x}{6}-\frac{y}{2})}dy\,dx$$

$$V=\int_0^6\int_0^{2(1-\frac{x}{6})}\left[5-\frac{5}{6}x-\frac{5}{2}y\right]dy\,dx \longrightarrow \int_0^6\left[5y-\frac{5}{6}xy-\frac{5}{4}y^2\right]_0^{2-\frac{1}{3}x}dx$$

$$V=\int_0^6\left[10-\frac{5}{3}x-\frac{5}{3}x+\frac{5}{9}x^2-\frac{5}{4}(2-\frac{1}{3}x)^2\right]dx \rightarrow \int_0^6\left[10-\frac{10}{3}x+\frac{5}{9}x^2-\frac{5}{4}(4-\frac{4}{3}x+\frac{1}{9}x^2)\right]dx$$

$$V=\int_0^6\left[\frac{5}{9}x^2-\frac{10}{3}x+10-5+\frac{5}{3}x-\frac{5}{36}x^2\right]dx \longrightarrow \int_0^6\left[\frac{5}{12}x^2-\frac{5}{3}x+5\right]dx$$

$$V=\left[\frac{5}{36}x^3-\frac{5}{6}x^2+5x\right]_0^6 \longrightarrow 30-30+30 \longrightarrow \boxed{\cancel{V=30}}$$

-1

-1

$X = 3 - \frac{3}{8}Y$

$\left(\frac{64}{9}x^2 - \frac{128}{3}x + 64\right)\left(-\frac{8}{3}x + 8\right) = -\frac{512}{27}x^3 + \frac{1024}{9}x^2 + \frac{512}{9}x^2 - \frac{512}{3}x - \frac{1024}{3}x + 512$

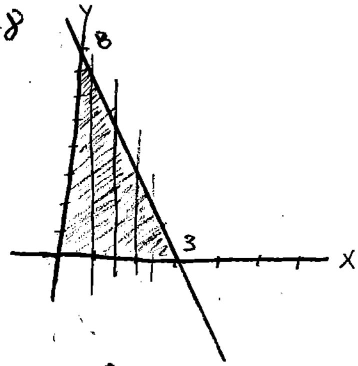
$= -\frac{512}{27}x^3 + \frac{512}{3}x^2 - 512x + 512$

Solve the problem.

7) Find the moment of inertia about the x–axis of a thin plane of constant density $\delta = 2$ bounded by the coordinate axes and the line $\frac{x}{3} + \frac{y}{8} = 1$. $\longrightarrow$ $y = -\frac{8}{3}x + 8$

$0 \le y \le -\frac{8}{3}x + 8$

$0 \le x \le 3$



$I_x = \iint y^2 \delta \, dA$

$I_x = \int_0^3 \int_0^{-8/3x+8} 2y^2 \, dy\, dx$

$1a^3 - 3a^2b + 3ab^2 - 1b^3$

$I_x = \int_0^3 \left[\frac{2}{3}y^3\right]_0^{-8/3x+8} dx \longrightarrow \int_0^3 \left[\frac{2}{3}\left(-\frac{8}{3}x+8\right)^3\right] dx \rightarrow \int_0^3 \left[\frac{2}{3}\left(-\frac{512}{27}x^3 + \frac{512}{3}x^2 - 512x + 512\right)\right] dx$

$I_x = \int_0^3 \left[-\frac{1024}{81}x^3 + \frac{1024}{9}x^2 - \frac{1024}{3}x + \frac{1024}{3}\right] dx \longrightarrow \frac{1024}{3}\left[-\frac{1}{108}x^4 + \frac{1}{9}x^3 - \frac{1}{2}x^2 + x\right]_0^3$

$I_x = \frac{1024}{3}\left[-\frac{3}{4} + 3 - \frac{9}{2} + 3\right] \longrightarrow \frac{1024}{3}\left[\frac{3}{4}\right]$

$\boxed{I_x = 256}$

E.C.

$f(3) = \frac{9}{63} = \frac{1}{7}$

$f(6) = \frac{36}{63} = \frac{4}{7}$

yes it's valid

why?

mean $= \int_3^6 \frac{x^3}{63} dx \longrightarrow \left[\frac{1}{252}x^4\right]_3^6$

~~+8~~ +4

$\mu = \left[\frac{1296}{252} - \frac{81}{252}\right]$

$\boxed{\mu = \frac{135}{28}}$

respect to x

$\int_0^{3-\frac{3}{8}y} 2y^2 = 2y^2 x$

$2y^2\left[3 - \frac{3}{8}y\right] = 6y^2 - \frac{6}{8}y^3$

$6\int_0^8 y^2 - \frac{1}{8}y^3 = 6\left[\frac{1}{3}y^3 - \frac{1}{32}y^4\right]_0^8$

$6\left[\frac{512}{3} - \frac{4096}{32}\right] = 6\left(\frac{128}{3}\right)$

$= 256$ +4

Steven
Romeiro

Practice Exam 3

(1) $\iint_R \sin(15x)\,dA$, $R: 0 \le x \le \frac{\pi}{15}$, $0 \le y \le \pi$

$$\int_0^{\pi}\int_0^{\pi/15} \sin(15x)\,dx\,dy \rightarrow \int_0^{\pi}\left[-\frac{1}{15}\cos(15x)\right]_0^{\pi/15} dy$$

$$\int_0^{\pi}\left[\left(-\frac{1}{15}\cos(\pi\right) + \frac{1}{15}\cos(0)\right] dy \rightarrow \int_0^{\pi}\left[\frac{1}{15} + \frac{1}{15}\right] dy$$

$$\int_0^{\pi} \frac{2}{15}\,dy \rightarrow \left[\frac{2}{15}y\right]_0^{\pi} \rightarrow \boxed{\frac{2\pi}{15}}$$

(2) $f(x,y) = 5x\sin xy$ over rectangle $0 \le x \le \pi$
$0 \le y \le 1$

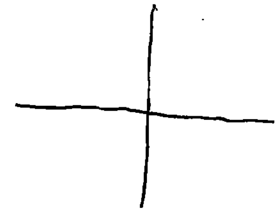
$$\int_0^{\pi}\int_0^{1} 5x\sin(xy)\,dy\,dx$$

$$\int_0^{\pi}\left[-\frac{5x}{x}\cos(xy)\right]_0^1 dx \rightarrow \int_0^{\pi}\left[-5\cos(xy)\right]_0^1$$

$$\int_0^{\pi}\left[-5\cos(x) + 5\cos(0)\right] \cancel{dx} \rightarrow \int_0^{\pi}\left[-5\cos x + 5\right] dx$$

$$\left[-5\sin x + 5x\right]_0^{\pi} = \left[-5\cancel{\sin(\pi)}^{0} + 5\pi\right] - \left[\cancel{-5\sin(0)+5(0)}^{0}\right]$$

$$= \boxed{5\pi}$$



(3) $Z = \frac{x}{y}$; $0 \le x \le 1$, $1 \le y \le e$

$$V = \int_1^e \int_0^1 \frac{x}{y}\,dx\,dy \longrightarrow \int_1^e \left[\frac{1}{2y}x^2\right]_0^1 dy \rightarrow \int_1^e \frac{1}{2y}(1)^2\,dy$$

$$\frac{1}{2}\int_1^e \frac{1}{y}\,dy \longrightarrow \frac{1}{2}\Big[\ln y\Big]_1^e \longrightarrow \frac{1}{2}\Big[\ln e - \ln 1\Big] = \boxed{\frac{1}{2}}$$

(4) $$\int_1^5 \int_0^{\ln x} e^y\,dy\,dx \longrightarrow \int_1^5 \Big[e^y\Big]_0^{\ln x} dx \longrightarrow \int_1^5 \Big[x-1\Big]dx$$

$$\left[\frac{1}{2}x^2 - x\right]_1^5 \longrightarrow \left[\frac{25}{2} - 5\right] - \left[\frac{1}{2} - 1\right]$$

$$\frac{15}{2} + \frac{1}{2} = \frac{16}{2} = \boxed{8}$$

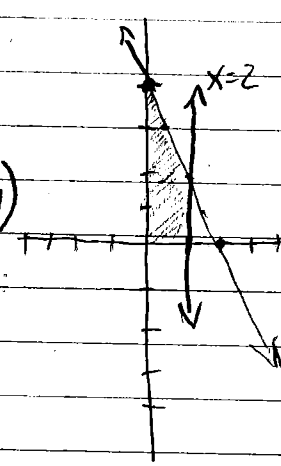
(5) $f(x,y) = \frac{x}{2} + \frac{y}{4}$, $x=0$, $y=0$, $x=2$, $y=-2x+8$

$$\int_0^2 \int_0^{-2x+8} \frac{x}{2} + \frac{y}{4}\,dy\,dx \longrightarrow \int_0^2 \left[\frac{xy}{2} + \frac{1}{8}y^2\right]_0^{-2x+8} dx$$

$$\int_0^2 \left[\frac{-2x^2+8x}{2} + \frac{1}{8}(-2x+8)^2\right] dx \longrightarrow \int_0^2 -x^2+4x+\frac{1}{8}(+4x^2-32x+64)$$

$$\int_0^2 \frac{-x^2}{2} + 8\,dx \longrightarrow \left[-\frac{x^3}{6} + 8x\right]_0^2 \longrightarrow -\frac{8}{6} + 16$$

$$= \boxed{\frac{44}{3}}$$



$0 \le y \le -2x+8$

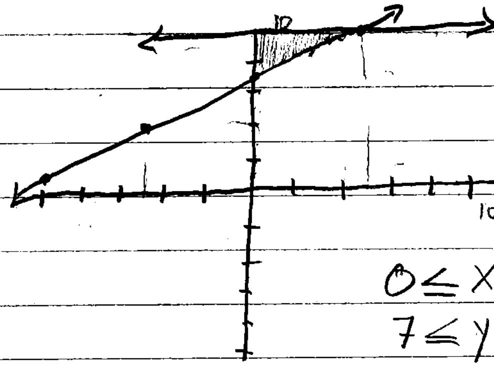
$0 \le x \le 2$

$$\frac{3x}{5}+7 \le y \le 10$$

$$0 \le x \le 5$$

$$y = \frac{3x}{5}+7$$

(6) $$\boxed{\int_0^5 \int_{\frac{3x}{5}+7}^{10} dy\,dx}$$



$$\frac{3y}{5} = y-7$$

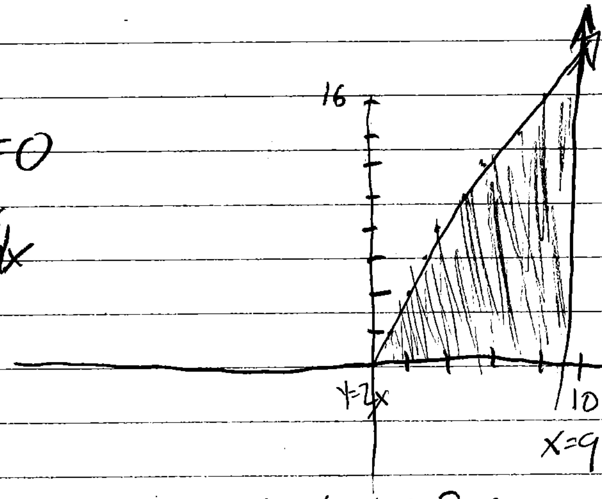
$$x = \frac{5y}{3} - \frac{35}{3}$$

$$\boxed{\int_7^{10} \int_0^{\frac{5(y-7)}{3}} dx\,dy}$$

$$0 \le x \le 5(y-7)/3$$

$$7 \le y \le 10$$

(7) $Z = x^2+y^2,\ x=9,\ y=2x,\ y=0$



$$\int_0^9 \int_0^{2x} x^2+y^2\,dy\,dx = \int_0^9 \left[x^2y + \frac{1}{3}y^3\right]_0^{2x} dx$$

$$\int_0^9 \left[2x^3 + \frac{8x^3}{3}\right] dx = \int_0^9 \frac{14}{3}x^3$$

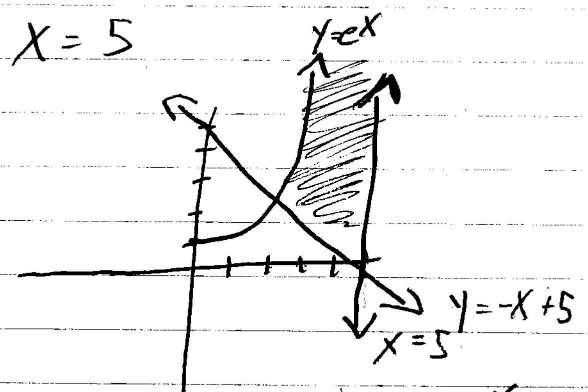
$$0 \le y \le 2x$$

$$0 \le x \le 9$$

$$V = \left[\frac{14}{12}x^4\right]_0^9 \qquad V = \left[\frac{7}{6}x^4\right]_0^9$$

$$\boxed{V = \frac{15{,}309}{2}}$$

(8) Area of $y = e^x$, $x + y = 5$, $x = 5$



$-x+5 \le y \le e^x$

(9) $\int_{\pi/4}^{\pi/2} \int_{6\cos x}^{6\sin x} dy\,dx \longrightarrow \int_{\pi/4}^{\pi/2} \left[y \right]_{6\cos x}^{6\sin x}$

$\int_{\pi/4}^{\pi/2} 6\sin x - 6\cos x \, dx \longrightarrow 6\int_{\pi/4}^{\pi/2} \sin x - \cos x \, dx$

$6\left[-\cos x - \sin x \right]_{\pi/4}^{\pi/2}$

$6\left[-\cos(\pi/2) - \sin(\pi/2) \right] - \left[-\cos(\pi/4) - \sin(\pi/4) \right]$

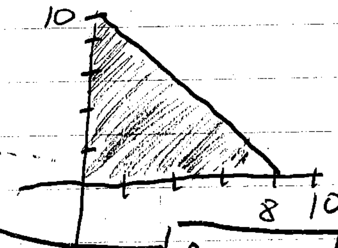
$6\left[0 - 1 \right] - \left[-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right]$

$\boxed{-6 + 6\sqrt{2}}$

?

(10) $f(x,y) = 8x + 3y$, $(0,0)(0,10),(8,0)$

$Area = \frac{1}{2}bh = \frac{1}{2}(10)(8) = 40$



$Av = \frac{1}{Area}\iint f(x,y)$

$Av = \frac{1}{40}\int_0^8\int_0^{10} 8x + 3y \, dy\,dx$

$0 \le y \le \boxed{-\frac{5}{4}x + 10}$

$0 \le x \le 8$

$Av = \frac{1}{40}\int_0^8 \left[8xy + \frac{3}{2}y^2\right]_0^{10} = \frac{1}{40}\int_0^8 \left[80x + 150\right] dx$

$Av = \frac{10}{40}\left[4x^2 + 15x\right]_0^8 = \frac{1}{4}\left[256 + 120\right] = \boxed{94}$?

(11) $\int_{-7}^{0}\int_{-\sqrt{49-x^2}}^{0} \frac{1}{1+\sqrt{x^2+y^2}}\,dy\,dx$

$x = r\cos\theta$
$y = r\sin\theta$
$x^2+y^2 = r^2$

$y = -\sqrt{49-x^2} \rightarrow y^2 + x^2 = 49$

$\int_{-7}^{0}\int_{-\sqrt{49-r^2\cos^2\theta}}^{0} \frac{1}{1+\sqrt{r^2}}\, r\,dr\,d\theta \rightarrow \int_{-7}^{0}\int_{-\sqrt{49-r^2\cos^2\theta}}^{0} \frac{r}{1+r}\,dr\,d\theta = \frac{r+1-1}{r+1}$

r'	s
r	$(1+r)^{-1}$
1	$\ln(1+r)$
0	$(1+r)(\ln(1+r)) - (1+r)$

$\int_{\pi}^{3\pi/2}\int_0^7 \frac{r}{1+r}\,dr\,d\theta \rightarrow \int_{-7}^{0}\left[r\ln(1+r) - \ln(1+r) + r\ln(1+r) - (1+r)\right]_{-7}^{0}$

$\int_{-7}^{0} -7\ln(-6)$

MAC2313 Practice Exam 3
Instructor: Wendy Pogoda

This practice exam covers Chapter 15 (15.1 - 15.6). Your actual exam will be different. Use this exam to evaluate the areas in which you need additional studying and practice. Your actual exam will have less multiple choice questions.

Evaluate the double integral over the given region.

1) $\iint_R \sin 15x \, dA$, R: $0 \le x \le \frac{\pi}{15}$, $0 \le y \le \pi$

A) π B) $\frac{\pi}{15}$ C) $\frac{2\pi}{15}$ D) $\frac{\pi}{30}$

Integrate the function f over the given region.

2) $f(x, y) = 5x \sin xy$ over the rectangle $0 \le x \le \pi$, $0 \le y \le 1$

A) $\frac{\pi}{5}$ B) $5\pi - 5$ C) π D) 5π

Find the volume under the surface z = f(x,y) and above the rectangle with the given boundaries.

3) $z = \frac{x}{y}$; $0 \le x \le 1$, $1 \le y \le e$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{6}$

Evaluate the integral.

4) $\int_1^5 \int_0^{\ln x} e^y \, dy \, dx$

A) 8 B) 18 C) 9 D) 4

Integrate the function f over the given region.

5) $f(x, y) = \frac{x}{2} + \frac{y}{4}$ over the trapezoidal region bounded by the x–axis, y–axis, line x = 2, and line y = - 2x + 8

A) $\frac{92}{3}$ B) $\frac{44}{3}$ C) 20 D) 12

Write an equivalent double integral with the order of integration reversed.

6) $\int_0^5 \int_{3x/5 + 7}^{10} dy \, dx$

A) $\int_7^5 \int_0^{5(y-7)/3} dx \, dy$ B) $\int_5^{10} \int_0^{5(y-7)/3} dx \, dy$

C) $\int_7^{10} \int_0^{5(y-7)/3} dx \, dy$ D) $\int_0^{10} \int_0^{5(y-7)/3} dx \, dy$

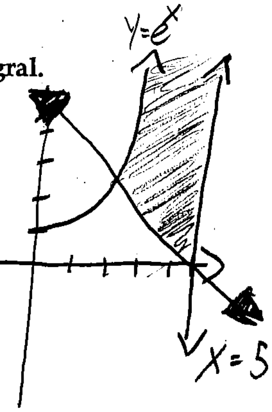
Find the volume of the indicated region.

7) the region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $x = 9$, $y = 0$, and $y = 2x$

A) 13,122 B) $\frac{15,309}{2}$ C) $\frac{189}{2}$ D) $\frac{1701}{2}$

Express the area of the region bounded by the given line(s) and/or curve(s) as an iterated double integral.

8) The curve ~~$y = e^x$ and the~~ lines $x + y = 5$ and $x = 5$



Law

$x=0$?

A) $\int_0^5 \int_{5-x}^{e^x} dy\,dx$ B) $\int_0^5 \int_5^{e^x - x} dy\,dx$

C) $\int_0^5 \int_{x-5}^{e^x} dy\,dx$ D) $\int_0^5 \int_x^{e^x - x} dy\,dx$

Find the area of the region specified by the integral(s).

9) $\int_{\pi/4}^{\pi/2} \int_{6 \cos x}^{6 \sin x} dy\,dx$

$-6 + 6\sqrt{2}$?

A) $6\sqrt{2}$ B) 6 C) 1 D) 12

Find the average value of the function f over the given region.

10) $f(x, y) = 8x + 3y$ over the triangle with vertices (0, 0), (8, 0), and (0, 10).

A) $\frac{64}{3}$ B) $\frac{67}{3}$ C) $\frac{94}{3}$ over 3 ? $= 94$ D) $\frac{74}{3}$

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

11) $\int_{-7}^{0} \int_{-\sqrt{49 - x^2}}^{0} \frac{1}{1 + \sqrt{x^2 + y^2}}\, dy\,dx$

A) $\frac{\pi(7 + \ln 8)}{2}$ B) $\frac{\pi(7 - \ln 8)}{4}$ C) $\frac{\pi(7 - \ln 8)}{2}$ D) $\frac{\pi(7 + \ln 8)}{4}$

Find the area of the region specified in polar coordinates.

12) one petal of the rose curve $r = 7 \cos 3\theta$

A) $\frac{49}{4}\pi$ B) $\frac{49}{2}\pi$ C) $\frac{49}{6}\pi$ D) $\frac{49}{12}\pi$

Solve the problem.

13) Find the average height of the paraboloid $z = 6x^2 + 2y^2$ above the annular region $9 \le x^2 + y^2 \le 64$ in the xy-plane.

A) $\frac{511}{2}$ B) $\frac{292}{3}$ C) 146 D) $\frac{365}{2}$

14) Write an iterated triple integral in the order dz dy dx for the volume of the tetrahedron cut from the first octant by the plane $\frac{x}{9} + \frac{y}{4} + \frac{z}{3} = 1$.

A) $\int_0^9 \int_0^{4(1 - x/9)} \int_0^{3(1 - x/9 - y/4)} dz\, dy\, dx$

B) $\int_0^9 \int_0^{1 - y/4} \int_0^{1 - x/9 - y/4} dz\, dy\, dx$

C) $\int_0^9 \int_0^{1 - x/9} \int_0^{1 - x/9 - y/4} dz\, dy\, dx$

D) $\int_0^9 \int_0^{9(1 - y/4)} \int_0^{3(1 - x/9 - y/4)} dz\, dy\, dx$

Find the volume of the indicated region.

15) the region bounded by the paraboloid $z = 1 - \frac{x^2}{49} - \frac{y^2}{64}$ and the xy-plane

A) 196π B) $\frac{56}{3}\pi$ C) 224π D) 28π

Solve the problem.

16) Find the center of mass of the thin semicircular region of constant density $\delta = 9$ bounded by the x-axis and the curve $y = \sqrt{196 - x^2}$.

A) $\bar{x} = 0, \bar{y} = \frac{14}{3\pi}$ B) $\bar{x} = 0, \bar{y} = \frac{112}{3\pi}$ C) $\bar{x} = 0, \bar{y} = \frac{28}{3\pi}$ answer D) $\bar{x} = 0, \bar{y} = \frac{56}{3\pi}$?

17) Find the moment of inertia about the x-axis of a thin plane of constant density $\delta = 4$ bounded by the coordinate axes and the line $\frac{x}{10} + \frac{y}{7} = 1$.

A) $\frac{7000}{3}$ B) $\frac{3430}{3}$ answer = 3430 C) $\frac{4900}{3}$ D) $\frac{2401}{3}$

18) Find the center of mass of a thin infinite region in the first quadrant bounded by the coordinate axes and the curve $y = e^{-6x}$ if $\delta(x, y) = xy$. too many terms

A) $\bar{x} = \frac{1}{9}, \bar{y} = \frac{8}{27}$ B) $\bar{x} = \frac{1}{9}, \bar{y} = \frac{2}{9}$ C) $\bar{x} = \frac{1}{6}, \bar{y} = \frac{8}{27}$ D) $\bar{x} = \frac{1}{6}, \bar{y} = \frac{2}{9}$

19) Find the moment of inertia I_z of a tetrahedron of constant density $\delta(x, y, z) = 1$ bounded by the coordinate planes and the plane $\frac{x}{9} + \frac{y}{10} + \frac{z}{7} = 1$. too many terms

A) 2415 B) $\frac{3129}{2}$ C) $\frac{3801}{2}$ D) 1365

20) Find the mass of a tetrahedron of density $\delta(x, y, z) = x + y + z$ bounded by the coordinate planes and the plane $\frac{x}{8} + \frac{y}{10} + \frac{z}{7} = 1$.

A) $\frac{7000}{9}$ B) $\frac{1750}{3}$ C) $\frac{2800}{3}$ D) 700

Answer Key
Testname: CALC3_PRACTICETEST3

1) C
2) D
3) B
4) A
5) B
6) C
7) B
8) A
9) B
10) C
11) C
12) D
13) C
14) A
15) D
16) D
17) B
18) C
19) C
20) B

MAC2313 P.T. #3

1) $\iint_R \sin 15x \, dA$, R: $0 \le x \le \frac{\pi}{15}$, $0 \le y \le \pi$

$$= \int_0^{\pi}\int_0^{\pi/15} \sin 15x \, dx\, dy = \int_0^{\pi} -\frac{\cos 15x}{15}\Big]_0^{\pi/15} dy$$

$$= \int_0^{\pi}\left[-\frac{\cancel{\cos \pi}^{-1}}{15} + \frac{\cancel{\cos 0}^{1}}{15}\right] dy$$

$$= \int_0^{\pi} \frac{2}{15}\, dy = \frac{2y}{15}\Big]_0^{\pi} = \frac{2\pi}{15} - \frac{0}{15} = \boxed{\frac{2\pi}{15}}$$

easier

2) $\int_0^{\pi}\int_0^{1} 5x\sin(xy)\,dy\,dx$ OR $\cancel{\int_0^{1}\int_0^{\pi} 5x\sin(xy)\,dx\,dy}$

$\int_0^{\pi} 5\cancel{x}\left(\frac{-\cos(xy)}{\cancel{x}}\right)\Big]_0^1 dx$

$= \int_0^{\pi} -5(\cos x - \cos 0)\,dx$

$= \int_0^{\pi} -5(\cos x - 1)\,dx$

$= \int_0^{\pi} (-5\cos x + 5)\,dx$

$= -5\sin x + 5x\Big]_0^{\pi}$

$= -5\,\underbrace{\cancel{\sin\pi}}_{0} + 5\pi - (-5\,\underbrace{\cancel{\sin 0}}_{0} + \underbrace{\cancel{5\cdot 0}}_{0})$

$= \boxed{5\pi}$

3) $z = \frac{x}{y}$, $0 \le x \le 1$, $1 \le y \le e$

$$V = \int_1^e \int_0^1 \frac{x}{y}\, dx\, dy \quad \text{or} \quad \int_0^1 \int_1^e \frac{x}{y}\, dy\, dx$$

$$= \int_1^e \frac{1}{y} \cdot \frac{1}{2} x^2 \Big]_0^1 dy$$

$$= \int_1^e \frac{1}{y} \cdot \frac{1}{2}(1^2 - 0^2)\, dy$$

$$= \int_1^e \frac{1}{2y}\, dy$$

$$= \frac{1}{2} \ln|y| \Big]_1^e$$

$$= \frac{1}{2}(\overset{1}{\cancel{\ln e}} - \overset{0}{\cancel{\ln 1}})$$

$$= \boxed{\frac{1}{2}}$$

4) $$\int_1^5 \int_0^{\ln x} e^y \, dy \, dx$$

$$= \int_1^5 e^y \Big]_0^{\ln x} dx$$

$$= \int_1^5 (e^{\ln x} - e^0) \, dx$$

$$= \int_1^5 (x-1) \, dx$$

$$= \frac{1}{2}x^2 - x \Big]_1^5$$

$$= \left(\frac{25}{2} - 5\right) - \left(\frac{1}{2} - 1\right)$$

$$= \frac{15}{2} + \frac{1}{2} = \frac{16}{2} = \boxed{8}$$

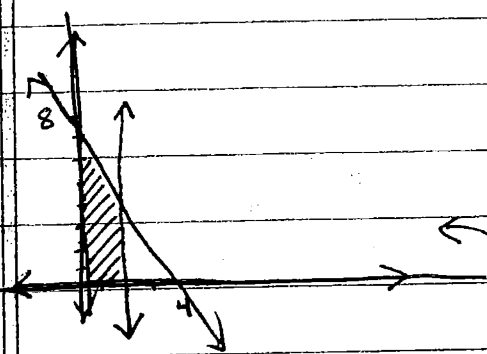
5) $f(x,y) = \frac{x}{2} + \frac{y}{4}$ over region bounded by:

x-axis

y-axis

$x = 2$

$y = -2x+8$



vertical cross sections easier (horiz would require $\iint + \iint$)

$$\int_0^2\int_0^{-2x+8} \left(\frac{x}{2} + \frac{y}{4}\right) dy\, dx$$

$$-2x+8 = -2(x-4)$$

$$= \int_0^2 \left(\frac{x}{2}\cdot y + \frac{y^2}{8}\right)\Big|_0^{-2x+8} dx$$

$$= \int_0^2 \left(\frac{x}{\cancel{2}}(-\cancel{2}x+\cancel{8}^{\,4}) + \frac{(-2)^{\cancel{2}}(x-4)^2}{\cancel{8}\,2}\right) dx$$

$$= \int_0^2 \left[-x^2 + 4x + \frac{x^2}{2} - \frac{8x}{2} + \frac{16}{2}\right] dx$$

$$= \int_0^2 \left[-x^2 + \cancel{4x} + \frac{x^2}{2} - \cancel{4x} + 8\right] dx$$

$$= \int_0^2 \left[-\frac{x^2}{2} + 8\right] dx$$

$$= -\frac{x^3}{6} + 8x \Big]_0^2$$

$$= -\frac{\cancel{8}^{\,4}}{\cancel{6}_{\,3}} + 16 - 0$$

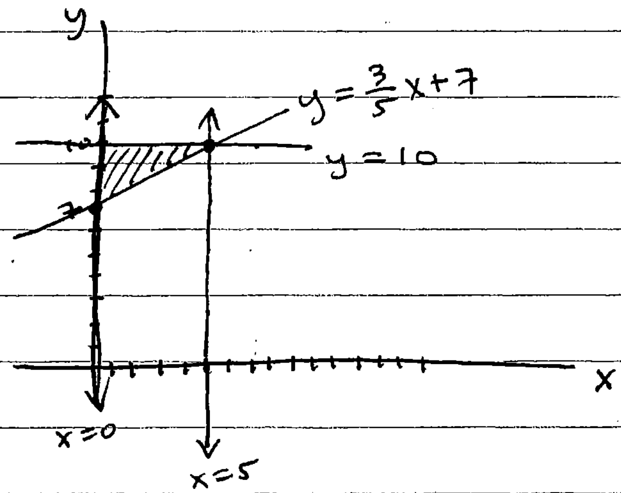
$$= \boxed{\frac{44}{3}}$$

Reverse order

6) $\int_0^5 \int_{\frac{3x}{5}+7}^{10} dy\, dx$

$\frac{3x}{5} + 7 \le y \le 10$, $0 \le x \le 5$

bounds: $y = \frac{3}{5}x + 7$, $y = 10$, $x = 0$, $x = 5$



For horiz cross sections:

$0 \le x \le$ line

line: $y = \frac{3}{5}x + 7$

$y - 7 = \frac{3}{5}x$

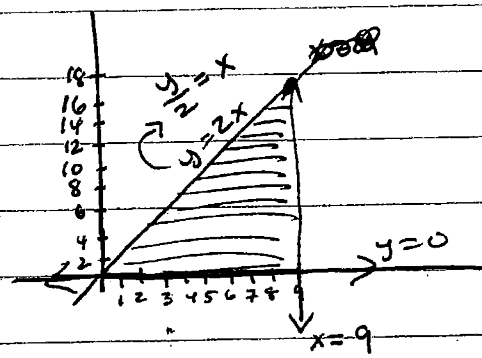
$x = \frac{5}{3}(y - 7)$

$0 \le x \le \frac{5}{3}(y-7)$

$7 \le y \le 10$

$$\boxed{\int_7^{10} \int_0^{\frac{5}{3}(y-7)} dx\, dy}$$

7) V of region under $z=x^2+y^2$ & above triangle enclosed by $x=9$, $y=0$, $y=2x$



$$V=\int_0^{18}\int_{y/2}^{9}(x^2+y^2)\,dx\,dy \quad \text{OR}$$

$$V=\int_0^{9}\int_0^{2x}(x^2+y^2)\,dy\,dx$$

$$V=\int_0^9\left[x^2y+\frac{y^3}{3}\right]_0^{2x}dx$$

$$=\int_0^9\left[x^2\cdot 2x+\frac{8x^3}{3}\right]dx$$

$$=\int_0^9\left[2x^3+\frac{8}{3}x^3\right]dx$$

$$=\int_0^9\frac{14}{3}x^3\,dx$$

$$=\frac{\cancel{14}^{\,7}}{3}\cdot\frac{1}{\cancel{4}_{\,2}}x^4\Big]_0^9$$

$$=\frac{7}{\cancel{6}_{\,2}}\left(\cancel{6561}^{\,2187}-0\right)$$

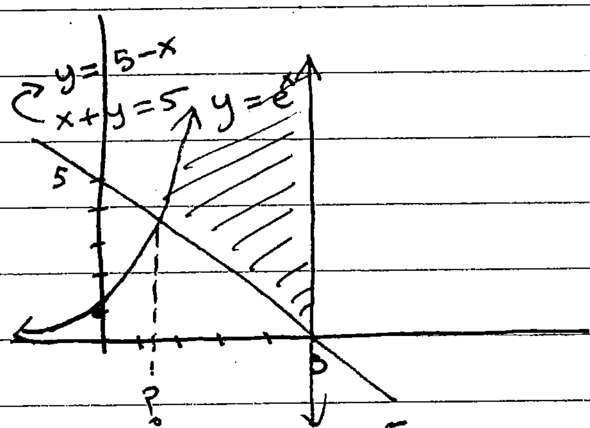
$$=\boxed{\frac{15{,}309}{2}}$$

! Answer NOT correct!

(on answer key → correct answer not a choice)

8) Express area as $\iint$

$y = e^x$ & $x+y=5$ & $x=5$



for vertical cross sections:

$5-x \le y \le e^x$

$? \le x \le 5$

$$A = \int_{?}^{5}\int_{5-x}^{e^x} dy\,dx$$

!

9

Correct answer not a choice
(answer on key is wrong)

9) $$\int_{\pi/4}^{\pi/2}\int_{6\cos x}^{6\sin x} dy\,dx$$

$$= \int_{\pi/4}^{\pi/2} y \Big]_{6\cos x}^{6\sin x} dx$$

$$= \int_{\pi/4}^{\pi/2} [6\sin x - 6\cos x]\,dx$$

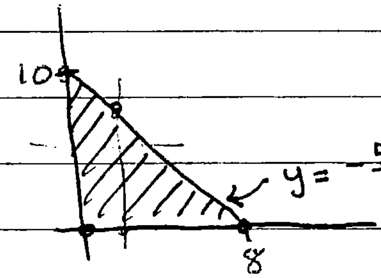
$$= -6\cos x - 6\sin x \Big]_{\pi/4}^{\pi/2}$$

$$= -6\overset{0}{\cancel{\cos}\left(\tfrac{\pi}{2}\right)} - 6\overset{1}{\cancel{\sin\left(\tfrac{\pi}{2}\right)}} - \left[-6\overset{\frac{\sqrt{2}}{2}}{\cancel{\cos\tfrac{\pi}{4}}} - 6\overset{\frac{\sqrt{2}}{2}}{\cancel{\sin\tfrac{\pi}{4}}}\right]$$

$$= \boxed{-6 + 6\sqrt{2}}$$

Avg Value?

10) $f(x,y)=8x+3y$ over Δ w/ vertices $(0,0),(8,0),(0,10)$



$A = \frac{1}{2}\cdot 8\cdot 10 = 40$

$$\text{Avg Value} = \frac{1}{40}\int_0^8\int_0^{-\frac{5}{4}x+10}(8x+3y)\,dy\,dx$$

$$= \frac{1}{40}\int_0^8\left[8xy+\frac{3}{2}y^2\right]_0^{10-\frac{5}{4}x}dx$$

$$= \frac{1}{40}\int_0^8\left[\overbrace{8x\left(10-\frac{5}{4}x\right)}^{80x-10x^2}+\frac{3}{2}\overbrace{\left(10-\frac{5}{4}x\right)^2}^{100-25x+\frac{25}{16}x^2}\right]dx$$

$$= \frac{1}{40}\int_0^8\left[80x-10x^2+150-\frac{75}{2}x+\frac{75}{32}x^2\right]dx$$

$$= \frac{1}{40}\int_0^8\left[-\frac{245}{32}x^2+\frac{85}{2}x+150\right]dx$$

$$= \frac{1}{40}\left[-\frac{245}{96}x^3+\frac{85}{4}x^2+150x\right]_0^8$$

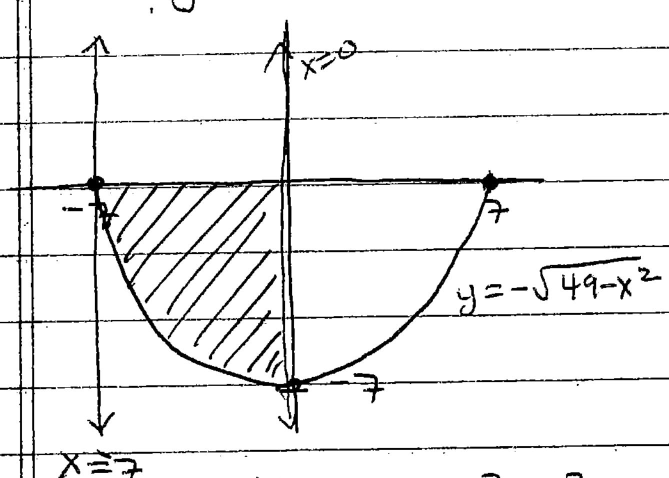
$$= \frac{1}{40}\left[\frac{-125,440}{96}+\frac{5440}{4}+1200\right]$$

$$= -\frac{3136}{96}+\frac{\cancel{136}^{34}}{\cancel{4}}+30$$

$$= -\frac{98}{3}+64\cdot\frac{3}{3} = \boxed{\frac{94}{3}}$$

11) $\displaystyle\int_{-7}^{0}\int_{-\sqrt{49-x^2}}^{0} \frac{1}{1+\sqrt{x^2+y^2}}\,dy\,dx$

Region: $-\sqrt{49-x^2} \le y \le 0$, $-7 \le x \le 0$



$0 \le r \le 7$

$\pi \le \theta \le \frac{3\pi}{2}$

NOTE: $x^2+y^2=r^2 \rightarrow \sqrt{x^2+y^2}=r$

$dy\,dx = r\,dr\,d\theta$

$$= \int_{\pi}^{\frac{3\pi}{2}}\int_{0}^{7} \frac{1}{1+r}\cdot r\,dr\,d\theta$$

$$\frac{r}{1+r} = \frac{r+1-1}{r+1} = \frac{r+1}{r+1} - \frac{1}{r+1} = 1 - \frac{1}{r+1}$$

$$= \int_{\pi}^{\frac{3\pi}{2}}\int_{0}^{7} \left[1 - \frac{1}{r+1}\right] dr\,d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} \Big[r - \ln|r+1|\Big]_0^7\,d\theta$$

$$= \int_{\pi}^{3\pi/2} \left(7 - \ln 8 - (0 - \cancel{\ln 1}^{\,0})\right) d\theta$$

$$= (7-\ln 8)\,\theta \Big]_{\pi}^{3\pi/2}$$

$$= (7-\ln 8)\left(\frac{3\pi}{2} - \pi\right)$$

$$= (7-\ln 8)\left(\frac{\pi}{2}\right)$$

$$\boxed{\frac{\pi}{2}(7-\ln 8)}$$

12) area of 1 petal of rose: $r = 7\cos 3\theta$

Note: when r goes from $0 \to 1 \to 0$ is 1 petal

$\cos 3\theta = 0 \Rightarrow \cos 3\theta = 1 \Rightarrow \cos 3\theta = 0$

$3\theta = -\frac{\pi}{2}$ $\quad 3\theta = 0 \quad$ $3\theta = \frac{\pi}{2}$

$\theta = -\frac{\pi}{6}$ $\quad \theta = 0 \quad$ $\theta = \frac{\pi}{6}$

optional { Since petal is symmetric, we can have θ go from 0 to $\frac{\pi}{6}$ and double area

$0 \le \theta \le \frac{\pi}{6}$

$0 \le r \le 7\cos 3\theta$

} bounds

NOTE $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$A = 2\int_0^{\pi/6}\int_0^{7\cos 3\theta} r\,dr\,d\theta$$

$$= \cancel{2}\int_0^{\pi/6} \left.\frac{r^2}{\cancel{2}}\right]_0^{7\cos 3\theta} d\theta$$

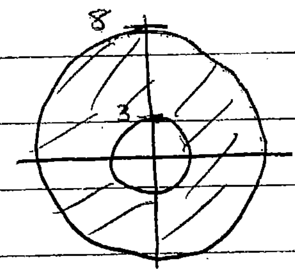
$$= \cancel{2}\int_0^{\pi/6} \frac{49\cos^2 3\theta}{\cancel{2}}\, d\theta$$

$$= \int_0^{\pi/6} 49\left(\frac{1+\cos 6\theta}{2}\right) d\theta$$

$$= \frac{49}{2}\left[\theta + \frac{\sin 6\theta}{6}\right]_0^{\pi/6}$$

$$= \frac{49}{2}\left[\frac{\pi}{6} + \frac{\cancel{\sin \pi}}{6} - 0\right] = \boxed{\frac{49\pi}{12}}$$

13) Avg height of $z = 6x^2 + 2y^2$ above $9 \le x^2 + y^2 \le 64$ in xy plane



$A = \pi(8)^2 - \pi(3)^2 = 64\pi - 9\pi$
$= 55\pi$

Avg height $= \frac{1}{55\pi} \iint_R (6x^2 + 2y^2)\, dA$

where, $dA = r\,dr\,d\theta$ $\quad R: \; 0 \le \theta \le 2\pi$
$3 \le r \le 8$

also $x = r\cos\theta$ & $y = r\sin\theta$

$\Rightarrow 6x^2 + 2y^2 = 2(3x^2 + y^2)$
$= 2(2x^2 + x^2 + y^2)$
$= 2[2r^2\cos^2\theta + r^2\cos^2\theta + r^2\sin^2\theta]$
$= 2[\cancel{2}r^2(\frac{1+\cos 2\theta}{\cancel{2}}) + r^2]$
$= 2r^2[2 + \cos 2\theta]$

Avg height $= \frac{1}{55\pi} \int_0^{2\pi}\int_3^8 2r^3(2 + \cos 2\theta)\, dr\, d\theta$

$= \frac{1}{55\pi} \int_0^{2\pi} \cancel{2}(2 + \cos 2\theta) \cdot \frac{1}{\cancel{4}_2} r^4 \Big]_3^8 d\theta$

$= \frac{1}{\cancel{110}_2\pi} \int_0^{2\pi} (2 + \cos 2\theta)\left(\frac{\cancel{8^4 - 3^4}}{\cancel{4015}_{73}}\right) d\theta$

$= \frac{73}{2\pi}\left[2\theta + \frac{\sin 2\theta}{2}\right]_0^{2\pi}$

$= \frac{73}{\cancel{2\pi}}(\cancel{4\pi}^{2} + 0 - (0+0)) = \boxed{146}$

dz, dy, dx order

14) V of tetra. cut from 1^{st} octant by plane $\frac{x}{9}+\frac{y}{4}+\frac{z}{3}=1$

NOTE: Since in 1^{st} octant, $x, y, z \geq 0$

Solve for z: $\frac{z}{3} = 1 - \frac{x}{9} - \frac{y}{4}$

$$z = 3\left(1-\frac{x}{9}-\frac{y}{4}\right)$$

bounds z: $\boxed{0 \leq z \leq 3\left(1-\frac{x}{9}-\frac{y}{4}\right)}$

solve for bounds for y: largest $y \to$ smallest z and smallest z can be is $0 \to$

$$\frac{x}{9}+\frac{y}{4}=1$$

$$\frac{y}{4} = 1-\frac{x}{9}$$

$$y = 4\left(1-\frac{x}{9}\right)$$

bounds y: $\boxed{0 \leq y \leq 4\left(1-\frac{x}{9}\right)}$

largest $x \to$ smallest z & $y \to$ smallest z & y can be is $0 \to$

$$\frac{x}{9}=1 \quad \to \quad x=9$$

bounds x: $\boxed{0 \leq x \leq 9}$

$$\boxed{V = \int_0^9 \int_0^{4(1-\frac{x}{9})} \int_0^{3(1-\frac{x}{9}-\frac{y}{4})} dz\, dy\, dx}$$

15

Note: This is harder than intended. I get a different answer.

15) Region bounded by

$1-\frac{x^2}{49}-\frac{y^2}{64}$ and xy plane. } Since bounded by xy plane, $\iint$ can be used.

$$\int_{-7}^{7}\int_{-\frac{8}{7}\sqrt{49-x^2}}^{\frac{8}{7}\sqrt{49-x^2}}\left(1-\frac{x^2}{49}-\frac{y^2}{64}\right)dy\,dx$$

$$=\int_{-7}^{7}\left[y-\frac{x^2y}{49}-\frac{y^3}{192}\right]_{-\frac{8}{7}\sqrt{49-x^2}}^{\frac{8}{7}\sqrt{49-x^2}}dx$$

Some steps skipped

$$=\int_{-7}^{7}\left[\frac{64\sqrt{49-x^2}}{21}-\frac{64x^2\sqrt{49-x^2}}{1029}\right]dx$$

$$=\frac{8\left[x(245-2x^2)\sqrt{49-x^2}+7203\sin^{-1}\left(\frac{x}{7}\right)\right]}{1029}\Bigg|_{-7}^{7}$$

$$=\boxed{56\pi}$$

NOTE on bounds:

bounded by xy plane $\rightarrow z\geq 0$

if $|y|>8$ or $|x|>7$ then $z<0$

$\Longrightarrow$

z: $0\leq z\leq 1-\frac{x^2}{49}-\frac{y^2}{64}$

y: $1-\frac{x^2}{49}-\frac{y^2}{64}\geq 0 \rightarrow \frac{y^2}{64}\leq 1-\frac{x^2}{49}$

$y^2\leq 64\left(1-\frac{x^2}{49}\right)$

$-\sqrt{64\left(1-\frac{x^2}{49}\right)}\leq y\leq\sqrt{64\left(1-\frac{x^2}{49}\right)} \rightarrow -\frac{8}{7}\sqrt{49-x^2}\leq y\leq\frac{8}{7}\sqrt{49-x^2}$

x: $1-\frac{x^2}{49}\geq 0 \rightarrow x^2\leq 49 \rightarrow -7\leq x\leq 7$

16) Center of mass of thin ⌓ w/ $\delta = 9$

bounded by x-axis & $y = \sqrt{196 - x^2}$

$\bar{x} = \frac{M_y}{M}$, $\bar{y} = \frac{M_x}{M}$ Bounds: $0 \le y \le \sqrt{196 - x^2}$

$-14 \le x \le 14$

$$M = \iint_R \delta \, dA = \int_{-14}^{14} \int_0^{\sqrt{196-x^2}} 9 \, dy \, dx$$

Used Int table # 45 w/ a = 14

$$= \int_{-14}^{14} 9\sqrt{196 - x^2} \, dx$$

$$= 9 \cdot \left[\frac{x}{2}\sqrt{196 - x^2} + \frac{196}{2} \sin^{-1} \frac{x}{14} \right]_{-14}^{14}$$

$$= \frac{9}{2} \left[14 \cdot 0 + \overset{196}{\cancel{98}} \sin^{-1} 1 - 0 - 196 \sin^{-1}(-1) \right]$$

$$= \frac{9}{2}\left(196 \cdot \frac{\pi}{2} - \left(\cancel{196} \; 196\left(-\frac{\pi}{2}\right)\right)\right)$$

$$= \frac{9}{2}(196\pi)$$

$$= 882\pi$$

Skipped work

$$M_y = \int_{-14}^{14} \int_0^{\sqrt{196-x^2}} 9x \, dy \, dx = -3(196 - x^2)^{3/2} \Big]_{-14}^{14} = 0$$

$$\bar{x} = \frac{M_y}{M} = \frac{0}{882\pi} = \boxed{0}$$

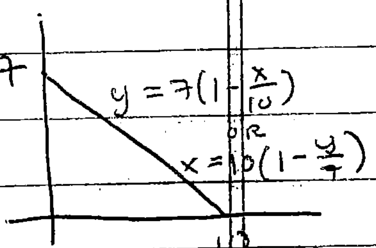
$$M_x = \int_{-14}^{14} \int_0^{\sqrt{196-x^2}} 9y \, dy \, dx = 882x - \frac{3x^3}{2} \Big]_{-14}^{14} = 16{,}464$$

$$\bar{y} = \frac{M_x}{M} = \frac{16{,}464}{882\pi} = \boxed{\frac{56}{3\pi}}$$

ctr: $\boxed{\left(0, \frac{56}{3\pi}\right)}$

17) $I_x = \iint y^2 \delta \, dA$, $\delta = 4$

$0 \le x \le 10(1-\frac{y}{7})$, $0 \le y \le 7$



$$I_x = \int_0^7 \int_0^{10(1-\frac{y}{7})} 4y^2 \, dx \, dy$$

$$= \int_0^7 4y^2 \left(10\left(1-\frac{y}{7}\right)\right) dy$$

$$= 40 \int_0^7 \left[y^2 - \frac{y^3}{7}\right] dy$$

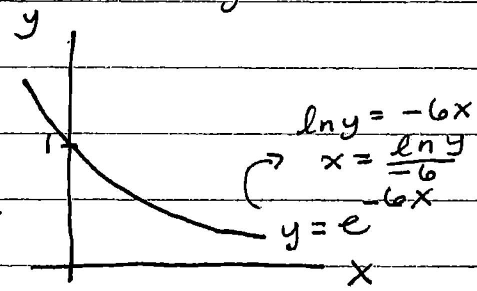
$$= 40 \left[\frac{y^3}{3} - \frac{y^4}{28}\right]_0^7$$

$$= 40 \left[\frac{343}{3} - \frac{\cancel{2401}^{343}}{\cancel{28}_{4}} - 0\right]$$

$$\cancel{40}^{\cancel{20}\,10} \cdot \frac{\cancel{1372 - 1029}^{343}}{\cancel{12}_{3}} = \boxed{\frac{3430}{3}}$$

18) Ctr of mass of thin infinite region in QI bounded by coordinate axes & $y=e^{-6x}$ if $\delta(x,y)=xy$

$$M=\iint_R \delta\, dA$$



$0\le x\le -\frac{\ln y}{6}$ OR $0\le y\le e^{-6x}$

$0<y\le 1$ $\quad$ $0\le x$

$$M=\int_0^1\int_0^{-\frac{\ln y}{6}} xy\,dx\,dy = \frac{1}{288}$$

$$M_y=\int_0^1\int_0^{-\frac{\ln y}{6}} xy\cdot x\,dx\,dy = \frac{1}{1728}$$

$$M_x=\int_0^1\int_0^{-\frac{\ln y}{6}} xy^2\,dx\,dy = \frac{1}{972}$$

$$\bar{x}=\frac{1/1728}{1/288}=\boxed{\frac{1}{6}}$$

$$\bar{y}=\frac{1/972}{1/288}=\boxed{\frac{8}{27}}$$

due to laziness, integrals we solved using mathematica

19) $I_z = ?$ $\quad \delta = 1$

Bounded by coordinate planes and

$$\frac{x}{9} + \frac{y}{10} + \frac{z}{7} = 1$$

$$I_z = \iiint (x^2+y^2)\,\delta\, dV$$

Bounds:

$0 \le z \le 7\left(1 - \frac{x}{9} - \frac{y}{10}\right)$, $0 \le y \le 10\left(1 - \frac{x}{9}\right)$, $0 \le x \le 9$

$$I_z = \int_0^9 \int_0^{10(1-\frac{x}{9})} \int_0^{7(1-\frac{x}{9}-\frac{y}{10})} (x^2+y^2)\, dz\, dy\, dx$$

$$= \int_0^9 \int_0^{10(1-\frac{x}{9})} 7\left(1 - \frac{x}{9} - \frac{y}{10}\right)(x^2+y^2)\, dy\, dx$$

Steps skipped

$$= \int_0^9 \frac{35(x-9)^2(4050 - 900x + 293x^2)}{19{,}683}\, dx$$

$$= \boxed{\frac{3801}{2}}$$

20) Mass of tetra. w/ density $\delta = x+y+z$
bounded by coordinate plane &

$$\frac{x}{8} + \frac{y}{10} + \frac{z}{7} = 1$$

bounds: $0 \le z \le 7\left(1-\frac{x}{8}-\frac{y}{10}\right)$, $0 \le y \le 10\left(1-\frac{x}{8}\right)$,
$0 \le x \le 8$

$$\text{Mass} = \int_0^8 \int_0^{10(1-\frac{x}{8})} \int_0^{7(1-\frac{x}{8}-\frac{y}{10})} (x+y+z)\,dz\,dy\,dx$$

$$= \int_0^8 \int_0^{10(1-\frac{x}{8})} \left[\frac{49}{2} + \frac{7x}{8} - \frac{63x^2}{128} + \frac{21y}{10} - \frac{77xy}{80} - \frac{91y^2}{200}\right] dy\,dx$$

steps skipped

$$= \int_0^8 \left[\frac{595}{3} - \frac{315x}{8} + \frac{35x^2}{64} + \frac{245x^3}{1536}\right] dx$$

$$= \boxed{\frac{1750}{3}}$$

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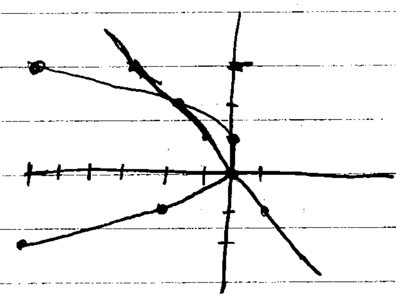
\# 11, 13, 23

15.6 Homework

(11) Find I_x of a thin plate bounded by the parabola $X = y - y^2$, $X + y = 0$ if $\delta(x,y) = X + Y$

$$I_x = \iint y^2 \delta \, dA$$



X	$y - y^2$
0	0
0	1
-2	2
-6	3
-2	-1
-6	-2

$$I_x = \int_0^{-2} \int_{-y}^{y-y^2} (y^2 x + y^3) \, dx \, dy$$

$$I_x = \int_0^{-2} \left[\tfrac{1}{2} y^2 x^2 + y^3 x\right]_{-y}^{y-y^2} dy$$

$$I_x = \int_0^{-2} \left[\tfrac{1}{2} y^2 (y - y^2)^2 + y^3 (y - y^2)\right] dy - \int_0^{-2} \left[\tfrac{1}{2} y^2 (-y)^2 + y^3 (-y)\right] dy$$

$$I_x = \int_0^{-2} \left[\tfrac{1}{2} y^2 (y^2 - 2y^3 + y^4) + y^4 - y^5\right] dy - \int_0^{-2} \left[\tfrac{1}{2} y^4 - y^4\right] dy$$

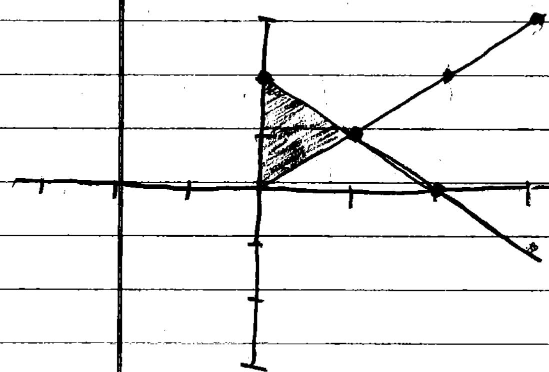
$$I_x = \int_0^{-2} \left[\frac{y^4 - 2y^5 + y^6}{2} + y^4 - y^5 - \frac{y^4}{2} + y^4\right]$$

$$I_x = \int_0^{-2} \frac{y^4}{2} - 2y^5 + \frac{y^6}{2} + \frac{3}{2} y^4 = \int_0^{-2} y^6/2 - 2y^5 + 2y^4$$

$$I_x = \left[\frac{y^7}{14} - \frac{y^6}{3} + \frac{2y^5}{5}\right]_0^{-2} = -\frac{128}{4} - \frac{64}{3} - \frac{64}{5}$$

$(4-4x+x^2)(2-x) = (8-8x+2x^2-4x+4x^2+x^3)$
$= -x^3+6x^2-12x+8$

(13) find Center of Mass bounded by
y-axis, $y=x$ &, $y=2-x$, if $\delta = 6x+3y+3$



$$m = \int_0^1 \int_x^{2-x} 6x+3y+3 \, dy\,dx$$

$$m = \int_0^1 \left[6xy + \frac{3}{2}y^2 + 3y\right]_x^{2-x} dx$$

$$m = \int_0^1 \left[6x(2-x) + \frac{3}{2}(4-4x+x^2) - 3x + 6 - 6x^2 - \frac{3}{2}x^2 - 3x\right] dx$$

$$m = \int_0^1 \cancel{12x} - 6x^2 + 6 - \cancel{6x} + \cancel{\frac{3}{2}x^2} - \cancel{3x} + 6 - 6x^2 - \cancel{\frac{3}{2}x^2} - \cancel{3x} \,] dx$$

$$m = \int_0^1 -12x^2 + 12 \, dx \longrightarrow -12\int_0^1 x^2 - 1 \, dx \longrightarrow -12\left[\frac{1}{3}x^3 - x\right]_0^1$$

$$m = -12\left[\frac{1}{3} - 1\right] = +\frac{24}{3} = \boxed{8} = m$$

$$m_x = \int_0^1 \int_x^{2-x} 6xy + 3y^2 + 3y \, dy\,dx = \int_0^1 \left[3xy^2 + y^3 + \frac{3}{2}y^2\right]_x^{2-x}$$

$$m_x = \int_0^1 \cancel{12x} - 12x^2 + 3x^3 - x^3 + \cancel{6x} - \cancel{12x} + 8 + \frac{12 - 12x + 3x^2}{2} - 3x^3 - x^3 - \frac{3}{2}x^2$$

$$m_x \int_0^1 -6x^2 + 2x^3 + 8 + \frac{3}{2}x^2 - 6x + 6 \longrightarrow \int_0^1 +2x^3 - \frac{9}{2}x^2 - 6x + 14 \, dx$$

$$m_x = \int_0^1 -2x^3 - 6x^2 - 6x + 14 \, dx \longrightarrow$$

$$m_x = \left[-\frac{1}{2}x^4 - 2x^3 - 3x^2 + 14x\right]_0^1 = \boxed{\frac{17}{2}}\ m_x$$

$$m_y = \int_0^1\int_x^{2-x} 6x^2 + 3yx + 3x\,dy\,dx = \int_0^1 \left[6x^2y + \frac{3}{2}y^2x + 3xy\right]_x^{2-x} dx$$

$$m_y = \int_0^1 6x^2(2-x) + \frac{3}{2}(x^2-4x+4)x + 3x(2-x) - 6x^3 - \frac{3}{2}x^3 - 3x^2\,dx$$

$$m_y = \int_0^1 -6x^3 + \cancel{12x^2} + \cancel{\frac{3}{2}x^3} - \cancel{6x^2} + 6x - 3x^2 + 6x - 6x^3 - \cancel{\frac{3}{2}x^3} - \cancel{3x^2}\,dx$$

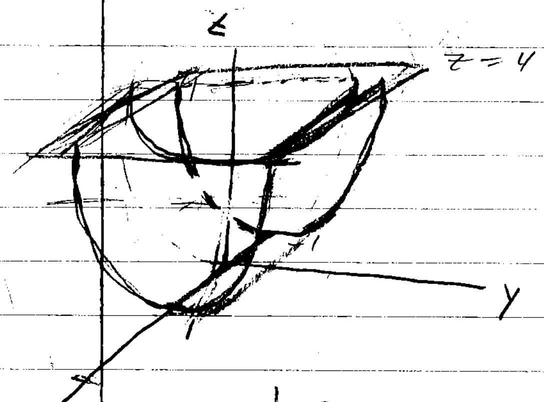
$$m_y = \int_0^1 -12x^3 + 12x\,dx = \left[-3x^4 + 6x^2\right]_0^1 = \boxed{m_y = 3}$$

$$\bar{X} = \frac{3}{8} \qquad \bar{Y} = \frac{\frac{17}{2}}{8} = \frac{17}{2}\cdot\frac{1}{8} = \frac{17}{16}$$

$$\boxed{\bar{X} = \frac{3}{8} \qquad \bar{Y} = \frac{17}{16}}$$

(23) $z = 4y^2, \quad z = 4, \quad x = 1, \quad x = -1$

Find CoM & Inertia of 3 axes $\quad \delta = 1$



$$m = \int_{-1}^{1}\int_{0}^{0}\int_{4y^2}^{4} \delta \, dz\, dy\, dx$$

$$m = \int_{-1}^{1}\int_{0}^{0} z\Big]_{4y^2}^{4} dy\, dx$$

$$m = \int_{-1}^{1}\int_{0}^{0} 4 - 4y^2 \, dy\, dx \longrightarrow \int_{-1}^{1} 4y - \frac{4}{3}y^3\Big]_0^0 dx = 0$$

$$I_x = \int_{0}^{0}\int_{-1}^{1}\int_{4y^2}^{4} (y^2 + z^2)\, dz\, dx\, dy$$

$$I_x = \int_{0}^{0}\int_{-1}^{1} \left(y^2 z + \frac{1}{3}z^3\right)_{4y^2}^{4} \rightarrow \int_0\int_{-1}^{1} 4y^2 + 64 - 4y^4 - \frac{64y^6}{3} \, dy$$

$$I_x = \int_0^0 4y^2 x + 64x - 4y^4 x - \frac{64}{3}y^6 x\Big]_{-1}^{1} =$$

$$I_x = \int_0^0 4y^2 + 64 - 4y^4 - \frac{64}{3}y^6$$

$$I_x = \left[\frac{4}{3}y^3 + 64y - \frac{4}{5}y^5 - \frac{64}{21}y^7\right]_0^0$$

$$\underline{I_x = 0}$$

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15.5 Triple Integrals Homework

(1) $\int_0^1\int_0^{1-x}\int_{x+z}^1 dy\,dz\,dx = \int_0^1\int_0^{1-x}\left[y\right]_{x+z}^1 dz\,dx$

$\int_0^1\int_0^{1-x}[1-x-z]\,dz\,dx \rightarrow \int_0^1\left[z-xz-\frac{1}{2}z^2\right]_0^{1-x} dx$

$\int_0^1(1-x)dx - \int_0^1(x-x^2)dx - \frac{1}{2}\int_0^1(1-2x+x^2)dx$

$\left[x-\frac{1}{2}x^2\right]_0^1 - \left[\frac{1}{2}x^2-\frac{1}{3}x^3\right]_0^1 - \frac{1}{2}\left[x-x^2+\frac{1}{3}x^3\right]_0^1$

$1-\frac{1}{2}-\frac{1}{2}+\frac{1}{3}-\frac{1}{2}+\frac{1}{2}-\frac{1}{6} \rightarrow \boxed{V = 1/6}$

(7) $\int_0^1\int_0^1\int_0^1(x^2+y^2+z^2)\,dz\,dy\,dx$

$\int_0^1\int_0^1\left[x^2z+y^2z+\frac{1}{3}z^3\right]_0^1 dy\,dx \rightarrow \int_0^1\int_0^1(x^2+y^2+\frac{1}{3})dy\,dx$

$\int_0^1\left[x^2y+\frac{1}{3}y^3+\frac{1}{3}y\right]_0^1 dx \rightarrow \int_0^1(x^2+\frac{1}{3}+\frac{1}{3})dx$

$\left[\frac{1}{3}x^3+\frac{1}{3}x+\frac{1}{3}x\right]_0^1 = \frac{1}{3}+\frac{1}{3}+\frac{1}{3}$

$= \boxed{1}$

11) $$\int_0^{\pi/6}\int_0^1\int_{-2}^3 y\sin z\,dx\,dy\,dz \rightarrow \int_0^{\pi/6}\int_0^1 \left[xy\sin z\right]_{-2}^3 dy\,dz$$

$$\int_0^{\pi/6}\int_0^1 (3y\sin z + 2y\sin z)\,dy\,dz$$

$$\int_0^{\pi/6}\left[\frac{3}{2}y^2\sin z + y^2\sin z\right]_0^1 = \int_0^{\pi/6}\left(\frac{3}{2}\sin z + \sin z\right)dz$$

$$\int_0^{\pi/6}\frac{5}{2}\sin z\,dz = \left[-\frac{5}{2}\cos z\right]_0^{\pi/6} = -\frac{5}{2}\left(\frac{\sqrt{3}}{2}\right)+\frac{5}{2}(1) = \boxed{.335}$$

15) $$\int_0^1\int_0^{2-x}\int_0^{2-x-y} dz\,dy\,dx = \int_0^1\int_0^{2-x}\left[z\right]_0^{2-x-y} dy\,dx$$

$$\int_0^1\int_0^{2-x}(2-x-y)\,dy\,dx \rightarrow \int_0^1\left[2y-xy-\frac{1}{2}y^2\right]_0^{2-x} dx$$

$$\int_0^1\left[4-2x-2x+x^2-\frac{1}{2}(4-4x+x^2)\right]dx$$

$$\int_0^1\left[4-4x+x^2-2+2x-x^2\right]dx \rightarrow \int_0^1\left[2-2x\right]dx$$

$$\left[2x-x^2\right]_0^1 = 2-1 = \boxed{1}$$

(19) $\int_0^{\pi/4}\int_0^{\ln(\sec v)}\int_{-\infty}^{2t} e^x \, dx\, dt\, dv$

$\int_0^{\pi/4}\int_0^{\ln(\sec v)} \left[e^x\right]_{-\infty}^{2t} dt\, dv \longrightarrow \int_0^{\pi/4}\int_0^{\ln \sec v}\left[e^{2t}-0\right]dt\,dv$

$\int_0^{\pi/4}\left[\frac{1}{2}e^{2t}\right]_0^{\ln \sec v} dv \longrightarrow \int_0^{\pi/4}\left[\frac{1}{2}e^{2\ln \sec v}-\frac{1}{2}\right]dv$

$\int_0^{\pi/4}\frac{1}{2}\sec^2 v\,dv - \int_0^{\pi/4}\frac{1}{2}\,dv = \left[\frac{1}{2}\tan v\right]_0^{\pi/4} - \left[\frac{1}{2}v\right]_0^{\pi/4}$

$\boxed{\frac{1}{2}-\frac{\pi}{8}}$

(23) $z = y^2$, $0 \le x \le 1$, $-1 \le y \le 1$,
$0 \le z \le 1$

$\int_0^1\int_{-1}^1\int_0^{y^2} dz\,dy\,dx \longrightarrow \int_0^1\int_{-1}^1\left[z\right]_0^{y^2}dy\,dx \longrightarrow \int_0^1\int_{-1}^1 y^2\,dy\,dx$

$\int_0^1\left[\frac{1}{3}y^3\right]_{-1}^1 dx \longrightarrow \int_0^1\left[\frac{1}{3}+\frac{1}{3}\right]dx \longrightarrow \int_0^1\left[\frac{2}{3}\right]dx$

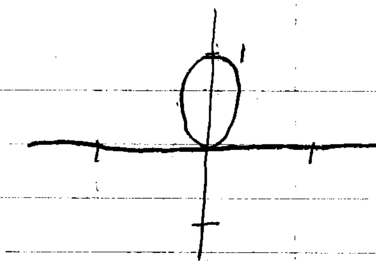
$= \left[\frac{2}{3}x\right]_0^1 \longrightarrow \boxed{V=\frac{2}{3}}$

"How to change

?

θ

(25) $\int_0^{\pi/4}\int_0^{2\sec\theta} r^5 \sin^2\theta \, dr\, d\theta$



$0 \le r \le 2\sec\theta$

$0 \le r \le \frac{2}{\cos\theta}$

$0 \le r \le$

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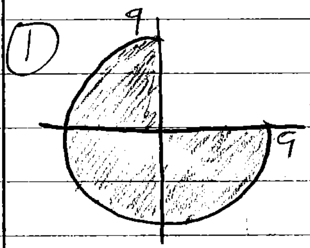
\# 1, 5, 11, 29

15.4 Homework

(1)

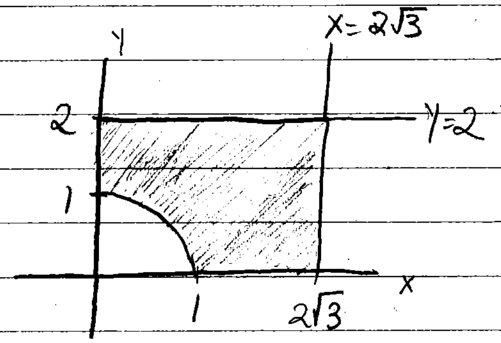


$0 \leq r \leq 3$

$\pi/2 \leq \theta \leq 2\pi$

$$A = \int_{\pi/2}^{2\pi}\int_0^3 r\,dr\,d\theta$$

(5)



$0 \leq x \leq 2\sqrt{3}$

$\sqrt{1-x^2} \leq y \leq 2$

$0 \leq y \leq 2$

$x = 2\sqrt{3}$

$r\cos\theta = 2\sqrt{3}$

$r = 2\sqrt{3}\sec\theta$

$y = 2$

$r\sin\theta = 2$

$r = 2\csc\theta$

$$\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$$

$$\boxed{\begin{array}{lcl} 1 \leq r \leq 2\sqrt{3}\sec\theta & \& & 1 \leq r \leq 2\csc\theta \\ 0 \leq \theta \leq \pi/6 & & \pi/6 \leq \theta \leq \pi/2 \end{array}}$$

//

(11) $\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2+y^2)\,dx\,dy$

$x^2+y^2=r^2$

$0 \le x \le \sqrt{4-y^2} \rightarrow x^2+y^2=4$

$r=2$

$0 \le y \le 2$

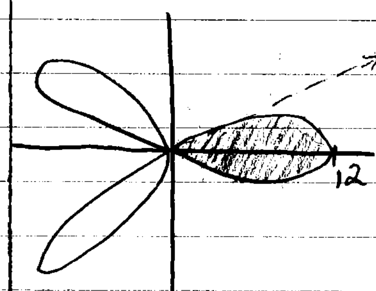
$0 \le r \le 2$

$0 \le \theta \le \pi/2$

$\int_0^{\pi/2}\int_0^2 r^2 \cdot r\,dr\,d\theta$

$\int_0^{\pi/2} \left[\frac{1}{4}r^4\right]_0^2 d\theta \longrightarrow \int_0^{\pi/2} 4\,d\theta \longrightarrow \left[4\theta\right]_0^{\pi/2} = \boxed{2\pi}$

(29) Area of one leaf $r = 12\cos 3\theta$



$0 \le r \le 12\cos 3\theta$

$-\pi/2 \le \theta \le \pi/2$

$\int_{-\pi/2}^{\pi/2}\int_0^{12\cos 3\theta} r\,dr\,d\theta \longrightarrow \int_{-\pi/2}^{\pi/2}\left[\frac{1}{2}r^2\right]_0^{12\cos 3\theta} d\theta$

$\int_{-\pi/2}^{\pi/2} \frac{1}{2}(12\cos 3\theta)^2 \longrightarrow \int_{-\pi/2}^{\pi/2} \frac{1}{2}144\cos^2 3\theta \longrightarrow \int_{-\pi/2}^{\pi/2} 72\cos^2 3\theta\,d\theta$

$72\int_{-\pi/2}^{\pi/2} \frac{1}{2}(1+\cos 6\theta)\,d\theta \longrightarrow 36\left[\theta + \frac{1}{6}\sin 6\theta\right]_{-\pi/2}^{\pi/2}$

$36\left\{\left[\frac{\pi}{2}+0\right]-\left[-\frac{\pi}{2}+0\right]\right\} \longrightarrow 36(\pi) = \boxed{36\pi}$

for θ

$12\cos 3\theta = 0$

$\cos 3\theta = 0$

$3\theta = \cos^{-1}(0) \longrightarrow 3\theta = \frac{\pi}{2} \longrightarrow \boxed{\theta = \frac{\pi}{6}}$

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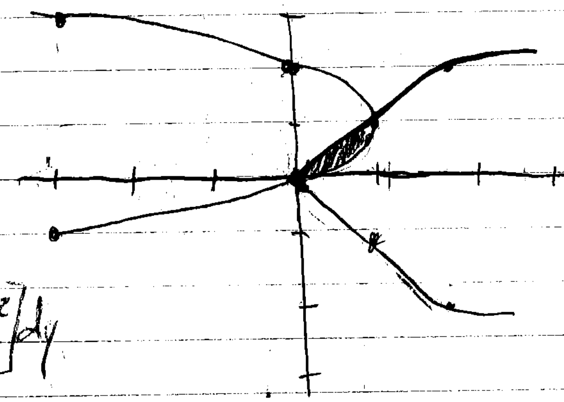
7, 13, 21

15.3 Homework

$A = \iint_R dA \qquad \text{Average} = \frac{1}{\text{area}\, R} \iint_R f\, dA$

? Sketch

(7) $x = y^2, \quad x = 2y - y^2$

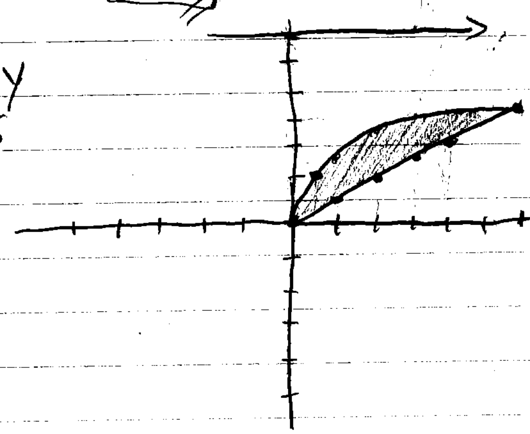


$$\int_0^1 \int_{y^2}^{2y-y^2} dx\, dy$$

$$A = \int_0^1 \left[x\right]_{y^2}^{2y-y^2} dy \longrightarrow \int_0^1 \left[2y - y^2 - y^2\right] dy$$

$$A = \int_0^1 (2y - 2y^2) \longrightarrow 2\left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_0^1 = \boxed{\frac{1}{3}}$$

(13) $\displaystyle\int_0^6 \int_{y^2/3}^{2y} dx\, dy \qquad \begin{matrix} y^2/3 \le x \le 2y \\ 0 \le y \le 6 \end{matrix}$



$$\int_0^6 \left[x\right]_{2y}^{y^2/3} dy \longrightarrow \int_0^6 \frac{y^2}{3} - 2y$$

$$\left[\frac{1}{9}y^3 - y^2\right]_0^6 \longrightarrow \boxed{12}$$

21) Average height of $Z = x^2 + y^2$
over square $0 \leq x \leq 2$, $0 \leq y \leq 2$

$$A = \int_0^2 \int_0^2 dx\,dy = \boxed{4}$$

$$\text{Average} = \frac{1}{4} \int_0^2 \int_0^2 (x^2 + y^2)\,dx\,dy$$

$$= \frac{1}{4} \int_0^2 \left[\frac{1}{3}x^3 + y^2 x\right]_0^2 dy \rightarrow \frac{1}{4} \int_0^2 \left(\frac{8}{3} + 2y^2\right) dy$$

$$\frac{1}{4}\left[\frac{8}{3}y + \frac{2}{3}y^3\right]_0^2 = \frac{1}{4}\left[\frac{8}{3}(2) + \frac{(2)(2)^3}{3}\right]$$

$$\text{Average} = \frac{1}{4}\left[\frac{16}{3} + \frac{16}{3}\right] =$$

$$\text{Average} = \frac{1}{4}\left[\frac{32}{3}\right]$$

$$\boxed{\text{Average} = \frac{8}{3}}$$

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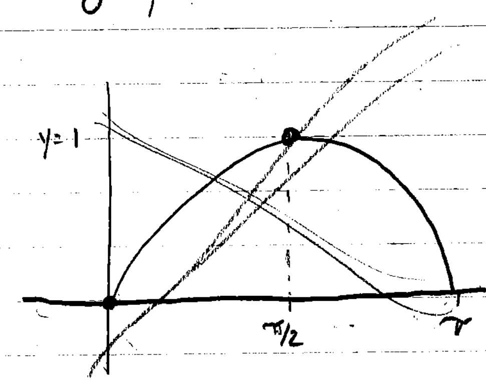
#9, 13, 19, 23, 33, 41, 57, 47

15.2 Homework

(9) $\int_0^2\int_{x^3}^8 dy\,dx$ b) $\int_0^8\int_0^{y^{1/3}} dx\,dy$

(13) $\int_0^9\int_0^{\sqrt{x}} dy\,dx$ b) $\int_0^3\int_{y^2}^9 dx\,dy$

(19) $\int_0^\pi\int_0^x x\sin y\,dy\,dx$



$$\int_0^\pi\left[-x\cos y\right]_0^x dx \longrightarrow \int_0^\pi\left[-x\cos x + x\cos 0\right]dx$$

f	$\int$
$-x$ +	$\cos x$
-1	$\sin x$
0	$-\cos x$

$$\int_0^\pi\left[-x\cos x + x\right]dx \longrightarrow \left[\left(-x\sin x + \cos x\right) + \frac{1}{2}x^2\right]_0^\pi$$

$$\left[-\pi\sin\pi + \cos\pi + \frac{1}{2}\pi^2\right] - \left[-0\sin 0 + \cos 0 + \frac{1}{2}(0)^2\right]$$

$$\left[0 - 1 + \frac{1}{2}\pi^2\right] - 1 \longrightarrow \boxed{-2 + \frac{1}{2}\pi^2}$$

(23) $\int_0^1\int_0^{y^2} 3y^3 e^{xy}\,dx\,dy \rightarrow \int_0^1 \left[3y^2 e^{xy}\right]_0^{y^2} dy$

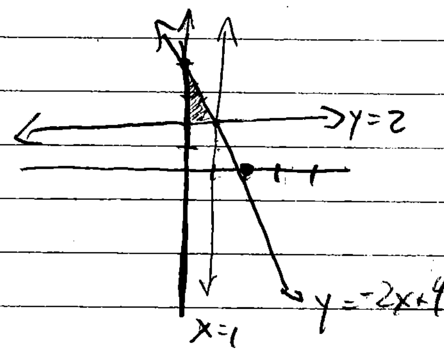
$e^{y^3} = 3y^2 e^{y^3}$

$\int_0^1 \left[3y^2 e^{y^3} - 3y^2\right] dy \rightarrow \left[e^{y^3} - y^3\right]_0^1$

$\left[e^1 - 1\right] - \left[e^0 - 0\right] \rightarrow \boxed{e - 2}$

(33) $\int_0^1\int_2^{4-2x} dy\,dx$

$2 \leq y \leq 4 - 2x$
$0 \leq x \leq 1$



vertical cross section

$0 \leq x \leq -y/2 + 2$
$2 \leq y \leq 4$

$\boxed{\int_2^4\int_0^{-y/2+2} dx\,dy}$

$-2x = y - 4$
$x = -\frac{y+4}{2}$
$x = -\frac{y}{2} + 2$

$x^2+y^2=1$

(41) $\displaystyle\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dx$

$-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}$

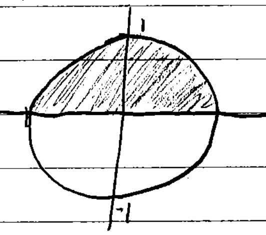
$0 \le y \le 1$

Vert cross sections

$0 \le y \le \sqrt{1-x^2}$

$-1 \le x \le 1$

$$\boxed{\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 3y\,dy\,dx}$$

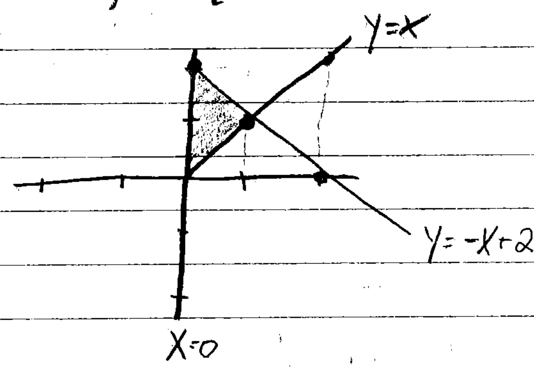


(57) $z = x^2+y^2, \quad y=x, \; x=0, \; x+y=2$

Vert cross sections

$-x+2 \le y \le x$

$0 \le x \le 1$



$$\int_0^1\int_{-x+2}^{x} (x^2+y)\,dy\,dx$$

$$\int_0^1 \left[x^2y+\frac{1}{2}y^2\right]_{-x+2}^{x} dx \longrightarrow \int_0 \left[x^3+\frac{1}{2}x^2-\left(-x^3+2x^2-\frac{1}{2}(x^2-4x+4)\right)\right]dx$$

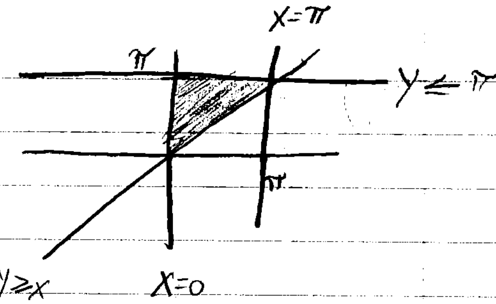
$$\int_0^1 \left[x^3+\frac{1}{2}x^2-\left(-x^3+2x^2-\frac{1}{2}x^2+2x-2\right)\right] \longrightarrow \int_0 \left[x^3+\frac{1}{2}x^2+x^3-2x^2+\frac{1}{2}x^2-2x+2\right]dx$$

$$\int_0^1 \left[2x^3-x^2-2x+2\right]dx \longrightarrow \left[\frac{1}{2}x^4-\frac{1}{3}x^3-x^2+2x\right]_0^1$$

$$= \frac{1}{2}-\frac{1}{3}-1+2 = \boxed{7/6}$$

$y \geq x$

(47) $\displaystyle\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dy\, dx$



$x \leq y \leq \pi$

$0 \leq x \leq \pi$

horizontal cross section

$0 \leq x \leq y$

$0 \leq y \leq \pi$

$\displaystyle\int_0^{\pi}\int_0^{y} \frac{\sin y}{y}\, dx\, dy$

$\displaystyle\int_0^{\pi}\left[\frac{\sin y}{y} x\right]_0^y dy \longrightarrow \int_0^{\pi}\left[\frac{\cancel{y}\sin y}{\cancel{y}} - \cancel{\frac{0\sin y}{y}}^{\,0}\right] dy$

$\displaystyle\int_0^{\pi} \sin y\, dy \longrightarrow \left[-\cos y\right]_0^{\pi}$

$-\cos\pi + \cos 0 \longrightarrow +1 + 1 = \boxed{2}$

Steven Romeiro

#1,9,25,15

15.1 Homework

(1) $\int_1^2\int_0^4 2xy\,dy\,dx \longrightarrow \int_1^2 \left[2x\cdot\frac{1}{2}y^2\right]_0^4 = \int_1^2 \left[xy^2\right]_0^4 dx$

$\int_1^2 \left[x4^2 - x(0)^2\right]dx \longrightarrow \int_1^2 16x\,dx \longrightarrow \left[\frac{16}{2}x^2\right]_1^2$

$= 8(2)^2 - 8(1)^2 \longrightarrow \boxed{24}$

(9) $\int_0^{\ln 2}\int_1^{\ln 5} e^{2x+y}\,dy\,dx \longrightarrow \int_0^{\ln 2}\left[e^{2x+y}\cdot 1\right]_1^{\ln 5} dx$

$\int_0^{\ln 2}\left[e^{2x+\ln 5} - e^{2x+1}\right]dx \longrightarrow \int_0^{\ln 2} e^{2x+\ln 5}\,dx - \int_0^{\ln 2} e^{2x+1}\,dx$

$\int_0^{\ln 2}(e^{2x}\cdot e^{\ln 5})dx - \int_0^{\ln 2}(e^{2x}\cdot e^1)dx \longrightarrow 5\int_0^{\ln 2} e^{2x}dx - e^1\int_0^{\ln 2} e^{2x}dx$

$5\left[2e^{2x}\right]_0^{\ln 2} - e^1\left[2e^{2x}\right]_0^{\ln 2} \longrightarrow 5\left[2(4)-2\right] - e^1\left[2(4)-2\right]$

$= 5(6) - e^1(6) \longrightarrow \boxed{35 - 6e}$

(25) Region of bounded above by $z = x^2 + y^2$ & below by $R: -1 \leq x \leq 1, -1 \leq y \leq 1$. Find Volume.

$$\iint_R (x^2+y^2)\,dA \longrightarrow \int_{-1}^{1}\int_{-1}^{1}(x^2+y^2)\,dy\,dx$$

$$\int_{-1}^{1}\left[x^2y + \frac{1}{3}y^3\right]_{-1}^{1} dx \rightarrow \int_{-1}^{1}\left[x^2 + \frac{1}{3} + x^2 + \frac{1}{3}\right]dx$$

$$\int_{-1}^{1}\left[2x^2 + \frac{2}{3}\right]dx \longrightarrow \left[\frac{2}{3}x^3 + \frac{2}{3}x\right]_{-1}^{1} \longrightarrow \frac{4}{3} + \frac{4}{3} \rightarrow \boxed{\frac{8}{3}}$$

(15) $$\iint_R (6y^2 - 2x)\,dA \longrightarrow \int_0^1\int_0^2 (6y^2 - 2x)\,dy\,dx$$

$$\int_0^1 \left[2y^3 - 2xy\right]_0^2 dx \longrightarrow \int_0^1 \left[16 - 4x\right]dx$$

$$\left[16x - 2x^2\right]_0^1 = 16 - 2 = \boxed{14}$$

MAC2313 - Test 2 - Take Home - Spring 2016

101

Name Steven Romeiro ☺ +1
Instructor: Wendy Pogoda

*Directions: This is a take home test. While I realize that you may work together to understand the concepts, the final work must be your own (i.e. do not have someone else take the test for you). **Please show your work** for all questions to receive credit. If you read these directions, put a smiley face next to your name for one bonus point. Test and HW is due on 3/23, but please try to complete most of the test by 3/21 if possible, so you don't fall behind in the class. This test covers Chapter 13 & 14.*

The position vector of a particle is r(t). Find the requested vector.

1) The velocity at t = 3 for $\mathbf{r}(t) = (9 - 1t^2)\mathbf{i} + (8t + 9)\mathbf{j} - e^{-6t}\mathbf{k}$ 1) C

A) $\mathbf{v}(3) = 6\mathbf{i} + 8\mathbf{j} + 6e^{-18}\mathbf{k}$
B) $\mathbf{v}(3) = -6\mathbf{i} + 8\mathbf{j} - 6e^{-18}\mathbf{k}$
C) $\mathbf{v}(3) = -6\mathbf{i} + 8\mathbf{j} + 6e^{-18}\mathbf{k}$
D) $\mathbf{v}(3) = -3\mathbf{i} + 8\mathbf{j} + 6e^{-18}\mathbf{k}$

$\frac{dr}{dt} = -2t\,i + 8j + 6e^{-6t}k$ @ $t=3$

$v'(t) = -2(3)i + 8j + 6e^{-6(3)} = \boxed{-6i + 8j + 6e^{-18}k}$

Solve the initial value problem.

2) Differential Equation: $\frac{d\mathbf{r}}{dt} = (\sec^2 t)\mathbf{i} + (4t^3 + 3)\mathbf{j}$ 2) C

Initial Condition: $\mathbf{r}(0) = -4\mathbf{j}$

A) $\mathbf{r}(t) = (\tan t)\mathbf{i} + (t^4)\mathbf{j}$
B) $\mathbf{r}(t) = (-\tan t)\mathbf{i} + (12t^2 - 4)\mathbf{j}$
C) $\mathbf{r}(t) = (\tan t)\mathbf{i} + (t^4 + 3t - 4)\mathbf{j}$
D) $\mathbf{r}(t) = (\tan t)\mathbf{i} + (t^4 - 4)\mathbf{j}$

$\int \frac{dr}{dt} = \int \sec^2 t\, dt + \int 4t^3 + \int 3$

$r(t) = \tan t\, i + (t^4 + 3t)j$

$r(0) = \tan t\, i + (t^4 + 3t - 4)j$

$r(0) = 0i + (0^4 + 3(0) - 4)j$

$r(0) = -4j$

$\boxed{r(t) = \tan t\,\vec{i} + (t^4 + 3t - 4)\vec{j}}$

Solve the problem. Unless stated otherwise, assume that the projectile flight is ideal, that the launch angle is measured from the horizontal, and that the projectile is launched from the origin over a horizontal surface

3) An athlete puts a 16-lb shot at an angle of 43° to the horizontal from 6.2 ft above the ground at an initial speed of 42 ft/sec. How far forward does the shot travel before it hits the ground? Round your answer to the nearest tenth. 3) A

A) 61 ft
B) 186 ft
C) 2 ft
D) 6 ft

$X = X_0 + V_0(\cos\alpha)t$

$X = 0 + 42(\cos 43)t$

$Y = Y_0 + V_0(\sin\alpha)t - \frac{1}{2}gt^2$

$Y = 6.4 + 42(\sin 43)t - \frac{1}{2}(32)t^2$

$Y = 6.4 + 28.644t - 16t^2$

$t = \frac{-28.644 \pm \sqrt{28.644^2 - (4)(-16)(6.4)}}{-32}$

$t = \frac{-28.644 - 35.672}{-32}$

$t = 1.99$

$X = 42(\cos 43)(1.99)$

$\boxed{X = 61.162\text{ ft}}$

Calculate the arc length of the indicated portion of the curve r(t).

4) $\mathbf{r}(t) = (5 - 6t)\mathbf{i} + (4 + 9t)\mathbf{j} + (2t - 6)\mathbf{k}$, $-6 \le t \le 10$

A) 44 B) 1936 C) 176 D) 484

4) C

$L = \int_a^b \left|\frac{dr}{dt}\right|$

$\frac{dr}{dt} = -6i + 9j + 2k$

$\left|\frac{dr}{dt}\right| = \sqrt{36+81+4} = \sqrt{121}$

$\left|\frac{dr}{dt}\right| = 11$

$L = \int_{-6}^{10} 11\,dt$

$L = [11t]_{-6}^{10}$

$L = [11(10)] - [11(-6)]$

$L = 110 + 66$

$\boxed{L = 176}$

Find the curvature of the space curve.

5) $\mathbf{r}(t) = 12t\mathbf{i} + \left(10 + 4\cos\frac{5}{4}t\right)\mathbf{j} + \left(9 + 4\sin\frac{5}{4}t\right)\mathbf{k}$

A) $\kappa = \frac{5}{169}$ B) $\kappa = \frac{25}{52}$ C) $\kappa = \frac{5}{13}$ D) $\kappa = \frac{25}{676}$

5) D

$K = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$

$\vec{v} = \frac{dr}{dt} = 12i + \left(-5\sin\left(\frac{5}{4}t\right)\right)j + \left(5\cos\left(\frac{5}{4}\right)t\right)k$

$|\vec{v}| = \sqrt{12^2 + (5\sin(\frac{5}{4}))^2 + (5\cos(\frac{5}{4}))^2}$

$|\vec{v}| = \sqrt{144 + 25\sin^2(\frac{5}{4}) + 25\cos^2(\frac{5}{4})}$

$|\vec{v}| = \sqrt{144 + 25(\sin^2(\frac{5}{4}) + \cos^2(\frac{5}{4})}$

$|\vec{v}| = \sqrt{144+25} \longrightarrow \sqrt{169} \longrightarrow 13$

$\vec{T} = \frac{12i - 5\sin(\frac{5}{4})tj + 5\cos(\frac{5}{4})tk}{13}$

$\frac{d\vec{T}}{dt} = \frac{12}{13}i - \frac{5}{13}\sin\left(\frac{5}{4}\right)t_j + \frac{5}{13}\cos\left(\frac{5}{4}\right)tk$

$\frac{d\vec{T}}{dt} = 0i - \frac{25}{52}\cos\left(\frac{5}{4}\right)j - \frac{25}{52}\sin\left(\frac{5}{4}\right)k$

$\frac{d\vec{T}}{dt} = -\frac{25}{52}\cos\left(\frac{5}{4}\right)j - \frac{25}{52}\sin\left(\frac{5}{4}\right)k$

$\left|\frac{d\vec{T}}{dt}\right| = \sqrt{\frac{625}{2704}\left(\cos^2\left(\frac{5}{4}\right) + \sin^2\left(\frac{5}{4}\right)\right)}$

$\left|\frac{d\vec{T}}{dt}\right| = \frac{25}{52}$

$K = \frac{1}{13}\left(\frac{25}{52}\right)$

$\boxed{K = \frac{25}{676}}$

1-0

Answer Key
Testname: CALC3_TEST2_SPRING2016

1) C
2) C
3) A
4) C
5) D
6) B
7) C
8) C
9) C
10) Answers will vary. One possibility is Path 1: $x = t$, $y = t$; Path 2: $x = t$, $y = t^{3/2}$
11) B
12) A
13) B
14) D
15) A
16) D
17) A
18) Absolute maximum: 88 at $(2, 4)$; absolute minimum: 0 at $(0, 0)$
19) Maximum: 324 at $(18, 18)$ and $(-18, -18)$; minimum: -324 at $(18, -18)$ and $(-18, 18)$
20) A

$\frac{\sec t \tan t}{\sec t} =$

Find the torsion of the space curve. (SHOW WORK OR NO CREDIT)

6) $\mathbf{r}(t) = (t - 9)\mathbf{i} + (\ln(\sec t) + 6)\mathbf{j} - 4\mathbf{k}, -\pi/2 < t < \pi/2$ 6) B

A) Undefined B) $\tau = 0$ C) $\tau = -1$ D) $\tau = 1$

$$\tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|\vec{v} \times \vec{a}|^2} \rightarrow \begin{array}{ccc} i & j & k \end{array} \begin{vmatrix} 1 & \tan t & 0 \\ 0 & \sec^2 t & 0 \\ 0 & 2\sec^2 t \tan t & 0 \end{vmatrix} = \frac{1(0-0) - 0(0-0) + 0(0-0)}{} = \frac{0}{|\vec{v} \times \vec{a}|^2}$$

$$\tau = \frac{0}{|\vec{v} \times \vec{a}|^2} \rightarrow \boxed{\tau = 0}$$

Find the limit.

7) $\lim\limits_{(x, y) \to (2, 4)} \left(\frac{1}{x} - \frac{4}{y}\right)$ 7) C

A) $\frac{1}{2}$ B) -4 C) $-\frac{1}{2}$ D) No limit

$$\lim_{(x,y)\to(2,4)} \left(\frac{1}{2} - \frac{4}{4}\right) \rightarrow \frac{1}{2} - 1 = \boxed{-\frac{1}{2}}$$

8) $\lim_{\substack{(x, y) \to (10, 9) \\ y \neq 9}} \frac{xy + 8y - 9x - 72}{y - 9}$

8) C

A) 2
B) 0
C) 18
D) 1

$\lim_{\substack{(x,y) \to (10,9) \\ y=x}} \frac{x^2 + 8x - 9x - 72}{x - 9} \longrightarrow \lim_{\substack{x,y \to (10,9) \\ y=x}} \frac{x^2 - x - 72}{x - 9} \longrightarrow \frac{(10)^2 - 10 - 72}{10 - 9}$

$\downarrow$

$\frac{18}{1} \longrightarrow \boxed{18}$

At what points is the given function continuous?

9) $f(x, y, z) = \frac{1}{|x - 4| + |y - 5| + |z + 7|}$

9) C

A) All (x, y, z)
B) All (x, y, z) $\neq$ (0, 0, 0)
C) All (x, y, z) $\neq$ (4, 5, -7)
D) All (x, y, z) $\neq \pm$ (4, 5, -7)

Find two paths of approach from which one can conclude that the function has no limit as (x, y) approaches (0, 0).

10) $f(x, y) = \frac{x^3 + y^6}{x^3}$

10) DNE

$\lim_{(x,y) \to (0,0)} \frac{x^3 + y^6}{x^3} \longrightarrow \lim_{x \to 0} \frac{x^3 + x^6}{x^3} \longrightarrow \lim_{x \to 0} \frac{\cancel{x^3}(1 + x^3)}{\cancel{x^3}} \longrightarrow \lim_{x \to 0} 1 + x^3 = 1 + 0 = \boxed{1}$

along y=x

$\lim_{(x,y) \to (0,0)} \frac{x^3 + y^6}{x^3} \longrightarrow \lim_{x \to 0} \frac{x^3 + (x^{1/2})^6}{x^3} \longrightarrow \lim_{x \to 0} \frac{x^3 + x^3}{x^3} \longrightarrow \lim_{x \to 0} \frac{2\cancel{x^3}}{\cancel{x^3}} = \boxed{2}$

along $y = x^{1/2}$

Limit DNE since $\lim_{\substack{(x,y) \to (0,0) \\ \text{along } y=x}} f(x) = 1$

and the $\lim_{\substack{(x,y) \to (0,0) \\ \text{along } y=x^{1/2}}} f(x) = 2$

$\ln(x) = \frac{1}{x}$ $\quad \ln(\frac{1}{x}) = \frac{1}{\frac{1}{x}} = x$

Find $\frac{\delta f}{\delta x}$ and $\frac{\delta f}{\delta y}$.

11) $f(x, y) = \ln\left(\frac{y^9}{x^8}\right)$ 11) B

A) $\frac{\partial f}{\partial x} = -\ln\left(\frac{8}{x}\right); \frac{\partial f}{\partial y} = \ln\left(\frac{9}{y}\right)$

B) $\frac{\partial f}{\partial x} = -\frac{8}{x}; \frac{\partial f}{\partial y} = \frac{9}{y}$

C) $\frac{\partial f}{\partial x} = -\ln\left(\frac{8y^9}{x^9}\right); \frac{\partial f}{\partial y} = \ln\left(\frac{9y^8}{x^8}\right)$

D) $\frac{\partial f}{\partial x} = \frac{9}{y}; \frac{\partial f}{\partial y} = \frac{8}{x}$

$\frac{\partial f}{\partial x} = \frac{1}{\frac{y^9}{x^8}} \cdot -8x^{-9}y^9 = -8x^{-9}y \cdot \frac{x^8}{y^9} \quad \frac{-8x^{-1}y^9}{\cancel{y^9}} = \boxed{\frac{-8}{x}}$

$\frac{\partial f}{\partial y} = \frac{1}{\frac{y^9}{x^8}} \cdot \frac{9y^8}{x^8} = \frac{\cancel{x^8}}{y^9} \cdot \frac{9y^8}{\cancel{x^8}} = \boxed{\frac{9}{y}}$

Provide an appropriate answer.

12) Find $\frac{\partial w}{\partial u}$ when $u = -2$ and $v = -5$ if $w(x, y, z) = \frac{xy^2}{z}$, $x = \frac{u}{v}$, $y = u + v$, and $z = u \cdot v$. 12) A

A) $\frac{\partial w}{\partial u} = -\frac{14}{25}$

B) $\frac{\partial w}{\partial u} = -\frac{28}{25}$

C) $\frac{\partial w}{\partial u} = \frac{35}{4}$

D) $\frac{\partial w}{\partial u} = -\frac{7}{25}$

w
x y z
u v u v u v

$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial u} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial u}$

$x = \frac{-2}{-5} = \frac{2}{5}$

$y = -2 - 5 = -7$

$z = (-2)(-5) = 10$

$\frac{\partial w}{\partial u} = \left(\frac{y^2}{z}\right)\left(\frac{1}{v}\right) + \left(\frac{2yx}{z}\right)(1) + \left(-\frac{xy^2}{z^2}\right)(v)$

$\frac{\partial w}{\partial u} = \left(\frac{49}{10}\right)\left(\frac{1}{-5}\right) + \left(\frac{2(-7)(\frac{2}{5})}{10}\right)(1) + \left(-\frac{\frac{2}{5}(49)}{100}\right)(-5)$

$\frac{\partial w}{\partial u} = \cancel{\frac{-49}{50}} + \left(-\frac{14}{25}\right) + \cancel{\frac{49}{50}}$

$\boxed{\frac{\partial w}{\partial u} = -\frac{14}{25}}$

Compute the gradient of the function at the given point.

13) $f(x, y, z) = -7x - 7y - 5z, \quad (4, 8, -10)$ 13) B

A) $\nabla f = -28\mathbf{i} + 56\mathbf{j} + 50\mathbf{k}$

B) $\nabla f = -7\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$

C) $\nabla f = -28\mathbf{i} - 56\mathbf{j} + 50\mathbf{k}$

D) $\nabla f = 4\mathbf{i} + 8\mathbf{j} - 10\mathbf{k}$

$\nabla F = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k \longrightarrow \boxed{\nabla f = -7i - 7j - 5k}$

$\frac{\partial f}{\partial x} = -7$

$\frac{\partial f}{\partial y} = -7$

$\frac{\partial f}{\partial z} = -5$

Find the derivative of the function at P_0 in the direction of u.

14) $f(x, y) = 5x^2 + 3y$, $P_0(-9, -10)$, $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j}$ 14) D

A) $-\frac{462}{5}$ B) $-\frac{552}{5}$ C) $-\frac{372}{5}$ D) $-\frac{282}{5}$

$D_{\vec{u}} f = \nabla f \cdot \frac{\vec{U}}{|\vec{U}|}$

$\nabla f = 10xi + 3j$

$\frac{\partial f}{\partial x} = 10x$

$\frac{\partial f}{\partial y} = 3$

$\nabla f|_{P_0} = 10(-9)i + 3j$

$\nabla f|_{P_0} = -90i + 3j$

$|\vec{U}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$\frac{\vec{U}}{|\vec{U}|} = \frac{3}{5}i - \frac{4}{5}j$

$D_{\vec{u}} f = (-90)(\frac{3}{5}) + (3)(-\frac{4}{5})$

$D_{\vec{u}} f = -54 + (-\frac{12}{5})$

$\boxed{D_{\vec{u}} f = -\frac{282}{5}}$

Solve the problem.

15) Find the equation for the tangent plane to the surface $9x + 5y - 4z = -4$ at the point $(1, -1, 2)$. 15) A

A) $9x + 5y - 4z = -4$ B) $9x - 5y - 8z = 10$

C) $9x + 5y - 4z = 10$ D) $9x - 5y - 8z = -4$

$f_x = 9$

$f_y = 5$

$f_z = -4$

$f_x|_{P_0}(x - x_0) + f_y|_{P_0}(y - y_0) + f_z|_{P_0}(z - z_0) = 0$

$9(x-1) + 5(y+1) - 4(z-2) = 0$

$9x - 9 + 5y + 5 - 4z + 8 = 0$

$\boxed{9x + 5y - 4z = -4}$

16) About how much will $f(x, y, z) = -6x + 10y + 9z$ change if the point (x, y, z) moves from $(-5, -6, 10)$ a distance of $ds = \frac{1}{10}$ unit in the direction $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$? 16) D

A) $-\frac{4}{35}$ B) $\frac{11}{35}$ C) $\frac{1}{35}$ D) $\frac{6}{35}$

$dF = \left(\nabla f|_{P_0} \cdot \frac{\vec{U}}{|\vec{U}|}\right) ds \longrightarrow df = \left(\langle -6, 10, 9\rangle \cdot \langle \frac{2}{7}, \frac{-3}{7}, \frac{6}{7}\rangle\right)\frac{1}{10}$

$f_x = -6$

$f_y = 10$

$f_z = 9$

$\nabla f = -6i + 10j + 9k$

$|\vec{U}| = \sqrt{2^2 + 3^2 + 6^2}$

$|\vec{U}| = \sqrt{49} = 7$

$\frac{\vec{U}}{|\vec{U}|} = \frac{2}{7}i - \frac{3}{7}j + \frac{6}{7}k$

$df = \frac{(-6)(\frac{2}{7}) + (10)(\frac{-3}{7}) + (9)(\frac{6}{7})}{10}$

$df = \frac{\frac{-12 - 30 + 54}{7}}{10}$

$df = \frac{12}{7} \cdot \frac{1}{10}$

$\boxed{df = \frac{6}{35}}$

Find all the local maxima, local minima, and saddle points of the function.

17) $f(x, y) = x^2 + 20x + y^2 + 2y - 6$ 17) A

A) $f(-10, -1) = -107$, local minimum
B) $f(10, -1) = 293$, saddle point
C) $f(-10, 1) = -103$, saddle point
D) $f(10, 1) = 297$, local maximum

$f_x = 2x + 20 \rightarrow 2x + 20 = 0 \rightarrow x = -10$

$f_y = 2y + 2 \rightarrow 2y + 2 = 0 \rightarrow y = -1$

$\boxed{f_{xx} = 2 > 0}$

$f_{xy} = 0$

$f_{yy} = 2$

$f_{xx} f_{yy} - (f_{xy})^2 > 0$

$\boxed{4 > 0}$

$f(-10, -1) = (-10)^2 + 20(-10) + (-1)^2 + 2(-1) - 6$

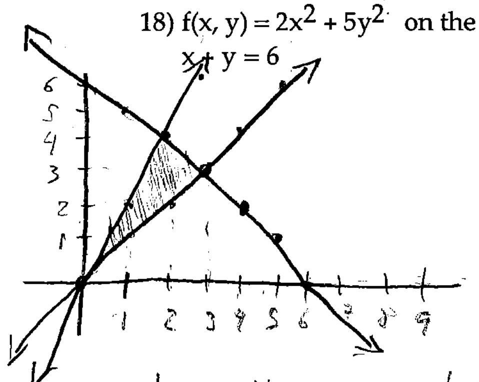
$f(-10, -1) = 100 - 200 + 1 - 2 - 6$

$f(-10, -1) = -107$

$\boxed{\text{Local Min}}$

Find the absolute maxima and minima of the function on the given domain.

18) $f(x, y) = 2x^2 + 5y^2$ on the closed triangular region bounded by the lines $y = x$, $y = 2x$, and $x + y = 6$ 18) ______



$f_x = 4x \rightarrow 4x = 0 \rightarrow x = 0$

$f_y = 10y \rightarrow 10y = 0 \rightarrow y = 0$

$f(0,0) = \boxed{0}$ Min

On line $y = 2x$

$f(x, 2x) = 2x^2 + 20x^2 = 22x^2$

$f'(x, 2x) = 44x = 0 \rightarrow x = 0$

$f(2,4) = 2(4) + 5(16) = \boxed{88}$ Max

on line $y = x$

$f(x,x) = 2x^2 + 5x^2 = 7x^2$

$f'(x,x) = 14x = 14x = 0 \quad x = 0$

$f(3,3) = 2(3)^2 + 5(3)^2$

$f(3,3) = 18 + 45 = \boxed{63}$

on line $y = -x + 6$

$f(x, -x+6) = 2x^2 + 5(-x+6)^2$

$f(x, -x+6) = 2x^2 + 5(x^2 - 12x + 36)$

$f(x, -x+6) = 7x^2 - 60x + 180$

$f'(x, -x+6) = 14x - 60 = 0$

$x = \frac{30}{7}$

$y = -\frac{30}{7} + 6$

$y = \frac{12}{7}$

$f(\frac{30}{7}, \frac{12}{7}) = 2(\frac{30}{7})^2 + 5(\frac{12}{7})^2$

$f(\frac{30}{7}, \frac{12}{7}) = \frac{360}{7} = \boxed{51.43}$

Absolute max of 88 at (2,4)

Absolute min of 0 at (0,0)

Find the extreme values of the function subject to the given constraint.

19) $f(x, y) = xy$, $x^2 + y^2 = 648$ 19) ______

$$\mathcal{L} = xy + \lambda(x^2 + y^2 - 648) = xy + x^2\lambda + y^2\lambda - 648\lambda$$

$$\mathcal{L}_\lambda = x^2 + y^2 - 648 = 0$$

$$\mathcal{L}_x = y + 2x\lambda = 0$$

$$\mathcal{L}_y = x + 2y\lambda = 0$$

$$\frac{y}{x} = \frac{2x\lambda}{2y\lambda} \qquad \frac{y}{x} = \frac{x}{y} \qquad y^2 = x^2$$

$$f(x,x) = x^2 + x^2 - 648 = 0$$

$$2x^2 = 648$$

$$x^2 = 324$$

$$x = \pm 18$$

$$x = y$$

$$y = \pm 18$$

$$\boxed{\begin{aligned} f(18,18) &= (18)(18) = 324 \\ f(-18,-18) &= (-18,-18) = 324 \\ f(-18,18) &= (-18)(18) = -324 \\ f(18,-18) &= (18)(-18) = -324 \end{aligned}}$$

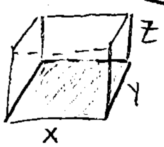
Maximum of 324 at (18,18) & (-18,-18)

Minimum of -324 at (-18,18) & (18,-18)

Solve the problem.

20) A rectangular box with square base and no top is to have a volume of 32 ft^3. What is the least amount of material required? 20) A

A) 48 ft^2 B) 36 ft^2 C) 40 ft^2 D) 42 ft^2



$$\text{Volume} = 32\,ft^3$$

$$32\,ft^3 = xyz \rightarrow z = \frac{32}{xy} \rightarrow z = \frac{32}{(4)(4)} \rightarrow \boxed{z = 2}$$

$$SA = xy + 2xz + 2yz \rightarrow (4)(4) + 2(4)(2) + 2(4)(2)$$

$$SA = xy + 2x\left(\frac{32}{xy}\right) + 2y\left(\frac{32}{xy}\right)$$

$$SA = 16 + 16 + 16$$

$$\boxed{SA = 48\,ft^2}$$

$$SA = xy + \frac{64}{y} + \frac{64}{x}$$

$$\frac{\partial SA}{\partial x} = y - \frac{64}{x^2} \rightarrow y - \frac{64}{x^2} = 0$$

$$\frac{\partial SA}{\partial y} = x - \frac{64}{y^2} \rightarrow x - \frac{64}{y^2} = 0$$

$$x = \frac{64}{y^2}, \quad y = \frac{64}{x^2}$$

$$y - \frac{64}{x^2} = 0$$

$$y - \frac{64}{\left(\frac{64}{y^2}\right)^2} = 0$$

$$y - \frac{64}{\frac{4096}{y^4}} = 0$$

$$y - \frac{64y^4}{4096} = 0$$

$$y - \frac{y^4}{64} = 0$$

$$y\left(1 - \frac{y^3}{64}\right) = 0$$

$$y = 0$$

$$1 - \frac{y^3}{64} = 0$$

$$y^3 = 64$$

$$\boxed{y = 4} \ \& \ \boxed{x = 4}$$

Steven Romeiro

$\cancel{\mathcal{L} = xy + 2xz + 2yz + \lambda(xyz - 32)}$

$\cancel{\mathcal{L} = xy + 2xz + 2yz + xyz\lambda - 32\lambda}$

$\cancel{\mathcal{L}_\lambda = xyz - 32 = 0}$

$\cancel{\mathcal{L}_x = y + 2z + yz\lambda = 0}$

$\cancel{\mathcal{L}_y = x + 2z + xz\lambda = 0}$

$\cancel{\frac{y}{x} = \frac{-2z - yz\lambda}{-2z - xz\lambda}}$

$\cancel{\frac{y}{x} = \frac{-y}{-x}}$

#20

$V = x^2h = x^2h - 32 = 0$

$SA = x^2 + 4xh$

$\mathcal{L} = x^2 + 4xh + \lambda(x^2h - 32)$

$= x^2 + 4xh + x^2h\lambda - 32\lambda$

$\mathcal{L}_\lambda = x^2h - 32 = 0$

$\mathcal{L}_x = 2x + 4h + 2xh\lambda = 0 \rightarrow 2x + 4h = -2xh\lambda$

$\mathcal{L}_h = 4x + x^2\lambda = 0 \rightarrow 4x = x^2\lambda$

$\frac{2x + 4h}{4x} = \frac{\cancel{+}2xh\cancel{\lambda}}{\cancel{+}\cancel{x^2}\cancel{\lambda}} \rightarrow \frac{x + 2h}{2x} = \frac{2h}{x}$

$x^2 + 2hx = 4xh$

$x^2 = 2xh$

$x = 2h$

$V = (2h)^2(h) \rightarrow 32 = 4h^3$

$h^3 = 8 \rightarrow h = 2$

$32 = x^2(2) \rightarrow x = 4$

$SA = 4^2 + 4(4)(2) \rightarrow SA = 48\,ft^2$

Steven Romeiro

1, 5, 9, 15, 19, 23

5 & 5

13.1 Homework

(1) $\vec{r}(t) = (t+1)i + (t^2-1)j$; $t=1$

$\vec{v} = \frac{d\vec{r}}{dt} = i + 2tj \rightarrow \boxed{\vec{v}(1) = i + 2j}$

$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = 2j}$

$X = t+1 \longrightarrow t = X-1$

$Y = t^2 - 1 \longleftarrow$

$\hookrightarrow y = (x-1)^2 - 1 \rightarrow y = x^2 - 2x + 1 - 1$

$\boxed{y = x^2 - 2x}$

(5) $x^2 + y^2 = 1$; $\vec{r}(t) = (\sin t)i + (\cos t)j$; $t = \frac{\pi}{4}$ & $\frac{\pi}{2}$

$\vec{v}(t) = \frac{d\vec{r}}{dt} = (\cos t)i - (\sin t)j$

$\vec{v}(\pi/4) = (\cos\frac{\pi}{4})i - (\sin\frac{\pi}{4})j = \frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2}j$

$\vec{a}(t) = \frac{d\vec{v}}{dt} = (-\sin t)i - (\cos t)j$

$\vec{a}(\pi/4) = (-\sin\frac{\pi}{4})i - (\cos\frac{\pi}{4})j = -\frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2}j$

(9) $\vec{r}(t) = (t+1)i + (t^2-1)j + 2tk$; $t=1$

$\vec{v}(t) = \frac{d\vec{r}}{dt} = i + 2tj + 2k$

$\vec{v}(1) = i + 2j + 2k$

$\vec{a}(t) = \frac{d\vec{v}}{dt} = 2j$

Speed $= |\vec{v}(t)| = \sqrt{1^2+2^2+2^2} = \boxed{3}$

direction $= \frac{\vec{v}}{|\vec{v}|} = \boxed{\frac{i+2j+2k}{3}}$

(15) $\vec{r}(t) = (3t+1)i + \sqrt{3}\,tj + t^2k$; $t=0$

$\vec{v}(t) = 3i + \sqrt{3}j + 2tk$
$\vec{a}(t) = 2k$

$\theta = \cos^{-1}\left(\frac{\vec{v}\cdot\vec{a}}{|\vec{v}||\vec{a}|}\right)$

$\vec{v}(t)\cdot\vec{a}(t) =$
$\vec{v}(0)\cdot\vec{a}(0) = (0)(3) + (0)(\sqrt{3}) + (0)(2) = 0$

$|\vec{v}| = \sqrt{3^2+3^2} = \sqrt{18} = 3\sqrt{2}$
$|\vec{a}| = \sqrt{2^2} = 2$

$\theta = \cos^{-1}\left(\frac{0}{(3\sqrt{2})(2)}\right) = \theta = \cos^{-1}(0)$

$\boxed{\theta = \pi/2}$

Steven
Romero

1, 5, 9, 11, 15, 23

13.2 Homework

(1) $\int_0^1 [t^3 i + 7j + (t+1)k]\,dt$

$\left[\frac{1}{4}t^4\right] i + 7t j + \left(\frac{1}{2}t^2 + t\right)k$

$\boxed{\frac{1}{4} i + 7j + \frac{3}{2}k}$

(5) $\int_1^4 \left[\frac{1}{t} i + \frac{1}{5-t} j + \frac{1}{2t} k\right] dt$

$\left[\ln t\right]_1^4 i + \left[\ln(5-t)\right]_1^4 j + \frac{1}{2}\left[\ln t\right]_1^4 k$

$= [\ln 4 - \ln 1] i + [\ln(5-4) - \ln(5-1)] j + \frac{1}{2}[\ln 4 - \ln 1] k$

$= \boxed{\ln 4 i - \ln 4 j + \ln 2 k}$

(9) $\int_0^{\pi/2} [\cos t i - \sin 2t j + \sin^2 t k]\,dt$

$\left[\sin t\right]_0^{\pi/2} i + \left[\frac{1}{2}\cos 2t\right]_0^{\pi/2} j + \int_0^{\pi/2} \left(\frac{1}{2} - \cos 2t\right) dt$

$\left[\sin t\right]_0^{\pi/2} i + \left[\frac{1}{2}\cos 2t\right]_0^{\pi/2} j + \left[\frac{1}{2}t - \frac{1}{2}\sin 2t\right]_0^{\pi/2} k$

$\boxed{i - j + \frac{\pi}{4}k}$

(11) $\frac{dr}{dt} = -ti - tj - tk$; $r(0) = i + 2j + 3k$

$$\int \frac{dr}{dt} = \int -ti - tj - tk$$

$$r(t) = -\frac{1}{2}t^2 i - \frac{1}{2}t^2 j - \frac{1}{2}t^2 k$$

$$\boxed{r(t) = \left(-\frac{1}{2}t^2 + 1\right)i + \left(-\frac{1}{2}t^2 + 2\right)j + \left(-\frac{1}{2}t^2 + 3\right)k}$$

(15) $\frac{d^2 r}{dt^2} = -32k$; $\left.\frac{dr}{dt}\right|_{t=0} = 8i + 8j$; $r(0) = 100k$

$$\int \frac{dr}{dt} = \int 8i + \int 8j = \boxed{r = 8ti + 8tj}$$

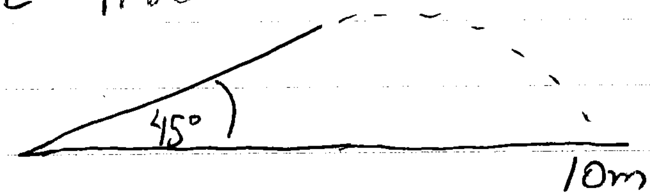
$$\int \frac{d^2 r}{dt^2} = \int -32k \longrightarrow \int \frac{dr}{dt} = \int -32tk \longrightarrow \boxed{r(t) = -16t^2 k}$$

$$\boxed{r(t) = 8ti + 8tj + \left(-16t^2 + 100\right)k}$$

?

13.2 HW

(23)



$X = V_0 \cos(\alpha) t$

Steven Romeiro

1, 3, 11, 13

13.3 Homework

(1) $r(t) = (2\cos t)i + (2\sin t)j + \sqrt{5}tK; \quad 0 \leq t \leq \pi$

$\vec{T} = \frac{\vec{V}}{|\vec{V}|}$: $\vec{V} = -2\sin t\, i + 2\cos t\, j + \sqrt{5}K$

$|\vec{V}| = \sqrt{(-2\sin t)^2 + (2\cos t)^2 + (\sqrt{5})^2}$

$|\vec{V}| = \sqrt{4(\sin^2 t + \cos^2 t) + 5}$

$|\vec{V}| = \sqrt{4(1) + 5} = 3$

$$\vec{T} = \frac{-2\sin t\, i + 2\cos t\, j + \sqrt{5}K}{3}$$

$$L = \int_a^b |\vec{V}|\, dt = \int_0^{\pi} 3\, dt = [3t]_0^{\pi} = \boxed{3\pi}$$

(3) $r(t) = ti + (2/3)t^{3/2}K, \quad 0 \leq t \leq 8$

$\vec{V} = i + t^{1/2}K$, $|\vec{V}| = \sqrt{(1)^2 + (t^{1/2})^2} = \sqrt{1+t}$

$$\vec{T} = \frac{i + t^{1/2}K}{\sqrt{1+t}}$$

$$L = \int_a^b |\vec{V}|\, dt \rightarrow \int_0^8 (\sqrt{1+t})dt \rightarrow \int_0^8 (1+t)^{1/2} dt$$ $u = 1+t$, $du = dt$

$$\int_0^8 u^{1/2} du \rightarrow \frac{2}{3}\left[(1+t)^{3/2}\right]_0^8 = \frac{2}{3}\left[(1+8)^{3/2} - (1+0)^{3/2}\right]$$

$$= \frac{2}{3}\left[9^{3/2} - 1\right] = \frac{2}{3}[27 - 1] = 17.3$$

11) $r(t) = (4\cos t)i + (4\sin t)j + 3tk; \quad 0 \leq t \leq \pi/2$

$\vec{v} = -4\sin t\, i + 4\cos t\, j + 3k$

$|\vec{v}| = \sqrt{16(1) + 3^2} = \sqrt{25} = 5$

$$S = \int_0^t |v\tau|\, d\tau = [5\tau]_0^t = [5t]_0^{\pi/2} = \boxed{5\pi/2}$$

13) $r(t) = (e^t\cos t)i + (e^t\sin t)j + e^t k, \quad -\ln 4 \leq t \leq 0$

$\vec{v} = (e^t\cos t - e^t\sin t)i + (e^t\sin t + e^t\cos t)j + e^t k$

$|\vec{v}| = \sqrt{e^t(\cos t - \sin t)^2 + e^t(\sin t + \cos t) + e^{2t}}$

$|\vec{v}| = \sqrt{e^{2t}(\cos^2 t - 2\cos t\sin t + \sin^2 t) + e^{2t}(\sin^2 t + 2\cos t\sin t + \cos^2 t) + e^{2t}}$

$|\vec{v}| = \sqrt{e^{2t}(\cos^2 t + \sin^2 t) + e^{2t}(\sin^2 t + \cos^2 t) + e^{2t}}$

$|\vec{v}| = \sqrt{e^{2t}(1) + e^{2t} + e^{2t}} = \sqrt{3e^{2t}} = e^t\sqrt{3}$

$$S = \int_0^t e^\tau \sqrt{3}\, d\tau = S = \sqrt{3}\int_0^t e^\tau\, d\tau$$

$$S = \sqrt{3}\,[e^t]_{-\ln 4}^{0} \longrightarrow \sqrt{3}\,[e^0 - e^{-\ln 4}]$$

$$S = \sqrt{3}\left[1 - \tfrac{1}{4}\right] \rightarrow S = \sqrt{3}\left[\tfrac{3}{4}\right]$$

$$S = \frac{3\sqrt{3}}{4}$$

(19) $\vec{r}(t) = \sin t\, i + (t^2 - \cos t)\, j + e^t K \; ; \; t = 0$

$\vec{n} = \vec{v}(t) = \frac{d\vec{r}}{dt} = \cos t\, i + (2t + \sin t)\, j + e^t K$

$\vec{r}(0) = \sin(0)\, i + (0^2 - \cos(0))\, j + e^0 K$

$\vec{r}(0) = \underset{x_0}{0i} + \underset{y_0}{1j} + \underset{z_0}{1K}$

$\vec{v}(0) = \cos(0)\, i + (2(0) + \sin(0))\, j + e^0 K$

$\vec{v}(0) = \underset{v_1}{1i} + \underset{v_2}{0j} + \underset{v_3}{1K}$

$x = x_0 + t v_1 \longrightarrow$ $\quad x = t$

$y = y_0 + t v_2 \longrightarrow$ $\quad y = 1$

$z = z_0 + t v_3 \longrightarrow$ $\quad z = 1 + t$

$$\boxed{\begin{aligned} x &= t \\ y &= 1 \\ z &= 1 + t \end{aligned}}$$

(23) a) $r(t) = (\cos t)\, i + (\sin t)\, j \quad ; \quad t \geq 0$

i) $\vec{v}(t) = -\sin t\, i + \cos t\, j$

$\vec{v}(0) = 0i + 1j \rightarrow |\vec{v}(0)| = \sqrt{1^2} = \boxed{1}$ yes constant

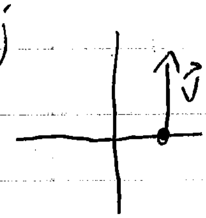
ii) $\vec{a}(t) = -\cos t\, i - \sin t\, j$

$\vec{v} \cdot \vec{a} = (-\sin)(-\cos) + (\cos)(-\sin)$

$\vec{v} \cdot \vec{a} = \sin\cos - \sin\cos = \boxed{0}$ Yes ortho

iii) $\vec{r}(0) = \cos 0\, i + \sin 0\, j = 1i + 0j$

$\vec{v}(0) = 0i + 1j = \boxed{\text{Counter-C}}$



e) $\vec{r}(t) = \cos(t^2)\,i + \sin(t^2)\,j \; ; \; t \geq 0$

i) $\vec{v}(t) = -2t\sin(t^2)\,i + 2t\cos(t^2)\,j$

$\vec{v}(0) = -2(0)\sin(0)\,i + 2(0)\cos(0)\,j = 0i + 0j$

$|\vec{v}| = \sqrt{(-2t\sin t)^2 + (2t\cos t)^2}$

$|\vec{v}| = \sqrt{4t^2\sin^2 t + 4t^2\cos^2 t}$

$|\vec{v}| = \sqrt{(4t^2)(\sin^2 t + \cos^2 t)}$

$|\vec{v}| = \sqrt{4t^2(1)} \rightarrow \boxed{2t}$ Yes const vel.

ii) Yes since particles moving in a circular pattern/motion are always accelerating towards the center as defined by centripetal acceleration

iii) $\vec{r}(0) = \cos(0)\,i + \sin(0)\,j = 1i + 0j$

$\vec{v}(0) = -2(0)\sin 0\; i + 2(0)\cos(0)\,j = 0i + 0j$

$\boxed{\text{Counter - C}}$

iv) $\vec{r}(0) = 1i + 0j = (1, 0)$

$\boxed{\text{Yes}}$

Steven Romeiro

#1, 9, 13

13.4 Homework

(1) $r(t) = ti + (\ln \cos t)j \ ; \ -\pi/2 < t < \pi/2$

Find $\vec{T}$, $\vec{N}$ & K

$\vec{T} = \frac{\vec{v}}{|\vec{v}|}$; $\vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|}$ $\quad K = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$

$\vec{v} = \frac{d\vec{r}}{dt} = i + \left(\frac{1}{\cos t}(-\sin t)\right)j = i - \tan(t)j$

$|\vec{v}| = \sqrt{1^2 + (-\tan(t))^2} = \sqrt{1^2 + \tan^2 t} = \sec(t)$

$\vec{T} = \frac{i - \tan(t)j}{\sec(t)}$ $\qquad \vec{T} = \frac{i}{\sec t} - \frac{\tan(t)j}{\sec t} = \cos t\, i - \tan(\cos t)j$

$\frac{d\vec{T}}{dt} = -\sin(t)i - (\sec^2 t \cos t + \tan(-\sin t))j$

$= -\sin(t)i - (\sec t - \sin t \tan t)j$

$= -\sin(t)i + (-\sec t + \sin t \tan t)j$

$= -\sin(t)i + \left(-\frac{1}{\cos t} + \frac{\sin^2 t}{\cos t}\right)j$

$= -\sin(t)i - \left(\frac{1 - \sin^2 t}{\cos t}\right)j$

$= -\sin(t)i - \left(\frac{\cos^2 t}{\cos t}\right)j = -\sin(t)i - \cos(t)j$

$\frac{d\vec{T}}{dt} = -\sin(t)i - \cos(t)j$

$\left|\frac{d\vec{T}}{dt}\right| = \sqrt{\sin^2 t + \cos^2 t} = \underline{1}$

#4,9,16

Shawn Ramirez

13.4 Homework

$$\vec{N} = \frac{\frac{d\vec{T}}{dt}}{|d\vec{T}/dt|} = \frac{-\sin(t)i - \cos(t)j}{1} = \boxed{-\sin(t)i - \cos(t)j}$$

$$K = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right| = \boxed{\cos(t)}$$

(9) $r(t) = (3\sin(t))\vec{i} + (3\cos(t))\vec{j} + 4t\vec{k}$

$\vec{T} = \frac{\vec{v}}{|\vec{v}|}$ $\vec{N} = \frac{d\vec{T}}{|d\vec{T}/dt|}$ $K = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$

$\vec{v} = 3\cos(t)i - 3\sin(t)j + 4K$

$|\vec{v}| = \sqrt{9(\cos^2 t + \sin^2 t) + 16} = \sqrt{25} = 5$

$$\boxed{\vec{T} = \frac{3\cos(t)i - 3\sin(t)j + 4k}{5}}$$

$$\frac{d\vec{T}}{dt} = \frac{-3\sin(t)i}{5} - \frac{3\cos(t)j}{5}$$

$$\left|\frac{d\vec{T}}{dt}\right| = \frac{3}{5}$$

$$\vec{N} = \frac{-\frac{3}{5}\sin t\, i - \frac{3}{5}\cos(t)\, j}{3/5} = \boxed{-\sin t\, i - \cos t\, j}$$

$$K = \frac{1}{5}\left(\frac{3}{5}\right) \longrightarrow \boxed{K = \frac{3}{25}}$$

(13) $r(t) = (t^3/3)i + \left(\frac{t^2}{2}\right)j \quad ; \quad t>0$

$$\vec{T} = \frac{\vec{v}}{|\vec{v}|} \qquad \vec{N} = \frac{d\vec{T}/dt}{|d\vec{T}/dt|} \qquad K = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right|$$

$$\vec{v} = t^2 i + tj$$

$$|\vec{v}| = \sqrt{t^4+t^2} = \sqrt{t^2(t^2+1)} = t\sqrt{t^2+1}$$

$$\vec{T} = \frac{t^2 i + tj}{t\sqrt{t^2+1}} \longrightarrow \frac{ti+j}{\sqrt{t^2+1}} = \frac{(ti+j)\sqrt{t^2-1}}{t^2+1}$$

$$\frac{d\vec{T}}{dt} = (ti+j)(t^2+1)^{-1/2} = \left[(t)(t^2+1)^{-1/2}\right]i + \left[(1)(t^2+1)^{-1/2}\right]j$$

$$\frac{d\vec{T}}{dt} = \left[(t^2+1)^{-1/2} + (t)(-t(t^2+1)^{-3/2}\right]i + \left[-t(t^2+1)^{-3/2}\right]j$$

$$\frac{d\vec{T}}{dt} = \left[\frac{1}{(t^2+1)^{1/2}} - \frac{t^2}{(t^2+1)^{3/2}}\right]i + \left[\frac{-t}{(t^2+1)^{3/2}}\right]j$$

$$\frac{d\vec{T}}{dt} = \left[\frac{(t^2+1)^{1/2} - t^2}{(t^2+1)^{3/2}}\right]i - \left[\frac{t}{(t^2+1)^{3/2}}\right]j$$

$$\left|\frac{d\vec{T}}{dt}\right| = \sqrt{\left(\frac{(t^2+1)^{1/2}-t^2}{(t^2+1)^{3/2}}\right)^2 + \left(\frac{-t}{(t^2+1)^{3/2}}\right)^2}$$

$$A = (t^2+1)^{1/2} \qquad b = t^2$$

$$(t^2+1) - 2(t^2+1)^{1/2}(t^2) + t^4$$

$$t^2+1 - 2t^2(t^2+1) + t^4 = t^2+1 - 2t^4 - 2t^2 + t^4$$

$$t^4 - 2t^2 + t^2 + 1$$

$$t^4 - t^2 + 1$$

$$\left|\frac{d\vec{T}}{dt}\right| = \frac{\sqrt{\left((t^2+1)^{1/2} - t^2\right)^2 + (-t)^2}}{\sqrt{(t^2+1)^{3/2}}}$$

$$\left|\frac{d\vec{T}}{dt}\right| = \sqrt{\frac{t^4 - t^2 + 1 + t^2}{(t^2+1)^3}}$$

$$\left|\frac{d\vec{T}}{dt}\right| = \frac{\sqrt{t^4+1}}{(t^2+1)^3} = \frac{\sqrt{(t^2+1)(t^2-1)}}{(t^2+1)^3}$$

$$\left|\frac{d\vec{T}}{dt}\right| = \frac{\sqrt{t^2-1}}{(t^2+1)^{5/2}}$$

Steven Romeiro

1, 9, 13

13.5 Homework

$a = a_T \vec{T} + a_N \vec{N}$

1) $r(t) = (a\cos t)i + (a\sin t)j + btk$

$a_T = \frac{d}{dt}|\vec{v}| \qquad a_N = k|\vec{v}|^2$

$a_T = \frac{d}{dt}(3) = \boxed{a_T = 0}$

$a_N = \sqrt{|\vec{a}|^2 - a_T^2} = \sqrt{|\vec{a}|^2 - 0} = \sqrt{|\vec{a}|^2} = |\vec{a}|$

$$\boxed{a = |\vec{a}|\vec{N}}$$

9) $r(t) = (3\sin t)i + (3\cos t)j + 4tk$

$\vec{B} = \vec{T} \times \vec{N} \qquad \tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|\vec{v} \times \vec{a}|}$

$\vec{T} = \frac{3\cos t i - 3\sin t j + 4k}{5}$

$\vec{N} = -\sin t i - \cos t j$

$$\vec{T} \times \vec{N} = \begin{array}{ccc} i & j & K \end{array}$$
$$\begin{vmatrix} \frac{3}{5}\cos t & \frac{-3}{5}\sin t & \frac{4}{5} \\ -\sin t & -\cos t & 0 \end{vmatrix} \longrightarrow$$

$$\vec{T}\times\vec{N} = \begin{vmatrix} -\frac{3}{5}\operatorname{Sin}t & \frac{4}{5} \\ -\operatorname{Cos}t & 0 \end{vmatrix} i - \begin{vmatrix} \frac{3}{5}\operatorname{Cos}t & \frac{4}{5} \\ -\operatorname{Sin}t & 0 \end{vmatrix} j + \begin{vmatrix} \frac{3}{5}\operatorname{Cos}t & -\frac{3}{5}\operatorname{Sin}t \\ -\operatorname{Sin}t & -\operatorname{Cos}t \end{vmatrix} K$$

$$\vec{T}\times\vec{N} = 4\operatorname{Cos}t\, i - 4\operatorname{Sin}t\, j + (-3\operatorname{Cos}^2 t - 3\operatorname{Sin}^2 t)K$$

$$\vec{T}\times\vec{N} = \frac{4}{5}\operatorname{Cos}t\, i - \frac{4}{5}\operatorname{Sin}t\, j - \frac{3}{5}K$$

$$\boxed{\vec{B} = \frac{4}{5}\operatorname{Cos}t\, i - \frac{4}{5}\operatorname{Sin}t\, j - \frac{3}{5}K}$$

$$\tau = \frac{\begin{array}{ccc} i & j & K \\ \end{array} \begin{vmatrix} 3\operatorname{Cos}t & -3\operatorname{Sin}t & 4 \\ -3\operatorname{Sin}t & -3\operatorname{Cos}t & 0 \\ -3\operatorname{Cos}t & 3\operatorname{Sin}t & 0 \end{vmatrix}}{|\vec{v}\times\vec{a}|^2}$$

$$\tau = 3\cos t(0) + 3\operatorname{Sin}t(0) + 4(-9\operatorname{Sin}^2 t - 9\operatorname{Cos}^2 t)$$

$$\tau = \frac{-36}{|\vec{v}\times\vec{a}|^2}$$

$$|\vec{v}\times\vec{a}| = \begin{array}{ccc} i & j & K \end{array} \begin{vmatrix} 3\operatorname{Cos}t & -3\operatorname{Sin}t & 4 \\ -3\operatorname{Sin}t & -3\operatorname{Cos}t & 0 \end{vmatrix}$$

$$|\vec{v}\times\vec{a}| = +12\operatorname{Cos}t\, i - 12\operatorname{Sin}t\, j + (-9\operatorname{Cos}^2 t - 9\operatorname{Sin}^2 t)K$$

$$|\vec{v}\times\vec{a}| = 12\operatorname{Cos}t\, i - 12\operatorname{Sin}t\, j - 9$$

$$|\vec{v}\times\vec{a}| = \sqrt{144+81} = \sqrt{225}$$

$$|\vec{v}\times\vec{a}|^2 = 225$$

$$\tau = \frac{-36}{225}$$

Steven Romeiro

#1, 5, 9

14.1 Homework

(1) $f(x,y) = x^2 + xy^3$

a) $f(0,0) = x(x+y^3) = \boxed{0}$

b) $f(-1,1) = x(x+y^3) = (-1)(-1+1) = \boxed{0}$

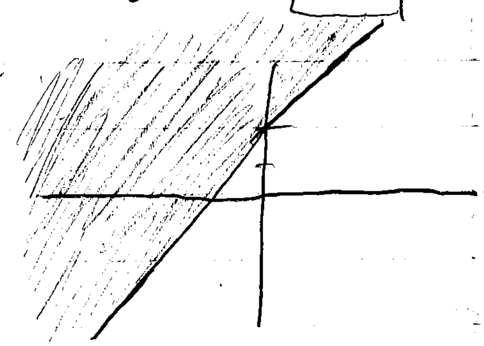
c) $f(2,3) = 2(2+(3)^3) = 2(2+27) = \boxed{58}$

d) $f(-3,-2) = -3(-3+(-2)^3) = -3(-3-8) = \boxed{-33}$

(5) $f(x,y) = \sqrt{y-x-2}$

$y - x - 2 \geq 0$

$y \geq x+2$



(13) $f(x,y) = x+y-1$, $c = -3, -2, -1, 0, 1, 2, 3$

$x+y-1 = c \longrightarrow x+y-1 = -3, \rightarrow y = -x-2$

$x+y-1 = -2 \rightarrow y = -x-1$

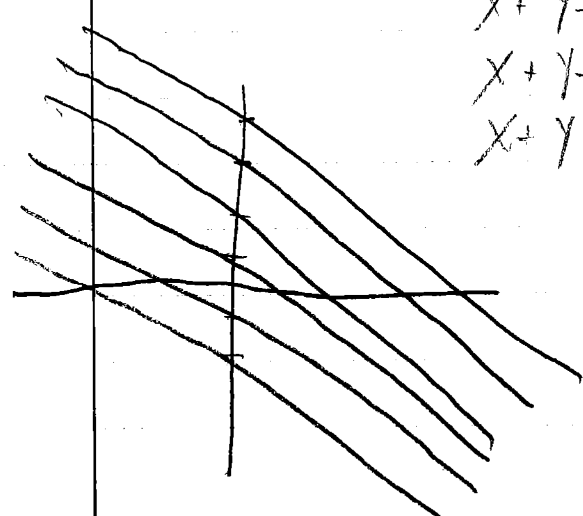
$x+y-1 = -1 \rightarrow y = -x$

$x+y-1 = 0 \rightarrow y = -x+1$

$x+y-1 = 1 \rightarrow y = -x+2$

$x+y-1 = 2 \rightarrow y = -x+3$

$x+y-1 = 3 \rightarrow y = -x+4$



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Steven Romeiro

1-29 EOO, 33, 35, 37, 41, 45, 49, 55, 57, 61

14.2 Homework

1) $\lim\limits_{xy \to (0,0)} \dfrac{3x^2 - y^2 + 5}{x^2 + y^2 + 2} = \dfrac{3(0) - 0 + 5}{0 + 0 + 2} = \boxed{\dfrac{5}{2}}$

5) $\lim\limits_{(x,y) \to (0,\pi/4)} \sec x \tan y = \dfrac{1}{\cos x}\left(\dfrac{\sin y}{\cos y}\right) = \dfrac{\sin y}{\cos x \cos y}$

$= \dfrac{\sqrt{3}/2}{(1)(\frac{\sqrt{2}}{2})} = \dfrac{\sqrt{2}}{2} \Big/ \dfrac{\sqrt{2}}{2} = \boxed{1}$

9) $\lim\limits_{x,y \to (0,0)} \dfrac{e^y \sin x}{x} = e^{(0)}\left(\dfrac{\sin x}{x}\right) = \boxed{1}$

13) $\lim\limits_{\substack{(x,y) \to (1,1) \\ x \neq y}} \dfrac{x^2 - 2xy + y^2}{x - y} = \dfrac{(x-y)^{\cancel{2}}}{\cancel{x-y}}$

$\lim\limits_{\substack{x,y \to (1,1) \\ x \neq y}} x - y = 1 - 1 = \boxed{0}$

17) $\lim\limits_{\substack{(x,y) \to (0,0) \\ x \neq y}} \dfrac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{(\sqrt{x}+\sqrt{y})\cancel{(\sqrt{x}-\sqrt{y})} + 2\sqrt{x} - 2\sqrt{y}}{\cancel{\sqrt{x}-\sqrt{y}}}$

$= \sqrt{x} + \sqrt{y} + 2\sqrt{x} - 2\sqrt{y} = 3\sqrt{x} - \sqrt{y}$

$= \boxed{0}$

(19) $$\lim_{\substack{x,y\to(2,0)\\ 2x-y\neq 4}} \frac{\sqrt{2x-y}-2}{2x-y-4} = \frac{\cancel{\sqrt{2x-y}-2}}{(\sqrt{2x-y}+2)\cancel{(\sqrt{2x-y}-2)}}$$

$$\lim_{x,y\to(2,0)} \frac{1}{\sqrt{2x-y}+2} = \frac{1}{\sqrt{(2)(2)-0}+2} = \boxed{\frac{1}{4}}$$

(27) $$\lim_{P\to(\pi,\pi,0)} (\sin^2 x + \cos^2 y + \sec^2 z)$$

$$= (\sin\pi)^2 + (\cos\pi)^2 + \left(\frac{1}{\cos 0}\right)^2 = 0 + 1 + \frac{1}{1} = \boxed{2}$$

(33) a. $g(x,y) = \sin\frac{1}{xy}$ $\boxed{\text{all } (x,y) \text{ where } x,y \neq 0}$

(35) $f(x,y,z) = x^2 + y^2 - 2z^2$ $\boxed{\text{all } (x,y,z)}$

(37) $h(x,y,z) = xy\sin\frac{1}{z} =$ $\boxed{\text{all } (x,y,z) \text{ except } z = 0}$

(41) $$f(x,y) = \frac{-x}{\sqrt{x^2+y^2}} = \lim_{\substack{x,y\to(0,0)\\ y=x,\ x>0}} \frac{-x}{\sqrt{x^2+y^2}}$$

$$\lim_{\substack{x,y\to(0,0)\\ y=x,\ x>0}} \frac{-x}{\sqrt{x^2+x^2}} = \frac{-x}{x\sqrt{2}} = \boxed{-\frac{1}{\sqrt{2}}}$$

(49) $\lim_{(x,y)\to(1,1)} \frac{xy^2-1}{y-1} = \lim_{(x,y)\to(1,1)} \frac{y(y^2-1)}{y-1} = \lim_{x,y\to(1,1)} \frac{y^2-1}{-1}$

$x = y$

$= \frac{1^2-1}{-1} = \boxed{0}$

Limit DNE

$\lim_{(x,y)\to(1,1)} \frac{(\frac{1}{y})(y^2-1)}{y-1} = \lim_{\substack{x,y\to(1,1)\\ x=\frac{1}{y}}} \frac{y-1}{y-1} = \boxed{1}$

$x = \frac{1}{y}$

(55) Knowing $1 - \frac{x^2y^2}{3} < \frac{\tan^{-1} xy}{xy} < 1$

tell about $\lim_{x,y\to(0,0)} \frac{\tan^{-1} xy}{xy}$?

$\lim_{x,y\to(0,0)} 1 - \frac{x^2y^2}{3} \rightarrow 1 - \frac{(0)^2(0)^2}{3} = 1$

$\therefore\ 1 < \frac{\tan^{-1} xy}{xy} < 1$ - Since Limit of $1 - \frac{x^2y^2}{3} = 1$ and our fxn is between 1 and 1. By Sandwich Theorem, Limit has to be 1.

(57) Knowing that $\left|\sin\frac{1}{x}\right| \leq 1$
tell about $\lim_{(x,y)\to(0,0)} y\sin\frac{1}{x}$?

$-1 \leq \sin\frac{1}{x} \leq \longrightarrow -y \leq y\sin\frac{1}{x} \leq y$

if $y > 0$
then $-y \leq y\sin\frac{1}{x} \leq y$

if $y < 0$
then $-y \geq y\sin\frac{1}{x} \geq y$

Since $-x \to 0$ & $x \to 0$ as $(x,y)\to(0,0)$
then $y\sin\frac{1}{x} = 0 \longrightarrow \lim_{x,y\to(0,0)} y\sin\frac{1}{x} = 0$

(61) $f(x,y) = \dfrac{x^3 - xy^2}{x^2+y^2}$

$$\lim_{x,y\to 0,0} \frac{x^3 - xy^2}{x^2+y^2} = \lim_{x,y\to 0,0} \frac{x(x^2-y^2)}{x^2+y^2} =$$

if $(x,y) \to 0$

then $r \to 0$

$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2+y^2$

$$\lim_{r\to 0} \frac{r\cos\theta(r^2\cos^2\theta - r^2\sin^2\theta)}{r^2}$$

$$\lim_{r\to 0} \frac{\cancel{r^2}\cos\theta(r\cos^2\theta - r\sin^2\theta)}{\cancel{r^2}}$$

$$\lim_{r\to 0} r(\cos^3\theta - \sin^2\theta\cos\theta)$$

$$0(\cos^3\theta - \sin^2\theta\cos\theta)$$

$$= \boxed{0}$$

Steven Romeiro

#1-21 EOO, 23, 27, 29, 33, 37, 39, 43
53, 65, 73, 81

14.3 Homework

(1) $f(x,y) = 2x^2 - 3y - 4$

$\boxed{\frac{\partial f}{\partial x} = 4x}$ $\boxed{\frac{\partial f}{\partial y} = -3}$

(5) $f(x,y) = (xy-1)^2$

$\frac{\partial f}{\partial x} = 2(xy-1)\cdot y \longrightarrow \boxed{\frac{\partial f}{\partial x} = 2y(xy-1)}$

$\frac{\partial f}{\partial y} = 2(xy-1)\cdot x \longrightarrow \boxed{\frac{\partial f}{\partial y} = 2x(xy-1)}$

(9) $f(x,y) = 1/(x+y)$

$\frac{\partial f}{\partial x} = (x+y)^{-1} \longrightarrow \boxed{-\frac{1}{(x+y)^2} = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}}$

(13) $f(x,y) = e^{(x+y+1)}$

$\boxed{\frac{\partial f}{\partial x} = e^{x+y+1}}$ $\boxed{\frac{\partial f}{\partial y} = e^{x+y+1}}$

(17) $f(x,y) = \sin^2(x-3y)$

$\boxed{\frac{\partial f}{\partial x} = 2\sin(x-3y)(\cos x-3y)}$

$\boxed{\frac{\partial f}{\partial y} = -6\sin(x-3y)\cos(x-3y)}$

(21) $f(x,y) = \int_x^y g(t)\,dt = \left[\frac{1}{2}t^2\right]_x^y$

$f(y,y) = \frac{1}{2}y^2 - \frac{1}{2}x^2$

$\boxed{\frac{\partial f}{\partial x} = -x}$ $\boxed{\frac{\partial f}{\partial y} = y}$

(23) $F(x,y,z) = 1 + xy^2 - 2z^2$

$\boxed{f_x = y^2}$ $\boxed{f_y = 2yx}$ $\boxed{f_z = -4z}$

(27) $f(x,y,z) = \sin^{-1}(xyz)$

$\boxed{\frac{\partial f}{\partial x} = \frac{yz}{\sqrt{1-x^2y^2z^2}}}$ $\boxed{\frac{\partial f}{\partial z} = \frac{xy}{\sqrt{1-x^2y^2z^2}}}$ $\boxed{\frac{\partial f}{\partial y} = \frac{xz}{\sqrt{1-x^2y^2z^2}}}$

(29) $f(x,y,z) = \ln(x+2y+3z)$

$\boxed{f_x = \frac{1}{x+2y+3z}}$ $\boxed{F_y = \frac{2}{x+2y+3z}}$ $\boxed{f_z = \frac{3}{x+2y+3z}}$

(33) $f(x,y,z) = \tanh(x+2y+3z)$

$$\boxed{\begin{aligned} f_x &= \operatorname{sech}^2(x+2y+3z) \\ f_y &= 2\operatorname{sech}^2(x+2y+3z) \\ f_z &= 3\operatorname{sech}^2(x+2y+3z) \end{aligned}}$$

(37) $h(\rho, \phi, \theta) = \rho \sin\phi \cos\theta$

$\boxed{h_\rho = \sin\phi \cos\theta}$ $\boxed{h_\phi = \rho \cos\phi \cos\theta}$

$\boxed{h_\theta = -\rho \sin\phi \sin\theta}$

(39) $W(P, V, \delta, v, g) = PV + \dfrac{V\delta v^2}{2g}$

$\boxed{W_P = V}$

$\boxed{W_V = P + \dfrac{\delta v^2}{2g}}$

$\boxed{W_g = -\dfrac{V\delta v^2}{2g^2}}$

$\boxed{W_\delta = \dfrac{Vv^2}{2g}}$ $\boxed{W_v = \dfrac{V\delta v}{g}}$

(43) $g(x, y) = x^2 y + \cos y + y \sin x$

$f_x = 2xy + y \cos x$

$\boxed{f_{xx} = 2y - y \sin x}$

$f_y = y - \sin y + 1$

$\boxed{f_{yy} = -\cos y}$

derivative $\frac{d}{dx}$ wrt "y" $\boxed{f_{xy} = 2x + \cos x}$

53) $w = xy^2 + x^2y^3 + x^3y^4$

$$\frac{\partial w}{\partial x} = y^2 + 2xy^3 + 3x^2y^4$$

$$\boxed{\frac{\partial^2 w}{\partial x \partial y} = 2y + 6xy^2 + 12x^2y^3 = \frac{\partial^2 w}{\partial y \partial x}}$$

65) $\frac{\partial z}{\partial x}$, point $(1,1,1)$ of $xy + z^3x - 2yz = 0$

$$\underbrace{\frac{\partial}{\partial x}(xy)} + \underbrace{\frac{\partial}{\partial x}(z^3x)} - \underbrace{\frac{\partial}{\partial x}(2yz)} = 0$$

$$\frac{\partial x}{\partial x} \cdot y + \left(\frac{\partial z}{\partial x} 3z^2x + \frac{\partial x}{\partial x} z^3\right) - \frac{\partial z}{\partial x} 2y = 0$$

$$(1) \cdot y + \frac{\partial z}{\partial x}(3z^2x) + (1) \cdot z^3 - \frac{\partial z}{\partial x}(2y) = 0$$

$$\frac{\partial z}{\partial x}(3z^2x) - \frac{\partial z}{\partial x}(2y) = -y - z^3$$

$$\frac{\partial z}{\partial x}(3z^2x - 2y) = -y - z^3$$

$$\frac{\partial z}{\partial x} = \frac{-y - z^3}{3z^2x - 2y}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)} = \frac{-1-1}{3(1)(1) - 2(1)} = \boxed{-2}$$

? (73) $f(x,y,z) = x^2 + y^2 - 2z^2$

Laplace = $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z} = 0$

$f_x = 2x \qquad f_y = 2y \qquad f_z = -4z$

$f_{xx} = 2 \qquad f_{yy} = 2 \qquad f_{zz} = -4$

$2 + 2 - 4 = 0$

$4 - 4 = 0$

$\boxed{0 = 0}$

(81) $w = \sin(x + ct)$

One dimensional wave equation

$= \frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$

$w_t = \cos(x + ct) \cdot c \quad | \quad w_x = \cos(x + ct)$

$w_{tt} = \underline{-\sin(x + ct) \cdot c^2} \quad | \quad w_{xx} = \underline{-\sin(x + ct)}$

$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$

$\boxed{-c^2 \sin(x + ct) = -c^2 \sin(x + ct)}$

Steven Romeiro

3, 7, 9, 11?, 13?, 17, 21, 23, 25, 27
29, 33, 41

14.4 Homework

(3) $w = \frac{x}{z} + \frac{y}{z}$, $x = \cos^2 t$ $\quad$ $t = 3$

$y = \sin^2 t$ $\quad$ $z = \frac{1}{t}$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial w}{\partial z} \cdot \frac{dz}{dt}$$

$$\frac{\partial w}{\partial x} = \frac{1}{z} \qquad \frac{\partial w}{\partial y} = \frac{1}{z} \qquad \frac{\partial w}{\partial z} = -\frac{x}{z^2} - \frac{y}{z^2}$$

$$\frac{dx}{dt} = -2\cos t \sin t \qquad \frac{dy}{dt} = 2\sin t \cos t$$

$$\frac{dz}{dt} = -\frac{1}{t^2}$$

$$\frac{dw}{dt} = \cancel{\frac{1}{z}(-2\cos t \sin t)} + \cancel{\frac{1}{z}(2\sin t \cos t)} + \left(\frac{-x}{z^2} - \frac{y}{z^2}\right)\left(-\frac{1}{t^2}\right)$$

$$\frac{dw}{dt} = \left(\frac{+\cos^2 t}{z^2} + \frac{\sin^2 t}{z^2}\right)\left(\frac{1}{t^2}\right)$$

$$\frac{dw}{dt} = \frac{+\cos^2 t (t)^2 + \sin^2 t (t)^2}{t^2} \longrightarrow \frac{\cancel{t^2}(+\cos^2 t + \sin^2 t)}{\cancel{t^2}}$$

$$\frac{dw}{dt} = \cos^2 t + \sin^2 t$$

$$\boxed{\frac{dw}{dt} = 1}$$

(7) $Z = 4e^x \ln y$, $X = \ln(u\cos v)$, $Y = u\sin v$
$(u,v) = (2, \pi/4)$

$$Z = 4e^{(\ln(u\cos v))} \ln(u\sin v)$$

$$Z = 4(u\cos v)\ln(u\sin v)$$

$$\frac{\partial Z}{\partial u} = (4\cos v)(\ln(u\sin v)) + (4u\cos v)\left(\frac{\sin v}{u\sin v}\right)$$

$$\frac{\partial Z}{\partial u} = 4\cos v \ln(u\sin v) + \frac{4u\cos v}{u}$$

$$\frac{\partial Z}{\partial u} = 4\cos v \ln(u\sin v) + 4\cos v$$

$$\frac{\partial Z}{\partial u} = 4\cos(\pi/4)\ln(2\sin \pi/4) + 4\cos\frac{\pi}{4}$$

$$\frac{\partial Z}{\partial u} = 4\frac{\sqrt{2}}{2}\ln(\sqrt{2}) + 4\frac{\sqrt{2}}{2}$$

$$\frac{\partial Z}{\partial u} = 2\sqrt{2}\,\frac{1}{2}\ln 2 + 2\sqrt{2}$$

$$\boxed{\frac{\partial Z}{\partial u} = \sqrt{2}(\ln 2 + 2)}$$

(9) $w = XY + YZ + XZ$

$X = U+V, \quad Y = U-V, \quad Z = UV, \quad (U,V) = (\tfrac{1}{2}, 1)$

$$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial u}$$

$$\frac{\partial w}{\partial x} = Y+Z \qquad \frac{\partial w}{\partial z} = Y+X \qquad \frac{\partial w}{\partial y} = X+Z$$

$$\frac{\partial x}{\partial u} = \frac{\partial y}{\partial u} = 1 \qquad \frac{\partial z}{\partial u} = V$$

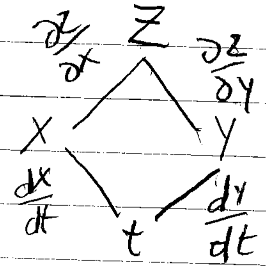
$$\frac{\partial w}{\partial u} = (Y+Z)(1) + (X+Z)(1) + (Y+X)(V)$$

$$\frac{\partial w}{\partial u} = U - V + UV + U + V + UV + UV - V^2 + UV + V^2$$

$$\frac{\partial w}{\partial u} = 2U + 4UV = 2(\tfrac{1}{2}) + 4(\tfrac{1}{2})(1)$$

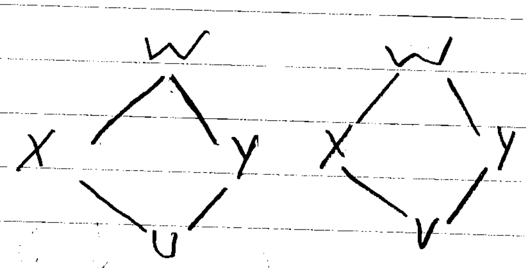
$$\boxed{\frac{\partial w}{\partial u} = 3}$$

(13) $\frac{dz}{dt}$ for $z = f(x,y)$, $x = g(t)$, $y = h(t)$

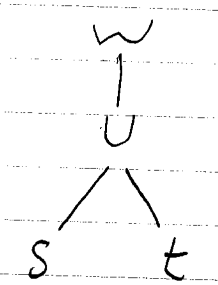


$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

(17) $\frac{\partial w}{\partial u}$ & $\frac{\partial w}{\partial v}$ for $w = g(x,y)$, $x = h(u,v)$, $y = k(u,v)$



(21) $\frac{\partial w}{\partial s}$ & $\frac{\partial w}{\partial t}$ for $w = g(u)$, $u = h(s,t)$



$$\frac{\partial w}{\partial s} = \frac{dw}{du} \cdot \frac{\partial u}{\partial s}$$

$$\frac{\partial w}{\partial t} = \frac{dw}{du} \cdot \frac{\partial u}{\partial t}$$

14.4 HW

41) $V = IR$ $\qquad \frac{dV}{dt} = \frac{\partial V}{\partial I} \cdot \frac{dI}{dt} + \frac{\partial V}{\partial R} \cdot \frac{dR}{dt}$, $R = 600$

$I = .04$

$\frac{dR}{dt} = .5$

$\frac{dV}{dt} = -.01$

$\frac{dI}{dt} = ?$ $\qquad \frac{\partial V}{\partial I} = R$

$\frac{\partial V}{\partial R} = I$ $\qquad -0.01 = R \cdot \frac{dI}{dt} + (.04)(.5)$

$$-.01 = 600\frac{dI}{dt} + .02$$

$$-.03 = 600\frac{dI}{dt} \rightarrow .00005 = \frac{dI}{dt}$$

(23) $\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$ for $w = f(x,y)$, $x = g(r)$, $y = h(s)$

$$\begin{tikzcd} & w \arrow[dl, dash] \arrow[dr, dash] & \\ x \arrow[d, dash] & & y \arrow[d, dash] \\ r & & s \end{tikzcd}$$

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dr}$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial y} \cdot \frac{dy}{ds}$$

(25) $x^3 - 2y^2 + xy = 0 \qquad (1,1)$

Theorem 8: $\frac{dy}{dx} = -\frac{F_x}{F_y}$

$F_x = 3x^2 + y = 0$

$F_y = -4y + x = 0$

$$\frac{dy}{dx} = \frac{-3x^2 - y}{-4y + x} = \frac{-3(1)^2 - 1}{-4(1) + 1} = \boxed{\frac{4}{3}}$$

(27) $x^2 + xy + y^2 - 7 = 0 \quad (1,2)$

Theorem 8: $\frac{dy}{dx} = \frac{-F_x}{F_y}$

$F_x = 2x + y$

$F_y = x + 2y$

$$-\frac{F_x}{F_y} = \frac{-2x - y}{x + 2y} = \frac{-2(1) - 2}{1 + 2(2)} = \boxed{\frac{-4}{5}}$$

(29) $Z^3 - XY + YZ + Y^3 - 2 = 0 \quad (1,1,1)$

$\frac{d}{dx}(Z)^3 - \frac{d}{dx}(XY) + \frac{d}{dx}(YZ) + \frac{d}{dx}(Y^3) - \frac{d}{dx}(2) = 0$

$\frac{dZ}{dx} 3Z^2 - \frac{dx}{dx} Y + \frac{dZ}{dx} = 0$

$\frac{dZ}{dx}(3Z^2 + Y) - Y = 0 \rightarrow \frac{dZ}{dx} = \frac{Y}{3Z^2+Y} = \frac{1}{3(1)+1} = \boxed{\frac{1}{4}}$

$\frac{d}{dy}(Z)^3 - \frac{d}{dy}(XY) + \frac{d}{dy}(YZ) + \frac{d}{dy}(Y^3) - \frac{d}{dy}(2) = 0$

$\frac{dZ}{dy}(3Z^2) - \frac{dy}{dy}(X) + \frac{dZ}{dy}(Y)(Y) + \frac{dy}{dy}(Z)(Z) + \frac{dy}{dy}(3y^2) = 0$

$\frac{dZ}{dy}(3Z^2 + Y^2) + \frac{dy}{dy}(-X + Z^2 + 3Y^2) = 0$

$\left.\frac{dZ}{dy}\right|_{(1,1,1)} = \frac{1 - 1 - 3(1)}{3(1)+1} = \boxed{\frac{-3}{4}}$

(41) $\frac{dV}{dt} = \frac{\partial V}{\partial I} \cdot \frac{dI}{dT} + \frac{\partial V}{\partial R} \cdot \frac{dR}{dt}$

$V = IR \qquad R = 600\,\Omega, \quad I = 0.04\,A \qquad V = 24v$

$\frac{dR}{dt} = 0.5 \frac{\Omega}{s}, \quad \frac{dV}{dt} = -0.01 \frac{V}{s}$

$I = \frac{V}{R}$ need $\frac{dI}{dt} = \dfrac{\frac{\partial I}{\partial V} \cdot \frac{dV}{dt}}{\frac{\partial I}{\partial R} \cdot \frac{dR}{dt}}$

$\frac{\partial I}{\partial R} = \ln(R) \qquad \frac{\partial I}{\partial V} = \frac{1}{R}$

$\frac{\partial I}{\partial R} = \ln(600) \qquad \frac{\partial I}{\partial V} = \frac{1}{600}$

$$\frac{dI}{dt} = \frac{\frac{\partial I}{\partial V} \cdot \frac{dV}{dt}}{\frac{\partial I}{\partial R} \cdot \frac{dR}{dt}}$$

$$\frac{dI}{dt} = \frac{\ln(R) \cdot (-.01)}{\frac{1}{R} \cdot (0.5)} = \frac{\ln(600)\,(-.01)}{\frac{1}{600} \cdot (0.5)}$$

$$\boxed{\frac{dI}{dt} = -76.763\ A/s}$$

Steven Romeiro

#1, 3, 7, 11, 15, 19, 21, 27

14.5 Homework

(1) $f(x,y) = y - x$, $(2,1)$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j$$

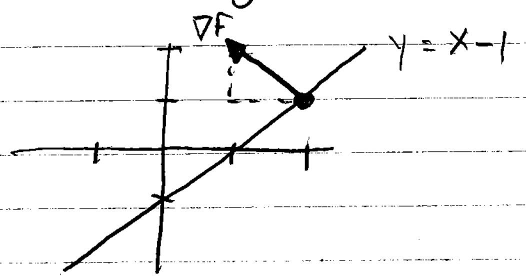
$f_x = -1 \quad f_y = 1$

$\nabla f = -i + j$

Level Curve:

$f(2,1) = 1 - 2 = -1$

$f(x,y) = y - x = -1 \rightarrow y = x - 1$



(3) $g(x,y) = xy^2$, $(2,-1)$

$\nabla f = f_x i + f_y j$

$\boxed{\nabla f = i - 4j}$

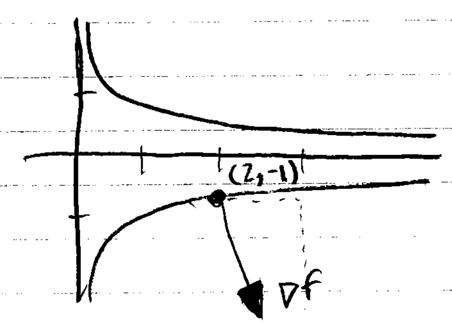
$f_x = y^2 \rightarrow f_x|_{(2,-1)} = 1$

$f_y = 2xy \rightarrow f_y|_{(2,-1)} = -4$

Level Curve:

$g(2,-1) = (2)(-1)^2 = 2$

$xy^2 = 2 \rightarrow y = \sqrt{\frac{2}{x}}$



7 $f(x,y,z) = x^2+y^2-2z^2+z\ln x$, $(1,1,1)$

$f_x = 2x + \frac{z}{x} \rightarrow f_x|_{(1,1,1)} = 2+1 = 3$

$f_y = 2y \longrightarrow f_y|_{(1,1,1)} = 2$

$f_z = -4z + \ln x \rightarrow f_z|_{(1,1,1)} = -4(1)+\ln(1) = -4$

$\boxed{\nabla F = 3i + 2j - 4k}$

11 $f(x,y) = 2xy - 3y^2$, $P_0(5,5)$, $\vec{u} = 4i+3j$

Directional Derivative

$D_{\vec{u}}F = \nabla F \cdot \frac{\vec{u}}{|\vec{u}|}$

$|\vec{u}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$ $\quad \frac{\vec{u}}{|\vec{u}|} = \frac{4}{5}i + \frac{3}{5}j$

$F_x = 2y \longrightarrow F_x|_{P_0} = 2(5) = 10$

$F_y = 2x - 6y \rightarrow F_y|_{P_0} = 2(5)-6(5) = -20$

$D_{\vec{u}}F = \langle 10, -20\rangle \cdot \langle \frac{4}{5}, \frac{3}{5}\rangle$

$D_{\vec{u}}F = (10)(\frac{4}{5}) + (-20)(\frac{3}{5}) \rightarrow 8 + (-12) = \boxed{-4}$

(15) $F(x,y,z) = xy + yz + zx$, $P_0(1,-1,2)$, $\vec{u} = 3i + 6j - 2k$

$D_{\vec{u}}F = \nabla F \cdot \frac{\vec{u}}{|\vec{u}|}$

$|\vec{u}| = \sqrt{9+36+4} = \sqrt{49} = 7$ $\quad \frac{\vec{u}}{|\vec{u}|} = \frac{3}{7}i + \frac{6}{7}j - \frac{2}{7}k$

$F_x = y + z \rightarrow F_x|_{P_0} = -1 + 2 = 1$

$F_y = x + z \rightarrow F_y|_{P_0} = 1 + 2 = 3$

$F_z = y + x \rightarrow F_z|_{P_0} = -1 + 1 = 0$

$\nabla F = 1i + 3j$

$D_{\vec{u}}F = \langle 1, 3, 0 \rangle \cdot \langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \rangle$

$D_{\vec{u}}F = (1)(3/7) + (3)(\frac{6}{7}) + (0)(-2/7) = \frac{3+18}{7} = \frac{21}{7} = \boxed{3}$

(19) $F(x,y) = x^2 + xy + y^2$, $P_0(-1,1)$

$\nabla F = F_x i + F_y j$

$F_x = 2x + y \rightarrow F_x|_{P_0} = 2(-1) + 1 = -1$

$F_y = x + 2y \rightarrow F_y|_{P_0} = (-1) + 2(1) = 1$

$\nabla F = -i + j$; $|\nabla F| = \sqrt{2}$

$\frac{\nabla F}{|\nabla F|} = -\frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$ increases rapidly

$\frac{-\nabla F}{|\nabla F|} = \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$ decrease rapidly

$$\frac{\nabla f}{|\nabla f|} = \text{direction of fastest increase}$$

(21) $F(X,Y,Z) = (X/Y) - YZ \quad , \quad P_0(4,1,1)$

$F_x = \frac{1}{Y} \longrightarrow F_x|_{P_0} = \frac{1}{1} = \boxed{1}$

$F_y = \frac{-X}{Y^2} - Z \longrightarrow f_y|_{P_0} = -\frac{4}{1} - 1 = -5$

$F_z = -Y \longrightarrow f_z|_{P_0} = -1$

$\nabla F = f_x i + f_y j + f_z K \longrightarrow 1\vec{i} - 5\vec{j} - 1\vec{K}$

$|\nabla F| = \sqrt{1^2 + 5^2 + 1^2} = \sqrt{27}$

$\vec{U} = \frac{\nabla f}{|\nabla f|} = \frac{\hat{i} - 5\hat{j} - \hat{K}}{\sqrt{27}}$

$\boxed{\text{fastest increase in } \frac{\hat{i} - 5\hat{j} - \hat{K}}{\sqrt{27}} \quad \text{fastest decrease in } \frac{-\hat{i} + 5\hat{j} + \hat{K}}{\sqrt{27}}}$

$D_{\vec{u}} f = |u||v| \cos\theta \quad , \quad \theta = 0$

$D_{\vec{u}} f = |U||M|(1) \longrightarrow \sqrt{27}(1) \longrightarrow \boxed{D_{\vec{u}} f = \sqrt{27}}$

$D_{-\vec{u}} f = -D_{\vec{u}} f \longrightarrow \boxed{D_{-\vec{u}} f = -\sqrt{27}}$

* OR $D_{\vec{u}} f = \nabla f \cdot \vec{u} \longrightarrow \langle 1, -5, -1 \rangle \cdot \langle \frac{1}{\sqrt{27}}, -\frac{5}{\sqrt{27}}, -\frac{1}{\sqrt{27}} \rangle$

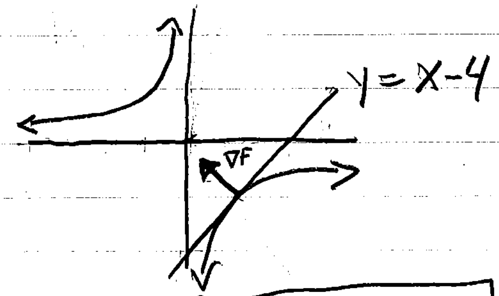
$D_{\vec{u}} f = (1)\left(\frac{1}{\sqrt{27}}\right) + (-5)\left(\frac{-5}{\sqrt{27}}\right) + (-1)\left(\frac{-1}{\sqrt{27}}\right)$

$D_{\vec{u}} f = \frac{1}{\sqrt{27}} + \frac{25}{\sqrt{27}} + \frac{1}{\sqrt{27}}$

$D_{\vec{u}} f = \frac{27}{\sqrt{27}} \longrightarrow \frac{27}{\sqrt{27}} \cdot \left(\frac{\sqrt{27}}{\sqrt{27}}\right) \longrightarrow \frac{\cancel{27}\sqrt{27}}{\cancel{27}} \longrightarrow \boxed{D_{\vec{u}} f = \sqrt{27}}$

27) $xy = -4$, $(2,-2)$

$y = \frac{-4}{x}$



$\nabla f = f_x\, i + f_y\, j \longrightarrow \boxed{\nabla f = -2i + 2j}$

$f_x = y \longrightarrow f_x|_{p_0} = -2$

$f_y = x \longrightarrow f_y|_{p_0} = 2$

Equation for tangent Line:

$$f_x|_{p_0}(x - x_0) + f_y|_{p_0}(y - y_0)$$

$$-2(x-2) + 2(y+2) = 0$$

$$(-2x + 4 + 2y + 4 = 0) \div 2$$

$$-x + 2 + y + 2 = 0$$

$\boxed{y = x - 4} \longrightarrow$ eq. to tangent line

8

Steven Romero

#1, 3, 9, 11, 19, 23, 29

14.6 Homework

① $x^2+y^2+z^2=3$, $P_0(1,1,1)$

a) Tangent plane: $f_x|_{p_0}(x-x_0)+f_y|_{p_0}(y-y_0)+f_z|_{p_0}(z-z_0)=0$

b) Normal line @ P_0: $x=x_0+f_x|_{p_0}(t)$, $y=y_0+f_y|_{p_0}(t)$

$f_x=2x \rightarrow f_x|_{p_0}=2$

$f_y=2y \rightarrow f_y|_{p_0}=2$

$f_z=2z \rightarrow f_z|_{p_0}=2$

Normal line

$$\boxed{\begin{array}{l} x=1+2t \\ y=1+2t \qquad -\infty<t<\infty \\ z=1+2t \end{array}}$$

Tangent plane

$2(x-1)+2(y-1)+2(z-1)=0$

$(2x-2+2y-2+2z-2=0)\div 2$

$x-1+y-1+z-1=0$

$x+y+z-3=0$

$$\boxed{x+y+z=3}$$

③ $2z-x^2=0$, $P_0(2,0,2)$

$f_x=-2x \rightarrow f_x|_{p_0}=-4$

$f_z=2 \rightarrow f_z|_{p_0}=2$

Normal line: $x=x_0+f_x|_{p_0}(t)$, $z=z_0+f_z|_{p_0}(t)$

$x=2+(-4)t \rightarrow$ $x=2-4t$ $\quad -\infty<t<\infty$

$z=2+2(t) \rightarrow$ $z=2+2t$

$y=0$

equation of tangent Plane:

$f_x|_{p_0}(x-x_0)+f_y|_{p_0}(y-y_0)+f_z|_{p_0}(z-z_0)=0$

$-4(x-2)+0+2(z-2)=0$

$(-4x+8+2z-4=0)\div 2$

$-2x+z=-2$

$$\boxed{z=2x-2 \quad @\ y=0}$$

$(y-x)^{1/2} = \frac{1}{2}(y-x)^{-1/2}$

(9) $z = \ln(x^2+y^2)$, $(1,0,0)$

equation tangent to plane surface $z = f(x,y)$

$f_x|_{p_0}(x-x_0) + f_y|_{p_0}(y-y_0) - (z-z_0)$

$f_x = \frac{2x}{x^2} = \frac{2}{1}$ $\quad f_y = \frac{2}{y} = \frac{2}{0} = 0$

eq: $2(x-1) + 0(y-0) - (z-0) = 0$

$2x - 2 + 0 - z = 0$

$\boxed{2x - z - 2 = 0}$ //

(11) $z = \sqrt{y-x}$, $(1,2,1)$

$f_x = (y-x)^{1/2-1}\left(\frac{1}{2}\right)(-1) = \frac{-1}{2\sqrt{y-x}} \rightarrow f_x|_{p_0} = \frac{-1}{2(1)} = -\frac{1}{2}$ //

$f_y = \frac{1}{2\sqrt{y-x}} \rightarrow f_y|_{p_0} = \frac{1}{2(1)} = \frac{1}{2}$ //

equation tangent to a surface:

$f_x|_{p_0}(x-x_0) + f_y|_{p_0}(y-y_0) - (z-z_0) = 0$

$-\frac{1}{2}(x-1) + \frac{1}{2}(y-2) - (z-1) = 0$

$-\frac{1}{2}x + \frac{1}{2} + \frac{1}{2}y - 1 - z + 1 = 0$

$\left(-\frac{1}{2}x + \frac{1}{2}y - z + \frac{1}{2} = 0\right) * 2$

$\boxed{-x + y - 2z + 1 = 0}$ //

By how much will

(19) $F(x,y,z) = \ln\sqrt{x^2+y^2+z^2} = \frac{1}{2}\ln(x^2+y^2+z^2)$
if point $P(x,y,z)$ moves from $P_0(3,4,12)$ a distance of $ds = 0.1$ in the direction of $\vec{u} = 3\vec{i} + 6\vec{j} - 2\vec{k}$

Change in fxn $= df = (\nabla f|_{p_0} \cdot \vec{u})\,ds$

$f_x = \dfrac{2x}{2(x^2+y^2+z^2)} = \dfrac{x}{x^2+y^2+z^2} \rightarrow \dfrac{3}{9+16+144} = \dfrac{3}{169}$ //

$f_y = \dfrac{2y}{2(x^2+y^2+z^2)} = \dfrac{y}{x^2+y^2+z^2} \rightarrow f_y|_{p_0} = \dfrac{4}{169}$ //

$f_z = \dfrac{2z}{2(x^2+y^2+z^2)} = \dfrac{z}{x^2+y^2+z^2} \rightarrow f_z|_{p_0} = \dfrac{12}{169}$ //

$\nabla f = \dfrac{3}{169}i + \dfrac{4}{169}j + \dfrac{12}{169}k$;

$\hat{u} = \dfrac{\vec{u}}{|\vec{u}|} = \dfrac{3i+6j-2k}{7}$

$df = \left(\left\langle \frac{3}{169}, \frac{4}{169}, \frac{12}{169}\right\rangle \cdot \left\langle \frac{3}{7}, \frac{6}{7}, \frac{-2}{7}\right\rangle\right)\left(\frac{1}{10}\right)$

$df = \left(\dfrac{9 + \cancel{24} - \cancel{24}}{1183}\right)\left(\dfrac{1}{10}\right)$

$$\boxed{df = \frac{9}{11830} \approx 0.0008}$$

?

(23) $T(x,y) = x\sin(2y)$ $\quad ds = 2\ m/s$

radius $= i + j$

a) $P(1/2, \sqrt{3}/2)$

(29) $f(x,y) = e^x \cos y$, $(0,0)$, $(0, \pi/2)$

$L(x,y) = f(x_0, y_0) + f_x|_{p_0}(x - x_0) + f_y|_{p_0}(y - y_0)$

$f(x_0, y_0) = e^0 \cos(0) = 1$

$f_x = e^x \cos y \rightarrow f_x|_{p_0} = e^0 \cos 0 = 1$

$f_y = -e^x \sin y \rightarrow f_y|_{p_0} = -e^0 \sin(0) = 0$

$L(x,y) = 1 + 1(x-0) + 0(y-0)$

$\boxed{L(x,y) = 1 + x}$

$L(x,y) = 0 + 0(x-0) - 1(y - \pi/2)$

$\boxed{L(x,y) = -y + \pi/2}$

Name Steven Romeiro

Instructor: Wendy Pogoda

104
-2

$102 + 5 = 107$

+5

MAC2313 - [redacted] Exam 1 - Spring 2016

Directions:Each question is worth 6 points unless specified otherwise. Show all major steps for partial credit and ~~mark your~~ final answer clearly. Good luck!

Find the center and radius of the sphere.

1) $x^2 + y^2 + z^2 - 14x - 6y - 14z = -91$

$$x^2 - 14x + y^2 - 6y + z^2 - 14z = -91$$

$$x^2 - 14x + \underline{49} + y^2 - 6y + \underline{9} + z^2 - 14z + \underline{49} = -91 + \underline{49} + \underline{9} + \underline{49}$$

$\frac{14}{2} = 7^2$ $\quad \frac{6}{2} = 3^2$ $\quad \frac{14}{2} = 7^2$

$$(x-7)^2 + (y-3)^2 + (z-7)^2 = 16$$

Center: $(7,3,7)$

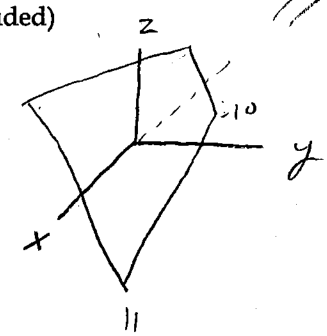
Radius = 4

$\boxed{\text{center}: (7,3,7) \quad r = 4}$

Write one or more inequalities that describe the set of points.

2) The slab bounded by the planes x = -10 and x = 11 (planes included)

$$-\infty < y < \infty$$
$$-\infty < z < \infty$$
$$-10 \leq x \leq 11$$



Calculate the direction of $\overrightarrow{P_1P_2}$ and the midpoint of line segment P_1P_2.

3) $P_1(7, 4, -6)$ and $P_2(13, 6, -9)$

$$\overrightarrow{P_1P_2} = \langle 6, 2, -\cancel{13} \rangle$$

$$|\overrightarrow{P_1P_2}| = \sqrt{6^2 + 2^2 + (-15)^2} = \sqrt{2\cancel{65}}$$

$$\boxed{\frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \left(\frac{6\vec{i}}{\sqrt{265}} + \frac{2\vec{j}}{\sqrt{265}} - \frac{15\vec{k}}{\sqrt{265}}\right) \text{ direction}}$$

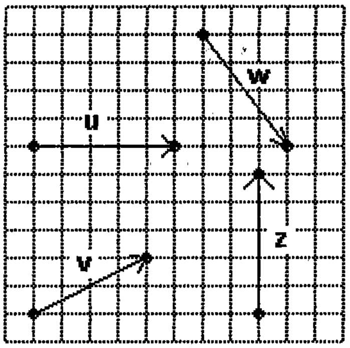
-1

$$\text{midpoint} = \left(\frac{7+13}{2}, \frac{4+6}{2}, \frac{-6-9}{2}\right)$$

$$\boxed{\text{midpoint} = \left(10, 5, \frac{-15}{2}\right)}$$

-1

Use the vectors u, v, w, and z head to tail as needed to sketch the indicated vector.

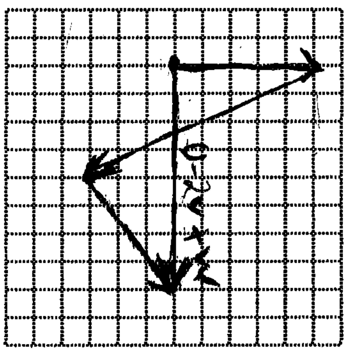


$U = 5\vec{i}$
$V = 4\vec{i} + 2\vec{j}$
$w = 3\vec{i} - 4\vec{j}$

$U - 2v + w = 5\vec{i} - 8\vec{i} + 3\vec{i}, -4\vec{j} - 4\vec{j}$

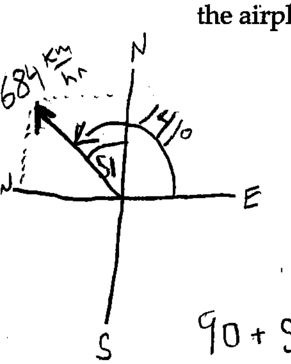
$U - 2v + w = 0\vec{i}, -8\vec{j}$

4) **u – 2v + w**



Solve the problem.

5) An airplane is flying in the direction 51° west of north at 684 km/hr. Find the component form of the velocity of the airplane, assuming that the positive x–axis represents due east and the positive y–axis represents due north.



$\langle 684\cos(141), 684\sin(141)\rangle$

$\boxed{\langle -531.6, 430.5\rangle}$

$90 + 51 = 141°$ from "E"

Find v · u.

6) **v** = 6i - 5j and **u** = -2i - 8j

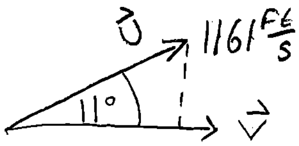
$\vec{V}\cdot\vec{U} = (6)(-2) + (-5)(-8)$

$\vec{V}\cdot\vec{U} = -12 + 40$

$\boxed{\vec{V}\cdot\vec{U} = 28}$

Solve the problem.

7) A bullet is fired with a muzzle velocity of 1161 ft/sec from a gun aimed at an angle of 11° above the horizontal. Find the horizontal component of the velocity.



horizontal component $= |proj_{\vec{V}} \vec{U}|$

$|proj_{\vec{V}} \vec{U}| = |\vec{U}| \cos\theta = 1161 \cos(11)$

$\boxed{|proj_{\vec{V}} \vec{U}| = 1139.7}$ ft/sec ✓

Find the acute angle, in degrees, between the lines.

8) $3x - y = -14$ and $2x + y = -13$ $x+3y$ $x-3y$

$\vec{U} = \langle 1, 3 \rangle$ $\quad \vec{V} = \langle 1, -2 \rangle$

$\vec{U} \cdot \vec{V} = (1)(1) + (3)(-2)$ $\quad |\vec{U}| = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\vec{U} \cdot \vec{V} = -5$ $\quad |\vec{V}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$\cos\theta = \frac{\vec{U} \cdot \vec{V}}{|\vec{U}||\vec{V}|} \longrightarrow \theta = \cos^{-1}\left(\frac{\vec{U} \cdot \vec{V}}{|\vec{U}||\vec{V}|}\right) \longrightarrow \theta = \cos^{-1}\left(\frac{-5}{\sqrt{10}\sqrt{5}}\right) \longrightarrow \boxed{\theta = 135°}$

45° is better answer

Find the length and direction (when defined) of u × v.

9) u = 2i + 2j – k, v = –i + k

$$\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ 2 & 2 & -1 \\ -1 & 0 & 1 \end{vmatrix}$$

$\vec{U} \times \vec{V} = (2-0)i - (2-1)j + (0-(-2))k$

$\vec{U} \times \vec{V} = 2i - 1j + 2k$

$|\vec{U} \times \vec{V}| = \sqrt{2^2 + (-1)^2 + 2^2} \longrightarrow \boxed{|\vec{U} \times \vec{V}| = 3}$ length

$\boxed{\frac{\vec{U} \times \vec{V}}{|\vec{U} \times \vec{V}|} = \frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}}$ direction ✓

Solve the problem.

10) Find the area of the parallelogram determined by the points P(7, –5, 5), Q(–7, 2, –2), R(10, 1, 3) and S(–4, 8, –4).

$\vec{PQ} = \langle -14, 7, -7 \rangle$

$\vec{PR} = \langle 3, 6, -2 \rangle$

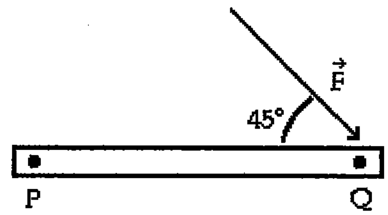
$$\text{Area} = |\vec{PQ} \times \vec{PR}| = \begin{vmatrix} i & j & k \\ -14 & 7 & -7 \\ 3 & 6 & -2 \end{vmatrix}$$

$|\vec{PQ} \times \vec{PR}| = (-14 + 42)i - (28 + 21)j + (-84 - 21)k$

$|\vec{PQ} \times \vec{PR}| = 28\vec{i} - 49\vec{j} - 105\vec{k} \longrightarrow |\vec{PQ} \times \vec{PR}| = \sqrt{28^2 + (-49)^2 + (-105)^2}$

$|\vec{PQ} \times \vec{PR}| = \sqrt{14210} \longrightarrow |\vec{PQ} \times \vec{PR}| = \sqrt{49 * 290} \longrightarrow \boxed{|\vec{PQ} \times \vec{PR}| = 7\sqrt{290}}$ ✓

11) Find the magnitude of the torque in foot-pounds at point P for the following lever:



$|\overrightarrow{PQ}| = 6$ in. and $|\mathbf{F}| = 10$ lb

$|\vec{PQ}| = \dfrac{6\text{ in}}{} \left|\dfrac{1\text{ ft}}{12\text{ in}}\right| = \dfrac{1}{2}\text{ ft}$

$|\text{Torque}| = |\vec{PQ}||\vec{F}|\sin\theta$

$|\text{Torque}| = (\tfrac{1}{2})(10)\sin(45°) \rightarrow \dfrac{\sqrt{2}}{2}$

$\boxed{|\text{Torque}| = \cancel{3.5}\text{ foot-pounds}}$

−1

Find parametric equations for the line described below.

12) The line through the point P(-3, 2, 3) and perpendicular to the vectors **u** = -7**i** - 4**j** + 2**k** and **v** = -2**i** + 4**j** - 6**k**

$$\vec{U} \times \vec{V} = \begin{array}{c} \begin{matrix} i & j & K \end{matrix} \\ \begin{vmatrix} -7 & -4 & 2 \\ -2 & 4 & -6 \end{vmatrix} \end{array} = (24-8)i - (42+4)j + (-28-8)k = \underbrace{16\vec{i}}_{V_1} - \underbrace{46\vec{j}}_{V_2} - \underbrace{36\vec{k}}_{V_3}$$

$P(\underset{X_0}{-3}, \underset{Y_0}{2}, \underset{Z_0}{3})$

$X = X_0 + tV_1 \longrightarrow X = -3 + t(16) \longrightarrow$

$Y = Y_0 + tV_2 \longrightarrow Y = 2 + t(-46) \longrightarrow$

$Z = Z_0 + tV_3 \longrightarrow Z = 3 + t(-36) \longrightarrow$

$$\boxed{\begin{aligned} X &= -3 + 16t \\ Y &= 2 - 46t \qquad -\infty < t < \infty \\ Z &= 3 - 36t \end{aligned}}$$

Find a parametrization for the line segment joining the points.

13) (-4, 0, 5), (0, 3, 0)

$P(\underset{X_0}{-4}, \underset{Y_0}{0}, \underset{Z_0}{5})$

$S(0, 3, 0)$

$\vec{PS} = \langle \underset{V_1}{4}, \underset{V_2}{3}, \underset{V_3}{-5} \rangle$

$X = X_0 + tV_1 \longrightarrow$

$Y = Y_0 + tV_2 \longrightarrow$

$Z = Z_0 + tV_3 \longrightarrow$

$$\boxed{\begin{aligned} X &= -4 + 4t \\ Y &= 3t \qquad 0 \leq t \leq 1 \\ Z &= 5 - 5t \end{aligned}}$$

$0 \leq Y \leq 3$

$0 \leq 3t \leq 3$

$0 \leq t \leq 1$

T−1

$X_0 \quad Y_0 \quad Z_0$ $\quad\quad \vec{n} = \langle 2, 3, 4\rangle$

Write the equation for the plane.

$v_1 \quad v_2 \quad v_3$

14) The plane through the point P(-4, -1, 9) and perpendicular to the line x = 1 + 2t, y = 8 + 3t, z = 6 + 4t

$Ax + by + Cz = Ax_0 + by_0 + Cz_0$

$2x + 3y + 4z = 2(-4) + 3(-1) + 4(9)$

$\boxed{2x + 3y + 4z = 25}$

✓

Calculate the requested distance.

15) The distance from the point S(-7, -5, -3) to the plane 11x + 10y + 2z = 9

$d = \left|\vec{PS} \cdot \frac{\vec{n}}{|\vec{n}|}\right|$ $\quad P = (1, 0, -1)$ $\quad \vec{PS} = \langle -8, -5, -2\rangle$ $\quad \vec{n} = \langle 11, 10, 2\rangle$ $\quad |\vec{n}| = \sqrt{11^2 + 10^2 + 2^2} = 15$

$\frac{\vec{n}}{|\vec{n}|} = \frac{11}{15}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{15}\vec{k}$

$d = \left|(-8)(\frac{11}{15}) + (-5)(\frac{2}{3}) + (-2)(\frac{2}{15})\right|$

$d = \left|(\frac{-88}{15}) + (\frac{-10}{3}) + (\frac{-4}{15})\right|$

$\rightarrow d = \left|-\frac{142}{15}\right|$

$\boxed{d \approx 9.47}$ ✓

Find the intersection.

16) -2x + 9y = 5, 9y - 5z = 9

$\vec{n}_1 = \langle -2, 9, 0\rangle$

$\vec{n}_2 = \langle 0, 9, -5\rangle$

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ -2 & 9 & 0 \\ 0 & 9 & -5 \end{vmatrix} = (-45-0)i - (10-0)j + (-18-0)k$

$\vec{n}_1 \times \vec{n}_2 = -45\vec{i} - 10\vec{j} - 18\vec{k}$

$\langle v_1, v_2, v_3\rangle$

$P(\overset{x_0}{2}, \overset{y_0}{1}, \overset{z_0}{0})$

$\begin{cases} -2x + 9y = 5 \\ 9y - 5z = 9 \end{cases}$

Let $z = 0$

$\begin{cases} -2x + 9y = 5 \\ 9y - 0 = 9 \end{cases}$

Solve for "y"

$y = 1$

$-2x + 9(1) = 5$

$-2x = 5 - 9$

$x = 2$

$X = X_0 + tV_1 \rightarrow$ $\quad X = 2 - 45t$

$Y = Y_0 + tV_2 \rightarrow$ $\quad Y = 1 - 10t$ $\quad -\infty < t < \infty$

$Z = Z_0 + tV_3 \rightarrow$ $\quad Z = -18t$

✓

$\frown$ ☺

Match the equation with the surface it defines. (4 points)

17) $\dfrac{y^2}{100} + \dfrac{z^2}{25} = 1$

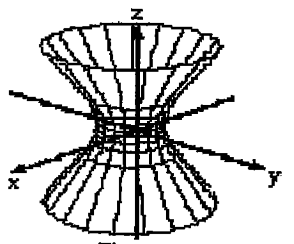


Figure 1

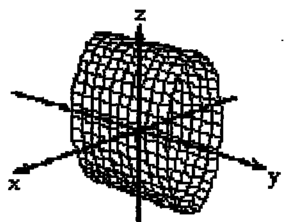


Figure 2

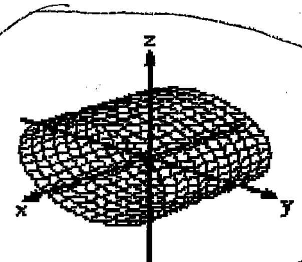


Figure 3

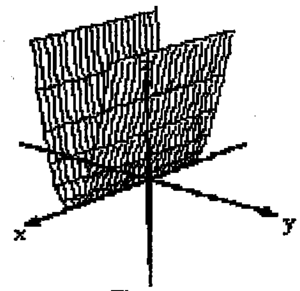


Figure 4

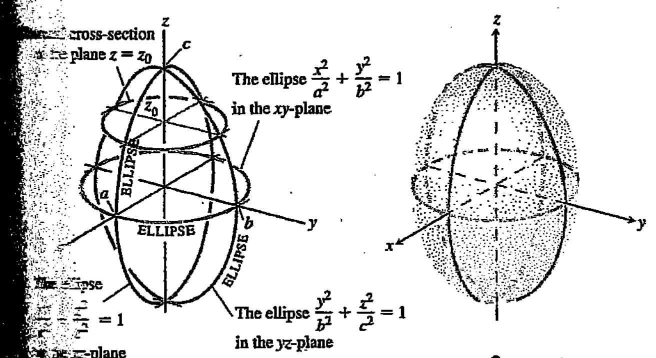
A) Figure 3 B) Figure 4 C) Figure 2 D) Figure 1

Identify the type of surface represented by the given equation. (4 points)

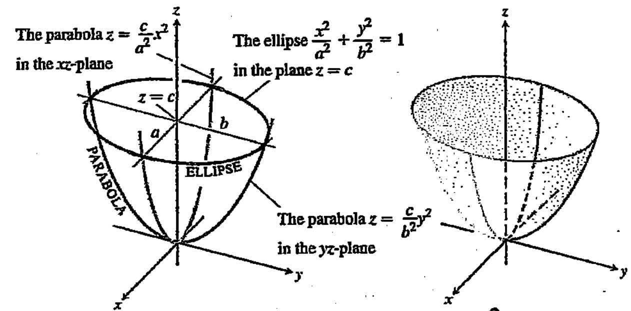
18) $\dfrac{x^2}{9} + \dfrac{y^2}{6} + \dfrac{z^2}{10} = 1$

A) Ellipsoid B) Elliptical cone C) Paraboloid D) Sphere

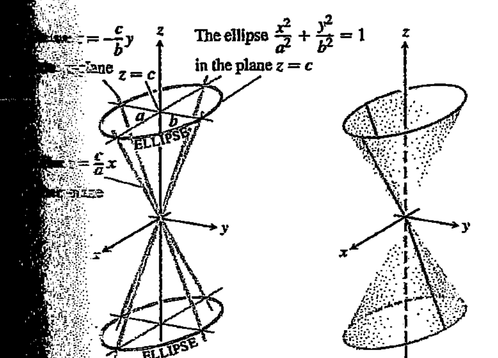
...LE 12.1 Graphs of Quadric Surfaces



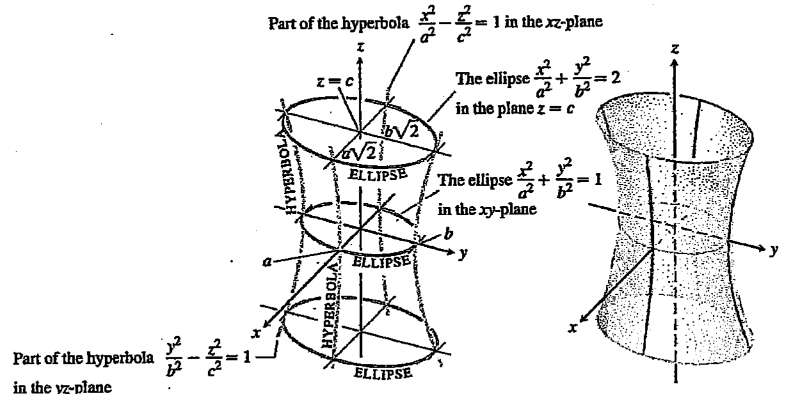
ELLIPSOID $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



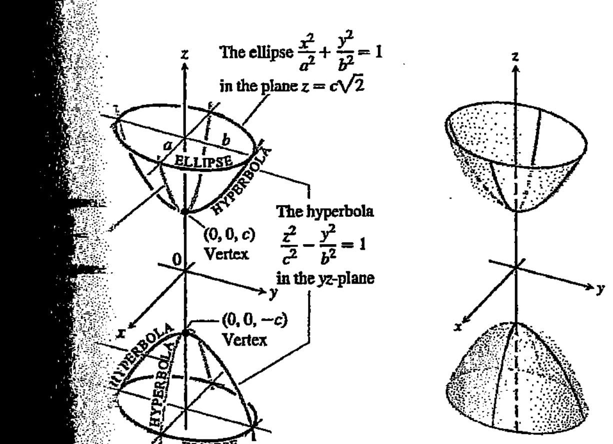
ELLIPTICAL PARABOLOID $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$



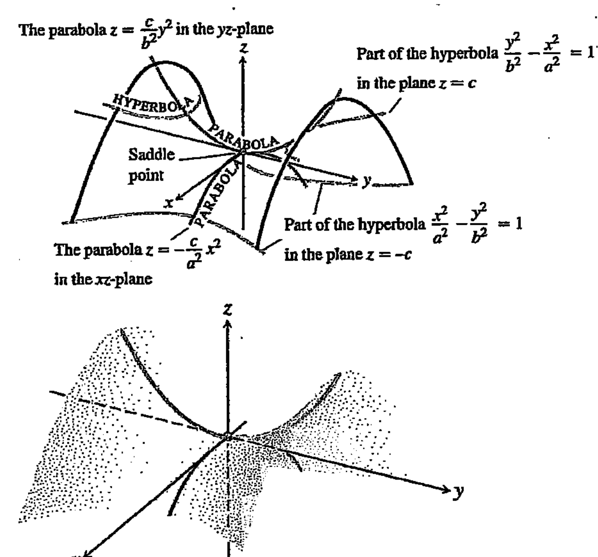
...IPTICAL CONE $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$



HYPERBOLOID OF ONE SHEET $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



...RBOLOID OF TWO SHEETS $\quad \frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



HYPERBOLIC PARABOLOID $\quad \frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, c > 0$

MAC2313 - Practice Exam 1

Directions: This is a practice test. Your actual exam will be different. Use this practice exam to evaluate the areas in which you need additional studying and practice. Exam 1 will cover Chapter 12 and will be shorter than the practice exam.

Describe the given set of points with a single equation or with a pair of equations.

1) The circle of radius 9 centered at the point (4, -6, 81) and lying in a plane parallel to the xy-plane

$r = 9$

Center $= (4, -6, 81)$

$(x-4)^2 + (y+6)^2 = 81, \quad z = 81$

Write one or more inequalities that describe the set of points.

2) The closed region bounded by the spheres of radius 4 and 9, both centered at the origin, and the planes x = 1 and x = 4

Find the distance between points P_1 and P_2.

3) $P_1(4, 7, -3)$ and $P_2(5, 8, -4)$

Find the center and radius of the sphere.

4) $3x^2 + 3y^2 + 3z^2 - 2x + 2y = 9$

$x^2 + y^2 + z^2 = r^2$

$(3x^2 - 2x + 3y^2 + 2y + 3z^2 = 9) \div 3$

$x^2 - \frac{2}{3}x + y^2 + \frac{2}{3}y + z^2 = 3$

$x^2 - \frac{2}{3}x + \frac{1}{9} + y^2 + \frac{2}{3}y + \frac{1}{9} + z^2 = 3 + \frac{1}{9} + \frac{1}{9}$

$(x - \frac{1}{3})^2 + (y + \frac{1}{3})^2 + z^2 = \frac{29}{9}$

$\boxed{\text{Center} = (\frac{1}{3}, -\frac{1}{3}, 0)}$

$\boxed{r = \frac{\sqrt{29}}{3}}$

Find the magnitude.

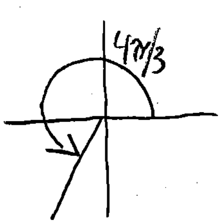
5) Let $\mathbf{u} = \langle -1, 3 \rangle$ and $\mathbf{v} = \langle 0, 1 \rangle$. Find the magnitude (length) of the vector: $-4\mathbf{u} - \mathbf{v}$.

Find the component form of the specified vector.

6) The unit vector that makes an angle $4\pi/3$ with the positive x-axis

$\langle \overset{x}{\cos \frac{4\pi}{3}}, \overset{y}{\sin \frac{4\pi}{3}} \rangle$

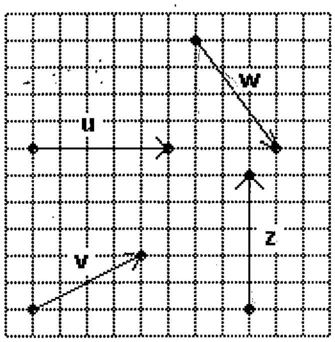
$\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \rangle$



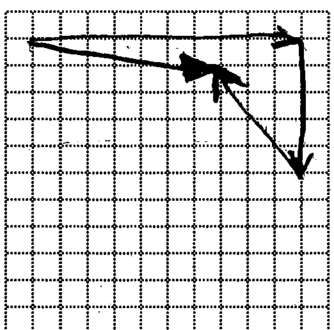
Express the vector in the form $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.

7) $\overrightarrow{AB}$ if A is the point (-1, -9, 1) and B is the point (4, -16, 4)

Use the vectors u, v, w, and z head to tail as needed to sketch the indicated vector.



8) $2\mathbf{u} - \mathbf{z} - \mathbf{w}$



Express the vector as a product of its length and direction.

9) $3\mathbf{j} + \frac{8}{5}\mathbf{k}$

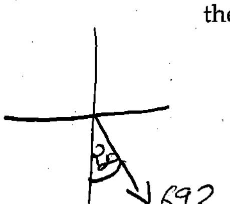
Calculate the direction of $\overrightarrow{P_1P_2}$ and the midpoint of line segment P_1P_2.

10) $P_1(2, 1, 6)$ and $P_2(8, 3, 9)$

Solve the problem.

11) An airplane is flying in the direction 28° east of south at 692 km/hr. Find the component form of the velocity of the airplane, assuming that the positive x–axis represents due east and the positive y–axis represents due north.

$\langle 692\cos \_\_ , 692\sin \_\_ \rangle \to \langle 324.9, -611 \rangle$



$360-62$

$\theta = 298$

$\langle 692\cos 298, 692\sin 298 \rangle$

Find v · u.

12) $\mathbf{v} = 9\mathbf{i} - 6\mathbf{j}$ and $\mathbf{u} = -2\mathbf{i} - 5\mathbf{j}$

Find the angle between u and v in radians.

13) $\mathbf{u} = 3\mathbf{j} - 9\mathbf{k}$, $\mathbf{v} = 5\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$

Find the vector $\text{proj}_{\mathbf{v}}\,\mathbf{u}$.

14) $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{u} = 4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}$

$$\text{proj}_{\vec v}\vec U = \frac{\vec U \cdot \vec V}{|\vec V|} = \frac{(4)(1) + 12(1) + 3(1)}{\sqrt{1^2+1^2+1^2}} = \frac{19}{\sqrt{3}} * \text{direction}$$

$$\frac{\vec V}{|\vec V|} = \frac{\langle 1, 1, 1\rangle}{\sqrt{3}}$$

$$= \frac{19}{\sqrt{3}}\left(\frac{1i + 1j + 1k}{\sqrt{3}}\right) = \boxed{\frac{19\vec i + 19\vec j + 19\vec k}{3}}$$

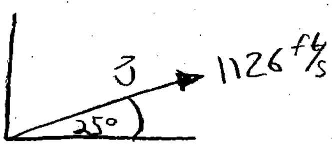
Find an equation for the line that passes through the given point and satisfies the given conditions.

15) $P = (5, 8)$; parallel to $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j}$

16) P = (10, 8); perpendicular to **v** = 8**i** - 5**j**

Solve the problem.

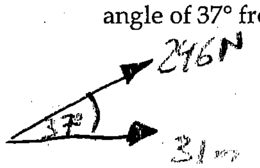
17) A bullet is fired with a muzzle velocity of 1126 ft/sec from a gun aimed at an angle of 25° above the horizontal. Find the horizontal component of the velocity.



$|Proj_{\vec{v}}\vec{U}| = |\vec{U}|\cos\theta$

$|Proj_{\vec{v}}\vec{U}| = 1126\cos 25°$ → $\boxed{1020.5 \text{ ft/s}}$

18) How much work does it take to slide a box 31 meters along the ground by pulling it with a 246 N force at an angle of 37° from the horizontal?



$\omega = \vec{F}\cdot\vec{d} = |\vec{F}||\vec{d}|\cos\theta$

$(246)(31)\cos 37$ → $\boxed{W = 6090 \text{ Nm}}$

Find the acute angle, in degrees, between the lines.

$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right)$

19) $x - \sqrt{3}\,y = 2$ and $\sqrt{3}\,x - y = -2$

take the original eq. & change the sign on one.

Ex: $2i + 7j$

Parall → $\vec{A} = \sqrt{3}i - j$

Parall → $\vec{B} = i + \sqrt{3}j$

$\theta = \cos^{-1}\left(\frac{2\sqrt{3}}{(2)(2)}\right)$

$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ → $\theta = 60°$

Find the length and direction (when defined) of u × v.

20) **u** = 2**i** + 2**j** – **k**, **v** = –**i** + **k**

Solve the problem.

21) Find the area of the triangle determined by the points P(1, 1, 1), Q(-4, -3, -6), and R(6, 10, -9).

22) Find a unit vector perpendicular to plane PQR determined by the points P(2, 1, 1), Q(1, 0, 0) and R(2, 2, 2).

Find the triple scalar product (u x v) · w of the given vectors.

23) **u** = -5**i** − 3**j** + 4**k**; **v** = 10**i** − 3**j** + 3**k**; **w** = 4**i** − 7**j** − 10**k**

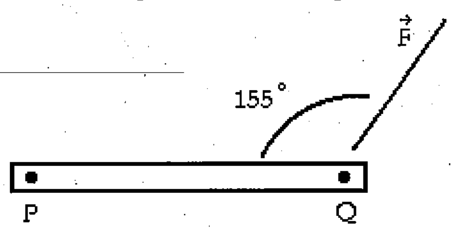
$$\begin{vmatrix} -5 & -3 & 4 \\ 10 & -3 & 3 \\ 4 & -7 & -10 \end{vmatrix} = -5(30+21) + 3(-100-12) + 4(-70+12)$$

$$-5(9) + 3(-112) + 4(-58)$$

$$= \boxed{-823}$$

Solve the problem.

24) Find the magnitude of the torque in foot–pounds at point P for the following lever:



$|\overrightarrow{PQ}| = 4$ in. and $|\mathbf{F}| = 25$ lb

Find parametric equations for the line described below.

25) The line through the point P(-5, -3, 4) parallel to the vector −3**i** + 2**j** − 5**k**

26) The line through the point P(-3, 2, 7) and perpendicular to the vectors **u** = 8**i** - 2**j** - 8**k** and **v** = −8**i** + 2**j** + 5**k**

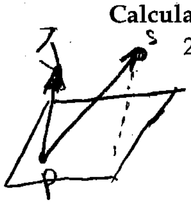
Find a parametrization for the line segment joining the points.

27) (-4, –6, -3), (0, –6, -5)

Write the equation for the plane.

28) The plane through the point P(3, 8, -7) and perpendicular to the line x = 5 + 8t, y = -5 + 3t, z = 4 – t.

Calculate the requested distance.



29) The distance from the point S(-4, -8, -7) to the plane 2x + 2y + z = 7

$\vec{v} \perp$ to plane $= \langle 2, 2, 1 \rangle$ $\quad \dfrac{|\vec{PS} \cdot \vec{v}|}{|\vec{v}|}$

$|\vec{v}| = \sqrt{9} = 3$

$\dfrac{\vec{v}}{|\vec{v}|} = \dfrac{2}{3}, \dfrac{2}{3}, \dfrac{1}{3}$

$\boxed{\dfrac{|\vec{PS} \cdot \vec{v}|}{|\vec{v}|} = \dfrac{38}{3}}$

$P = (0, 0, 7)$

$\vec{PS} = \langle -4, -8, -14 \rangle$

$\vec{PS} \cdot \vec{v} = (-4)(2) + (-8)(2) + (-14)(1)$

$\vec{PS} \cdot \vec{v} = -8 + (-16) - 14$

$|\vec{PS} \cdot \vec{v} = -38|$

30) The distance from the point S(1, 5, -9) to the plane 11x + 2y + 10z = 1

Find the intersection.

31) 9x - 4y - 3z = 4, 8x - 5y - 7z = 8 Find $\vec{n}$ to each line $\vec{n} = \langle A, B, C \rangle$

$\vec{n}_1 \times n_2$

then

Let $z = 0$

& solve system

of eq. to

find pt in

common

Match the equation with the surface it defines.

32) $\frac{x^2}{9} + \frac{z^2}{9} = \frac{y}{8}$

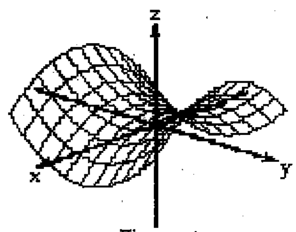


Figure 1

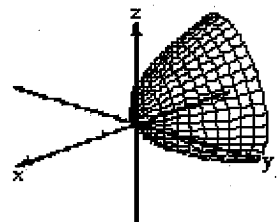


Figure 2

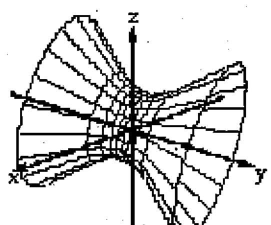


Figure 3

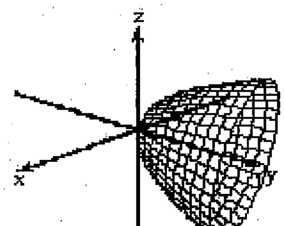


Figure 4

A) Figure 3 B) Figure 1 C) Figure 4 D) Figure 2

Identify the type of surface represented by the given equation.

33) $x = -2z^2$, no limit on y

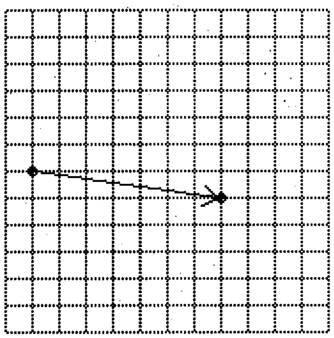
A) Hyperboloid of two sheets
B) Sphere
C) Cylinder
D) Parabolic cylinder

34) $\frac{x^2}{10} + \frac{y^2}{4} - \frac{z^2}{6} = 1$

A) Hyperboloid of two sheets
B) Elliptical cone
C) Hyperboloid of one sheet
D) Ellipsoid

Answer Key
Testname: CALC3_PRACTICETEST1

1) $(x - 4)^2 + (y - 6)^2 = 81$ and $z = 81$
2) $16 \le x^2 + y^2 + z^2 \le 81$ and $1 \le x \le 4$
3) $\sqrt{3}$
4) $C\left(\frac{1}{3}, -\frac{1}{3}, 0\right)$, $a = \frac{\sqrt{29}}{3}$
5) $\sqrt{185}$
6) $\left\langle \frac{-1}{2}, \frac{-\sqrt{3}}{2} \right\rangle$
7) $\mathbf{v} = 5\mathbf{i} - 7\mathbf{j} + 3\mathbf{k}$
8)



9) $\frac{17}{5}\left(\frac{15}{17}\mathbf{j} + \frac{8}{17}\mathbf{k}\right)$
10) $\frac{6}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{3}{7}\mathbf{k}$; $\left(5, 2, \frac{15}{2}\right)$
11) $\langle 324.9, -611.0 \rangle$
12) 12
13) 1.35
14) $\frac{19}{3}\mathbf{i} + \frac{19}{3}\mathbf{j} + \frac{19}{3}\mathbf{k}$
15) $5x - 4y = -7$
16) $8x - 5y = 40$
17) 1021 ft/sec
18) 6090 joules
19) 30°
20) 3; $\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$
21) $\frac{3\sqrt{2051}}{2}$
22) $\frac{1}{\sqrt{2}}(\mathbf{j} - \mathbf{k})$
23) −823
24) 3.52 ft–lb
25) $x = -3t - 5$, $y = 2t - 3$, $z = -5t + 4$
26) $x = 6t - 3$, $y = 24t + 2$, $z = 0t + 7$
27) $x = 4t - 4$, $y = -6$, $z = -2t - 3$, $0 \le t \le 1$
28) $8x + 3y - z = 55$
29) $\frac{38}{3}$
30) $\frac{14}{3}$
31) $x = 13t - \frac{12}{13}$, $y = 39t - \frac{40}{13}$, $z = -13t$
32) C
33) D
34) C

MAC2313 - Practice Exam 1

Directions: This is a practice test. Your actual exam will be different. Use this practice exam to evaluate the areas in which you need additional studying and practice. Exam 1 will cover Chapter 12 and will be shorter than the practice exam.

Describe the given set of points with a single equation or with a pair of equations.

1) The circle of radius 9 centered at the point (4, -6, 81) and lying in a plane parallel to the xy–plane

$(x-4)^2 + (y+6)^2 = 9^2, \quad z = 81$

Write one or more inequalities that describe the set of points.

2) The closed region bounded by the spheres of radius 4 and 9, both centered at the origin, and the planes x = 1 and x = 4

$4^2 \le x^2 + y^2 + z^2 \le 9^2$ and $1 \le x \le 4$

Find the distance between points P_1 and P_2.

3) $P_1(4, 7, -3)$ and $P_2(5, 8, -4)$

$d = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Find the center and radius of the sphere.

4) $3x^2 + 3y^2 + 3z^2 - 2x + 2y = 9$

$3x^2 - 2x + \_\_ + 3y^2 + 2y + \_\_ + 3z^2 + 0z + \_\_ = 9$

$3[x^2 - \frac{2}{3}x + \frac{1}{9}] + 3[y^2 + \frac{2}{3} + \frac{1}{9}] + 3z^2 = 9 + \frac{1}{3} + \frac{1}{3}$

$3(x - \frac{1}{3})^2 + 3(y + \frac{1}{3})^2 + 3z^2 = \frac{29}{3}$

$(x - \frac{1}{3})^2 + (y + \frac{1}{3})^2 + z^2 = \frac{29}{9}$

$\boxed{\text{ctr: } (\frac{1}{3}, -\frac{1}{3}, 0) \quad r = \frac{\sqrt{29}}{3}}$

Find the magnitude.

5) Let $\mathbf{u} = \langle -1, 3 \rangle$ and $\mathbf{v} = \langle 0, 1 \rangle$. Find the magnitude (length) of the vector: $-4\mathbf{u} - \mathbf{v}$.

$-4\langle -1, 3\rangle - \langle 0, 1\rangle$

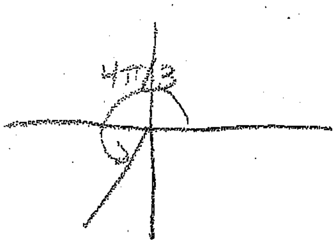
$\langle 4, -12\rangle + \langle 0, -1\rangle$

$= \langle 4, -13\rangle$

$|-4\vec{u} - \vec{v}| = \sqrt{16 + 169} = \boxed{\sqrt{185}}$

Find the component form of the specified vector.

6) The unit vector that makes an angle $4\pi/3$ with the positive x–axis



$\langle \cos\frac{4\pi}{3}, \sin\frac{4\pi}{3} \rangle$

$= \boxed{\langle -\frac{1}{2}, -\frac{\sqrt{3}}{2} \rangle}$

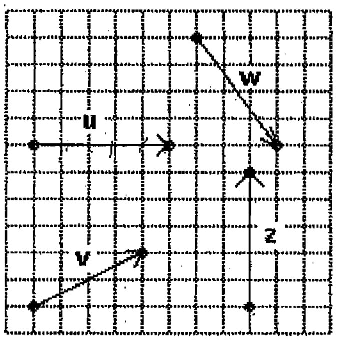
Express the vector in the form $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$.

7) $\overrightarrow{AB}$ if A is the point (-1, -9, 1) and B is the point (4, -16, 4)

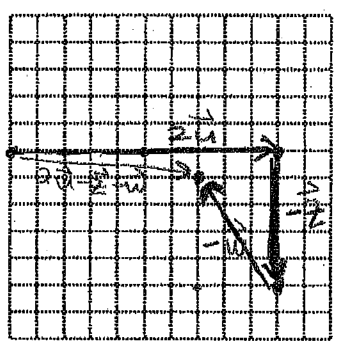
$\langle 5, -7, 3 \rangle$

$\boxed{5\vec{i} - 7\vec{j} + 3\vec{k}}$

Use the vectors u, v, w, and z head to tail as needed to sketch the indicated vector.



8) 2u - z - w $= \langle 7, -1 \rangle$



Express the vector as a product of its length and direction.

9) $3\mathbf{j} + \frac{8}{5}\mathbf{k}$

$$\left|3\vec{j} + \frac{8}{5}\vec{k}\right| = \sqrt{9 + \frac{64}{25}} = \sqrt{\frac{225+64}{25}} = \frac{\sqrt{289}}{5} = \frac{17}{5}$$

$$= \frac{17}{5}\left\langle \frac{3}{\left(\frac{17}{5}\right)}, \frac{8/5}{(17/5)} \right\rangle$$

$$= \boxed{\frac{17}{5}\left\langle \frac{15}{17}, \frac{8}{17} \right\rangle}$$

Calculate the direction of $\overrightarrow{P_1P_2}$ and the midpoint of line segment P_1P_2.

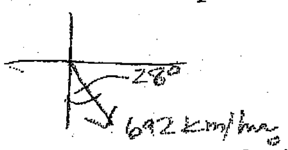
10) $P_1(2, 1, 6)$ and $P_2(8, 3, 9)$

$\overrightarrow{P_1P_2} = \langle 6, 2, 3 \rangle$ $\quad |\overrightarrow{P_1P_2}| = \sqrt{36+4+9} = 7$

$\dfrac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \boxed{\langle \frac{6}{7}, \frac{2}{7}, \frac{3}{7} \rangle}$ $\quad$ midpt $= \boxed{(5, 2, \frac{15}{2})}$

Solve the problem.

11) An airplane is flying in the direction 28° east of south at 692 km/hr. Find the component form of the velocity of the airplane, assuming that the positive x-axis represents due east and the positive y-axis represents due north.



$\langle 692 \cdot \cos(-62°), 692 \cdot \sin(-62°) \rangle$

$-90° + 28° = -62°$ $\quad \approx \langle 324.9, -611.0 \rangle$

Find v · u.

12) $\mathbf{v} = 9\mathbf{i} - 6\mathbf{j}$ and $\mathbf{u} = -2\mathbf{i} - 5\mathbf{j}$

$\vec{v} \cdot \vec{u} = -18 + 30 = \boxed{12}$

Find the angle between u and v in radians.

13) $\mathbf{u} = 3\mathbf{j} - 9\mathbf{k}$, $\mathbf{v} = 5\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$

$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta \rightarrow \theta = \cos^{-1}\left[\dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right]$

$\vec{u} \cdot \vec{v} = 0 + (-18) + 36 = 18$

$|\vec{u}| = \sqrt{9+81} = 3\sqrt{10}$

$|\vec{v}| = \sqrt{25+36+16} = \sqrt{77}$

$\theta = \cos^{-1}\left[\dfrac{\cancel{18}\,6}{\cancel{3}\sqrt{770}}\right] \approx \boxed{1.353}$

Find the vector $\text{proj}_{\mathbf{v}}\,\mathbf{u}$.

14) $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{u} = 4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}$

$|\text{proj}_{\vec{v}}\,\vec{u}| = |\vec{u}|\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|} = \dfrac{4+12+3}{\sqrt{1+1+1}} = \dfrac{19}{\sqrt{3}}$

$\dfrac{\vec{v}}{|\vec{v}|} = \langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \rangle$ $\quad \text{proj}_{\vec{v}}\,\vec{u} = \boxed{\langle \frac{19}{3}, \frac{19}{3}, \frac{19}{3} \rangle}$

Find an equation for the line that passes through the given point and satisfies the given conditions.

15) $P = (5, 8)$; parallel to $\mathbf{v} = 4\mathbf{i} + 5\mathbf{j}$

$5x - 4y = C$ $\quad \boxed{5x - 4y = -7}$

$25 - 32 = C$

$C = -7$

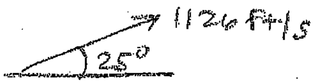
16) P = (10, 8); perpendicular to v = 8i - 5j

$$8x-5y=C$$
$$80-40=C$$
$$C=40$$

$\boxed{8x-5y=40}$

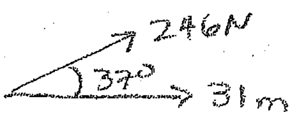
Solve the problem.

17) A bullet is fired with a muzzle velocity of 1126 ft/sec from a gun aimed at an angle of 25° above the horizontal. Find the horizontal component of the velocity.



$(\cos 25^\circ)(1126) \approx \boxed{1020.5 \text{ ft/s}}$

18) How much work does it take to slide a box 31 meters along the ground by pulling it with a 246 N force at an angle of 37° from the horizontal?

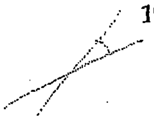


$W=\vec{F}\cdot\vec{d}=|\vec{F}||\vec{d}|\cos 37^\circ$

$=(246)(31)\cos 37^\circ \approx \boxed{6090.4 \text{ Nm}}$ J

Find the acute angle, in degrees, between the lines.

19) x − √3 y = 2 and √3 x − y = -2



$A\cdot B=|A||B|\cos\theta \rightarrow \theta=\cos^{-1}\dfrac{A\cdot B}{|A||B|}$

$\vec{A}=\sqrt{3}\,\vec{i}+\vec{j}$

$\vec{B}=\vec{i}+\sqrt{3}\,\vec{j}$

$\vec{A}\cdot\vec{B}=\sqrt{3}+\sqrt{3}=2\sqrt{3}$

$|\vec{A}|=\sqrt{3+1}=2$

$|\vec{B}|=2$

$\theta=\cos^{-1}\left(\dfrac{\cancel{2}\sqrt{3}}{\cancel{4}_2}\right)$

$\boxed{\cancel{\frac{\pi}{6}} \text{ or } \boxed{30^\circ}}$

Find the length and direction (when defined) of u × v.

20) u = 2i + 2j − k, v = -i + k

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & -1 \\ -1 & 0 & 1 \end{vmatrix} = 2\vec{i}-(2-1)\vec{j}+(0+2)\vec{k} = \langle 2,-1,2\rangle$$

$\boxed{\begin{array}{l} |\vec{u}\times\vec{v}|=\sqrt{4+1+4}=3 \\ \text{direction} = \langle \frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\rangle \end{array}}$

Solve the problem.

21) Find the area of the triangle determined by the points P(1, 1, 1), Q(-4, −3, -6), and R(6, 10, -9).

$\overrightarrow{PQ}=\langle -5,-4,-7\rangle$

$PR=\langle 5,9,-10\rangle$

$$\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & -4 & -7 \\ 5 & 9 & -10 \end{vmatrix} = (40+63)\vec{i}-(50+35)\vec{j}+(-45+20)\vec{k}$$
$$=\langle 103,-85,-25\rangle$$

$|\overrightarrow{PQ}\times\overrightarrow{PR}|=\sqrt{10609+7225+625}=\sqrt{18459}=3\sqrt{2051}$

$\boxed{A=\frac{1}{2}\cdot 3\sqrt{2051}}$

22) Find a unit vector perpendicular to plane PQR determined by the points P(2, 1, 1), Q(1, 0, 0) and R(2, 2, 2).

$\vec{PQ} = \langle -1, -1, -1 \rangle$

$\vec{PR} = \langle 0, 1, 1 \rangle$

$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = (-1+1)\vec{i} - (-1-0)\vec{j} + (-1-0)\vec{k} = \vec{j} - \vec{k}$

$\therefore |\vec{PQ} \times \vec{PR}| = \sqrt{1+1} = \sqrt{2}$

unit vector $= \boxed{\frac{1}{\sqrt{2}}\vec{j} - \frac{1}{\sqrt{2}}\vec{k}}$

Find the triple scalar product (u x v) · w of the given vectors.

23) u = –5i – 3j + 4k; v = 10i – 3j + 3k; w = 4i – 7j – 10k

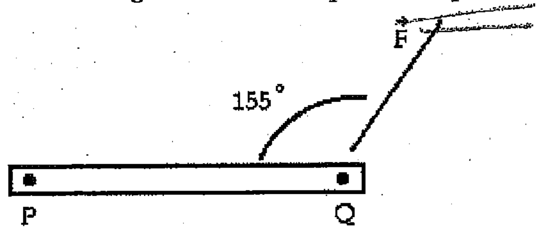
$\begin{vmatrix} -5 & -3 & 4 \\ 10 & -3 & 3 \\ 4 & -7 & -10 \end{vmatrix} = -5(30+21) + 3(-100-12) + 4(-70+12)$

$= -5 \cdot 51 + 3(-112) + 4(-58)$

$= -255 - 336 - 232 = \boxed{-823}$

Solve the problem.

24) Find the magnitude of the torque in foot–pounds at point P for the following lever:



$|\vec{PQ}| = 4$ in. and $|F| = 25$ lb

torque $= |\vec{PQ}|\,|\vec{F}| \sin\theta$

$= \left(\frac{4}{12}\right)(25)\sin 155^\circ$

$\approx \boxed{3.52 \text{ ft lb}}$

Find parametric equations for the line described below.

25) The line through the point P(-5, -3, 4) parallel to the vector –3i + 2j – 5k

$x = -5 - 3t$

$y = -3 + 2t \qquad -\infty < t < \infty$

$z = 4 - 5t$

26) The line through the point P(-3, 2, 7) and perpendicular to the vectors **u** = 8i - 2j - 8k and **v** = –8i + 2j + 5k

$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & -2 & -8 \\ -8 & 2 & 5 \end{vmatrix} = (-10+16)\vec{i} - \overbrace{(40-64)}^{-24}\vec{j} + (16-16)\vec{k}$

$= \langle 6, 24, 0 \rangle$

$x = -3 + 6t$

$y = 2 + 24t \qquad -\infty < t < \infty$

$z = 7$

Find a parametrization for the line segment joining the points.

27) (-4, -6, -3), (0, -6, -5) $\quad \vec{v} = \langle 4, 0, -2 \rangle$

$$\boxed{\begin{aligned} x &= -4+4t \\ y &= -6 \qquad\qquad 0 \le t \le 1 \\ z &= -3-2t \end{aligned}}$$

Write the equation for the plane.

28) The plane through the point P(3, 8, -7) and perpendicular to the line x = 5 + 8t, y = -5 + 3t, z = 4 - t.

// vector to line $\langle 8,3,-1 \rangle$

$\langle 8,3,-1 \rangle \perp$ to plane, so

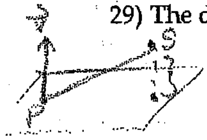
plane: $8x+3y-z=C$

$8(3)+3(8)-(-7)=C \rightarrow C=55$

$\boxed{8x+3y-z=55}$

Calculate the requested distance.

29) The distance from the point S(-4, -8, -7) to the plane 2x + 2y + z = 7



$\vec{v} = \langle 2,2,1 \rangle \qquad \frac{\vec{v}}{|\vec{v}|} = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle \qquad$ arbitrary $P=(0,0,7)$

$|\vec{v}| = \sqrt{9} = 3$

$\vec{PS} = \langle -4,-8,-14 \rangle \qquad$ distance $= \left| \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle \cdot \langle -4,-8,-14 \rangle \right|$

$= \left| -\frac{8}{3} - \frac{16}{3} - \frac{14}{3} \right| = \left| -\frac{38}{3} \right| = \boxed{\frac{38}{3}}$

Same question

30) The distance from the point S(1, 5, -9) to the plane 11x + 2y + 10z = 1

$\vec{v} = \langle 11,2,10 \rangle \qquad \frac{\vec{v}}{|\vec{v}|} = \langle \frac{11}{15}, \frac{2}{15}, \frac{10}{15} \rangle \qquad$ arbitrary $P=(1,0,-1)$

$|\vec{v}| = \sqrt{121+4+100} = \sqrt{225} = 15$

$\vec{PS} = \langle 0,5,-8 \rangle \quad \pm d = \left| \langle 0,5,-8 \rangle \cdot \langle \frac{11}{15}, \frac{2}{15}, \frac{10}{15} \rangle \right| = 0 + \frac{\cancel{10}^{2}}{\cancel{15}_{3}} - \frac{\cancel{80}^{16}}{\cancel{15}_{3}} = -\frac{14}{3}$

$d = \boxed{\frac{14}{3}}$

Find the intersection. awful!!

31) 9x - 4y - 3z = 4, 8x - 5y - 7z = 8

$\vec{n}_1 = \langle 9,-4,-3 \rangle \qquad \vec{n}_2 = \langle 8,-5,-7 \rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & -4 & -3 \\ 8 & -5 & -7 \end{vmatrix} = (+28-15)\vec{i} - \underbrace{(-63+24)}_{-39}\vec{j} + \underbrace{(-45+32)}_{-13}\vec{k}$$

$$= 13\vec{i} + 39\vec{j} - 13\vec{k}$$

intersection // to $\langle 13, 39, -13 \rangle$

let $z=0$:

$(-5)(9x-4y)=(4)(-5)$

$(4)(8x-5y)=(8)4$

$-45x+20y=-20$

$32x-20y=32$

$-13x=12$

$x=-\frac{12}{13}$

$8\left(-\frac{12}{13}\right)-5y=8$

$-\frac{96}{13}-5y=8$

$-5y=\frac{200}{13}$

$y=-40/13$

$$\boxed{\begin{aligned} x &= -\frac{12}{13}+13t \\ y &= -\frac{40}{13}+39t \\ z &= -13t \end{aligned}}$$

Match the equation with the surface it defines.

32) $\frac{x^2}{9} + \frac{z^2}{9} = \frac{y}{8}$

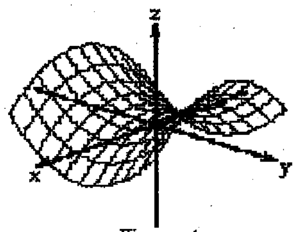


Figure 1

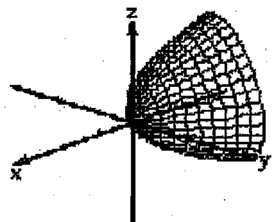


Figure 2

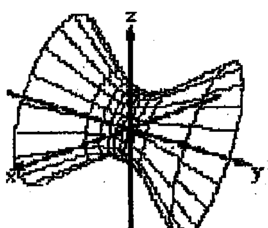


Figure 3

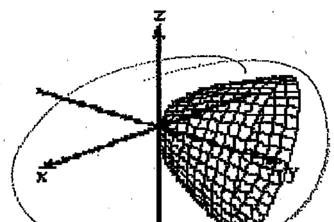


Figure 4

A) Figure 3   B) Figure 1   C) Figure 4   D) Figure 2

Identify the type of surface represented by the given equation.

33) $x = -2z^2$, no limit on y

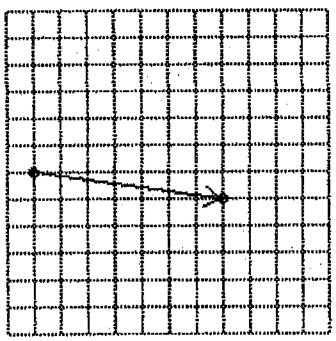
A) Hyperboloid of two sheets
B) Sphere
C) Cylinder ← not wrong
D) Parabolic cylinder

34) $\frac{x^2}{10} + \frac{y^2}{4} - \frac{z^2}{6} = 1$

A) Hyperboloid of two sheets
B) Elliptical cone
C) Hyperboloid of one sheet
D) Ellipsoid

Answer Key
Testname: CALC3_PRACTICETEST1

1) $(x-4)^2+(y-6)^2=81$ and $z=81$
2) $16 \le x^2+y^2+z^2 \le 81$ and $1 \le x \le 4$
3) $\sqrt{3}$
4) $C\left(\frac{1}{3},-\frac{1}{3},0\right), a=\frac{\sqrt{29}}{3}$
5) $\sqrt{185}$
6) $\left\langle\frac{-1}{2},\frac{-\sqrt{3}}{2}\right\rangle$
7) $\mathbf{v}=5\mathbf{i}-7\mathbf{j}+3\mathbf{k}$
8)



9) $\frac{17}{5}\left(\frac{15}{17}\mathbf{j}+\frac{8}{17}\mathbf{k}\right)$
10) $\frac{6}{7}\mathbf{i}+\frac{2}{7}\mathbf{j}+\frac{3}{7}\mathbf{k};\left(5,2,\frac{15}{2}\right)$
11) $\langle 324.9,-611.0\rangle$
12) 12
13) 1.35
14) $\frac{19}{3}\mathbf{i}+\frac{19}{3}\mathbf{j}+\frac{19}{3}\mathbf{k}$
15) $5x-4y=-7$
16) $8x-5y=40$
17) 1021 ft/sec
18) 6090 joules
19) 30°
20) $3;\frac{2}{3}\mathbf{i}-\frac{1}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$
21) $\frac{3\sqrt{2051}}{2}$
22) $\frac{1}{\sqrt{2}}(\mathbf{j}-\mathbf{k})$
23) -823
24) 3.52 ft–lb
25) $x=-3t-5, y=2t-3, z=-5t+4$
26) $x=6t-3, y=24t+2, z=0t+7$
27) $x=4t-4, y=-6, z=-2t-3, 0\le t\le 1$
28) $8x+3y-z=55$
29) $\frac{38}{3}$
30) $\frac{14}{3}$
31) $x=13t-\frac{12}{13}, y=39t-\frac{40}{13}, z=-13t$
32) C
33) D
34) C

Steven Romeiro

1, 5, 9, 15, 17, 19, 21, 25, 27, 29, 37
43, 45, 47, 53, 57, 59, 61

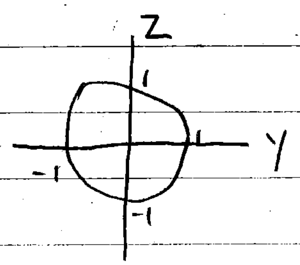
12.1 Homework

+5

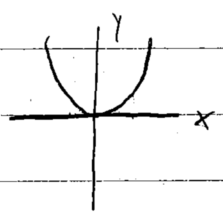
(1) $X=2,\ Y=3$ — A line through the point $(2,3,0)$

(5) $X^2+Y^2=4,\ Z=0$ — A circle centered at the origin with $r=2$

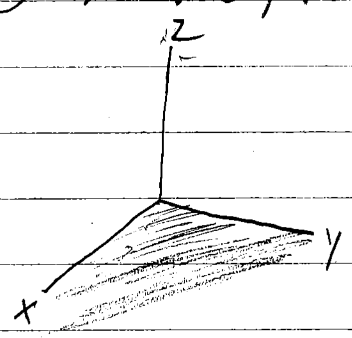
(9) $X^2+Y^2+Z^2=1,\ X=0$ — A circle in the ZY-Plane centered at the origin, $r=1$



(15) $Y=X^2,\ Z=0$ — A parabola in the XY-Plane with it's vertex at the origin. Opening upwards in Y.



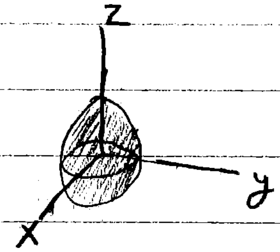
(17) a) $X \geq 0,\ Y \geq 0,\ Z=0$



The entire first quadrant of the XY-Plane

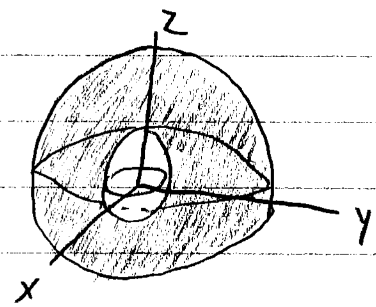
-1

(19) $x^2+y^2+z^2 \leq 1$



A solid sphere (ball) with radius 1

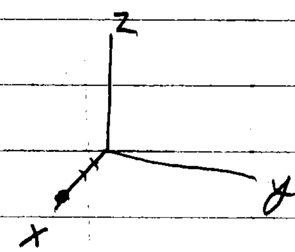
(21) $1 \leq x^2+y^2+z^2 \leq 4$



A ball centered at the origin $r=2$ without a ball centered at origin $r=1$

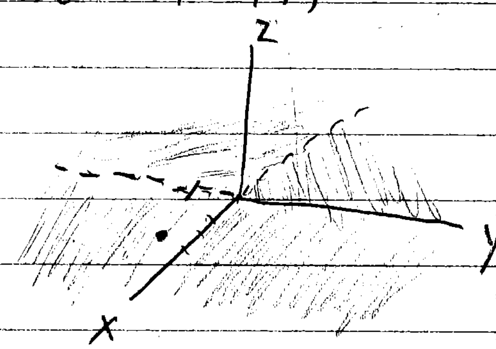
(25) Plane perpendicular to x-axis at $(3,0,0)$

$x=3$



(27) Plane through the point $(3,-1,1)$ parallel to the a xy-plane

$z=1$



29 Circle of radius 2 centered at $(0,2,0)$ lying in the XY - Plane

$$x^2 + (y-2)^2 = 4, \quad z = 0$$

37 The half - space consisting of the points on & below the XY - Plane

$$z \leq 0$$

43 Distance between $P_1(1,4,5)$ & $P_2(4,-2,7)$

$$d = \sqrt{(4-1)^2 + (-2-4)^2 + (7-5)^2} = d = \sqrt{9+36+4}$$

$$d = \sqrt{49} \longrightarrow \boxed{d = 7}$$

45 $P_1(0,0,0)$ $P_2(2,-2,-2)$

$$d = \sqrt{2^2 + 2^2 + 2^2} = d = \sqrt{12} = \boxed{d = 2\sqrt{3}}$$

47 $(x+2)^2 + y^2 + (z-2)^2 = 8$

Center $(-2,0,2)$ radius $= 2\sqrt{2}$

53 $\left(-1, \frac{1}{2}, -\frac{2}{3}\right)$ $r = \frac{4}{9}$

$$(x+1)^2 + \left(y - \frac{1}{2}\right)^2 + \left(z + \frac{2}{3}\right)^2 = \frac{16}{81}$$

(57) $2x^2 + 2y^2 + 2z^2 + x + y + z = 9$

$2x^2 + x + 2y^2 + y + 2z^2 + z = 9$

$2(x^2 + \frac{1}{2}x + y^2 + \frac{1}{2}y + z^2 + \frac{1}{2}z) = 9$

$2(x^2 + \frac{1}{2}x + \frac{1}{16} + y^2 + \frac{1}{2}y + \frac{1}{16} + z^2 + \frac{1}{2}z + \frac{1}{16}) = 9 + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$2[(x^2 + 1/4)^2 + (y^2 + 1/4)^2 + (z^2 + 1/4)^2] = 9 + 3/8$

$\boxed{(x^2 + 1/4)^2 + (y^2 + 1/4)^2 + (z^2 + 1/4)^2 = \frac{75}{16}}$

(59) Formula for distance from pt (x, y, z) to the x-axis

$d = \sqrt{(x-x)^2 + (y-0)^2 + (z-0)^2}$

$\boxed{d = \sqrt{y^2 + z^2}}$

(61) Perimeter of triangle with vertices $A(-1,2,1)$, $B(1,-1,3)$, $C(3,4,5)$

$d_{AB} = \sqrt{(1+1)^2 + (-1-2)^2 + (3-1)^2} = d_{AB} = \sqrt{17}$

$d_{BC} = \sqrt{(3-1)^2 + (4+1)^2 + (5-3)^2} = d_{BC} = \sqrt{33}$

$\underset{4}{} \quad \underset{25}{} \quad \underset{4}{}$

$d_{AC} = \sqrt{(3+1)^2 + (4-2)^2 + (5-1)^2} = d_{AC} = 6$

$\underset{16}{} \quad \underset{4}{} \quad \underset{16}{}$

$\sqrt{17} + \sqrt{33} + 6 = \boxed{15.87}$

Steven
Romeiro

$\# 1, 3, 5, 9, 15, 17, 21, 25, 33, 35$
$39, 41, 43, 47$

12.2 Homework

$U = (3, -2) \quad V = (-2, 5)$

(1) a) Component Form of $3u$

$3u = (3 \cdot 3,\ 3 \cdot -2) = \boxed{\langle 9, -6 \rangle}$

b) mag of $3u$

$3u = \langle 9, -6 \rangle$

$\text{mag} = \sqrt{9^2 + (-6)^2} = \text{mag} = \sqrt{117} = \boxed{3\sqrt{13}}$

(3) a) Component of $U + V$

$U + V = (3 - 2,\ -2 + 5) = \boxed{U + V = \langle 1, 3 \rangle}$

b) mag of $U + V = \sqrt{1^2 + 3^2} = \boxed{\sqrt{10}}$

(5) a) $2u - 3v = \langle 2(3), 2(-2) \rangle - \langle 3(-2), 3(5) \rangle$

$2u - 3v = \langle 6, -4 \rangle - \langle -6, 15 \rangle$

$2u - 3v = \langle 6 + 6, -4 - 15 \rangle = \boxed{\langle 12, -19 \rangle}$

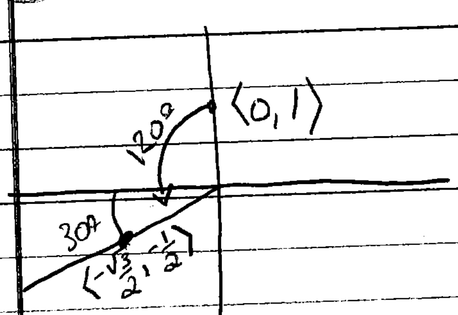
b) mag $2u - 3v = \sqrt{12^2 + (-19)^2}$

$\boxed{\text{mag} = \sqrt{505}}$

(9) Component of $\overrightarrow{PQ}$, $P = (1,3)$ & $Q = (2,-1)$

$\langle -1 - 3, 2 - 3 \rangle = \boxed{\langle 1, -4 \rangle}$

(15) Vector $\langle 0, 1 \rangle$ rotated $120°$ counterclock



$\boxed{\text{Unit vector} = \langle -\frac{\sqrt{3}}{2}, \frac{-1}{2} \rangle}$

(17) $\overrightarrow{P_1P_2}$ if $P_1\ (5, 7, -1)$ & $P_2\ (2, 9, -2)$

$\overrightarrow{P_1P_2}\ \langle 2-5\ \ 9-7\ \ -2+1\rangle$

$\overrightarrow{P_1P_2}\ \langle -3\hat{i}, 2\hat{j}, -1\hat{k}\rangle = \boxed{-3\hat{i}+2\hat{j}-1\hat{k}}$

(21) $5u - v$ if $u = \langle 1, 1, -1\rangle$ & $v = \langle 2, 0, 3\rangle$

$5u - v = \langle 5-2, 5-0, -5-3\rangle$

$5u - v = \langle 3, 5, -8\rangle = \boxed{3\hat{i} + 5\hat{j} - 8\hat{k}}$

(25) $2\hat{i} + \hat{j} - 2\hat{k}$

$mag = \sqrt{2^2\hat{i} + 1^2\hat{j}\ (-2)^2\hat{k}} = \sqrt{4\hat{i} + 1\hat{j} + 4^2\hat{k}}$

$mag = \sqrt{9} = 3$

Unit vector $= \boxed{\left\langle \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k}\right\rangle\ 3}$

(33) Find vector of mag 7 in direction $V = 12i - 5k$

$mag\ \sqrt{12^2 + (-5)^2} = mag = \sqrt{144 + 25}$

$mag = \sqrt{169} \rightarrow mag = 13$

$\left(\frac{12}{13}\hat{i} - \frac{5}{13}\hat{k}\right) 7 \longrightarrow \boxed{(12\hat{i} - 5\hat{k})\ \frac{7}{13}}$

(35) a) $\overrightarrow{P_1P_2}$ $P_1(-1,1,5)$ & $P_2(2,5,0)$

direction $\overrightarrow{P_1P_2} = \langle 2+1, 5-1, 0-5 \rangle = 3\hat{i} + 4\hat{j} - 5\hat{k}$

mag $= \sqrt{3^2+4^2+(-5)^2} = \sqrt{50} = 5\sqrt{2}$

direction $= \boxed{\frac{3}{5\sqrt{2}}\hat{i} + \frac{4}{5\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}}$

b) midpoint of $\overrightarrow{P_1P_2}$

$\frac{-1+2}{2}, \frac{1+5}{2}, \frac{5+0}{2} = \boxed{\left(\frac{1}{2}, 3, 5\right)}$

(39) $\overrightarrow{AB} = \vec{i} + 4\vec{j} - 2\vec{k}$, $B = (5,1,3)$

Find A $= (5-A, 1-A, 3-A)$

$5-A = 1\hat{i} \rightarrow -A = 1-5 \rightarrow A = 4$

$1-A = 4\hat{j} \rightarrow -A = 4-1 \rightarrow A = -3$

$3-A = -2\hat{k} \rightarrow -A = -2-3 \rightarrow A = 5$

$\boxed{A = (4,-3,5)}$

(41) $\vec{u} = 2\hat{i} + \hat{j}$, $\vec{v} = \hat{i} + \hat{j}$, $\vec{w} = \vec{i} - \vec{j}$

Find scalars so $\vec{u} = a\vec{v} + b\vec{w}$

$2i + j = a(i + j) + b(i - j)$

$2i + j = ai + bi + aj - bj$

$2i + j = (a+b)i + (a-b)j$

$$\begin{cases} 2 = a + b \\ 1 = a - b \end{cases}$$

$3 = 2a + 0 \longrightarrow \boxed{a = \frac{3}{2}}$

$2 = \frac{3}{2} + b \longrightarrow \boxed{b = \frac{1}{2}}$

#1, 3, 9, 13, 21, 23, 31, 33, 37
41, 43

12.3 Homework

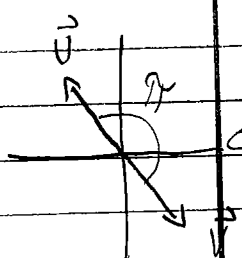
① a) $\vec{v}\cdot\vec{u}$, $|\vec{v}|$, $|\vec{u}|$ $\quad \vec{v} = 2\vec{i} - 4\vec{j} + \sqrt{5}\vec{k}$, $\vec{u} = -2\vec{i} + 4\vec{j} - \sqrt{5}\vec{k}$

$|\vec{u}| = \sqrt{2^2+4^2+(\sqrt{5})^2} = \boxed{5}$

$|\vec{v}| = \sqrt{2^2+4^2+(-\sqrt{5})^2} = \boxed{5}$

$\vec{v}\cdot\vec{u} = (2)(-2)+(-4)(4)+(\sqrt{5})(-\sqrt{5}) = -4-16-5 = \boxed{-25}$

b) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-25}{25} = \boxed{\cos\theta = -1}$



c) $|\text{Proj}_{\vec{v}}\vec{u}| = |\vec{u}|\cos\theta = 5(-1) = \boxed{-5}$

↳ or $= \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \dfrac{-25}{5} = \boxed{-5}$

↳ length of $\vec{u}$ in dir of $\vec{v}$ $\quad$ mag of both vec $\quad$ unit vec of $\vec{v}$ = direction

d) $\text{Proj}_{\vec{v}}\vec{u} = \dfrac{\boxed{\vec{u}\cdot\vec{v}}}{\boxed{|\vec{v}|}} \cdot \boxed{\dfrac{\vec{v}}{|\vec{v}|}}$

length of $\vec{v}$

$\text{proj}_{\vec{v}}\vec{u} = \dfrac{(\vec{u}\cdot\vec{v})\vec{v}}{|\vec{v}|^2} = \dfrac{(-\cancel{25})(2\vec{i}-4\vec{j}+\sqrt{5}\vec{k})}{\cancel{25}}$

$$\boxed{\text{proj}_{\vec{v}}\vec{u} = -2\vec{i}+4\vec{j}-\sqrt{5}\vec{k}}$$

(3) $\vec{V} = 10\vec{i} + 11\vec{j} - 2\vec{K}$, $\vec{U} = 0\vec{i} + 3\vec{j} + 4\vec{K}$

a) $|\vec{U}| = \sqrt{0^2 + 3^2 + 4^2} = \sqrt{25} = \boxed{5}$

$|\vec{V}| = \sqrt{10^2 + 11^2 + (-2)^2} = \sqrt{225} = \boxed{15}$

$\vec{V} \cdot \vec{U} = 10(0) + (11)(3) + (-2)(4) = \boxed{25}$

b) $\vec{U} \cdot \vec{V} = |\vec{U}||\vec{V}| \cos\theta = \cos\theta = \dfrac{\vec{U} \cdot \vec{V}}{|\vec{V}||\vec{U}|}$

$\cos\theta = \dfrac{25}{(5)(15)} = \boxed{\dfrac{1}{3}}$

c) $|\text{proj}_{\vec{V}} \vec{U}| \Rightarrow |\vec{U}||\vec{V}| \cos\theta = \vec{U} \cdot \vec{V} \rightarrow \dfrac{\vec{U} \cdot \vec{V}}{|\vec{V}|} = |\vec{U}| \cos\theta$

$|\text{Proj}_{\vec{V}} \vec{U}| =$ either $|\vec{U}| \cos\theta$ or $\dfrac{\vec{U} \cdot \vec{V}}{|\vec{V}|} = \dfrac{25}{15} = \boxed{\dfrac{5}{3}}$

d) $\text{proj}_{\vec{V}} \vec{U} = \dfrac{(\vec{U} \cdot \vec{V})}{|\vec{V}|} * \dfrac{\vec{V}}{|\vec{V}|} = \dfrac{\vec{V}(\vec{U} \cdot \vec{V})}{|\vec{V}|^2}$

$\text{proj}_{\vec{V}} \vec{U} = \dfrac{25}{15}\left(\dfrac{10i + 11j - 2K}{15}\right) = \boxed{\dfrac{1}{9}(10i + 11j - 2K)}$

(9) $\vec{U} = 2i + j + 0K$
$\vec{V} = i + 2j - K$

$\vec{U} \cdot \vec{V} = |\vec{U}||\vec{V}| \cos\theta$

$\cos\theta = \dfrac{\vec{U} \cdot \vec{V}}{|\vec{U}||\vec{V}|} \longrightarrow \theta = \cos^{-1}\left(\dfrac{\vec{U} \cdot \vec{V}}{|\vec{U}||\vec{V}|}\right)$

$|\vec{U}| = \sqrt{2^2 + 1^2} = \sqrt{5}$
$|\vec{V}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

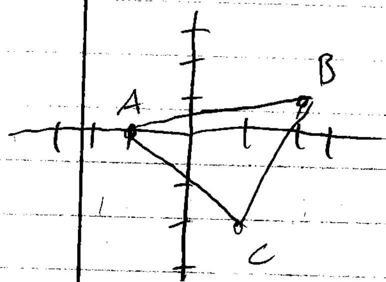
$\vec{U} \cdot \vec{V} = (2)(1) + (1)(2) + (0)(-1) = 4$

$\theta = \cos^{-1}\left(\dfrac{4}{\sqrt{5}\sqrt{6}}\right)$

$\theta = \cos^{-1}\left(\dfrac{4}{\sqrt{30}}\right)$

$\boxed{\theta = 0.75}$

(13) $A = (-1, 0)$, $B = (2, 1)$, $C = (1, -2)$



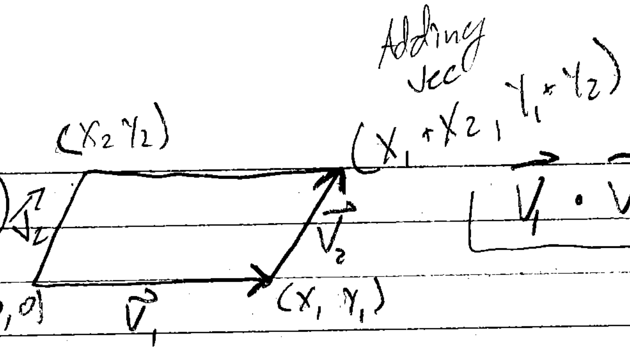
$\overrightarrow{AC}$ = mag of A to C

$\overrightarrow{AC} = \langle 1+1, -2-0 \rangle = \langle 2, -2 \rangle$

$\overrightarrow{BC} =$

$\overrightarrow{AB} = \langle 2+1, 1-0 \rangle = \langle 3, 1 \rangle$

21



$\boxed{\vec{V_1} \cdot \vec{V_2} = 0}$ means it's orthogonal

$\vec{V_1} = \langle X_1, Y_1 \rangle \qquad V_1 \cdot V_2 = X_1X_2 + Y_1Y_2$

$V_2 = \langle X_2, Y_2 \rangle \qquad \boxed{X_1X_2 + Y_1Y_2 = 0}$

if $\| \langle X_1+X_2, Y_1+Y_2 \rangle \| = \| \langle X_2-X_1, Y_2-Y_1 \rangle \|$

$= \sqrt{(X_1+X_2)^2 + (Y_1+Y_2)^2} = \sqrt{(X_2-X_1)^2 + (Y_2-Y_1)^2}$

Cancel out

$= \cancel{X_1^2} + 2X_1X_2 + \cancel{Y_1^2} + 2Y_1Y_2 + \cancel{Y_2^2} = \cancel{X_2^2} - 2X_2X_1 + \cancel{X_1^2} - 2Y_2Y_1 + \cancel{Y_1^2}$

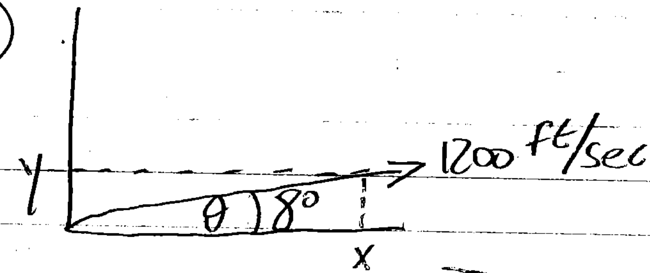
$= 2X_1X_2 + 2Y_1Y_2 = -2X_1X_2 - 2Y_1Y_2$

$= 4X_1X_2 + 4Y_1Y_2 = 0$

$= \boxed{X_1X_2 + Y_1Y_2 = 0 \longrightarrow \vec{V_1} \cdot \vec{V_2} = 0}$

orthogonal.

(23)



$X = ?$

$Y = ?$

$\sin\theta = \frac{opp}{hyp}$

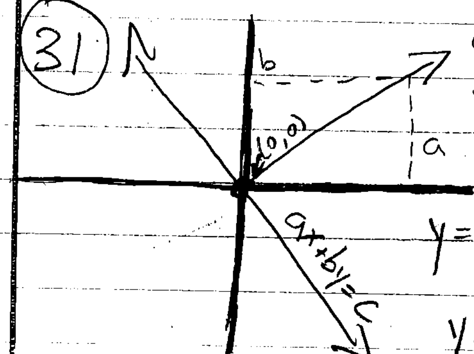
$hyp \sin\theta = opp$

$hyp \cos\theta = adj$

$1200 \sin 8 = Y \longrightarrow$ $\boxed{Y = 167 \text{ ft/sec}}$

$1200 \sin 8 = X \longrightarrow$ $\boxed{X = 1188 \text{ ft/sec}}$

(31)



$a\vec{i} + b\vec{j}$

Slope of $\vec{v} = \frac{b-0}{a-0} = \boxed{\frac{b}{a}}$ → negative reciprocals

$y = \frac{c - ax}{b}$

slope

$y = \frac{-ax}{b} + \frac{c}{b} \longrightarrow y = \boxed{-\frac{a}{b}} x + \frac{c}{b}$ constant

(33) $P(2,1)$, $\vec{V} = \vec{i} + 2\vec{j}$

$ax + by = c$ & $\vec{V} = a\vec{i} + b\vec{j}$ ∴ $a = 1$ & $b = 2$

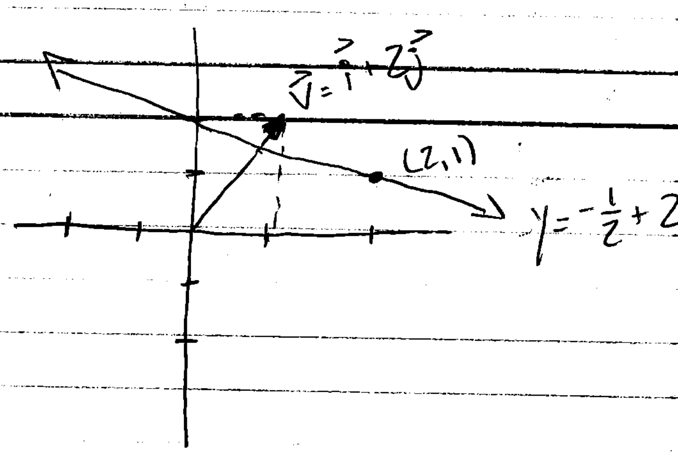
$1x + 2y = C$ $\qquad$ $P(2,1) = X = 2$ & $Y = 1$

$1(2) + 2(1) = C$

$C = 4$

$\boxed{x + 2y = 4}$

$\boxed{Y = -\frac{1}{2} + 2}$



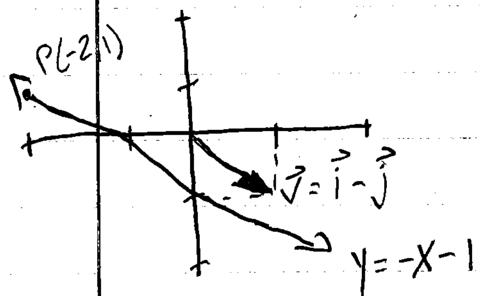
(37) $P(-2,1)$, $\vec{V} = \vec{i} - \vec{j}$ where $P \parallel \vec{V}$

$\vec{V} = a\vec{i} + b\vec{j}$ $\therefore$ $a = 1$ & $b = -1$

$bx - ay = c$ (from #32 given)

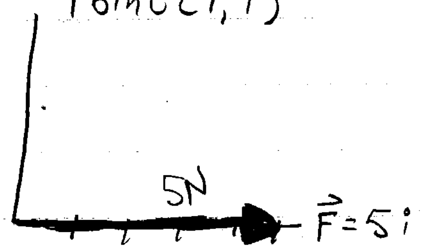
$(-1)x - (1)y = c \longrightarrow c = (-1)(-2) - (1)(1) \longrightarrow c = 1$

$-x - y = 1 \longrightarrow -y = x + 1 \longrightarrow \boxed{y = -x - 1}$



? (41) $\vec{F} = 5i$ $|\vec{F}| = 5N$ Distance = 1 ?

Point(1,1)



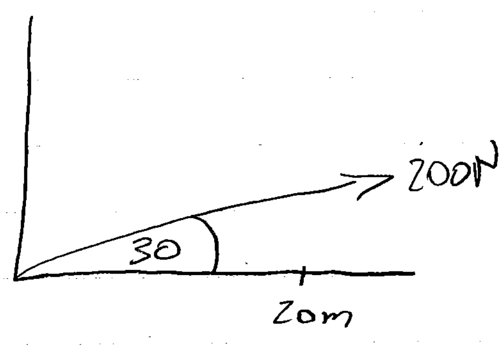
$W = F * D * \cos\theta$

$\theta = 0°$ $\cos\theta = 1$

$W = F * D * 1$

$W = (5)(1)(1) \longrightarrow \boxed{W = 5J}$

(43) $|\vec{F}| = 200N$, $X = 20m$, $\theta = 30°$, $w = ?$



$W = |\vec{F}| D \cos\theta$

$W = 200(20)\cos(30°)$

$\boxed{W = 3464 J}$

Steven Romeiro

1, 5, 7, 9, 11, 15, 21, 25, 37, 39, 41, 45

12.4 Homework

(1) $\vec{U} = 2\vec{i} - 2\vec{j} - \vec{k}$, $\vec{V} = \vec{i} - \vec{k}$

$$\vec{U} \times \vec{V} = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} 2 & -2 & -1 \\ 1 & 0 & -1 \end{vmatrix} \end{array}$$

$$\vec{U} \times \vec{V} = \begin{vmatrix} -2 & -1 \\ 0 & -1 \end{vmatrix} \vec{i} - \begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} \vec{j} + \begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} \vec{k}$$

$$\vec{U} \times \vec{V} = (2-0)\vec{i} - (-2-(-1))\vec{j} + (0-(-2))\vec{k}$$

$$\vec{U} \times \vec{V} = 2\vec{i} + \vec{j} + 2\vec{k}$$

$$|\vec{U} \times \vec{V}| = \sqrt{2^2+1^2+2^2} = \sqrt{9} = \boxed{3 \text{ (length)}}$$

$$\boxed{\text{direction} = \frac{2\vec{i} + \vec{j} + 2\vec{k}}{3}}$$

(5) $\vec{U} = 2\vec{i}$, $\vec{V} = -3\vec{j}$

$$\vec{U} \times \vec{V} = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \end{vmatrix} \end{array}$$

$$\vec{U} \times \vec{V} = (0-0)i - (0-0)j + (-6-0)\vec{k}$$

$$\vec{U} \times \vec{V} = -6\vec{k} \rightarrow |\vec{U} \times \vec{V}| = \sqrt{36} = \boxed{6 \text{ (length)}}$$

$$\text{direction} = \frac{-6\vec{k}}{-6} = \boxed{-\vec{k}}$$

(7) $\vec{U} = -8i - 2j - 4k \qquad \vec{V} = 2i + 2j + k$

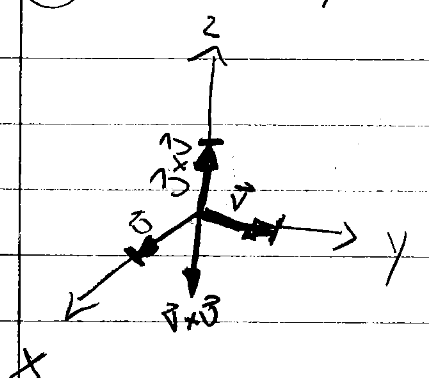
$\vec{U} \times \vec{V} = (-2 - (-8))i - (-8 - (-8))j + (-16 - (-4))k$

$\vec{U} \times \vec{V} = 6\vec{i} + 0j - 12k$

$|\vec{U} \times \vec{V}| = \sqrt{180} = \boxed{6\sqrt{5} \text{ length}}$ direction $= \dfrac{6i - 12k}{6\sqrt{5}} = \boxed{\dfrac{i - 2k}{\sqrt{5}}}$

(9) $\vec{U} = \vec{i}$, $\vec{V} = \vec{j}$

$$\vec{U} \times \vec{V} = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \end{array}$$



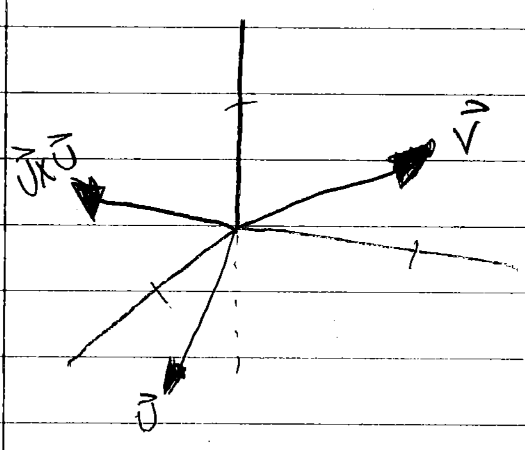
$\vec{U} \times \vec{V} = 0\vec{i} - 0j + 1\vec{k}$

direction $= 1\vec{k}$

(11) $\vec{U} = \vec{i} - \vec{k}$, $\vec{V} = \vec{j} + \vec{k}$

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{vmatrix} = (0 - (-1))i - (1 - 0)j + (1 - 0)k$$

$$= \langle 1\vec{i} - 1\vec{j} + 1\vec{k} \rangle$$



(15) $P(1,-1,2)$, $Q(2,0,-1)$, $R(0,2,1)$

a) Area of Δ

$\vec{PQ} = \langle 1, 1, -3 \rangle$, $\vec{PR} = \langle -1, 1, -1 \rangle$

$$\vec{PQ} \times \vec{PR} \quad \begin{array}{ccc} i & j & k \end{array}$$
$$\begin{vmatrix} 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix} = (-1-(-3))i - (-1-3)j + (1-(-1))k$$
$$= (2\vec{i} + 4\vec{j} + 2\vec{k})$$

$$|\vec{PQ} \times \vec{PR}| = \sqrt{2^2 + 4^2 + 2^2} = \sqrt{24} = 2\sqrt{6}$$

* Right Answ: $(-1-(-9))i - (-1-(3))j + (3-(-1))k$

$$\vec{U} \times \vec{V} = \langle 8\vec{i} + 4\vec{j} + 4\vec{k} \rangle = |\vec{U} \times \vec{v}| = \sqrt{8^2 + 4^2 + 4^2}$$

$$|\vec{U} \times \vec{v}| = \sqrt{96} \rightarrow \text{Area of } \Delta = \frac{1}{2} |\vec{U} \times \vec{v}|$$

$$\text{Area} = \frac{1}{2}\sqrt{96} = \boxed{\text{Area} = 2\sqrt{6}}$$

b) Unit vector $= \dfrac{\vec{U} \times \vec{V}}{|\vec{U} \times \vec{V}|}$

$$= \frac{8\vec{i} + 4\vec{j} + 4\vec{k}}{2\sqrt{6}} = \boxed{\frac{4\vec{i} + 2\vec{j} + 4\vec{k}}{\sqrt{6}}}$$

(21) $\vec{u} = 2\vec{i} + \vec{j}$, $\vec{v} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{w} = \vec{i} + 2\vec{k}$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = \begin{array}{ccc} i & j & k \\ \end{array}$$
$$\begin{vmatrix} 2 & \boxed{1} & 0 \\ 2 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix}$$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = 2\begin{vmatrix} -1 & 1 \\ 0 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 0\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}$$

$$(\vec{u} \times \vec{v}) \cdot \vec{w} = 2(-2-0) - (4-1) + 0$$

Absolute $|(\vec{u} \times \vec{v}) \cdot \vec{w}| = |2(-2) - (3)| \longrightarrow \boxed{(\vec{u} \times \vec{v}) \cdot \vec{w} = +7}$

(25) $\vec{PQ} = 8$, $|\vec{F}| = 30$

mag of torque $= |\vec{PQ} \times \vec{F}| = |\vec{PQ}||\vec{F}| \sin\theta$

$|\vec{PQ}| = \sqrt{8^2} = \underline{\underline{8}}$ $\therefore$ mag of torque $= \left(\frac{8}{12}\right)(30)\sin 60$

mag of torque $= 10\sqrt{3} = \boxed{17.32}$

$$\frac{8\,\cancel{\text{inch}}}{} \Big| \frac{1\text{ foot}}{12\,\cancel{\text{inch}}} = \frac{8}{12}\text{ foot}$$

?

37 $A(-1,2)$, $B(2,0)$, $C(7,1)$, $D(4,3)$

Area of Parallelogram : $|\vec{U} \times \vec{V}|$

$\vec{U} = \vec{AB} = \langle 3, -2 \rangle$ $\quad \vec{AD} = \langle 5, 1 \rangle$

$\vec{V} = \vec{AC} = \langle 8, -1 \rangle$

$$\vec{AB} \times \vec{AC} \quad \begin{array}{ccc} i & j & k \end{array} \atop \begin{vmatrix} 3 & -2 & 0 \\ 8 & -1 & 0 \end{vmatrix}$$

$\vec{AB} \times \vec{AC} = (0-0)i - (0-0)j + (-3-(-16))k$

$\vec{AB} \times \vec{AC} = 0i + 0j + 13\vec{k}$

$|\vec{AB} \times \vec{AC}| = \sqrt{13^2} = \boxed{13}$

39 $A(0,0,0)$, $B(3,2,4)$, $C(5,1,4)$, $D(2,-1,0)$

$\vec{AB} = \langle 3, 2, 4 \rangle$ $\quad \vec{AB} \times \vec{AC} = \begin{array}{ccc} i & j & k \end{array} \atop \begin{vmatrix} 3 & 2 & 4 \\ 5 & 1 & 4 \end{vmatrix}$

$\vec{AC} = \langle 5, 1, 4 \rangle$

$\vec{AB} \times \vec{AC} = (8-4)i - (12-20)j + (3-10)k$

$\vec{AB} \times \vec{AC} = 4i + 8j - 7k$

$|\vec{AB} \times \vec{AC}| = \sqrt{4^2+8^2+7^2}$

$\boxed{|\vec{AB} \times \vec{AC}| = \sqrt{129}}$

41) $A(0,0)$, $B(-2,3)$, $C(3,1)$

$\vec{AB} = \langle -2, 3 \rangle$

$\vec{AC} = \langle 3, 1 \rangle$ $\quad \vec{AB} \times \vec{AC}$ $\begin{array}{ccc} i & j & k \end{array}$ $\begin{vmatrix} -2 & 3 & 0 \\ 3 & 1 & 0 \end{vmatrix}$

$\vec{AB} \times \vec{AC} = (0)i - (0)j + (-2-9)k$

$\vec{AB} \times \vec{AC} = -11k \cdot \frac{1}{2} \longrightarrow \vec{AB} \times \vec{AC} = -\frac{11}{2}k$

$|\vec{AB} \times \vec{AC}| = \frac{1}{2}\sqrt{11^2} = \boxed{\pm \frac{11}{2}}$

45) $A(1,0,0)$, $B(0,2,0)$, $C(0,0,-1)$

$\vec{AB} = \langle -1, 2, 0 \rangle$

$\vec{AC} = \langle -1, 0, -1 \rangle$ $\quad \vec{AB} \times \vec{AC}$ $\begin{array}{ccc} i & j & k \end{array}$ $\begin{vmatrix} -1 & 2 & 0 \\ -1 & 0 & -1 \end{vmatrix}$

$\vec{AB} \times \vec{AC} = (-2-0)i - (1-0)j + (0+2)k$

$\vec{AB} \times \vec{AC} = -2i - j + 2k$

$|\vec{AB} \times \vec{AC}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3 \cdot \frac{1}{2} = \boxed{\frac{3}{2}}$

Steven Romeiro

1, 3, 13, 19, 21, 23, 33, 37, 39, 53, 59

12.5 Homework

① $P(3,-4,-1)$ // to vector $(\vec{i}+\vec{j}+\vec{k})$

$$x = x_0 + tv_1 \longrightarrow \quad x = 3 + t$$
$$y = y_0 + tv_2 \longrightarrow \quad y = -4 + t$$
$$z = z_0 + tv_3 \longrightarrow \quad z = -1 + t$$

③ $P(-2, 0, 3)$, $Q(3, 5, -2)$

$$\overrightarrow{PQ} = \langle \overset{v_1}{5}, \overset{v_2}{5}, \overset{v_3}{-5} \rangle \qquad \begin{aligned} x &= -2 + t(5) \\ y &= 0 + t(5) \\ z &= 3 + t(-5) \end{aligned}$$

$$\boxed{x = -2 + 5t, \quad y = 5t, \quad z = 3 - 5t}$$

⑬ $P(0,0,0)$, $Q(1, 1, 3/2)$

$$\overrightarrow{PQ} = \langle 1, 1, 3/2 \rangle$$

$$\begin{aligned} x &= 0 + t(1) \\ y &= 0 + t(1) \\ z &= 0 + t(3/2) \end{aligned} \qquad \boxed{\begin{aligned} x &= t \qquad 0 \le x \le 1 \\ y &= t \qquad 0 \le t \le 1 \\ z &= 3/2\,t \end{aligned}}$$

(19) $P(2,0,2)$, $Q(0,2,0)$

$\vec{PQ} = \langle -2, 2, -2 \rangle$

$x = 2 + t(-2) \rightarrow$
$y = 0 + t(2) \rightarrow$
$z = 2 + t(-2) \rightarrow$

$$\boxed{\begin{aligned} x &= 2-2t \\ y &= 2t \\ z &= 2-2t \\ 0 &\le t \le 1 \end{aligned}}$$

$0 \le y \le 2$
$0 \le 2t \le 2$

(21) Plane thru $P(0, 2, -1)$
normal to $\vec{n} = \langle 3i - 2j - k \rangle$
$\vec{n} = (A, B, C) = \langle 3, -2, -1 \rangle$
$x_0 = 0$, $y_0 = 2$, $z_0 = -1$

$Ax + By + Cz = Ax_0 + By_0 + Cz_0$
$3x - 2y - 1z = 3(0) + (-2)(2) + (-1)(-1)$
$3x - 2y - 1z = -4 + 1$
$$\boxed{3x - 2y - z = -3}$$

23) $P(1,1,-1)$, $Q(2,0,2)$, $R(0,-2,1)$

$\vec{PQ} = \langle 1,-1,3\rangle$, $\vec{PR}\langle -1,-3,2\rangle$

$$\vec{PQ}\times\vec{PR} = \begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix} \end{array} =$$

$\vec{PQ}\times\vec{PR} = (-2-(-9))i - (2-(-3))j + (-3-1)k$

$\vec{PQ}\times\vec{PR} = 7i - 5j - 4k$

$n = \langle 7,-5,-4\rangle$

$Ax + By + Cz = Ax_0 + By_0 + Cz_0$

$7x - 5y - 4z = 7(1) - 5(1) - 4(-1)$

$7x - 5y - 4z = 7 - 5 + 4$

$$\boxed{7x - 5y - 4z = 6}$$

33) $P(0,0,12)$ $x = 4t$, $y = -2t$, $z = 2t$

$P_0 = (0,0,0)$

$\vec{P_0P} = \langle 0,0,12\rangle$ $\qquad \vec{P_0P}\times\vec{v}$ $\begin{array}{c} \begin{matrix} i & j & k \end{matrix} \\ \begin{vmatrix} 0 & 0 & 12 \\ 4 & -2 & 2 \end{vmatrix} \end{array}$

$\vec{v} = \langle 4,-2,2\rangle$

$\vec{P_0P}\times\vec{v} = (0+24)i - (0-48)j + 0k$

$\vec{P_0P}\times\vec{v} = 24i + 48j$

$$\frac{|\vec{P_0P}\times\vec{v}| = \sqrt{24^2+48^2}}{|\vec{v}| = \sqrt{4^2+2^2+2^2}} = \frac{\sqrt{2880}}{\sqrt{24}} = \boxed{d = 10.954}$$

37) $P(3,-1,4)$ $x = 4-t$, $y = 3+2t$, $z = -5+3t$

$P_0(4,3,-5)$ $\vec{v}_1 = -1$ $\vec{v}_2 = 2$ $\vec{v}_3 = 3$

$\vec{P_0P} = \langle -1,-4,9\rangle$ $\vec{v} = \langle -1,2,3\rangle$

$$\vec{P_0P} \times \vec{v} = \begin{array}{ccc} i & j & k \\ \hline -1 & -4 & 9 \\ -1 & 2 & 3 \end{array}$$

$\vec{P_0P} \times \vec{v} = (-12-18)i - (-3+9)j + (-2-4)k$

$\vec{P_0P} \times \vec{v} = -30i - 6j - 6k$

$|\vec{P_0P} \times \vec{v}| = \sqrt{30^2+6^2+6^2} = \sqrt{972}$

$|\vec{v}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$$d = \frac{|\vec{P_0P} \times \vec{v}|}{|\vec{v}|} = \frac{\sqrt{972}}{\sqrt{14}} \longrightarrow \boxed{d = 8.33}$$

39) $P(2,-3,4)$, $x + 2y + 2z = 13$

$\vec{n} = \langle 1,2,2\rangle$

$P_0 = (13,0,0)$ $\vec{P_0P} \cdot \vec{n} = (-11)(1) + (-3)(2) + (4)(2)$

$\vec{P_0P} = \langle -11,-3,4\rangle$ $\vec{P_0P} \cdot \vec{n} = -11 - 6 + 8 =$

$\vec{P_0P} \cdot \vec{n} = -9$

$$\frac{\vec{P_0P} \cdot \vec{n}}{|\vec{n}|} = \frac{|-9|}{\sqrt{1^2+2^2+2^2}} = \frac{|-9|}{\sqrt{9}} = \frac{9}{3} = \boxed{3}$$

Dreamspark = Free Microsoft

(53) $x = 1-t$, $y = 3t$, $z = 1+t$; $2x - y + 3z = 6$

$2(1-t) - 3t + 3(1+t) = 6$

$2 - 2t - 3t + 3 + 3t = 6$

$-2t + 5 = 6$

$-2t = 1 \longrightarrow t = -\frac{1}{2}$

$x = 1 + \frac{1}{2} = \frac{3}{2}$

$y = 3(\frac{-1}{2}) = -3/2$

$z = 1 - \frac{1}{2} = \frac{1}{2}$

$$\boxed{\begin{aligned} x &= \tfrac{3}{2} \\ y &= -3/2 \\ z &= 1/2 \end{aligned}}$$

(59) $x - 2y + 4z = 2$, $x + y - 2z = 5$

$\vec{n}_1 = \langle 1, -2, 4 \rangle$ $\vec{n}_2 = \langle 1, 1, -2 \rangle$

$\vec{n}_1 \times \vec{n}_2$ $\begin{vmatrix} 1 & -2 & 4 \\ 1 & 1 & -2 \end{vmatrix} = ((4-4)i - (-2-4) + (1+2)$

$= 0i + 6j + 3k$

Let $z = 0$

$\begin{cases} x - 2y = 2 \\ x + y = 5 \longrightarrow x = 5 - y \end{cases}$

$(5-y) - 2y = 2$

$-3y = -3$

$y = 1$ $x = 4$ $z = 0$

$x = 4$

$y = 1$

$x = 4 + t(0) \longrightarrow x = 4$

$y = 1 + t(6) \longrightarrow y = 1 + 6t$

$z = 0 + t(3) \longrightarrow z = 3t$

Steven
Romeiro

1-11 odd

12.6 Homework

1. d - Ellipsoid
3. a - Cylinder
5. i - Hyperbolic Paraboloid
7. b - Cylinder
9. k - Hyperbolic Paraboloid
11. h - Cone

Steven Romeiro

9, 13, 35

16.1 Homework

(9) Evaluate $\int_c (x+y)\,ds$ where C is the straight line segment, $x=t$, $y=1-t$, $z=0$ from $(0,1,0)$ to $(1,0,0)$.

$r(t) = t\,i + (1-t)\,j \qquad 0 \le t \le 1$

$\int (x+y)\,|v(t)|\,dt \rightarrow \int_0^1 (t+1-t)(\sqrt{2})\,dt$

$|v(t)| = i - j = \sqrt{1^2+(-1)^2} = \sqrt{2}$

$\sqrt{2}\int_0^1 1\,dt \rightarrow \sqrt{2}\,[t]_0^1 = \boxed{\sqrt{2}}$

(13) Find line integral of $f(x,y,z) = x+y+z$ over the straight line segment of $(1,2,3)$ to $(0,-1,1)$

$\int_c (x+y+z)\,ds \qquad r(t) = (1-t)i + (2-3t)j + (3-2t)k$

$|v(t)| = -i - 3 - 2 = |v(t)| = \sqrt{14}$

$x = 1-t,\ y = 2-3t,\ z = 3-2t$ bounds? Solve for "t"

$\int_0^1 [(1-t)+(2-3t)+(3-2t)]\sqrt{14}\,dt \rightarrow \int_0^1 [-6t+6]\sqrt{14}\,dt$

$-6\sqrt{14}\int_0^1 (t-1)\,dt \rightarrow -6\sqrt{14}\left[\frac{t^2}{2} - t\right]_0^1$

$-6\sqrt{14}\left[-\frac{1}{2}\right] \rightarrow \boxed{3\sqrt{14}}$

(35) Mass of a thin wire lying along curve $r(t) = \sqrt{2}t\,i + \sqrt{2}t\,j + (4-t^2)k$, $0 \le t \le 1$ density is a) $\delta = 3t$ and b) $\delta = 1$

$$M = \int_C \delta \, ds \qquad ds = |V(t)| = \sqrt{2 + 2 + 4t^2} = 2\sqrt{1+t^2}$$

$$M = \int_0^1 3t\left(2\sqrt{1+t^2}\right)dt \rightarrow \int_0^1 6t\sqrt{1+t^2}\,dt \qquad \begin{aligned} u &= 1+t^2 \\ du &= 2t\,dt \end{aligned}$$

$$M = \int 3 \cdot 2t\sqrt{1+t^2}\,dt \rightarrow \int_0^1 3u^{1/2}\,du$$

$$M = 3\int_0^1 u^{1/2}\,du \rightarrow 3\left[\frac{2}{3}u^{3/2}\right]_0^1 \rightarrow \left[2(1+t^2)^{3/2}\right]_0^1$$

$$\boxed{M = 4\sqrt{2} - 2}$$

Steven Romeiro

#3, 7, 21, 23, 29

16.2 Homework

(3) $g(x,y,z) = e^{z} - \ln(x^2+y^2)$

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} k$$

$$\frac{\partial f}{\partial x} = -\frac{1}{x^2+y^2} \cdot 2x = -\frac{2x}{x^2+y^2}$$

$$\frac{\partial f}{\partial y} = -\frac{2y}{x^2+y^2} \qquad \frac{\partial f}{\partial z} = e^{z}$$

$$\boxed{\nabla f = -\left(\frac{2x}{x^2+y^2}\right)\vec{i} - \left(\frac{2y}{x^2+y^2}\right)\vec{j} + e^{z}\vec{k}}$$

(7) $\vec{F} = 3y\,i + 2x\,j + 4z\,k$ from $(0,0,0)$ to $(1,1,1)$

a) $C_1: \vec{r}(t) = t\,i + t\,j + t\,k, \quad 0 \leq t \leq 1$

$x = t, \ y = t, \ z = t$

$$\vec{F} = 3t\,i + 2t\,j + 4t\,k \cdot r'(t) = i + j + k$$

$$\int_0^1 9t\,dt \rightarrow \left[\frac{9}{2}t^2\right]_0^1 \rightarrow \boxed{\frac{9}{2}}$$

(21) $\vec{F} = zi + xj + yK$

$r(t) = \sin t\, i + \cos t\, j + tK \qquad 0 \le t \le 2\pi$

$x = \sin t, \quad y = \cos t, \quad z = t$

$F = ti + \sin t\, j + \cos t\, K$

$W = \int F(r(t)) \cdot r'(t)\, dt \qquad r'(t) = \cos t\, i - \sin t\, j + K$

$F \cdot r'(t) = t\cos t - \sin^2 t + \cos t$

$$W = \int_0^{2\pi} t\cos t - \left(\frac{1-\cos 2t}{2}\right) + \cos t\, dt$$

F'	$\int$
t	$\cos t$
1	$\sin t$
0	$-\cos t$

$$W = \int_0^{2\pi} t\cos t - \frac{1}{2}\int_0^{2\pi} 1-\cos 2t + \int_0^{2\pi} \cos t$$

$$W = \Big[t\sin t + \cos t\Big]_0^{2\pi} - \frac{1}{2}\Big[t - \tfrac{1}{2}\sin t\Big]_0^{2\pi} + \Big[\sin t\Big]_0^{2\pi}$$

$$W = [0+1] - 1 - \frac{1}{2}[2\pi] + 0 \rightarrow \boxed{W = -\pi}$$

(23) Evalate $\int_c xy\,dx + (x+y)\,dy$ along curve $y = x^2$

from $(-1,1)$ to $(2,4)$

$y = 9t^2 - \cancel{6}t + 1$

$x = -1 + 3t, \quad y = 1 + 3t, \quad r(t) = 3i + j$

$dx = 3dt \quad dy = 3dt$

$\cancel{3}t = \frac{x+1}{3}, \quad 0 \le t \le 1$

$$\int_0^1 (-1+3t)(1+3t)(3dt) + (-1+3t+1+3t)dt$$

$$\int_0^1 (27t^2 - 3)dt + (18t)dt = \int_0^1 [27t^2 + 18t - 3]\,dt$$

$[9t^3$

(29) Find circulation & flux of fields

$F_2 = -yi + xj$, The circle $r(t) = \cos t\, i + \sin t\, j$

& The ellipse $r(t) = \cos t\, i + 4\sin t\, j$

a) flow $\int F \cdot dr$ $\qquad x = \cos t,\ y = \sin t$

$F = -\sin t\, i + \cos t \qquad d\vec{r} = r'(t) = -\sin t\, i + \cos t\, j$

$F \cdot dr = \sin^2 t + \cos^2 t$

$$\int_0^{2\pi} \sin^2 t + \cos^2 t\, dt \rightarrow \int_0^{2\pi} dt \rightarrow [t]_0^{2\pi} \rightarrow \boxed{2\pi}$$

b) flux $\int_0^{2\pi} M\,dy - N\,dx \qquad M = \cos t,\ N = 4\sin t$

$dx = 0,\ dy = 0$

$$\int \cos t(0) - 4\sin t(0) = 0$$

$F_1 = xi + yj$ $\quad = \int_0^{2\pi} \cos t(-\sin t) - \sin t(\cos t)\, dt \qquad M = \cos t,\ N = \sin t$

$dx = -\sin t,\ dy = \cos t$

$u = \cos t$

$du = -\sin t$

$$\int_0^{2\pi} -2\cos t \sin t\, dt \rightarrow 2\int_0^{2\pi} u\,du$$

$$2\left[\frac{1}{2}u^2\right]_0^{2\pi} \quad [\cos^2 t]_0^{2\pi} \rightarrow \boxed{1}$$

Steven Romeiro

1, 3, 13, 19

16.3 Homework

(1) $F = yz\,i + xz\,j + xy\,K$

$\frac{\partial M}{\partial z} = y$, $\frac{\partial P}{\partial x} = y$

$\frac{\partial N}{\partial x} = z$, $\frac{\partial M}{\partial y} = z$

$\frac{\partial N}{\partial z} = x$, $\frac{\partial P}{\partial y} = x$

$$\boxed{\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \qquad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y} \qquad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}}$$

Conservative

(3) $F = y\,i + (x+z)\,j - y\,K$

$\frac{\partial M}{\partial y} = 1$ $\quad \frac{\partial N}{\partial x} = z$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \text{Not conservative}}$$

(13) $$\int_{(0,0,0)}^{(2,3,-6)} 2x\,dx + 2y\,dy + 2z\,dz$$

$M_y = 0$, $N_x = 0$, $P_x = 0$, $M_z = 0$, $N_z = 0$, $P_y = 0$ $\boxed{\text{Exact}}$

$$\left[x^2 + y^2 + z^2\right]_0^{(2,3,-6)} \rightarrow 4 + 9 + 36$$

$$\boxed{= 49}$$

Steven Romeiro

5, 21, 27

16.4 Homework

(5) $F = (x-y)i + (y-x)j$

$C: x=0, x=1, y=0, y=1$

circ $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$ $\quad \frac{\partial N}{\partial x} = -1, \ \frac{\partial M}{\partial y} = -1$

$\int_0^1\int_0^1 -1+1\, dx\,dy \longrightarrow \boxed{0}$

flux $\int_0^1\int_0^1 \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy$ $\quad \frac{\partial M}{\partial x} = 1, \ \frac{\partial N}{\partial y} = 1$

$\int_0^1\int_0^1 2\, dx\,dy \rightarrow \int_0^1 2\,dy \longrightarrow \boxed{2}$

(21) $\oint (y^2 dx + x^2 dy)$ $\quad C$: triangle $x=0, x+y=1, y=0$

circ $\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy \longrightarrow$ $\quad \frac{\partial N}{\partial x} = 2x, \ \frac{\partial M}{\partial y} = 2y$

$\int_0^1\int_0^{-x+1} (2x-2y)\,dy\,dx \rightarrow 2\int_0^1 \left[xy - \frac{1}{2}y^2\right]_0^{-x+1}$

$2\int_0^1 x(-x+1) - \frac{1}{2}(-x+1)^2 \longrightarrow$

$$2\int_0^1 -x^2+x-\frac{1}{2}(x^2-2x+1)\,dx$$

$$\int_0^1 -2x^2+2x-\frac{1}{2}x^2+x-\frac{1}{2}\,dx$$

$$\int_0^1 -\frac{5}{2}x^2+3x-\frac{1}{2}\,dx \longrightarrow \left[-\frac{5}{6}x^3+\frac{3}{2}x^2-\frac{1}{2}x\right]_0^1$$

$$-\frac{5}{6}+\frac{3}{2}-\frac{1}{2} = \boxed{\frac{1}{6}}$$ bounds should have been both 0 -> 1

(27) The astroid $r(t)=\cos^3 t\,i+\sin^3 t\,j \quad 0\le t\le 2\pi$

Area $\frac{1}{2}\oint x\,dy-y\,dx$

$x=\cos^3 t$

$dx=3\cos^2 t(-\sin t)$

$y=\sin^3 t$

$dy=3\sin^2 t\cos t$

$$\frac{1}{2}\int_0^{2\pi} 3\cos^4 t\sin^2 t+3\sin^4 t\cos^2 t\,dt$$

$$\frac{3}{2}\int_0^{2\pi} \cos^4 t\,\frac{1}{2}(1-\cos(2t))+\sin^4 t\,\frac{1}{2}(1+\cos 2t)$$

$$\frac{3}{4}\int_0^{2\pi} \cos^4 t-\cos^4 t\cos 2t+\sin^4 t+\sin^4 t\cos 2t$$

$$\frac{3}{2}\left[\frac{1}{32}(4t-\sin 4t)\right]_0^{2\pi} \longrightarrow \frac{3}{64}(8\pi-0)$$

$$\boxed{\frac{3\pi}{8}}$$

Steven Romero

3, 7, 21, 39

16.5 Homework

(3) The first octant of the cone $Z = \frac{\sqrt{x^2+y^2}}{2}$ between planes $0 \le Z \le 3$

$Z = \frac{r}{2}$ $\qquad 0 \le \frac{r}{2} \le 3 \rightarrow 0 \le r \le 6$

$$\boxed{\vec{r}(r,\theta) = r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j} + \frac{r}{2}\vec{K} \qquad 0 \le \theta \le \pi/2}$$

(7) $x^2 + y^2 + z^2 = 3$ $\qquad -\frac{\sqrt{3}}{2} \le z \le \frac{\sqrt{3}}{2}$

Spherical $(\overset{a}{\rho}, \overset{\phi}{\psi}, \overset{\theta}{\theta})$

$x = a\sin\phi\cos\theta$, $y = a\sin\phi\sin\theta$, $z = a\cos\phi$

$a = r = \sqrt{3}$

$\vec{r}(\phi,\theta) = \sqrt{3}\sin\phi\cos\theta\,\vec{i} + \sqrt{3}\sin\phi\sin\theta\,\vec{j} + \sqrt{3}\cos\phi\,\vec{K}$

$\pi/3 \le \phi \le \frac{2\pi}{3}$

$-\frac{\sqrt{3}}{2} \le \sqrt{3}\cos\phi \le \frac{\sqrt{3}}{2} \rightarrow -\frac{1}{2} \le \cos\phi \le \frac{1}{2}$

$\cos^{-1}(-\frac{1}{2}) \le \phi \le \cos^{-1}(\frac{1}{2})$

$$\boxed{\begin{array}{c} \frac{\pi}{3} \le \phi \ \frac{2\pi}{3} \\ 0 \le \theta \le 2\pi \end{array}}$$

(21) Portion of cylinder $x^2+y^2=1$, $1 \leq z \leq 4$

$z = x^2 + y^2 - 1$

$$A = \iint \sqrt{f_x^{\,2} + f_y^{\,2} + 1}\, dx\,dy$$

$f_x = 2x$, $f_y = 2y$

$$A = \iint \sqrt{4x^2 + 4y^2 + 1}\, dx\,dy$$

$z = r^2 + 1 \rightarrow 1 \leq r^2 + 1 \leq 4 \rightarrow 0 \leq r^2 \leq 3$

$0 \leq r \leq \sqrt{3}$

$$\int_0^{\sqrt{3}}\int_0^{2\pi} \sqrt{4r^2(\cos^2\theta + \sin^2\theta) + 1}\; r\, d\theta\, dr$$

$$\rightarrow \int_0^{\sqrt{3}}\int_0^{2\pi} r\sqrt{4r^2+1}\, d\theta\, dr \rightarrow \int_0^{\sqrt{3}} 2\pi r\sqrt{4r^2+1}\, dr \qquad u = 4r^2+1 \quad du = 8r\,dr$$

$$\frac{\pi}{4}\int_0^{\sqrt{3}} 8r\sqrt{4r^2+1}\, dr \longrightarrow \frac{\pi}{4}\left[\frac{2}{3}u^{3/2}\right]_0^{\sqrt{3}}$$

$$\frac{\pi}{6}\left[\left(4(\sqrt{3})^2+1\right)^{3/2} - \left(4(0)+1\right)^{3/2}\right]$$

$$\frac{\pi}{6}\left[(13)^{3/2} - 1\right] \longrightarrow$$

$$\boxed{6\pi}$$

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16.6 Homework

7. $H(X,Y,Z) = X^2(5-4z)^{1/2}$

$Z = 1 - x^2 - y^2$; $Z \geq 0$

$Z = f(X,Y)$ = explicit $\therefore \iint G(X,Y,f(x,y))\sqrt{f_x^2 + f_y^2 + 1}\, dxdy$

$H(X,Y,f(x,y)) = X^2(5-4(1-x^2-y^2))^{1/2}$

$= X^2(5-4+4x^2+4y^2)^{1/2} \rightarrow X^2(1+4x^2+4y^2)^{1/2}$

$f_x = -2x = f_x^2 = 4x^2$

$f_y = -2y = f_y^2 = 4y^2$

$\iint x^2(1+4x^2+4y^2)^{1/2}(4x^2+4y^2+1)^{1/2}\, dxdy$

$\iint x^2(1+4x^2+4y^2)\, dxdy$

bands $Z = 1 - x^2 - y^2$

$Z = 0 \rightarrow 0 = 1 - x^2 - y^2$

$x^2 + y^2 = 1$

$0 \leq x \leq 1$

$0 \leq y \leq 1$

$\iint x^2 + 4x^4 + 4y^2x^2\, dx\, dy$

$\int_0^1\int_0^1 x^2 + 4x^4 + 4y^2x^2\, dy\, dx \longrightarrow \int_0 \left[x^2y + 4x^4y + \frac{4}{3}y^3x^2\right]_0^1$

$\int_0^1 x^2 + 4x^4 + \frac{4}{3}x^2 \longrightarrow \int_0^1 4x^4 + \frac{7}{3}x^2\, dx \longrightarrow \left[\frac{4}{5}x^5 + \frac{7}{9}x^3\right]_0^1$

$\frac{4}{5} + \frac{7}{9} = \boxed{\frac{71}{45}}$

(13) $G(x,y,z) = x+y+z$ over $2x+2y+z=2$

implicitly $G(x,y,z) = C$

$$\iint G(x,y,z) \frac{|\nabla f|}{|\nabla f \cdot p|} \qquad p = \vec{K}$$

$$\frac{\partial f}{\partial x} = 2, \quad \frac{\partial f}{\partial y} = 2, \quad \frac{\partial f}{\partial z} = 1$$

$$\nabla f = 2i + 2j + K \rightarrow |\nabla f| = \sqrt{4+4+1} = 3$$

$$\nabla f \cdot p = K \rightarrow |\nabla f \cdot p| = 1$$

$$\iint (x+y+z)3 \, dx\,dy = \iint 3x + 3y + 3(2-2x-2y)\,dx\,dy$$

$$\iint 3x + 3y + 6 - 6x - 6y \, dx\,dy \rightarrow \iint -3x-3y+6\,dx\,dy$$

$$\int_0^1 \left[-\frac{3}{2}x^2 - 3yx + 6x\right]_0^1 = \int_0^1 -\frac{3}{2} - 3y + 6$$

$$\left[-\frac{3}{2}y - \frac{3}{2}y^2 + 6y\right]_0^1 \rightarrow -\frac{3}{2} - \frac{3}{2} + 6$$

$$\boxed{= 3}$$

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16.7 Homework

(5) $F = (y^2+z^2)\vec{i} + (x^2+y^2)\vec{j} + (x^2+y^2)\vec{k}$

C: square band by $x = \pm 1$ & $y = \pm 1$ in XY plane

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right) - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right) + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$$

$$= (2y - 0)i - (2x - 2z)j + (2x - 2y)k$$

$$\nabla \times F = 2y\,i - (2x - 2z)j + (2x - 2y)k$$

$$n = \frac{\nabla f}{|\nabla f|} \rightarrow \frac{\nabla f}{|\nabla f|} = \frac{i + j + k}{1}$$

$$\iint 2y - 2x - 2z + 2x - 2y \rightarrow \iint -2z\,dx\,dy$$

$$\int_0^1 \left[2\frac{x^2}{2}\right]_0^1 dy$$

$$\int_0^1 1\,dy \sim [y]_0^1 = \boxed{1}$$

$$\int F(x,y)(|v(t)|)\,dt$$

(3) $f(x,y) = x^2 + y^2$, $C: y = 2x - 3$, $0 \le x \le 3$

let $x = t$

$y(0) = -3$, $y(3) = 3$

$-3 \le y \le 3$

$f(t) = t^2 + (2t-3)^2$

$f(t) = t^2 + 4t^2 - 12t + 9$

$f(t) = 5t^2 - 12t + 9$

$r(t) = ti + (2t-3)j$

$v(t) = i + 2j$

$|v(t)| = \sqrt{5}$

$$\int (5t^2 - 12t + 9)(\sqrt{5})\,dt$$

$$\int_0^3 5\sqrt{5}t^2 - 12\sqrt{5}t + 9\sqrt{5}\,dt$$

$$\left[\frac{5\sqrt{5}}{3}t^3 - \frac{12\sqrt{5}}{2}t^2 + 9\sqrt{5}t\right]_0^3$$

$$\sqrt{5}\left[\frac{5}{3}(3)^3 - 6(3)^2 + 9(3)\right]$$

$$\sqrt{5}\left[45 - 54 + 27\right] = \boxed{18\sqrt{5}}$$

(4) $f(x,y,z) = x^4 e^{5x} + y^6 z^5$

$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z}$

$\frac{\partial f}{\partial x} = 4x^3 e^{5x} + x^4 5e^{5x} = x^3 e^{5x}(4+5x)$

$\frac{\partial f}{\partial y} = 6y^5 z^5$ $\quad$ $\frac{\partial f}{\partial z} = 5z^4 y^6$

$\boxed{\nabla f = x^3 e^{5x}(4+5x) i + 6y^5 z^5 j + 5z^4 y^6 k}$

(5) $F = -7y i + 7x j + 4z^6 k$, $C: r(t) = \cos t\, i + \sin t\, j$, $0 \leq t \leq 3$

$x = \cos t$, $y = \sin t$,

$W = \int F(r(t)) \cdot V(t)\, dt$

$F(r(t)) = -7\sin t\, i + 7\cos t\, j + 4z^6 k$

$V(t) = -\sin t\, i + \cos t\, j$

$F(r(t)) \cdot V(t) = 7\sin^2 t + 7\cos^2 t$

$\int_0^3 7(\sin^2 t + \cos^2 t)\, dt \longrightarrow \int_0^3 7\, dt \longrightarrow \left[7t\right]_0^3$

$W = \boxed{21}$

7) $F = xi + yj$ around $x^2 + y^2 = 9$

$r(t) = r\cos t\, i + r\sin t\, j$ where $r = 3$

$\oint M\,dy - N\,dx$ $0 \le t \le 2\pi$

$r(t) = \underbrace{3\cos t}_{M}\, i + \underbrace{3\sin t}_{N}\, j$

$x = M = 3\cos t$ $y = N = 3\sin t$

$dx = -3\sin t$ $dy = 3\cos t$

$$\oint 3\cos t(3\cos t)\,dt - \oint 3\sin t(-3\sin t)\,dt$$

$$\oint 9\cos^2 t + \int 9\sin^2 t\,dt \rightarrow \int_0^{2\pi} 9(\cos^2 t + \sin^2 t)\,dt$$

$$\int_0^{2\pi} 9\,dt \rightarrow \left[9t\right]_0^{2\pi} \rightarrow \boxed{\text{Flux} = 18\pi}$$

8) $F = \underbrace{5x^4y^5z^5}_{M}\, i + \underbrace{5x^5y^4z^5}_{N}\, j + \underbrace{5x^5y^5z^4}_{P}\, k$

$\frac{\partial M}{\partial y} = 25x^4y^4z^5$ $\frac{\partial N}{\partial x} = 25x^4y^4z^5$ $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$

$\frac{\partial P}{\partial x} = 25x^4y^5z^4$ $\frac{\partial M}{\partial z} = 25x^4y^5z^4$ $\boxed{\frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}}$

$\frac{\partial P}{\partial y} = 25x^5y^4z^4$ $\frac{\partial N}{\partial z} = 25x^5y^4z^4$ $\boxed{\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}}$

$\boxed{\text{Conservative}}$

Practice Test 4 - MAC2313
Instructor: Wendy Pogoda

Directions: This is a practice test, your actual test will be different. Use this exam to evaluate the areas in which you need additional studying and practice. Your exam will cover Chapter 16 (Sections 1-7)

Evaluate the line integral along the curve C.

1) $\int_C (y + z)\, ds$, C is the straight-line segment $x = 0$, $y = 4 - t$, $z = t$ from $(0, 4, 0)$ to $(0, 0, 4)$

skip 2) $\int_C \left(\frac{4}{z}\right)^{5/7} ds$, C is the curve $\mathbf{r}(t) = (4t^7 \cos t)\mathbf{i} + (4t^7 \sin t)\mathbf{j} + 4t^7\mathbf{k}$, $0 \le t \le 7\sqrt{2}$

Evaluate the line integral of f(x,y) along the curve C.

3) $f(x, y) = x^2 + y^2$, C: $y = 2x - 3$, $0 \le x \le 3$

Find the gradient field of the function.

4) $f(x, y, z) = x^4e^{5x} + y^6z^5$

Find the work done by F over the curve in the direction of increasing t.

5) $\mathbf{F} = -7y\mathbf{i} + 7x\mathbf{j} + 4z^6\mathbf{k}$; C: $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j}$, $0 \le t \le 3$

Calculate the circulation of the field F around the closed curve C.

skip 6) $\mathbf{F} = xy\mathbf{i} + 2\mathbf{j}$, curve C is $\mathbf{r}(t) = 3 \cos t\mathbf{i} + 3 \sin t\mathbf{j}$, $0 \le t \le 2\pi$

Calculate the flux of the field F across the closed plane curve C.

7) $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$; the curve C is the counterclockwise path around the circle $x^2 + y^2 = 9$

Test the vector field F to determine if it is conservative.

8) $\mathbf{F} = 5x^4y^5z^5\mathbf{i} + 5x^5y^4z^5\mathbf{j} + 5x^5y^5z^4\mathbf{k}$

Find the potential function f for the field F.

9) $\mathbf{F} = \frac{1}{z}\mathbf{i} - 6\mathbf{j} - \frac{x}{z^2}\mathbf{k}$

Evaluate. The differential is exact.

10) $\int_{(0,0,0)}^{(8,3,2)} (2xy^2 - 2xz^2)\, dx + 2x^2y\, dy - 2x^2z\, dz$

Using Green's Theorem, compute the counterclockwise circulation of F around the closed curve C.

11) $\mathbf{F} = \sin 3y\mathbf{i} + \cos 8x\mathbf{j}$; C is the rectangle with vertices at $(0, 0)$, $\left(\frac{\pi}{8}, 0\right)$, $\left(\frac{\pi}{8}, \frac{\pi}{3}\right)$, and $\left(0, \frac{\pi}{3}\right)$

Using Green's Theorem, find the outward flux of F across the closed curve C.

12) $\mathbf{F} = (x^2 + y^2)\mathbf{i} + (x - y)\mathbf{j}$; C is the rectangle with vertices at $(0, 0)$, $(1, 0)$, $(1, 4)$, and $(0, 1)$

Apply Green's Theorem to evaluate the integral.

13) $\oint_C (7y + x)\,dx + (y + 4x)\,dy$

C: The circle $(x - 2)^2 + (y - 4)^2 = 4$

Parametrize the surface S.

14) S is the portion of the cone $\frac{x^2}{64} + \frac{y^2}{64} = \frac{z^2}{25}$ that lies between $z = 6$ and $z = 8$.

Calculate the area of the surface S.

15) S is the portion of the cone $\frac{x^2}{25} + \frac{y^2}{25} = \frac{z^2}{16}$ that lies between $z = 2$ and $z = 5$.

Evaluate the surface integral of G over the surface S.

16) S is the portion of the cone $z = 5\sqrt{x^2 + y^2}$, $0 \le z \le 2$; $G(x, y, z) = z - y$

Evaluate the surface integral of the function g over the surface S.

17) $G(x,y,z) = x^2 + y^2 + z^2$; S is the surface of the cube formed from the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$

Use Stokes' Theorem to calculate the circulation of the field F around the curve C in the indicated direction.

18) $\mathbf{F} = 5y\mathbf{i} + 6x\mathbf{j} + z^3\mathbf{k}$; C: the counterclockwise path around the perimeter of the triangle in the x–y plane formed from the x–axis, y–axis, and the line $y = 5 - 4x$