Steven
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Homework #1

2.1: 7, 15, 24, 30, 31, 33, 39, 42, 54

(7) Juan is a math major but not Computer Science major.

m: Juan is a math major
c: Computer Science major

$m \wedge \sim c$

(15) $P \wedge (\sim q \vee r)$

P	q	r	~q	~q ∨ r	P ∧ (~q ∨ r)
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

(24) $(P \vee q) \vee (P \wedge r)$ and $(P \vee q) \wedge r$

$(P \vee q) \vee (P \wedge r) \equiv (P \vee q) \wedge r$

P	q	r	P ∨ q	P ∧ r	(P ∨ q) ∨ (P ∧ r)	(P ∨ q) ∧ r
T	T	T	T	T	T	T
T	T	F	T	F	T	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

$(P \vee q) \vee (P \wedge r) \not\equiv (P \vee q) \wedge r$ They are not logically equivalent since their truth values aren't equivalent

(30) The dollar is at an all-time high (P) and the stock market is at a record low (q).

$\sim(P \wedge q)$ Demorgan's Law

$\boxed{\sim P \vee \sim q}$

The dollar isn't at an all-time high or the stock market isn't at a record low.

(31) The train is late (P) or my watch is fast (q)

$\sim(P \vee q) \longrightarrow$ DeMorgans Law $\longrightarrow \boxed{\sim P \wedge \sim q}$

The train is not late and my watch is not fast

(33) $-10 < x < 2 \rightarrow$ X is greater than -10 and less than 2

DeMorgans $\longrightarrow$ X is not greater than -10 or not less than 2

$= (-10 < x) \wedge (x < 2) \rightarrow \sim((-10 < x) \wedge (x < 2))$

$= (\sim(-10 < x) \vee \sim(x < 2)) \longrightarrow (-10 \nless x) \vee (x \nless 2)$

$= \boxed{(-10 \geq x) \vee (x \geq 2)}$

P q

(39) (num_orders < 50 and num_instock > 300)

or (50 ≤ num_orders < 75 and num_instock > 500)

r s

$(P \wedge q) \vee (r \wedge s) \Rightarrow \sim((P \wedge q) \vee (r \wedge s))$

$\sim(P \wedge q) \wedge \sim(r \wedge s) \Rightarrow (\sim P \vee \sim q) \wedge (\sim r \vee \sim s)$

$\sim P \vee \sim q$ = num_orders ≥ 50 OR num_instock ≤ 300

$\sim r \vee \sim s$ = ~[(50 ≤ num_orders) AND (num_orders < 75)] OR (num_instock ≤ 500)

$\sim r \vee \sim s$ = (num_orders < 50) OR (num_orders ≥ 75) OR (num_instock ≤ 500)

= (no ≥ 50 OR ni ≤ 300) AND (no < 50 OR no ≥ 75 OR ni ≤ 500)

(42) $((\sim P \wedge q) \wedge (q \wedge r)) \wedge \sim q$

P	q	r	~P	~P∧q	q∧r	(~P∧q)∧(q∧r)	~q	((~P∧q)∧(q∧r)) ∧~q
T	T	T	F	F	T	F	F	F
T	T	F	F	F	F	F	F	F
T	F	T	F	F	F	F	T	F
T	F	F	F	F	F	F	T	F
F	T	T	T	T	T	T	F	F
F	T	F	T	T	F	F	F	F
F	F	T	T	F	F	F	T	F
F	F	F	T	F	F	F	T	F

Contradiction

(54) $(P \wedge (\sim(\sim P \vee q))) \vee (P \wedge q) \equiv P$

$(P \wedge (\sim(\sim P \vee q))) \vee (P \wedge q)$	De Morgans
$(P \wedge (P \wedge \sim q)) \vee (P \wedge q)$	Distributive
$(P \wedge (P \wedge (\sim q \vee q)))$	Negation Law.
$(P \wedge (P \wedge t))$	Identity
$(P \wedge t)$	Identity
P	

OR

$(P \wedge (P \wedge \sim q)) \vee (P \wedge q)$	Assosiative
$((P \wedge P) \wedge \sim q) \vee (P \wedge q)$	Idempotent
$(P \wedge \sim q) \vee (P \wedge q)$	Distributive
$P \wedge (\sim q \vee q)$	Negation
$P \wedge t$	Identity
P	

Homework 2.2

Steven Romeiro

6, 15, 30, 48

⑥ $(P \vee q) \vee (\sim P \wedge q) \rightarrow q$

P	q	$P \vee q$	$\sim P$	$\sim P \wedge q$	$(P \vee q) \vee (\sim P \wedge q)$	$(P \vee q) \vee (\sim P \wedge q) \rightarrow q$
T	T	T	F	F	T	T
T	F	T	F	F	T	F
F	T	T	T	T	T	T
F	F	F	T	F	F	T

⑮ $P \rightarrow (q \rightarrow r)$ and $(P \rightarrow q) \rightarrow r$

r	P	q	$q \rightarrow r$	$P \rightarrow q$	$P \rightarrow (q \rightarrow r)$	$(P \rightarrow q) \rightarrow r$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	T	F	T	F	T	T
F	F	T	F	T	T	F
F	F	F	T	T	T	F

Not logically equivalent

(30) $\underbrace{P \wedge (q \vee r)}_{X} \equiv \underbrace{(P \wedge q) \vee (P \wedge r)}_{Y}$

P	q	r	(q∨r)	(P∧q)	(P∧r)	P∧(q∨r)	(P∧q)∨(P∧r)	X↔Y
T	T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T	T
T	F	T	T	F	T	T	T	T
T	F	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F	F
F	T	F	T	F	F	F	F	F
F	F	T	T	F	F	F	F	F
F	F	F	F	F	F	F	F	F

$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$ therefore $P \wedge (q \vee r) \leftrightarrow (P \wedge q) \vee (P \wedge r)$

(48) $\underbrace{P \vee \sim q}_{X} \rightarrow \underbrace{r \vee q}_{Y}$

Know: $p \rightarrow q \equiv \sim p \vee q$
& $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

$X \rightarrow Y \equiv \sim X \vee Y$

$(P \vee \sim q) \rightarrow (r \vee q) \equiv \sim (P \vee \sim q) \vee (r \vee q)$

a) $(\sim P \wedge q) \vee (r \vee q)$

Know: $p \vee q \equiv \sim(\sim p \wedge \sim q)$

b) $P \vee \sim q \rightarrow r \vee q \equiv \sim (P \vee \sim q) \vee (r \vee q)$

$\sim (P \vee \sim q) \vee (r \vee q) \equiv \sim (\sim (P \vee \sim q) \wedge \sim (r \vee q))$

$\sim ((\sim P \wedge q) \wedge (\sim r \wedge \sim q))$

Know: $\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim ((\sim P \wedge q) \wedge (\sim r \wedge \sim q))$

Homework 2.3

Steven Romeiro

9, 12, 28-30, 39, 42

(9) $P \wedge q \rightarrow \sim r$

$P \vee \sim q$

$\sim q \rightarrow P$

$\therefore \sim r$

P	q	r	$P \wedge q$	$P \wedge q \rightarrow \sim r$	$P \vee \sim q$	$\sim q \rightarrow P$	$\sim r$
T	T	T	T	F	T	T	F
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	F
T	F	F	F	T	T	T	T
F	T	T	F	T	F	T	F
F	T	F	F	T	F	T	T
F	F	T	F	T	T	F	F
F	F	F	F	T	T	F	T

Premises | Conclusion

Invalid

Argument is Invalid

(12) $P \rightarrow q$

q

$\therefore P$

P	q	$P \rightarrow q$	P
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	F

Premises | Conclusion

* Invalid row

Arguments invalid

$P \rightarrow q$

$\sim P$

$\therefore \sim q$

P	q	$P \rightarrow q$	$\sim P$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Premises | Conclusion

* Invalid row

Arguments invalid

(28) if there are as many rational numbers as there are irrational numbers, then the set of all irrational numbers is infinite

p → q
q
∴ p

p	q	p → q	p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	F

* Invalid row

Premises Conclusion

Argument Invalid Converse Error

(29) If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.

p → q
~p
∴ ~q

p	q	p → q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

* Invalid row

premises Conclusion

Arguments invalid by inverse error

P

(30) If this computer program is correct, then it produces the correct output when run with the test data my teacher sent.

Q

$P \to Q$
Q
$\therefore P$

P	Q	P→Q	P
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	F

* invalid row

Argument invalid by converse error

(39) a) Murdered by candlestick
b) LH ∨ maids in dining
c) CK → BKill ~CK
d) LH → Chauf Kill
e) CNK → SND
f) SD → WSKill

① Cook in Kitchen
∴ Butler killed him by poison (Modus ponens by ⓐ)
② Cook not in kitchen ⟶ Sara not in dining by ⓔ (ponens)
③ Lady H ∨ Sara in dining by ⓑ
∴ Lady H in dining by ③ (elimination)
④ Lady H in dining ⟶ chauffer killed Lord H ⓓ
∴ chauffer killed Lord H by ponens ④

(42) a) $P \vee q$

b) $q \rightarrow r$

c) $P \wedge S \rightarrow t$

d) $\sim r$

e) $\sim q \rightarrow U \wedge S$

f) $\therefore t$

rewrite

b) $q \rightarrow r$

d) $\sim r$

① $\therefore \sim q$ (tollens)

e) $\sim q \rightarrow U \wedge S$

$\therefore U \wedge S$ (Ponens by ①)

$U \wedge S$

② $\therefore S$ by (specification)

$\sim q$

$P \vee q$

$\therefore P$ (elimination)

① and e)

$\therefore U \wedge S$ true (ponens)

$P \vee q$

$\sim q$

$\therefore P$ elimination

④ $P \wedge S$ true by conjugation

$\therefore t$ by ponens