Steven
Romeiro

\# 1, 3, 5, 10, 21, 25, 31, 37
47, 52

$\frac{34}{34}$ 100%

7.1 Homework

(1) $\int_{-3}^{-2} \frac{dx}{x} = \int_{-3}^{-2} \frac{1}{x}\,dx = \ln|x| \Big|_{-3}^{-2}$

$= (\ln|-2|) - (\ln|-3|) = (\ln 2) - (\ln 3) = \boxed{-.405}$

(3) $\int \frac{2y\,dy}{y^2-25} = \begin{array}{l} U = y^2-25 \\ du = 2y \end{array} = \int \frac{1}{U}\,du$

$= \ln|U| + e = \boxed{\ln|y^2-25| + C}$

(5) $\int \frac{3\sec^2(t)}{6+3\tan(t)}\,dt = \begin{array}{l} U = 6+3\tan(t) \\ du = \end{array}$

(10) $\int 8e^{(x+1)}\,dx \quad \begin{array}{l} U = x+1 \\ dU = 1\,dx \end{array} = \frac{1}{8}\int 8e^{u}\,du$

$= e^{U} + C = \boxed{e^{(x+1)} + C}$

(21) $\int e^{(\sec \pi t)} \cdot \sec \pi t \cdot \tan \pi t \, dt$

$U = \sec \pi t$

$du = \sec \pi t \cdot \tan \pi t \, dt = \int e^{u} du = e^{u} + c = \boxed{e^{\sec \pi t} + C}$

(25) $\int \frac{e^{r}}{1+e^{r}} dr \quad U = 1+e^{r}, \ du = e^{r} dr = \int \frac{1}{U} du$

$= \ln |u| + c = \boxed{\ln |1+e^{r}| + C}$

(31) $\int_0^{\pi/2} 7^{\cos t} \sin t \, dt \quad a = 7 \quad U = \cos t, \ du = -\sin t \, dt$

$-\int_0^{\pi/2} 7^{\cos t} \cdot (-\sin t) dt = -\int_0^{\pi/2} a^{u} du$

$= \frac{a^{u}}{\ln a} = \frac{7^{\cos t}}{\ln 7} \Big|_0^{\pi/2} = \boxed{-3.083}$

(37) $\int \frac{\log_{10} x}{x} dx = \log_{10} x = \frac{\ln x}{\ln 10} = \int \frac{\frac{\ln x}{\ln 10}}{x} dx$

$= \int \frac{\ln x}{\ln 10} \cdot \frac{1}{x} dx = \int \frac{\ln x}{x \ln 10} = \frac{1}{\ln 10} \int \frac{\ln x}{x} dx$

$U = \ln x$

$du = \frac{1}{x} dx \quad \frac{1}{\ln 10} \int U du = \frac{1}{\ln 10} \cdot \frac{U^2}{2} + C = \frac{1}{\ln 10} \cdot \frac{(\ln x)^2}{2} = \boxed{\frac{(\ln x)^2}{2 \ln 10}}$

47 $\frac{dy}{dt} = e^t \sin(e^t - 2) \qquad y(\ln 2) = 0$

$dy = e^t \sin(e^t + 2)\, dt$

$\int dy = \int e^t \sin(e^t - 2)\, dt \qquad U = e^t - 2$

$du = e^t\, dt$

$\int dy = \int \sin u\, du = y = -\cos u + C$

$= 0 = -\cos(e^{\ln 2} - 2) + C$

$0 = -\cos(2-2) + C$

$0 = -1 + C \qquad \boxed{C = 1}$

52 $\frac{d^2y}{dx^2} = \sec^2 x \qquad y(0) = 0, \quad y'(0) = 1$

$\int d^2y = \int \sec^2 x\, dx^2$

$dy = (\tan x + c)\, dx$

$1 = \tan 0 + C$

$1 = C \longrightarrow \quad dy = \tan x + 1 \rightarrow \int dy = \int \tan x + 1\, dx$

$y = -\ln|\cos x| + 1 + C$

$0 = -\ln|\cos 0| + 0 + C$

$0 = C$

$y = -\ln|\cos x| +$

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\# 9, 13, 17, 19, 23, 25, 43

7.2 Homework

(9) $2\sqrt{xy}\,\frac{dy}{dx} = 1$ $\qquad x, y > 0$

$dy = \frac{1}{2\sqrt{xy}}\,dx \rightarrow \sqrt{y}\,dy = \frac{1}{2\sqrt{x}}\,dx$

$y^{1/2}\,dy = \frac{1}{2}x^{-1/2}\,dx \rightarrow \frac{2}{3}y^{3/2} = \frac{1}{2}\cdot\frac{2}{1}x^{1/2} \rightarrow \frac{2}{3}y^{3/2} = x^{1/2} + c$

$\left(y^{3/2}\right)^{2/3} = \left(\frac{2}{3}x^{1/2}\right)^{2/3} \longrightarrow \boxed{y = \sqrt[3]{\frac{4}{9}x}}$ or $\boxed{\frac{2}{3}y^{3/2} - x^{1/2} = C}$

(13) $\frac{dy}{dx} = \sqrt{y}\cos^2\sqrt{y} \longrightarrow dy = y^{1/2}\cos^2(y^{1/2})\,dx$

$\frac{dy}{y^{1/2}\cos^2(y^{1/2})} = dx \longrightarrow \int \frac{1}{y^{1/2}\cdot\cos^2(y^{1/2})}\,dy = \int dx$

$u = y^{1/2}$

$du = \frac{1}{2}y^{-1/2}\,dy$

$\rightarrow \int y^{-1/2}\cdot\frac{1}{\cos^2(y^{1/2})}\,dy = \int dx \rightarrow 2\int \frac{1}{2}y^{-1/2}\,dy\cdot\frac{1}{\cos(y^{1/2})} = \int dx$

$2\int \frac{1}{\cos^2(u)}\,du = x + c \longrightarrow 2\int \sec^2(u)\,du = x + c$

$2\tan(u) = x + c \longrightarrow \boxed{2\tan(y^{1/2}) - x = c}$

(17) $\frac{dy}{dx} = 2x\sqrt{1-y^2}$ $\quad -1 < y < 1$

$dy = 2x\sqrt{1-y^2} \cdot dx \rightarrow \frac{dy}{(1-y^2)^{1/2}} = 2x\,dx$

$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int 2x\,dx \longrightarrow \quad \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

$\therefore = \sin^{-1}(y) = x^2 + c \longrightarrow \sin^{-1}(y) - x^2 = c$

$\cancel{\sin}(\cancel{\sin^{-1}}(y)) = \sin(x^2+c) \rightarrow \boxed{y = \sin(x^2+c)}$

(19) $y^2\frac{dy}{dx} = 3x^2y^3 - 6x^2 \longrightarrow y^2\frac{dy}{dx} = 3x^2(y^3-2)$

$\frac{y^2}{(y^3-2)}dy = 3x^2dx \longrightarrow \int \frac{y^2}{(y^3-z)}dy = \int 3x^2dx$

$u = y^3 - z$
$du = 3y^2dy$

$\frac{1}{3}\int \frac{3y^2\,dy}{(y^3-z)} = \int 3x^2dx \longrightarrow \frac{1}{3}\int \frac{1}{u}du = \int 3x^2dx$

$\frac{1}{3}\ln|u| = x^3 + c \longrightarrow \boxed{\frac{1}{3}\ln|y^3-z| = x^3 + c}$

$y =$ tooth size

(23) a) $y = 0.99 y_0$

$y = y_0 e^{kt}$

$\frac{.99 \cancel{y_0}}{\cancel{y_0}} = \frac{\cancel{y_0} e^{k(1000)}}{\cancel{y_0}}$

$.99 = e^{1000k}$

$\ln .99 = 1000k \ln(e)$

$\ln .99 = 1000k \rightarrow k = \frac{\ln .99}{1000} = \boxed{-1.005E-5}$

b.) $y = 1.8,\ y_0 = 2,\ k = -1.005E-5$

$2 = 1.8e^{(-1.005E-5)t} \rightarrow \frac{1.8}{2} = e^{(-1.005E-5)t}$

$\ln\left(\frac{1.8}{2}\right) = (-1.005E-5)t \rightarrow \boxed{t = 10,536}$

c) $y_0 = 1,\ k = -1.005E-5,\ t = 20,000$

$y = 1e^{(-1.005E-5)(20,000)}$

$y = .8179 * 100$

$\boxed{y = 81.79\%}$

(25) $\frac{dy}{dt} = -0.6y$, $y = y_0 e^{kt}$

$dy = -.6y\,dt$

$\int \frac{dy}{y} = \int -.6\,dt = \ln|y| = -.6t + c$

$e^{\ln y} = e^{(-.6t+c)}$

$y = e^{(-.6t+c)}$

$y = e^{(-.6t)} \cdot e^{(c)}$

$y = e^{(-.6t)} \cdot c$

$100 = 1 \cdot c$

$c = 100 \rightarrow y = e^{(-.6t)} \cdot 100$

$y = e^{(-.6)} \cdot 100$

$\boxed{y = 54.88g}$

(43) $Water_i = 46°C$ $t = 10m$ $water_1 = 39°C$

$water_F = 33°C$

?

$$H - H_s = (H_0 - H_s)e^{-kt}$$

#1. $39 - H_s = (46 - H_s)e^{-k10}$

#2. $33 - H_s = (46 - H_s)e^{-k20}$

Solve e^{-kt} #1. $\frac{39 - H_s}{46 - H_s} = e^{-k10}$ #2. $\frac{33 - H_s}{46 - H_s} = e^{-k(20)}$

#2. $\frac{33 - H_s}{46 - H_s} = e^{-k(10)(2)}$

$\therefore \frac{33 - H_s}{46 - H_s} = \left(\frac{39 - H_s}{46 - H_s}\right)^2$

#2. $\frac{33 - H_s}{46 - H_s} = \left(e^{-k10}\right)^2$

$$\frac{33 - H_s}{46 - H_s} = \frac{(39 - H_s)^2}{(46 - H_s)^2}$$

easier way ?

$$\frac{(33 - H_s)\cancel{(46 - H_s)}^2}{\cancel{(46 - H_s)}} = (39 - H_s)^2 \rightarrow (33 - H_s)(46 - H_s) = (39 - H_s)^2$$

$1518 - 33H_s - 46H_s + H_s^2 = 1521 - 78H_s + H_s^2$

#2 $1518 - 79H_s + H_s^2 = 1521 - 78H_s + H_s^2$ #1

$1518 - 79H_s + \cancel{H_s^2} - \cancel{H_s^2} = 1521 - 78H_s$

$1518 - 1521 = 79H_s - 78H_s$

$\boxed{-3 = H_s}$

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1, 7, 13, 17, 27, 35, 37, 39, 41, 43, 67, 71

7.3 Homework

(1) $\sinh x = -\frac{3}{4} \rightarrow \cosh x = -\frac{3}{4}$?

(7) $\cosh 5x + \sinh 5x \rightarrow \sinh x = \frac{e^{x} - e^{-x}}{2}, \cosh x = \frac{e^{x} + e^{-x}}{2}$

$\therefore \frac{e^{5x} + e^{-5x}}{2} + \frac{e^{5x} - e^{-5x}}{2} \rightarrow \frac{2e^{5x}}{2} \rightarrow \boxed{e^{5x}}$

(13) $y = 6 \sinh \frac{x}{3} \rightarrow y' = 6 \cdot \frac{1}{3} \cosh\left(\frac{x}{3}\right)$

$\boxed{y' = 2 \cosh\left(\frac{x}{3}\right)}$

(17) $y = \ln(\sinh(z)) \rightarrow y' = \ln(u)du \quad u = \sinh(z)$

$du = \cosh(z)dz$

$y' = \frac{1}{u} du \rightarrow y' = \frac{1}{\sinh(z)} \cdot \cosh(z)dz$

$y' = \frac{\cosh(z)}{\sinh(z)} dz \rightarrow \boxed{y' = \coth(z)\, dz}$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2} = \tan^2 + 1 = \sec^2$$

(27) $y = (1-\theta)\tanh^{-1}(\theta)$

$$y' = (-1)(\tanh^{-1}(\theta)) + (1-\theta)\cdot\left(\frac{1}{1-\theta^2}\right)$$

$$y' = \tanh^{-1}(\theta) + \frac{\cancel{(1-\theta)}}{\cancel{(1-\theta)}(1+\theta)}$$

$$\boxed{y' = \frac{1}{1+\theta} - \tanh^{-1}(\theta)}$$

(35) $y = \sinh^{-1}(\tan x)$

$$y' = \frac{1}{\sqrt{1+\tan^2 x}}\cdot \sec^2 x \Rightarrow y' = \frac{1}{\sqrt{1+\tan^2 x}}\cdot(\sec x)(\sec x)$$

* identity
$1+\tan^2 x = \sec^2 x$

$$y' = \frac{(\sec x)(\sec x)}{\sqrt{\sec^2 x}} = \frac{(\sec x)^2}{(\sec^2 x)^{1/2}} = \frac{(\sec x)^2}{\sec x}$$

$$\boxed{y' = \sec x}$$

$\cosh^2 x - \sinh^2 x = 1$

37 $\int \operatorname{sech}(x)\,dx = \tan^{-1}(\sinh x) + C$

Derive right side

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} \quad \therefore \frac{d}{dx}\tan^{-1}(\sinh x) + C = \frac{1}{1+\sinh^2 x}\cdot\cosh x$

$= \frac{\cosh x}{1+\sinh^2 x} = \frac{\cosh x}{\cosh^2 x} = \frac{1}{\cosh x} = \boxed{\int \operatorname{sech} x\,dx}$

39 $\int x\coth^{-1}x\,dx = \frac{x^2-1}{2}\coth^{-1}x + \frac{x}{2} + C$

$= \frac{1}{2}(x^2-1)\cdot\coth^{-1}(x) + \frac{1}{2}x$

$= \frac{1}{\cancel{2}}(\cancel{2}x)\cdot\coth^{-1}(x) + \frac{1}{2}(x^2-1)\left(\frac{1}{1-x^2}\right) + \frac{1}{2}$

$= x\coth^{-1}x + \left[-\frac{1}{2}(\cancel{1-x^2})\left(\frac{1}{\cancel{1-x^2}}\right)\right] + \frac{1}{2}$

$= x\coth^{-1}x - \frac{1}{2} + \frac{1}{2}$

$= \boxed{\int x\coth^{-1}x\,dx}$

$u = 2x$
$du = 2\,dx$

(41) $\int \sinh 2x\,dx = \frac{1}{2}\int \sinh(2x)\,2\,dx$

$= \frac{1}{2}\int \sinh u\,du = \frac{1}{2}\cosh u + C$

$= \boxed{\frac{\cosh 2x}{2} + C}$

$u = \frac{x}{2} - \ln 3$
$du = \frac{1}{2}\,dx$

(43) $\int 6\cosh\left(\frac{x}{2} - \ln 3\right)dx = 12\int \frac{6}{12}\cosh\left(\frac{x}{2} - \ln 3\right)dx$

$= 12\int \cosh\left(\frac{x}{2} - \ln 3\right)\frac{1}{2}\,dx = 12\int \cosh u\,du$

$= 12\sinh u + C = \boxed{12\sinh\left(\frac{x}{2} - \ln 3\right) + C}$

(67) $\displaystyle\int_0^{2\sqrt{3}} \frac{1}{\sqrt{4+x^2}}\,dx = \frac{d}{dx}\sinh^{-1}\left(\frac{u}{a}\right) = \int \frac{1}{\sqrt{a^2+u^2}}\,\frac{du}{dx}$

$u^2 = x^2 \quad a^2 = 4$

$u = x \quad a = 2 \quad \therefore \displaystyle\int_0^{2\sqrt{3}} \frac{1}{\sqrt{a^2+u^2}}\,du = \sinh^{-1}\left(\frac{u}{a}\right)\Bigg|_0^{2\sqrt{3}}$

$du = dx$

$$= \sinh^{-1}\left(\frac{x}{2}\right)\Bigg|_0^{2\sqrt{3}} = \sinh^{-1}\left(\frac{2\sqrt{3}}{2}\right) - \sinh^{-1}\left(\frac{0}{2}\right)$$

$$= \sinh^{-1}(\sqrt{3}) - \sinh^{-1}(0)$$

$$= \sinh^{-1}(\sqrt{3}) - 0$$

$$\boxed{= \sinh^{-1}(\sqrt{3})}$$

$$* \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{Sech}^{-1}\left(\frac{u}{a}\right)+C$$

(71) $$\int_{1/5}^{3/13} \frac{dx}{x\sqrt{1-16x^2}} =$$

$u^2 = 16x^2 \quad a^2 = 1$

$u = 4x \quad a = 1$

$du = 4dx \quad dx = \frac{1}{4}du$

$$= \int_{1/5}^{3/13} \frac{1\,du}{4x\sqrt{1-16x^2}} = \int_{1/5}^{3/13} \frac{du}{u\sqrt{a^2-u^2}}$$

$$= -\frac{1}{a}\operatorname{Sech}^{-1}\left(\frac{u}{a}\right)\Bigg|_{1/5}^{3/13} = -\frac{1}{1}\operatorname{Sech}^{-1}\left(\frac{4x}{1}\right)\Bigg|_{1/5}^{3/13}$$

$$= -\operatorname{Sech}^{-1}\left(4\cdot\frac{3}{13}\right) + \operatorname{Sech}^{-1}\left(4\cdot\frac{1}{5}\right)$$

$$= -\operatorname{Sech}^{-1}\left(\frac{12}{13}\right) + \operatorname{Sech}^{-1}\left(\frac{4}{5}\right)$$

$$= \boxed{-\operatorname{Sech}^{-1}\left(\frac{12}{13}\right) + \operatorname{Sech}^{-1}\left(\frac{4}{5}\right)}$$

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1,3,7,9,17

7.4 Homework

1) $\lim_{x\to\infty} \frac{e^x}{x-3} = \frac{\infty}{\infty}$

L'Hopitals $\lim_{x\to\infty} \frac{e^x}{1} = \boxed{\infty}$ $\therefore$ $\boxed{e^x \text{ grows faster than } x-3}$

b. $\lim_{x\to\infty} \frac{e^x}{x^3+\sin x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{e^x}{3x^2+\cos x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{e^x}{6x-\sin x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{e^x}{6-\cos x} = \boxed{\infty}$ $\therefore$ $\boxed{e^x \text{ grows faster than } x^3+\sin x}$

c. $\lim_{x\to\infty} \frac{e^x}{\sqrt{x}} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{e^x}{\frac{1}{2}x} = \frac{2e^x}{x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2e^x}{1} = \boxed{\infty}$ $\therefore$ $\boxed{e^x \text{ grows faster than } \sqrt{x}}$

Steven Tamura

7.4 Homework

1, 3, 7, 9, 17

$$\frac{d}{du}(a^u) = a^u \cdot \ln(u) \cdot du$$

? d. $\lim_{x \to \infty} \frac{e^x}{4^x} = \frac{\infty}{\infty}$

Why 4^x faster than e^x ? e & 4 are both constants raised to a Variable power of the same magnitude

? e. $\lim_{x \to \infty} \frac{e^x}{(3/2)^x}$ e^x grows faster ?

f. $\lim_{x \to \infty} \frac{e^x}{e^{x/2}} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{e^x}{\frac{1}{2}e^{x/2}}$ e^x grows faster because it's a larger constant ?

Same? g. $\lim_{x \to \infty} \frac{e^x}{e^{\cos x}}$ $\frac{\infty}{\text{small constant}} = \infty$

h. $\lim_{x \to \infty} \frac{e^x}{\log_{10} x} = \frac{e^x}{\frac{\ln x}{\ln 10}} = \frac{e^x}{\frac{1}{\ln 10} \cdot \ln x} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{e^x}{c + \frac{1}{x}}$ $\frac{e^x}{c + 0} = \boxed{\infty}$ $\boxed{e^x \text{ grows faster than } \log_{10} x}$

3) a. $\lim_{x\to\infty} \frac{x^2}{x^2+4x} = \frac{\infty}{\infty} = \boxed{1}$

$\lim_{x\to\infty} \frac{2x}{2x+4} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2}{2} = \boxed{1}$

$\boxed{x^2 \text{ grows at the same rate as } x^2+4x}$

b. $\lim_{x\to\infty} \frac{x^2}{x^5-x^2} = \frac{\infty}{\infty}$

$\lim_{x\to\infty}'' \frac{2}{20x^3-2} = \frac{2}{\infty} = \boxed{0}$

$\boxed{x^2 \text{ grows slower than } x^5-x^2}$

c. $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+x^3}} = \frac{x^2}{x^2+x^{3/2}} = \boxed{1}$

$\boxed{x^2 \text{ grows at the same rate}}$

d. $\lim_{x\to\infty} \frac{x^2}{(x+3)^2} = \frac{x^2}{x^2+6x+9} = \boxed{1}$

$\boxed{x^2 \text{ grows at the same rate as } (x+3)^2}$

e. $\lim_{x\to\infty} \frac{x^2}{x\ln x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2x}{\ln x + x(\frac{1}{x})} = \frac{2x}{\ln x+1} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2}{\frac{1}{x}} = 2x = \boxed{\infty}$

$\boxed{x^2 \text{ grows faster than } x\ln x}$

f. $\lim_{x\to\infty} \frac{x^2}{2^x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2x}{2^x \cdot \ln x} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2}{(2^x \cdot \ln x)(\ln x) + (2^x)(\frac{1}{x})} = \boxed{0}$

$\boxed{x^2 \text{ grows slower than } 2^x}$

g. $\lim_{x\to\infty} \frac{x^2}{x^3 e^{-x}} = \frac{x^2}{x^3 \cdot (\frac{1}{e^x})} = \frac{x^2}{\frac{x^3}{e^x}} = \frac{x^2 e^x}{x^3} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2xe^x + x^2e^x}{3x^2} = \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{2e^x + 2xe^x + 2xe^x + x^2e^x}{6x} = \boxed{\infty}$

$\boxed{x^2 \text{ grows faster than } x^3e^{-x}}$

h. $\lim_{x\to\infty} \frac{x^2}{8x^2} = \frac{1}{8}$

$\boxed{x^2 \text{ grows at the same rate as } 8x^2}$

(7) d, a, c, b

(9) a. $x = o(x)$ $f(x)$ is little-oh of $g(x)$

$\lim_{x\to\infty} \frac{x}{x} = \boxed{1}$ $\therefore$ $\boxed{\text{x grows at the same rate as x. } x = o(x) \text{ is false}}$

b. $x = o(x+5)$ $f(x)$ is little-oh of $g(x)$

$\lim_{x\to\infty} \frac{x}{x+5} = \boxed{1}$ $\boxed{\text{x grows at the same rate as } x+5. \ x = o(x+5) \text{ is false}}$

c. $x = O(x+5)$ $f(x)$ is big-oh of $g(x)$

$\lim_{x\to\infty} \frac{x}{x+5} = \boxed{1}$ $\boxed{\text{x grows at the same rate as } x+5 \ \therefore x = O(x+5) \text{ is true}}$

d. $x = O(2x)$ $f(x)$ is big-oh of $g(x)$

$\lim_{x\to\infty} \frac{x}{2x} = \boxed{1}$ $\boxed{\text{x grows at the same rate as } 2x \ \therefore x = O(2x) \text{ is true}}$

e. $e^x = o(e^{2x})$ $f(x)$ is little-oh of $g(x)$

$$\lim_{x\to\infty} \frac{e^x}{e^{2x}} = \frac{e^x}{(e^x)^2} = \frac{e^x}{2e^x \cdot e^x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{e^x}{2e^{2x}} = \frac{1}{2}\cdot\frac{e^x}{e^{2x}} = \frac{1}{2}\cdot e^{x-2x} = \frac{1}{2}\cdot e^{-x} = \frac{1}{2e^x}$$

$= \boxed{0}$ $e^x = o(e^{2x})$ is true

F. $x + \ln x = O(x)$ f(x) grows @ same rate as G(x)

$$\lim_{x\to\infty} \frac{x+\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{1+\frac{1}{x}}{1} = \boxed{1} \therefore$$

$\boxed{\begin{array}{l} x+\ln x \text{ grows at the same rate as } x \\ x+\ln x = O(x) \text{ is true} \end{array}}$

G. $\ln x = o(\ln 2x)$ f(x) is little-oh of G(x)

$$\lim_{x\to\infty} \frac{\ln x}{\ln 2x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{2\frac{1}{x}} = \frac{x}{2x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \boxed{\frac{1}{2}}$$ $\therefore$ $\ln x$ grows at the same rate as $\ln 2x$. $\ln x = o(\ln 2x)$ false

H. $\sqrt{x^2+5} = O(x)$ f(x) is big-oh of G(x)

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{2}(x^2+5)^{-\frac{1}{2}}\cdot 2x}{1} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}x}{(x^2+5)^{1/2}} = \frac{x}{(x^2+5)^{1/2}}$$

$$\lim_{x\to\infty} \frac{1}{\frac{1}{2}\cdot(x^2+5)^{3/2}\cdot 2x} = \frac{2}{(x^2+5)^{3/2}\cdot\cancel{2}x} = \frac{1}{(x^2+5)^{3/2}\cdot x} = \boxed{0}$$

(17) $\sqrt{10x+1} = O(\sqrt{x+1})$ & $O(\sqrt{x})$

$$\lim_{x\to\infty} \frac{(10x+1)^{1/2}}{(x)^{1/2}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \left(\frac{10x+1}{x}\right)^{1/2} = \left(\frac{10x}{x} + \frac{1}{x}\right)^{1/2} = \left(10 + \frac{1}{x}\right)^{1/2}$$

$$\lim_{x\to\infty} \left(10 + \frac{1}{\infty}\right)^{1/2} = (10+0)^{1/2} = \boxed{\sqrt{10}} \therefore \boxed{\sqrt{10x+1} \text{ grows @ same rate as } \sqrt{x}}$$

$$\lim_{x\to\infty} \frac{\sqrt{x+1}}{\sqrt{x}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \left(\frac{x+1}{x}\right)^{1/2} = \left(\frac{x}{x} + \frac{1}{x}\right)^{1/2} = \left(1 + \frac{1}{x}\right)^{1/2}$$

$$= \left(1 + \frac{1}{\infty}\right)^{1/2} = (1+0)^{1/2} = \sqrt{1} = \boxed{1} \therefore \boxed{\sqrt{x+1} \text{ grows @ same rate as } \sqrt{x}}$$

$\boxed{\text{By equivalency, } \sqrt{10x+1} \text{ grows at the same rate as } \sqrt{x+1}}$

Steven Romeiro

3, 9, 13, 19, 23, 27, 31, 33, 37, 39, 55, 63, 65

LIPET 66/66 100

L: Logs, I: Inverse, P: Polynomial, E: Exponential, T: Trig

8.2 Homework

(1) $\int x \sin\frac{x}{2}\,dx$

$u = x \quad dv = \sin\frac{x}{2}\,dx$

$du = dx \quad v = -2\cos\frac{x}{2}$

$\int u \cdot dv = u \cdot v - \int v \cdot du$

$= x \cdot -2\cos\frac{x}{2} - \int -2\cos\frac{x}{2}\,dx$

$= -2x\cos\frac{x}{2} + 2\int \cos\frac{x}{2}\,dx \qquad u = \frac{x}{2} \quad du = \frac{1}{2}dx$

$= -2x\cos\frac{x}{2} + 2 \cdot 2\int \cos\frac{x}{2} \cdot \frac{1}{2}\,dx$

$= -2x\cos\frac{x}{2} + 4\int \cos u\,du$

$= \boxed{-2x\cos\frac{x}{2} + 4\sin\frac{x}{2} + C}$

(3) $\int t^2 \cos t\,dt$

$u = t^2 \quad dv = \cos t\,dt$

$du = 2t\,dt \quad v = \sin t$

$= t^2 \cdot \sin t - \int \sin t \cdot 2t\,dt \qquad u = 2t \quad dv = \sin t\,dt$

$du = 2dt \quad v = -\cos t$

$= t^2\sin t - \left(-2t\cos t - \int -\cos t \cdot 2dt\right) \qquad u = t \quad du = dt$

$= t^2\sin t + 2t\cos t - 2\int \cos t\,dt$

$= \boxed{t^2\sin t + 2t\cos t - 2\sin t + C}$

L I P E T

o n o x r
g v l p i
  y   g

Homework

(13) $\int X \sec^2 x \, dx$ $\qquad U = X \quad dV = \sec^2 x \, dx$

$\qquad\qquad\qquad\qquad dU = dx \quad V = \tan x$

$= X \tan X - \int \tan x \, dx$

$= X \tan x - \ln \sec x + C$

$= X \tan x + \ln \frac{1}{\cos x^{-1}} = \boxed{X \tan x + \ln \cos X + C}$

(19) $\int x^5 e^x \, dx$ $\qquad U = X^5 \quad dV = e^x \, dx$

$\qquad\qquad\qquad\qquad dU = 5x^4 \, dx \quad V = e^x$

$= x^5 e^x - \int e^x 5x^4 \, dx$

X^5	+	e^x
$5X^4$	-	e^x
$20x^3$	+	e^x
$60X^2$	-	e^x
$120X$	+	e^x
120		$e^x + C$

$\boxed{X^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120xe^x - 120e^x + C}$

$\int u \cdot dv = u \cdot v - \int v\,du$ $\qquad \frac{1}{3}\int \cos 3x\,dx \cdot 3 \quad u = 3x,\ du = 3dx$

(23) $\boxed{\int e^{2x}\cos 3x\,dx}$ $\qquad u = e^{2x} \quad dv = \cos 3x\,dx$
$\qquad du = 2e^{2x}dx \quad v = \frac{1}{3}\sin 3x$

$= e^{2x}\cdot\frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x \cdot 2e^{2x}dx$

$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx$ $\qquad u = e^{2x} \quad dv = \sin 3x\,dx$
$\qquad du = 2e^{2x}dx \quad v = -\frac{1}{3}\cos 3x$

$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[e^{2x}\cdot -\frac{1}{3}\cos 3x - \int -\frac{1}{3}\cos 3x\, 2e^{2x}dx\right]$

$= \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \boxed{\frac{4}{9}\int e^{2x}\cos 3x\,dx}$

$\frac{4}{9}\int e^{2x}\cos 3x\,dx + \int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$

$\frac{13}{9}\int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$

$\int e^{2x}\cos 3x\,dx = \frac{9}{13}\cdot\frac{1}{3}e^{2x}\sin 3x + \frac{9}{13}\cdot\frac{2}{9}e^{2x}\cos 3x$

$\int e^{2x}\cos 3x\,dx = \frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x$

$$\int e^{2x}\cos 3x\,dx = \frac{1}{13}e^{2x}\left(3\sin 3x + 2\cos 3x + C\right)$$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2} \longrightarrow \tan^2 x = \sec^2 x - 1$$

(27) $\displaystyle\int_0^{\pi/3} x\tan^2 x\,dx$

$u = x \qquad dv = \sec^2 x - 1$

$du = dx \qquad v = \tan x - x$

$$= x \cdot (\tan x - x) - \int (\tan x - x)\,dx$$

$$= x\tan x - x^2 - \int \tan x\,dx + \int x\,dx$$

$$= x\tan x - x^2 - \ln \sec x + \frac{1}{2}x^2$$

$$= x\tan x - \frac{1}{2}x^2 + \ln \cos x \Big|_0^{\pi/3}$$

$$\left[\frac{\pi}{3}\tan\frac{\pi}{3} - \frac{1}{2}\left(\frac{\pi}{3}\right)^2 + \ln\cos\left(\frac{\pi}{3}\right)\right] - \left[\cancel{0\tan(0)}^{0} - \cancel{\frac{1}{2}(0)^2}^{0} + \cancel{\ln\cos 0}^{0}\right]$$

$$= \frac{\pi}{3}(\sqrt{3}) - \frac{\pi^2}{18} + \ln\left(\frac{1}{2}\right)$$

$$\boxed{= \frac{\pi\sqrt{3}}{3} - \frac{\pi^2}{18} + \ln\left(\frac{1}{2}\right)}$$

(31) $\int x \sec x^2 \, dx \qquad u = x^2 \quad du\ 2x\,dx$

$$\frac{1}{2}\int \sec(x^2)\cdot 2x\,dx = \frac{1}{2}\int \sec u\,du$$

$$= \frac{1}{2}\ln\left(\sec u + \tan u\right) + C$$

$$\boxed{= \frac{1}{2}\ln\left(\sec(x^2) + \tan(x^2)\right) + C}$$

(33) $\int x(\ln x)^2\,dx = \qquad u = (\ln x)^2 \quad dv = x\,dx$

$\qquad du = \frac{2\ln x}{x} \quad v = \frac{1}{2}x^2$

$$= (\ln x)^2\cdot\frac{1}{2}x^2 - \int \frac{1}{2}x^2\,\frac{2\ln x}{x}\,dx$$

$$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}\int \cancel{x^2}\,\frac{2\ln x}{\cancel{x}}\,dx$$

$$= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}\int x\cdot 2\ln x\,dx$$

$$= \frac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx \qquad u = \ln x \quad dv = x\,dx$$

$\qquad du = \frac{1}{x} \quad v = \frac{1}{2}x^2$

$$= \frac{1}{2}x^2(\ln x)^2 - \left[\ln x\cdot\frac{1}{2}x^2 - \int \frac{1}{2}x^2\cdot\frac{1}{x}\,dx\right]$$

$$\frac{1}{2}x^2(\ln x)^2 - \left[\ln x \cdot \frac{1}{2}x^2 - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx\right]$$

$$= \frac{1}{2}x^2(\ln x)^2 - \left[\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx\right]$$

$$= \frac{1}{2}x^2(\ln x)^2 - \left[\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2\right]$$

$$\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2$$

$$\boxed{\frac{1}{2}x^2\left((\ln x)^2 - \ln x + \frac{1}{2}\right)}$$

37) $\int x^3 e^{x^4}\,dx$ $\quad U = x^4$
$du = 4x^3\,dx \quad dx = \frac{1}{4x^3}\,du$

$$= \frac{1}{4}\int e^u \cdot du = \frac{1}{4}e^u = \boxed{\frac{1}{4}e^{x^4} + C}$$

(39) $\int x^3\sqrt{x^2+1}\,dx$ $\qquad u = x^2 \qquad dv = x(x^2+1)^{1/2}dx$

$du = 2x\,dx \qquad v = \frac{1}{2}\frac{(x^2+1)^{3/2}}{3/2}$

$\int x^2 \cdot x\sqrt{x^2+1}\,dx$

$v = \frac{1}{3}(x^2+1)^{3/2}$

$\frac{1}{3}x^2(x^2+1)^{3/2} - \int \frac{2}{3}x(x^2+1)^{3/2}$

$u = x^2+1 \quad du = 2x$

or $\int x^2 \cdot \sqrt{x^2+1}\cdot x\,dx \qquad u = x^2+1 \qquad x^2 = u-1$

$du = 2x\,dx$

$\int x^2 \cdot (u)^{1/2} \cdot \frac{1}{2}du = \frac{1}{2}\int (u-1)(u^{1/2})\,du$

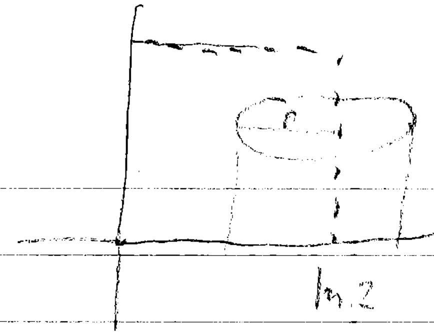
$= \frac{1}{2}\int u^{3/2} - u^{1/2}\,du = \frac{1}{2}\int u^{3/2}du - \frac{1}{2}\int u^{1/2}du$

$= \frac{1}{2}\cdot\frac{2}{5}u^{5/2} - \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$\boxed{\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C}$

53) $r = (\ln 2) - x$

$h = e^{x}$ $\quad th = dx$



$$\int_0^{\ln 2} 2\pi\, r \cdot h \cdot th$$

23) $\int e^{2x} \cos 3x\, dx$ $\quad U = e^{2x}$ $\quad dV = \cos 3x\, dx$

$du = 2e^{x} dx$ $\quad V = \frac{1}{3} \sin 3x$

$$\frac{1}{3} \int 3 \cos 3x\, dx$$

$$\int e^{2x} \cos 3x\, dx = \frac{1}{3} e^{2x} \sin 3x - \int \frac{2}{3} e^{2x} \sin 3x\, dx$$

One more time

$$= \frac{13}{9} \int e^{2x} \cos 3x\, dx = \frac{1}{3} e^{2x} \sin 3x + \frac{2}{9} e^{2x} \cos 3x$$

$$\int e^{2x} \cos 3x\, dx = \frac{3}{13} e^{2x} \sin 3x + \frac{2}{13} e^{2x} \cos 3x + C$$

39) $\int x^3\sqrt{x^2+1}\,dx = \int x^2 \cdot x\sqrt{x^2+1}\,dx$

$U = x^2$ $\quad \int dV = \int x\sqrt{x^2+1}\,dx =$ $\quad U = x^2+1$ $\quad = \frac{1}{2}\int (x^2+1)^{1/2}\,2x\,dx$

$dU = 2x\,dx$ $\quad\quad dU = 2x\,dx$

$\int dV = \frac{1}{2}\int U^{1/2} = V = \frac{1}{2}\cdot\frac{2}{3}(U)^{3/2} = \boxed{V = \frac{1}{3}(x^2+1)^{3/2}}$

$= x^2\cdot\frac{1}{3}(x^2+1)^{3/2} - \int \frac{1}{3}(x^2+1)^{3/2}\,2x\,dx$

$= \frac{1}{3}x^2(x^2+1)^{3/2} - \frac{2}{3}\int x(x^2+1)^{3/2}\,dx$ $\quad U = x^2+1$

$du = 2x\,dx$

$= \frac{1}{3}x^2(x^2+1)^{3/2} - \frac{1}{3}\int U^{3/2}\,du \rightarrow \frac{1}{3}\cdot\frac{2}{5}(x^2+1)^{5/2}$

$= \frac{1}{3}x^2(x^2+1)^{3/2} - \frac{2}{15}(x^2+1)^{5/2} + C$

$= \frac{1}{15}(x^2+1)^{3/2}\left[5x^2 - 2(x^2+1)\right] = \frac{1}{15}(x^2+1)^{3/2}\left[5x^2-2x^2-2\right]$

$$\boxed{\frac{1}{15}(x^2+1)^{3/2}(3x^2-2)+C}$$

$$\boxed{= \frac{1}{15}(x^2+1)^{3/2}(3x^2-2)+C}$$

(55) $r = (\ln 2) - x$

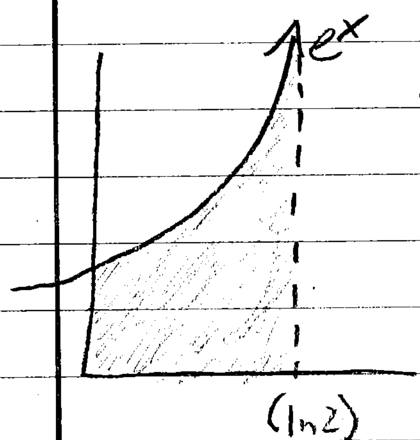
$h = e^x \qquad th = dx \qquad \int 2\pi r \cdot h \cdot th$

$$\int_0^{\ln 2} 2\pi(\ln 2 - x)e^x\,dx = 2\pi\int_0^{\ln 2}(\ln 2 - x)e^x\,dx \qquad \begin{matrix} u = (\ln 2 - x) & dv = e^x dx \\ du = -dx & v = e^x \end{matrix}$$

$$= 2\pi\left[(\ln 2 - x)e^x - \int_0^{\ln 2} e^x - dx\right] = 2\pi\left[(\ln 2 - x)e^x + \int_0^{\ln 2} e^x dx\right]$$

?

$$= 2\pi\left[(\ln 2 - x)e^x + e^x\Big|_0^{\ln 2}\right] = e^{\ln 2} - e^0 = 2 - 1$$



$2\pi[(\ln 2 - \ln 2) + \ln 2 - 0$

$\cancel{2\pi\left[(\ln 2 - [\ln 2 - 0])(2-1) + 2 - 1\right] = 2\pi\left[(\ln 2 - \ln 2)(1) + 1\right]}$

$(\ln 2 - (\ln$

$$= \boxed{2\pi(1 - \ln 2)} \quad ?$$

(63) $\int x^n \cos x \, dx = x^n \sin x - n\int x^{n-1} \sin x \, dx$

$u = x^n \qquad dv = \cos x \, dx$

$du = x^{n-1}(n)dx \qquad v = \sin x$

$x^n \cdot \sin x - \int \sin x \cdot (nx^{n-1})dx$

$= x^n \sin x - n\int \sin x \cdot x^{n-1} dx$

$= \boxed{x^n \sin x - n\int x^{n-1} \sin x \, dx}$

$$(65) \int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax}\,dx \qquad a \neq 0$$

$u = x^n$ $\quad dv = e^{ax}\,dx$ $\quad ax = 3x$ $\quad e^{3x}\,dx$

$du = nx^{n-1}\,dx$ $\quad v = \frac{1}{a}e^{ax}$ $\quad u = 3x$

$du = 3\,dx$

$$= x^n \cdot \frac{1}{a}e^{ax} - \int \frac{1}{a}e^{ax} \cdot nx^{(n-1)}\,dx$$

$$= \frac{x^n e^{ax}}{a} - \frac{n}{a}\int e^{ax} \cdot x^{n-1}\,dx$$

$$= \boxed{\frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{(n-1)} e^{ax}\,dx}$$

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$\#1, 3, 7, 11, 17, 19, 23, 27, 33, 35, 37$
$41, 51, 57$

8.3 Homework

(1) $\int \cos 2x \, dx$ $\quad u = 2x$
$\quad du = 2\,dx$

$\frac{1}{2}\int \cos 2x \, 2dx \rightarrow \frac{1}{2}\int \cos u \, du$

$= \boxed{\frac{1}{2}\sin 2x + C}$

(3) $\int \cos^3 x \sin x \, dx = \int \cos^2 x \cdot \cos x \sin x \, dx$

$= \int (1-\sin^2 x)\cos x \sin x \, dx$

$\int = \sin x - \sin^3 x \cos x \, dx$

$\int \sin x \cos x \, dx - \int \sin^3 x \cos x \, dx$

$u = \sin x$ $\quad u = \sin x$

$du = \cos x \, dx$ $\quad du = \cos x \, dx$

$\int u \, du - \int u^3 \, du = \boxed{\frac{1}{2}\sin^2 x - \frac{1}{4}\sin^4 x + C}$

(7) $\int \sin^5 x\,dx = \int \sin^2 x \sin^2 x \sin x\,dx$

$\int (1-\cos^2 x)(1-\cos^2 x)\sin x\,dx$

$\int (1-2\cos^2 x+\cos^4 x)\sin x\,dx$

$\int \sin x\,dx - 2\int \cos^2 x \sin x\,dx + \int \cos^4 x \sin x\,dx$

$u = \cos x \qquad\qquad u = \cos x$

$du = -\sin x\,dx \qquad du = -\sin x\,dx$

$= -\cos x + 2\int \cos^2 x \cdot -\sin x\,dx - \int \cos^4 x \cdot -\sin x\,dx$

$= -\cos x + 2\int u^2\,du - \int u^4\,du$

$= \boxed{-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C}$

(11) $\int \sin^3 x \cos^3 x = \int \sin^2 x \cdot \sin x \cos^3 x\,dx$

$\int (1-\cos^2 x)\cos^3 x \sin x\,dx$

$\int (\cos^3 x - \cos^5 x)\sin x\,dx$

$\int \cos^3 x \sin x\,dx - \int \cos^5 x \sin x\,dx$

$u = \cos x$ $\quad$ $u = \cos x$

$du = \sin dx$ $\quad$ $du = -\sin x$

$-\int u^3\,du + \int u^5\,du \rightarrow \boxed{-\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C}$

(17) $8\int_0^{\pi} \sin^2 x \sin^2 x\,dx = 8\int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$

$2\int (1 - 2\cos 2x + \cos^2 2x)\,dx = 2\int dx - 2\int 2\cos 2x\,dx + 2\int \frac{1+\cos 4x}{2}$

$u=2x$ $\quad$ $du = 2dx$

$2x\Big|_0^{\pi} - 2\sin 2x\Big|_0^{\pi} + \int (1+\cos 4x)\,dx$

$2\pi - 0 - 2\cancel{\sin(2\pi)} - (-2\cancel{\sin}\,2(0)) + \int dx + \frac{1}{4}\int 4\cos 4x\,dx$

$2\pi + x\Big|_0^{\pi} + \frac{1}{4}\sin 4x\Big|_0^{\pi} = \boxed{3\pi}$

(19) $16\int\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$

$4\int(1-\cos 2x)(1+\cos 2x) = 4\int(1-\cos^2 2x)\,dx$

$4\int dx - 4\int \cos^2 2x\,dx = 4x - 4\int \frac{1+\cos 2(2x)}{2}\,dx$

$4x - 2\int(1+\cos 4x)\,dx$

$= 4x - 2\int dx - 2\,\frac{1}{4}\int 4\cos 4x\,dx$ $\quad U = 4x \quad du = 4dx$

$= 4x - 2x - \frac{1}{2}\int \cos u\,du \rightarrow 2x - \frac{1}{2}\sin U$

$$\boxed{2x - \frac{1}{2}\sin 4x + C}$$

23) $\displaystyle\int_0^{2\pi}\sqrt{\frac{1-\cos x}{2}}\,dx$

$\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\sin^2\left(\frac{x}{2}\right) = \dfrac{1-\cos 2\left(\frac{x}{2}\right)}{2}$

$\sin^2\left(\frac{x}{2}\right) = \dfrac{1-\cos x}{2}$

$\displaystyle\int_0^{2\pi}\sqrt{\sin^2\left(\frac{x}{2}\right)}\,dx$

$u = \frac{1}{2}x$

$du = \frac{1}{2}dx \longrightarrow 2\displaystyle\int_0^{2\pi}\sin\left(\frac{x}{2}\right)\cdot\frac{1}{2}dx$

$2\displaystyle\int_0^{\pi}\sin u\,du \longrightarrow -2\cos\left(\frac{x}{2}\right)\Big|_0^{\pi} = 0+2 = \boxed{2}$

27) $\displaystyle\int_{\pi/3}^{\pi/2}\frac{\sin^2 x}{\sqrt{1-\cos x}}\,dx = \int \frac{\sin^2 x}{(1-\cos x)^{1/2}}\cdot\frac{(1+\cos x)^{1/2}}{(1+\cos x)^{1/2}}$

$\displaystyle\int\frac{\sin^2 x\,(1+\cos x)^{1/2}}{(1-\cos^2 x)^{1/2}} = \int\frac{\sin^2 x\,(1+\cos x)^{1/2}}{\sqrt{\sin^2 x}}$

$u = 1+\cos x \quad du = -\sin x$

$\displaystyle\int \sin x\,(1+\cos x)^{1/2} \quad -\int u^{1/2}du = -\frac{2}{3}u^{3/2}\Big|_0^{\pi}$

$\dfrac{-2}{3}\left[(1+\cos x)^{3/2}\right]\Big|_0^{\pi} = \begin{matrix}(1+\cos\pi)^{3/2}-(1+\cos(0))^{3/2}\\ (1-1)^{3/2}-(1+1)^{3/2}\end{matrix} = \frac{2}{3}(\sqrt{8})$

$= \frac{2}{3}2\cdot\sqrt{2} = \boxed{\frac{4}{3}\sqrt{2}}$

(33) $\int \sec^2 x \tan x \, dx = \int \sec x \cdot \sec x \tan x \, dx$

$u = \sec x$

$dx = \sec x \tan x \, dx$

$\int u \, du = \frac{1}{2} u^2 + C = \boxed{\frac{1}{2} \sec^2 x + C}$

(41) $\int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx$

$\int (\tan^2 x + 1) \sec^2 x \, dx = \int \tan^2 x \sec^2 x + \sec^2 x \, dx$

$\int \tan^2 x \sec^2 x \, dx + \int \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x$

$\int u^2 du + \tan x + C$

$\boxed{\frac{1}{3} \tan^3 x + \tan x + C}$

(51) $\int \sin 3x \cos 2x\,dx = \int \frac{1}{2}\left[\sin(3-2)x + \sin(3+2)x\right]dx$

$\frac{1}{2}\int \sin x + \sin 5x\,dx = \frac{1}{2}\int \sin x + \frac{1}{2}\cdot\frac{1}{5}\int \sin 5x\, 5dx$

$= \boxed{-\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C}$

(57) $\int \sin^2\theta \cos 3\theta\,d\theta = \int (1-\cos^2\theta)\cos 3\theta\,d\theta$

$\int \cos 3\theta - \cos 3\theta \cos^2\theta\,d\theta = \frac{1}{3}\sin 3\theta - \int \cos 3\theta \cdot \frac{1}{2}(1-\cos 2\theta)$

$= -\frac{1}{2}\int \cos 3\theta(1-\cos 2\theta) = -\frac{1}{2}\int \cos 3\theta - \cos 3\theta \cos 2\theta\,d\theta$

$-\frac{1}{2}\int \cos 3\theta\,d\theta - \frac{1}{2}\int \cos 3\theta \cos 2\theta\,d\theta$

$= -\frac{1}{2}\cdot\frac{1}{3}\sin 3\theta - \frac{1}{2}\int \frac{1}{2}\left[\cos(3-2) + \cos(3+2)\right]dx$

$= -\frac{1}{6}\sin 3\theta - \frac{1}{4}\int \cos x + \cos 5x\,dx$ $\longrightarrow$

$$= -\frac{1}{4}\int \cos x\,dx - \frac{1}{4}\int \cos 5x\,dx$$

$$= -\frac{1}{4}\sin x - \frac{1}{4}\cdot\frac{1}{5}\int \cos u\,du$$

$$= -\frac{1}{20}\sin 5x + c$$

$$= \frac{1}{3}\sin 3\theta - \frac{1}{6}\sin 3\theta - \frac{1}{4}\sin\theta - \frac{1}{20}\sin 5\theta + c$$

$$\boxed{\frac{1}{6}\sin 3\theta - \frac{1}{4}\sin\theta - \frac{1}{20}\sin 5\theta + c}$$

Steven Romero

* Bonus Questions

\# 3, 5, 7, 9, 11, 13, 35, 37, 43

$\underbrace{35, 37}_{\text{Bonus}}$

8.4 Homework

(3) $$\int \frac{dx}{4+x^2} = \frac{1}{a^2+u^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$$

$$a^2=4 \quad a=2 \quad u^2=x^2 \quad u=x$$

$$= \boxed{\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C}$$

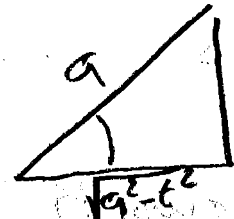
(5) $$\int \frac{dx}{\sqrt{9-x^2}}$$

$$x = a\sin\theta = x = 3\sin\theta$$
$$\sin\theta = \frac{x}{a} = \sin\theta = \frac{x}{3}$$
$$\theta = \sin^{-1}\left(\frac{x}{3}\right)$$
$$dx = 3\cos\theta\, d\theta$$

$$\int \frac{3\cos\theta\, d\theta}{\sqrt{9-(3\sin\theta)^2}} \rightarrow \int \frac{3\cos\theta\, d\theta}{3\sqrt{1-\sin^2\theta}}$$

$$\int \frac{\cos\theta\, d\theta}{\sqrt{\cos^2\theta}} = \int \frac{\cos\theta\, d\theta}{\cos\theta} = \int d\theta$$

$$= \theta = \boxed{\sin^{-1}\left(\frac{x}{3}\right)}$$



(7) $\int \sqrt{25-t^2}\,dt$ $\quad t = a\sin\theta \;=\; t = 5\sin\theta$

$\theta = \sin^{-1}\left(\frac{t}{a}\right) \quad dt = 5\cos\theta\,d\theta$

$$\int \sqrt{25-(5\sin\theta)^2}\cdot 5\cos\theta\,d\theta$$

$$\int 5\sqrt{1-\sin^2\theta}\cdot 5\cos\theta\,d\theta = 25\int\sqrt{\cos^2\theta}\cdot\cos\theta\,d\theta$$

$$= 25\int \cos^2\theta\,d\theta \longrightarrow 25\int \frac{1}{2}(1+\cos 2\theta)\,d\theta$$

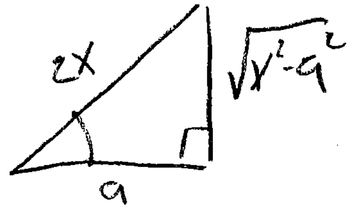
$$\frac{25}{2}\int d\theta + \frac{25}{2}\int \cos 2\theta\,d\theta \longrightarrow \frac{25}{2}\theta + \frac{25}{4}\int \cos u\,du$$

$$= \boxed{\frac{25}{2}\theta + \frac{25}{4}\sin 2\theta + C} = \frac{25}{4}\sin\left(\frac{\sin^{-1}\left(\frac{t}{5}\right)}{2}\right) ?$$

=

$X^2 = 4x^2$
$X = 2x$
x^2, a^2

$\frac{2x}{7}$



$\tan\theta = \frac{\sqrt{x^2-a^2}}{a}$

9) $\int \frac{dx}{\sqrt{4x^2-49}}$ $\quad X = a\sec\theta \quad a^2 = 49 \quad a = 7$
$X = 7\sec\theta \quad dx = 7\sec\theta\tan\theta\,d\theta$

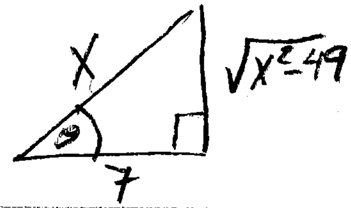
$$\int \frac{7\sec\theta\tan\theta\,d\theta}{\sqrt{4(7\sec\theta)^2-49}} = \int \frac{7\sec\theta\tan\theta\,d\theta}{\sqrt{784\sec^2\theta-49}}$$

$$\int \frac{7\sec\theta\tan\theta\,d\theta}{\sqrt{49(16\sec^2\theta-1)}} = \int \frac{7\sec\theta\tan\theta\,d\theta}{7\sqrt{16\sec^2\theta-1}}$$

$$= \int \frac{\sec\theta\tan\theta\,d\theta}{\sqrt{16\tan^2\theta}} = \int \frac{\sec\theta\tan\theta\,d\theta}{4\tan\theta}$$

$$\frac{1}{4}\int \sec\theta\,d\theta = \frac{1}{4}\ln(\sec\theta + \tan\theta) + C$$

$$\boxed{\frac{1}{4}\ln\left(\frac{2x}{7} + \frac{\sqrt{4x^2-49}}{7}\right) + C}$$



$\theta = \tan^{-1}\left(\frac{\sqrt{x^2-49}}{7}\right)$

(11) $\int \frac{\sqrt{x^2-49}}{x}\,dx$ $\quad x = 7\sec\theta \quad \theta = \sec^{-1}\left(\frac{x}{7}\right)$

$dx = 7\sec\theta\tan\theta\,d\theta$

$$\int \frac{\sqrt{49\sec^2\theta - 49}}{7\cancel{\sec\theta}} \cdot 7\cancel{\sec\theta}\tan\theta\,d\theta$$

$$= \int 7\sqrt{\tan^2\theta}\cdot\tan\theta\,d\theta = 7\int \tan^2\theta\,d\theta$$

$$= 7\int \sec^2 - 1\,d\theta = 7\int \sec^2\theta\,d\theta - 7\int d\theta$$

$$= 7\tan\theta - 7\theta = \frac{7\sqrt{x^2-49}}{7} - 7\sec^{-1}\left(\frac{x}{7}\right)$$

$$\boxed{\sqrt{x^2-49} - 7\sec^{-1}\left(\frac{x}{7}\right) + C}$$

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3, 7, 11, 15, 19, 21, 27, 33, 37, 39, 41, 51

8.5 Homework

(3) $\frac{x+4}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} = \frac{1}{(x+1)} + \frac{3}{(x+1)^2}$

$x+4 = A(x+1) + B$ Let $x=-1$

$\therefore -1+4 = A\cancel{(-1+1)}^0 + B \quad B=3$

Let $x=0$ $\therefore 0+4 = A(0+1)+3 \quad 4 = A+3 \quad A=1$

(7) $\frac{t^2+8}{t^2-5t+6} = (t-2)(t-3)$

$\frac{t^2+8}{t^2-5t+6} = \frac{A}{(t-2)} + \frac{B}{(t-3)} = \boxed{\frac{-12}{(t-2)} + \frac{17}{(t-3)}}$

$\therefore t^2+8 = A(t-3) + B(t-2)$ Let $x=3$

$3^2+8 = A(\cancel{3-3})^0 + B(3-2) = 17 = B$

Let $x=2$ $\therefore 2^2+8 = A(2-3) + B(\cancel{2-2})^0 = 12 = -A \quad A=-12$

(11) $\int \frac{x+4}{x^2+5x-6}dx \to (x-1)(x+6) = \frac{A}{(x-1)} + \frac{B}{(x+6)}$

?

$= x+4 = A(x+6) + B(x-1)$ Let $x=+1$

$\therefore 1+4 = A(1+6) + B(\cancel{1-1})^0 = 5 = A7 \quad A = \frac{5}{7}$

Let $x=-6$ $\therefore -6+4 = A(\cancel{-6+6})^0 + B(-6-1) \to B = \frac{2}{7}$

$\frac{5}{7}\int \frac{1}{(x-1)}dx + \frac{2}{7}\int \frac{1}{(x+6)}dx = \boxed{\frac{5}{7}\ln|x-1| + \frac{2}{7}\ln|x+6| + C}$

(15) $\displaystyle\int \frac{dt}{t^3+t^2-2t} \rightarrow t(t-1)(t+2) = \frac{A}{t} + \frac{B}{(t-1)} + \frac{C}{(t+2)}$

$1 = A(t-1)(t+2) + B(t)(t+2) + C(t)(t-1)$ Let $t=0$

$\therefore \; 1 = A(0-1)(0+2) + \cancel{B(0)(0+2)} + \cancel{C(0)(0-1)} = A = -\frac{1}{2}$

Let $t=-2 \therefore \; 1 = \cancel{A(-2-1)(-2+2)} + \cancel{B(-2)(-2+2)} + C(-2)(-2-1) = C = \frac{1}{6}$

Let $t=+1 \therefore \; 1 = \cancel{A(+1-1)(+1+2)} + B(1)(1+2) + \cancel{C(1)(1-1)} = B = \frac{1}{3}$

$-\frac{1}{2}\displaystyle\int \frac{1}{t}\,dt + \frac{1}{3}\int \frac{1}{(t-1)}\,dt + \frac{1}{6}\int \frac{1}{(t+2)}\,dt =$

$= \boxed{-\frac{1}{2}\ln|t| + \frac{1}{3}\ln|t-1| + \frac{1}{6}\ln|t+2| + C}$

(19) $\displaystyle\int \frac{dx}{(x^2-1)^2} \rightarrow [(x+1)(x-1)]^2 = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x+1)^2} + \frac{D}{(x-1)^2}$

$1 = A(x+1)(x-1)^2 + B(x+1)^2(x-1) + C(x-1)^2 + D(x+1)^2$

$x=-1 \therefore 1 = \cancel{A(-1+1)(-1-1)^2} + \cancel{B(-1+1)^2(-1-1)} + C(-1-1)^2 + \cancel{D(-1+1)^2}$ $\quad C = \frac{1}{4}$

$x=+1 \therefore 1 = \cancel{A(1+1)(1-1)^2} + \cancel{B(1+1)^2(1-1)} + \cancel{C(1-1)^2} + D(1+1)^2$ $\quad D = \frac{1}{4}$

$x=2 \therefore 1 = A(2+1)(2-1)^2 + B(2+1)^2(2-1) + \frac{1}{4}(2-1)^2 + \frac{1}{4}(2+1)^2$

$1 = 3A + 9B + 1/4 + 9/4 \Rightarrow 1 - 5/2 = 3A + 9B \rightarrow -3 = A + 3B$

$x=0 \therefore 1 = A(0+1)(0-1)^2 + B(0+1)^2(0-1) + 1/4(0-1)^2 + 1/4(0+1)^2$

$1 = A + B + 1/4 + 1/4 \rightarrow \frac{1}{2} = A + B$

$A + B - \frac{1}{2} = 0 \quad \& \quad A + 3B + 3 = 0$

$A + B - \frac{1}{2} - A - 3B - \frac{1}{2} = 0$

$-2B - 1 = 0 \quad B = -\frac{1}{2}$

$X=0$: $1 = A(0+1)(0-1)^2 + B(0+1)^2(0-1) + 1/4(0-1)^2 + 1/4(0+1)^2$

$1 = A + B + 1/4 + 1/4 \rightarrow 1 = A - \frac{1}{2} + \frac{1}{2} \quad A = 1$

$$\int \frac{1}{X+1}dx - \frac{1}{2}\int \frac{1}{X-1}dx + \frac{1}{4}\int \frac{1dx}{(X+1)^2} + \frac{1}{4}\int \frac{1}{(X-1)^2}dx$$

$U=X+1 \quad U=X-1 \quad U=X+1 \quad U=X-1$

?

$$\int \frac{1}{U}dU - \frac{1}{2}\int \frac{1}{U}du + \frac{1}{4}\int \frac{1du}{U^2} + \frac{1}{4}\int \frac{1}{U^2}du$$

$$\boxed{\ln|x+1| - \frac{1}{2}\ln|X-1| - \frac{1}{4(X+1)} - \frac{1}{4(X-1)} + C}$$

21) $$\int_0^1 \frac{dx}{(X+1)(X^2+1)} = \frac{A}{(X+1)} + \frac{Bx+C}{(X^2+1)}$$

$1 = A(X^2+1) + (Bx+C)(X+1) \quad$ Let $X=-1$

$\therefore 1 = A((-1)^2+1) + \cancel{(-B+C)(-1+1)}^{0} \rightarrow A = 1/2$

Let $X=0$: $1 = 1/2(0+1) + (B(0)+C)(0+1) \rightarrow 1 = 1/2 + C \rightarrow C = 1/2$

Let $X=1$: $1 = 1/2(1+1) + (B+\frac{1}{2})(1+1) \rightarrow 1 = 1 + 2B + 1 \rightarrow B = -1/2$

$$\frac{1}{2}\int_0^1 \frac{1}{(x+1)}dx + \int_0^1 \frac{(-1/2)x + \frac{1}{2}}{(x^2+1)}dX \Rightarrow \frac{1}{2}\int_0^1 \frac{1}{U}du - \frac{1}{2}\int_0^1 \frac{x-1}{(x^2+1)}$$

$U = x^2+1$

$du = 2xdx$

$$\frac{1}{2}\int_0^1 \frac{1}{U}du - \frac{1}{2}\cdot\frac{1}{2}\int_0^1 \frac{2xdx}{x^2+1} + \frac{1}{2}\int_0^1 \frac{1}{x^2+1}dx$$

$\frac{1}{2}\ln|x+1|\Big|_0^1 - \frac{1}{4}\int_0^1 \frac{1}{u}du + \tan^{-1}x\Big|_0^1$

$\frac{1}{2}[\ln 2 - \ln 1] - \frac{1}{4}[\ln 2 - \ln 1] + [\tan^{-1}(1) - \tan^{-1}(0)]$

? $\rightarrow$ $\frac{1}{2}\ln 2 - \frac{1}{4}\ln 2 - \frac{1}{2}\ln 1 + \frac{1}{4}\ln 1 + \frac{1}{2}\cdot\left(\frac{\pi}{4}\right)$

$\rightarrow$ $\boxed{\frac{1}{4}\ln 2 - \frac{1}{4}\ln 1 + \frac{\pi}{8}}$

?

(27) $\int \frac{x^2 - x + 2}{x^3 - 1}dx = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+x+1}$

$x^2 - x + 2 = A(x^2+x+1) + (Bx+C)(x-1)$

$x=0 \therefore 0-0+2 = A(0+0+1) + (0+C)(-1) \rightarrow 2 = A - C$ $\quad A = 2/3$

$x=1 \therefore 1-1+2 = A(1+1+1) + (0) \rightarrow 2/3 = A$ $\quad B = 1/3$

$x=-1 \therefore -1-1+2 = A(1-1+1) + (-B+C)(-2) \rightarrow 4 = A + 2B - 2C$ $\quad C = -4/3$

$\int \frac{2/3}{x-1} + \int \frac{1/3x - 4/3}{x^2+x+1} = \frac{1}{3}\int \frac{2}{x-1} + \frac{1}{3}\int \frac{x}{x^2+x+1} - \frac{1}{3}\int \frac{4}{x^2+x+1}$

$= \frac{2}{3}\int \frac{1}{u} +$

(33) $\int \frac{2x^3-2x^2+1}{x^2-x}$

$$\begin{array}{r} 2x \\ x^2-x \overline{\big)\, 2x^3-2x^2+1} \\ -2x^3 \pm 2x^2 \\ \hline 0 \quad +1 \end{array}$$

$\int 2x + \frac{1}{x^2-x} = \int 2x\,dx + \int \frac{1}{x^2-x} = \frac{A}{(x)} + \frac{B}{(x-1)}$

$1 = A(x-1) + B(x)$

$x=0$ $\quad 1 = A(0-1)+0 \rightarrow A = -1$

$x=1$ $\quad 1 = A(0) + B \rightarrow B = 1$

$\int 2x + \int \frac{-1}{x} + \int \frac{1}{x-1} = \int 2x - \int \frac{1}{x} + \int \frac{1}{x-1}$

$\boxed{x^2 - \ln|x| + \ln|x-1| + C}$

(37) $\int \frac{x^4+x^2-1}{x^3+x}\,dx = \quad x^3+x \overline{)\,x^4+x^2-1}$ quotient x

x^4+x^2

$0 \quad 0 \quad -1$

$\int x - \frac{1}{x^3+x} = \int x - \int \frac{1}{x^3+x} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$

$1 = A(x+1)^2 + B(x)(x+1) + C(x)$

$x=0$ $\quad 1 = A(0+1)^2 + \cancel{B(0)(0+1)} + \cancel{C(0)} \rightarrow A=1$

$x=-1$ $\quad 1 = \cancel{A(-1+1)} + \cancel{B(-1)(-1+1)} + C(-1) \rightarrow C=-1$

$x=1$ $\quad 1 = 1(1+1)^2 + B(1)(1+1) - 1 \rightarrow 1 = 4 + 2B - 1 \rightarrow B = -1$

$\int x - \int \frac{1}{x} - \int \frac{1}{x+1} - \int \frac{1}{(x+1)^2}$

?

$\boxed{\frac{1}{2}x^2 - \ln|x| - \ln|x+1| + \frac{1}{x+1} + C}$

39) $\int \frac{e^t\,dt}{e^{2t}+3e^t+2}$ $\quad u=e^t$, $du=e^t\,dt$

$$\int \frac{1}{u^2+3u+2}\,du = \frac{A}{u+1}+\frac{B}{u+2}$$

$1 = A(u+2)+B(u+1)$

$x=-2$ $\quad 1 = A(0)+B(-2+1) \qquad B=-1$

$x=-1$ $\quad 1 = A(-1+2)+B(0) \qquad A=1$

$$\int \frac{1}{u+1}\,du + \int \frac{-1}{u+2} = \ln|u+1| - \ln|u+2| + C$$

$$\ln\left|\frac{u+1}{u+2}\right| + C = \boxed{\ln\left|\frac{e^t+1}{e^t+2}\right| + C}$$

(41) $$\int \frac{\cos y \, dy}{\sin^2 y + \sin y - 6}$$ $U = \sin y$, $dU = \cos y \, dy$

$$\int \frac{1}{U^2+U-6} dU \rightarrow (U+3)(U-2) = \frac{A}{(U+3)} + \frac{B}{(U-2)}$$

$1 = A(U-2) + B(U+3) \rightarrow U = 2 \therefore 1 = \cancel{A(2-2)} + B(5) = \boxed{B = \frac{1}{5}}$

$U = -3 \therefore$ $1 = A(-3-2) + \cancel{B(-3+3)} \rightarrow 1 = A(-5) \rightarrow \boxed{A = -\frac{1}{5}}$

$$-\frac{1}{5}\int \frac{1}{U+3} dU + \frac{1}{5}\int \frac{1}{U-2} dU \rightarrow -\frac{1}{5}\ln|U+3| + \frac{1}{5}\ln|U-2| + C$$

$$\frac{1}{5}\left(\ln|\sin y - 2| - \ln|\sin y + 3|\right) + C$$

$$= \boxed{\frac{1}{5}\left(\frac{\ln|\sin y - 2|}{\ln|\sin y + 3|}\right) + C}$$

51) $(t^2-3t+2)\frac{dx}{dt}=1 \qquad x(3)=0$

$\frac{dx}{dt}=\frac{1}{t^2-3t+2} \longrightarrow dx=\frac{1}{t^2-3t+2}dt$

$\int dx=\int\frac{1}{t^2-3t+2}dt \longrightarrow x=\int\frac{A}{(t-2)}+\frac{B}{(t-1)}dt$

$x=1=A(t-1)+B(t-2)$ Let $t=2$

$\therefore x=1=A(2-1)+\cancel{B(2-2)}^{0} \longrightarrow \boxed{1=A}$ Let $t=1$

$\therefore x=1=\cancel{A(1-1)}^{0}+B(1-2) \longrightarrow \boxed{B=-1}$

$x=\int\frac{1}{t-2}dt-\int\frac{1}{t-1}dt \longrightarrow x=\ln|t-2|-\ln|t-1|+C$

$x(3)=0 \Rightarrow 0=\ln|3-2|-\ln|3-1|+C$

$0=\ln 1-\ln 2+C \longrightarrow 0=0-\ln 2+C$

$C=\ln 2$

$$\boxed{x=\ln|t-2|-\ln|t-1|+\ln 2}$$

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3, 9, 13, 17, 31, 34, 41, 47

8.6 Homework

T-1
#22

(3) $\int \frac{x}{\sqrt{x-2}}\,dx = \int x(x-2)^{-1/2}\,dx$

$a=1,\ b=-2$
$n=-\frac{1}{2}$

$$= \frac{(ax+b)^{n+1}}{a^2}\left[\frac{ax+b}{n+2} - \frac{b}{n+1}\right] + C$$

$$\frac{(x-2)^{-\frac{1}{2}+1}}{1^2}\left[\frac{x-2}{-\frac{1}{2}+2} - \frac{(-2)}{-\frac{1}{2}+1}\right] + C$$

$$\frac{(x-2)^{1/2}}{1}\left[\frac{x-2}{3/2} + \frac{2}{1/2}\right] + C$$

$$\boxed{(x-2)^{1/2}\left[\frac{2(x-2)}{3} + 4\right] + C}$$

Simplified

$$(x-2)^{1/2}\left[\frac{2x-4}{3} + 4\right] + C$$

$$(x-2)^{1/2}\left(\frac{2x-4+12}{3}\right) + C$$

$$(x-2)^{1/2}\left(\frac{2x+8}{3}\right) + C$$

⑨ $\int x\sqrt{4x-x^2}\,dx =$ T-5 $a=2$ #124

$$= \frac{(x+a)(2x-3a)\sqrt{2ax-x^2}}{6} + \frac{a^3}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$$

$$= \frac{(x+2)(2x-3(2))\sqrt{2(2x)-x^2}}{6} + \frac{2^3}{2}\sin\left(\frac{x-2}{2}\right) + C$$

$$\boxed{\frac{(x+2)(2x-6)\sqrt{4x-x^2}}{6} + 4\sin\left(\frac{x-2}{2}\right) + C}$$

⑬ $\int \frac{\sqrt{4-x^2}}{x}\,dx$ T-2 $a=2$ #47 $a^2=4$

$$= \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$$

$$= \boxed{\sqrt{4-x^2} - 2\ln\left|\frac{2+\sqrt{4-x^2}}{x}\right| + C}$$

(17) $\int x\cos^{-1}x\,dx$ T-4 #107 $n=1$ $a=1$

$$= \frac{x^{n+1}}{n+1}\cos^{-1}ax + \frac{a}{n+1}\int \frac{x^{n+1}}{\sqrt{1-a^2x^2}}dx$$

$$= \frac{x^2}{2}\cos^{-1}x + \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}dx$$ T-2 #49 $a=1$ $a^2=1$

$$= \frac{x^2}{2}\cos^{-1}x + \frac{1}{2}\left[\frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{1}{2}x\sqrt{a^2-x^2} + C\right]$$

$$\frac{x^2}{2}\cos^{-1}x + \frac{1}{2}\left[\frac{1}{2}\sin^{-1}x - \frac{1}{2}\sqrt{1-x^2} + C\right]$$

$$\boxed{\frac{x^2}{2}\cos^{-1}x + \frac{1}{4}\sin^{-1}x - \frac{1}{4}\sqrt{1-x^2} + C}$$

(31) $$\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx = \int \frac{x^{1/2}}{\sqrt{1-x}}\,dx$$

$x = u^2$
$u = \sqrt{x}$
$u^2 = x$
$du = \frac{1}{2\sqrt{x}}\,dx$
$\cancel{dx}\ 2\sqrt{x}\,du$

$$= \int \frac{\sqrt{x}}{\sqrt{1-x}} \cdot 2\sqrt{x}\,du$$

$$= 2\int \frac{u}{\sqrt{1-u^2}} \cdot u\,du = 2\int \frac{u^2}{\sqrt{1-u^2}}\,du$$

T-2
#49

$a^2 = 1$
$a = 1$

$$= \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{1}{2}x\sqrt{a^2-x^2} + C$$

$$2\left[\frac{1}{2}\sin^{-1}\frac{u}{1} - \frac{1}{2}u\sqrt{1^2-u^2} + C\right]$$

$$2\left[\frac{1}{2}\sin^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x}\sqrt{1^2-x} + C\right]$$

$$2\left[\frac{1}{2}\sin^{-1}\sqrt{x} - \frac{1}{2}\sqrt{x-x^2} + C\right]$$

$$\boxed{\sin^{-1}\sqrt{x} - \sqrt{x-x^2} + C}$$

(34) $$\int \frac{dt}{\text{Tan} t\sqrt{4-\sin^2 t}} = \int \frac{dt}{\frac{\sin t}{\cos t}\sqrt{4-\sin^2 t}}$$

$$\int \frac{\cos t\, dt}{\sin t\sqrt{4-\sin^2 t}}$$

$x = \sin t$
$dx = \cos t\, dt$

$$\int \frac{dx}{x\sqrt{4-x^2}}$$

T-2 $\quad a = 2$
#50 $\quad a^2 = 4$

$$= -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$$

$$-\frac{1}{2}\ln\left|\frac{2+\sqrt{4-x^2}}{x}\right| + C$$

$$\boxed{-\frac{1}{2}\ln\left|\frac{2+\sqrt{4-\sin^2 t}}{\sin t}\right| + C}$$

(41) $\int \sin^5 2x\,dx$ T-3 #67 $n=5$, $a=2$

$$= -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} ax\,dx$$

$$= -\frac{\sin^4 2x \cos 2x}{10} + \frac{4}{5}\int \sin^3 2x\,dx$$ T-3 #67

$n=3$, $a=2$

$$= \frac{4}{5}\left[-\frac{\sin^2 2x \cos 2x}{6} + \frac{2}{3}\int \sin 2x\,dx\right]$$ $u=2x$, $du=2dx$

$$\frac{4}{5}\left[-\frac{\sin^2 2x \cos 2x}{6} + \frac{2}{3}\cdot\frac{1}{2}\int \sin 2x \cdot 2dx\right]$$

$$\frac{4}{5}\left[-\frac{\sin^2 2x \cos 2x}{6} + \frac{1}{3}\int \sin u\,du\right]$$

$$\frac{4}{5}\left[-\frac{\sin^2 2x \cos 2x}{6} + \frac{1}{3}(-\cos 2x)\right]$$

$$-\frac{2\sin^2 2x \cos 2x}{15} - \frac{4}{15}\cos 2x$$

$$\boxed{-\frac{\sin^4 2x \cos 2x}{10} - \frac{2\sin^2 2x \cos 2x}{15} - \frac{4\cos 2x}{15} + C}$$

(47) $\int 2\sec^3 \pi x\,dx = 2\int \sec^3 \pi x\,dx$ T-4 #99 $a=\pi$ $n=3$

$a=\pi$
$n=3$

$$= \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2} ax\,dx$$

$$= 2\left[\frac{\sec \pi x \tan \pi x}{2\pi} + \frac{1}{2}\int \sec \pi x\,dx\right]$$ T-4 # 95

$a=\pi$

$$= \frac{1}{2}\left[\frac{1}{a}\ln|\sec ax + \tan ax|\right]$$

$$= \frac{1}{2}\left[\frac{1}{\pi}\ln|\sec \pi x + \tan \pi x|\right]$$

$$= \frac{1}{2\pi}\ln|\sec \pi x + \tan \pi x|$$

$$= 2\left[\frac{\sec \pi x \tan \pi x}{2\pi} + \frac{1}{2\pi}\ln|\sec \pi x + \tan \pi x|\right] + C$$

$$= \boxed{\frac{\sec \pi x \tan \pi x}{\pi} + \frac{\ln|\sec \pi x \tan \pi x|}{\pi} + C}$$

Steven Romeiro

1, 3, 5, 7, 9

8.7 Homework

1. $\int_1^2 x\,dx$ $n=4$, $a=1, b=2$ $\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$

$x = 1, 1.25, 1.50, 1.75, 2.0$

$= \frac{\frac{1}{4}}{2}(1 + 2(1.25) + 2(1.50) + 2(1.75) + 2)$

$= \frac{1}{8}(1 + 2.5 + 3 + 3.50 + 2)$

$\frac{1}{8}(12) = \frac{3}{2} = \boxed{1.5}$

$E_T = \frac{M(b-a)^3}{12n^2} =$ $f'(x) = 1$, $f''(x) = 0$

I

$E_T = \frac{0(2-1)^3}{12(4)^2} = 0$

$\boxed{a.\ f''(x) = 0 = M}$

$\boxed{b.\ 1.5 \text{ error} = 0}$

$0 \times 100 = 0\%$ $\boxed{c.\ 0\%}$

II $\frac{1/4}{3}(1 + 4(1.25) + 2(1.50) + 4(1.75) + 2)$

$\frac{1}{12}(1 + 5 + 3 + 7 + 2) = \frac{1}{12}(18) = \frac{3}{2}$

$= \boxed{1.5}$

$$|E_s| \le \frac{M(b-a)^5}{180 n^4} \qquad f''''(x) = 0 \quad M = 0$$

$$|E_s| \le \frac{0(2-1)^5}{180(4)^4} = \boxed{0} \times 100 = \boxed{0\%}$$

$\boxed{\text{b. } 1.5 \text{ error } 0}$ $\boxed{\text{c. } 0\%}$ $\boxed{\text{a. } f''''(x) = 0 = M}$

(3) $\displaystyle\int_{-1}^{1} (x^2+1)\,dx \qquad n = 4$

$$\Delta x = \frac{b-a}{4} = \frac{1+1}{4} = \boxed{\frac{1}{2}}$$

$$x = -1, -0.5, 0, 0.5, 1$$

$$\frac{1/2}{2}\left(2 + 2(1.25) + 2(1) + 2(1.25) + 2\right)$$

$$\frac{1}{4}\left(2 + 2.5 + 2 + 2.5 + 2\right) = \frac{1}{4}(11) = \boxed{2.75} \text{ A.}$$

$$|E_T| \le \frac{M(b-a)^3}{12 n^2} \qquad f'(x) = 2x \quad f''(x) = 2 = M$$

$$|E_T| \le \frac{2(1+1)^3}{12(4)^2} = \frac{16}{192} = \boxed{\frac{1}{12} \text{ or } 0.083}$$

$$\frac{1}{3}x^3 + x \Big|_{-1}^{1} = \left(\frac{1}{3}(1)^3 + 1\right) - \left(\frac{1}{3}(-1)^3 + (-1)\right)$$

$$= \frac{4}{3} - \left(-\frac{4}{3}\right) = \boxed{\frac{8}{3} \text{ or } 2.67} \text{ B.}$$

$$|E_s| \le \frac{M(b-a)^5}{180n^4}$$

$f''(x) = 2$
$f'''(x) = 0$
$f''''(x) = 0 = M$

$$|E_s| \le \frac{0(1+1)^5}{180(4)^4} = 0$$

$\boxed{c.\ 0\%}$

(5) $\int_0^2 (t^3+t)\,dt$ $n=4$ $\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \boxed{\frac{1}{2}}$

$x = 0, 0.5, 1, 1.5, 2$

$$\frac{\frac{1}{2}}{2}\left(0 + 2\left(\frac{5}{8}\right) + 2(2) + 2\left(\frac{39}{8}\right) + 10\right)$$

$$\frac{1}{4}\left(0 + \frac{5}{4} + 4 + \frac{39}{4} + 10\right) = \frac{1}{4}(25) = \boxed{6.25}$$

$$|E_T| \le \frac{M(b-a)^3}{12n^2}$$

$f'(t) = 3t^2+1$
$f''(t) = 6t$
$f'''(t) = 6$ no c.p

$$|E_T| \le \frac{12(2-0)^3}{12(4)^2}$$

$f''(0) = 6(0) = 0$, $f(2) = 6(2) = \boxed{12 = M}$ A

$$|E_T| \le \frac{12(8)}{12(16)} = \frac{1}{2} = \boxed{0.5}$$ A

$$\left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]\Big|_0^2 = \left[\frac{1}{4}(2)^4 + \frac{1}{2}(2)^2\right] - \cancel{\left[\frac{1}{4}(0) + \frac{1}{2}(0)\right]}$$

$$= 4 + 2 = \boxed{6}$$ B.

$$\frac{0.5}{6} \times 100 = \boxed{8.33\%}$$ C.

$$\frac{\frac{1}{2}}{3}\left(0+4\left(\frac{5}{8}\right)+2(2)+4\left(\frac{39}{8}\right)+10\right)$$

$$\frac{1}{6}(36) = \boxed{6}$$

$$|E_s| \le \frac{M(b-a)^5}{180n^4}$$

$f''(t) = 6t$

$f'''(t) = 6$

$f''''(t) = \boxed{0 = M}$ A.

$$|E_s| \le \frac{0(2-0)^5}{180(4)^4} = \boxed{0}$$

$$\frac{0}{6} \times 100 = \boxed{0\%}$$ C.

(7) $\int_1^2 \frac{1}{s^2}\,ds$ $n=4$, $\Delta x = \frac{2-1}{4} = \boxed{\frac{1}{4}}$

$x = (1, 1.25, 1.50, 1.75, 2)$

$\frac{\frac{1}{4}}{2}\left(1 + 2\left(\frac{16}{25}\right) + 2\left(\frac{4}{9}\right) + 2\left(\frac{16}{49}\right) + \frac{1}{4}\right)$

$= \frac{1}{8}(4.072) = \boxed{.509}$

$|E_T| \leq \frac{M(b-a)^3}{12n^2}$ $f'(s) = \frac{-2}{s^3}$

$|E_T| \leq \frac{6(2-1)^3}{12(4)^2}$ $f''(s) = \frac{6}{s^4}$

$f'''(s) = -\frac{24}{s^5} = 0$

$s=0$ C.P out of bounds $1 \to 2$

$|E_T| \leq \frac{6}{256} = \boxed{.0234}$ $f''(1) = \frac{6}{1} = 6$

$f''(2) = \frac{6}{2^4} = \frac{6}{16} = \frac{3}{8}$

$\boxed{M=6}$ A.

$\int_1^2 s^{-2}\,ds = -\frac{1}{s}\Big|_1^2 = -\frac{1}{2} + \frac{1}{1} = \boxed{0.5}$ B.

$\frac{.0234}{0.5} \times 100 = \boxed{4.69\%}$

II

$$\frac{\frac{1}{4}}{3}\left(1+4\left(\frac{16}{25}\right)+2\left(\frac{4}{9}\right)+4\left(\frac{16}{49}\right)+\frac{1}{4}\right)$$

$$\frac{1}{12}(6.005) = .5004$$

$$|E_s| \le \frac{M(b-a)^5}{180n^4} \qquad f'''(s) = \frac{-24}{s^5}$$

$$f''''(s) = \frac{120}{s^6}$$

no c.p
@ 6th deriv.

$$|E_s| \le \frac{120(2-1)^5}{180(4)^4} \qquad f''''(1) = \boxed{120 = M}\ A.$$

$$|E_s| \le \boxed{.002604}\ B.$$

$$\frac{.002604}{0.5} \times 100 = \boxed{0.520\%}\ C.$$

(9) $\int_0^{\pi} \sin t \, dt$ $\quad n=4, \Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4}$

$t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$

$$\frac{\frac{\pi}{4}}{2}\left(0 + 2\left(\frac{\sqrt{2}}{2}\right) + 2(1) + 2\left(\frac{\sqrt{2}}{2}\right) + 0\right)$$

$$\frac{\pi}{8}\left(\sqrt{2} + 2 + \sqrt{2}\right) = \boxed{0.60355}$$

$$|E_T| \leq \frac{M(b-a)}{12n^2} \quad f'(t) = \cos t$$

$$f''(t) = -\sin t$$

$$|E_T| \leq \frac{1(\pi - 0)}{12(4)^2} \quad f'''(t) = -\cos t = 0 = t = -\cos^{-1}(0)$$

$$f''(0) = 0 \quad f''(\pi) = 0 \quad = -\frac{\pi}{2}$$

$$\boxed{|E_T| \leq .0163} \text{ B.} \quad f''\left(-\frac{\pi}{2}\right) = \boxed{1 = M}$$

$$\int_0^{\pi} \sin t \, dt = -\cos t \Big|_0^{\pi} = -\cos \pi + \cos(0) = \boxed{2} \text{ B}$$

$$\frac{.0163}{2} \times 100 = \boxed{.8181\%} \text{ C.}$$

II

$$\frac{\frac{\pi}{4}}{3}\left(0+4\left(\frac{\sqrt{2}}{2}\right)+2(1)+4\left(\frac{\sqrt{2}}{2}\right)+0\right)$$

$$\frac{\pi}{12}(7.6569) = \boxed{2.005}$$

$$|E_s| \le \frac{M(b-a)^5}{180n^4} \qquad f'''(t) - \cos t$$

$$f''''(t) = \sin t$$

$$|E_s| \le \frac{1(\pi)^5}{180(4)^4} \qquad f'''''(t) = \cos t = 0$$

$$f''''(0) = 0 \qquad t = \frac{\pi}{2} \text{ C.P}$$

$$f''''(\pi/2) = \boxed{1 = M} \text{ A.}$$

$$|E_s| \le \boxed{0.006641} \text{ B} \qquad f''''(\pi) = 0$$

$$\frac{.006641}{2} \times 100 = \boxed{.332\%} \text{ C.}$$

Steven Romeiro

\# 3, 7, 15, 21, 25, 35, 39, 43

8.8 Homework

(3) $\int_0^1 \frac{dx}{\sqrt{x}}$ Integrand is infinite @ $x=0$

$$\lim_{t\to 0^+} \int_t^1 \frac{dx}{\sqrt{x}} = \lim_{t\to 0^+} \int_t^1 x^{-1/2}\,dx = \frac{x^{-1/2+1}}{-\frac{1}{2}+1} = \frac{x^{1/2}}{\frac{1}{2}}$$

$$\lim_{t\to 0^+} 2x^{1/2}\Big|_t^1 = \lim_{t\to 0^+} 2\left[\sqrt{1} - \sqrt{t}\right]$$

$$= 2(1-0) = \boxed{2}$$

finite limit
∴ Converges to 2

(7) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ integrand is infinite @ $x=1$

$$\lim_{t\to 1^-} \int_0^t \frac{1}{\sqrt{1-x^2}}dx = \lim_{t\to 1^-} \sin^{-1}x\Big|_0^t$$

$$= \lim_{t\to 1^-} \left(\sin^{-1}t - \sin^{-1}0\right) = \sin^{-1}1 - \sin^{-1}0$$

$$= \frac{\pi}{2} - 0 = \boxed{\frac{\pi}{2}}$$

limit is finite
∴ Converges at $\frac{\pi}{2}$

(15) $\displaystyle\int_0^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\,d\theta$ integrand is infinite @ $\theta=0$

$$\lim_{x\to 0^+}\int_x^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\,d\theta = \begin{array}{l} u=\theta^2+2\theta \\ du = 2\theta+2\,d\theta \end{array} \ \frac{1}{2}\int_x^1 \frac{2\theta+2\,d\theta}{\sqrt{\theta^2+2\theta}}$$

$$=\frac{1}{2}\int_x^1 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_x^1 u^{-1/2}\,du = \frac{2}{2}\sqrt{u}\,\Big|_x^1 = \sqrt{\theta^2+2\theta}\,\Big|_x^1$$

$$\lim_{x\to 0^+}\left[\sqrt{1^2+2(1)}-\sqrt{x^2+2x}\right] = \left[\sqrt{3}-\sqrt{0}\right]$$

$= \boxed{\sqrt{3}}$ limit is finite $\therefore$ Converges at $\sqrt{3}$

(21) $\displaystyle\int_{-\infty}^0 \theta e^\theta\,d\theta = \lim_{x\to-\infty}\int_x^0 \theta e^\theta\,d\theta =$

y'	$\int$
θ (+)	e^θ
1 (−)	e^θ
0	e^θ

$$\lim_{x\to-\infty}\left(\theta e^\theta - 1e^\theta\right)\Big|_x^0 = \lim_{x\to-\infty}\left[\left(0e^0 - xe^x\right)-\left(e^0-e^x\right)\right]$$

$$(0-0)-(1-0) = \boxed{-1}$$

(25) $\int_0^1 x\ln x$ infinite Integrand @ $x=0$

$\lim_{t\to 0^+}\int_t^1 x\ln x =$ $\quad u=\ln x \quad dv = x\,dx$

$\quad du=\frac{1}{x}dx \quad v=\frac{1}{2}x^2$

$= \ln x\cdot\frac{1}{2}x^2 - \int\frac{1}{2}x^2\cdot\frac{1}{x}dx$

$= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx = \left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]\Big|_t^1$

$\lim_{t\to 0^+}\frac{1}{2}x^2\ln x\Big|_t^1 - \lim_{t\to 0^+}\frac{1}{4}x^2\Big|_t^1$

? $\to$ $\left(\frac{1}{2}(1)^2\ln 1 - \frac{1}{2}(0)^2\ln(0)\right) - \left(\frac{1}{4}(1)^2 - \frac{1}{4}(0)^2\right)$

$(0-0)-\frac{1}{4} = \boxed{\frac{1}{4}}$

limit is finite
$\therefore$ Converges at $\frac{1}{4}$

Correction $\frac{1}{2}(1)^2\ln 1 - \underbrace{\frac{1}{2}(t)^2\ln(t)}$ $\to$ getting closer & closer to - infinity

$0 - \infty \;\therefore\to\; \frac{t^2\ln(t)}{2} = \frac{\ln(t)}{2t^{-2}} = \frac{\ln(t)}{\frac{2}{t^2}} = \frac{\infty}{\infty}$

now use L'hopital's $\frac{\frac{1}{t}}{-4t^{-3}} = \frac{1}{t}\cdot\frac{t^3}{-4} = -\frac{t^2}{4} = \frac{0^2}{4} = \boxed{0}$

(35) $\displaystyle\int_0^{\pi/2} \tan\theta\, d\theta$ integrand is infinite @ $\theta = \frac{\pi}{2}$

$$\lim_{x\to\pi/2^-} \int_0^x \tan\theta\, d\theta = \ln|\sec\theta| \Big|_0^x$$

$$\lim_{x\to\pi/2^-} \left(\ln|\sec x| - \ln|\sec 0|\right)$$

? $= \left(\ln|\sec\tfrac{\pi}{2}| - \ln 1\right) = \left(\ln\left|\frac{1}{\cos\frac{\pi}{2}}\right| - 0\right)$

as it approach $\pi/2 \to$ zero $= \ln(\cos\pi/2)^{-1} = -\ln\cos\frac{\pi}{2} = -\ln 0$

(39) $\displaystyle\int_0^{\ln 2} x^{-2} e^{-1/x}$ integrand infinite @ $x = 0$

$$\lim_{t\to 0^+} \int_t^{\ln 2} x^{-2} e^{-1/x} \quad \begin{matrix} u = -x^{-1} \\ du = x^{-2} \end{matrix} \quad = \lim_{t\to 0^+} \int_t^{\ln 2} e^u\, du$$

$$\lim_{t\to 0^+} e^u \Big|_t^{\ln 2} = e^{-1/x} \Big|_t^{\ln 2} = \lim_{t\to 0^+} \left(e^{-1/\ln 2} - e^{-1/t}\right)$$

$$e^{-1/\ln 2} - \left(e^{-\infty}\right) = e^{-1/\ln 2} - 0$$

$$= \boxed{e^{-\frac{1}{\ln 2}}}$$

limit is finite

$\therefore$ it converges at $e^{-\frac{1}{\ln 2}}$

T-2 #42

43 $$\int_0^2 \frac{dx}{1-x^2} = \int_0^1 \frac{dx}{1-x^2} + \int_1^2 \frac{dx}{1-x^2}$$

$$\lim_{t\to 1^-} \int_0^t \frac{dx}{1-x^2} + \lim_{t\to 1^+} \int_t^2 \frac{dx}{1-x^2}$$

$$\lim_{t\to 1^-} \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|\Bigg|_0^t + \lim_{t\to 1^+} \frac{1}{2}\ln\left|\frac{x+1}{x-1}\right|\Bigg|_t^2$$

$$\frac{1}{2}\ln\left|\begin{matrix}\text{large}\\ \text{value}\end{matrix}\right| = \infty$$

one of the limits is infinity $\therefore$ it diverges

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#5, 9, 11, 15, 17, 19, 23, 31, 39

84/86 98 b 45, 53, 63, 92

10.1 Homework

(5) $a_n = \frac{2^n}{2^{n+1}}$ $a_1 = \frac{2}{2^2} = \boxed{\frac{1}{2}}$

$a_2 = \frac{4}{8} = \boxed{\frac{1}{2}}$ $a_3 = \frac{8}{16} = \boxed{\frac{1}{2}}$ $a_4 = \frac{16}{32} = \boxed{\frac{1}{2}}$

? Values right?

(9) $a_1 = 2$ $a_{n+1} = (-1)^{n+1} a_n/2$

$a_2 = (-1)^3 \cdot 2/2 = \boxed{-1}$ $a_3 = \boxed{1.5}$ $a_4 = \boxed{-2}$

$a_5 = \boxed{2.5}$ $a_6 = \boxed{-3}$ $a_7 = \boxed{3.5}$ $a_8 = \boxed{-4}$

$a_9 = \boxed{4.5}$ $a_{10} = (-1)^{11} \cdot 10/2 = \boxed{-5}$

(11) $a_1 = a_2 = 1$ $a_{n+2} = a_{n+1} + a_n$

$a_{1+2} = a_{1+1} + a_1 = a_3 = a_2 + a_1 = a_3 = 1 + 1 = \boxed{2}$

$a_4 = a_3 + a_2 = 2 + 1 = \boxed{3}$ $a_5 = a_4 + a_3 = 2 + 3 = \boxed{5}$

$a_6 = a_5 + a_4 = 5 + 3 = \boxed{8}$ $a_7 = a_6 + a_5 = 8 + 5 = \boxed{13}$

$a_8 = a_7 + a_6 = 13 + 8 = \boxed{21}$ $a_9 = 21 + 13 = \boxed{34}$ $a_{10} = 34 + 21 = \boxed{55}$

(15) $\{\overset{1}{1}, \overset{2}{-4}, \overset{3}{9}, \overset{4}{-16}, \overset{5}{25} \ldots\}$ $\boxed{a_n = (-1)^{n+1}(a_n)^2}$

(17) $\left\{\overset{1}{\frac{1}{9}}, \overset{2}{\frac{2}{12}}, \overset{3}{\frac{2^2}{15}}, \overset{4}{\frac{2^3}{18}}, \overset{5}{\frac{2^4}{21}}\right\}$ $\boxed{a_n = \frac{(2)^{n-1}}{(3)(n+2)}}$

(19) $\{\overset{1}{0}, \overset{2}{3}, \overset{3}{8}, \overset{4}{15}, \overset{5}{24}\}$ $\boxed{a_n = (n+1)(n-1)}$

$2+1+3$
$2+0+3$

1 2 3 4 5

(23) $\frac{5}{1}, \frac{8}{2}, \frac{11}{6}, \frac{14}{24}, \frac{17}{120}$ $\boxed{a_n = \frac{3n+2}{n!}}$

(31) $a_n = \frac{1-5n^4}{1+2n} = \lim_{n\to\infty} \frac{1-5n^4}{1+2n} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{20n^3}{2} = \boxed{\infty}$ Diverge

(39) $a_n = \frac{(-1)^{n+1}}{2n-1} = \lim_{n\to\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{(-1)^{\infty}}{2\infty} = \frac{\pm 1}{\text{large}}$

$\lim_{n\to\infty} = \boxed{0}$ Converges

(45) $a_n = \frac{\sin(n)}{n} = \lim_{n\to\infty} \frac{\sin(n)}{n} = \frac{(\pm 1)}{\text{Large}}$

$\lim_{n\to\infty} \frac{\sin(n)}{n} = \boxed{0}$ Converges

? e^7 ? (53) $a_n = \left(1+\frac{7}{n}\right)^n = \lim_{n\to\infty} \left(1+\frac{7}{n}\right)^n = \boxed{1}$ Converges

Theorem 5. $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$

?

Compare with $1/n$?

(63) $a_n = \frac{n!}{n^n} = \lim_{n \to \infty} \frac{n!}{n^n} = \frac{\infty}{\infty}$

Comparisson

$\frac{n!}{n^n} \leq \frac{1}{n}$

Theorem 6. $\lim_{n \to \infty} \frac{x^n}{n!} = 0$

?

(92) $a_1 = -1$, $a_{n+1} = \frac{a_n + 6}{a_n + 2}$

$a_2 = \frac{7}{3}$

$\lim_{n \to \infty} \frac{n+6}{n+2} = \frac{\infty}{\infty} \therefore \lim_{n \to \infty} = \boxed{1}$ Converges

$\lim_{n \to \infty} a_n = L \longrightarrow \lim_{n \to \infty} a_{n+1} = \lim_{n \to \infty} \frac{a_n + 6}{a_n + 2}$

$= \frac{\lim_{n \to \infty} (a+6)}{\lim_{n \to \infty} (a+2)} = \frac{\lim_{n \to \infty} a_n + \lim_{n \to \infty} 6}{\lim_{n \to \infty} a_n + \lim_{n \to \infty} 2}$

$= \frac{L+6}{L+2} \longrightarrow \lim_{n \to \infty} a_{n+1} = L$

$L = \frac{L+6}{L+2} \longrightarrow L(L+2) = L+6$

now solve for quadratic L

Steven Romero

\# 9, 13, 19, 21, 24, 27, 31, 35
38, 49, 53, 63, 83

10.2 Homework

(9) $\sum_{n=1}^{\infty}\left(1-\frac{7}{4^n}\right)$ $a_1 = \left(1-\frac{7}{4^1}\right) = \boxed{-\frac{3}{4}}$

$a_2 = \boxed{9/16}$ $a_3 = \boxed{\frac{57}{64}}$ $a_4 = \boxed{\frac{249}{256}}$ $a_5 = \boxed{\frac{1017}{1024}}$

$a_6 = \boxed{4089/4096}$ $a_7 = \boxed{.99957}$ $\boxed{a_8 = .99989}$

$\lim_{n\to\infty}\left(1-\frac{7}{4^n}\right) = \left(1-\cancel{\frac{7}{4^\infty}}^{0}\right) = \boxed{1}$ converge

(13) $\sum_{n=0}^{\infty}\left(\frac{1}{2^n}+\frac{(-1)^n}{5^n}\right) = a_1 = \left(\frac{1}{2^1}+\frac{(-1)^1}{5^1}\right) = \boxed{\frac{3}{10}}$

$a_2 = \boxed{\frac{29}{100}}$ $a_3 = \boxed{\frac{117}{1000}}$ $a_4 = \boxed{.0641}$ $a_5 = \boxed{.03093}$

$a_6 = \boxed{.015689}$ $a_7 = \boxed{.0078}$ $a_8 = \boxed{.00391}$

$\lim_{n\to\infty}\left(\frac{1}{2^n}+\frac{(-1)^n}{5^n}\right) = \left(\frac{1}{2^\infty}+\frac{(-1)^\infty}{5^\infty}\right) = \left(0+\cancel{\pm 1}^{0}\right)$

$= \boxed{0 \text{ converges}}$

(19) $0.\overline{23} = 0.232323 = \boxed{\frac{23}{99}}$ $= \frac{23}{100}\cdot\frac{1}{100}+\frac{23}{100}\cdot\left(\frac{1}{100}\right)^2 \cdots S = \frac{a}{1+r}$

(21) $0.\overline{7} = 0.7777\ldots = \boxed{7/9}$

(24) $1.\overline{414} = 1.414141\ldots$ $\boxed{140/99}$

(27) $\sum_{n=1}^{\infty} = \frac{n}{n+10} = \lim_{n\to\infty}\frac{n}{n+10} = \frac{\infty}{\infty} = \lim_{n\to 10} = \boxed{1}$ Diverges

nth test

(31) $\sum_{n=1}^{\infty} \cos\frac{1}{n} = \lim_{n\to\infty} \cos\frac{1}{\cancel{n}} = \cos(0) = \boxed{1}$ Diverges

? (35) $\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) = \sum_{n=1}^{\infty}\frac{1}{n} - \sum_{n=1}^{\infty}\frac{1}{n+1}$

$= \lim_{n\to\infty}\frac{1}{n} - \lim_{n\to\infty}\frac{1}{n+1} = 0 - 0 = \boxed{0}$

? (38) $\sum_{n=1}^{\infty}(\tan(n) - \tan(n-1))$

$\sum_{n=1}^{\infty}\tan(n) - \sum_{n=1}^{\infty}\tan(n-1)$

$=$

(49) $\sum_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n = \lim_{n\to\infty}\left(\frac{1}{\sqrt{2}}\right)^n = \boxed{\infty}$ Diverges

(53) $\sum_{n=0}^{\infty}\cos\left(\frac{n\pi}{2}\right) = \lim_{n\to\infty}\cos\left(\frac{n\pi}{2}\right) = \boxed{1}$ Diverges

(63) $\sum_{n=1}^{\infty}\frac{2^n + 3^n}{4^n} = \sum_{n=1}^{\infty}\left(\frac{2+3}{4}\right)^n$

$= \sum_{n=1}^{\infty}\left(\frac{5}{4}\right)^n = \lim_{n\to\infty}\left(\frac{5}{4}\right)^n = \boxed{\infty}$ Diverges

? 83 $\sum (a_n/b_n) = \left(\frac{a_n}{b_n}\right) = \left(\frac{1}{2}\right)^n$

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10.3 Homework

(1) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ continuous, positive + decreasing

p-Series, $p = 2 > 1$ $\therefore$ Series converge

Integral Test

$$\int_1^{\infty} \frac{1}{n^2}\,dn = \lim_{b\to\infty} \int_1^b \frac{1}{n^2}\,dn = \left[-\frac{1}{n}\right]\Big|_1^b$$

$$\lim_{b\to\infty}\left[-\frac{1}{b} - \left(-\frac{1}{1}\right)\right] = [0+1] = \boxed{1 \text{ Converges}}$$

(5) $\sum_{n=1}^{\infty} e^{-2n} = \sum_{n=1}^{\infty} \frac{1}{e^{2n}} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^{2n}$

$$= \lim_{b\to\infty} \int_1^b \left(\frac{1}{e}\right)^{2n} dn = \int_1^b e^{-2n}\,dn \quad u = -2n \quad du = -2dn$$

$$= -\frac{1}{2}\int_1^b e^{-2n}\cdot -2dn = -\frac{1}{2}\int_1^b e^{u}\,du = -\frac{1}{2}\left[e^{-2n}\right]\Big|_1^b$$

$$-\frac{1}{2}\lim_{b\to\infty}\left[e^{-2(\infty)} - e^{-2(1)}\right] = -\frac{1}{2}\left[\cancel{\frac{1}{e^{\infty}}}^{\,0} - \frac{1}{e^2}\right]$$

$$= \boxed{\frac{1}{2e^2} \text{ Converges}}$$

(9) $$\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}} = \int_7^{\infty} \frac{n^2}{e^{n/3}}\,dn = \lim_{b\to\infty} \int_7^{b} \frac{n^2}{e^{n/3}}\,dn$$

Continuous, positive & decreasing only after $n=7$

$$\int_1^b n^2 \cdot e^{-n/3}\,dn$$

f'	$\int$
n^2 +	$e^{-n/3}$
$2n$ −	$-\frac{1}{3}e^{-n/3}$
2 +	$+\frac{1}{9}e^{-n/3}$
0	$-\frac{1}{27}e^{-n/3}$

$$\left[-\frac{1}{3}n^2 e^{-n/3} - \frac{1}{9}\cdot 2n e^{-n/3} - \frac{2}{27}e^{-n/3}\right]\Bigg|_7^b$$

$$= \left[-\frac{1}{3}e^{-n/3}\left(n^2 + \frac{2}{3}n + \frac{2}{9}\right)\right]\Bigg|_7^b$$

$$\lim_{b\to\infty} \cancel{\left[-\frac{1}{3e^{b/3}}\left(b^2 + \frac{2}{3}b + \frac{2}{9}\right)\right]}^{\to 0} - \left[-\frac{1}{3e^{7/3}}\left(7^2 + \frac{2}{3}(7) + \frac{2}{9}\right)\right]$$

$$0 - \left[-\frac{1}{3e^{7/3}}\left(\frac{485}{9}\right)\right] = 0 + \frac{485}{27e^{7/3}}$$

$$\boxed{\text{Series Converges at } \frac{485}{27e^{7/3}}}$$

Geometric Series

(11) $\sum_{n=1}^{\infty} \frac{1}{10^n} = \sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n = \frac{1}{10} + \frac{1}{10} \cdot \frac{1}{10} + \frac{1}{10} \cdot \left(\frac{1}{10}\right)^2$

$a = \frac{1}{10}$ & $r = \frac{1}{10}$ $\quad |r| < 1$ $\therefore$ Series Converges

(14) $\sum_{n=1}^{\infty} \frac{5}{n+1}$ Continuous, positive & decreasing
Integral test

$\int_1^{\infty} \frac{5}{n+1} dn = \lim_{b \to \infty} \int_1^b \frac{5}{n+1} dn$ $\quad$ $u = n+1$, $du = dn$

$5 \lim_{b \to \infty} \int_1^b \frac{1}{u} du = 5 \lim_{b \to \infty} \left[\ln(n+1) \right] \Big|_1^b$

$5 \lim_{b \to \infty} \left[\ln(b+1) - \ln(1+1) \right] = 5\left[\ln(\infty) - \ln(2) \right]$

$= \boxed{\infty \therefore \text{Series diverges}}$

(17) $\sum_{n=1}^{\infty} \frac{1}{8^n} = \sum_{n=1}^{\infty} \left(\frac{1}{8}\right)^n = \frac{1}{8} + \frac{1}{8} \cdot \frac{1}{8} + \frac{1}{8} \cdot \left(\frac{1}{8}\right)^2 \ldots$

Geometric Series $\therefore$ $a = \frac{1}{8}$ & $r = \frac{1}{8}$

$\boxed{|r| < 1 \therefore \text{Series Converges}}$

(23) $\sum_{n=1}^{\infty} \frac{-2}{n+1} = -2\sum_{n=1}^{\infty} \frac{1}{n+1}$ Continuous Positive Decreasing

Integral test

$-2\int_1^{\infty} \frac{1}{n+1}\,dn = -2\lim_{b\to\infty} \int_1^b \frac{1}{n+1}\,dn = -2\lim_{b\to\infty} \left[\ln(n+1)\right]\Big|_1^b$

$-2\lim_{b\to\infty} \left[\ln(b+1) - \ln(1+1)\right] = -2\left[\infty - \ln 2\right]$

$\boxed{= \infty \therefore \text{Series diverge}}$

(27) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln(n)} =$ Continuous Positive Decreasing $\left\{\begin{array}{l}\text{integ}\\ \text{test}\end{array}\right. = \int_2^{\infty} \frac{\sqrt{n}}{\ln(n)}\,dn$

$\lim_{b\to\infty} \int_2^{\infty} n^{1/2} \cdot (\ln n)^{-1}\,dn$

f'	$\frac{1}{\ln(n)}$	n	0
$\int$	$n^{1/2}$	$\frac{2n^{3/2}}{3}$	$\frac{4}{15}n^{5/2}$

Tab method

$\lim_{b\to\infty} \left[\frac{1}{\ln(n)} \cdot \frac{2}{3}n^{3/2} - \frac{4}{15}n^{7/2}\right]\Big|_2^{\infty}$

$\lim_{b\to\infty} \left[\left(\frac{2(b)^{3/2}}{3\ln(b)} - \frac{4}{15}b^{7/2}\right) - \left(\frac{2(2)^{3/2}}{3(\ln 2)} - \frac{4}{15}\cdot(2)^{7/2}\right)\right]$

$\left[(\infty) - \left(\frac{2(2)^{3/2}}{3\ln 2} - \frac{4}{15}(2)^{7/2}\right)\right] = \boxed{\infty \text{ Series Diverge}}$

Geometric Series

(29) $\sum_{n=1}^{\infty} \frac{1}{(\ln(2))^n} = \sum_{n=1}^{\infty} \left(\frac{1}{\ln 2}\right)^n = \frac{1}{\ln 2} + \frac{1}{\ln 2}\cdot\frac{1}{\ln 2} + \frac{1}{\ln 2}\cdot\left(\frac{1}{\ln 2}\right)^2 \cdots$

$a = \frac{1}{\ln 2}$, $r = \frac{1}{\ln 2}$ $\boxed{|r| \geq 1 \therefore \text{Series Diverges}}$

(33) $\sum_{n=1}^{\infty} n \cdot \sin \frac{1}{n}$ nth test for Divergence

$\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right) = \infty \cdot 0$ $\cancel{\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{n}}$

$= \lim_{n\to\infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} = \frac{0}{0} = \lim_{n\to\infty} \frac{-\frac{1}{n^2}\cos\frac{1}{n}}{-\frac{1}{n^2}}$

$= \lim_{n\to\infty} \cos\frac{1}{n} = 1 \neq 0$

(35) $\sum_{n=1}^{\infty} \frac{e^n}{1+e^{2n}} = e\sum_{n=1}^{\infty}\left(\frac{1}{1+e^2}\right)^n$

Geometric Series

$= \frac{e}{1+e^2} + \frac{e}{1+e^2}\cdot\frac{e}{1+e^2} + \frac{e}{1+e^2}\left(\frac{e}{1+e^2}\right)^2 \cdots$

$a = \frac{e}{1+e^2}$, $r = \frac{e}{1+e^2} < 1$ $\therefore$ Series Converges

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10.4 Homework

① $\sum_{n=1}^{\infty} \frac{1}{n^2+30}$ $\quad U_n = \frac{1}{n^2+30} \quad V_n = \frac{1}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2} =$ p-Series, $p = 2 > 1$ Converges

$0 < U_n < V_n \therefore U_n$ Series Converges

③ $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ $\quad U_n = \frac{1}{n^{1/2}-1} \quad V_n = \frac{1}{n^{1/2}}$

$\sum_{n=2}^{\infty} \frac{1}{n^{1/2}}$ p-series, $p = \frac{1}{2} < 1 \therefore$ Diverges

$0 < V_n < U_n \therefore U_n$ Series will Diverge

⑤ $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{3/2}}$ $\quad \cos^2 n \to$ always $= 1$ $\quad U_n = \frac{\cos^2 n}{n^{3/2}} \quad V_n = \frac{1}{n^{3/2}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ p-Series, $p = \frac{3}{2} > 1$ Converges

$0 \leq U_n \leq V_n \therefore$ Converges

⑨ $\sum_{n=1}^{\infty} \frac{n-2}{n^3-n^2+3} =$ $\quad U_n = \frac{n-2}{n^3-n^2+3} \quad V_n = \frac{1}{n^3}$

?
$V_n = \frac{n}{n^3}$
or
$V_n = \frac{1}{n^3}$

$\sum_{n=1}^{\infty} \frac{1}{n^3} =$ p-Series, $p = 3 > 1$ Converges

$\lim_{n\to\infty} \frac{\frac{n-2}{n^3-n^2+3}}{\frac{1}{n^3}} = \lim_{n\to\infty} \frac{n^4-2n^3}{n^3-n^2+3} = \frac{4n^3-6n^2}{3n^2-2n} = \frac{12n^2-12n}{6n-2}$

$\lim_{n\to\infty} \frac{24n-12}{6} = \boxed{\infty}$?

(13) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n^{1/2} \cdot 4^n}$ $\quad U_n = \dfrac{5^n}{n^{1/2} \cdot 4^n} \quad V_n = \dfrac{5^n}{4^n}$

$\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{4}\right)^n = \frac{5}{4} + \frac{5}{4} \cdot \left(\frac{5}{4}\right) \ldots$ Geometric Series $|r| > 1$ Converges

$$\lim_{n\to\infty} \frac{\frac{5^n}{n^{1/2} 4^n}}{\frac{5^n}{4^n}} = \lim_{n\to\infty} \frac{\cancel{5^n}}{n^{1/2}\cancel{4^n}} \cdot \frac{\cancel{4^n}}{\cancel{5^n}} = \lim_{n\to\infty} \frac{1}{n^{1/2}} = \boxed{0}$$

$\boxed{L = 0 \quad V_n \text{ Series Converge} \therefore U_n \text{ Series Converges}}$

? how to pick V_n here

(17) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^{1/2} + n^{3/2}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}(2+n)}$

$U_n = \dfrac{1}{n^{1/2}(2+n)} \qquad V_n = \dfrac{1}{n^{1/2}}$ because this V_n is larger & easy p-series

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ p-series, $p = 1/2 \leq 1 \therefore$ Divergent

?

$$\lim_{n\to\infty} \frac{\frac{1}{2n^{1/2} + n^{3/2}}}{\frac{1}{n^{1/2}}} = \lim_{n\to\infty} \frac{1}{\cancel{n^{1/2}}(2+n)} \cdot \cancel{(n)^{1/2}}$$

$$\lim_{n\to\infty} \frac{1}{2+n} = \boxed{0}$$

OR $\displaystyle\lim_{n\to\infty} \frac{n^{1/2}}{n^{1/2}(2+n)} =$ leading terms $= n^{1/2}$

$\displaystyle\lim_{n\to\infty} = \boxed{1}$ $\quad L = 1, \; 0 < 1 < \infty$

V_n diverges $\therefore$ U_n diverges

(23) $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$ $U_n = \frac{10n+1}{n(n^2+3n+2)}$ $V_n = \frac{10n}{n^3} = \frac{10}{n^2}$

$\sum_{n=1}^{\infty} \frac{10}{n^2}$ p-series, $p = 2 > 1$ Converges

$$\lim_{n\to\infty} \frac{\frac{10n+1}{n(n^2+3n+2)}}{\frac{10}{n^2}} = \lim_{n\to\infty} \frac{10n+1}{\cancel{n}(n^2+3n+2)}\left(\frac{n^{\cancel{2}}}{\cancel{10}}\right)$$

$$\lim_{n\to\infty} \frac{(n+1)n}{n^2+3n+2} = \lim_{n\to\infty} \frac{n^2+n}{n^2+3n+2} = \boxed{1}$$

$\boxed{L=1, \quad 0<1<\infty \quad V_n \text{ Converges } \therefore U_n \text{ Converges}}$

(25) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$ $U_n = \left(\frac{n}{3n+1}\right)^n$ $V_n = \left(\frac{1}{3}\right)^n$

$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{3} + \frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\left(\frac{1}{3}\right)^2 \ldots$ Geometric Series $|r| < 1$ Converges

$0 < U_n < V_n$ V_n Converges $\therefore U_n$ Converges

or $$\lim_{n\to\infty} \frac{\frac{n^n}{3n^n+1^n}}{\frac{1}{3^n}} = \left(\frac{n}{3n+1}\cdot\frac{3}{1}\right)^n = \left(\frac{n}{3n+1}\right)^n$$

$$\lim_{n\to\infty} \frac{n^n}{3n^n+1} = \boxed{\frac{1}{3}} \quad \text{Converges}$$

$L = \frac{1}{3}$ $0 < 1 < \infty$ V_n Converges $\therefore U_n$ Converges

?
What is V_n ?

(27) $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)} = U_n = \frac{1}{\ln(\ln n)} \quad V_n = \frac{1}{\ln n}$

$\sum_{n=3}^{\infty} \frac{1}{\ln n}$ Continuous positive & Decreasing
Use integral test

$\lim_{b\to\infty} \int_3^{\infty} \frac{1}{\ln n} dx$

$\ln n > \ln(\ln(n))$

$n > \ln n > \ln(\ln n)$

$\frac{1}{n} < \frac{1}{\ln n} < \frac{1}{\ln(\ln n)} \quad \therefore \quad V_n = \frac{1}{n}$

$\sum_{n=3}^{\infty} \frac{1}{n}$ = Harmonic Series Divergent

Because $V_n < U_n$ & V_n is Divergent
$\therefore$ U_n is Divergent

?

Converges but $L = \infty$

(30) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$ $\quad V_n = \frac{1}{n^{3/2}}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ p-series, $p = 3/2 > 1$ Converges

$\displaystyle\lim_{n\to\infty} \frac{(\ln n)^2}{n^{3/2}} \cdot n^{3/2} = \lim_{n\to\infty} (\ln n)^2 = \infty$?

?

?

(41) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n - n}{n 2^n}$ $\quad V_n = \frac{2^n}{2^n} = \left(\frac{2}{2}\right)^n = (1)^n$

Geometric Series

$\displaystyle\sum_{n=1}^{\infty} 1^n = 1 + 1 \cdot 1 + 1 \cdot (1)^2 \cdots$ $|r| \geq 1$ Diverges

lim

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#7 is on last page

10.5 Homework

(3) $\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2}$ $\quad a_{n+1} = \frac{(n+1-1)!}{(n+1+1)^2} = \frac{n!}{(n+2)^2}$

$$\lim_{n\to\infty} \left| \frac{\frac{n!}{(n+2)^2}}{\frac{(n-1)!}{(n+1)^2}} \right| = \lim_{n\to\infty} \frac{(n!)(n+1)^2}{(n+2)^2(n-1)!} = \frac{n\cancel{(n-1)!}(n+1)^2}{(n+2)^2\cancel{(n-1)!}}$$

$$\lim_{n\to\infty} \frac{n(n+1)^2}{(n+2)^2} = \infty \quad \boxed{\text{Diverges}} \quad p = \infty > 1$$

(5) $\sum_{n=1}^{\infty} \frac{n^4}{(-4)^n}$ $\quad a_{n+1} = \frac{(n+1)^4}{(-4)^{n+1}}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^4}{(-4)^{n+1}}}{\frac{n^4}{(-4)^n}} \right| = \lim_{n\to\infty} \frac{(n+1)^4(-4)^n}{(n)^4(-4)^{n+1}} = \frac{(n+1)^4\cancel{(-4)^n}}{n^4\cancel{(-4)^n}(-4)^1}$$

$$\lim_{n\to\infty} \left| \frac{(n+1)^4}{-4n^4} \right| = \boxed{\frac{1}{4}} \quad \boxed{p = \frac{1}{4} < 1 \therefore \text{Converges}}$$

(9) $\sum_{n=1}^{\infty} \frac{7}{(2n+5)^n} = \lim_{n\to\infty} \sqrt[n]{\frac{7}{(2n+5)^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{7}}{\sqrt[n]{(2n+5)^n}}$

$$\lim_{n\to\infty} \frac{7^{\frac{1}{n}}}{2n+5} = \frac{1}{2(\infty)+5} = 0$$

$$\boxed{p = 0 < 1 \therefore \text{Converges}}$$

$$\sum_{n=1}^{\infty}\left(\frac{4n+3}{3n-5}\right)^n = \lim_{n\to\infty}\sqrt[n]{\left(\frac{4n+3}{3n-5}\right)^n}$$

$$\lim_{n\to\infty}\frac{4n-3}{3n-5} = \frac{4}{3} \quad \boxed{p=\frac{4}{3}>1 \therefore \text{Diverges}}$$

$$\sum_{n=1}^{\infty} n!(-e)^{-n} = \frac{n!}{-e^n} \quad a_{n+1} = \frac{(n+1)!}{-e^{n+1}}$$

$$\lim_{n\to\infty}\left|\frac{\frac{(n+1)!}{-e^{n+1}}}{\frac{n!}{-e^n}}\right| = \lim_{n\to\infty}\frac{-e^n(n+1)!}{-e^{n+1}n!} = \lim_{n\to\infty}\frac{\cancel{-e^n}(n+1)\cancel{n!}}{\cancel{-e^n}\cdot(-e)^1\cdot\cancel{n!}}$$

$$\lim_{n\to\infty}\frac{n+1}{-e} = \infty \quad \boxed{p=\infty>1 \therefore \text{Diverges}}$$

$$\sum_{n=1}^{\infty}\left(1-\frac{1}{3n}\right)^n = \quad \text{Deffinition of } e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$$

$$= \lim_{n\to\infty}\left(1-\frac{1}{3n}\right)^n = \lim_{n\to\infty}\left(1+\left(-\frac{1}{3}\right)\cdot\frac{1}{n}\right)^n$$

$$\lim_{n\to\infty}\left(1+\frac{\left(-\frac{1}{3}\right)}{\frac{n}{1}}\right)^n = \lim_{n\to\infty}\left(1+\frac{\left(-\frac{1}{3}\right)}{n}\right)^n \quad x=-\frac{1}{3}$$

$$\therefore \lim_{n\to\infty}\left(1+\frac{(-1/3)}{n}\right)^n = \boxed{e^{(-1/3)} \text{ Converges}}$$

?

Theorem 5 $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

yes

Definition Theorem 5

(27) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ $\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$ converges

$\therefore \lim_{n\to\infty} \frac{\ln(n)}{n} \cdot \frac{1}{n^2} = \boxed{0 \text{ converges}}$

OR $\therefore$ Comparisson Test

$U_n = \frac{\ln(n)}{n^3} < V_n = \frac{n}{n^3} = \frac{1}{n^2}$

$\ln(n) < n$

$\sum_{n=1}^{\infty} \frac{1}{n^2} =$ p-series, $p = 2 > 1$ Converges

Because $U_n < V_n$, & V_n Converges
$\therefore U_n$ must also Converge

Root test

(28) $\sum_{n=1}^{\infty} \frac{(-\ln n)^n}{n^n} = \lim_{n\to\infty} \left| \sqrt[n]{\frac{(-\ln n)^n}{n^n}} \right|$

$\lim_{n\to\infty} \left| \frac{\sqrt[n]{(-\ln n)^n}}{\sqrt[n]{n^n}} \right| = \lim_{n\to\infty} \frac{\ln n}{n} = \frac{\infty}{\infty}$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$

$\boxed{\rho = 0 < 1 \text{ Series Converge}}$

$(3+1)! = (3+1)\cdot 3! = 4\cdot 3! = 4!$

$(3+3)! = (3+3)(3!) = (6)(3!)$

$3\cdot 2+3$

?

35 $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\cdot n!\cdot 3^n} = \frac{(n+3)!}{6\cdot n!\cdot 3^n}$ $\quad V_n = \dfrac{(n+1+3)!}{3!\,(n+1)!\,3^{n+1}}$

$$\lim_{n\to\infty} \frac{\dfrac{(n+4)!}{3!(n+1)!\,3^n\cdot 3}}{\dfrac{(n+3)!}{3!\,n!\,3^n}} = \lim_{n\to\infty} \frac{\dfrac{\cancel{(n+3)!}(n+4)}{\cancel{3!}\,\cancel{n!}\,(n+1)\cancel{3^n}\cdot 3}}{\dfrac{\cancel{(n+3)!}}{\cancel{3!}\,\cancel{n!}\,\cancel{3^n}}}$$

$$\lim_{n\to\infty} \frac{n+4}{(n+1)3} \to \lim_{n\to\infty} = \frac{1}{3} \quad \boxed{\rho\ \tfrac{1}{3} < 1 \text{ Converge}}$$

39 $\displaystyle\sum_{n=2}^{\infty} \frac{-n}{(\ln(n))^n}$ = Root test

$$\lim_{n\to\infty} \left| \sqrt[n]{\frac{n}{(\ln n)^n}} \right| = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{\sqrt[n]{(\ln n)^n}}$$

$$\lim_{n\to\infty} \frac{1}{\ln n} = 0 \quad \boxed{\rho = 0 < 1 \text{ Converges}}$$

7 $\displaystyle\sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n!\,3^{2n}}$ $\qquad V_n = \dfrac{(n+1)^2(n+3)!}{(n+1)!\,3^{2(n+1)}}$

$$\lim_{n\to\infty} \frac{\dfrac{(n+1)^2(n+3)!}{(n+1)!\,3^{2n+2}}}{\dfrac{n^2(n+2)}{n!\,3^{2n}}} = \frac{\cancel{n!}\,\cancel{3^{2n}}\,(n+1)^{\cancel{2}}\,\cancel{(n+2)!}\,(n+3)}{\cancel{n!}\,\cancel{(n+1)}\,n^2\,\cancel{(n+2)!}\,\cancel{3^{2n}}\,3^2}$$

$$\lim_{n\to\infty} \frac{(n+1)(n+3)}{n^2\cdot 3^2} = \lim_{n\to\infty} \frac{n^2+4n+3}{9n^2} = \frac{1}{9}$$

$$\boxed{\lim = \frac{1}{9} < 1 \ \therefore \text{ Converges}}$$

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10.6 HomeworK

1) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

1) $a_n > 0$ ✓
2) $a_{n+1} \leq a_n$ ✓
3) $\lim_{n\to\infty} a_n = 0$ ✓

$\boxed{\therefore \text{ Alternating Series is Convergent}}$

5) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$

1) $a_n > 0$ ✓
2) $a_{n+1} \leq a_n$ ✓
3) $\lim_{n\to\infty} a_n = 0$ ✓

$\boxed{\therefore \text{ Series is Convergent}}$

7) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^n}{n^2} =$ Root test $\lim_{n\to\infty} \sqrt[n]{\frac{2^n}{n^2}}$

$\lim_{n\to\infty} \frac{\sqrt[n]{2^n}}{\sqrt[n]{n \cdot n}} = \lim_{n\to\infty} \frac{2}{\sqrt[n]{n} \cdot \sqrt[n]{n}} = \lim_{n\to\infty} \frac{2}{1 \cdot 1}$

?
no alt test & root diverges ?

$\lim_{n\to\infty} = 2$ $\boxed{p = 2 > 1 \text{ Divergent}}$?

$a_n = \frac{2^n}{n^2}$

1) $a_n > 0$ ✓
2) $a_{n+1} \leq a_n$ X

(11) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$ Theorem 5 Definition

$\lim_{n\to\infty} \frac{\ln n}{n} = 0$ Convergent

$\therefore$ $\boxed{\text{Series absolutely converges}}$

Can't do that

(17) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

1) $a_n > 0$ ✓
2) $a_{n+1} \leq a_n$ ✓
3) $\lim_{n\to\infty} a_n = 0$ ✓

$\therefore$ Convergent

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ = p-series, $p = \frac{1}{2} < 1$ Series Diverge

$\boxed{\therefore \text{ Alternating Series Conditionally Convergent}}$

(21) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n+3}$ $U_n = \frac{1}{n+3} < V_n = \frac{1}{n}$

1) $a_n > 0$ ✓
2) $a_{n+1} \leq a_n$ ✓
3) $\lim_{n\to\infty} a_n = 0$ ✓

$\sum_{n=1}^{\infty} \frac{1}{n}$ = Harmonic Series Diverge

$\lim_{n\to\infty} \frac{\frac{1}{n+3}}{\frac{1}{n}} = \frac{n}{n+3} = 1$

$L = 1$, $0 < 1 < \infty$
V_n is Divergent $\therefore$ U_n is Divergent

$\boxed{\text{Series is Conditionally Convergent}}$

? (29) $\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1} n}{n^2+1}$

1) $a_n > 0$ ✓
2) $a_{n+1} \leq a_n$ ✓
3) $\lim_{n\to\infty} a_n = 0$ ✓? $= \frac{\pi/2}{\infty} = 0$

Int test

$\lim_{b\to\infty} \int_1^b \frac{\tan^{-1} n}{n^2+1} dn = \begin{matrix} U = \tan^{-1} n \\ du = \frac{1}{n^2+1} dn \end{matrix} \rightarrow \lim_{b\to\infty} \int_1^b U\, du$

$\left[\frac{U^2}{2}\right]\Big|_1^b = \frac{1}{2}\left[\left(\frac{\pi}{2}\right)^2 - \left(\frac{\pi}{2}\right)^2\right] = \boxed{\text{finite conv}}$

(39) $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$ Ratio Test $a_{n+1} = \frac{2(n+1)!}{2^{n+1} \cdot (n+1)!(n+1)}$

?
factorials

$\lim_{n\to\infty} \left| \frac{\frac{2(n+1)!}{2^{n+1}(n+1)!(n+1)}}{\frac{2(n)!}{2^n n! n}} \right| = \lim_{n\to\infty} \frac{\frac{(2n+2)!}{2^n \cdot 2(n+1)(n+1)!}}{\frac{(2n)!}{2^n \cdot n! \cdot n}}$

$\lim_{n\to\infty} \frac{\cancel{2^n} \cancel{n!}\, n (2n+2)!}{\cancel{2^n} \cdot 2 (2n)! (n+1)(n+1)\cancel{n!}} = \frac{n (\cancel{2n+2})(2n+1)\cancel{2n!}}{\cancel{2} \cdot \cancel{2n!} (\cancel{n+1})}$

$\lim_{n\to\infty} \frac{4n^2}{2n^2} = \infty$ ∴ nth term test for div

Diverges

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#1, 3, 7, 13, 19, ~~55~~

#55 not needed

10.7 Homework

① $\sum_{n=0}^{\infty} x^n$ Root test $\lim_{n\to\infty} \sqrt[n]{x^n}$

?

$\lim_{n\to\infty} x = x$

$x = a \therefore R = 0$

?

Converges when $x < 1$

Radius $= 1$

interval of conv $-1 < x < 1$

$x = -1$ $\sum_{n=0}^{\infty} (-1)^n$ nth test for Div

$\lim_{n\to\infty} 1^n =$ DNE $\therefore$ Diverges

$x = -1$ cannot be included

$x = 1$ Nth test for Div.

$\sum_{n=0}^{\infty} 1^n$ $\lim_{n\to\infty} 1^n = 1 \therefore$ Diverges

& cannot be inc

$\boxed{\text{interval for convergence is } -1 < x < 1}$

3 $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$ Root Test

$$\lim_{n\to\infty} \left|\sqrt[n]{(-1)^n(4x+1)^n}\right| = \lim_{n\to\infty} \left|(-1)(4x+1)\right|$$

$$\lim_{n\to\infty} 4|x+1| = |4x+1| < 1$$

$$\left|x+\frac{1}{4}\right| < \boxed{\frac{1}{4}} \qquad \boxed{R = \frac{1}{4}}$$

Interval of Convergence

$$-\frac{1}{4} < x+\frac{1}{4} < \frac{1}{4} \longrightarrow \boxed{\cancel{-\frac{5}{4} < x < \frac{3}{4}}} \to \boxed{-\frac{1}{2} < x < 0}$$

Test endpoints: $x = -\frac{5}{4}$

$$\sum_{n=0}^{\infty} (-1)^n\left(4\left(-\frac{5}{4}\right)+1\right)^n \longrightarrow \sum_{n=0}^{\infty} (-1)^n(-5+1)^n$$

$$\sum_{n=0}^{\infty} (-1)^n(-4)^n \quad \text{Root test} \quad \lim_{n\to\infty} \sqrt[n]{(-1)^n(-4)^n}$$

$$\lim_{n\to\infty} \left|(-1)(-4)\right| =$$

4 $\rho = 4 > 1$ diverges
$-\frac{5}{4}$ cannot be included in the interval

Test endpoints: $X = -\frac{3}{4}$

$$\sum_{n=0}^{\infty} (-1)^n \left(4\left(-\frac{3}{4}\right)+1\right)^n = \sum_{n=0}^{\infty} (-1)^n (-3+1)^n$$

$$\sum_{n=0}^{\infty} (-1)^n (-2)^n \quad \text{Root test} \quad \lim_{n\to\infty} \sqrt[n]{(-1)^n(-2)^n}$$

$$\lim_{n\to\infty} \left|(-1)(-2)\right| = 2$$

$\boxed{\rho = 2 > 1 \therefore \text{diverges}}$ endpoint $-\frac{3}{4}$ cannot be included in the interval

$\therefore$ The interval of convergence for this series is:

$$\boxed{-\frac{5}{4} < X < -\frac{3}{4}}$$

(7) $\sum_{n=0}^{\infty} \frac{nx^n}{n+2}$ Ratio test $\quad a_{n+1} = \frac{(n+1)x^{n+1}}{(n+1+2)} = \frac{(n+1)x^{n+1}}{n+3}$

?

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)x^{n+1}}{n+3}}{\frac{nx^n}{n+2}} \right| = \lim_{n\to\infty} \frac{(n+1)\cancel{x^n}\cdot x\,\cancel{(n+2)}}{n\cancel{x^n}(\cancel{n+2}+1)}$$

$$\lim_{n\to\infty} \frac{\cancel{(n+1)}x}{\cancel{n+1}} = \lim_{n\to\infty} x = x < 1$$

Radius $= \boxed{1}$

Interval of Convergence

$-1 < x < 1$

Test endpoints : $x = -1$

$x = -1$

$$\sum_{n=0}^{\infty} \frac{n(-1)^n}{n+2} = \lim_{n\to\infty} \frac{n(-1)^n}{n+2} = (-1) = \text{DNE} \neq 0$$

$$\sum_{n=0}^{\infty} \frac{n(1)^n}{n+2} = \lim_{n\to\infty} \frac{n}{n+2} = 1 \neq 0 \text{ DIV}$$

Absolutely Converges $-1 < x < 1$

(13) $\sum_{n=0}^{\infty} \frac{4^n x^{2n}}{n}$ Root test $\lim_{n\to\infty} \left| \sqrt[n]{\frac{4^n x^{2n}}{n}} \right|$

$\lim_{n\to\infty} 4x^2 =$ $4x^2 < 1 \to x^2 < \frac{1}{4}$

$x < \boxed{\frac{1}{2}} = R$

interval of Convergence

$-\frac{1}{2} < x < \frac{1}{2}$

Test endpoints: $x = -\frac{1}{2}$

$\sum_{n=0}^{\infty} \frac{4^n(-\frac{1}{2})^{2n}}{n} = \frac{4^n\left(-\frac{1^{2n}}{2^{2n}}\right)}{n} = \frac{4}{n} \cdot \left(\frac{-1^{2n}}{2^{2n}}\right)$

$\sum_{n=0}^{\infty} \frac{4^n(-1^{2n})}{n(2^{2n})} = \sum_{n=0}^{\infty} \frac{\cancel{2^{2n}}(-1^{2n})}{n(\cancel{2^{2n}})} = \sum_{n=0}^{\infty} \frac{(-1)^{2n}}{n}$

?

$\sum_{n=0}^{\infty} \frac{[(-1)^n]^2}{n} =$ alternating Harmonic Series ?

$x = -\frac{1}{2}$ Converges

Test endpoints : $X = \frac{1}{2}$

$$\sum_{n=0}^{\infty} \frac{4^n (\frac{1}{2})^{2n}}{n} = \frac{4^n}{n} \cdot \left(\frac{1}{2}\right)^{2n} = \frac{4^n (1^{2n})}{n(2^{2n})}$$

$$\sum_{n=0}^{\infty} \frac{(\cancel{2^{2n}})(1^{2n})}{n(\cancel{2^{2n}})} = \sum_{n=0}^{\infty} \frac{1^{2n}}{n}$$ nth test for Div.

$$\lim_{n \to \infty} \frac{1^{2n}}{n} = \frac{\infty}{\infty} = \frac{2}{1} = 2$$

$X = 1$ diverges & cannot be used
Interval is $-1 < X < 1$

(19) $\sum_{n=0}^{\infty} \frac{\sqrt{n}\,x^n}{3^n}$ Ratio test $a_{n+1} = \frac{\sqrt{n+1}\,x^{n+1}}{3^{n+1}}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^{1/2} x^{n+1}}{3^{n+1}}}{\frac{(n)^{1/2} x^n}{3^n}} \right| = \lim_{n\to\infty} \left| \frac{(n+1)^{1/2} \cancel{x^n} \cdot x \cdot \cancel{3^n}}{\cancel{3^n} \cdot 3 \cdot (n)^{1/2} \cdot \cancel{x^n}} \right|$$

$$\lim_{n\to\infty} \frac{(n+1)^{1/2} \cdot x}{3n^{1/2}} = \frac{1}{3}x = \text{Limit}$$

Radius

$$\frac{1}{3}x < 1 \to x < \boxed{3} = R$$

Interval of convergence

$$-3 < x < 3$$

Test endpoints: $x = -3$

$$\sum_{n=0}^{\infty} \frac{(n)^{1/2}(-3)^n}{3^n} = \sum_{n=0}^{\infty} \frac{n^{1/2}(-1)^n \cancel{(3)^n}}{\cancel{3^n}} = \sum_{n=0}^{\infty} n^{1/2}(-1)^n$$

Alternating Series test

1) $a_n \geq 0$ ✓

2) $a_{n+1} \leq a_n$ ✗

n^{th} test for div

$$\lim_{n\to\infty} n^{1/2} = \infty$$

$$\lim_{n\to\infty} (-1)^n = \text{DNE}$$

∴ $x = -3$ cannot be included in the interval of convergence test enpoints: $X = 3$

$$\sum_{n=0}^{\infty} \frac{\sqrt{n}\,\cancel{(3)^n}}{\cancel{3^n}} = \sum_{n=0}^{\infty} n^{1/2}$$ nth test for Div.

$$\lim_{n\to\infty} n^{1/2} = \infty$$

$\therefore$ $X=3$ cannot be included in interval

Interval of Convergence for this series is $-3 < X < 3$

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#1, 5, 25, 29

10.8 Homework

(1) $f(x) = e^{2x}$ $\qquad f(0) = 1$

$f'(x) = 2e^{2x}$ $\qquad f'(0) = 2$

$f''(x) = 4e^{2x}$ $\qquad f''(0) = 4$

$f'''(0)\ 8e^{2x}$ $\qquad f'''(0) = 8$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n = f(0) + f(0)x + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3 +$$

$$\boxed{e^{2x} = 1 + 2x + 2x^2 + \frac{4}{3!}x^3}$$

(5) $f(x) = 1/x$ $\qquad f(2) = 1/2$

$f'(x) = -1/x^2$ $\qquad f'(2) = -1/4$

$f''(x) = 2/x^3$ $\qquad f''(2) = 1/4$

$f'''(x) = -6/x^4$ $\qquad f'''(2) = -3/8$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$$\boxed{\frac{1}{x} = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{4}(x-2)^2 - 3/8(x-2)^3}$$

(25) $f(x) = x^4 + x^2 + 1$ $\quad f(-2) = 21$

$f'(x) = 4x^3 + 2x$ $\quad f'(-2) = -36$

$f''(x) = 12x^2 + 2$ $\quad f''(-2) = 50$

$f'''(x) = 24x$ $\quad f'''(-2) = -48$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$$\boxed{x^4 + x^2 + 1 = 21 - 36(x+2) + 25(x+2) - 16(x+2) + \dots}$$

(29) $f(x) = e^x$ $\quad f(2) = e^2$

$f'(x) = e^x$ $\quad f'(2) = e^2$

$f''(x) = e^x$ $\quad f''(2) = e^2$

$f'''(x) = e^x$ $\quad f'''(2) = e^2$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$$\boxed{e^x = e^2 + e^2(x-2) + \frac{e^2}{2!}(x-2)^2 + \frac{e^2}{3!}(x-2)^3}$$

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#1, 5, 11, 13, 21, 29

10.9 Homework

① $f(x) = e^{-5x}$

Use known series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$x = -5x$

$$e^{-5x} \sum_{n=0}^{\infty} \frac{(-5x)^n}{n!} = \boxed{e^{-5x} \sum_{n=0}^{\infty} \frac{(-1)^n (5x)^n}{n!}}$$

⑤ $f(x) = \cos 5x^2$

$$\cos x \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$x = 5x^2$

$$\cos 5x^2 \sum_{n=0}^{\infty} \frac{(-1)^n (5x^2)^{2n}}{(2n)!}$$

$$\boxed{\cos 5x^2 \sum_{n=0}^{\infty} \frac{(-1)^n (5x)^{4n}}{(2n)!}}$$

(11) $f(x) = xe^x$ $\qquad f(0) = 0$

$f'(x) = e^x + xe^x$ $\qquad f'(0) = 1$

$f''(x) = e^x + e^x + xe^x$ $\qquad f''(0) = 2$

$f'''(x) = 3e^x + xe^x$ $\qquad f'''(0) = 3$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$\boxed{xe^x = 0 + x + x^2 + x^3 + \ldots}$$

(13) $\frac{x^2}{2} - 1 + \cos x$ $\qquad$ Use a Known Series

$f(x) = \frac{x^2}{2} - 1$ $\qquad \cos x \; \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$f'(x) = x$

$f''(x) = 1$

?

$f(0) = -1$

$f'(0) = 0$

$f''(0) = 0$

$$\boxed{\frac{x^2}{2} - 1 + \cos x = 0 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \frac{1}{8!}}$$

(21) $f(x) = \frac{1}{(1-x)^2} = \frac{1}{(1-x)} = (1-x)^{-1}$

Use Known Series

$\frac{1}{1-x} \sum_{n=0}^{\infty} x^n$

$x = (1-x)^2$

$\frac{d}{dx} = (1-x)^{-1} = -1(1-x)^{-2}(-1)$

$\frac{1}{1-x} = 1 + 2x + 3x^2 + 4x^3$

$$f(x) = \frac{1}{(1-x)^2} = \sum_{n=\cancel{0}}^{\infty} n(x^{(n-1)}) = \sum_{n=0}^{\infty} (n+1)\, x^n$$

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1, 5, 11, 41, 45, 67, 69

10.10 Homework

(1) $(1+x)^{1/2} = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k \qquad m = 1/2 \quad x = x$

$\binom{1/2}{1} = \frac{1}{2} \qquad \binom{1/2}{2} = \frac{1/2(1/2-1)}{2!} = -1/8 \qquad \binom{1/2}{3}\ \frac{1/2(1/2-1)(1/2-2)}{3!} = 1/16$

$(1+x)^{1/2} = 1 + \binom{1/2}{1}x + \binom{1/2}{2}x^2 + \binom{1/2}{3}x^3 + \dots$

$\boxed{(1+x)^{1/2} = 1 + \frac{1}{2}x - 1/8x^2 + 1/16x^3 + \dots}$

(5) $(1+\frac{x}{2})^{-2} = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad m = -2, \quad x = \frac{x}{2}$

$\binom{-2}{1} = -2 \qquad \binom{-2}{2}\ \frac{-2(-2-1)}{2!} = 3 \qquad \binom{-2}{3}\ \frac{-2(-2-1)(-2-2)}{3!} = -4$

$(1+\frac{x}{2})^{-2} = 1 + \binom{-2}{1}(\frac{x}{2}) + \binom{-2}{2}(\frac{x}{2})^2 + \binom{-2}{3}(\frac{x}{2})^3 + \dots$

$\boxed{(1+\frac{x}{2})^{-2} = 1 - x + \frac{3}{4}x^2 - \frac{1}{2}x^3 + \dots}$

(11) $(1+x)^4 = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad m = 4, \quad x = x$

$\binom{4}{1} = 4 \qquad \binom{4}{2} = \frac{4(4-1)}{2!} = 6 \qquad \binom{4}{3}\ \frac{4(4-1)(4-2)}{3!} = 4 \qquad \binom{4}{4} = 1$

$(1+x)^4 = 1 + \binom{4}{1}x + \binom{4}{2}x^2 + \binom{4}{3}x^3 + \binom{4}{4}x^4 + \binom{4}{5}x^5 + \dots$

$\boxed{(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4 + 0}$

(41) $1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$

Known Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\boxed{\begin{array}{l}\text{when } x = 1 \\ e^1 = \sum_{n=0}^{\infty} \frac{1^n}{n!} = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots\end{array}}$$

(45) $\frac{\pi}{3} - \frac{\pi^3}{3^3 \cdot 3!} + \frac{\pi^5}{3^5 \cdot 5!} - \frac{\pi^7}{3^7 \cdot 7!} + \ldots \quad \sum_{n=0}^{\infty} \frac{(-1)^n (\pi)^{2n+1}}{3^{2n+1} \cdot (2n+1)!}$

Known Series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n (x)^{2n+1}}{(2n+1)!}$$

$$\boxed{\begin{array}{l}\text{when } x = \frac{\pi}{3} \\ \sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - \frac{\pi^3}{3^3 \cdot 3!} + \frac{\pi^5}{3^5 \cdot 5!} - \frac{\pi^7}{3^7 \cdot 7!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{3}\right)^{2n+1}}{(2n+1)!}\end{array}}$$

(67) Use eq. $e^{i\theta} = \cos\theta + i\sin\theta$ (Euler's Identity) to write powers of e in $a+bi$ form

a) $e^{-i\pi} = \quad e^{i\theta} = \cos\theta + i\sin\theta, \quad \theta = -\pi$

$e^{-i\pi} = \cos(-\pi) + i\sin(-\pi)$

$e^{-i\pi} = -1 + i(0)$

$\boxed{e^{-i\pi} = -1}$

b) $e^{i\pi/4}, \quad e^{i\theta} = \cos\theta + i\sin\theta, \quad \theta = \pi/4$

$e^{i\pi/4} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\pi/4\right)$

$\boxed{e^{i\pi/4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}(1+i)}$

c) $e^{-i\pi/2} \quad e^{i\theta} = \cos\theta + i\sin\theta, \quad \theta = -\frac{\pi}{2}$

$e^{-i\pi/2} = \cos(-\pi/2) + i\sin(-\pi/2)$

$e^{-i\pi/2} = 0 - i1$

$\boxed{e^{-i\pi/2} = -i1}$

69

Steven Romein

#1, 5, 9, 11, 17, 22, 25

$\frac{67}{69}$ 97%

11.1 Homework

1) $x = 3t, \quad y = 9t^2 \qquad -\infty < t < \infty$

$t = \frac{x}{3}, \quad y = 9\left(\frac{x}{3}\right)^2$

$\boxed{y = x^2}$

5) $x = \cos 2t, \quad y = \sin 2t \qquad 0 \le t \le \pi$

$\cancel{\cos^{-1}(x) = 2t}$

$\cancel{t = \frac{\cos^{-1}(x)}{2}}$

$x^2 = \cos^2 2t \qquad y^2 = \sin^2 2t$

$x^2 + y^2 = \cos^2 2t + \sin^2 2t$

$\boxed{x^2 + y^2 = 1}$

9) $x = \sin t, \quad y = \cos 2t, \quad -\pi/2 \le t \le \pi/2$

$x^2 = \sin^2 t \quad y^2 = \cos^2 2t$

? why not $\longrightarrow$ $x^2 + y^2 = \sin^2 t + \cos^2 2t$

$y = \cos 2t \longrightarrow y = 1 - 2\sin^2 t \longrightarrow$ why this id?

$\boxed{y = 1 - x^2} \longrightarrow$ How?

(11) $x = t^2$, $y = t^6 - 2t^4$, $-\infty < t < \infty$

$y = (\sqrt{x})^6 - 2(\sqrt{x})^4$

$\boxed{y = x^3 - 2x^2}$

(17) $x = -\cosh t$, $y = \sinh t$, $-\infty < t < \infty$

$x^2 = \cosh^2 t$, $y^2 = \sinh^2 t$

$x^2 - y^2 = \cosh^2 t - \sinh^2 t$

$\boxed{x^2 - y^2 = 1}$

(22) the line segment with endpoints $(-1,3)$ & $(3,-2)$

$m = \frac{-2-3}{3+1} = \frac{-5}{4}$

$y - y_1 = m(x - x_1)$

$y + 2 = -\frac{5}{4}(x-3)$ Let $t = x - 3$

$y + 2 = \frac{-5}{4}t \longrightarrow \boxed{y = \frac{-5}{4}t - 2}$

Domain of t

$x = -1$ $x = 3$

$t = -1-3$ $t = 3-3$

$t = -4$ $t = 0$

$-4 \leq t \leq 0$

25 half Line, initial point (2,3) that passes through (-1,-1)

$m = \frac{-1-3}{-1-2} = \boxed{\frac{4}{3}}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{4}{3}(x-2)$ $\quad t = x - 2$

$y - 3 = \frac{4}{3}t$ $\quad$ Domain of t

$\boxed{y = \frac{4}{3}t + 3}$

$x = 2$	$x = -1$
$t = 2-2$	$t = -1-2$
$t = 0$	$t = -3$

$-3 \leq t \leq 0 \longrightarrow$?

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3, 5, 9, 15, 21, 23, 25, ~~27~~, 31, 33

11.2 Homework

(3) $X = 4\sin t$, $Y = 2\cos t$, $t = \pi/4$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{dy}{dt} = -2\sin t \qquad \frac{dx}{dt} = 4\cos t$$

$$\frac{dy}{dx} = \frac{-2\sin t}{4\cos t} = \frac{-1}{2}\tan t \qquad \frac{dy}{dx} = m$$

$$X\left(\tfrac{\pi}{4}\right) = 4\sin\left(\tfrac{\pi}{4}\right) = 4\left(\tfrac{\sqrt{2}}{2}\right) = 2\sqrt{2}$$

$$y\left(\tfrac{\pi}{4}\right) = 2\cos\left(\pi/4\right) = 2\left(\tfrac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$$\frac{dy}{dx}\left(\tfrac{\pi}{4}\right) = -\tfrac{1}{2}\tan\left(\tfrac{\pi}{4}\right) = -\tfrac{1}{2}(1) = -\tfrac{1}{2}$$

$$y - y_1 = m(x - x_1) \rightarrow y - \sqrt{2} = \frac{-1}{2}(x - 2\sqrt{2})$$

$$y - \sqrt{2} = \frac{-1}{2}x + \sqrt{2}$$

$$y = \frac{-1}{2}x + \sqrt{2} + \sqrt{2} \rightarrow \boxed{y = \frac{-1}{2}x + 2\sqrt{2}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\cdot\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{-1}{2}\tan t\right) = \frac{-1}{2}\sec^2 t$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{-\frac{1}{2}\sec^2 t}{4\cos t} = -\frac{1}{8}\sec^3 t$$

$$@\ t = \frac{\pi}{4} = \frac{-1}{8\cos^3\left(\frac{\pi}{4}\right)} = \frac{-1}{8\left(\frac{\sqrt{2}}{2}\right)^3} = \frac{-1}{(\sqrt{2})^3} = \frac{-1}{\sqrt{8}}$$

$$= \frac{-1}{\sqrt{4\cdot 2}} = \frac{-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2\cdot 2} = \boxed{\frac{-\sqrt{2}}{4}}$$

(5) $x = t, \quad y = \sqrt{t}, \quad t = 1/4$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{dy}{dt} = \frac{1}{2\sqrt{t}} \qquad \frac{dx}{dt} = 1$$

$$\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{t}}}{1} = \frac{1}{2\sqrt{t}}$$

$$x\left(\tfrac{1}{4}\right) = \frac{1}{4}$$

$$y\left(\tfrac{1}{4}\right) = \sqrt{\tfrac{1}{4}} = \frac{1}{2}$$

$$\frac{dy}{dx}\left(\tfrac{1}{4}\right) = \frac{1}{2\sqrt{\frac{1}{4}}} = \frac{1}{1} = 1 = m$$

$$y - y_1 = m(x - x_1) \longrightarrow y - \frac{1}{2} = 1\left(x - \frac{1}{4}\right)$$

$$y = x - \frac{1}{4} + \frac{1}{2} \longrightarrow \boxed{y = x + \frac{1}{4}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{dy'}{dt} = \frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{1}{2\sqrt{t}}\right) =$$

$$\frac{d}{dt}\left(\frac{1}{2}(t)^{-1/2}\right) = \frac{-1}{4\sqrt[3]{t^2}} = \boxed{\frac{-1}{4}t^{-3/2} = \frac{d^2y}{dx^2}}$$

$$\frac{d^2y}{dx^2}\left(\tfrac{1}{4}\right) = \frac{-1}{4}\left(\tfrac{1}{4}\right)^{-3/2} = \frac{-1}{4\left(\frac{1}{\sqrt{64}}\right)} = \frac{-1}{\frac{4}{8}} = -\frac{8}{4}$$

$$\boxed{\frac{d^2y}{dx^2}\left(\tfrac{1}{4}\right) = -2}$$

(9) $x = 2t^2 + 3$, $y = t^4$ $t = -1$

$\frac{dx}{dt} = 4t$, $\frac{dy}{dt} = 4t^3$

$\frac{dy}{dx} = \frac{4t^3}{4t} = \frac{dy}{dx} = t^2$

$x(-1) = 2(-1)^2 + 3 = 5$

$y(-1) = (-1)^4 = 1$

$\frac{dy}{dx}(-1) = 4(-1)^2 = 1$

$y - 1 = 1(x - 5) \longrightarrow \boxed{y = x - 4}$

$\frac{d^2y}{dx^2}$ $\frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{dy'}{dt} = \frac{d}{dt} \cdot \frac{dy}{dx} = \frac{d}{dt} t^2 = 2t$

$\frac{d^2y}{dx^2} = \frac{2t}{4t} = \boxed{\frac{1}{2}}$

(15) $x^3 + 2t^2 = 9$, $2y^3 - 3t^2 = 4$, $t = 2$

$3x^2 \frac{dx}{dt} + 4t \frac{dt}{dt} = 0$ $6y^2 \frac{dy}{dt} - 6t = 0$

$3x^2 \frac{dx}{dt} = -4t$ $6y^2 \frac{dy}{dt} = 6t$

$\frac{dx}{dt} = \frac{-4t}{3x^2}$ $\frac{dy}{dt} = \frac{6t}{6y^2}$

$\frac{dy}{dt} = \frac{t}{y^2}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{t}{y^2}}{\frac{-4t}{3x^2}} = \frac{\cancel{t}}{y^2} \cdot \frac{3x^2}{-4\cancel{t}} = \frac{-3x^2}{4y^2}$$

$$x^3 + 2(2)^2 = 9 \rightarrow x^3 = 1 \rightarrow \boxed{x = 1}$$

$$2y^3 - 3(2)^2 = 4 \rightarrow 2y^3 = 16 \rightarrow y^3 = 8 \rightarrow y = 2$$

$$\frac{dy}{dx} = \frac{-3(1)^2}{4(2)^2} = \boxed{\frac{-3}{16}}$$

(21) $x = a(t - \sin t)$, $y = a(1 - \cos t)$, $0 \le t \le 2\pi$

$$A = \int_0^{2\pi} y\,dx \qquad dx = a(1-\cos t)dt$$

$$A = \int_0^{2\pi} a(1-\cos t) \cdot a(1-\cos t)dt$$

$$A = a^2\int_0^{2\pi} (1-\cos t)^2 dt \longrightarrow a^2\int_0^{2\pi}(1 - 2\cos t + \cos^2 t)dt$$

$$a^2\int_0^{2\pi} 1\,dt - a^2\int_0^{2\pi} -2\cos t + a^2\int_0^{2\pi}\cos^2 t\,dt$$

$$a^2(t)_0^{2\pi} + 2a^2(\sin t)_0^{2\pi} + a^2\int_0^{2\pi}\frac{1+\cos(2t)}{2}dt \qquad \begin{matrix} u = 2t \\ du = 2dt \end{matrix}$$

$$a^2(t)_0^{2\pi} + 2a^2(\sin t)_0^{2\pi} + \frac{1}{4}a^2\int_0^{2\pi} 1 + \cos u\,du$$

$$a^2(t)_0^{2\pi} + 2a^2(\sin t)_0^{2\pi} + \frac{1}{4}a^2(t + \sin(2t))_0^{2\pi} \longrightarrow$$

$a^2(t)_0^{2\pi} + 2a^2(\sin t)_0^{2\pi} + \frac{1}{4}a^2(t+\sin(2t))_0^{2\pi}$

$2\pi a^2 + 2a^2(0) + 1/4\, a^2(2\pi+0)$

$2\pi a^2 + 2\pi/4\; a^2 \longrightarrow 2\pi a^2 + \pi/2\, a^2$

$\boxed{2.5\pi a^2}$

23) $X = a\cos t\ ,\ Y = b\sin t\ ,\ 0 \le t \le 2\pi$

$A = \int_0^{2\pi} Y\,dx\ ,\qquad dx = a(-\sin t\,dt)$

$A = \int_{2\pi}^{0} b\sin t \cdot a(-\sin t\,dt) \longrightarrow ab\int_0^{2\pi} -\sin^2 t\,dt$

$A = -ab\int_0^{2\pi} \frac{1-\cos(2t)}{2}\,dt \longrightarrow -\frac{ab}{2}\int_0^{2\pi} 1-\cos(2t)\,dt$

$= -\frac{ab}{2}\int_0^{2\pi} 1\,dt + \frac{ab}{2}\int_0^{2\pi}\cos(2t)\,dt \qquad \begin{array}{l} u = 2t \\ du = 2dt \end{array}$

$-\frac{ab}{2}(t)_0^{2\pi} + \frac{ab}{4}\left[\sin(2t)\right]_0^{2\pi}$

$A = -\frac{ab}{2}(2\pi) + \frac{ab}{4}(0)$

$A = |-\pi ab|$

Cannot have neg area
reverse boundries

(25) $X = \cos t \;, \quad Y = t + \sin t \;, \qquad 0 \leq t \leq \pi$

$$S = \int_a^b \sqrt{(x')^2 + (y')^2}\, dt$$

$$\frac{dx}{dt} = -\sin t \;, \quad \frac{dy}{dt} = 1 + \cos t$$

$$(x')^2 = \sin^2 t \;, \quad (y')^2 = (1 + \cos t)^2$$

$$S = \int_0^\pi \sqrt{\sin^2 t + (1 + \cos t)^2}\, dt$$

$$S = \int_0^\pi (\sin^2 t + 1 + \cos t)\, dt$$

$$S = \int_0^\pi \sin^2 t\, dt + \int_0^\pi 1\, dt + \int_0^\pi \cos t\, dt$$

$$S = \frac{1}{2}\int_0^\pi 1 - \cos 2t\, dt + (t)_0^\pi + (\sin t)_0^\pi$$

$$S = \frac{1}{2}(t - \sin 2t)_0^\pi + \pi + 0$$

$$S = \left(\frac{\pi}{2} - 0\right) + \pi$$

$$\boxed{S = \frac{3\pi}{2}}$$

27) $x = \frac{t^2}{2}$, $y = \frac{(2t+1)^{3/2}}{3}$ $\quad 0 \le t \le 4$

$$S = \int_a^b \sqrt{(x')^2 + (y')^2}\, dt$$

$x' = t$
$(x')^2 = t^2$

$y' = \frac{\cancel{3}}{\cancel{2}} \cdot \frac{(2t+1)^{1/2}}{\cancel{3}} \cdot \cancel{2} = (2t+1)^{1/2}$

$(y')^2 = [(2t+1)^{1/2}]^2 = 2t+1$

$$S = \int_0^4 \sqrt{t^2+2t+1}\, dt$$

$$S = \int_0^4 \sqrt{(t+1)^2}\, dt \longrightarrow \int_0^4 (t+1)\, dt$$

$$S = \int_0^4 t\, dt + \int_0^4 1\, dt$$

$$S = \frac{1}{2}(t^2)_0^4 + (t)_0^4$$

$$S = \frac{1}{2}(16) + 4$$

$$\boxed{S = 12}$$

(31) $X = \cos t, \quad Y = 2 + \sin t, \quad 0 \le t \le 2\pi$

$$SA = \int_a^b 2\pi y \, ds \qquad X' = -\sin t;\ (X')^2 = \sin^2 t$$
$$Y' = \cos t,\ (Y')^2 = \cos^2 t$$

$$ds = \sqrt{(X')^2 + (Y')^2}\, dt \rightarrow ds = \sqrt{\sin^2 t + \cos^2 t} = 1\, dt$$

$$SA = \int_0^{2\pi} 2\pi (2 + \sin t)\, dt \Rightarrow 2\pi \int_0^{2\pi} 2 + \sin t\, dt$$

$$SA = 2\pi \int_0^{2\pi} 2\, dt + 2\pi \int_0^{2\pi} \sin t\, dt$$

$$SA = 2\pi (2t)\Big|_0^{2\pi} + 2\pi(-\cos t)\Big|_0^{2\pi}$$

$$SA = 2\pi(2\pi) + 2\pi(-1-1)$$
$$SA = 4\pi^2 + 2\pi(-2)$$
$$SA = 4\pi^2 - 4\pi$$
$$\boxed{SA = 4\pi(\pi - 1)}$$

33 $x = t + \sqrt{2}$, $y = (t^2/2) + \sqrt{2}t$, $-\sqrt{2} \leq t \leq \sqrt{2}$

$x' = 1$ $\quad y' = t + \sqrt{2}$

$(x')^2 = 1$ $\quad (y')^2 = (t+\sqrt{2})^2$

$$ds = \sqrt{1 + (t+\sqrt{2})^2}\,dt \longrightarrow ds = 1 + t + \sqrt{2}\,dt$$

$$SA = 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} \left(\frac{t^2}{2} + \sqrt{2}t\right)(1 + t + \sqrt{2})\,dt$$

$$SA = 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} (t^2 + \sqrt{t} + 1)\,dt$$

$$SA = 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} t^2 dt + 2\pi \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{t}\,dt + 2\pi \int 1\,dt$$

$$SA = 2\pi\left(\frac{1}{3}t^3\right) + 2\pi\left(\frac{2}{3}t^{3/2}\right) + 2\pi(t)$$

$$\boxed{SA = \frac{52\pi}{3}}$$

Steven Romeiro

#1, 5, 7, 13, 17, 23, 29, 33, 35, 39
43 45, 49, 51, 53, 57, 59, 61, 65

11.3 Homework –

(1) $x = r\cos\theta$ $(3, 0)$

$r^2 = x^2 + y^2 \longrightarrow r = \sqrt{3^2 + 0} \longrightarrow \boxed{r = 3}$

$x = r\cos\theta$

$\theta = \cos^{-1}\left(\frac{3}{3}\right) \longrightarrow \boxed{\theta = 0}$ $\boxed{(3, \theta)}$

c) $(2, 2\pi/3)$

$\boxed{a, e, b, g, c, h, d, f}$

(5) $\left(2, \frac{2\pi}{3}\right)$

$x = r\cos\theta$ $y = r\sin\theta$

$x = 2\cos\left(\frac{2\pi}{3}\right)$ $y = 2\sin\left(\frac{2\pi}{3}\right)$

$x = 2\left(-\frac{1}{2}\right) \longrightarrow \boxed{x = -1}$ $\boxed{y = \sqrt{3}}$

$(-3, \pi)$

$x = r\cos\theta$ $y = r\sin\theta$

$x = -3\cos(\pi)$ $y = -3\sin(\pi)$

$x = -3(-1)$ $y = -3(0)$

$\boxed{x = 3}$ $\boxed{y = 0}$

$(-3, 2\pi)$

$x = r\cos\theta$ $y = r\sin\theta$

$x = -3\cos(2\pi)$ $y = -3\sin(2\pi)$

$x = -3(1)$ $y = -3(0)$

$\boxed{x = -3}$ $\boxed{y = 0}$

$(-2, -\pi/3)$

$x = r\cos\theta$ $\quad$ $y = r\sin\theta$

$x = -2\cos(\frac{-\pi}{3})$ $\quad$ $y = -2\sin(-\frac{\pi}{3})$

$x = -2(\frac{1}{2})$ $\quad$ $y = -2(-\frac{\sqrt{3}}{2})$

$\boxed{x = -1}$ $\quad$ $\boxed{y = \sqrt{3}}$

7) $0 \leq \theta \leq 2\pi$ $\quad$ $r \geq 0$

a) $(1,1)$

$r^2 = x^2 + y^2$ $\quad$ $\tan\theta = \frac{1}{1}$

$r = \sqrt{1+1}$ $\quad$ $\theta = \tan^{-1}(1)$

$\boxed{r = \sqrt{2}}$ $\quad$ $\boxed{\theta = \pi/4}$

b) $(-3, 0)$

$r = \sqrt{9+0}$ $\quad$ $\tan\theta = \frac{0}{-3}$ $\quad$ $x = r\cos\theta$

$\boxed{r = 3}$ $\quad$ $\theta = \tan^{-1}(0)$ $\quad$ $\theta = \cos^{-1}(\frac{x}{r})$

$\theta = 0$ $\quad$ $\boxed{\theta = \pi}$

c) $(\sqrt{3}, -1)$

$r = \sqrt{3+1}$ $\quad$ $x = r\cos\theta$

$r = \sqrt{4}$ $\quad$ $\theta = \cos^{-1}(\frac{x}{r})$

$\boxed{r = 2}$ $\quad$ $\theta = \cos^{-1}(\frac{\sqrt{3}}{2}) = \boxed{\pi/6}$

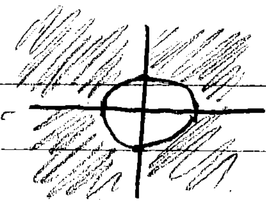
d) $(-3, 4)$

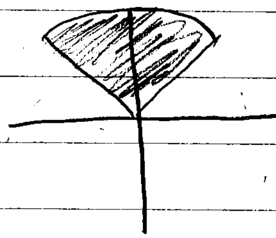
$r = \sqrt{9+16}$ $\quad$ $x = r\cos\theta$

$\boxed{r = 5}$ $\quad$ $\theta = \cos^{-1}(\frac{x}{r})$

$\boxed{\theta = \cos^{-1}(-\frac{3}{5})}$

17

13 

23 

29 $r\sin\theta = 0$

$y = r\sin\theta \quad \therefore \quad \boxed{y = 0}$

33 $r\cos\theta + r\sin\theta = 1$

$\boxed{x + y = 1}$ $\quad x^2 + y^2 = r^2$

$\boxed{x = 1}, \boxed{y = 1}$

35 $r^2 = 1$

$r^2 = x^2 + y^2$

$\boxed{x^2 + y^2 = 1}$

39 $r = \cot\theta \csc\theta$

$$r = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta}$$

$$r = \frac{\cos\theta}{\sin^2\theta}$$

$r\sin^2\theta = \cos\theta$

$r^2\sin^2\theta = r\cos\theta$

$y^2 = x \longrightarrow \boxed{y = \sqrt{x}}$

43

45 $r^2 = -4r\cos\theta$

$r^2 = -4x$

$x^2+y^2 = r^2 \longrightarrow x^2+y^2 = -4x$

$x^2+4x+y^2=0$

$x^2+4x+\_\_\_ + y^2 = 0$

$\hookrightarrow$ Complete the Square

$x^2+4x+4+y^2=4$

$$\boxed{(x+2)^2+y^2=4}$$

49 $r = 2\cos\theta + 2\sin\theta$

$r^2 = 2r\cos\theta + 2r\sin\theta$

$r^2 = 2x+2y$

$x^2+y^2=r^2$

$x^2+y^2 = 2x+2y$

$x^2-2x+\_\_\_ + y^2-2y+\_\_\_ = 0$

$x^2-2x+1+y^2-2y+1 = 1+1$

$$\boxed{(x-1)^2+(y-1)^2=2}$$

51 $r\sin\left(\theta + \frac{\pi}{6}\right) = 2$

$\sin(\theta+\pi/6) = (\sin\theta\cos\pi/6 + \cos\theta\sin\frac{\pi}{6})r = 2$

$\frac{\sqrt3}{2}r\sin\theta + \frac12 r\cos\theta = 2$

$\frac{\sqrt3}{2}y + \frac12 x = 2 \longrightarrow \sqrt3 y + x = 4$

$\sqrt3 y = -x+4$

$$\boxed{y = \frac{-x}{\sqrt3} + \frac{4}{\sqrt3}}$$

53 $x=7$

$x=r\cos\theta$

$\boxed{7=r\cos\theta}$

57 $x^2+y^2=4$

$x^2+y^2=r^2$

$r^2=4$

$\boxed{r=2}$ or $\boxed{r=-2}$

59 $\frac{x^2}{9}+\frac{y^2}{4}=1$

$r=6$

$4x^2+9y^2=36$

$4x^2+9y^2=36$

$\boxed{4r^2\cos^2\theta+9r^2\sin^2\theta=6^2}$

61 $y^2=4x$

$y^2=4r\cos\theta$

$r^2\sin^2\theta=4r\cos\theta$

$r\sin^2\theta=4\cos\theta$

$r=4\frac{\cos\theta}{\sin^2\theta}$

$\boxed{r=4\cot\theta\csc\theta}$

(65) $(x-3)^2+(y+1)^2=4$

$x^2-6x+9+y^2+2y+1=4$

$x^2-6x+y^2+2y+10=4$

$r^2\cos^2\theta-6r\cos\theta+r^2\sin^2\theta+2r\sin\theta+6=0$

$\cancel{r\cos^2\theta-6\cos\theta+r\sin^2\theta+2\sin\theta+6=0}$

$r^2(\cos^2\theta+\sin^2\theta)-6r\cos\theta+2r\sin\theta=-6$

$r^2(1)-6r\cos\theta+2r\sin\theta=-6$

$\boxed{r^2=6r\cos\theta-2r\sin\theta-6}$

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1, 5, 9, 13, 17, 21, 25, 29

11.4 Homework

1) $r = 1 + \cos\theta$

* X-axis Sym $(r, -\theta)$

$r(\frac{\pi}{3}) = 1 + \cos(\frac{\pi}{3}) = \boxed{3/2}$, $r(-\frac{\pi}{3}) = 1 + \cos(-\frac{\pi}{3}) = \boxed{3/2}$

Y-axis Sym $(r, \pi - \theta)$

$r(\frac{\pi}{3}) = 3/2$, $r(\pi - \frac{\pi}{3}) = 1 + \cos(\pi - \frac{\pi}{3}) = \frac{1}{2}$

Origin-axis Sym $(r, \pi + \theta)$

$r(\frac{\pi}{3}) = 3/2$, $r(\pi + \pi/3) = 1 + \cos(\pi + \frac{\pi}{3}) = \frac{1}{2}$

5) $r = 2 + \sin\theta$

X-axis Sym $(r, -\theta)$

$r(\pi/6) = 2 + \sin(\pi/6) = 5/2$, $r(-\pi/6) = 2 + \sin(-\pi/6) = 3/2$

* Y-axis Sym $(r, \pi - \theta)$

$r(\pi/6) = \boxed{5/2}$, $r(\pi - \pi/6) = 2 + \sin(\pi - \pi/6) = \boxed{5/2}$

Origin-axis Sym $(r, \pi + \theta)$

$r(\pi/6) = 5/2$, $r(\pi + \pi/6) = 2 + \sin(\pi + \pi/6) = 3/2$

9) $r^2 = \cos\theta \longrightarrow r = \pm\sqrt{\cos\theta}$

* X-axis Sym $(r, -\theta)$

$r(\pi/3) = \sqrt{\cos(\pi/3)} = \boxed{\frac{\sqrt{2}}{2}}$, $r(-\frac{\pi}{3}) = \sqrt{\cos(-\pi/3)} = \boxed{\frac{\sqrt{2}}{2}}$

Y-axis Sym $(r, \pi - \theta)$

$r(\pi/3) = \frac{\sqrt{2}}{2}$, $r(\pi - \frac{\pi}{3}) = \sqrt{\cos(\pi - \frac{\pi}{3})} =$ imaginary

Origin-axis Sym $(r, \pi + \theta)$

$r(\pi/3) = \frac{\sqrt{2}}{2}$, $r(\pi + \pi/3) = \sqrt{\cos(\pi + \pi/3)} =$ Imaginary

?

(13) $r^2 = 4\cos 2\theta \longrightarrow r = 2\sqrt{\cos 2\theta}$

* x-axis Sym $(r, -\theta)$

$r(\pi/3)^2 = 4\cos(\frac{2\pi}{3}) = \boxed{-2}$, $r(\frac{-\pi}{3})^2 = 4\cos(\frac{-2\pi}{3}) = \boxed{-2}$

y-axis Sym $(r, \pi - \theta)$ or $(-r, -\theta)$

$r(\pi/3)^2 = -2$, $r(\pi - \pi/3)^2 = 4\cos(\pi - \frac{2\pi}{3}) = 2$

Origin Symm $(r, \pi + \theta)$ or $(-r, \theta)$

$r(\pi/3)^2 = -2$, $r(\pi + \pi/3)^2 = 4\cos(\pi + 2\pi/3) = 2$

* $r(\pi - \pi/3)^2 = 4\cos(2\pi - 2\pi/3) = \boxed{-2}$

* $r(\pi + \pi/3)^2 = 4\cos(2\pi + 2\pi/3) = \boxed{-2}$

(17) $r = -1 + \cos\theta$; $\theta = \pm \pi/2$

$$\text{Slope} = m = \frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$r' = -\sin\theta$

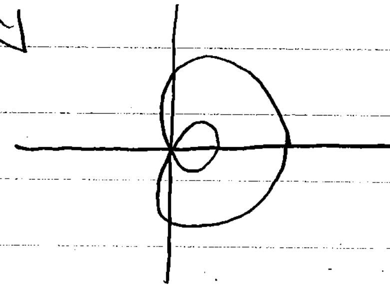
$r'(\pm\pi/2) = -\sin\frac{\pm\pi}{2} = \pm 1$

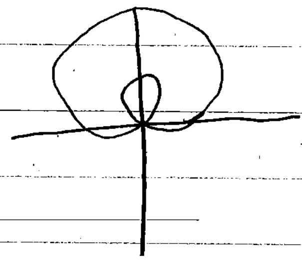
$r(\pm\pi/2) = -1 + \cos(\pm\pi/2) = -1$

$$\frac{dy}{dx} = \frac{\pm 1(\pm 1) + (-1)(0)}{\pm 1(0) - (-1)(\pm 1)} = \frac{1}{\pm 1} = \boxed{\pm 1}$$

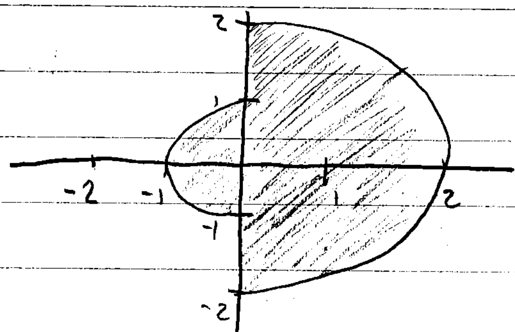
21 a) $r = \frac{1}{2} + \cos\theta$

b) $r = \frac{1}{2} + \sin\theta$



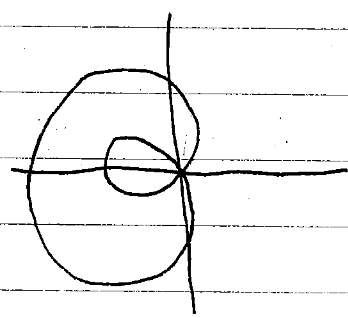


? 25 $-1 \leq r \leq 2$ & $-\pi/2 \leq \theta \leq \pi/2$



? 29 $r = 1 - \cos\theta$

a) $r = -1 - \cos\theta$, b) $r = 1 + \cos\theta$



Steven Romeiro

1, 5, 9, 13, 17, 21, 25

11.5 Homework

(1) $r = \theta$, $0 \le \theta \le \pi$

$$A = \int_0^{\pi} \frac{1}{2}\theta^2 \, d\theta \rightarrow \frac{1}{2}\int_0^{\pi} \theta^2 \, d\theta = \boxed{\frac{1}{6}\pi^3 \text{ or } 5.168}$$

(5) $r = \cos 2\theta$

$$A = \frac{1}{2}\int_0^{2\pi} \cos^2(2\theta) \qquad \begin{matrix} u = 2\theta \\ du = 2d\theta \end{matrix}$$

$$A = \frac{1}{4}\int_0^{2\pi} \cos^2 u \, du \longrightarrow \frac{1}{4}\int_0^{2\pi} \frac{1}{2}(1+\cos 2u)$$

$A = \frac{\pi}{2}$ for all four

$A = \frac{\frac{\pi}{2}}{4} = \boxed{\frac{\pi}{8} \text{ for one petal}}$

How to pick r_1 or r_2

(9) $r = 2\cos\theta$ & $r = 2\sin\theta$

$2\cos\theta = 2\sin\theta$

$$\frac{2\sin\theta}{2\cos\theta} = 1 \longrightarrow \tan\theta = 1 \rightarrow \theta = \pi/4$$

$$A = \int_0^{\pi/4} \frac{1}{2}\left[(2\cos\theta)^2 - (2\sin\theta)^2\right]$$

$\boxed{A = 1 \text{ or } -1 \text{ ?}}$

(13) $r^2 = 6\cos 2\theta$, $r = \sqrt{3}$

$r^2 = 3$

? Not Same Area

$6\cos 2\theta = 3$

$\cos 2\theta = \frac{1}{2} \longrightarrow 2\theta = \cos^{-1}(\frac{1}{2})$

$2\theta = \frac{\pi}{3}$ & $\frac{5\pi}{3}$ $\quad \theta = \frac{\pi}{6}$ & $\frac{5\pi}{6}$

wrong limits its from $0 \to \pi/6$ then * 4 for all petals

$A = \int_{\pi/6}^{5\pi/6} \frac{1}{2}(r_2^2 - r_1^2)\,d\theta \longrightarrow \int_{\pi/6}^{5\pi/6} \frac{1}{2}(6\cos 2\theta - 3)\,d\theta$

$\boxed{A = 5.7397}$

(17) Inside $r = 4\cos\theta$, right of $r = \sec\theta$

$4\cos\theta = \sec\theta \longrightarrow 4\cos\theta = \frac{1}{\cos\theta}$

$4\cos^2\theta = 1 \longrightarrow 2\cos\theta = 1$

$\cos\theta = \frac{1}{2} \longrightarrow \theta = \pi/3,\ 5\pi/3 = $ Only $\pi/3$

?

$\cancel{A = \int_{5\pi/3}^{\pi/3} \frac{1}{2}(16\cos^2\theta - \sec^2\theta)\,d\theta}$

Cant divide by zero?

$A =$

$2\int_0^{\pi/3} \frac{1}{2}\,(4\cos^2\theta) -$

(21) $r = \theta^2, \quad 0 \le \theta \le \sqrt{5}$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \qquad \frac{dr}{d\theta} = 2\theta$$

$$L = \int_0^{\sqrt{5}} \sqrt{\theta^4 + 4\theta^2}\, d\theta \rightarrow \boxed{L = 6.333}$$

(25) $r = \dfrac{6}{1+\cos\theta} \qquad 0 \le \theta \le \pi/2$

$$\frac{dr}{d\theta} = \frac{(1+\cos\theta)0 - 6(-\sin\theta)}{(1+\cos\theta)^2} = \frac{6\sin\theta}{(1+\cos\theta)^2}$$

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{\pi/2} \sqrt{\left(\frac{6}{1+\cos\theta}\right)^2 + \left(\frac{6\sin\theta}{(1+\cos\theta)^2}\right)^2}\, d\theta$$

$$\boxed{L = 6.8868}$$

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\# 11, 12, 19, 23, 29, 31, 43, 57, 59, 65

11.6 Homework

(11) $x^2 = -8y$ → Parabola opens down

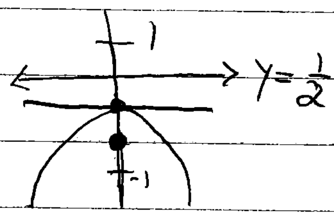
Find p

Vertex $= (0,0)$ $\quad -8 = 4p \rightarrow p = -\frac{1}{2}$

Focus $(0, 0+p)$

$(0, 0-\frac{1}{2}) \rightarrow (0, -\frac{1}{2})$

directrix $=$ $y = 0 - p \rightarrow y = +\frac{1}{2}$



(12) $y^2 = -2x$ → half-parabola opens left

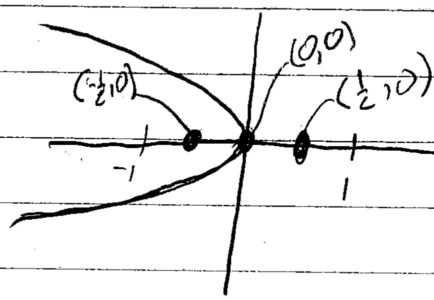
Finp P $=$ $4p = -2 \rightarrow p = -\frac{1}{2}$

Vertex $(0,0)$, Focus $(0+P, 0)$

$(-\frac{1}{2}, 0)$

Directrix $=$ $x = 0 - P$

$x = 0 - (-\frac{1}{2}) \rightarrow x = \frac{1}{2}$



(19) $2x^2 + y^2 = 2 \rightarrow x^2 + \frac{y^2}{2} = 1$

$a^2 = 2$ $\therefore$ major axis is along y-axis

Vertices of major axis $= a = \pm\sqrt{2}$

$(0, \sqrt{2}+0)$ & $(0, -\sqrt{2}+0)$ $=$ $(0, \sqrt{2})$ & $(0, -\sqrt{2})$

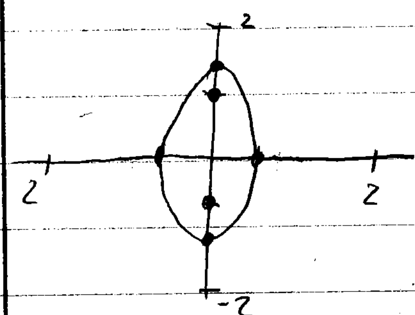
Vertices of minor axis $= b^2 = 1 = b = \pm 1$

$(0+1, 0)$ & $(0-1, 0)$ $=$ $(1, 0)$ & $(-1, 0)$

Focci $a^2 - b^2 = c^2$

$c^2 = 2 - 1 \longrightarrow c^2 = 1 \rightarrow c = \pm 1$

$(0, 0+1)$ & $(0, 0-1) \longrightarrow (0, 1)(0, -1)$



(23) $6x^2 + 9y^2 = 54 \rightarrow \frac{x^2}{9} + \frac{y^2}{6} = 1$

$a^2 = 9$ $\therefore$ major axis is X-axis.

Vertices of major axis $a = \pm 3$

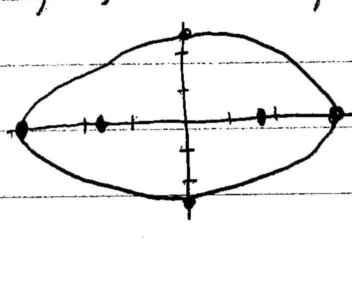
$(0+3, 0)$ & $(0-3, 0)$ $=$ $(3, 0)$ & $(-3, 0)$

Vertices of minor axis $b = \pm\sqrt{6}$

$(0, 0+\sqrt{6})$ & $(0, 0-\sqrt{6}) =$ $(0, \sqrt{6})$ & $(0, -\sqrt{6})$

Focci $c^2 = a^2 - b^2 \longrightarrow c^2 = 9 - 6 \longrightarrow c = \pm\sqrt{3}$

$(0+\sqrt{3}, 0)$ & $(0-\sqrt{3}, 0) \longrightarrow (\sqrt{3}, 0)$ & $(-\sqrt{3}, 0)$



(29) $y^2 - x^2 = 8 \rightarrow \frac{y^2}{8} - \frac{x^2}{8} = 1$

Hyperbola that opens lef/light (0,0)

$a^2 = 8 = a = \pm 2\sqrt{2}$

Vertices of transverse axis

$(0, 0+a)$ & $(0, 0-a) = (0, 2\sqrt{2})$ & $(0, -2\sqrt{2})$

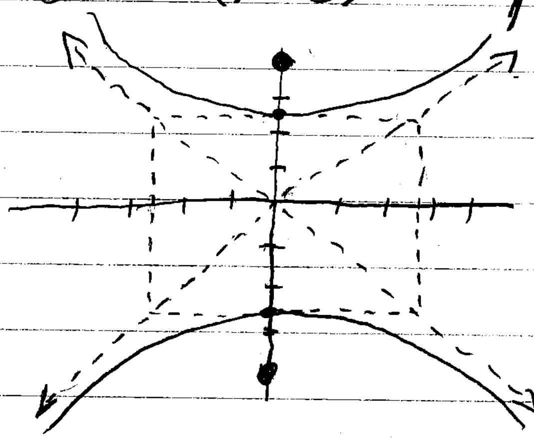
Focci: $b^2 = 8 = b = \pm 2\sqrt{2}$ $(2\sqrt{2}, 0)$ & $(-2\sqrt{2}, 0)$

$c^2 = a^2 + b^2 \rightarrow c^2 = 16 \rightarrow c = \pm 4$

$(0, 0+4)$ & $(0, 0-4) = (0, 4)$ & $(0, -4)$

Assymptotes: $y - y_1 = \pm a/b (x - x_1)$

$y - 0 = \pm 1(x - 0) \rightarrow y = \pm x$



(31) $8x^2 - 2y^2 = 16 \rightarrow \frac{x^2}{2} - \frac{y^2}{8} = 1$

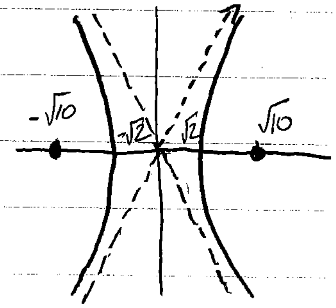
opens up/down center (0,0)

Vertex: $a^2 = 2 = a = \pm\sqrt{2}$ $\therefore$ $(\sqrt{2}, 0)$ & $(-\sqrt{2}, 0)$

Focci: $b^2 = 8 = b = \pm 2\sqrt{2}$

$c^2 = a^2 + b^2 \rightarrow c^2 = 10 \rightarrow c = \pm\sqrt{10}$

Assymptote: $y - 0 = \pm 4(x - 0) \rightarrow y = \pm 4x$

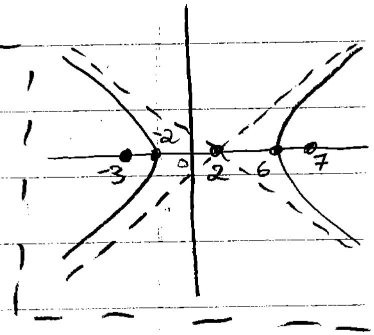


(43) $\frac{(x-2)^2}{16} - \frac{y^2}{9} = 1$ opens up/down center $(2,0)$

Vertices of Transverse: $a = \pm 4$ ∴ $(6,0)$ & $(-2,0)$

Focci: $b = \pm 3$; $c^2 = 16 + 9 \rightarrow c = \pm 5$ ∴ $(7,0)$ & $(-3,0)$

Assym: $y - 0 = 3/4\,(x-2) \rightarrow y = \pm 3/4\,(x-2)$



(57) $x^2 + 4x + y^2 = 12$

$x^2 + 4x + 4 + y^2 = 12 + 4$

$(x^2+2)^2 + y^2 = 16$ Circle center $(-2,0)$

radius = 4

(59) $x^2 + 2x + 4y - 3 = 0$

$x^2 + 2x + 1 + 4y = 3 + 1$

$(x+1)^2 = -4y + 4 \rightarrow (x+1)^2 = -4(y-1)$

Parabola opens up center $(-1,1)$

Find p:

$4p = 4y \rightarrow 4p = 4 \rightarrow p = 1$

Focus: $(-1+p, 1) \rightarrow (-1+1, 1) \rightarrow (0,1)$

(65) $x^2 - y^2 - 2x + 4y = 4 \rightarrow x^2 - 2x + 1 - (y^2 - 4y + 4) = 4 - 4 + 1$

$(x-1)^2 + (y-2)^2 = 1$

Circle with center at $(1,2)$

radius of 1

Steven Romeiro

\# 29, 31, 33, 35, 37, 39, 41, 43

11.7 Homework

(29) $e = 1$, $x = 2 \longrightarrow$ directrix

$\therefore k = 2$ $\quad$ & $\quad r = \dfrac{ke}{1 + e\cos\theta}$

$$r = \frac{(2)(1)}{1 + 1\cos\theta} = \boxed{r = \frac{2}{1 + \cos\theta}}$$

(31) $e = 5$, $y = -6 \longrightarrow$ directrix

$\therefore k = 6$ $\quad$ $r = \dfrac{ke}{1 - e\sin\theta}$

$$r = \frac{(6)(5)}{1 - 5\sin\theta} \longrightarrow \frac{(6)(\cancel{5})}{\cancel{5}(\frac{1}{5} - \sin\theta)} \longrightarrow \boxed{r = \frac{6}{\frac{1}{5} - \sin\theta}}$$

(33) $e = 1/2$, $x = 1 \longrightarrow$ directrix

$\therefore k = 1$ $\quad$ $r = \dfrac{ke}{1 + e\cos\theta}$

$$r = \frac{(1)(1/2)}{1 + 1/2\cos\theta} \longrightarrow r = \frac{(1)(\cancel{1/2})}{\cancel{1/2}(2 + \cos\theta)}$$

$$\boxed{r = \frac{1}{2 + \cos\theta}}$$

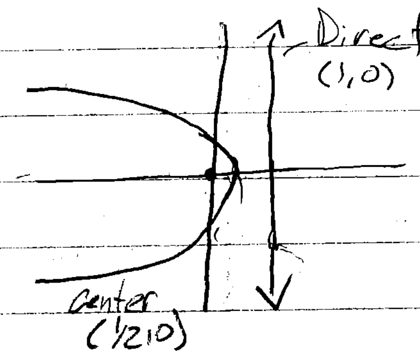
(35) $e = 1/5$, $y = -10 \longrightarrow$ directrix

$\therefore K = 10$, $r = \dfrac{Ke}{1 - e\sin\theta}$

$r = \dfrac{(10)(1/5)}{1 - 1/5 \sin\theta} \longrightarrow \dfrac{(10)(\cancel{1/5})}{(\cancel{1/5})(5 - \sin\theta)} = \boxed{r = \dfrac{10}{5 - \sin\theta}}$

(37) $r = \dfrac{1}{1 + \cos\theta}$

$Ke = 1$, $K = 1$ & $e = 1$



?

(39) $r = \dfrac{25}{10 - 5\cos\theta} = \dfrac{25}{10(1 - \frac{1}{2}\cos\theta)} = \dfrac{5}{2(1 - \frac{1}{2}\cos\theta)} = \dfrac{2.5}{1 - 1/2\cos\theta}$

$Ke = 2.5$

$e = 1/2$

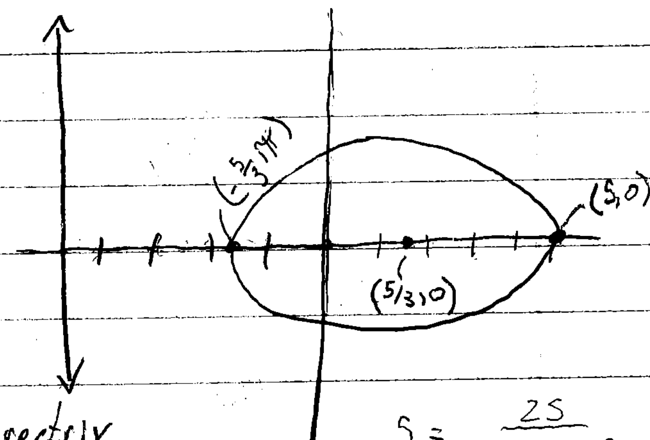
$K(1/2) = 2.5 \longrightarrow K = 5$

directrix $= x = -5$

$Ke = a(1 - e^2)$

$2.5 = a(1 - 1/2^2) = 2.5 = a(3/4)$

$a = \frac{5}{2} \cdot \frac{4}{3} = \boxed{\frac{10}{3}}$



directrix

(-5,0)

$5 = \dfrac{25}{10 - 5\cos\theta}$

$10 - 5\cos\theta = 5$

$2 - \cos\theta = 1 \longrightarrow \cos\theta = 1$

$\theta = 0$ & $\theta = \pi$

?

$\dfrac{a}{e} = \dfrac{10/3}{1/2} = \dfrac{10}{3} \cdot 2 = \dfrac{20}{3}$

$K - \frac{a}{e}$ or $\frac{a}{e} - K$

center: $\dfrac{a}{e} - K = \dfrac{20}{3} - 5 = \boxed{\dfrac{5}{3}}$

vertices: $\frac{5}{3} \pm a \longrightarrow 5/3 \pm \frac{10}{3} \longrightarrow (5,0)$ & $(-5/3, 0)$

(41) $r = \dfrac{400}{16+8\sin\theta} \longrightarrow \dfrac{400}{16(1+\frac{1}{2}\sin\theta)} \longrightarrow r = \dfrac{25}{1+\frac{1}{2}\sin\theta}$

$e = 1/2$ ellipse open up/down

$ke = 25 \rightarrow k(\frac{1}{2}) = 25 \rightarrow k = 50$ & direc: $y = 50$

$ke = a(1-e^2) \rightarrow 25 = a(1-\frac{1}{2}^2) \rightarrow 25 = a\,\frac{3}{4} \rightarrow a = \frac{100}{3}$

$\frac{a}{e} = \dfrac{100/3}{1/2} = \frac{100}{3}\cdot 2 \rightarrow \frac{a}{e} = \frac{200}{3}$

Center: $\frac{a}{e} - k = \frac{200}{3} - 50 \rightarrow \frac{50}{3}$

radius from focus/pole to center: $ea \rightarrow \frac{1}{2}\cdot\frac{100}{3} \rightarrow ea = \frac{50}{3}$

? angle for center $\rightarrow r = \frac{400}{16+8\sin\theta} \rightarrow \frac{50}{3} = \frac{400}{16+8\sin\theta} \rightarrow 50(16+8\sin\theta) = 1200$

$16+8\sin\theta = 24 \rightarrow 2+\sin\theta = 3 \rightarrow \sin\theta = 1 \rightarrow \theta = \frac{3\pi}{2}$?

Vertices: Center $\pm a$

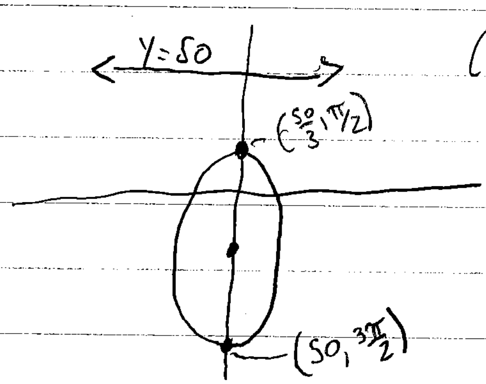
$\frac{50}{3} + \frac{100}{3} = \frac{150}{3} = 50$ & $\frac{50}{3} - \frac{100}{3} = \frac{-50}{3}$

$50 = \dfrac{400}{16+8\sin\theta}$ | $\frac{50}{3} = \dfrac{400}{16+8\sin\theta}$

$50(16+8\sin\theta) = 400$
$16+8\sin\theta = 8$
$2+\sin\theta = 1$
$\sin\theta = -1$
$\theta = \frac{3\pi}{2}$
$(50, \frac{3\pi}{2}$

?

$50(16+8\sin\theta) = 1200$
$16+8\sin\theta = 24$
$2+\sin\theta = 3$
$\sin\theta = 1$
$\theta = \pi/2$
$(-\frac{50}{3}, \frac{\pi}{2})$



(43) $r = \dfrac{8}{2-2\sin\theta} \rightarrow \dfrac{8}{2(1-\sin\theta)} \longrightarrow r = \dfrac{4}{1-\sin\theta}$

$ke = 4$, $e = 1$

? can't use

$k(1) = 4 \rightarrow k = 4$: directrix $= y = -4$

$ke = a(1-e^2) \rightarrow 4 = a(1-1^2) = 4 = a(0)$

Vertex: half-way point b/w focus & directrix

vertex: $(0,-2)$ cartesian

$2 = \dfrac{8}{2-2\sin\theta} \longrightarrow 2(2-2\sin\theta) = 8$

$2-2\sin\theta = 4 \rightarrow 1-\sin\theta = 2$

$\sin\theta = -1 \longrightarrow \theta = \frac{3\pi}{2} \quad (2, \frac{3\pi}{2})$