48 T1 SU10 **Name** _____

For credit, all work must be shown clearly.

1a) What is the SI unit for mass?

kg

1b) What are the dimensions of C if $C = \frac{B}{2VL^2}$

(where B is a mass, L is a length and V is a velocity).

$$\frac{[M]}{\frac{[L]}{[T]} \cdot [L]^2} = \frac{[M]}{[L]^2} \cdot \frac{[T]}{[L]} = \frac{[M][T]}{[L]^3}$$

1c) Does it ever make sense for you to divide parameters with different dimensions?

Yes $\frac{[L]}{[T]}$ = Velocity

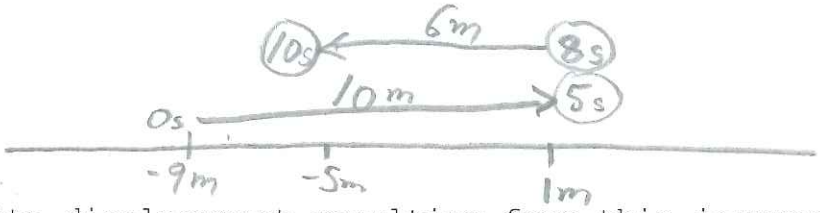
1d) Add these two numbers, quoting your answer with the appropriate rounding and significant digits. (4.326m and 1.3m)

$\boxed{5.6\,m}$

1e) Convert 3.4 naps/jup into cm/min. (1jup=7s; 5nap=8cm)

$$\frac{3.4\,naps}{jup} \cdot \frac{1\,jup}{7\,s} \cdot \frac{8\,cm}{5\,naps} \cdot \frac{60\,s}{1\,min} = \boxed{46.6\,\frac{cm}{min}}$$

2a) An object starts at position -9m. It moves to the right at 2m/s for 5s. It is then stationary for 3s. It then moves 6m to the left in another 2s. (Right is positive and left is negative).



2ai) What is its displacement resulting from this journey?

$\Delta X = X_f - X_i = -5m - (-9m) = \boxed{\Delta X = 4m}$

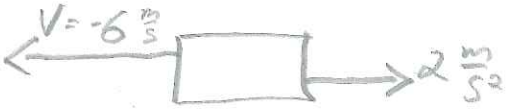
2aii) What is its position (address) at the end of the journey?

position = -5m

2aiii) What is its average velocity during the last 5s of this journey?

$\bar{V} = \frac{\Delta X}{\Delta T} = \frac{X_f - X_i}{T_f - T_i} = \frac{-5m - 1m}{10s - 5s} = \frac{-6m}{5s} = \boxed{\bar{V} = -1.2\frac{m}{s}}$

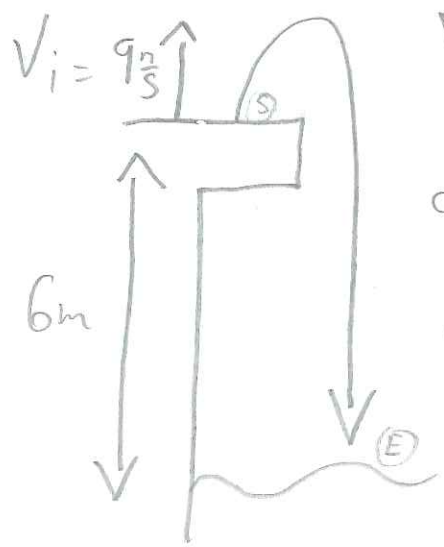
2b) A particle is passing +6m. It has an instantaneous velocity of -3m/s. It has an instantaneous acceleration of +2m/s². What is its direction of travel and is it speeding up or slowing down?



Neg direction

Slowing down

2c) A student jumps vertically up, off of a diving board, with a speed of 9m/s. The diving board is 6m above the water.
What is her hang time for this journey?



$V_i = 9\frac{m}{s}$

$\Delta X = -6m$

$a = -10\frac{m}{s^2}$

$\Delta T = ?$

$\Delta X = V_i \cdot \Delta T + \frac{1}{2} a \Delta T^2$

$-6 = 9T - 5T^2 \longrightarrow -5T^2 + 9T + 6 = 0$

$T = \frac{-9 \pm \sqrt{81 - 4(-5)(6)}}{2(-5)} = \frac{-9 \pm \sqrt{201}}{-10}$ $T = -.518$ $\boxed{T = 2.318}$

$\Delta V = a \cdot \Delta T = V_f - V_i = a\Delta T = V_f = V_i + a\Delta T$

$V_f = 9 + (-10)(\Delta T) \longrightarrow \sqrt{201} = 9 - 10T$

$V_f^2 = V_i^2 + 2a\Delta X$

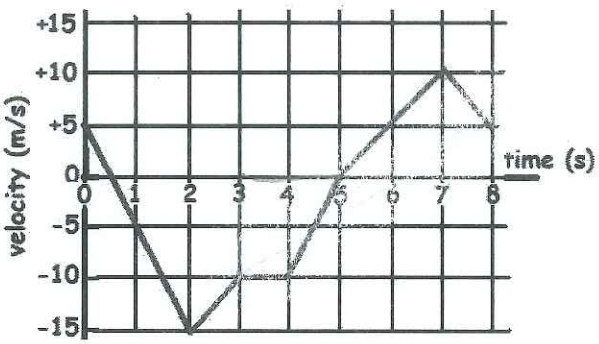
$V_f^2 = 81 + 120$

$V_f = \sqrt{201}$

$T = \frac{9 \pm \sqrt{201}}{-10}$

$T =$

2d) Consider the graph, which depicts the journey of a particle that is moving on the X-axis.



2di) In which direction (positive or negative) is the particle traveling at t=4.5s?

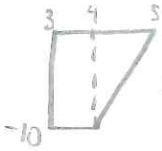
Negative

2dii) What is the particle's average acceleration between t=3s and t=8s?

$\bar{a} = \frac{\Delta V}{\Delta T} = \frac{5-(-10)}{8-3} = \frac{15}{5} = \boxed{3 \frac{m}{s^2}}$

2diii) When does the particle first change direction?

$\boxed{0.5s}$



$= (-10 \cdot 1)$

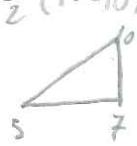
$+ \frac{1}{2}(1 \cdot -10)$

$= -15$

$+$

10

2div) What is the particle's displacement between t=3s and t=7s?



$\frac{1}{2}(2 \cdot 10) =$ $\boxed{= -5m}$

2e) A particle moves along the x axis according to the equation: $x=3t^2+4t-8$ m (where x is in meters and t is in seconds).

2ei) Using calculus determine the objects position at t=3s.

$3(9)+4(3)-8$ $\boxed{X = 31m}$

2eii) Using calculus determine the objects instantaneous velocity at t=3s.

$X' = 6t+4$ @ $t=3s$ $X' = 6(3)+4$ $\boxed{X' = 22 \frac{m}{s}}$

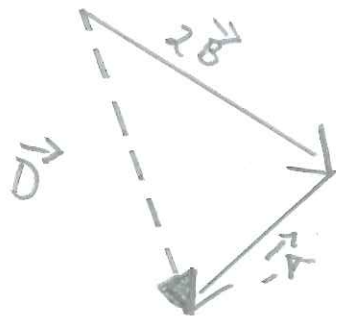
2eiii) Using calculus determine the objects instantaneous acceleration at t=3s.

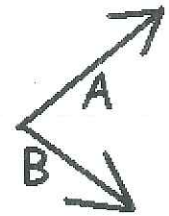
$\boxed{X'' = 6 \frac{m}{s^2}}$

3a) Circle the vectors in the following list?

velocity, displacement, time, temperature, mass

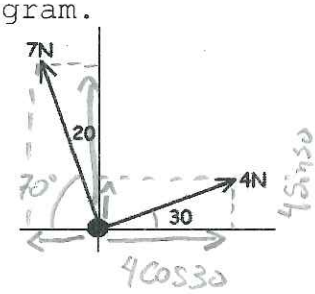
3b) Draw a vector diagram (to scale using a straight edge) to determine **D** if **D**=2**B**-**A**. Clearly indicate vector **D** in your diagram.





3c) Two forces act on particle P as shown in the vector diagram.

3ci) Express the 7N vector in 'ijk' vector notation.



$-7\cos 70°\,\hat{i} + 7\sin 70°\,\hat{j} + 0\hat{k}$

$\boxed{(-2.394\hat{i} + 6.578\hat{j} + 0k)N}$

3cii) Using 'ijk' vector notation, determine the resultant force that results from adding these two forces together.

$(4\cos 30\,\hat{i} + 4\sin 30\,\hat{j} + 0\hat{k})$

$= (3.464\hat{i} + 2.000\hat{j} + 0\hat{k})N \rightarrow$

i	j	k
-2.394	6.578	0
3.464	2.000	0

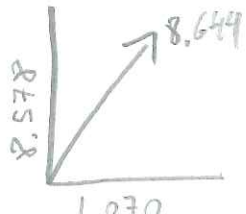
$\boxed{(1.070\hat{i} + 8.578\hat{j} \quad 0\hat{k})N}$

3ciii) Determine the magnitude of the resultant vector.

$Mag = \sqrt{(1.070)^2 + (8.578)^2} \rightarrow mag = \sqrt{74.727}$

$\boxed{mag = 8.644\ N}$

3ciiii) Determine the direction of the resultant expressed as an angle to the positive X-axis.

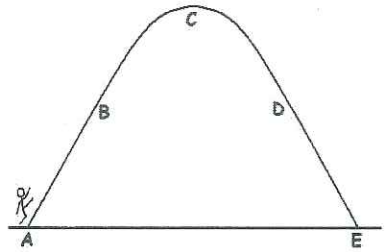


$8.644\cos\theta = 1.070$

$\theta = \cos^{-1}\left(\frac{1.070}{8.644}\right)$

$\boxed{\theta = 83°}$

4a) Consider the diagram. It represents the freefall path of a ball that has been kicked on a level field. The journey starts at point A and ends at point E. The ball was kicked from A at a velocity of 9m/s at and angle of 40° elevation above the horizontal. Points B and D are level with each other and at half the maximum height. Air resistance can be ignored.



4ai) What is the ball's horizontal component of velocity at D?

$9 \cos 40$

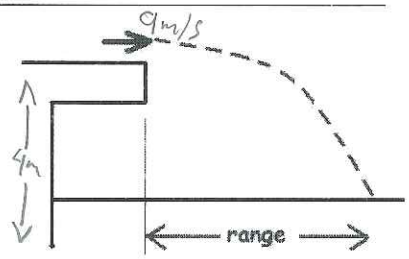
4aii) What is the ball's horizontal component of acceleration at B?

0

4aii) At which point does the ball have least speed?

At $C = 0$

4b) A ball is rolling with a horizontal speed of 9m/s. It rolls off a diving board that is 4m above the water. Using some of the four kinematic equations, determine its range?



$\Delta X = ?$
$\Delta T = ?$
$a = 0$
$V_i = 9 \frac{m}{s}$

$\Delta X_r = V_i \Delta T + \frac{1}{2} a \Delta T^2$

$\Delta X = 9(0.894) + 0$

$\boxed{\Delta X = 8.05\ m}$

find ΔT with vert info

$\Delta X = -4m$
$V_i = 0 \frac{m}{s}$
$a = -10 \frac{m}{s^2}$
$\Delta T = ?$
$V_f = ?$

$V_f = V_i + a\Delta T$

$\frac{V_f - V_i}{a} = \Delta T$

$\Delta T = \frac{-\sqrt{80} - 0}{-10}$

$\boxed{\Delta T = 0.894\ s}$

$V_f^2 = V_i^2 + 2a\Delta X$

$V_f^2 = 0^2 + 2(-10)(-4)$

$V_f^2 = 80$

$\boxed{V_f = \pm\sqrt{80}}$

4c) At t=5s, a 5kg particle (moving with constant acceleration of (2i-3j+4k) m/s^2) has a velocity of (2**i**-4**j**+6**k**) *m/s*. What <u>was</u> the velocity vector of the particle at t=2s?

4d) The velocity of Bill relative to Alex is (7**i**-2**j**+0**k**) **m/s**. The velocity of Bill relative to Colin is (5**i**+2**j**-6**k**). What is the velocity vector of Colin relative to Alex?

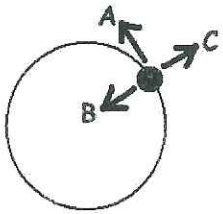
4e) A 7kg particle moves in a counter-clockwise circular orbit with a steady speed. The orbit has a radius of 6m. The particle has a centripetal acceleration of 1.5m/s^2.

i) What is the speed of the particle?

$a_c = \frac{v^2}{r} \rightarrow v^2 = a_c r$, $v = \sqrt{a_c \cdot r}$ $\rightarrow v = \sqrt{1.5 \cdot 6}$

$\boxed{v = 3}$

ii) In which direction does the centripetal acceleration act? (see diagram)



Towards $\boxed{B}$

Answers (If you do not get the correct answers you need to see me).

1a) kg

1b) $\frac{[M][T]}{[L^3]}$

1c) yes

1d) 5.6 m

1e) 46.6 cm/min

2ai) +4 m

2aii) -5 m

2aiii) -1.2 m/s

2b) travelling in a negative direction while slowing down

2c) 2.3 s

2di) negative (to the left)

2dii) +3 m/s^2

2diii) 0.5 s

2div) -5 m

2ei) +31 m

2eii) +22 m/s

2eiii) +6 m/s^2

3a) velocity displacement

3b) see DrB

3ci) (-2.4i+6.6j+0k) N

3cii) (1.1i+8.6j+0k) N

3ciii) 8.7 N

3civ) 83°

4ai) 6.9 m/s

4aii) 0 m/s^2

4aiii) C

4b) 8.05 m

4c) (-4i+5j-6k) m/s

4d) (+2i-4j+6k) m/s

4ei) 3 m/s^2

4eii) B

2048 T1 SU15 name_____

There is no equation sheet for this test. For credit, all work must be shown clearly. State answers with appropriate units. Use the standard form for equations. Put you final answer in a box. Missing units will reduce your score.

1) Express 1.2μg in standard units.

1.2×10^{-9} kg

2) Express 7.4km^2 in standard units.

$7.4 \times 10^{+6}$ m^2

3) Express 8.5nm^3 in standard units.

8.5×10^{-27} m^3

4) Given: $J = \frac{3m^2x}{4v}$, what are the dimensions of J, (m is mass, v is velocity, x is length)?

$$\frac{\frac{[M]^2[L]}{[L]}}{[T]} = \boxed{[M]^2[T]}$$

5) If a rock's linear dimensions are tripled, by what factor will its volume change?

if L is x3

V = x27

6) Using the factor label method convert 9 peck/nap into cm/min (where 4 peck equals 1 meter and 1 nap equals 5 second).

7) How many significant figures are there in 1700 kg?

2

8) How many significant figures are there in 0.01700 kg?

4

9) Calculate 2.50 kg plus 29.703 kg. State the answer with appropriate precision.

32.20 kg

10) What is the surface area of a sphere that has a radius of 8m?

8m

$V = \frac{4}{3}\pi(8)^3 = 2145\,m^3$ $SA = 4\pi(8)^2$

$\boxed{SA = 804.25\,m^2}$

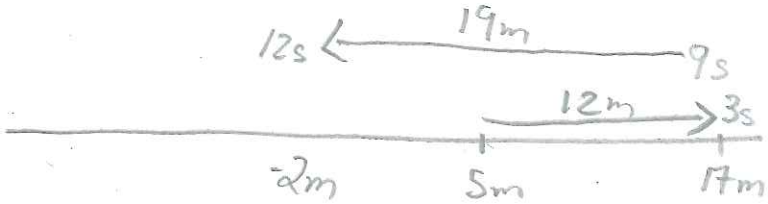
11) What is the surface area of a right cylinder that has a radius of 2m and a height of 6m?

$SA = 2\pi r \cdot h + 2\pi r^2$

$= 2\pi(2)(6) + 2\pi(2)^4 = \boxed{100.53\,m^2}$

12) A runner starts at a coordinate of +5m. She runs at 4m/s for 3s to the right. The rests for 6s. She then takes 3s to move to position -2m. What is her average velocity during this journey?

$\bar{V} = \frac{x_f - x_i}{T_f - T_i} = \frac{-2-5}{12-0} = \boxed{-.583\,\frac{m}{s}}$



13) A particle has a position versus time relationship given by $y=3t^2-9t+6$. What is its instantaneous acceleration at t=2s?

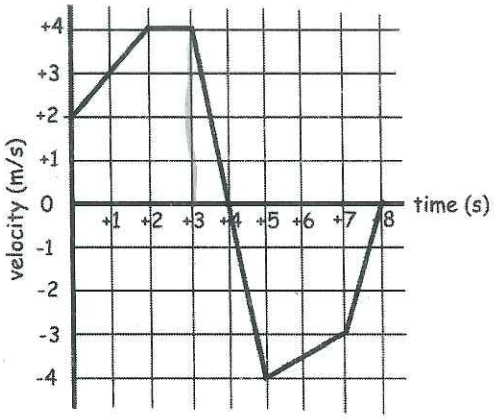
$\boxed{y'' = 6\,\frac{m}{s^2}}$

14) A runner has a velocity of +2m/s and an acceleration of -3m/s^2. Is she travelling in a positive or negative direction and is she speeding up or slowing down?

positive
Slowing

15) Consider this graph.

- What is the displacement of the particle after the first three seconds of its journey?

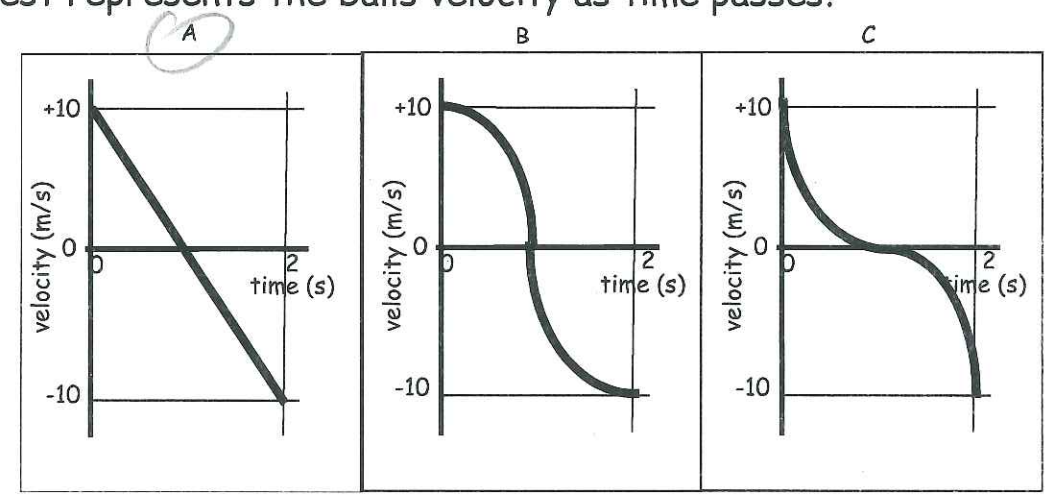


$(4) + \frac{1}{2}(2+4)2 = \boxed{10m}$

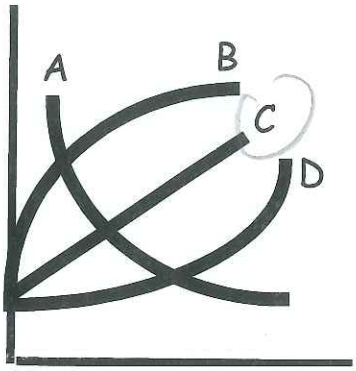
- What is the particle's average acceleration for the entire journey?

$\bar{a} = \frac{V_F - V_i}{T_F - T_i} = \frac{0-2}{8-0} = \boxed{-.25 \frac{m}{s^2}}$

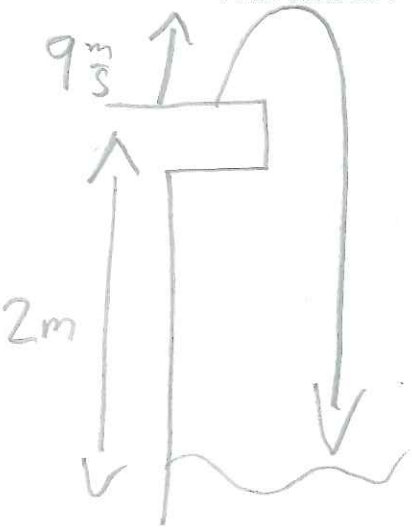
16) A student throws a ball upwards with a speed of 10m/s. Circle the graph that best represents the balls velocity as time passes?



17) A car is travelling in a positive direction along a straight and level road. At a certain time the car starts to move with a constant positive acceleration. Which trace (A, B, C or D) best represents the resulting velocity versus time graph for this part of the journey?



18) A diver jumps upwards from a diving board with a speed of 9m/s. The board is 2m above the water. How long does it take the diver to reach the water?



$V_i = 9$

$\Delta x = -2$

$a = -10$

$\Delta T = ?$

$\Delta x = V_i \Delta T + \frac{1}{2} a \Delta T^2$

$-2 = 9T - 5T^2$

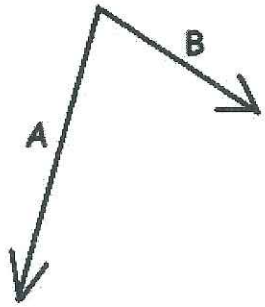
$-5T^2 + 9T + 2 = 0$

$T = \frac{-9 \pm \sqrt{81 - 4(-5)(2)}}{2(-5)} = \frac{-9 \pm \sqrt{121}}{-10}$

$T = -.2$

$\boxed{T = 2s}$

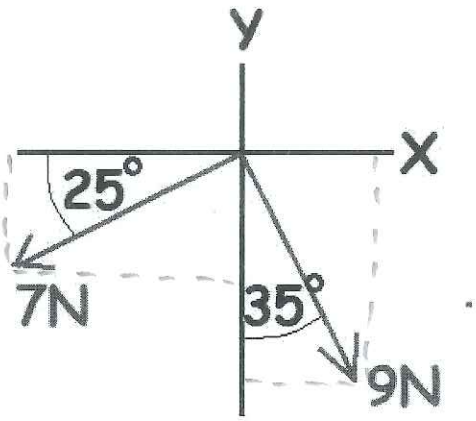
19) Consider these force vectors. Draw a vector diagram to determine vector **2B-A**



20) Consider this vector diagram.

- What is x component of the 7N vector?

$-7\cos 25° = \boxed{-6.34\,N}$



- What is the magnitude of the resultant vector? (Must show clear work)

$-7\cos 25° - 7\sin 25° + 0K$

$9\sin 35° - 9\cos 35° + 0K$

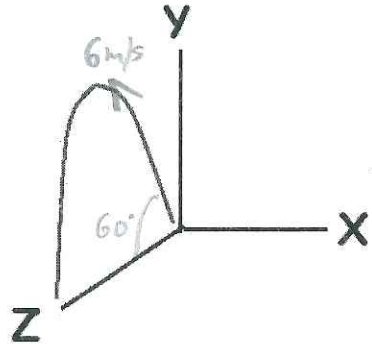
$(-1.18\hat{i} - 10.33\hat{j} + 0\hat{k})N$

$mag = \sqrt{(-1.18)^2 + (-10.33)^2}$

$\boxed{mag = 10.40\,N}$

21) A football's is kicked at 6m/s in a direction that is inclined at 60° elevation above the positive Z axis.

- Express this initial velocity as an 'ijk' vector.



- Express the acceleration due to gravity as an 'ijk' vector.

22) Given that $A=(3i+4j-5k)m$

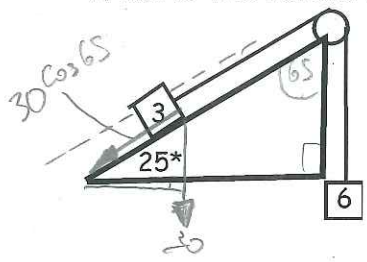
- Write an expression for the magnitude of this vector.

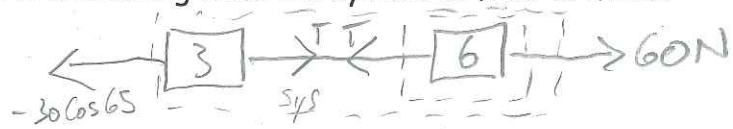
- Write an expression for the unit vector that has the same direction as this vector.

48 T2 SU12 Name _____

For credit: All working must be shown clearly. Circle your final answer.

5a) A 3kg block rests on a smooth table that is inclined at 25* above the horizontal. A 6kg load is connected to the block by a string that passes over a massless and frictionless pulley. What is the tension in the string once the system is free to move?





$\Sigma F_6 = m_6 \cdot a_6$

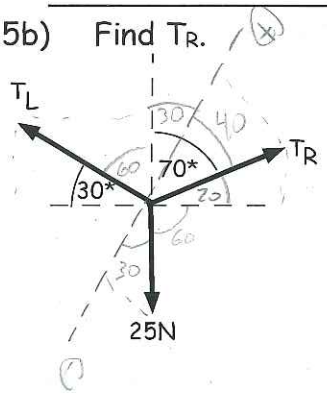
$-T + 60N = 6a \rightarrow T = 60N - 6(5.26) = \boxed{T = 28.45}$

$\Sigma F_{sys} = m_{sys}\, a_{sys}$

$-30\cos 65 + 60N = 9(a)$

$a_{sys} = 5.26$

5b) Find T_R.



$\Sigma F_h = 0$

$T_R \cos 20 - T_L \cos 30 = 0$

$+T_L = \frac{+T_R \cos 20}{\cos 30}$

$\Sigma F_v = 0$

$T_L \sin 30 + T_R \sin 20 - 25 = 0$

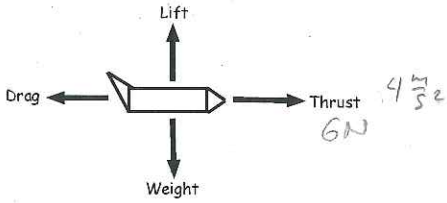
$\left(\frac{T_R \cos 20}{\cos 30} \cdot \sin 30\right) + T_R \sin 20 = 25$

$T_R = \frac{25}{.5425 + .3420}$ $\boxed{T_R = 28.26}$

$T_R \sin 40 - 25 \sin 30 = 0$

$T_R = \frac{25 \sin 30}{\sin 40}$

5c) An 5kg airplane flies straight and level to the right while accelerating at +4m/s^2 to the right. The engines provide a thrust of +6N to the right.



i) What is the net horizontal force?

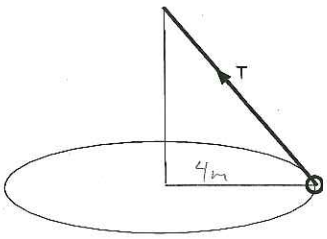
$\Sigma_h = m \cdot a$

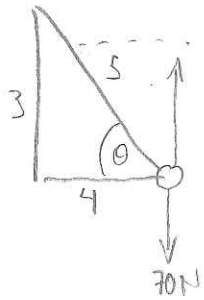
$\Sigma F_h = 5 \cdot 4 = \boxed{20N}$

ii) What is the Lift Force?

$= 50N$

6a) A conical pendulum is made from a 7kg bob and a 5m long string. The bob moves in a horizontal circle that has a radius of 4m. What is the tangential speed of the bob?





$\Sigma F_h = m\frac{v^2}{r}$

$(T\cos\theta) = \frac{7v^2}{4}$

$\frac{4}{5}T = \frac{7}{4}v^2$

$v^2 = \left(\frac{4}{5}T\right)\frac{4}{7}$

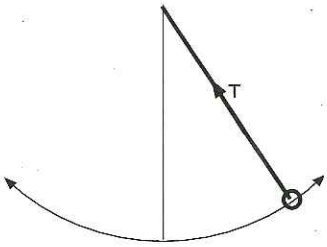
$\Sigma F_v = 0$

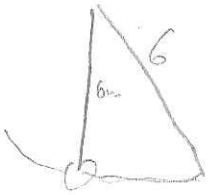
$T\sin\theta - 70 = 0$

$\frac{3}{5}T = 70$

$T = \frac{70 \cdot 5}{3}$

6b) A 3kg pendulum bob swings in an arc as shown below. The length of the pendulum is 6m. The bob's maximum speed is 24m/s. What is the maximum tension in the string?



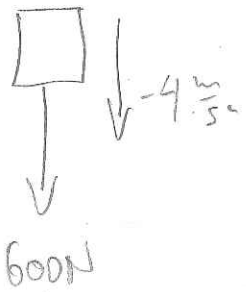


$a_c = \frac{v^2}{r}$ $\Sigma F = \frac{m \cdot a_c^2}{r}$

$T = \frac{3 \cdot 576}{6}$

$T = 288 + 30$ $T = 318$

6c) A 60kg student is in an elevator that is travelling upwards. As she reaches her floor, the elevator slows, having an acceleration whose magnitude is $4m/s^2$. What is her apparent weight at this instant?



$\Sigma F_v = ma$

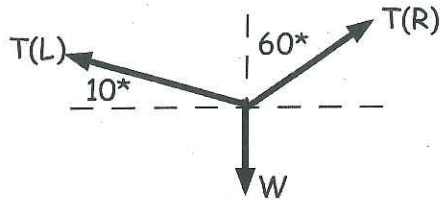
$F_{aw} - 600N = 60 \cdot (-4)$

$F_{aw} = 360N$

48 T2 SU11 **Name** _____

For credit: All working must be shown clearly. Circle your final answer.

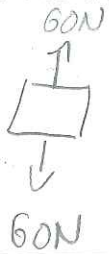
5a) A 12kg traffic light is supported by two strings as shown in the diagram. What is the force of tension in the left hand rope?



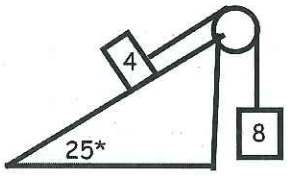
5b)A 6kg airplane flies at a constant 34m/s straight and level. Its engine provides a thrust force of 9N.

i) What is the airplane's lift force?

ii) What is the airplane's vertical acceleration?

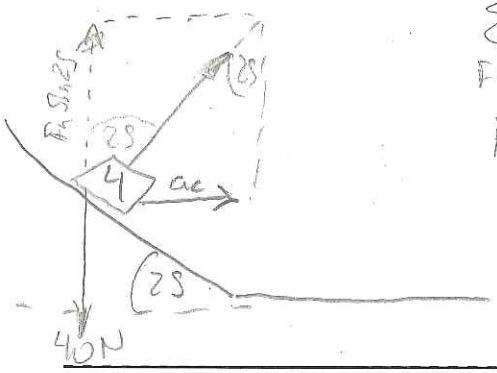


5c) A 4kg block rests on a smooth 25° incline. What is the tension in the string that connects the two masses when the system is released?



4kg

6a) A racecar drives around a circular banked track. It moves in a circle of 7m radius. The bank is inclined at 25° above the horizontal. What is the tangential speed of the car if no frictional force occurs between the tires and the track?



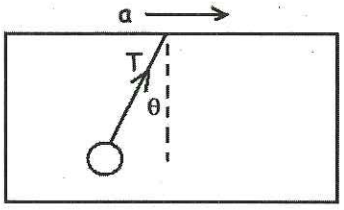
$\Sigma F = ma$

$F_n \sin 25 - 40 = 0$

$F_n =$

6b)A car is driven round a circular track. The track has a radius of 9m. The driver speeds up steadily, going from a speed of 3m/s to a speed of 7m/s in 12s. What is the magnitude of the <u>total</u> acceleration of the car when its speed is 5m/s?

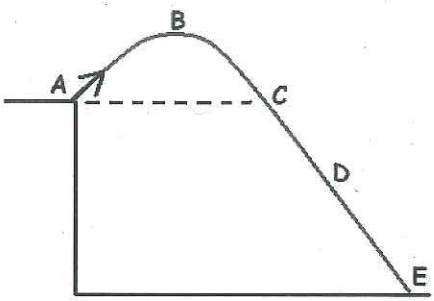
6c)A 6kg object is suspended from the ceiling of an accelerating boxcar. The acceleration of the boxcar is $+5m/s^2$. What is the angle between the string and the vertical axis?



48 T2 FA14 Name _____

For credit: All working must be shown clearly. Circle your final answer.

Ei) A ball is fired at 6m/s muzzle velocity at 35* elevation above the horizontal, from the edge of a cliff that is 4m above a lake.



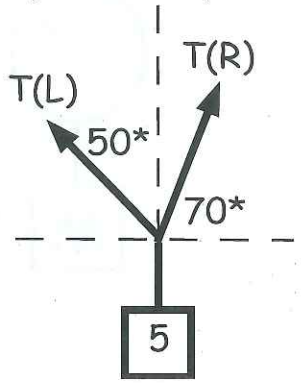
- What is the horizontal component of the ball's acceleration at D?
- What is the vertical component of the ball's velocity at point C?
- What is the vertical component of the ball's acceleration at B?
- What is the vertical component of the ball's displacement at E?
- What is the horizontal range of the ball?

Eii) At t=5s, a red, 4kg ball is moving with constant acceleration of (-1**i**+2**j**+5**k**) *m/s*^{2}, and has a velocity of (+2**i**-1**j**+2**k**) *m/s*. What is the particle's velocity vector at t=9s?

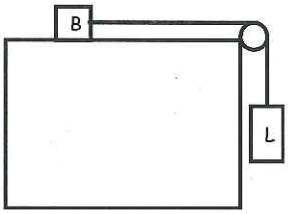
Eiii) Particle A has a velocity of (+1i+2j+4k) m/s relative to particle B. Particle C has a velocity of (-2i-3j+4k) m/s relative to particle A. What is the velocity of particle B relative to particle C?

Fi) A student pushes a car, that has run out of gas, at a steady speed, along a level road. Given that the 'action' force is the force of the student's foot on the road. What is the corresponding reaction force?

Fii) Find T_L (the tension in the left hand string).



Gi) A 3kg block rests on a rough table. There is a 0.2 coefficient of kinetic friction between the block and the table. A 6kg load is connected to the block by a string that passes over a massless and frictionless pulley. What is the tension in the string once the system is free to move?

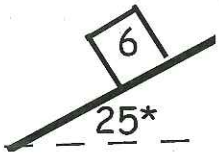


Gii) A 3 kg fish hangs from a newton meter while being carried upwards in an elevator. The elevator has a upwards acceleration of 6 m/s^2.

- Is the elevator speeding up or slowing down?

- What is the apparent weight of the fish as indicated by the scale?

Giii) A 6kg block moves at constant speed down a 'rough' slope, that is inclined at 25* above the horizontal.

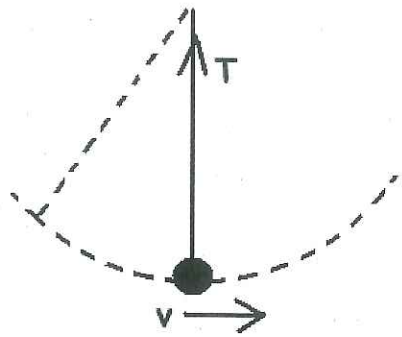


- Is the block in equilibrium along the slope?

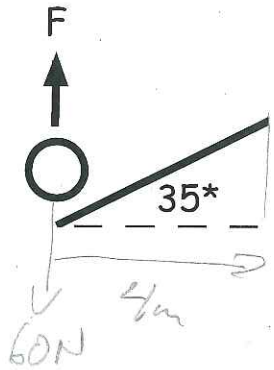
- What is the force of friction between the block and the slope?

Hi) A 6kg model car drives at a constant speed of 9m/s around a level, (unbanked), circular track, of 4m radius. If the car is driving as fast as it can without skidding, what is the coefficient of friction between the car tires and the track?

Hii) A ball is attached to a branch by a rope that is 3m long. The ball is swung in an arc forming a pendulum. The speed of the ball at its lowest point is 6 m/s. The tension in the rope at its lowest point is 24N. What is the mass of the ball?



Ii) A 6kg block is slowly carried 4m along a ramp that in inclined at 35* above the horizontal. How much work is done by the lifting force?



$W = F \cdot d \cdot \cos\theta$

$W = 60N \cdot 4m \cdot \cos 0°$

$W = 240\,J$

Iii) A force acts on a 2kg particle. The force varies with position. The force is defined by $F=(6x^2-6x+5)$, where F is in Newton and x is in meters. The force and the displacement are in the same direction. How much work is done by this force between x=3m and x=5m?

$W = \int F_x \, dx$

$W = \int_3^5 6x^2 - 6x + 5 \, dx$

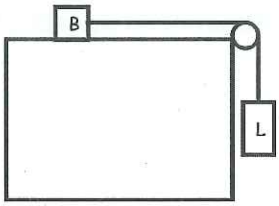
$W = \left[2x^3 - 3x^2 + 5x\right]_3^5 \rightarrow W = \left[(5(5)^3 - 3(5)^2 + 5(5)) - (2(3)^3 - 3(3)^2 + 5(3))\right]$

$W = 575 - 123$

$\boxed{W = 452\,J}$

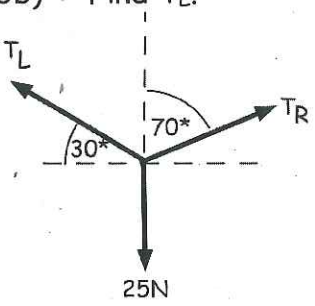
48 T2 FA11 Name _____

For credit: All working must be shown clearly. Circle your final answer.

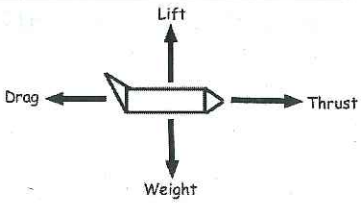


5a) A 6kg block rests on a rough table. There is a 0.5 coefficient of kinetic friction between the block and the table. A 9kg load is connected to the block by a string that passes over a massless and frictionless pulley. What is the tension in the string once the system is free to move?

5b) Find T_L.



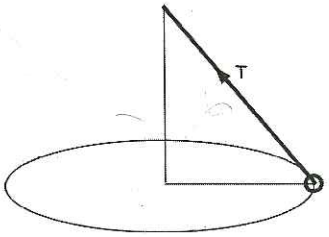
5c) An 5kg airplane flies straight and level to the right while accelerating at $+4m/s^2$ to the right.



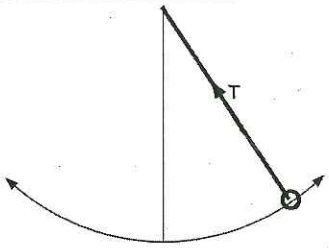
i) What is the net horizontal force?

ii) What is the Lift Force?

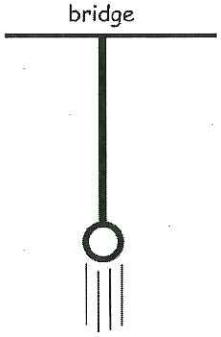
6a) A conical pendulum is made from a 6kg bob and a 5m long string. The bob moves in a horizontal circle of 3m radius. What is the tangential speed of the bob?



6b) A 4kg pendulum bob swings in an arc as shown below. The length of the pendulum is 3m. The bob's maximum kinetic energy is 160J. What is the maximum tension in the string?



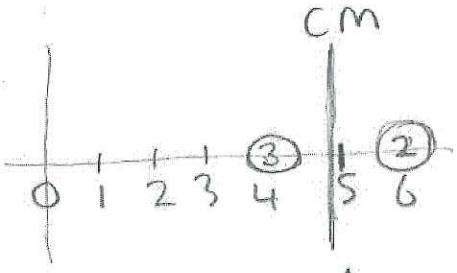
6c) A bungee jumper jumps off a bridge. At the instant shown she is being pulled upwards by the bungee cord. She has a mass of 6kg. Her acceleration is $+9m/s^2$. Her velocity is +6m/s. What is the tension in the bungee cord at this instant?



2048 Torque Homework

Center of Mass

1) A 3kg mass is located at position X=4m along the X-axis. A 2kg mass is located at position X=6m along the X-axis. What is the position along the X-axis of the center of mass? (4.8m)



3 kg (i j k) + 2 kg (

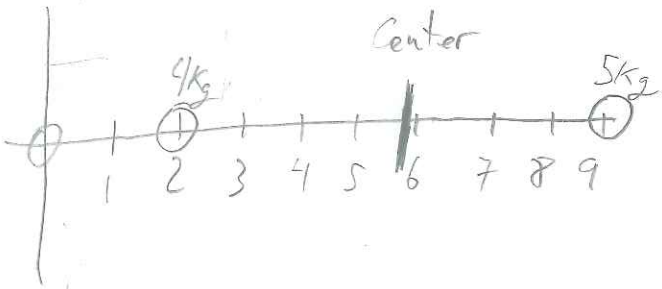
$$X_{cm} = \frac{\sum x_i m_i}{\sum m_i} = \frac{(3)(4) + (2)(6)}{(3+2)}$$

$$X_{cm} = \frac{(+4)(3) + (+6)(2)}{(3) + (2)}$$

$$X_{cm} = \frac{(+12) + (+12)}{5} = \frac{24}{5}$$

$$X_{cm} = \boxed{+4.8m}$$

2) A 5kg mass is located at position X=9m along the X-axis. A 4kg mass is located at position X=2m along the X-axis. What is the position along the X-axis of the center of mass? (5.9m)



$$X_{cm} = \frac{\sum m \cdot x}{\sum m}$$

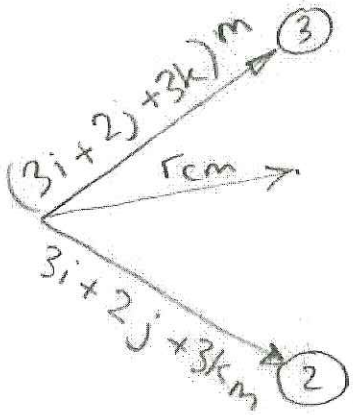
$$X_{cm} = \frac{(4)(2) + (5)(9)}{4+5}$$

$$X_{cm} = \frac{8 + 45}{9}$$

$$X_{cm} = \frac{53}{9}$$

$$\boxed{X_{cm} = 5.9m}$$

3) A 3kg mass is located at position (3i+2j+4k)m. A 2kg mass is located at the position (-3i+2j+3k)m. What is the position vector to the center of mass?



$$r_{cm} = \frac{\sum m_i r_i}{\sum m_i}$$

i	j	k	
9	6	9	$m_1 r_1$
6	4	6	$m_2 r_2$
15	10	15	$\sum m_i r_i$

$$\frac{15}{5}\hat{i} + \frac{10}{5}\hat{j} + \frac{15}{5}\hat{k} \qquad \frac{\sum m_i r_i}{\sum m_i}$$

$$\boxed{(3\hat{i} + 2\hat{j} + 3\hat{k})\ m}$$

4) A 5kg mass is located at position (-2i-3j+4k)m. A 4kg mass is located at (+6i-2j+0k)m. What is the position vector to the center of mass?

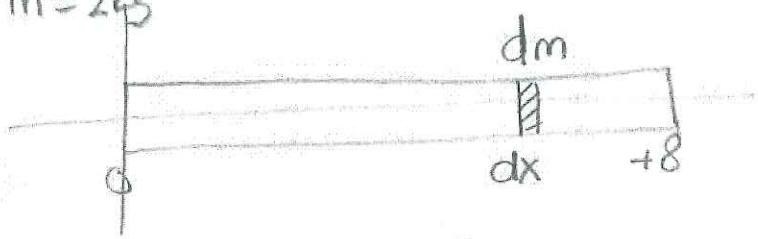
$$X_{cm} = \frac{\sum m \cdot r}{\sum m} = \frac{(5)(-2i-3j+4k) + (4)(6-2+0)}{(5+4)}$$

$$X_{cm} = \frac{(-10i-15j+20k) + (24i-8j+0k)}{9}$$

$$X_{cm} = (14i-23j$$

5) Using the method of calculus shown in the physics lecture class, determine the position of the center of mass of a long rod that is positioned with one end on the origin and its other end at +8m on the x-axis. The rod is uniform and has a mass of 2kg. (+4m on the x-axis)

$M = 2kg$



$$x_{cm} = \frac{\int x\,dm}{M}$$

cannot integrate $\int x\,dm$ so convert dm into dx. We do this using linear density.

$$dm = \rho_{LIN}\,dx = \frac{M}{L}\,dx$$

$$dm = \frac{M}{L}\,dx = \frac{2}{8}\,dx$$

$$x_{cm} = \frac{\int_0^8 x\left(\frac{2}{8}dx\right)}{2}$$

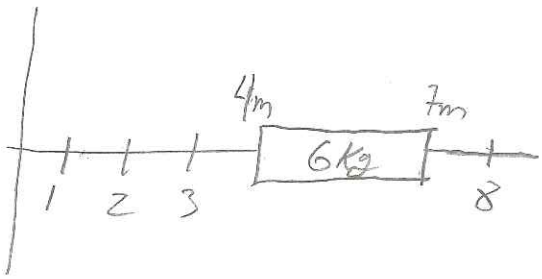
$$x_{cm} = \frac{(2)}{(8)(2)}\int_0^8 x\,dx$$

$$x_{cm} = \frac{(2)}{(16)}\left[\frac{x^2}{2}\right]_0^8$$

$$x_{cm} = \frac{(2)}{(16)}\left\{\frac{64}{2} - \frac{0}{2}\right\}$$

$$x_{cm} = \frac{64}{16} = \boxed{4m}$$

6) Using the method of calculus shown in the physics lecture class, determine the position of the center of mass of a 3m long rod that is positioned with one end at +4m on the X-axis and its other end at +7m on the x-axis. The rod is uniform and has a mass of 6kg. (+5.5m on the x-axis)



$$X_{cm} = \frac{1}{M}\int x\,dm$$

$$X_{cm} = \frac{1}{6}\int x\,dm$$

$$X_{cm} = \frac{1}{6}\int x \cdot 2dx$$

$$X_{cm} = \frac{1}{3}\int_4^7 x\,dx$$

$$X_{cm} = \frac{1}{3}\left[\frac{1}{2}x^2\right]_4^7$$

$$X_{cm} = \frac{1}{3}\left[\frac{1}{2}(7)^2 - \frac{1}{2}(4)^2\right]$$

$$X_{cm} = \frac{1}{3}\left[\frac{49}{2} - \frac{16}{2}\right]$$

$$X_{cm} = \frac{1}{3}\left[\frac{33}{2}\right]$$

$$\boxed{X_{cm} = 5.5m}$$

$$dm = \rho_{Linear} \cdot dx$$

$$dm = \left(\frac{M}{L}\right) \cdot dx$$

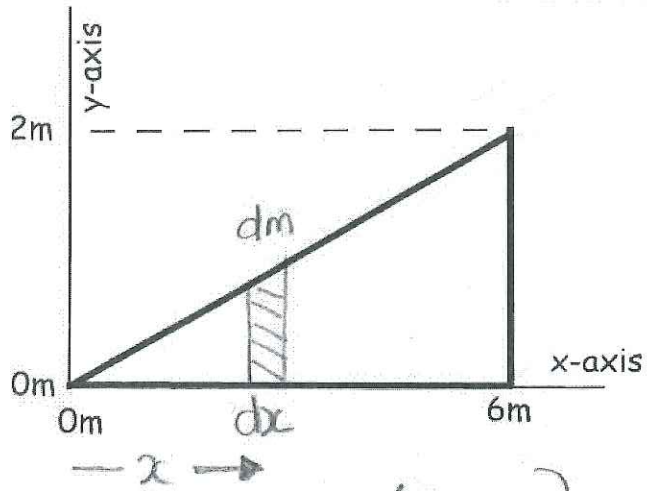
$$dm = \frac{6}{3}dx$$

$$dm = 2dx$$

7) Using the method of calculus shown in the physics lecture class, determine the position of the center of mass of a long rod that is positioned with one end at -3m on the X-axis and its other end at +7m on the x-axis. The rod is uniform and has a mass of 2kg. (+2m on the x-axis)

8) Using the method of calculus shown in the physics lecture class, determine the position of the center of mass of a long rod that is positioned with one end at +6m on the X-axis and its other end at +12m on the x-axis. The rod is uniform and has a mass of X kg. (+9m on the x-axis)

9) For credit use the method of calculus shown in the physics lecture class. An 8kg triangular sheet of metal is positioned as shown. Find the x coordinate of its center of mass. (+4m on the x-axis)



$$x_{cm} = \frac{\int x\, dm}{M}$$

Cannot integrate $\int x\, dm$ so convert dm into dx. We do this using area density.

$$dm = \rho_{AREA}\, Vol_{dm}$$

$$dm = \left(\frac{M_{SHEET}}{A_{SHEET}}\right) Vol_{dm}$$

$$dm = \frac{8}{\left(\frac{1}{2}(6)(2)\right)}(dx\, h)$$

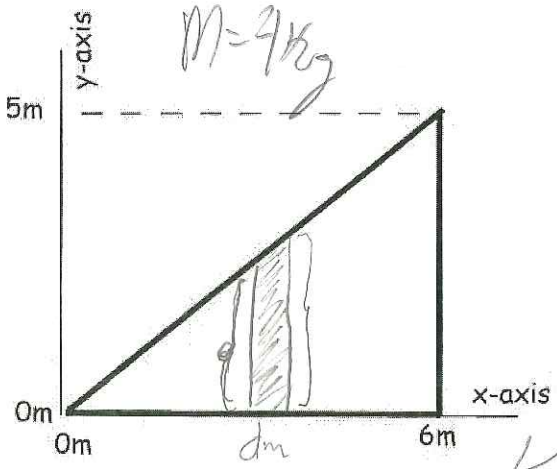
$$dm = \frac{8}{6}\left((dx)((slope)(x))\right)$$

$$dm = \frac{8}{6}\, dx\left(\frac{2}{6}\right)x = \frac{16}{36}\, x\, dx$$

$$x_{cm} = \frac{\int x\, dm}{M} = \frac{\int_0^6 x\left(\frac{16}{36}\, x\, dx\right)}{8} = \frac{(16)}{(36)(8)}\int_0^6 x^2\, dx$$

$$x_{cm} = \frac{1}{18}\left[\frac{x^3}{3}\right]_0^6 = \frac{1}{18}\left\{72 - 0\right\} = \frac{72}{18} = \frac{8}{2} = \boxed{4m}$$

10) For credit use the method of calculus shown in the physics lecture class. A 4kg triangular sheet of metal is positioned as shown. The metal is 10cm thick. Find the x coordinate of its center of mass. (+4m on the x-axis)



$M = 4kg$

$$X_{cm} = \frac{1}{M}\int x\,dm$$

$$X_{cm} = \frac{1}{4}\int x\,dm$$

$$dm = \rho_{Area} \cdot dA$$

$$dm = \left(\frac{M}{A}\right)\cdot dA$$

$$dm = \left(\frac{4}{\frac{1}{2}(5)(6)}\right)dA$$

$$dm = \frac{4}{15}\cdot dA \rightarrow dA = \text{height}\,x\,dx$$

$$dA = \text{Slope of } x\ dx$$

$$dA = \frac{5}{6}x\,dx$$

$$dm = \frac{4}{15}\left(\frac{5}{6}\right)x\,dx$$

$$dm = \frac{2}{3}\left(\frac{1}{3}\right)x\,dx$$

$$dm = \frac{2}{9}x\,dx$$

$$X_{cm} = \frac{1}{4}\int x\,\frac{2}{9}x\,dx$$

$$X_{cm} = \frac{1}{4}\cdot\frac{2}{9}\int x^2\,dx$$

$$X_{cm} = \frac{1}{18}\int_0^6 x^2\,dx$$

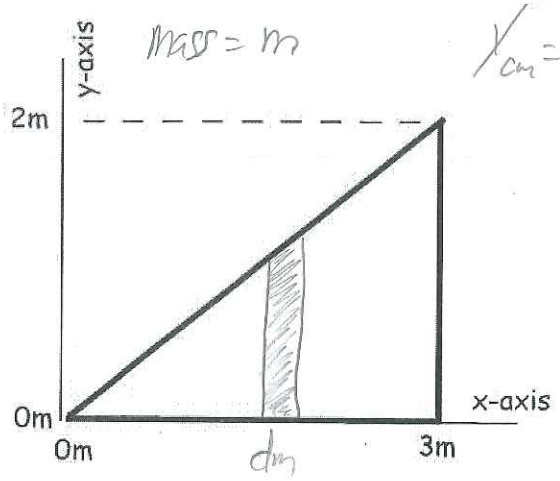
$$X_{cm} = \frac{1}{18}\left[\frac{1}{3}x^3\right]\Big|_0^6$$

$$X_{cm} = \frac{1}{18}\left[\frac{1}{3}(6)^3 - \frac{1}{3}(0^3)\right]$$

$$X_{cm} = \frac{1}{18}\left[\frac{1}{3}(216) - 0\right]$$

$$\boxed{X_{cm} = 4m}$$

11) For credit use the method of calculus shown in the physics lecture class. A uniform triangular sheet of metal is positioned as shown. Find the x coordinate of its center of mass. (+2m on the x-axis)



mass = m

$$X_{cm} = \frac{1}{m}\int x\,dm$$

$$X_{cm} = \frac{1}{m}\int x\,dm$$

$$X_{cm} = \frac{1}{m}\int x\,\frac{2m}{9}x\,dx$$

$$dm = \rho_{Area}\cdot dA$$

$$dm = \left(\frac{m}{A}\right)\cdot dA$$

$$dm = \frac{m}{\frac{1}{2}(3)(2)}\cdot dA$$

$$dm = \frac{m}{3}\cdot dA$$

$$dA = \frac{2}{3}x\,dx$$

$$dm = \frac{2m}{9}x\,dx$$

$$X_{cm} = \frac{1}{\cancel{m}}\cdot\frac{2m}{9}\int x^2\,dx$$

$$X_{cm} = \frac{2}{9}\int_0^3 x^2\,dx$$

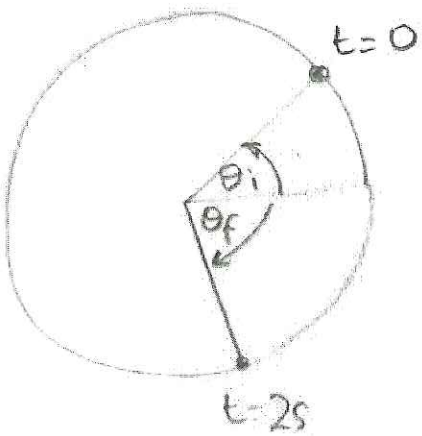
$$X_{cm} = \frac{2}{9}\left[\frac{1}{3}x^3\right]\Big|_0^3$$

dm

=

Angular Kinematics

12) A point starts at an angular displacement of +5 radians. After 2 seconds it has an angular displacement of -7 radians. What is its average angular velocity during this journey? (-6rad/s)



$\bar{\omega} = ?$

$\theta_i = +5\,rad$

$\theta_f = -7\,rad$

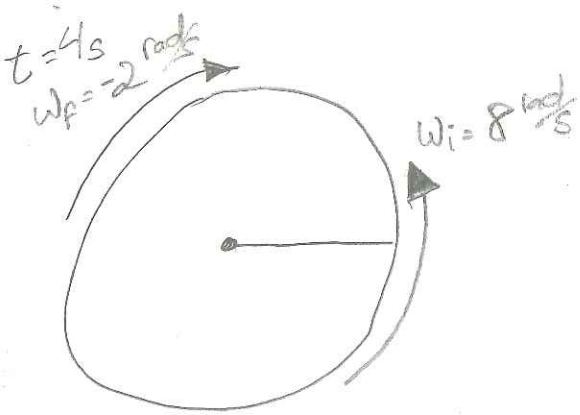
$\Delta t = t_f - t_i = 2 - 0 = 2s$

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

$$\bar{\omega} = \frac{(-7) - (+5)}{(2 - 0)}$$

$$\bar{\omega} = \frac{-12}{2} = \boxed{-6\,rad/s}$$

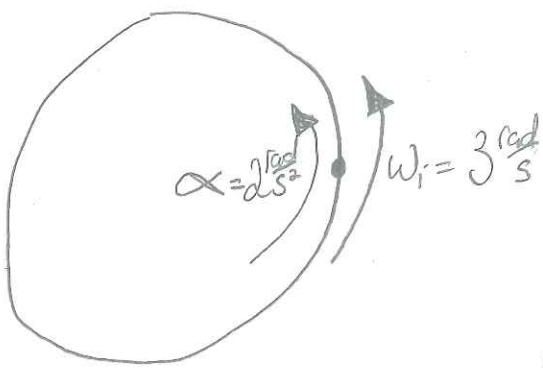
13) A point starts at an angular velocity of +8 rad/s. After 4 seconds it has an angular velocity of -2 rad/s radians. What is its average angular acceleration during this journey? (-2.5rad/s)



$$\alpha = \frac{\Delta\omega}{t} = \frac{-2 - 8}{4} = \frac{-5}{2} = \boxed{-2.5\,\frac{rad}{s^2}}$$

14) A point travels around the circumference of a circle. It has an initial velocity of +3 rad/s. It has a constant angular acceleration of +2 rad/s^2. What is its angular velocity after it has moved through a displacement of 6 radians?
(+5.7rad/s)

$\Delta\theta = 6$ radians



$V_i = 3$
$a = 2$
$\Delta x = 6$

$a = \frac{\Delta v}{t}$

$V_f^2 = V_i^2 + 2a\Delta x$

$\omega_f^2 = \omega_i + 2\alpha\Delta\theta$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

$\omega_f^2 = 9 + 4(6)$

$\omega_f^2 = 9 + 24$

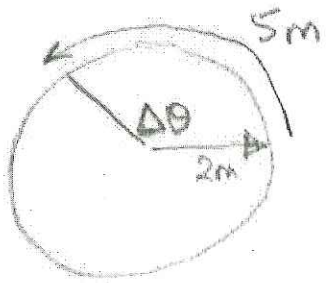
$\omega_f^2 = 33$

$\omega_f = \sqrt{33}$

$\boxed{\omega_f = 5.7\frac{rad}{s}}$

Angular and linear conversions

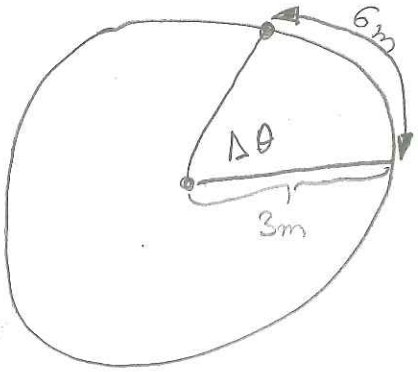
15) A point moves 5 m around the circumference of a 2 m radius circle. What is its angular displacement? (2.5rad)



$$\Delta\theta = \theta_f - \theta_i = \frac{arc_f}{r} - \frac{arc_i}{r}$$

$$\Delta\theta = \frac{5}{2} - \frac{0}{2} = \boxed{2.5 \text{ rad}}$$

16) A point moves 6 m around the circumference of a 3 m radius circle. What is its angular displacement? (2.0rad)



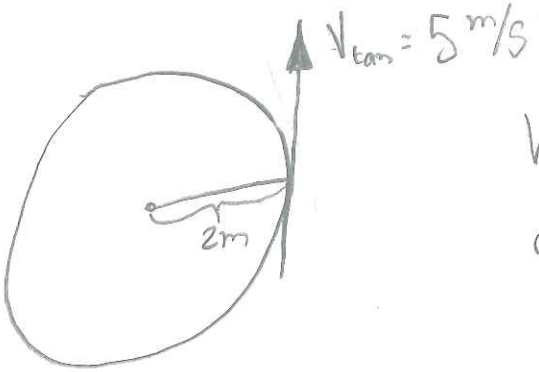
$$\Delta\theta = \theta_f - \theta_i$$

$$\theta = \frac{s}{r}$$

$$\Delta\theta = \frac{6m}{3m} - \frac{0m}{3m}$$

$$\boxed{\Delta\theta = 2 \text{ rad}}$$

17) A point moves at 5 m/s around the circumference of a 2 m radius circle. What is its angular velocity? (2.5rad/s)

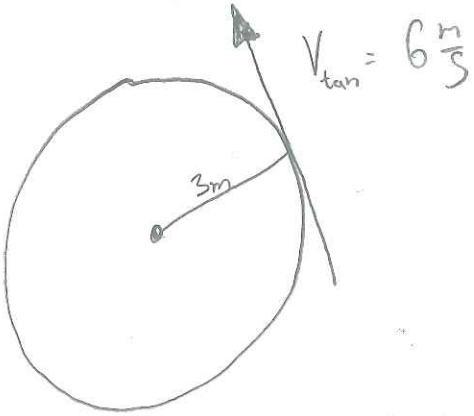


$$V_{tan} = \omega r$$

$$\omega = \frac{V_{tan}}{r}$$

$$\omega = \frac{5\ m/s}{2m} = \boxed{2.5\ \frac{rad}{sec}}$$

18) A point moves 6 m/s around the circumference of a 3 m radius circle. What is its angular velocity? (2.0rad/s)

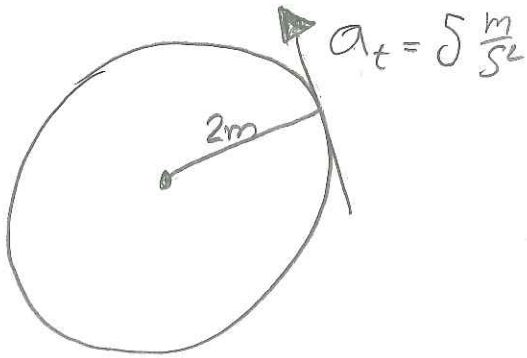


$$V_{tan} = \omega r$$

$$\omega = \frac{V_{tan}}{r}$$

$$\omega = \frac{6}{3} = \boxed{2\ \frac{rad}{sec}}$$

19) A point moves with a tangential acceleration of 5 m/s^2 around the circumference of a 2 m radius circle. What is its angular acceleration? (2.5rad/s^2)



$a_t = 5 \frac{m}{s^2}$

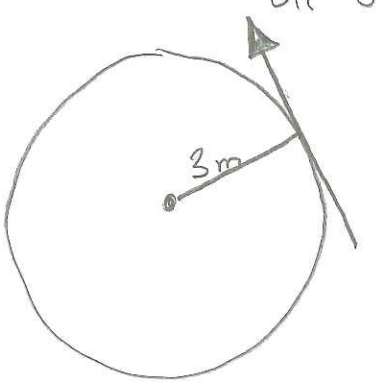
$a_t = \alpha r$

$\alpha = \frac{a_t}{r}$

$\alpha = \frac{5}{2} = \boxed{\alpha = 2.5 \frac{rad}{s^2}}$

20) A point moves with a tangential acceleration of 6 m/s^2 around the circumference of a 3 m radius circle. What is its angular acceleration? (2.0rad/s^2)

$a_t = 6 \, m/s^2$



$a_t = \alpha r$

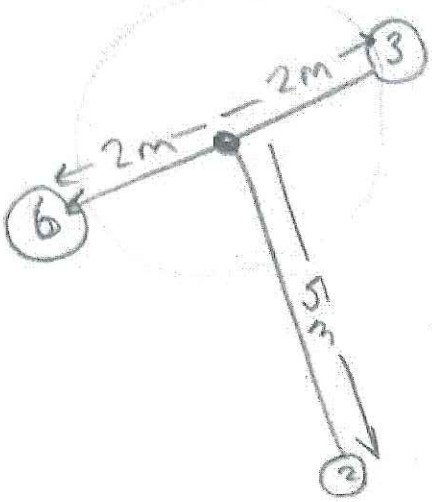
$\alpha = \frac{a_t}{r}$

$\alpha = \frac{6}{3}$

$\boxed{\alpha = 2 \frac{rad}{s^2}}$

Moment of inertia of a population of masses

21) A 3kg mass is located 2m from the axis of rotation. A 2kg mass is located 5m from the axis of rotation. A 6kg mass is located 2m from the axis of rotation. What is the moment of inertia about this axis of rotation? (86 kgm^2)



$$I = \sum m_i r_i^2$$

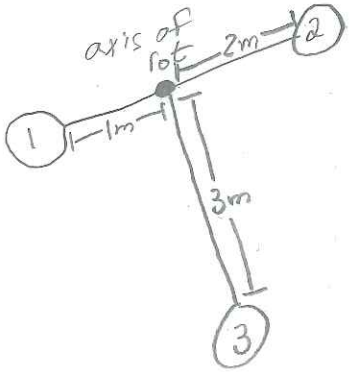
$$I = (3)(2^2) + (2)(5^2) + (6)(2^2)$$

$$I = (3)(4) + (2)(25) + (6)(4)$$

$$I = 12 + 50 + 24$$

$$I = \boxed{86 \text{ kgm}^2}$$

22) A 2kg mass is located 2m from the axis of rotation. A 3kg mass is located 3m from the axis of rotation. A 1kg mass is located 1m from the axis of rotation. What is the moment of inertia about the axis of rotation? (36 kgm^2)



moment of Inertia

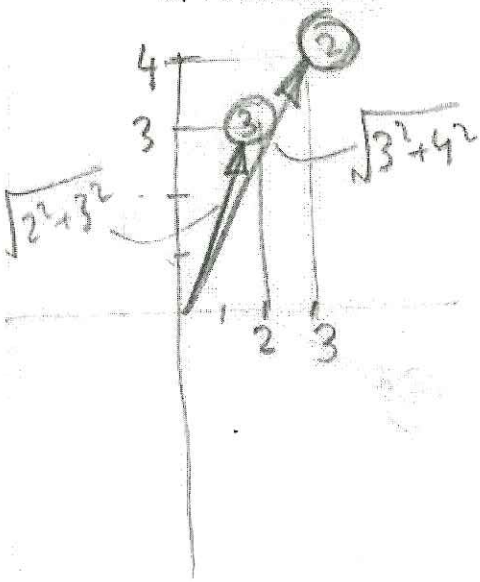
$$I = \sum m_i r_i^2$$

$$I = (2)(2)^2 + (1)(1)^2 + (3)(3)^2$$

$$I = 8 + 1 + 27$$

$$\boxed{I = 36 \text{ kgm}^2}$$

23) A 2kg mass is located at (3i+4j+0k)m. A 3kg mass is located at (2i+3j+0k)m. The axis of rotation is at the origin, along the Z axis. What is the moment of inertia about the axis of rotation?



$$I = \sum m_i r_i^2$$

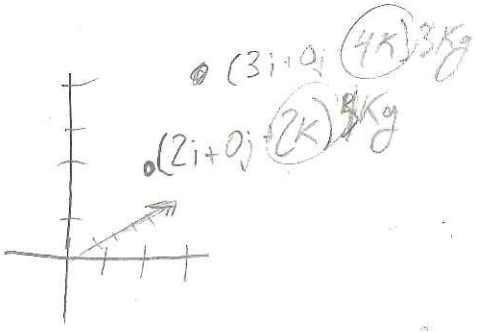
$$I = (2)(3^2+4^2) + (3)(2^2+3^2)$$

$$I = (2)(25) + (3)(13)$$

$$I = 50 + 39$$

$$I = \boxed{89\ kgm^2}$$

24) A 4kg mass is located at (2i+0j+2k)m. A 3kg mass is located at (3i+0j+4k)m. The axis of rotation is at the origin, along the Y axis. What is the moment of inertia about the axis of rotation?



$$I = \sum m_i r_i^2$$

$$I = (4)(2+2)^2 + (3)(3+4)^2$$

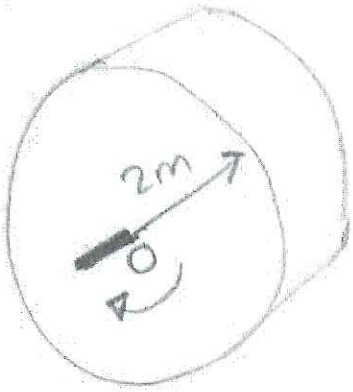
$$I = (4)(8) + (3)(25)$$

$$I = 32 + 75$$

$$\boxed{I = 107\ Kgm^2}$$

Moment of inertia of regularly shaped objects

25) A solid cylinder has a mass of 5 kg and a radius of 2 m. What is its moment of inertia about its axis of symmetry? (10 kgm^2)



$M = 5\,kg$

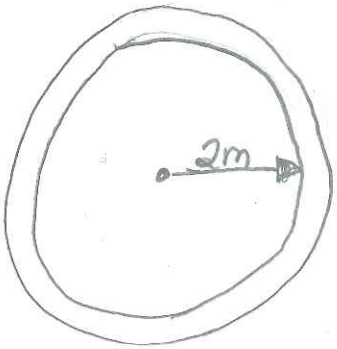
$I_{SS} = \frac{1}{2} MR^2$

$I_{SS} = \frac{1}{2}(5)(2^2)$

$I_{SS} = \frac{1}{2} 20 = \boxed{10\ kgm^2}$

26) A hollow hoop has a mass of 5 kg and a radius of 2 m. What is its moment of inertia about its axis of symmetry? (20 kgm^2)

$M = 5\,Kg$



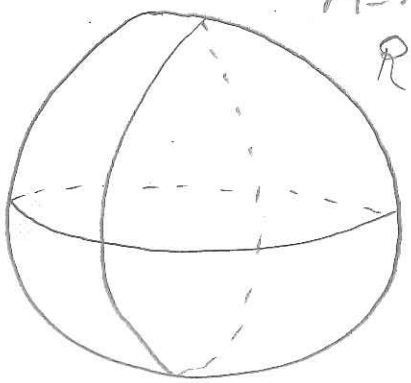
$I = MR^2$

$I = 5(2)^2$

$I = 5 \cdot 4$

$\boxed{I = 20\ Kgm^2}$

27) A solid sphere has a mass of 5 kg and a radius of 2 m. What is its moment of inertia about its axis of symmetry? (8 kgm^2)



$M = 5 kg$

$R = 2m$

$I = \frac{2}{5} MR^2$

$I = \frac{2}{5}(5)(2)^2$

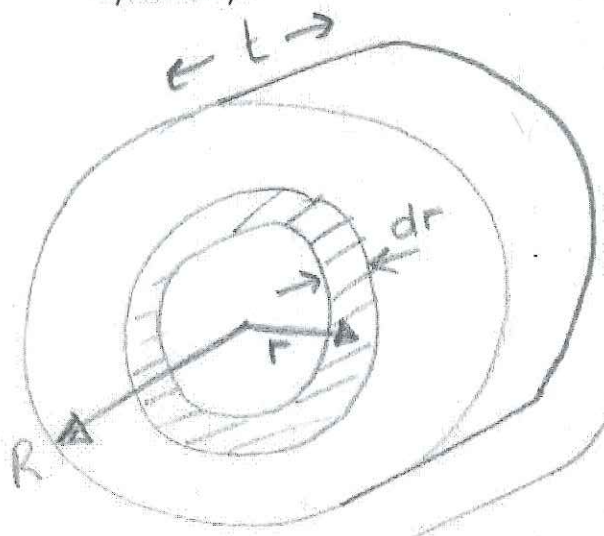
$I \frac{2}{5}(5)(4)$

$I = 2(4)$

$\boxed{I = 8 kgm^2}$

Moment of inertia of regularly shaped objects using Calculus

28) Using the method of calculus shown the our physics lecture class determine the moment of inertia about its axis of circular symmetry of a solid cylinder has a mass of 4 kg and a radius of 3 m and a thickness of 6 m. What is its moment of inertia about its axis of symmetry?



$$I = \int r^2 \, dm$$

$$dm = \rho_{vol} \, (\text{Vol of sleeve})$$

$$dm = \frac{M_{ss}}{V_{ss}} \, (\text{Vol of sleeve})$$

$$dm = \frac{M}{\pi R^2 t} \left((2\pi r)(t)(dr) \right)$$

$M = 4\,kg$

$R = 3\,m$

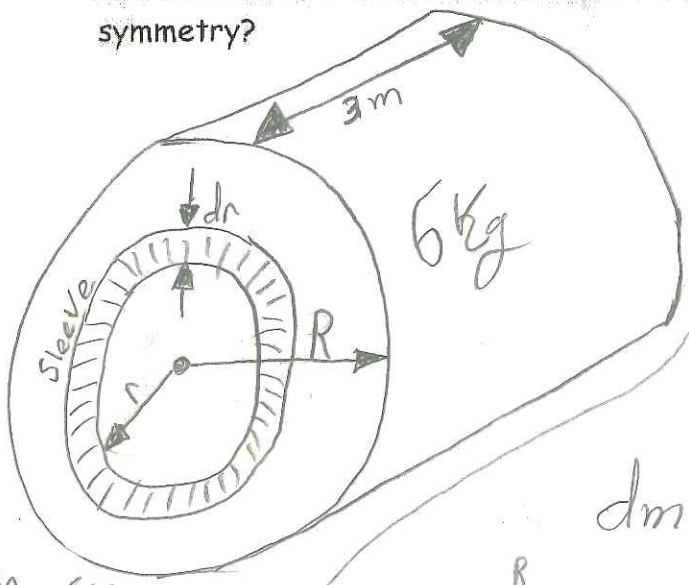
$$dm = \frac{2M}{R^2} r \, dr$$

$$I = \int_0^R r^2 \, dm = \int_0^R r^2 \left(\frac{2M}{R^2} r \, dr \right)$$

$$I = \frac{2M}{R^2} \int_0^R r^3 \, dr = \frac{2M}{R^2} \left[\frac{r^4}{4} \right]_0^R = \frac{2M}{R^2} \left\{ \frac{R^4}{4} - \frac{0^4}{4} \right\}$$

$$I = \frac{2MR^4}{4R^2} = \frac{1}{2} MR^2 = \frac{1}{2} \left[(4)(3^2) \right] = \boxed{18 \text{ kgm}^2}$$

29) Using the method of calculus shown the our physics lecture class determine the moment of inertia about its axis of circular symmetry of a solid cylinder has a mass of 6 kg and a radius of 2 m and a thickness of 3 m. What is its moment of inertia about its axis of symmetry?



$M = 6Kg$

$R = 2m$

$$I = \int r^2 dm$$

$$dm = \rho_{vol} \text{ (Volume of Sleeve)}$$

$$dm = \frac{M}{V} \cdot \overbrace{\left((2\pi r)(th)(dr)\right)}$$

$$dm = \frac{M}{\pi R^2 \cdot th} \cdot \left[(2\pi r)(th)(dr)\right]$$

$$dm = \frac{2M\,r\,dr}{R^2}$$

$$I = \int_0^R r^2\left(\frac{2Mr}{R^2}dr\right) \rightarrow I = \frac{2M}{R^2}\int_0^R r^2(r\,dr)$$

$$I = \frac{2M}{R^2}\int_0^R r^3 dr \longrightarrow I = \frac{2M}{R^2}\left[\frac{1}{4}r^4\right]_0^R$$

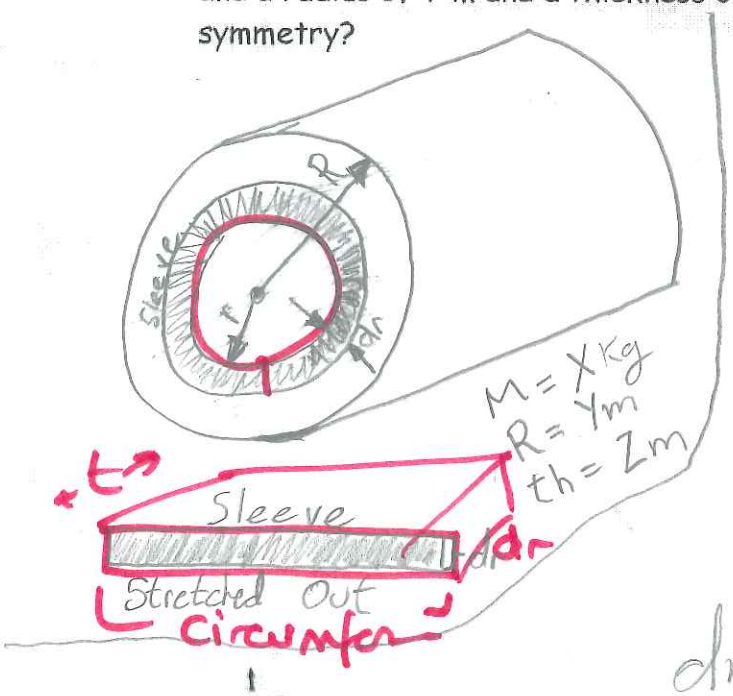
$$I = \frac{2M}{4R^2}\left[R^4 - 0\right] \rightarrow I = \frac{1M}{2R^2} \cdot R^4$$

$$I = \frac{1}{2}MR^2 \longrightarrow I = \frac{1}{2}(6)(2)^2$$

$$\boxed{I = 12\,Kgm^2}$$

?

30) Using the method of calculus shown the our physics lecture class determine the moment of inertia about its axis of circular symmetry of a solid cylinder has a mass of X kg and a radius of Y m and a thickness of Z m. What is its moment of inertia about its axis of symmetry?



$$I = \int_0^R r^2\,dm$$

$$dm = \rho\,\text{Vol} \cdot (\text{Vol of Sleeve})$$

$$dm = \frac{M}{V} \cdot \underbrace{(2\pi r \cdot th \cdot dr)}_{\text{why } 2\pi r th?}$$

when $\text{Vol} = \pi r^2 th$

$$dm = \frac{M}{\cancel{\pi} R^2 \cancel{th}} \cdot (2\cancel{\pi} r \cdot \cancel{th} \cdot dr)$$

$$dm = \frac{2Mr}{R^2}\,dr$$

$$I = \int_0^R r^2\left(\frac{2Mr}{R^2}\,dr\right) = \quad I = \frac{2M}{R^2}\int_0^R r^3\,dr$$

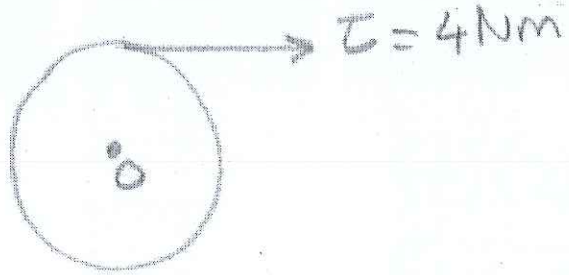
$$I = \frac{2M}{R^2}\left[\frac{1}{4}r^4\right]_0^R \rightarrow I = \frac{2M}{4R^2}\left[R^4 - 0^4\right]$$

$$I = \frac{1M}{2\cancel{R^2}} \cdot \cancel{R^4} \rightarrow I = \frac{1}{2}MR^2$$

$$\boxed{I = \frac{1}{2}XY^2\,Kgm^2}$$

Torque, Moment of inertial and Angular acceleration

31) A solid cylinder is acted on by a torque of 4 Nm. The disk has a moment of inertia of 5 kgm^2. What is the resulting angular acceleration of the disk? (0.8 rad/s^2)



$\tau = I\alpha$

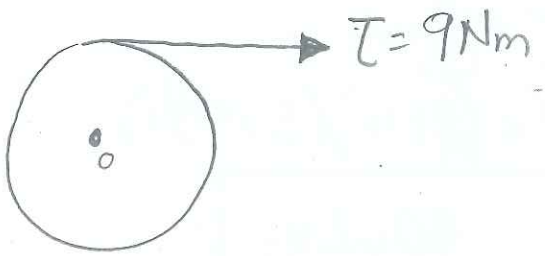
$4 = 5\alpha$

$\alpha = \frac{4}{5} = \boxed{0.8\ rad/s^2}$

$I = 5\ kgm^2$

$\alpha = ?$

32) A solid cylinder is acted on by a torque of 9 Nm. The disk has a moment of inertia of 18 kgm^2. What is the resulting angular acceleration of the disk? (0.5 rad/s^2)



$\tau = I\alpha$

$\alpha = \frac{\tau}{I}$

$\alpha = \frac{9Nm}{18kgm^2}$

$\boxed{\alpha = \frac{1}{2}\ rad/s^2}$

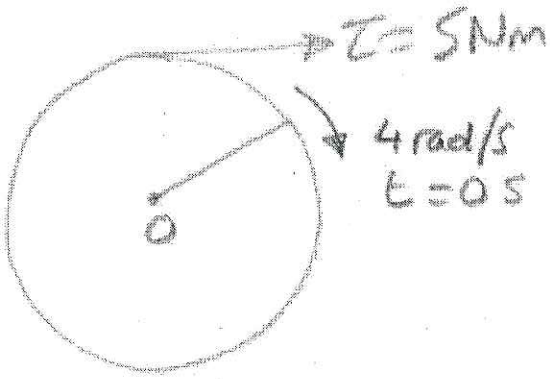
$\tau = 9Nm$

$I = 18\ kgm^2$

$\alpha = ?$

Power, Torque and Angular velocity

33) A solid cylinder is acted on by a torque of 5 Nm. At an instant in time it has an angular velocity of 4 rad/s. What power is dissipated at this instant? (20W)

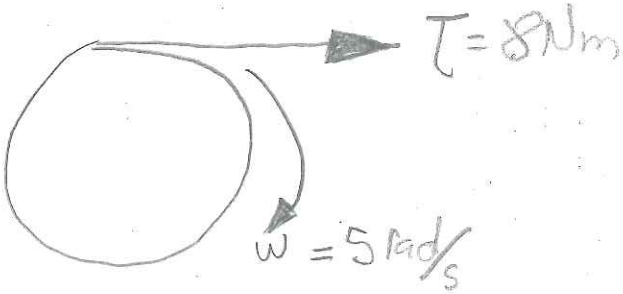


$P = \tau \cdot \omega$ (Like $P = F \cdot v$)

$P = (5)(4)$

$P = \boxed{20W}$

34) A solid cylinder is acted on by a torque of 8 Nm. At an instant in time it has an angular velocity of 5 rad/s. What power is dissipated at this instant? (40W)



$P = \tau \cdot \omega$

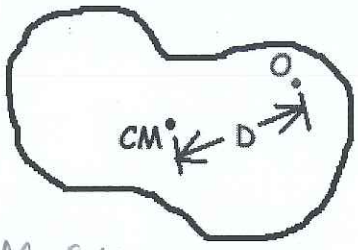
$P = (8Nm)(5\ rad/s)$

$\boxed{P = 40\ Watts}$

48 Assignments Chapter 11

Parallel axis Theorem

1) A 6kg object's moment of inertia about an axis that passes through its center of mass is 12kgm^2. What is the objects moment of inertia about a parallel axis that is 2m away from the original axis? (36kgm^2)



$M = 6\,Kg$

$I_{cm} = 12\,Kgm^2$

$D = 2m$

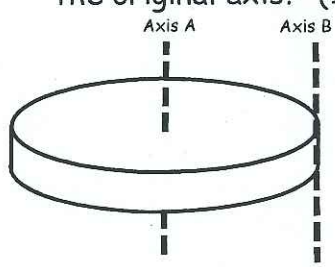
$I_o = I_{cm} + MD^2$

$I_o = 12 + 6(2)^2$

$I_o = 12 + 24$

$I_o = 36\,Kgm^2$

2) A 4kg object's moment of inertia about an axis that passes through its center of mass is 9kgm^2. What is the objects moment of inertia about a parallel axis that is 5m away from the original axis? (109kgm^2)



$I_{cm} = 9\,Kgm^2 = I_A$

$M = 4\,Kg$

$D = 5m$

$I_o = I_{cm} + MD^2$

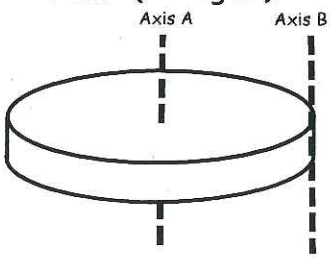
$I_B = I_A + MD^2$

$I_B = 9 + 4(5)^2$

$I_B = 9 + 100$

$I_B = 109\,Kgm^2$

3) A 9kg solid cylinder has a radius of 4m. What is its moment of inertia about a point on its rim? (216kgm^2)



$M = 9\,kg$

$R = 4m$

$I_{cm}\ Cyl = \frac{1}{2}MR^2$

$D = R = 4M$

$I_o = I_{cm} + MD^2$

$I_o = \left(\frac{1}{2}MR^2\right) + MR^2$

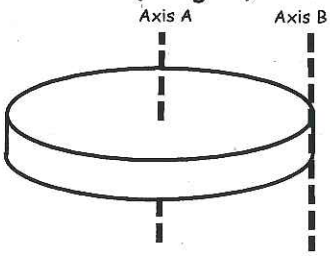
$I_o = \frac{1}{2}(9)(4)^2 + (9)(4)^2$

$I_o = \frac{1}{2}(9)(16) + (9)(16)$

$I_o = 72 + 144$

$\boxed{I_o = 216\,kgm^2}$

4) A 3kg solid cylinder has a radius of 2m. What is its moment of inertia about a point on its rim? (18kgm^2)



$M = 3\,kg$

$R = 2m$

$I_o = I_{cm} + MD^2$

$I_o = \left(\frac{1}{2}MR^2\right) + MR^2$

$I_o = \frac{1}{2}(3)(2)^2 + (3)(2)^2$

$I_o = 6 + 12$

$\boxed{I_o = 18\,kgm^2}$

Rolling Motion of a Rigid Object

5) A 3kg object rolls along a surface. Its center of mass has a translational velocity of 4 m/s. The object has a moment of inertia of 6 kgm^2 about its center of mass. It rolls with an angular velocity of 5 radians/s about its center of mass. What is its total kinetic energy? (99 J)

$M = 3Kg$

$V = 4 m/s$

$I = 6Kgm^2$

$\omega = 5 \, rad/s$

$KE_T = ?$

$KE_T = KE_{Tran} + KE_{Rot}$

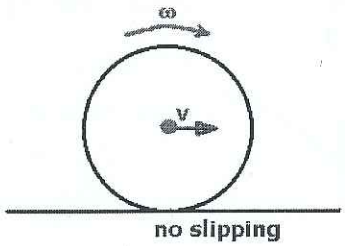
$KE_T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$KE_T = \frac{1}{2}(3)(4)^2 + \frac{1}{2}(6)(5)^2$

$KE_T = \frac{1}{2}(48) + \frac{1}{2}(150$

$KE_T = 99 J$

6) A 2 kg object rolls along a surface. Its center of mass has a translational velocity of 6 m/s. The object has a moment of inertia of 3 kgm^2 about its center of mass. It rolls with an angular velocity of 2 radians/s. about its center of mass. What is its total kinetic energy? (42 J)



$M = 2Kg$

$V = 6 m/s$

$I_{cm} = 3Kgm^2$

$\omega = 2 \, rad/s$

$KE_T = ?$

$KE_T = KE_{Trans} + KE_{rot}$

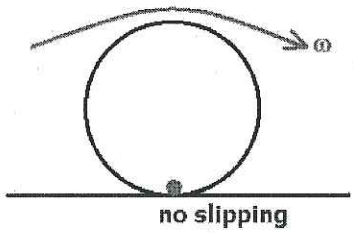
$KE_T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$KE_T = \frac{1}{2}(2)(6)^2 + \frac{1}{2}(3)(2)^2$

$KE_T = 36 + 6$

$KE_T = 42 J$

7) An object rolls along a surface. The object has a moment of inertia of 8kgm^2 about the point where it touches the surface. The object has an angular velocity of 5radians/s about the point where it touches the surface. What is its total kinetic energy? (100J)



$I = 8 kgm^2$

$\omega = 5 rad/s$

$KE = \frac{1}{2} I \omega^2$

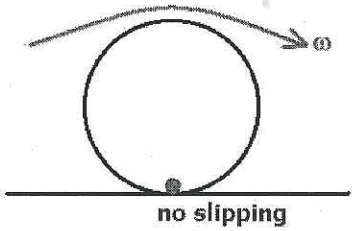
$KE = \frac{1}{2}(8)(5)^2$

$KE = \frac{1}{2}(8)(25)$

$KE = \frac{1}{2}(200)$

$KE = 100 J$

8) An object rolls along a surface. The object has a moment of inertia of 4kgm^2 about the point where it touches the surface. The object has an angular velocity of 2radians/s about the point where it touches the surface. What is its total kinetic energy? (8J)



$I = 4 kgm^2$

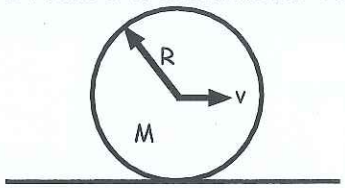
$\omega = 2 rad/s$

$KE = \frac{1}{2}(4)(2)^2$

$KE = \frac{1}{2}(4)(4)$

$KE = 8 J$

9) A solid cylinder rolls along a surface. The cylinder has a mass of 4kg and a radius of 3m. It has a translational velocity of 8m/s. What is its total kinetic energy? (192J)



$M = 4kg$

$R = 3m$

$V = 8 m/s$

$$KE_T = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$KE_T = \frac{1}{2}(4)(8)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{V}{R}\right)^2$$

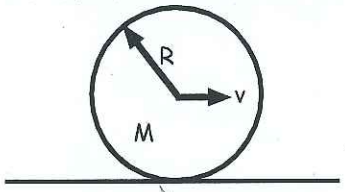
$$KE_T = \frac{1}{2}(4)(64) + \frac{1}{4}(MR^2)\left(\frac{V^2}{R^2}\right)$$

$$KE_T = \frac{1}{2}MV^2 + \frac{1}{4}MV^2$$

$$KE_T = \frac{1}{2}(4)(64) + \frac{1}{4}(4)(64)$$

$$\boxed{KE_T = 192J}$$

10) A solid cylinder rolls along a surface. The cylinder has a mass of 6kg and a radius of 2m. It has a translational velocity of 4m/s. What is its total kinetic energy? (72J)



$M = 6kg$

$R = 2m$

$V = 4 m/s$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$KE = \frac{1}{2}6 \cdot 16 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{V^2}{R^2}\right)$$

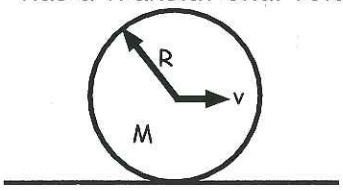
$$KE = 48 + \frac{1}{4}MV^2$$

$$KE = 48 + 1.5(16)$$

$$KE = 48 + 24$$

$$\boxed{KE = 72J}$$

11) A hollow hoop rolls along a surface. The hoop has a mass of 4kg and a radius of 3m. It has a translational velocity of 8m/s. What is its total kinetic energy? (256J)



$M = 4KG$

$R = 3m$

$V = 8\frac{m}{s}$

$KE_T = KE_{ROT} + KE_{TRAN}$

$KE_T = \frac{1}{2}I\omega^2 + \frac{1}{2}MV^2$

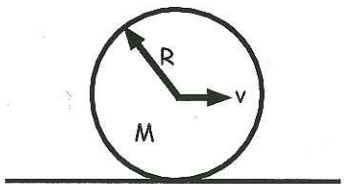
$KE_T = \frac{1}{2}(MR^2)(\frac{V}{R})^2 + \frac{1}{2}MV^2$

$KE_T = \frac{1}{2}\frac{M\cancel{R^2}V^2}{\cancel{R^2}} + \frac{1}{2}MV^2$

$KE_T = \frac{1}{2}MV^2 + \frac{1}{2}MV^2$

$KE_T = MV^2 \longrightarrow \boxed{256\ J}$

12) A hollow hoop rolls along a surface. The hoop has a mass of 6kg and a radius of 2m. It has a translational velocity of 4m/s. What is its total kinetic energy? (96J)



$M = 6Kg$

$R = 2m$

$V = 4m/s$

$KE_T = KE_{Tran} + KE_{Rot}$

$KE_T = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$

$KE_T = \frac{1}{2}MV^2 + \frac{1}{2}(MR^2)(\frac{V}{R})^2$

$KE_T = \frac{1}{2}MV^2 + \frac{1}{2}\frac{M\cancel{R^2}V^2}{\cancel{R^2}}$

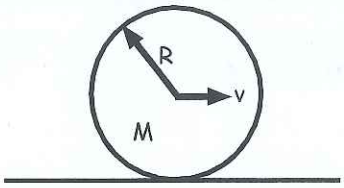
$KE_T = \frac{1}{2}MV^2 + \frac{1}{2}MV^2$

$KE_T = MV^2$

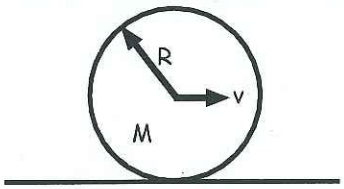
$KE_T = (6)(16)$

$\boxed{KE_T = 96\ J}$

13) A solid sphere rolls along a surface. The sphere has a mass of 4kg and a radius of 3m. It has a translational velocity of 8m/s. What is its total kinetic energy? (179.2J)

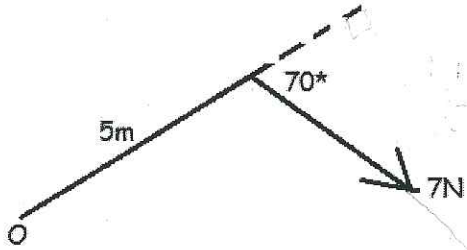


14) A solid sphere rolls along a surface. The sphere has a mass of 6kg and a radius of 2m. It has a translational velocity of 4m/s. What is its total kinetic energy? (67.2J)



Torque, Force and lever arm

15) What is the torque about the turning axis o shown below? (-32.9NM - negative because turning tendency is clockwise)



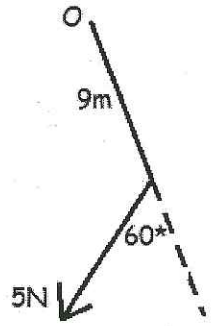
$\tau = F \cdot r \cdot \sin\phi$

$\tau = (7)(5)\sin 70$

$\tau = 35 \sin 70$

$\boxed{\tau = -32.9 \text{ Nm}}$

16) What is the torque about the turning axis o shown below? (-39.0 Nm - negative because clockwise)

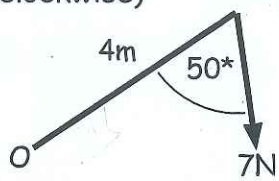


$\tau = Fr \sin\phi$

$\tau = (-5)(9)\sin 60$
clock

$\boxed{\tau = -39 \text{ Nm}}$

17) What is the torque about the turning axis o shown below? (-21.5 Nm – negative because clockwise)



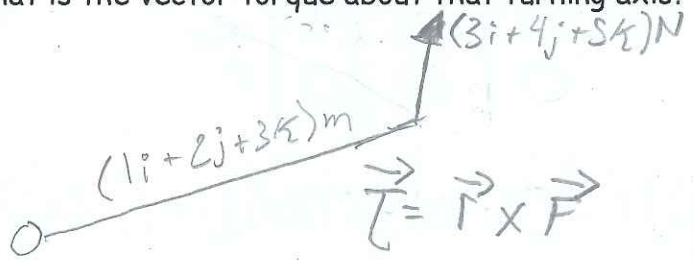
$\tau = Fr \sin\phi$

$\tau = (-7)(4)\sin 50$

clock

$\boxed{\tau = -21.5 \text{ Nm}}$

18) A force of (3i+4j+5k) N acts at a displacement of (1i+2j+3k) m from a turning axis O. What is the vector torque about that turning axis? ((-2i+4j-2k) Nm)



$\vec{\tau} = \vec{r} \times \vec{F}$

i	j	k	
1	2	3	$\vec{r}$
3	4	5	$\vec{F}$

$\vec{\tau} = [(2)(5)-(3)(4)]i - [(1)(5)-(3)(3)]j + [(1)(4)-(2)(3)]k$

$\vec{\tau} = [10-12]i - [5-9]j + [4-6]k$

$\boxed{\vec{\tau} = (-2i+4j-2k)\text{ Nm}}$

19) A force of (0i+4j+0k) N acts at a displacement of (1i+0j+0k) m from a turning axis O. What is the vector torque about that turning axis? ((0i+0j+4k) Nm)

20) A force of 6N acts in the positive Y direction. The force acts at a displacement of (2i+3j+4k) m from a turning axis O. What is the vector torque about that turning axis? ((-24i+0j+12k) Nm)

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\begin{array}{ccc|c} i & j & k & \\ 2 & 3 & 4 & \vec{r} \\ 0 & 6 & 0 & \vec{F} \end{array}$$

$$\vec{\tau} = [(3)(0) - (4)(6)]i - [(2)(0) - (4)(0)]j + [(2)(6) - (3)(0)]k$$

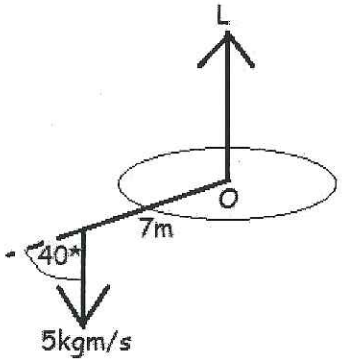
$$\vec{\tau} = [0 - 24]i - [0 - 0]j + [12 - 0]k$$

$$\boxed{\vec{\tau} = (-24i + 0j + 12k)\,Nm}$$

21) A force of (3i+2j+1k) N acts a displacement of 6m along the positive X-axis from a turning axis O. What is the vector torque about that turning axis? ((0i-6j+12k) Nm)

Angular Momentum of a Block

22) A block moves across a frictionless table. It is tethered to a turning axis O by a 7m length of massless string. It has a momentum of 5kgm/s in the plane of the table. What is this block's angular momentum about the turning axis O? (+22.5 positive because it moves clockwise)



$R = 7m$

$p = 5 \frac{kgm}{s}$

$L = I\omega \rightarrow (MR^2)(\frac{V}{R}) \rightarrow MRV$

$L = MRV \rightarrow R \cdot MV \rightarrow R \cdot P$

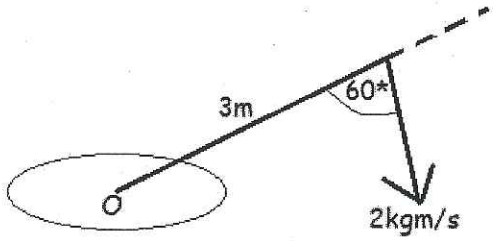
$L = R \cdot P(\sin\phi)$

$L = (7m)(5 \frac{kgm}{s}) \sin 40$

c-clock

$L = 22.5 \, kg \frac{m^2}{s}$

23) A block moves across a frictionless table. It is tethered to a turning axis O by a 3m length of massless string. It has a momentum of 2kgm/s in the plane of the table. What is this block's angular momentum about the turning axis O? (-5.2 negative because it moves clockwise)



$r = 3m$

$p = 2 kgm/s$

$L = r p \sin\phi$

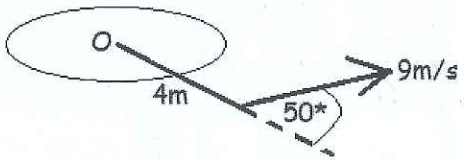
$L = (3)(-2) \sin 60°$

clock

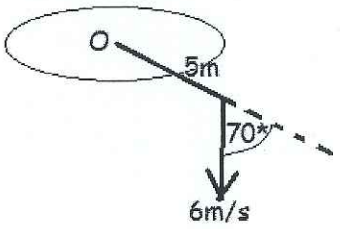
$L = (-6) \sin 60°$

$L = -5.2 \, kg \frac{m^2}{s}$

24) A 3kg block moves across a frictionless table. It is tethered to a turning axis O by a 4m length of massless string. It has a velocity of 9m/s in the plane of the table. What is this block's angular momentum about the turning axis O? (+82.7 positive because it moves counter clockwise)



25) A 4kg block moves across a frictionless table. It is tethered to a turning axis O by a 5m length of massless string. It has a velocity of 6m/s in the plane of the table. What is this block's angular momentum about the turning axis O? (-112.8 negative because it moves clockwise)



26) A particle has a linear momentum of (2i+3j+4k) kgm/s. It has a displacement of (3i+4j+5k)m from a turning axis O. What is the particle's angular momentum about the turning axis? (1i-2j+1k)

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\begin{array}{ccc|c} i & j & k & \\ 3 & 4 & 5 & \vec{r} \\ 2 & 3 & 4 & \vec{p} \end{array}$$

$$\vec{L} = [(4)(4)-(5)(3)]i - [(3)(4)-(5)(2)]j + [(3)(3)-(4)(2)]k$$

$$\vec{L} = [16-15]i - [12-10]j + [9-8]k$$

$$\boxed{\vec{L} = (1i - 2j + 1k)\ \frac{kg\,m^2}{s}}$$

27) A particle has a linear momentum of (0i+2j+0k) kgm/s. It has a displacement of (3i+0j+0k)m from a turning axis O. What is the particle's angular momentum about the turning axis? (0i-0j+6k)

Net Torque and Angular Momentum

28) A solid cylinder has an angular momentum of 5 kgm^2/s. After 8 seconds it has an angular momentum of 9 kgm^2/s. What net external torque caused this change? (0.5 Nm)

$$\sum \tau = \frac{dL}{dt}$$

$L = 5 \frac{kgm^2}{s}$

$T = 8s$

$\downarrow$

$L = 9 \frac{kgm^2}{s}$

$$\sum \tau = \frac{L_F - L_i}{T_F - T_i} = \frac{9-5}{8-0}$$

$$\sum \tau = \frac{4}{8} \rightarrow \boxed{\sum \tau = \frac{1}{2} Nm}$$

29) A solid cylinder has an angular momentum of 9 kgm^2/s. After 6 seconds it has an angular momentum of 15 kgm^2/s. What net external torque caused this change? (1.0 Nm)

Torque, moment of inertia and angular acceleration

30) A solid cylinder is acted on by a net external torque of 6 Nm. The solid cylinder has a moment of inertia of 4 kgm^2. What is the angular acceleration of the disk? (1.5 rad/s^2)

$\sum \tau = 6 Nm$

$I = 4 kgm^2$

$\alpha = ?$

$\sum F = M \cdot a \quad \therefore \quad \sum \tau = I \cdot \alpha$

$\alpha = \frac{\sum \tau}{I}$

$\alpha = \frac{6 Nm}{4 \frac{kgm^2}{s}}$

$\boxed{\alpha = 1.5 \frac{rad}{s^2}}$

31) A solid cylinder is acted on by a net external torque of 8 Nm. The solid cylinder has a moment of inertia of 2 kgm^2. What is the angular acceleration of the disk? (4 rad/s^2)

Angular Momentum

32) A skater with moment of inertia of 6 kgm^2 spins at a rate of 5 rad/s. What is her angular momentum? (30 kgm^2/s)

$I = 6\ kgm^2$

$\omega = 5\ \frac{rad}{s}$

$L = ?$

$p = m \cdot v \therefore L = I \cdot \omega$

$L = (6)(5)$

$L = 30\ \frac{kgm^2}{s}$

33) A 6kg skater with moment of inertia of 9 kgm^2 spins at a rate of 2 rad/s. What is her angular momentum? (18 kgm^2/s)

$M = 6\ kg$

$I = 9\ kgm^2$

$\omega = 2\ \frac{rad}{s}$

Conservation of angular momentum

34) A skater with a moment of inertia of 4 kgm^2 spins with a rate of 6 rad/s. She reduces her moment of inertia to 3 kgm^2 by pulling her arms into her body. What is her new rate of rotation? (8 rad/s)

$$L_i = L_f \rightarrow I_i \cdot \omega_i = I_f \cdot \omega_f$$

$I_i = 4\ kgm^2$

$\omega_i = 6\ rad/s$

$I_f = 3\ kgm^2$

$\omega_f = ?$

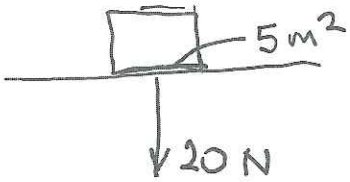
35) A skater with a moment of inertia of 8 kgm^2 spins with a rate of 2 rad/s. She reduces her moment of inertia to 4 kgm^2 by pulling her arms into her body. What is her new rate of rotation? (4 rad/s)

36) A skater with a moment of inertia of 2 kgm^2 spins with a rate of 3 rad/s. She increases her moment of inertia to 4 kgm^2 by pushing her arms away from her body. What is her new rate of rotation? (1.5 rad/s)

Fluids Homework

Pressure

1) A 20 N weight rests on a table. It has a base area of 5 m^2. What pressure does it exert on the table? (4Pa)



$$P = \frac{F}{A} = \frac{20}{5} = \boxed{4\ Pa}$$

2) A 40 N weight rests on a table. It has a base area of 2 m^2. What pressure does it exert on the table?. (20Pa)

3) A 60 N weight rests on a table. It has a base area of 10 m^2. What pressure does it exert on the table?. (6Pa)

4) A 4kg mass rests on a table. It has a base area of 5 m^2. What pressure does it exert on the table?. (8Pa)

5) A 20kg mass rests on a table. It has a base area of 2 m^2. What pressure does it exert on the table?. (100Pa)

6) A 35kg mass rests on a table. It has a base area of 5 m^2. What pressure does it exert on the table?. (70Pa)

7) A 5N weight rests on a table. It has a square base of 2m side. What pressure does it exert on the table?. (1.25 Pa)

8) A 10 N weight rests on a table. It has a square base of 2m side. What pressure does it exert on the table?. (2.5 Pa)

9) A 45 N weight rests on a table. It has a square base of 3m side. What pressure does it exert on the table?. (5 Pa)

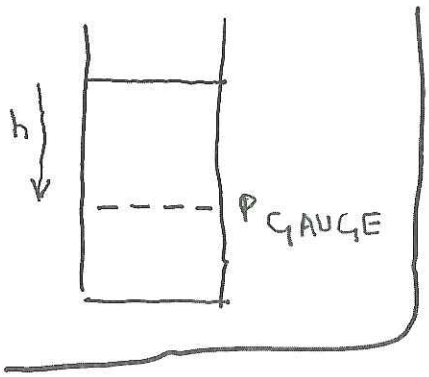
10) A 6 kg mass rests on a table. It has a square base of 2m side. What pressure does it exert on the table?. (15Pa)

11) A 40 kg mass rests on a table. It has a square base of 5m side. What pressure does it exert on the table?. (16Pa)

12) A 90 kg mass rests on a table. It has a square base of 3m side. What pressure does it exert on the table?. (100Pa)

Variation of gauge pressure with depth

13) What is the gauge pressure at a point 3 m below the surface of a liquid that has a density of 5 kg/m^3? (150Pa)



$$P_{GAUGE} = \rho g h$$

$$P_{GAUGE} = (5)(10)(3)$$

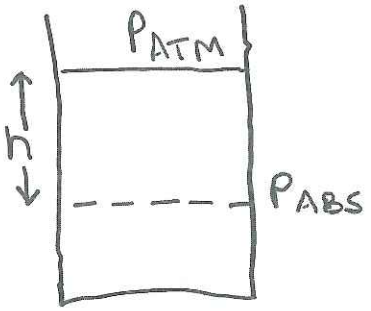
$$P_{GAUGE} = \boxed{150\ Pa}$$

14) What is the gauge pressure at a point 4 m below the surface of a liquid that has a density of 2 kg/m^3? (80Pa)

15) What is the gauge pressure at a point 9 m below the surface of a liquid that has a density of 3 kg/m^3? (270Pa)

Variation of absolute pressure with depth

16) What is the absolute pressure at a point 3 m below the surface of a liquid that has a density of 5 kg/m^3? The atmospheric pressure above the liquid is 40 Pa. (190Pa)



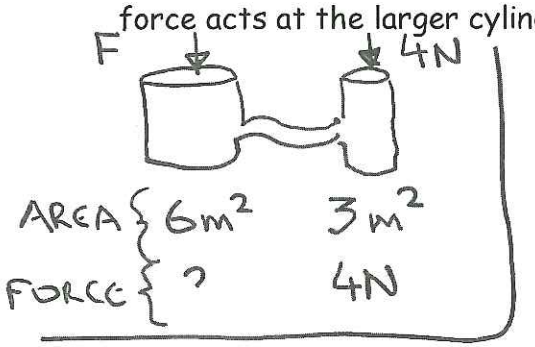
$$P_{ABS} = P_{ATM} + P_{GAUGE}$$
$$P_{ABS} = P_{ATM} + \rho g h$$
$$P_{ABS} = (40) + (5)(10)(3)$$
$$P_{ABS} = (40) + (150) = \boxed{190\ Pa}$$

17) What is the absolute pressure at a point 4 m below the surface of a liquid that has a density of 2 kg/m^3? The atmospheric pressure above the liquid is 25 Pa. (105Pa)

18) What is the absolute pressure at a point 9 m below the surface of a liquid that has a density of 3 kg/m^3? The atmospheric pressure above the liquid is 60 Pa. (330Pa)

Pascal's Law

19) A hydraulic system consists of one cylinder that has an area of $3m^2$, connected to another cylinder that has an area of $6m^2$. A force of 4N is applied to the smaller cylinder. What force acts at the larger cylinder? (8N)



$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{F}{6} = \frac{4}{3}$$

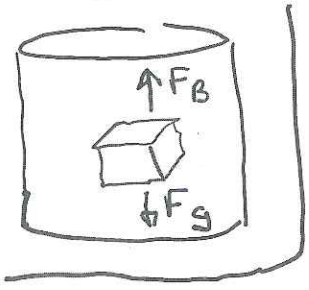
$$F = \frac{(4)(6)}{(3)} = \boxed{8N}$$

20) A hydraulic system consists of one cylinder that has an area of $4m^2$, connected to another cylinder that has an area of $6m^2$. A force of 8N is applied to the smaller cylinder. What force acts at the larger cylinder? (12N)

21) A hydraulic system consists of one cylinder that has an area of $8m^2$, connected to another cylinder that has an area of $2m^2$. A force of 6N is applied to the smaller cylinder. What force acts at the larger cylinder? (24N)

Buoyant force

22) A body that has a volume of 3 m^3 is submerged in a liquid that has a density of 5 kg/m^3. What buoyant force does the body experience? (150N)



$$F_B = \rho_F g V_{DP}$$
$$F_B = (5)(10)(3)$$
$$F_B = \boxed{150\ N}$$

23) A body that has a volume of $6m^3$ is submerged in a liquid that has a density of $4kg/m^3$. What buoyant force does the body experience? (240N)

24) A body that has a volume of $2m^3$ is submerged in a liquid that has a density of $8kg/m^3$. What buoyant force does the body experience? (160N)

25) A body that has a volume of $4m^3$ is submerged in a liquid that has a weight density of $5N/m^3$. What buoyant force does the body experience? (20N)

26) A body that has a volume of $3m^3$ is submerged in a liquid that has a weight density of $4N/m^3$. What buoyant force does the body experience? (12N)

27) A body that has a volume of $7m^3$ is submerged in a liquid that has a weight density of $8N/m^3$. What buoyant force does the body experience? (56N)

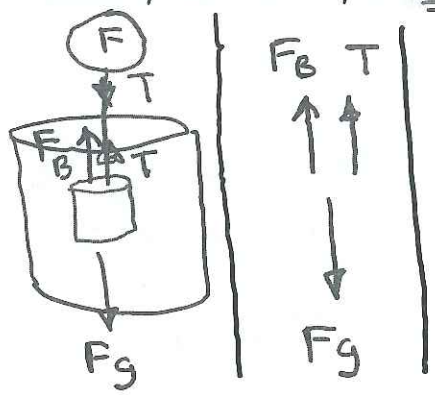
28) Which has the greater buoyant force, a $1m^3$ block of Styrofoam floating on a surface, or a $1m^3$ block of lead that is submerged? (The lead)

weight density mass density

ρ in N/m^3 is like ρg in $\frac{kg}{m^3}\frac{m}{s^2} \Rightarrow N/m^3$

Apparent Weight

29) A body that has a volume of $4m^3$ is submerged in a liquid that has a density of $5N/m^3$. The body has a density of $8N/m^3$. What apparent weight does the body experience? (12N)



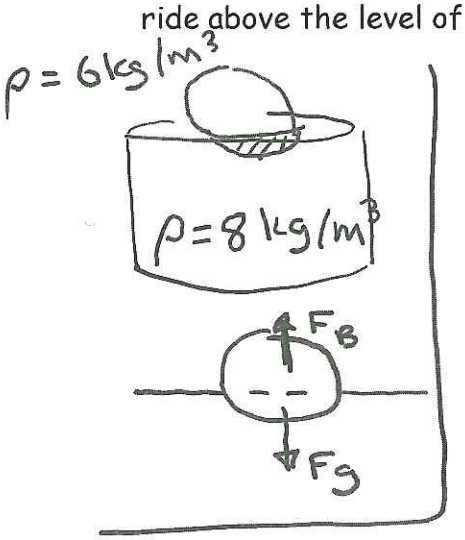
$$\Sigma F_y = 0$$
$$(F_B) + (T) + (-F_g) = 0$$
$$(\rho_F g V_{DF}) + (T) - (\rho_o g V_o) = 0$$
$$T = (\rho_o g V_o - \rho_F g V_{DF})$$
$$T = (8-5)(4) = \boxed{12\ N}$$

30) A body that has a volume of $2m^3$ is submerged in a liquid that has a density of $3N/m^3$. The body has a density of $4N/m^3$. What apparent weight does the body experience? (2N)

31) A body that has a volume of $9m^3$ is submerged in a liquid that has a density of $5N/m^3$. The body has a density of $7N/m^3$. What apparent weight does the body experience? (18N)

Floating

32) A sphere of material floats on the surface of a fluid. The sphere material has a density of 6kg/m^3. The fluid has a density of 8kg/m^3. What percentage of the sphere will ride above the level of the fluid's surface? (25%)



FOR FLOATING $F_B = F_g$

$$\rho_F g V_{DF} = \rho_o g V_o$$

$$\rho_F V_{DF} = \rho_o V_o$$

% BELOW WATER IS V_{DF}/V_o

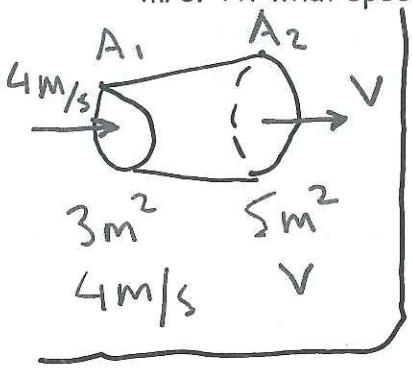
$$\frac{V_{DF}}{V_o} = \frac{\rho_o}{\rho_F} = \frac{6}{8} = 0.75 = 75\%$$

% ABOVE = 100 - % BELOW = 100 - 75 = $\boxed{25\%}$

33) A sphere of material floats on the surface of a fluid. The sphere material has a density of 2kg/m^3. The fluid has a density of 10kg/m^3. What percentage of the sphere will ride above the level of the fluid's surface? (80%)

Continuity Equation

34) A pipe has an entrance of $3m^2$. It has an exit of $5m^2$. Water flows into the pipe at 4 m/s. At what speed does water flow out of the pipe? (2.4m/s)



$$A_1 v_1 = A_2 V_2$$

$$(3)(4) = (5)V$$

$$V = \frac{(3)(4)}{(5)}$$

$$V = \boxed{2.4\ m/s}$$

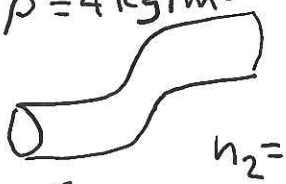
35) A pipe has an entrance of $7m^2$. It has an exit of $2m^2$. Water flows into the pipe at 3m/s. At what speed does water flow out of the pipe? (10.5m/s)

36) A pipe has an entrance of $8m^2$. It has an exit of $20m^2$. Water flows into the pipe at 2m/s. At what speed does water flow out of the pipe? (0.8m/s)

Bernoulli's Equation

37) A pipe has an entrance that has a height of 5m above a table. The pressure of liquid at the pipes entrance is 60 Pa. The speed of the liquid at the pipe's entrance is 3m/s. The liquid's density is 4kg/m^3. The pipe's exit is 6m above the table. The pressure of liquid at the pipes exit is 20Pa. What is the speed of the liquid at the pipe's exit? (3m/s)

$\rho = 4\ kg/m^3$



$h_1 = 5m$ $\quad h_2 = 6m$

$P_1 = 60Pa$ $\quad P_2 = 20Pa$

$V_1 = 3m/s$ $\quad V_2 = ?$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$(60) + \frac{1}{2}(4)(3^2) + (4)(10)(5) = (20) + \frac{1}{2}(4)v^2 + (4)(10)(6)$$

$$(60) + (18) + (200) = (20) + 2v^2 + (240)$$

$$\frac{278 - 260}{2} = v^2 = 9$$

$$v = \boxed{3\ m/s}$$

38) A pipe has an entrance that has a height of 2m above a table. The pressure of liquid at the pipes entrance is 90 Pa. The speed of the liquid at the pipe's entrance is 8m/s. The liquid's density is 5kg/m^3. The pipe's exit is 6m above the table. The pressure of liquid at the pipes exit is 10Pa. What is the speed of the liquid at the pipe's exit? (4m/s)

39) A pipe has an entrance that has a height of 2m above a table. The pressure of liquid at the pipes entrance is 90 Pa. The speed of the liquid at the pipe's entrance is 8m/s. The liquid's density is $5kg/m^3$. The pipe's exit is 6m above the table. The pressure of liquid at the pipes exit is 10Pa. What is the speed of the liquid at the pipe's exit? (4m/s)

Physics & Measurement Homework

Units

1) What is the standard unit of mass? Kilogram

2) What is the standard unit of length? Meter

3) What is the standard unit of time? Second

4) What is the standard unit of velocity? meter per Second

5) What is the standard unit of area? $meter^2$

6) What is the standard unit of volume? $meter^3$

Prefixes

7) Express 4mm in standard units. 4×10^{-3} m

8) Express 5μm in standard units. 5×10^{-6} m

9) Express 3μg in standard units. 3×10^{-6} g $= 3 \times 10^{-9}$ Kg

10) Express 12ns in standard units. 12×10^{-9} s

11) Express 7mg in standard units. 7×10^{-6} Kg

12) Express $7km^2$ in standard units. 7×10^{6} m^2

13) Express $7cm^2$ in standard units. 7×10^{-4} m^2

14) Express $12Mm^3$ in standard units. 12×10^{18} m^3

15) Express $5\mu m^3$ in standard units. 5×10^{-18} m^3

16) Express $3nm^3$ in standard units. 3×10^{-27} m^3

Dimensional Analysis

17) Given: $J = \frac{5mv}{x}$, what are the dimensions of J, (m is mass, v is velocity, x is length)?

$$J = \frac{[M][L]}{[T]} \Big/ [L] = \frac{[M]\cancel{[L]}}{[T]} \cdot \frac{1}{\cancel{[L]}} = \boxed{\frac{[M]}{[T]}}$$

18) Given: $J = \frac{m^2 v}{3t}$, what are the dimensions of J, (m is mass, v is velocity, x is length)?

$$J = \frac{[M]^2 \cdot [L]}{[T]} \Big/ [T] = \frac{[M]^2 \cdot [L]}{[T]} \cdot \frac{1}{[T]} = \boxed{\frac{[M]^2 \cdot [L]}{[T]^2}}$$

19) Given: $J = \sqrt{\frac{1}{2}\frac{mv}{t^2}}$, what are the dimensions of J, (m is mass, v is velocity, x is length)?

$$J = \left(\frac{1}{2} \cdot \frac{[M][L]}{[T]} \Big/ [T]^2\right)^{1/2} = \left(\frac{1}{2}\frac{[M][L]}{[T]^3}\right)^{1/2} = \boxed{\left(\frac{1}{2}\right)^{1/2} \frac{[M]^{1/2}[L]^{1/2}}{[T]^{3/2}}}$$

20) Given: $J = \frac{m}{v\sqrt{t}}$, what are the dimensions of J, (m is mass, v is velocity, x is length)?

$$J = \frac{[M]}{\frac{[L]}{[T]} \cdot [T]^{1/2}} = \frac{[M]}{\frac{[L]}{[T]^{1/2}}} = [M] \cdot \frac{[T]^{1/2}}{[L]} = \boxed{\frac{[M][T]^{1/2}}{[L]}}$$

21) If a tomato's linear dimensions are tripled, by what factor will it's volume change?

tomato's factor will change by x8

22) If an asteroid's linear dimension is doubled, by what factor will its surface area change?

Asteroid factor will change by x4

23) If a cylinder of rock's linear dimension is tripled, by what factor will its mass change?

factor will increase by x8

24) If a spider's linear dimension is halved, by what factor will its volume change?

Lin dim. $(\frac{1}{2})$

Area $(\frac{1}{2})(\frac{1}{2})$

Volume $(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$ → factor will change by $\times \frac{1}{8}$

Conversion of Units

25) Using the factor label method convert 5 peck/nap into m/s (where 4 peck equals 1 meter and 2 nap equals 1 second).

$$\frac{5\ \text{peck}}{\text{nap}} \cdot \frac{2\ \text{nap}}{1\ \text{sec}} \cdot \frac{1\ \text{m}}{4\ \text{peck}} \cdot \frac{10\ \text{m}}{4\ \text{sec}} \cdot \frac{5\ \text{m}}{2\ \text{sec}} = \boxed{2.5\ \frac{\text{m}}{\text{sec}}}$$

26) Using the factor label method convert 7 peck/nap into m/s (where 9 meter equals 1 peck and 3 nap equals 1 second).

$$\frac{7\ \text{peck}}{\text{nap}} \cdot \frac{9\ \text{m}}{1\ \text{peck}} \cdot \frac{3\ \text{nap}}{1\ \text{sec}} \cdot \frac{189\ \text{m}}{1\ \text{sec}} = \boxed{189\ \frac{\text{m}}{\text{sec}}}$$

27) Using the factor label method convert 8 peck/nap into m/s (where 4 meter equals 1 peck and 5 nap equals 2 second).

$$\frac{8\ \text{peck}}{\text{nap}} \cdot \frac{4\ \text{m}}{\text{peck}} \cdot \frac{5\ \text{nap}}{2\ \text{sec}} \cdot \frac{160\ \text{m}}{2\ \text{sec}} = \boxed{60\ \frac{\text{m}}{\text{s}}}$$

28) Using the factor label method convert 8 peck/nap into mm/min (where 1 meter equals 4 peck and 1 nap equals 12 second).

$$\frac{8\ \text{peck}}{\text{nap}} \cdot \frac{1\ \text{m}}{4\ \text{peck}} \cdot \frac{1\ \text{nap}}{12\ \text{sec}} \cdot \frac{60\ \text{sec}}{1\ \text{min}} \cdot \frac{1000\ \text{mm}}{1\ \text{m}} = \frac{480{,}000}{48}\ \frac{\text{mm}}{\text{min}} = \boxed{10{,}000\ \frac{\text{mm}}{\text{min}}}$$

Significant Figures

29) How many significant figures are there in 1002 kg?
4 sf

30) How many significant figures are there in 1002.0 kg?
5 sf

31) How many significant figures are there in 0.01002 kg?
4 sf

32) How many significant figures are there in 0.010020 kg?
5 sf

33) How many significant figures are there in 1002000 kg?
4 sf

Significant Digit Calculations

34) Calculate 542.35 m divided by 41.2 seconds, stating the answer with appropriate precision.

$$\frac{542.35\ \text{m}}{41.2\ \text{s}} = \boxed{13.2\ \frac{\text{m}}{\text{s}}}$$

35) Calculate 542.350 m divided by 41.2 seconds, stating the answer with appropriate precision.

$$\frac{542.350\,m}{41.2\,s} = \boxed{13.2\ \frac{m}{s}}$$

36) Calculate 542.35 m divided by 41.20 seconds, stating the answer with appropriate precision.

$$\frac{542.35\,m}{41.20\,s} \quad \boxed{13.16\ \frac{m}{s}}$$

37) Calculate 625.4 kg plus 29.76 kg. State the answer with appropriate precision.

$$625.4\,kg + 29.76\,kg = \boxed{655.2\,kg}$$

38) Calculate 625.4 kg plus 29.760 kg. State the answer with appropriate precision.

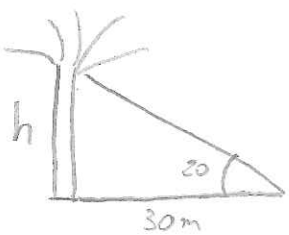
$$625.4\,kg + 29.760 = \boxed{655.2\,kg}$$

39) Calculate 625.40 kg plus 29.76 kg. State the answer with appropriate precision.

$$625.40\,kg + 29.76 = \boxed{655.16\,kg}$$

Tigonometry

40) What is the height of a tree that subtends an angle of 20 degrees at a point on the ground that is 30 m away from its base?

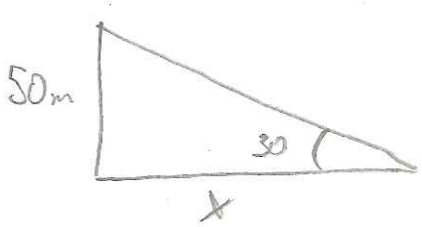


$$\tan\theta = \frac{h}{30m}$$

$$(30\,m)(\tan 20) = h$$

$$h = 10.9$$

41) How far away from point P is a 50 m tall tree that subtends an angle of 30 degrees with a point P which is on the ground?

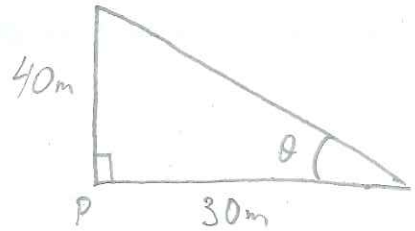


$$\tan(30) = \frac{50\,m}{X}$$

$$X = \frac{50\,m}{\tan(30)}$$

$$\boxed{X = 86.6\,m}$$

42) A near side of a building is 40 m tall. The distance from point P, which is on level ground, to the building's base, is 30m. What angle does the side of the building subtend at point P?



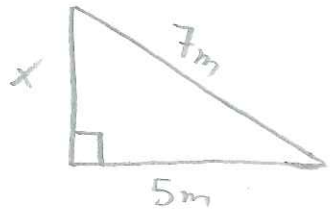
$$\tan(\theta) = \frac{40\,m}{30\,m}$$

$$\tan^{-1}(\tan(\theta)) = \tan^{-1}\left(\frac{40\,m}{30\,m}\right)$$

$$\boxed{\theta = 53.1^\circ}$$

<u>Pythagoras</u>

43) What is the length of the missing side of a right triangle if the 'adjacent' is 5 m and the 'hypotenuse' is 7 m?



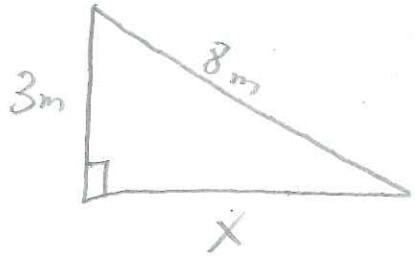
$$x^2 + 5^2 = 7^2$$

$$x^2 = 49 - 25$$

$$x = \sqrt{24}$$

$$\boxed{x = 4.9\,m}$$

44) What is the length of the missing side of a right triangle if the 'opposite' is 3 m and the 'hypotenuse' is 8 m?



$$x^2 + 3^2 = 8^2$$

$$x^2 = 64 - 9$$

$$x = \sqrt{55}$$

$$\boxed{x = 7.4\,m}$$

Geometery

45) What is the area of a circle that has a radius of 3m?

$Area = \pi r^2$

$Area = \pi 9$

$A = 28.3 m^2$

46) What is the circumference of a circle that has a radius of 5m?

$C = 2\pi r$

$C = 2\pi 5$

$C = 31.4 m$

47) What is the volume of a sphere that has a radius of 4m?

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi 64$

$V = 268.1 m^3$

48) What is the surface area of a sphere that has a radius of 2m?

$SA = 4\pi r^2$

$SA = 4\pi 4$

$SA = 50.3 m^2$

49) What is the volume of a right cylinder that has a radius of 3m and a height of 4m?

$V = \pi r^2 h$

$V = \pi 3^2 \cdot 4$

$V = 113.1 m^3$

50) What is the surface area of a right cylinder that has a radius of 2m and a height of 6m?

$SA = 2\pi r h + 2\pi r^2$

$SA = 2\pi 2 \cdot 6 + 2\pi 4 =$ $SA = 100.5 m^2$

Chapter 1 Answers

1) kg
2) m
3) s,
4) m/s
5) m^2
6) m^3
7) $4x10^{-3}m$
8) $5x10^{-6}m$
9) $3x10^{-9}kg$
10) $12x10^{-9}s$
11) $7x10^{-6}kg$
12) $7x10^{6}m^2$
13) $7x10^{-4}m^2$
14) $12x10^{18}m^3$
15) $5x10^{-18}m^3$
16) $3x10^{-27}m^3$
17) $\frac{[M]}{[T]}$
18) $\frac{[M]^2[L]}{[T]^2}$
19) $\frac{[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}}{[T]^{\frac{3}{2}}}$
20) $\frac{[M][T]^{\frac{1}{2}}}{[L]}$
21) The tomato's volume will be 8 times greater.
22) The asteroid's surface area will be 4 times greater.
23) The cylinder of rock's mass will be 8 times greater.
24) The spider's volume will be $1/8^{th}$ of its original volume.
25) 2.5 m/s
26) 189m/s
27) 80m/s
28) 10,000mm/min
29) four
30) five
31) four
32) five
33) four
34) 13.2m/s
35) 13.2m/s
36) 13.16m/s
37) 655.2kg
38) 655.2kg
39) 655.16kg
40) 10.9m
41) 86.6m
42) 53.1°
43) 4.9m
44) 7.4m
45) $28.3m^2$
46) 31.4m
47) $268.1m^3$
48) $50.3m^2$
49) $113.1m^3$
50) $100.5m^2$

Dimensional Analysis

17) Given: $J = \frac{5mv}{x}$, what are the dimensions of J, (m is mass, v is velocity, x is length)?

$$\frac{[M]\frac{[L]}{[T]}}{[L]} = \frac{[M][L]}{[T][L]} = \boxed{\frac{[M]}{[T]}}$$

18) Given: $J = \frac{m^2 v}{3t}$, what are the dimensions of J, (m is mass, v is velocity, t is time)?

$$\frac{[M]^2\frac{[L]}{[T]}}{[T]} = \frac{[M]^2[L]}{[T][T]} = \boxed{\frac{[M]^2[L]}{[T]^2}}$$

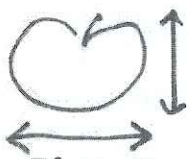
19) Given: $J = \sqrt{\frac{1}{2}\frac{mv}{t^2}}$, what are the dimensions of J, (m is mass, v is velocity, t is time)?

$$\sqrt{\frac{[M]\frac{[L]}{[T]}}{[T]^2}} = \sqrt{\frac{[M][L]}{[T][T]^2}} = \boxed{\frac{[M]^{\frac{1}{2}}[L]^{\frac{1}{2}}}{[T]^{\frac{3}{2}}}}$$

20) Given: $J = \frac{m}{v\sqrt{t}}$, what are the dimensions of J, (m is mass, v is velocity, t is time)?

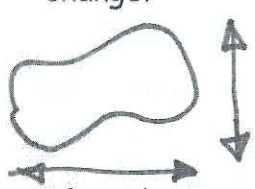
$$\frac{[M]}{\frac{[L]}{[T]}[T]^{\frac{1}{2}}} = \frac{[M][T]}{[L][T]^{\frac{1}{2}}} = \boxed{\frac{[M][T]^{\frac{1}{2}}}{[L]}}$$

21) If a tomato's linear dimensions are tripled, by what factor will it's volume change?



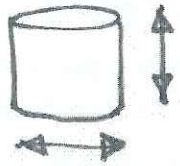
Lin dim x 3
Area x (3)(3)
Volume x (3)(3)(3) → **27 times**

22) If an asteroid's linear dimension is doubled, by what factor will its surface area change?



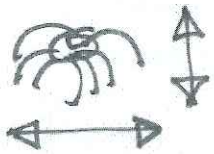
linear dim x 2
Area x (2)(2) → **4 times.**
Volume x (2)(2)(2)

23) If a cylinder of rock's linear dimension is tripled, by what factor will its mass change?



lin. dim x 3
Area x (3)(3)
Vol x (3)(3)(3) → $M \propto Vol$ → x **27 times**

24) If a spider's linear dimension is halved, by what factor will its volume change?



lin dim x $(\frac{1}{2})$
Area $(\frac{1}{2})(\frac{1}{2})$
Vol $(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$ → **x $\frac{1}{8}$ times**

Conversion of Units

25) Using the factor label method convert 5 peck/nap into m/s (where 4 peck equals 1 meter and 2 nap equals 1 second).

$$\frac{5\ \text{peck}}{1\ \text{nap}} \cdot \frac{1\ \text{m}}{4\ \text{peck}} \cdot \frac{2\ \text{nap}}{1\ \text{s}} = \frac{10}{4}\ \text{m/s} = \boxed{2.5\ \text{m/s}}$$

26) Using the factor label method convert 7 peck/nap into m/s (where 9 meter equals 1 peck and 3 nap equals 1 second).

$$\frac{7\ \text{peck}}{1\ \text{nap}} \cdot \frac{9\ \text{m}}{1\ \text{peck}} \cdot \frac{3\ \text{nap}}{1\ \text{s}} = \boxed{189\ \text{m/s}}$$

27) Using the factor label method convert 8 peck/nap into m/s (where 4 meter equals 1 peck and 5 nap equals 2 second).

$$\frac{8\ \text{peck}}{1\ \text{nap}} \cdot \frac{4\ \text{m}}{1\ \text{peck}} \cdot \frac{5\ \text{nap}}{2\ \text{s}} = \boxed{80\ \text{m/s}}$$

28) Using the factor label method convert 8 peck/nap into mm/min (where 1 meter equals 4 peck and 1 nap equals 12 second).

$$\frac{8\ \text{peck}}{1\ \text{nap}} \cdot \frac{1\ \text{m}}{4\ \text{peck}} \cdot \frac{1\ \text{nap}}{12\ \text{s}} \cdot \frac{1000\ \text{mm}}{1\ \text{m}} \cdot \frac{60\ \text{s}}{1\ \text{min}} = \boxed{10{,}000\ \text{mm/min}}$$

Significant Figures

29) How many significant figures are there in 1002 kg? $\boxed{4}$

30) How many significant figures are there in 1002.0 kg? $\boxed{5}$

31) How many significant figures are there in 0.01002 kg? $\boxed{4}$

32) How many significant figures are there in 0.010020 kg? $\boxed{5}$

33) How many significant figures are there in 1002000 kg? $\boxed{4}$

Significant Digit Calculations

34) Calculate 542.35 m divided by 41.2 seconds, stating the answer with appropriate precision.

5 3

$$\frac{542.35}{41.2} = 13.163835 \xrightarrow[\text{Sig dit}]{} \boxed{13.2\ \text{m/s}}$$

35) Calculate 542.350 m divided by 41.2 seconds, stating the answer with appropriate precision.

$$\frac{542.350}{41.2} = 13.163835 \longrightarrow \boxed{13.2\ m/s}$$

36) Calculate 542.35 m divided by 41.20 seconds, stating the answer with appropriate precision.

$$\frac{542.35}{41.20} = 13.163835 \longrightarrow \boxed{13.16\ m/s}$$

37) Calculate 625.4 kg plus 29.76 kg. State the answer with appropriate precision.

$$\begin{array}{r} 625.4 \\ +\ 29.76 \end{array} = 655.16 \xrightarrow[\text{d place}]{} \boxed{655.2\ kg}$$

38) Calculate 625.4 kg plus 29.760 kg. State the answer with appropriate precision.

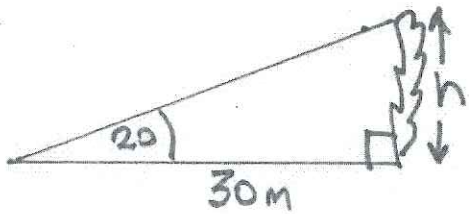
$$\begin{array}{r} 625.4 \\ +\ 29.760 \end{array} = 655.160 \longrightarrow \boxed{655.2\ kg}$$

39) Calculate 625.40 kg plus 29.76 kg. State the answer with appropriate precision.

$$\begin{array}{r} 625.40 \\ +\ 29.76 \end{array} = 655.160 \longrightarrow \boxed{655.16\ kg}$$

Tigonometry

40) What is the height of a tree that subtends an angle of 20 degrees at a point on the ground that is 30 m away from its base?



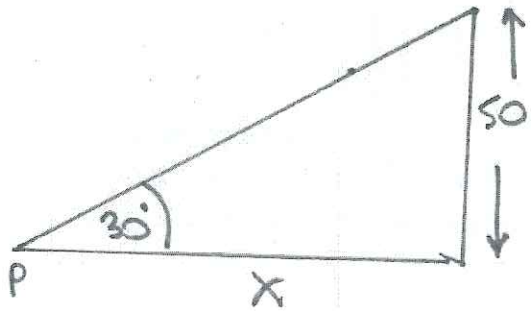
$$\tan\theta = \frac{OPP}{ADJ}$$

$$OPP = ADJ \tan\theta$$

$$OPP = 30 \tan 20^\circ$$

$$\boxed{h = 10.919\ m}$$

41) How far away from point P is a 50 m tall tree that subtends an angle of 30 degrees with a point P which is on the ground?

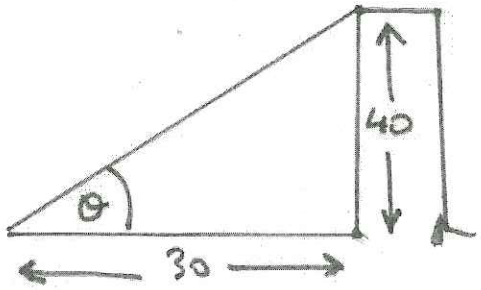


$$\tan\theta = \frac{OPP}{ADJ}$$

$$\cancel{ADJ} = \frac{\cancel{ADJ}\ OPP}{\tan\theta}$$

$$x = \frac{50}{\tan 30} = \boxed{86.603\ m}$$

42) A near side of a building is 40 m tall. The distance from point P, which is on level ground, to the building's base, is 30m. What angle does the side of the building subtend at point P?



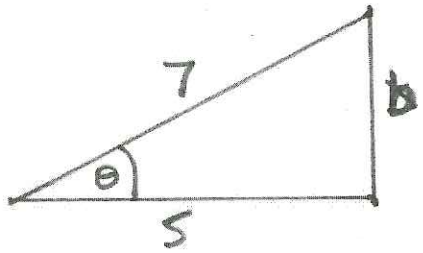
$$\tan\theta = \frac{OPP}{ADJ}$$

$$\theta = \tan^{-1}\left(\frac{OPP}{ADJ}\right) = \tan^{-1}\left(\frac{40}{30}\right)$$

$$\theta = \boxed{53.130^\circ}$$

Pythagoras

43) What is the length of the missing side of a right triangle if the 'adjacent' is 5 m and the 'hypotenuse' is 7 m?



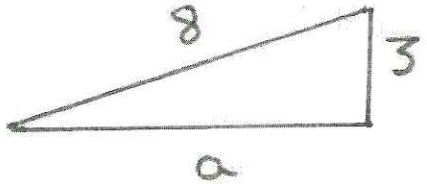
$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = 7^2$$

$$b^2 = 49 - 25 = 24$$

$$b = \sqrt{24} = \boxed{4.899\text{ m}}$$

44) What is the length of the missing side of a right triangle if the 'opposite' is 3 m and the 'hypotenuse' is 8 m?



$$c^2 = a^2 + b^2$$

$$8^2 = a^2 + 3^2$$

$$a^2 = 64 - 9 = 55$$

$$a = \sqrt{55} = \boxed{7.416\text{ m}}$$

Geometery

45) What is the area of a circle that has a radius of 3m?

$r = 3m$

$A = \pi r^2 = \pi(3^2) = 9\pi = \boxed{28.274\ m^2}$

46) What is the circumference of a circle that has a radius of 5m?

$r = 5m$

$C = 2\pi r = 2(\pi)(5) = 10\pi = \boxed{31.416\ m}$

47) What is the volume of a sphere that has a radius of 4m?

$r = 4m$

$V_{sph} = \frac{4}{3}\pi r^3 = \frac{4}{3}(\pi)(4^3) = \frac{256\pi}{3} = 85.33\pi$

$= \boxed{268.083\ m^3}$

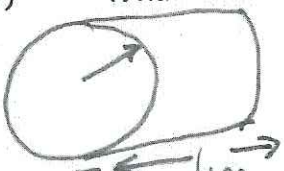
48) What is the surface area of a sphere that has a radius of 2m?

$r = 2m$

$SA_{sph} = 4\pi r^2 = 4(\pi)(2^2) = 16\pi =$

$= \boxed{50.265\ m^2}$

49) What is the volume of a right cylinder that has a radius of 3m and a height of 4m?

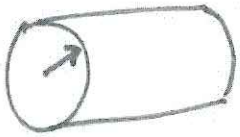


$r = 3m$ $4m$

$V = (\pi r^2)(h) = (\pi)(3^2)(4) = 36\pi$

$= \boxed{113.097\ m^3}$

50) What is the surface area of a right cylinder that has a radius of 2m and a height of 6m?



$r = 2m$

$h = 6m$

$SA_{cyl} = 2(\pi r^2) + (2\pi r)(h)$

$SA_{cyl} = 2(\pi)(2^2) + (2)(\pi)(2)(6)$

$SA_{cyl} = 8\pi + 24\pi = 32\pi = \boxed{100.531\ m^2}$