Steven Romeiro

50 pts

Study Guide 1

1) $A = w \cdot h$

$w = \frac{\pi}{4}$

$h = y = \sin x$

$A = w(h_1 + h_2 + h_3 + h_4)$

$A = w\left(\sin\frac{\pi}{4} + \sin\frac{\pi}{2} + \sin\frac{3\pi}{4} + \sin\pi\right)$

$A = \frac{\pi}{4}\left(\frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2} + 0\right)$

$A = \frac{\pi}{4}(\sqrt{2} + 1)$ $\boxed{A = 1.8961}$

2) $\lim\limits_{x \to 0} \frac{\cos(3x) - 1}{3x}$

x	$-.1$	$-.01$	$-.001$	$.001$	$.01$	$.1$
$f(x)$	$.14888$	$.015$	$.0015$	$-.0015$	$-.015$	$-.1489$

$\boxed{L = 0}$

$y = \frac{13}{x}$

$w = 3$

3) a) $A = w \cdot h$

$A = 3(h_1 + h_2 + h_3 + h_4)$

$A = 3\left(\frac{13}{1} + \frac{13}{4} + \frac{13}{7} + \frac{13}{10}\right)$

$A = 3\left(13 + 3.25 + \frac{13}{7} + 1.3\right)$

$A = 3(19.407143)$

$\boxed{A = 58.2214}$

3) b) $A = 1.5\left(\frac{13}{2.5} + \frac{13}{4} + \frac{13}{5.5} + \frac{13}{7} + \frac{13}{8.5} + \frac{13}{10} + \frac{13}{11.5} + \frac{13}{1}\right)$

$A = 1.5\,(29.63062577)$

$\boxed{A = 44.4459}$

4) $\lim_{x \to -7} \frac{\sqrt{5x+40} - \sqrt{5}}{x+7}$

x	-7.1	-7.01	-7.001	-6.999	-6.99	-6.9
$f(x)$	1.1475	1.1208	1.1183	1.1178	1.1153	1.0914

$\boxed{L = 1.118}$

5) $\lim_{x \to 1} \frac{\frac{1}{x+8} - \frac{1}{9}}{x-1}$

x	.9	.99	.999	1.001	1.01	1.1
$f(x)$	-.0125	-.0124	-.0123	-.0123	-.0123	-.0122

$\boxed{L = -.0123}$

6) $\lim_{x \to \frac{\pi}{4}} \sin x = \sin \frac{\pi}{4}$

$\boxed{L = \frac{\sqrt{2}}{2}}$

7) $\lim_{x \to 1} \cos\left(\frac{\pi x}{3}\right) = \cos\left(\frac{\pi}{3}\right)$

$\boxed{L = \frac{1}{2}}$

8) $\lim_{x \to 4} 5 + 3x^2 = 5 + 3(4)^2$

$\boxed{L = 53}$

b) $\lim_{x \to 3} \sqrt{x+5} = \sqrt{3+5} = \sqrt{8}$

$\boxed{L = 2\sqrt{2}}$

c) $\lim_{x \to 2} g(f(x)) = g(5 + 3(2)^2)$

$= g(17) = \sqrt{17+5} =$

$\boxed{L = \sqrt{22}}$

9) a) $\lim_{x \to 2} -x^2 - 4 = -(2)^2 - 4$

$\boxed{L = -8}$

b) $\lim_{x \to -5} 2x = 2(-5) =$

$\boxed{L = -10}$

c) $\lim_{x \to 5} g(-x^2 - 4) = g(-(5^2) - 4)$

$= g(-29) = 2(-29) =$

$\boxed{L = 58}$

10) $\lim_{x \to 14} \frac{x^3 - 2744}{x - 14} = \lim_{x \to 14} \frac{(x-14)^3}{(x-14)}$

$= \lim_{x \to 14} \frac{(x-14)(x^2 + 14x + 196)}{(x-14)}$

$= \lim_{x \to 14} x^2 + 14x + 196$

$= (14)^2 + 14(14) + 196$

$\boxed{L = 588}$

11) $\lim_{x \to -8} \frac{x+8}{x^2-64} = \lim_{x \to -8} \frac{(x+8)}{(x+8)(x-8)} =$

$= \lim_{x \to -8} \frac{1}{x-8} = \frac{1}{-8-8} = \boxed{-\frac{1}{16}}$

12) $\lim_{x \to 0} \frac{\sin x\,(1-\cos x)}{2x^6}$

$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1-\cos x}{x} \cdot \frac{1}{2x^4}$

$= (1)\,(0)\,(\infty) =$ $\boxed{\text{Does not exist}}$

13) $\lim_{x \to 1} \ln\left(\frac{x}{e^{-3x}}\right) = \ln\left(\lim_{x \to 1} \left(\frac{x}{e^{-3x}}\right)\right)$

$= \ln\left[\frac{1}{e^{-3}}\right] = \ln\left[\frac{1}{\frac{1}{e^3}}\right] = \ln\left(e^3\right) = 3$

$\boxed{L = 3}$

14) $\lim_{x \to 0} e^{4x} \cos(\pi x) = e^{4(0)} \cos(\pi 0)$

$= 1(1) = \boxed{L = 1}$

15) a) $\lim_{x \to 4^+} f(x) = \boxed{1}$ b) $\lim_{x \to 4^-} f(x) = \boxed{1}$

c) $f(c) = f(4) = \boxed{2}$

Since $\lim_{x \to 4} f(x) \neq f(c)$

then this function is not continuous

16) $\lim_{x \to -5^-} \ln(x+5) = \ln\left[\lim_{x \to -5^-} (x+5)\right]$

$= \ln[-5+5] = \ln(0) =$ $\boxed{\text{Does not exist}}$

17) $f(x) = \dfrac{x}{x^2+49}$

$$\lim_{x \to 0} \frac{x}{x^2+49} = \frac{0}{0^2+49} = \frac{0}{49} = 0$$

(continuous everywhere)

18) $f(x) = \begin{cases} -15\left(\dfrac{\sin x}{x}\right), & x < 0 \\ a + 4x, & x \geq 0 \end{cases}$

$$\lim_{x \to 0^-} -15\left(\frac{\sin x}{x}\right) = -15(1) = \boxed{L = -15}$$

$$\lim_{x \to 0^+} a + 4x = a + 4(0) = \boxed{a}$$

Since $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

then $\boxed{a = -15}$

19) $f(x) = \dfrac{x^6}{x^2-25}$

x	-5.5	-5.1	-5.01	-5.001
$f(x)$	5272.5	17422	157976	1.565...

x	-4.999	-4.99	-4.9	-4.5
$f(x)$	-1.656	-1.555	-13981	-1748

$\lim\limits_{x \to -5^-} f(x) = \infty$ $\lim\limits_{x \to -5^+} f(x) = -\infty$

20) $f(x) = \dfrac{x^2-16}{x^2+2x-8} = \dfrac{\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x-2)}$

$\lim\limits_{x \to -4} \dfrac{(x-4)}{(x-2)} = \dfrac{-4-4}{-4-2} = \dfrac{-8}{-6} = \boxed{\dfrac{4}{3}}$

Since $\lim\limits_{x \to -4} \neq \infty$ then $x = -4$
is not a V.A + is removable

$\lim\limits_{x \to 2} \dfrac{x-4}{x-2} = \dfrac{2-4}{2-2} = \dfrac{-2}{0}$

$\lim\limits_{x \to 2} f(x) = \infty$ and is non-removable
so $x = 2$ is a V.A.

? 21) $f(x) = \operatorname{Tan}(-13x) = \dfrac{\operatorname{Sin}(-13x)}{\cos(-13x)}$

$\cos -13x = 0$

$13x = \dfrac{\pi}{2} + k\pi$

$x = \dfrac{\frac{\pi}{2}}{13} + \dfrac{k\pi}{13}$

$\to x = \dfrac{\pi}{26} + \dfrac{k\pi \cdot 2}{13 \cdot 2}$

$x = \dfrac{\pi + 2k\pi}{26}$

$\boxed{x = \pi\left(\dfrac{1+2k}{26}\right)}$

22) $f(x) = \dfrac{\ln(x-7)}{x^2-64} = \dfrac{\ln(x-7)}{(x+8)(x-8)}$

$\lim\limits_{x \to 8} \dfrac{\ln(x-7)}{(x+8)(x-8)} = \ln\left[\dfrac{(8-7)}{(8+8)(8-8)}\right] = \ln \dfrac{1}{0}$

$\lim\limits_{x \to 7} \dfrac{\ln(x-7)}{(x+8)(x-8)} = \ln\left[\dfrac{(7-7)}{(7+8)(7-8)}\right] = \ln\left(\dfrac{0}{-15}\right)$

$x = 8$ and $x = 7$ are both V.A

23) $f(x) = \dfrac{\ln(x^2-25)}{x^2+36}$

$$\lim_{x \to 5} \frac{\ln(x^2-25)}{x^2+36} = \ln\left[\frac{(25-25)}{61}\right] = \ln\left(\frac{0}{61}\right)$$

$x = \pm 5$ is a V.A.

24) $f(x) = \dfrac{x^2-1}{e^{7x}-6} =$

$e^{7x} - 6 = 0$

$e^{7x} = 6$

$\ln e^{7x} = \ln 6$

$7x = \ln 6$

$\boxed{x = \dfrac{\ln 6}{7}}$

25) $f(x) = \dfrac{8}{e^{2x} - 11e^x + 30}$

$e^x = u$

$(e^x)^2 - 11e^x + 30 = 0$

$= u^2 - 11u + 30 = 0$

$= (u-6)(u-5) = 0$

$u = 6 \qquad u = 5$

$e^x = 6 \qquad e^x = 5$

$\ln e^x = 6 \qquad \ln e^x = 5$

$\boxed{x = \ln 6} \qquad \boxed{x = \ln 5}$

26) $f(x) = \dfrac{6}{e^{2x} + 4e^{x} - 21}$ $\qquad$ $\underline{\underline{e^x = U}}$

$(e^x)^2 + 4e^x - 21 = 0$
$U^2 + 4U - 21 = 0$
$(U+7)(U-3) = 0$
$U = -7 \qquad U = 3$

$\to e^x = -7 \qquad e^x = 3$
$\ln e^x = \ln -7 \qquad \ln e^x = \ln 3$
$x = \ln -7 \qquad \boxed{x = \ln 3}$
Cannot exist

27) $f(x) = \dfrac{2}{e^{2x} + 9e^{x} + 20}$ $\qquad$ $\underline{\underline{U = e^x}}$

$(e^x)^2 + 9e^x + 20 = 0$
$U^2 + 9U + 20 = 0$
$(U+4)(U+5) = 0$
$U = -4 \qquad U = -5$

$\to e^x = -4 \qquad e^x = -5$
$\ln e^x = \ln -4 \qquad \ln e^x = \ln -5$
$x = \ln -4 \qquad x = \ln -5$
$\boxed{\text{Both do not exist}}$

28) $\lim_{x \to 8^+} \dfrac{x+10}{x-8} = \dfrac{8+10}{8-8}$

$= \dfrac{18}{0} = \quad L = \infty$

29) $\lim_{x \to -10} \frac{x^2+10x}{(x^2+100)(x+10)} = \frac{x(x+10)}{(x^2+100)(x+10)}$

$= \lim_{x \to -10} \frac{x}{x^2+100} = \frac{-10}{(-10)^2+100} = \frac{-10}{200} =$

$\boxed{L = -\frac{1}{20}}$

30) $\lim_{x \to 0} \frac{\sin^{10} x}{x^{10}} =$

$\lim_{x^{10} \to 0} \frac{\sin^{10} x}{x^{10}} = \lim_{x \to \theta} \frac{\sin \theta}{\theta}$

$\boxed{L = 1}$

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Pg 67 # 1 - 11

Section 2.1

1) Precalc

$T = 15\,s$ $\quad D = R \cdot T$

$R = 20$ feet per sec $\quad D = 15(20)$

$\boxed{D = 300\,ft}$

5) Precalc

$A = \frac{1}{2} b \cdot h \qquad b = X,\ h = Y$

$A = \frac{1}{2} 5 \cdot 3$

$\boxed{A = 7.5 \text{ or } \frac{15}{2} \text{ square units}}$

9) a) $A = W \cdot L \qquad W = 1$

$A = W(L_1 + L_2 + L_3 + L_4)$

$A = 1(5 + 2.5 + 1.75 + 1.25)$

$\boxed{A = \sim 10.5}$

$Y = \frac{5}{X}$

$A = 1(1 + 2 + 2.9 + 4)$

$\boxed{A = \sim 9.9}$

b = $A = W \cdot L \qquad W = .5$

$A = .5\left(\frac{5}{1} + \frac{5}{1.5} + \frac{5}{2} + \frac{5}{2.5} + \frac{5}{3} + \frac{5}{3.5} + \frac{5}{4} + \frac{5}{4.5} + \frac{5}{5}\right)$

$A = .5(5 + 3.3 + 2.5 + 1.7 + 1.4 + 1.3 + 1.1 + 1)$

$\boxed{A = \sim 8.65}$

$$\begin{array}{cccc} x_1 & y_1 & x_2 & y_2 \end{array}$$

11) a) $(1, 5)$ $(5, 1)$

$D = \sqrt{(5-1)^2 + (1-5)^2}$

$D = \sqrt{16+16} = D = \sqrt{32} = \boxed{\sim 5.66}$

b) $(1, 5)$ $(2, 2.5)$

$D_1 = \sqrt{(2-1)^2 + (2.5-5)^2}$

$D_1 = \sqrt{7.25}$ or $\sqrt{\frac{7}{4}} = \boxed{\frac{\sqrt{7}}{2}}$

$D_2 = (2, 2.5)$ $(3, 1.5)$

$D_2 = \sqrt{(3-2)^2 + (1.5-2.5)^2}$

$D_2 = \boxed{\sqrt{2}}$

$D_3 = (3, 1.5)$ $(4, 1.25)$

$D_3 = \sqrt{(4-3)^2 + (1.25-1.5)^2}$

$D_3 = \frac{\sqrt{17}}{4}$

$\frac{\sqrt{17}}{4} + \frac{\sqrt{7}}{2} + \sqrt{32} = \boxed{8.01051}$

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Pg 74 # 1-27, 31-49, 65, 67, 69

Section 2.2

1) $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-x-2} = \boxed{0.33\bar{3}}$

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	0.34483	0.33445	0.33344	0.33322	0.33223	0.32258

5) $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = \boxed{1}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.99833	0.99998	1	1	0.99998	0.99833

9) $\lim\limits_{x \to 0} \dfrac{\ln(x+1)}{x} = \boxed{1}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	1.0536	1.005	1.0005	0.9995	0.99503	0.9531

13) $\lim\limits_{x \to 3} \dfrac{|x-3|}{x-3} =$ Does not exist Since lim from the right = 1 & lim from the left = -1

17) $\lim_{x \to \frac{\pi}{2}} \tan x =$ Does not exist
Since from Right $= \infty$
& from Left $= -\infty$

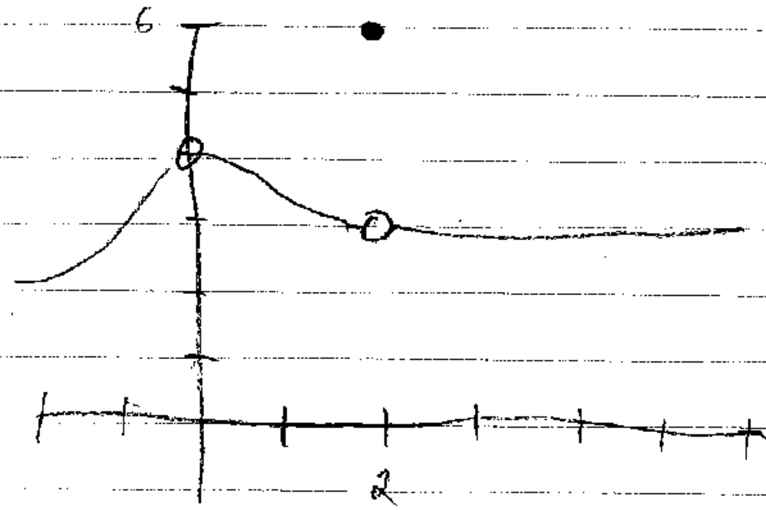
21) a) $f(1) = \boxed{2}$

b) $\lim_{x \to 1} f(x) =$ Does not exist since
from left $= 3.5 \neq$
from right $= 1$

c) $f(4) =$ Does not exist since
there is a hole

d) $\lim_{x \to 4} f(x) = \boxed{2}$

27)



31) $|f(x) - 3| < 0.4$

$|(x+1) - 3| < 0.4$

$|x-2| < 0.4$ Let $\boxed{\delta = 0.4}$

35) $\lim_{x \to 2} (3x+2)$ $L = (3 \cdot 2 + 2)$ $\boxed{L = 8}$

$c = 2$
$f(x) = (3x+2)$
$L = 8$

So $\lim_{x\to 2} (3x+2)$ means there is

an $\varepsilon > 0$ such that $\delta > 0$, $|f(x) - L| < \varepsilon$

$|(3x+2) - 8| < 0.01 = |3x - 6| < 0.01$

$= 3|x-2| < 0.01 = |x-2| < \frac{1}{300}$ $\boxed{\text{let } \delta = \frac{1}{300}}$

39) $\lim_{x\to 2} (x+3)$ $L = 2+3 = L = 5$

$c = 2$
$f(x) = (x+3)$
$L = 5$

$\lim_{x\to 2} (x+3)$ means there is an $\varepsilon > 0$ for a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$

$|(x+3) - 5| < \varepsilon$

$|x-2| < \varepsilon$ So let $\boxed{\varepsilon = \delta}$

43) $\lim_{x \to 6} 3$ $\qquad \boxed{L = 3}$

Since $\lim_{x \to 6} 3$ it means there is a $\varepsilon > 0$ for a $\delta > 0$ such that

$$|f(x) - L| < \varepsilon$$

$$|3 - 3| < \varepsilon$$

$$\boxed{0 < \varepsilon}$$

47) $\lim_{x \to 0} \sqrt[3]{x}$ $\qquad L = \sqrt[3]{0} = L = 0$

$\lim_{x \to 0} \sqrt[3]{x} = 0$ means that there is a $\varepsilon > 0$ for a $\delta > 0$ such that

$$|f(x) - L| < \varepsilon$$

$$|\sqrt[3]{x} - 0| < \varepsilon$$

$$|x - 0| < \varepsilon^3 \quad \text{let } \boxed{\delta = \varepsilon^3}$$

49) $\lim_{x \to 1} (x^2+1) \quad L = (1^2+1) = L = 2$

$\lim_{x \to 1} (x^2+1) = 2$ means that there is a $\varepsilon > 0$ for a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$

$|(x^2+1) - 2| < \varepsilon$

$|x^2 - 1| < \varepsilon$

$|(x+1)(x-1)| < \varepsilon$

$(x+1)|x-1| < \varepsilon$

$|x-1| < \frac{\varepsilon}{x+1}$ let $\boxed{\delta = \frac{\varepsilon}{x+1}}$

65) True

67) True

69) a) $\sqrt{\frac{1}{4}} = \frac{1}{2}$

$\frac{1}{4} = \frac{1}{4}$ True ✓

b) $\sqrt{x} = 0$

$\sqrt{0} = 0$

False since $\sqrt{x} > 0$

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Pg 87 # 1-123

Section 2.3

1) $h(x) = x^2 - 5x$

a) $\lim_{x \to 5} h(x) = \boxed{0}$

b) $\lim_{x \to -1} h(x) = \boxed{6}$

5) $\lim_{x \to 2} x^4 = \quad L = (2)^4 = 16$

$\boxed{L = 16}$

9) $\lim_{x \to -3} (2x^2 + 4x + 1)$

$L = (2(-3)^2 + 4(-3) + 1) = (18 - 12 + 1)$

$\boxed{L = 7}$

13) $\lim_{x \to 1} \frac{x - 3}{x^2 + 4} = \quad L = \frac{1 - 3}{1^2 + 4} = -\frac{2}{5}$

$\boxed{L = -\frac{2}{5}}$

17) $\lim_{x \to 3} \sqrt{x+1} = L = \sqrt{3+1}$

$L = \sqrt{4} = \boxed{L=2}$

21) $\lim_{x \to 2} \cos\left(\frac{\pi x}{3}\right) = L = \cos\left(\frac{2\pi}{3}\right)$

$\boxed{L = -\frac{1}{2}}$

25) $\lim_{x \to \frac{5\pi}{6}} \sin x = L = \sin\left(\frac{5\pi}{6}\right) =$

$\boxed{L = \frac{1}{2}}$

29) $\lim_{x \to 0} e^x \cdot \cos 2x = L = e^0 \cdot \cos(0)$

$L = 1 \cdot 1 \quad \boxed{L=1}$

33) $f(x) = 5 - x \quad G(x) = x^3$

a) $\lim_{x \to 1} f(x) = L = 5 - 1$

$\boxed{L=4}$

b) $\lim_{x \to 4} G(x)$

$L = (4)^3$

$\boxed{L = 64}$

c) $\lim_{x \to 1} G(f(x)) = L = G(4)$

$L = (4)^3 = 64$ $\boxed{L = 64}$

37) $\lim_{x \to c} f(x) = 2$ $\lim_{x \to c} G(x) = 3$

a) $\lim_{x \to c} [5G(x)]$ $L = [5(3)]$

$\boxed{L = 15}$

b) $\lim_{x \to c} [f(x) + G(x)] =$ $L = 2 + 3$

$\boxed{L = 5}$

c) $\lim_{x \to c} [f(x) \cdot G(x)] =$ $L = (2)(3)$

$\boxed{L = 6}$

d) $\lim_{x \to c} \frac{f(x)}{G(x)} =$ $\boxed{L = \frac{2}{3}}$

41) $G(x) = \dfrac{-2x^2+x}{x}$ $f(x) = -2x+1$

a) $\lim_{x\to 0} g(x) =$ $G(x) = f(x)$ so $\lim_{x\to 0} f(x) = -2(0)+1$

$\boxed{L=1}$

b) $\lim_{x\to -1} G(x) =$ $G(x) = f(x)$ so $\lim_{x\to -1} f(x) = -2(-1)+1$

$\boxed{L=3}$

43) $G(x) = \dfrac{x^3-x}{x-1}$ $f(x) = x(x+1)$?

a) $\lim_{x\to 1} G(x) = f(x) = G(x) = \lim_{x\to 1} f(x)$

$L = 1(1+1) =$

$= \boxed{L=2}$

b) $\lim_{x\to -1} g(x) = G(x) = f(x) = \lim_{x\to 1} f(x)$

$L = -1(-1+1)$

$\boxed{L=0}$

47) $\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \frac{\cancel{(x-2)}(x^2 + 2x + 4)}{\cancel{(x-2)}}$

$\lim_{x \to 2} x^2 + 2x + 4 = (2)^2 + 2(2) + 4$

$= 4 + 4 + 4 = \boxed{12}$

51) $\lim_{x \to 5} \frac{x - 5}{x^2 - 25} = \quad L = \frac{5 - 5}{25 - 25} =$

$L = \frac{\cancel{5-5}}{(5+5)\cancel{(5-5)}} = \frac{1}{5+5} = \boxed{L = \frac{1}{10}}$

55) $\lim_{x \to 0} \frac{\sqrt{x+5} - \sqrt{5}}{x} =$

$L = \lim_{x \to 0} \frac{\sqrt{x+5} - \sqrt{5}}{x} \cdot \frac{\sqrt{x+5} + \sqrt{5}}{\sqrt{x+5} + \sqrt{5}}$

$L = \lim_{x \to 0} \frac{(x+5) - 5}{x(\sqrt{x+5} + \sqrt{5})} = \frac{\cancel{x}}{\cancel{x}(\sqrt{x+5} + \sqrt{5})}$

$L = \frac{1}{\sqrt{5} + \sqrt{5}} = \frac{1}{2\sqrt{5}} \cdot \frac{2\sqrt{5}}{2\sqrt{5}} = \frac{\cancel{2}\sqrt{5}}{\cancel{4} \cdot 5} = \frac{\sqrt{5}}{2 \cdot 5}$

$\boxed{L = \frac{\sqrt{5}}{10}}$

59) $\lim_{x \to 0} \dfrac{[1/(3+x)] - (\frac{1}{3})}{x}$

$= \lim_{x \to 0} \dfrac{\dfrac{3-(3+x)}{3(3+x)}}{x}$

$= \lim_{x \to 0} \dfrac{3-(3+x)}{3(3+x)} \cdot \dfrac{1}{x}$

63) $\lim_{\Delta x \to 0} \dfrac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{\cancel{x^2} + 2x\Delta x + \Delta x^2 \cancel{-2x} - 2\Delta x \cancel{+1} \cancel{-x^2} \cancel{+2x} \cancel{-1}}{\Delta x}$

$\lim_{\Delta x \to 0} \dfrac{\cancel{\Delta x}^2 + 2x\cancel{\Delta x} - 2\cancel{\Delta x}}{\cancel{\Delta x}}$

$= \lim_{\Delta x \to 0} \Delta x + 2x - 2 = (0) + 2x - 2$

$= \boxed{2x - 2}$

69) $\lim\limits_{x \to 0} \dfrac{\sin x}{5x}$

$\lim\limits_{x \to 0} \left[\left(\dfrac{\sin x}{x}\right)\left(\dfrac{1}{5}\right)\right] = 1\left(\dfrac{1}{5}\right) = \boxed{1}$

73) $\lim\limits_{x \to 0} \dfrac{\sin^2 x}{x} = \lim\limits_{x \to 0} \dfrac{\sin x}{x} \cdot \sin x$

$= (1) \sin 0 = \boxed{0}$

77) $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos x}{\dfrac{\cos x}{\sin x}} = \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cancel{\cos x}}{1} \cdot \dfrac{\sin x}{\cancel{\cos x}}$

$= \lim\limits_{x \to \frac{\pi}{2}} \sin x = \sin \dfrac{\pi}{2} = \boxed{1}$

81) $\lim_{t \to 0} \frac{\sin 3t}{2t} = \lim_{t \to 0} \left[\left(\frac{\sin t}{t}\right)\left(\frac{3}{2}\right)\right]$

$= 1\left(\frac{3}{2}\right) = \boxed{\frac{3}{2}}$

85) $f(x) = 2x + 3$

1) $f(x + \Delta x) = 2(x + \Delta x) + 3 = \boxed{2x + 2\Delta x + 3}$

2) $f(x + \Delta x) - f(x) = 2x + 2\Delta x + 3 - (2x + 3)$

$= 2x + 2\Delta x + 3 - 2x - 3$

$= 2\Delta x$

3) $\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{2\cancel{\Delta x}}{\cancel{\Delta x}} = 2$

$= \lim_{\Delta x \to 0} 2 = \boxed{2}$

89) $f(x) = \frac{4}{x}$

1) $f(x + \Delta x) = \frac{4}{x + \Delta x} = \frac{4}{x + \Delta x}$

2) $f(x + \Delta x) - f(x) = \frac{4}{x + \Delta x} - \frac{4}{x} = \frac{4x - 4x - 4\Delta x}{x(x + \Delta x)} = \frac{-4\Delta x}{x(x + \Delta x)}$

3) $\frac{\frac{-4\Delta x}{x(x + \Delta x)}}{\Delta x} = \frac{-4\cancel{\Delta x}}{x(x + \Delta x)} \cdot \frac{1}{\cancel{\Delta x}} = \frac{-4}{x(x + \Delta x)}$

$$\lim_{\Delta x \to 0} \frac{-4}{x(x+\Delta x)} = \frac{-4}{x(x+0)} = \boxed{\frac{-4}{x^2}}$$

93) $C=0 \quad 4-x^2 \leq f(x) \leq 4+x^2$

$\lim_{x \to c} f(x) = ?$

$$\lim_{x \to 0} 4-x^2 = \lim_{x \to 0} 4+x^2$$

$= 4-0^2 = \boxed{4}$ and $4+0^2 = \boxed{4}$

Therefore $\boxed{\lim_{x \to 0} f(x) = 4}$

97) if $h(x) \leq f(x) \leq G(x)$ for all x and $\lim_{x \to c} h(x) = L = \lim_{x \to c} G(x)$

then $\lim_{x \to c} f(x) = L$

so, $\boxed{\lim_{x \to 0} |x| \sin x = 0}$

Steven Romeiro
Pg 98 # 1-69, 91, 93, 95

Section 2.4

1) a) $\lim_{x \to 3^+} f(x) = 1$

b) $\lim_{x \to 3^-} f(x) = 1$

c) $\lim_{x \to 3} f(x) = \boxed{1}$

Since $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$ then the $\lim_{x \to 3} f(x) = 1$ and is continuous

5) a) $\lim_{x \to 4^-} f(x) = -2$

b) $\lim_{x \to 4^+} f(x) = 2$

Since $\lim_{x \to 4^-} f(x) \neq \lim_{x \to 4^+} f(x)$, then the limit does not exist

9) $\lim\limits_{x \to -3^-} \frac{x}{\sqrt{x^2-9}} = \frac{x}{\sqrt{x^2-9}} \cdot \frac{\sqrt{x^2-9}}{\sqrt{x^2-9}}$

$= \frac{x\sqrt{x^2-9}}{x^2-9} = \frac{x\sqrt{x^2-9}}{(x+3)(x-3)} = \frac{-3}{0}$

$\boxed{\lim\limits_{x \to -3^-} \frac{x}{\sqrt{x^2-9}} \text{ does not exist}}$

13) $\lim\limits_{\Delta x \to 0^-} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x} = \frac{\frac{1x - 1x - 1\Delta x}{x(x+\Delta x)}}{\Delta x}$

$= \lim\limits_{\Delta x \to 0^-} \frac{\frac{-\Delta x}{x(x+\Delta x)}}{\Delta x} = \frac{-\Delta x}{x(x+\Delta x)} \cdot \frac{1}{\Delta x} = \frac{-1}{x(x+\Delta x)}$

$= \frac{-1}{x(x+0)} = \boxed{\frac{-1}{x^2}}$

17) $\lim\limits_{x \to 1} f(x) \qquad f(x) \begin{cases} x^3+1 & < 1 \\ x+1 & \geq 1 \end{cases}$

$\lim\limits_{x \to 1^-} f(x) = x^3+1 = (1)^3+1 = \boxed{2}$

$\lim\limits_{x \to 1^+} f(x) = x+1 = 1+1 = \boxed{2}$

21) $\lim_{x\to 4^-} (3[\![x]\!] - 5) = \lim_{x\to 4^-} 3[\![x]\!] - \lim_{x\to 4^-} 5$

$= \lim_{x\to 4^-} 3[\![x]\!] - 5 = 3\left(\lim_{x\to 4^-}[\![x]\!]\right) - 5 = 3(3) - 5$

$= 9 - 5 = \boxed{4}$

25) $\lim_{x\to 3^+} \ln(x-3) = \ln\left[\lim_{x\to 3^+}(x-3)\right]$

$= \ln(3-3) = \boxed{\ln(0) \text{ does not exist}}$

29) $f(x) = \dfrac{1}{x^2-4}$

f is continuous everywhere except at 2 and -2 because $f(2)$, $f(-2)$ does not exist

33) $g(x) = \sqrt{25 - x^2}$ $[-5, 5]$

g is continuous a $[-5, 5]$

37) $f(x) = x^2 - 2x + 1$

f is continuous everywhere and there are no discontinuities

41) $f(x) = \frac{\cancel{x}}{x^2 - \cancel{x}} = \frac{x}{x(x-1)} = \frac{1}{x-1}$

f is continuous every where except at $x=0$, $x=1$. The discont $x=0$ is removable while $x=1$ is non-removable

45) $f(x) = \frac{x+2}{x^2 - 3x - 10} = \frac{\cancel{x+2}}{\cancel{(x+2)}(x-5)}$

$= \frac{1}{x-5}$ f is continuous everywhere except at $x=-2$, $x=5$. The discontinuity $x=-2$ is removable while $x=5$ is non-removable

49) $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

1) $\lim_{x \to 1^-} x = \boxed{1}$ $\lim_{x \to 1^+} x^2 = \boxed{1}$

$$\boxed{\begin{array}{l} \text{Since } \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \\ \text{then } \lim_{x \to 1} f(x) = 1 \end{array}}$$

2) $f(c) = f(\cancel{x}) = f(1) = \boxed{1}$

3) Since $f(c) = 1$ and $\lim_{x \to c} f(x) = 1$
then f is continuous
everywhere

53) $f(x) = \begin{cases} \tan \frac{\pi x}{4}, & |x| < 1 \\ x, & |x| \ge 1 \end{cases}$

1) $f(c) = f(x) = |x| = |1| = \boxed{1}$

2) $\lim_{x \to 1^-} \tan \frac{\pi x}{4} = \tan \frac{\pi}{4} = \boxed{1}$

$\lim_{x \to 1^+} x = \boxed{1}$

Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

then $\lim_{x \to 1} f(x) = \boxed{1}$

3) Since $f(c) = \lim_{x \to 1} f(x)$

then fxn f is continuous everywhere

57) $f(x) = \csc 2x$

$= f(x) = \frac{1}{\sin 2x}$ $\quad \sin 2x \neq 0$

$\sin \frac{\pi}{2} = 0$ so $x \neq \frac{\pi}{2}$

f has non-removable discontinuities at $x = k\frac{\pi}{2}$

63) $f(x) \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$

$\lim_{x \to 2^-} x^3 = (2)^3 = \boxed{8}$

$\lim_{x \to 2^+} ax^2 = a(2)^2 = \boxed{4a}$ $\quad 4a = 8$ $\quad \boxed{a = 2}$

Since $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

then $a = 2$

67) $f(x) = x^2, \quad g(x) = x - 1$

$h(x) = f(g(x)) = f(x-1) = \boxed{x^2 - 1}$

$\boxed{h(x) \text{ is continuous everywhere}}$

69) $f(x) = \frac{1}{x-6}, \quad g(x) = x^2 + 5$

$h(x) = f(x^2+5) = \left[\left(\frac{1}{x-6}\right)^2 + 5\right] = \left[\left(\frac{1}{(x-6)^2}\right) + 5\right]$

$= 1 + 5(x-6)^2 = 1 + 5(x^2 - 12x + 36) =$

$= \cancel{1 + 5x^2 - 60x + 180 = 5x^2 - 60x + 181}$

$\boxed{h(x) \text{ is continuous everywhere except at } x = \pm 1}$

91) $f(x) = x^2 + x - 1 \quad [0,5], \; f(c) = 11$

$f(0) = 0^2 + 0 - 1 = \boxed{-1} = a$

$f(5) = (5)^2 + 5 - 1 = \boxed{29} = b$

$f(a) = -1 < c = -1 < 11$

$f(b) = 29 > c = 29 > 11$

$f(c) = c^2 + c - 1 = 11$

$= c^2 + c - 12 = 0$

$(c+4)(c-3) = 0$

$c = -4 \quad \boxed{c = 3}$

93) $f(x) = x^3 - x^2 + x - 2 \quad [0,3]$

$f(c) = 4$

$f(0) = \boxed{-2}$

$f(3) = 3^3 - 3^2 + 3 - 2 = \boxed{19}$

$f(a) = -2 < 4$

$f(b) = 19 > 4$

$f(c) = c^3 - c^2 + c - 2 = 4$

$= c^3 - c^2 + c - 6 = (c-2)(c^2 + c + 3) = 0$

$\boxed{c = 2}$

95) a) $\lim_{x \to c} f(x)$ does not exist, so not continuous

b) $\lim_{x \to c} f(x)$ is not defined so not continuous

c) $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$ so $f(x)$ is not continuous

d) $\lim_{x \to c} f(x)$ does not exist so not continuous

Steven Romero
Pg 108 # 1-51, 65, 69

Section 2.5

1) $f(x) = 2\left|\frac{x}{x^2-4}\right|$

$\lim_{x \to -2^-} f(x) = -\infty$

$\lim_{x \to -2^+} f(x) = \infty$

5) $f(x) = \frac{1}{x^2-9}$

x	-3.5	-3.1	-3.01	-3.001
$f(x)$	.30769	1.6393	16.639	166.64

x	-2.999	-2.99	-2.9	-2.5
$f(x)$	-166.7	-16.69	-1.695	-.3636

$\lim_{x \to 3^-} f(x) = \infty$

$\lim_{x \to 3^+} f(x) = -\infty$

9) $f(x) = \frac{1}{x^2}$

$\lim_{x \to 0} f(x) \frac{1}{x^2} = \frac{1}{0^2}$ $\boxed{x = 0, \text{ V.A}}$

13) $f(x) = \frac{x^2}{x^2 - 4}$

$\lim_{x \to \pm 2} \frac{x^2}{x^2 - 4} = \frac{(\pm 2)^2}{(\pm 2)^2 - 4} = \frac{4}{4 - 4} = \frac{4}{0}$

$\boxed{x = \pm 2, \text{ V.A}}$

17) $f(x) = \tan 2x = \frac{\sin 2x}{\cos 2x}$

$= \cos 2x = 0$

$2x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{4}$ or $x = \frac{n\pi}{2} + \frac{\pi}{4}$

21) $f(x) = \dfrac{x}{x^2+x-2} = \dfrac{x}{(x+2)(x-1)}$

$\lim\limits_{x\to -2} \dfrac{x}{(x+2)(x-1)} = \dfrac{-2}{(-2+2)(-2-1)} = \dfrac{-2}{0}$ $\boxed{x=-2, \text{ V.A}}$

$\lim\limits_{x\to 1} \dfrac{x}{(x+2)(x-1)} = \dfrac{1}{(1+2)(1-1)} = \dfrac{1}{0}$ $\boxed{x=1, \text{ V.A}}$

25) $f(x) = \dfrac{e^{-2x}}{x-1}$

$\lim\limits_{x\to 1} \dfrac{e^{-2x}}{x-1} = \dfrac{e^{-2(1)}}{1-1} = \dfrac{e^{-2}}{0}$ $\boxed{x=1, \text{ V.A}}$

29) $f(x) = \dfrac{1}{e^x-1} =$

$\lim\limits_{x\to 0} \dfrac{1}{e^x-1} = \dfrac{1}{e^0-1} = \dfrac{1}{1-1} = \dfrac{1}{0}$ $\boxed{x=0, \text{ V.A}}$

33) $f(x) = \dfrac{x^2-1}{x+1}$

$\lim\limits_{x\to -1} \dfrac{x^2-1}{x+1} = \dfrac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} = x-1 = \boxed{-2}$

discontinuity at $x=-1$ that is removable

39) $\lim_{x \to 2^+} \frac{x-3}{x-2} = \frac{-1}{0} = \boxed{-\infty}$

43) $\lim_{x \to -3^-} \frac{x^2+2x-3}{x^2+x-6} = \frac{(x+3)(x-1)}{(x+3)(x-2)}$

$\lim_{x \to -3^-} \frac{x-1}{x-2} = \frac{-4}{-5} = \boxed{\frac{4}{5}}$

49) $\lim_{x \to 0^+} \frac{2}{\sin x} = \frac{2}{\sin(0)} = \frac{2}{0} = \boxed{\infty}$

51) $\lim_{x \to (\frac{\pi}{2})^+} \ln(\cos x)$

$= \ln\left[\lim_{x \to \frac{\pi}{2}^+} \cos x\right] = \ln(0) = \boxed{\infty}$

$r = 50\pi \sec^2\theta$

65) a) $r = 50\pi \sec^2 \frac{\pi}{6}$

$$r = 50\pi \left(\frac{1}{\cos\frac{\pi}{6}}\right)^2$$

$$r = 50\pi \left(\frac{1}{\frac{\sqrt{3}}{2}}\right)^2$$

$$r = \frac{200\pi}{3} \text{ ft/sec}$$

b) $r = 50\pi \left(\frac{1}{\cos\frac{\pi}{3}}\right)^2$

$$r = 50\pi \left(\frac{1}{\frac{1}{2}}\right)^2 = \quad r = 50\pi(2)^2$$

$$r = 200\pi \text{ ft/sec}$$

c) $\lim\limits_{\theta \to \left(\frac{\pi}{2}\right)^-} 50\pi \left(\frac{1}{\cos\theta}\right)^2 = 50\pi \lim\limits_{\theta \to \left(\frac{\pi}{2}\right)^-} \left(\frac{1}{\cos\theta}\right)^2$

$\cos\theta \to 0^+$ as $\theta \to \frac{\pi}{2}^-$

So $\lim\limits_{\theta \to \left(\frac{\pi}{2}\right)^-} r = \boxed{\lim\limits_{\theta \to \left(\frac{\pi}{2}\right)^-} (50\pi \sec^2\theta) = \infty}$

69) a) $y = \dfrac{25x}{x-25}$

$\lim_{x \to 25} \dfrac{25x}{x-25} = \dfrac{625}{0}$ $\boxed{x = 25,\ \text{V.A}}$

$\boxed{\text{domain } x > 25}$

b)

x	30	40	50	60
y	150	66.667	50	42.857

Steven Romeiro

50 pts

Study Guide 2

1) $f(x) = \frac{7}{x-5}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

1) $f(x+h) = \frac{7}{x+h-5}$

2) $f(x+h) - f(x) = \frac{7}{x+h-5} - \frac{7}{x-5} = \frac{7x-35-(7x+7h-35)}{(x+h-5)(x-5)}$

$= \frac{7h}{(x+h-5)(x-5)}$

3) $\frac{f(x+h)-f(x)}{h} = \frac{-7\cancel{h}}{(x+h-5)(x-5)} \cdot \frac{1}{\cancel{h}} =$

$= \frac{-7}{(x+h-5)(x-5)}$

4) $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$f'(x) = \lim_{h \to 0} \frac{-7}{(x+h-5)(x-5)}$

$f'(x) = \frac{7}{(x-5)(x-5)} = \boxed{f'(x) = \frac{-7}{(x-5)^2}}$

2) $f(x) = \sqrt{5x-8}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = \sqrt{5(x+h)-8} = \sqrt{5x+5h-8}$

2) $f(x+h) - f(x) = \sqrt{5x+5h-8} - \sqrt{5x-8}$

3) $$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{5x+5h-8} - \sqrt{5x-8}}{h}$$

$$= \frac{\sqrt{5x+5h-8} - \sqrt{5x-8}}{h} \cdot \frac{\sqrt{5x+5h-8} + \sqrt{5x-8}}{\sqrt{5x+5h-8} + \sqrt{5x-8}}$$

$$= \frac{5x+5h-8-5x+8}{h\left(\sqrt{5x+5h-8} + \sqrt{5x-8}\right)} = \frac{5h}{h\left(\sqrt{5x+5h-8} + \sqrt{5x-8}\right)}$$

$$= \frac{5}{\sqrt{5x+5h-8} + \sqrt{5x-8}}$$

4) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) \lim_{h \to 0} \frac{5}{\sqrt{5x+5h-8} + \sqrt{5x-8}}$$

$$f'(x) = \frac{5}{\sqrt{5x-8} + \sqrt{5x-8}} = \boxed{f'(x) = \frac{5}{2\sqrt{5x-8}}}$$

3) $f(x) = 3x^2 - 3x - 3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = 3(x+h)^2 - 3(x+h) - 3$

$= 3(x^2 + 2xh + h^2) - 3x - 3h - 3$

$= 3x^2 + 6xh + 3h^2 - 3x - 3h - 3$

2) $f(x+h) - f(x) = \cancel{3x^2} + 6xh + 3h^2 - \cancel{3x} - 3h - \cancel{3} - (\cancel{3x^2} - \cancel{3x} - \cancel{3})$

$= 6xh + 3h^2 - 3h$

3) $\frac{f(x+h) - f(x)}{h} = \frac{6xh + 3h^2 - 3h}{h}$

$= 6x + 3h - 3$

4) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \to 0} 6x + 3h - 3$

$\boxed{f'(x) = 6x - 3}$

4) $f(x) = -5x + 2e^x$ when Slope $= 0$

$F'(x) = 2e^x - 5$ $\rightarrow$ $\ln(e^x) = \ln\left(\frac{5}{2}\right)$

$2e^x - 5 = 0$

$2e^x = 5$

$e^x = \frac{5}{2}$

$\boxed{x = \ln \frac{5}{2}}$

5) $y(x) = x^3 + 15x^2 + 6$ when Slope $= 0$

$y'(x) = 3x^2 + 30x$ $\rightarrow$ $3x = 0$ / $(x+10) = 0$

$\boxed{x = 0}$ / $\boxed{x = -10}$

$y'(x) = 3x(x + 10)$

$3x(x+10) = 0$

6) $y(x) = x^4 - 256x + 5$ when Slope $= 0$

$y'(x) = 4x^3 - 256$

$y'(x) = 4(x^3 - 64)$

$4(x^3 - 64) = 0$

$x^3 - 64 = 0$

$x = \sqrt[3]{64}$

$\boxed{x = 4}$

7) $S(t) = -4.9t^2 + V_0 t + S_0$

$v(t) = S'(t)$

$S'(t) = -9.8t + 122$ / $S'(t) = -9.8t + 122$

$S'(3) = -29.4 + 122$ / $S'(11) = -107.8 + 122$

$\boxed{S'(3) = 92.6}$ / $\boxed{S'(11) = 14.2}$

8) $Q(x) = x\left(3 - \dfrac{2}{x+5}\right)$

$$Q(x) = \left(3x - \frac{2x}{x+5}\right)$$

$$Q(x) = \frac{3x^2 + 15x - 2x}{x+5}$$

$$Q(x) = \frac{3x^2 + 13x}{x+5}$$

$$Q'(x) = \frac{(6x+13)(x+5) - 1(3x^2+13x)}{(x+5)^2}$$

$$Q'(x) = \frac{6x^2 + 30x + 13x + 65 - 3x^2 - 13x}{(x+5)^2}$$

$$\boxed{Q'(x) = \frac{3x^2 + 30x + 65}{(x+5)^2}}$$

9) $P(x) = (x^4+5)^5$

$p'(x) = 4x^3 \cdot 5(x^4+5)^4$

$\boxed{p'(x) = 20x^3(x^4+5)^4}$

10) $f(v) = 5v \sin v + 3\cos v$

$f'(v) = 5\cdot(\sin v) + 5v(\cos v) + 3(-\sin v)$

$f'(v) = 5\sin v + 5v\cos v - 3\sin v$

$\boxed{f'(v) = 5v\cos v + 2\sin v}$

11) $f(t) = (t-3)(t^2-5)$; $(2,1)$

$f'(t) = 1(t^2-5) + (t-3)(2t)$

$f'(t) = t^2 - 5 + 2t^2 - 6t$

$f'(t) = 3t^2 - 6t - 5$

$f'(2) = 3(2)^2 - 6(2) - 5$

$f'(2) = -5$

$y - y_1 = m(x - x_1)$

$y - 1 = -5(x-2)$

$y - 1 = -5x + 10$

$\boxed{y = -5t + 11}$

12) $R(t) = 30t^5 + 6\sec(t)$

$$\boxed{R'(t) = 150t^4 + 6\sec t \tan t}$$

13) $L = 4t+5 \quad h = t^5$

$A = h \cdot w = (4t+5)(t^5)$

$A = 4t^6 + 5t^5$

$A' = 24t^5 + 25t^4$

$$\boxed{A' = t^4(24t+25)}$$

14) $R = \sqrt{5t+4} \qquad h = t^5$

$V = \pi r^2 h = \pi(\sqrt{5t+4})^2(t^5)$

$V = \pi(5t+4)(t^5)$

$V = \pi(5t^6+4t^5)$

$V' = 0(5t^6+4t^5) + \pi(30t^5+20t^4)$

$$\boxed{V' = 10\pi t^4(3t+2)}$$

15) $g(x) = x^5 \operatorname{Sec} x$

$g'(x) = 5x^4(\operatorname{Sec} x) + x^5(\operatorname{Tan} x \operatorname{Sec} x)$

$g'(x) = 5x^4 \operatorname{Sec} x + x^5 \operatorname{Tan} x \operatorname{Sec} x$

$g''(x) = [20x^3(\operatorname{Sec} x) + 5x^4(\operatorname{Tan} x \operatorname{Sec} x)] +$

$[x^5((\operatorname{Sec}^2 x)(\operatorname{Sec} x) + (\operatorname{Tan} x)(\operatorname{Tan} x \operatorname{Sec} x))]$

$g''(x) = [20x^3(\operatorname{Sec} x) + 5x^4(\operatorname{Tan} x \operatorname{Sec} x)]$

$+ [x^5(\operatorname{Sec}^3 x) + (\operatorname{Tan}^2 x \operatorname{Sec} x)]$

$\boxed{g''(x) = x^3 \operatorname{Sec} x [20 + 5x \operatorname{Tan} x + x^2 \operatorname{Sec}^2 x + \operatorname{Tan}^2 x}$

16) $f''(x) = 5x^{\frac{6}{7}}$ find $f^{iv}(x)$

$f'''(x) = \frac{6}{7}(5)x^{-\frac{1}{7}}$

$f'''(x) = \frac{30}{7}x^{-\frac{1}{7}}$

$f^{iv}(x) = \left(\frac{30}{7}\right)\left(-\frac{1}{7}\right)x^{-\frac{8}{7}}$

$\boxed{f^{iv}(x) = \frac{-30}{49x^{8/7}}}$

17) $f(t) = (8+4t)^{5/3}$

$f'(t) = \frac{5}{3}(8+4t)^{2/3} \cdot 4$

$$\boxed{f'(t) = \frac{20}{3}(8+4t)^{\frac{2}{3}}}$$

18) $f(x) = x^5\sqrt{1-9x}$

$f(x) = x^5(1-9x)^{1/2}$

$f'(x) = 5x^4(1-9x)^{\frac{1}{2}} + x^5\left(\frac{1}{2}(1-9x)^{-\frac{1}{2}} \cdot (-9)\right)$

$$f'(x) = 5x^4(1-9x)^{\frac{1}{2}} + x^5\left(\frac{-9}{2(1-9x)^{\frac{1}{2}}}\right)$$

$$f'(x) = 5x^4(1-9x)^{\frac{1}{2}} - \frac{9x^5}{2(1-9x)^{\frac{1}{2}}}$$

LCD =
$2(1-9x)^{\frac{1}{2}}$

$$f'(x) = \frac{5x^4(2(1-9x)) - 9x^5}{2(1-9x)^{\frac{1}{2}}}$$

$$f'(x) = \frac{5x^4(2-18x) - 9x^5}{2(1-9x)^{\frac{1}{2}}}$$

$$f'(x) = \frac{10x^4 - 90x^5 - 9x^5}{2(1-9x)^{\frac{1}{2}}}$$

$$\boxed{f'(x) = \frac{x^4(10-99x)}{2(1-9x)^{\frac{1}{2}}}}$$

19) $y = 9\cos 2x$

$y' = -9\sin 2x \cdot 2$

$\boxed{y' = -18\sin 2x}$

20) $y = \cos(3x^6 + 3)$

$y' = -\sin(3x^6+3) \cdot 18x^5$

$\boxed{y' = -18x^5 \sin(3x^6+3)}$

21) $f(\theta) = \frac{4}{5}\sin^2 4\theta$

$f'(\theta) = \frac{4}{5}\sin^2(4\theta)$

$f'(\theta) = 2\left(\frac{4}{5}\right)\sin(4\theta) \cdot \cos(4\theta) \cdot 4$

$\boxed{f'(\theta) = \frac{32}{5}\sin(4\theta)\cos(4\theta)}$

22) $f(x) = \ln\left(\frac{13x}{x^2+8}\right)$

$$f(x) = \ln 13x - \ln(x^2+8)$$

$$f'(x) = \frac{13}{13x} - \frac{2x}{(x^2+8)}$$

$$f'(x) = \frac{13(x^2+8) - 2x(13x)}{13x(x^2+8)}$$

$$f'(x) = \frac{13x^2+104 - 26x^2}{13x(x^2+8)}$$

$$f'(x) = \frac{-13x^2+104}{13x(x+8)}$$

$$f'(x) = \frac{13x(-x+8)}{13x(x+8)}$$

$$\boxed{f'(x) = \frac{(8-x)}{(x+8)}}$$

23) $f(x) = \ln(5x+4)$

$$\boxed{f'(x) = \frac{5}{5x+4}}$$

24) $f(x) = \sin 3x^4$ Second derivative

$f'(x) = \sin(3x^4) \longrightarrow$ Chain Rule

$f'(x) = \cos 3x^4 \cdot 12x^3$

$f'(x) = 12x^3 \cos(3x^4) \longrightarrow$ Product Rule

$f''(x) = 36x^2(\cos(3x^4)) + (12x^3)(-\sin(3x^4))(12x^3)$

$$\boxed{f''(x) = 36x^2 \cos 3x^4 - 144x^6 \sin 3x^4}$$

25) $y = 7^{-x}$; $(-1, 7)$

$y' = -1(\ln 7)7^{-x}$

$y' = -7^{-x}(\ln 7)$

$y' = -7^{-(-1)}(\ln 7)$

$y' = -7(\ln 7) \rightarrow$ Slope

$y - y_1 = m(x - x_1)$

$y - 7 = (-7\ln 7)(x+1)$

$y = -7\ln 7x - 7\ln 7 + 7$

$y = 7(-\ln 7x - \ln 7 + 1)$

$\boxed{y = 7[1 - \ln 7(x+1)]}$

26) $f(x) = \tan^5 x$; $\left(\frac{7\pi}{5}, 276.134\right)$

$f'(x) = 5\tan^4 x \cdot \sec^2 x$

$f'(x) = 5\tan^4\left(\frac{7\pi}{5}\right) \cdot \sec^2\left(\frac{7\pi}{5}\right)$

Calculator $= f'(x) = 4697.87 \rightarrow$ Slope

$y - y_1 = m(x - x_1)$

$y - 276.134 = 4697.87(x - 4.398229715)$

$y - 276.134 = 4697.87x - 20662.44338$

$\boxed{y = 4697.87x - 20386.18}$

$$\frac{d}{dx} = \frac{1}{u} \cdot u'$$

27) $y = \ln(x^4)$; $(1, 0)$

$$y' = \frac{1}{x^4} \cdot 4x^3$$

$$y' = \frac{4x^3}{x^4}$$

$$y' = \frac{4}{x} \longrightarrow \boxed{y' = 4} \text{ Slope}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = 4(x - 1)$$

$$\boxed{y = 4(x - 1)}$$

28) $x^{3/4} + y^{4/5} = 4$

$$\frac{d}{dx}\left(x^{3/4} + y^{4/5}\right) = \frac{d}{dx}(4)$$

$$\frac{3}{4}x^{-1/4} + \frac{4}{5}y^{-1/5}\frac{dy}{dx} = 0$$

$$\frac{4}{5}y^{-1/5}\frac{dy}{dx} = -\frac{3}{4x^{1/4}}$$

$$\frac{4}{5}y^{-\frac{1}{5}}\frac{dy}{dx} = \frac{-3}{4x^{1/4}}$$

$$\frac{dy}{dx} = \frac{-3}{4x^{\frac{1}{4}}} \cdot \frac{5y^{\frac{1}{5}}}{4}$$

$$\boxed{\frac{dy}{dx} = -\frac{15y^{\frac{1}{5}}}{16x^{\frac{1}{4}}}}$$

29) $x^7 + 9x + x^3y - y^7 = 9$

$$\frac{d}{dx}\left(x^7 + 9x + x^3y - y^7\right) = \frac{d}{dx}(9)$$

$$7x^6 + 9 + \left(3x^2y + \frac{dy}{dx} \cdot x^3\right) - 7y^6\frac{dy}{dx} = 0$$

$$x^3\frac{dy}{dx} - 7y^6\frac{dy}{dx} = -7x^6 - 3x^2y - 9$$

$$\frac{dy}{dx}\left(x^3 - 7y^6\right) = -7x^6 - 3x^2y - 9$$

$$\frac{dy}{dx} = \frac{-7x^6 - 3x^2y - 9}{x^3 - 7y^6}$$

$$\boxed{\frac{dy}{dx} = \frac{7x^6 + 9 + 3x^2y}{7y^6 - x^3}}$$

30) $\sin x + 2\cos 5y = 2$

$$\frac{d}{dx}\left(\sin x + 2\cos 5y\right) = \frac{d}{dx}(2)$$

$$\cos x - 2\sin 5y \cdot 5\frac{dy}{dx} = 0$$

$$-10\sin 5y\frac{dy}{dx} = -\cos x$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{10\sin 5y}}$$

31) $x^6 + 4\ln y = 9$

$$\frac{d}{dx}\left(x^6 + 4\ln y\right) = \frac{d}{dx}(9)$$

$$6x^5 + 4\frac{y'}{y} = 0$$

$$4\frac{y'}{y} = -6x^5$$

$$\frac{y'}{y} = -\frac{6x^5}{4}$$

$$y' = \frac{-6x^5 y}{4}$$

$$\boxed{y' = -\frac{3x^5 y}{2}}$$

32) $y = \dfrac{\sqrt{4x^4-25}}{x-1}$

$$\ln y = \ln\left(\frac{\sqrt{4x^4-25}}{x-1}\right)$$

$$\ln y = \frac{1}{2}\ln(4x^4-25) - \ln(x-1)$$

$$\frac{y'}{y} = \frac{1(16x^3)}{2(4x^4-25)} - \frac{1(1)}{(x-1)}$$

$$\frac{y'}{y} = \frac{16x^3}{8x^4-50} - \frac{1}{(x-1)}$$

$$\frac{y'}{y} = \frac{16x^3(x-1) - 1(8x^4-50)}{(8x^4-50)(x-1)}$$

$$\frac{y'}{y} = \frac{16x^4-16x^3-8x^4+50}{(8x^4-50)(x-1)}$$

$$\frac{y'}{y} = \frac{8x^4-16x^3+50}{(8x^4-50)(x-1)}$$

$$y' = \left[\frac{8x^4-16x^3+50}{(8x^4-50)(x-1)}\right]\cdot\left[\frac{(4x^4-25)^{\frac{1}{2}}}{(x-1)}\right]$$

$$y' = \frac{\cancel{2}(4x^4-8x^3+25)(4x^4-25)^{\frac{1}{2}}}{\cancel{2}(4x^4-25)(x-1)^2}$$

$$\boxed{y' = \frac{(4x^4-8x^3+25)(4x^4-25)^{\frac{1}{2}}}{(4x^4-25)(x-1)^2}}$$

33) $y = \dfrac{x^{56}\sqrt{25x-18}}{(x-1)^{39}}$

$$\ln y = \ln\left(\frac{x^{56}(25x-18)^{\frac{1}{2}}}{(x-1)^{39}}\right)$$

$$\ln y = 56\ln x + \frac{1}{2}\ln(25x-18) - 39\ln(x-1)$$

$$\frac{y'}{y} = \frac{56}{x} + \frac{1(25)}{2(25x-18)} - \frac{39}{x}$$

$$y' = \left[\frac{56}{x} + \frac{25}{2(25x-18)} - \frac{39}{x}\right]\cdot y$$

$$\boxed{y' = \left[\frac{56}{x} + \frac{25}{2(25x-18)} - \frac{39}{x}\right]\cdot\left[\frac{x^{56}\sqrt{25x-18}}{(x-1)^{39}}\right]}$$

34) $Y = X^{\sin(4x)}$

$\ln Y = \sin(4x) \ln x$ → Product & Chain Rule

$\frac{Y'}{Y} = \cos 4x \cdot 4 \cdot \ln x + \frac{1}{X} \cdot \sin(4x)$

$\frac{Y'}{Y} = 4\cos 4x (\ln x) + \frac{\sin(4x)}{X}$

$$\boxed{Y' = \left[4\cos 4x(\ln x) + \frac{\sin(4x)}{x}\right] \cdot X^{\sin(4x)}}$$

35) $f(x) = x^3 + 5x \quad (f^{-1})'(6)$?

$x^3 + 5x = 6$

$x^3 + 5x - 6 = 0$ → Graph

$X_{int} = 1$

So, $\frac{1}{f'(1)} = \frac{1}{3x^2+5} = \frac{1}{3(1)^2+5}$

$$\boxed{(f^{-1})'(6) = \frac{1}{8}}$$

? 36) $f(x) = \sin 5x$; $(f^{-1})'\left(\frac{1}{2}\right)$

$\sin \frac{1}{2} = \frac{\pi}{6}$ $\quad 5x = \frac{\pi}{6}$ $\quad f'(x) = 5\cos 5x$

$\boxed{x = \frac{\pi}{30}}$

$$(f^{-1})'\left(\frac{1}{2}\right) = \frac{1}{f'(f^{-1}(\frac{1}{2}))} = \frac{1}{5\cos 5(\frac{\pi}{30})} = \frac{1}{5\cos\frac{\pi}{6}}$$

$$(f^{-1})'\left(\frac{1}{2}\right) = \frac{1}{5\frac{\sqrt{3}}{2}} = \frac{2}{5\sqrt{3}} = \frac{2}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{(f^{-1})'\left(\frac{1}{2}\right) = \frac{2\sqrt{3}}{15}}$$

37) $y = \sin^{-1} 4x \quad ; \quad \left(\frac{1}{4\sqrt{2}}, \frac{\pi}{4}\right)$

$$y' = \frac{4}{\sqrt{1-(4x)^2}}$$

$$y' = \frac{4}{\sqrt{1-16x^2}}$$

$$y' = \frac{4}{\sqrt{1-16\left(\frac{\sqrt{2}}{8}\right)^2}}$$

$$y' = \frac{4}{\sqrt{1-16\left(\frac{1}{32}\right)}}$$

$$y' = \frac{4}{\sqrt{1-\frac{1}{2}}}$$

$$y' = \frac{4}{\sqrt{\frac{1}{2}}}$$

$$y' = \frac{4}{\frac{\sqrt{2}}{2}}$$

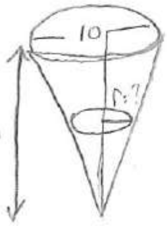
$$\boxed{y' = \frac{8}{\sqrt{2}}}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{\pi}{4} = \frac{8}{\sqrt{2}}\left(x - \frac{\sqrt{2}}{8}\right)$$

$$y - \frac{\pi}{4} = \frac{8}{\sqrt{2}}x - 1$$

$$\boxed{y = \frac{8}{\sqrt{2}}x - 1 + \frac{\pi}{4}}$$



Similar Δ's = $\frac{h}{13} = \frac{r}{5}$

$$\boxed{r = \frac{5h}{13}}$$

38) $r = \frac{5h}{13}$ $h = 13$ $\frac{dV}{dt} = 12$ at $h = 8$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \frac{25h^2}{169} \cdot h$

$V = \frac{1}{3}\pi \frac{25h^3}{169}$

$V = \frac{25}{507}\pi h^3$

$\frac{dV}{dt} = \frac{75}{507}\pi h^2 \cdot \frac{dh}{dt}$

$12 = \frac{75}{507}\pi (8)^2 \cdot \frac{dh}{dt}$

$12 = \frac{4800\pi}{507} \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = 12 \cdot \frac{507}{4800\pi}$

$$\boxed{\frac{dh}{dt} = \frac{507}{400\pi}}$$

39) $f(x) = x^2 - 7$ when $x_1 = 2.6$

$f'(x) = 2x$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 2.6 - \frac{f(2.6)}{f'(2.6)}$ $\boxed{x_2 = 2.646154}$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 2.646154 - \frac{f(2.646154)}{f'(2.646154)}$

$$\boxed{x_3 = 2.645751}$$

40) $f(x) = 2 - x$ at $x_1 = 1.5$

?

$g(x) = \frac{1}{\sqrt{x^2+5}}$

$$\frac{1}{\sqrt{x^2+5}} = 2 - x$$

$$\boxed{h(x) = \frac{1}{\sqrt{x^2+5}} + x - 2 = 0}$$

$$h'(x) = \frac{-(x^2+5)^{-\frac{1}{2}} \cdot 2x}{2(x^2+5)} + 1 = \frac{-2x}{2(x^2+5)^{\frac{3}{2}}} + 1$$

$$\boxed{h'(x) = 1 - \frac{x}{2(x^2+5)^{\frac{3}{2}}}}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x)} = 1.5 - \frac{f(1.5)}{f(1.5)} = \boxed{x_2 = 1.633748}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6337 - \frac{f(1.6337)}{f'(1.6337)} = \boxed{x_3 = 1.639107}$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.639107 - \frac{f(1.639107)}{f'(1.639107)} = \boxed{x_4 = 1.639321}$$

Zero is 1.639

Steven Romeiro

Pg 123 #1-23, 57, 59, 81, 83, 85

Section 3.1

1) a) = (x_1, y_1) slope = 0

b) = (x_2, y_2) Slope = ~ $\frac{5}{2}$

5) $f(x) = 3 - 2x$, $(-1, 5)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1) $f'(x+h) = 3 - 2(x+h) = 3 - 2x - 2h$

2) $f(x+h) - f(x) = 3 - 2x - 2h - (3 - 2x)$

$= 3 - 2x - 2h - 3 + 2x$

3) $\frac{f(x+h) - f(x)}{h} = \frac{\cancel{3} - \cancel{2x} - 2h - \cancel{3} + \cancel{2x}}{h}$

$= -\frac{2h}{h} = \boxed{-2}$

4) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \boxed{f'(x) = -2}$

$m = f'(-1) = -2(-1) = \boxed{2}$? Answer = -2

9) $f(x) = 3x - x^2 \quad (0,0)$

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = 3(x+h) - (x+h)^2$

$= 3x + 3h - (x^2 + 2xh + h^2)$

$= 3x + 3h - x^2 - 2xh - h^2$

2) $f(x+h) - f(x) = \cancel{3x} + 3h - \cancel{x^2} - 2xh - h^2 - \cancel{3x} + \cancel{x^2}$

$= 3h - 2xh - h^2$

3) $$\frac{f(x+h) - f(x)}{h} = \frac{3\cancel{h} - 2x\cancel{h} - h^{\cancel{3}}}{\cancel{h}} = 3 - 2x - h$$

4) $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 3 - 2x - h$$

$\boxed{f'(x) = 3 - 2x}$

$m = f'(0)$

$m = 3 - 2(0)$

$\boxed{m = 3}$

13) $f(x) = -5x$

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = -5(x+h) = -5x - 5h$

2) $f(x+h) - f(x) = \cancel{-5x} - 5h + \cancel{5x} = -5h$

3) $\frac{f(x+h) - f(x)}{h} = \frac{-5\cancel{h}}{\cancel{h}} = -5$

4) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} -5$ $\boxed{f'(x) = -5}$

17) $f(x) = 2x^2 + x - 1$

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = 2(x+h)^2 + (x+h) - 1$

$= 2(x^2 + 2xh + h^2) + x + h - 1$

$= 2x^2 + 4xh + 2h^2 + x + h - 1$

2) $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 + x + h - 1 - (2x^2 + x - 1)$

$= \cancel{2x^2} + 4xh + 2h^2 + \cancel{x} + h - \cancel{1} - \cancel{2x^2} - \cancel{x} + \cancel{1}$

$= 4xh + 2h^2 + h$

3) $\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2 + h}{h} = 4x + 2h + 1$

$\longrightarrow$

4) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} 4x + 2h + 1 = 4x + 2(0) + 1$

$\boxed{f'(x) = 4x + 1}$

?

21) $f(x) = \frac{1}{x-1}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

1) $f(x+h) = \frac{1}{(x+h)-1} = \frac{1}{x+h-1}$

2) $f(x+h) - f(x) = \frac{1}{x+h-1} - \frac{1}{x-1} = \frac{x-1-x-h+1}{(x+h-1)(x-1)}$

$= \frac{-h}{(x+h+1)(x-1)}$

3) $= \frac{f(x+h)-f(x)}{h} = \frac{\frac{-\cancel{h}}{(x+h+1)(x-1)}}{\cancel{h}}$

$= \frac{-1}{(x+h-1)(x-1)}$

4) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{-1}{(x+h-1)(x-1)} = \frac{-1}{(x+0-1)(x-1)}$

$= \frac{-1}{(x-1)(x-1)} = \boxed{f'(x) = -\frac{1}{(x-1)^2}}$

23) $f(x) = \sqrt{x+1}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

1) $f(x+h) = \sqrt{(x+h)+1} = \sqrt{x+h+1}$

2) $f(x+h) - f(x) = \sqrt{x+h+1} - \sqrt{x+1}$

3) $\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$

4) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$

$= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$

$$\lim_{h\to 0} \frac{x+h+1-(x+1)}{h(\sqrt{x+h+1}+\sqrt{x+1})} =$$

$$\lim_{h\to 0} \frac{h}{h(\sqrt{x+h+1}+\sqrt{x+1})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$$

$$= \frac{1}{\sqrt{x+0+1}+\sqrt{x+1}} = \frac{1}{\sqrt{x+1}+\sqrt{x+1}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x+1}}}$$

? 57) $f(x) = 4x - x^2 \qquad (2,5), (x_0, y_0)$

Dont understad how this works

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - y_0}{2 - x_0} = \frac{5-3}{2-x_0} = \frac{2}{2-x_0}$$

$\underline{m = f'(x_0)}$

$$f'(x_0) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = 4(x+h) - (x+h)^2$
$= 4x + 4h - (x^2 + 2xh + h^2)$
$= 4x + 4h - x^2 - 2xh - h^2$

$$2)\ f(x+h) - f(x) = 4\cancel{x} + 4h - \cancel{x^2} - 2xh - h^2 - 4\cancel{x} + \cancel{x^2}$$
$$= 4h - 2xh - h^2$$

$$3)\ \frac{f(x+h) - f(x)}{h} = \frac{4h - 2xh - h^2}{h} = \boxed{4 - 2x - h}$$

$$4)\ f'(x_0) = \lim_{h \to 0} 4 - 2x - h = \boxed{4 - 2x}$$

$$m = f'(x_0) = 4 - 2x_0$$

$$\frac{2}{2 - x_0} = 4 - 2x_0$$

$$2 = (4 - 2x_0)(2 - x_0)$$
$$2 = 8 - 4x_0 - 4x_0 + 2x_0^2$$
$$2x_0^2 - 8x_0 + 6 = 0$$
$$(2x_0 - 2)(x_0 - 3) = 0$$
$$\boxed{x_0 = 1} \qquad \boxed{x_0 = -3}$$

$$y_0 = 4x_0 - x_0^2 \qquad\qquad y_0 = 4x_0 - x^2$$
$$y_0 = 4(1) - 1 \qquad\qquad y_0 = 4(-3) - 9$$
$$y_0 = 3 \quad (1, 3) \qquad\qquad y_0 = -21 \quad (-3, -21)$$

$m = f'(X_0)$
$X_0 = 1$
$4 - 2(1) = \boxed{2}$
$(1,3)$ $m = 2$

$m = f'(X_0)$
$X_0 = -3$
$4 - 2(-3) = \boxed{10}$
$m = 10$ $(-3, -21)$

$Y - Y_1 = m(X - X_1)$
$Y - 3 = 2(X - 1)$
$Y - 3 = 2x - 2$
$\boxed{Y = 2x + 1}$

$Y - Y_1 = m(X - X_1)$
$Y + 21 = 10(X + 3)$
$Y + 21 = 10x + 30$
$\boxed{Y = 10x + 9}$? $Y = -2x + 9$

? 59) a) $G'(0) = -3$
b) $G'(+3) = 0$

c) That the graph is decreasing at $x = 1$

d) that the graph is increasing at $x = -4$

e) both are positive so $g(6) > g(4)$

f) not possible to find $g(2)$

81) $f(x) = \dfrac{1}{x+1}$

$x + 1 = 0$

$x = -1$ $\quad$ x cannot equal -1

$f(x)$ is differentiable everywhere except at -1

83) $f(x) = (x-3)^{\frac{2}{3}}$

$f(x) = \sqrt[3]{(x^2-6x+9)}$

$f(x)$ is differentiable everywhere except at 3 because of the sharp turn

85) $f(x) = \sqrt{x-1}$

$x - 1 \geqslant 0$

$x \geqslant 1$

$f(x)$ is differentiable everwhere where $x \geqslant 1$ since $x < 1$ is not defined.

Steven Romeiro

Pg 136 - # 1-65, 73, 75, 113

Section 3.2

1) a) $y' = x^{\frac{1}{2}} = \frac{1}{2}x^{(\frac{1}{2}-1)} = \frac{1}{2}x^{-\frac{1}{2}}$

$m = y'(1) = \frac{1}{2}(1)^{-\frac{1}{2}}$ $\boxed{m = \frac{1}{2}}$

b) $y = x^3$ $y' = 3x^{(3-1)} = y' = 3x^2$

$m = y'(1) = 3(1)^2 =$ $\boxed{m = 3}$

5) $y = x^6$

$y' = 6x^{(6-1)} =$ $\boxed{y' = 6x^5}$

9) $f(x) = x + 1$

$f'(x) = 1 + 0$

$\boxed{f'(x) = 1}$

13) $g(x) = x^2 + 4x^3$

$g'(x) = 2x + 4(3)x^2$

$\boxed{\begin{array}{c} g'(x) = 2x + 12x^2 \\ g'(x) = 2x(6x+1) \end{array}}$

17) $f(x) = 6x - 5e^x$

$\boxed{f(x) = 6 - 5e^x}$

21) $y = x^2 - \frac{1}{2}\cos x$

$y' = 2x^{(2-1)} - \frac{1}{2}(-\sin x)$

$\boxed{y' = 2x + \frac{1}{2}\sin x}$

25) $y = \frac{5}{2x^2} = y = \frac{5x^{-2}}{2}$

$y' = \frac{5}{2}(-2)x^{-3} = -5x^{-3} = \boxed{y' = -\frac{5}{x^3}}$

29) $y = \frac{\sqrt{x}}{x} = y = x^{\frac{1}{2}} \cdot x^{-1} = y = x^{-\frac{1}{2}}$

$y' = -\frac{1}{2}x^{-\frac{3}{2}} = \boxed{y' = \frac{-1}{2x^{\frac{3}{2}}}}$

33) $f(x) = -\frac{1}{2} + \frac{7}{5}x^3 \quad ; \quad (0, -\frac{1}{2})$

$f'(x) = -0 + \frac{7}{5}(3)x^2 = \boxed{\frac{21}{5}x^2}$

$m = f'(x_1)$

$f'(0) = \frac{21}{5}(0)^2$

$\boxed{m = f'(0) = 0}$

37) $f(t) = \frac{3}{4}e^t$; $(0, \frac{3}{4})$

$f'(t) = \frac{3}{4}e^t$ $m = f'(x_1)$

$f'(0) = \frac{3}{4}e^0 = \frac{3}{4}\cdot(1)$ $\boxed{m = f'(0) = \frac{3}{4}}$

41) $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$

$f(x) = x - 3 + 4x^{-2}$

$f'(x) = 1 - 0 - 8x^{-3}$

$f'(x) = 1 - 8x^{-3}$

$\boxed{f'(x) = 1 - \frac{8}{x^3}}$

45) $f(x) = \sqrt{x} - 6\sqrt[3]{x}$

$f(x) = x^{\frac{1}{2}} - 6x^{\frac{1}{3}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-\frac{2}{3}}$

$\boxed{f'(x) = \frac{1}{2x^{\frac{1}{2}}} - \frac{2}{x^{2/3}}}$

49) $f(x) = 6\sqrt{x} + 5\cos x$

$f(x) = 6x^{\frac{1}{2}} + 5\cos x$

$f'(x) = 3x^{-\frac{1}{2}} + 5(-\sin x)$

$$\boxed{f'(x) = \frac{3}{x^{\frac{1}{2}}} - 5\sin x}$$

53) $y = x^4 - x \quad ; \quad (-1, 2)$

$y' = 4x^3 - 1$

$m = f'(x_1) = f'(-1) = 4(-1)^3 - 1$

$\boxed{m = -5}$

$y - y_1 = m(x - x_1)$

$y - 2 = -5(x + 1)$

$y - 2 = -5x - 5 \qquad \boxed{y = -5x - 3}$

57) $y = x^4 - 8x^2 + 2$

$y' = 4x^3 - 16x$

Tangent is horizontal at $4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$4x(x-2)(x+2) = 0$

$4x = 0 \, , \quad x - 2 = 0 \quad , \quad x + 2 = 0$

$\boxed{x = 0} \, , \quad \boxed{x = 2} \quad , \quad \boxed{x = -2}$

61) $y = -4x + e^x$

$y' = -4 + e^x$

$-4 + e^x = 0$

$e^x = +4$

$\ln e^x = \ln 4$

$x = \ln 4$

65) $f(x) = \frac{K}{x}$; $y = -\frac{3}{4}x + 3$

$\left(\frac{K}{x}\right)' = \left(-\frac{3}{4}x + 3\right)'$

$(Kx^{-1})' = -Kx^{-2} = \frac{-K}{x^2}$

$\left(-\frac{3}{4}x + 3\right)' = -\frac{3}{4}$

$-\frac{K}{x^2} = -\frac{3}{4}$

$-3x^2 = -4K$

$-K = -\frac{3}{4}x^2$

$K = \frac{3}{4}x^2$

$\frac{\frac{3}{4}x^2}{x} = -\frac{3}{4}x + 3$

$\frac{3}{4}x = -\frac{3}{4}x + 3$

$x = -\frac{3}{4} \cdot \frac{4}{3} + 3$

$x = -1 + 3$

$\boxed{x = 2}$

$\frac{K}{2} = -\frac{3}{4}(2) + 3$

$\frac{K}{2} = \frac{-3}{2} + 3$

$\frac{K}{2} = \frac{3}{2}$

$\boxed{K = 3}$

$(x_1\ y_1)\quad(x_2\ y_2)$

? $73)\quad y = x^2 \quad ; \quad y = -x^2 + 6x - 5$

$y_1' = 2x_1 \quad ; \quad y_2' = -2x_2 + 6$

$2x_1 = -2x_2 + 6$

$\boxed{x_1 = -x_2 + 3}$

Since $y_1 = x_1^2$ and $y_2 = -x_2^2 + 6x_2 - 5$

So $y_1 = (-x_2 + 3)^2$

$y_1 = x_2^2 - 6x_2 + 9$

then $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(-x_2^2 + 6x_2 - 5) - (x_2^2 - 6x_2 + 9)}{x_2 - (-x_2 + 3)}$

$= -2x_2 + 6 = \dfrac{-x_2^2 + 6x_2 - 5 - x_2^2 - 6x_2 + 9}{2x_2 - 3}$

$(-2x_2 + 6)(2x_2 - 3) = -2x_2^2 + 12x_2 - 14$

$-4x_2^2 + 12x_2 + 6x_2 - 18 = -2x_2^2 + 12x_2 - 14$

$-4x_2^2 + 18x_2 - 18 = -2x_2^2 + 12x_2 - 14$

$2x_2^2 - 6x_2 + 4 = 0$

$x_2^2 - 3x_2 + 2 = 0$

$x_2 = 1$ or 2

for $X_2 = 1$

$y_2 = -1 + 6 - 5 = 0$

$X_1 = -1 + 3 = 2$ & $y_1 = 2^2 = 4$

$(1,0)$ $(2,4)$ → Tangent line points

eq: = $y - 0 = \left(\frac{4-0}{2-1}\right)(x-1)$

$$\boxed{y = 4x - 4}$$

for $x_2 = 2$, $y_2 = -2^2 + 6(2) - 5 = -4 + 12 - 5 = 3$

$x_1 = -2 + 3 = 1$, $y_1 = 1^2 = 1$

$(2,3)$ $(1,1)$ → Tangent line points

eq: $y - 1 = \frac{(3-1)}{(2-1)}(x-1)$

$y - 1 = 2x - 2$

$$\boxed{y = 2x - 1}$$

?

75) $f(x) = 3x + \sin x + 2$

$f'(x) = 3 + \cos x$

$f'(x) = 0$ for horizontal Tangent

$3 + \cos x = 0$

$\cos x = -3$

$x = \cos^{-1}(-3)$ cannot exist

because $-1 \leq \cos x \leq 1$

?

113) $f(x) = \begin{cases} ax^3, & x \leq 2 \\ x^2+b, & x > 2 \end{cases}$

$\lim\limits_{x \to 2} a(x^3) = \lim\limits_{x \to 2} x^2 + b$

$a(2^3) = 2^2 + b$

$8a = 4 + b$

$\boxed{b = 8a - 4}$

$f'(x) \begin{cases} 3ax^2, & x < 2 \\ 2x, & x > 2 \end{cases}$

$3ax^2 = 2x$

$3a(2^2) = 2(2)$

$3a(4) = 4$

$12a = 4$

$\boxed{a = \frac{1}{3}}$

$b = 8(\frac{1}{3}) - 4$

$b = \frac{8-12}{3}$

$\boxed{b = -\frac{4}{3}}$

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Pg 147 # 1-57, 77, 79, 81, 97 - 107

Section 3.3

1) $g(x) = (x^2+1)(x^2-2x)$

$U' = 2x \quad V' = 2x-2$

$g'(x) = 2x(x^2-2x) + 2x-2(x^2+1)$

$= 2x^3 - 4x^2 + 2x - 2x^2 - 2$

$\boxed{g'(x) = 2x^3 - 6x^2 + 2x - 2}$

$\boxed{g'(x) = 2(x^3 - 3x^2 + x - 1)}$

5) $f(x) = x^3 \cos x$

$U = x^3 \quad V = \cos x$

$U' = 3x^2 \quad V' = -\sin x$

$f'(x) = 3x^2(\cos x) + -\sin x(x^3)$

$\boxed{f'(x) = 3x^2 \cos x - x^3 \sin x}$

9) $h(x) = \dfrac{\sqrt[3]{x}}{x^3+1} \quad U = x^{\frac{1}{3}} \quad V = x^3+1$

$U' = \frac{1}{3}x^{-\frac{2}{3}} \quad V' = 3x^2$

$$\frac{\frac{1}{3}x^{-2/3}(x^3+1) - 3x^2(x^{\frac{1}{3}})}{(x^3+1)^2} = \frac{\frac{1}{3}x^{-2/3}(x^3+1) - 3x^{\frac{7}{3}}}{(x^3+1)^2}$$

? $\rightarrow \dfrac{x^3+1 - 3x^{\frac{7}{3}}(3x^{\frac{2}{3}})}{(3x^{\frac{2}{3}})(x^3+1)^2} = \dfrac{x^3+1-9x^3}{(3x^{\frac{2}{3}})(x^3+1)^2}$

$= \boxed{\dfrac{(1-8x^3)}{(3x^{\frac{2}{3}})(x^3+1)^2}}$

$U = X^3 - 3X \qquad V = 2x^2 + 3x + 5$

$U' = 3X^2 - 3 \qquad V' = 4x + 3$

13) $f(x) = (X^3 - 3x)(2x^2 + 3x + 5)$; $c = 0$

$f'(x) = (3x^2 - 3)(2x^2 + 3x + 5) + (4x + 3)(X^3 - 3x)$

1) $6x^3 + 9x^2 + 15x - 6x^2 - 9x - 15 = \boxed{6x^3 + 3x^2 + 6x - 15}$

2) $\boxed{4x^4 - 12x^2 + 3x^3 - 9x}$

3) $6x^3 + 3x^2 + 6x - 15 + 4x^4 - 12x^2 + 3x^3 - 9x$

$\boxed{f'(x) = 4x^4 + 9x^3 - 9x^2 - 3x - 15}$

$\boxed{f'(0) = -15}$

17) $f(x) = X \cos X \qquad c = \frac{\pi}{4}$

$U = X \qquad V = \cos x$

$U' = 1 \qquad V' = -\sin X$

$f'(x) = 1(\cos x) + X(-\sin x)$

$\boxed{f'(x) = \cos x - X \sin x}$

$f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) - \left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$

$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} - \frac{\pi\sqrt{2}}{8}$

$= \frac{4\sqrt{2}}{8} - \frac{\pi\sqrt{2}}{8} = \frac{\sqrt{2}}{8}(4 - \pi)$

$$\boxed{f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{8}(4 - \pi)}$$

21) $y = \dfrac{x^2 + 2x}{3} = \quad y = \dfrac{1}{3}\left(x^2 + 2x\right)$

$y' = \dfrac{1}{3}(2x+2) \quad = \quad \boxed{y' = \dfrac{2x+2}{3}}$

25) $y = \dfrac{4x^{3/2}}{x}$

$y = 4\left(x^{\frac{3}{2}-1}\right) \qquad y = 4\left(x^{\frac{1}{2}}\right)$

$y' = 4\left(\frac{1}{2}x^{-\frac{1}{2}}\right) \qquad y' = 4\left(\dfrac{1}{2x^{\frac{1}{2}}}\right)$

$\boxed{y' = \dfrac{2}{\sqrt{x}}}$

29) $f(x) = x\left(1 - \dfrac{4}{x+3}\right)$

$f(x) = x\left(\dfrac{x+3-4}{x+3}\right) = x\left(\dfrac{x-1}{x+3}\right)$

$f(x) = \left(\dfrac{x^2 - 1x}{x+3}\right) = \qquad U = x^2 - x \qquad V = x+3$

$U' = 2x - 1 \qquad V' = 1$

$f'(x) = \dfrac{2x-1(x+3) - (x^2 - x)(1)}{(x+3)^2}$

$$\frac{2x^2+6x-x-3-x^2+x}{(x+3)^2}$$

$$\boxed{f'(x) = \frac{x^2+6x-3}{(x+3)^2}}$$

33) $h(x) = (x^3-2)^2$

$h(x) = x^6 - 4x^3 + 4$

$h'(x) = 6x^5 - 12x^2$

$\boxed{h'(x) = 6x^2(x^3-2)}$

37) $f(x) = (3x^3+4x)(x-5)(x+1)$

$f(x) = 3x^4 - 15x^3 + 3x^4 + 3x^3 + 4x^2 - 20x + 4x^2 + 4x$

$+ x^2 + x - 5x - 5$

$f(x) = 6x^4 - 12x^3 + 9x^2 - 20x - 5$

$\boxed{f'(x) = 24x^3 - 36x^2 + 18x - 20}$

41) $f(x) = x^2 \sin x$

$u = x^2$ $\quad v = \sin x$

$u' = 2x$ $\quad v' = \cos x$

$f'(x) = 2x(\sin x) + x^2(\cos x)$

$= 2x\sin x + x^2\cos x$

$\boxed{f'(x) = x(2\sin x + x\cos x)}$

45) $f(x) = -e^x + \tan x$

$$\boxed{f'(x) = -e^x + \sec^2 x}$$

49) $\dfrac{3(1-\sin x)}{2\cos x}$

$U = 1-\sin x$

$U' = -\cos x$

$V = \cos x$

$V' = -\sin x$

$$y = \frac{3-3\sin x}{2\cos x} \qquad y = \frac{3}{2} \cdot \left(\frac{1-\sin x}{\cos x}\right)$$

$$y' = \frac{3}{2}\left[\frac{(-\cos x)(\cos x) - (1-1\sin x)(-\sin x)}{(\cos x)^2}\right]$$

$$y' = \frac{3}{2}\left[\frac{-\cos^2 x - \sin^2 x + \sin x}{\cos^2 x}\right]$$

$$y' = \frac{3}{2}\left[\frac{-(\cos^2 x + \sin^2 x) + \sin x}{\cos^2 x}\right]$$

because $\sin^2 x + \cos^2 x = 1$

$$y' = \frac{3}{2}\left[\frac{-1 + \sin x}{\cos^2 x}\right]$$

$$y' = \frac{3}{2}\left[\frac{-1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right]$$

$$y' = \frac{3}{2}\left[\frac{-1}{\cos x} + \frac{\sin x}{\cos x}\right] \cdot \left(\frac{1}{\cos x}\right)$$

$$\boxed{y' = \frac{3}{2}(\sec x)(\tan x - \sec x)}$$

53) $f(x) = x^2 \tan x$ $\qquad U = x^2$

$U' = 2x$

$f'(x) = 2x(\tan x) + x^2(\sec^2 x)$ $\qquad V = \tan x$

$\boxed{f'(x) = 2\tan x + x^2 \sec^2 x}$ $\qquad V' = \sec^2 x$

57) $y = \dfrac{e^x}{4\sqrt{x}}$ $\qquad U = e^x \quad V = 4x^{\frac{1}{2}}$

$U' = e^x \quad V' = 2x^{-\frac{1}{2}}$

$$y = \frac{e^x}{4x^{\frac{1}{2}}}$$

$$y' = \frac{e^x(4x^{\frac{1}{2}}) - e^x(2x^{-\frac{1}{2}})}{(4x^{\frac{1}{2}})^2}$$

$$y' = \frac{e^x 4x^{\frac{1}{2}} - e^x 2x^{-\frac{1}{2}}}{(4x^{\frac{1}{2}})^2}$$

$$y' = \frac{e^x(4x^{\frac{1}{2}} - 2x^{-\frac{1}{2}})}{(4x^{\frac{1}{2}})^2}$$

$$y' = \frac{e^x(4x^{\frac{1}{2}} - 2x^{-\frac{1}{2}})}{16x}$$

$$y' = \frac{e^x(2x^{-\frac{1}{2}} - x^{-\frac{3}{2}})}{8}$$

$$y' = \frac{e^x}{8}(2x^{-\frac{1}{2}} - x^{-\frac{3}{2}})$$

$$\boxed{y' = \frac{e^x}{8}\left(\frac{2}{\sqrt{x}} - \frac{1}{\sqrt{x^3}}\right)}$$

$U = x^2 \quad V = x-1$
$U' = 2x \quad V' = 1$

77) $F(x) = \dfrac{x^2}{x-1}$

$f'(x) = \dfrac{2x(x-1) - x^2(1)}{(x-1)^2}$

$f'(x) = \dfrac{2x^2 - 2x - x^2}{(x-1)^2}$

$f'(x) = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2} = 0$

$x(x-2) = 0 \qquad \boxed{x = 0 \text{ or } x = 2}$

$x = 0$
$F(x) = \dfrac{(0)^2}{0-1} = 0 \quad \boxed{(0,0)}$

$x = 2$
$f(2) = \dfrac{(2)^2}{2-1} = 4 \quad \boxed{(2,4)}$

$U = 8x - 16 \qquad V = e^x$
$U' = 8 \qquad V' = e^x$

79) $g(x) = \dfrac{8(x-2)}{e^x}$

$g(x) = \dfrac{8x - 16}{e^x}$

$g'(x) = \dfrac{8(e^x) - (8x-16)e^x}{(e^x)^2}$

$g'(x) = \dfrac{8e^x - 8xe^x + 16e^x}{(e^x)^2}$

$g'(x) = \dfrac{8 - 8x + 16}{e^x}$

$g'(x) = \dfrac{-8x + 24}{e^x} = 0$

$-8x + 24 = 0$
$-8(x-3) = 0$
$\boxed{x = 3}$

$g(3) = \dfrac{8(3-2)}{e^3} = \boxed{\dfrac{8}{e^3}} \qquad \left(3, \dfrac{8}{e^3}\right)$

$U = x+1 \quad V = x-1$
$U' = 1 \quad V' = 1$

$m = -\frac{1}{2}$

81) $f(x) = \frac{x+1}{x-1}$ $\parallel$ $y = -\frac{1}{2}x + 3$

$f'(x) = \frac{1(x-1) - (x+1)1}{(x-1)^2}$

$f'(x) = \frac{x-1-x-1}{(x-1)^2}$

$f'(x) = \frac{-2}{(x-1)^2} = -\frac{1}{2}$

$-(x-1)^2 = -4$

$-(x^2-2x+1) = -4$

$-x^2+2x-1+4 = 0$

$-x^2+2x+3 = 0$

$(x+1)(-x+3) = 0$

$x = -1$ or $x = 3$

$x = -1$

$f(x) = \frac{(-1)+1}{-1-1} = \frac{0}{-2} = 0 \quad (-1, 0)$

$x = 3$

$f(x) = \frac{3+1}{3-1} = \frac{4}{2} = 2 \quad (3, 2)$

$y - 0 = -\frac{1}{2}(x+1)$

$\boxed{y = -\frac{1}{2}x - \frac{1}{2}}$

$y - 2 = -\frac{1}{2}(x-3)$

$y - 2 = -\frac{1}{2}x + \frac{3}{2}$

$\boxed{y = -\frac{1}{2} + \frac{7}{2}}$

97) $f(x) = 4x^{\frac{3}{2}}$

$f'(x) = 4\left(\frac{3}{2}\right)x^{\frac{1}{2}}$

$f'(x) = 6x^{\frac{1}{2}}$

$f''(x) = 6\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$

$\boxed{f''(x) = 3x^{-\frac{1}{2}}}$

101) $f(x) = 3\sin x$

$f'(x) = 3\cos x$

$f''(x) = 3(-\sin x)$

$f''(x) = -3\sin x$

105) $f'(x) = x^2$

$f''(x) = 2x$

107) $f'''(x) = 2x^{\frac{1}{2}}$

$f^4(x) = 2\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$

$\boxed{f^4(x) = \frac{1}{\sqrt{x}}}$

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Pg 161 # 1-35, 47-111, 123-127

Section 3.4

1) $y = (6x-5)^4$ $u = g(x) = 6x-5$

$f(u) = u^4$

5) $y = \csc^3 x$ $u = g(x) = \csc x$

$f(u) = u^3$

9) $y = (2x-7)^3$

$y' = (2)^3$

$y' = 2 \cdot 3(2x-7)^2$

$\boxed{y' = 6(2x-7)^2}$

13) $f(x) = (9-x^2)^{\frac{2}{3}}$

$f'(x) = (-2x)^{\frac{2}{3}}$

$f'(x) = -2x \cdot \frac{2}{3}(9-x^2)^{\frac{1}{3}}$

$\boxed{f'(x) = -\frac{4}{3}x(9-x^2)^{1/3}}$

17) $y = \sqrt[3]{9x^2+4}$

$y = (9x^2+4)^{\frac{1}{3}}$

$y' = (18x)^{\frac{1}{3}}$

$y' = 18x \cdot \frac{1}{3}(9x^2+4)^{-\frac{2}{3}}$

$y' = 6x\,(9x^2+4)^{-\frac{2}{3}}$

$\boxed{y' = \dfrac{6x}{(9x^2+4)^{2/3}}}$

21) $y = \dfrac{1}{x-2}$

$y = (x-2)^{-1}$

$y' = (1)^{-1}$

$y' = 1 \cdot -1(x-2)^{-2}$

$y' = \dfrac{-1}{(x-2)^2}$

25) $y = \dfrac{1}{\sqrt{x+2}}$ $\qquad y = (x+2)^{-\frac{1}{2}}$

$y' = (1)^{-\frac{1}{2}}$ $\qquad y' = 1 \cdot -\frac{1}{2}(x+2)^{-\frac{3}{2}}$

$\boxed{y' = \dfrac{-1}{2(x+2)^{3/2}}}$ or $\boxed{y' = \dfrac{-1}{2(x+2)(x+2)^{\frac{1}{2}}}}$

$U = x \quad V = (1-x^2)^{\frac{1}{2}}$
$U' = 1 \quad V' = (-2x) \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}}$

29) $y = x\sqrt{1-x^2}$

$y = x(1-x^2)^{1/2}$

$y' = 1\left((1-x^2)^{\frac{1}{2}}\right) + x\left[-2x \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}}\right]$

$y' = (1-x^2)^{\frac{1}{2}} + x\left[-x(1-x^2)^{-\frac{1}{2}}\right]$

$y' = (1-x^2)^{\frac{1}{2}} - x^2(1-x^2)^{-1/2}$

Factor? $y' = (1-x^2)^{-1/2}\left[-x^2 + (1-x^2)\right]$

$y' = \dfrac{-x^2 + 1 - x^2}{(1-x^2)^{1/2}}$

$$\boxed{y' = \frac{-2x^2 + 1}{(1-x^2)^{1/2}}}$$

35) $f(x) = \left(\dfrac{1-2x}{1+x}\right)^3 \qquad U = 1-2x \quad U' = -2$
$V = 1+x \quad V' = 1$

$f'(x) = \left(\dfrac{-2(1+x) - (1-2x)(1)}{(1+x)^2}\right)^3$

$f'(x)\left(\dfrac{-2x-2+2x-1}{(1+x)^2}\right)^3 \qquad f'(x) = \left(\dfrac{-3}{(1+x)^2}\right)^3$

$f'(x) = -\dfrac{3}{(1+x^2)} \cdot 3\left(\dfrac{1-2x}{1+x}\right)^2$

$$f'(x) = \left(\frac{-9}{(1+x)^2}\right)\left(\frac{1-2x}{1+x}\right)^2$$

$$\boxed{f'(x) = \frac{-9(1-2x)^2}{(1+x)^4}}$$

47) a) $y = \sin x$

$y' = \cos x$

$y' = \cos(0)$

$y' = \boxed{1}$

b) $y = \sin 2x$

$y' = \cos 2x \cdot 2$

$y' = 2\cos 2x$

$y' = 2\cos 2(0)$

$y' = 2\cos(0) = \boxed{2}$

51) $y = \ln x^3$

$y = 3\ln x$

$y' = \frac{3(1)}{x} = \boxed{y' = \frac{3}{x}}$ $y'(1) = \frac{3}{1}$

$$\boxed{y = 3}$$

55) $y = \cos 3x$ $\quad U = 3x$

$y = \cos U$

$y' = -\sin U \cdot U'$

$y' = -\sin 3x \cdot 3$

$\boxed{y' = -3\sin 3x}$

59) $f(\theta) = \frac{1}{4}\sin^2 2\theta$ $\quad U = 2\theta$

$f'(\theta) = \frac{1}{4}\sin^2 U$

$f'(\theta) = \frac{1}{4}(\sin U)^2$

$f'(\theta) = 2\left(\frac{1}{4}\right)(\sin U) \cdot \cos U \cdot U'$

$f'(\theta) = \frac{1}{2}\sin U \cdot \cos U \cdot U'$

$f'(\theta) = \frac{1}{2}\sin 2\theta \cdot \cos 2\theta \cdot (2\theta)'$

$f'(\theta) = \frac{1}{2}\sin 2\theta \cdot \cos 2\theta \cdot 2$

$f'(\theta)$ $\frac{1}{2}\left[2\sin 2\theta \cdot \cos 2\theta\right]$ $\quad * \sin 2\theta = 2\sin\theta\cos\theta$

$\sin 4\theta = 2\sin 2\theta\cos 2\theta$

$\boxed{f'(\theta) = \frac{1}{2}\sin 4\theta}$

$\frac{1}{4}\sin 4x^2$

$\frac{1}{4}\cos 4x^2 \cdot 8x$

61) $y = \sqrt{x} + \frac{1}{4}\sin((2x)^2)$ $\quad 4x^2 \quad 8x$

$y = x^{\frac{1}{2}} + \frac{1}{4}\sin 4x^2$

$y' = \frac{1}{2\sqrt{x}} + \frac{1}{4} 8x \cdot \cos(2x)^2$

$\boxed{y' = \frac{1}{2\sqrt{x}} + 2x\cos(2x)^2}$

67) $y = e^{\sqrt{x}}$ $\quad U = \sqrt{x}$

$y = e^{U}$ $\quad U' = \frac{1}{2}x^{-\frac{1}{2}}$

$y' = e^{U}$

$y' = U' \cdot e^{U}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} \cdot e^{x^{\frac{1}{2}}}$

$\boxed{y' = \frac{1e^{\sqrt{x}}}{2\sqrt{x}}}$

71) $y = \ln(e^{x^2})$

$y = \ln(e^{2x})$ $\quad U = e^{x^2} \quad U' = 2x \cdot e^{x^2}$

$y = \ln(U)$

$y' = \frac{U'}{U} = \frac{2x \cdot e^{x^2}}{e^{x^2}}$

$\boxed{y' = 2x}$

75) $y = x^2e^x - 2xe^x + 2e^x$

$y = e^x(x^2 - 2x + 2)$

$y' = e^x(x^2-2x+2) + e^x(2x-2)$

$y' = x^2e^x - \cancel{2xe^x} + \cancel{2e^x} + \cancel{2xe^x} - \cancel{2e^x}$

$\boxed{y' = x^2e^x}$

79) $y = e^x(\sin x + \cos x)$

$y' = e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$

$\boxed{y' = 2e^x \cos x}$

83) $y = (\ln x)^4$

$y' = \frac{1}{x} \cdot 4(\ln x)^3$

$\boxed{y' = \frac{4}{x}(\ln x)^3}$

87) $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

$U = \frac{x}{x^2+1}$

$U' = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2}$

$U' = \frac{x^2+1-x(2x)}{(x^2+1)^2}$

$f(x) = \ln(u)$

$f'(x) = \frac{u'}{u} =$

$f'(x) = \frac{x^2+1-x(2x)}{\cancel{(x^2+1)^2}} \cdot \frac{x^2+1}{x} = \frac{x^2+1-2x^2}{x(x^2+1)}$

$\boxed{f'(x) = \frac{1-x^2}{x(x^2+1)}}$

$U = \left(\frac{x+1}{x-1}\right) \quad U' = \left(\frac{x-1-x+1}{(x-1)^2}\right) = \frac{-2}{(x-1)^2}$

91) $y = \ln\sqrt{\frac{x+1}{x-1}}$

$y = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

$y = \frac{1}{2}\ln(U)$

$y' = \frac{1}{2}\left(\frac{U'}{U}\right) = \frac{1}{2}\left(\frac{-2}{(x-1)^2}\cdot\frac{x-1}{x+1}\right) = \frac{1}{2}\left(\frac{-2}{x^2-1}\right)$

$\boxed{y' = \frac{-1}{x^2-1}}$

95) $y = \ln(\sin x) \qquad U = \sin x \quad U' = \cos x$

$y = \ln U$

$y' = \frac{U'}{U} = \frac{\cos x}{\sin x}$

$\boxed{y' = \cot x}$

101) $f(x) = 2(x^2-1)^3$

$f'(x) = 2x \cdot 6(x^2-1)^2$

$f'(x) = 12x(x^2-1)^2$

$f''(x) = 12x \cdot 2x \cdot 2(x^2-1) + (x^2-1)^2 \cdot 12$

$f''(x) = 48x^2(x^2-1) + 12(x^2-1)^2$

$f''(x) = 12(4x^2(x^2-1) + (x^2-1)^2$

$f''(x) = 12(4x^4 - 4x^2 + x^4 - 2x^2 + 1)$

$\boxed{f''(x) = 12(5x^4 - 6x^2 + 1)}$

105) $f(x) = (3+2x)e^{-3x}$ $\quad U = 3+2x \quad U' = 2$

$f'(x) = 2(e^{-3x}) + (3+2x)(-3e^{-3x})$ $\quad V = e^{-3x} \quad V' = -3 \cdot e^{-3x}$

$f'(x) = 2e^{-3x} - 9e^{-3x} - 6xe^{-3x}$

$f'(x) = -7e^{-3x} - 6xe^{-3x}$

$f'(x) = -e^{-3x}(7+6x)$ $\quad U = -e^{-3x} \quad U' = 3e^{-3x}$

$f''(x) = 3e^{-3x}(7+6x) + (-e^{-3x})(6)$ $\quad V = 7+6x \quad V' = 6$

$f''(x) = 21e^{-3x} + 18xe^{-3x} + (-6e^{-3x})$

$f''(x) = 15e^{-3x} + 18xe^{-3x}$

$\boxed{f''(x) = 3e^{-3x}(5+6x)}$

109) $f(x) = \dfrac{3}{x^3-4}$; $\left(-1, -\dfrac{3}{5}\right)$

$$f'(x) = \frac{0(x^3-4) - 3(3x^2)}{(x^3-4)^2}$$

$$f'(x) = \frac{-9x^2}{(x^3-4)^2} = f'(x) = -\left[\frac{3x}{(x^3-4)}\right]^2$$

$$f'(-1) = -\left[\frac{-3}{(-1-4)}\right]^2 = -\left[\frac{-3}{-5}\right]^2$$

$$\boxed{f'(-1) = -\frac{9}{25}}$$

$U = 3x+2 \quad U' = 3$
$V = x-1 \quad V' = 1$

111) $f(x) = \dfrac{3x+2}{x-1}$; $(0,-2)$

$f'(x) = \dfrac{3(x-1) - (3x+2)(1)}{(x-1)^2}$

$f'(x) = \dfrac{3x-3-3x-2}{(x-1)^2}$

$f'(x) = \dfrac{-5}{(x-1)^2}$ $\quad f'(0) = \dfrac{-5}{(0-1)^2}$ $\quad \boxed{f'(0) = -5}$

123) $f(x) = 4^x$
$\boxed{f'(x) = (\ln 4)4^x}$

127) $g(x) = x^2 \cdot 2^x$

$U = x^2$; $U' = 2x$ $\quad V = 2^x$; $V' = (\ln 2)2^x$

$g'(x) = 2x(2^x) + x^2((\ln 2)2^x)$

$g'(x) = 2x2^x + (x^2\ln 2)2^x$
$g'(x) = 2^x(2x + x^2\ln 2)$

$\boxed{g'(x) = x2^x(2 + x\ln 2)}$

Steven Romeiro
Pg 171 # 1-45, 65-73

Section 3.5

1) $x^2+y^2=36$

$\frac{d}{dx}[x^2+y^2]=\frac{d}{dx}(36)$

$2x+2y\frac{dy}{dx}=0$

$2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{-2x}{2y}$

$\boxed{\frac{dy}{dx}=-\frac{x}{y}}$

5) $x^3-xy+y^2=4$

$\frac{d}{dx}(x^3-xy+y^2)=\frac{d}{dx}(4)$

$3x^2-(1\cdot y+\frac{dy}{dx}\cdot x)+2y\frac{dy}{dx}=0$

$3x^2-y-x\frac{dy}{dx}+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}(-x+2y)=y-3x^2$

$\boxed{\frac{dy}{dx}=\frac{y-3x^2}{-x+2y}}$

9) $x^3y^3 - y = x$

$\frac{d}{dx}(x^3y^3 - y) = \frac{d}{dx}x$

$(3x^2y^3 + 3y^2\frac{dy}{dx}x^3) - \frac{dy}{dx} = 1$

$3x^2y^3 + 3x^3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1$

$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$

$\boxed{\frac{dy}{dx} = \frac{1-3x^2y^3}{3x^3y^2-1}}$

13) $\sin x + 2\cos 2y = 1$

$\frac{d}{dx}(\sin x + 2\cos 2y) = 1$

$\cos x - 2\sin 2y \frac{dy}{dx} \cdot 2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-2\sin 2y \cdot 2) = -\cos x$

$\boxed{\frac{dy}{dx} = \frac{\cos x}{4\sin 2y}}$

17) $y = \sin(xy)$

$$\frac{d}{dx}(y) = \frac{d}{dx}\left[\sin(xy)\right]$$

$$\frac{dy}{dx} = \cos xy \cdot \left(1y + \frac{dy}{dx}x\right)$$

$$\frac{dy}{dx} = \cos xy\left(y + x\frac{dy}{dx}\right)$$

$$\frac{dy}{dx} = y\cos xy + x\cos xy\frac{dy}{dx}$$

$$\frac{dy}{dx} - x\cos xy\frac{dy}{dx} = y\cos xy$$

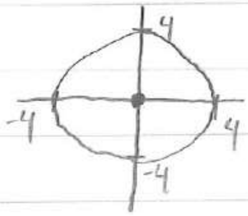
$$\frac{dy}{dx}\left(1 - x\cos xy\right) = y\cos xy$$

$$\boxed{\frac{dy}{dx} = \frac{y\cos xy}{1 - x\cos xy}}$$

21) $x^2 + y^2 = 16$

$y^2 = -x^2 + 16$

$\boxed{y = \pm\sqrt{-x^2+16}}$

b) 

c) $y = \pm\sqrt{-x^2+16}$

$y' = \pm(-x^2+16)^{\frac{1}{2}}$

$y' = \pm\frac{1}{2}(-x^2+16)^{-\frac{1}{2}} \cdot -2x$

$y' = \pm\frac{-2x}{2(-x^2+16)^{\frac{1}{2}}}$

$\boxed{y' = \pm\frac{x}{\sqrt{-x^2+16}}} \rightarrow \boxed{y' = \pm\frac{x}{y}}$

d) $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(16)$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{2x}{2y} \rightarrow \boxed{\frac{dy}{dx} = -\frac{x}{y}}$

or

$\boxed{y' = -\frac{x}{y}}$

25) $xy = 4$; $(-4, -1)$

$$\frac{d}{dx}(xy) = \frac{d}{dx}(4)$$

$$\left(1y + \frac{dy}{dx}x\right) = 0$$

$$x\frac{dy}{dx} = -y$$

$$\boxed{\frac{dy}{dx} = -\frac{y}{x}}$$

$$\frac{dy}{dx} = -\left(\frac{-1}{-4}\right)$$

$$\boxed{\frac{dy}{dx} = -\frac{1}{4}}$$

29) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$; $(8, 1)$

$$\frac{d}{dx}\left(x^{\frac{2}{3}} + y^{\frac{2}{3}}\right) = \frac{d}{dx}5$$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$$

$$\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = -\frac{2}{3}x^{-\frac{1}{3}}$$

$$\frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = -\frac{2}{3x^{\frac{1}{3}}}$$

$$\frac{dy}{dx} = -\frac{2}{3x^{\frac{1}{3}}} \cdot \frac{3y^{\frac{1}{3}}}{2}$$

$$\frac{dy}{dx} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}}$$

$$\frac{dy}{dx} = \sqrt[3]{\frac{1}{8}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{2}}$$

33) $3e^{xy} - x = 0$

$$\frac{d}{dx}\left(3e^{xy} - x\right) = \frac{d}{dx}(0)$$

$$3\left(1y + x\frac{dy}{dx}\right) \cdot e^{xy} - 1 = 0$$

$$3\left(y + x\frac{dy}{dx}\right)e^{xy} = 1$$

$$ye^{xy} + xe^{xy}\frac{dy}{dx} = \frac{1}{3}$$

$$xe^{xy}\frac{dy}{dx} = \frac{1}{3} - ye^{xy}$$

$$\frac{dy}{dx} = \frac{\frac{1}{3} - ye^{xy}}{xe^{xy}}$$

$$\frac{dy}{dx} = \frac{\frac{1-3ye^{xy}}{3}}{xe^{xy}}$$

$$\boxed{\frac{dy}{dx} = \frac{1-3ye^{xy}}{3xe^{xy}}}$$

$(3, 0)$

$$\frac{dy}{dx} = \frac{1-3(0)e^{(3)(0)}}{3(3)e^{(3)(0)}}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{9}}$$

$$37)\ f(x) = (x^2+y^2)^2 = 4x^2y \quad ; \quad (1,1)$$

$$\frac{d}{dx}(x^2+y^2)^2 = \frac{d}{dx}(4x^2y)$$

$$2(x^2+y^2)\cdot\left(2x+2y\frac{dy}{dx}\right) = 4\left(2x\cdot y + x\cdot\frac{dy}{dx}\right)$$

$$\rightarrow \frac{2}{2}\left(2x^3+2x^2y\frac{dy}{dx}+2y^2x+2y^3\frac{dx}{dy}\right) = \frac{4}{2}\left(2xy + x\frac{dy}{dx}\right)$$

$$2x^3+2y^2x-2xy = 4x\frac{dy}{dx} - 2x^2y\frac{dy}{dx} - 2y^3\frac{dx}{dy}$$

$$2x^3+2y^2x-4xy = (4x-2x^2y-2y^3)\frac{dx}{dy}$$

$$\frac{dx}{dy} = \frac{2x^3+2y^2x-4xy}{4x-2x^2y-2y^3}$$

$$\frac{dx}{dy} = \frac{x^3+y^2x-2xy}{2x-x^2y-y^3}$$

$$(1,1)\quad \frac{dy}{dx} = \frac{(1)^3+(1)^2(1)-2(1)(1)}{2(1)-(1)^2(1)}$$

$$\frac{dy}{dx} = \frac{1+1-2}{2-1}$$

$$\frac{dy}{dx} = \frac{0}{1} = \boxed{0}$$

41) $xy = 1$; $(1,1)$

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$1 \cdot y + x\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} = -y$$

$$\boxed{\frac{dy}{dx} = -\frac{y}{x}}$$

$$\frac{dy}{dx} = -\frac{1}{1}$$

$$\boxed{\frac{dy}{dx} = -1}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -1(x - 1)$$

$$y - 1 = -x + 1$$

$$\boxed{y = -x + 2}$$

45) $3(x^2+y^2)^2 = 100(x^2-y^2)$; $(4,2)$

$$\frac{d}{dx}\left(3(x^2+y^2)^2\right) = \frac{d}{dx}\left(100(x^2-y^2)\right)$$

$$3\left(2(x^2+y^2)\cdot\left(2x+2y\frac{dy}{dx}\right)\right) = 100\left(2x - 2y\frac{dy}{dx}\right)$$

$$\frac{6}{6}\left(2x^3 + 2x^2y\frac{dy}{dx} + 2y^2x + 2y^3\frac{dy}{dx}\right) = \frac{100}{6}\left(2x - 2y\frac{dy}{dx}\right)$$

$$2x^3 + 2x^2y\frac{dy}{dx} + 2y^2x + 2y^3\frac{dy}{dx} = \frac{50}{3}\left(2x - 2y\frac{dy}{dx}\right)$$

$$2x^2y\frac{dy}{dx} + 2y^3\frac{dy}{dx} + \frac{100}{3}y\frac{dy}{dx} = \frac{100}{3}x - 2x^3 - 2y^2x$$

$$\frac{dy}{dx}\left(2x^2y + 2y^3 + \frac{100}{3}\right) = \frac{100}{3}x - 2x^3 - 2y^2x$$

$$\frac{dy}{dx} = \frac{\frac{100}{3}x - 2x^3 - 2y^2x}{2x^2y + 2y^3 + \frac{100}{3}}$$

$$\frac{dy}{dx} = \frac{\frac{100x - 6x^3 - 6y^2x}{3}}{\frac{6x^2y + 6y^3 + 100}{3}} = \frac{100x - 6x^3 - 6y^2x}{6x^2y + 6y^3 + 100}$$

$$\boxed{\frac{dy}{dx} = \frac{50x - 3x^3 - 3y^2x}{3x^2y + 3y^3 + 50}}$$

$$(4,2)\ \frac{dy}{dx} = \frac{50(4) - 3(4)^3 - 3(2)^2(4)}{3(4)^2(2) + 3(2)^3 + 50} = \frac{-40}{220} = \boxed{-\frac{2}{11}}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{2}{11}(x - 4)$$

$$y - 2 = -\frac{2}{11}x + \frac{8}{11}$$

$$\boxed{y = -\frac{2}{11}x + \frac{30}{11}}$$

65) $y = x\sqrt{x^2-1}$

$y = x(x^2-1)^{\frac{1}{2}}$

$\ln y = \ln\left[x(x^2-1)^{\frac{1}{2}}\right]$

$\frac{y'}{y} = \frac{1}{2}\ln\left[x(x^2-1)\right]$

$\frac{y'}{y} = \ln x + \frac{1}{2}\ln(x^2-1)$

$\frac{y'}{y} = \frac{1}{x} + \frac{1(2x)}{2(x^2-1)}$

$\frac{y'}{y} = \frac{1}{x} + \frac{x}{(x^2-1)} = \frac{1(x^2-1)}{x(x^2-1)} + \frac{x(x)}{(x^2-1)(x)}$

$\rightarrow \frac{y'}{y} = \frac{x^2-1+x^2}{x(x^2-1)}$

$y' = \left[\frac{2x^2-1}{x(x^2-1)}\right] \cdot y$

$y' = \left[\frac{2x^2-1}{x(x^2-1)}\right] \cdot \left[x(x^2-1)^{\frac{1}{2}}\right]$

$y' = \frac{(2x^2-1)x}{x(x^2-1)\cdot(x^2-1)^{-\frac{1}{2}}}$

$y' = \frac{2x^2-1}{(x^2-1)^{\frac{1}{2}}}$

$\boxed{y' = \frac{2x^2-1}{\sqrt{x^2-1}}}$

69) $y = \dfrac{x(x-1)^{\frac{3}{2}}}{\sqrt{x+1}}$

$$y = x(x-1)^{\frac{3}{2}} \cdot (x+1)^{-\frac{1}{2}}$$

$$\ln y = \ln\left[x(x-1)^{\frac{3}{2}} \cdot (x+1)^{-\frac{1}{2}}\right]$$

$$\ln y = \ln x + \ln(x-1)^{\frac{3}{2}} + \ln(x+1)^{-\frac{1}{2}}$$

$$\ln y = \ln x + \frac{3}{2}\ln(x-1) - \frac{1}{2}\ln(x+1)$$

$$\frac{y'}{y} = \frac{1}{x} + \frac{3}{2(x-1)} - \frac{1}{2(x+1)}$$

$$\frac{y'}{y} = \frac{1}{2}\left(\frac{1}{x} + \frac{3}{x-1} - \frac{1}{x+1}\right)$$

$\boxed{\begin{array}{c}LCD \\ 2x(x+1)(x-1)\end{array}}$

$$\frac{y'}{y} = \frac{2(x+1)(x-1) + 3x(x+1) - x(x-1)}{2x(x-1)(x+1)}$$

$$\frac{y'}{y} = \frac{2x^2 - 2 + 3x^2 + 3x - x^2 + x}{2x(x+1)(x-1)}$$

$$\frac{y'}{y} = \frac{4x^2 + 4x - 2}{2x(x+1)(x-1)}$$

$$\frac{y'}{y} = \frac{2(2x^2 + 2x - 1)}{2x(x+1)(x-1)}$$

$$\frac{y'}{y} = \frac{2x^2+2x-1}{x(x+1)(x-1)}$$

$$y' = \left[\frac{2x^2+2x-1}{\cancel{x}(x+1)(x-1)}\right] \cdot \left[\frac{\cancel{x}(x-1)^{3/2}}{(x+1)^{1/2}}\right]$$

$$\boxed{y' = \frac{(2x^2+2x-1)(x-1)^{\frac{1}{2}}}{(x+1)^{3/2}}}$$

73) $y = (x-2)^{x+1}$

$\ln y = \ln (x-2)^{x+1}$

$\ln y = (x+1) \ln (x-2)$ $\longrightarrow$ Product Rule

$$\frac{y'}{y} = 1\left(\ln(x-2)\right) + \frac{1}{(x-2)} \cdot (x+1)$$

$$\frac{y'}{y} = \ln(x-2) + \frac{(x+1)}{(x-2)}$$

$$y' = \left[\ln(x-2) + \left(\frac{x+1}{x-2}\right)\right] \cdot y$$

$$\boxed{y' = \left[\ln(x-2) + \left(\frac{x+1}{x-2}\right)\right] \cdot (x-2)^{x+1}}$$

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Section 3.6

19) $f(x) = 2\sin^{-1}(x-1)$

$$f'(x) = \frac{2}{\sqrt{1-(x-1)^2}}$$

$$f'(x) = \frac{2}{\sqrt{1-(x^2+2x+1)}}$$

$$\boxed{f'(x) = \frac{2}{\sqrt{x^2+2x}}}$$

23) $f(x) = \tan^{-1}\left(\frac{x}{a}\right)$

$$f'(x) = \frac{\frac{1(a)-x(0)}{a^2}}{1+\left(\frac{x}{a}\right)^2} = \frac{\frac{a}{a^2}}{1+\frac{x^2}{a^2}}$$

$$= \frac{\frac{1}{a}}{1+\frac{x^2}{a^2}} = \frac{\frac{1}{a}}{\frac{a^2+x^2}{a^2}} = \frac{a^2}{a^3+ax^2}$$

$$\boxed{f'(x) = \frac{a}{a^2+x^2}}$$

27) $f(x) = \dfrac{\cos^{-1}(x)}{x+1}$

$$f'(x) = \frac{\frac{-1}{\sqrt{1-x^2}}(x+1) - 1(\cos^{-1}(x))}{(x+1)^2}$$

$$f'(x) = \frac{\frac{-(x+1)}{\sqrt{1-x^2}} - \cos^{-1}(x)}{(x+1)^2}$$

$$f'(x) = \frac{\frac{-(x+1) - \cos^{-1}(x)\sqrt{1-x^2}}{\sqrt{1-x^2}}}{(x+1)^2}$$

$$\boxed{f'(x) = \frac{-(x+1) - \cos^{-1}(x)\sqrt{1-x^2}}{\sqrt{1-x^2}\,(x+1)^2}}$$

31) $f(t) = \sin(\cos^{-1} t)$

? $f'(t) = \cos(\cos^{-1} t) \cdot \dfrac{-1}{\sqrt{1-t^2}}$ $\qquad \cos^{-1} t = \theta$ $\quad \boxed{\cos\theta = t}$

$f'(t)\ \cos t \cdot \dfrac{-1}{\sqrt{1-t^2}} = \dfrac{-\cos t}{\sqrt{1-t^2}}$?

$$\boxed{f'(x) = \frac{-t}{\sqrt{1-t^2}}}$$

35) $y = \frac{1}{2}\left(\frac{1}{2}\ln\frac{x+1}{x-1} + \tan^{-1}x\right)$

$$y = \frac{1}{2}\left(\frac{1}{2}\ln x+1 - \frac{1}{2}\ln x-1 + \tan^{-1}x\right)$$

$$y' = \frac{1}{2}\left(\frac{1}{2(x+1)} - \frac{1}{2(x-1)} + \frac{1}{1+x^2}\right)$$

$$y' = \frac{1}{2}\left(\frac{2(x-1)(1+x^2) - 2(x+1)(1+x^2) + 2(x+1)2(x-1)}{2(x+1)2(x-1)(1+x^2)}\right)$$

$$y' = \frac{1}{2}\left(\frac{\cancel{2x} + \cancel{2x^3} - 2 - \cancel{2x^2} - \cancel{2x} - \cancel{2x^3} - 2 - \cancel{2x^2} + \cancel{4x^2} - 4}{2(x+1)2(x-1)(1+x^2)}\right)$$

$$y' = \frac{1}{2}\left(\frac{-6}{2(x+1)2(x-1)(1+x^2)}\right) = \frac{1}{2}\left(\frac{-6}{(4x^2-4)(1+x^2)}\right)$$

$$= \frac{1}{2}\left(\frac{-6}{\cancel{4x^2} + 4x^4 - 4 - \cancel{4x^2}}\right) = \frac{1}{2}\left(\frac{-6}{4x^4-4}\right)$$

$$\boxed{y' = \frac{3}{4}\left(\frac{-1}{x^4-1}\right)}$$

43) $y = \tan^{-1} x + \dfrac{x}{1+x^2}$

$$y' = \frac{1}{1+x^2} + \frac{(1+x^2) - x(2x)}{(1+x^2)^2}$$

$$y' = \frac{1}{1+x^2} + \frac{1+x^2-2x^2}{(1+x^2)^2}$$

$$y' = \frac{1}{1+x^2} + \frac{1-x^2}{(1+x^2)^2}$$

$$y' = \frac{1+x^2+1-x^2}{(1+x^2)^2}$$

$$\boxed{y' = \frac{2}{(1+x^2)^2}}$$

47) $y = \operatorname{Tan}^{-1}\frac{x}{2}$; $\left(2, \frac{\pi}{4}\right)$

$$y' = \frac{\frac{2}{4}}{1+\left(\frac{x}{2}\right)^2} = \frac{\frac{2}{4}}{1+\frac{x^2}{4}} = \frac{\frac{2}{4}}{\frac{4+x^2}{4}} = \frac{8}{4(4+x^2)}$$

$$y' = \frac{8}{16+4x^2} = \boxed{\frac{2}{4+x^2}}$$

$$y' = \frac{2}{4+(2)^2} = \frac{2}{8} = \boxed{\frac{1}{4}} \rightarrow \text{Slope}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{\pi}{4} = \frac{1}{4}x - \frac{2}{4}$$

$$\boxed{y = \frac{1}{4}x - \frac{1}{2} + \frac{\pi}{4}}$$

51) $f(x) = \cos^{-1} x$ $\quad$ Slope = -2

$f'(x) = \dfrac{-1}{\sqrt{1-x^2}} = -2$

$f'(x) = -1 = -2\sqrt{1-x^2} \rightarrow \dfrac{1}{2} = \sqrt{1-x^2}$

$= \dfrac{1}{4} = 1 - x^2 \rightarrow -x^2 = -1 + \dfrac{1}{4}$

$-x^2 = -\dfrac{3}{4} \rightarrow \boxed{x = \pm \dfrac{\sqrt{3}}{2}}$

$f\left(\dfrac{\sqrt{3}}{2}\right) = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \boxed{\dfrac{\pi}{6}}$

$f\left(-\dfrac{\sqrt{3}}{2}\right) = \cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = \boxed{\dfrac{5\pi}{6}}$

$y - \dfrac{\pi}{6} = -2\left(x - \dfrac{\sqrt{3}}{2}\right)$

$\boxed{y = -2x + \left(\dfrac{\pi}{6} + \sqrt{3}\right)}$

$y - \dfrac{5\pi}{6} = -2\left(x + \dfrac{\sqrt{3}}{2}\right)$

$\boxed{y = -2x + \left(\dfrac{5\pi}{6} - \sqrt{3}\right)}$

Steven Romeiro
Pg 187 # 1-39, 43

Section 3.7

1) $y=\sqrt{x}$ $\quad \frac{dy}{dt}=\frac{1}{2}x^{-\frac{1}{2}}\cdot\frac{dx}{dt}=\frac{1}{2\sqrt{x}}\cdot\frac{dx}{dt}$

$$\frac{dy}{dt}=\frac{1}{2\sqrt{4}}\cdot 3=\boxed{\frac{3}{4}}$$

5) $y=x^2+1$ $\quad \frac{dx}{dt}=2$

$\frac{dy}{dt}=2x\frac{dx}{dt}$ $\quad$ a) $=2(-1)\cdot 2=\boxed{-4}$

b) $=2(0)\cdot 2=\boxed{0}$

c) $=2(1)\cdot 2=\boxed{4}$

9) $y=x^2+1$ $\quad \frac{dx}{dt}=2$

$$\frac{dy}{dt}=2x\frac{dx}{dt}$$

$$\frac{dy}{dt}=2x\cdot 2$$

$$\boxed{\frac{dy}{dt}=4x}$$

13) $A = \pi r^2$ $\qquad \frac{dr}{dt} = 3$

a) $r = 6$

$$\frac{da}{dt} = \pi 2r \cdot \frac{dr}{dt}$$

$$\frac{da}{dt} = \pi 12 \cdot 3$$

$$\boxed{\frac{da}{dt} = 36\pi}$$

b) $r = 24$

$$\frac{da}{dt} = \pi 2r \cdot \frac{dr}{dt}$$

$$\frac{da}{dt} = \pi 48 \cdot 3$$

$$\boxed{\frac{da}{dt} = 144\pi}$$

17) $V = \frac{1}{3}\pi h(108-h^2)$ $h=2$, $r=6$, $\frac{dv}{dt}=3$

$$\frac{dv}{dt} = \frac{1}{3}\pi \frac{dh}{dt}(108-h^2)\cdot(-2h)$$

$$\frac{dv}{dt} = \frac{1}{3}\pi \left(-2h\frac{dh}{dt}(108-h^2)\right)$$

$$3 = \frac{1}{3}\pi\left(-2(2)\frac{dh}{dt}(108-4)\right)$$

$$\frac{1}{\pi} = -4\frac{dh}{dt}(104)$$

$$\frac{1}{\pi} = -416\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{1}{\pi}\cdot\frac{1}{416}$$

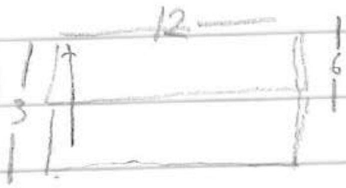
$$\boxed{\frac{dh}{dt} = \frac{1}{416\pi}}$$

21) $SA = 6s^2$ $\quad s = 1 \quad \frac{dsa}{dt} = 3$

$\frac{dsa}{dt} = 6 \cdot 2s \cdot \frac{ds}{dt}$

$3 = 6 \cdot 2(1) \cdot \frac{ds}{dt}$

$12\frac{ds}{dt} = 3$ $\quad \boxed{\frac{ds}{dt} = \frac{1}{4}}$

25) 

$\frac{dv}{dt} = \frac{1}{4} \quad h = 1$

$V = \frac{1}{2} \cdot 6 \cdot x \cdot h$

$\frac{x}{h} = \frac{12}{2} \Rightarrow \boxed{x = 6h}$

$V = \frac{1}{2} \cdot 6 \cdot 6h \cdot h$

$V = 18h^2$

$\frac{dv}{dt} = 36\frac{dh}{dt}$ $\quad \boxed{\frac{dh}{dt} = \frac{1}{144}}$

$\frac{dh}{dt} = \frac{1}{4 \cdot 36}$

29) $x^2+y^2=12^2$

$y=6$

$x^2=144-36$

$x=\sqrt{144-36}$

$x=\sqrt{108}$

$x=6\sqrt{3}$

$S^2=x^2+(12+y)^2$

$S^2=108+(12-6)^2$

$S=12$

$\frac{d}{dt}=x^2+y^2=12^2$

$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$

$\frac{dx}{dt}=\frac{-y}{x}\frac{dy}{dt}$

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Pg. 195 #1-15

Section 3.8

1) $f(x) = x^2 - 3$, $x_1 = 1.7$ $f'(x) = 2x$

$X_2 = X_1 - \frac{f(x_1)}{f'(x_1)} = 1.7 - \frac{f(1.7)}{f'(1.7)}$ $\boxed{X_2 = 1.7935}$

$X_3 = X_2 - \frac{f(x_2)}{f'(x_2)} = 1.7935 - \frac{f(1.7935)}{f'(1.7935)}$

$\boxed{X_3 = 1.599226}$

5) $f(x) = x^3 + x - 1$ $X_1 = 0.5$

$f'(x) = 3x^2 + 1$

$X_2 = X_1 - \frac{f(x_1)}{f(x_2)} \rightarrow 0.5 - \frac{f(.5)}{f'(.5)} =$ $\boxed{X_2 = .714286}$

$X_3 = X_2 - \frac{f(x_2)}{f'(x_2)} \rightarrow .714286 - \frac{f(.714286)}{f'(.714286)} =$ $\boxed{X_3 = .6831797}$

$X_4 = X_3 - \frac{f(X_3)}{f'(x_3)} \rightarrow .6831797 - \frac{f(.6831797)}{f'(.6831797)} =$ $\boxed{X_4 = .682328}$

$X_5 = X_4 - \frac{f(X_4)}{f'(X_4)} \rightarrow .682328 - \frac{f(.682328)}{f'(.682328)} =$ $\boxed{X_5 = .682328}$

$\boxed{\text{Zero is } .682}$

9) $f(x) = x - e^{-x}$ $\quad x_1 = .5$

$f'(x) = e^{-x} + 1$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \rightarrow .5 - \frac{f(.5)}{f'(.5)} = \boxed{x_2 = .566311}$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \rightarrow .566311 - \frac{f(.566311)}{f'(.566311)} = \boxed{x_3 = .567143}$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \rightarrow .567143 - \frac{f(.567143)}{f'(.567143)} = \boxed{x_4 = .567143}$

$\boxed{\text{Zero is } .567}$

13) $f(x) = x^3 - 3.9x^2 + 4.79x - 1.881$ $\quad x_1 = 2$

$f'(x) = 3x^2 - 7.8x + 4.79$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \rightarrow 2 - \frac{f(2)}{f'(2)} = \boxed{x_2 = 1.916807}$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \rightarrow 1.916807 - \frac{f(1.916807)}{f'(1.916807)} = \boxed{x_3 = 1.900601}$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \rightarrow 1.900601 - \frac{f(1.900601)}{f'(1.900601)} = \boxed{x_4 = 1.900006}$

$\boxed{\text{Zero is } 1.9}$

15) $f(x) = x + \sin(x+1)$ $\quad x_1 = -0.5$

$f'(x) = 1 + \cos(x+1) \cdot 1$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \rightarrow -0.5 - \frac{f(-0.5)}{f'(-.5)} = \boxed{x_2 = -.489042}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \rightarrow -.489042 - \frac{f(-.489042)}{f'(-.489042)} = \boxed{x_3 = -.489026}$$

$\boxed{\text{Zero is } -.489}$

Steven Romeiro

50 pt

Study Guide 3

1) $f(\theta) = 2\sec\theta + \tan\theta$

$f'(\theta) = 2(\sec\theta\tan\theta) + \sec^2\theta$

$2(\sec\theta\tan\theta) + \sec^2\theta = 0$

$\sec\theta\,[2(\tan\theta) + \sec\theta] = 0$

$\frac{\sec\theta}{\sec\theta}\left(\frac{2\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right) = \frac{0}{\sec\theta}$

$\frac{2\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = 0$

$\frac{2\sin\theta}{\cos\theta} = -\frac{1}{\cos\theta}$

$-2\sin\theta\cos\theta = -\cos\theta$

$\cos\theta(-2\sin\theta) = -\cos\theta$

$-2\sin\theta = -1$

$\sin\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{6}}$

2) $g(t) = t\sqrt{14-t}$ $\quad t < 14$

$g'(t) = 1(14-t)^{\frac{1}{2}} + t \cdot \frac{1}{2(14-t)^{\frac{1}{2}}}$

$g'(t) = (14-t)^{\frac{1}{2}} + \frac{t}{2(14-t)^{\frac{1}{2}}}$

$(14-t)^{\frac{1}{2}} + \frac{t}{2(14-t)^{\frac{1}{2}}} = 0$

$\frac{t}{2(14-t)^{\frac{1}{2}}} = -(14-t)^{\frac{1}{2}}$

$-t = -2(14-t)$

$-t = -28 + 2t$

$-3t = -28$

$\boxed{t = \frac{28}{3}}$

3) $f(x) = 8x + 1$

a) $[0, 4]$

$f'(x) = 8$

$f'(x) = x \neq 0$

$f(0) = 8(0) + 1 = \boxed{1} \rightarrow$ Absolute Min

$f(4) = 8(4) + 1 = \boxed{33} \rightarrow$ Absolute Max

b) $[0, 4)$

$f(0) = 8(0) + 1 = \boxed{1} \rightarrow$ Absolute Min

↳ No absolute Max

c) $(0, 4]$

$f(4) = 8(4) + 1 = \boxed{33} \rightarrow$ Abolute Max

↳ No Absolute Min

d) $(0, 4) \rightarrow$ No Absolute Max or Min

4) $f(x) = x\ln(x+15)$ $\left[\frac{15}{2}, 15\right]$

$f\left(\frac{15}{2}\right) = \frac{15}{2}\ln\left(\frac{15}{2} + \frac{30}{2}\right) = \boxed{\frac{15}{2}\ln\left(\frac{45}{2}\right)}$ Min

$f(15) = 15\ln(15+15) = 15\ln(30)$

5) $f(x) = x \ln(x+5)$ $\quad [\frac{5}{2}, 5]$

$$f'(x) = x\left(\frac{1}{x+5}\right) + 1\left(\ln(x+5)\right)$$

$$f'(x) = \frac{x}{x+5} + \ln(x+5)$$

$$f\left(\frac{5}{2}\right) = \frac{5}{2}\ln\left(\frac{5}{2}+\frac{10}{2}\right) = \frac{5}{2}\ln\left(\frac{15}{2}\right)$$

$$f(5) = 5\ln(5+5) = \boxed{5\ln(10)}$$

Max

6) $f(x) = \cos \pi x \quad \left[\frac{1}{6}, \frac{11}{6}\right]$

$f(a) = f(b)$?

$f\left(\frac{1}{6}\right) = \cos \frac{\pi}{6} = \boxed{\frac{\sqrt{3}}{2}}$ ✓

$f\left(\frac{11}{6}\right) = \cos \frac{11\pi}{6} = \boxed{\frac{\sqrt{3}}{2}}$ ✓

?

$f'(x) = -\sin \pi x \,(\pi \cdot 1)$

$f'(x) = -\pi \sin \pi x$

$-\pi \sin \pi x = 0$

$\pi(-\sin x) = 0$

$-\sin x = 0$

$x = 0$?

7) $f(x) = (x-2)(x-3)^2 \quad [2, 3]$

$f(a) = f(b)$?

$f(2) = (2-2)(2-3)^2 = \boxed{0}$ ✓

$f(3) = (3-2)(3-3)^2 = \boxed{0}$ ✓

$f'(x) = 1(x-3)^2 + (x-2)\cdot(2(x-3))\cdot(1)$

$f'(x) = (x-3)^2 + (x-2)(2x-6)$

$f'(x) = x^2 - 6x + 9 + 2x^2 - 6x - 4x + 12$

$f'(x) = 3x^2 - 16x + 21$

$f'(x) = (3x-7)(x-3)$

$(3x-7) = 0 \qquad (x-3) = 0$

$3x = 7 \qquad \boxed{x = 3} \longrightarrow$ boundry pt not included

$\boxed{x = \frac{7}{3}}$ ✓

8) $f(x) = x^3$ $\qquad [0,2]$

$f(a) = (0)^3 = \boxed{0}$

$f(b) = (2)^3 = \boxed{8}$

$\frac{f(b) - f(a)}{b - a} = \frac{8-0}{2-0} = \boxed{4}$

$f'(x) = 3x^2 \qquad f'(\cancel{x}) = \frac{f(b) - f(a)}{b - a}$

$3x^2 = 4$

$x^2 = \frac{4}{3} = \frac{\sqrt{4}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

9) $f'(x) = 6x + 2 \qquad (-6, -4)$

$f(x) = 3x^2 + 2x + C$

$f(a) = 3(-6)^2 + 2(-6) + C$

$-4 = 3(-6)^2 + 2(-6) + C$

$-4 = 3(36) - 12 + C$

$\boxed{C = 100}$

$\boxed{f(x) = 3x^2 + 2x + 100}$

10) $f(x) = x\sqrt{24-x^2}$

$f'(x) = x(24-x^2)^{\frac{1}{2}}$

$f'(x) = 1(24-x^2)^{\frac{1}{2}} + x\left(\frac{1}{2}(24-x^2)^{-\frac{1}{2}} \cdot (-2x)\right)$

$f'(x) = (24-x^2)^{\frac{1}{2}} + (-2x^2)\left(\frac{1}{2(24-x^2)^{\frac{1}{2}}}\right)$

$f'(x) = (24-x^2)^{\frac{1}{2}} - \frac{2x^2}{2(24-x^2)^{\frac{1}{2}}}$

$f'(x) = (24-x^2)^{\frac{1}{2}} - \frac{x^2}{(24-x^2)^{\frac{1}{2}}}$

$(24-x^2)^{\frac{1}{2}} - \frac{x^2}{(24-x^2)^{1/2}} = 0$

$(24-x^2)^{1/2} = \frac{x^2}{(24-x^2)^{1/2}}$

$x^2 = 24 - x^2$

$2x^2 = 24$

$x^2 = 12$

$x = \pm 2\sqrt{3}$

$-\infty$ — $-2\sqrt{3}$ + $2\sqrt{3}$ — ∞

f increases @ $(-2\sqrt{3}, 2\sqrt{3})$

F decreases @ $(-\infty, -2\sqrt{3})$ + $(2\sqrt{3}, \infty)$

11) $f(x) = e^{3x} - e^{6x}$

$f'(x) = e^{3x} \cdot 3 - e^{6x} \cdot 6$

$f'(x) \ 3e^{3x} - 6e^{6x} =$

$3e^{3x} - 6e^{6x} = 0$

$3e^{3x}(1 - 2e^{3x}) = 0$

$3e^{3x} = 0 \qquad 1 - 2e^{3x} = 0$

$e^{3x} = 0 \qquad 2e^{3x} = 1$

$3x = 0 \qquad e^{3x} = \frac{1}{2}$

$x = 0 \qquad 3x = \ln \frac{1}{2}$

$x = \frac{1}{3} \ln \frac{1}{2}$

$\frac{1}{3}\ln\frac{1}{2} \qquad 0$

$-\infty \quad + \quad - \quad - \quad \infty$

f increases from $(-\infty, \frac{1}{3}\ln\frac{1}{2})$

12) $f(x) = \frac{x}{6} - \ln x$

$f'(x) = \frac{1}{6} - \frac{1}{x}$

$\frac{1}{6} - \frac{1}{x} = 0$

$\frac{1}{6} = \frac{1}{x}$

$x = 6$

0 — 6 + ∞

f decreases from $(0, 6)$

13) $f(x) = (x-1)^{14/15}$

$f'(x) = \frac{14}{15}(x-1)^{-1/15} \cdot (1)$

a) $f'(x)\ \frac{14}{15(x-1)^{1/15}}$

$\frac{14}{15(x-1)^{1/15}} = 0$

$\frac{1}{(x-1)^{1/15}} \cdot \frac{14}{15} = 0$

$\frac{1}{(x-1)^{1/15}} = 0$

$\frac{1}{(x-1)^{\frac{1}{15}}} \cdot \frac{(x-1)^{15}}{(x-1)^{15}} = 0$

$\frac{(x-1)^{15}}{x-1} = \frac{0}{x-1}$

$(x-1)^{15} = 0$

$x - 1 = 0$

$\boxed{x = 1}$

b) — 1 +

f increases on $(1, \infty)$
decreases on $(-\infty, 1)$

c) $f(1) = (1-1)^{14/15} = \boxed{0}$

f has a rel Min at $(1, 0)$

14) $F(x) = x^2 - 3x - 4$

a) $f'(x) = 2x - 3$

$2x - 3 = 0$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

b) $-\infty \quad - \quad \frac{3}{2} \quad + \quad \infty$

f increases on $(\frac{3}{2}, \infty)$

F decreases on $(-\infty, \frac{3}{2})$

c) $F\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 4 = \boxed{-6.25}$

f has a Rel Min at $(\frac{3}{2}, -6.25)$

15) $F(x) = 3^{-1/2} \sin x - \cos x \quad (0, 2\pi)$

a) $f(x) = \frac{1}{\sqrt{3}} \sin x - \cos x$

$f'(x) = \frac{1}{\sqrt{3}} \cos x + \sin x = 0$

$\frac{1}{\sqrt{3}} \cos x + \sin x = 0$

$\frac{1}{\sqrt{3}} \cos x = -\sin x$

$-\frac{1}{\sqrt{3}} = \tan x \qquad x = \frac{5\pi}{6}, \frac{11\pi}{6}$

$0 \quad + \quad \frac{5\pi}{6} \quad - \quad \frac{11\pi}{6} \quad + \quad 2\pi$

$F\left(\frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}} \sin\left(\frac{5\pi}{6}\right) - \cos\left(\frac{5\pi}{6}\right) = \boxed{.788675}$

$F\left(\frac{11\pi}{6}\right) = \frac{1}{\sqrt{3}} \sin\left(\frac{11\pi}{6}\right) - \cos\left(\frac{11\pi}{6}\right) = \boxed{-.788675}$

16) $f(x) = 26x - x^2$

a) $\boxed{f'(x) = 26 - 2x} \to 26 - 2x = 0$

$2x = 26$

$x = 13$

b)

$$\begin{array}{ccccc} & & 13 & & \\ \hline -\infty & + & & - & \infty \end{array}$$

c) positive from $(-\infty, 13)$, negative from $(13, \infty)$

d) change direction @ $x = 13$

17) $f(x) = x^2 - 6x + 2$

a) $f'(x) = 2x - 6$

$2x - 6 = 0 \longrightarrow \boxed{x = 3}$

b)

$$\begin{array}{ccccc} & & 3 & & \\ \hline -\infty & - & & + & \infty \end{array}$$

c) decreasing on $(-\infty, 3)$ & increasing $(3, \infty)$

d) changes direction at $x = 3$

18) $f(x) = -2x - \cos x$ $\quad [0, 2\pi]$

$f'(x) = -2 + \sin x$

$f''(x) = \cos x$

$\frac{\pi}{2}$ $\quad \frac{3\pi}{2}$

$0 \quad + \quad - \quad + \quad 2\pi$

$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{\pi}{2}$ $\quad \frac{3\pi}{2}$

$+ \quad - \quad +$

f concaves up on $(0, \frac{\pi}{2})$ & $(\frac{3\pi}{2}, 2\pi)$

f concaves down on $(\frac{\pi}{2}, \frac{3\pi}{2})$

f has an inflection pt @ $x = \frac{\pi}{2}$ & $\frac{3\pi}{2}$

19) $f(x) = x\sqrt{x+3}$

$f'(x) = x(x+3)^{\frac{1}{2}} = 1(x+3)^{\frac{1}{2}} + x\left(\frac{1}{2}(x+3)^{-\frac{1}{2}}\right)$

LCD First

$\boxed{f'(x) = (x+3)^{\frac{1}{2}} + \frac{x}{2(x+3)^{1/2}}} = (x+3)^{\frac{1}{2}} + \frac{1}{2}\left(x(x+3)^{-\frac{1}{2}}\right)$

$f''(x) = \frac{1}{2(x+3)^{\frac{1}{2}}} + \frac{1}{2}\left(1(x+3)^{-\frac{1}{2}} + x\left(-\frac{1}{2(x+3)^{\frac{3}{2}}}\right)\right)$

$f''(x) = \frac{1}{2(x+3)^{\frac{1}{2}}} + \frac{1}{2(x+3)^{\frac{1}{2}}} - \frac{x}{4(x+3)^{\frac{3}{2}}}$

$f''(x) = \frac{2}{2(x+3)^{\frac{1}{2}}} - \frac{x}{4(x+3)^{\frac{3}{2}}}$

$f''(x) = \frac{1}{(x+3)^{\frac{1}{2}}} - \frac{x}{4(x+3)^{\frac{3}{2}}}$

$\hookrightarrow$

$$f''(x) = \frac{1}{(x+3)^{\frac{1}{2}}} - \frac{x}{4(x+3)^{3/2}} =$$

$$\frac{4(x+3)^{3/2}}{4[(x+3)^{1/2}(x+3)^{3/2}]} - \frac{x(x+3)^{1/2}}{4(x+3)^{1/2}(x+3)^{3/2}} = \frac{4(x+3)^{3/2} - x(x+3)^{1/2}}{4(x+3)^2}$$

$$= \frac{(x+3)^{\frac{1}{2}}[4(x+3) - x]}{4(x+3)^2} = \frac{4(x+3) - x}{4(x+3)^{3/2}} = \frac{4x+12-x}{4(x+3)^{3/2}}$$

$$f''(x) \boxed{= \frac{3(x+4)}{4(x+3)^{3/2}}} \qquad 3(x+4) = 0 \qquad \boxed{x = -4}$$

$-\infty$ undef. -4 $+ + +$ ∞

f concaves up from $(-3, \infty)$ & no inflection Pt.

20) $f(x) = -2x^3 - 3x^2 + 4x - 2$

$f'(x) = -6x^2 - 6x + 4$

$f''(x) = -12x - 6$

$-12x - 6 = 0$

$-6(2x+1) = 0 \longrightarrow x = -\frac{1}{2}$

$-\infty$ $+$ $-1/2$ $-$ ∞

f concaves up on $(-\infty, -1/2)$

f concaves down on $(-1/2, \infty)$

f has an inflection point on $x = -\frac{1}{2}$

21) $\lim\limits_{x \to \infty} \dfrac{7x}{\sqrt{16x^2 - 7}}$

$\lim\limits_{x \to \infty} \dfrac{\frac{7x}{x}}{\sqrt{\frac{16x^2}{x^2} - \frac{7}{x^2}}} = \lim\limits_{x \to \infty} \dfrac{7}{\sqrt{16 - \frac{7}{x^2}}}$

$\dfrac{7}{\sqrt{16-0}} = \boxed{\dfrac{7}{4}}$

22) $\lim\limits_{x \to \infty} \left(-\dfrac{5x}{7} - \dfrac{6}{x^8}\right)$

$\lim\limits_{x \to \infty} \left(-\dfrac{5x^9}{7x^8} - \dfrac{42}{7x^8}\right) = \lim\limits_{x \to \infty} \left(-\dfrac{\frac{5x^9}{x^8}}{\frac{7x^8}{x^8}} - \dfrac{\frac{42}{x^8}}{\frac{7x^8}{x^8}}\right)$

$\lim\limits_{x \to \infty} \left(\dfrac{-5x - \frac{42}{x^8}}{7}\right) = \dfrac{-5(\infty) - 0}{7} = \boxed{-\infty}$

23) $\lim\limits_{x \to \infty} \dfrac{-x-7}{-3x^2-8}$

$\lim\limits_{x \to \infty} \dfrac{\frac{-x}{x^2} - \frac{7}{x^2}}{\frac{-3x^2}{x^2} - \frac{8}{x^2}} = \lim\limits_{x \to \infty} \dfrac{-\frac{1}{x} - \frac{7}{x^2}}{-3 - \frac{8}{x^2}}$

$= \dfrac{-0-0}{-3-0} = \dfrac{0}{-3} = \boxed{0}$

Domain $(0, \infty)$

24) $Xy^2 = 4$

$y = \pm \frac{2}{\sqrt{X}}$ $\quad X\left(\pm \frac{2\sqrt{X}}{X}\right) = 4 \rightarrow \pm 2\sqrt{X} = 4$

$\boxed{y = \pm \frac{2\sqrt{X}}{X}}$ $\quad = \pm\sqrt{X} = 2 \longrightarrow \boxed{X = 4}$

$\longrightarrow y = \frac{2\sqrt{4}}{4} = \frac{4}{4} = \boxed{y = 1}$

$y' = \frac{2}{X^{\frac{1}{2}}} = 2(X)^{-\frac{1}{2}} = -1(X)^{-\frac{3}{2}} = \frac{-1}{X^{3/2}}$

$-\frac{1}{X^{3/2}} = 0 \longrightarrow \boxed{X = 0}$ crit point not in domain

$y(0) = \frac{2}{\sqrt{0}} =$ undefined

$-\infty$ ERR 0 $+$ ∞

$\lim\limits_{X \to \pm\infty} \frac{2}{\sqrt{X}} = \lim\limits_{X \to \pm\infty} \frac{\frac{2}{X^{\frac{1}{2}}}}{\sqrt{\frac{X}{X}}} = \lim\limits_{X \to \pm\infty} \frac{\frac{2}{X^{1/2}}}{1}$

$= \frac{0}{1} = \boxed{0}$ $\quad \boxed{y = 0 \text{ H.A}}$

$\lim\limits_{X \to 0} \frac{2}{\sqrt{X}} = \frac{2}{0} =$ undefined

$\boxed{X = 0, \text{ V.A}}$

$y'' = \frac{-1}{X^{3/2}} = -1(X)^{-3/2}$

$y'' = \frac{3}{2}(X)^{-5/2} \longrightarrow y'' = \frac{3}{2X^{5/2}}$

$\frac{3}{2X^{5/2}} = 0$ $\quad \boxed{X = 0}$ not in domain

25) $f(x) = \dfrac{x^2}{x^2-1}$

① Domain $x^2-1=0 \rightarrow \boxed{x=\pm 1}$ $\left\{x \mid x \neq \pm 1\right\}$

② $\displaystyle\lim_{x\to\pm 1} \frac{x^2}{x^2-1} = \frac{1}{1-1} = \frac{1}{0} =$ undefined

Since $\displaystyle\lim_{x\to\pm 1} f(x) =$ undefined, $x=\pm 1$, V.A.

③ $\displaystyle\lim_{x\to\pm\infty} \frac{x^2}{x^2-1}$

$$\lim_{x\to\pm\infty} \frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}} = \lim_{x\to\pm\infty} \frac{1}{1-\frac{1}{x^2}} = \frac{1}{1-0}$$

Since $\displaystyle\lim_{x\to\pm\infty} f(x) = 1$, then $\boxed{y=1,\ H.A}$

④ $f'(x) = \dfrac{2x(x^2-1) - x^2(2x)}{(x^2-1)^2}$

$f'(x) = \dfrac{2x^3-2x-2x^3}{(x^2-1)^2} = \dfrac{-2x}{(x^2-1)^2}$

$\dfrac{-2x}{(x^2-1)^2} = 0 \quad -2x = 0 \rightarrow \boxed{x=0}$

$-\infty$		-1		0		1		∞
	$+$		$-$		$+$		$+$	
	↗		↘		↘		↗	

⑤ x-int $\frac{x^2}{x^2-1} = 0$ Y-int $f(0) = \frac{x^2}{x^2-1}$

$\boxed{x=0}$ $\frac{0}{0-1} = \boxed{0}$

$(0,0)$

⑥ $f''(x) = \frac{-2x}{(x^2-1)^2} \longrightarrow \frac{-2(x^2-1)^2 + 2x(2(x^2-1)\cdot(2x)}{(x^2-1)^4}$

$f''(x) = \frac{-2(x^4-2x^2+1)+2x(4x^3-4x)}{(x^2-1)^4}$

$f''(x) = \frac{-2x^4+4x^2-2+8x^4-8x^2}{(x^2-1)^4}$

$f''(x) = \frac{6x^4-4x^2-2}{(x^2-1)^4} = \frac{2(3x^2+1)(x^2-1)}{(x^2-1)^4}$

$f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3} \longrightarrow 2(3x^2+1)=0$

$x^2 = -\frac{1}{3}$

$x =$ undefined

$-\infty$ + -1 — 1 + ∞

f concaves up on $(-\infty,-1)$ & $(1,\infty)$
f concaves down on $(-1,1)$
f has no inflection point ?

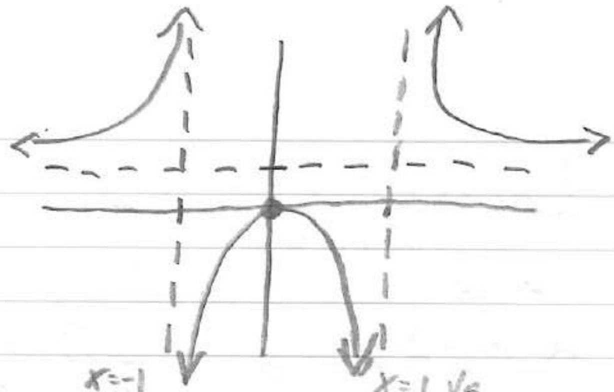
$f''(0) = \frac{2(3(0)^2+1)}{(0^2-1)^3} = \frac{2}{-1} = -2$ Max here

$f(0) = 0$

Since $f''(0) < 0$, f has a

Rel Max at $(0,0)$

⑦



26) $f(x) = \frac{2+x}{2-x}$

① Domain $2-x=0 \to x=2$ $\{x \mid x \neq 2\}$

② $\lim\limits_{x\to 2} \frac{2+x}{2-x} = \frac{2+2}{2-2} = \frac{4}{0}$ = undefined

Since $\lim\limits_{x\to 2} f(x)$ = undef $\boxed{x=2, \text{ V.A.}}$

③ $\lim\limits_{x\to\pm\infty} \frac{2+x}{2-x} = \lim\limits_{x\to\pm\infty} \frac{\frac{2}{x}+\frac{x}{x}}{\frac{2}{x}-\frac{x}{x}} =$

$\lim\limits_{x\to\pm\infty} \frac{\frac{2}{x}+\frac{\cancel{x}}{\cancel{x}}}{\frac{2}{x}-\frac{\cancel{x}}{\cancel{x}}} = \lim\limits_{x\to\pm\infty} \frac{\frac{2}{x}+1}{\frac{2}{x}-1} = \boxed{-1}$

Since $\lim\limits_{x\to\pm\infty} f(x) = -1$, then $\boxed{y=-1, \text{ H.A}}$

④ x-int

$2+x=0$

$\boxed{x=-2}$

y-int

$f(0) = \frac{2+x}{2-x} = \frac{2+0}{2-0}$

$\boxed{y=1}$

(5)

$$f'(x) = \frac{1(2-x) - (2+x)(-1)}{(2-x)^2}$$

$$f'(x) = \frac{2-x+2+x}{(2-x)^2} = \frac{4}{(2-x)^2}$$

$\frac{4}{(2-x)^2} = 0 \rightarrow$ No Solution

$$\begin{array}{ccccc} & & 2 & & \\ -\infty & + & | & + & \infty \end{array}$$

f increasing from $(-\infty, 2)$ & $(2, \infty)$

$$f''(x) = 4(2-x)^{-2} = 4(-2(2-x)^{-3} \cdot (-1))$$

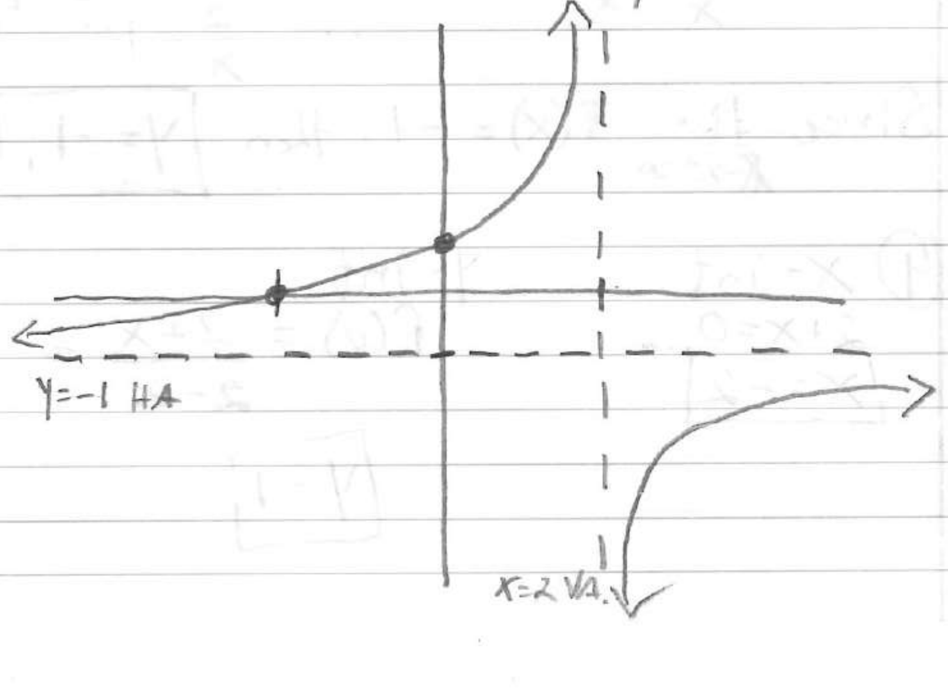
$$f''(x) = 4\left(\frac{2}{(2-x)^3}\right) = f''(x) = \frac{8}{(2-x)^3}$$

$\frac{8}{(2-x)^3} = 0 \rightarrow$ No Solution

$$\begin{array}{ccccc} & & 2 & & \\ -\infty & + & | & - & \infty \end{array}$$

f concaves up on $(-\infty, 2)$, down on $(2, \infty)$

f has no inflection point



27) $f(x) = \dfrac{x}{x+1}$

(1) Domain $x+1=0 \rightarrow \boxed{x=-1}$ $\{x \mid x \neq -1\}$

(2) $\lim\limits_{x\to -1} \dfrac{x}{x+1} = \dfrac{-1}{-1+1} = \dfrac{-1}{0}$ = undefined

Since $\lim\limits_{x\to -1} f(x)$ = undefined, $\boxed{x=-1,\ \text{V.A}}$

(3) $\lim\limits_{x\to \pm\infty} \dfrac{x}{x+1} = \lim\limits_{x\to \pm\infty} \dfrac{\cancel{\frac{x}{x}}}{\frac{x}{x}+\frac{1}{x}} = \lim\limits_{x\to \pm\infty} \dfrac{1}{1+\frac{1}{x}}$

$= \dfrac{1}{1+0} = \boxed{1}$ Since $\lim\limits_{x\to \pm\infty} f(x) = 1$

then, $\boxed{y=1,\ \text{H.A.}}$

(4) x-int

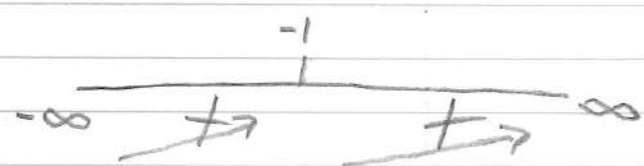
$\dfrac{x}{x+1} = 0$ $\boxed{x=0}$

Y-int $f(0) = \dfrac{x}{x+1}$

$\dfrac{0}{0+1} = 0$ $\boxed{y=0}$

(5) $f'(x) = \dfrac{1(x+1) - x(1)}{(x+1)^2} = \dfrac{x+1-\cancel{x}}{(x+1)^2}$

$\dfrac{1}{(x+1)^2} = 0 \rightarrow$ No Solution



f is constantly increasing from $(-\infty,-1)$ & $(-1,\infty)$

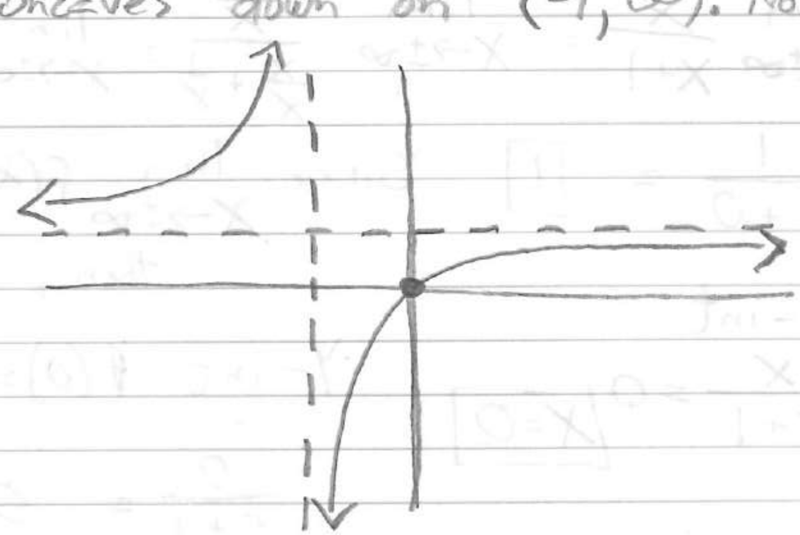
(6) $f''(x) = 1(x+1)^{-2} = -2(x+1)^{-3}$

$f''(x) = \dfrac{-2}{(x+1)^3} \longrightarrow \dfrac{-2}{(x+1)^3} = 0 \longrightarrow$ No Solution

$-\infty \quad + \quad -1 \quad - \quad \infty$

f concaves up on $(-\infty, -1)$
f concaves down on $(-1, \infty)$. No inflection pt.

(7)



28) $f(x) = -4x + \ln(x+2)$

1) Domain $x + 2 > 0 \longrightarrow x > -2 \quad (-2, \infty)$

2) $\lim\limits_{x \to -2} -4 + \ln(-2+2) = -4 + \ln(0) = \text{undefined}$

Since $\lim\limits_{x \to -2} f(x) = \text{undefined}$, $\boxed{x = -2,\ \text{V.A.}}$

3) H.A.

No H.A. or O.A.

4) x-int

$-4 + \ln(x+2) = 0$

$\ln(x+2) = 4$

$e^4 = x + 2$

$\boxed{x = e^4 - 2}$

y-int $f(0) = -4 + \ln(x+2)$

$f(0) = -4 + \ln(0+2)$

$f(0) = -4 + \ln(2)$

?

5) $f'(x) = \dfrac{1}{(x+2)} \longrightarrow \dfrac{1}{x+2} = 0 \rightarrow$ No Solution

$-2 \quad \nearrow \quad \nearrow \quad \infty$

f is increasing on $(-2, \infty)$

(6) $f''(x) = 1(x+2)^{-1} = -1(x+2)^{-2}$

$f''(x) = \frac{-1}{(x+2)^2} = 0 \longrightarrow$ No Solution

$-2 \quad + \quad \infty$

f concaves up $(-2, \infty)$ & has no inflection pt.

(7)

29) $f(x) = \dfrac{-5x^2 + 9x - 3}{x - 1}$

$$\begin{array}{r} -5x + 4 \\ x - 1 \overline{)\, -5x^2 + 9x - 3} \\ -(\cancel{-5x^2} + 5x) \\ \hline 0 \quad 4x - 3 \\ 4x - 4 \\ \hline -7 \end{array}$$

$\boxed{y = -5x + 4}$

30) $f(x) = e^{-3x^2}$ $\quad f(0) = e^0 = 1$

$\boxed{y = 1}$

$A = 2x \cdot y$

$e^{-3x^2} = 0$

$A = 2x \cdot e^{-3x^2}$

$A' = 2(e^{-3x^2}) + 2x(e^{-3x^2} \cdot -6x)$

$A' = e^{-3x^2}\left[2 - 12x^2\right] = 0$

$e^{-3x^2} \neq 0 \qquad 2 - 12x^2 = 0$

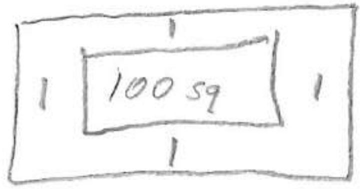
$x^2 = \dfrac{1}{6} = \boxed{\pm\sqrt{\dfrac{1}{6}}}$

$-\sqrt{\tfrac{1}{6}} \quad \sqrt{\tfrac{1}{6}}$

$- \quad + \quad -$

$\text{Max } A = 2\sqrt{\dfrac{1}{6}} \cdot e^{-3(\frac{1}{6})} = 2\sqrt{\dfrac{1}{6}} \cdot \dfrac{1}{\sqrt{e}}$

$\text{Max } A = 2 \cdot \dfrac{1}{\sqrt{6e}} = \dfrac{2}{\sqrt{2}\sqrt{3e}} = \boxed{\dfrac{2\sqrt{2}}{2\sqrt{3e}}}$



$A = x \cdot y$

$100 = x \cdot y$

$\boxed{y = \frac{100}{x}}$

31) $p = 2x + 2y$

$p = 2x + 2\left(\frac{100}{x}\right)$

$p = 2x + 200x^{-1}$

$p' = 2 + (-1(200x^{-2}))$

$p' = 2 - \frac{200}{x^2} \longrightarrow 2 - \frac{200}{x^2} = 0 \longrightarrow \frac{2x^2 - 200}{x^2}$

$2x^2 - 200 = 0$

$x^2 = 100$

$\boxed{x = 10} \rightarrow$ add 2 inches $x = 10 + 2 = \boxed{x = 12}$

$y = \frac{100}{10} \longrightarrow \boxed{y = 10}$ add 2 inches $\boxed{y = 12}$

32) $y = \frac{4 - x}{2}$ $\qquad A = x \cdot y$

$A = x\left(\frac{4-x}{2}\right)$ $\qquad A = \frac{4x - x^2}{2}$ $\qquad y = \frac{4-2}{2}$

$\boxed{y = 2}$

$A'(x) = \frac{1}{2}(4 - 2x) \longrightarrow \frac{1}{2}(4 - 2x) = 0$

$2 - 1x = 0$

$-x = -2$

$\boxed{x = 2}$

$A''(x) = \frac{1}{2}(-2)$

$\boxed{A''(x) = -1}$

$A''(x) < 0$ so it is concave down with a Rel Max @ $(2, 1)$

$f(2) = \frac{4-2}{2} = \boxed{1}$

33) $P = 32\,m$

$2x + 2y = 32\,m$

$2y = 32 - 2x$

$\boxed{y = 16 - x}$

$A = x \cdot y$

$A = x(16 - x)$

$A = 16x - x^2$

$A' = 16 - 2x$

$16 - 2x = 0$

$2x = 16$

$\boxed{x = 8}$

$y = 16 - 8$

$\boxed{y = 8}$

$A = 8 \cdot 8$

$\boxed{A = 64}$

34) Differential dy $\quad y = 3x^{5/11}$

$(y') = \frac{dy}{dt}$ or y'

$(x') = \frac{dx}{dt}$ or x'

$y = \frac{5}{11}(3)x^{-6/11}\ \frac{dx}{dt}$

$\frac{dy}{dt} = \frac{15}{11}x^{-6/11}\ \frac{dx}{dt}$

$\boxed{dy = \frac{15}{11}x^{-6/11}\ dx}$

35) Differential of dy $y = x \sin x$

$$\frac{dy}{dt} = (1)\frac{dx}{dt}(\sin x) + x\left(\cos x \frac{dx}{dt}\right)$$

$$dy = dx(\sin x) + x(\cos x\, dx)$$

$$\boxed{dy = dx\left(\sin x + x\cos x\right)}$$

36) $r = 12$ inches Err: 0.04

a) $V_{\text{Spere}} = \frac{4}{3}\pi r^3$

$$\frac{dV}{dr} = 4\pi r^2 \cdot dr = 4\pi(12)^2 \cdot (\pm .04)$$

$$\boxed{\frac{dV}{dr} = \pm 72.38}$$

b) $S = 4\pi r^2$

$$\frac{dS}{dr} = 8\pi r \cdot (\pm .04)$$

$$\frac{dS}{dr} = 8\pi(12)(\pm .04)$$

$$\boxed{\frac{dS}{dr} = \pm 12.06}$$

c) $$\frac{72.38}{\frac{4}{3}\pi r^3} = \frac{72.38}{\frac{4}{3}\pi(12)^3}$$

$$= \frac{72.38}{7238.23} = \boxed{.001}$$

c) $$\frac{12.06}{4\pi r^2} = \frac{12.06}{4\pi(12)^2} = \frac{12.06}{1809.56} = \boxed{.007}$$

37) $f(4) = \sqrt[3]{512.75} \;=\; f(x) = \sqrt[3]{x}$

$\Delta y = f'(x)\,dx$ Approximation of $X = 512$

So $\sqrt[3]{512} = \boxed{8}$

$dX = .75$

$$\Delta y = \frac{1}{3}x^{-\frac{2}{3}} \cdot \frac{3}{4}$$

$$\Delta y = \frac{1}{3x^{2/3}} \cdot \frac{3}{4} = \frac{1}{4x^{2/3}} = \frac{1}{4(\sqrt[3]{512})^2}$$

$\Delta y = .0039$

$8 + .0039 = 8.0039$

$(8.0039)^3 \approx 512.7491 \approx 512.75$

Steven Romeiro
Pg. 209 #1-39

Section 4.1

1) A = Rel Min
B = Absolute Max
C = Neither
D = None
E = Rel Max
F = Rel Min
G = Rel Max

5) $f(x) = x + \frac{27}{2x^2}$; $(3, \frac{9}{2})$

$f'(x) = 1 + \left(\frac{-4x \cdot 27}{(2x^2)^2}\right)$

$f'(x) = 1 + \left(-\frac{108x}{4x^4}\right)$

$f'(x) = 1 - \frac{108x}{4x^4} = 1 - \frac{27}{x^3}$

$x = 3$

$f'(3) = 1 - \frac{27}{3^3}$

$f'(3) = 1 - 1$

$\boxed{f'(3) = 0}$

9) The critical # is at $X=2$ because according to the definition of critical #'s, slope or $f'(c)=0$ or "f" is not differentiable at c.

At $X=2$, slope is 0, so 2 is the critical number and both the rel/absolute Max

13) $f(X) = X^2 (X-3)$

$f'(X) = 2X(X-3) + X^2(1)$

$f'(X) = 2X^2 - 6X + X^2$

$f'(X) = 3X^2 - 6X$

$f'(X) = 3X(X-2)$

$3X(X-2)=0$

$3X=0$ / $X-2=0$

$\boxed{X=0}$ / $\boxed{X=2}$

17) $h(X) = (\sin X)^2 + \cos X$

$h'(X) = 2(\sin X)\cdot \cos X + (-\sin X)$

$h'(X) = 2\sin X \cos X - \sin X$

$h'(X) = \sin X(2\cos X - 1)$

$\sin X(2\cos X - 1) = 0$

$\sin X = 0$ / $2\cos X - 1 = 0$

$X = 0$ / $2\cos X = 1$

$\boxed{X = \pi}$ / $\cos X = \frac{1}{2}$

$\boxed{X = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}}$

21) $f(x) = 2(3-x)$; $[-1, 2]$

a) $f'(x) = 2(-1)$
$f'(x) = -2$
Since $f'(x) \neq 0$ for all x, there is no critical number
b) no critical #'s to evalucte

c) $f(-1) = 2(3+1)$
$f(-1) = 2(4) = \boxed{8}$ $(-1, 8)$

$f(2) = 2(3-2)$
$f(2) = \boxed{2}$ $(2, 2)$

d) Absolute Max @ $(-1, 8)$
Absolute Min @ $(2, 2)$

25) $f(x) = x^3 - \frac{3}{2}x^2$, ; $[-1, 2]$

a) $f'(x) = 3x^2 - 3x$
$f'(x) = 3x(x-1)$
$3x(x-1) = 0$
$3x = 0$ / $x - 1 = 0$
$\boxed{x = 0}$ / $\boxed{x = 1}$

$\longrightarrow$

b) $f(0) = 0^3 - \frac{3}{2}(0)^2 = \boxed{0}$

$f(1) = 1^3 - \frac{3}{2}(1)^2 = \boxed{-\frac{1}{2}}$

c) $f(-1) = -1^3 - \frac{3}{2}(-1)^2 = \boxed{-\frac{5}{2}}$

$f(2) = 2^3 - \frac{3}{2}(2)^2 = \boxed{2}$

d) Absolute Max @ $(2, 2)$
Absolute Min @ $(-1, -5/2)$

29) $f(x) = \frac{x^2}{x^2+3}$; $[-1, 1]$

a) $f'(x) = \frac{2x(x^2+3) - x^2(2x)}{(x^2+3)^2}$

$f'(x) = \frac{2x^3+6x-2x^3}{(x+3)^2}$

$f'(x) = \frac{6x}{(x+3)^2}$

$\frac{6x}{(x+3)^2} = 0$

$6x = 0(x+3)^2$

$6x = 0$

$\boxed{x = 0}$

b) $f(0) = \frac{0^2}{0^2+3} = \boxed{0}$

c) $f(-1) = \frac{(-1)^2}{(-1)^2+3} = \boxed{\frac{1}{4}}$

$f(1) = \frac{1^2}{1^2+3} = \boxed{\frac{1}{4}}$

d) Absolute Max @ $(1, \frac{1}{4})(-1, \frac{1}{4})$
Absolute Min @ $(0, 0)$

31) $h(x) = \frac{1}{x-2}$; $[0, 1]$

a) $h'(x) = (x-2)^{-1}$

$h'(x) = -1(x-2)^{-2}$

$\frac{-1}{(x-2)^2} = 0$

$(x-2)^2 = 0$

$\boxed{x = 2}$

Not in domain

b) No crit #

c) $f(0) = \frac{1}{0-2} = \boxed{-\frac{1}{2}}$

$f(1) = \frac{1}{1-2} = \boxed{-1}$

d) Absolute Max @ $(0, -\frac{1}{2})$

Absolute Min @ $(1, -1)$

35) $f(x) = \cos \pi x$; $[0, \frac{1}{6}]$

a) $f'(x) = \pi(-\sin \pi x)$

$-\pi \sin \pi x = 0$

$\pi x = 0$

$\boxed{x = 0}$

b) $f(0) = \cos \pi(0) = \cos(0) = \boxed{1}$

c) $f(\frac{1}{6}) = \cos \pi (\frac{1}{6}) = \cos \frac{\pi}{6} = \boxed{\frac{\sqrt{3}}{2}}$

d) Absolute Max @ $(0, 1)$

Absolute Min @ $(\frac{1}{6}, \frac{\sqrt{3}}{2})$

39) $f(x) = 2x - 3$

a) $f'(x) = 2$

b) No crit # since $f(x) \neq 0$

c) $[0,2]$

$f(0) = 2(0) - 3 = \boxed{-3}$

$f(2) = 2(2) - 3 = \boxed{1}$

d) Absolute Max @ $(2,1)$, Min @ $(0,-3)$

e) $[0,2) \rightarrow$ Absolute min still @ $(0,-3)$
No Absolute Max

c) $(0,2] \rightarrow$ Absolute max still @ $(2,1)$
No Absolute Min

c) $(0,2) \rightarrow$ No Extremas, No absolute Max or Min since both ends go to $\pm\infty$

Steven Romeiro

Pg: 216 # 1-23, 33, 37, 43-51, 63

Section 4.2

1) $f(x) = 1 - |x-1|$

Rolles Theorem does not apply here because f has a sharp turn making it not differentiable

5) $f(x) = x^2 - x - 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2 \quad x = -1}$

$f'(x) = 2x - 1$

$2x - 1 = 0$

$\boxed{x = \frac{1}{2}}$

$\boxed{f'\left(\frac{1}{2}\right) = 0}$

9) $f(x) = x^2 + 3x - 4$

$f'(x) = 2x + 3$

$f'(x) = 0$

$2x + 3 = 0$

$\boxed{x = -\frac{3}{2}}$

$\boxed{f'\left(-\frac{3}{2}\right) = 0}$

13) $f(x) = (x-1)(x-2)(x-3)$; $[1,3]$

$f(a) = f(b)$?

$f(1) = (1-1)(1-2)(1-3) = \boxed{0}$ ✓

$f(3) = (3-1)(3-2)(3-3) = \boxed{0}$ ✓

$f(x) = x^3 - 6x^2 + 11x - 6$

$f'(x) = 3x^2 - 12x + 11$

$3x^2 - 12x + 11 = 0$

$$x = \frac{12 \pm \sqrt{12^2 - 4(3)(11)}}{2(3)} = \frac{12 \pm \sqrt{144 - 132}}{6}$$

$$x = \frac{12 \pm \sqrt{12}}{6} = \frac{12 \pm 2\sqrt{3}}{6} = \frac{6 \pm \sqrt{3}}{3}$$

$\boxed{c = \frac{6 \pm \sqrt{3}}{3}}$ so $\boxed{f'\left(\frac{6 \pm \sqrt{3}}{3}\right) = 0}$

17) $f(x) = \dfrac{x^2 - 2x - 3}{x+2}$ $[-1, 3]$

$f(a) = f(b)$?

$f(-1) = \dfrac{(-1)^2 - 2(-1) - 3}{-1+2} = \dfrac{1+2-3}{-1+2} = \boxed{0}$ ✓

$f(3) = \dfrac{(3)^2 - 2(3) - 3}{3+2} = \dfrac{9-9}{6} = \boxed{0}$ ✓

$f'(x) = \dfrac{(2x-2)(x+2) - (1)(x^2-2x-3)}{(x+2)^2}$

$f'(x) = \dfrac{2x^2+4x-2x-4-x^2+2x+3}{(x+2)^2}$

$f'(x) = \dfrac{x^2+4x-1}{(x+2)^2}$

$x^2+4x-1=0$

$x = \dfrac{-4 \pm \sqrt{4^2-4(1)(-1)}}{2(1)}$

$x = \dfrac{-4 \pm \sqrt{16+4}}{2} \rightarrow \dfrac{-4 \pm \sqrt{20}}{2}$

$\dfrac{-4 \pm 2\sqrt{5}}{2}$

$-2 \pm \sqrt{5} \rightarrow -2+\sqrt{5} = \boxed{.236}$

$-2-\sqrt{5} = -4.236$

Not in domain

21) $f(x) = \sin x$ $\quad [0, 2\pi]$

$f(a) = f(b)$?

$f(0) = \sin 0 = \boxed{0}$ ✓

$f(2\pi) = \sin 2\pi = \boxed{0}$

$f'(x) = \cos x$

$\cos x = 0$

$x = \boxed{\frac{\pi}{2}}$ or $\boxed{\frac{3\pi}{2}}$

23) $f(x) = \frac{6x}{\pi} - 4\sin^2 x$; $\left[0, \frac{\pi}{6}\right]$

$f(0) = \frac{6(0)}{\pi} - 4\sin^2(0) = \boxed{0}$ ✓

$f\left(\frac{\pi}{6}\right) = \frac{6\left(\frac{\pi}{6}\right)}{\pi} - \underset{4\left(\frac{1}{2}\right)^2 = 4\left(\frac{1}{4}\right)}{4\sin^2\left(\frac{\pi}{6}\right)} = 1 - 1 = \boxed{0}$ ✓

Quotient Rule

$f'(x) = \frac{6}{\pi} - 4(2\sin x \cdot \cos x)$

$f'(x)\ \frac{6}{\pi} - 8\sin x \cos x$

$\frac{6}{\pi} - 8\sin x \cos x = 0$

$\frac{6}{\pi} = 8\sin x \cos x$

$\pi(8\sin x \cos x) = 6$

$4\pi(2\sin x \cos x) = 6$

$4\pi(\sin 2x) = 6$

$\sin 2x = \frac{6}{4\pi}$

$\sin 2x = \frac{3}{2\pi}$

$2x = \sin^{-1}\left(\frac{3}{2\pi}\right)$

$2x = .497767$

$x = \frac{.497767}{2}$

$\boxed{x = 0.248884}$

33) $F(t) = -16t^2 + 48t + 32$

a) $f(1) = -16(1)^2 + 48(1) + 32 = \boxed{64}$ ✓

$f(2) = -16(2)^2 + 48(2) + 32 = \boxed{64}$ ✓

b) $f'(t) = -32t + 48$

$-32t + 48 = 0$

$-32t = -48$

$\boxed{t = \frac{3}{2}}$

$\boxed{f'\left(\frac{3}{2}\right) = 0}$

37) Mean Value Theorem does not apply because the Fxn is not continuous on [0,6] with $x=2$ being undefined

43) $f(x) = x^2 \qquad [-2, 1]$

$f(-2) = (-2)^2 = \boxed{4}$

$f(1) = 1^2 = \boxed{1}$

$f'(x) = 2x$

$\frac{f(b) - f(a)}{b - a} = f'(x)$

$\frac{1 - 4}{1 + 2} = 2x$

$2x = \frac{-3}{3}$ $\rightarrow$ $\boxed{x = -\frac{1}{2}}$

let $\boxed{c = -\frac{1}{2}}$

47) $F(x) = \sqrt{2-x}$; $[-7, 2]$

$f(-7) = \sqrt{2+7} = \sqrt{9} = \boxed{3}$

$f(2) = \sqrt{2-2} = \sqrt{0} = \boxed{0}$

$$\frac{f(b)-f(a)}{b-a} = \frac{0-3}{2+7} = -\frac{3}{9} = \boxed{-\frac{1}{3}}$$

$$f'(x) = (2-x)^{\frac{1}{2}} = \frac{1}{2}(2-x)^{-\frac{1}{2}} \cdot (-1) = \boxed{\frac{-1}{2(2-x)^{\frac{1}{2}}}}$$

$$f'(x) = \frac{f(b)-f(a)}{b-a}$$

$$\frac{-1}{2(2-x)^{\frac{1}{2}}} = -\frac{1}{3}$$

$$3 = 2(2-x)^{\frac{1}{2}}$$

$$(2-x)^{\frac{1}{2}} = \frac{3}{2}$$

$$(2-x) = \frac{9}{4}$$

$$-x+2 = \frac{9}{4}$$

$$-x = \frac{9}{4} - 2$$

$$-x = \frac{9-8}{4}$$

$\boxed{x = \frac{-1}{4}}$

So let $\boxed{c = \frac{-1}{4}}$

51) $f(x) = x \log_2 x \qquad [1, 2]$

?how $\rightarrow f(x) = x \dfrac{\ln x}{\ln 2}$

$f(1) = 1 \dfrac{\ln(1)}{\ln 1} = 1 \dfrac{0}{\ln 2} = \boxed{0}$

$f(2) = 2 \dfrac{\ln 2}{\ln 2} = \boxed{2}$

$\dfrac{f(b) - f(a)}{b - a} = \dfrac{2 - 0}{2 - 1} = \boxed{2}$

$f'(x) = x\left(\dfrac{\frac{1}{x}}{\ln 2}\right) + 1\left(\dfrac{\ln x}{\ln 2}\right) = x\left(\dfrac{1}{x \ln 2}\right) + \dfrac{\ln x}{\ln 2}$

$= \dfrac{1}{\ln 2} + \dfrac{\ln x}{\ln 2} = \boxed{\dfrac{1 + \ln x}{\ln 2}}$

$f'(x) = \dfrac{f(b) - f(a)}{b - a}$

$\dfrac{1 + \ln x}{\ln 2} = 2$

$1 + \ln x = 2 \ln 2$

$\ln x = 2 \ln 2 - 1$

? $x = \dfrac{2 \ln 2 - 1}{\ln}$

63) Let $t = 0$ @ 2:00 PM
and $t = 5.5$ @ 7:30 PM

When $t=0$ distance traveled by plane $S(0) = 0$

$t = 5.5$ distance traveled $S(5.5) = 2500$

$S(t)$ is continuos + differen. on $(0, 5.5)$

$$S'(t_0) = \frac{S(5.5) - S(0)}{5.5 - 0} = \text{average Speed}$$

$$S'(t_0) = \frac{2500 - 0}{5.5} = \boxed{454.5 \text{ mph}}$$

Since plane speed @ $t_0 = V(t_0) = S'(t_0)$ then time t_0 there is a time where speed must equal average speed 454.5 mph

Since plane starts @ $t = 0h$ and stops @ $t = 5.5h$ it means speed accelerates then decelerates. When we break the time interval into two subintervals $[0, t_0]$ + $[t_0, 5.5]$

Since $V(0) < 400 < V(t_0)$ + $t_0 < 400 < (5.5)$ then by Interm Val Theo there is a time t_1 in $[0, t_0]$ and t_2 in $[t_0, 5.5]$ such that $V(t_1) = V(t_2) = 400$ mph

So there are at least 2 times when speed = 400 mph

Steven Romeiro
Pg 226 #1-53, 63-71

Section 4.3

1) a) [0,6]
b) [6,8]

5) $y = \frac{x^3}{4} - 3x$

$y' = \frac{3x^2}{4} - 3$ $\quad \frac{3x^2}{4} - 3 = 0$

$\frac{3x^2}{4} = 3$

$\frac{x^2}{4} = 1$

$x^2 = 4$

$\boxed{x = \pm 2}$

$-\infty \quad + \quad -2 \quad - \quad 2 \quad + \quad \infty$

f is increasing @ $(-\infty,-2)$ & $(2,\infty)$
f is decreasing @ $(-2,2)$

9) $f(x) = \frac{1}{x^2}$

$f'(x) = -\frac{2}{x^3}$

$-\frac{2}{x^3} = 0$

$-x^3 = 0$

$\boxed{x = 0}$

$-\infty \quad + \quad 0 \quad - \quad \infty$

f increasing @ $(-\infty, 0)$
f decreasing @ $(0, \infty)$

13) $y = x\sqrt{16-x^2}$

$$y = 1(16-x^2)^{\frac{1}{2}} + x\frac{1}{2}(16-x^2)^{-\frac{1}{2}} \cdot (2x)$$

$$y = (16-x^2)^{\frac{1}{2}} + \frac{2x^2}{2(16-x^2)^{\frac{1}{2}}} = 0$$

$$\frac{2x^2}{2(16-x^2)^{\frac{1}{2}}} = (16-x^2)^{\frac{1}{2}}$$

$$2x^2 = 2(16-x^2)$$

$$2x^2 = -2x^2 + 36$$

$$4x^2 = 36$$

$$x^2 = 9$$

$$\boxed{x = \pm 3}$$

$-\infty$ $-$ -3 $+$ 3 $-$ ∞

f increasing on $(-3, 3)$

f decreasing on $(-\infty, -3)(3, \infty)$

17) $f(x) = x^2 - 6x$

$f'(x)\ 2x - 6$

$2x - 6 = 0$

$\boxed{x = 3}$

$-\infty$ $-$ 3 $+$ ∞

$$f(3) = (3)^2 - 6(3) = 9 - 18 = \boxed{-9}$$

f increases on $(3, \infty)$ decreases on $(-\infty, 3)$

fc has a Rel Min on $(3, -9)$

21) $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12$

$6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0$

$(x+2)(x-1) = 0$

$\boxed{x = -2}$ $\boxed{x = 1}$

$-2 \qquad 1$

$-\infty \quad + \quad - \quad + \quad \infty$

$f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) = \boxed{20}$

$f(1) = 2(1)^3 + 3(1)^2 - 12(1) = \boxed{-7}$

F increases on $(-\infty, -2)$ & $(1, \infty)$, decreases on $(-2, 1)$

F has Rel Max on $(-2, 20)$, Rel Min on $(1, -7)$

25) $f(x) = \dfrac{x^5 - 5x}{5}$

$f'(x) = \dfrac{1}{5}(5x^4 - 5) = \dfrac{5}{5}(x^4 - 1) = \boxed{x^4 - 1}$

$x^4 - 1 = 0$

$x^4 = 1$

$\boxed{x = \pm 1}$

$-1 \qquad 1$

$-\infty \quad + \quad - \quad + \quad \infty$

$f(1) = \dfrac{1^5 - 5(1)}{5} = \boxed{-\dfrac{4}{5}}$

$f(-1) = \dfrac{-1^5 - 5(-1)}{5}$ $\boxed{\dfrac{4}{5}}$

f increases on $(-\infty, -1)$ & $(1, \infty)$, decreases on $(-1, 1)$

f has Rel Max on $(-1, 4/5)$, Rel Min on $(1, -4/5)$

29) $f(x) = (x-1)^{\frac{2}{3}}$

$f'(x) = \frac{2}{3}(x-1)^{-\frac{1}{3}} \cdot (1)$

$f'(x) = \frac{2}{3(x-1)^{\frac{1}{3}}} = 0$

$-\infty \quad - \quad 1 \quad + \quad \infty$

$3(x-1)^{\frac{1}{3}} = 0$

$\boxed{x = 1}$

$f(1) = (1-1)^{\frac{2}{3}} = \boxed{0}$

f increases on $(1, \infty)$, decreases $(-\infty, 1)$

f has Rel Min on $(1, 0)$

33) $f(x) = x + \frac{1}{x}$

$f'(x) = 1 + \left(\frac{-1}{x^2}\right) = 1 - \frac{1}{x^2}$

$-\frac{1}{x^2} = -1$

$\boxed{x = \pm 1}$

$-\infty \quad + \quad -1 \quad - \quad 0 \quad - \quad 1 \quad + \quad \infty$

f increases on $(-\infty, -1)$ & $(1, \infty)$ decreases $(-1, 0)(0, 1)$

$f(1) = 1 + \frac{1}{1} = \boxed{2}$

$f(-1) = 1 + \frac{1}{-1} = \boxed{0}$

f has Rel Max $(1, 2)$ & Rel Min $(-1, 0)$

37) $f(x) = \dfrac{x^2 - 2x + 1}{x + 1}$

$$f'(x) = \frac{(2x-2)\cdot(x+1) - (1)(x^2 - 2x + 1)}{(x+1)^2}$$

$$f'(x) = \frac{\cancel{2}x^2 + \cancel{2x} - \cancel{2x} - 2 - \cancel{x^2} + 2x - 1}{(x+1)^2}$$

$$f'(x) = \frac{x^2 + 2x - 3}{(x+1)^2} = \frac{(x+3)(x-1)}{(x+1)^2}$$

$$\frac{(x+3)(x-1)}{(x+1)^2} = 0$$

$x + 3 = 0$ / $x - 1 = 0$ / Discontinuity @

$\boxed{x = -3}$ / $\boxed{x = 1}$ / $(x+1)^2 = 0$

$x + 1 = 0$

$\boxed{x = -1}$

$-\infty$ | -3 | -1 | 1 | ∞

$+$ | $-$ | $-$ | $+$

$$f(-3) = \frac{(-3)^2 - 2(-3) + 1}{-3 + 1} = \boxed{-8}$$

$$f(1) = \frac{1^2 - 2(1) + 1}{1 + 1} = \boxed{0}$$

f increases @ $(-\infty, -3)$ + $(1, \infty)$ + decreases on $(-3, -1)$ $(-1, 1)$

f has a RelMax on $(1, 0)$ + RelMin on $(-3, -8$

?

6

41) $f(x) = 4(x - \sin^{-1} x)$

$f'(x) = 4\left(1 - \dfrac{1}{\sqrt{1-x^2}}\right)$

$f'(x) = 4 - \dfrac{4}{\sqrt{1-x^2}}$

$4 = \dfrac{4}{\sqrt{1-x^2}} \longrightarrow 16 = \dfrac{16}{1-x^2}$

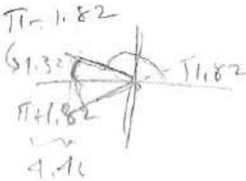
$16 - 16x^2 = 16$

$-16x^2 = 0$

$\boxed{x=0}$

$-\infty$ Error ? 0 Error ? ∞

3.



$\sin x + 2\sin 2x = 0$

$\sin x + 4\sin x \cos x = 0$

$\sin x (1 + 4\cos x) = 0$

$\sin x = 0 \qquad 1 + 4\cos x = 0$

$0, \pi, 2\pi \qquad \cos x = -\frac{1}{4}$

$x = 1.823$

$= 4.46$

$3(x+3)^{\frac{1}{2}} - 3x - 6 = 0$

$3(x+3)^{\frac{1}{2}} - 3(x+2) = 0$

$\cancel{3}(x+3)^{\frac{1}{2}} = \cancel{3}(x+2)$

$(\sqrt{x+3})^2 = (x+2)^2$

$x + 3 = x^2 + 4x + 4$

$0 = x^2 + 3x + 1$

45) $f(x) = x - \log_4 x$

$f'(x) = 1 - \frac{1}{(\ln 4)x}$

$1 - \frac{1}{(\ln 4)x} = 0$

$\frac{1}{\ln 4}$

$-\infty \quad + \quad + \quad \infty$

$\frac{1}{(\ln 4)x} = 1$

$(\ln 4)x = 1$

$\boxed{x = \frac{1}{\ln 4}}$

f is increasing from $\left(\frac{1}{\ln 4}, \infty\right)$

?

$f\left(\frac{1}{\ln 4}\right) = \frac{1}{\ln 4} - \log_4\left(\frac{1}{\ln 4}\right)$

?

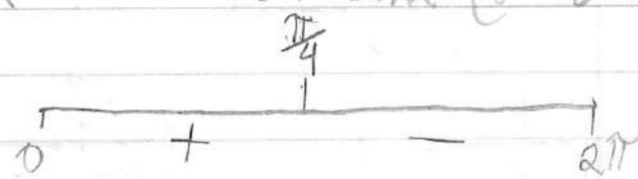
49) $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

$\cos x - \sin x = 0$

$\cos x = \sin x$ $\qquad$ $\cos x = \sin x$ @ 45°

crit # = $x = \frac{\pi}{4}$ or $\boxed{\frac{\sqrt{2}}{2}}$



f is Increasing on $(0, \frac{\pi}{4})$

f is decreasing on $(\frac{\pi}{4}, 2\pi)$

$f(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) + \cos(\frac{\pi}{4})$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \boxed{\sqrt{2}}$

53) $f(x) = \sin^2 x + \sin x$

$f'(x) = 2\sin x(\cos x) + \cos x$

$2\sin x \cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0 \qquad 2\sin x + 1 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$f\left(\frac{\pi}{2}\right) = \sin^2\frac{\pi}{2} + \sin\frac{\pi}{2} = \boxed{2}$

$f\left(\frac{7\pi}{6}\right) = \sin^2\frac{7\pi}{6} + \sin\frac{7\pi}{6} = \frac{1}{4} + \left(-\frac{1}{2}\right) = \boxed{-\frac{1}{4}}$

$f\left(\frac{11\pi}{6}\right) = \sin^2\frac{11\pi}{6} + \sin\frac{11\pi}{6} = \frac{1}{4} - \frac{1}{2} = \boxed{-\frac{1}{4}}$

$f\left(\frac{3\pi}{2}\right)\ \sin^2\frac{3\pi}{2} + \sin\frac{3\pi}{2} = +1 - 1 = \boxed{0}$

$\frac{\pi}{2} \quad \frac{7\pi}{6} \quad \frac{3\pi}{2} \quad \frac{11\pi}{6}$

$0 \quad + \quad - \quad + \quad - \quad + \quad 2\pi$

f increases on $\left(0, \frac{\pi}{2}\right)$ + $\left(\frac{7\pi}{6}, \frac{3\pi}{2}\right)$ & $\left(\frac{11\pi}{6}, 2\pi\right)$

f decreases on $\left(\pi/2, 7\pi/6\right)$ + $\left(3\pi/2, \frac{11\pi}{6}\right)$

f has Rel Max on $\left(\frac{\pi}{2}, 2\right)$ $\left(\frac{3\pi}{2}, 0\right)$

f has Rel Min on $\left(\frac{7\pi}{6}, -\frac{1}{4}\right)$ + $\left(\frac{11\pi}{6}, -\frac{1}{4}\right)$

63) $f(x) = 3$

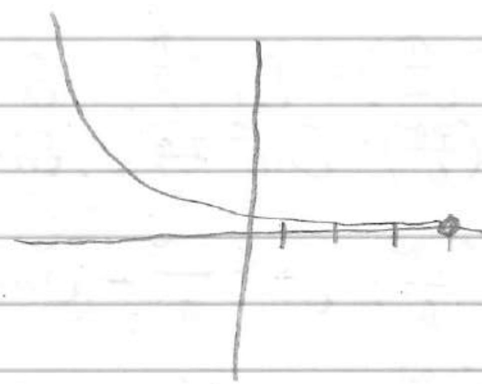
$f'(x) = 0$

67) $f(x) = \sqrt{x+4}$

$f'(x) = (x+4)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x+4)^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2\sqrt{x+4}}$



71) f changes from + to − at $x=0$

f has Rel Max @ $(x=0)$

f changes from − to + at $x=1$

f has Rel Min @ $(x=1)$

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pg235 #1-53, 67, 69, 73

4.4 Section

1) $y = x^2 - x - 2$

$y' = 2x - 1$ y is concave up on $(-\infty, \infty)$

$y'' = 2$

$2 > 0$

5) $f(x) = \dfrac{x^2+1}{x^2-1}$

$f'(x) = \dfrac{2x(x^2-1) - (x^2+1)(2x)}{(x^2-1)^2}$

$f'(x) = \dfrac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$

$f'(x) = \dfrac{-4x}{(x^2-1)^2} = \dfrac{-4x}{x^4-2x^2+1}$

$f''(x) = \dfrac{-4(x^4-2x^2+1) - (-4x)(4x^3-4x)}{(x^4-2x^2+1)^2}$

$f''(x) = \dfrac{-4x^4 + 8x^2 - 4 + 16x^4 - 16x^2}{(x^4-2x^2+1)^2}$

$f''(x) = \dfrac{12x^4 - 8x^2 - 4}{(x^4-2x^2+1)^2} = \dfrac{4(3x^4 - 2x^2 - 1)}{(x^2-2x^2+1)^2}$

$f''(x) = \dfrac{4(3x^2+1)(x^2-1)}{(x^2-2x^2+1)^2} = \dfrac{4(3x^2+1)(x^2-1)}{(x^2-1)^4}$

(12x+4)/((x^2-1)^3)

$$f'' = \frac{4(3x^2+1)}{(x^2-1)^3}$$

$$\frac{4(3x^2+1)}{(x^2-1)^3} = 0$$

$$\frac{12x^2+4}{(x^2-1)^3} = 0 \longrightarrow (x^2-1)^3\left(\frac{12x+4}{(x^2-1)^3}\right) = 0(x^2-1)^3$$

$12x^2+4=0$

$12x^2=-4$

$$\boxed{\begin{matrix} x = \pm\frac{i}{\sqrt{3}} \\ x \neq \pm 1 \end{matrix}}$$

-1 +1

+ — + ∞

f concaves up $(-\infty,-1)$ & $(1,\infty)$

f concave down $(-1,1)$

9) $y = 2x - \tan x$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y' = 2 - \sec^2 x$

$y'' = 0 - 2\sec x \cdot (\tan x \sec x)$

$y'' = -2\sec^2 x(\tan x)$

$-2\sec^2 x \tan x = 0$

$-2\sec^2 x = 0$

$-\sec^2 x = 0$

$-\frac{1}{\cos^2 x} = 0$

$x \neq 0$

$$\boxed{\begin{matrix} \tan x = 0 \\ x = 0 \end{matrix}}$$

$-\frac{\pi}{2}$ + 0 — $\frac{\pi}{2}$

f concaves up $\left(-\frac{\pi}{2}, 0\right)$

f concaves down $\left(0, \frac{\pi}{2}\right)$

13) $f(x) = \frac{1}{4}x^4 - 2x^2$

$f'(x) = x^3 - 4x$

$f''(x) = 3x^2 - 4$

$3x^2 - 4 = 0$

$3x^2 = 4$

$x = \sqrt{\frac{4}{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

$-1 \quad \frac{2\sqrt{3}}{3} \quad 1$

$-\infty \quad + \quad - \quad - \quad + \quad \infty$

f concaves up $(-\infty, -1)$ & $(1, \infty)$

f concaves down $(-1, 1)$

$f\left(\frac{2\sqrt{3}}{3}\right) = \frac{1}{4}\left(\frac{2\sqrt{3}}{3}\right)^4 - 2\left(\frac{2\sqrt{3}}{3}\right)^2 = \boxed{-2.222\ldots}$

Point of inflection $\left(\frac{2\sqrt{3}}{3}, -2.222\ldots\right)$

17) $f(x) = x\sqrt{x+3}$

$f'(x) = x(x+3)^{\frac{1}{2}}$

$f'(x) = 1(x+3)^{\frac{1}{2}} + x\left(\frac{1}{2}(x+3)^{-\frac{1}{2}}\right)$

$f'(x) = (x+3)^{\frac{1}{2}} + \frac{x}{2(x+3)^{\frac{1}{2}}} = \frac{(x+3)^{\frac{1}{2}}(2(x+3)^{\frac{1}{2}}) + x}{2(x+3)^{\frac{1}{2}}}$

$f'(x) = \frac{2(x+3) + x}{2(x+3)^{\frac{1}{2}}} = \frac{2x+6+x}{2(x+3)^{\frac{1}{2}}}$

$f'(x) = \frac{3x+6}{2(x+3)^{\frac{1}{2}}}$

$\underline{\underline{x = -4}}$?

$$f''(x) = \frac{3x+6}{2(x+3)^{\frac{1}{2}}}$$

$$f''(x) = \frac{1}{2}\left[\frac{3x+6}{(x+3)^{\frac{1}{2}}}\right]$$

$$f''(x) = \frac{1}{2}\left[\frac{3(x+3)^{\frac{1}{2}} - (3x+6)\cdot\frac{1}{2}(x+3)^{-\frac{1}{2}}}{(x+3)}\right]$$

$$f''(x) = \frac{1}{2}\left[\frac{3(x+3)^{\frac{1}{2}} - 3x - 6}{2(x+3)(x+3)^{\frac{1}{2}}}\right]$$

$$f''(x) = \frac{1}{2}\left[\frac{3(x+3)^{\frac{1}{2}} - 3x - 6}{2(x+3)^{3/2}}\right]$$

? $\searrow$

$$f''(x) = \frac{3(x+3)^{\frac{1}{2}} - 3x - 6}{4(x+3)^{3/2}} =$$

$$f''(x) = \frac{(3 - 3x - 6)(x+3)}{4} = \frac{-3x^2 - 9x - 3x + 9}{4}$$

$$f''(x) = \frac{-3x^2 - 12x + 9}{4} = \frac{(-3x-9)(x-1)}{4}$$

$(-3x-9) = 0$ / $x - 1 = 0$

$-3x = 9$ / $\boxed{x = 1}$

$\boxed{x = -3}$

21) $f(x) = \sin\frac{x}{2}$ $[0, 4\pi]$

$f'(x) = \frac{1}{2}\cos\frac{x}{2}$

$f(2\pi) = \sin\frac{2\pi}{2}$

$f(2\pi) = \sin\pi$

$f(2\pi) = \boxed{0}$

$f''(x) = \frac{1}{4} - \sin\frac{x}{2}$

$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$

$-\frac{1}{4}\sin\frac{x}{2} = 0$

$\sin\frac{x}{2} = 0$

2π

0 $-$ $+$ 4π

$\frac{x}{2} = \sin^{-1}(0)$

$\frac{x}{2} = 0$ or π

$x = 2\pi$

f concaves up $(2\pi, 4\pi)$

f concaves down $(0, 2\pi)$

Point of inflection $(2\pi, 0)$

25) $f(x) = 2\sin x + \sin 2x \quad [0, 2\pi]$

$f'(x) = 2\cos x + 2\cos 2x$

$f''(x) = -2\sin x - 4\sin 2x$

$f''(x) = -2(\sin x + 2\sin 2x)$

$-2(\sin x + 2\sin 2x) = 0$

$\sin x + 2\sin 2x = 0$

$2\sin 2x = -\sin x$

$2(2\sin x\cos x) = -\sin x$

$4\sin x(\cos x) = -\sin x$

$\cos x = -\frac{1}{4}$ $\quad \boxed{x = 1.823476582}$ $\quad \pi - 1.823 = \underline{1.319}$

$1.823 \quad \pi \quad 4.460$

$0 \quad - \quad + \quad - \quad + \quad 2\pi$ $\qquad \pi + 1.319$

f concaves down $(0, 1.823)(\pi, 4.460) = \boxed{4.460} \rightarrow$ 2rd cos point

f concaves up $(1.823, \pi)(4.460, 2\pi)$

29) $f(x) = x^4 - 4x^3 + 2$

$f'(x) = 4x^3 - 12x^2$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0 \qquad x = 3$

$f''(x) = 12x^2 - 24x$

$f''(0) = 12(0)^2 - 24(0) = \boxed{0}$ Max here

$f''(3) = 12(3)^2 - 24(3) = \boxed{39}$ Min here

$f(0) = 0^4 - 4(0)^3 + 2 = \boxed{2}$

$f(3) = 3^4 - 4(3)^3 + 2 = \boxed{-25}$

Since $f'(0) = 0$ f has no max

Since $f''(3) > 0$ f has a Min @ $(3, -25)$

33) $f(x) = x^3 - 3x^2 + 3$

$f'(x) = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0 \;/\; x - 2 = 0$

$\boxed{x = 0} \;/\; \boxed{x = 2}$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6 = \boxed{-6}$

$f''(2) = 6(2) - 6 = \boxed{6}$

$f(0) = 0^3 - 3(0)^2 + 3 = \boxed{3}$

$f(2) = 2^3 - 3(2)^2 + 3 = \boxed{-1}$

Since $f''(0) < 0$, f has a Max at $(0, 3)$

Since $f''(2) > 0$, f has a Min at $(2, -1)$

37) $f(x) = x^{2/3} - 3$

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3x^{1/3}} = 0 \to x = 0$

$f''(x) = \frac{-2}{9x^{4/3}}$

$f''(0) =$ undef, so we must use first derivative test

$$\begin{array}{ccccc} & & 0 & & \\ -\infty & - & \text{min} & + & \infty \end{array}$$

$f(0) = 0^{2/3} - 3 = -3$

f increases on $(-\infty, 0)$, Decreses $(0, \infty)$

f has a Min on $(0, -3)$

41) $f(x) = \cos x - x$ $\quad [0, 4\pi]$

$f'(x) = -\sin x - 1$

$-\sin x = 1$

$\boxed{x = \frac{3\pi}{2} \text{ and } \frac{7\pi}{2}}$

$f''(x) = -\cos x$

$f''\left(\frac{3\pi}{2}\right) = -\cos\left(\frac{3\pi}{2}\right) = \boxed{0}$

$f''\left(\frac{7\pi}{2}\right) = -\cos\left(\frac{7\pi}{2}\right) = \boxed{0}$

f has no relative Extrema

45) $y = \dfrac{x}{\ln x}$

$$y' = \frac{1(\ln x) - x\left(\frac{1}{x}\right)}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2}$$

$$\frac{\ln x - 1}{\ln x^2} = 0 \rightarrow \ln x - 1 = 0 \rightarrow \ln x = 1 \rightarrow \boxed{e = x}$$

$$y'' = \frac{\ln x - 1}{(\ln x)^2} \rightarrow \frac{\frac{1}{x}(\ln x)^2 - (\ln x - 1)(2(\ln x)(\frac{1}{x}))}{(\ln x)^4}$$

$$y'' = \frac{\cancel{x} \cdot \frac{(\ln x)^2}{\cancel{x}} - \frac{(\ln x - 1)(2\ln x)}{\cancel{x}} \cdot \cancel{x}}{x \cdot (\ln x)^4}$$

$$y'' = \frac{(\ln x)^2 - (\ln x - 1)(2\ln x)}{x(\ln x)^4}$$

$$y'' = \frac{\cancel{\ln x}[\ln x - 2(\ln x - 1)]}{x(\ln \cancel{x})^4}$$

$$y'' = \frac{\ln x - 2\ln x + 2}{x(\ln x)^3} \rightarrow \frac{2 - \ln x}{x(\ln x)^3}$$

$$y''(e) = \frac{2-\ln e}{e(\ln e)^3} = \frac{2-1}{e(1)} = \boxed{\frac{1}{e} > 0} \text{ Min here}$$

$$y(e) = \frac{e}{\ln e} = \frac{e}{1} = \boxed{e}$$

Since $y''(e) >$, we have a min @ (e, e)

49) $f(x) = x^2 e^{-x}$

$f'(x) = 2x(e^{-x}) + x^2(-1(1)e^{-x})$

$f'(x) = 2xe^{-x} - x^2 e^{-x}$

$f'(x) = xe^{-x}(2-x)$

$xe^{-x}(2-x) = 0$

$xe^{-x} = 0 \qquad 2-x=0$

$\frac{x}{e^{x}} = 0 \quad \boxed{x=0} \quad \boxed{x=2}$

$f''(x) = e^{-x}(2x-x^2)$

$f''(x) = -e^{-x}(2x-x^2) + e^{-x}(2-2x)$

$f''(x) = e^{-x}[-2x + x^2 + 2 - 2x]$

$f''(x) = e^{-x}[x^2 - 4x + 2]$

$f''(0) = e^0[0^2 - 4(0) + 2] = \boxed{2} > 0$

$f''(2) = \frac{1}{e^2}[4 - 8 + 2] = \frac{1}{e^2}(-2) = \boxed{-\frac{2}{e^2}} < 0$

$f(0) = 0^2 e^0 = \boxed{0} \rightarrow$ Rel Min

$f(2) = 2^2 e^{-2} = \boxed{\frac{4}{e^2}}$ Rel Max

$$\sec^{-1}(u) = \frac{u'}{|u|\sqrt{u^2-1}}$$

53) $f(x) = \sec^{-1}x - x$

$$f'(x) = \frac{1}{|x|\sqrt{x^2-1}} - 1$$

$$\frac{1}{|x|\sqrt{x^2-1}} - 1 = 0$$

$$\frac{1}{|x|\sqrt{x^2-1}} = 1 \longrightarrow 1 = |x|\sqrt{x^2-1}$$

$$(1)^2 = \left(|x|\sqrt{x^2-1}\right)^2$$

$$1 = x^2(x^2-1)$$

$$x^4 - x^2 - 1$$

$$x^2 = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$$

$$x^2 = \frac{1 \pm \sqrt{5}}{2} = \boxed{x = \pm\sqrt{\frac{1+\sqrt{5}}{2}}}$$

$$f''(x) = \frac{0\left(|x|\sqrt{x^2-1}\right) - 1\left(\frac{1}{|x|} \cdot \frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot (2x)\right)}{\left(|x|\sqrt{x^2-1}\right)^2}$$

$$f''(x) = \frac{-1\left(\frac{x}{|x|\sqrt{x^2-1}}\right)}{\left(|x|\sqrt{x^2-1}\right)^2} \longrightarrow -\frac{x}{\left(|x|\sqrt{x^2-1}\right)^3}$$

$x = 1.618$

$$f''(1.618) = -\frac{1.618}{\left(1.618\sqrt{(1.618)^2-1}\right)^3} \approx 0.1856 > 0$$

$$f''(-1.618) = \frac{1.618}{\left((-1.618)\sqrt{(-1.618)^2-1}\right)^3} = -0.1856 < 0$$

$f(\pm 1.618)$ = Not defined since $-1 < \cos x < 1$

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Section 4.5

1) 4 is a horizontal assymptote

5) $f(x) = \frac{x}{x^2+2} = \lim_{x\to\pm\infty} \frac{x}{x^2+2}$

$f(x) = \lim_{x\to\infty} \frac{\frac{x}{x^2}}{\frac{x^2+2}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{2}{x^2}} = \boxed{0}$

graph $\boxed{d}$

9) $f(x) = \frac{4x+3}{2x-1}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	7	2.263	2.025	2.002	2.000	2	2

13) a) $\lim_{x\to\infty} \frac{x^2+2}{x^3-1} = \lim_{x\to\infty} \frac{\frac{x^2}{x^3}+\frac{2}{x^3}}{\frac{x^3}{x^3}-\frac{1}{x^3}} =$

$= \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^3}}{1-\frac{1}{x^3}} = \frac{0+0}{1-0} = \frac{0}{1} = \boxed{0}$

b) $\lim_{x\to\infty} \frac{x^2+2}{x^2-1} = \frac{\frac{x^2}{x^2}+\frac{2}{x^2}}{\frac{x^2}{x^2}-\frac{1}{x^2}} = \frac{1+\frac{2}{x^2}}{1-\frac{1}{x^2}} = \frac{1+0}{1-0} = \boxed{1}$

e) $\lim\limits_{x\to\infty} \dfrac{x^2+2}{x-1} = \lim\limits_{x\to\infty} \dfrac{\frac{x^2}{x^2}+\frac{2}{x^2}}{\frac{x}{x^2}-\frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{1-\frac{2}{x^2}}{\frac{1}{x}-\frac{1}{x^2}}$

$= \dfrac{1-0}{0-0} = \boxed{\text{Undefined}}$

21) $\lim\limits_{x\to\infty} \dfrac{2x-1}{3x+2} = \lim\limits_{x\to\infty} \dfrac{\frac{2x}{x}-\frac{1}{x}}{\frac{3x}{x}+\frac{2}{x}} = \lim\limits_{x\to\infty} \dfrac{2-\frac{1}{x}}{3+\frac{2}{x}}$

$\dfrac{2-0}{3+0} = \boxed{\dfrac{2}{3}}$

25) $\lim\limits_{x\to-\infty} \dfrac{6x^2}{x+3} = \lim\limits_{x\to-\infty} \dfrac{\frac{6x^2}{x}}{\frac{x}{x}+\frac{3}{x}} = \lim\limits_{x\to-\infty} \dfrac{6x}{1+\frac{3}{x}}$

$\dfrac{6(-\infty)}{1+0} = \boxed{-\infty}$

29) $\lim\limits_{x\to-\infty} \dfrac{2x+1}{\sqrt{x^2-x}} = \lim\limits_{x\to-\infty} \dfrac{\frac{2x}{x}+\frac{1}{x}}{-\sqrt{\frac{x^2}{x^2}-\frac{x}{x^2}}}$

$\lim\limits_{x\to-\infty} \dfrac{2+\frac{1}{x}}{-\sqrt{1-\frac{1}{x}}} = \dfrac{2+0}{-\sqrt{1-0}} = \dfrac{2}{-1} = \boxed{-2}$

33) $\lim_{x\to\infty} \frac{1}{2x+\sin x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{2x}{x}+\frac{\sin x}{x}}$

$\frac{0}{2+0} = \boxed{0}$

37) $\lim_{x\to-\infty} \frac{3}{1+2e^{x}} = \frac{3}{1+2(0)} = \frac{3}{1} = \boxed{3}$

? 47) $\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{x\to\infty} \frac{1}{t}\sin\frac{1}{\frac{1}{t}}$

?

$\lim_{x\to\infty} \frac{1}{t}\sin t = \lim_{x\to\infty} \frac{\sin t}{t} = \boxed{0}$ $\boxed{\text{actually } 1}$

51) $\lim_{x\to\infty} \left(x-\sqrt{x^2+x}\right) = \lim_{x\to\infty} \frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}}$

$\lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+x}} = \lim_{x\to\infty} \frac{\frac{-x}{x}}{\frac{x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{x}{x^2}}}$

$\lim_{x\to\infty} \frac{-1}{1+\sqrt{1+\frac{1}{x}}} = \frac{-1}{1+\sqrt{1+0}} = \frac{-1}{1+1} = \boxed{-\frac{1}{2}}$

57) $F(x) = x\sin\frac{1}{2x}$

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$	.4794	.4997	.5	.5	.5	.5	.5

$\lim\limits_{x\to\infty} x\sin\frac{1}{2x} =$ $\quad x = \frac{1}{t}$? why

?

$\lim\limits_{x\to\infty} \frac{1}{t}\sin\frac{1}{2\left(\frac{1}{t}\right)} = \lim\limits_{x\to\infty} \frac{1}{t}\sin\frac{t}{2}$

$\lim\limits_{x\to\infty} \frac{\sin\frac{t}{2}}{t}$

63) $y = \dfrac{2+x}{1-x}$

1) Domain $\{x \mid x \neq 1\}$

2) V.A. $\lim_{x \to 1} f(x) = \lim_{x \to 1} \dfrac{2+x}{1-x} = \dfrac{2+1}{1-1} = \dfrac{3}{0}$

$\boxed{\text{Undefined}}$ Since $\lim_{x \to 1} f(x)$ is undefined

$x = 1$, V.A.

3) H.A. $\lim_{x \to \pm\infty} \dfrac{2+x}{1-x} = \dfrac{\frac{2}{x} + \frac{x}{x}}{\frac{1}{x} - \frac{x}{x}} = \dfrac{0+1}{0-1} = \boxed{-1}$

Since $\lim_{x \to \pm\infty} f(x) = -1$ then $y = -1$, H.A.

4) x-int, y-int

$2 + x = 0$ $\qquad$ $f(0) = 2 + 0$

$\boxed{x = -2}$ $\qquad$ $\boxed{f(0) = 2}$

5) $f'(x) = \dfrac{1(1-x) - (2+x)(-1)}{(1-x)^2} = \dfrac{1 - x + 2 + x}{(1-x)^2}$

$f'(x) = \dfrac{3}{(1-x)^2} \longrightarrow 1 - x = 0$

$\boxed{x = 1}$

$-\infty \quad + \quad 1 \quad + \quad \infty$

f has no extrema & increases everywhere

⑥ $f''(x) = \frac{3}{(1-x)^2} = f''(x) = 0(1-x)^2 + 3\left[-2(1-x)^{-3}(-1)\right]$

$F''(x) = 3\left[\frac{2}{(1-x)^3}\right] = F''(x) = \frac{6}{(1-x)^3} \rightarrow \frac{1-x=0}{\boxed{x=1}}$ inflect point

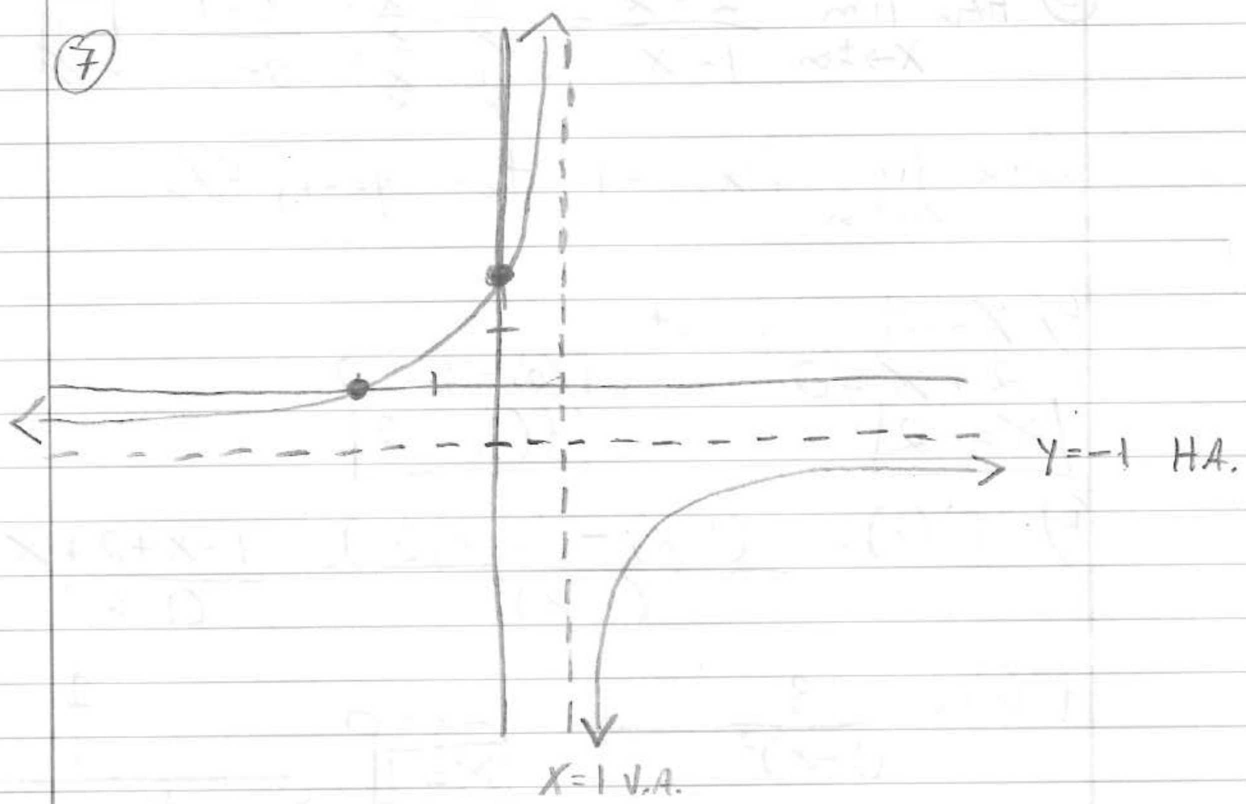
$-\infty \quad + \quad | \quad - \quad \infty$

F concaves up @ $(-\infty, 1)$

F Concaves down @ $(1, \infty)$

f has no inflection point since $x=1$ is not part of the domain

⑦



73) $F(x) = \frac{2x}{1-x}$

(1) Domain $\{x \setminus x \neq 1\}$

(2) V.A. $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{2x}{1-x} = \frac{2(1)}{1-1} = \boxed{\frac{2}{0}}$

Undefined. Since the $\lim_{x\to 1} f(x)$ is undefined

$x = 1$, V.A.

(3) HA $\lim_{x\to \pm\infty} \frac{2x}{1-x} = \lim_{x\to\pm\infty} \frac{\frac{2x}{x}}{\frac{1}{x} - \frac{x}{x}} = \frac{2}{0-1} = \boxed{-2}$

Since $\lim_{x\to\pm\infty} F(x) = -2$ then $y = -2$, HA.

(4) X-int Y-int

$2x = 0$

$\boxed{x = 0}$

$F(0) = \frac{2(0)}{1-0} = \boxed{0}$

(5) $f'(x) = \frac{2x}{(1-x)} = f'(x) = \frac{2(1-x) - 2x(-1)}{(1-x)^2}$

$F'(x) = \frac{2 - 2x + 2x}{(1-x)^2} = f'(x) = \frac{2}{(1-x)^2}$

$\frac{2}{(1-x)^2} = 0$

$1 - x = 0$

$\boxed{x = 1}$

$-\infty$ + (1) + ∞

No extrema, F increases everywhere

(6) $f'(x) = \dfrac{2}{(1-x)^2} = 2(1-x)^{-2}$

$F''(x) = 0(1-x)^{-2} + 2(-2(1-x)^{-3}(-1))$

$F''(x) = 2[2(1-x)^{-3}]$

$F''(x) = \dfrac{4}{(1-x)^3} \longrightarrow -x+1=0$

$\boxed{x=1}$

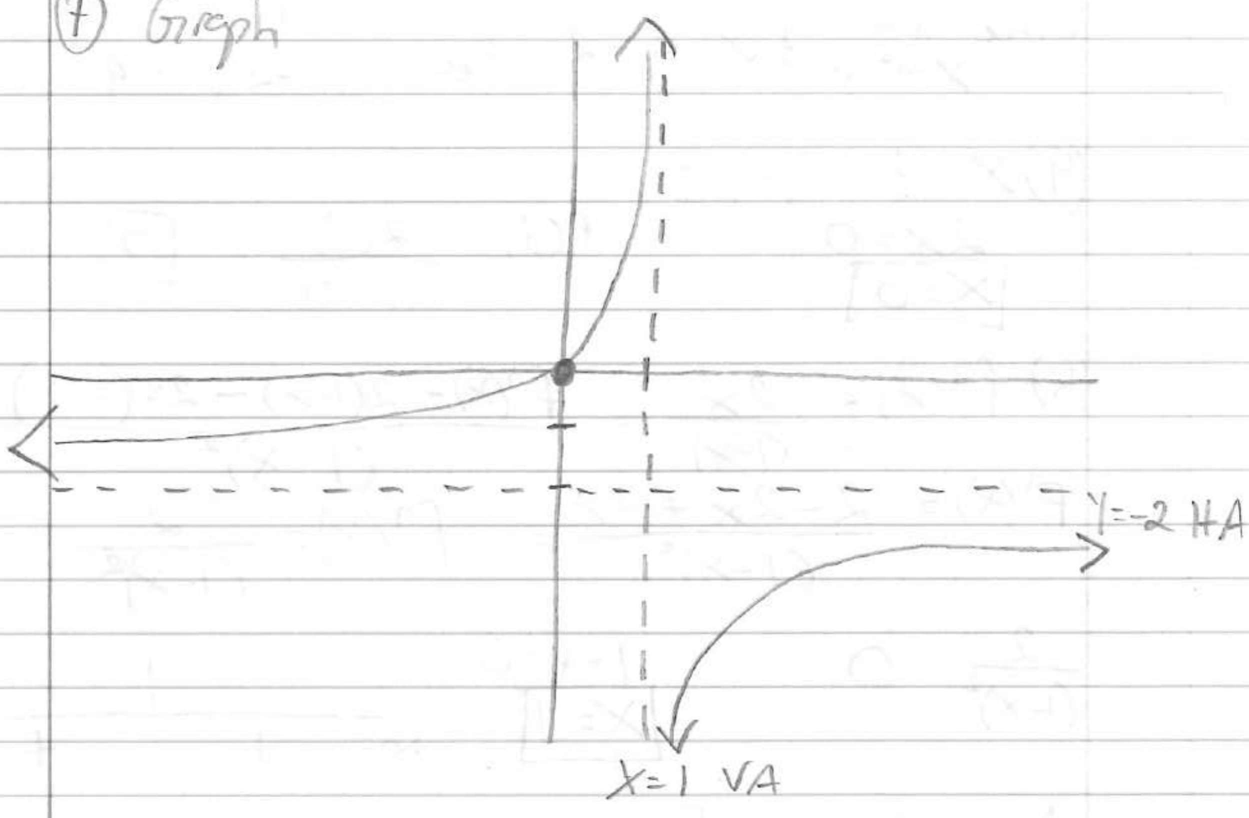
$-\infty \quad + \quad 1 \quad - \quad \infty$

f concaves up @ $(-\infty, 1)$

f concaves down @ $(1, \infty)$

f has no inflection point since $x=1$ is not part of the domain

(7) Graph



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Section 4.6

1) $f(x) = -x$
$f'(x) = -1 \rightarrow \boxed{D}$

5) a) $f'(x)=0$ @ $x=2$, $x=-2$
b) $f''(x)=0$ @ $x=0$
Concave down $(-\infty, 0)$
Concave up $(0, \infty)$
c) $f''(x) > 0$ so f increases on $(0,\infty)$
d) $f''(x)$ changes from neg to positive @ $x=0$
so $f'(x)$ = Minimum on $x=0$

9) $f(x) = \frac{1}{x-2} - 3$

① Domain $\{x \mid x \neq 2\}$

② V.A. $\lim_{x\to2} f(x) = \lim_{x\to2} \frac{1}{x-2} - 3 = \frac{1}{0} - 3$

$\boxed{\text{Undefined}}$ Since $\lim_{x\to2} f(x)$ is undefined then $x=2$ V.A.

③ HA $\lim_{x\to\pm\infty} \frac{1}{x-2} - 3 = \lim_{x\to\pm\infty} \frac{\frac{1}{x}}{\frac{x}{x} - \frac{2}{x}} - 3$

$= \frac{0}{1-0} - 3 = -3$, Since $\lim_{x\to\pm\infty} f(x) = -3$, then $y=-3$, HA.

④ X-Int, Y-Int

$\frac{1}{x-2} - 3 = 0$
$\frac{1}{x-2} = 3$
$3(x-2) = 1$
$x-2 = \frac{1}{3}$
$x = \frac{1}{3} + 2$
$\boxed{x = \frac{7}{3}}$

$f(0) = \frac{1}{0-2} - 3$
$f(0) = -\frac{1}{2} - 3$
$\boxed{f(0) = -\frac{7}{2}}$

(5) $f'(x) = \frac{1}{x-2} - 3 = 1(x-2)^{-1} - 3$

$f'(x) = 0(x-2)^{-1} + 1(-1(x-2)^{-2}(1))$

$f'(x) = \frac{1(-1)}{(x-2)^2} = \boxed{\frac{-1}{(x-2)^2}} \longrightarrow x-2=0 \quad \boxed{x=2}$

$-\infty \quad - \quad 2 \quad - \quad \infty$ f is constantly decreasing & has no extrema

(6) $f''(x) = -1(x-2)^{-2}$

$f''(x) = 0(x-2)^{-2} + (-1)(-2(x-2)^{-3}(1))$

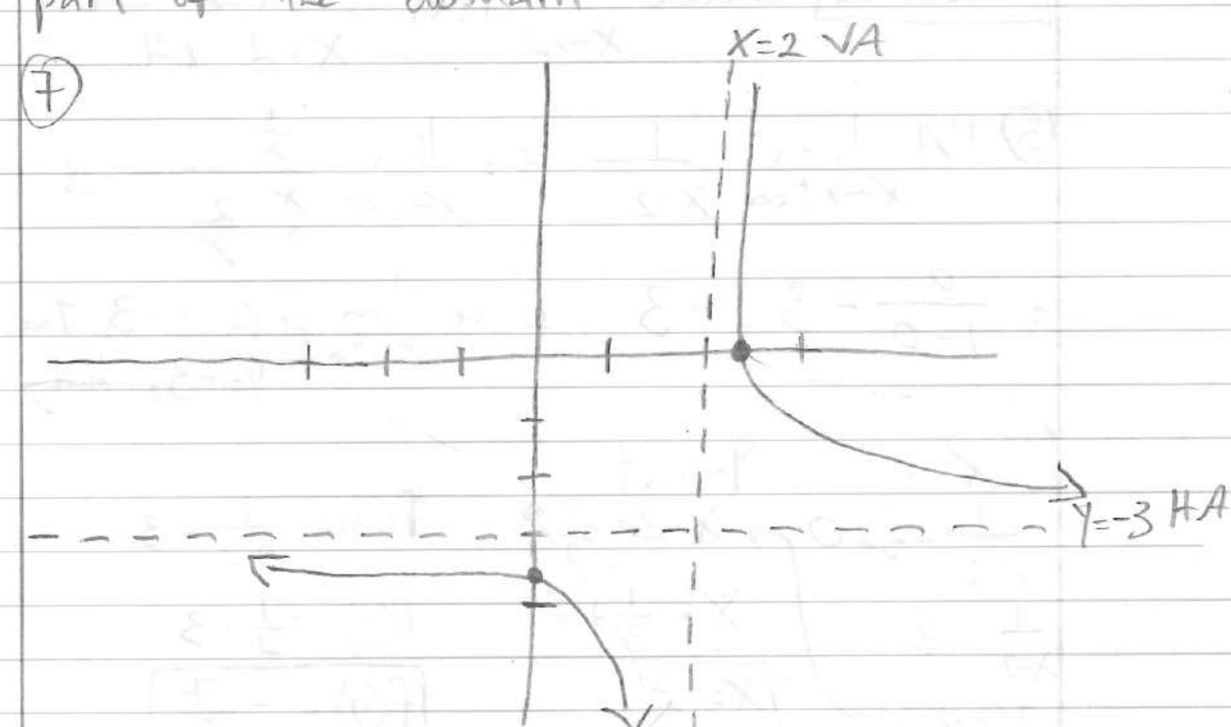
$f''(x) = -1[-2(x-2)^{-3}]$

$f''(x) = \frac{2}{(x-2)^3} \longrightarrow x-2=0 \quad \boxed{x=2}$

$-\infty \quad - \quad 2 \quad + \quad \infty$ f concaves up @ $(2, \infty)$
f concaves down @ $(-\infty, 2)$

f has no inflection point since $x=2$ is not a part of the domain

(7)



13) $f(x) = x + \dfrac{4}{x^2+1}$

1) Domain all real numbers

2) V.A. No V.A.

3) HA is slant so combine terms with LCD

O.A $\longrightarrow$

$$\frac{x(x^2+1)+4}{(x^2+1)} = \frac{x^3+x+4}{x^2+1}$$

$$\begin{array}{r} x \\ (x^2+1)\overline{)\,x^3+0x^2+x+4} \\ -(x^3+0x^2+x) \\ \hline 0 \ \boxed{+4} \end{array}$$

Slant Asymptote = $\boxed{y = x}$

on $y = x + \dfrac{4}{x^2+1}$

4) x-int y-int

$$x + \frac{4}{x^2+1} = 0 \rightarrow \frac{4}{x^2+1} + \frac{x(x^2+1)}{x^2+1} = 0$$

$$\frac{x(x^2+1)+4}{x^2+1} = \quad x^3+x+4 = 0$$

$$y\text{-int} = f(0) = 0 + \frac{4}{0^2+1} = \boxed{4}$$

5) $f'(x) = 1 + 4(x^2+1)^{-1}$

$f'(x) = 1 + 0(x^2+1)^{-1} + 4(-1(x^2+1)^{-2}(2x))$

$f'(x) = 1 - 8x(x^2+1)^{-2}$

$f'(x) = 1 - \dfrac{8x}{(x^2+1)^2} = 0 \rightarrow \dfrac{8x}{(x^2+1)^2} = 1$

? $8x = (x^2+1)^2 \rightarrow 8x = x^4 + 2x^2 + 1 \rightarrow x^4 + 2x^2 - 8x + 1 = 0$

HOW? $x \approx 0.1292$ or $x \approx 1.6085$

(6) $f'(x) = 1 - \dfrac{8x}{(x^2+1)^2} \longrightarrow 1 - 8\left[\dfrac{x}{(x^2+1)^2}\right]$

$f''(x) = 0 - 8\left[\dfrac{1(x^2+1)^2 - x\left(2(x^2+1)^1 \cdot (2x)\right)}{(x^2+1)^4}\right]$

$f''(x) = -8\left[\dfrac{(x^2+1)^2 - 2x\left[(x^2+1)\cdot(2x)\right]}{(x^2+1)^4}\right]$

$f''(x) = -8\left[\dfrac{(x^2+1)^2 - 4x(x^2+1)}{(x^2+1)^4}\right]$

? $f''(x) = -8\left[\dfrac{1 - 4x^2}{(x^2+1)^3}\right]$ Answer $\boxed{\dfrac{8(3x^2-1)}{(x^2+1)^3}}$?

$f''(x) = \dfrac{8(4x^2-1)}{(x^2+1)^3}$

17) $f(x) = \dfrac{x^2 - 6x + 12}{x - 4}$

1) Domain $\{x \mid x \neq 4\}$

2) VA $\lim\limits_{x \to 4} \dfrac{x^2 - 6x + 12}{x - 4} = \dfrac{4^2 - 6(4) + 12}{4 - 4} = \dfrac{4}{0} = \boxed{\text{undefined}}$

Since $\lim\limits_{x \to 4} f(x)$ does not exist then $x = 4$, VA.

3) HA Slant

$$\begin{array}{r} x - 2 \\ (x-4)\overline{\big)\,x^2 - 6x + 12} \\ -(x^2 - 4x) \\ \hline -2x + 12 \\ -(-2x + 8) \\ \hline \boxed{3} \end{array}$$

$\boxed{y = x - 2 + \boxed{\dfrac{3}{x-4}}}$

4) x-int y-int

$x^2 - 6x + 12 = 0$

No real solution

since $b^2 - 4ac = -12$

$f(0) = \dfrac{0^2 - 6(0) + 12}{0 - 4} = \dfrac{12}{-4}$

$f(0) = \boxed{-3}$

5) $f'(x) = \dfrac{(2x - 6)(x - 4) - (x^2 - 6x + 12)(1)}{(x-4)^2}$

$f'(x) = \dfrac{2x^2 - 14x + 24 - x^2 + 6x - 12}{(x-4)^2}$

$f'(x) = \dfrac{x^2 - 8x + 12}{(x-4)^2} \longrightarrow \dfrac{(x-2)(x-6)}{(x-4)^2}$

$f'(x) = 0 \longrightarrow (x-2)(x-6) = 0$

$\boxed{x = 2}$ $\boxed{x = 6}$

2 6

$-\infty$ + − + ∞

max min

$$f(2) = \frac{(2)^2-6(2)+12}{2-4} = \frac{4}{2} = \boxed{-2}$$

$$f(6) = \frac{(6)^2-6(6)+12}{6-4} = \frac{12}{2} = \boxed{6}$$

f increases on $(-\infty,2)(6,\infty)$ with Rel Max @ $(2,-2)$
f decreases on $(2,4)(4,6)$ with Rel Min @ $(6,6)$

(6) $$F''(X) = \frac{x^2-8x+12}{(X-4)^2} \longrightarrow \frac{(x^2-8x+12)}{(X-4)^2}$$

$$f''(x) = \frac{(2x-8)(x-4)^2-(x^2-8x+12)(2(x-4))}{(x-4)^4}$$

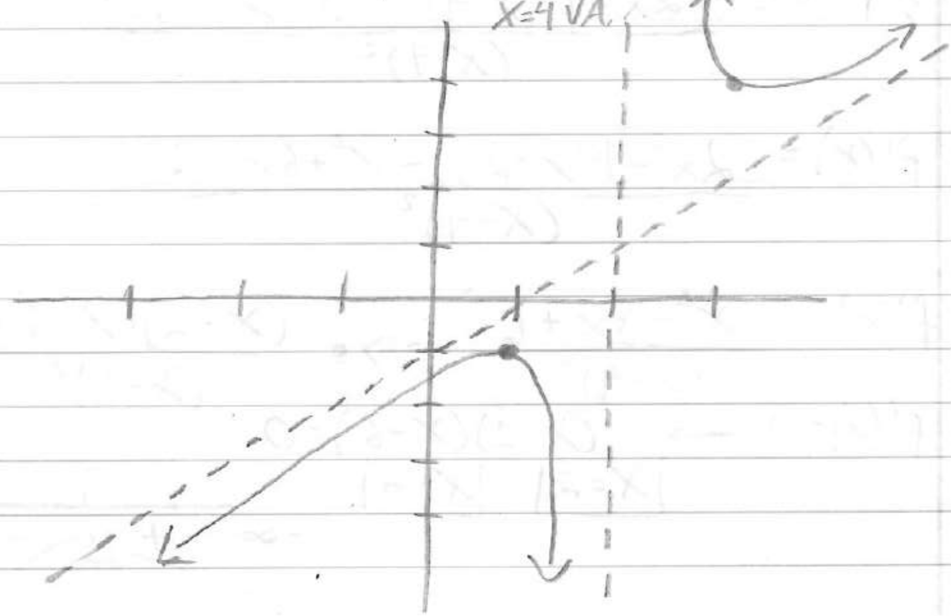
$$f''(x) = \frac{(\cancel{2x-8})(\cancel{x^2-8x}+16)-(\cancel{x^2-8x}+12)(\cancel{2x-8})}{(x-4)^4}$$

$$f''(x) = \frac{4}{(x-4)^4} \longrightarrow \frac{4}{(x-4)^4} = 0$$

$= X-4 = 0 \longrightarrow X=4$

$-\infty$ ——— 4 ——— ∞
$-$ $+$

f Concaves down @ $(-\infty,4)$ + up on $(4,\infty)$
f has no inflection point since $x=4$ is not on domain



21) $h(x) = x\sqrt{9-x^2}$

① Domain $\quad 9-x^2 \geq 0$

$x \geq \pm 3$

$-\infty \quad \text{ERR} \quad -3 \quad + \quad 3 \quad \text{ERR} \quad \infty$

Domain $= [-3, 3]$

② V.A.

No VA since its not a rational fxn

③ HA

No HA Since its not a rational fxn

④ $f'(x) = x(9-x^2)^{\frac{1}{2}}$

$f'(x) = 1(9-x^2)^{\frac{1}{2}} + x\left(\frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)\right)$

$f'(x) = (9-x^2)^{\frac{1}{2}} - 2x^2\left(\frac{1}{2}(9-x^2)^{-\frac{1}{2}}\right)$

$f'(x) = (9-x^2)^{\frac{1}{2}} - \dfrac{x^2}{(9-x^2)^{\frac{1}{2}}}$

$f'(x) = \dfrac{9-x^2-x^2}{(9-x^2)^{\frac{1}{2}}} \longrightarrow \dfrac{9-2x^2}{(9-x^2)^{\frac{1}{2}}}$

$\dfrac{9-2x^2}{(9-x^2)^{\frac{1}{2}}} = 0 \longrightarrow 9-2x^2 = 0$

$2x^2 = 9$

$x = \pm\dfrac{3}{\sqrt{2}}$

$-3 \quad - \quad -\frac{3}{\sqrt{2}} \quad + \quad \frac{3}{\sqrt{2}} \quad - \quad 3$

min $\quad$ Max

$f\left(-\frac{3}{\sqrt{2}}\right) = -\frac{3}{\sqrt{2}}\sqrt{9+\left(\frac{3}{\sqrt{2}}\right)^2}$

$= \boxed{-4.5}$

$f\left(\frac{3}{\sqrt{2}}\right) = \frac{3}{\sqrt{2}}\sqrt{9-\left(\frac{3}{\sqrt{2}}\right)^2} = \boxed{4.5}$

F decreases from $(-3, -\frac{3}{\sqrt{2}})$ & $(\frac{3}{\sqrt{2}}, 3)$ with a Rel Min @ $(-\frac{3}{\sqrt{2}}, -4.5)$

F increases from $(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$ with a Rel Max @ $(\frac{3}{\sqrt{2}}, 4.5)$

⑥ $f''(x) = \dfrac{9-2x^2}{(9-x^2)^{\frac{1}{2}}} =$

$$f''(x) = \frac{-4x(9-x^2)^{\frac{1}{2}} - (9-2x^2)\left(\frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x)\right)}{(9-x^2)}$$

$$f''(x) = \frac{(9-x^2)^{-\frac{1}{2}}\left[-4x(9-x^2) - (9-2x^2)(-x)\right]}{(9-x^2)}$$

$$f''(x) = \frac{x(9-x^2)^{-\frac{1}{2}}\left[-36+4x^2+9-2x^2\right]}{(9-x^2)}$$

$$f''(x) = \frac{x(9-x^2)^{-\frac{1}{2}}\left[2x^2-27\right]}{(9-x^2)}$$

$$f''(x) = \frac{x(2x^2-27)}{(9-x^2)^{3/2}}$$

$x(2x^2-27)=0$

$x=0 \quad 2x^2-27=0$

$2x^2=27$

$\boxed{x = \pm\dfrac{3\sqrt{3}}{\sqrt{2}}} \longrightarrow$ not in domain

$$f''\left(\pm\frac{3}{\sqrt{2}}\right) = \frac{\pm\frac{3}{\sqrt{2}}\left(2\left(\pm\frac{3}{\sqrt{2}}\right)^2-27\right)}{\left(9-\left(\pm\frac{3}{\sqrt{2}}\right)^2\right)^{3/2}}$$

Since $f''\left(\frac{3}{\sqrt{2}}\right) < 0$, then f concaves down

Since $f''\left(-\frac{3}{\sqrt{2}}\right) > 0$, then f concaves up

29) $f(x) = 3x^3 - 9x + 1$

(1) Domain $(-\infty, \infty)$

(2) VA. No VA since its not a rational fxn

(3) No HA or OA since its not a rational fxn

(4) X-int Y-int $f'(x) = 9x^2 - 9$

$3x^3 - 9x + 1 = 0$ $\quad X_1 = \pm 2,$

$$X_2 = X_1 - \frac{f(x_1)}{f'(x_1)} = \pm 2 - \frac{f(\pm 2)}{f'(\pm 2)} = \boxed{\begin{array}{l} X_2 = 1.740741 \\ X_2 = -1.814815 \end{array}}$$

$$X_3 = 1.740741 - \frac{f(1.740741)}{f'(1.740741)} = \boxed{X_3 = 1.677386}$$

$$X_3 = -1.814815 - \frac{f(-1.814815)}{f'(-1.804815)} = \boxed{X_3 = -1.785833}$$

$$X_4 = 1.677586 - \frac{f(1.677586)}{f'(1.677586)} = \boxed{X_4 = 1.673581}$$

$$X_4 = -1.785833 - \frac{f(-1.785833)}{f'(-1.785833)} = \boxed{X_4 = -1.785141}$$

(5) $f'(x) = 9x^2 - 9 \longrightarrow 9x^2 - 9 = 0$

$\boxed{X = \pm 1}$

$-\infty$		-1		1		∞
	$+$	max	$-$	min	$+$	

$f(1) = 3(1)^3 - 9(1) + 1 = \boxed{-5}$

$f(-1) = 3(-1)^3 - 9(-1) + 1 = \boxed{7}$

F Increases on $(-\infty,-1)$ & $(1,\infty)$ with a Rel Max @ $(-1,7)$. F decreases on $(-1,1)$ with a Rel Min on $(1,-5)$

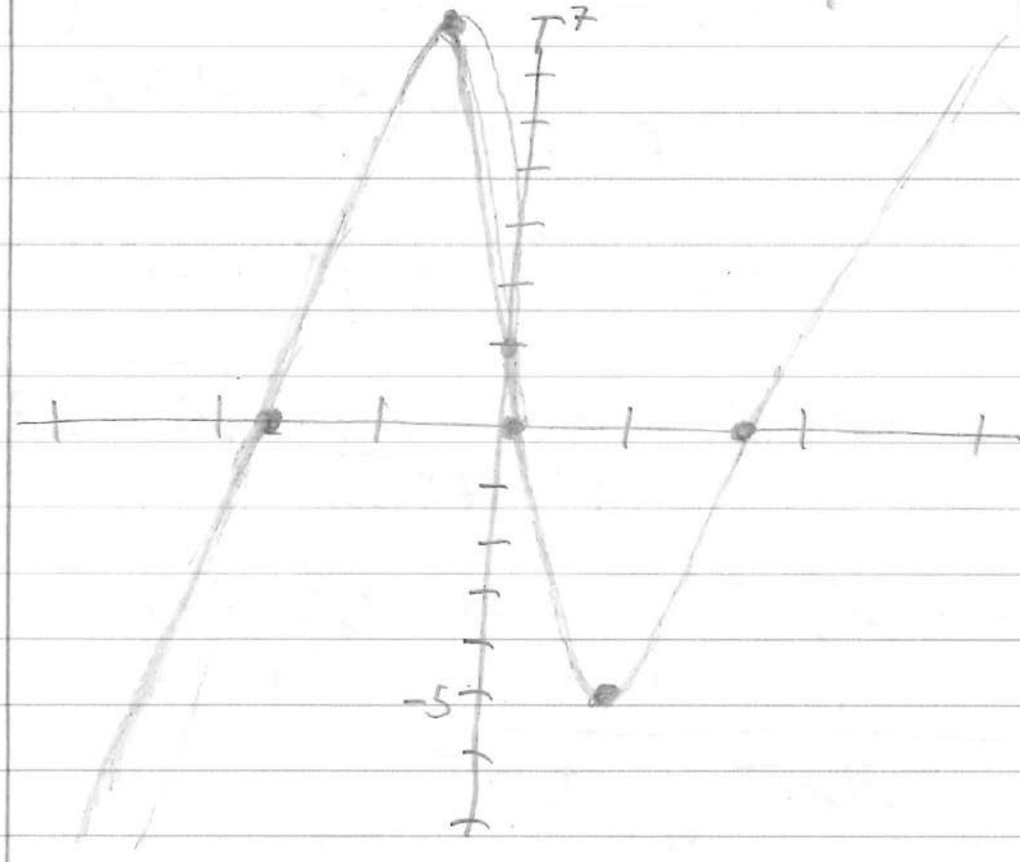
⑥ $F''(X) = 18x - 0 \longrightarrow 18x = 0$

$\boxed{X = 0}$

$$\begin{array}{ccccc} & & 0 & & \\ -\infty & - & | & + & \infty \end{array}$$

$f(0) = 3(0)^3 - 9(0) + 1 = \boxed{1}$

F concaves up on $(0,\infty)$ & concaves down on $(-\infty,0)$. f has inflection pt on $(0,1)$



33) $f(x) = x^4 - 4x^3 + 16x$

① Domain $(-\infty, \infty)$

② VA No VA since not rational fxn

③ No HA or OA since no rational fxn

④ x-int

$x^4 - 4x^3 + 16x = 0$

$f'(x) = 4x^3 - 12x^2 + 16$

Newtons Method $\boxed{x_1 = -1.5}$

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$x_2 = -1.5 - \dfrac{f(-1.5)}{f'(-1.5)} = \boxed{x_2 = -1.679}$

$\boxed{x = 0}$, $\boxed{x = -1.679}$

$\boxed{f(0) = 0}$

⑤ $f'(x) = 4x^3 - 12x^2 + 16 \rightarrow 4x^3 - 12x^2 + 16 = 0$

$\dfrac{P}{Q} = \dfrac{\pm 1, \pm 2, \pm 4, \pm 8, \pm 16}{\pm 1, \pm 2, \pm 4}$

$\boxed{\text{Zeroes } -1, 2}$

$$\begin{array}{r|rrrr} -1 & 4 & -12 & 0 & 16 \\ & & -4 & 16 & -16 \\ \hline & 4 & -16 & 16 & 0 \end{array}$$

$$\begin{array}{r|rrr} 2 & 4 & -16 & 16 \\ & & 8 & -16 \\ \hline & 4 & -8 & 0 \end{array}$$

$4x - 8 = 0$

$4x = 8$

$\boxed{x = 2}$

$\boxed{x-2}, \boxed{x+1}, \boxed{x-2} = 0$

-1, 2

$-\infty$ − + + ∞

$f(-1) = (-1)^4 - 4(-1)^3 + 16(-1)$,

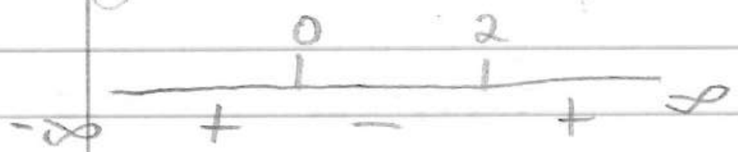
$\boxed{f(-1) = -11}$

F decreases on $(-\infty, -1)$ with a Rel Min on $(-1, -11)$. F increases on $(-1, \infty)$

(6) $f''(x) = 12x^2 - 24x \longrightarrow 12x^2 - 24x = 0$

$12x(x-2) = 0$

$\boxed{x=0}$ $\boxed{x=2}$



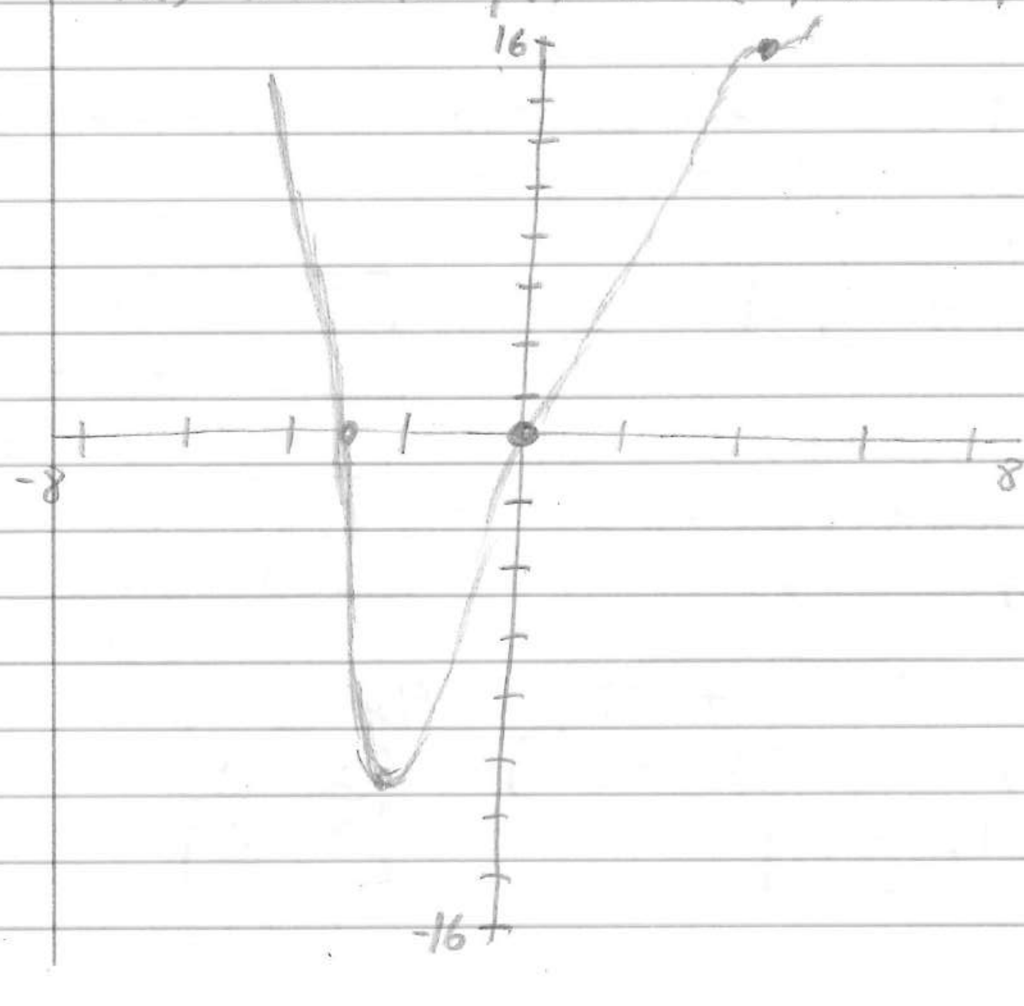
$F(0) = (0)^4 - 4(0)^3 + 16(0) = \boxed{0}$

$F(2) = (2)^4 - 4(2)^3 + 16(2) = \boxed{16}$

F concaves up on $(-\infty, 0)$ & $(2, \infty)$

F concaves down on $(0, 2)$

F has inflection pts on $(0,0)$ & $(2, 16)$



4) 1) $f(x) = \frac{10}{1+4e^{-x}} = \frac{10e^{x}}{1+4} = \frac{10e^{x}}{5}$

(1) Domain $(-\infty, \infty)$

(2) No V.A. since no values would make this fxn undefined

? ⇒ (3) HA $\lim_{x \to \pm\infty} \frac{10e^{x}}{5} = \quad y = 10$

(4) x-int

$\frac{10e^{x}}{5} = 0$

$\boxed{x = 1.3863}$

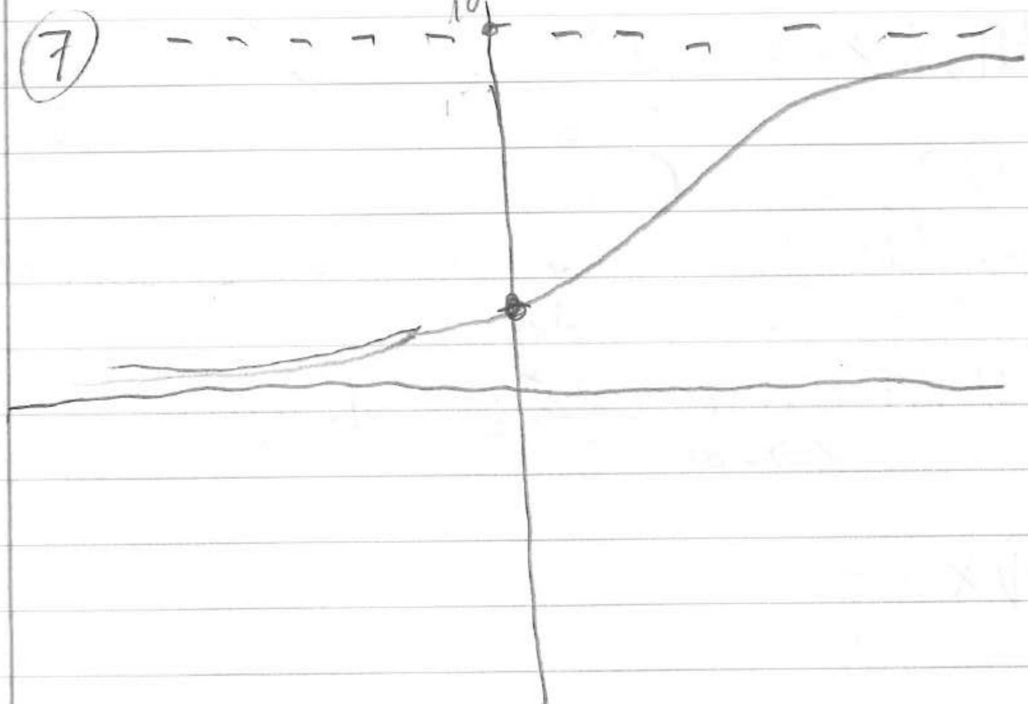
(5) $f'(x) = \frac{40e^{-x}}{(1+4e^{-x})^2}$

Since $f'(x) > 0$ then f is increasing

(6) $f''(x) = \frac{40e^{-x}(1+4e^{-x})^2 - e^{-x}(1+4e^{-x})^2}{(1+4e^{-x})^4}$

$f''(x) = \frac{40e^{-x}(4e^{-x} - 1)}{(1+4e^{-x})^3}$

7



53) $f(x) = 2x - \tan x$ $\quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

① Domain $2x - \frac{\sin x}{\cos x}$ $\quad \boxed{x \neq \frac{\pi}{2}}$ $\boxed{x \neq \frac{3\pi}{2}}$

② $VA = \lim_{x \to \pm\frac{\pi}{2}} 2x - \frac{\sin x}{\cos x} = 2\left(\pm\frac{\pi}{2}\right) - \frac{(\pm 1)}{0} = \boxed{\text{Undefined}}$

Since $\lim_{x \to \pm\frac{\pi}{2}} f(x)$ does not exist $x = \frac{\pi}{2}, \frac{3\pi}{2}$, V.A.

? → ③ HA. No HA. or OA.

? → ④ x-int $\quad$ y-int

$2x - \tan x = 0$

$f(0) = 2(0) - \tan(0)$

$\boxed{f(0) = 0}$

⑤ $f'(x) = 2 - \sec^2 x$

$2 - \sec^2 x = 0 \rightarrow \sec^2 x = 2 \rightarrow \sec x = \pm\sqrt{2}$

$\frac{1}{\cos x} = \pm\sqrt{2} \rightarrow \pm\sqrt{2}\cos x = 1 \rightarrow \cos x = \frac{1}{\pm\sqrt{2}}$

$\boxed{x = \pm\frac{\pi}{4}}$

$-\frac{\pi}{2}$ $\quad -\frac{\pi}{4}$ $\quad \frac{\pi}{4}$ $\quad \frac{\pi}{2}$

$-$ $\quad$ $+$ $\quad$ $-$

min $\quad$ max

$f\left(-\frac{\pi}{4}\right) = 2\left(-\frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{4}\right) = \boxed{-\frac{\pi}{2} + 1}$ → Rel Min

$f\left(\frac{\pi}{4}\right) = 2\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right) = \boxed{\frac{\pi}{2} - 1}$ → Rel Max

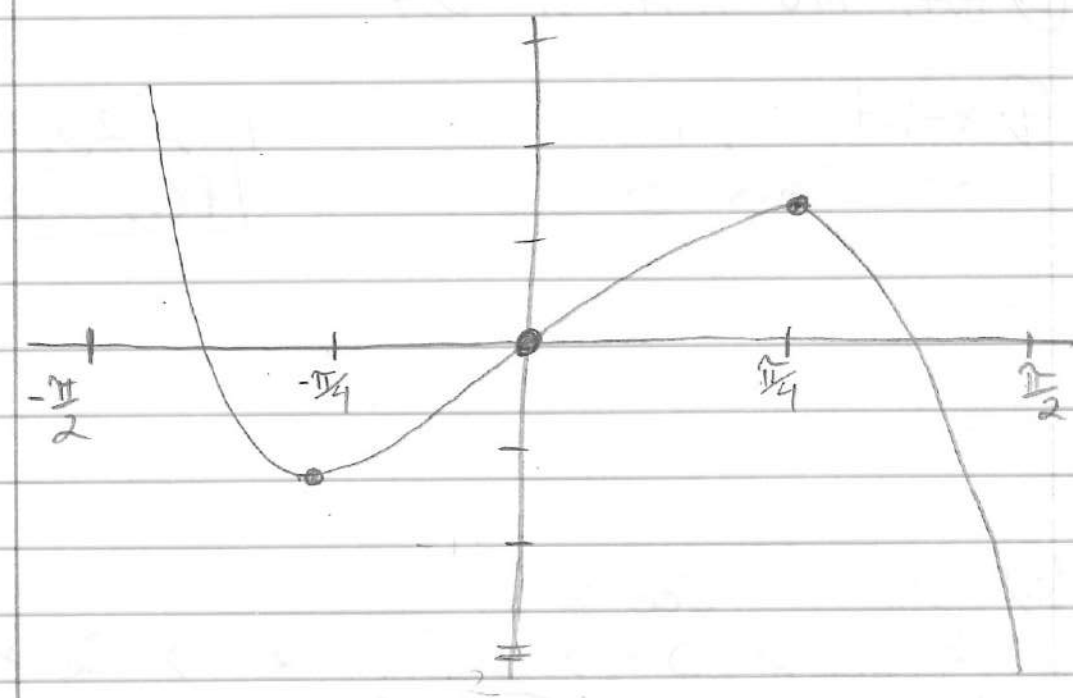
⑥ $f''(x) = 0 - 2\sec x(\sec x \tan x)$

$f''(x) = -2\sec^2 x(\tan x)$

$f''(x) = -2(1+\tan^2 x)(\tan x)$

$-2(1+\tan^2 x)(\tan x) = 0$

$-2(1+\tan^2(0))\tan(0) = 0 \longrightarrow -2(1+0)(0) = \boxed{0}$



57) $f(x) = x\tan x$

① Domain $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

② V.A. $\lim\limits_{x \to \pm\frac{\pi}{4}} \frac{x\sin x}{\cos x} = \frac{\left(\pm\frac{\pi}{4}\right)\left(\sin\frac{\pi}{4}\right)}{\cos\left(\pm\frac{\pi}{4}\right)} = \frac{\left(\pm\frac{\pi}{4}\right)(\pm 1)}{0} = \boxed{\text{undef}}$

Since $\lim\limits_{x \to \pm\frac{\pi}{4}} f(x)$ is undefined, $x = \pm\frac{\pi}{4}$, V.A.

③ HA : No HA or O.A for Tan function

④ $x\tan x = 0$ $\qquad f(0) = 0\tan 0$

$\tan x = 0 \rightarrow \frac{\sin x}{\cos x} = 0$ $\qquad \boxed{f(0) = 0}$

$x = 0$ $\qquad x = \pi, -\pi$

⑤ $f'(x) = 1(\tan x) + x(\sec^2 x)$

$\tan x + x(\tan^2 x + 1) = 0$

$x\tan^2 x + \tan x + x = 0$ $\qquad$ when $x = 0$ $f'(x) = 0$

$-\frac{3\pi}{2}$ — Min + $\frac{3\pi}{2}$ $\qquad f(0) = 0\tan 0$

$f(0) = 0$

f decreases on $\left(-\frac{3\pi}{2}, 0\right)$ with a Rel Min @ $(0,0)$

f increases on $\left(0, \frac{3\pi}{2}\right)$

⑥ $f''(x) = \sec^2 x + 1(\sec^2 x) + x(2(\sec x)(\sec x\tan x))$

$f''(x) = \sec^2 x + \sec^2 x + 2x(\sec^2 x\tan x)$

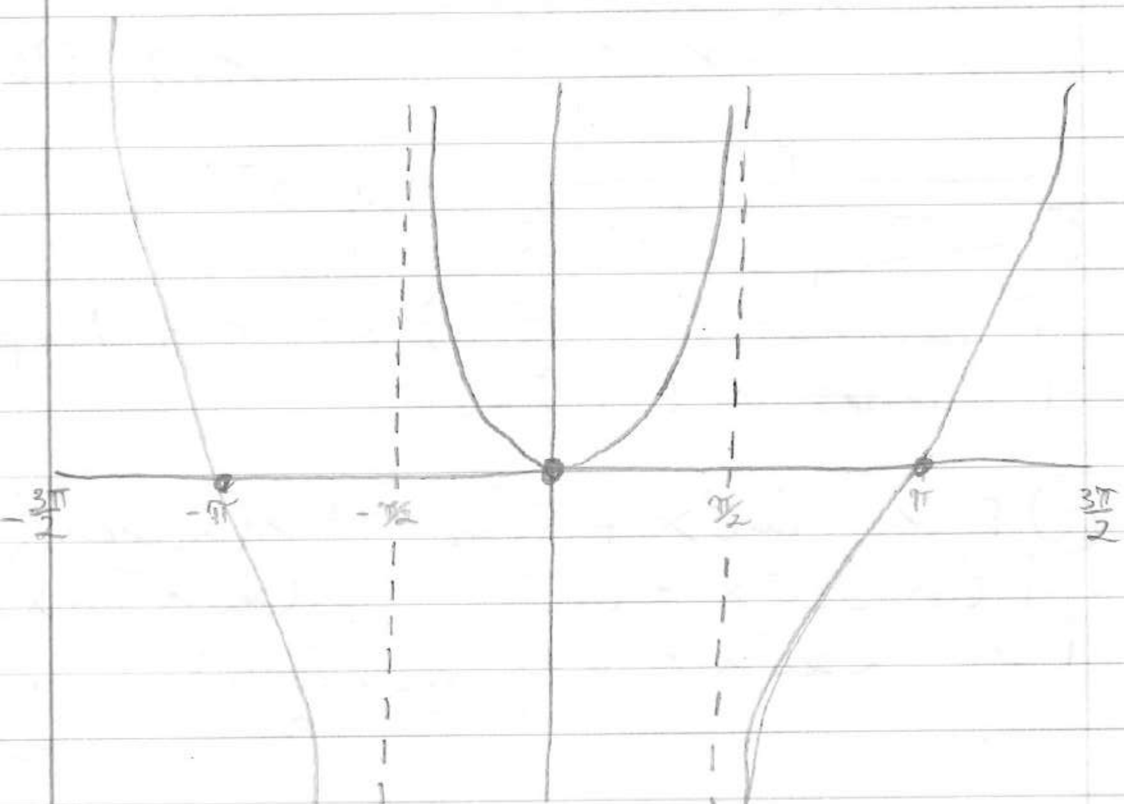
$f''(x) = 2\sec^2 x + 2x\left(\frac{1}{\cos^2 x} \cdot \frac{\sin x}{\cos x}\right)$

$$f''(x) = 2\sec^2 x + 2x\left(\frac{\sin x}{\cos^3 x}\right)$$

$$f''(x) = 2\sec^2 x\,(\pi\tan x + 1)$$

$$f''(x) = \frac{2(\cos x + \pi \sin x)}{\cos^3 x}$$

7



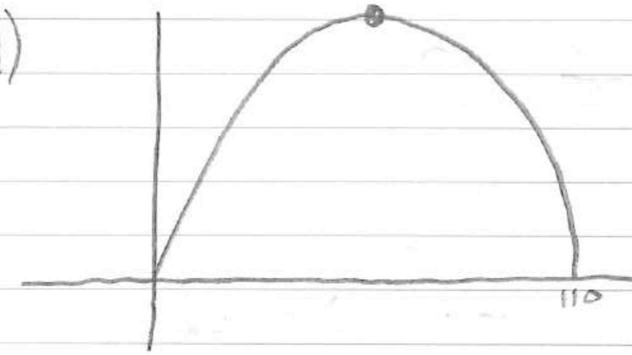
Steven Romeiro
Pg 265 #1-29

Section 4.7

1) c) $p = x(110 - x)$

$p = 110x - x^2$

d)



E) $\frac{dp}{dx} = 110 - 2x$

$\frac{dp}{dx} = 0$ $\quad 2x = 110$

$\boxed{x = 55}$

P has a max at $x = 55$

First # = 55 $\quad$ Second # = 110 − 55
Second # = 55

5) $xy = 192$

$y = \frac{192}{x}$

$S = x + 3\left(\frac{192}{x}\right)$

$S = x + \frac{576}{x}$

$S' = 1 - \frac{576}{x^2} = 0$

$\frac{576}{x^2} = 1$

$x^2 = 576 \longrightarrow \boxed{x = \pm 24}$

$S'' = \frac{1152}{x^3}$

$y = \frac{192}{24} = \boxed{y = 8}$

9) $P = 2(x+y) = 100\text{ m}$

$2(x+y) = 100$

$x + y = 50$

$\boxed{y = 50 - x}$

$A = x \cdot y$

$A = x(50 - x)$

$A = 50x - x^2$

$\longrightarrow d'a = 50 - 2x$

$50 - 2x = 0$

$\boxed{x = 25}$

$d''a = -2 < 0$ Rel Min

$y = 50 - 25$

$\boxed{y = 25}$

13) $f(x) = \sqrt{x}$ so $y = \sqrt{x}$

points $(x, \sqrt{x})$ & $(4, 0)$

$$d = \sqrt{(x-4)^2 + (\sqrt{x} - 0)^2}$$

$$d = \sqrt{x^2 + 16 - 8x + x}$$

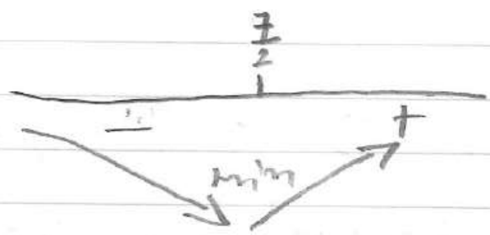
$$f(x) = (x^2 + 16 - 7x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(x^2 - 7x + 16)^{-\frac{1}{2}} \cdot (2x - 7)$$

$$f'(x) = \frac{(2x - 7)}{2\sqrt{x^2 - 7x + 16}}$$

$2x - 7 = 0$

$\boxed{x = \frac{7}{2}}$



$x = \frac{7}{2}$ then $y = \sqrt{\frac{7}{2}}$

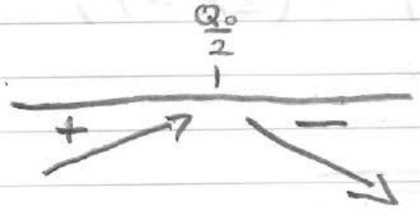
$\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$ closest to $(4,0)$

17) $\frac{dQ}{dx} = Kx(Q_0 - x)$

$$\frac{dQ}{dx} = KQ_0 - Kx^2$$

$$\frac{d''Q}{dx} = KQ_0 - 2Kx = K(Q_0 - 2x) = 0$$

$$Q_0 - 2x = 0 \rightarrow 2x = Q_0 \qquad \boxed{x = \frac{Q_0}{2}}$$



$X = \frac{Q_0}{2}$ is a Max

21) $S = 4(11 \cdot 3) + 2 \cdot (3 \cdot 3)$

a) $S = 132 + 18 = \boxed{150}$

b) $S = 4(5 \cdot 5) + 2 \cdot (5 \cdot 5)$

$S = 100 + 50 = \boxed{150}$

c) $S = 4(3.25 \cdot 6) + 2(6 \cdot 6)$

$S = 78 + 72 = \boxed{150}$

a) $V = 11 \cdot 3 \cdot 3 = \boxed{99}$

b) $V = 5 \cdot 5 \cdot 5 = \boxed{125}$

c) $V = 3.25 \cdot 6 \cdot 6 = \boxed{117}$

$S = 4xy + 2x^2 = 150$

$$y = \frac{150 - 2x^2}{4x} = \frac{75 - x^2}{2x}$$

$V = \pi \cdot \pi \cdot y$

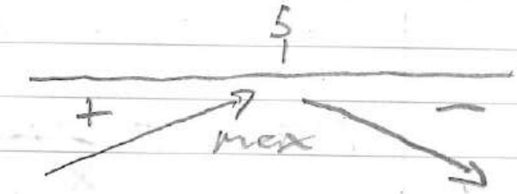
$$= x^2\left(\frac{75 - x^2}{2x}\right) = \frac{1}{2}\left(75x - x^3\right)$$

$$\frac{dV}{dx} = \frac{1}{2}\left(75 - 3x^2\right) = 0$$

$75 - 3x^2 = 0$

$3x^2 = 75$

$\boxed{x = 5}$



$x = 5$, $y = 5$ Volume will be max when dimensions are $5 \times 5 \times 5$

25) a) $\frac{y-2}{0-1} = \frac{0-2}{x-1}$

$$y = 2 + \frac{2}{x-1}$$

$$L = \sqrt{x^2 + y^2}$$

$$L = \sqrt{x^2 + \left(2 + \frac{2}{x-1}\right)^2}$$

$$L = \sqrt{x^2 + 4 + \frac{8}{x-1} + \frac{4}{(x-1)^3}}$$

e) $A = \frac{1}{2} b \cdot h$

$$A = \frac{1}{2} x \left(2 + \frac{2}{x-1}\right)$$

$$A = x + \frac{x}{x-1}$$

$$A' = 1 + \frac{(x-1) - x}{(x-1)^2}$$

2

1

min

$$A' = 1 - \frac{1}{(x-1)^2} = \frac{(x-1)^2 - 1}{(x-1)^2} = 0$$

$$(x-1)^2 = 1 \rightarrow x - 1 = \pm 1$$

$$x = 2 \quad \text{or} \quad x = 0$$

$y = 2 + 2$

$\boxed{y = 4}$ $(2,0), (0,4), (0,0)$

2a) $A = x \cdot y$

$A = (x+2)(y+2)$

$30 = xy$ $y = \frac{30}{x}$

$A = (x+2)\left(\frac{30}{x} + 2\right)$

$A = 30 + \frac{60}{x} + 2x + 4$

$A = \frac{60}{x} + 2x + 34$

$da \; -\frac{60}{x^2} + 2 = \frac{2x^2 - 60}{x^2}$

$\frac{2x^2 - 60}{x^2} = 0 \longrightarrow 2x^2 - 60 = 0$

$2x^2 = 60$

$\boxed{x = \sqrt{30}}$

$\sqrt{30}$

min

$y = \frac{30}{\sqrt{30}} = \frac{30\sqrt{30}}{30} = \boxed{y = \sqrt{30}}$

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Section 4.8

1) $f(x) = x^2$ $(2, 4)$
$f'(x) = 2x$

$y - f(2) = f'(2)(x-2)$
$y - 4 = 4(x-2)$
$y = 4x - 4$

x	1.9	1.99	2	2.01	2.1
$f(x) = x^2$	3.6100	3.9601	4	4.0401	4.4100
$T(x) = 4x-4$	3.600	3.9600	4	4.0400	4.4000

5) $f(x) = \sin x$ $(2, \sin 2)$
$f'(x) = \cos x$

$y - y_1 = f'(x)(x - x_1)$
$y - \sin 2 = \cos 2 (x-2)$
$y = \cos 2 (x-2) + \sin 2$

x	1.9	1.99	2	2.01	2.1
$f(x) = \sin x$	.9463	.9134	.9093	.9051	.8632
$T(x) = \cos 2(x-2) + \sin 2$	.9509	.9135	.9093	.9051	.8677

9) $y = x^4 + 1$ $x = -1$ $\Delta x = dx = 0.01$

$y = f(x)$
$f(x) = x^4 + 1$
$f'(x) = 4x^3$
$dy = f'(x)\,dx$
$dy = 4x^3 \cdot 0.01$
$dy = 4(-1)^3 \cdot 0.01$

$\Delta y = f(x + \Delta x) - f(x)$
$\Delta y = f(-1 + 0.01) - f(-1)$
$\Delta y = f(-.99) - f(-1)$
$\Delta y = 0.03940 - 0$
$\boxed{\Delta y = .03940}$

$\boxed{dy = -.04}$

13) $y = \frac{x+1}{2x-1}$ find dy

$dy = f'(x)\,dx$

$dy = \frac{1(2x-1) - (x+1)(2)}{(2x-1)^2}(dx)$

$dy = \frac{2x-1-2x-2}{(2x-1)^2}(dx)$

$$\boxed{dy = \frac{-3}{(2x-1)^2}\,dx}$$

17) $y = \ln\sqrt{4-x^2}$

$dy = f'(x)\,dx$

$dy = \ln(4-x^2)^{\frac{1}{2}}\,dx$

$dy = \frac{1}{2}\ln(4-x^2)\,dx$

$dy = \frac{1}{2}\cdot\frac{-2x}{4-x^2}\,dx$

$$\boxed{dy = \frac{-x}{2-x^2}\,dx}$$

21) $y = \frac{1}{3}\cos\left(\frac{6\pi x - 1}{2}\right)$

$dy = f'(x)\,dx$

$dy = \frac{1}{3}\left[-\sin\left(\frac{6\pi x - 1}{2}\right)\left(\frac{6\pi}{2}\right)\right]$

$dy = \frac{1}{3}\left(\frac{6\pi}{2}\right)\left[-\sin\left(\frac{6\pi x - 1}{2}\right)\right]$

$\boxed{dy = -\pi \sin\left(\frac{6\pi x - 1}{2}\right)}$

25) a) $f(1.9)$ b) $f(2.04)$

$f(x) = \sqrt{x}$ $dx = 0.9$, $x = 1$

$f(x + \Delta x) \approx f(x) + f'(x) \cdot dx$

$= \sqrt{x} + \frac{1}{2x^{1/2}} \cdot dx = \sqrt{1} + \frac{1}{2\sqrt{1}} \cdot (0.9)$

$= 1 + \frac{1}{2} \cdot (0.9) = 1 + \frac{9}{20} = \boxed{1.45}$

B) $f(x + \Delta x) \approx f(x) + f'(x) \cdot dx$

$= f(2) + f'(2) \cdot (0.04) = 1 + 1(0.04)$

$= \boxed{1.04}$

29) a) $g(2.93) = g(3-0.07)$

$\approx g(3) + g'(3)(-0.07) = 8 + \left(-\frac{1}{2}\right)(-0.07)$

$= \boxed{8.035}$

B) $g(3.1) = g(3+0.1)$

$\approx g(3) + g'(3)(0.1)$

$\approx 8 + \left(-\frac{1}{2}\right)(0.1) = \boxed{7.95}$

31) $A = x^2 \qquad x = 12 \qquad dx = \pm \frac{1}{64}$

$dA = 2x\,dx = 2(12)\left(\pm\frac{1}{64}\right)$

$= \boxed{\pm \frac{3}{8}}$

Steven Romeiro

50 pt

Study Guide 4

1) $\int 5\sin t + 2\cos t \, dt$

$\int 5\sin t + 2\int \cos t \, dt$

$\boxed{= -5\cos t + 2\sin t + C}$

2) $\int \frac{6x^2+8x-15}{x^4} dx$

$= \int \frac{6x^2}{x^4} + \int \frac{8x}{x^4} - \int \frac{15}{x^4} dx$

$= \int \frac{6}{x^2} + \int \frac{8}{x^3} - \int \frac{15}{x^4}$

$= 6\int x^{-2} + 8\int x^{-3} - 15\int x^{-4}$

$= 6\left(\frac{1}{-x}\right) + 8\left(\frac{-2}{x^2}\right) - 15\left(\frac{-3}{x^3}\right)$

$= \boxed{-\frac{6}{x} - \frac{16}{x^2} + \frac{45}{x^3} + C}$

3) $\frac{dG}{dZ} = -20z^4 - 5$ $\quad G(2) = -3$

$\int -20z^4 - 5\, dx = -4G^5 - 5G + C$

$-4(2)^5 - 5(2) + C = -3$

$-128 - 10 + C = -3$ $\quad \boxed{G(2) = -4G^5 - 5G + 135}$

$C = -3 + 138$

$\boxed{C = 135}$

4) $S_0 = 8ft$ $\quad V_0 = 60$ $\quad$ Gravity $= a(t) = -32$

$S'(t) = \int a(t) = \int -32 \rightarrow -32t + C$

$S'(t) = V(t)$ $\quad V(0) = -32(0) + C = \boxed{C = 60}$

$S'(t) = -32t + 60$

$S(t) = \int S'(t) = \int -32t + 60 = -16t^2 + 60t + C$

$S(0) = -16(0)^2 + 60(0) + C = C = 8$

$S(t) = -16t^2 + 60t + 8$

$-32t + 60 = 0 \longrightarrow \boxed{t = \frac{15}{8}}$

$S(\frac{15}{8}) = -16(\frac{15}{8})^2 + 60(\frac{15}{8}) + 8 = \boxed{S(\frac{15}{8}) = 64.25}$

5) $\sum_{j=2}^{8} (2^j - j)$

$= (2^2-2)+(2^3-3)+(2^4-4)+(2^5-5)+(2^6-6)+(2^7-7)+(2^8-8)$

$= \boxed{473}$

Sum(Seq(2^X - X, X, 2, 8, 1))

6) $\sum_{i=1}^{32} (i-6)^2 = \sum_{i=1}^{32} (i^2 - 12i + 36)$

$= \sum_{i=1}^{32} i^2 - 12\sum_{i=1}^{32} i + \sum_{i=1}^{32} 36$

$= \frac{2n^3+3n^2+n}{6} + \frac{n^2+n}{2} + 36n$

$= \frac{2(32)^3+3(32)^2+32}{6} + 12\left(\frac{32^2+32}{2}\right) + 36(32)$

$= \boxed{6256}$

7) $\displaystyle\sum_{i=4}^{30}(4i^2+8) = 4\sum_{i=4}^{30} i^2 + \sum_{i=4}^{30} 8$

$$= 4 \cdot \frac{2n^3+3n^2+n}{6} + 8n$$

$$= 4 \cdot \frac{2(30)^3+3(30)^2+30}{6} + 8(30)$$

$$= 38060 - \sum_{i=1}^{3}(4i^2+8)$$

$$= 38060 - \left[4 \cdot \frac{2(3)^3+3(3)^2+3}{6} + 8(3)\right]$$

$$38060 - 80$$

$$= \boxed{37980}$$

$8)\ f(x) = \sqrt{x} + 1 \qquad [0, 1]$

$\Delta x = \frac{1}{4}$

$$S = \Delta x\left[(\sqrt{.25}+1) + (\sqrt{.5}+1) + (\sqrt{.75}+1) + (\sqrt{1}+1)\right]$$

$$S = (1/4)\cdot\left[\frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + 2\right]$$

$$\boxed{S = 1.768}$$

$$S = \Delta x\left[(\sqrt{0}+1) + (\sqrt{.25}+1) + (\sqrt{.5}+1) + (\sqrt{.75}+1)\right]$$

$$S = (1/4)\cdot\left[1 + \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right]$$

$$\boxed{S = 1.518}$$

9) $\displaystyle\int_4^{11} -6s^2\,ds = \left[-2s^3\right]\Big|_4^{11}$

$\left[-2(11)^3\right] - \left[-2(4)^3\right] = \boxed{-2534}$

10) $\displaystyle\int_{-4}^{-2} (-9z^2+2)\,dz = \left[-3z^3 + 2z\right]\Big|_{-4}^{-2}$

$= \left[-3(-2)^3 + 2(-2)\right] - \left[-3(-4)^3 + 2(-4)\right]$

$= \boxed{-164}$

11) $\displaystyle\lim_{\|\Delta x\| \to 0} \sum_{i=1}^{n} -5c_i\,(5 + c_i^{-1})\,\Delta x_i$ $\quad [4, 8]$

$a = 4 \quad b = 8$

$\displaystyle\int_4^8 -5x\,(5 + x^{-1})\,dx$

$= \displaystyle\int_4^8 -25x - 5\,dx \longrightarrow \left[-\frac{25}{2}x^2 - 5x\right]\Big|_4^8$

$= \left[-\frac{25}{2}(8)^2 - 5(8)\right] - \left[-\frac{25}{2}(4)^2 - 5(4)\right] = \boxed{-620}$

12) $\int_0^{\pi} 6e^x + 2\sin x \, dx$

$6\int_0^{\pi} e^x + 2\int_0^{\pi} \sin x \, dx$

$= \left[6e^x - 2\cos x\right]\Big|_0^{\pi}$

$= \left[6e^{\pi} - 2\cos \pi\right] - \left[6e^0 - 2\cos 0\right]$

$= 6e^{\pi} + 2 - 6 + 2 = \boxed{6e^{\pi} - 2}$

13) $\int_0^{6\pi} (6u + 5\cos u)\, du = 6\int_0^{6\pi} u \, du + 5\int_0^{6\pi} \cos u \, du$

$= \left[3u^2 + 5\sin u\right]\Big|_0^{6\pi}$

$= \left[3(6\pi)^2 + 5\sin(6\pi)\right] - \left[3(0)^2 + 5\sin(0)\right]$

$= 108\pi^2 + 0 - 0 = \boxed{108\pi^2}$

14) $\int_0^{3\pi} (-4\sin x + 2\cos x)\,dx$

$= \left[4\cos x + 2\sin x\right]\Big|_0^{3\pi}$

$= \left[4\cos 3\pi + 2\sin 3\pi\right] - \left[4\cos 0 + 2\sin 0\right]$

$= -4 + 0 - 4 + 0 = \boxed{-8}$

15) $y = x(1-x)$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x \qquad [0,1]$

$a=0 \quad b=1$

$\Delta x = \frac{b-a}{n} = \boxed{\frac{1}{n}}$

$c_i = a + \Delta x i = 0 + \frac{1}{n} i = \boxed{\frac{1}{n} i}$

$f(c_i)\Delta x = \left[\frac{1}{n} i \left(1 - \frac{1}{n} i\right)\right]\left(\frac{1}{n}\right)$

$= \left[\frac{1}{n} i \left(\frac{n-i}{n}\right)\right]\left(\frac{1}{n}\right) = \left(\frac{in - i^2}{n^2}\right)\left(\frac{1}{n}\right) = \boxed{\frac{ni - i^2}{n^3}}$

$$\sum_{i=1}^{n} f(c_i)\Delta x = \sum_{i=1}^{n} \frac{ni-i^2}{n^3}$$

$$\frac{1}{n^2}\sum_{i=1}^{n} i - \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{1}{n^2}\left(\frac{n^2+n}{2}\right) - \frac{1}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right) = \boxed{\frac{1}{2}+\frac{1}{2n}-\frac{1}{3}-\frac{1}{2n}-\frac{1}{n^2}}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x = \lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2n}-\frac{1}{3}-\frac{1}{2n}-\frac{1}{n^2}\right.$$

$$\frac{1}{2}-\frac{1}{3} = \boxed{\frac{1}{6}}$$

16) $y = \frac{1}{2}e^{x}$ $[0, 1]$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$ $a=0,\ b=1$

$\Delta x = \frac{b-a}{n} = \boxed{\frac{1}{n}}$

$c_i = a + \Delta x i = 0 + \frac{1}{n}i = \boxed{\frac{1}{n}i}$

$f(c_i) = \frac{1}{2}e^{\frac{1}{n}i} = \boxed{\frac{1}{2}e^{\frac{1}{n}i}}$

$f(c_i)\Delta x = \frac{1}{2}e^{\frac{1}{n}i}\left(\frac{1}{n}\right) = \boxed{\frac{1}{2n}e^{\frac{1}{n}i}}$

$\sum_{i=1}^{n} f(c_i)\Delta x = \sum_{i=1}^{n} \frac{1}{2n}e^{\frac{1}{n}i} = \frac{1}{2n}\sum_{i=1}^{n} e^{\frac{1}{n}i}$

$= \frac{1}{2n}\sum_{i=1}^{n} e^{\frac{1}{n}\left(\frac{n^2+n}{2}\right)} = \frac{1}{2n}\sum_{i=1}^{n} e^{\frac{n+1}{2}}$

?

17) $f(t) = 36 - 9t^2$ $\qquad [-5, 5]$

$$\text{Average} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{5+5}\int_{-5}^{5}(36-9t^2)\,dt = \frac{1}{10}\int_{-5}^{5}(36-9t^2)\,dt$$

$$= \frac{1}{10}\left[36t - 3t^3\right]\Big|_{-5}^{5}$$

$$= \frac{1}{10}\left[\left(36(5)-3(5)^3\right) - \left(36(-5)-3(-5)^3\right)\right]$$

$$= \frac{1}{10}\left[-195 - 195\right] = \boxed{-39}$$

$u = -4x$
$du = -4dx$

$$-\frac{1}{4}\int e^{-4x}\cdot(-4dx) = -\frac{1}{4}\int e^{u}du = -\frac{1}{4}e^{u} + C$$

18) $$\int e^{-4x}dx \longrightarrow \boxed{-\frac{1}{4}e^{-4x} + C}$$

19) $$\int 5xe^{-4x^2}dx$$

$u = -4x^2$
$du = -8x\,dx$

$$\int e^{-4x^2}\cdot 5x\,dx \longrightarrow -\frac{5}{8}\int e^{-4x^2}\cdot -8x\,dx$$

$$-\frac{5}{8}\int e^{u}du \longrightarrow -\frac{5}{8}e^{u} \longrightarrow \boxed{-\frac{5}{8}e^{-4x^2} + C}$$

20) $$\int \cos 6x \cdot e^{\sin 6x}dx$$

$u = \sin 6x$
$du = 6\cos 6x\,dx$

$$\frac{1}{6}\int e^{\sin 6x}\cdot(6\cos 6x)dx \longrightarrow \frac{1}{6}\int e^{u}du$$

$$\frac{1}{6}e^{u} + C \longrightarrow \boxed{\frac{1}{6}e^{\sin 6x} + C}$$

21) $\int 3^{7x}\,dx$ $\qquad U = 7x$, $du = 7\,dx$

$\frac{1}{7}\int 3^{7x}\cdot(7\,dx) \longrightarrow \frac{1}{7}\int 3^{u}\,du$

$= \left(\frac{1}{7}\right)\left(\frac{1}{\ln 3}\right)\cdot 3^{u} + C = \boxed{\left(\frac{1}{7\ln 3}\right)3^{7x} + C}$

22) $\int_{0}^{4\sqrt{\pi}} z\sin(7z^2)\,dz$ $\qquad U = 7z^2$, $du = 14z\,dz$

$\frac{1}{14}\int_{0}^{4\sqrt{\pi}} \sin(7z^2)\cdot(14z)\,dz = \frac{1}{14}\int_{0}^{112\pi}\sin u\,du$

$U = 7(0)^2 = 0$ / $U = 7(4\sqrt{\pi})^2 = 7(16\pi) = 112\pi$

$\frac{1}{14}\left[(-\cos u)\right]\Big|_{0}^{112\pi} \rightarrow \frac{1}{14}\left\{[-\cos 112\pi] - [-\cos 0]\right\}$

$\frac{1}{14}(-1+1) = \frac{1}{14}(0) = \boxed{0}$

23) $\int_3^6 \sqrt[5]{u}\,du \longrightarrow \int_3^6 u^{\frac{1}{5}}\,du$

$n = 16$
$a = 3 \quad b = 6$

Trap Rule

$\Delta x = \frac{b-a}{n} = \frac{6-3}{16} = \boxed{\frac{3}{16}}$

$x_0 = 3$
$x_1 = \frac{51}{16}$
$x_2 = \frac{27}{8}$
$x_3 = \frac{57}{16}$
$x_4 = 15/4$
$x_5 = 63/16$
$x_6 = 33/8$
$x_7 = 69/16$
$x_8 = 9/2$
$x_9 = 75/16$
$x_{10} = 39/8$
$x_{11} = 81/16$
$x_{12} = 21/4$
$x_{13} = 87/16$
$x_{14} = 45/8$
$x_{15} = 93/16$
$x_{16} = 6$

$$\int_3^6 u^{\frac{1}{5}}du \approx \frac{3}{2(16)}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + 2f(x_5) + 2f(x_6) + 2f(x_7) + 2f(x_8) + 2f(x_9) + 2f(x_{10}) + 2f(x_{11}) + 2F(x_{12}) + 2f(x_{13}) + 2f(x_{14}) + 2f(x_{15}) + f(x_{16})\Big]$$

$$\frac{3}{32}\Big[1.245731 + 2.521855 + 2.550849 + 2.578582 + 2.605171 + 2.630717 + 2.655307 + 2.679019 + 2.70192 + 2.72407 + 2.745522 + 2.766324 + 2.786518 + 2.806143 + 2.825235 + 2.843823 + 1.430969\Big] = \boxed{4.040125}$$

Simpson Rule

$$\frac{3}{48}\Big[1.245731 + 5.043709 + 2.550849 + 5.157164 + 2.605171 + 5.261434 + 2.655307 + 5.358038 + 2.70192 + 5.44814 + 2.745522 + 5.532647 + 2.786518 + 5.612287 + 2.82323 + 5.687676 + 1.430969\Big] = \boxed{4.040554}$$

24) $\int_1^4 \frac{1}{\sqrt{5+z^4}}\,dz \longrightarrow \int_1^4 (5+z^4)^{-\frac{1}{2}}\,dz$ $\quad n=4$

$a=1 \quad b=4$

$\Delta x = \frac{b-a}{n} = \boxed{\frac{3}{4}} \quad \boxed{x_0 = 1} \quad x_1 = 1 + \frac{3}{4} = \boxed{\frac{7}{4}}$

$x_2 = \frac{7}{4} + \frac{3}{4} = \boxed{\frac{5}{2}} \quad x_3 = \frac{5}{2} + \frac{3}{4} = \boxed{\frac{13}{4}} \quad x_4 = \frac{13}{4} + \frac{3}{4} = \boxed{4}$

Trap Rule $\int_1^4 (5+z^4)^{-\frac{1}{2}}\,dz \approx \frac{3}{8}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$\frac{3}{8}\left[.40825 + 2(.26372) + 2(.15065) + 2(.09262) + .0619\right]$

$= \boxed{.556549}$

Simpson Rule $\int_1^4 (5+z^4)^{-\frac{1}{2}} \approx \frac{3}{12}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$

$\frac{3}{12}\left[.40825 + 4(.26372) + 2(.15065) + 4(.09262) + .0619\right]$

$\approx \boxed{.549203}$

$X_0 = 4$
$X_1 = 67/16$
$X_2 = 35/8$
$X_3 = 73/16$
$X_4 = 19/4$
$X_5 = 79/16$
$X_6 = 41/8$
$X_7 = 85/16$
$X_8 = 11/2$
$X_9 = 91/16$
$X_{10} = 47/8$
$X_{11} = 97/16$
$X_{12} = 25/4$
$X_{13} = 103/16$
$X_{14} = 53/8$
$X_{15} = 109/16$
$X_{16} = 7$

25) $\int_4^7 \frac{1}{\sqrt{u}\sqrt{9-u}}\,du$

$n = 16$

$a = 4 \quad b = 7$

$\Delta X = \frac{b-a}{n} = \frac{3}{16}$

Trap

$= \frac{3}{32}\left[.223607 + .44552 + .444616 + .444487 + .445132 + .44656 + .448794 + .451871 + .451842 + .42391 + .466768 + .473931 + .482418 + .492424 + .504203 + .518087 + .267261\right]$

$\approx \boxed{.7005}$

Simpson

$\frac{3}{48}\left[.223607 + .89104 + .444616 + .888975 + .445132 + .89312 + .448794 + .903742 + .455842 + .921558 + .466768 + .947862 + .482418 + .984849 + .504203 + 1.036174 + .26726\right]$

$\boxed{\approx .7804}$

26) $\int_2^4 (2x+3)\,dx$ $\quad n=10 \quad a=2 \quad b=4$

$f'(x)=2$

$f''(x)=0$

$f^{IV}=0$

Trap = $E \leq \frac{(b-a)^3}{12n^2}\left[\max|F''(x)|\right]$

$\frac{(2)^3}{1200}\left[\max|0)\right] = \boxed{.0000}$

Simpson = $E \leq \frac{(b-a)^5}{180n^4}\left[\max|F^{IV}(x)|\right]$

$= \frac{2^5}{180(10)^5}\left[0\right] = \boxed{0.000}$

27) $\int \frac{5\sec^2 2x}{\tan 2x+9}\,dx$

$u = \tan 2x + 9$

$du = 2\sec^2 2x\,dx$

$\frac{5}{2}\int \frac{1}{\tan 2x+9}\cdot(2\sec^2 2x\,dx)$

$= \frac{5}{2}\int \frac{1}{u}\,du$

$= \frac{5}{2}\ln u + C \longrightarrow \boxed{\frac{5}{2}\ln(\tan 2x+9)+C}$

28) $\int \frac{\sin x}{2\cos x+3}\,dx$ $\qquad U = 2\cos x + 3$

$\qquad\qquad\qquad\qquad du = -2\sin x\,dx$

$-\frac{1}{2}\int \frac{1}{2\cos x+3}\cdot(-2\sin x)\,dx$

$= -\frac{1}{2}\int \frac{1}{U}\,du = -\frac{1}{2}\ln(U) + C$

$\boxed{= -\frac{1}{2}\ln(2\cos x+3) + C}$

29) $\int_1^4 \left(\frac{7}{x} - \frac{3}{x^2}\right)dx$

$= 7\int_1^4 \frac{1}{x}\,dx - 3\int_1^4 \frac{1}{x^2}\,dx = 7\int_1^4 \frac{1}{x}\,dx - 3\int_1^4 x^{-2}\,dx$

$= \left[7\ln x + \frac{3}{x}\right]\Big|_1^4 = \left[7\ln|4| + \frac{3}{4}\right] - \left[7\ln 1 + \frac{3}{1}\right]$

$= 7\ln|4| + \frac{3}{4} - 3 = \boxed{-\frac{9}{4} + 7\ln|4|}$

30) $\int \frac{e^{-6x}}{\sqrt{1-e^{-12x}}}\,dx$ $\qquad U = e^{-6x}$

$du = -6e^{-6x}\,dx$

$\int \frac{e^{-6x}}{\sqrt{1^2-(e^{-6x})^2}} = -\frac{1}{6}\int \frac{-6e^{-6x}}{\sqrt{1^2-(e^{-6x})^2}}\,dx$

$-\frac{1}{6}\int \frac{du}{\sqrt{1^2-(u)^2}} = \boxed{-\frac{1}{6}\sin^{-1}e^{-6x}+C}$

31) $\frac{1}{11}\int \frac{1(11)}{\sqrt{7^2-(11x)^2}}$ $\qquad U = 11x$

$du = 11\,dx$

$= \boxed{\frac{1}{11}\sin^{-1}\frac{11x}{7}+C}$

32) $\frac{1}{10}\int \frac{1(10)}{x\sqrt{(10x)^2-7^2}}\,dx$ $\qquad U = 10x$

$du = 10\,dx$

$\frac{1}{10}\cdot\frac{1}{7}\sec^{-1}\frac{10x}{7}+C \Rightarrow \boxed{\frac{1}{70}\sec^{-1}\frac{10x}{7}+C}$

33) $\frac{1}{7}\int \frac{(7)e^{7x}}{1+(e^{7x})^2}$ $\quad U = e^{7x}$

$du = 7e^{7x}\,dx$

$\frac{1}{7}\left(\frac{1}{1}\operatorname{Tan}^{-1}\frac{e^{7x}}{1}\right) + C = \boxed{\frac{1}{7}\operatorname{Tan}^{-1}e^{7x} + C}$

34) $\cosh(\ln(4))$ $\quad \operatorname{Tanh}(\ln(8))$

$\cosh = \frac{e^{\ln 4} + e^{-\ln 4}}{2} = \boxed{\frac{17}{6} \text{ or } 2.125}$

$\operatorname{Tanh} = \frac{\frac{e^{\ln 8} - e^{-\ln 8}}{2}}{\frac{e^{\ln 8} + e^{-\ln 8}}{2}} = \boxed{\frac{63}{65} \text{ or } .969231}$

35) $\sinh(\ln(\frac{1}{4})) = \frac{e^{\ln\frac{1}{4}} - e^{-\ln\frac{1}{4}}}{2} = \boxed{\frac{-15}{8} \text{ or } -1.875}$

$\cosh(\ln(\frac{1}{4})) = \frac{e^{\ln\frac{1}{4}} + e^{-\ln\frac{1}{4}}}{2} = \boxed{\frac{17}{8} \text{ or } 2.125}$

36) $y = \operatorname{Sech}(3x+7)$

$$f'(x) = 3\left|-(\operatorname{Sech}(3x+7)\operatorname{Tan}(3x+7))\right|$$

$$= \boxed{-3\operatorname{Sech}(3x+7)\operatorname{Tan}(3x+7)}$$

37) $y = \cosh(10x)$

$$f'(x) = 10(-\operatorname{Sec}^2 h(10x))$$

$$= \boxed{-10\operatorname{Sech}^2(10x)}$$

38) $y = \frac{1}{16}\sinh(8x) - 10\coth\left(\frac{x}{5}\right) + \frac{x}{16}$

$$F'(t) = \frac{8}{16}\cosh(8x) - 10\left(\frac{1}{5}\right) - \operatorname{Sech}^2\left(\frac{x}{5}\right) + \frac{1}{16}$$

$$= \boxed{\frac{1}{2}\cosh(8x) + 2\operatorname{Sech}^2\left(\frac{x}{5}\right) + \frac{1}{16}}$$

39) $\int x^3 \operatorname{csch}^2\left(\frac{x^4}{4}\right) dx$ $\qquad U = \frac{1}{4}x^4$

$du = x^3 dx$

$\boxed{-\coth\left(\frac{x^4}{4}\right) + C}$

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Section 5.1

1) $\int \left(-\frac{9}{x^4}\right) dx = \frac{3}{x^3} + C$

$= 3x^{-3} + C$

$= 9x^{-4} + 0$

$= \boxed{\frac{-9}{x^4}}$

5) $\frac{dy}{dt} = 3t^2$

$y = \frac{3}{2+1} t^{(2+1)} + C = y = \frac{3}{3} t^3 + C = \boxed{y = t^3 + C}$

$y' \frac{d}{dt} = t^3 = \boxed{\frac{dy}{dt} = 3t^2}$

9) $\int \sqrt[3]{x}\, dx$

Rew- $\int x^{\frac{1}{3}}\, dx$

Integ- $\frac{1}{\frac{1}{3}+1} x^{\left(\frac{1}{3}+1\right)} + C$

Simp- $\frac{1}{\frac{4}{3}} x^{\left(4/3\right)} = \boxed{\frac{3}{4} x^{4/3} + C}$

13) $\int \frac{1}{2x^3}\,dx$

Rew $- \ \frac{1}{2} x^{-3}\,dx$

Integ $- \ \frac{1}{2}\left(\frac{1}{-3+1} x^{(-3+1)}\right) + c$

Simp $- \ \frac{1}{2}\left(-\frac{1}{2} x^{-2}\right) + c = \boxed{\frac{-1}{4x^2} + C}$

17) $\int (x^3+5)\,dx$

$= \frac{1}{3+1} x^{(3+1)} + 5x + c$

$= \boxed{\frac{1}{4}x^4 + 5x + C}$

$\boxed{\frac{d}{dx} = x^3 + 5}$ ✓

21) $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx$

$= \frac{1}{-3+1} x^{(-3+1)} + C$

$= -\frac{1}{2} x^{-2} + C = \boxed{-\frac{1}{2x^2} + C}$

$\frac{d}{dx} = \frac{-1}{-2} x^{-2} + C = x^{-3} + 0 = \boxed{\frac{1}{x^3}}$

25) $\int (x+1)(3x-2)\,dx = \int 3x^2 - 2x + 3x - 2$

$\int 3x^2 + x - 2 = \frac{3}{2+1} x^{2+1} + \frac{1}{1+1} x^{1+1} - 2x + C$

$= \frac{3}{3} x^3 + \frac{1}{2} x^2 - 2x + C = \boxed{x^3 + \frac{1}{2} x^2 - 2x + C}$

$\boxed{\frac{d}{dx} = 3x^2 + x - 2}$ ✓

29) $\int dx = \int 1 \cdot dx$

$= \boxed{x + c}$ $\quad \frac{d}{dx}(1+0) = \boxed{1}$ ✓

33) $\int (1 - \csc t \cot t)\, dt$

$= \boxed{1t + \csc t + c} \rightarrow \frac{d}{dt}(t + \csc t + c)$

$= \boxed{1 - \csc t \operatorname{Tan} t}$ ✓

37) $\int (\sec^2\theta - \sin\theta)\, d\theta$

$= \boxed{\operatorname{Tan}\theta + \cos\theta + c} \rightarrow \frac{d}{d\theta}(\operatorname{Tan}\theta + \cos\theta + c)$

$= \boxed{\sec^2\theta - \sin\theta}$ ✓

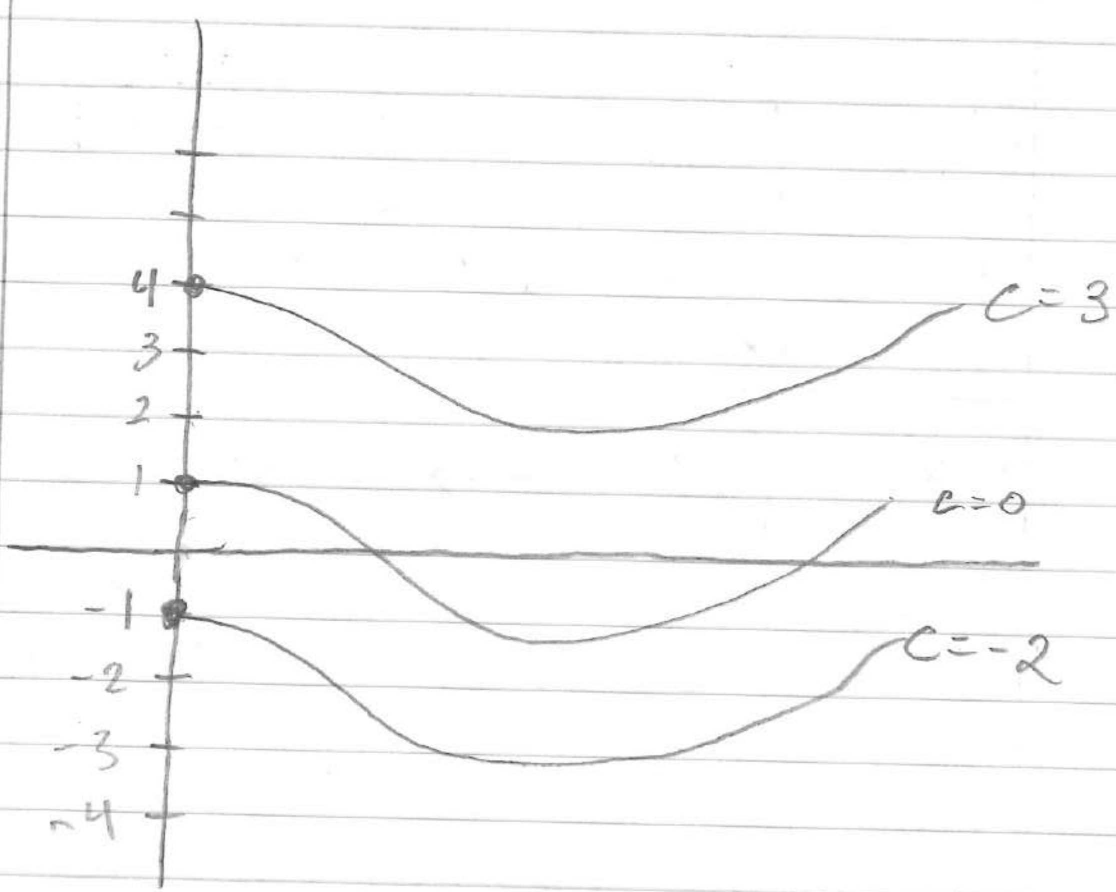
41) $\int (2x - 4^x)\,dx$

$= \frac{2}{1+1}x^{1+1} - \left(\frac{1}{\ln 4}\right)4^x + C$

$= \boxed{x^2 - \frac{4^x}{\ln 4} + C} = 2x - \frac{1 \cdot \cancel{(\ln 4)} \cdot 4^x}{\cancel{\ln 4}} + 0$

$= \boxed{2x - 4^x}$ ✓

45) $g(x) = f(x) + C$, $C = -2$, $C = 0$
$f(x) = \cos x$ & $C = 3$



63) $f'(x) = 4x$ $\qquad f(0) = 6$

$f(x) = \int 4x\,dx$

$= \frac{4}{1+1}x^{1+1} + c = \frac{4}{2}x^2 + c = \boxed{2x^2 + c}$

$f(x) = 2x^2 + c, \qquad f(0) = 2(0)^2 + c = 6$

$\boxed{c = 6}$

$\boxed{f(x) = 2x^2 + 6}$

67) $f''(x) = 2, \qquad f'(2) = 5$

$f(2) = 10$

$f'(x) = \int 2\,dx = \boxed{f'(x) = 2x + c}$

$f'(2) = 2(2) + c = 5 \longrightarrow 4 + c = 5$

$\boxed{f'(x) = 2x + 1} \qquad \boxed{c = 1}$

$f(x) = \int 2x + 1 \longrightarrow \frac{2}{1+1}x^2 + 1x + c$

$\boxed{f(x) = x^2 + x + c} \longrightarrow f(2) = (2)^2 + 2 + c = 10$

$c = 10 - 6 \longrightarrow \boxed{c = 4}$

$\boxed{f(x) = x^2 + x + 4}$

77) $a(t) = -32$ $\quad S_0 = 6, \ V_0 = 60$

$V(t) = \int -32\,dt \longrightarrow V(t) = -32t + C$

$V(0) = -32(0) + C = 60$

$\boxed{C = 60}$

$S(t) = \int (-32t + 60)\,dt$ $\quad \boxed{V(t) = -32t + 60}$

$S(t) = -16t^2 + 60t + C$

$S(0) = -16(0)^2 + 60(0) + C = 6$

$\boxed{C = 6} \longrightarrow \boxed{S(t) = -16t^2 + 60t + 6}$

Max height $= S'(t) = V(t) = 0$

$V(t) = -32t + 60 = 0 \longrightarrow -32t = -60$

$\boxed{t = \frac{15}{8}}$ seconds

Position of ball at $\frac{15}{8}$ seconds is

$S\left(\frac{15}{8}\right) = -16\left(\frac{15}{8}\right)^2 + 60\left(\frac{15}{8}\right) + 6$

$\boxed{S\left(\frac{15}{8}\right) = \frac{294}{4}}$ feet

81) $a(t) = -9.8$

$\boxed{F(t) = -4.9t^2 + V_0 t + S_0}$ ✓

$a(t) = v'(t) = -9.8$

$v(t) = \int -9.8\,dt \longrightarrow \boxed{-9.8t + c_1}$

$c_1 = v(0) = V_0$

$v(t) = S'(t) = -9.8t + V_0 \quad \boxed{v(t) = -9.8t + V_0}$

$S(t) = \int -9.8t + V_0 \longrightarrow \frac{-9.8}{2}t^2 + V_0 t + c_2$

$c_2 = S(0) = S_0$

$\boxed{S(t) = -4.9t^2 + V_0 t + S_0}$ //

85) $a(t) = -1.6$

$a(t) = v'(t) = -1.6$

$v(t) = \int -1.6\,dt \longrightarrow -1.6t + C_1$

$V(0) = -1.6(20) + C_1 = 0$

$C = 0$

$\boxed{V(t) = -1.6t}$

?

How I thought $V(t) = -1.6 + 32$

$V(20) = 0$

$S(20) = 0$

$S'(t) = v(t) = -1.6t$

$S(t) = \int -1.6t \longrightarrow -0.8t^2 + C_2$

$S(20) = -0.8(20)^2 + C_2 = 0$

$C_2 = 320 = \boxed{S_0 = 320}$

$\boxed{S(t) = -0.8t^2 + 320}$

$V(20) = 0$

$V(20) = -1.6(20) = 0$

$\boxed{V(20) = -32}$

*car at rest

Truck

$V_0 = 30$

93) $a(t) = 6$

$V(0) = 0 \quad \& \; S(0) = 0$

$V(t) = \int 6\,dt \longrightarrow V(t) = 6t + C_1$

$V(0) = 6(0) + C_1 = 0$

$\boxed{V(t) = 6t}$

$S(t) = \int 6t\,dt \longrightarrow S(t) = 3t^2 + C_2$

$S(0) = 3(0) + C_2 = 0$

$\boxed{S(t) = 3t^2}$ car

* Truck constant

$S(t) = \int 30\,dt \longrightarrow S(t) = 30t + C_1$

when $t = 0,\ S(t) = 0$

$S(0) = 30(0) + C_1 = 0$

$\boxed{S(t) = 30t}$ truck

* Both vehicles

meet at $S_{car}(t) = S_{truck}(t)$

$3t^2 = 30t \longrightarrow \frac{3t^2}{t} = \frac{30t}{t} \longrightarrow 3t = 30$

$= t = \frac{30}{3} \longrightarrow \boxed{t = 10\text{ sec}}$

How fast when pass? $V(10) = 6(10)$

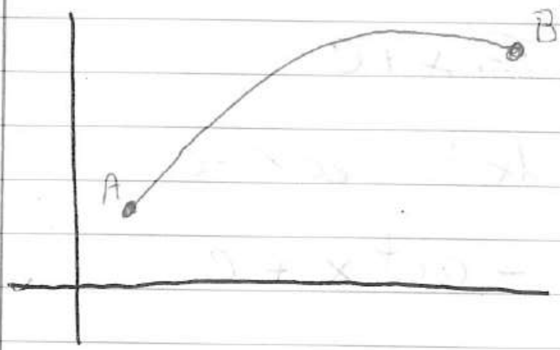
$\boxed{V(10) = 60 \text{ feet per sec}}$

? distance at $t = 10 \longrightarrow S_{car}(10) = S_{truck}(10)$?

$30 \cdot 10 \longrightarrow \boxed{300 \text{ feet}}$

5.1 Antiderivatives & Indefinite integrations

Area under the curve (integration of fxn)



$\int$ = big "S" that means integration or area under the fxn

$F(x) + C$ = The full derivative

$$\int f(x)\,dx = F(x) + C$$

1) $\int K\,dx = Kx + C$
2) $\int ax\,dx = \frac{a}{2}x^2 + C$
3) $\int ax^n\,dx = \frac{a}{(n+1)}x^{(n+1)} + C \quad n \neq -1$
4) $\int \frac{1}{x}\,dx = \ln x + C$

5) $\int \cos x \, dx = \sin x + C$

6) $\int \sin x \, dx = -\cos x + C$

7) $\int \sec^2 x \, dx = \tan x + C$

8) $\int \sec x \tan x \, dx = \sec x + C$

9) $\int \csc^2 x \, dx = -\cot x + C$

10) $\int \csc x \cot x = -\csc x + C$

11) $\int e^x \, dx = e^x + C$

12) $\int a^x \, dx = \left(\frac{1}{\ln a}\right) a^x + C$

13) $\int k \cdot f(x) \, dx = k \int f(x) \, dx \longrightarrow$ just factor out the constant

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5.1 Examples

Take derive of this to find the integral

2) $\int \left(4x^3 - \frac{1}{x^2}\right) dx = \underbrace{x^4 + \frac{1}{x} + c}$

$= 4x^3 - x^{-2} + 0$

$\boxed{= 4x^3 - \frac{1}{x^2}}$

8) $\frac{dy}{dx} = 2x^{-3}$ → find antiderive and you'll get y

$y = \frac{2}{(-3+1)} x^{(-3+1)} + c$

$y = \frac{2}{-2} x^{-2} + c = x^{-2} + c$

$\boxed{y = -\frac{1}{x^2} + c}$

12) $\int x(x^2+3)\, dx$

Rewrite $= \int x^3 + 3x\, dx$

Integrate $=$ #3 $\left(\frac{1}{(3+1)} x^{(3+1)} + \frac{3}{(1+1)} x^{(1+1)} + c\right)$

Simplify $= \boxed{\frac{1}{4}x^4 + \frac{3}{2}x^2 + c}$

14) $\int \frac{1}{(3x)^2}\,dx$

Rew $= \int \frac{1}{9x^2}\,dx = \int \frac{1}{9}x^{-2}\,dx$

int: $\frac{1}{9}\left(\frac{1}{(-2+1)}x^{(-2+1)}\right) + C$

Simp: $-\frac{1}{9}x^{-1} + C = \boxed{-\frac{1}{9}x + C}$

24) $\int \frac{x^2+2x-3}{x^4}\,dx$

$\int \left(\frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{3}{x^4}\right)dx$

$\int \left(x^{-2} + 2x^{-3} - 3x^{-4}\right)\,dx$

$= \frac{1}{(-2+1)}x^{(-2+1)} + \frac{2}{(-3+1)}x^{(-3+1)} - \frac{3}{(-4+1)}x^{(-4+1)} + C$

$-x^{-1} - x^{-2} + x^{-3} + C$

$\frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$

26) $\int (2t^2 - 1)^2 \, dt$

$\int (4t^4 - 4t^2 + 1) \, dt$

#1

$= \frac{4}{4+1} t^{(4+1)} - \frac{4}{2+1} t^{(2+1)} + t + c$

$\boxed{\frac{4}{5} t^5 - \frac{4}{3} t^3 + t + c}$

38) $\int \sec y (\tan y - \sec y) \, dy$

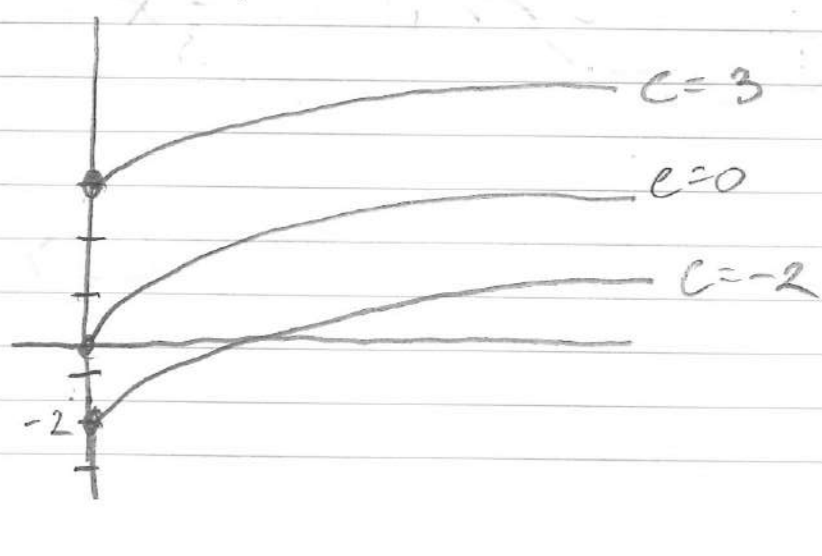
$\int (\underbrace{\sec y \tan y}_{\#8} - \underbrace{\sec^2 y}_{\#7}) \, dy$

$= \sec y - \tan y + c$

46) $f(x) = \sqrt{x}$ when $c = 0$

$g(x) = f(x) + c$

$c = 0$, $c = 2$, $c = 3$



gives precise value of c

64) $g'(x) = 6x^2$, $g(0) = -1$

$g(x) = \int 6x^2 dx$

$= \frac{6}{2+1} x^{(2+1)} + c$

$= 2x^3 + c$

$g(0) = 2(0)^3 + c = -1$

$c = -1$

$\boxed{g(x) = 2x^3 - 1}$

68) $f''(x) = x^2$, $f'(0) = 6$; $f(0) = 3$

$f'(x) = \int x^2 dx = \frac{1}{2+1} x^{2+1} + c$

$f'(x) = \frac{1}{3} x^3 + c$

$f'(0) = \frac{1}{3} 0^3 + c = 6$

$\boxed{c = 6}$

$\boxed{f'(x) = \frac{1}{3} x^3 + 6}$

$$f(x) = \int \frac{1}{3}x^3 + 6$$

$$= \frac{1}{3}\left(\frac{1}{3+1}x^{3+1}\right) + 6x + c$$

$$F(x) = \frac{1}{12}x^4 + 6x + C$$

$$f(0) = \frac{1}{12}0^4 + 6(0) + c = 3$$

$$\boxed{c = 3}$$

$$\hookrightarrow \boxed{f(x) = \frac{1}{12}x^4 + 6x + 3}$$

80) $V_0 = 8 ft/s$ $\quad f(t) = -16t^2 + V_0 t + S_0$

$S_0 = 64 ft$

a) $f(t) = -16t^2 + 8t + 64$

when is $S(t) = 0$

$$f(t) = \frac{-16t^2}{-8} + \frac{8t}{-8} + \frac{64}{-8} = \frac{0}{-8}$$

$$2t^2 - t - 8 = 0$$

$a = 2$
$b = -1$
$c = -8$

$$\frac{1 \pm \sqrt{1 - 4(2)(-8)}}{4}$$

$$\frac{1 \pm \sqrt{65}}{4}$$

$$\frac{1 + \sqrt{65}}{4} = \boxed{2.265564}$$

$$\frac{1 - \sqrt{65}}{4} = -1.655644$$

b) $V(t) = S'(t)$

$V(t) = -32t + 8$

$V\left(\frac{1 \pm \sqrt{65}}{4}\right) = -32\left(\frac{1+\sqrt{65}}{4}\right) + 8$

$\boxed{V\left(\frac{1+\sqrt{64}}{4}\right) = -64.9806}$

88) $x(t) = (t-1)(t-3)^2 \qquad 0 \leq t \leq 5$

$x(t) = (t-1)(t^2 - 6t + 9)$

$x(t) = t^3 - 7t^2 + 15t - 9$

a) $v(t) = x'(t)$

$= \boxed{3t^2 - 14t + 15}$

$a(t) = v'(t)$

$= \boxed{6t - 14}$

b)

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5.2 Area

1) $\sum_{i=1}^{5}(2i+1) = 2\sum_{i=1}^{5} i + \sum_{i=1}^{5} 1$

$= 2\frac{(5^2+5)}{2} + 5 = \boxed{35}$

5) $\sum_{i=1}^{4} c = \boxed{c(4)}$

9) $\boxed{\sum_{i=1}^{8}\left[5\left(\frac{i}{8}\right)+3\right]}$

13) $\boxed{\sum_{i=1}^{N}\left[2\left(1+\frac{3i}{n}\right)^2\right]\left(\frac{3}{n}\right)}$

? 17) $\sum_{i=1}^{20} (i-1)^2 = \sum_{i=1}^{20} i^2 - 2\sum_{i=1}^{20} i + \sum_{i=1}^{20} 1$

0 $= \frac{2n^3+3n^2+n}{6} - 2\left(\frac{n^2+n}{2}\right) + n$

$= \frac{2(20)^3+3(20)^2+20}{6} - 20^2+20+20$

$= 2870 - 400 + 40 = \boxed{2510}$

21) $\sum_{i=1}^{20} (i^2+3) =$

$\sum_{i=1}^{20} i^2 + \sum_{i=1}^{20} 3 = \frac{2n^3+3n^2+n}{6} + 3(n)$

$= \frac{2(20)^3+3(20)^2+20}{6} + 3(20)$

$= 2870 + 60 = \boxed{2930}$

Calculator = Sum(Seq(X^2+3, X, 1, 20, 1) = $\boxed{2930}$

25) $w = 1$ upper $= 3, 3, 5$

Lower $= 2, 2, 3$

$S = 1(3+3+5)$ $s = 1(2+2+3)$

$S = \boxed{11}$ $s = \boxed{7}$

29) $y = \frac{1}{x}$

if $n = \#$ of rectangles then $\boxed{n=5}$

$$\Delta X = \frac{2-1}{n} = \boxed{\frac{1}{5}}$$

$$S = \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} \approx \boxed{0.7456}$$

$$s = \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{5} \approx \boxed{0.6456}$$

33) $S(n) = \frac{18}{n^2}\left[\frac{n^2+n}{2}\right] \quad n \to \infty$

$= \frac{18}{2}\left[\frac{n^2+n}{n^2}\right] = 9\left(1+\frac{1}{n}\right)$

$\lim_{n\to\infty} 9\left(1+\frac{1}{n}\right) = 9(1+0) = \boxed{9}$

37) $\sum_{k=1}^{n} \frac{6k(k-1)}{n^3}$

$\frac{6}{n^3}\sum_{k=1}^{n} k^2 - k$

$= \frac{6}{n^3}\left[\sum_{k=1}^{n} k^2 - \sum_{k=1}^{n} k\right]$

$= \frac{6}{n^3}\left[\frac{2n^3+3n^2+n}{6} - \frac{n^2+n}{2}\right]$

$\frac{6}{n^3}\left[\frac{4n^3+6n^2+2n-(6n^2+6n)}{12}\right]$

$\frac{6}{12}\left[\frac{4n^3-4n}{n^3}\right] = \frac{1}{2}\left[4 - \frac{4}{n^2}\right]$

$= 2 - \frac{2}{n^2} \rightarrow n=10 \rightarrow 1.98$

$= n = 100 \rightarrow 1.9998$

$n = 1000 \rightarrow 2.0$

$n = 10000 \rightarrow 2.0$

4II) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n^3}(i-1)^2$

$\frac{i^2 - 2i + 1}{n^3} =$

$= \frac{1}{n^3}\left[\sum_{i=1}^{n} i^2 - 2\sum_{i=1} i + \sum_{i=1}^{n} 1\right]$

$= \frac{1}{n^3}\left[\frac{2n^3+3n^2+n}{6} - 2\left(\frac{n^2+n}{2}\right) + n\right]$

$\left[\frac{2n^3}{6n^3} + \frac{3n^2}{6n^3} + \frac{n}{6n^3} - \frac{n^2}{n^3} - \frac{n}{n^3} + \frac{n}{n^3}\right]$

$\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} - \frac{1}{n} - \frac{1}{n^2} + \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} - \frac{1}{n} - \frac{1}{n^2} + \frac{1}{n^2}$

$= \frac{1}{3} + 0 + 0 - 0 - 0 + 0$

$= \boxed{\frac{1}{3}}$

43) $\lim_{n \to \infty} \sum_{i=1}^{n} \left(1+\frac{i}{n}\right)\left(\frac{2}{n}\right)$

$= \left(\frac{n+i}{n}\right)\left(\frac{2}{n}\right) = \frac{2n+2i}{n^2} = \frac{2n}{n^2} + \frac{2i}{n^2}$

$= \sum_{i=1}^{n} \frac{2n+2i}{n^2}$

$= \frac{2}{n^2}\left[\sum_{i=1}^{n} n + \sum_{i=1}^{n} i\right]$

$= \frac{2}{n^2}\left[n^2 + \frac{n^2+n}{2}\right]$

$= \frac{2n^2}{n^2} + \frac{2n^2+2n}{2n^2} = 2+1+\frac{1}{n}$

$= 3+\frac{1}{n}$

$\lim_{n \to \infty} 3+\frac{1}{n} = 3+0 = \boxed{3}$

47) $y = -2x + 3$ $\quad [0, 1]$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \boxed{\frac{1}{n}}$$

$$c_i = a + \Delta x i = 0 + \frac{1}{n} = \boxed{\frac{1}{n} i}$$

$$f(c_i)\Delta x = \left[-2\left(\frac{1}{n} i\right) + 3\right]\left(\frac{1}{n}\right)$$

$$= \boxed{\frac{-2i}{n^2} + \frac{3}{n}}$$

$$\sum_{i=1}^{n} f(c_i)\Delta x = \sum_{i=1}^{n} \frac{-2i}{n^2} + \frac{3}{n}$$

$$= \frac{-2}{n^2}\sum_{i=1}^{n} i + \frac{1}{n}\sum_{i=1}^{n} 3$$

$$= \frac{-2}{n^2}\left[\frac{n^2+n}{2}\right] + \frac{1}{n}\left[3n\right]$$

$$= -1\left(\frac{n^2+n}{n^2}\right) + 3$$

$\rightarrow$

$-1\left(1+\frac{1}{n}\right)+3$

$=-1-\frac{1}{n}+3$

$\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$

$=\lim_{n\to\infty} -1-\frac{1}{n}+3$

$-1-0+3= \boxed{A=2}$

51) $y=16-x^2$ $[1,3]$

$A=\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$

1) $\Delta x=\frac{b-a}{n}=\frac{3-1}{n}=\boxed{\frac{2}{n}}$

2) $c_i=a+\Delta x i=\boxed{1+\frac{2}{n}i}$

3+4) $f(c_i)\Delta x=\left[16-\left(1+\frac{2}{n}i\right)^2\right]\left(\frac{2}{n}\right)$

$=\left[16-\left(\frac{4}{n^2}i^2+\frac{4}{n}i+1\right)\right]\left(\frac{2}{n}\right)$ $\longrightarrow$

$$\left[15 - \frac{4}{n^2} i^2 - \frac{4}{n} i\right]\left(\frac{2}{n}\right)$$

$$= \frac{30}{n} - \frac{8}{n^3} i^2 - \frac{8}{n^2} i$$

5) $$\sum_{i=1}^{n} \left(\frac{-8}{n^3} i^2 - \frac{8i}{n^2} + \frac{30}{n}\right)$$

$$= \frac{1}{n}\left[\frac{-8}{n^2}\sum_{i=1}^{n} i^2 - \frac{8}{n}\sum_{i=1}^{n} i + \sum_{i=1}^{n} 30\right]$$

$$= \frac{1}{n}\left[-\frac{8}{n^2}\left(\frac{2n^3+3n^2+n}{6}\right) - \frac{8}{n}\left(\frac{n^2+n}{2}\right) + 30n\right]$$

$$= \frac{-8}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right) - \frac{8}{n^2}\left(\frac{n^2+n}{2}\right) + \frac{30n}{n}$$

$$= -\frac{4}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - 4\left(1 + \frac{1}{n}\right) + 30$$

6) $$\lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x$$

$$= \lim_{n \to \infty}\left[-\frac{4}{3}\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) - 4 - \frac{4}{n} + 30\right]$$

$$= -\frac{4}{3}(2+0+0) - 4 - 0 + 30$$

$$= -\frac{4}{3}(2) + 26 = -\frac{8}{3} + 26 = \boxed{A = \frac{70}{3}}$$

Steven Romero

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Section 5.3

1) $\lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \Delta x_i$

$c_i = \frac{3i^2}{n^2}$

$f(x) = \sqrt{x}$

$y = 0, x = 0, x = 3$

(1) $\Delta x_i = c_i - \frac{3(i-1)^2}{n^2} = \frac{3i^2}{n^2} - \frac{3(i-1)^2}{n^2}$

$= \boxed{\frac{3}{n^2}(2i-1)}$

(3) $f(c_i) = \boxed{\sqrt{\frac{3i^2}{n^2}}}$

(4) $f(c_i)\Delta x_i = \boxed{\sqrt{\frac{3i^2}{n^2}}\left(\frac{3}{n^2}(2i-1)\right)}$

(5) $\sum_{i=1}^{n} f(c_i)\Delta x_i = \sum_{i=1}^{n} \frac{\sqrt{3}}{n}\left[\frac{3}{n^2}(2i^2 - i)\right]$

$\frac{3\sqrt{3}}{n^3} \sum_{i=1}^{n} (2i^2 - i) = \frac{3\sqrt{3}}{n^3}\left[2 \cdot \left(\frac{2n^3 + 3n^2 + n}{6}\right) - \left(\frac{n^2 + n}{2}\right)\right]$

$\lim_{n \to \infty} = 3\sqrt{3}\left[\frac{2n^3 + 3n^2 + n}{3n^3} - \frac{(n^2 + n)}{2n^3}\right]$

$\lim_{n \to \infty} = 3\sqrt{3}\left[\frac{2}{3} + \frac{1}{n} + \frac{1}{n^2} - \frac{1}{2n} - \frac{1}{2n^2}\right]$

$3\sqrt{3}\left[\frac{2}{3} + 0\right] = \boxed{2\sqrt{3}}$

$a = -1 \quad b = 1$

5) $\int_{-1}^{1} x^3 dx \qquad A = \lim_{n \to \infty} \sum_{i=1}^{n} f(c_i) \Delta x$

(1) $\Delta x = \frac{b-a}{n} = \frac{1+1}{n} = \boxed{\frac{2}{n}}$

(2) $c_i = a + \Delta x i = -1 + \frac{2i}{n} = \boxed{\frac{-n+2i}{n}}$

(3) $f(c_i) = \left(\frac{-n+2i}{n}\right)^3 = \frac{(-n+2i)^3}{n^3} = \frac{-n^3 + 6n^2 i - 12n i^2 + 8i^3}{n^3}$

$= -1 + \frac{6i}{n} - \frac{12i^2}{n^2} + \frac{8i^3}{n^3}$

(4) $f(c_i) \Delta x = \left[-1 + \frac{6i}{n} - \frac{12i^2}{n^2} + \frac{8i^3}{n^3}\right]\left(\frac{2}{n}\right)$

$-\frac{2}{n} + \frac{12i}{n^2} - \frac{24i^2}{n^3} + \frac{16i^3}{n^4}$

(5) $\sum_{i=1}^{n} -\frac{2}{n} + \sum_{i=1}^{n} \frac{12i}{n^2} - \sum_{i=1}^{n} \frac{24i^2}{n^3} + \sum_{i=1}^{n} \frac{16i^3}{n^4}$

$= -\frac{2}{n}(n) + \frac{12}{n^2}\left(\frac{n^2+n}{2}\right) - \frac{24}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right) + \frac{16}{n^4}\left(\frac{n^4+2n^3+n^2}{4}\right)$

$= -2 + 6\left(1 + \frac{1}{n}\right) - 4\left(2 + \frac{3}{n} + \frac{1}{n^2}\right) + 4\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)$

(6) $\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$

$A = \lim_{n\to\infty} -2 + 6 + \frac{6}{n} - 8 - \frac{12}{n} - \frac{4}{n^2} + 4 + \frac{8}{n} + \frac{4}{n^2}$

$A = -2 + 6 + 0 - 8 - 0 - 0 + 4 + 0 + 0$

$A = 8 - 8$ $\boxed{A = 0}$

9) $\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} (3c_i + 10)\Delta x_i$ $[-1, 5]$

$\boxed{\int_{-1}^{5} (3x + 10)\,dx}$

$a = -1$
$b = 5$
$\Delta x_i = dx$
$c_i = x$

13) $\lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} \left(1 + \frac{3}{c_i}\right)\Delta x_i$ $[1, 5]$

$\boxed{\int_{1}^{5} \left(1 + \frac{3}{x}\right)dx}$

$a = 1$ $b = 5$
$\Delta x_i = dx$
$c_i = x$

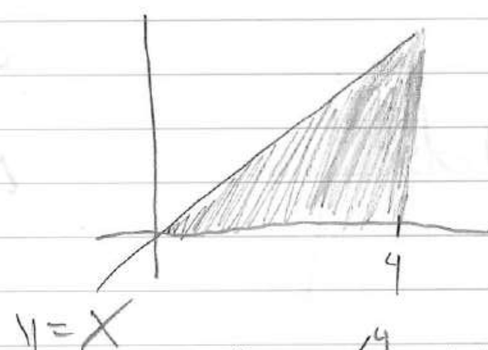
17) $f(x)=\frac{2}{x}$ $\quad a=1,\ b=4$

$$\int_1^4 \frac{2}{x}\,dx$$

21) $g(y)=y^3$ $\quad a=0 \quad b=8$

$$\int_0^8 y^3\,dy$$

25)

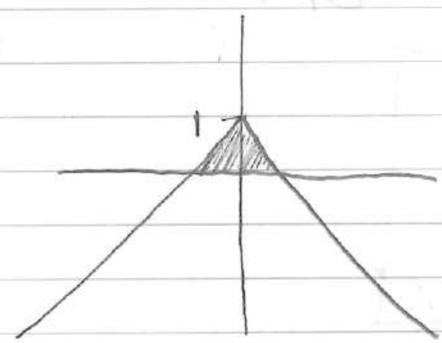


$f(x)=x$
$a=0\quad b=4$

base $=4$
$h=f(4)=4$

$$A=\int_0^4 \frac{1}{2}bh=\frac{1}{2}4(4)=\frac{1}{2}8=\boxed{4}$$

29)



base $=1$
$h=1$

$$A=\int_{-1}^{1}\frac{1}{2}bh=\frac{1}{2}(1)(1)=\boxed{\frac{1}{2}}*2=\boxed{1}$$

33) $\int_4^2 x\,dx$

$-\int_2^4 x\,dx = 6 = \boxed{-6}$

37) $\int_2^4 (x-8)\,dx =$

$\int_2^4 x\,dx - \int_2^4 8\,dx =$

$\int_2^4 x\,dx - 8\int_2^4 dx = 6 - 8(2) = \boxed{-10}$

41) a) $\int_0^7 f(x)\,dx =$

$\int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx = 10 + 3 = \boxed{13}$

b) $\int_5^0 f(x)\,dx = -\int_0^5 f(x)\,dx = \boxed{-10}$

↳

c) $\int_{5}^{5} f(x)\,dx = 0$

d) $\int_{0}^{5} 3f(x)\,dx = 3\int_{0}^{5} f(x)\,dx = 3(10) = \boxed{30}$

43) a) $\int_{2}^{6} [f(x) + G(x)]\,dx = \int_{2}^{6} f(x) + \int_{2}^{6} G(x)$

$= 10 + (-2) = \boxed{8}$

b) $\int_{2}^{6} [G(x) - f(x)]\,dx = \int_{2}^{6} g(x) - \int_{2}^{6} f(x)\,dx$

$= -2 - 10 = \boxed{-12}$

c) $\int_{2}^{6} 2g(x)\,dx = 2\int_{2}^{6} g(x)\,dx = 2(-2)$

$= \boxed{-4}$

d) $\int_{2}^{6} 3f(x)\,dx = 3\int_{2}^{6} f(x)\,dx = 3(10) = \boxed{30}$

Steven Romeiro

Pg 327 # 5-61, 75, 87-105

Section 5.4

5) $\int_0^1 2x\,dx = x^2 \Big|_0^1$

$(1)^2 - (0)^2 = \boxed{1}$

9) $\int_{-1}^{1} (t^2 - 2)\,dt = \left[\frac{t^3}{3} - 2t\right]_{-1}^{1}$

$\left[\frac{(-1)^3}{3} - 2(-1)\right] - \left[\frac{(1)^3}{3} - 2(1)\right] = \frac{5}{3} + \frac{5}{3} = \boxed{\frac{10}{3}}$

13) $\int_1^2 \left(\frac{3}{x^2} - 1\right)dx = \int_1^2 (3x^{-2} - 1)\,dx$

$= \left[-\frac{3}{x} - 1x\right]\Big|_1^2 = \left[-\frac{3}{2} - (2)\right] - \left[-\frac{3}{1} - 1\right]$

$= -\frac{7}{2} + 4 = \boxed{\frac{1}{2}}$

17) $\int_{-1}^{1} (\sqrt[3]{t} - 2)\, dt = \int_{-1}^{1} (t^{1/3} - 2)\, dt$

$= \left[\frac{3}{4}t^{4/3} - 2t\right]\Big|_{-1}^{1} = \left[\frac{3}{4}(1)^{4/3} - 2(1)\right] - \left[\frac{3}{4}(-1)^{4/3} - 2(-1)\right]$

$= -\frac{5}{4} - \frac{11}{4} = \boxed{-4}$

21) $\int_{-1}^{0} (t^{1/3} - t^{2/3})\, dt$

$= \left[\frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3}\right]\Big|_{-1}^{0}$

$= \left[\frac{3}{4}(0)^{4/3} - \frac{3}{5}(0)^{5/3}\right] - \left[\frac{3}{4}(-1)^{4/3} - \frac{3}{5}(-1)^{5/3}\right]$

$= 0 - \frac{3}{4} - \frac{3}{5} = \boxed{-\frac{27}{20}}$

ON Exam

25) $\int_0^3 |x^2-4|\,dx$

Put fxn without abs symbol in calc to find x-int = -2 & 2 & solve for insides but only from 0→3

below x — above x

$-\int_0^2 x^2-4\,dx + \int_2^3 x^2-4\,dx$

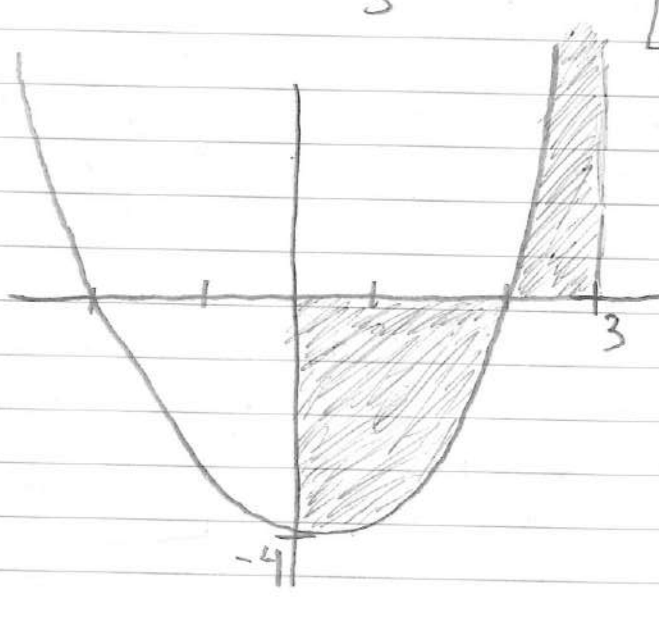
$= \left(-\frac{x^3}{3}+4x\right)\Big|_0^2 + \left[\left(\frac{x^3}{3}-4x\right)\right]\Big|_2^3$

$= \left[-\frac{0^3}{3}+4(0)-\left(-\frac{(2)^3}{3}+4(2)\right)\right] + \left[\left(\frac{x^3}{3}-4x\right)\right]\Big|_2^3$

$= \left[0+\frac{16}{3}\right] + \left[\frac{(3)^3}{3}-4(3)-\left(\frac{2^3}{3}-4(2)\right)\right]$

$= \left[\frac{16}{3}\right] + \left[(9-12)-(-16/3)\right]$

$= \frac{16}{3}+\frac{16}{3}-3 = \frac{32}{3}-3 = \boxed{\frac{23}{3}}$



29) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec^2 x\,dx = \left[\tan x\right]\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$

$\left[\tan\left(\frac{\pi}{6}\right)\right] - \left[\tan\left(-\frac{\pi}{6}\right)\right]$

$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \boxed{\frac{2}{\sqrt{3}} \text{ or } \frac{2\sqrt{3}}{3}}$

33) $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 4\sec\theta\tan\theta\,d\theta$

is in 4th Quadrant so pos 2

$= 4\sec\theta\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} = 4\sec\left(\frac{\pi}{3}\right) - 4\sec\left(-\frac{\pi}{3}\right)$

$= 4(2) - 4(2) = 8 - 8 = \boxed{0}$

37) $\int_{-1}^{1} (e^{\theta} + \sin\theta)\,d\theta$

$\int_{-1}^{1} e^{\theta}\,d\theta + \int_{-1}^{1} \sin\theta\,d\theta = \left[e^{\theta}\right]\Big|_{-1}^{1} + \left[-\cos\theta\right]\Big|_{-1}^{1}$

$= \left[e^{1} - (e^{-1})\right] + \left[(-\cos 1) - (-\cos(-1))\right]$

$\left[e - \frac{1}{e}\right] + 0 - 0 = \boxed{e - \frac{1}{e}}$

41) $y = (3-x)\sqrt{x} \longrightarrow y = (3x^{\frac{1}{2}} - x^{\frac{3}{2}})$

$$\int_0^3 (3x^{\frac{1}{2}} - x^{\frac{3}{2}}) = \left[2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]\Big|_0^3$$

$$= \left[2(3)^{\frac{3}{2}} - \frac{2}{5}(3)^{\frac{5}{2}}\right] - 0$$

$$= 2\sqrt{27} - \frac{2}{5}\sqrt{243} = 2\sqrt{3\cdot 9} - \frac{2}{5}\sqrt{81\cdot 3}$$

$$= 3\cdot 2\sqrt{3} - 9\cdot\frac{2}{5}\sqrt{3} = 6\sqrt{3} - \frac{18}{5}\sqrt{3} = \frac{30\sqrt{3} - 18\sqrt{3}}{5}$$

$$= \boxed{\frac{12\sqrt{3}}{5}}$$

45) $y = 3x^2 + 1 \longrightarrow \int_0^2 (3x^2+1)\,dx$

$$= \left[x^3 + x\right]\Big|_0^2 \longrightarrow (2^3 + 2) - (0^3 + 0)$$

$$= \boxed{10}$$

49) $y = \frac{4}{x} \rightarrow \int_1^e \frac{4}{x}\,dx \rightarrow 4\int_1^e \frac{1}{x}\,dx$

$= (4\ln x)\Big|_1^e \rightarrow 4\ln e - 4\ln(1)$

$= 4 - 4(0) = \boxed{4}$

53) $f(x) = 2\sec^2 x \qquad \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$\int_a^b f(x)\,dx = f(c)(b-a)$

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 2\sec^2 x\,dx = 2\tan x\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$2\tan\frac{\pi}{4} - 2\tan\left(-\frac{\pi}{4}\right) = 2(1) - 2(-1) = \boxed{4}$

$f(c)(b-a) = 2\sec^2 c\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)2\sec^2 c$

$\pi\sec^2 c = 4$

$\sec^2 c = \frac{4}{\pi}$

$\sec c = \frac{2}{\sqrt{\pi}}$

$\boxed{c = \sec^{-1}\left(\frac{2}{\sqrt{\pi}}\right)}$

57) $f(x) = 4 - x^2$ $[-2, 2]$

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{2+2}\int_{-2}^{2} 4-x^2\,dx = \frac{1}{4}\left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2}$$

$$\frac{1}{4}\left[4(2) - \frac{2^3}{3} - 4(-2) + \frac{(-2)^3}{3}\right]$$

$$\frac{1}{4}\left(\frac{32}{3}\right) = \boxed{\frac{8}{3}}$$

61) $f(x) = \sin x$ $[0, \pi]$

$$\frac{1}{\pi-0}\int_0^{\pi} \sin x\,dx = \frac{1}{\pi}(\cos x)\Big|_0^{\pi}$$

$$\frac{1}{\pi}(\cos\pi - \cos 0)$$

$$= \frac{1}{\pi}(-1-1) = \boxed{\frac{2}{\pi}}$$

75) $f(x) = (.1729t + .1522t^2 - .0374t^3)$ $[0,5]$

$$\frac{1}{5-0}\int_0^5 (.1729t + .1522t^2 - .0374t^3)\,dt$$

$$\frac{1}{5}\left(.08654t^2 + .05073t^3 - .00935t^4\right)\Big|_0^5$$

$$\approx \boxed{0.5318 \text{ liters}}$$

87) $F(x) = \int_0^x (t+2)\,dt$

$$= \left(\frac{t^2}{2} + 2t\right)\Big|_0^x \longrightarrow \boxed{\frac{x^2}{2} + 2x}$$

$$F'(x) = \frac{x^2}{2} + 2x \longrightarrow \boxed{x + 2}$$

95) $F(x) = \int_{-2}^x (t^2 - 2t)\,dt$

$$= \left(\frac{t^3}{3} - t^2\right)\Big|_{-2}^x \rightarrow \left(\frac{x^3}{3} - x^2\right) - \left(\frac{(-2)^3}{3} - (-2)^2\right)$$

$$\boxed{\frac{x^3}{3} - x - \frac{4}{3}}$$

Steven Romeiro
Pg 340 #1-33, 47-83, 87, 95-109

Section 5.5

1) $\int (5x^2+1)^2 (10x)\,dx$

$U = 5x^2+1$

$du = 10x\,dx$

5) $\int \tan^2 x \sec^2 x\,dx$

$= \int \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\,dx$

$= \int \frac{\sin^2 x}{\cos^4 x}\,dx \rightarrow \int \sin^2 x \cdot \cos^{-4} x\,dx$

$U = \cos^{-4} x$
$du = 4\sin^{-5} x$

9) $\int \sqrt{9-x^2}\,(-2x)\,dx \longrightarrow \int (9-x^2)^{\frac{1}{2}} (-2x)\,dx$

$U = 9-x^2$
$du = -2x\,dx$ $\longrightarrow \int U^{\frac{1}{2}}\,du$

$= \frac{2}{3}U^{\frac{3}{2}} + C \longrightarrow \boxed{\frac{2}{3}(9-x^2)^{\frac{3}{2}} + C}$

$\frac{d}{dx}\left(\frac{2}{3}(9-x^2)^{\frac{3}{2}} + C\right) \longrightarrow \boxed{(9-x^2)^{\frac{1}{2}} \cdot (-2x)}$

13) $\int x^2(x^3-1)^4\,dx$ $\qquad u = x^3-1$, $du = 3x^2\,dx$

$\frac{1}{3}\int (x^3-1)^4 \cdot (3x^2)\,dx \longrightarrow \frac{1}{3}\int u^4\,du$

$= \left(\frac{1}{3}\right)\frac{u^5}{5} + c \longrightarrow \boxed{\frac{(x^3-1)^5}{15} + c}$

$\frac{d}{dx}\left[\frac{1}{15}(x^3-1)^5 + c\right] = \boxed{\frac{1}{3}(x^3-1)^4 \cdot (3x^2)}$

17) $\int 5x\sqrt[3]{1-x^2}\,dx$ $\qquad u = 1-x^2$, $du = -2x\,dx$

$-\frac{5}{2}\int (1-x^2)^{\frac{1}{3}} \cdot (-2x)\,dx \longrightarrow -\frac{5}{2}\int u^{\frac{1}{3}}\,du$

$= \left(-\frac{5}{2}\right) \cdot \frac{3}{4}u^{4/3} + c \longrightarrow \boxed{-\frac{15}{8}u^{4/3} + c}$

$\boxed{-\frac{15}{8}(1-x^2)^{4/3} + c} \longrightarrow \frac{d}{dx}\left[-\frac{15}{8}(1-x^2)^{4/3} + c\right]$

$= -\frac{15}{8} \cdot \frac{4}{3}(1-x^2)^{\frac{1}{3}} \cdot (-2x)$

$= \boxed{-\frac{5}{2}\left[(1-x^2)^{\frac{1}{3}} \cdot (-2x)\right]}$

21) $\int \frac{x^2}{(1+x^3)^2}\,dx \rightarrow \int x^2(1+x^3)^2\,dx$

$u = 1+x^3$
$du = 3x^2\,dx \longrightarrow \frac{1}{3}\int (1+x^3)^2 \cdot (3x^2)\,dx$

$\frac{1}{3}\int u^2\,du \longrightarrow \frac{1}{3} \cdot \frac{u^3}{3} + C = \frac{u^3}{9} + C$

$\frac{d}{dx} \cdot \boxed{\frac{1}{9}(1+x^3)^3 + C} \longrightarrow \boxed{\frac{1}{3}(1+x^3)^2 \cdot (3x^2)}$

25) $\int \left(1+\frac{1}{t}\right)^3 \cdot \left(\frac{1}{t^2}\right) dt \qquad u = 1+t^{-1}$
$\qquad du = -t^{-2}\,dt$

$-\int (1+t^{-1})^3 \cdot (-t^{-2})\,dt \longrightarrow -\int u^3\,du$

$= -\frac{u^4}{4} + C \longrightarrow \boxed{-\frac{1}{4}(1+t^{-1})^4 + C} \cdot \frac{d}{dx}$

$= -\frac{1}{4} \cdot 4(1+t^{-1})^3 \cdot (-t^{-2}) = \boxed{-(1+t^{-1})^3 \cdot (-t^{-2})}$

29) $\displaystyle\int \frac{x^2+3x+7}{x^{\frac{1}{2}}}\,dx$

$\displaystyle\int (x^2+3x+7)\cdot(x^{-\frac{1}{2}})\,dx$

$\displaystyle\int \left(x^{\frac{3}{2}}+3x^{\frac{1}{2}}+7x^{-\frac{1}{2}}\right)dx$

$= \boxed{\frac{2}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+14x^{\frac{1}{2}}+C}$

33) $\displaystyle\int (9-y)\sqrt{y}\,dy \longrightarrow \int (9-y)(y)^{\frac{1}{2}}\,dy$

$\boxed{\left(-\frac{y^2}{2}+9y\right)\left(\frac{2}{3}y^{\frac{3}{2}}\right)+C}$

47) $\int \pi \sin \pi x \quad dx$ $\qquad U = \pi x$
$\qquad du = \pi dx$

$= \int \sin \pi x \; \pi dx \longrightarrow \int \sin u \; du$

$= -\cos U \longrightarrow \boxed{-\cos \pi x}$

51) $\int \frac{1}{\theta^2} \cos \frac{1}{\theta} \, d\theta \longrightarrow \int \theta^{-2} \cos \theta^{-1} \; d\theta$

$\qquad U = \theta^{-1}$
$\qquad du = -\theta^{-2} d\theta$

$-\int \cos \theta^{-1} \cdot -\theta^{-2} d\theta$

$-\int \cos u \, du \longrightarrow -\sin U + C$

$= -\sin \theta^{-1} \longrightarrow \boxed{-\sin \frac{1}{\theta} + C}$

55) $\int x^2 e^{-x^3}\,dx$ $\quad u = -x^3$, $du = -3x^2\,dx$

$-\frac{1}{3}\int e^{-x^3} \cdot -3x^2\,dx \longrightarrow -\frac{1}{3}\int e^{u}\,du$

$-\frac{1}{3}e^{u} + C = \boxed{-\frac{1}{3}e^{-x^3} + C}$

59) $\int \sin 2x \cos 2x\,dx$ $\quad u = \sin 2x$, $du = 2\cos 2x\,dx$

$\frac{1}{2}\int \sin 2x \cdot 2\cos 2x\,dx$

$\frac{1}{2}\int u\,du = \frac{1}{2} \cdot \frac{u^2}{2} + C = \frac{1}{4}u^2 + C$

$= \boxed{\frac{1}{4}\sin^2 2x + C}$

Since $\csc^2 = 1 + \cot^2 \longrightarrow \cot^2 = \csc^2 - 1$

63) $\int \cot^2 x \, dx \longrightarrow \int (\csc^2 - 1)\, dx$

$= \int \csc^2 dx - \int 1\, dx$

$= \boxed{-\cot x - x + C}$

67) $\int e^x \sqrt{1 - e^x}\, dx$ $\quad U = 1 - e^x$
$du = -e^x dx$

$-\int (1 - e^x)^{\frac{1}{2}} \cdot (-e^x)\, dx$

$-\int u^{\frac{1}{2}} du = -\frac{2}{3} u^{\frac{3}{2}} + C$

$\boxed{-\frac{2}{3}(1 - e^x)^{\frac{3}{2}}}$

71) $\int e^{\sin \pi x} \cos \pi x \, dx$ $\quad U = \sin \pi x$
$du = \pi \cos \pi x \, dx$

$\frac{1}{\pi} \int e^{\sin \pi x} \cdot \pi \cos \pi x \, dx$

$\frac{1}{\pi} \int e^u du = \frac{1}{\pi} e^u + C \longrightarrow \boxed{\frac{1}{\pi} e^{\sin \pi x} + C}$

75) $\int 3^{x/2}\,dx$

$u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2\int 3^{x/2} \cdot \frac{1}{2}dx$

$2\int 3^u du = 2\left(\frac{1}{\ln 3}\right)3^u + C$

$\frac{2}{\ln 3} \cdot 3^u + C = \frac{6^u}{\ln 3} + C$

$\boxed{\frac{6^{x/2}}{\ln 3} + C}$ or $\boxed{2 \cdot \frac{3^{x/2}}{\ln 3} + C}$

79) $f'(x) = x\sqrt{4-x^2}$ $(2, 2)$

$f'(x) = \int x\sqrt{4-x^2}\,dx \longrightarrow \int (4-x^2)^{\frac{1}{2}} \cdot x\,dx$

$u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}\int (4-x^2)^{\frac{1}{2}} \cdot (-2x)\,dx$

$-\frac{1}{2}\int u^{\frac{1}{2}} du = -\frac{1}{2} \cdot \frac{2}{3}u^{\frac{3}{2}} + C$

$f(x) = -\frac{1}{3}(4-x^2)^{\frac{3}{2}} + C \longrightarrow 2 = -\frac{1}{3}(4-2^2)^{\frac{1}{2}} + C$

$\boxed{C = 2}$ $\boxed{f(x) = -\frac{1}{3}(4-x^2)^{\frac{1}{2}} + 2}$

83) $f'(x) = 2e^{-x/4}$ $(0, 1)$

$f(x) = \int 2e^{-x/4}\,dx$

$U = -x/4$

$du = -\frac{1}{4}\,dx$

$-8\int e^{-x/4} \cdot \left(-\frac{2}{8}\right) dx$

$-8\int e^{u}\,du = -8e^{u} + C = -8e^{-x/4} + C$

$1 = -8e^{-0/4} + C \rightarrow \boxed{C = 9}$ $\boxed{f(x) = -8e^{-x/4} + 9}$

On Final

87) $\int x(x+2)^{\frac{1}{2}}\,dx$

$\boxed{\begin{array}{l} U = x+2 \\ du = 1\,dx \end{array}}$

$* \boxed{x = U - 2}$

$\int (U-2)\,U^{\frac{1}{2}}\,du$

$\int U^{\frac{3}{2}} - 2U^{\frac{1}{2}}\,du = \frac{2}{5}U^{\frac{5}{2}} - \frac{4}{3}U^{\frac{3}{2}} + C$

$$\boxed{\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C}$$

when $x=-1$
$U=(-1)^2+1$ $\boxed{U=2}$
when $x=1$
$U=(1)^2+1 = \boxed{U=2}$

95) $\int_{-1}^{1} x(x^2+1)^3 dx$

$U=x^2+1$
$du=2x\,dx$

$\frac{1}{2}\int_{-1}^{1}(x^2+1)^3 \cdot 2x\,dx \longrightarrow \frac{1}{2}\int_{2}^{2} U^3 \cdot du$

$\boxed{\frac{1}{2}\int_{2}^{2} U^3 du = 0}$

99) $\int_{0}^{4} \frac{1}{\sqrt{2x+1}} dx \longrightarrow \int_{0}^{4} (2x+1)^{-\frac{1}{2}} dx$

$U=2x+1$
$du=2dx$
$U=2(0)+1$
$\boxed{U=1}$
$U=2(4)+1$
$\boxed{U=9}$

$\frac{1}{2}\int_{1}^{9} U^{-\frac{1}{2}} \cdot 2du = \frac{1}{2} \cdot \frac{2}{1} U^{\frac{1}{2}} + C$

$\left(U^{\frac{1}{2}} + C\right)\Big|_{1}^{9}$

$(\sqrt{9}+\cancel{C}) - (\sqrt{1}+\cancel{C})$

$= 3-1$

$= \boxed{2}$

$U = \frac{3}{1}$ $\quad$ $U = \frac{3}{3}$

$\boxed{U = 3}$ $\quad$ $\boxed{U = 1}$

103) $\int_1^3 \frac{e^{3/x}}{x^2}\,dx \longrightarrow \int_1^3 e^{3/x} \cdot x^{-2}\,dx$

$U = \frac{3}{x}$

$du = -\frac{3}{x^2}\,dx$

$-\frac{1}{3}\int_1^3 e^{3/x} \cdot -\frac{3}{x^2}\,dx$

$-\frac{1}{3}\int_3^1 e^u\,du \longrightarrow -\left(-\frac{1}{3}\right)\int_1^3 e^u\,du$

$\left(\frac{1}{3}e^u\right)\Big|_1^3 = \left(\frac{1}{3}e^3\right) - \left(\frac{1}{3}e^1\right) = e\left(\frac{1}{3}e^2 - \frac{1}{3}\right)$

$\boxed{\frac{e}{3}\left(e^2 - 1\right)}$

109) $\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right)dx$

$U = \frac{2x}{3}$

$du = \frac{2}{3}\,dx$

$\frac{3}{2}\int_0^{\pi/2} \cos\left(\frac{2x}{3}\right) \cdot \frac{2}{3}\,dx$

$U = \frac{2(0)}{3} = \boxed{0}$

$U = \frac{2\left(\frac{\pi}{2}\right)}{3} = \boxed{\frac{\pi}{3}}$

$$\frac{3}{2}\int_0^{\pi/2} \cos\left(\frac{2\pi}{3}\right)\cdot\frac{2}{3}\,dx$$

$$\frac{3}{2}\int_0^{\pi/3} \cos u\,du = \frac{3}{2}(\sin u)\Big|_0^{\frac{\pi}{3}}$$

$$\frac{3}{2}\left(\frac{\sqrt{3}}{2}\right)-\left(-0\right) = \frac{3}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\frac{3\sqrt{3}}{4}}$$

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Section 5.6

1) $\int_0^2 x^2 \, dx$ $\quad n = 4$

$a = 0 \quad b = 2$

Trap Rule = $\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + \dots + f(x_n)\right]$

$\Delta x = \frac{b-a}{n} = \boxed{\frac{1}{2}}$

$x_0 = a = 0$

$x_1 = 0 + \frac{1}{2} = \frac{1}{2} \qquad x_3 = 1 + \frac{1}{2} = \frac{3}{2}$

$x_2 = \frac{1}{2} + \frac{1}{2} = 1 \qquad x_4 = \frac{3}{2} + \frac{1}{2} = 2$

$\frac{2-0}{2(4)}\left[f(0) + 2f(\tfrac{1}{2}) + 2f(1) + 2f(\tfrac{3}{2}) + f(2)\right]$

$\frac{1}{4}\left[0 + 2(\tfrac{1}{4}) + 2(1) + 2(\tfrac{5}{2}) + 4\right]$

$\approx \boxed{2.750}$

Simpson Rule $\int_a^b f(x)dx \approx \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \dots + f(x_n)\right]$

$\approx \frac{2-0}{3(4)}\left[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\right]$

$\approx \frac{1}{6}\left[0 + 4(\tfrac{1}{4}) + 2(1) + 4(\tfrac{5}{2}) + 4\right]$

$\approx \boxed{2.667}$

5) $\int_0^2 x^3\,dx$ $\quad n=8$, $a=0$ $\;b=2$

$\Delta x = \frac{2-0}{8} = \boxed{\frac{1}{4}}$ $\quad x_0 = \boxed{0}$

$x_1 = \boxed{\frac{1}{4}}$ $\quad x_2 = \frac{1}{4}+\frac{1}{4} = \boxed{\frac{1}{2}}$ $\quad x_3 = \frac{1}{4}+\frac{1}{2} = \boxed{\frac{3}{4}}$

$x_4 = \frac{3}{4}+\frac{1}{4} = \boxed{1}$ $\quad x_5 = \frac{1}{4}+1 = \boxed{\frac{5}{4}}$

$x_6 = \frac{5}{4}+\frac{1}{4} = \boxed{\frac{3}{2}}$ $\quad x_7 = \frac{3}{2}+\frac{1}{4} = \boxed{\frac{7}{4}}$ $\quad x_8 = \boxed{2}$

TRap RULE $\frac{2}{2(8)}\left[0+2\left(\frac{1}{4}\right)^3+2\left(\frac{1}{2}\right)^3+2\left(\frac{3}{4}\right)^3+2(1)^3+2\left(\frac{5}{4}\right)^3+2\left(\frac{3}{2}\right)^3+2\left(\frac{7}{4}\right)^3+2^3\right]$

$\approx \boxed{4.0625}$

Simpson Rule $\frac{2}{3(8)}\left[0+4\left(\frac{1}{4}\right)^3+2\left(\frac{1}{2}\right)^3+4\left(\frac{3}{4}\right)^3+2(1)^3+4\left(\frac{5}{4}\right)^3+2\left(\frac{3}{2}\right)^3+4\left(\frac{7}{4}\right)^3+2^3\right]$

$\approx \boxed{4.0000}$

9) $\int_1^2 \frac{1}{(x+1)^2}\,dx$ $n=4$
$a=1$ $b=2$

$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \boxed{\frac{1}{4}}$

$x_0 = 1$ $x_1 = 1 + \frac{1}{4} = \boxed{\frac{5}{4}}$

$x_2 = \frac{5}{4} + \frac{1}{4} = \boxed{\frac{3}{2}}$ $x_3 = \frac{3}{2} + \frac{1}{4} = \boxed{\frac{7}{4}}$ $x_4 = \frac{7}{4} + \frac{1}{4} = \boxed{2}$

TRAP Rule $\approx \frac{2-1}{2(4)}\left[f(1) + 2f(\tfrac{5}{4}) + 2f(\tfrac{3}{2}) + 2f(\tfrac{7}{4}) + f(2)\right]$

$\approx \frac{1}{8}\left[.25 + 2(.19753) + 2(.16) + 2(.13223) + .11111\right]$

$\approx \boxed{.1676}$

Simpson Rule $\frac{2-1}{3(4)}\left[.25 + 4(.19753) + 2(.16) + 4(.13223) + .1111\right]$

$\approx \boxed{.16668}$

13) $\int_0^1 \sqrt{x}\,\sqrt{1-x}\,dx$ $\qquad n=4$ $\quad a=0 \quad b=1$

$\Delta x = \boxed{\frac{1}{4}} \qquad x_0 = 0 \qquad x_1 = 0 + \frac{1}{4} = \frac{1}{4}$

$x_2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \quad x_3 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \quad x_4 = \frac{3}{4} + \frac{1}{4} = 1$

Trap Rule $\approx \frac{1-0}{2(4)}\left[f(0) + 2f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 2f(\tfrac{3}{4}) + f(1)\right]$

$\frac{1}{8}\left[0 + 2(.43301) + 2(.5) + 2(.43301) + 0\right]$

$\approx \boxed{.341505}$

Simpson Rule $\approx \frac{1-0}{3(4)}\left[0 + 4(.43301) + 2(.5) + 4(.43301) + 0\right]$

$\approx \boxed{.3721}$

17) $\int_{1}^{1.1} \sin x^2 \, dx$

$n = 4$

$a = 1 \quad b = 1.1$

$\Delta X = \frac{.1}{4} \qquad X_0 = \boxed{1} \qquad X_1 = 1 + \frac{.1}{4} = \boxed{\frac{41}{40}}$

$X_2 = \frac{41}{40} + \frac{.1}{4} = \boxed{\frac{21}{20}} \qquad X_3 = \frac{21}{20} + \frac{.1}{4} = \boxed{\frac{43}{40}} \qquad X_4 = \frac{43}{40} + \frac{.1}{4} = \boxed{\frac{11}{10}}$

Trap Rule $\approx \frac{1.1 - 1}{2(4)} \left[(.84147) + 2(.86773) + 2(.89234) + 2(.91505) + (.93562) \right]$

$\approx \boxed{.089092}$

Simpson Rule $\approx \frac{1.1 - 1}{3(4)} \left[(.84147) + 4(.86773) + 2(.89234) + 4(.91505) + (.93562) \right]$

$\approx \boxed{.089107}$

21) $\int_0^{\pi/4} x \tan x \, dx$

$n = 4$

$a = 0 \quad b = \frac{\pi}{4}$

$\Delta x = \frac{\frac{\pi}{4}}{4} = \boxed{\frac{\pi}{16}} \quad x_0 = \boxed{0} \quad x_1 = \boxed{\frac{\pi}{16}}$

$x_2 = \frac{\pi}{16} + \frac{\pi}{16} = \boxed{\frac{\pi}{8}} \quad x_3 = \frac{\pi}{8} + \frac{\pi}{16} = \boxed{\frac{3\pi}{16}} \quad x_4 = \frac{\pi}{16} + \frac{3\pi}{16} = \boxed{\frac{\pi}{4}}$

Trap Rule $\approx \frac{b-a}{2(n)}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$\approx \frac{\pi}{32}\left[(0) + 2(.03906) + 2(.16266) + 2(.39359) + (.7854)\right]$

$\approx \boxed{.193995}$

Simpson Rule $\int_a^b f(x)\,dx \approx \frac{b-a}{3(n)}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$

$\approx \frac{\pi}{48}\left[0 + 4(.03906) + 2(.16266) + 4(.39359) + (.7854)\right]$

$\approx \boxed{.185964}$

29) $\int_0^{\pi} \cos x \, dx$ $\quad n=4$, $a=0$ $\quad b=\pi$

Trap Rule

$$E \leq \frac{(b-a)^3}{12n^2}\left[\max|f''(x)|\right]$$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x \longrightarrow f''(0) = -\cos 0 = \boxed{-1}$$

$$f''(\pi) = -\cos\pi = \boxed{+1}$$

$$E \leq \frac{(\pi-0)^3}{12(4)^2}\left[1\right] \longrightarrow E \leq \frac{\pi^3}{192}\left[1\right]$$

$$\boxed{E \leq 0.161491}$$

Simpson Rule

$$E \leq \frac{(b-a)^5}{180(n)^4}\left[\max|f^{IV}(x)|\right]$$

$$f'''(x) = \sin x$$

$$f^{IV}(x) = \cos x \longrightarrow f^{IV}(0) = \cos(0) = \boxed{1}$$

$$f^{IV}(\pi) = \cos(\pi) = \boxed{-1}$$

$$E \leq \frac{(\pi-0)^5}{180(4)^4}\left[1\right]$$

$$E \leq \frac{\pi^5}{46080}(1)$$

$$\boxed{E \leq .006641}$$

33) $\int_0^1 \cos(\pi x)\,dx$ $\quad n = ? \quad a=0 \quad b=1$

$E < .00001$

$f'(x) = -\pi \sin \pi x \qquad \max f''(x) = -\pi^2 \cos(\pi(0)) = \boxed{-9.8696}$

$f''(x) = -\pi^2 \cos \pi x$

$\max f''(x) = -\pi^2 \cos(\pi) = \boxed{+9.8696}$

Trap Rule $\quad E \leq \frac{(b-a)^3}{12n^2}\left[\max\left|f''(x)\right|\right] \leq 10^{-5}$

$.00001 \gtreqless \frac{(1-0)^3}{12n^2}\left[9.8696\right]$

Why does the sign switch?

$12n^2(.00001) \gtreqless 9.8696$

$n^2 \leq \frac{9.8696}{12(.00001)} \longrightarrow \boxed{n \gtreqless \pm\sqrt{82246.7} \text{ or } n \gtreqless 286.8}$ should be $n \geq$

Simpson Rule $\quad f'''(x) = \pi^3 \sin \pi x$

$f^{IV}(x) = \pi^4 \cos \pi x \qquad \max f^{IV}(x) = \pi^4 \cos(\pi 0) = \boxed{\pi^4}$

$E \leq \frac{(b-a)^5}{180n^4}\left[\max\left|f^{IV}(x)\right|\right] < 10^{-5}$

$10^{-5} \gtreqless \frac{1}{180n^4}(\pi^4) \longrightarrow 10^{-5} \gtreqless \frac{\pi^4}{180n^4}$

$180n^4(10^{-5}) \gtreqless \pi^4 \longrightarrow n^4 \gtreqless \frac{\pi^4}{180(10^{-5})}$

$n^4 \gtreqless \frac{\pi^4(10^5)}{180} = \boxed{n \gtreqless \sqrt[4]{\frac{\pi^4(10^5)}{180}}}$

37) $\int_0^1 \operatorname{Tan} x^2 \, dx$ $\quad E = 10^{-5}$

$a = 0 \quad b = 1$

$f'(x) = 2x \operatorname{Sec}^2 x$

$f''(x) = x(\operatorname{Sec}^2 x) + 2x(2\operatorname{Sec} x \cdot \operatorname{Sec} x \operatorname{Tan} x)$

$f'''(x) = 2(2x \operatorname{Tan} x + 1)\operatorname{Sec}^2 x$

$f^{iv}(x) = 8\operatorname{Sec}^2 x\,[12x^2 + (3 + 32x^4)\operatorname{Tan}(x^2) + 36x^2 \operatorname{Tan}^2(x^2) + 48x^4 \operatorname{Tan}^3(x^2)]$

Trap Rule $\quad E \leq \frac{(1-0)^3}{12n^2}[49.5305] < 10^{-5}$

$n^2 > 412{,}754.17 \rightarrow \boxed{\begin{array}{l} n > 642.46 \\ n > 643 \end{array}}$

Simpson Rule $\quad E \leq \frac{(1-0)^5}{180n^4}[9184.4734] < 10^{-5}$

$n^4 > 5{,}102{,}485.22$

$\boxed{\begin{array}{l} n > 47.53 \\ n > 48 \end{array}}$

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Section 5.7

1) $\int \frac{5}{x}\,dx = 5\int \frac{1}{x}\,dx = \boxed{5\ln|x| + c}$

5) $\int \frac{1}{3-2x}\,dx$ $\quad U = 3-2x$
$\quad du = -2$

$-\frac{1}{2}\int \frac{1}{3-2x}(-2)\,dx \longrightarrow -\frac{1}{2}\int \frac{1}{u}\,du$

$= -\frac{1}{2}\ln|u| + c \longrightarrow \boxed{-\frac{1}{2}\ln|(3-2x)| + c}$

9) $\int \frac{x^2-4}{x}\,dx \longrightarrow \int \frac{x^2}{x} - \frac{4}{x}\,dx$

$= \int x - \frac{4}{x}\,dx = \int x\,dx - 4\int \frac{1}{x}\,dx$

$= \boxed{\frac{x^2}{2} - 4\ln|x| + c}$

13) $\int \frac{x^2-3x+2}{x+1}\,dx$

$$\begin{array}{r l}
 & x-4 \\
x+1 & \overline{)\,x^2-3x+2} \\
 & -(x^2+x) \\
 & -4x+2 \\
 & -(-4x-4) \\
 & \boxed{6}
\end{array}$$

$\int\left(x-4+\frac{6}{x+1}\right)dx = \int x-4 + 6\int \frac{1}{x+1}\,dx$

$= \boxed{\frac{x^2}{2}-4x+6\ln|(x+1)|+C}$

17) $\int \frac{x^4+x-4}{x^2+2}\,dx$

$$\begin{array}{r l}
 & x^2-2 \\
x^2+2 & \overline{)\,x^4+0x^3+0x^2+x-4} \\
 & -(x^4+0+2x^2) \\
 & -2x^2+x \\
 & -(2x^2+0-4) \\
 & \boxed{x}
\end{array}$$

$\int\left(x^2-2+\frac{x}{x^2+2}\right)dx$

$\int x^2-2\,dx + \int \frac{x}{x^2+2}\,dx$ $\qquad u=x^2+2$, $du = 2x\,dx$

$\int x^2-2\,dx + \frac{1}{2}\int \frac{1}{x^2+2}\cdot 2dx$

can you substitute U terms
In only one of the integrals?

$$\int x^2 - 2\,dx + \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{x^3}{3} - 2x + \frac{1}{2}\ln|u| + C$$

$$= \boxed{\frac{x^3}{3} - 2x + \frac{1}{2}\ln|(x^2+2)| + C}$$

21) $\int \frac{1}{\sqrt{x+1}}\,dx$

$u = (x+1)$
$du = 1\,dx$

$$\int \frac{1}{u^{\frac{1}{2}}}\,du \longrightarrow \int u^{-\frac{1}{2}}\,du$$

$$= 2u^{\frac{1}{2}} + C \longrightarrow 2(x+1)^{\frac{1}{2}} + C$$

$$= \boxed{2\sqrt{x+1} + C}$$

25) $\int \frac{1}{1+\sqrt{2x}}\,dx$

$U = 1+\sqrt{2x} = 1+(2x)^{\frac{1}{2}}$

$dU = \frac{1}{2}(2x)^{-\frac{1}{2}} \cdot 2 = \frac{1}{\sqrt{2x}}\,dx$

$dx = \sqrt{2x}\,du$

$U = 1+\sqrt{2x}$

$\boxed{\sqrt{2x} = U-1}$

$dx = (U-1)\,du$

$\int \frac{1}{U}\cdot(U-1)\,du$

$\int \frac{U-1}{U}\,du \longrightarrow \int\left(\frac{U}{U}-\frac{1}{U}\right)du$

$= \int\left(1-\frac{1}{U}\right)du = \int 1\,du - \int\frac{1}{U}\,du$

$= U - \ln|U| + C$

$= \boxed{1+\sqrt{2x} - \ln\left|(1+\sqrt{2x})\right| + C}$

Should be

$\sqrt{2x} - \ln(1+\sqrt{2x}) + C$?

29) $\int \frac{\cos\theta}{\sin\theta}\, d\theta$ $\quad U = \sin\theta$
$du = \cos\theta\, d\theta$

$\int \frac{1}{U}\, du = \ln|U| + C$

$= \boxed{\ln|\sin\theta| + C}$

33) $\int \frac{\cos t}{1+\sin t}\, dt =$ $\quad U = 1+\sin t$
$dU = \cos t\, dt$

$\int \frac{1}{1+\sin t} \cdot \cos t\, dt \longrightarrow \int \frac{1}{U}\, du$

$\ln|U| + C = \boxed{\ln|1+\sin t| + C}$

37) $\int e^{-x} \tan(e^{-x})\, dx$ $\quad U = e^{-x}$
$du = -e^{-x} dx$
$dx = \frac{du}{-e^{-x}} = -e^{x} du$

$\int \tan(e^{-x}) \cdot e^{-x} dx$

$-\int \tan(e^{-x}) \cdot (-e^{-x})\, dx$

$\longrightarrow$

$$-\int \tan u \, du \longrightarrow -\int \frac{\sin u}{\cos u}\, du \qquad \begin{aligned} v &= \cos u \\ dv &= -\sin u\, du \end{aligned}$$

$$= \int \frac{1}{\cos u} \cdot (-\sin u)\, du$$

$$= \int \frac{1}{v}\, dv = \ln|v| + C$$

$$= \ln|\cos u| + C$$

$$= \boxed{\ln|\cos(e^{-x})| + C}$$

49) $$\int_0^4 \frac{5}{3x+1}\, dx = \int_0^4 \frac{1}{3x+1} \cdot 5\, dx$$

$$\begin{aligned} u &= 3x+1 \\ du &= 3\, dx \end{aligned} \qquad \frac{5}{3}\int_0^4 \frac{1}{3x+1} \cdot 3\, dx$$

$$\begin{aligned} u &= 3(0)+1 = \boxed{1} \\ u &= 3(4)+1 = \boxed{13} \end{aligned} \qquad \frac{5}{3}\int_1^{13} \frac{1}{u}\, du = \frac{5}{3}\ln|u|\Big|_1^{13}$$

$$= \frac{5}{3}\ln|3x+1|\Big|_0^4 = \frac{5}{3}\Big[\ln|3(4)+1| - \ln|3(0)+1|\Big]$$

$$= \frac{5}{3}\Big[\ln 13 - \ln 1\Big] = \boxed{\frac{5}{3}\ln 13}$$

53) $\int_0^2 \frac{x^2-2}{x+1}\,dx =$

$$\begin{array}{r} x-1 \\ x+1 \overline{\smash{)}\,x^2+0x-2} \\ -(x^2+x) \\ \hline -x-2 \\ -(-x-1) \\ \hline \boxed{-1} \end{array}$$

$$= \int_0^2 \left(x-1+\frac{-1}{x+1}\right)dx$$

$$= \left[\frac{x^2}{2} - x - \ln|x+1|\right]\Big|_0^2$$

$$= \left[\frac{1}{2}(2)^2 - 2 - \ln|2+1|\right] - \left[\frac{1}{2}(0)^2 - 0 - \ln|0+1|\right]$$

$$= \left[0 - \ln 3\right] - \left[\ln 1\right] = \boxed{-\ln 3}$$

55) $\int_1^2 \frac{1-\cos\theta}{\theta-\sin\theta}\,d\theta$

$u = \theta - \sin\theta$
$du = 1-\cos\theta\, d\theta$

$$\int_1^2 \frac{1}{\theta-\sin\theta}\cdot(1-\cos\theta)\,d\theta$$

$u = 1-\sin(1) =$
$u = .158529$
$u = 2-\sin(2)$
$u = 1.090703$

$$\int_{.158529}^{1.090703} \frac{1}{u}\,du = \ln u = \left[\ln|\theta-\sin\theta|\right]\Big|_1^2$$

$$= \left[\ln(1.090703)\right] - \left[\ln(.158529)\right]$$

$$= \boxed{1.928640}$$

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Section 5.8

1) $\int \frac{5}{\sqrt{9-x^2}}dx = 5\int \frac{1}{\sqrt{3^2-x^2}}dx$

$= \boxed{5\sin^{-1}\frac{x}{3} + C}$

5) $\int \frac{1}{x\sqrt{4x^2-1}}dx = \int \frac{1}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$

$2\int \frac{1}{2x\sqrt{(2x)^2-1^2}} = \frac{1}{1}\sec^{-1}\frac{2x}{1} + C$

$= \boxed{\sec^{-1}2x + C}$

9) $\int \frac{1}{\sqrt{1-(x+1)^2}}dx$

$u = x+1$
$du = 1dx$

$\int \frac{1}{\sqrt{1^2-u^2}}du = \sin^{-1}\frac{u}{1} + C$

$\boxed{\sin^{-1}(x+1) + C}$

$$\frac{x}{x^2+1} \longrightarrow \frac{1}{x+1} \text{ ? why not}$$

13) $\displaystyle\int \frac{e^{2x}}{4+e^{4x}} = \int \frac{e^{2x}}{(2)^2+(e^{2x})^2}\,dx$

$u = e^{2x}$
$du = 2e^{2x}$

$\displaystyle\frac{1}{2}\int \frac{1}{2^2+(e^{2x})^2}\cdot(2e^{2x})\,dx$

$\displaystyle=\frac{1}{2}\int \frac{1}{2^2+u^2}\,du = \frac{1}{2}\left[\frac{1}{a}\operatorname{Tan}^{-1}\frac{u}{a}+C\right]$

$\displaystyle\frac{1}{2}\left[\frac{1}{2}\operatorname{Tan}^{-1}\frac{u}{2}+C\right] = \boxed{\frac{1}{4}\operatorname{Tan}^{-1}\frac{e^{2x}}{2}+C}$

17) $\displaystyle\int \frac{x-3}{x^2+1}\,dx = \int \frac{x}{x^2+1}\,dx - 3\int \frac{1}{x^2+1}\,dx$

$u = x^2+1$
$du = 2x\,dx$

$\displaystyle\frac{1}{2}\int \frac{1}{x^2+1}(2dx) - 3\int \frac{1}{x^2+1}\,dx$

$\displaystyle\frac{1}{2}\int \frac{1}{u}\,du - 3\int \frac{1}{x^2+1^2}\,dx$

$\displaystyle= \frac{1}{2}\ln|u| - 3\operatorname{Tan}^{-1}\frac{x}{1} + C$

$\displaystyle= \boxed{\frac{1}{2}\ln|x^2+1| - 3\operatorname{Tan}^{-1}x + C}$

why transform in terms of u?

21) $\int_0^{\frac{1}{6}} \frac{1}{\sqrt{1-9x^2}}\,dx = \int_0^{\frac{1}{6}} \frac{1}{\sqrt{1^2-(3x)^2}}\,dx$

$u = 3x$

$du = 3dx$

$\frac{1}{3}\int \frac{1}{\sqrt{1^2-(3x)^2}} \cdot 3dx$

$\frac{1}{3}\int \frac{1}{\sqrt{1^2-u^2}}\,du = \left[\frac{1}{3}\sin^{-1}3x\right]\Big|_0^{\frac{1}{6}}$

$= \frac{1}{3}\left[\sin^{-1}\left(3\cdot\frac{1}{6}\right)\right] - \left[\sin^{-1}(3\cdot 0)\right]$

$= \frac{1}{3}\left[\sin\theta = \frac{1}{2}\right] - \left[\sin\theta = 0\right]$

$= \frac{1}{3}\left(\frac{\pi}{6} - 0\right) = \boxed{\frac{\pi}{18}}$

25) $\int_0^{\frac{1}{2}} \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$u = \sin^{-1}x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \int \frac{1}{\sqrt{1-x^2}}\,dx \cdot \sin^{-1}x$

$= \int u\,du = \frac{u^2}{2}$

$$\frac{(\sin^{-1}x)^2}{2} = \left[\frac{1}{2}(\sin^{-1}x)^2\right]\Big|_0^{\frac{1}{2}}$$

$$\frac{1}{2}\left(\sin^{-1}\frac{1}{2}\right)^2 - \left(\sin^{-1}0\right)^2$$

$$= \boxed{\frac{\pi^2}{32}}$$

29) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$ $\qquad u = \cos x$, $du = -\sin x\,dx$

$$- \int \frac{1}{1+\cos^2 x} \cdot (-\sin x\,dx)$$

$$- \int \frac{1}{1^2+u^2}\,du = -\left(\operatorname{Tan}\frac{u}{1}\right)$$

$$= \left(-\operatorname{Tan}\cos x\right)\Big|_{\frac{\pi}{2}}^{\pi}$$

$$= -\operatorname{Tan}(\cos\pi) - \operatorname{Tan}\left(\cos\frac{\pi}{2}\right)$$

$$- \operatorname{Tan}(-1) - \operatorname{Tan}(0)$$

$$\boxed{\frac{\pi}{4}}$$

33) $\int \frac{2x}{x^2+6x+13}\,dx \longrightarrow \int \frac{2x+6\ -6}{x^2+6x+13}$

$\int \frac{2x+6}{x^2+6x+13}\,dx - 6\int \frac{1}{x^2+6x+13}$ $\quad U = x^2+6x+13$
$du = 2x+6$

$\int \frac{1}{U}\,du - 6\int \frac{1}{4+(x+3)^2}\,dx$

$= \ln|U| - 6\left[\frac{1}{2}\tan^{-1}\frac{(x+3)}{2}\right] + C$

$= \boxed{\ln|x^2+6x+13| - 3\tan^{-1}\frac{(x+3)}{2} + C}$

37) $\int \frac{x+2}{\sqrt{-x^2-4x}} =$ $\quad U = -x^2-4x$
$du = -2x-4\,dx$

$-\frac{1}{2}\int \frac{1}{\sqrt{-x^2-4x}}\cdot(-2x-4)\,dx = -\frac{1}{2}\int \frac{1}{\sqrt{U}}\,du$

$-\frac{1}{2}\int U^{-\frac{1}{2}}\,du = -\frac{1}{2}\left(2U^{\frac{1}{2}}\right) + C$

$= -U^{\frac{1}{2}} + C = \boxed{-\sqrt{x^2+4x} + C}$

41) $\int \frac{x}{x^4+2x^2+2}\,dx = -\int \frac{x}{(x^2+1)^2+1}\,dx$

$U = x^2+1$

$du = 2x\,dx$ $\quad -\frac{1}{2}\int \frac{1}{(x^2+1)^2+1}\cdot(2x\,dx)$

$-\frac{1}{2}\int \frac{1}{U^2+1^2}\,du \longrightarrow -\frac{1}{2}\left[\tan^{-1}U + c\right]$

$\boxed{-\frac{1}{2}\tan^{-1}(x^2+1)+c}$

45) $\int_1^3 \frac{1}{\sqrt{x}(1+x)}\,dx$ $\qquad U = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$U^2 = x$

$2\int \frac{1}{2\sqrt{x}\,(1^2+x^2)}\,dx = 2\int \frac{1}{1^2+x^2}\cdot\frac{1}{2\sqrt{x}}\,dx$

$2\int \frac{1}{1^2+U^2}\,du = 2\tan^{-1}U + C$

$= \left[2\tan^{-1}\sqrt{x}+c\right]\Big|_1^3$

$(2\tan^{-1}\sqrt{3}+\cancel{c}) - (\tan^{-1}1+\cancel{c})$

$2\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = 2\left(\frac{\pi}{12}\right) = \boxed{\frac{\pi}{6}}$

Steven Romeiro

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Section 5.9

1) $\sinh 3 = \dfrac{e^{3} - e^{-3}}{2} \approx \boxed{10.018}$

$\tanh -2 = \dfrac{\dfrac{e^{-2} - e^{2}}{2}}{\dfrac{e^{-2} + e^{2}}{2}} \approx \boxed{-0.964}$

5) $\cosh^{-1} 2 = \ln(2+\sqrt{3}) = \boxed{1.317}$

$\operatorname{sech}^{-1} \frac{2}{3} = \ln\left(\dfrac{1 + \sqrt{1-(4/9)}}{2/3}\right)$

$= \boxed{0.962}$

9) $K = \left(\frac{e^{x}-e^{-x}}{2}\right)\left(\frac{e^{y}+e^{-y}}{2}\right)+\left(\frac{e^{x}+e^{-x}}{2}\right)\left(\frac{e^{y}-e^{-y}}{2}\right)$

$K = \frac{1}{4}\left[2\left(e^{x+y}-e^{-(x+y)}\right)\right]$

$K = \frac{e^{(x+y)}-e^{-(x+y)}}{2}$

$K = \sinh(x+y)$

$\boxed{K=K}$

12) $f(x) = \ln(\sinh x)$

$f'(x) = \frac{1}{\sinh x} \cdot (\cosh x)$

$\boxed{f'(x) = \coth x}$

21) $f(x) = \frac{1}{4}\sinh 2x - \frac{x}{2}$

$f'(t) = \frac{1}{2}\cosh 2x - \frac{1}{2}$

$f'(t) = \frac{\cosh(2x) - 1}{2}$

$\boxed{f'(t) = \sinh^2 x}$

25) $y = \sinh(1 - x^2)$

$y = f(t)$

$f(t) = \sinh(1 - x^2) \qquad (1, 0)$

$f'(t) = \cosh(1 - x^2)(-2x)$

$f'(1) = \cosh(1 - 1^2)(-2)$

$f'(1) = -2$

$y - 0 = -2(x - 1)$

$\boxed{y = -2x + 2}$

33) $y = a \sinh x \quad = \quad y''' - y' = 0$

$y' = a \cosh x$

$y'' = a \sinh x$

$y''' = a \cosh x$

$$\boxed{y''' - y' = a \cosh x - a \cosh x = 0}$$

37) $y = 10 + 15 \cosh \frac{x}{15} \qquad [-15, 15]$

$x = \pm 15$

$y = 10 + 15 \cosh \frac{15}{15}$

$y = 10 + 15 \cosh (1) = \boxed{33.146}$

$x = 0$

$y = 10 + 15 \cosh(0) = \boxed{25}$

$y' = \sinh \frac{x}{15}$

$x = 15$

$y' = \sinh(1)$

$= \boxed{1.175}$

41) $\int \cosh^2(x-1) \sinh(x-1)\, dx$

$u = \cosh(x-1)$
$du = \sinh(x-1)\,dx$

$\int u^2\, du = \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}\cosh^3(x-1) + C}$

49) $\int \frac{x}{x^4+1}\, dx$

$u = x^2$
$du = 2x\, dx$

$\frac{1}{2}\int \frac{2x}{(x^2)^2 + 1^2}\, dx = \boxed{\frac{1}{2}\tan^{-1}(x^2) + C}$

57) $y = \cosh^{-1}(3x)$

$y' = \boxed{\frac{3}{\sqrt{9x^2-1}}} = \frac{u'}{\sqrt{u^2-1}}$

$u = 3x$
$du = 3dx$

61) $y = \tan^{-1}(\sin 2x)$

$$y' = \frac{U'}{1-U^2}$$

$U = \sin 2x$
$du = 2\cos 2x$

$$y' = \frac{1}{1-\sin^2 2x} \cdot (2\cos 2x)\,dx$$

$$\boxed{y' = 2\sec 2x}$$

71) $\lim_{x\to 0} \frac{\sinh x}{x}$

$\sinh(u) = \frac{e^u - e^{-u}}{2}$

$$= \lim_{x\to 0} \frac{e^x - e^{-x}}{2x}$$

$$\frac{e^0 - e^{-0}}{2x} = \frac{1-1}{2(0)} = \boxed{1}$$

79) $\displaystyle\int \frac{1}{1-4x-2x^2}\,dx$

$$\int \frac{1}{1-4x-2x^2}\,dx = \int \frac{1}{3-2(x+1)^2}\,dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{\sqrt{2}}{(\sqrt{3})^2 - [\sqrt{2}(x+1)]^2}\,dx$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2\sqrt{3}} \ln\left(\frac{\sqrt{3}+\sqrt{2}(x+1)}{\sqrt{3}-\sqrt{2}(x+1)}\right) + C$$

$$= \boxed{\frac{1}{2\sqrt{6}} \ln\left(\frac{\sqrt{2}(x+1)+\sqrt{3}}{\sqrt{2}(x+1)-\sqrt{3}}\right) + C}$$