HW 2 Solutions

1. (2 points) Convert the decimal number 25.84375 to binary with a maximum of six places to the right of the binary point.

   11001.11011.

2. (2 points) Convert the hexadecimal number $AC12_{16}$ to binary.

   1010 1100 0001 0010_2.

3. (6 points) Represent the following decimal numbers in binary using 8-bit signed magnitude, one's complement, and two's complement representations: a) 60 b) -60

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a. Signed magnitude: 00111100
   One's complement: 00111100
   Two's complement: 00111100

b. Signed magnitude: 10111100
   One's complement: 11000011
   Two's complement: 11000100

4. (4 points) What decimal value does the 8-bit binary number 10011110 have if:
   a) it is interpreted as an unsigned number?
   b) it is on a computer using signed-magnitude representation?
   c) it is on a computer using ones complement representation?
   d) it is on a computer using twos complement representation?

   $a)158.\ b)-30.\ c)-97.\ d)-98.$

5. (4 points) Given the following two binary numbers: 11111100 and 01110000.
   a) Which of these two numbers is the larger unsigned binary number?
   b) Which of these two is the larger when it is being interpreted on a computer using signed-two's complement representation?
   c) Which of these two is the smaller when it is being interpreted on a computer using signed-magnitude representation?

Ans.

a. 11111100 b. 01110000 c. 11111100

6. (3 points) Using a "word" of 4 bits, list all of the possible signed binary numbers and their decimal equivalents that are representable in:
   a) Signed magnitude b) One's complement c) Two's complement

Ans.

a. 0111 to 1111, or +7 to −7
b. 0111 to 1000, or +7 to −7
c. 0111 to 1000, or +7 to −8

7. (3 points) From the results of the previous two questions, generalize the range of values (in decimal) that can be represented in any given x number of bits using:
a) Signed magnitude b) One's complement c) Two's complement

Ans.

a. $-(2^{x-1}-1)$ to $+(2^{x-1}-1)$
b. $-(2^{x-1}-1)$ to $+(2^{x-1}-1)$
c. $-(2^{x-1})$ to $+(2^{x-1}-1)$

8. (3 points) Using arithmetic shifting, perform the following (assume the binary strings are in 2's complement format):
a) double the value 00010101_2
b) quadruple the value 01110111_2
c) divide the value 11001010_2 in half

Ans.

a. 0010 1010 b. Error (sign bit changed) c. 1110 0101

9. (5 points) Decode the following ASCII message, assuming 7-bit ASCII characters and no parity: 1001010 1001111 1001000 1001110 0100000 1000100 1001111 1000101

Ans.

100 1010 = J
100 1111 = O
100 1000 = H
100 1110 = N
010 0000 = space
100 0100 = D
100 1111 = O
100 0101 = E

10. (5 points) Assume we wish to create a code using 3 information bits, 1 parity bit (appended to the end of the information), and odd parity. List all legal code words in this code.

Ans.

The legal code words are:

0001	1000
0010	1011
0100	1101
0111	1110

11. (3 points) Suppose we are given the following subset of codewords, created for a 7-bit memory word with one parity bit: 11100110, 00001000, 10101011, and 11111110. Does this code use even or odd parity? Explain.

Ans.

Each codeword has an odd numbers of pits so odd parity is used.

12. (10 points) Perform the following unsigned operations.

(a) $100111_2 + 111001_2$,

(b) $10110_2 - 101_2$,

(a) $100111_2 + 111001_2$,

$$\begin{array}{cccccccc}
\textit{carry} & 1 & 1 & 1 & 1 & 1 & 1 & \\
 & & 1 & 0 & 0 & 1 & 1 & 1 \\
+ & & 1 & 1 & 1 & 0 & 0 & 1 \\
\hline
 & & 1 & 0 & 0 & 0 & 0 & 0
\end{array}$$

(b) $10110_2 - 101_2$,

$$\begin{array}{cccccc}
\textit{borrow} & & & & -1 & 2 \\
 & 1 & 0 & 1 & 1 & 0 \\
- & & & 1 & 0 & 1 \\
\hline
 & 1 & 0 & 0 & 0 & 1
\end{array}$$

13. (10 points) Let $x = 87$ and $y = 25$. Complete 2's complement operations for the following. Use 8 bits for binary numbers.

   (a) $x + y$
   (b) $x - y$
   (c) $-x + y$
   (d) $-x - y$

(a) $x + y$

The 2's complement representations of x and y are $0101\ 0111_2$ and $0001\ 1001_2$, respectively. $x + y$, which is $0111\ 0000_2$, is calculated as follows.

$$
\begin{array}{cccccccccc}
\textit{carry} & & & & 1 & 1 & 1 & 1 & 1 & \\
 & & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\
+ & & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
\hline
 & & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}
$$

(b) $x - y$

$x - y$ is equivalent to $x + (-y)$. To perform $x + (-y)$, we need to find the 2's complement representations of x and $-y$, which are $0101\ 0111_2$ and $1110\ 0111_2$, respectively. Then $x + (-y)$, which is $0011\ 1110_2$, is calculated as follows.

$$
\begin{array}{cccccccccc}
\textit{carry} & 1 & 1 & & & & 1 & 1 & 1 & \\
 & & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\
+ & & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\
\hline
 & & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0
\end{array}
$$

(c) $-x + y$

To perform $(-x) + y$, we need to find the 2's complement representations of $-x$ and y, which are $1010\ 1001_2$ and $0001\ 1001_2$, respectively. Then $(-x) + y$, which is $1100\ 0010_2$, is calculated as follows.

$$
\begin{array}{cccccccccc}
\textit{carry} & & & 1 & 1 & 1 & & & 1 & \\
 & & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
+ & & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
\hline
 & & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

(d) $-x - y$

$-x - y$ is equivalent to $(-x) + (-y)$. To perform $(-x) + (-y)$, we need to find the 2's complement representations of $-x$ and $-y$, which are $1010\ 1001_2$ and $1110\ 0111_2$, respectively. Then $(-x) + (-y)$, which is $1001\ 0000_2$, is calculated as follows.

$$
\begin{array}{cccccccccc}
\textit{carry} & 1 & 1 & 1 & & 1 & 1 & 1 & 1 & \\
 & & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
+ & & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\
\hline
 & & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{array}
$$