MAC 2312 - Calculus w/Analytic Geometry II

Name____________________________

Review for Test #1 - Chapter 7

Date____________________________

Evaluate the integral.

1) $\int_2^3 \frac{x^4+1}{x^5+5x}\,dx$

A) $\frac{1}{5}\ln\left|\frac{38}{249}\right|$

[✓ B) $\frac{1}{5}\ln\left|\frac{43}{7}\right|$]

C) $\frac{1}{5}\ln\left|\frac{2}{3}\right|$

D) $\frac{2}{3}\ln\left|\frac{3}{2}\right|$

2) $\int_0^{\pi/24} \frac{\sec^2 6x}{6+\tan 6x}\,dx$

A) $\ln\left|\frac{7}{6}\right|$

B) $e^{7/6}$

C) $\frac{1}{6}\ln\left|\frac{1}{6}\right|$

[✓ D) $\frac{1}{6}\ln\left|\frac{7}{6}\right|$]

3) $\int \frac{7e^{(7\sin 2x)}}{\sec 2x}\,dx$

A) $\frac{1}{2}\ln(\sec 2x)+C$

[✓ B) $\frac{1}{2}e^{(7\sin 2x)}+C$]

C) $7\ln(\sec 2x)+C$

D) $e^{(7\sin 2x)}+C$

4) $\int \frac{35e^{\sqrt{5x}}}{2\sqrt{x}}\,dx$

A) $\frac{35}{2}e^{\sqrt{5x}}+C$

B) $\sqrt{5}\,e^{\sqrt{5x}}+C$

[✓ C) $7\sqrt{5}\,e^{\sqrt{5x}}+C$]

D) $35\,e^{\sqrt{5x}}+C$

5) $\int (e^x+e^{-x})^2\,dx$

A) $\frac{1}{2}(e^{2x}+e^{-2x})+2x+C$

B) $\frac{1}{2}(e^{2x}+e^{-2x})+C$

C) $\frac{1}{2}(e^{2x}-e^{-2x})+C$

[✓ D) $\frac{1}{2}(e^{2x}-e^{-2x})+2x+C$]

6) $\int \frac{e^{3\theta}}{1+e^{3\theta}}\,d\theta$

A) $\ln(1+e^{3\theta})+C$

[✓ B) $\frac{\ln(1+e^{3\theta})}{3}+C$]

C) $\frac{\ln(1+3e^{\theta})}{3}+C$

D) $3\ln(1+e^{3\theta})+C$

7) $\int_1^{\sqrt{2}} x8^{x^2}\,dx$

[✓ A) $\frac{28}{\ln 8}$]

B) $\frac{8\sqrt{2}-8}{2\ln 8}$

C) 28

D) $\frac{8}{\ln 8}$

8) $\int \frac{\log_3 x}{x}\,dx$

A) $3^x \ln 3 + C$ [✓ B) $\frac{(\ln x)^2}{2 \ln 3} + C$] C) $\frac{\ln 3\,(\ln x)^2}{2} + C$ D) $\frac{\ln x}{\ln 3} + C$

Solve the initial value problem.

9) $\frac{dy}{dt} = e^t \sin(e^t - 10),\ y(\ln 10) = 0$

A) $y = \sin e^t - \sin 2$ B) $y = e^t \cos(e^t - 10) - 10$
[✓ C) $y = -\cos(e^t - 10) + 1$] D) $y = \cos(e^t - 10) - 1$

10) $\frac{dy}{dx} = -8e^{-x} \sec e^{-x} \tan e^{-x},\ y(0) = 8 \sec 1 + 8$

A) $y = -8 \sec x + 1$ [✓ B) $y = 8 \sec e^{-x} + 8$] C) $y = 8 \tan e^{-x} + 8$ D) $y = -8 \sec e^{-x} + 1$

11) $\frac{d^2y}{dx^2} = 9e^{-x},\ y(0) = 1, y'(0) = 0$

[✓ A) $y = 9e^{-x} + 9x - 8$] B) $y = 9e^{-x} + 1$ C) $y = 9e^{-x} - 9x + 10$ D) $y = -9e^{-x} + C$

Solve the problem.

12) The region between the curve $y = \frac{1}{x^2}$ and the x-axis from $x = \frac{1}{3}$ to $x = 3$ is revolved about the y-axis to generate a solid. Find the volume of the solid.

A) $2\pi \ln 3 - \pi$ [✓ B) $4\pi \ln 3$] C) $\pi \ln 3 - \pi$ D) $2\pi \ln 3$

Solve the differential equation.

13) $\frac{dy}{dx} = \frac{2y^2}{x}$

A) $y = -2\ln x + C$ B) $y = 6\ln x + C$ [✓ C) $y = \frac{-1}{2\ln x + C}$] D) $y = \frac{1}{2\ln x + C}$

14) $\frac{dy}{dx} = 7x^6 e^{-y}$

[✓ A) $y = \ln(x^7 + C)$] B) $y = \ln(7x^7 + C)$ C) $y = x^7 + C$ D) $y = C \ln(x^7)$

15) $\frac{dy}{dx} = 6x^5 \cos^2 y$

A) $y = \tan(x^6 + C)$ B) $y = x^6 + C$ [✓ C) $y = \tan^{-1}(x^6 + C)$] D) $y = \tan^{-1}(x^5 + C)$

Solve the problem.

16) A loaf of bread is removed from an oven at 350° F and cooled in a room whose temperature is 70° F. If the bread cools to 210° F in 20 minutes, how much longer will it take the bread to cool to 190° F.

A) 24 min B) 5 min C) 17 min [✓ D) 4 min]

17) Find the half-life of the radioactive element radium, assuming that its decay constant is $k = 4.332 \times 10^{-4}$, with time measured in years.

A) 800 years B) 1400 years [✓ C) 1600 years] D) 2308 years

18) The charcoal from a tree killed in a volcanic eruption contained 64.9% of the carbon-14 found in living matter. How old is the tree, to the nearest year? Use 5700 years for the half-life of carbon-14.

[✓ A) 3555 years] B) 2464 years C) 5700 years D) 1708 years

A value of sinh x or cosh x is given. Use the definitions and the identity $\cosh^2 x - \sinh^2 x = 1$ to find the value of the other indicated hyperbolic function.

19) $\sinh x = \frac{5}{12}$, $\cosh x =$

A) $\frac{169}{144}$ ~~B) $\frac{12}{13}$~~ [✓ C) $\frac{13}{12}$] D) $-\frac{13}{12}$

Rewrite the expression in terms of exponentials and simplify the results.

20) $\cosh 4x + \sinh 4x$

A) $e^{4x} - e^{-4x}$ B) $4x$ C) $2e^{4x}$ [✓ D) e^{4x}]

Find the derivative of y.

21) $y = \cosh x^7$

A) $-\sinh x^7$ B) $\sinh x^7$ C) $-7x^6 \sinh x^7$ [✓ D) $7x^6 \sinh x^7$]

22) $y = \ln (\sinh 5x)$

A) $5 \operatorname{csch} 5x$ B) $\coth 5x$ C) $\frac{1}{\sinh 5x}$ [✓ D) $5 \coth 5x$]

Find the derivative of y with respect to the appropriate variable.

23) $y = \sinh^{-1} \sqrt{11x}$

A) $\frac{1}{2\sqrt{11x}(1 + 11x)}$ [✓ B) $\frac{11}{2\sqrt{11x}(1 + 11x)}$] C) $\frac{11}{2\sqrt{11x}(11x - 1)}$ D) $\frac{1}{\sqrt{1 + 11x}}$

24) $y = (\theta^2 + 9\theta) \tanh^{-1} (\theta + 8)$

A) $-\frac{\theta}{\theta + 7}$

B) $(2\theta + 9) \tanh^{-1} (\theta + 8) - \frac{\theta^2 + 9\theta}{1 + (\theta + 8)^2}$

[✓ C) $(2\theta + 9) \tanh^{-1} (\theta + 8) - \frac{\theta}{\theta + 7}$]

D) $(2\theta + 9) - \frac{1}{\theta + 63}$

25) $y = \sinh^{-1} (\cos x)$

A) $\frac{1}{\sqrt{1 + \cos^2 x}}$ B) $\frac{-\sin x}{\sqrt{1 + x^2}}$ C) $\frac{-\sin x}{\sqrt{1 + \cos^2 x}}$ D) $-\sin x$

Answer Key
Testname: MAC 2312 - REV T1 - CH 7

1) B
2) D
3) B
4) C
5) D
6) B
7) A
8) B
9) C
10) B
11) A
12) B
13) C
14) A
15) C
16) D
17) C
18) A
19) C
20) D
21) D
22) D
23) B
24) C
25) C
26) B
27) B
28) B
29) B
30) FALSE
31) FALSE

Evaluate the integral.

26) $\int \cosh \frac{x}{9}\, dx$

A) $\sinh \frac{x}{9} + C$ [✓ B) $9 \sinh \frac{x}{9} + C$] C) $-9 \sinh \frac{x}{9} + C$ D) $\sin^{-1} \frac{x}{9} + C$

27) $\int 5 \sinh (4x - \ln 5)\, dx$

A) $20 \cosh (4x - \ln 5) + C$ [✓ B) $\frac{5}{4} \cosh (4x - \ln 5) + C$]

C) $5 \cosh (4x - \ln 5) + C$ D) $\frac{1}{4} \cosh 4x + C$

28) $\int \operatorname{csch}^2\left(2 - \frac{x}{2}\right) dx$

A) $-\coth\left(2 - \frac{x}{2}\right) + C$ B) $2 \coth\left(2 - \frac{x}{2}\right) + C$ C) $2 \tanh\left(2 - \frac{x}{2}\right) + C$ D) $\frac{2}{3} \operatorname{csch}^3\left(2 - \frac{x}{2}\right) + C$

Find the slowest growing and the fastest growing functions as x→∞.

29) $y = 2x^2 + 4x$ mid

$y = e^x$

$y = e^{x/7}$ } fastest

$y = \log_7 x$ slowest

A) Slowest: $2x^2 + 4x$
Fastest: $y = e^x$

[✓ B)] Slowest: $y = \log_7 x$
Fastest: $y = e^x$ and $y = e^{x/7}$ grow at the same rate.

C) Slowest: $y = \log_7 x$
Fastest: $y = e^x$

D) Slowest: $y = e^{x/7}$
Fastest: $2x^2 + 4x$

Determine if the statement is true or false as x→∞.

30) $\ln x = o(\ln 4x)$

31) $2x^3 + \cos x = O(2x^2)$

Answer Key
Testname: MAC 2312 - REV T1 - CH 7

1) B
2) D
3) B
4) C
5) D
6) B
7) A
8) B
9) C
10) B
11) A
12) B
13) C
14) A
15) C
16) D
17) C
18) A
19) C
20) D
21) D
22) D
23) B
24) C
25) C
26) B
27) B
28) B
29) B
30) FALSE
31) FALSE

MAC 2312 - Calculus with Analytic Geometry II

Name______________________________

Final Exam Review

Date______________________________

Solve the initial value problem.

1) $\frac{dy}{dx} + xy = 3x;\ y(0) = -4$

A) $y = 3e^{-x^2/2} - 7$
B) $y = -7e^{x^2/2} + 3$
C) $y = 3e^{x^2/2} - 7$
D) $y = -7e^{-x^2/2} + 3$

Evaluate the integral.

2) $\int \cos^{-1} x\, dx$

A) $x \cos^{-1} x + \sqrt{1 - x^2} + C$
B) $x \cos^{-1} x - 2\sqrt{1 - x^2} + C$
C) $x \cos^{-1} x - \sqrt{1 - x^2} + C$
D) $x \cos^{-1} x - \frac{1}{\sqrt{1 - x^2}} + C$

3) $\int -8x \cos 2x\, dx$

A) $- 4 \cos 2x - 8x \sin 2x + C$
B) $- 2 \cos 2x - 4x \sin 2x + C$
C) $- 2 \cos 2x - 4 \sin 2x + C$
D) $- 2 \cos 2x - 4x \sin 8x + C$

4) $\int 7 \cos^3 5x\, dx$

A) $\frac{7}{5} \sin 5x - \frac{7}{15} \sin^3 5x + C$
B) $7 \sin 5x - \frac{7}{3} \sin^3 5x + C$
C) $\frac{7}{5} \sin 5x + \frac{7}{15} \sin^3 5x + C$
D) $\frac{7}{5} \sin 5x - \frac{7}{15} \cos^3 5x + C$

5) $\int 4 \cos^4 6x\, dx$

A) $\frac{3}{2}x + \frac{1}{6} \sin 12x + \frac{1}{48} \sin 24x + C$
B) $3x + \frac{1}{6} \sin 6x + \frac{1}{12} \sin 24x + C$
C) $3x + \frac{1}{6} \sin 6x + \frac{1}{48} \sin 12x + C$
D) $\frac{3}{2}x + \frac{1}{3} \sin 12x + \frac{1}{8} \sin 24x + C$

Express the integrand as a sum of partial fractions and evaluate the integral.

6) $\int \frac{8x - 16}{x^2 - 4x - 5} dx$

A) $\ln|4(x - 5) + 4(x + 1)| + C$
B) $4\ln|x - 5| + 4\ln|x + 1| + C$
C) $5\ln|x - 5| - 4\ln|x + 1| + C$
D) $4\ln|x + 5| + 4\ln|x - 1| + C$

7) $\int \frac{x^3}{x^2 + 10x + 25} dx$

A) $\frac{x^2}{2} - 25x + 75 \ln|x + 5| + \frac{125}{x + 5} + C$

B) $75 \ln|x - 25| + \frac{75}{x + 5} - \frac{125}{(x + 5)^2} + C$

C) $\frac{x^2}{2} - 25x - 75 \ln|x + 5| + \frac{125}{(x + 5)^2} + C$

D) $\frac{x^2}{2} - 25x + 15 \ln|x + 5| - \frac{25}{x + 5} + C$

Use the integral test to determine whether the series converges.

8) $\sum_{n=1}^{\infty} \frac{15}{\sqrt{n}}$

A) diverges

B) converges

9) $\sum_{n=1}^{\infty} \frac{\cos 1/n}{n^2}$

A) converges

B) diverges

Use the limit comparison test to determine if the series converges or diverges.

10) $\sum_{n=1}^{\infty} \frac{5\sqrt{n}}{2n^{3/2} + 5n + 8}$

A) Diverges

B) Converges

Use the ratio test to determine if the series converges or diverges.

11) $\sum_{n=1}^{\infty} \frac{(2n)!}{4^n n!}$

A) Diverges

B) Converges

12) $\sum_{n=1}^{\infty} \frac{5n!}{n^n}$

Just Set up ratio test
Don't Solve

A) Diverges

B) Converges

Use the root test to determine if the series converges or diverges.

13) $\sum_{n=1}^{\infty} \left(\frac{\ln n}{3n - 4}\right)^n$

A) Diverges

B) Converges

Determine if the series converges absolutely, converges, or diverges.

14) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{4n^8 + 4}{8n^9 + 2}\right)$

A) Converges absolutely

B) Converges conditionally

C) Diverges

15) $\sum_{n=1}^{\infty} \frac{(-2)^n}{8n^6 + 8^n}$

A) Diverges B) converges conditionally C) Converges absolutely

Find the interval of convergence of the series.

Possible //

16) $\sum_{n=0}^{\infty} \frac{(x - 4)^n}{8 + 5n}$

A) $-1 < x < 9$ B) $3 \leq x < 5$ C) $\frac{19}{8} < x < \frac{45}{8}$ D) $-1 \leq x \leq 9$

Find the first four nonzero terms in the Maclaurin series for the function.

not tst qst

17) $f(x) = e^{2x}\sqrt{1 + x}$

Big product rule for 2nd & 3rd deriv.

A) $1 + \frac{3}{2}x + \frac{7}{8}x^2 + \frac{17}{48}x^3 + \ldots$ B) $1 - \frac{5}{2}x + \frac{23}{8}x^2 - \frac{103}{48}x^3 + \ldots$

C) $2 + \frac{5}{2}x + \frac{15}{8}x^2 + \frac{67}{48}x^3 + \ldots$ D) $1 + \frac{5}{2}x + \frac{23}{8}x^2 + \frac{103}{48}x^3 + \ldots$

Find the Taylor polynomial of order 3 generated by f at a.

test qst //

18) $f(x) = \ln(x + 1)$, $a = 8$

A) $P_3(x) = \ln 9 + \frac{x - 8}{9} - \frac{(x - 8)^2}{162} + \frac{(x - 8)^3}{2187}$ B) $P_3(x) = \ln 7 + \frac{x - 8}{7} + \frac{(x - 8)^2}{98} + \frac{(x - 8)^3}{1029}$

C) $P_3(x) = \ln 7 - \frac{x - 8}{7} + \frac{(x - 8)^2}{98} - \frac{(x - 8)^3}{1029}$ D) $P_3(x) = \ln 9 - \frac{x - 8}{9} + \frac{(x - 8)^2}{162} - \frac{(x - 8)^3}{2187}$

Find an equation for the line tangent to the curve at the point defined by the given value of t.

19) $x = \csc t$, $y = 12 \cot t$, $t = \frac{\pi}{3}$

A) $y = -24x + 12\sqrt{3}$ B) $y = 4\sqrt{3}x - 24$ C) $y = 24x + 4\sqrt{3}$ [✓ D) $y = 24x - 12\sqrt{3}$]

Find the value of d^2y/dx^2 at the point defined by the given value of t.

20) $x = \tan t$, $y = 9 \sec t$, $t = \frac{3\pi}{4}$

A) $-\frac{9\sqrt{2}}{4}$ B) $9\sqrt{2}$ C) $-\frac{9}{2}$ D) $\frac{\sqrt{2}}{4}$

Find the area of the specified region. Show the integrals used and the integration.

21) Inside the outer loop and outside the inner loop of the limacon $r = 6 \sin \theta - 3$

A) $\frac{3}{2}(4\pi - 3\sqrt{3})$ B) $\frac{9}{2}(2\pi - 3\sqrt{3})$ C) $\frac{9}{2}(4\pi + 3\sqrt{3})$ D) $9(\pi + 3\sqrt{3})$

Find the length of the curve. Show the integral.

22) The parabolic segment $r = \dfrac{5}{1 + \cos \theta}$, $0 \leq \theta \leq \dfrac{\pi}{2}$

A) $\dfrac{50}{3}\pi$

B) $\dfrac{5}{2}(\sqrt{2} + \ln(\sqrt{2} + 1))$

C) $\dfrac{25}{3}$

D) $\dfrac{5}{2}(\sqrt{2} - \ln(\sqrt{2} - 1))$

Find the vertices and foci of the ellipse.

23) $36x^2 + 121y^2 = 4356$

A) Vertices: $(0, \pm 11)$; Foci $(0, \pm\sqrt{85})$

B) Vertices: $(\pm 6, 0)$; Foci: $(\pm\sqrt{85}, 0)$

C) Vertices: $(0, \pm 6)$; Foci $(0, \pm\sqrt{85})$

D) Vertices: $(\pm 11, 0)$; Foci: $(\pm\sqrt{85}, 0)$

Solve the problem.

24) Find the foci and asymptotes of the following hyperbola:

$25x^2 - y^2 = 25$

A) Foci: $(5, 0)$, $(-5, 0)$; Asymptotes: $y = \dfrac{1}{5}x$, $y = -\dfrac{1}{5}x$

B) Foci: $(\sqrt{26}, 0)$, $(-\sqrt{26}, 0)$; Asymptotes: $y = \dfrac{1}{5}x$, $y = -\dfrac{1}{5}x$

C) Foci: $(\sqrt{26}, 0)$, $(-\sqrt{26}, 0)$; Asymptotes: $y = 5x$, $y = -5x$

D) Foci: $(0, \sqrt{26})$, $(0, -\sqrt{26})$; Asymptotes: $y = 5x$, $y = -5x$

If the equation represents a hyperbola, find the center, foci, and asymptotes. If the equation represents an ellipse, find the center, vertices, and foci. If the equation represents a circle, find the center and radius. If the equation represents a parabola, find the focus and directrix.

25) $7x^2 - y^2 - 56x + 6y - 9 = 0$

A) C: $(4, 3)$; F: $(4 + 8\sqrt{2}, 3)$, $(4 - 8\sqrt{2}, 3)$; A: $y - 3 = 7(x - 4)$, $y - 3 = -7(x - 4)$

B) C: $(0, 3)$; F: $(8\sqrt{2}, 0)$, $(-8\sqrt{2}, 0)$; A: $y = 7x$, $y = -7x$

C) C: $(4, 3)$; F: $(4 + 8\sqrt{2}, 3)$, $(4 - 8\sqrt{2}, 3)$; A: $y - 3 = \sqrt{7}(x - 4)$, $y - 3 = -\sqrt{7}(x - 4)$

D) C: $(4, 3)$; F: $(8\sqrt{2}, 0)$, $(- 8\sqrt{2}, 0)$; A: $y - 3 = \sqrt{7}(x - 4)$, $y - 3 = -\sqrt{7}(x - 4)$

The eccentricity is given of a conic section with one focus at the origin, along with the directrix corresponding to that focus. Find a polar equation for the conic section.

26) $e = \dfrac{1}{3}$, $y = -6$

A) $r = \dfrac{6}{1 + 3 \cos \theta}$

B) $r = \dfrac{6}{3 - \sin \theta}$

C) $r = \dfrac{6}{3 + \sin \theta}$

D) $r = \dfrac{6}{3 - \cos \theta}$

Answer Key
Testname: MAC 2312 - FINAL EXAM REVIEW

1) D
2) C
3) B
4) A
5) A
6) B
7) A
8) A
9) A
10) A
11) A
12) B
13) B
14) B
15) C
16) B
17) D
18) A
19) D
20) A
21) D
22) B
23) D
24) C
25) C
26) B

order of mag

n	$f^{(n)}(x)$	$f^{(n)}(a)$	$\frac{f^{(n)}(a)}{n!}(x-a)^n$
0			
1			
2			
3			

Final Review

① $\frac{dy}{dx} = 3x - xy \rightarrow \frac{dy}{dx} = x(3-y)$

$-\int \ominus \frac{1}{3-y}\,dy = \int x\,dx$ $\quad u = 3-y$

$\quad du = -dy$

$-\int \frac{1}{u}\,du = \frac{1}{2}x^2 + C \longrightarrow -\ln|3-y| = \frac{1}{2}x^2 + C$

$-\ln|3-(-4)| = \frac{1}{2}(0)^2 + C \longrightarrow C = -\ln|7|$

$-\ln|3-y| = \frac{1}{2}x^2 - \ln 7 \longrightarrow \ln|3-y| = -\frac{1}{2}x^2 + \ln 7$

$3 - y = e^{(-\frac{1}{2}x^2 + \ln 7)}$

$-y = e^{-\frac{1}{2}x^2} \cdot e^{\ln 7} - 3 \longrightarrow -y = e^{-\frac{1}{2}x^2} \cdot 7 - 3$

$$\boxed{y = -7e^{-\frac{1}{2}x^2} + 3}$$

② $\int \cos^{-1}x\,dx$ $\quad u = \cos^{-1}x \quad dv = dx$

$\quad du = \frac{-1}{\sqrt{1-x^2}}\,dx \quad v = x$

$x\cos^{-1}x - \frac{1}{2}\int \frac{\textcircled{2}-x}{\sqrt{1-x^2}}\,dx$ $\quad u = 1-x^2$

$\quad du = -2x\,dx$

$x\cos^{-1}x - \frac{1}{2}\int u^{-\frac{1}{2}}\,du \longrightarrow x\cos^{-1}x - \frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$

$$\boxed{x\cos^{-1}x - (1-x^2)^{\frac{1}{2}} + C}$$

③ $\int -8x\cos 2x\,dx$ $\qquad u=-8x \quad dv=\cos 2x\,dx$

$du=-8\,dx \quad v=\frac{1}{2}\sin 2x$

$(-8x)(\frac{1}{2}\sin 2x)-\int -8\cdot\frac{1}{2}\sin 2x\,dx$ $\qquad u=2x$

$du\ 2dx$

$-4x\sin 2x+2\int 4\sin 2x\,dx$

$-4x\sin 2x+2\int \sin u\,du \to \boxed{-4x\sin 2x-2\cos 2x+C}$

④ $\int \cos^2 5x\cos 5x\,dx \to \int (1-\sin^2 5x)\cos 5x\,dx$

$7\frac{1}{5}\int 5\cos 5x\,dx - 7\frac{1}{5}\int 5\sin^2 5x\cos 5x\,dx$

$u=5x$ $\qquad u=\sin 5x \quad du=5\cos 5x$

$du=5dx$

$\frac{7}{5}\int \cos u\,du-\frac{7}{5}\int u^2\,du \to \frac{7}{5}\sin u-\frac{7}{5}\frac{u^3}{3}+C$

$$\boxed{\frac{7}{5}\sin 5x-\frac{7}{15}\sin^3 5x+C}$$

⑤ $\int 4\cos^2 6x \cos^2 6x \, dx$

$4\int \frac{1}{2}(1+\cos 12x)\cdot\frac{1}{2}(1+\cos 12x)\cancel{dx}$

$\int (1+2\cos 12x+\cos^2 12x)\cancel{dx}$

$x + \left(\frac{1}{6}\right)\int 6\cdot 2\cos 12x\,dx + \int \frac{1}{2}(1+\cos 24x)\cancel{dx}$

$u=12x \quad du=12dx \qquad u=24x \quad du=\cancel{24dx}$

$x+\frac{1}{6}\sin u+\frac{1}{2}x+\frac{1}{48}\sin u + C$

$\boxed{\frac{3}{2}x+\frac{1}{6}\sin 12x+\frac{1}{48}\sin 24x + C}$

⑥ $\frac{8x-16}{(x+1)(x-5)} \quad \frac{A}{x+1}+\frac{B}{x-5}$

$8x-16 = A(x-5)+B(x+1)$

$8(-1)-16 = A(-1-5)+\cancel{B(-1+1)}^{0}$

$A=4 \qquad B=4$

$\int\frac{4}{x+1}dx + \int\frac{4}{x-5}dx \rightarrow \boxed{4\ln|x+1|+4\ln|x-5|+C}$

⑦
$$\begin{array}{r} x-10 \\ x^2+10x+25\;\overline{)\,x^3+0x^2+0x+0} \\ \ominus\; x^3 \overset{\ominus}{+} 10x^2 \overset{\ominus}{+} 25x \end{array} \longrightarrow 75x+250$$

$$\int x-10+\frac{75x+250}{x^2+10x+25}\,dx \longrightarrow \frac{75y+250}{(x+5)(x+5)} = \frac{A}{x+5}+\frac{B}{(x+5)^2}$$

$$75(-5)+250 = A(-5+5)+B \quad , \quad B=-125$$

$$75(0)+250 = A(0+5)-125 \longrightarrow A=75$$

$$\int x-10+\frac{75}{(x+5)}-\frac{125}{(x+5)^2}\,dx \qquad \begin{array}{l} u=x+5 \\ du=dx \end{array}$$

$$\frac{1}{2}x^2-10x+75\ln|x+5|-125\int u^{-2}\,du$$

$$\boxed{\frac{1}{2}x^2-10x+75\ln|x+5)+\frac{125}{x+5}+C}$$

⑧
$$\lim_{b\to\infty}\int_1^b 15n^{-1/2}\,dn \longrightarrow \lim_{b\to\infty} 15\frac{n^{1/2}}{\frac{1}{2}}\Bigg|_1^b$$

$$30n^{1/2}\Big|_1^b \longrightarrow \lim_{b\to\infty} 30b^{1/2}-30$$

$$\boxed{\infty-30}$$ Diverges

⑨ $\lim_{b \to \infty} \ominus \int_1^b \ominus \cos n^{-1}(n^{-2})\,dn$ $\quad u = n^{-1}$, $du = -n^{-2}\,dn$

$\lim_{b \to \infty} -\int_1^b \cos u\,du \longrightarrow -\sin u \Big|_1^b$

$\lim_{b \to \infty} -\sin \frac{1}{n} \Big|_1^b \longrightarrow -\sin \frac{1}{b} - (-\sin \frac{1}{1})$

$\boxed{0 + \sin 1}$ // Conv...

⑩ $\sum v_n = \sum \frac{5n^{\frac{1}{2}}}{2n^{3/2}} = \sum \frac{5}{2}\frac{1}{n}$ Harmonic ∴ div

$\lim_{n \to \infty} \frac{5n^{1/2}}{2n^{3/2}+5n+8} \cdot \frac{2n}{5} \longrightarrow \frac{10n^{3/2}}{10n^{3/2}+25n+40} = \boxed{1}$ //

$0 < 1 < \infty$ $\boxed{\sum v_n \text{ div } \therefore \sum u_n \text{ div.}}$ //

⑪ $a_{n+1} = \frac{(2(n+1))!}{4^{n+1}(n+1)!} = \frac{(2n+2)!}{4^n \cdot 4(n+1)!} = \frac{(2n+2)(2n+1)(2n)!}{4^n \cdot 4(n+1)n!}$

$\lim_{n \to \infty} \frac{(2n+2)(2n+1)\cancel{(2n)!}}{\cancel{4^n} \cdot 4(n+1)\cancel{n!}} \cdot \frac{\cancel{4^n}\,\cancel{n!}}{\cancel{(2n)!}}$

$\lim_{n \to \infty} \frac{(2n+2)(2n+1)}{4n+4} \longrightarrow \lim_{n \to \infty} \frac{4n^2+6n+2}{4n+4} = \infty$

$\boxed{\therefore \text{Div}}$ //

⑬ $\lim\limits_{n\to\infty} \sqrt[n]{\left(\frac{\ln n}{3n-4}\right)^n} \longrightarrow \lim\limits_{n\to\infty} \frac{\ln n}{3n-4} = \frac{\infty}{\infty}$

$\lim\limits_{n\to\infty} \frac{\frac{1}{n}}{3} = \frac{1}{3n} = \boxed{0}$ // CONV...

⑭ $a_n > 0$, $a_{n+1} \leq a_n$, $\lim\limits_{n\to\infty} a_n = 0$ $\therefore$ Conv

Abs Conv. Limit Comp

$V_n = \frac{4n^8}{8n^9} = \frac{1}{2n}$ Harmonic $\therefore$ Div

V_n div $\therefore$ U_n Div.

$\lim\limits_{n\to\infty} \frac{4n^8+4}{8n^9+2} \cdot \frac{2n}{1} \longrightarrow \lim\limits_{n\to\infty} \frac{8n^9+8n}{8n^9+2} = \boxed{1 \therefore \text{Cond Conv}}$

⑮ $\sum (-1)^n \frac{2^n}{8n^6+8^n}$

$a_n > 0$ ✓

$a_{n+1} \leq a_n$ ✓ $\therefore$ Conv

$\lim = 0$ ✓

Ratio test

$a_{n+1} = \frac{2^{n+1}}{8(n+1)^6 8^{n+1}} = \frac{2^n \cdot 2}{8(n+1)^6 + 8^n \cdot 8} = \frac{2^n \cdot 2}{8((n+1)^6 + 8^n)}$

$\lim\limits_{n\to\infty} \frac{\cancel{2^n} \cdot 2}{8((n+1)^6 + 8^n)} \cdot \frac{8n^6 + 8^n}{\cancel{2^n}} \longrightarrow \lim\limits_{n\to\infty} \frac{2(8n^6 + 8^n)}{8((n+1)^6 + 8^n)}$

$= \frac{1}{4} < 1$ $\boxed{\therefore \text{ConV + Abs ConV}}$

⑯ Ratio test $A_{n+1} = \frac{(X-4)^{n+1}}{8+5(n+1)} = \frac{(X-4)^n(X-4)}{8+5n+5}$

$$\lim_{n\to\infty}\left|\frac{\cancel{(X-4)^n}(X-4)}{5n+13}\cdot\frac{5n+8}{\cancel{(X-4)^n}}\right|$$

$$\lim_{n\to\infty}\left|\frac{(5n+8)(X-4)}{5n+13}\right| \longrightarrow \lim_{n\to\infty}|X-4| < 1$$

$$-1 \leq X-4 < 1$$

$$3 < X < 5$$

$X=3$ $\sum \frac{(3-4)^n}{8+5n} \rightarrow \sum (-1)^n \frac{1}{8+5n}$

AST ✓

$a_n \geq 0$

$a_{n+1} \leq a_n$ ✓ ∴ Conv

$\lim = 0$ ✓ ∴ $X=3$

$X=5$ $\sum \frac{(5-4)^n}{8+5n} \rightarrow \sum \frac{(1)^n}{8+5n} = \sum \frac{1}{8+5n}$

Limit Comp Test $V_n = \frac{1}{5n}$ Harmonic ∴ Div

$\lim_{n\to\infty} \frac{5n}{5n+8} = 1$ $0 < 1 < \infty$ V_n Div ∴ U_n div

$$X \neq 5$$

$$\boxed{\text{Interval } 3 \leq X < 5}$$

⑰ $f(x) = e^{2x}(1+x)^{1/2}$

$f'(x) = e^{2x} \cdot \frac{1}{2}(1+x)^{-1/2} + 2e^{2x}(1+x)^{1/2}$

$f''(x) =$ Big product rule

$f(0) = 1$

$f'(0) = 5/2$ $\qquad = 1 + \frac{5}{2}x + \frac{\frac{23}{4}}{2}x^2 + \frac{\frac{103}{8}}{3!}x^3 + \ldots$

$f''(0) = \frac{23}{4}$

$f'''(0)\ \frac{103}{8}$ $\qquad = 1 + \frac{5}{2}x + \frac{23}{8}x^2 + \frac{103}{48}x^3 + \ldots$

⑱ $f'(x) = \frac{1}{x+1} = (x+1)^{-1}$ $\qquad f(8) = \ln 9$

$f'(x) = -(x+1)^{-2}$ $\qquad f'(8) = \frac{1}{9}$

$f'''(x) = 2(x+1)^{-3}$ $\qquad f''(8) = \frac{-1}{81}$

$f'''(8) = \frac{2}{729}$

$= \ln 9 + \frac{1}{9}(x-8) - \frac{\frac{1}{81}}{2!}(x-8)^2 + \frac{\frac{2}{729}}{3!}(x-8)^3 + \ldots$

$= \ln 9 + \frac{1}{9}(x-8) - \frac{1}{162}(x-8)^2 + \frac{1}{2187}(x-8)^3 + \ldots$

(19) $\frac{dY}{dX} = -\csc t \cot t \qquad \frac{dY}{dt} = -12\csc^2 t$

$\frac{dY}{dX} = \frac{-12\csc^2 t}{-\csc \cot t} = \frac{12\csc t}{\cot t} = \frac{12 \sin t}{\sin t \cos t}$

$\frac{dY}{dX} = \frac{12}{\cos t} = \frac{12}{\cos \pi/3} = \frac{12}{\frac{1}{2}} = 24$

$X = \csc \pi/3 = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$Y = 12 \cot \pi/3 = 4\sqrt{3}$

$\boxed{y - 4\sqrt{3} = 24\left(x - \frac{2\sqrt{3}}{3}\right)}$ ✓

(20) $\frac{dX}{dt} = \sec^2 t \qquad \frac{dY}{dt} = 9\sec t \tan t$

$\frac{dY}{dX} = \frac{9\sec t \tan t}{\sec^2 t} = \frac{9\tan t}{\sec t} = \frac{9\sin t}{\cos t} \cdot \cos t$

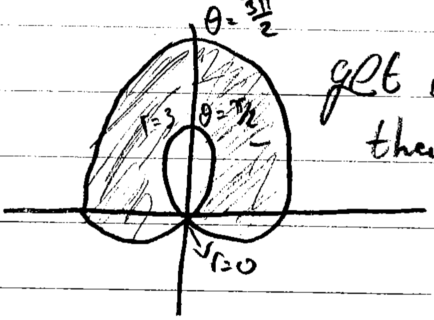
$\frac{dY}{dX} = 9\sin t$

$\frac{d^2Y}{dX} = \frac{9\cos t}{\sec^2 t} = 9\cos^3 t$

(a) $\frac{3\pi}{4}$ $\quad 9\left(-\frac{\sqrt{2}}{2}\right)^3 \rightarrow 9\left(-\frac{2\sqrt{2}}{8}\right) = \boxed{-\frac{9\sqrt{2}}{4}}$

(21) when $\theta = 0$ $r = -3$ from graph

$r = -3 + 6\sin\theta$



get one half then double it

when $r = 0 = -3 + 6\sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$2\int_{\frac{5\pi}{6}}^{3\pi/2} \frac{1}{2}(-3+6\sin\theta)^2 d\theta - \int_{\pi/6}^{5\pi/6} \frac{1}{2}(-3+6\sin\theta)^2 d\theta$$

Big loop − Small loop

$79.931 - 4.892 = 75.039 = \boxed{(75.040)}$

(22) $r = 5(1+\cos\theta)^{-1}$

$r' = -5(1+\cos\theta)^{-2}(-\sin\theta) = \frac{5\sin\theta}{(1+\cos\theta)^2}$

$$L = \int_0^{\pi/2} \sqrt{\left(\frac{5}{1+\cos\theta}\right)^2 + \left(\frac{5\sin\theta}{(1+\cos\theta)^2}\right)^2}\, d\theta$$

$\boxed{5.739}$

(23) $\frac{36x^2}{4356}+\frac{121y^2}{4356}=1 \longrightarrow \frac{x^2}{121}+\frac{y^2}{36}=1$ X-Axis major

$a^2=121$, $a=11$
$b^2=36$, $b=6$
$c^2=121-36$
$c=\pm\sqrt{85}$

$\boxed{\begin{array}{l} V=(-11,0),(11,0) \\ F=(-\sqrt{85},0)(\sqrt{85},0) \end{array}}$

(24) $\frac{25x^2}{25}-\frac{y^2}{25}=1 \longrightarrow x^2-\frac{y^2}{25}=1$ X-axis major

$a^2=1$, $a=1$
$b^2=25$, $b=5$
$c^2=25+1$, $c=\pm\sqrt{26}$

$F=(-\sqrt{26},0)(\sqrt{26},0)$
Asym: $m=\pm\frac{b}{a}=\pm\frac{5}{1}=\pm5$
$\underline{y=\pm5x}$

(25) $7x^2-56x-y^2+6y=9$
$7(x^2-8x)-(y^2-6y)=9$
$7(x^2-8x+\underline{16})-(y^2-6y+\underline{9})=9-9+112$
$7(x-4)^2-(y-3)^2=112$
$\frac{7(x-4)^2}{112}-\frac{(y-3)^2}{112}=1$ Transverse axis is x-axis
$\frac{(x-4)^2}{16}-\frac{(y-3)^2}{112}=1$

$a^2=16$, $a=4$
$b^2=112$, $b=\pm\sqrt{112}=\pm4\sqrt{7}$
$c^2=128$, $c=8\sqrt{2}$

Center $(4,3)$
V: $(0,3)(8,3)$
F: $(4-8\sqrt{2},3)(4+8\sqrt{2},3)$

Assymp: $m=\pm\frac{b}{a}=\frac{4\sqrt{7}}{4}=\sqrt{7}$
$\boxed{y-3=\pm\sqrt{7}(x-4)}$

㉕ $e = \frac{1}{3}$, $y = -6$

$$r = \frac{ke}{1 - e\sin\theta} \longrightarrow r = \frac{6\left(\frac{1}{3}\right)}{\frac{1}{3}(3 - \sin\theta)} = \boxed{r = \frac{6}{3 - \sin\theta}}$$

MAC 2312 - Calculus w/Analytic Geometry II

Name__________________________________

Review for Test 4 - Chapter 11

Date___________________________________

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the Cartesian equation given the parametric equations.

1) $x = 2 \sin t$, $y = 4 \cos t$, $0 \le t \le 2\pi$

2) $x = 9t^2$, $y = 3t$, $-\infty \le t \le \infty$

Find an equation for the line tangent to the curve at the point defined by the given value of t.

3) $x = \sin t$, $y = 6 \sin t$, $t = \frac{\pi}{3}$

4) $x = t + \cos t$, $y = 2 - \sin t$, $t = \frac{\pi}{6}$

Find the value of d^2y/dx^2 at the point defined by the given value of t.

5) $x = 9 \sin t$, $y = 9 \cos t$, $t = \frac{3\pi}{4}$

6) $x = \tan t$, $y = 9 \sec t$, $t = \frac{3\pi}{4}$

Find the Cartesian coordinates of the given point.

7) $\left(-1, \frac{1}{2}\pi\right)$

8) $(3, 4\pi/3)$

Replace the polar equation with an equivalent Cartesian equation.

9) $r = \dfrac{1}{2 \cos \theta - 3 \sin \theta}$

10) $r = 4 \cot \theta \csc \theta$

11) $r^2 \sin 2\theta = 30$

Replace the Cartesian equation with an equivalent polar equation.

12) $x^2 + y^2 - 4x = 0$

13) $xy = 1$

14) $(x - 19)^2 + (y + 3)^2 = 361$

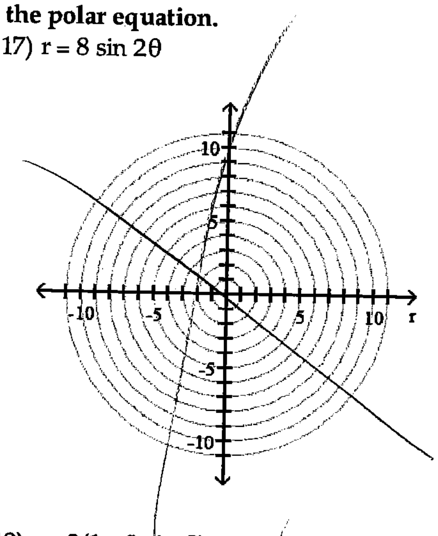
Determine the symmetries of the curve.

15) $r = -3 - 2 \sin \theta$

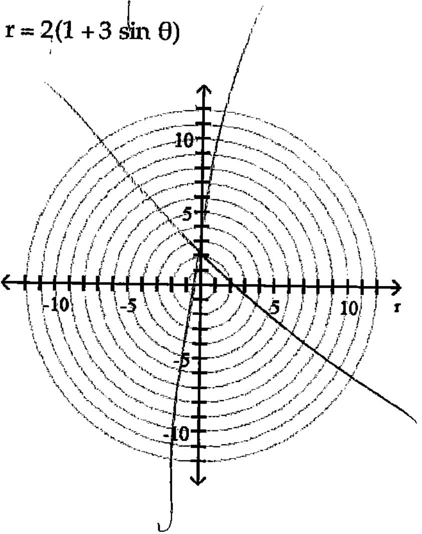
16) $r = 6 \cos 3\theta$

Graph the polar equation.

17) $r = 8 \sin 2\theta$



18) $r = 2(1 + 3 \sin \theta)$



Find the slope of the polar curve at the indicated point.

19) $r = 1 - \sin \theta, \theta = \pi$

20) $r = 8(1 + \cos\theta), \theta = \frac{\pi}{4}$

Find the area of the specified region.

21) Inside the limacon $r = 9 + 4 \sin \theta$

22) Inside the smaller loop of the limacon $r = 2 + 4 \sin \theta$

23) Inside the three- leaved rose $r = 8 \cos 3\theta$

24) Inside the limacon $r = 3 + 2 \sin \theta$

25) Inside the smaller loop of the limacon $r = 4 + 8 \sin \theta$

26) Inside the circle $r = -4 \cos \theta$ and outside the circle $r = 2$

Find the length of the curve.

27) The spiral $r = 3\theta^2, 0 \le \theta \le 2\sqrt{3}$

28) The parabolic segment $r = \frac{3}{1 + \sin \theta}, 0 \le \theta \le \frac{\pi}{2}$

Find the focus and directrix of the parabola.

29) $x^2 = 20y$

30) $y^2 = 36x$

Find the vertices and foci of the ellipse.

31) $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Solve the problem.

32) Find the foci and asymptotes of the following hyperbola:
$x^2 - y^2 = 72$

33) Find the vertices and asymptotes of the following hyperbola:
$\frac{y^2}{100} - \frac{x^2}{16} = 1$

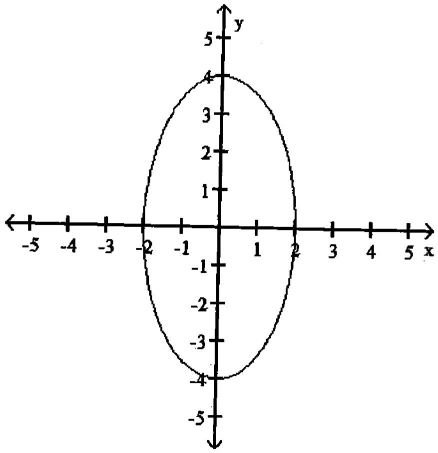
The eccentricity is given of a conic section with one focus at the origin, along with the directrix corresponding to that focus. Find a polar equation for the conic section.

34) $e = 3, y = 10$

35) $e = \frac{1}{5}, x = 7$

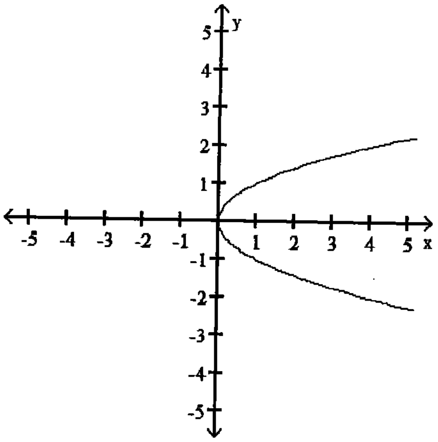
Answer Key
Testname: MAC 2312 - REV T4 - CH 11

1) $\frac{x^2}{4} + \frac{y^2}{16} = 1$



Counterclockwise from (0, 4) to (0, 4), one rotation

2) $x = y^2$



Entire parabola, bottom to top (from fourth quadrant to origin to first quadrant)

3) $y = 6x$

4) $y = -\sqrt{3}x + \frac{\sqrt{3}}{6}\pi + 3$

5) $-\frac{2\sqrt{2}}{9}$

6) $-\frac{9\sqrt{2}}{4}$

7) $(0, -1)$

8) $\left(\frac{-3}{2}, \frac{-3\sqrt{3}}{2}\right)$

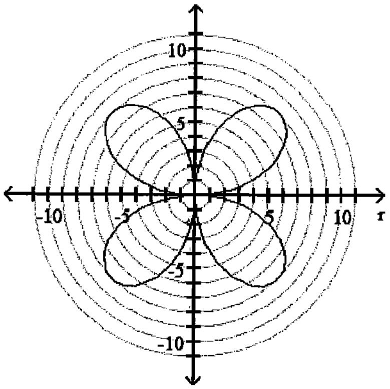
9) $2x - 3y = 1$

10) $y^2 = 4x$

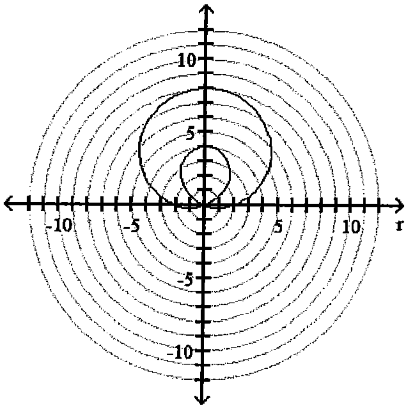
11) $y = \frac{15}{x}$

12) $r = 4 \cos \theta$

13) $r^2 \sin 2\theta = 2$
14) $r^2 = 38r \cos \theta - 6r \sin \theta - 9$
15) y- axis only
16) x- axis only
17)



18)



19) 1
20) $1 - \sqrt{2}$
21) 89π
22) $2(2\pi - 3\sqrt{3})$
23) 16π
24) 11π
25) $8(2\pi - 3\sqrt{3})$
26) $\frac{2}{3}(2\pi + 3\sqrt{3})$
27) 56
28) $\frac{3}{2}(\sqrt{2} - \ln(\sqrt{2} - 1))$
29) (0, 5); $y = -5$
30) (9, 0); $x = -9$
31) Vertices: $(\pm 5, 0)$; Foci: $(\pm 3, 0)$
32) Foci: (12, 0), (- 12, 0); Asymptotes: $y = x$, $y = - x$
33) Vertices: (0, 10), (0, - 10); Asymptotes: $y = \pm \frac{5}{2}x$

34) $r = \frac{30}{1 + 3 \sin \theta}$

35) $r = \frac{7}{5 + \cos \theta}$

Steven
Romeiro

Review for test 1

① $\int_2^3 \frac{x^4+1}{x^5+5x}dx$ $u = x^5+5x$
$du = 5x^4+5\,\cancel{dx}$

$$\frac{1}{5}\int_2^3 \frac{5x^4+5\cancel{dx}}{x^5+5x} = \frac{1}{5}\int_2^3 \frac{1}{u}du$$

$$= \frac{1}{5}\ln|u|\Big|_2^3 = \frac{1}{5}\ln|x^5+5x|\Big|_2^3$$

$$= \frac{1}{5}\ln|3^5+5(3)| - \frac{1}{5}\ln|2^5+5(2)|$$

$$= \frac{1}{5}\ln|258| - \frac{1}{5}\ln|42|$$

$$= \frac{1}{5}\ln\left(\frac{258}{42}\right) = \boxed{\frac{1}{5}\left(\frac{43}{7}\right)}$$

② $\displaystyle\int_0^{\frac{\pi}{24}} \frac{\sec^2 6x}{6+\tan 6x}\,dx$ $\qquad U = 6 + \tan 6x$

$\qquad du = 6\sec^2 6x\,dx$

$$= \frac{1}{6}\int_0^{\pi/24} \frac{6\sec^2 6x\,dx}{6+\tan 6x} = \frac{1}{6}\int_6^{\pi/24} \frac{1}{U}\,dU$$

$$= \frac{1}{6}\ln|U|\Big|_0^{\pi/24} = \frac{1}{6}\ln|6+\tan 6x|\Big|_0^{\pi/24}$$

$$= \frac{1}{6}\ln\left|6+\tan 6\left(\frac{\pi}{24}\right)\right| - \frac{1}{6}\ln|6+\tan 6(0)|$$

$$= \frac{1}{6}\ln|6+1| - \frac{1}{6}\ln|6+0|$$

$$\frac{1}{6}\ln|7| - \frac{1}{6}\ln|6|$$

$$= \boxed{\frac{1}{6}\ln\left(\frac{7}{6}\right)}$$

③ $\int \frac{7e^{(7\sin 2x)}}{\sec 2x}dx$ $U=7\sin 2x$
$du = 14\cos 2x\,dx$

$$= \int \frac{7e^{(7\sin 2x)}}{\frac{1}{\cos 2x}}dx = \int 7e^{(7\sin 2x)}\cdot \cos 2x\,dx$$

$$= \frac{1}{14}\int 7e^{(7\sin 2x)} \cdot 14\cos 2x\,dx$$

$$= \frac{1}{2}\int e^{u}\,du = \frac{1}{2}e^{u} = \boxed{\frac{1}{2}e^{(7\sin 2x)} + C}$$

④ $\int \frac{35e^{(\sqrt{5x})}}{2\sqrt{x}} = \frac{35}{2}\int \frac{e^{(\sqrt{5x})}}{\sqrt{x}}$ $U=(5x)^{\frac{1}{2}}$
$du = \frac{5}{2}(5x)^{-\frac{1}{2}} = \frac{5}{2(5x)^{1/2}}$

$$= \frac{35}{2}\int \frac{\sqrt{5}\cdot e^{(\sqrt{5x})}}{\sqrt{5}\cdot\sqrt{x}} = \frac{35}{2}\int \frac{\sqrt{5}e^{(\sqrt{5x})}}{\sqrt{5x}} = \frac{2}{5}\cdot\frac{35}{2}\int \frac{5\sqrt{5}e^{(\sqrt{5x})}}{2\sqrt{5x}}$$

$$= \frac{2}{5}\cdot\frac{35}{2}\cdot\sqrt{5}\int e^{(\sqrt{5x})}\cdot\frac{5}{2\sqrt{5x}} = \frac{35\cdot\sqrt{5}}{5}\int e^{u}\,du$$

$$= 7\sqrt{5}\,e^{u} + C = \boxed{7\sqrt{5}\,e^{\sqrt{5x}} + C}$$

⑤ $\int (e^{x}+e^{-x})^{2}dx = \int (e^{x}+e^{-x})(e^{x}+e^{-x})dx$

$= \int e^{2x}+2+e^{-2x} = \int e^{2x} + \int 2 + \int e^{-2x}$

$u = 2x$
$du = 2dx$
$u = -2x$
$du = -2dx$

$= \frac{1}{2}\int e^{u}du + \int 2dx - \frac{1}{2}\int e^{u}du$

$= \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + c$

$= \boxed{\frac{1}{2}\left(e^{2x} - e^{-2x}\right) + 2x + C}$

⑥ $\int \frac{e^{3\theta}}{1+e^{3\theta}} \, d\theta$

$U = 1+e^{3\theta}$

$dU = 3e^{3\theta} \, d\theta$

$$\frac{1}{3}\int \frac{3e^{3\theta}}{1+e^{3\theta}} \, d\theta = \frac{1}{3}\int \frac{1}{U} \, dU$$

$$= \frac{1}{3}\ln(U) + C = \boxed{\frac{1}{3}\ln(1+e^{3\theta}) + C}$$

⑦ $\int_1^{\sqrt{2}} x8^{x^2} \, dx = \int_1^{\sqrt{2}} x \cdot 8^{x^2}$

$U = x^2$

$dU = 2x \, dx$

$$\frac{1}{2}\int_1^{\sqrt{2}} 8^{x^2} \cdot 2x \, dx = \frac{1}{2}\int_1^{\sqrt{2}} 8^U \, dU = \frac{1}{2}\frac{8^U}{\ln(8)}\Bigg|_1^{\sqrt{2}}$$

$$= \frac{1}{2} \cdot \frac{8^{x^2}}{\ln 8}\Bigg|_1^{\sqrt{2}} = \frac{1}{2\ln 8} \cdot 8^{x^2}\Bigg|_1^{\sqrt{2}}$$

$$= \frac{1}{2\ln 8}\left[8^{(\sqrt{2})^2} - 8^{(1)^2}\right] = \frac{1}{2\ln 8}\left[8^2 - 8^1\right]$$

$$\frac{1}{2\ln 8}(56) = \boxed{\frac{28}{\ln 8}}$$

⑧ $\int \frac{\log_3 x}{x}\,dx = \int \frac{\frac{\ln x}{\ln 3}}{x}\,dx$

$\int \frac{\ln x}{x \ln 3}\,dx = \frac{1}{\ln 3}\int \frac{\ln x}{x}\,dx$ $U = \ln x$, $du = \frac{1}{x}\,dx$

$\frac{1}{\ln 3}\int \ln x \cdot \frac{1}{x}\,dx = \frac{1}{\ln 3}\int U\,dU = \frac{1}{\ln 3}\cdot\frac{1}{2}U^2 + C$

$= \frac{1}{2\ln 3}(\ln x)^2 + C = \boxed{\frac{(\ln x)^2}{2\ln 3} + C}$

⑨ $\frac{dy}{dt} = e^t \sin(e^t - 10)$, $y(\ln 10) = 0$

$\therefore x = \ln 10$

$\int dy = \int e^t \sin(e^t - 10)\,dt$ $u = e^t - 10$, $du = e^t\,dt$

$y = \int \sin u\,du \rightarrow y = -\cos u + c$

$y = -\cos(e^t - 10) + c \rightarrow 0 = -\cos(e^{\ln 10} - 10) + C$

$0 = -\cos(10 - 10) + c \rightarrow 0 = -\cos(0) + c$

$0 = -1 + C \rightarrow C = 1$

$\boxed{y = -\cos(e^t - 10) + 1}$

10) $\int dy = \int -8^{-x} \sec e^{-x} \tan e^{-x} dx$ $\quad u = e^{-x}$, $du = -e^{-x} dx$

$y = 8 \int -e^{-x} \sec e^{-x} \tan e^{-x} dx$

$y = 8 \int \sec u \tan u \, du \rightarrow 8 \sec u + C$

$y = 8 \sec(e^{-x}) + C$

$8\sec 1 + 8 = 8 \sec(e^{0}) + C \rightarrow 8\sec 1 + 8 = 8 \sec 1 + C$

$8\sec 1 - 8 \sec 1 + 8 = C \quad C = 8$

$\boxed{y = 8\sec(e^{-x}) + 8}$

⑪ $\frac{d^2y}{d^2x} = 9e^{-x}$ $\quad y(0) = 1$, $y'(0) = 0$

$\int d^2y = \int 9e^{-x} d^2x \rightarrow dy = \int 9e^{-x} d^2x$ $\quad u = -x$, $du = -dx$

$dy = -9 \int -e^{-x} dx^2 \rightarrow dy = -9 \int e^{u} du \, (dx)$

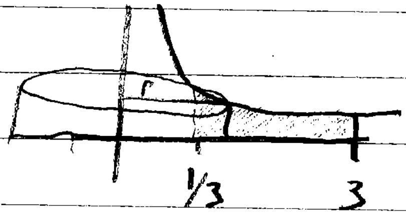
$\frac{dy}{dx} = -9e^{-x} + C \rightarrow 0 = -9e^{-0} + c \rightarrow c = 9$

$\frac{dY}{dx} = -9e^{-x} + 9 = \int dy = \int -9e^{-x} + 9 \, dx$ $\quad u = -x$, $du = -dx$

$y = 9 \int -e^{-x} dx + \int 9 dx \rightarrow y = 9e^{u} + 9x + C$

$Y = 9e^{-x} +$

12) $y = \frac{1}{x^2}$, $y = 0$, $x = \frac{1}{3}$, $x = 3$ rotate about y-axis



$r = x \quad ht = \frac{1}{x^2} - 0 = \frac{1}{x^2}$

$$V = \int_a^b 2\pi r \cdot ht \cdot th$$

$$V = \int_{1/3}^{3} 2\pi x \cdot \frac{1}{x^2} \cdot dx$$

$$V = 2\pi \int_{1/3}^{3} x \cdot \frac{1}{x^2} dx \rightarrow 2\pi \int_{1/3}^{3} \frac{1}{x} dx = 2\pi \ln(x) \Big|_{1/3}^{3}$$

$$= 2\pi \left(\ln 3 - \ln 1/3 \right) = 2\pi \left(\ln 3 - (\ln 1 - \ln 3) \right)$$

$$= 2\pi \left(\ln 3 - \ln 1 + \ln 3 \right) = 2\pi \, 2 \ln 3$$

$$= \boxed{4\pi \ln 3}$$

⑬ $\frac{dy}{dx} = \frac{2y^2}{x} \rightarrow y^{-2}dy = \frac{2}{x}dx$

$\int y^{-2}dy = 2\int \frac{1}{x}dx \rightarrow \frac{y^{-1}}{-1} = 2\ln|x|+C$

$-\frac{1}{y} = 2\ln|x|+C \rightarrow \boxed{y = -\frac{1}{2\ln|x|+C}}$

⑭ $\frac{dy}{dx} = 7x^6 e^{-y} \rightarrow e^y dy = 7x^6 dx$

$\int e^y dy = \int 7x^6 dx \rightarrow e^y = \frac{7x^7}{7} + C$

$e^y = x^7 + C \rightarrow \boxed{y = \ln(x^7+C)}$

⑮ $\frac{dy}{dx} = 6x^5 \cos^2 y \rightarrow \cos^{-2}y\, dy = 6x^5 dx$

$\frac{1}{\cos^2 y}dy = 6x^5 dx \Rightarrow \sec^2 dy = 6x^5 dx$

$\int \sec^2 dy = \int 6x^5 dx \rightarrow \tan y = x^6 + C$

$\tan^{-1}(\tan y) = \tan^{-1}(x^6+C) \rightarrow \boxed{y = \tan^{-1}(x^6+C}$

* ⑯ $H - H_s = (H_0 - H_s)e^{-kt}$

$210 - 70 = (350 - 70)e^{-k20}$

$140 = 280e^{-20K} \rightarrow \frac{140}{280} = e^{-20K} \rightarrow \frac{1}{2} = e^{-20K}$

$\ln\left(\frac{1}{2}\right) = -20K \rightarrow \boxed{K = 0.034657}$

$190 - 70 = (350 - 70)e^{-0.034657t}$

$\frac{120}{280} = e^{-0.034657t} \rightarrow \ln\left(\frac{3}{7}\right) = -0.034657t$

$t = 24.448\text{min}$

$24.448 - 20\text{min} = \boxed{4.448\text{min}}$

* ⑰ $K = -4.332 \times 10^{-4}$. find 1/2 life

$A = A_0e^{Kt} \rightarrow 5 = 10e^{-0.0004332t}$

$\ln(.5) = -0.0004332t$ $\boxed{t = 1600.063}$

* ⑱ 64.9% of C_{14} $\frac{1}{2}$ life $C_{14} = 5700$ yrs

$A = A_0e^{kt} \rightarrow 1 = 2e^{k(5700)}$

$\ln(.5) = 5700K \rightarrow K = -0.000122$

$64.9 = 100e^{-0.000122t} \rightarrow \ln(.649) = -0.000122t$

$t = 3543.6276$

$\boxed{t = 3544\text{ yrs}}$

using $K = 0.0001216$, $\boxed{t = 3555\text{ yrs}}$

⑲ $\sinh x = \frac{5}{12}$, $\cosh x =$

$$\cosh^2 - \sinh^2 = 1 \rightarrow \cosh^2 = 1 + \left(\frac{5}{12}\right)^2$$

$$\cosh^2 = 1 + \frac{25}{144}$$

$$\cos^2 h = \frac{169}{144} = \cosh = \sqrt{\frac{169}{144}} = \cosh = \boxed{\frac{13}{12}}$$

⑳ $\cosh 4x + \sinh 4x$

$$\frac{e^{4x} + e^{4x}}{2} + \frac{e^{4x} - e^{-4x}}{2} = \frac{2e^{4x}}{2} = \boxed{e^{4x}}$$

㉑ $y = \underline{\cosh x^7 \rightarrow} \; y' = \sinh x^7 \cdot (7x^6)$

$$\boxed{y' = 7x^6 \sinh(x^7)}$$

㉒ $y = \ln(\sinh 5x)$ $\qquad y' = \frac{1}{\sinh 5x} \cdot \cosh(5x) \cdot 5$

$$y' = \frac{5\cosh(5x)}{\sinh(5x)} = \boxed{y' = 5\coth(5x)}$$

(23) $y = \sinh^{-1}(\sqrt{11x}) \rightarrow y' = \frac{1}{\sqrt{1+((11x)^{1/2})^2}} \cdot \frac{1}{2}(11x)^{-1/2} \cdot 11$

$$y' = \frac{11}{2\sqrt{11x}\sqrt{1+11x}} \rightarrow \boxed{\frac{11}{2\sqrt{11x(1+11x)}}}$$

(24) $y = (\theta^2+9\theta)\tanh^{-1}(\theta+8)$

$$y' = (\theta^2+9\theta) \cdot \frac{1}{1-(\theta+8)^2} + (2\theta+9)\tanh^{-1}(\theta+8)$$

$$y' = \frac{\theta^2+9\theta}{1-(\theta^2+16\theta+64)} + (2\theta+9)(\tanh^{-1}(\theta+8))$$

$$y' = \frac{\theta^2+9\theta}{1-\theta^2-16\theta-64} + (2\theta+9)\tanh^{-1}(\theta+8)$$

$$y' = \frac{\theta^2+9\theta}{-\theta^2-16\theta-63} + (2\theta+9)[\tanh^{-1}(\theta+8)]$$

$$y' = -\frac{\theta^2+9\theta}{\theta^2+16\theta+63} + (2\theta+9)\tanh^{-1}[\theta+8]$$

$$y' = \frac{-\theta\cancel{(\theta+9)}}{(\theta+7)\cancel{(\theta+9)}} + (2\theta+9)\tanh^{-1}(\theta+8)$$

$$y' = \frac{-\theta}{(\theta+7)} + (2\theta+9)\tanh^{-1}(\theta+8)$$

25) $Y = \mathrm{Sinh}^{-1}(\cos x)$

$$Y' = \frac{1}{\sqrt{1+(\cos x)^2}} \cdot (-\sin x) \rightarrow \boxed{Y' = -\frac{\sin x}{\sqrt{1+\cos^2 x}}}$$

㉖ $\int \cosh \frac{x}{9}\, dx$ $\qquad U = \frac{1}{9}x$

$\qquad du = \frac{1}{9} dx$

$$9\int \frac{1}{9} \cosh \frac{x}{9}\, dx \longrightarrow 9\int \cosh u\, du$$

$$9 \sinh u + C \rightarrow \boxed{9 \sinh(\tfrac{1}{9}x) + C}$$

㉗ $\int 5 \sinh(4x - \ln 5)\, dx$ $\qquad U = 4x - \ln 5$

$\qquad du = 4\, dx$

$$5 \cdot \frac{1}{4} \int 4 \sinh(4x - \ln 5)\, dx$$

$$\frac{5}{4}\int \sinh u\, du \longrightarrow \frac{5}{4} \cosh u + C$$

$$\boxed{\frac{5}{4} \cosh(4x - \ln 5) + C}$$

㉘ $\int \operatorname{csch}^2\left(2-\frac{x}{2}\right)dx$ $\quad U=2-\frac{1}{2}x$

$du=-\frac{1}{2}dx$

$-2\int -\frac{1}{2}\operatorname{csch}\left(2-\frac{1}{2}x\right)dx \to -2\int \operatorname{csch}^2 U\,du$

$-2\left(-\coth U\right)+c \to \boxed{2\coth\left(2-\frac{1}{2}x\right)+c}$

㉙ $Y=2x^2+4x,\ Y=e^x,\ Y=\frac{e^x}{7},\ Y=\log_7 X=\frac{\ln x}{\ln 7}$

$\lim_{x\to\infty}\frac{2x^2+4x}{e^x}=\frac{\infty}{\infty}\to\frac{4x+4}{e^x}=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{4}{e^x}=0$ $\quad e^x$ faster than $2x^2+4x$

$\lim_{x\to\infty}\frac{e^x}{\frac{e^x}{7}}=\frac{\infty}{\infty}\to\frac{7e^x}{e^x}=7\ \therefore$ Same rate

e^x & $\frac{e^x}{7}$ grow faster than $2x^2+4x$

$\lim_{x\to\infty}\frac{\frac{e^x}{7}}{\frac{\ln x}{\ln 7}}=\frac{e^x\ln 7}{7\ln x}=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{e^x\ln 7}{7\cdot\frac{1}{x}}=\frac{xe^x\ln 7}{7}=\infty$ $\quad\frac{e^x}{7}$ faster

(30) $\overbrace{\ln x}^{\text{slower}} = o\overbrace{(\ln 4x)}^{\text{faster}}$

$$\lim_{x\to\infty} \frac{\ln x}{\ln 4x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{1/x}{\frac{1}{4x}\cdot 4} = \frac{0}{0} \qquad \lim_{x\to\infty} 1 = 1$$

$$\boxed{\begin{array}{c}\ln x = o(\ln 4x)\\ \text{false}\end{array}}$$

(31) $2x^3 + \cos x = O(2x^2)$

$$\lim_{x\to\infty} \frac{2x^3 + \cos x}{2x^2} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{6x^2 - \sin x}{4x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{12x - \cos x}{4} = \infty \;\therefore\; \boxed{\begin{array}{c}2x^3 + \cos x = O(2x^2)\\ \text{false}\end{array}}$$

Test Review #4

① $\frac{x}{2} = \sin t \qquad \frac{y}{4} = \cos t$

$\frac{x^2}{4} = \sin^2 t \qquad \frac{y^2}{16} = \cos^2 t$

$$\frac{x^2}{4} + \frac{y^2}{16} = \sin^2 t + \cos^2 t$$

$$\boxed{\frac{x^2}{4} + \frac{y^2}{16} = 1}$$

② $y = 3t$

$\frac{y}{3} = t \qquad x = 9\left(\frac{y}{3}\right)^2$

$x = 9 \cdot \frac{y^2}{9}$

$$\boxed{x = y^2}$$

③ $\frac{dx}{dt} = \cos t \qquad \frac{dy}{dt} = 6\cos t \qquad \frac{dy}{dx} = \frac{6\cos t}{\cos t} = \boxed{6}$

$x = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \;, \qquad y = 6\sin\frac{\pi}{3} = 6\,\frac{\sqrt{3}}{2} = \boxed{3\sqrt{3}}$

$\left(\frac{\sqrt{3}}{2}, 3\sqrt{3}\right) m = 6$

$y - 3\sqrt{3} = 6\left(x - \frac{\sqrt{3}}{2}\right) \longrightarrow y - 3\sqrt{3} = 6x - 3\sqrt{3}$

$$\boxed{y = 6x}$$

④ $\frac{dx}{dt} = 1 - \sin t$ $\quad \frac{dy}{dt} = -\cos t$ $\quad \frac{dy}{dx} = \frac{-\cos\frac{\pi}{6}}{1-\sin\pi/6} = \frac{-\frac{\sqrt{3}}{2}}{1-\frac{1}{2}}$

$\boxed{m = -\sqrt{3}}$

$x = \frac{\pi}{6} + \cos\frac{\pi}{6} = \frac{\pi}{6} + \frac{\sqrt{3}}{2} = \frac{\pi + 3\sqrt{3}}{6}$

$y = 2 - \sin\frac{\pi}{6} = 2 - \frac{1}{2} = \frac{3}{2}$ $\quad \left(\frac{\pi + 3\sqrt{3}}{6}, \frac{3}{2}\right)$

$y - \frac{3}{2} = -\sqrt{3}\left(x - \frac{\pi + 3\sqrt{3}}{6}\right) \longrightarrow \boxed{y = -\sqrt{3}x + \frac{\pi\sqrt{3}}{6} + 3}$

⑤ $\frac{dx}{dt} = 9\cos t$ $\quad \frac{dy}{dt} = -9\sin t$

$\frac{dy}{dx} = \frac{-9\sin t}{9\cos t} = -\tan t$

$\frac{d^2y}{dx^2} = \frac{-\sec^2 t}{9\cos t} = \frac{-1}{9\cos^3 t}$

ⓐ $\frac{3\pi}{4} \rightarrow \frac{-1}{9(\cos\frac{3\pi}{4})^3} = \frac{-1}{9(-\frac{\sqrt{2}}{2})} = \frac{-1}{-9\frac{\sqrt{8}}{8}}$

$\frac{-8}{-9(2\sqrt{2})} = \frac{4}{9\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{9\cdot 2} = \boxed{\frac{2\sqrt{2}}{9}}$

⑥ $\frac{dx}{dt} = \sec^2 t \qquad \frac{dy}{dt} = 9\sec t \tan t$

$\frac{dy}{dx} = \frac{9\sec t \tan t}{\sec^2 t} = \frac{9\tan t}{\sec t} = \frac{9\sin t}{\cos t} \cdot \cos t = \underline{9\sin t}$

$\frac{d^2y}{dx^2} = \frac{9\cos t}{\sec^2 t} = \underline{9\cos^3 t}$ @ $t = \frac{3\pi}{4}$

$9(\cos\frac{3\pi}{4})^3 = 9(-\frac{\sqrt{2}}{2})^3 = -9\frac{\sqrt{8}}{\sqrt{8}} = -9\frac{2\sqrt{2}}{8} = \boxed{-\frac{9\sqrt{2}}{4}}$

⑦ $x = r\cos\theta \qquad y = r\sin\theta$

$x = -1\cos\frac{\pi}{2} \qquad y = -1\sin\frac{\pi}{2}$

$x = 0 \qquad y = -1 \qquad \boxed{(0, -1)}$

⑧ $x = r\cos\theta \qquad y = r\sin\theta$

$x = 3\cos\frac{4\pi}{3} \qquad y = 3\sin\frac{4\pi}{3}$

$x = 3(-\frac{1}{2}) \qquad y = 3(-\frac{\sqrt{3}}{2})$

$x = -\frac{3}{2} \qquad y = -\frac{3\sqrt{3}}{2} \qquad \boxed{(-\frac{3}{2}, \frac{-3\sqrt{3}}{2})}$

⑨ $r = (2\cos\theta - 3\sin\theta) = 1$

$2r\cos\theta - 3r\sin\theta = 1$

$2x - 3y = 1$

$-3y = -2x + 1$

$\boxed{y = \frac{2}{3}x - \frac{1}{3}}$

⑩ $r = 4\,\frac{\cos\theta}{\sin\theta} \,,\, \frac{1}{\sin\theta}$

$r\sin\theta = 4\cot\theta \qquad \tan\theta = \frac{y}{x}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y}$

$y = 4 \cdot \frac{x}{y} \longrightarrow \boxed{y^2 = 4x}$

⑪ $r^2\, 2\sin\theta\cos\theta = 30$

$2r\sin\theta \cdot r\cos\theta = 30$

$yx = 15 \longrightarrow \boxed{y = \frac{15}{x}}$

⑫ $x^2 + y^2 - 4x = 0$

$r^2 - 4r\cos\theta = 0$

$r^2 = 4r\cos\theta$

$\boxed{r = 4\cos\theta}$

⑬ $xy = 1$

$r\cos\theta\, r\sin\theta = 1$

$r^2(\cos\theta\sin\theta) = 1$

$r^2(2\cos\theta\sin\theta) = 2$

$r^2\sin 2\theta = 2$

$\boxed{r^2 = \frac{2}{\sin 2\theta}}$

⑭ $x^2 - 38x + 361 + y^2 + 6y + 9 = 361$

$x^2 - 38x + y^2 + 6y - 9$

$x^2 + y^2 - 38x + 6y = -9$

$r^2 - 38r\cos\theta + 6r\sin\theta = -9$

$\boxed{r^2 = 38r\cos\theta - 6r\sin\theta - 9}$ //

⑮ $r = -3 - 2\sin\theta$

$r(\frac{\pi}{6}) = -4 \rightarrow r(-\frac{\pi}{6}) = -2$ ∴ no X axis sym

$r(\pi - \frac{\pi}{6}) = -4$ ∴ y-axis sym

$r(\pi + \pi/6) = -2$ ∴ no origin sym

⑯ $r(\frac{\pi}{18}) = 5.196$

$r(-\frac{\pi}{18}) = 5.196$ = X-axis sym

$r(\pi - \frac{\pi}{18}) = -5.196$ = no Y-axis

$r(\pi + \pi/18) = -5.196$ = no origin sym

⑲ $r' = -\cos\theta$

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{-\cos\theta\sin\theta + (1-\sin\theta)\cos\theta}{-\cos\theta\cos\theta - (1-\sin\theta)\sin\theta}$$

$$= \frac{-\cos\pi\sin\pi + (1-\sin\pi)\cos\pi}{-\cos\pi\cos\pi - (1-\sin\pi)\sin\pi} = \frac{-1}{-1-0} = \boxed{1}$$ //

⑳ $r' = -8\sin\theta$

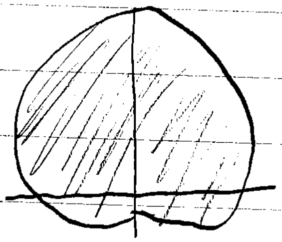
$$\frac{dx}{dy} = \frac{-8\sin\theta\sin\theta + (8+8\sin\theta)\cos\theta}{-8\sin\theta\cos\theta - (8+8\sin\theta)\sin\theta}$$

$$= \frac{-8\left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{2}\right) + \left(8+8\left(\frac{\sqrt2}{2}\right)\right)\left(\frac{\sqrt2}{2}\right)}{-8\left(\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{2}\right) - \left(8+8\frac{\sqrt2}{2}\right)\left(\frac{\sqrt2}{2}\right)}$$

$$= \frac{-2(2)+4\sqrt2+2(2)}{-2(2)-(4\sqrt2)-2(2)} = \frac{4\sqrt2}{-4-4\sqrt2-4} = \frac{4\sqrt2}{-4(2+\sqrt2)}$$

$$= \frac{-\sqrt2}{2+\sqrt2} = \frac{2\sqrt2}{2\sqrt2} = \frac{-2\sqrt2+2}{4-2} = \frac{2(-\sqrt2+1)}{2} = \boxed{1-\sqrt2}$$

㉑

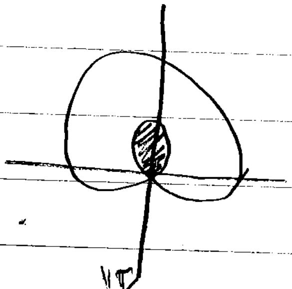


0 to 2π

$$A = \int_0^{2\pi} \frac12(4+4\sin\theta)^2 d\theta$$

$A = 279.602$

㉒



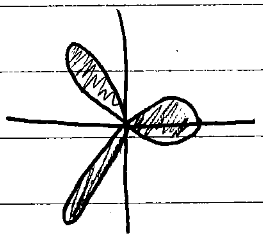
$r = 0 = 2 + 4\sin\theta$

$-2 = 4\sin\theta$

$-\frac12 = \sin\theta \rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$$A = \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \frac12(2+4\sin\theta)^2 d\theta \qquad \boxed{A = 2.174}$$

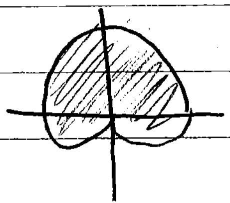
㉓



$$A = \int_0^{\pi} \tfrac{1}{2}(8\cos 3\theta)^2 \, d\theta$$

$$\boxed{A = 50.265} \; //$$

㉔



$$A = \int_0^{2\pi} \tfrac{1}{2}(3+2\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (9+12\sin\theta+4\sin^2\theta)\,d\theta$$

$$\frac{1}{2}\left[9\theta - 12\cos\theta + 4\int \frac{1-\cos 2\theta}{2}\,d\theta\right]_0^{2\pi}$$

$$\frac{1}{2}\left[9\theta - 12\cos\theta + 2\theta - \int 2\cos 2\theta\,d\theta)\right]_0^{2\pi}$$

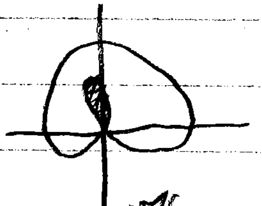
$$\frac{1}{2}\left[11\theta - 12\cos\theta - \int \cos u\,du\right]_0^{2\pi}$$

$$\frac{1}{2}\left[11\theta - 12\cos\theta - \sin 2\theta\right]_0^{2\pi}$$

$$\frac{1}{2}\left[11(2\pi) - 12(\cos 2\pi) - \sin 2(2\pi)\right] - \left[11(0) - 12\cos(0) - \sin(0)\right]$$

$$\boxed{A = 34.558} \; //$$

㉕



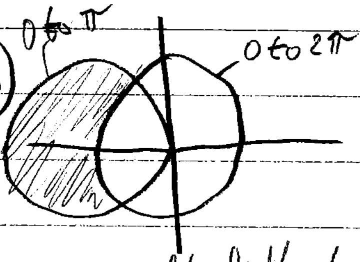
$r = 0 = 4 + 8\sin\theta$

$-4 = 8\sin\theta$

$\sin\theta = -\frac{1}{2} = \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$$A = \int_{7\pi/6}^{11\pi/6} \frac{1}{2}(4+8\sin\theta)^2 d\theta = \boxed{A = 8.696}$$

㉖



$r_1 = r_2$ for POI

$-4\cos\theta = 2$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$A = \overset{\text{need to double it}}{2}\int_{\frac{2\pi}{3}}^{\pi} \frac{1}{2}(-4\cos\theta)^2 - (2)^2 d\theta$$

or

$\boxed{A = 7.653}$

$$A = \int_{\frac{2\pi}{3}}^{4\pi/3} \frac{1}{2}\left((-4\cos\theta)^2 - (2)^2\right) d\theta$$

(27) $\frac{dr}{d\theta} = 6\theta$

$$AL = \int_0^{2\sqrt{3}} \sqrt{(3\theta^2)^2 + (6\theta)^2}\, d\theta = \int_0^{2\sqrt{3}} \sqrt{9\theta^4 + 36\theta^2}\, d\theta$$

$$= \int_0^{2\sqrt{3}} \sqrt{9\theta^2} \cdot \sqrt{\theta^2+4}\, d\theta = \int_0^{2\sqrt{3}} 3\theta(\theta^2+4)^{1/2}\, d\theta \quad \begin{matrix} u = \theta^2+4 \\ du = 2\theta d\theta \end{matrix}$$

$$= \frac{3}{2}\int_0^{2\sqrt{3}} \frac{2}{3} \cdot 3\theta(\theta^2+4)^{1/2}\, d\theta = \frac{3}{2}\int_0^{2\sqrt{3}} 2\theta(\theta^2+4)^{1/2}\, d\theta$$

$$= \frac{3}{2}\int u^{1/2}\, du \longrightarrow (\theta^2+4)^{3/2}\Big|_0^{2\sqrt{3}} = \boxed{56}$$

(28) $\frac{dr}{d\theta} = -3(1+\sin\theta)^{-2}\cos\theta = \frac{-3\cos\theta}{(1+\sin\theta)^2}$

$$AL = \int_0^{\pi/2} \sqrt{\left(\frac{3}{1+\sin\theta}\right)^2 + \left(\frac{-3\cos\theta}{(1+\sin\theta)^2}\right)}\, d\theta$$

$$\boxed{AL = 3.443}$$

㉙ $x^2 = 4py$

$4p = 20$

$\underline{\underline{p = 5}}$

$\boxed{\begin{array}{l}\text{focus: } (0, 5) \\ \text{Directrix: } y = -5\end{array}}$

㉚ $y^2 = 4px$

$4p = 36$

$p = 9$

$\boxed{\begin{array}{l}\text{focus: } (9, 0) \\ \text{Directrix: } x = -9\end{array}}$

㉛ $a^2 = 25$ $b^2 = 16$ center: $(0, 0)$

$a = 5$ $b = 4$

foci: $c = 3$ $\boxed{(-3, 0) \,\&\, (3, 0)}$

$\boxed{\text{Vertices: } (-5, 0) \,\&\, (5, 0)}$

㉜ $\frac{x^2}{72} - \frac{y^2}{72} = 1$

$a^2 = 72$ $a = 6\sqrt{2}$

$b^2 = 72$ $b = 6\sqrt{2}$

center $(0, 0)$ $c = \sqrt{a^2 + b^2}$

$c = 12$

$\boxed{\text{Foci: } (-12, 0) \,\&\, (12, 0)}$

$m = \pm \frac{b}{a} = \pm \frac{6\sqrt{2}}{6\sqrt{2}} = \pm 1$

$\boxed{y = \pm x}$

㉝ $\frac{y^2}{100} - \frac{x^2}{16} = 1$ $a^2 = 100$ $a = 10$

$b^2 = 16$ $b = 4$

$\boxed{\text{Vertices } (0,-10) \,\&\, (0,10)}$ Center $(0,0)$

Assymptotes: $m = \pm \frac{a}{b} = \pm \frac{10}{4} = \boxed{\pm\frac{5}{2}}$

$$\boxed{y = \pm\frac{5}{2}x}$$

㉞ $e > 1$ ∴ hyperbola

$y = 10$ ∴ $K = 10$ $r = \frac{Ke}{1 + e\sin\theta}$

$r = \frac{10(3)}{1 + 3\sin\theta} =$

$$\boxed{r = \frac{30}{1 + 3\sin\theta}}$$

㉟ $e < 1 \therefore$ ellipse

$x = 7 \therefore K = 7$

$$r = \frac{7(\frac{1}{5})}{1 + \frac{1}{5}\cos\theta} = \frac{7(\frac{1}{5})}{5(1 + \frac{1}{5}\cos\theta)}$$

$$\boxed{r = \frac{7}{5 + \cos\theta}} \; //$$

MAC 2312 - Calculus w/Analytic Geometry II

Name____________________________________

Review for Test 3 - Chapter 10

Date____________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the limit of the sequence if it converges; otherwise indicate divergence.

1) $a_n = \frac{5+3n}{3+2n}$

A) $\frac{5}{3}$ [✓ B) $\frac{3}{2}$] C) 1 D) Diverges

2) $a_n = \frac{7n+6}{2+1\sqrt{n}}$

A) 6 B) 7 C) $\frac{7}{2}$ [✓ D) Diverges]

Find the sum of the series.

3) $\sum_{n=1}^{\infty} \frac{7}{2^n}$

A) $\frac{7}{3}$ [✓ B) 7] C) 14 D) $\frac{14}{3}$

4) $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{4^n}\right) = \sum_{n=0}^{\infty} \frac{1}{2^n} - \sum_{n=0}^{\infty} \frac{1}{4^n}$ = Two Geometrics

A) 2 B) - 2 C) - $\frac{2}{3}$ [✓ D) $\frac{2}{3}$]

Determine if the series converges or diverges. If the series converges, find its sum.

5) $\sum_{n=1}^{\infty} \left(\frac{1}{\ln(n+3)} - \frac{1}{\ln(n+4)}\right)$

A) converges; $\frac{1}{\ln 3}$ B) diverges C) converges; ln 4 [✓ D) converges; $\frac{1}{\ln 4}$]

6) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n+1}} - \frac{1}{\sqrt{n+3}}\right)$

A) converges; $\frac{1}{\sqrt{3}} + \frac{1}{2}$ B) diverges

[✓ C) converges; $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}$] D) converges; $\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}}$

Use the nth-Term Test for divergence to show that the series is divergent, or state that the test is inconclusive.

7) $\sum_{n=1}^{\infty} \cos \frac{8}{n}$

A) inconclusive [✓ B) diverges] C) converges, 8 D) converges, 1

Use the integral test to determine whether the series converges.

8) $\sum_{n=1}^{\infty} \frac{4n}{n^2+2}$

A) converges [✓ B) diverges]

9) $\sum_{n=1}^{\infty} \frac{\cos 1/n}{n^2}$

A) diverges [✓ B) converges]

Use the limit comparison test to determine if the series converges or diverges.

10) $\sum_{n=1}^{\infty} \frac{2}{4n - 5 \ln n + 4}$

A) Converges [✓ B) Diverges]

11) $\sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$

[✓ A) Converges] B) Diverges

Use the ratio test to determine if the series converges or diverges.

12) $\sum_{n=1}^{\infty} \frac{5^n}{n!}$

[✓ A) Converges] B) Diverges

13) $\sum_{n=1}^{\infty} \frac{9(n!)^2}{(2n)!}$

[✓ A) Converges] B) Diverges

14) $\sum_{n=1}^{\infty} \frac{6^n}{n!}$

A) Diverges [✓ B) Converges]

Use the root test to determine if the series converges or diverges.

15) $\sum_{n=1}^{\infty} \frac{n^n}{5n^2}$

[✓ A) Converges] B) Diverges

Determine convergence or divergence of the alternating series.

16) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3 + 8}$

A) Diverges [✓ B) Converges]

17) $\sum_{n=1}^{\infty} (-1)^n \ln\left[\frac{7n+3}{6n+2}\right]$

[✓ A) Diverges] B) Converges

18) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$

A) Converges B) Diverges

Determine if the series converges absolutely, converges, or diverges.

19) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{4n^8 + 4}{8n^9 + 2}\right)$

A) Converges absolutely B) Diverges [✓ C) Converges conditionally]

20) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{7}{4} - \frac{5}{n}\right)^n$

A) converges conditionally B) Converges absolutely [✓ C) Diverges]

21) $\sum_{n=1}^{\infty} (-9)^{-n}$

A) diverges [✓ B) converges absolutely] C) converges conditionally

Find the interval of convergence of the series.

22) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^4 6^n}$

A) $1 \le x \le 3$ [✓ B) $-4 \le x \le 8$] C) $-8 < x < 8$ D) $x < 8$

23) $\sum_{n=1}^{\infty} \frac{(x-4)^n}{(2n)!}$

A) $3 \le x \le 5$ B) $x \le 5$ [✓ C) $-\infty < x < \infty$] D) $2 \le x \le 6$

For what values of x does the series converge absolutely?

24) $\sum_{n=0}^{\infty} (-1)^n (6x+9)^n$

A) $-\frac{5}{3} < x \le -\frac{4}{3}$ [✓ B) $-\frac{5}{3} < x < -\frac{4}{3}$] C) $-\frac{7}{9} \le x < -\frac{5}{9}$ D) $-\frac{7}{9} < x < -\frac{5}{9}$

25) $\sum\limits_{n=1}^{\infty} \frac{(x-8)^n}{\sqrt{n}}$

A) $x = 8$
B) $x = \pm 8$
C) $x = -8$
[✓ D) $7 < x < 9$]

Find the Taylor polynomial of order 3 generated by f at a.

26) $f(x) = \ln x, a = 6$

A) $P_3(x) = \frac{\ln 6}{6} + \frac{x-6}{36} + \frac{(x-6)^2}{216} + \frac{(x-6)^3}{1296}$
B) $P_3(x) = \ln 6 - \frac{x-6}{6} + \frac{(x-6)^2}{72} - \frac{(x-6)^3}{648}$
C) $P_3(x) = \frac{\ln 6}{6} - \frac{x-6}{36} + \frac{(x-6)^2}{216} - \frac{(x-6)^3}{1296}$
D) $P_3(x) = \ln 6 + \frac{x-6}{6} - \frac{(x-6)^2}{72} + \frac{(x-6)^3}{648}$

27) $f(x) = e^{-2x}, a = 0$

A) $P_3(x) = 1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{18}$
B) $P_3(x) = 1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{6}$
C) $P_3(x) = 1 - 4x + \frac{16x^2}{2} - \frac{64x^3}{12}$
D) $P_3(x) = 1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{3}$

Use power series operations to find the Taylor series at x = 0 for the given function.

28) $f(x) = x^6 e^{4x}$

A) $\sum\limits_{n=0}^{\infty} \frac{4^{n+6} x^{n+6}}{n!}$
B) $\sum\limits_{n=0}^{\infty} \frac{4^{n+6} x^{n+6}}{(n+6)!}$
C) $\sum\limits_{n=0}^{\infty} \frac{4^{n} x^{n+6}}{(n+6)!}$
D) $\sum\limits_{n=0}^{\infty} \frac{4^{n} x^{n+6}}{n!}$

29) $f(x) = x^3 \sin x$

A) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+4)!}$
B) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n}}{(2n+4)!}$
C) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+1)!}$
D) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n 3^n x^{2n}}{(2n+1)!}$

Find the first four terms of the binomial series for the given function.

30) $(1 + 10x)^{1/2}$

A) $1 - 5x + \frac{25}{2}x^2 - \frac{125}{4}x^3$
B) $1 + 5x - \frac{25}{2}x^2 + \frac{125}{4}x^3$
C) $1 - 5x + \frac{25}{2}x^2 - \frac{125}{2}x^3$
D) $1 + 5x - \frac{25}{2}x^2 + \frac{125}{2}x^3$

31) $\left(1 + \frac{x}{8}\right)^{-2}$

A) $1 - \frac{1}{4}x + \frac{3}{64}x^2 - \frac{3}{128}x^3$
B) $1 - \frac{1}{4}x + \frac{1}{16}x^2 - \frac{3}{256}x^3$
C) $1 - \frac{1}{4}x + \frac{3}{64}x^2 - \frac{1}{128}x^3$
D) $1 - \frac{1}{4}x + \frac{1}{16}x^2 - \frac{1}{64}x^3$

Answer Key
Testname: MAC 2312 - REV T3 - CH 10

1) B
2) D
3) B
4) D
5) D
6) C
7) B
8) B
9) B
10) B
11) A
12) A
13) A
14) B
15) A
16) B
17) A
18) A
19) C
20) C
21) B
22) B
23) C
24) B
25) D
26) D
27) B
28) D
29) C
30) D
31) C

Test Reviews

① $a_n = \frac{5+3n}{3+2n} = \lim_{n\to\infty} \frac{5+3n}{3+2n} = \boxed{\frac{3}{2}}$

② $a_n = \frac{7n+6}{2+1n^{1/2}} = \lim_{n\to\infty} \frac{7n+6}{2+n^{1/2}} = \frac{\infty}{\infty} \quad \frac{d}{dn} \cdot \frac{1}{2n^{1/2}}$

$\lim_{n\to\infty} \frac{7}{\frac{1}{2n^{1/2}}} = \lim_{n\to\infty} 7 \cdot \frac{2n^{1/2}}{1} = \boxed{\infty}$ Diverges

③ $\sum_{n=1}^{\infty} \frac{7}{2^n} = 7 \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 7\left[\frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right)^2 + \dots\right]$

$a = \frac{7}{2} \quad r = \frac{1}{2} \leq 1$ Converges

$S_n = \frac{\frac{7}{2}}{1-\frac{1}{2}} = \frac{\frac{7}{2}}{\frac{2-1}{2}} = \frac{7/2}{\frac{1}{2}} = \frac{7}{\cancel{2}} \cdot \frac{2}{1} = \boxed{7}$

④ $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{4^n}\right) = 0 + \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{16}} + \frac{1}{8} - \frac{1}{64} + \cancel{\frac{1}{16}} - \frac{1}{256}$

$\sum_{n=0}^{\infty} \left(\frac{1}{2} - \frac{1}{4}\right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{4} + \frac{1}{4}\left(\frac{1}{4}\right) + \frac{1}{4}\left(\frac{1}{4}\right)^2$

$a = \frac{1}{4}, \quad r = \frac{1}{4} < 1$ Converges

$S_n = \left(\frac{\frac{1}{4}}{1-\frac{1}{4}} + 1\right)$

④ $\sum_{n=0}^{\infty}\left(\frac{1}{2^n}-\frac{1}{4^n}\right) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$

$a=\frac{1}{2}$ $r=\frac{1}{2}<1$ conv. $a=\frac{1}{4}$ $r=\frac{1}{4}<1$ conv.

$$\text{Sum} = \left(1+\frac{\frac{1}{2}}{1-\frac{1}{2}}\right) - \left(1+\frac{\frac{1}{4}}{1-\frac{1}{4}}\right) = \boxed{\frac{2}{3}}$$

⑤ $\sum_{n=1}^{\infty}\left(\frac{1}{\ln(n+3)}-\frac{1}{\ln(n+4)}\right) = \frac{1}{\ln 4} - \cancel{\frac{1}{\ln 5}} + \cancel{\frac{1}{\ln 5}} - \cancel{\frac{1}{\ln 6}} + \cancel{\frac{1}{\ln 6}} -$

$$\boxed{\text{Converges} = \frac{1}{\ln 4}}$$

⑥ $\sum_{n=1}^{\infty}\left(\frac{1}{(n+1)^{1/2}}-\frac{1}{(n+3)^{1/2}}\right)$

$$\frac{1}{(2)^{1/2}} - \cancel{\frac{1}{(4)^{1/2}}} + \frac{1}{\sqrt{3}} - \cancel{\frac{1}{\sqrt{5}}} + \cancel{\frac{1}{\sqrt{4}}} - \cancel{\frac{1}{\sqrt{6}}} + \cancel{\frac{1}{\sqrt{5}}} - \frac{1}{\sqrt{7}} + \cancel{\frac{1}{\sqrt{6}}} - \cancel{\frac{1}{\sqrt{8}}} + \ldots$$

$$\boxed{\text{Converges } \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}}$$

⑦ $\sum_{n=1}^{\infty} \cos\frac{8}{n}$ nth term test

$\lim_{n\to\infty} \cos\frac{8}{n} = \cos\frac{8}{\infty} = \boxed{\begin{array}{l}\cos(0) = 1 \neq 0 \\ \text{diverges}\end{array}}$

⑧ $\sum_{n=1}^{\infty} \frac{4n}{n^2+2} = \lim_{b\to\infty} \int_1^b \frac{4n}{n^2+2}\,dn$ $u = n^2+2$

$du = 2n\,dn$

$\lim_{b\to\infty} 2\int_1^b \frac{\frac{1}{2}\cdot 4n\,dn}{n^2+2} = \lim_{b\to\infty} 2\int_1^b \frac{1}{u}\,du$

$\lim_{b\to\infty} 2\left[\ln(n^2+2)\right]\Big|_1^b = \lim_{b\to\infty} 2\left[\ln(b^2+2) - \ln(1^2+2)\right]$

$2\left[\ln(\infty) - \ln 3\right] = 2\left[\frac{\ln(\infty)}{\ln 3}\right] = \boxed{\infty}$ diver...

⑨ $\sum_{n=1}^{\infty} \frac{\cos 1/n}{n^2}$ $\lim_{b\to\infty}\int_1^b \frac{\cos 1/n}{n^2}$ $U=\frac{1}{n}=n^{-1}$

$dU=-\frac{1}{n^2}$

$$\lim_{b\to\infty} -\int_1^b -\frac{1dn}{n^2}\cdot\cos\frac{1}{n} = \lim_{b\to\infty} -\int_1 \cos U$$

$$\lim_{b\to\infty} -\left[\sin\frac{1}{n}\right]_1^b = \lim_{b\to\infty} -\left[\sin\frac{1}{b}-\sin 1\right]$$

$$-\left[\sin 0 - \sin 1\right] = 0+\sin 1 = \boxed{\sin 1 \text{ Converges}}$$

⑩ $\sum_{n=1}^{\infty} \frac{2}{4n-5\ln(n)+4}$ $V_n = \frac{2}{4n} = \frac{1}{2n}$ Harmonic Div.

$$\lim_{n\to\infty}\frac{V_n}{V_n} = \frac{\frac{2}{4n-5\ln(n)+4}}{\frac{1}{2n}} = \frac{4n}{4n-5\ln n+4} = \frac{\infty}{\infty}$$

$$= \lim_{n\to\infty}\frac{4}{4-\frac{5}{n}} = 1 \quad \boxed{\begin{array}{c}0<1<\infty\\ \underline{V_n}\text{ div} \therefore U_n\text{ div}\end{array}}$$

⑪ $\sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$ $\quad V_n = \frac{2^n}{4^n} = \left(\frac{1}{2}\right)^n$

Geometric $r = \frac{1}{2} < 1 \therefore V_n$ Conv.

$$\lim_{n\to\infty} \frac{\frac{2^n}{3+4^n}}{\frac{1}{2^n}} = \frac{2^n \cdot 2^n}{3+4^n} = \lim_{n\to\infty} \frac{2^{2n}}{3+4^n}$$

$$\lim_{n\to\infty} \frac{(2^2)^n}{3+4^n} = \lim_{n\to\infty} \frac{4^n}{3+4^n} = 1$$

$$\boxed{0<1<\infty \quad V_n \text{ Conv} \therefore U_n \text{ Conv}}$$

⑫ $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ $\quad a_{n+1} = \frac{5^{n+1}}{(n+1)!}$

$$\lim_{n\to\infty} \frac{\frac{5^{n+1}}{(n+1)!}}{\frac{5^n}{n!}} = \frac{\cancel{5^n} \cdot 5 \cdot \cancel{n!}}{(n+1)\cancel{n!}\,\cancel{5^n}} = \frac{5}{n+1} = 0$$

$$\boxed{0<1 \therefore \text{Converges}}$$

⑬ $\sum_{n=1}^{\infty} \frac{9(n!)^2}{(2n)!}$ $a_n = \frac{9((n+1)!)^2}{(2(n+1))!}$

$$\lim_{n\to\infty} \frac{9((n+1)n!)^2}{(2n+2)!} \div \frac{9(n!)^2}{(2n)!}$$

$$\lim_{n\to\infty} \frac{9((n+1)n!)^2}{(2n+2)(2n+1)(2n)!} \cdot \frac{(2n)!}{9(n!)^2}$$

$$\lim_{n\to\infty} \frac{\cancel{9}(n+1)^2\cancel{(n!)^2}}{(2n+2)(2n+1)\cancel{(2n)!}} \cdot \frac{\cancel{(2n)!}}{\cancel{9(n!)^2}} -$$

$$\lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{n^2}{4n^2} = \boxed{\frac{1}{4} < 1 \text{ Conv}}$$

⑭ $\sum_{n=1}^{\infty} \frac{6^n}{n!}$ $a_n = \frac{6^{n+1}}{(n+1)!}$

$$\lim_{n\to\infty} \frac{6^n \cdot 6}{(n+1)n!} \div \frac{6^n}{n!} = \lim_{n\to\infty} \frac{\cancel{6^n} \cdot 6}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{\cancel{6^n}}$$

$$\lim_{n\to\infty} \frac{6}{n+1} = \boxed{0 < 1 \therefore \text{ Converges}}$$

⑮ $\sum_{n=1}^{\infty} \frac{n^n}{5^{n^2}}$ $\lim_{n \Rightarrow \infty} \sqrt[n]{\frac{n^n}{5^{n^2}}}$

$$\lim_{n \to \infty} \frac{(n^n)^{1/n}}{(5^{n^2})^{1/2}} = \frac{n}{5^{n^2/n}} = \frac{n}{5^n} = \frac{\infty}{\infty}$$

$$\lim_{n \to \infty} \frac{1}{5^n \ln 5} = \boxed{0 < 1 \therefore \text{Converges}}$$

⑯ $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3+8}$

$a_n > 0$ ✓

$a_{n+1} \leq a_n$ ↓ ✓

$\lim_{n \to \infty} \frac{1}{2n^3+8} = 0$ ✓ $\therefore$ Converges

⑰ $\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{7n+3}{6n+2}\right)$

$a_n > 0$ ✓

$a_{n+1} \leq a_n$ ↓

$\lim_{n \to \infty} a_n = 0$?

$$\lim_{n \to \infty} \ln\left(\frac{7n+3}{6n+2}\right) = \ln\left(\lim_{n \to \infty}\left(\frac{7n+3}{6n+2}\right)\right) = \frac{\infty}{\infty}$$

$$\ln\left(\frac{7}{6}\right) = \ln \frac{7}{6} \neq 0$$ inconclusive

n^{th} term test $\boxed{\lim_{n \to} (-1)^n = DNE \cdot \ln \frac{7}{7} \quad \therefore \text{Diverg}}$

⑱ $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^{1/3}}$

$$\boxed{\begin{array}{l} a_n > 0 \checkmark \\ a_{n+1} \le a_n \downarrow \checkmark \\ \lim_{n\to\infty} \frac{1}{n^{1/3}} = 0 \checkmark \quad \therefore \text{Converges} \end{array}}$$

⑲ $\sum_{n=1}^{\infty} (-1)^n \left(\frac{4n^8+4}{8n^9+2}\right)$

$a_n > 0 \checkmark$

$a_{n+1} \le a_n \downarrow \checkmark$

$\lim_{n\to} \frac{4n^8+4}{8n^9+2} = 0 \checkmark \therefore$ Conv.

Limit Comparison test

$V_n = \frac{4n^8}{8n^9} = \frac{1}{2n} =$ Harmonic Series Div..

$$\lim_{n\to\infty} \frac{4n^8+4}{8n^9+2} \div \frac{1}{2n} = \lim_{n\to\infty} \frac{4n^8+4}{8n^9+2} \cdot 2n$$

$$\lim_{n\to\infty} \frac{(8n^9+8n)}{8n^9+2} = 1$$

$$0 < 1 < \infty$$

$\therefore V_n$ Diverges $\therefore U_n$ Diverges

$$\boxed{\therefore \text{Conditionally Conv.}}$$

⑳ $\sum_{n=1}^{\infty} (-1)^n \left(\frac{7}{4} - \frac{5}{n}\right)^n$ $a_n > 0$ ✓

$a_{n+1} \leq a_n$ X

n^{th} term test

$\lim_{n\to\infty} (-1)^n = DNE$ $\lim_{n\to\infty} \left(\frac{7}{4} - \frac{5}{n}\right)^n = DIV$

$\boxed{(DNE)(Div) = DNE}$

㉑ $\sum_{n=1}^{\infty} (-9)^{-n} = \sum_{n=1}^{\infty} (-1)^n (9)^{-n}$

$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{9^n} = \sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{9}\right)^n$ $a_n > 0$ ✓

$a_{n+1} \leq a_n$ ✓

$\lim_{n\to\infty} \left(\frac{1}{9}\right)^n = 0$ ✓

CONV.

Absolute Conv?

$\sum_{n=1}^{\infty} \left(\frac{1}{9}\right)^n$ Geometric

$a = \frac{1}{9}$ $r = \frac{1}{9} < 1$ Converges

$\boxed{\text{Converges Absolutely}}$

(22) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^4 6^n}$ Ratio test

$$\lim_{n\to\infty} \left| \frac{\cancel{(x-2)}^{n+1}}{(n+1)^4 \cancel{6^{n+1}}} \cdot \frac{n^4 \cancel{6^n}}{\cancel{(x-2)^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{n^4}{(n+1)^4} \cdot \frac{x-2}{6} \right| \to \left| \frac{x-2}{6} \right| < 1$$

$$-1 < \frac{x-2}{6} < 1 \to -6 < x-2 < 6$$

$$-4 < x < 8$$

Test End Points $x = -4$

$$\sum_{n=0}^{\infty} \frac{(-4-2)^n}{n^4 6^n} = \sum_{n=0}^{\infty} \frac{(-6)^n}{n^4 6^n} = \sum_{n=0}^{\infty} (-1)^n \frac{\cancel{6^n}}{n^4 \cancel{6^n}}$$

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n^4}$$

$a_n > 0$

$a_{n+1} \leq a_n$ ✓

$\lim_{n\to\infty} a_n = 0$ ✓ CONV.

㉒ Cont 19.1

$x=8$

$$\sum_{n=0}^{\infty}\frac{(8-2)^n}{n^4 6^n} = \sum_{n=0}^{\infty}\frac{\cancel{6^n}}{n^4\cancel{6^n}} = \sum_{n=0}^{\infty}\frac{1}{n^4}$$

p-series, $p=4>1$ Conv.

$$\boxed{\begin{gathered}\text{Interval of Convergence}\\ -4\le x\le 8\end{gathered}}$$

㉓ $\sum_{n=1}^{\infty}\frac{(x-4)^n}{(2n)!}$ Ratio Test

$$\lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{(2n+1)!}\cdot\frac{(2n)!}{(x-4)^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{\cancel{(x-4)^n}(x-4)^1}{(2n+2)(2n+1)\cancel{(2n)!}}\cdot\frac{\cancel{(2n)!}}{\cancel{(x-4)^n}}\right|$$

$$\lim_{n\to\infty}\frac{x-4}{(2n+2)(2n+1)} = \boxed{0 \;\therefore\; \text{Interval of Convergence} \quad -\infty < x < \infty}$$

㉔ $\sum_{n=6}^{\infty} (-1)^n (6x+9)^n$ Ratio test

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)}^{n+1} (6x+9)^n \cancel{(6x+9)}^1}{\cancel{(-1)}^n \cancel{(6x+9)}^1} \right|$$

$$\lim_{n\to\infty} \left| (-1)(6x+9) \right| = |6x+9| < 1$$

$$-1 < 6x+9 < 1$$

$$-5/3 < x < -4/3$$

Test End Points

$x = -5/3$

$$\sum_{n=0}^{\infty} (-1)^n (6(-5/3)+9)^n \rightarrow \sum_{n=0} (-1)^n (-1)^n$$

Absolute Convergence ∴

$$\sum_{n=0}^{\infty} \left| (-1)^n (-1)^n \right| = \sum_{n=0}^{\infty} 1$$ nth term test for Div.

$$\lim_{n\to\infty} 1 \neq 0$$ ∴ Diverges & Cannot use $X = -5/3$

㉔ Cont...

$X = -4/3$

$$\sum_{n=0}^{\infty} (-1)^n \left(6\left(-4/3\right)+9\right)^n = \sum_{n=0}^{\infty} \left|(-1)^n\right| = \sum_{n=0}^{\infty} 1$$

nth term test for Div.

$\lim_{n\to\infty} 1 \neq 0$ Diverges ∴ Cannot use $X = -4/3$

$$-5/3 < X < -4/3$$

㉕ $\sum_{n=1}^{\infty} \frac{(X-8)^n}{\sqrt{n}}$ Ratio test

$$\lim_{n\to\infty} \left| \frac{(X-8)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(X-8)^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{\sqrt{n}}{\sqrt{n+1}} \cdot (X-8) \right| = |X-8| < 1$$

$$-1 < X-8 < 8$$

$$7 < X < 9$$

$X = 7$

$$\sum_{n=1}^{\infty} \frac{(7-8)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \left|(-1)^n \frac{1}{\sqrt{n}}\right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$$

$p = \frac{1}{2} < 1$ Div

㉕ Cont...

$x = 9$ $\sum_{n=1}^{\infty} \frac{(9-8)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$

$p = \frac{1}{2} < 1$ Div.

$\boxed{\text{Interval of Convergence is } 7 < x < 9}$

㉖ $f(x) = \ln x$ $a = 6$ order 3

$f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f''(x) = -x^{-2}$

$f'''(x) = 2x^{-3}$

$f'(6) = 1/6$

$f''(6) = -1/36$

$f'''(6) = 2/216 = \frac{1}{108}$

$f(9) + f'(9)(x-9) + f''(9)(x-9)^2 + f'''(9)(x-9)^3 + \dots$

$f(x) = \ln 6 + 1/6 \cdot (x-6) - \frac{\frac{1}{36}}{2!}(x-6)^2 + \frac{\frac{1}{108}}{3!}(x-6)^3 + \dots$

$\ln 6 + \frac{1}{6}(x-6) - \frac{1}{72}(x-6)^2 + \frac{1}{648}(x-6)^3 + \dots$

(27) $f(x) = e^{-2x}$, $a = 0$

$f(x) = e^{-2x}$ $\qquad f(0) = 1$

$f'(x) = -2e^{-2x}$ $\qquad f'(0) = -2$

$f''(x) = 4e^{-2x}$ $\qquad f''(0) = 4$

$f'''(x) = -8e^{-2x}$ $\qquad f'''(0) = -8$

$$1 + (-2)(x-0) + \frac{4}{2!}(x-0)^2 + \frac{(-8)}{3!}(x-0)^3 + \dots$$

$$\boxed{1 - 2x + 2x^2 - 4/3\,x^3 + \dots}$$

(28) $f(x) = x^6 e^{4x}$

$e^x = 1 + x + x^2/2! + x^3/3! + \dots$

$e^{4x} = 1 + 4x + 16x^2/2! + 64x^3/3! + \dots$

$e^{4x} = 1 + 4x + 8x^2 + 32/3\,x^3 + \dots$

$$x^6(e^{4x}) = x^6\left(1 + 4x + 8x^2 + 32/3\,x^3 + \dots\right)$$

$$x^6(e^{4x}) = x^6 + 4x^7 + 8x^8 + \frac{32}{3}x^9 + \dots$$

$$= x^6 + 4x^7 + \frac{16x^8}{2!} + \frac{64x^9}{3!} + \dots$$

$$= \overset{0}{x^6} + \overset{1}{4x^7} + \overset{2}{\frac{4^2x^8}{2!}} + \overset{3}{\frac{4^3x^9}{3!}} + \dots$$

$$\boxed{\sum_{n=0}^{\infty} \frac{4^n x^{n+6}}{n!}}$$

㉙ $f(x) = x^3 \sin x$

$\sin x = x - x^3/3! + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$

$x^3 \sin x = \underset{0}{x^4} - \underset{1}{x^6/3!} + \underset{2}{x^8/5!} + \underset{3}{\frac{x^{10}}{7!}} + \dots$

$$\boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+4}}{(2n+1)!}}$$

㉚ $(1+10x)^{1/2}$ $m = 1/2$ $x = 10x$

$$1 + \sum_{n=1}^{\infty} \binom{m}{n} x^n$$

$\binom{m}{1} = \frac{1}{2}$ $\binom{m}{2} = \frac{1/2(1/2-1)}{2!} = -\frac{1}{8}$ $\binom{m}{3} = \frac{1/2(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!} = \frac{1}{16}$

$(1+10x)^{1/2} \approx 1 + \frac{1}{2}(10x) + (-\frac{1}{8})(10x)^2 + \frac{1}{16}(10x)^3$

$(1+10x)^{1/2} \approx 1 + 5x - \frac{25}{2}x^2 + \frac{125}{2}x^3 + \dots$

(31) $\left(1+\frac{x}{8}\right)^{-2}$ $\quad m=-2 \quad X=\frac{x}{8}$

$\binom{m}{1}\ -2 \qquad \binom{m}{2}\ \frac{-2(-2-1)}{2!}=3 \qquad \binom{m}{3}\ \frac{-2(-2-1)(-2-2)}{3!}=-4$

$$\approx 1+(-2)\left(\frac{x}{8}\right)+3\left(\frac{x}{8}\right)^2+(-4)\left(\frac{x}{8}\right)^3$$

$$\left(1+\frac{x}{8}\right)^{-2}\approx 1-\frac{1}{4}x+\frac{3}{64}x^2-\frac{1}{128}x^3$$

if it were $\left(2+\frac{x}{8}\right)^{-2}$

$$\boxed{2}\left(1+\frac{x}{8}\right)^{-2}$$

$$2\left[1-\frac{1}{4}x+\dots\right]$$

MAC 2312 - Calculus w/Analytic Geometry II	Name________________________

Review for Test 2 - Chapter 8	Date________________________

Evaluate the integral using integration-by-parts.

1) $\int -8x\cos 2x\,dx = -8x\cdot\frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x\cdot(-8dx) = -4x\sin 2x + 4\int \sin 2x\,dx$

$u = 2x$
$du = 2dx$

$u = -8x$ $dv = \cos 2x\,dx$
$du = -8dx$ $v = \frac{1}{2}\sin 2x$

$= -4x\sin 2x + 4\cdot\frac{1}{2}\int \sin 2x\cdot 2dx = -4x\sin 2x + 2\int \sin u\,du$

$= -4x\sin 2x + 2(-\cos u) + C = \boxed{-4x\sin 2x - 2\cos 2x + C}$

2) $\int x\csc^2 3x\,dx = x\cdot -\frac{1}{3}\cot 3x - \int -\frac{1}{3}\cot 3x\,dx = -\frac{1}{3}x\cot 3x + \frac{1}{3}\int \cot 3x\,dx$

$U = x$ $dV = \csc^2 3x\,dx$
$du = dx$ $V = -\frac{1}{3}\cot 3x$

$= -\frac{1}{3}x\cot 3x + \frac{1}{3}\int \frac{\cos 3x}{\sin 3x}dx = -\frac{1}{3}x\cot 3x + \frac{1}{3}\cdot\frac{1}{3}\int \frac{3\cos 3x\,dx}{\sin 3x}$

$u = \sin 3x$
$du = 3\cos 3x\,dx$

$= -\frac{1}{3}x\cot 3x + \frac{1}{9}\int \frac{1}{u}du = \boxed{-\frac{1}{3}x\cot 3x + \frac{1}{9}\ln|\sin 3x| + C}$

3) $\int 2xe^x\,dx$

f'	$\int$
$2x$ +	e^x
2 −	e^x
0	e^x

$= \boxed{2xe^x - 2e^x + C}$

4) $\int x^3\ln 4x\,dx = \ln 4x\cdot\frac{1}{4}x^4 - \int \frac{1}{4}x^4\cdot\frac{1}{x}dx = \frac{1}{4}x^4\ln 4x - \frac{1}{4}\int x^3\,dx$

$U = \ln 4x$ $dV = x^3\,dx$
$du = \frac{4}{4x} = \frac{1}{x}dx$ $V = \frac{1}{4}x^4$

$= \frac{1}{4}x^4\ln 4x - \frac{1}{4}\left(\frac{1}{4}x^4\right) + C$

$= \boxed{\frac{1}{4}x^4\ln 4x - \frac{1}{16}x^4 + C}$

5) **Evaluate the following trigonometric integrals.**

$$\int 4\cos^4 6x\,dx = 4\int \cos^2 6x\cdot \cos^2 6x = 4\int\left(\frac{1+\cos 12x}{2}\right)\cos^2 6x\,dx = 4\int\left(\frac{1+\cos 12x}{2}\right)\left(\frac{1+\cos 12}{2}\right)$$

$$= 4\cdot\frac{1}{4}\int(1+\cos 12x)(1+\cos 12x)\,dx = \int 1+2\cos 12x+\cos^2 12x\,dx = \int dx + \int 2\cos 12x\,dx + \int \cos^2 12x\,dx$$

$$= \int\frac{1+\cos 24x}{2}dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 24x\,dx = \boxed{\frac{1}{2}x+\frac{1}{48}\sin 24x} \quad = x+\frac{1}{6}\sin 12x + \frac{1}{2}x+\frac{1}{48}\sin 24x + C$$

$$= \boxed{\frac{3}{2}x+\frac{1}{6}\sin 12x+\frac{1}{48}\sin 24x + C}$$

6) $\int 7\cos^3 5x\,dx$

$$= 7\int \cos^2 5x\cdot\cos 5x\,dx = 7\int(1-\sin^2 5x)\cos 5x\,dx = 7\int \cos 5x - \sin^2 5x\cos 5x\,dx$$

$$= 7\int\cos 5x\,dx - 7\int \sin^2 5x\cos 5x\,dx = \frac{7}{5}\sin 5x - \frac{7}{5}\int u^2\,du = \frac{7}{5}\sin 5x - \frac{7}{15}u^3 + C$$

$u = \sin 5x \quad du = 5\cos 5x$

$$= \boxed{\frac{7}{5}\sin 5x - \frac{7}{15}\sin^3 5x + C}$$

7) $\int \csc^3 6t\,dt = \int \csc^2 6t\cdot\csc 6t\,dt =$ $\quad U=\csc 6t \quad dV = \csc^2 6t\,dt$

$du = -6\csc 6T\cot 6T \quad V = -\frac{1}{6}\cot 6T$

$$= -\frac{1}{6}\cot 6T\cdot\csc 6T - \int -\frac{1}{6}\cot 6T\cdot(-6\csc 6T\cot 6T)\,dT = -\frac{1}{6}\cot 6T\csc 6T - \int \csc 6T\cot^2 6T\,dt$$

?

8) $\int \tan^4 6t\,dt = \int \tan^2 6T\tan^2 6T\,dt = \int(\sec^2-1)\tan^2\,dt = \int \tan^2\sec^2 - \tan^2\,dt$

$$= \int \tan^2\sec^2\,dt - \int\tan^2\,dt = \frac{1}{6}\int u^2 - \int \sec^2 - 1\,dt = \frac{1}{18}\tan^3 6t - \left[\int -dt + \int \sec^2\,dt\right] = \frac{1}{18}\tan^3 6t + t - \frac{1}{6}\tan 6t$$

$u = \tan 6t$

$du = 6\sec^2 6t\,dt$

$$= \boxed{\frac{1}{18}\tan^3 6t - \frac{1}{6}\tan 6t + t + C}$$

9) $\int \sin 7x\cos 4x\,dx = \frac{1}{2}\int \sin(7-4)+\sin(7+4)\,dx = \frac{1}{2}\int \sin 3x + \sin 11x\,dx$

$$= \frac{1}{2}\int \sin 3x\,dx + \frac{1}{2}\int \sin 11x\,dx = \boxed{-\frac{1}{6}\cos 3x - \frac{1}{22}\cos 11x + C}$$

10) $\int \sin 8t\sin 5t\,dt = \frac{1}{2}\int \cos(8-5)x - \cos(8+5)x\,dx = \frac{1}{2}\int\cos 3x - \frac{1}{2}\int \cos 13x\,dx$

$$= \boxed{\frac{1}{6}\sin 3x - \frac{1}{26}\sin 13x + C}$$

$Sin^2 + Cos^2 = 1$

$Cos^2 = 1 - Sin^2$

Integrate the function using trig substitution.

11) $\int \sqrt{36 - x^2}\,dx = \int \sqrt{36-(6Sin\theta)^2}\,(6Cosd\theta) = \int \sqrt{36(1-Sin^2\theta)}\,(6Cosd\theta)$

$a^2 = 36$

$a = 6$

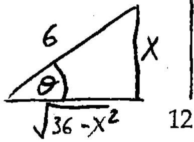
$x = aSin\theta$

$x = 6Sin\theta$

$dx = 6Cosd\theta$

$Sin\theta = \frac{x}{6}$

$\theta = Sin^{-1}\frac{x}{6}$



$= \int 6\sqrt{Cos^2\theta}\cdot(6Cos\theta\, d\theta) = \int 36Cos^2\theta d\theta = 36\int \frac{1}{2}(1+Cos2\theta)d\theta$

$= 18\int d\theta + 18\cdot\frac{1}{2}\int Cos2\theta = 18\theta + 9Sin2\theta = 18Sin^{-1}\frac{x}{6} + 9\cdot 2Sin\theta Cos\theta$

$= \boxed{18Sin^{-1}\frac{x}{6} + 18\frac{x\sqrt{36-x^2}}{6(6)} + C}$

12) $\int \frac{1}{t^2\sqrt{4-t^2}}\,dt$

$a^2 = 4 \quad a = 2$

$x = 2Sin\theta$

$dx = 2Cos\theta d\theta$

$Sin\theta = \frac{x}{2}$

$\theta = Sin^{-1}(\frac{x}{2})$

$\int \frac{2Cos\theta\, d\theta}{4Sin^2\theta\sqrt{4-4Sin^2\theta}} = \frac{1}{2}\int \frac{Cos\theta d\theta}{Sin^2\theta\sqrt{4(1-Sin^2\theta)}} = \frac{1}{2}\int \frac{Cos\theta d\theta}{Sin^2\theta\cdot 2\sqrt{Cos^2\theta}}$

$= \frac{1}{4}\int \frac{1}{Sin^2\theta}d\theta = \frac{1}{4}\int Csc^2\theta d\theta = \frac{-1}{4}Cot\theta + C = \frac{-1}{4}\left(\frac{\sqrt{4-t^2}}{t}\right) = \boxed{-\frac{\sqrt{4-t^2}}{4t} + C}$

13) $\int \frac{40\,dx}{x^2\sqrt{x^2+64}} = 40\int \frac{8Sec^2\theta d\theta}{64Tan^2\theta\sqrt{64Tan^2\theta+64}} = 40\int \frac{8\,Sec^2\theta\, d\theta}{64Tan^2\theta\cdot 8\sqrt{Tan^2\theta+1}}$

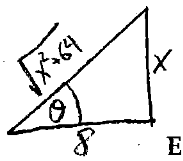
$a^2 = 64 \quad a = 8$

$x = 8Tan\theta$

$dx = 8Sec^2\theta d\theta$

$Tan\theta = \frac{x}{8}$

$\theta = Tan^{-1}(\frac{x}{8})$



$= \frac{40}{64}\int \frac{Sec^2\theta d\theta}{Tan^2\theta\sqrt{Sec^2\theta}} = \frac{5}{8}\int \frac{Sec^2\theta d\theta}{Tan^2\theta\cdot Sec\theta} = \frac{5}{8}\int \frac{Sec\theta}{Tan^2\theta}d\theta = \frac{5}{8}\int \frac{1}{Cos\theta}\cdot\frac{Cos^2\theta}{Sin^2\theta}d\theta$

$= \frac{5}{8}\int \frac{Cos\theta}{Sin^2\theta}d\theta$ $\quad U = Sin\theta \quad du = Cos\theta d\theta$ $= \frac{5}{8}\int \frac{1}{U^2} = \frac{5}{8}(-\frac{1}{U}) + C = \boxed{\frac{-5}{8Sin\theta} + C}$

Sub is $Sin\theta$ to finish

Express the integrand as a sum of partial fractions and evaluate the integral.

14) $\int \frac{x+9}{x^2+5x}\,dx$ $\quad x(x+5) = \frac{A}{x} + \frac{B}{x+5} = x+9 = A(x+5) + Bx$

let $x = 0 \therefore 0+9 = 5A + 0 = \boxed{A = 9/5}$

let $x = -5 \therefore 4 = 0 - 5B = \boxed{B = -4/5}$

$\int \frac{9/5}{x}dx + \int \frac{-4/5}{x+5}dx = \frac{9}{5}\int\frac{1}{x}dx - \frac{4}{5}\int\frac{1}{U}du$ $\quad U = x+5 \quad du = dx$

$= \boxed{\frac{9}{5}\ln|x| - \frac{4}{5}\ln|x+5| + C} = \frac{1}{5}\left[9\ln|x| - 4\ln|x+5|\right] + C = \frac{1}{5}\ln\left|\frac{x^9}{(x+5)^4}\right| + C$

15) $\int \frac{36\,dx}{x^3 - 9x} = \frac{A}{x} + \frac{B}{(x-3)} + \frac{C}{(x+3)} = 36 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$

$x(x^2-9)$

$x(x-3)(x+3)$

Let $x = 0 \therefore 36 = A(0-3)(0+3) = \boxed{A = -4}$

Let $x = 3 \therefore 36 = A(\cancel{3-3})(3+3) + 3B(3+3) + 3C(\cancel{3-3})$

$36 = 18B = \boxed{B = 2}$

Let $x = -3 \therefore 36 = 0 + 0 - 3C(-6)$

$36 = 18C = \boxed{C = 2}$

$\int \frac{-4}{x} + \int \frac{2}{x-3} + \int \frac{2}{x+3} = -4\int\frac{1}{x} + 2\int\frac{1}{x-3} + 2\int\frac{1}{x+3}$

$= \boxed{-4\ln|x| + 2\ln|x-3| + 2\ln|x+3| + C}$

$(X+5)(x+5)$

$(X+5)^2$

Long Division Not on Test

16) $\int \cancel{\frac{x^3}{x^2+10x+25}}dx = \frac{A}{(x+5)} + \frac{B}{(x+5)^2} = A(x+5)+B$

Let $x = -5 \therefore (-5)^3 = A(-5+5)^0 + B$

$\boxed{B = -125}$

Let $x = 0 \therefore 0 = A(0+5) - 125$

$A = \frac{125}{5} = \boxed{A = 25}$

$\int \frac{25}{x+5} + \int \frac{-125}{(x+5)^2} = 25\int \frac{1}{x+5} - 125\int \frac{1}{(x+5)^2}$ $\quad U = x+5$

$= 25\int \frac{1}{U}du - 125\int \frac{1}{U^2}du$

$= 25\ln|x+5| + 125\frac{1}{x+5} = \boxed{25\ln|x+5| + \frac{125}{x+5} + C}$

17) $\int \frac{8x^2+x+252}{x^3+36x}dx = \frac{A}{x} + \frac{Bx+C}{x^2+36} = \int \frac{7}{x}dx + \int \frac{1x+1}{x^2+36}dx \rightarrow 7\int \frac{1}{x}dx + \int \frac{x\,dx}{x^2+36} + \int \frac{1\,dx}{x^2+36}$ $\quad U = x+3$ $\quad du = d$

$x(x^2+36)$

$8x^2+x+252 = A(x^2+36) + (Bx+C)(x)$

Let $x=0 \therefore 252 = A(36) + 0 \rightarrow \boxed{A=7}$

Let $x=1 \therefore 8+1+252 = 7(37) + B + C$

$261 = 259 + B + C \rightarrow B+C = 2$

Let $x=-1 \therefore 8-1+252 = 7(37) + B - C$

$259 - 259 = B - C \rightarrow B = C$

Let $x=1 \therefore B+C = 2 \rightarrow B+B = 2 \rightarrow \boxed{B = 1 = C}$

$= 7\ln|x| + \frac{1}{2}\int \frac{2x\,dx}{x^2+36} + \frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right)$

$= 7\ln|x| + \frac{1}{2}\ln$

Use the Table of Integrals to evaluate the integral.

#30 18) $\int \frac{\sqrt{4x-7}}{x^2}dx = -\frac{\sqrt{ax-b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax-b}} = -\frac{\sqrt{4x-7}}{x} + 2\int \frac{dx}{x\sqrt{4x-7}}$ #29(b)

$= -\frac{\sqrt{4x-7}}{x} + 2\left[\frac{2}{\sqrt{b}}\tan^{-1}\sqrt{\frac{ax-b}{b}}\right] = \boxed{-\frac{\sqrt{4x-7}}{x} + \frac{4}{\sqrt{7}}\tan^{-1}\sqrt{\frac{4x-7}{7}} + C}$

#47 19) $\int \frac{\sqrt{9-x^2}}{x}dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

$a = 3$

$a^2 = 9$

$= \boxed{\sqrt{9-x^2} - 3\ln\left|\frac{3+\sqrt{9-x^2}}{x}\right| + C}$

#43 20) $\int \frac{dx}{(25-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{4a^3}\ln\left|\frac{x+a}{x-a}\right|$

$a^2 = 25$

$a = 5$

$= \frac{x}{2(25)(25-x^2)} + \frac{1}{4(125)}\ln\left|\frac{x+5}{x-5}\right| + C$

$= \boxed{\frac{x}{50(25-x^2)} + \frac{1}{500}\ln\left|\frac{x+5}{x-5}\right| + C}$

#78

a=2

b=5

c=13

21) $\int \frac{dx}{5 + 13 \sin 2x} = = \frac{-1}{a\sqrt{c^2-b^2}} \ln\left|\frac{c+b\sin ax + \sqrt{c^2-b^2}\cos ax}{b+c\sin ax}\right| + C$

$= \frac{-1}{2\sqrt{169-25}} \ln\left|\frac{13+5\sin 2x + \sqrt{169-25}\cos 2x}{5+13\sin 2x}\right| + C$

Use the Trapezoidal Rule with n = 4 steps to estimate the integral.

22) $\int_0^2 2x^2\,dx$

23) $\int_{-1}^1 (x^2 + 4)\,dx$

24) $\int_{-\pi}^0 -\sin x\,dx$

Use Simpson's Rule with n = 4 steps to estimate the integral.

25) $\int_0^2 3x^2\,dx$

26) $\int_{-1}^{1} (x^2 + 7)\, dx$

27) $\int_{-\pi}^{0} \sin x\, dx$ $\quad \Delta x = \frac{-\pi - 0}{4} = \frac{-\pi}{4}$

Evaluate the improper integral or state that it is divergent.

28) $\int_{1}^{\infty} \frac{dx}{x^{2.955}} = \lim_{t \to \infty} \int_{1}^{t} \frac{dx}{x^{2.955}} = \lim_{t \to \infty} \int_{1}^{t} x^{-2.955}\, dx = \lim_{t \to \infty}$

29) $\int_{7}^{\infty} \frac{dx}{(x - 6)(x - 5)}$

30) $\int_{0}^{\infty} 10xe^{2x}\, dx = \lim_{t \to \infty} \int_{0}^{t} 10xe^{2x}\, dx = 10x2e^{2x} - 10 \cdot 4e^{2x}$

f'	∫
10x	e^{2x}
10	$2e^{2x}$
0	$4e^{2x}$

$= \lim_{t \to \infty} \left[10x2e^{2x} - 40e^{2x} \right] \Big|_{0}^{t} = \lim_{t \to \infty} \left[(0 - 40) - (10(\infty)2e^{2(\infty)} - 40e^{\infty}) \right]$

$= (0 - 40) - \infty = \infty = \text{Div.}$

31) $\int_0^{64} \frac{dx}{\sqrt{64 - x}}$

32) $\int_4^{16} \frac{dt}{t\sqrt{t^2 - 16}}$ Cant get answer

1) $-2\cos 2x - 4x\sin 2x + C$

2) $-\frac{1}{3}x\cot 3x + \frac{1}{9}\ln|\sin 3x| + C$

3) $2xe^{x} - 2e^{x} + C$

4) $\frac{1}{4}x^4\ln 4x - \frac{1}{16}x^4 + C$

5) $\frac{3}{2}x + \frac{1}{6}\sin 12x + \frac{1}{48}\sin 24x + C$

6) $\frac{7}{5}\sin 5x - \frac{7}{15}\sin^3 5x + C$

7) $-\frac{1}{12}\csc 6t\cot 6t - \frac{1}{12}\ln|\csc 6t + \cot 6t| + C$

8) $\frac{1}{18}\tan^3 6t - \frac{1}{6}\tan 6t + t + C$

9) $-\frac{1}{22}\cos 11x - \frac{1}{6}\cos 3x + C$

10) $\frac{1}{6}\sin 3t - \frac{1}{26}\sin 13t + C$

11) $18\sin^{-1}\left(\frac{x}{6}\right) + \frac{x\sqrt{36 - x^2}}{2} + C$

12) $-\frac{\sqrt{4 - t^2}}{4t} + C$

13) $-\frac{5\sqrt{x^2 + 64}}{8x} + C$

14) $\frac{1}{5}\ln\left|\frac{x^9}{(x + 5)^4}\right| + C$

15) $-4\ln|x| + 2\ln|x - 3| + 2\ln|x + 3| + C$

16) $\frac{x^2}{2} - 25x + 75\ln|x + 5| + \frac{125}{x + 5} + C$

17) $7\ln|x| + \frac{1}{2}\ln|x^2 + 36| + \frac{1}{6}\tan^{-1}\left(\frac{x}{6}\right) + C$

18) $-\frac{\sqrt{4x - 7}}{x} + \frac{4\sqrt{7}}{7}\tan^{-1}\sqrt{\frac{4x - 7}{7}} + C$

19) $\sqrt{9 - x^2} - 3\ln\left|\frac{3 + \sqrt{9 - x^2}}{x}\right| + C$

20) $\frac{1}{50}\left(\frac{x}{25 - x^2} + \frac{1}{10}\ln\left|\frac{x + 5}{x - 5}\right|\right) + C$

21) $-\frac{1}{24}\ln\left|\frac{13 + 5\sin 2x + 12\cos 2x}{5 + 13\sin 2x}\right| + C$

22) $\frac{11}{2}$

23) $\frac{35}{4}$

24) $-\frac{1+\sqrt{2}}{4}\pi$

25) 8

26) $\frac{44}{3}$

27) $-\frac{1+2\sqrt{2}}{6}\pi$

28) $\frac{1}{1.955}$

29) ln 2

30) Divergent

31) 16

32) $\frac{\pi}{24}$

⑰ $X(X^2+36) = \dfrac{A}{X} + \dfrac{Bx+C}{X^2+36}$

$A = 7$

$X=1 = 8+1+252 = 7(37)+B+C$

$261 = 259 + B + C$

$2 = B + C$

$X=-1 \therefore 8-1+252 = 7(37)+B-C = 259-259 = B-C$

$B = C$

$$\int\frac{7}{X} + \int\frac{X+1}{X^2+36} = 7\ln|X| + \frac{1}{2}\int\frac{②X}{X^2+36} + \int\frac{1}{X^2+36} = \text{Tan}^{-1}$$

$$7\ln X + \frac{1}{2}\int U + \frac{1}{6}\text{Tan}^{-1}\frac{X}{6} + C$$

㉒ $A_T = \frac{1}{4}\left[0+2\left(\frac{1}{2}\right)+2(2)+2\left(\frac{9}{2}\right)+8\right]$

$= \frac{1}{4}(22) = \boxed{\frac{11}{2}}$

㉔ $\Delta x = \frac{\pi}{4}$? $X = -\pi, \frac{-3\pi}{4}, \frac{-\pi}{2}, \frac{-\pi}{4}, 0$

$A_T = \frac{\pi}{8}\left[f(-\pi)+2f\left(-\frac{3\pi}{4}\right)+2f\left(-\frac{\pi}{2}\right)+2f\left(\frac{-\pi}{4}\right)+f(0)\right]$

$A_T = [0-1.414-2-1.414+0]$

$A_T = -1.896$

$x = -\pi, -\frac{3}{4}\pi, -\frac{\pi}{2}, -\frac{\pi}{4}, 0$

(27) $A_s = \frac{\frac{-\pi}{4}}{3}\left[f(-\pi) + 4f(-\frac{3\pi}{4}) + 2f(-\frac{\pi}{2}) + 4f(-\frac{\pi}{4}) + f(0)\right]$

$A_s = -\frac{\pi}{12}\left[-7.656\right] = 2.004$

(28) $\lim_{x\to\infty}\int_1^t x^{-2.955} = \lim_{x\to\infty}\frac{x^{-1.955}}{-1.955}\Big|_1^t$

$\frac{1}{-1.955}\lim_{x\to\infty}\frac{1}{x^{1.955}}\Big|_1^t = -\frac{1}{1.955}\lim_{x\to\infty}\left[\frac{1}{t^{1.955}} - \frac{1}{1^{1.955}}\right]$

$= \cancel{\frac{1}{\infty}}\nearrow^{0} - \frac{1}{1^{1.955}} = \frac{-1}{1.955}\cdot(-1) = \boxed{\frac{1}{1.955}}$ Conv

(29) $\frac{1}{(x-6)(x-5)} = \frac{A}{x-6} + \frac{B}{x-5}$ $B = -1$ $A = 1$

$= \int_7^\infty\left(\frac{1}{x-6} - \frac{1}{x-5}\right) = \lim_{x\to\infty}\int_7^t\left(\frac{1}{x-6} - \frac{1}{x-5}\right)$

$\lim_{t\to\infty}\left[\ln|x-6| - \ln|x-5|\right]_7^t$

$\lim_{t\to\infty}\left[\ln\left|\frac{x-6}{x-5}\right|\right]_7^t = \lim_{t\to\infty}\left[\ln\left|\underset{\infty/\infty}{\frac{t-6}{t-5}}\right| - \ln\left|\frac{7-6}{7-5}\right|\right]$

$\lim_{t\to\infty}\left[\ln\frac{1}{1} - \ln\frac{1}{2}\right] = -\ln\frac{1}{2} = -\left[\ln 1 - \ln 2\right]$

$\boxed{\ln 2}$

30 $u = 10x \quad dv = e^{2x}$

$du = 10 \quad v = \frac{1}{2}e^{2x}$

$$5xe^{2x} - \frac{1}{2}\int 5e^{2x}\,2dx$$

$$\lim_{t\to\infty} 5xe^{2x} - \frac{5}{2}e^{2x}\Big|_0^t$$

$$\lim_{t\to\infty}\left[5e^{2x}\left(x - \frac{1}{2}\right)\right]_0^t$$

$$\lim_{t\to\infty}\left[5e^{2t}\left(t-\frac{1}{2}\right) - 5e^{(2\cdot 0)}\left(0 - \frac{1}{2}\right)\right]$$

$$\infty + \frac{5}{2} \quad \boxed{\text{Div.}}$$

$$\textcircled{31}\ \lim_{t\to 64^-} -\int_0^t \ominus\frac{1}{\sqrt{64-x}} \qquad \begin{matrix} U=64-x \\ du=-dx \end{matrix}$$

$$\lim_{t\to 64^-} -\int_0^t \frac{1}{U^{1/2}} = \lim_{t\to 64^-} -\frac{U^{1/2}}{\frac{1}{2}}\Big|_0^t$$

$$\lim_{t\to \cancel{64}64^-} -2\sqrt{64-x}\Big|_0^t = \lim_{t\to 64^-}\left[-2\sqrt{64-t} - -2\sqrt{64-0}\right]$$

$$0+2(8) = \boxed{16} \text{ conv.}$$

$$\textcircled{32}\ \lim_{b\to 4^+}\int_b^{16}\frac{1}{t\sqrt{t^2-16}} \qquad \begin{matrix} a^2=16 \\ a=4 \end{matrix}$$

$$\lim_{b\to 4^+} \frac{1}{4}\sec^{-1}\left|\frac{x}{4}\right|\Big|_b^{16} = \lim_{b\to 4^+}\frac{1}{4}\cos^{-1}\left|\frac{4}{x}\right|\Big|_b^{16}$$

$$\lim_{b\to 4^+}\left[\frac{1}{4}\cos^{-1}\left|\frac{4}{16}\right| - \cancelto{1:0}{\frac{1}{4}\cos^{-1}\left|\frac{4}{b}\right|}\right]$$

$$\frac{1}{4}\cos^{-1}\left|\frac{1}{4}\right| = \boxed{0.330}$$