$= 99$

Please Show All Your Work For Credit!

89 pts $+10$

Name: Steven Romeiro Date: ____________

$w = \frac{\pi}{4}$

1. Use the rectangles in the following graph to approximate the area of the region bounded by $y = \cos x$, $y = 0$, $x = -\frac{\pi}{2}$, and $x = \frac{\pi}{2}$.

$+7$

$A = w \cdot h$

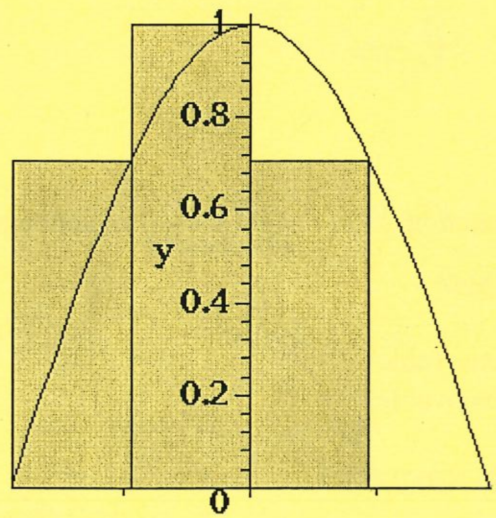
$A = w(h_1 + h_2 + h_3 + h_4)$

$A = \frac{\pi}{4}\left(\cos -\frac{\pi}{2} + \cos -\frac{\pi}{4} + \cos 0 + \cos \frac{\pi}{4}\right)$

$A = \frac{\pi}{4}\left(0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right)$

$A = \frac{\pi}{4}(\sqrt{2} + 1)$

$\boxed{A = 1.8961}$



A) 3.7922
B) 2.5282
C) 0.9481
D) 1.4221
E) 1.8961

2. Complete the table and use the result to estimate the limit.

$$\lim_{x \to 0} \frac{\sin^2 x}{x^2}$$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	.9967	.99997	1	1	.99997	.9967

A) 1
B) 0.5
C) -1
D) -0.5
E) 0

$\boxed{L = 1}$

3. Find the limit (if it exists):

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + (x+\Delta x) - 4 - (x^2 + x - 4)}{\Delta x}$$

A) $\frac{1}{3}x^3 + \frac{1}{2}x^2 - 4x$
B) $x^3 + x^2 - 4x$
C) 0
D) $2x+1$
E) $x^2 + x - 4$

1

Scratch Paper

4. Find the x-values (if any) at which the function $f(x) = \dfrac{x-6}{x^2-4x-12}$ is not continuous.

Which of the discontinuities are removable?
A) No points of discontinuity.
B) $x=6$ (Not removable), $x=-2$ (Removable)
C) $x=6$ (Removable), $x=-2$ (Not removable)
D) No points of continuity.
E) $x=6$ (Not removable), $x=-2$ (Not removable)

$\lim_{x \to -2} \frac{x-6}{(x+2)(x-6)} = \lim_{x \to -2} \frac{1}{(x+2)}$

Discontinuities at $x=-2$ and $x=6$ where $x=6$ is removable and $x=-2$ is non-removable

5. Find constants a and b such that the function

$$f(x) = \begin{cases} 4, & x \le -7 \\ ax+b, & -7 < x < 1 \\ -4, & x \ge 1 \end{cases}$$

is continuous on the entire real line.
A) $a = 1, b = 0$
B) $a = 1, b = -3$
C) $a = 1, b = 3$
D) $a = -1, b = 3$
E) $a = -1, b = -3$

4

$\lim_{x \to -7} 4 = 4$ $\quad \lim_{x \to -7^-} f(x) = 4$

$\lim_{x \to -7^+} f(x) = \lim_{x \to -7^+} (ax+b) = -7a+b$

$\lim_{x \to 1^+} -4 = -4$

$-7a + b = 4$ ①

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (ax+b)$
$= a(1)+b = a+b$

$x \to -7 < x < 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (-4) = -4$

$\lim_{x \to -1} ax+b = a(-1)$ ②

$a+b=-4$

Multiply ② by -1 and add it to ①

$-a-b=4$
$-7a+b=4$
$-8a = 8 \Rightarrow \boxed{a=-1}$

Replace a in ②
$-1+b=-4$
$\boxed{b=-3}$

-6

3) $$\lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 + \cancel{x} + \Delta x - \cancel{4} - \cancel{x^2} - \cancel{x} + \cancel{4}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{2x\cancel{\Delta x} + \cancel{\Delta x}^2 + \cancel{\Delta x}}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} 2x + \Delta x = \boxed{2x}$$

$(x+4)(x+6)$

6. Find the vertical asymptotes (if any) of the function $f(x)=\dfrac{x^2+10x+24}{x^3-6x^2-16x+96}$.

A) $x=6$
B) $x=4$
C) $x=-6$
D) $x=-4$
E) A and B
F) C and D

$\lim\limits_{x\to 6}\dfrac{(x+4)(x+6)}{(x-6)(x^2+4x+16)} = \dfrac{10\cdot 12}{(6-6)(36+24+16)} = \dfrac{120}{0}$

$\lim\limits_{x\to 4}\dfrac{(x+4)(x+6)}{(x-4)(x^2+6x+36)} = \dfrac{8\cdot 10}{(4-4)(16+24+36)} = \dfrac{80}{0}$

Both $x=4$ and $x=6$ are V.A.

7. Find all values of x (if any) at which the graph of the function $f(x)=\dfrac{\ln(x^2+6)}{x^2+1}$ has vertical asymptotes.

A) 6, 1, and –1
B) 6 and 1
C) 6, –6, and 1
D) 6, –6, 1, and –1
E) no vertical asymptotes

$\lim\limits_{x\to -1}\dfrac{\ln(x^2+6)}{x^2+1} = \ln\left[\lim\limits_{x\to -1}\dfrac{x^2+6}{x^2+1}\right] = \ln\left(\dfrac{1+6}{1+1}\right)$

$= \ln\dfrac{7}{2}$

there are no V.A.

8. Find the limit:

$$\lim_{x\to 5}\frac{x^2-5x}{(x^2+25)(x-5)}$$

A) –10
B) $-\dfrac{1}{10}$
C) 10
D) $\dfrac{1}{10}$
E) 5

$\lim\limits_{x\to 5}\dfrac{x(x-5)}{(x^2+25)\cancel{(x-5)}} = \lim\limits_{x\to 5}\dfrac{x}{(x^2+25)} = \dfrac{5}{(5^2+25)}$

$= \dfrac{5}{50}$

$\boxed{L=\dfrac{1}{10}}$

9. Determine the limit (if it exists):

$$\lim_{x\to 0}\frac{14(1-\cos x)}{x^2}$$

A) 7
B) 0
C) 1
D) Does not exist
E) 11

$= \lim\limits_{x\to 0}\dfrac{14}{x}\cdot\dfrac{(1-\cos x)}{x} = \left(\dfrac{14}{0}\right)\cdot(0)$

Does not exist

10. Find all values of x (if any) at which the graph of $f(x) = \dfrac{x^2 - 4}{e^{-8x} + 6}$ has vertical asymptotes.

A) no vertical asymptotes

B) -2, 2, and $\ln\left(\dfrac{4}{3}\right)$

C) -2, 2, and $\dfrac{\ln(6)}{8}$

D) $\dfrac{3}{4}$

E) $\dfrac{\ln(6)}{8}$

$e^{-8x} + 6 = 0$

$e^{-8x} = -6$

$\ln e^{-8x} = \ln -6$

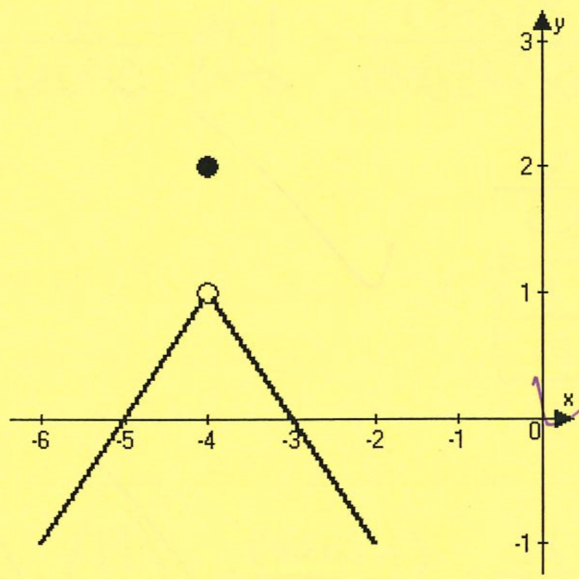
$-8x = \ln -6$

$\boxed{x = \dfrac{\ln 6}{8}}$

-3

11. Use the graph as shown to determine the following limits, and discuss the continuity of the function at $x = -4$.

(i) $\lim_{x \to -4^+} f(x)$ (ii) $\lim_{x \to -4^-} f(x)$ (iii) $\lim_{x \to -4} f(x)$



A) 1 , 1 , 1 , Not continuous
B) 1 , 1 , 1 , Continuous
C) 2 , 2 , 2 , Not continuous
D) 2 , 2 , 2 , Continuous
E) -4 , -4 , -4 , Continuous

$\lim_{x \to -4^+} f(x) = 1$

$\lim_{x \to -4^-} f(x) = 1$

$\lim_{x \to -4} f(x) = 1$

$f(c) = f(-4) = 2$

Since $\lim_{x \to -4} f(x) \neq f(c)$ then the fxn is not continuous

12. Find the limit:

$$\lim_{x\to\pi} \tan\left(\frac{2x}{3}\right)$$

A) $-3^{1/2}$
B) $3^{1/2}$
C) $6^{-1/2}$
D) Does not exist
E) $-6^{-1/2}$

$\lim_{x\to\pi} \frac{\sin\left(\frac{2x}{3}\right)}{\cos\left(\frac{2x}{3}\right)} = \frac{\sin\left(\frac{2\pi}{3}\right)}{\cos\left(\frac{2\pi}{3}\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$L = \sqrt{3}$ or $3^{\frac{1}{2}}$

13. Let $f(x) = 4x^2 - 5x - 1$ and $g(x) = \sqrt[3]{x+3}$. Find the limits:

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 3} g(x)$ (c) $\lim_{x\to 4} g(f(x))$

A) $-2, -\sqrt[3]{6}, -\sqrt[3]{46}$
B) $0, \sqrt[3]{6}, -\sqrt[3]{46}$
C) $0, -\sqrt[3]{6}, \sqrt[3]{46}$
D) $-2, \sqrt[3]{6}, \sqrt[3]{46}$
E) None of the above

a) $\lim_{x\to 1} 4x^2 - 5x - 1 = 4 - 5 - 1 = \boxed{L = -2}$

b) $\lim_{x\to 3} \sqrt[3]{x+3} = \sqrt[3]{3+3} = \boxed{L = \sqrt[3]{6}}$

c) $\lim_{x\to 4} g(4x^2 - 5x - 1) = g(4(16) - 5(4) - 1) = g(43)$

$= \sqrt[3]{43+3} = \boxed{\sqrt[3]{46}}$

Name: Steven Romeo Date: 9-29-11

89pts +7

$-15x^2-12x$

1. Find the derivative of the following function using the limiting process.

Scratch Paper ok!

$f(x) = -5x^3 - 6x^2 - 5$

A) $f'(x) = -15x^2 - 12x - 5$
B) $f'(x) = -10x^2 - 12x$
C) $f'(x) = -15x^2 - 12x$
D) $f'(x) = -10x^2 + 12x$
E) $f'(x) = -15x^2 + 12x$

2. Find all values of x (if any) at which the graph of the function $4x + 5e^x$ has a horizontal tangent.

A) no horizontal tangents
B) $\ln\frac{4}{5}$
C) $\ln\frac{5}{4}$
D) 1
E) 0

$f'(x) = 5e^x + 4 = 0$

$5e^x = -4$

$e^x = -\frac{4}{5}$

$\ln e^x = \ln\left(-\frac{4}{5}\right)$

$x = \ln -\frac{4}{5}$

$\ln \neq -\frac{4}{5}$

No horizontal tangents

3. Use the quotient rule to differentiate the following function and evaluate $g'(3)$.

$g(t) = \frac{6t}{t^5 + 6}$

A) $g'(3) = \frac{644}{6889}$
B) $g'(3) = -\frac{1932}{83}$
C) $g'(3) = \frac{1932}{83}$
D) $g'(3) = -\frac{644}{6889}$
E) $g'(3) = -\frac{644}{1715361}$

$g'(t) = \frac{6(t^5+6) - 6t(5t^4)}{(t^5+6)^2}$

$g'(t) = \frac{6t^5 + 36 - 30t^5}{(t^5+6)^2}$

$g'(t) = \frac{-24t^5 + 36}{(t^5+6)^2}$

$g'(3) = \frac{-24(3)^5 + 36}{(3^5+6)^2}$

$g'(3) = \frac{-5796}{62001} = \boxed{-\frac{644}{6889}}$

$v(t) = s'(t)$
$s'(t) = -32t + V_0$

4. A ball is thrown straight down from the top of a 250-ft building with an initial velocity of –26 ft per second.

$s'(1) = -32(1) + (-26)$

What is its velocity after 1 seconds? $\boxed{s'(1) = -58 \text{ ft/s}}$

What is its velocity after falling 110 ft? $-58(110)$

The position function is $s(t) = -16t^2 + v_0 t + s_0$.

A) Its velocity after 1 seconds is –6 ft per second. After falling 110 ft its velocity is about 87.84 ft per second.
B) Its velocity after 1 seconds is –58 ft per second. After falling 110 ft its velocity is about 87.84 ft per second.
C) Its velocity after 1 seconds is –58 ft per second. After falling 110 ft its velocity is about –87.84 ft per second.
D) Its velocity after 1 seconds is –6 ft per second. After falling 110 ft its velocity is about –87.84 ft per second.
E) None of the above

5. Find the derivative of the function.

$f(v) = 3v^3 \sin v + 4v^5 \cos v$

A) $f'(v) = (9v^2 - 4v^5)\sin v + (3v^3 + 20v^4)\cos v$
B) $f'(v) = (3v^3 - 20v^4)\sin v + (9v^2 - 4v^5)\cos v$
C) $f'(v) = -(9v^2 - 4v^5)\sin v + (3v^3 + 20v^4)\cos v$
D) $f'(v) = (9v^5 - 4v^2)\sin v + (3v^3 + 20v^4)\cos v$
E) $f'(v) = (9v^2 - 4v^5)\sin v - (3v^3 + 20v^4)\cos v$

$f'(v) = 9v^2(\sin v) + \cos v(3v^3) + 20v^4(\cos v) - \sin v(4v^5)$
$f'(v) = 9v^2 \sin v + 3v^3 \cos v + 20v^4 \cos v - 4v^5 \sin v$
$f'(v) = 9v^2 \sin v - 4v^5 \sin v + 3v^3 \cos v + 20v^4 \cos v$
$f'(v) = \sin v(9v^2 - 4v^5) + \cos v(3v^3 + 20v^4)$

① $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$1)\ f(x+h) = -5(x+h)^3 - 6(x+h)^2 - 5 = -5(x+h)(x^2 - xh + h^2) - 6(x^2 + 2xh + h^2) - 5$

$= -(75x^2 + 75h^2 + 150xh) - (6x^2 + 12xh + 6h^2) - 5 = -(81x^2 + 81h^2 + 162xh)$

$1)\ f(x+h) = -5(x+h)^3 - 6(x+h)^2 - 5 = -5(x+h)(x^2 - xh + h^2) - 6(x^2 + 2xh + h^2)$

$2)\ f(x+h) - f(x) = -5(x+h)(x^2 - xh + h^2) - 6(x^2 + 2xh + h^2) - 5 + 5x^3 + 6x^2 + 5$

$= -5(x+h)(x^2 - xh + h^2) - 6(x^2 + 2xh + h^2) + 5x^3 + 6x^2$

$3)\ \frac{f(x+h)-f(x)}{h} = \frac{-5(x+h)(x^2 - xh + h^2) - 6(x^2 + 2xh + h^2) + 5x^3 + 6x^2}{h}$

$= -5(x)(x^2 - x + h) - 6(x^2 + 2x + h) + 5x^3 + 6x^2$

$= \cancel{-5x^3} + 5x^2 - 5xh \cancel{-6x^2} - 12x - 6h \cancel{+5x^3} \cancel{+6x^2}$

$= 5x^2 - 5xh - 12x - 6h$

$4)\ \lim_{h \to 0} 5x^2 - 5xh - 12x - 6h = 5x^2 - 12x$

$h = t^3$
$V = \pi r^2 h$
$r = \sqrt{5t+5}$

6. The radius of a right circular cylinder is $\sqrt{5t+5}$ and its height is t^3, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume of the cylinder, V, with respect to time.

A) $\frac{dV}{dt} = \pi t^3(15+20t)$ cubic inches/second

B) $\frac{dV}{dt} = \pi t^2(15+20t)$ square inches/second

C) $\frac{dV}{dt} = \pi t^2(15+15t)$ cubic inches/second

D) $\frac{dV}{dt} = \pi t^2(15+20t)$ cubic inches/second

E) $\frac{dV}{dt} = \pi t^2(5+20t)$ inches/second

$V = \pi(\sqrt{5t+5})^2 \cdot (t^3)$
$V = \pi\, 5t+5 \cdot (t^3)$
$V = \pi\, 5t^4 + 5t^3$
$\frac{dV}{dt} = \pi\, 20t^3 + 15t^2$
$\boxed{\frac{dv}{dt} = \pi t^2(20t+15)}$

7. Find the second derivative of the function.

$$g(x) = \frac{6x^2+3x-4}{x}$$

A) $g''(x) = \frac{8}{x^3}$

B) $g''(x) = -\frac{8}{x^3}$

C) $g''(x) = \frac{4}{x^3}$

D) $g''(x) = -\frac{8}{x^2}$

E) $g'(x) = -\frac{x+8}{x^3}$

$g'(x) = \frac{(12x+3)(x) - (6x^2+3x-4)(1)}{x^2}$
$g'(x) = \frac{12x^2 + \cancel{3x} - 6x^2 - \cancel{3x} + 4}{x^2}$
$g'(x) = \frac{6x^2+4}{x^2}$
$g''(x) = \frac{12x(x^2) - (6x^2+4)(2x)}{x^4}$
$g''(x) = \frac{12x^3 - 12x^3 - 8x}{x^4}$
$g''(x) = \frac{-8x}{x^4}$
$\boxed{g''(x) = -\frac{8}{x^3}}$

8. Find the derivative of the function.

$$g(x)=\left(\frac{x+5}{x^2+9}\right)^5$$

A) $g'(x)=\dfrac{5(9-10x+x^2)}{(5+x)(9+x^2)}\left(\dfrac{(5+x)}{(9+x^2)}\right)^5$

B) $g'(x)=\dfrac{5(9+10x-x^2)(5+x)^4}{(9+x^2)^6}$

C) $g'(x)=\dfrac{5(9-10x-x^2)(5+x)^6}{(9+x^2)^4}$

D) $g'(x)=-\dfrac{5(9-10x-x^2)(5+x)^4}{(9+x^2)^6}$

E) $g'(x)=\dfrac{5(9-10x-x^2)(5+x)^4}{(9+x^2)^6}$

$$g'(x)=5\left(\frac{x+5}{x^2+9}\right)^4\cdot\frac{1(x^2+9)-2x(x+5)}{(x^2+9)^2}$$

$$g'(x)=5\left(\frac{x+5}{x^2+9}\right)^4\cdot\frac{x^2+9-2x^2-10x}{(x^2+9)^2}$$

$$g'(x)=5\left(\frac{x+5}{x^2+9}\right)^4\cdot\frac{-x^2-10x+9}{(x^2+9)^2}$$

$$g'(x)=\left(\frac{x+5}{x^2+9}\right)^4\cdot\frac{5(9-10x-x^2)}{(x^2+9)^2}$$

$$\boxed{g'(x)=\frac{5(9-10x-x^2)(x+5)^4}{(x^2+9)^6}}$$

9. Find the derivative of the function $y=\ln\left(x\sqrt{x^2+11}\right)$.

A) $\dfrac{1}{x}+\dfrac{1}{\sqrt{x^2+11}}$

B) $\dfrac{x}{x+\sqrt{x^2+11}}$

C) $\dfrac{1}{x\sqrt{x^2+11}}$

D) $\dfrac{x}{\sqrt{x^2+11}}\ln(x)+\dfrac{1}{x}\ln\left(\sqrt{x^2+11}\right)$

E) $\dfrac{1}{x}+\dfrac{x}{x^2+11}$

$$y=\ln\left(x(x^2+11)^{\frac{1}{2}}\right)$$

$$y=\ln x+\frac{1}{2}\ln(x^2+11)$$

$$y'=\frac{1}{x}+\frac{1(2x)}{2(x^2+11)}$$

$$y'=\frac{1}{x}+\frac{2x}{2(x^2+11)}$$

$$\boxed{y'=\frac{1}{x}+\frac{x}{x^2+11}}$$

$y' = \frac{1}{(\ln 3)x}$

10. Find an equation of the tangent line to the graph of $y = \log_3 x$ at the point $(243, 5)$.

A) $y = 5 + \frac{1}{\ln 3}(x - 243)$

B) $y = 5 + \frac{1}{3 \ln 243}(x - 243)$

C) $y = 5 + \frac{1}{243 \ln 3}(x - 243)$

D) $y = 5 + \frac{1}{243}(x - 243)$

E) None of the above

$y' = \frac{1}{(\ln 3)(243)}$ $\quad y' = \frac{1}{243(\ln 3)}$ Slope

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{243(\ln 3)}(x - 243)$

$\boxed{y = 5 + \frac{1}{243(\ln 3)}(x - 243)}$

11. Find *dy/dx* by implicit differentiation.

$x^5 + 6x + 7xy - y^8 = 4$

A) $\frac{dy}{dx} = \frac{5x^4 + 6 - 7y}{8y^7 - 7x}$

B) $\frac{dy}{dx} = \frac{4x^4 + 6 + 7y}{8y^7 - 7x}$

C) $\frac{dy}{dx} = \frac{5x^4 + 6 + 7y}{7y^7 - 7x}$

D) $\frac{dy}{dx} = -\frac{5x^4 + 6 + 7y}{8y^7 - 7x}$

E) $\frac{dy}{dx} = \frac{5x^4 + 6 + 7y}{8y^7 - 7x}$

$\frac{d}{dx}\left[x^5 + 6x + 7xy - y^8\right] = \frac{d}{dx}[4]$

$5x^4 + 6 + \left(7(y) + 7x\frac{dy}{dx}\right) - 8y^7\frac{dy}{dx} = 0$

$5x^4 + 6 + 7y + 7x\frac{dy}{dx} - 8y^7\frac{dy}{dx} = 0$

$7x\frac{dy}{dx} - 8y^7\frac{dy}{dx} = -5x^4 - 7y - 6$

$\frac{dy}{dx}(7x - 8y^7) = -5x^4 - 7y - 6$

$\frac{dy}{dx} = \frac{-5x^4 - 7y - 6}{7x - 8y^7}$

$\boxed{\frac{dy}{dx} = -\frac{5x^4 + 7y + 6}{8y^7 - 7x}}$

$y = \frac{x^4(3x^2+8)^{\frac{1}{2}}}{(x-1)^2}$

12. Use logarithmic differentiation, the derivative of $y = \dfrac{x^4\sqrt{3x^2+8}}{(x-1)^2}$.

A) $\dfrac{dy}{dx} = \dfrac{x^4\sqrt{3x^2+8}}{(x-1)^2}\left(\dfrac{4}{x} + \dfrac{3}{2(3x^2+8)}\right)$

B) $\dfrac{dy}{dx} = \dfrac{x^4\sqrt{3x^2+8}}{(x-1)^2}\left(\dfrac{4}{x} + \dfrac{3x}{3x^2+8} - \dfrac{2}{x-1}\right)$

C) $\dfrac{dy}{dx} = \dfrac{4x^3\sqrt{3x^2+8}}{(x-1)^4}$

D) $\dfrac{dy}{dx} = \dfrac{x^4\sqrt{3x^2+8}}{(x-1)^4}\left(\dfrac{4}{x} + \dfrac{3}{2(3x^2+8)}\right)$

E) None of the above

$\ln y = \ln\left(\frac{x^4(3x^2+8)^{\frac{1}{2}}}{(x-1)^2}\right)$

$\ln y = 4\ln x + \frac{1}{2}\ln(3x^2+8) - 2\ln(x-1)$

$\frac{y'}{y} = \frac{4}{x} + \frac{1(6x)}{2(3x^2+8)} - \frac{2}{(x-1)}$

$\boxed{y' = \frac{x^4\sqrt{3x^2+8}}{(x-1)^2}\left[\frac{4}{x} + \frac{3x}{(3x^2+8)} - \frac{2}{(x-1)}\right]}$

13. Let $f(x) = \sin 11x,\ 0 \le x \le \dfrac{\pi}{11}$. Find $\left(f^{-1}\right)'\left(\dfrac{1}{2}\right)$.

A) $\dfrac{22\sqrt{3}}{3}$

B) $\dfrac{\sqrt{3}}{33}$

C) $\dfrac{2}{33}$

D) $\dfrac{2}{11}$

E) $\dfrac{2\sqrt{3}}{33}$

$f'(x) = 11\cos 11x$

$\sin 11x = \frac{1}{2}$

$\sin \frac{1}{2} = \frac{\pi}{6}$

$11x = \frac{\pi}{6}$

$x = \frac{\pi}{66}$

$\frac{1}{f'(f^{-1}(\frac{1}{2}))}$

$\frac{1}{11\cos 11\left(\frac{\pi}{66}\right)} = \frac{1}{11\cos\frac{\pi}{6}} = \frac{1}{11\frac{\sqrt{3}}{2}}$

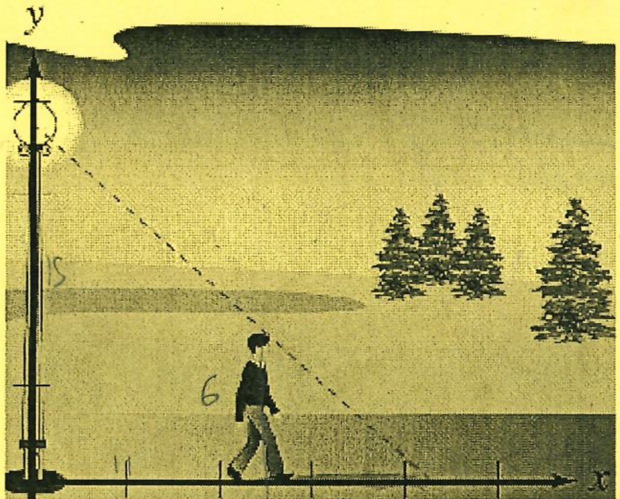
$= \frac{2}{11\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{33}}$

14. ***Shadow Length*** A man 6 feet tall walks at a rate of 2 ft per second away from a light that is 15 ft above the ground (see figure). When he is 11 ft from the base of the light find the following.

(a) The rate the tip of the shadow is moving.
(b) The rate the length of his shadow is changing.

$\frac{dx}{dt} = 2$

15



11 X?

$\frac{6}{15} = \frac{x}{11}$

$15x = 66$

$x = 4.4$

$4.4 + 11 = 15.4$

$a^2 + b^2 = c^2$

$15^2 + 15.4^2 = c^2$

4

	(a)	(b)
A)	$\frac{5}{3}$ ft per minute	$\frac{16}{3}$ ft per minute
B)	$\frac{5}{3}$ ft per minute	$\frac{4}{3}$ ft per minute
C)	$\frac{10}{3}$ ft per minute	$\frac{16}{3}$ ft per minute
D)	$\frac{10}{3}$ ft per minute	$\frac{4}{3}$ ft per minute
E)	None of the above	

15. Use Newton´s Method to approximate the x-value of the indicated point intersection of the two graphs accurate to three decimal places.

$X_1 = 0.6$

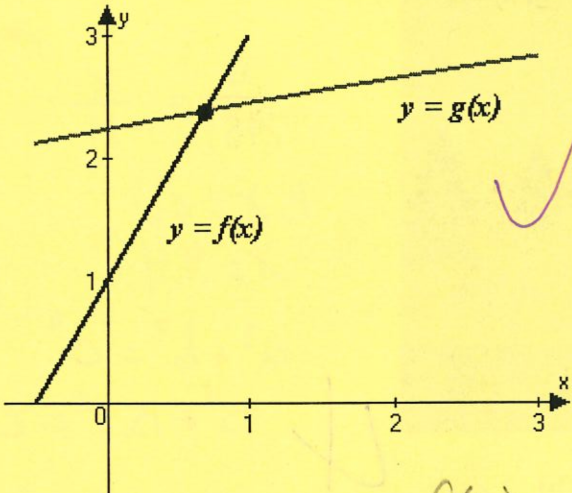
$f(x) = 2x+1$

$g(x) = \sqrt{x+5}$

$2x+1 = \sqrt{x+5}$

$h(x) = 2x+1-(x+5)^{\frac{1}{2}} = 0$

$h'(x) = 2 - \frac{1}{2(x+5)^{\frac{1}{2}}}$



A) 0.695
B) 0.693
C) 0.692
D) 0.691
E) 0.69

$X_2 = X_1 - \frac{f(x_1)}{f'(x_1)} = .6 - \frac{f(.6)}{f'(.6)} = X_2 = .693046$

$X_3 = X_2 - \frac{f(x_2)}{f'(x_2)} = .693046 - \frac{f(.693046)}{f'(.693046)} = X_3 = .693000$

$\boxed{\text{Zero is } .693}$

Please Show All Your Work For Credit!

83 pts

+7

Name: Steven Romeiro Date: 10-27-11

1. Find all critical numbers of the function $f(x) = \sin^2 3x + \cos 3x$, $0 < x < \frac{2\pi}{3}$.

A) $\frac{\pi}{18}, \frac{\pi}{3}, \frac{4\pi}{9}$

B) $\frac{\pi}{9}, \frac{\pi}{3}, \frac{5\pi}{9}$

C) $\frac{\pi}{9}, \frac{\pi}{6}, \frac{2\pi}{9}, \frac{\pi}{2}$

D) $\frac{\pi}{12}, \frac{\pi}{3}, \frac{\pi}{1}$

E) $\frac{\pi}{3}, \frac{5\pi}{12}, \frac{7\pi}{12}$

$f'(x) = 2(\sin 3x)\cdot\cos^2 3x - \sin 3x$

$f'(x) = \sin 3x\left[2\cos^2 3x - 1\right]$

$f'(x) = 0 \to 2\cos^2 3x - 1 = 0 \to \cos^2 3x = \frac{1}{2}$

$\to \sin 3x = 0 \to \boxed{x = 0, \text{ Not in domain}}$

4

2. Locate the absolute extrema of the function $f(x) = 7x + 1$ over the following intervals:

(a) [2,3] (b) [2,3) (c) (2,3] (d) (2,3).

A)

	Absolute Maximum	Absolute Minimum
(a)	22	15
(b)	None	15
(c)	22	None
(d)	None	None

Scratch Paper ok!

B)

	Absolute Maximum	Absolute Minimum
(a)	22	None
(b)	22	15
(c)	None	None
(d)	None	15

C)

	Absolute Maximum	Absolute Minimum
(a)	22	15
(b)	22	15
(c)	22	None
(d)	22	None

D)

	Absolute Maximum	Absolute Minimum
(a)	None	15
(b)	None	None
(c)	22	None
(d)	None	None

E) None of the above.

3. Determine whether Rolle´s Theorem can be applied to the function $f(x) = \dfrac{(x-15)(x+13)}{(x+16)^2}$ on the closed interval $[-13,15]$. If Rolle´s Theorem can be applied, find all numbers c in the open interval $(-13,15)$ such that $f'(c)=0$.

A) Rolle´s Theorem applies; $\dfrac{179}{17}$

B) Rolle´s Theorem applies; $-\dfrac{179}{17}$

C) Rolle´s Theorem applies; $-\dfrac{179}{34}$

D) Rolle´s Theorem applies; $\dfrac{179}{34}$

E) Rolle's Theorem does not apply

F) Both C and D

3

Scratch paper ok!

4. Determine whether the Mean Value Theorem can be applied to the function $f(x) = 2\sin x + \sin 2x$ on the closed interval $[4\pi, 5\pi]$. If the Mean Value Theorem can be applied, find all numbers c in the open interval $(4\pi, 5\pi)$ such that $f'(c) = \dfrac{f(5\pi) - f(4\pi)}{5\pi - 4\pi}$.

Yes MVT applies

A) MVT applies; $\dfrac{11\pi}{3}$

B) MVT applies; $\dfrac{11\pi}{4}$

C) MVT applies; $\dfrac{9\pi}{2}$

D) MVT applies; $\dfrac{11\pi}{6}$

E) MVT does not apply

3

$f(5\pi) = 2\sin(5\pi) + \sin 2(5\pi) =$
$= 2\sin(5\pi) + 2\sin 5\pi \cos 5\pi = 2(0) + 2(0) = 0$

$f(4\pi) = 2\sin(4\pi) + \sin 2(4\pi)$
$= 4\sin(4\pi) + 2\sin 4\pi \cos 4\pi$

$f'(x) = 2\cos x + 2\cos 2x = \dfrac{f(5\pi) - f(4\pi)}{5\pi - 4\pi}$

-6

② $f(x) = 7x+1$

a) $[2,3]$

$f'(x) = 7 \neq 0$

$f(2) = 14+1 = \boxed{15} \rightarrow$ Absolute Minimum $(2,15)$

$f(3) = 21+1 = \boxed{22} \rightarrow$ Absolute Maximum $(3,22)$

b) $[2,3)$

$f(2) = 14+1 = \boxed{15} \rightarrow$ Absolute Minimum $(2,15)$

$f(3)$ No absolute Maximum

c) $(2,3]$

$f(2) \rightarrow$ No absolute Minimum

$f(3) = 21+1 = \boxed{22} \rightarrow$ Absolute Maximum $(3,22)$

d) $(2,3)$

$f(2)$ - No absolute Minimum

$f(3)$ - No absolute Maximum

---

③ $f(x) = \dfrac{(x-15)(x+13)}{(x+16)^2}$ $[-13,15]$

$f(-13) = \dfrac{(-13-15)(-13+13)}{(-13+16)^2} = \dfrac{0}{3^2} = \boxed{0}$

$f(15) = \dfrac{(15-15)(15+13)}{(15+16)^2} = \dfrac{0}{31^2} = \boxed{0}$

$f(a) = f(b)$

$f'(x) = \dfrac{(x^2-2x-195)}{(x^2+32x+256)} =$

$f'(x) = \dfrac{(2x-2)(x^2+32x+256) - (x^2-2x-195)(2x+32)}{(x^2+32x+256)^2}$

$f'(x) = \dfrac{\cancel{2x^3}+\cancel{64}x^2+512x-2x^2-\cancel{64}x-512-(\cancel{2x^3}-4x^2-454x+32x^2-6240}{(x+16)^4}$

$f'(x) = \dfrac{34x^2+902x+5728}{(x+16)^4} = 34\left(x^2+\frac{451}{17}x+\frac{2864}{17}\right)$

7 $f(x) = -\sin x - \cos x$ $(0, 2\pi)$

$f'(x) = -\cos x + \sin x$

$f''(x) = \sin x + \cos x$

$f''(x) = 0 \rightarrow \sin x + \cos x = 0$

$\sin x = -\cos x \rightarrow \frac{3\pi}{4}, \frac{7\pi}{4}$

$\frac{3\pi}{4}$

$+$ $-$

f concaves up on $(0, \frac{3\pi}{4})$

f concaves down on $(\frac{7\pi}{4}, 2\pi)$

a) $f'(x) = \frac{4}{7}(x-1)^{-3/7} = \frac{4}{7(x-1)^{3/7}} = 0$

5. For the function $f(x) = (x-1)^{4/7}$:

$X \neq 1 \rightarrow$ critical value

(a) Find the critical numbers of f (if any);
(b) Find the open intervals where the function is increasing or decreasing; and
(c) Apply the First Derivative Test to identify all relative extrema.

1

− +

min

Use a graphing utility to confirm your results.

A) (a) $x = 1$
   (b) Increasing: $(-\infty, 1)$; Decreasing: $(1, \infty)$
   (c) Relative max: $f(1) = 0$

B) (a) $x = 1$
   (b) Decreasing: $(-\infty, 1)$; Increasing: $(1, \infty)$
   (c) Relative min: $f(1) = 0$

$f(1) = (1-1)^{4/7} = (0)^{4/7} = \boxed{0}$

Rel min $(1, 0)$

C) (a) $x = 0, 1$
   (b) Decreasing: $(-\infty, 0) \cup (1, \infty)$; Increasing: $(0, 1)$
   (c) Relative min: $f(0) = 1$; Relative max: $f(1) = 0$.

f decreases on $(-\infty, 1)$

f increases on $(1, \infty)$

D) (a) $x = 0$
   (b) Decreasing: $(-\infty, 0)$; Increasing: $(0, \infty)$
   (c) Relative min: $f(0) = 1$

E) (a) $x = 0$
   (b) Increasing: $(-\infty, 0)$; Decreasing: $(0, \infty)$
   (c) Relative max: $f(0) = 1$

6. The function $s(t) = t^3 - 21t^2 + 144t - 6$ describes the motion of a particle moving along a line.

   (a) Find the velocity function of the particle at any time t;
   (b) Identify the time intervals when the particle is moving in a positive direction;
   (c) Identify the time intervals when the particle is moving in a negative direction; and
   (d) Identify the times when the particle changes its direction.

A) (a) $v(t) = 3t^2 - 42t + 144$;
   (b) $(0,6) \cup (8,\infty)$;
   (c) $(6,8)$;
   (d) $t = 6$ and $t = 8$.

B) (a) $v(t) = 3t^2 - 42t + 144$;
   (b) $(6,8)$;
   (c) $(0,6) \cup (8,\infty)$;
   (d) $t = 6$ and $t = 8$.

C) (a) $v(t) = \frac{1}{4}t^4 - 7t^3 + 72t^2 - 6t$
   (b) $(-\infty,\infty)$;
   (c) Particle is never moving in a negative direction;
   (d) Particle never changes direction.

D) (a) $v(t) = \frac{1}{4}t^4 - 7t^3 + 72t^2 - 6t$
   (b) Particle is never moving in a positive direction;
   (c) $(-\infty,\infty)$;
   (d) Particle never changes direction.

E) None of the above.

a) Velocity = $V'(t) = 3t^2 - 42t + 144$

$V(t) = 3(t^2 - 14 + 48)$

$V(t) = 3(t-8)(t-6) = 0$

$\boxed{t=8}$ $\boxed{t=6}$

6 8

+ max — min +

B) S increases (positive) from $(-\infty, 6)$ and $(8,\infty)$

c) S decreases (Negative) from $(6,8)$

d) particle changes direction on $t=8$ & $t=6$

7. Find the points of inflection and discuss the concavity of the function $f(x) = -\sin x - \cos x$ on the interval $(0, 2\pi)$.

A) Concave downward on $\left(0, \frac{3}{2}\pi\right)$; concave upward on $\left(\frac{7}{2}\pi, 2\pi\right)$. Inflection point at $\left(0, \frac{3}{2}\pi\right)$

B) Concave upward on $\left(0, \frac{3}{2}\pi\right)$; concave downward on $\left(\frac{7}{2}\pi, 2\pi\right)$. Inflection point at $\left(\frac{7}{2}\pi, 2\pi\right)$

3

C) No inflection points. Concave up on $(0, 2\pi)$

D) No inflection points. Concave down on $(0, 2\pi)$

E) None of the above

scratch paper

8. Find the limit.

$$\lim_{x\to\infty} \frac{-6x+7}{\sqrt{64x^2+2x}}$$

A) $-\frac{3}{4}$

B) $-\frac{3}{32}$

C) -6

D) 1

E) $-\infty$

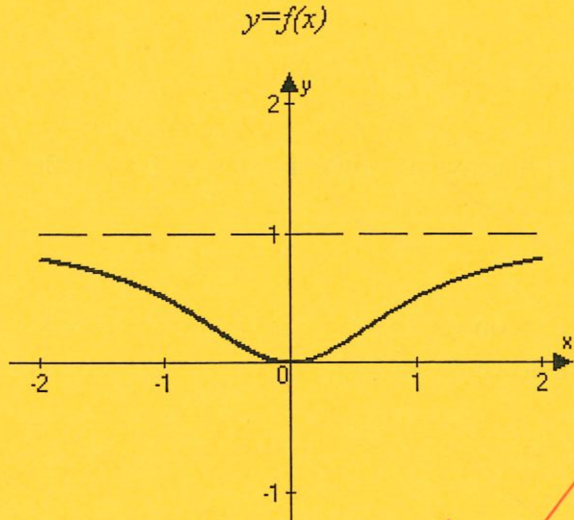
$$\lim_{x\to\infty} \frac{-\frac{6x}{x} + \frac{7}{x}}{\sqrt{\frac{64x^2}{x^2} + \frac{2x}{x^2}}} = \lim_{x\to\infty} \frac{-6 + \frac{7}{x}}{\sqrt{64 + \frac{2}{x}}}$$

$$= \frac{-6+0}{\sqrt{64+0}} = \frac{-6}{\sqrt{64}} = \frac{-6}{8} = \boxed{-\frac{3}{4}}$$

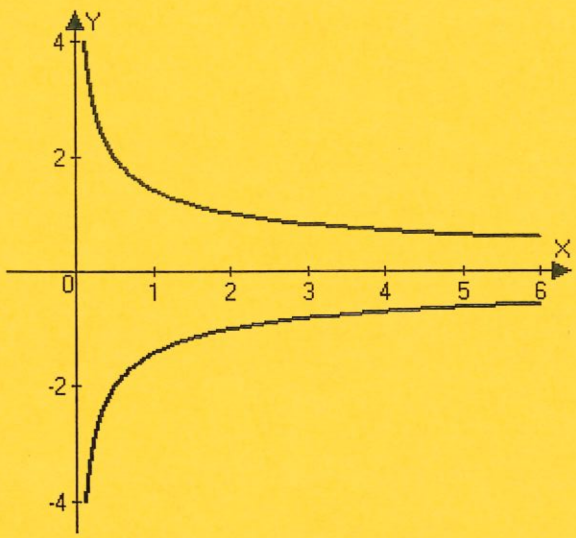
$y = f(x)$

9. Sketch the graph of the relation $x^2 y = 2$ using any extrema, intercepts, symmetry, and asymptotes.

A)



B)



$y = \frac{2}{x^2}$

$y' = -\frac{4}{x^3}$

$-\frac{4}{x^3} = 0 \rightarrow \boxed{x \neq 0}$

$\lim_{x \to 0} y = \lim_{x \to 0} \frac{2}{x^2}$

$\lim_{x \to 0} \frac{2}{0} =$ undefined

Since $\lim_{x \to 0} y =$ undefined $\boxed{x = 0,\ V.A}$

$\lim_{x \to \infty} y = \lim_{x \to \infty} \frac{2}{x^2} =$

$= \lim_{x \to \infty} \frac{\frac{2}{x^2}}{\frac{x^2}{x^2}} = \lim_{x \to \infty} \frac{\frac{2}{x^2}}{1} = \frac{0}{1} = \boxed{0}$

Since $\lim_{x \to \pm\infty} y = 0$, then $\boxed{y = 0,\ HA}$

$y\text{-int} = f(0) = \frac{2}{0} =$ undef

No x-int or y-int

Sign of 1st deriv

$+$	0	$-$

f(x) is decreasing from $(0, \infty)$

f(x) is increasing from $(-\infty, 0)$

$y'' = \frac{12}{x^4}$

$\frac{12}{x^4} \neq 0 \rightarrow$ No inflection point

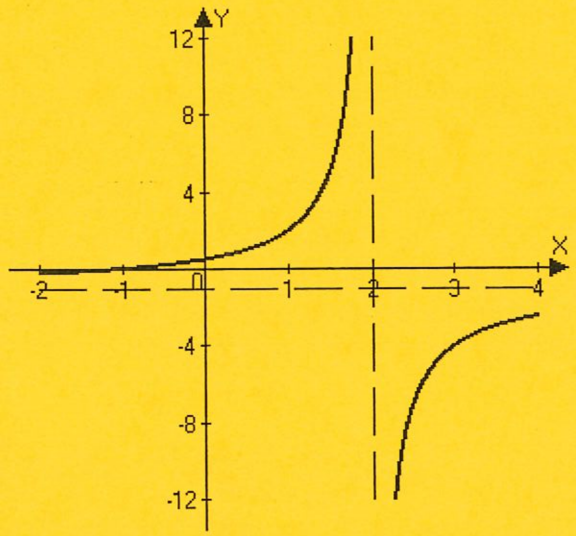
Sign of 2nd deriv

$+$	0	$+$

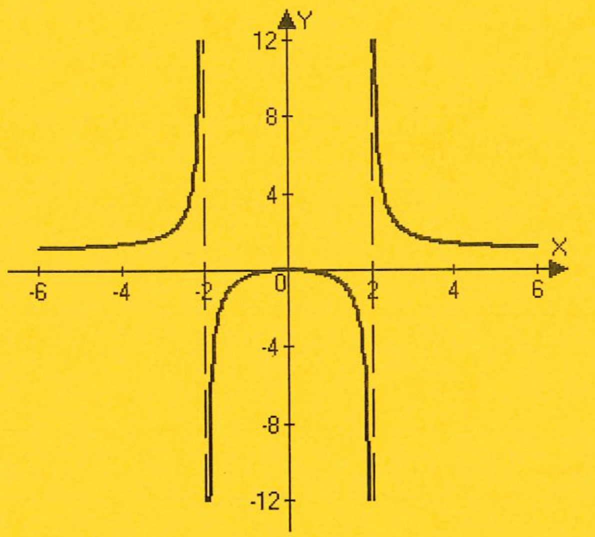
f(x) concave up from $(-\infty, 0)(0, \infty)$

GRAPH $\longrightarrow$

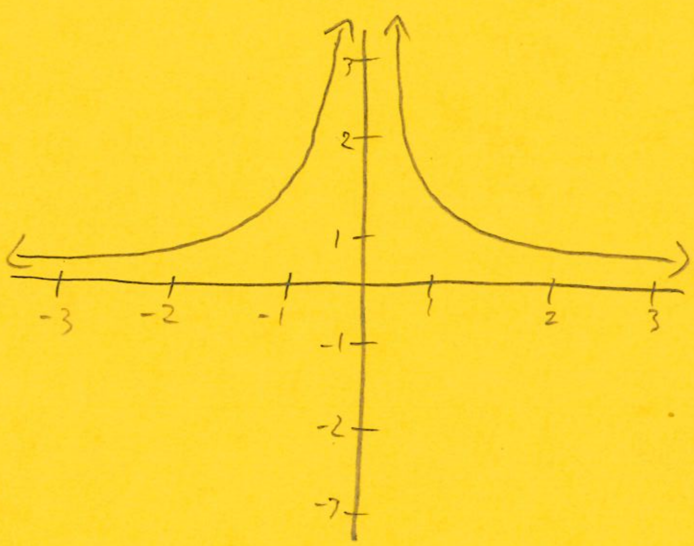
C)



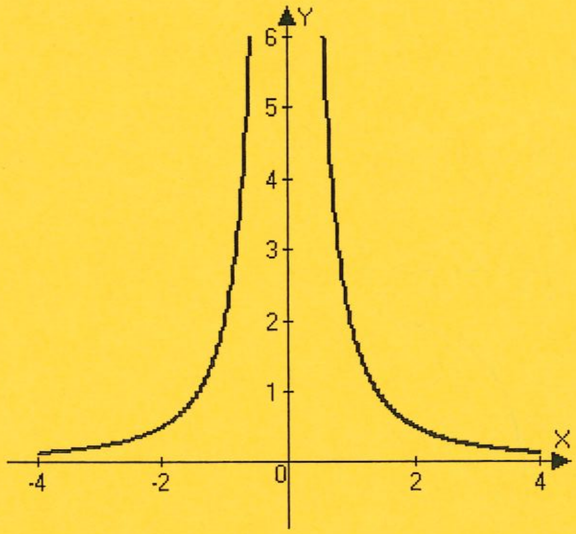
D)



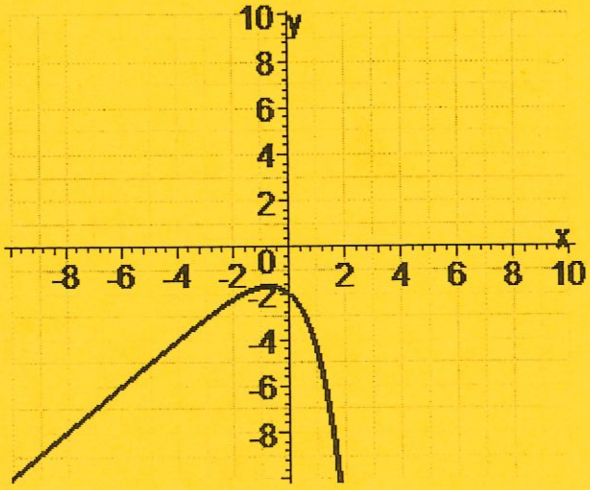
Graph



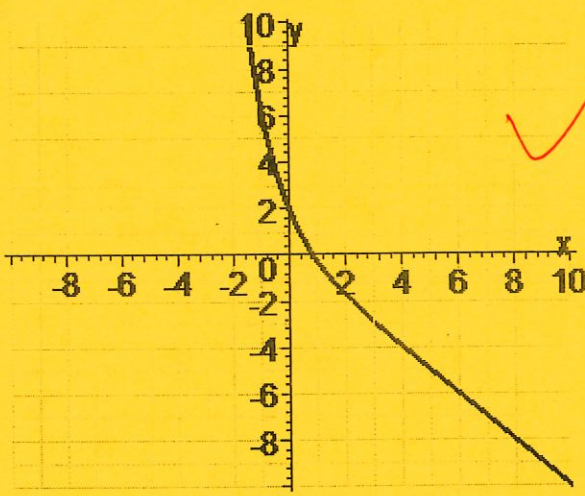
E)



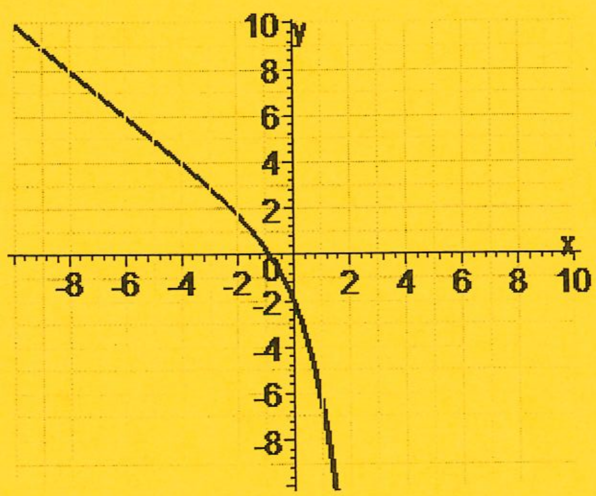
10. Analyze the graph of the function $f(x) = x - 2e^{-x}$ Determine any intercepts, relative extrema, points of inflection and asymptotes. Also determine where the graph is increasing or decreasing and concave up or concave down. Then identify the graph from the choices below.



A)



B)



C)

No VA or H.A since No x value will make this undefined & its not a rational fxn

X-int: $x - 2e^{-x} = 0$

$2e^{-x} = x$

$e^{-x} = \frac{x}{2}$

$\ln e^{-x} = \ln\left(\frac{x}{2}\right)$

$\boxed{x = -\ln\left(\frac{x}{2}\right)}$

$x =$

Y-int: $f(0) = 0 - 2e^{-0}$

$f(0) = 0 - 2$

$f(0) = -2$

$\boxed{y = -2}$

$f'(x) = 1 + 2e^{-x}$

$1 + 2e^{-x} \neq 0$ No vales will make this 0

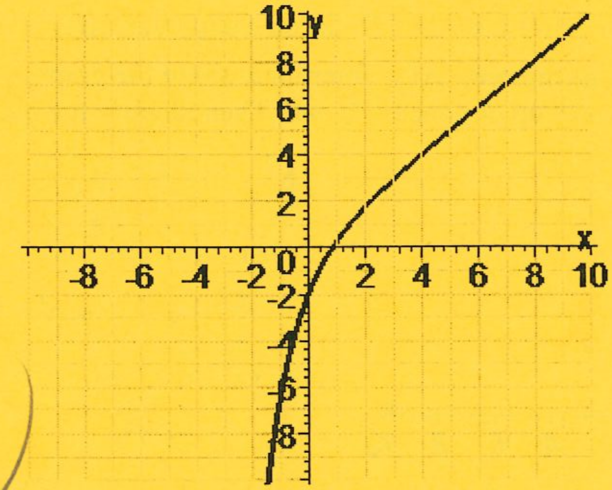
Sign 1st deriv: $1 + 2e^{-x}$: + +

f is increasing everywhere with no crit #

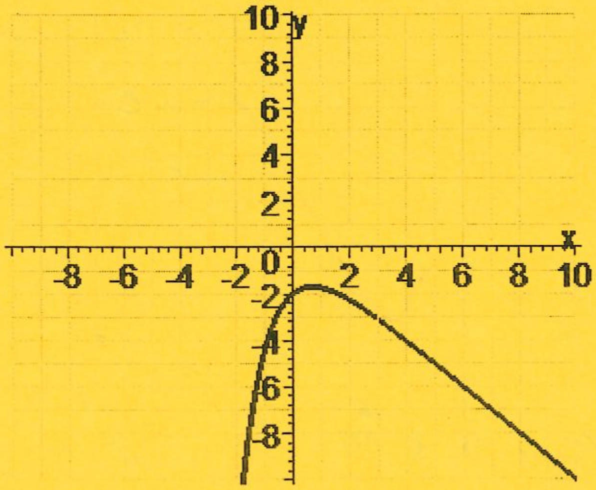
$f''(x) = -2e^{-x} < 0$

f is concave down everywhere with no inflection point

Graph ⟶

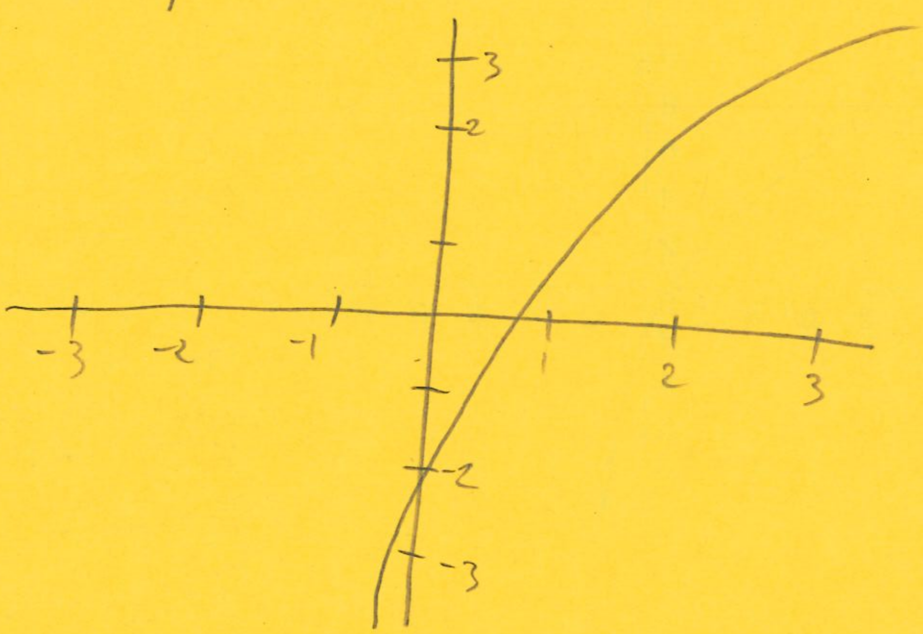

10 y
8
6
4
2
x
-8 -6 -4 -2 0 2 4 6 8 10
-2
-4
-6
-8

D)


10 y
8
6
4
2
x
-8 -6 -4 -2 0 2 4 6 8 10
-2
-4
-6
-8

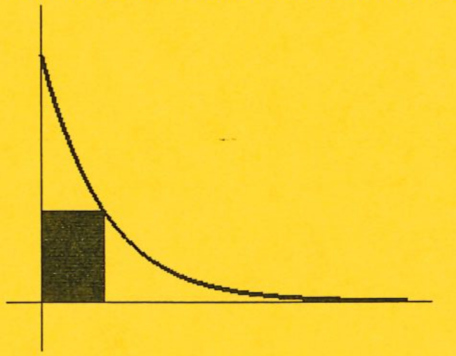
E)

Graph


3
2
-3 -2 -1
1 2 3
-2
-3

$f(x) = y$

11. A rectangle is inscribed between the graph of $f(x) = e^{-7x}$ and the x-axis as shown below. Determine the area of the largest such rectangle.



A) $\frac{1}{14e}$
B) $\frac{1}{7e}$
C) $\frac{7}{e}$
D) $\frac{1}{7e^2}$
E) $\frac{1}{7}$

$A = x \cdot y$

$A = x \cdot (e^{-7x})$

$A' = 1(e^{-7x}) + x(-7e^{-7x})$

$A' = e^{-7x}[1-7x]$

$e^{-7x} \neq 0 \qquad 1-7x=0$

$7x = 1 \rightarrow \boxed{x = \frac{1}{7}}$

$y = e^{-7(\frac{1}{7})} = e^{-1} = \frac{1}{e}$

$\boxed{y = \frac{1}{e}}$

$A = x \cdot y = \frac{1}{7} \cdot \frac{1}{e} = \boxed{A = \frac{1}{7e}}$

12. A rectangular page is to contain 36 square inches of print. The margins on each side are 1 inch. Find the dimensions of the page such that the least amount of paper is used.

A) 8, 8
B) 9, 9
C) 10, 10
D) 7, 7
E) 13, 13

1 1 36 1 1

$A = x \cdot y \rightarrow 36 = x \cdot y \rightarrow y = \frac{36}{x} \rightarrow y' = -\frac{36}{x^2}$

-4

13. Use differentials to approximate the value of $\sqrt{35.8}$. Round your answer to four decimal places.

A) 5.9833
B) 5.9733
C) 5.9933
D) 6.0033
E) 6.0133

$f'(x) = \frac{1}{2x^{\frac{1}{2}}}$

Let $f(x) = \sqrt{x}$, $x = 36$ $\quad \Delta x = -.2$

$dy = f'(x) \cdot dx \rightarrow dy = \frac{1}{2\sqrt{x}} \cdot dx$

$dy = \frac{1}{2\sqrt{36}} \cdot (-.2) = \boxed{dy = -0.017} + 6$

$\boxed{= 5.9833}$

572 so far

get 80 on nxt exam

then only need 74 on final

not counting extra credit on

Final

81 pts

Name: Steven Romero Date: 12-1-11

1. ***Acceleration*** The manufacturer of an automobile advertises that it takes 14 seconds to accelerate from 22 kilometers per hour to 75 kilometers per hour.

Assuming constant acceleration compute the following.

(a) The acceleration in meters per second per second.

(b) The distance traveled during the 14 seconds.

Give your answers to two decimal places.

A) (a) 1.05 m/sec^2 (b) 18569.44 meters
B) (a) 3.79 m/sec^2 (b) 188.61 meters
C) (a) 3.79 m/sec^2 (b) 18569.44 meters
D) (a) 1.05 m/sec^2 (b) 188.61 meters
E) (a) 1.05 m/sec^3 (b) 188.61 meters

5

Scratch Paper

2. Find the indefinite integral and check the result by differentiation.

$\int 5\tan^2 t + 10\ dt$

A) $5\tan t + 5t + C$
B) $5\tan t + 7t + C$
C) $5\tan t - 5t + C$
D) $\frac{5}{3}\tan^3 t + 10t + C$
E) None of the above

$\tan^2 t = \sec^2 t - 1$

$\int 5(\sec^2 t - 1) + 10\,dt$

$= 5\tan t - 5t + 10t + C$

$= \boxed{5\tan t + 5t + C}$

3. Use the properties of summation and Theorem 4.2 to evaluate the sum.

$\sum_{i=2}^{31} i(i^2+5)$

A) 10565
B) 248490
C) 10410
D) 218544
E) 248472

$\sum_{i=2}^{31} i^3 + 5\sum_{i=2}^{31} i = \frac{n^4+2n^3+n^2}{4} + 5\left(\frac{n^2+n}{2}\right)$

$= \frac{(31)^4+2(31)^3+31^2}{4} + \frac{5(31)^2+5(31)}{2} = 246016 + 2480$

$= \boxed{248496}$

$248496 - \left\{\left[\frac{(1)^4+2(1^3)+1^2}{4}\right] + \left[\frac{5(1)^2+5}{2}\right]\right\} = 248496 - 5 = \boxed{248491}$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 + b^3$

$(5n-3i)^3 = 125n^3 - 225n^2 i + 135 n i^2 + 27 i^3$

4. Evaluate the following definite integral by the limit definition.

$a = -5$
$b = -2$

$$\int_{-5}^{-2} -6x^3\,dx$$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x$

A) -609
B) $-\dfrac{349}{2}$
C) $\dfrac{1829}{2}$
D) $-\dfrac{351}{2}$
E) $\dfrac{1827}{2}$

1) $\Delta x = \frac{b-a}{n} = \frac{-2+5}{n} = \boxed{\frac{3}{n}}$

2) $c_i = a + \Delta x\, i = \boxed{-5 + \frac{3}{n} i} = \boxed{\frac{-5n+3i}{n}}$

3) $f(c_i) = -6\left(\frac{-5n+3i}{n}\right)^3$

4) $f(c_i)\Delta x = -6\left(\frac{-5n+3i}{n}\right)^3\left(\frac{3}{n}\right)$
$= \boxed{-\frac{18}{n}\left(\frac{-5n+3i}{n}\right)^3}$

5) $\sum_{i=1}^{n} f(c_i)\Delta x = \sum_{i=1}^{n} -\frac{18}{n}\left(\frac{-5n+3i}{n}\right)^3$
$= \sum_{i=1}^{n}$

-4

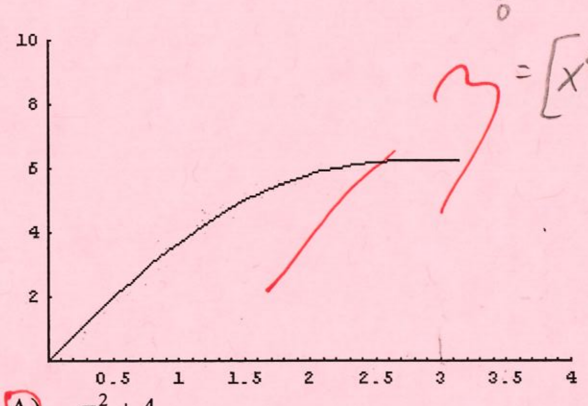
5. Evaluate the definite integral $\int_0^{\pi} 7e^x + 4\cos x\,dx$.

A) $7e^{\pi} - 15$
B) $7e^{\pi} - 7$
C) $7e^{\pi} + 8$
D) $7e^{\pi} + 1$
E) $7e^{\pi} - 3$

$7\int_0^{\pi} e^x\,dx + 4\int_0^{\pi} \cos x\,dx = \left[7e^x + 4\sin x\right]\Big|_0^{\pi} =$
$= \left[7e^{\pi} + 4\sin\pi\right] - \left[7e^0 + 4\sin 0\right] = 7e^{\pi} + 4(0) - 7(1) + 4(0)$
$= \boxed{7e^{\pi} - 7}$

6. Determine the area of the given region.

$y = 2x + 2\sin x,\quad 0 \le x \le \pi$



A) $\pi^2 + 4$
B) $4\pi^2 + 1$
C) $\pi^2 + 5$
D) $\pi^2 + 3$
E) $\pi^2 + 7$

$\int_0^{\pi} 2x + 2\sin x = \int_0^{\pi} 2x\,dx + \int_0^{\pi} 2\sin x\,dx$
$= \left[x^2 - 2\cos x\right]\Big|_0^{\pi}$

-3

$\left[\pi^2 - 2\cos\pi\right] - \left[0^2 - 2\cos 0\right]$
$\pi^2 + 2 - 0 + 1$
$\boxed{\pi^2 + 3}$

-7

1) a) $a_0 = 22$ Km/h $a(14) = 75$ Km/h

75 Km = .075 m

1 hour = 3600 sec

$\int 22\,dt = v(t) = 22t + C$

$V_0 = 22$

10) Trapezoidal Rule $n=4$ $a=0$ $b=1$ $\Delta X = \frac{1}{4}$

$X_0 = a = \boxed{0}$

$X_1 = 0 + \frac{1}{4} = \boxed{\frac{1}{4}}$

$X_2 = \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$

$X_3 = \frac{1}{2} + \frac{1}{4} = \boxed{\frac{3}{4}}$

$X_4 = \frac{3}{4} + \frac{1}{4} = \boxed{1}$

$$\int_0^1 \sqrt{6+s^6}\,ds \approx \frac{(b-a)}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$\approx \frac{1}{8}\left[2.4495 + 2(2.4495396) + 2(2.4527) + 2(2.4856) + 2.6458\right]$$

$$\approx \boxed{2.4838724}$$

Simpson Rule

$$\int_0^1 \sqrt{6+s^6}\,ds \approx \frac{(b-a)}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$\approx \frac{1}{12}\left[2.4495 + 4(2.4495396) + 2(2.4527) + 4(2.4856) + 2.6458\right]$$

$$\approx \boxed{2.47843382}$$

$\frac{1}{2}\left\{\left[5-\frac{2}{5}\right]-\left[3-\frac{2}{3}\right]\right\} = \frac{1}{2}\left[\frac{23}{5}-\frac{7}{3}\right] = \frac{1}{2}\left(\frac{34}{15}\right) = \boxed{\frac{17}{15}}$

7. Find the average value of the function over the given interval and all values z in the interval for which the function equals its average value.

$a=3$
$b=5$

$$f(z) = \frac{z^2+2}{z^2}, \quad 3 \le z \le 5$$

$\frac{1}{b-a}\int f(z)\,dz = \frac{1}{5-3}\int \frac{z^2+2}{z^2}\,dz = \frac{1}{2}\int \frac{z^2}{z^2}\,dz + 2\int z^{-2}\,dz$

$= \frac{1}{2}\int 1\,dz + \int 2z^{-2}\,dz = \frac{1}{2}\left[z - \frac{2}{z}\right]\Big|_3^5$

Use a graphing utility to verify your results.

A) The average is $\frac{17}{10}$ and the point at which the function is equal to its mean value is $\sqrt{15}$.

B) The average is $\frac{17}{15}$ and the point at which the function is equal to its mean value is $\sqrt{15}$.

C) The average is $\frac{17}{10}$ and the point at which the function is equal to its mean value is $\sqrt{15}$ and $-\sqrt{15}$.

D) The average is $\frac{17}{15}$ and the point at which the function is equal to its mean value is $\sqrt{15}$ and $-\sqrt{15}$.

E) The average is $\frac{17}{10}$ and the point at which the function is equal to its mean value is $-\sqrt{15}$.

$\frac{z^2+2}{z^2} = \frac{17}{15} = 17z^2 = 15z^2+30$
$2z^2 = 30 \longrightarrow \boxed{z=\sqrt{15}}$

8. Find the indefinite integral.

$\int e^{7x}\sec\left(e^{7x}\right)\tan\left(e^{7x}\right)dx \quad \frac{1}{7}\int \sec(e^{7x})\tan(e^{7x})\cdot(7e^{7x})\,dx$

$U = e^{7x}$
$du = 7e^{7x}dx$

A) $\frac{1}{7}\tan\left(e^{7x}\right)+C$

B) $\frac{1}{7}\sec\left(e^{7x}\right)+C$

C) $7\sec\left(e^{7x}\right)+C$

D) $\frac{e^{7x}}{7}\sec\left(e^{7x}\right)+C$

E) $\frac{e^{7x}}{7}\tan\left(e^{7x}\right)+C$

$\frac{1}{7}\int \sec U \tan U\,du = \frac{1}{7}\sec U + C$

$= \boxed{\frac{1}{7}\sec(e^{7x})+C}$

@ $x=0$
$U=e^{11(0)}+8=\boxed{9}$

@ $x=1$
$U=e^{11(1)}+8$
$U=\boxed{e^{11}+8}$

9. Evaluate the definite integral.

$\int_0^1 e^{11x}\sqrt{e^{11x}+8}\,dx$

A) $\left(e^{11}+8\right)^{3/2}-(9)^{3/2}$

B) $\dfrac{2\left(e^{11}+7\right)^{3/2}}{33}$

C) $\dfrac{2\left[\left(e^{11}+8\right)^{3/2}-(9)^{3/2}\right]}{33}$

D) $\dfrac{2\left[\left(e^{11}+8\right)^{3/2}-(8)^{3/2}\right]}{33}$

E) $\left(e^{11}+7\right)^{3/2}$

$\int_0^1 (e^{11x}+8)^{\frac{1}{2}}\cdot e^{11x}\,dx$

$U=e^{11x}+8$
$du=11e^{11x}dx$

$\frac{1}{11}\int_0^1 (e^{11x}+8)^{\frac{1}{2}}\cdot(11e^{11x})\,dx=\frac{1}{11}\int_9^{e^{11}+8} U^{\frac{1}{2}}\,du$

$=\frac{1}{11}\cdot\left[\frac{2}{3}U^{\frac{3}{2}}\right]\Big|_9^{e^{11}+8}=\frac{1}{11}\left\{\left[\frac{2}{3}(e^{11}+8)^{\frac{3}{2}}\right]-\left[\frac{2}{3}(9)^{\frac{3}{2}}\right]\right\}$

$=\frac{2}{33}\left[(e^{11}+8)^{\frac{3}{2}}-(9)^{\frac{3}{2}}\right]=\boxed{\frac{2\left[(e+8)^{\frac{3}{2}}-(9)^{\frac{3}{2}}\right]}{33}}$

10. Apply the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral using 4 subintervals. Round your answer to six decimal places and compare the result with the exact value of the definite integral.

$\int_0^1 \sqrt{6+s^6}\,ds$

Scratch paper

A) The Trapezoidal rule gives 2.483848 and Simpson's rule gives 2.478414.
B) The Trapezoidal rule gives 2.483848 and Simpson's rule gives 2.608040.
C) The Trapezoidal rule gives 2.500149 and Simpson's rule gives 2.608040.
D) The Trapezoidal rule gives –2.483848 and Simpson's rule gives –2.478414.
E) The Trapezoidal rule gives 2.500149 and Simpson's rule gives 2.478414.

11. Find the indefinite integral.

$\int \frac{e^{5x}}{8+e^{5x}}\,dx$

A) $\frac{1}{5}\ln\left|8e^{5x}\right|+C$

B) $\ln\left|8x+e^{5x}\right|+C$

C) $\frac{1}{5}\ln\left|8+e^{5x}\right|+C$

D) $\frac{1}{5}\ln\left|8x+e^{5x}\right|+C$

E) $\ln\left|8+e^{5x}\right|+C$

$\int\frac{1}{8+e^{5x}}\cdot e^{5x}\,dx$

$U=8+e^{5x}$
$du=5e^{5x}dx$

$\frac{1}{5}\int\frac{1}{U}\,du=\frac{1}{5}\ln|U|+C$

$=\boxed{\frac{1}{5}\ln\left|8+e^{5x}\right|+C}$

12. Find the indefinite integral.

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\operatorname{Tan}^{-1}\left(\frac{u}{a}\right)+C$$

$$\int \frac{1}{9+(x-2)^2}dx$$

$u=(x-2)$
$du=dx$

$$= \int \frac{du}{3^2+u^2} =$$

$a=3$
$u=(x-2)$

A) $\frac{1}{9}\arctan\left(\frac{x-2}{3}\right)+C$

B) $3\arctan\left(\frac{x-2}{3}\right)+C$

C) $\frac{1}{9}\arctan\left(\frac{x-2}{9}\right)+C$

D) $9\arctan\left(\frac{x-2}{9}\right)+C$

E) $\frac{1}{3}\arctan\left(\frac{x-2}{3}\right)+C$

$$\frac{1}{3}\operatorname{Tan}^{-1}\left(\frac{u}{3}\right)+C = \boxed{\frac{1}{3}\operatorname{Tan}^{-1}\left(\frac{x-2}{3}\right)+C}$$

13. Evaluate $\cosh(\ln(4))$ and $\tanh(\ln(4))$ in that order.

A) $\frac{17}{8}, -\frac{15}{17}$

B) $-\frac{17}{8}, \frac{15}{17}$

C) $-\frac{17}{8}, -\frac{15}{17}$

D) $\frac{17}{8}, \frac{15}{17}$

E) None of the above

$$\cosh(\ln(4)) = \frac{e^{\ln 4}+e^{-\ln 4}}{2} = \boxed{\frac{17}{8} \text{ or } 2.125}$$

$$\operatorname{Tanh}(\ln(4)) = \frac{\frac{e^{\ln 4}-e^{-\ln 4}}{2}}{\frac{e^{\ln 4}+e^{-\ln 4}}{2}} = \boxed{.882353 \text{ or } \frac{15}{17}}$$

~~14.~~ Find the derivative of the function $y=\ln\left(\cosh^7(8x)\right)$.

A) $\frac{dy}{dx}=7\sinh(8x)$

B) $\frac{dy}{dx}=7\operatorname{sech}(8x)$

C) $\frac{dy}{dx}=56\cosh(8x)$

D) $\frac{dy}{dx}=7\operatorname{csch}(8x)$

E) $\frac{dy}{dx}=56\tanh(8x)$

$y=$

MAC 2311 **Final Exam Review**

Show all work for full credit.

Find the following limits:

1. $\lim_{x \to 2} \frac{2x^2 + x - 1}{2x - 1}$ 2. $\lim_{x \to 4} \frac{x - 4}{2 - \sqrt{x}}$ 3. $\lim_{x \to \infty} \frac{5 - 2x - 3x^2}{4x - 2x^2}$

4. $\lim_{x \to 0} \frac{2 \sin (4x)}{7x}$

5. Given $f(x) = \begin{cases} 2x + 5, & x < 2 \\ x^3 - 2x, & x \geq 2 \end{cases}$, find

a) $f(2) =$ b) $\lim_{x \to 2^-} f(x) =$ c) $\lim_{x \to 2^+} f(x) =$ d) $\lim_{x \to 2} f(x) =$

e) Is f(x) continuous at $x = 2$? Give the reason to support your answer.

6. Given $f(x) = \frac{8x^2}{x^2 - 4}$, find the intervals where the function is increasing and decreasing. Then find all relative maximum and minimum.

7. Find the derivative for the following:
a) $f(x) = 2\cos^2 x - 3\sin^2 x$ b) $f(x) = 3\cos^{-1}(4x)$

8. Find the derivative for the following:
a) $f(x) = \ln\left(\frac{x^2 - 5x}{10x + 4}\right)$ b) $f(x) = (5x^2 + 3x + 7)^{(3x+8)}$

9. Find the position function s(t) if the velocity is $a(t) = 3t^2 - 12t$, $v(0) = 3$ and $s(0) = -2$.

? 10. Given $f(x) = \frac{|x - 1|}{x - 1}$, find the following:
a) $\lim_{x \to 1^+} f(x)$ b) $\lim_{x \to 1^-} f(x)$ c) $\lim_{x \to 1} f(x)$ d) Is f(x) continuous at x = 1? Why or why not?

11. Find the derivative of $f(x) = -x^2 + 4x - 3$ via the limit definition of a derivative.

12. Given $x^3 - 3x^2y + 3xy^2 = 26$, find y'. Then find an equation of the tangent line to the graph of the equation at the point (2, 3).

13. Given $y \ln x + y^2 = 0$, find y'. Then find an equation of the tangent line to the graph of the equation at the point $(e, -1)$.

14. Find the rate of change of the volume of a weather balloon when its radius is 10 cm if its radius is increasing at a rate of 0.3 cm/sec.

15. Find the intervals on which f(x) is increasing and decreasing and then find all relative extrema of f(x). $f(x) = \frac{80x}{x^2 + 4}$.

16. Find the intervals on which f(x) is concave upward and concave downward. Then find the inflection point(s) of f(x). $f(x) = x e^{2x}$.

Find the first and second derivatives of the following:

17. $f(x) = x^2 \ln(3x)$ 18. $f(x) = 2\sin^3(2x)$

Find the following integrals:

19. $\int \frac{3x^2 + 5x - 10}{x^3}\,dx$ 20. $\int_0^2 \frac{3x}{\sqrt{1 + 2x^2}}\,dx$ 21. $\int x\sqrt{x + 2}\,dx$

? 22) $\int_0^7 |2x - 5|\,dx$ 23. $\int \frac{6x + 5}{2x + 1}\,dx$ 24. $\int \frac{5x + 16}{x^2 + 9}\,dx$

25. $\int \tan t\,dt$ 26. $\int x \sin(x^2)\,dx$

27. Given $\frac{dy}{dx} = \frac{2000}{1 + 0.2x}$, $y(0) = 1000$, solve for y. Then find y(10).

28. Find the area of the region under the graph of the function $f(x) = x(x^2 - 4)$ on the interval [-2, 1].

29. Find the area of the region under the graph of the function $f(x) = x\,e^{(x^2 + 2)}$ on the interval [1, 4].

Sketch the graphs of the region for the following and then use a geometry formula to evaluate each integral.

30. $\int_0^6 |x - 2| + 3\,dx$ 31. $\int_{-3}^{3} \sqrt{9 - x^2}\,dx$

32. A spherical hot-air balloon is being filled. If the radius of the balloon *de*creases at the rate of 5 ft/min, at what rate does the volume *de*crease when the radius is 12 ft? (the volume of a sphere: $V = \frac{4}{3}\pi r^3$)

33. $\int \cos 2x \ \sin^2 2x\,dx$ Review

34. A population of a small town is changing at the rate of $\frac{dP}{dt} = 100t^{\frac{1}{2}} + 210t^{3/4}$, where t is the time in years. When t = 0, the population is 15000.

a) Write an equation that gives the population P at any time t.

b) Find the population of the town when t = 25.

35. Find the area of the region bounded by the graph on the given interval

$f(x) = \sqrt{3x + 7}$; [− 1, 10]

36. a) Use the **Trapezoidal Rule** to approximate the value of the definite integral for the indicated value of n. Round your answer to four decimal places and

$\int_0^{\pi/4} x \tan x\,dx$, n = 4

? b) Find the same integration with an integration rule and **compare the result with part (a).**

37. a) Use the **Trapezoidal Rule** to approximate the value of the definite integral for the indicated value of n. Round your answer to four decimal places and

$\int_0^2 x \ln(x + 1)\,dx$, n = 4

? b) Find the same integration with an integration rule and **compare the result with part (a).**

38. *Sketch the region bounded by the graphs of the equations, and determine its area.*
$y = 1 + e^x,\ y = 0,\ x = 0,\ x = 2$

39. *Sketch the region bounded by the graphs of the equations, and determine its area.*
$y = \frac{2}{x},\ y = 0,\ x = 1,\ x = 3$

40. *Evaluate the following definite integral :*

$$2\pi \int_0^1 (y-1)\sqrt{1-y}\ dy$$

Steven Romeiro

Final Exam Review

1) $\lim_{x \to 2} \frac{2x^2+x-1}{2x-1} = \frac{2(2)^2+2-1}{2(2)-1} = \frac{9}{3} = \boxed{3}$

2) $\lim_{x \to 4} \frac{x-4}{2-\sqrt{x}} = \lim_{x \to 4} \frac{x-4}{2-x^{\frac{1}{2}}} = \lim_{x \to 4} \frac{x^{\frac{1}{2}}-4}{2}$

$= \frac{\sqrt{4}-4}{2} = \frac{2-4}{2} = \frac{-2}{2} = \boxed{-1}$

3) $\lim_{x \to \infty} \frac{5-2x-3x^2}{4x-2x^2} = \lim_{x \to \infty} \frac{\frac{5}{x^2}-\frac{2x}{x^2}-\frac{3x^2}{x^2}}{\frac{4x}{x^2}-\frac{2x^2}{x^2}}$

$\lim_{x \to \infty} \frac{\frac{5}{x^2}-\frac{2}{x}-3}{\frac{4}{x}-2} = \frac{0-0-3}{0-2} = \boxed{\frac{3}{2}}$

4) $\lim_{x \to 0} \frac{2\sin(4x)}{7x} = \frac{f'(A)\,8\sin(x)\cdot 2\cos(4x)}{f'(x) = 7}$

$\lim_{x \to 0} \frac{8\sin x\, 2\cos 4x}{7} = \boxed{0}$

5) $f(x) = \begin{cases} 2x+5 & x<2 \\ x^3-2x & x \geq 2 \end{cases}$

a) $f(2) = 2^3 - 2(2) = \boxed{4}$

b) $\lim_{x\to 2^-} f(x) = \lim_{x \to 2^-} 2x+5 = 2(2)+5 = \boxed{9}$

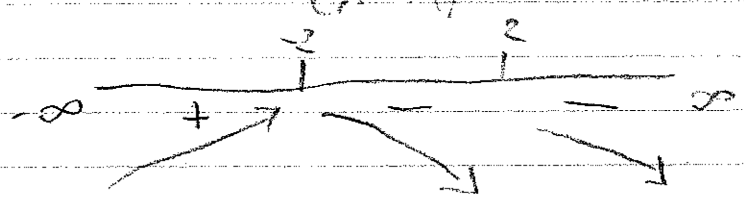
c) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^3-2x = (2)^3-2(2) = \boxed{4}$

d) $\lim_{x\to 2} f(x) = \lim_{x\to 2} x^3-2x = (2)^3-2(2) = \boxed{4}$

6) $f(x) = \dfrac{8x^2}{x^2-4}$

$f'(x) = \dfrac{16x(x^2-4) - 8x^2(2x)}{(x^2-4)^2}$

$f'(x) = \dfrac{16x^3 - 64x - 16x^3}{(x^2-4)^2} = \quad f'(x) \quad \dfrac{-64x}{(x^2-4)^2}$



f is increasing on $(-\infty, -2)$
f is decreasing on $(-2, 2)$ & $(2, \infty)$

7) a) $F(x) = 2\cos^2 x - 3\sin^2 x$

$f'(x) = 2(2\cos x)\cdot(-2\sin^2 x) - 2(3\sin x)\cdot(3\cos^2 x)$

$f'(x) = 4\cos x(-2\sin^2 x) - 6\sin x(3\cos^2 x)$

b) $f(x) = (5x^2 + 3x + 7)^{(5x+8)}$

$f'(x) = 3(5x^2 + 3x + 7)\cdot(5x + 3)$

$f'(x) = 15x^2 + 9x + 21(5x + 3)$

$f'(x) = 75x^3 + 45x^2 + 45x^2 + 27x + 105x + 63$

$f'(x) = 75x^3 + 90x^2 + 132x + 63$

8) a) $f(x) = \ln\left(\frac{x^2 - 5x}{10x + 4}\right)$

$= \ln(x^2 - 5x) - \ln(10x + 4)$

$f'(x) = \left(\frac{2x - 5}{x^2 - 5x}\right) - \left(\frac{10}{10x + 4}\right)$

$f'(x) = \frac{(2x-5)(10x+4) - (10)(x^2 - 5x)}{(x^2 - 5x)(10x + 4)}$

$f'(x) = \frac{20x^2 + 8x - 50x - 20 - 10x^2 + 50x}{(x^2 - 5x)(10x + 4)}$

$$\boxed{f'(x) = \frac{10x^2 + 8x - 20}{(x^2 - 5x)(10x + 4)}}$$

b) $f(x) = \cancel{3}\cos^{-1}(4x) = \frac{u'}{\sqrt{1-u^2}}$

$= \boxed{\frac{12}{\sqrt{1-16x^2}}}$

9) $a(t) = 3t^2 - 12 \qquad S(0) = -2 \qquad V(0) = 3$

$V(t) = \int 3t^2 - 12\, dt = t^3 - 12t + C$

$V(0) = 0^3 - 12(0) + C$

$V(t) = t^3 - 12t + 3 \qquad C = 3$

$S(t) = \int t^3 - 12t + 3 = \frac{t^4}{4} - 6t^2 + 3t + C$

$S(0) = \frac{0^4}{4} - 6(0)^2 + 3(0) + C$

$C = -2$

$S(t) = \frac{t^4}{4} - 6t^2 + 3t - 2$

10) $f(x) = \frac{|x-1|}{x-1} =$

a) $\lim_{x \to 1^+} f(x) = \quad y = \frac{|1.01-1|}{1.01-1} = \frac{0.1}{0.1} = \boxed{1}$

b) $\lim_{x \to 1^-} f(x) = \quad y = \frac{|.99-1|}{.99-1} = \frac{.01}{-.01} = \boxed{-1}$

c) $\lim_{x \to 1} f(x) = \frac{|1-1|}{1-1} = \frac{0}{0}$

d) $f(x)$ is not continuous
Since $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$

11) $f(x) = -x^2 + 4x - 3$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1) $f(x+h) = -(x+h)^2 + 4(x+h) - 3$

$= -(x^2 + 2xh + h^2) + 4x + 4h - 3$

$= -x^2 - 2xh - h^2 + 4x + 4h - 3$

$= -x^2 - 2xh - h^2 + 4x + 4h - 3$

2) $-f(x) = x^2 - 4x + 3$

$f(x+h) - f(x) = \cancel{-x^2} - 2xh - h^2 \cancel{+4x} + 4h \cancel{-3} \cancel{+x^2} \cancel{-4x} \cancel{+3}$

$= -2xh - h^2 + 4h$

3) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-2xh - h^2 + 4h}{h}$

$= -2x - h + 4$

4) $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} -2x - h + 4$

$= \boxed{-2x + 4}$

12) $x^3 - 3x^2y + 3xy^2 = 26 \qquad (2, 3)$

$$\frac{d}{dx}\left[x^3 - 3x^2y + 3xy^2\right] = \frac{d}{dx}[26]$$

$$3x - \left(6xy + 3x^2\frac{dy}{dx}\right) + \left(3y^2 + 3x\,2y\frac{dy}{dx}\right) = 0$$

$$3x - 6xy - 3x^2\frac{dy}{dx} + 3y^2 + 3x2y\frac{dy}{dx} = 0$$

$$3x^2\frac{dy}{dx} + 3x2y\frac{dy}{dx} = -3x + 6xy - 3y^2$$

$$\frac{dy}{dx}\left(3x^2 - 3x2y\right) = -3x + 6xy - 3y^2$$

$$\frac{dy}{dx} = \frac{-3x + 6xy - 3y^2}{3x^2 + 6xy}$$

$$\frac{dy}{dx} = \frac{-3(x - 2xy + y^2)}{3(x^2 + 2xy)}$$

$$\frac{dy}{dx} = \frac{-x - y^2}{x^2} = \text{Slope}$$

$x = 2,\ y = 3$

$$f'(x) = \frac{-2 - 3^2}{2^2} = \frac{-2 - 9}{4} = \boxed{-\frac{11}{4}} = m$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{11}{4}(x - 2)$$

$$y = -\frac{11}{4}x + \frac{11}{2} + 3$$

$$\boxed{y = -\frac{11}{4}x + \frac{17}{2}}$$

13) $y \ln x + y^2 = 0$ $\qquad (e, -1)$

$$\frac{dy}{dx}(\ln x) + y\left(\frac{1}{x}\right) + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(\ln x + 2y) = -\frac{y}{x}$$

$$\frac{dy}{dx} = -\frac{y}{x} \cdot \frac{1}{\ln x + 2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-1}{x \ln x + 2x}} = \text{slope}$$

$$y' = \frac{1}{e \cdot \ln e + 2e} = \boxed{\frac{1}{e + 2e}}$$

Eq: $y + 1 = \frac{1}{e + 2e}(x - e) = \quad y + 1 = \frac{\cancel{x}}{e + 2e} - \frac{1}{e + 2}$

$$Y = \frac{X}{e+2e} - \frac{1}{e+2} - 1$$

$$Y = \frac{x}{e+2e} - \frac{1-e-2}{e+2}$$

$$\boxed{Y = \frac{x}{e+2e} - \frac{3}{2}}$$

14) Rate of change $r = 10cm$ increasing $.3cm/s$

$\frac{dr}{dt} = 0.3 cm/s$

$$V_{Sphere} = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi (10)^2 \cdot (0.3)$$

$$\boxed{\frac{dv}{dt} = 376.99}$$

15) $f(x) = \dfrac{80x}{x^2+4}$

$f'(x) = \dfrac{80(x^2+4) - 80x(2x)}{(x^2+4)^2}$

$f'(x) = \dfrac{80x^2+320-160x^2}{(x^2+4)^2}$

$f'(x) = \dfrac{-80(x^2-4)}{(x^2+4)^2}$

Critical $\rightarrow$ $\dfrac{-80(x^2-4)}{(x^2+4)^2} = 0 \rightarrow -80(x^2-4) = 0$

$\boxed{x = \pm 2}$

$-\infty$		-2		2		∞
	$-$		$+$		$-$	
		Min		Max		

$f(-2) = \dfrac{80(-2)}{(-2)^2+4} = \dfrac{-160}{16} = \boxed{-10}$

$f(2) = \dfrac{80(2)}{(2)^2+4} = \dfrac{160}{16} = \boxed{10}$

f increases on $(-2, 2)$ & decreases on $(-\infty, -2)$ & $(2, \infty)$

f has a rel max $(2, 10)$

f has a rel min $(-2, -10)$

16) $f(x) = xe^{2x}$

$f'(x) = e^{2x} + x(2e^{2x})$

$f''(x) = 2e^{2x} + 2e^{2x} + x(4e^{2x})$

$f''(x) = 4e^{2x} + 4xe^{2x}$

$f''(x) = 4e^{2x}(1+x)$

possible inflection points= $4e^{2x}(1+x) = 0$

$4e^{2x} = 0$ $\quad\quad$ $x+1=0$

$x \neq 0$ $\quad\quad$ $x = -1$ $\quad\quad$ $f(-1) = -e^{-2} = -\frac{1}{e^2}$

$-\infty$ ——— -1 ——— ∞

$-$ $\quad$ $+$

~~c~~down $\quad$ cup

f concaves down on $(-\infty, -1)$
f concaves up on $(-1, \infty)$
f has an inflection point on $(-1, -\frac{1}{e^2})$

17) $f(x) = x^2 \ln(3x)$

$f'(x) = 2x(\ln 3x) + x^2\left(\frac{3}{3x}\right)$

$\boxed{f'(x) = 2x\ln 3x + x}$

$f''(x) = 2(\ln 3x) + 2x\left(\frac{3}{3x}\right) + 1$

$f''(x) = 2\ln 3x + 2 + 1$

$\boxed{f''(x) = 2\ln 3x + 3}$

18) $f(x) = 2\sin^3(2x)$

$f'(x) = 2\left[3\sin^2(2x)\cdot\cos(2x)\cdot 2\right]$

$\boxed{f'(x) = 12\sin^2(2x)\cdot\cos(2x)}$

$f''(x) = 12\left[\underbrace{4\sin(2x)\cdot\cos(2x)}_{u'}\cdot\underbrace{\cos(2x)}_{v} + \underbrace{12\sin^2(2x)}_{u}\cdot\right.$

$\left.\underbrace{-\sin(2x)\cdot 2}_{v'}\right]$

$f''(x) = 12\left[4\sin(2x)\cos^2(2x) - 24\sin^3(2x)\right]$

$f''(x) = 48\sin(2x)\cos^2(2x) - 288\sin^3(2x)$

$\boxed{f''(x) = 48\sin(2x)\left[\cos^2(2x) - \frac{1}{6}\sin^2(2x)\right]}$

19) $\int \frac{3x^2+5x-10}{x^3}\,dx$

$\int \frac{3x^2}{x^3}\,dx + \int \frac{5x}{x^3}\,dx - \int \frac{10}{x^3}\,dx$

$3\int \frac{x^2}{x^3}\,dx + 5\int \frac{x}{x^3}\,dx - 10\int \frac{1}{x^3}\,dx$

$3\int \frac{1}{x}\,dx + 5\int \frac{1}{x^2}\,dx - 10\int \frac{1}{x^3}\,dx$

$3\ln x + 5\left(-\frac{1}{x}\right) - 10\left(-\frac{1}{x^2}\right) + C$ $\rightarrow$

$$\boxed{3\ln x - \frac{5}{x} + \frac{10}{x^2} + C}$$

20) $\displaystyle\int_0^2 \frac{3x}{\sqrt{1+2x^2}}\,dx = \int_0^2 \frac{3x}{(1+2x^2)^{\frac{1}{2}}}\,dx$

$$\int_0^2 (1+2x^2)^{-\frac{1}{2}} \cdot 3x\,dx$$

$U = 1+2x^2$
$du = 4x\,dx$

$x=0 \rightarrow U = 1+2(0)^2 = \boxed{1}$
$x=2 \rightarrow U = 1+2(2)^2 = \boxed{7}$

$$\frac{3}{4}\int_0^2 (1+2x^2)^{-\frac{1}{2}} \cdot (4x\,dx)$$

$$\frac{3}{4}\int_1^7 U^{-\frac{1}{2}}\,du \longrightarrow \frac{3}{4}\left[2U^{\frac{1}{2}}\right]\Big|_1^7$$

$$= \frac{3}{4}\left[\left(2\cdot(7)^{\frac{1}{2}}\right) - \left(2\cdot(1)^{\frac{1}{2}}\right)\right] = \frac{3}{4}\left[2\sqrt{7} - 2\right]$$

$$\boxed{\frac{3\sqrt{7} - 3}{2}}$$

21) $\int x\sqrt{x+2}\,dx = \int x(x+2)^{\frac{1}{2}}dx$

$u = x+2$
$du = dx$
$x = u-2$

$\int (u^{\frac{1}{2}})\cdot(u-2)\,du$

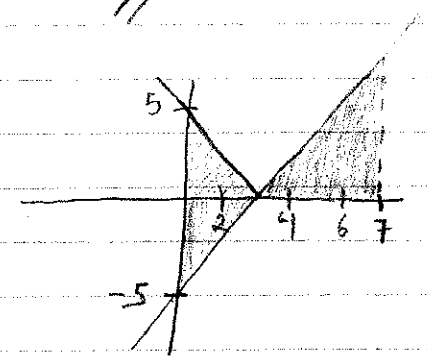
$\int u^{\frac{3}{2}} - 2u^{\frac{1}{2}}\,du = \frac{2}{5}u^{5/2} - \frac{4}{3}u^{\frac{3}{2}} + C$

$= \boxed{\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C}$

22) $\int_0^7 |2x-5|\,dx$

$2x-5=0$

$\boxed{x = \frac{5}{2} = 2.5}$



$\int_{\frac{5}{2}}^{7} 2x-5\,dx - \int_0^{5/2} 2x-5\,dx$

$= \left[x^2-5x\right]_{\frac{5}{2}}^{7} - \left[x^2-5x\right]_0^{5/2}$

$\left\{\left[(7)^2-5(7)\right] - \left[(\tfrac{5}{2})^2-5(\tfrac{5}{2})\right]\right\} - \left\{\left[(\tfrac{5}{2})^2-5(\tfrac{5}{2})\right] - 0\right\}$

$\frac{81}{4} - \left(-\frac{25}{4}\right) = \boxed{\frac{53}{2}}$

23) $\int \frac{6x+5}{2x+1}\,dx$

$$2x+1 \overline{)\,6x+5\,}^{\,3}$$
$$\underline{-(6x+3)}$$
$$2$$

$\int \left(3+\frac{2}{2x+1}\right)dx \longrightarrow \int 3\,dx + \int \frac{1}{2x+1}\,dx$

$u = 2x+1$
$du = 2dx$

$= \int 3\,dx + \frac{1}{2}\int \frac{1}{2x+1}\cdot(2dx) \longrightarrow \int 3\,dx + \frac{1}{2}\int \frac{1}{u}\,du$

$= 3x + \frac{1}{2}\ln u + C \longrightarrow \boxed{3x+\frac{1}{2}\ln(2x+1)}$

24) $\int \frac{5x+16}{x^2+9}\,dx = \int \frac{5x}{x^2+9}\,dx + \int \frac{16}{x^2+9}\,dx$

$= \int \frac{5}{x+9}\,dx + \int \frac{16}{x^2+9}\,dx$

$u = x+9$
$du = dx$

$5\int \frac{1}{x+9}\cdot\frac{1}{\cancel{5}}\cdot\cancel{5}dx + \int \frac{16}{x^2+9}\,dx$

$= 5\int \frac{1}{u}\,du + 16\int \frac{1}{x^2+9}\,dx$

$\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$

$u = x \quad a = 3$
$du = dx$

$= 5\ln u + \frac{16}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

$= \boxed{5\ln(x+9) + \frac{16}{3}\tan^{-1}\left(\frac{x}{3}\right)+C}$

25) $\int \tan t \, dt = \int \frac{\sin t}{\cos t} dt$

$u = \cos t$
$du = -\sin t \, dt$

$-\int \frac{1}{\cos t} \cdot (-\sin t) dt = -\int \frac{1}{u} du$

$= -\ln u + C \longrightarrow \boxed{-\ln(\cos t) + C}$

26) $\int x \sin(x^2) dx$

$u = x^2$
$du = 2x \, dx$

$\frac{1}{2} \int \sin(x^2) \cdot (2x) dx = \frac{1}{2} \int \sin u \, du$

$= -\frac{1}{2} \cos u + C \longrightarrow \boxed{-\frac{1}{2} \cos x^2 + C}$

27) $\frac{dy}{dx} = \frac{2000}{1+0.2x}$, $y(0) = 1000$ solve y, find y(10)

$y = \int \frac{2000}{1+.2x} dx$

$u = 0.2x + 1$
$du = 0.2 dx$

$\frac{2000}{.2} \int \frac{1}{1+.2x} \cdot (.2) dx$

$\frac{2000}{.2} \int \frac{1}{u} du \longrightarrow \frac{2000}{.2} \ln u + C = \boxed{\frac{2000}{0.2} \ln(0.2x+1) + C}$

$y(0) = \frac{2000}{.2} \ln(.2(0)+1) + C$ $\boxed{C = 1000}$

$\boxed{y(10) = 11986.1229}$ $\boxed{y = 10,000 \ln(0.2x+1) + 1000}$

28) $f(x) = x(x^2-4) \quad [-2, 1]$

$f(x) = x^3 - 4x$

$$\int_{-2}^{1} x^3 - 4x = \left[\frac{x^4}{4} - 2x^2\right]\Big|_{-2}^{1}$$

$$\left[\frac{1^4}{4} - 2(1)^2\right] - \left[\frac{(-2)^4}{4} - 2(-2)^2\right]$$

$$-\frac{7}{4} + 12 = \boxed{\frac{9}{4}}$$

29) $f(x) = xe^{(x^2+2)} \quad [1, 4]$

$$\int_{1}^{4} xe^{(x^2+2)} \qquad u = x^2+2 \qquad du = 2x\,dx \qquad \frac{1}{2}\int_{1}^{4} e^{(x^2+2)} \cdot (2x)\,dx$$

$x = 1,\ u = \boxed{3} \qquad x = 4,\ u = \boxed{18}$

$$\frac{1}{2}\int_{3}^{18} e^{u}\,du \longrightarrow \left[\frac{1}{2}e^{u}\right]\Big|_{3}^{18} = \left[\frac{1}{2}e^{18}\right] - \left[\frac{1}{2}e^{3}\right]$$

$$\boxed{\frac{1}{2}\left[e^{18} - e^{3}\right]} \text{ or } \boxed{32,829,974.53}$$

30) $\int_0^6 |x-2|+3\,dx$ $\qquad f(x)=|x-2|+3$

$$\int_0^6 3\,dx + \left[\int_2^6 (x-2)+3\,dx - \int_0^2 (x-2)+3\,dx\right]$$

$$\int_0^6 3\,dx + \left[\int_2^6 x+1\,dx - \int_0^2 x+1\,dx\right]$$

$$[3x]\Big|_0^6 + \left\{\left[\frac{x^2}{2}+x\right]\Big|_2^6 - \left[\frac{x^2}{2}+x\right]\Big|_0^2\right\}$$

$$\left[(3\cdot6)-(3\cdot0)\right] + \left\{\left[\frac{6^2}{2}+6\right]-\left[\frac{2^2}{2}+2\right]\right\} - \left\{\left[\frac{2^2}{2}+2\right]-\left[\frac{0^2}{0}+0\right]\right\}$$

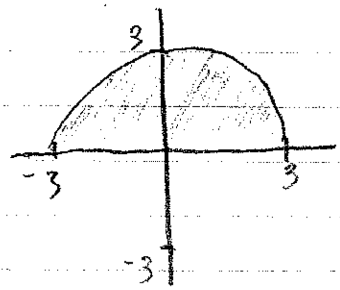
$$18+(24-4)-(4-0)$$

$$= 18+20-4$$

$$= \boxed{34}$$

31) $\int_{-3}^{3} \sqrt{9-x^2}\, dx$ $f(x) = 9 - x^2$

$x^2 + y^2 = 9$, $r = 3$



Area Circle $= \pi r^2$

$\frac{1}{2}$ Area $= \frac{1}{2}\pi r^2$

$$\int_{-3}^{3} \sqrt{9-x^2}\, dx = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi (3)^2 = \boxed{14.137167 \text{ or } \frac{9}{2}\pi}$$

32) $V = \frac{4}{3}\pi r^3$ $\frac{dr}{dt} = 5 \text{ ft/min}$ $r = 12 \text{ ft}$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = \quad \frac{dV}{dt} = 4\pi(12)(5)$$

$$\boxed{\frac{dV}{dt} = 240\pi}$$

33) $\int \cos 2x \ \sin^2 2x \, dx$ $\qquad U = 2x$, $du = 2dx$

$\frac{1}{2}\int \cos U \cdot \sin^2 U \cdot dU$ $\qquad V = \sin U$, $dV = \cos U \, dU$

$\frac{1}{2}\int \sin^2 U \cdot \cos U \, dU \longrightarrow \frac{1}{2}\int V^2 dV$

$\frac{1}{2} \cdot \frac{V^3}{3} + C \longrightarrow \frac{1}{6} V^3 + C \longrightarrow \frac{1}{6} \sin^3 U + C$

$= \boxed{\frac{1}{6} \sin^3 2x + C}$

34) $\frac{dP}{dt} = 100t^{\frac{1}{2}} + 210t^{3/4}$ $\qquad t(0) = 15,000$

a) $P(t) = \int 100t^{\frac{1}{2}} + 210t^{3/4} dt$

$= \int 100t^{\frac{1}{2}} dt + \int 210t^{3/4} dt = 100 \cdot \frac{2}{3}t^{3/2} + 210 \cdot \frac{4}{7}t^{7/4} + C$

$= \frac{200}{3}t^{3/2} + 120t^{7/4} + C \longrightarrow t(0) = \frac{200}{3}(0) + 120(0) + C$

$\boxed{C = 15,000}$

$\boxed{P(t) = \frac{200}{3}t^{3/2} + 120t^{7/4} + 15,000}$

b) $P(25) = \frac{200}{3}(25)^{3/2} + 120(25)^{7/4} + 15,000 = \boxed{56,874.353}$

$x = -1$
$U = 3(-1)+7$
$\boxed{U = 4}$

$x = 10$
$U = 3(10)+7$
$\boxed{U = 37}$

35) $f(x) = \sqrt{3x+7}$ $[-1, 10]$

$$\int_{-1}^{10} \sqrt{3x+7}\, dx = \int_{-1}^{10} (3x+7)^{\frac{1}{2}} dx$$

$U = 3x+7$
$du = 3\,dx$

$$\frac{1}{3}\int_{-1}^{10} (3x+7)^{\frac{1}{2}} \cdot (3)\, dx \longrightarrow \frac{1}{3}\int_{4}^{37} U^{\frac{1}{2}}\, du$$

$$= \frac{1}{3}\left[\frac{2}{3}U^{3/2}\right]\Big|_4^{37} = \frac{1}{3}\left[\frac{2}{3}(37)^{3/2} - \frac{2}{3}(4)^{3/2}\right]$$

$$\frac{1}{3}(144.708142) = \boxed{48.236047}$$

36) $\int_0^{\pi/4} x \tan x \, dx$ $\quad n=4, \; a=0, \; b=\frac{\pi}{4}$

a) $\Delta x = \frac{b-a}{n} = \frac{\frac{\pi}{4}-0}{4} = \boxed{\frac{\pi}{16}}$ $\quad x_0 = a = \boxed{0}$

$x_1 = x_0 + \Delta x = 0 + \frac{\pi}{16} = \boxed{\frac{\pi}{16}}$ $\quad x_2 = \frac{\pi}{16} + \frac{\pi}{16} = \boxed{\frac{\pi}{8}}$

$x_3 = \frac{\pi}{8} + \frac{\pi}{16} = \boxed{\frac{3\pi}{16}}$ $\quad x_4 = \frac{3\pi}{16} + \frac{\pi}{16} = \boxed{\frac{\pi}{4}}$

Trap Rule $\frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right] \approx \int_a^b f(x)\,dx$

$\approx \frac{\frac{\pi}{4}-0}{2(4)}\left[0 + 2(.039056) + 2(.162661) + 2(.393590) + .785398\right]$

$\approx \frac{\pi}{32}\left[1.976012\right] \approx \boxed{.1940}$

b) $\int_0^{\pi/4} x \tan x \, dx \rightarrow \int_0^{\pi/4} -\ln(\cos u) \cdot \frac{u}{\tan} \, du$ $\quad u = \tan x$, $du = -\ln(\cos u)\,dx$, $x = \frac{u}{\tan}$

$= \frac{1}{8}\left(4C - \pi \ln(2)\right) \approx \boxed{0.1858}$

37) a) $\int_0^2 x \ln(x+1)\,dx \qquad n=4,\ a=0,\ b=2$

$\Delta x = \frac{b-a}{n} = \frac{2-0}{4} = \boxed{\frac{1}{2}} \qquad x_0 = a = \boxed{0} \qquad x_1 = 0 + \frac{1}{2} = \boxed{\frac{1}{2}}$

$x_2 = \frac{1}{2} + \frac{1}{2} = \boxed{1} \qquad x_3 = 1 + \frac{1}{2} = \boxed{\frac{3}{2}} \qquad x_4 = \frac{3}{2} + \frac{1}{2} = \boxed{2}$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$\approx \frac{1}{4}\left[0 + 2(.202733) + 2(.693147) + 2(1.374436) + 2.197225\right]$$

$$\approx \boxed{1.6845}$$

b) $\int_0^2 x \ln(x+1)\,dx$

$u = x+1$
$du = dx$
$x = u-1$

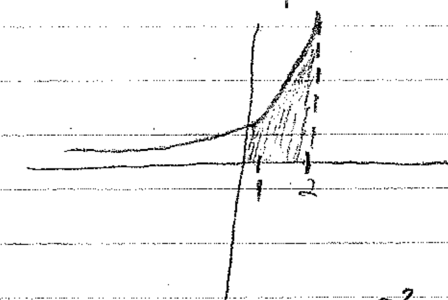
$u = \boxed{1}$
$u = \boxed{3}$

$$\int_1^3 (u-1)(\ln u)\,du = \int_1^3 u\ln u - \ln u\,du$$

$$\int_1^3 u\ln u\,du - \int_1^3 \ln u\,du$$

$$= \frac{3\log(3)}{2} \approx \boxed{1.6479}$$

38) $Y = 1 + e^x \quad y=0,\ x=0,\ x=2 \quad [0,2]$



$$\int_0^2 1 + e^x\,dx \longrightarrow \int_0^2 1\,dx + \int_0^2 e^x\,dx$$

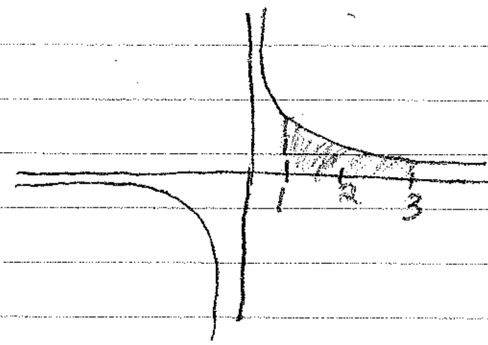
$$= \left[x + e^x\right]\Big|_0^2$$

$$= \left[(2+e^2) - (0+e^0)\right]$$

$$= 2 + e^2 + 1$$

$$= \boxed{3 + e^2}$$

39) $y = \frac{2}{x}$, $y=0$, $x=1$, $x=3$ $[1,3]$



$$\int_1^3 \frac{2}{x}\,dx \rightarrow 2\int_1^3 \frac{1}{x}\,dx$$

$$= \left[2\ln(x)\right]\Big|_1^3$$

$$= \left[2\ln(3) - 2\ln(1)\right]$$

$$= 2\ln(3) - 0$$

$$= \boxed{2\ln(3)}$$

40) $2\pi \int_0^1 (y-1)\sqrt{1-y}\, dy$

$2\pi \int_0^1 (y-1)(1-y)^{\frac{1}{2}} dy$

$u = 1-y$

$du = -dy$

$y = 1-u$

$-2\pi \int_0^1 (1-u-1)(u)^{\frac{1}{2}} du$

$u = \boxed{1}$

$u = \boxed{0}$

$-2\pi \int_1^0 -u\,(u)^{\frac{1}{2}} du \longrightarrow -2\pi \int_1^0 -u^{3/2} du$

$2\pi \int_0^1 -u^{3/2} du \longrightarrow -2\pi \int_0^1 u^{3/2} du$

$-2\pi \left[(1)^{3/2} - (0)^{3/2} \right]$

$= -2\pi(1)$

$\boxed{= -2\pi}$