Steven Romeiro
U1680 3837

Homework #2

Section 3.1: 4, 8, 18, 23, 24, 26, 30, 33
Section 3.2: 17, 19, 27, 29, 44, 46
Section 3.3: 16, 17, 24, 35, 36
Section 3.4: 4, 6, 11, 12, 15

Section 3.1:

(4) Let $Q(n)$ be predicate $n^2 \leq 30$

a) Write $Q(2)$, $Q(-2)$, $Q(7)$, $Q(-7)$

$Q(2) = 2^2 \leq 30 \longrightarrow 4 \leq 30$ T

$Q(-2) = (-2)^2 \leq 30 \longrightarrow 4 \leq 30$ T

$Q(7) = 7^2 \leq 30 \longrightarrow 49 \leq 30$ F

$Q(-7) = (-7)^2 \leq 30 \longrightarrow 49 \leq 30$ F

b) Find Truth set of $Q(n)$ for domain $\mathbb{Z}$

$\forall n \in \mathbb{Z}$, such that $Q(n)$ is $-5 \leq n \leq 5$

c) Truth set for $Q(n)$ for domain $\mathbb{Z}^+$

$\forall n \in \mathbb{Z}$, such that $Q(n)$ is $1 \leq n \leq 5$

(8) $B(x) = -10 < x < 10$ for domains

a) $\mathbb{Z} = \{-9 \ldots, 0 \ldots 9\}$

b) $\mathbb{Z}^+ = \{1 \ldots 8, 9\}$

c) Even $\mathbb{Z} = \{-8, -6, -4, -2, 0, 2, 4, 6, 8\}$

(18) $M(s)$ = Math major, $C(s)$ = Comp Sci
$E(s)$ = Engineer

a) $\exists s \in D$, such that $E(s) \wedge M(s)$

b) $\forall s \in D$ if $C(s)$ then $E(s)$

c) $\forall s \in D$ such that if $C(s)$ then $\sim E(s)$

d) $\exists s \in D$ such that if $C(s)$ then $M(s)$

e) $(\exists s \in D$, such that $C(s) \wedge E(s)) \wedge (\exists s \in D, C(s) \sim E(s))$

(23) a) $\forall x \in D$, such that if X is an equilateral triangle, then it is also isoceles

$\forall$ Equilateral triangles X, X is an isoceles

b) $\forall x \in D$, such that if x is a Computer Science student, then x needs to take data structures

$\forall$ Computer Science students X, X needs to take data structures

(24) a) $\exists x$ such that x is a hatter
and x is mad
$\exists$ hatters x such that x are mad

b) $\exists x$ such that x is a question
and x is easy
$\exists$ a question x such that x is easy

(26) $\forall x$, if x is an integer number, then x is a rational number, but $\exists x$ such that x is a rational number and x is not an integer

$$(\forall x, Int(x) \rightarrow Ratl(x)) \wedge (\exists x \text{ such that } Ratl(x) \wedge \sim Int(x))$$

(30) a) There exist a number X such that X is a prime number and X is not an odd integer. True because 2 is prime 2 ~odd

b) for All numbers X, if X is a prime integer then X is not a perfect square

True because prime cannot be perfect square

c) There are some odd integer X that are also perfect squares

True, 25 is such a number

(33) a) $a>0$ & $b>0 \rightarrow ab>0$ True

b) False. Ex: $-1<0$ & $-2<0$, $(-1)(-2)=\boxed{2}$

c) False. $a=0$ & $b=0 \therefore ab=0$

d) False: $a=-1, b=1, c=-2, d=2$

$ac=2$ & $bd=2$ $\quad 2 \not> 2$

Section 3.2

(17) $\forall$ integers d, if $6/d$ is integer then $d=3$

$\sim$($\forall$ integers d, if $6/d$ is integer then $d=3$)

$\boxed{\exists \text{ integer } d \text{ such that } 6/d \text{ is integer} \wedge d \neq 3}$

(19) $\forall n \in \mathbb{Z}$, if n is prime, then n is odd or $n=2$

$\sim$($\forall n \in \mathbb{Z}$, if n is prime, then n is odd or $n=2$)

$\exists n \in \mathbb{Z}$ such that n is prime but n is even and $n \neq 2$

(27) $\forall d \in \mathbb{Z}$, if $6/d$ is integer then $d=3$

- Contrapositive:

$\forall d \in \mathbb{Z}$, if $d \neq 3$ then $6/d$ is not an integer

- Converse:

$\forall d \in \mathbb{Z}$, if $d=3$ then $6/d$ is an integer

- Inverse:

$\forall d \in \mathbb{Z}$, if $6/d$ is not an integer then $d \neq 3$

(29) $\forall n \in \mathbb{Z}$, n is prime then n is odd or $n=2$

Inverse:

$\forall n \in \mathbb{Z}$, if n is not prime, then n is even and $n \neq 2$

Contrapositive:

$\forall n \in \mathbb{Z}$, if n is even and $n \neq 2$, then n isn't prime

Converse:

$\forall n \in \mathbb{Z}$, if n is odd or $n=2$, then n is prime

(44) ~(Having a large income is necessary condition for a person to be happy)

~(if a person is happy, then they have a large income)

There is a person who is happy and does not have a large income

(46) ~(Being a polynomial is a sufficient condition for a function to have a real root)

~(If a function is a polynomial, then the function has a real root)

There exists a function that is a polynomial and does not have a real root

Section 3.3

(16) $\exists$ a real number u such that
$\forall$ real numbers V, $UV = V$

a) There exist a real number, such that for all real numbers the product of the first number and the second equals the second

b) Negation: $\forall$ real numbers U, $\exists$ a real number V such that $UV \neq V$

- For every real number, there is a real number such that their products do not equal the second number

(17) $\forall r \in Q$, $\exists$ integers a and b such that $r = a/b$

a) For all rational numbers, there exists two integers such that the rational number equals the ratio of the integers

b) $\exists r \in Q$, $\forall$ integers a and b such $r \neq a/b$

There exists a rational number for all pair of integers such that the rational number does not equal their ratios

(24) a) $\sim(\forall_x \in D(\forall_y \in E(P(x,y))))$

$\equiv \exists_x \in D(\exists_y \in E(\sim P(x,y)))$

derivation: $\exists x \in D \sim(\forall_y \in E(P(x,y)))$

$\boxed{\exists_x \in D(\exists_y \in E(\sim P(x,y)))}$

There exists a number such that for a particular second number from another set, the function of both numbers is not satisfied.

b) $\sim(\exists_x \in D(\exists_y \in E(P(x,y)))$

$\equiv \forall x \in D(\forall_y \in E(\sim P(x,y)))$

derivation: $\forall x \in D \sim(\exists_y \in E(P(x,y)))$

$\boxed{\forall x \in D(\forall_y \in E(\sim P(x,y)))}$

For all numbers of the first set, All numbers of a second set do not satisfy a function of both these numbers

(35) Everybody trusts sombody

a) $\forall$ people x, $\exists$ a person y such that $\overbrace{x \text{ trusts } y}^{trust(x,y)}$

b) $\exists$ a person x, $\forall$ people y such that $\sim trust(x,y)$

(36) Somebody trusts everybody, $trust(x,y)$

a) $\exists$ a person x such that $\forall$ people y, $trust(x,y)$

b) $\forall$ people in x, $\exists$ a person y such that, $\sim trust(x,y)$

Section 3.4

(4) if $r^{+} \rightarrow (r^{a})^{b} = r^{ab}$

$r = 3,\ a = 1/2,\ b = 6$

$\therefore\ (3^{1/2})^{6} = 3^{3} = \boxed{27}$

(6) if program is correct, then compilation does not produce errors

Compilation produces errors

$\therefore$ $\boxed{\text{Computer program is not correct}}$

(11) All cheaters sit in back rows

Monty sits in the back row

$\therefore$ Monty is a cheater

Invalid by $\boxed{\text{converse error}}$

Since $P(x)$ = cheaters, $Q(x)$ back row

$Q(m)$ for monty $\nRightarrow$ $P(m)$ cheating Monty

(12) $\forall x$, if $\overbrace{x \text{ is honest}}^{P(x)}$, then $\overbrace{x \text{ pays taxes}}^{Q(x)}$

Darth is not honest $= \sim P(D)$

$\therefore$ Darth does not Pay taxes $= \sim Q(D)$

Invalid by $\boxed{\text{inverse error}}$

$P(x) \rightarrow Q(x)$

$\sim P(x)$

$\therefore \sim Q(x)$ inverse error

(15) Any sum of two rational numbers is rational
Sum r+s is rational
$\therefore$ Both r & s are rational

$\equiv$ $\forall x$, rational numbers, $\forall y$ rational numbers $P(x,y)$ = rational
$P(x,y)$ = rational
$\therefore$ x and y = rational

$\equiv$ if $\overbrace{\text{x and y are rational}}^{P(x)}$, then $\overbrace{x+y = \text{rational}}^{Q(x)}$
r+s = rational $] \longrightarrow Q(x)$
$\therefore$ r and s are rational $] \longrightarrow P(x)$
$\boxed{\text{Converse error}}$

$P(x) \longrightarrow Q(x)$
$Q(x)$
$\therefore P(x)$ converse error

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

$p \to q \equiv \sim p \vee q$

$\sim(p \to q) \equiv p \wedge \sim q$

necessary

if $q \to p$ or $\sim p \to \sim q$

Sufficient

if $p \to q$

$$A + (AB) \equiv A \circ (A + B) \equiv A$$
$$A + (A'B) \equiv A + B$$
$$A + (BC) \equiv (A+B)(A+C)$$
$$A + (AB) \equiv A \circ (A+B) \equiv A$$
$$A + (A'B) \equiv A + B$$
$$A + (BC) \equiv (A+B)(A+C)$$
$$A + (AB) \equiv A \circ (A+B) \equiv A$$
$$A + (A'B) \equiv A + B$$
$$A + (BC) \equiv (A+B)(A+C)$$

Steven Romeiro
U16803837

Homework #3

4.1: 5, 13, 28, 30
4.2: 18, 25
4.3: 3, 5, 13, 26, 39
4.4: 8, 9, 25, 30, 37

4.1

5) $m = 2,\ n = -2 \therefore \frac{1}{m} + \frac{1}{n} = \frac{1}{2} - \frac{1}{2} = \boxed{0}$

$\boxed{0 \in \mathbb{Z}}$

13) $\forall m,n \in \mathbb{Z}$, if $2m+n$ is odd, then m & n are odd

$\forall m,n \in \mathbb{Z}$, $odd(2m+n) \rightarrow odd(m) \wedge odd(n)$

Counterexample: $m = 2,\ n = 1$

$\therefore 2m + n =$

$= 2(2) + 1$

$= \boxed{\begin{array}{l} 5 \text{ is odd} \\ m = 2 \text{ is even} \end{array}}$

28) $\forall n \in \mathbb{Z}$, if $\overbrace{n \text{ is odd}}^{P}$, then $\overbrace{n^2 \text{ is odd}}^{Q}$

Suppose n is random & specifically chosen

n is odd $\equiv n = 2(K)+1 \ (K \in \mathbb{Z})$

$n^2 = (2k+1)^2$

$n^2 = 4K^2 + 4K + 1$

$n = 2(2K^2 + 2K) + 1$

Let $t = 2K^2 + 2K$

$t \in \mathbb{Z}$ since $2(K^2+K)$ is integer

$n^2 = 2t+1$ is odd by definition

$\therefore \boxed{n = 2K+1 \ \& \ n^2 = 2t+1 \ \forall K, t \in \mathbb{Z}}$

30) $\forall m \in \mathbb{Z}$, if m is even, then $3m+5$ is odd

Suppose m is random & specifically chosen

m is even $\equiv m = 2K \ (K \in \mathbb{Z})$

$3m+5$ is odd $\equiv 3m+5 = 2L+1 \ (L \in \mathbb{Z})$

$3(2K)+5$

$6K+5$

$2(3K+2)+1$

Let $t = 3K+2 \ (t \in \mathbb{Z})$

t is integer since product & sum of integers is an integer

$2t+1$ is odd by definition

$\therefore \boxed{3m+5 = 2t+1}$

4.2

(18) If r & s are rational, then $\frac{r+s}{2}$ is rational

r & s are rational

$r = \frac{a}{b}$ & $s = \frac{c}{d}$, $b \neq 0$, $d \neq 0$ $(a,b,c,d \in \mathbb{Z})$

$$\frac{r+s}{2} = \frac{\frac{a}{b}+\frac{c}{d}}{2} = \frac{\frac{ad+bc}{bd}}{2} = \frac{ad+bc}{2bd}$$

Let $X = ad+bc$ and $Y = 2bd$, $Y \neq 0$

X & Y are integers since sum & product of integers are integers $(X, Y \in \mathbb{Z})$

$\boxed{\text{TRUE} \therefore \frac{r+s}{2} = \frac{X}{Y} \text{ rational by definition}}$

(25) If r is any rational number, then $3r^2-2r+4$ is rational

r is a rational number

$r = \frac{a}{b}$, $b \neq 0$ $(a, b \in \mathbb{Z})$

$$3\left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right) + 4 = 3r^2 - 2r + 4$$

$$3\,\frac{a^2}{b^2} - 2\,\frac{a}{b} + 4$$

$$\frac{3a^2}{b} - \frac{2a}{b} + 4$$

$$\frac{3a^2 - 2a}{b} + 4$$

$$\frac{3a \cdot a - 2a + 4b}{b}$$

Product and difference of integers are integer

$t = 3a^2 - 2a + 4b$ $(t \in \mathbb{Z})$

$\frac{t}{b}$ is rational by definition

$\boxed{\therefore 3r^2 - 2r + 4 = \frac{t}{b} \quad \text{TRUE}}$

4.3

(3) Does $5|0$?

Does 5 divide 0? $\equiv \frac{0}{5}$

$0 = 5K$ for some integer K?

$\boxed{\text{Yes}}$, $K = \frac{0}{5} \equiv \boxed{K=0}$

(5) Is $6m(2m+10)$ divisible by 4?

$$\frac{12m^2 + 60m}{4} = 3m^2 + 15m$$

Let $t = 3m^2 + 15m$,

t is an integer since sum & product of integers is an integer ($t \in Z$)

So to show $4 \mid 6m(2m+10) = 4 \mid 12m^2 + 60m$

$12m^2 + 60m = 4t$

$\therefore$ $\boxed{\text{Yes by definition of divisibility}}$

$$\overbrace{\text{if } n = 4K+3}^{P}, \text{ does } \overbrace{8 \text{ divide } n^2 - 1}^{Q}$$

(13) if $n = 4K+3$, does 8 divide $n^2 - 1$

If $n = 4K+3$, does $8 \mid n^2 - 1$

$n = 4K+3$, by substitution $(4K+3)^2 - 1$

$(4K+3)^2 - 1 = 16K^2 + 24K + 9 - 1 = 16K^2 + 24K + 8$

$= 8(2K^2 + 3K + 1)$

Let $t = 2K^2 + 3k + 1$, So t is Integer

because sum & product of integers is integer ($t \in \mathbb{Z}$)

Does $8 \mid 8t$?

$8t = 8x \quad (x \in \mathbb{Z})$

$\boxed{\therefore\ t = x \text{ by definition, Yes}}$

(26) For all integers a & b & c, if $ab \mid c$, then $a \mid c$ and $b \mid c$.

$c = ab(K) \quad (K \in \mathbb{Z})$ by definition

$c = aQ \quad (Q \in \mathbb{Z})$ by definition

$c = bT \quad (T \in \mathbb{Z})$ by definition

$ab = aQ$ and $ab = bT$ by subst.

$b = Q$ and $a = T$

$c = a \cdot a$ and $c = b \cdot b$

Since product of of integers is integer

'a' and 'b' are integers

$\boxed{\therefore\ a \mid c \text{ and } b \mid c}$

39 a) std factored form for a^3

$$a^3 = (p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdot \ldots \cdot p_k^{e_k})$$

$$a^3 = (p_1^{e_1})^3 \cdot (p_2^{e_2})^3 \cdot (p_3^{e_3})^3$$

$$\boxed{a^3 = (p_1^{3e_1}) \cdot (p_2^{3e_2}) \cdot (p_3^{3e_3}) \cdot \ldots \cdot (p_k^{3e_k})}$$

4.4

(8) $50 \text{ div } 7 = \frac{50}{7}$ $n=50,\ d=7$

$n = dq + r$, $50 = 7(q) + r$

$\frac{50}{7} = 49 \ r1$, $q=7$, $r=1$

$n = (7)(7)+1$ $\boxed{50 \text{ div } 7 = 7}$

b) $50 \bmod 7 \implies n \bmod d = r$
$n = dq + r$, $n = 50$, $d = 7$

$50 = 7(q) + r$, $50 = 7(7) + 1$

$\boxed{50 \bmod 7 = 1}$

(9) a) $28 \text{ div } 5$, $n \text{ div } d = q$
$n = dq + r \implies 28 = 5(q) + r$
$28 = 5(5) + 3$
$\boxed{28 \text{ div } 5 = 5}$

b) $28 \bmod 5$, $n \bmod d = r$
$28 = 5(5) + 3$
$\therefore \boxed{28 \bmod 5 = 3}$

(25) if $a \bmod 7 = 5$ and $b \bmod 7 = 6$,

then $ab \bmod 7 = 2$

$n = dq + r$, $n \bmod d = r$

$a = 7(q) + 5$, $b = 7(t) + 6$

$(7q+5)(7t+6) = 7x + 2$ $(q, t, x \in \mathbb{Z})$

$49qt + 42t + 35q + 30 = 7x + 2$

$7(7qt + 6t + 5q + 4) + 2 = 7x + 2$

Let $Z = 7qt + 6t + 5q + 4)$, Since sum & products of integers are integer, $Z \in \mathbb{Z}$ integer set

$\therefore \boxed{7z + 2 = 7x + 2}$

$ab \bmod 7 = 2$

(30) $d = 3$, $n = dq + r$, $0 \leq r < d$

so $r = 0, 1, 2$

$n = 3q$, $n = 3q + 1$, $n = 3q + 2$

product of consecutive integers is $n(n+1)$

$n(n+1) = (3q)(3q+1)$

$= 9q^2 + 3q \implies 3(3q^2 + q)$

Let $t = 3q^2 + q$, sum & product of integer is integer $t \in \mathbb{Z}$

$\boxed{n(n+1) = 3t}$

$n = 3q+1$

$n(n+1) = (3q+1)(3q+2)$

$= 9q^2 + 9q + 2 = 3(3q^2+3q)+2$

Let $U = 3q^2 + 3q$, since sum & products of integers is integer, $U \in \mathbb{Z}$

$\therefore \boxed{n(n+1) = 3U+2}$

$n = 3q+2$

$n(n+1) = (3q+2)(3q+3) = 3(3q+2)(q+1)$

Let $V = (3q+2)(q+1)$, since sum & products of integer is integer, $V \in \mathbb{Z}$

$\therefore \boxed{n(n+1) = 3V}$

product of two consecutive integers are either $3K$ or $3K+2$ $(K \in \mathbb{Z})$

b) for $n \in \mathbb{Z}$ $\quad n(n+1) \bmod 3 = 0$ or

$n(n+1) \bmod 3 = 2$

(37) if n is even, $n = 2K$ $(K \in Z)$

$n^2 = (2K)^2 = 4K^2 = 4K \cdot K$

product of integers are integer

therefore $t = K^2$ $t \in Z$

$n^2 = \boxed{4t}$

if n is odd, $n = 2K+1$ $(K \in Z)$

$n^2 = (2K+1)^2 = 4K^2 + 4K + 1$

$n^2 = 4(K^2 + K) + 1$

product & sum of integers are integer

so $t = K^2 + K$ where $t \in Z$

$\boxed{n^2 = 4t + 1}$

Homework 4

Steven Romeiro
U16803837

Section 4.5: 21, 24, 28, 29
Section 4.6: 4, 6, 12, 17, 20, 28
Section 4.7: 4, 12, 19
Section 4.8: 15, 16

Section 4.5

(21) For all odd integers n, $\lceil n/2 \rceil = (n+1)/2$

odd$(n) = 2K+1$ $(K \in \mathbb{Z})$

Show: $\lceil n/2 \rceil = \frac{2k+1+1}{2} = \frac{2k+2}{2} = \frac{2(k+1)}{2} = k+1$ //

$\lceil n/2 \rceil = \left\lceil \frac{2k+1}{2} \right\rceil = \left\lceil \frac{2K}{2} + \frac{1}{2} \right\rceil$

$\lceil n/2 \rceil = \lceil k + \frac{1}{2} \rceil$

Ceiling $\lceil n \rceil = K$ $(K \in \mathbb{Z}) \longleftrightarrow K-1 < n \leq K$

$\therefore \lceil n/2 \rceil = K-1 < n/2 \leq K$

$k < k + 1/2 \leq k+1$ by defn

$\lceil n/2 \rceil = K+1$

Since $n = 2K+1$

$n+1 = 2K+2$

$n+1 = 2(K+1)$

$\frac{n+1}{2} = K+1$ $\quad \therefore \boxed{\lceil n/2 \rceil = \frac{(n+1)}{2}}$

(24) For any integer m & any real number x if x is not an integer, then $\lfloor x \rfloor + \lfloor m-x \rfloor = m-1$

$$\text{Floor} = \lfloor x \rfloor \longleftrightarrow n \le x < n+1$$
$$\lfloor x \rfloor = n$$
$$\lfloor -x \rfloor = -n-1$$
$$\lfloor m-x \rfloor = \lfloor m+(-x) \rfloor = m + \lfloor -x \rfloor$$

$$\lfloor x \rfloor + \lfloor m-x \rfloor = n+m-(n+1) = m-1$$
$$\boxed{\lfloor x \rfloor + \lfloor m-x \rfloor = m-1}$$

(28) For any odd integer n

$$\left\lfloor \frac{n^2}{4} \right\rfloor = \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)$$

LHS: $(K \in \mathbb{Z})$ $\left\lfloor \frac{n^2}{4} \right\rfloor = \left\lfloor \frac{(2K+1)^2}{4} \right\rfloor = \left\lfloor \frac{4K^2+4K+1}{4} \right\rfloor = \left\lfloor K^2+K+\frac{1}{4} \right\rfloor$

integer $\left\lfloor (K^2+K)+\frac{1}{4} \right\rfloor = \boxed{(K^2+K)}$

Since $(K^2+K) \le (K^2+K)+\frac{1}{4} < (K^2+K)+1$

RHS: $(K \in \mathbb{Z})$ $\left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right) = \left(\frac{(2K+1)-1}{2}\right)\left(\frac{(2K+1)+1}{2}\right)$

$$= \left(\frac{2K}{2}\right)\left(\frac{2K+2}{2}\right) = K(K+1) = \boxed{(K^2+K)}$$

$$\therefore \left\lfloor \frac{n^2}{4} \right\rfloor = \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)$$

Section 4.6

(4) $\forall m \in Z$, $7m+4$ not divisible by 7

by contradiction

$\sim(\forall m \in Z,\ 7 \nmid (7m+4))$

$\exists m \in Z,\ 7 \mid (7m+4)$

$7 \mid (7m+4) \equiv 7m+4 = 7K \quad (K \in Z)$

$4 = 7K - 7m$

$4 = 7(K-m)$ (integer − integer) = integer

$4 = 7t$ Let $t = (K-m)$ $(t \in Z)$

$Q = 4$ Let $Q = 7t$ $(Q \in Z)$

$\therefore 7 \mid 4$

$4/7$, Since $a \mid b$, $a \le b$

$\therefore \boxed{7 \mid 4,\ 7 \le 4 \ \text{False}}$

$\therefore 7m+4$ is not divisible by 7

(6) There is no greatest negative real number

Contradiction:

There is a greatest negative real number

Greatest negative real number = R

Let $y = \frac{R}{2}$, $R/2 > R$

$\boxed{y > R \ \therefore \text{There is no greatest neg real number}}$

(29) For odd integer n

$$\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2+3}{4}$$

LHS: $\left\lceil \frac{n^2}{4} \right\rceil = \left\lceil \frac{(2k+1)^2}{4} \right\rceil = \left\lceil \frac{4k^2+4k+1}{4} \right\rceil = \left\lceil k^2+k+\frac{1}{4} \right\rceil$

$(k \in \mathbb{Z})$

$\left\lceil k^2+k+\frac{1}{4} \right\rceil = \boxed{k^2+k+1}$ by deffinition

RHS: $\left(\frac{n^2+3}{4}\right) = \left(\frac{(2k+1)^2+3}{4}\right) = \left(\frac{4k^2+4k+4}{4}\right) = \boxed{k^2+k+1}$

$(k \in \mathbb{Z})$

$\therefore \left\lceil n^2/4 \right\rceil = \left(\frac{n^2+3}{4}\right)$

(12) if a and b are rational numbers, $b \neq 0$ and r is an irrational number, then $a+br$ is irrational

Contradiction : $a+br$ is rational

$br = (a+br) - a$

$r = \frac{br}{b}$, division of rational numbers is rational

r is irrational

$\therefore$ $\boxed{a+br \text{ is irrational}}$

(17) For all integers, if $a \bmod 6 = 3$, then $a \bmod 3 \neq 2$

Contradiction:

$\exists a \in \mathbb{Z},\ a \bmod 6 = 3 \wedge a \bmod 3 = 2$

$a = 6q + 3 \quad (q \in \mathbb{Z})$

$a = 3x + 2 \quad (x \in \mathbb{Z})$

$6q + 3 = 3x + 2$

$6q - 3x = -1$

$3(2q - x) = -1$

$2q - x = -\frac{1}{3}$, $2q - x$ is integer but $-\frac{1}{3}$ is not integer

$\therefore$ $\boxed{a \bmod 3 \neq 2}$

(20) if sum of two real numbers is less than 50, then at least one of the numbers is less than 25

Contraposition:

$\exists x, y \in R$, if $x \geq 25$ and $y \geq 25$, then $x+y \geq 50$

$r+s \geq 25+25$

$r+s \geq 50$

$\boxed{\therefore \text{ if } r+s < 50, \text{ then } r < 25 \text{ or } s < 25}$

(28) $\forall m,n \in Z$, if mn is even, then m or n is even

Contradiction:

$\exists m,n \in Z$, $even(mn) \land odd(m) \land odd(n)$

$m \cdot n = 2k \quad (k \in Z)$

$m = 2r+1, \quad n = 2p+1 \quad (r, p \in Z)$

$m \cdot n = 4rp + 2p + 2r + 1$

$m \cdot n = 2(2rp+p+r)+1$, Let $t = (2rp+p+r) \ (t \in Z)$

$m \cdot n = 2t+1$, but $m \cdot n = 2k$

$\boxed{\therefore even(m) \text{ or } even(n)}$

Contraposition: if m and n is odd, then mn is odd

$mn = (2k+1) \quad (k \in Z)$

$m = 2r+1, \quad n = 2p+1 \quad (r, p \in Z)$

$m \cdot n = (2r+1)(2p+1)$

$m \cdot n = 4rp + 2r + 2p + 1 = 2(2rp+r+p)+1$

Let $t = (2rp+r+p) \ (t \in Z)$

$mn = 2t+1$, $mn = 2k+1$

$\boxed{\therefore m \text{ or } n \text{ is even}}$

Section 4.7

(4) $3\sqrt{2} - 7$ is irrational

Contradiction: $3\sqrt{2} - 7$ is rational

$3\sqrt{2} - 7 = \frac{a}{b}$ $(a, b \in \mathbb{Z})$

$3\sqrt{2} = a/b + 7 \Rightarrow \sqrt{2} = \frac{a}{3b} + \frac{7}{3} \Rightarrow \sqrt{2} = \frac{a+7b}{3b}$

Let $t = a + 7b$ and $r = 3b$ $(t, r \neq 0 \in \mathbb{Z})$

$\sqrt{2} = \frac{t}{r}$, ratio of integers is rational

yet $\sqrt{2}$ is irrational

$\therefore$ $\boxed{3\sqrt{2} - 7 \text{ is irrational}}$ True

(12) Product of two irrational numbers is irrational

$\sqrt{2}$ is irrational as seen in problem 4

$\sqrt{2} \cdot \sqrt{2} = 2$

$2 = \frac{2}{1}$ $\therefore$ rational

$\therefore$ $\boxed{\text{False, Product of two irrational numbers is not irrational}}$

(19) $\sqrt{5}$ is irrational

Contradiction: $\sqrt{5}$ is rational, $\sqrt{5} = \frac{a}{b}$ $(a, b \in \mathbb{Z})$

$b \neq 0$ and a, b have no common factors

$5 = \frac{a^2}{b^2} \Rightarrow b^2 5 = a^2 \Rightarrow b^2 = \frac{a^2}{5} \Rightarrow 5 \mid a^2$

$a = 5k$ $(k \in \mathbb{Z}) \Rightarrow a^2 = 25k^2$

$b^2 = \frac{25k^2}{5} \Rightarrow b^2 = 5k^2 \Rightarrow 5 \mid b^2$

a and b have 5 as common factor

yet a and b have no common factor

$\therefore$ $\boxed{\sqrt{5} \text{ is irrational}}$

Section 4.8

(15) Euclidian Algorithm 832 and 10,933

$\frac{10,933}{832}$

$$\begin{array}{r} 13 \\ 832\overline{)10933} \\ -10816 \\ \hline 117 \end{array}$$

$10,933 = (832 \cdot 13) + 117$

$GCD(10933, 832) = GCD(832, 117)$

$\frac{117}{13} = 9$ & no remainder

$\boxed{GCD(10933, 832) = GCD(832, 117) = GCD(117, 13) = GCD(13, 0) = 13}$

(16) 4131 and 2431

$$\begin{array}{r} 1 \\ 2431\overline{)4131} \\ -2431 \\ \hline 1700 \end{array}$$

$GCD(4131, 2431) = GCD(2431, 1700)$

$$\begin{array}{r} 1 \\ 1700\overline{)2431} \\ -1700 \\ \hline 731 \end{array}$$

$GCD(2431, 1700) = GCD(1700, 731)$

$$\begin{array}{r} 2 \\ 731\overline{)1700} \\ 1462 \\ \hline 238 \end{array}$$

$GCD(1700, 731) = GCD(731, 238)$

$\frac{731}{238} = 3 + 17r$

$GCD(731, 238) = GCD(238, 17)$

$\frac{238}{17} = 14 + 0r$

$\boxed{GCD(4131, 2431) = GCD(2431, 1700) = GCD(1700, 731) = GCD(731, 238) = GCD(238, 17) = GCD(17, 0) = 17}$

Esthevan Romeiro
U16803837

Homework #5

Section 5.1: 4, 15, 25, 30, 38, 41, 44, 50, 54, 57, 78
Section 5.2: 4, 7, 11, 23, 27
Section 5.3: 9, 17, 20

Section 5.1

(4) $d_m = 1 + \left(\frac{1}{2}\right)^m$ for all integers $m \geq 0$

$m=0: \quad d_0 = 1 + \left(\frac{1}{2}\right)^0 = 1 + 1 = \boxed{2}$
$m=1: \quad d_1 = 1 + \left(\frac{1}{2}\right)^1 = 1 + \frac{1}{2} = \boxed{3/2}$
$m=2: \quad d_2 = 1 + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4} = \boxed{5/4}$
$m=3: \quad d_3 = 1 + \left(\frac{1}{2}\right)^3 = 1 + \frac{1}{8} = \boxed{9/8}$

(15) $a_1 = \overset{①}{0}, \ a_2 = \overset{②}{-\frac{1}{2}}, \ a_3 = \overset{③}{\frac{2}{3}}, \ a_4 = \overset{④}{-\frac{3}{4}}, \ a_5 = \overset{5}{\frac{4}{5}}, \ a_6 = \overset{⑥}{-\frac{5}{6}}, \ a_7 = \overset{⑦}{\frac{6}{7}}$

$a_m = \frac{\pm \text{number}}{m}, \quad m \geq 1$
number $= m-1$

$\therefore \boxed{a_m = (-1)^{m-1}\left(\frac{m-1}{m}\right)}$

(25) $\prod_{K=2}^{2}\left(1 - \frac{1}{K}\right) = 1 - \frac{1}{2} = \frac{2}{2} - \frac{1}{2} = \boxed{\frac{1}{2}}$

(30) $\sum_{J=1}^{n} J(J+1) = 1(1+1) + 2(2+1) + 3(3+1) + \ldots + n(n+1)$

$$\boxed{\sum_{J=1}^{n} J(J+1) = 1(2) + 2(3) + 3(4) + \ldots + n(n+1)}$$

(38) $\sum_{K=1}^{m+1} K^2$

$$\sum_{K=1}^{m+1} K^2 = (1)^2+(2)^2+(3)^2+\ldots+m^2+(m+1)^2$$

$$\boxed{\sum_{K=1}^{m} K^2+(m+1)^2 = (m+1)^2+\sum_{K=1}^{m} K^2}$$

(41) $\sum_{K=1}^{m} \frac{K}{K+1} + \frac{m+1}{m+2}$

$$\sum_{K=1}^{m} \frac{K}{K+1} = \frac{1}{1+1}+\frac{2}{2+1}+\frac{3}{3+1}+\ldots+\frac{m}{m+1}+\frac{m+1}{m+2}$$

$$\therefore \boxed{\sum_{K=1}^{m+1} \frac{K}{K+1}}$$

(44) $(1^3-1)-(2^3-1)+(3^3-1)-(4^3-1)+(5^3-1)$

$m=$ 1 2 3 4 5

$$\boxed{\sum_{m=1}^{5} (-1)^{m+1}(m^3-1)}$$

(50) $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}$

terms: 1st, 2nd, 3rd ... nth

$$\sum_{K=1}^{n} \frac{K}{(K+1)!} = \frac{1}{(1+1)!} + \frac{2}{(2+1)!} + \frac{3}{(3+1)!} + \dots + \frac{n}{(n+1)!}$$

$$\boxed{\sum_{K=1}^{n} \frac{K}{(K+1)!} = \frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}}$$

(54) $\prod_{K=1}^{n} \frac{K}{K^2+4}$, $i = K+1$

when $K=1$, $i = 1+1$, $i = 2$ when $K=n$, $i = n+1$, $i = n+1$

when $\frac{K}{K^2+4} = \frac{(i-1)}{(i-1)^2+4}$

$$\boxed{\therefore \prod_{i=2}^{n+1} \frac{(i-1)}{(i-1)^2+4}}$$

(57) $\sum_{i=1}^{n-1} \frac{i}{(n-i)^2}$, $J = i-1 \Rightarrow i = J+1$

when $i=1$, $J = 1+1$, $J = 2$, when $i = n-1$, $J = n-1-1$, $J = n-2$

when $\frac{i}{(n-i)^2} = \frac{J+1}{(n-(J+1))^2} = \frac{J+1}{(n-J-1)^2}$

$$\boxed{\therefore \sum_{J=2}^{n-2} \frac{J+1}{(n-J-1)^2}}$$

(78) $r+1 \leq n$, $\binom{n}{r+1} = \frac{n-r}{r+1}\binom{n}{r}$

All integers $0 \leq r+1 \leq n$

$$\binom{n}{r+1} = \frac{n!}{(r+1)!(n-(r+1))!}$$

RHS: $$\frac{n!}{(r+1)!(n-(r+1))!} \cdot \frac{(n-r)}{(n-r)}$$

$$= \frac{(n-r)(n!)}{(r+1)(r)!(n-r)((n-r)-1)!}$$

$$= \left(\frac{n-r}{r+1}\right)\frac{n!}{(r)!(n-r)!}$$

$$\boxed{\binom{n}{r+1} = \frac{n-r}{r+1}\binom{n}{r}}$$

Section 5.2

(4) $\underbrace{\sum_{i=1}^{n-1} i(i+1) - \frac{n(n-1)(n+1)}{3}}_{P(n)}$ $\quad n \geq 2$

a) $P(2) \equiv n=2$

$$\sum_{i=1}^{2-1} i(i+1) = \frac{2(2-1)(2+1)}{3}$$

$$\sum_{i=1}^{1} i(i+1) = \frac{6}{3} \Rightarrow \boxed{\sum_{i=1}^{1} i(i+1) = 2 \quad \hookrightarrow 1(1+1)=2}$$ True

b) $P(n)$ where $n=k \equiv P(k)$

$$\boxed{\sum_{i=1}^{k-1} i(i+1) = \frac{k(k-1)(k+1)}{3}}$$

c) $P(n)$ where $n = k+1$

$$\sum_{i=1}^{k+1-1} i(i+1) = \frac{(k+1)(k+1-1)(k+1+1)}{3}$$

$$\boxed{\sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}}$$

d) if $P(n)$ is true for $n=k$, then it's true for $n=k+1$

if $\sum_{i=1}^{k-1} i(i+1) = \frac{k(k-1)(k+1)}{3} \longrightarrow \sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$

? alternate way to solve

(7) For all integers $n \geq 1$

$$1+6+11+16+\dots+(5n-4) = \frac{n(5n-3)}{2}$$

Step 1: Show P(1) True

$P(n)$, $n=1$

LHS: $(5n-4) \Rightarrow (5(1)-4) \Rightarrow 5-4 \Rightarrow \boxed{1}$

RHS: $\frac{n(5n-3)}{2} \Rightarrow \frac{(1)(5\cdot 1-3)}{2} \Rightarrow \frac{2}{2} = \boxed{1}$ True

Step 2: Suppose P(K) is true

$$n=K;\quad 1+6+11+16+\dots+(5K-4) = \frac{K(5K-3)}{2}$$

Step 3: Prove P(K+1) is true, $n=K+1$

$$1+6+11+16+\dots+(5(K+1)-4) = \frac{(K+1)(5(K+1)-3)}{2}$$

LHS: $\underbrace{1+6+11+16+\dots+(5K-4)}_{P(K)}+(5(K+1)-4)$

$$= \frac{K(5K-3)}{2} + (5(K+1)-4)$$

$$= \frac{K(5K-3)}{2} + (5K+5-4)$$

$$= \frac{K(5K-3)}{2} + (5K+1) \Longrightarrow \frac{K(5K-3)+2(5K+1)}{2}$$

$$= \frac{K(5K-3)+(10K+2)}{2} \Longrightarrow \frac{5K^2-3K+10K+2}{2}$$

$$= \frac{5K^2+7K+2}{2} \Longrightarrow \frac{5K^2+5K+2K+2}{2}$$

Cont. of 7

$$\frac{5K(K+1) + 2(K+1)}{2} \Longrightarrow \frac{(K+1)(5K+2)}{2}$$

$$\frac{(K+1)(5K+2+3-3)}{2} \Longrightarrow \frac{(K+1)(5(K+1)-3)}{2}$$

$$\boxed{\begin{array}{l} LHS: \dfrac{(K+1)(5(K+1)-3)}{2} \\ RHS: \dfrac{(K+1)(5(K+1)-3)}{2} \end{array}}$$

P(K+1) True ✓

(11) $1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2, \quad n \geq 1$

Step 1: Show P(1) is true, $n=1$

TRUE

LHS: $n^3 = 1^3 = \boxed{1}$

RHS: $\left[\frac{n(n+1)}{2}\right]^2 = \left[\frac{1(1+1)}{2}\right]^2 = \left[\frac{2}{2}\right]^2 = (1)^2 = \boxed{1}$

Step 2: Assume P(K) is true, $n = K$

$$1^3 + 2^3 + \ldots + K^3 = \left[\frac{K(K+1)}{2}\right]^2$$

Step 3: Show that P(K+1) is true, $n = K+1$

Add a K+1 term to both sides of P(K)

$$\underbrace{1^3 + 2^3 + \ldots + K^3}_{P(K)} + (K+1)^3 = \left[\frac{K(K+1)}{2}\right]^2 + (K+1)^3$$

Arrive at $\left[\frac{(K+1)(K+2)}{2}\right]^2$ $\hookrightarrow$

RHS: $\dfrac{K^2(K+1)^2}{4} + (K+1)^3$

$$\frac{K^2(K+1)^2 + 4(K+1)^3}{4} \Rightarrow \frac{K^2(K+1)^2 + 4(K+1)^2(K+1)}{4}$$

$$\frac{(K+1)^2(K^2+4(K+1))}{4} \Rightarrow \frac{(K+1)^2(K^2+4K+4)}{4}$$

$$\frac{(K+1)^2(K+2)^2}{4} \Rightarrow \boxed{\left[\frac{(K+1)(K+2)}{2}\right]^2} \leftarrow \text{TRUE}$$

$$P(K) = \left[\frac{K(K+1)}{2}\right]^2, \quad \boxed{P(K+1) = \left[\frac{(K+1)(K+2)}{2}\right]^2}$$

23) $7+8+9+10+\ldots+600$

Sum of 1st integers $1+2+3+\ldots+n = \dfrac{n(n+1)}{2}$

$7+8+9+\ldots+600 = (1+2+3+4+\ldots+600)-(1+2+3+4+5+6)$

Sum of first 600 is: $1+2+3+4+\ldots+600 = \dfrac{600(601)}{2}$

$= 300(601)$

Sum of series is: $7+8+9+\ldots+600 = (1+2+3+4+\ldots+600)-(1+2+3+4+5+6)$

$$= \frac{600(601)}{2} - 21 \Rightarrow 300(601)-21$$

$$= \boxed{180{,}279}$$

? not seen in class

27) $5^3+5^4+5^5+\ldots+5^K \quad , \quad K \geq 3$

$5^3+5^4+5^5+\ldots+5^K = 5^3(1+5+5^2+5^3+\ldots+5^{K-3})$

Geometric Series is: $\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1} \quad , \quad n \geq 1, \; r=3$

$5^3+5^4+5^5+\ldots+5^K = 5^3(1+5+5^2+5^3+\ldots+5^{K-3})$

$= 5^3\left(\frac{5^{K-3+1}-1}{5-1}\right)$ where $r=5$

$= 5^3\left(\frac{5^{K-2}-1}{4}\right) \Rightarrow \boxed{\frac{125}{4}(5^{K-2}-1)}$

Section 5.3

(9) $7^n - 1$ divisible by 6 for $n \geq 0$

Step 1: Show $P(0)$ to be true

$7^0 - 1 = 1 - 1 = 0$

$6 | 0 \Rightarrow 0 = 6(k) \quad (k \in Z)$

$k = 0 \therefore 6|0$ True

Step 2: Assume $P(k)$ is true

$7^k - 1$ divisible by 6 $\equiv 7^k - 1 = 6(r) \quad (r \in Z)$

Step 3: Prove that $P(k+1)$ is true

$7^{k+1} - 1 \Rightarrow 7^{k+1} - 1 - 6 + 6 \Rightarrow 7^{k+1} - 7 + 6$

$= 7^k \cdot 7 - 7 + 6 \Rightarrow 7\underbrace{(7^k - 1)}_{P(k)} + 6$ substitute

$7^{k+1} - 1 = 7(P(x)) + 6 \Rightarrow 7^{k+1} - 1 = 7(6r) + 6$

$7^{k+1} - 1 = 6(7r + 1) \Rightarrow$ Let $t = 7r + 1, (t \in Z)$

$7^{k+1} - 1 = 6t$

$\boxed{\therefore 6 | (7^{k+1} - 1)}$ True

? How to solve Step 3

(17) $1+3n \leq 4^n$, $n \geq 0$

Step 1: Show P(0) to be true

$1+3(0) \leq 4^0 \Rightarrow 1+0 \leq 1 \Rightarrow \boxed{1 \leq 1}$ True

Step 2: Assume P(K) is true

$1+3K \leq 4^K$

Step 3: Prove that P(K+1) is true

$1+3(K+1) \leq 4^{(K+1)}$

LHS $1+3(K)+3(1) \Rightarrow 1+3K+3$

$1+3(K+1) = 1+3K+3$

$1+3(K+1) \leq 4^K+3$

$\leq 4^K+3\cdot 1 \Rightarrow \leq 4\cdot 4^K$ since $4^K \geq 1$

$= \boxed{4^{K+1}}$

(20) $2^0 < (n+2)!$, $n \geq 0$

Step 1: Show P(0) true

$2^0 < (0+2)! \Rightarrow 1 < 2! \Rightarrow \boxed{1<2}$ True

Step 2: Assume P(K) to be true

$2^K < (K+2)!$

Step 3: Prove P(K+1) to be true

$2^{K+1} < (K+1+2)! \Rightarrow 2^{K+1} < (K+3)!$

How? → $2^{K+1} < (K+3)(K+2)! \Rightarrow 2\cdot 2^K < (K+3)(K+2)!$

$$\boxed{\begin{aligned} 2\cdot(K+2)! &< (K+3)(K+2)! \\ 2^{K+1} &< (K+3)! \quad \text{TRUE} \end{aligned}}$$

Homework #6 Steven Romeiro U16803837

Section 5.6: 10, 14, 28
Section 5.7: 4, 7, 26
Section 5.8: 10, 12, 15

Section 5.6

(10) $b_n = 4^n$, $n \geq 0$

Satisfies recurrence relation $b_k = 4b_{k-1}$, $k \geq 1$

~~Find first four, b_1, b_2, b_3, b_4~~

Purpose: to show that the sequence $b_n = 4^n$ satisfies the recurrence $b_k = 4b_{k-1}$ for all $k \geq 1$

$b_n = 4^n \rightarrow$ equation (1)

Substitute $n = k$ into (1) } Step 1

$b_k = 4^k \rightarrow$ equation (2)

Now solve by substituting in $n = k-1$ in (1)

$b_{k-1} = 4^{k-1}$

$(4) \cdot b_{k-1} = 4^{k-1} \cdot (4)$

$4b_{k-1} = \frac{4^k}{4} \cdot (4)$ or $4b_{k-1} = 4^{k-1+1}$

Step 2

$4b_{k-1} = 4^k \} \rightarrow$ Sub in from (2)

$4b_{k-1} = b_k \rightarrow \boxed{b_k = 4b_{k-1}}$ $k \geq 1$

Satisfies recurrence relation

$b_k = 4b_{k-1}$, $k \geq 1$

(14) $d_n = 3^n - 2^n,\ n \geq 0$

Show that it satisfies $d_k = 5d_{k-1} - 6d_{k-2}$

Substitute $n = k$

$d_k = 3^k - 2^k$ equation (1)

Substitute $n = k-1$

$d_{k-1} = \underbrace{3^{k-1} - 2^{k-1}}$

also need $n = k-2$ Sub in

$d_{k-2} = \underbrace{3^{k-2} - 2^{k-2}}$

Original equation $d_k = 5d_{k-1} - 6d_{k-2}$

Substitute:

$$d_k = 5(3^{k-1} - 2^{k-1}) - 6(3^{k-2} - 2^{k-2})$$

$$d_k = 5\left(\frac{3^k}{3} - \frac{2^k}{2}\right) - 6\left(\frac{3^k}{3^2} - \frac{2^k}{2^2}\right)$$

$$d_k = \left(5 \cdot \frac{3^k}{3} - 5 \cdot \frac{2^k}{2}\right) - \left(\frac{6 \cdot 3^k}{3^2} - 6 \cdot \frac{2^k}{2^2}\right)$$

$$d_k = 3^k\left(\frac{5}{3} - \frac{6}{3^2}\right) - 2^k\left(\frac{5}{2} - \frac{6}{2^2}\right)$$

$$d_k = 3^k\left(\frac{5}{3} - \frac{2}{3}\right) - 2^k\left(\frac{5}{2} - \frac{3}{2}\right)$$

$$d_k = 3^k\left(\frac{3}{3}\right) - 2^k\left(\frac{2}{2}\right)$$

$$\boxed{\begin{array}{l} d_k = 3^k - 2^k \\ 5d_{k-1} - 6d_{k-2} = 3^k - 2^k = d_k \end{array}}$$

(28) Prove that $F_{K+1}^2 - F_K^2 - F_{K-1}^2 = 2F_K F_{K-1}$, $K \geq 1$

LHS: $F_{K+1}^2 - F_K^2 - F_{K-1}^2$

LHS: $\underbrace{F_{K+1}^2 - F_{K-1}^2}_{a^2 - b^2 = (a+b)(a-b)} - F_K^2$

LHS: $(F_{K+1} + F_{K-1})\underbrace{(F_{K+1} - F_{K-1})}_{F_K = F_{K+1} - F_{K-1}} - F_K^2$

only → wors for Fibonnaci sequences →

LHS: $(F_{K+1} + F_{K-1})(F_K) - F_K^2$

LHS: $F_K(F_{K+1} + F_{K-1} - F_K)$

LHS: $F_K(\underbrace{F_{K+1} - F_K}_{F_{K-1} = F_{K+1} - F_K} + F_{K-1})$

LHS: $F_K(F_{K-1} + F_{K-1})$

LHS: $F_K(2F_{K-1}) \Longrightarrow 2F_K F_{K-1} = \text{RHS}$

Section 5.7

(4) $b_k = \dfrac{b_{k-1}}{1+b_{k-1}}$ for $k \geq 1$, $b_0 = 1$

$$b_1 = \frac{b_{1-1}}{1+b_{1-1}} = \frac{b_0}{1+b_0} = \frac{1}{1+1} = \frac{1}{2} = \frac{1}{1+1}$$

$$b_2 = \frac{b_{2-1}}{1+b_{2-1}} = \frac{b_1}{1+b_1} = \frac{\frac{1}{1+1}}{1+\frac{1}{1+1}} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3} = \frac{1}{1+2}$$

$$b_3 = \frac{b_{3-1}}{1+b_{3-1}} = \frac{b_2}{1+b_2} = \frac{1/3}{1+1/3} = \frac{1/3}{4/3} = \frac{1}{4} = \frac{1}{1+3}$$

$$b_n = \frac{1}{n+1}$$

(7) $e_k = 4e_{k-1} + 5$ $k \geq 1$, $e_0 = 2$

$$e_1 = 4e_{1-1} + 5 = 4e_0 + 5 = 4(2)+5 = 4 \cdot 2 + 5 = 2^3 + 5 = 2+11$$

$$e_2 = 4e_{2-1} + 5 = 4e_1 + 5 = 4(2+11)+5 = 57 = 2+11+44 = 2+11+(11 \cdot 4)$$

$$e_3 = 4e_{3-1} + 5 = 4e_2 + 5 = 4(57)+5 = 2+11+220 = 2+11(1+11 \cdot 4)+(11 \cdot 4^2)$$

$$e_4 = 4e_{4-1} + 5 = 4e_3 + 5 = 937 = 2+11(1+11 \cdot 4)+(11 \cdot 4^2)+(11 \cdot 4^3)$$

$$e_n = 2+11(1+4+4^2+4^3+\ldots 4^{n-1}) = \quad \frac{4^{n-1+1}-1}{4-1} = \frac{4^n-1}{3}$$

$$e_n = 2+11\left(\frac{4^n-1}{3}\right) = \frac{11 \cdot 4^n - 5}{3}$$

Section 5.8

(10) $C_K = C_{K-1} + 6C_{K-2} \quad K \geq 2, \; C_0 = 0, \; C_1 = 3$

$t^2 = t + 6$

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

Roots: $t = 3$ & $t = -2$

$C_n = C(3)^n + D(-2)^n, \quad n \geq 0$

$C_0 = C(3)^0 + D(-2)^0 \longrightarrow 0 = C + D$

$C_1 = C(3)^1 + D(-2)^1 \longrightarrow 3 = 3C - 2D$

$-\; 3 \cdot 0 = -\cancel{3C} + (-3D)$

$3 = \cancel{3C} - 2D$

$3 = -5D \longrightarrow D = -3/5$

$C = 3/5$

$C_n = 3/5(3)^n - 3/5(-2)^n$

$C_n = 3/5(3^n - 2^n)$

(12) $e_K = 9e_{K-2} \quad K \geq 2, \; e_0 = 0, \; e_1 = 2$

$t^2 = 9 \longrightarrow t^2 - 9 = 0$

$(t+3)(t-3) = 0$

Roots: $t = 3$ & $t = -3$

$e_n = C(3)^n + D(-3)^n$

$e_0 = C(3)^0 + D(-3)^0 \longrightarrow 0 = C + D$

$e_1 = C(3)^1 + D(-3)^1 \longrightarrow 2 = 3C - 3D$

$3 \cdot 0 = 3C + \cancel{3D}$

$2 = 3C - \cancel{3D}$

$6C = 2 \longrightarrow C = 1/3$

$D = -1/3$

$e_n = \frac{1}{3} 3^n + (-\frac{1}{3})(-3^n)$

$e_n = 3^{n-1} - 3^{n-1}$

(26) a) initial = 1,000 , rate = 3%

$A_0 = 1000$

$A_1 = A_0 + 0.0025A_0 + 200 = 1.0025A_0 + 200$

$A_2 = A_1 + 0.0025A_1 + 200 = 1.0025A_1 + 200$

$A_n = 1.0025A_{n-1} + 200$

b) $A_1 = 1.0025A_0 + 200$

$A_2 = 1.0025A_1 + 200 = 1.0025^2A_0 + 200(1+1.0025)$

$A_3 = 1.0025A_2 + 200 = 1.0025^3A_0 + 200(1+1.0025+1.0025^2)$

$A_n = 1.0025^nA_0 + 200(1+1.0025+1.0025^2+\ldots+1.0025^{n-1})$

$r = 1.0025 > 1 \Rightarrow \dfrac{a(r^n-1)}{r-1} = \dfrac{1(1.0025^n-1)}{1.0025-1} = 400(1.0025^n-1)$

$A_n = 1.0025^n \cdot 81000 - 8000$

c) $n=0$ for A_n

$A_0 = 1.0025^0 \cdot 1000 + 80000(1.0025^0-1) = 1000 + 80000(0)$

$A_0 = 1000$

$n = k$ for A_n

$A_k = 1.0025^k \cdot 1000 + 80000(1.0025^k-1) = 1.0025A_{k-1} + 200$

$n = k+1$ for A_n

$A_{k+1} = 1.0025^{k+1} \cdot 1000 + 80000(1.025^{k+1}-1) =$

$A_{k+1} = 1.0025^{k+1} \cdot 1000 + 80000(1.0025^{k+1} - 1.0025 + 0.0025)$

$A_{k+1} = 1.0025^{k+1} \cdot 1000 + 80000(1.0025^{k+1} - 1.0025) + 80000 \cdot 0.0025$

$A_{k+1} = 1.0025^{k+1}(1.0025^k + 80000(1.0025^k-1)) + 200 = 1.0025A_k + 200$

$A_{k+1} = 1.0025A_k + 200$

d) $A_{240} = (1.0025^{240} \cdot 1000) + 80000(1.0025^{240}-1 = 67481.15$

$A_{480} = (1.0025^{480} \cdot 1000) + 80000(1.0025^{480}-1) = 188527.04$

e) $10000 = (1.0025^n \cdot 1000) + 80000(1.0025^n-1)$

$= 90000 = 1.0025^n \cdot 81000 = 1.0025^n = \frac{10}{9} = n\ln 1.0025 = \log 10 - \log 9$

$n = \dfrac{1-\log 9}{\log 1.0025} = \quad n = 42.19$

(15) $t_k = 6t_{k-1} - 9t_{k-2} \quad k \geq 2, t_0 = 1, t_1 = 3$

$t^2 = 6t - 9 \longrightarrow t^2 - 6t + 9$

$(t-3)^2$

Roots: 3^n & $n \cdot 3^n$

$t_n = C3^n + Dn3^n$

$t_0 = C3^0 + D(0)3^0 \longrightarrow 1 = C$

$t_1 = C3^1 + D(1)3^1 \longrightarrow 3 = 3C + 3D \longrightarrow 3D = 0 \rightarrow D = 0$

$t_n = 3^n + 0 3^n \longrightarrow t_n = 3^n \quad n \geq 0$

Homework #7 Esthevan Romeiro U16803837

Section 6.1 : 7, 12, 17, 27(b, c, e), 35

Section 6.2 : 10, 17, 31

Section 6.3 : 2, 35, 38

$A = \{\dots 4, 10, 16, 22 \dots\}$

Section 6.1

$B = \{\dots -2, 16, \dots\}$

(7) Let $A = [x \in \mathbb{Z} \mid x = 6a + 4 \text{ for } a \in \mathbb{Z}]$

$B = \{y \in \mathbb{Z} \mid y = 18b - 2 \text{ for } b \in \mathbb{Z}\}$

$C = \{z \in \mathbb{Z} \mid z = 18c + 16 \text{ f } c \in \mathbb{Z}\}$

a) $A \subseteq B$

Let $x \in A$ there is an integer a such that

$x = 6a + 4$ ①

$x \in B$, there is an integer b such that

$x = 18b - 2$ ②

So $x = 6a + 4$ and $x = 18b - 2$

$6a + 4 = 18b - 2$

$6a + 6 = 18b$

$6(a+1) = 6(3b)$

$a + 1 = 3b$

$b = \frac{a+1}{3}$

$\boxed{x = 4 \in A \text{ and } 4 \notin B \therefore A \nsubseteq B}$

b) $B \subseteq A$

Let $x \in B$, there is an integer b such that

$x = 18b - 2$ ①

Let $x \in A$, there is an integer a such that

$x = 6a + 4$

therefore $6a + 4 = 18b - 2$

$6a + 6 = 18b$

$3b = a +$

For any integer value b we can choose an integer value of a that would satisfy

$3b = a + 1$, example $3(1) = 2 + 1$ choose 2

therefore $\boxed{B \subseteq A}$ True

c) $B = C$

Assume $B=C$

then $\forall x \in \mathbb{Z}$, such $x \in B \wedge x \in C$

$x \in B$, there is an integer b such that

$x = 18b - 2$

$x \in C$, there is an integer c such that

$x = 18c + 16$

So $18b - 2 = 18c + 16$

$18b = 18c + 18$

$18(b) = 18(c+1)$

$b = c+1$

for all b values we can choose an integer c to satisfy $b = c+1$ $\therefore$ $B \subseteq C$

Similarly for $c+1 = b$

for all values of c, we can choose an integer b such that the equation $c+1=b$ is satisfied

$\therefore$ $C \subseteq B$

$$\boxed{\begin{array}{c}\text{Since } B \subseteq C \wedge C \subseteq B \\ B = C\end{array}}$$

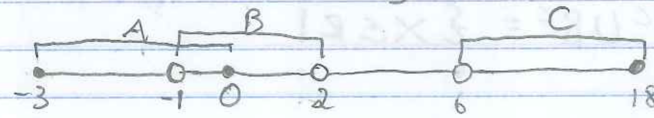
(12) Universal Set is $\mathbb{R}$

Let $A = \{x \in \mathbb{R} \mid -3 \leq x \leq 0\}$

$B = \{x \in \mathbb{R} \mid -1 < x < 2\}$

$C = \{x \in \mathbb{R} \mid 6 < x \leq 18\}$

a) $A \cup B$



$A \cup B = \{x \in \mathbb{R} \mid x \in [-3,0] \text{ or } x \in (-1,2)\}$

$A \cup B = \{x \in \mathbb{R} \mid x \in [-3,2)\}$

$A \cup B = [-3,2)$

b) $A \cap B = \{x \in \mathbb{R} \mid x \in [-3,0] \text{ and } x \in (-1,2)\}$

$A \cap B = \{x \in \mathbb{R} \mid x \in (-1,0]\}$

$A \cap B = (-1,0]$

c) $A^c = \{x \in \mathbb{R} \mid x \notin [-3,0]\}$

$A^c = \{x \in \mathbb{R} \mid x \in (-\infty,-3) \cup (0,\infty)\}$

d) $A \cup C = \{x \in \mathbb{R} \mid x \in [-3,0] \text{ or } x \in (6,18]\}$

$A \cup C = \{x \in \mathbb{R} \mid x \in [-3,0] \cup (6,18]\}$

$A \cup C = [-3,0] \cup (6,18]$

e) $A \cap C = \{x \in \mathbb{R} \mid x \in [-3,0] \text{ and } x \in (6,18]\}$

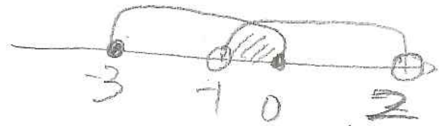
$A \cap C =$ there is no common element that belongs to both A and C

$\therefore A \cap C = \emptyset$

f) $B^c = \{x \in \mathbb{R} \mid x \notin (-1,2)\}$

$B^c = \{x \in \mathbb{R} \mid x \in (-\infty,-1] \cup [2,\infty)\}$

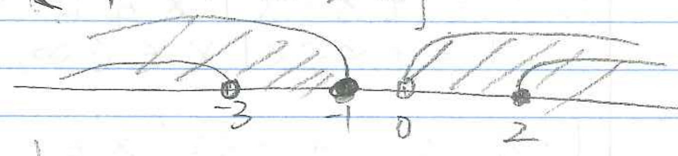
$B^c = (-\infty,-1] \cup [2,\infty)$



g) $A^c \cap B^c = \{x \in R \mid x \notin [-3,0] \text{ and } x \notin (-1,2)\}$
$A^c \cap B^c = \{x \in R \mid x \in (-\infty,3) \cup [2,\infty)\}$

h) $A^c \cup B^c = \{x \in R \mid x \notin [-3,0] \text{ or } x \notin (-1,2)\}$
$A^c \cup B^c = \{x \in R \mid$

i) $(A \cap B)^c = A^c \cup B^c$ by De-Morgans
$A \cap B = \{x \in R \mid -1 < x \leq 0\}$
$(A \cap B)^c = \{x \in R \mid x \leq -1 \text{ or } x > 0\}$
$A^c = \{x \in R \mid x < -3 \text{ or } x > 0\}$
$B^c = \{x \in R \mid x \leq -1 \text{ or } x \geq 2\}$



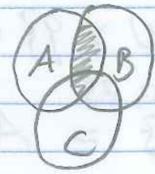
$A^c \cup B^c = \{x \in R \mid x \leq -1 \text{ or } x > 0\}$

h) $A^c \cup B^c = \{x \in R \mid x \in (-\infty,-1) \cup (0,\infty)\}$
$A^c \cup B^c = x < -1 \text{ or } x > 0$

$A = \{ \cdots\ -8, -2, 4, 10, 16, 22\ \cdots \}$

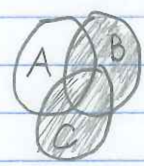
$B = \{ -2, 16, \}$

17 a)



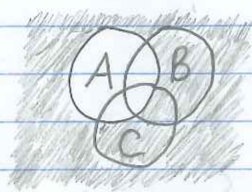
$= A \cap B$

b)



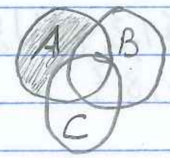
$= B \cup C$

c)



$= A^c$

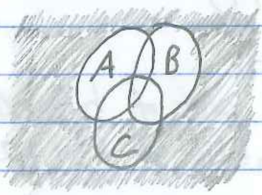
d)



$A - (B \cup C)$

$= A - (B \cup C) = \{x \in A \wedge x \notin (B \cup C)\}$

e)



$= (A \cup B)^c$

f)

from A^c in part C, do the same for B^c

$= A^c \cap B^c$

(27) b) $A_1 \cup A_2 \cup A_3 = \{w, x, v\} \cup \{u, y, q\} \cup \{p, z\}$
$= \{w, x, v, u, y, q\} \cup \{p, z\}$
$= \{w, x, v, u, y, q, p, z\} = A$

$A_1 \cap A_2 = \{w, x, v\} \cap \{u, y, q\} = \emptyset$

$A_1 \cap A_3 = \{w, x, v\} \cap \{p, z\} = \emptyset$

$A_2 \cap A_3 = \{u, y, q\} \cap \{p, z\} = \emptyset$

A_1, A_2, A_3 is partition of A
$\{\{w, x, v\}, \{u, y, q\}, \{p, z\}\}$ partition of $\{p, q, u, v, w, x, y, z\}$

c) $A_1 \cup A_2 \cup A_3 \cup A_4 = \{5, 4\} \cup \{7, 2\} \cup \{1, 3, 4\} \cup \{6, 8\}$
$A_1 \cup A_2 \cup A_3 \cup A_4 = \{5, 4, 7, 2, 1, 3, 4, 6, 8\} = A$

$A_1 \cap A_2 = \{5, 4\} \cap \{7, 2\} = \emptyset$
$A_1 \cap A_3 = \{5, 4\} \cap \{1, 3, 4\} = 4$
Not distinct since 4 is part of both A_1 and A_3
$\{\{5, 4\}, \{7, 2\}, \{1, 3, 4\}, \{6, 8\}\}$ <u>not</u> a patition of $\{5, 4, 7, 2, 1, 3, 4, 6, 8\}$

e) $A_1 \cup A_2 \cup A_3 = \{1, 5\} \cup \{4, 7\} \cup \{2, 8, 6, 3\}$
$A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6, 7, 8\} = A$

$A_1 \cap A_2 = \{1, 5\} \cap \{4, 7\} = \emptyset$
$A_1 \cap A_3 = \{1, 5\} \cap \{2, 8, 6, 3\} = \emptyset$
$A_2 \cap A_3 = \{4, 7\} \cap \{2, 8, 6, 3\} = \emptyset$

So $\{\{1, 5\}, \{4, 7\}, \{2, 8, 6, 3\}\}$ is a partition of $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(35) a) $B \cup C = \{1,2,3\}$
$A \times (B \cup C) = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$

b) $A \times B = \{(a,1),(a,2),(b,1),(b,2)\}$
$A \times C = \{(a,2),(a,3),(b,2),(b,3)\}$
$(A \times B) \cup (A \times C) = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3)\}$

c) $B \cap C = \{2\}$
$A \times (B \cap C) = \{(a,2),(b,2)\}$

d) $(A \times B) \cap (A \times C) = \{(a,2),(b,2)\}$

Section 6.2

(10) $x \in A - B \wedge x \in C - B$
$x \in A - B = x \in A \wedge x \notin B$
$x \in C - B = x \in C \wedge x \notin B$
$x \in A$ and $x \in C$ so $x \in A \cap C$
$x \in A \cap C \wedge x \notin B = x \in (A \cap C) - B$
$x \in (A-B) \cap (C-B) \rightarrow x \in (A \cap C) - B$
$(A-B) \cap (C-B) \subseteq (A \cup C) - B$

$x \in A \cup C$ and $x \notin B$
if $x \in A$ and $x \notin B$, then $x \in A - B$
if $x \in C$ and $x \notin B$, then $x \in C - B$
So $x \in (A-B) \cap (C-B)$
therefore $(A \cap C) - B \subseteq (A-B) \cap (C-B)$

$$(A-B) \cap (C-B) = (A \cap C) - B$$

(17) $x \in A \cup B$ and $A \subseteq B$, $A \subseteq C$

$x \in A$ or $x \in B$

$x \in A$ and $A \subseteq C$, $x \in C$

$x \in B$ and $B \subseteq C$, $x \in C$

if $x \in A \cup B$, then $x \in C$

$A \cup B \subseteq C$

(31) $x \in A \cap B$ so $x \in A$ and $x \in B$

$x \in B \land B \subseteq A^c \therefore x \in A^c \Rightarrow x \notin A$

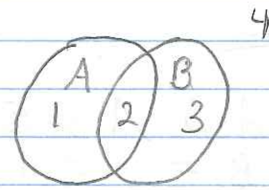
$x \in B \land x \notin A$ yet $x \in A$ is a contradiction

$A \cap B = \emptyset$

Section 6.3

(2) Counter $A = \{1, 2\}$

$B = \{2, 3\}$



$(A \cup B)^c = \{4\}$

$A^c = \{3\}$

$B^c = \{1\}$

$A^c \cup B^c = \{1, 3\}$

$\{4\} \neq \{1,3\} \therefore (A \cup B)^c \neq A^c \cup B^c$

(35) $A - (A - B) = A - (A \cap B^c)$ diff law

$A \cap (A \cap B^c)$ diff law

$A \cap (A^c \cup (B^c)^c)$ demorgan law

$A \cap (A^c \cup B$ double complement

$(A \cap A^c) \cup (A \cap B)$ distrib law

$\emptyset \cup (A \cap B)$ Complement law

$A \cap B$ identity law

$A - (A - B) = A \cap B$

(38) $A-(A\cap B) = A\cap(A\cap B)^c$ diff law

$A\cap(A^c\cup B^c)$ demorgans law

$(A\cap A^c)\cup(A\cap B^c)$ distrib law

$\emptyset \cup (A\cap B^c)$ complement law

$(A\cap B^c)$ identity law

$(A-B)$ set diff law

$A-(A\cap B) = A-B$

(5) integers m & n, $m > 1$, $n > 1$ $\wedge$ $\frac{1}{m} + \frac{1}{n} = int$

$m = 2$, $n = -2$, are distinct

$$\frac{1}{2} + \frac{1}{-2} = \frac{1}{2} - \frac{1}{2} = 0 = int$$

(13) $\forall_{m,n} \in \mathbb{Z}$ If $2m + n$ odd $\rightarrow$ odd(m) $\wedge$ odd(n)

$2m + n = 2K + 1 \quad (K \in \mathbb{Z})$

$m = 2t + 1$

$n = 2r + 1$

$\cancel{2(2t+1) + (2r+1) = 2K+1}$

$\cancel{4t + 2 = 2K - 2r}$

$\cancel{2(2t+1) = 2(K - r)}$

$\cancel{2t + 1 = K - r}$

$$\begin{aligned} 2m + n &= 2(2t+1) + (2r+1) \\ &= 4t + 2 + 2r + 1 \\ &= 4t + 2r + 3 \\ &= 4t + 2r + 2 + 1 \\ &= 2(2t + r + 1) + 1 \\ &= 2x + 1 \end{aligned}$$

4.3 #3 Does $5 | 0$

$0 = 5K \quad (K \in \mathbb{Z})$

$K = \frac{0}{5} \Rightarrow K = 0$

$5^K + 9 < 6^K$

$5^{K+1} + 9 < 6^{K+1} = 5^K \cdot 5 + 9 < 6^{K+1}$

$5^K \cdot 5 + 5 + 4 = 5(5^K + 1) + 4 =$

LHS: $5^{K+1} + 9$ RHS: $6^{K+1} = 6 \cdot 6^K$

Starting from $P(K)$ LHS $5^K + 9 \Rightarrow 6(5^K + 9) < 6^K \cdot 6$

$35^K + 54 < 6^{K+1} \Rightarrow 6 \cdot 5^K + 54$

$= 5(5^K) + 5^K + 54 > 5^{K+1} + 9$