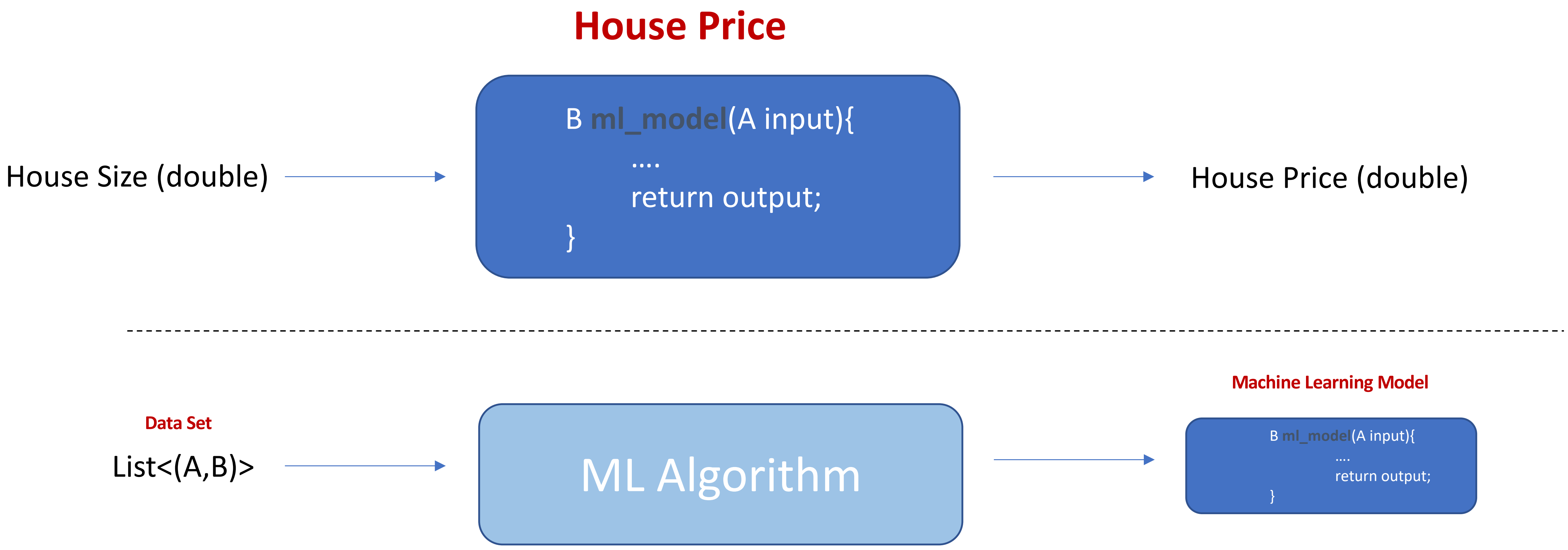
Linear Regression Multiple Variables

Andrew Ng's Machine Learning Course

Predict House Price given House Size



Multiple Features

Size in feet^2	Number of bedrooms	Number of floors	Age of home in years	Price ($) in $1000's
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
…	…	…	…	…

Multiple Features

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…	…	…	…	…

x_j = j^{th} feature

n = number of features

$\vec{x}^{(i)}$ = features of i^{th} training example

$x_j^{(i)}$ = value of feature j in i^{th} training example

Multi Linear Regression

Model:

$$f_{w,b}(\mathrm{x}) = w_1x_1 + w_2x_2 + \cdots + w_nx_n + b$$

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$$f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \vec{\mathrm{w}} \cdot \vec{\mathrm{x}} + b =$$

Gradient Descent for Multi Linear Regression

Parameters $w_1, \cdots, w_n$

b

Model $f_{\vec{w},b}(\vec{x}) = w_1 x_1 + \cdots + w_n x_n + b$

Cost function $J(w_1, \cdots, w_n, b)$

Gradient descent

repeat {

$$w_j = w_j - \alpha \frac{\partial}{\partial w_j} J(w_1, \cdots, w_n, b)$$

$$b = b - \alpha \frac{\partial}{\partial b} J(w_1, \cdots, w_n, b)$$

}

Gradient Descent for Multi Linear Regression

One feature

repeat {

$$w = w - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{w,b}(x^{(i)}) - y^{(i)}) x^{(i)}$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{w,b}(x^{(i)}) - y^{(i)})$$

simultaneously update w, b

}

n features ($n \geq 2$)

repeat {

$$w_1 = w_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) x_1^{(i)}$$

}

Gradient Descent for Multi Linear Regression

One feature

repeat {

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repeat {

$$w_1 = w_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) x_1^{(i)}$$

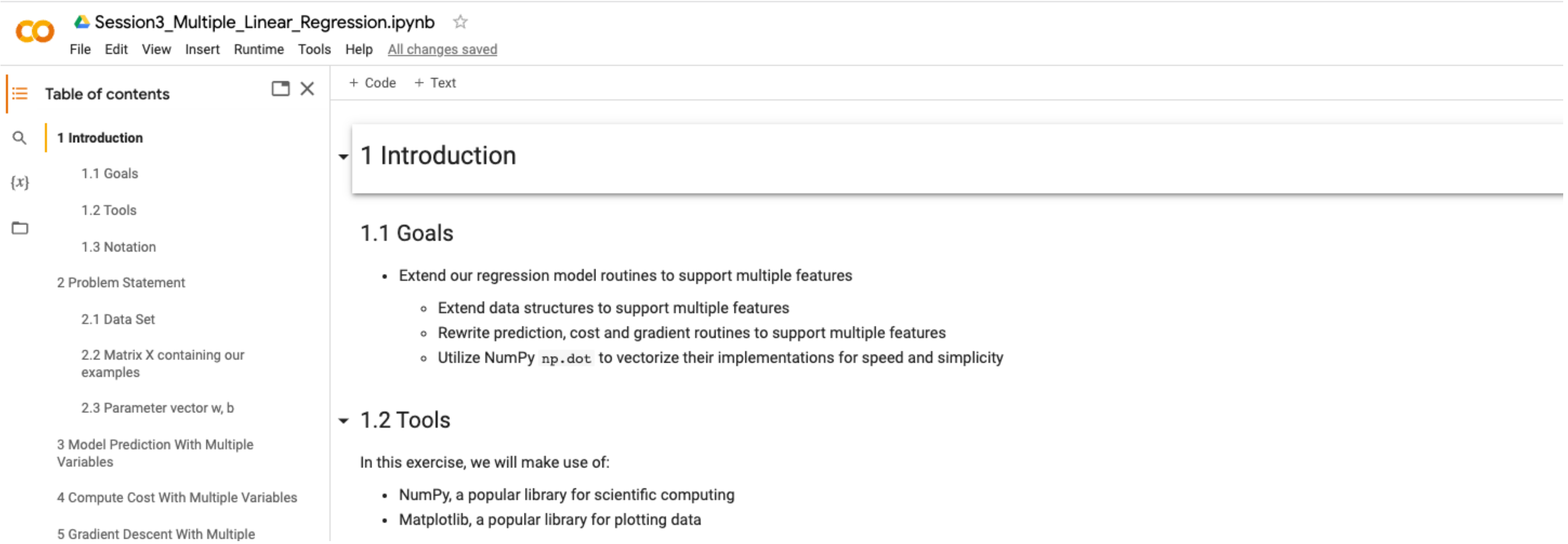
$$w_n = w_n - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}) x_n^{(i)}$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^{m} (f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)})$$

simultaneously update w_j (for $j = 1, \cdots, n$) and b

}

Notebook: Multiple Linear Regression



Feature Values and Parameter Values

$\widehat{price} = w_1x_1 + w_2x_2 + b$

x_1: size (feet^2)
range: 300 – 2,000

x_2: # bedrooms
range: 0 – 5

House: $x_1 = 2000$, $x_2 = 5$, $price = \$500k$

size of the parameters w_1, w_2?

$w_1 = 50, \quad w_2 = 0.1, \quad b = 50$

$w_1 = 0.1, \quad w_2 = 50, \quad b = 50$

Gradient Descent for Multi Linear Regression

n features ($n \geq 2$)

repeat {

$$w_1 = w_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right) x_1^{(i)}$$

$$w_n = w_n - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right) x_n^{(i)}$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)$$

}

Exercise 1: Feature Scaling

6 Feature Scaling

6.1 Runing Gradient Descent without Feature Scaling

```
# initialize parameters
initial_w = np.zeros_like(w_init)
initial_b = 0.
# some gradient descent settings
iterations = 10000
alpha = 0.0000001
# run gradient descent
w_final, b_final, J_hist = gradient_descent(X_train, y_train, initial_w, initial_b,
                                                    compute_cost, compute_gradient,
                                                    alpha, iterations)
print(f"b,w found by gradient descent: {b_final:0.2f},{w_final} ")
m,_ = X_train.shape
for i in range(3):
    print(f"prediction: {np.dot(X_train[i], w_final) + b_final:0.2f}, target value: {y_train[i]}")
```

Exercise 1: Explore the effect of feature scaling in Gradient Descent

Perform the following experiments:

- Observe how the learning rate has different effects before and after normalization.
- Observe how quickly gradient descent converges depending of the learning rate before and after normalization.

Feature Engineering

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + b$$

Feature Engineering

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + b$$

$$area = frontage \times depth$$

$$x_3 = x_1 x_2$$

$$f_{\vec{w},b}(\vec{x}) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

Feature engineering:
Using intuition to design new features, by transforming or combining original features.

Exercise 2: Feature Engineering

7 Feature Engineering

Let us introduce in our training data sample a new feature "size_per_bedrrom":

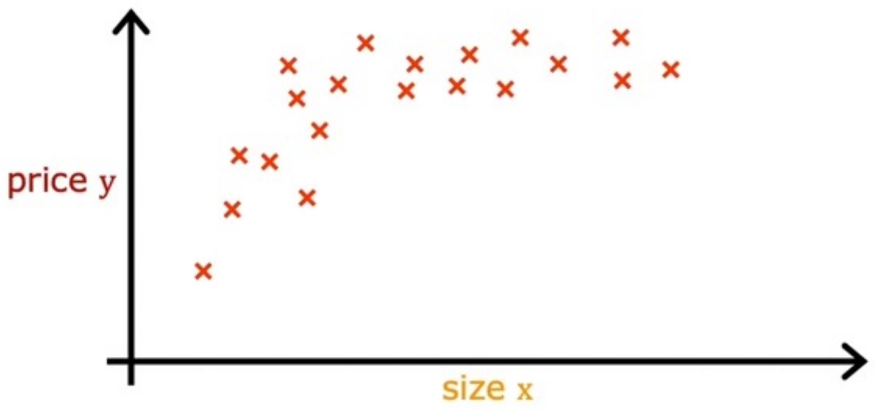
$$\text{size\_per\_bedrrom} = \frac{\text{size}}{\text{number of bedrooms}}$$

```python
# Create a larger matrix
X_train_extended = np.zeros((X_train.shape[0],X_train.shape[1]+1))

# Copy the training data to the new matrix
X_train_extended[:,:-1] = X_train

#Add the new colum for "size_per_bedroom"
X_train_extended[:,-1] = X_train[:,0]/(X_train[:,1])

#Create new names for the variable
X_features_extended = ['size','bedrooms','floors','age', 'sizer_per_bedroom']
```

Exercise 2: Add a new feature and see what happens

- Add a new feature you think it could help to better predict the price of a house, following the same approach used to add "size_per_bedroom" feature. (i) Justify why did you decided to use this feature? (ii) Explore the training cost of the new solution an compare it with the cost of the previous solutions. Is the cost smaller?

Polyomial Regression



Exercise 3: Polynomial Features

8.1 Adding Polynomial Features

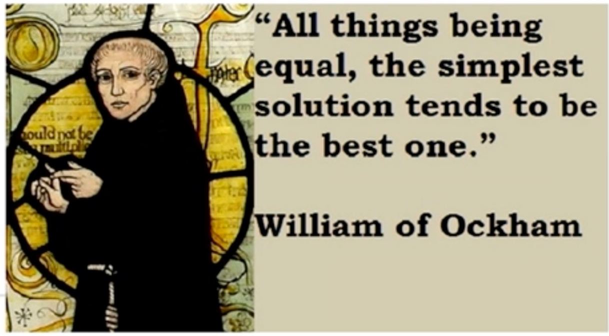
Let us add new polynomial features (i.e. $size^2$, $size^3$, etc) to our training data set and see what happens with the cost funciton after running gradient descent.

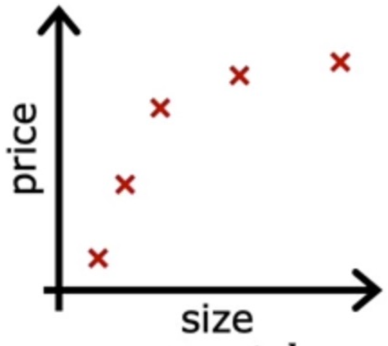
```
# Enter the degree of the new polynomial features.
degree = 2
# Create a larger matrix
X_train, y_train = load_house_data()
X_train, _, _ = zscore_normalize_features(X_train)
X_train_extended = np.zeros((X_train.shape[0],X_train.shape[1]+degree-1))
```

Exercise 3: Polynomial Features

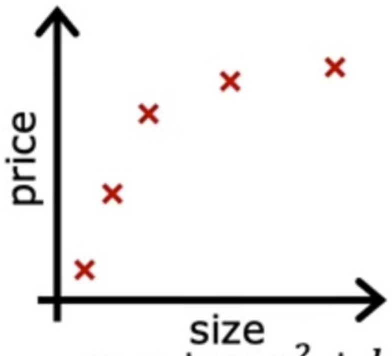
- Add polynomial features using the code above. Start with degree=2 and move to degree=20.
- Look how the Cost function is reduced.
- Do you think we are doing better by having more polynomial functions?

The problem of Overfitting

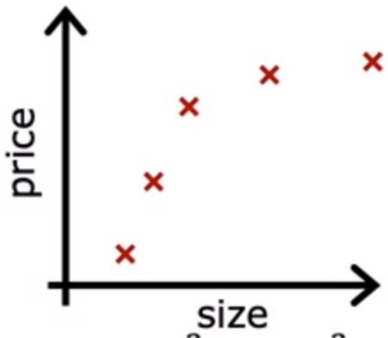




$w_1 x + b$

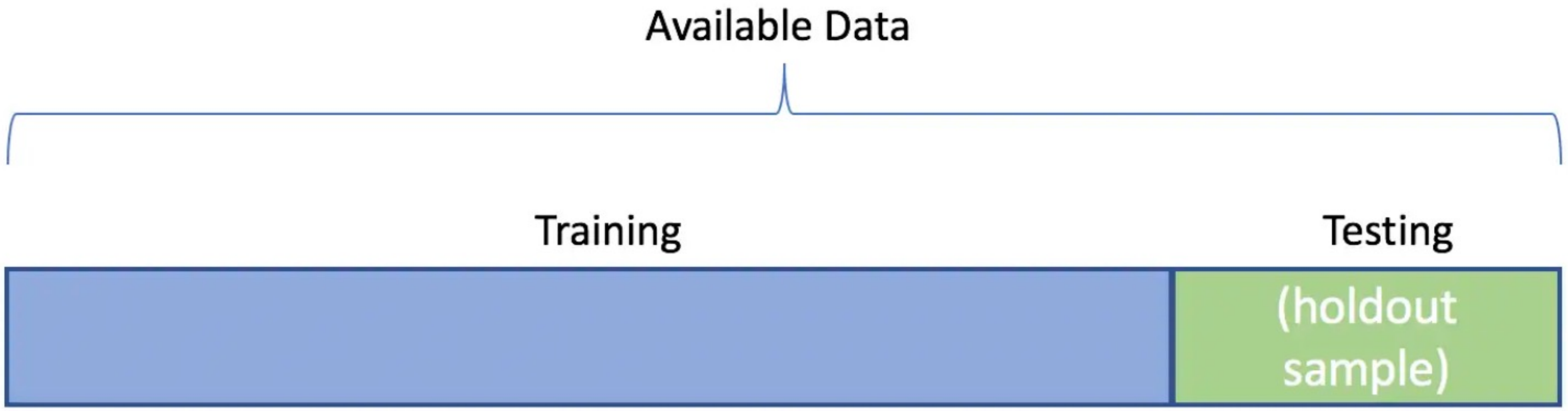


$w_1 x + w_2 x^2 + b$



$w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + b$

Measuring Overfitting



Exercise 4: Measuring Overfitting

8.2 Measuring Overfitting

We now split the data set in training data set and validation data set.

```
#Permutate the data
np.random.seed(123)
perm = np.random.permutation(X_train_extended.shape[0])
X_train_extended = X_train_extended[perm,:]
y_train = y_train[perm]

#Split the data
size_train = 66
X_split_train, y_split_train = X_train_extended[0:size_train,:], y_train[0:size_train]
X_split_val, y_split_val = X_train_extended[size_train:,:], y_train[size_train:]
```

Exercise 4: Measuing Training and Test Error

- Add polynomial features using the code above. Start with degree=2 and move to degree=20.
- Look how the Cost function for the training and the test cost evolves.
- Increase and decrease the number of iterations gradient descent is run (move it from 10000 to 20000) and observed what happen with the the training and the test cost.
- What do you think is happening?
- Which should be the degree of the included polynomial features?

Kaggle Competition

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Overview

Description

Evaluation

Bike sharing systems are a means of renting bicycles where the process of obtaining membership, rental, and bike return is automated via a network of kiosk locations throughout a city. Using these systems, people are able rent a bike from a one location and return it to a different place on an as-needed basis. Currently, there are over 500 bike-sharing programs around the world.

The data generated by these systems makes them attractive for researchers because the duration of travel, departure location, arrival location, and time elapsed is explicitly recorded. Bike sharing systems therefore function as a sensor network, which can be used for studying mobility in a city. In this competition, participants are asked to combine historical usage patterns with weather data in order to forecast bike rental demand in the Capital Bikeshare program in Washington, D.C.