

A STATISTICAL TREATMENT OF THE GAMMA-RAY BURST “NO HOST GALAXY” PROBLEM. I. METHODOLOGY

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ABSTRACT

If gamma-ray bursts originate in galaxies at cosmological distances, the host galaxy should be detected if a burst error box is searched deeply enough; are the host galaxies present? We present and implement a statistical methodology that evaluates whether the observed galaxy detections in a burst’s error box are consistent with the presence of the host galaxy, or whether all the detections can be attributed to unrelated background galaxies. This methodology requires the model-dependent distribution of host-galaxy fluxes. While our methodology was derived for galaxies in burst error boxes, it can be applied to other candidate host objects (e.g., active galaxies) and to other types of error boxes. As examples, we apply this methodology to two published studies of burst error boxes. We find that the nine error boxes observed by Larson and McLean are too large to discriminate between the presence or absence of host galaxies, while the absence of bright galaxies in the four significantly smaller error boxes observed by the *Hubble Space Telescope* (Schaefer et al.) does confirm that there is a “no host galaxy” problem within the “minimal” host-galaxy model.

Subject headings: galaxies: general — gamma rays: bursts — methods: statistical

1. INTRODUCTION

If gamma-ray bursts originate at cosmological distances then they most likely occur in (or near) galaxies; are the host galaxies present in burst error boxes? There have been various claims as to whether the error boxes that have been searched in the optical band contain galaxies bright enough to be the expected hosts of the burst sources. The question is whether the observations—the galaxies observed above the detection threshold—are consistent with the presence of the expected host galaxy when unrelated “background” galaxies are also be present. Here we present and implement a methodology to evaluate this question.

The “no host galaxy” problem was first raised by Schaefer (1992) who presented a compendium of brightness upper limits for galaxies in the error boxes of eight classical bursts, the soft gamma repeater associated with the 1979 March 5 event and four optical transients. For each error box Schaefer (1992) calculated the distance to a galaxy with the luminosity of M31 if the galaxy were as bright as the detection threshold for that error box. The distances were typically a few Gpc (a Gpc corresponds to $z = 0.25$ for $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$), requiring isotropic radiated energies greater than 10^{53} ergs in some cases. Note that Schaefer did not claim that there are no galaxies in the error boxes, just that there are no bright galaxies. Indeed, he used the brightest object in his field as the upper limit for the brightest galaxy.

Subsequently Fenimore et al. (1993) analyzed the *PVO-BATSE* cumulative burst intensity distribution under the assumption that bursts are cosmological, and assigned distances to the eight classical bursts in Schaefer’s sample based on the bursts’ intensities. Using the distance estimate and Schaefer’s brightness limit for each error box, Fenimore et al. (1993) calculated first the upper limit to the host galaxy’s luminosity and then the fraction of the host-galaxy

luminosity function that is fainter than this limit. They derived the host-galaxy luminosity function as the normal galaxy luminosity function weighted by the luminosity, since the number of potential burst sources in a galaxy presumably scales with the number of stars in the galaxy and therefore with the luminosity (for a constant mass-to-light ratio). If only host galaxies are present, the fraction of the host-galaxy luminosity function less than the observed host galaxy’s luminosity should be distributed uniformly between 0 and 1, with an average of $\frac{1}{2}$. Indeed, the analysis of Fenimore et al. (1993) gives an average value of 0.44 ± 0.10 for the upper limits calculated from Schaefer’s compendium. However, the average value of this fraction for the host galaxies’ actual luminosities is undoubtedly smaller (at least some of the host galaxies must be fainter than the upper limits), and Fenimore et al. (1993) concluded that the observations were only marginally consistent with bursts occurring in galaxies.

Vrba, Hartmann, & Jennings (1995) monitored the error boxes of seven classical bursts and one optical transient for 5 yr; many of the burst error boxes were included in the compendium of Schaefer (1992). Vrba et al. (1995) searched for, but did not find, unusual objects that varied or had bizarre colors. Many galaxies were identified based on morphology (for $V < 21.6$) and color (for fainter objects)—the number of galaxies was consistent with galaxy counts—but whether the error boxes contained a galaxy bright enough to be the expected host galaxy was not considered.

Larson & McLean (1997) observed in the infrared nine of the smallest error boxes of classical bursts localized by the third Interplanetary Network (Larson, McLean, & Becklin 1996 presented a preliminary report on six error boxes); these error boxes are typically $\sim 8 \text{ arcmin}^2$. In or near all but one error box they found at least one bright galaxy ($K \leq 15.5$). The fraction of the host-galaxy luminosity func-

tion fainter than the brightest galaxy in each error box, the statistic introduced by Fenimore et al. (1993), has an average of 0.47 ± 0.10 , consistent with the value 0.5 which is expected if only host galaxies are present (Larson 1997). Larson & McLean (1997) recognize that the error boxes are too large to discern between the host galaxy and unrelated background galaxies. However, they report that the surface density of bright galaxies is approximately a factor of 2 larger than the average, which they interpret as possible evidence for clustering around the host galaxy.

Schaefer et al. (1997) searched the error boxes of five classical bursts with the *Hubble Space Telescope* (*HST*); four of the error boxes are small (of order ~ 1 arcmin²) and three are in the compendium of Schaefer (1992). This study also looked for, but did not find, unusual objects with ultraviolet excesses, variability, parallax, or proper motion. Galaxies are present, but faint. Following Fenimore et al. (1993), Schaefer et al. (1997) estimated the distance to the host galaxies using the bursts' peak flux, with the distance scale determined from the burst cumulative intensity distribution. The brightest object in each field that could be a galaxy sets the upper limit on the host galaxy's brightness. An upper limit on the host galaxy's luminosity is derived from this observed brightness upper limit and the estimated distance; for the four small error boxes the luminosity upper limits are 10–100 times smaller than the luminosity of an L_* galaxy.

Thus the issue is not whether there are galaxies in burst error boxes, but whether the observed galaxies are likely host-galaxy candidates. Galaxies may be present, but they may be fainter than the expected host-galaxy brightness. Alternatively, the error box may be so large that the host galaxy is only one of the many expected unrelated background galaxies. The statistical analyses of the observations have thus far treated the brightest object within the error box as the host galaxy's brightness upper limit, even though many galaxies were detected. These analyses have not considered the size of the error box or the effect of the expected background galaxies. In addition, most analyses of error boxes treat the region within a certain confidence contour, typically 99%, as equally likely, whereas in reality the location probability density peaks within the error box.

Therefore, to evaluate whether the host galaxy is present we have developed and implemented a methodology that considers the location probability density (and thus the size of the error box), all the detected galaxies, and the presence of background galaxies. While this methodology was derived to determine whether host galaxies are present in burst error boxes, it can be applied to other candidate host objects, such as active galaxies (Luginbuhl et al. 1996). Of course, the methodology can also be used in other astrophysical contexts where a counterpart in one wavelength band is sought for a source observed in a second band. Although the methodology is derived within a Bayesian framework, the resulting Bayesian "odds ratio" can be understood intuitively, and thus can be treated as a non-Bayesian statistic. The ultimate purpose of the odds ratio is to determine whether host galaxies are present, but it can also be used to determine which error boxes will have the power to answer this question. We use the standard notation where $p(a|b)$ is the conditional probability of proposition a given proposition b . Propositions are simple statements, the validity of which may or may not be in question.

This methodology deals with error boxes that have a finite size and assumes that bursts occur within visible host galaxies. If the optical transients in the *BeppoSAX* error boxes do indicate the locations of the GRB 970228 and GRB 970508 bursts, then these bursts (and subsequent similarly localized bursts) can be treated as having much smaller error boxes than previous bursts. Our methodology can be applied to these bursts by using as the error box the region within which a galaxy would be acceptable as a host. The GRB 970228 optical transient appears to sit on a slightly extended source that may or may not be a galaxy (Djorgovski et al. 1997 report that an $R = 25.5$ source is still present half a year after the burst). No extended source has been associated with the GRB 970508 optical transient. This has revived suggestions that the burst source is expelled from the host galaxy (e.g., Lipunov et al. 1995; Bloom, Tanvir, & Wijers 1997). Source ejection can be treated within our methodology by expanding a burst's error box by the (model-dependent) angular distance the burster would have traveled before bursting.

Any analysis of the host-galaxy issue depends on the expected host-galaxy distribution. The examples presented here, the analyses published elsewhere, and indeed most studies of the possible cosmological properties of burst ensembles use a "minimal" cosmological model. In this model bursts are assumed to be standard candles that do not evolve in comoving density or luminosity. Modeling the intensity distribution gives a unique mapping between a burst's intensity and its distance. Bursts occur in galaxies at a rate proportional to a galaxy's mass and (assuming a constant mass-to-light ratio for all galaxies) therefore luminosity. This is most likely overly simplistic, and in the future we will relax various assumptions of this minimal model. However here, to demonstrate our methodology, we will test the minimal cosmological model.

In § 2.1 we develop the methodology, which is dependent on the model host-galaxy distribution, as discussed in § 2.2. The ability of the observations of a given error box to discriminate between the presence or absence of the host galaxy can be evaluated using this methodology (§ 2.3). Using the minimal cosmological model, we apply this methodology in § 3 to the *K*-band observations of Larson & McLean (1997) and the *HST* observations of Schaefer et al. (1997). Finally in § 4 we discuss important issues raised by our methodology.

2. METHODOLOGY

2.1. Likelihood Ratio

The basic strategy for evaluating whether the host galaxy may be present uses the three-dimensional space consisting of the two sky coordinates Ω and the optical flux f at each position. The detection threshold f_{lim} may vary over the searched region; for example, a bright star reduces the detectability of faint galaxies in the star's immediate vicinity. The searched region may not include the entire burst error box and may extend beyond it. Assume that n_d galaxies are detected, each with a flux f_i located at Ω_i . An observed galaxy is either the host galaxy or an unrelated background galaxy. The searched portion of the flux-position space should be visualized as little bins, most of which are empty; expressions are calculated for finite-sized bins that are subsequently reduced to infinitesimal dimensions. The probability for obtaining the observations is the

product of the probabilities for the observed detection or nondetection in each bin.

Let the distribution of background galaxies be $dN = \phi(f)df d\Omega$ where we assume that these galaxies are distributed uniformly without any clustering. If it exists, the burst's host galaxy is drawn from $\Psi(f)$, which must be normalized to 1 since there can only be one host galaxy per error box. The burst localization results in a probability density $\rho(\Omega)$ for the burst's position on the sky; ρ is also normalized to 1. Therefore, the probability that the host galaxy is at position Ω with flux f is $p(f, \Omega) = \Psi(f)\rho(\Omega)$.

The three-dimensional flux-position space is broken into bins with dimensions $\Delta\Omega$ and Δf . The probability of finding a background galaxy in a bin is governed by Poisson statistics; the expected number of galaxies in a bin centered on f_j is $n_j = \phi(f_j)\Delta f \Delta\Omega$. The detection probabilities are

$$p_j(n=0) = e^{-n_j} = e^{-\phi(f_j)\Delta f \Delta\Omega}$$

and

$$p_j(n=1) = n_j e^{-n_j} = \phi(f_j)\Delta f \Delta\Omega e^{-\phi(f_j)\Delta f \Delta\Omega}. \quad (1)$$

The bin volumes $\Delta f \Delta\Omega$ are assumed to be small enough that no more than one background galaxy per bin need be considered, particularly since eventually $\Delta f \Delta\Omega \rightarrow 0$. The probability that the host galaxy is found in the j th bin is

$$q_j = \Psi(f_j)\rho(\Omega_j)\Delta f \Delta\Omega. \quad (2)$$

Note that the occurrence of the host galaxy is *not* governed by Poisson statistics since (by assumption) there is only one host galaxy.

Now let H_{hg} be the hypothesis that the error box contains the burst's host galaxy in addition to background galaxies, while H_{bg} is the hypothesis that only background galaxies are present. First we calculate the probability $p(D|H_{\text{bg}})$ of obtaining the observed n_d galaxies and not observing galaxies at all other values of f and Ω (the data proposition D) assuming hypothesis H_{bg} . This probability $p(D|H_{\text{bg}})$, the likelihood for H_{bg} , is the product of the probabilities of the observations in each bin. Thus

$$\begin{aligned} p(D|H_{\text{bg}}) &= \prod_i^{n_d} p_i(n=1) \prod_{j \neq i} p_j(n=0) \\ &= \prod_i^{n_d} \phi(f_i)\Delta f \Delta\Omega \prod_{\text{all } j} \exp[-\phi(f_j)\Delta f \Delta\Omega] \\ &= \exp\left[-\sum_{\text{all } j} \phi(f_j)\Delta f \Delta\Omega\right] \prod_i^{n_d} \phi(f_i)\Delta f \Delta\Omega \\ &= \exp\left[-\int d\Omega \int_{f_{\text{lim}}(\Omega)}^{\infty} df \phi(f)\right] \prod_i^{n_d} \phi(f_i)\Delta f \Delta\Omega, \end{aligned} \quad (3)$$

where in the argument of the exponential we have let Δf and $\Delta\Omega$ go to zero so that the sum becomes an integral. Note that

$$\langle n_d \rangle = \int d\Omega \int_{f_{\text{lim}}(\Omega)}^{\infty} df \phi(f) \quad (4)$$

is the number of background galaxies expected to be observed within the searched region. The probability $p(D|H_{\text{hg}})$, the likelihood for H_{hg} , is

$$\begin{aligned} p(D|H_{\text{hg}}) &= p(D|H_{\text{bg}}) \\ &\times \left[\int d\Omega \int_0^{f_{\text{lim}}(\Omega)} df \Psi(f)\rho(\Omega) + \sum_{i=1}^{n_d} \frac{\Psi(f_i)\rho(\Omega_i)\Delta f \Delta\Omega}{\phi(f_i)\Delta f \Delta\Omega} \right], \end{aligned} \quad (5)$$

where the first term in the brackets is the probability that the host galaxy was unobservable, and the terms in the sum are the probabilities that the i th observed galaxy is the host galaxy and not a background galaxy [the denominator cancels the factor in $p(D|H_{\text{bg}})$ that described the host galaxy as a background galaxy]. If there are galaxies in the error box, then we have to consider the possibility that any one of them (or none of them) might be the host galaxy; thus $p(D|H_{\text{hg}})$ is a sum of the probabilities for each possible occurrence of the host galaxy. If part of the error box is not observed, as often occurs in searches for contemporaneous counterparts (e.g., by GROCE—Lee et al. 1997; Park et al. 1997a—or LOTIS—Park et al. 1997b), then f_{lim} is effectively infinite for the unobserved portion; the integral over Ω in the first term should be over the entire error box.

In standard “frequentist” statistics the ratio $p(D|H_{\text{hg}})/p(D|H_{\text{bg}})$ can be used as a measure of how well the presence of a host galaxy explains the data. In Bayesian statistics the odds ratio

$$O(H_{\text{hg}}, H_{\text{bg}}) = \frac{p(H_{\text{hg}}|D)}{p(H_{\text{bg}}|D)} = \frac{p(H_{\text{hg}})}{p(H_{\text{bg}})} \frac{p(D|H_{\text{hg}})}{p(D|H_{\text{bg}})} \quad (6)$$

updates the ratio of the “priors” $p(H_{\text{hg}})/p(H_{\text{bg}})$, the probabilities that H_{hg} and H_{bg} are true based on information available before obtaining the new data D , using the “Bayes factor” $p(D|H_{\text{hg}})/p(D|H_{\text{bg}})$, the ratio of the likelihoods. The values of the priors $p(H_{\text{hg}})$ and $p(H_{\text{bg}})$ depend on our assessment of the validity of the hypotheses, and in the absence of a strong preference for one hypothesis over the other, it is best to set $p(H_{\text{hg}})/p(H_{\text{bg}}) = 1$. A value of the odds ratio much larger than 1 favors the presence of a host galaxy, while a value much less than 1 indicates that no host galaxy is present; the observations cannot discriminate between the two hypotheses for a value of order unity. Clearly the search for host galaxies in multiple error boxes can be treated by the product of the likelihood ratios for each error box. In our case the ratio of the likelihoods for one error box is

$$\begin{aligned} \frac{p(D|H_{\text{hg}})}{p(D|H_{\text{bg}})} &= \int d\Omega \int_0^{f_{\text{lim}}(\Omega)} df \Psi(f)\rho(\Omega) \\ &+ \sum_{i=1}^{n_d} \frac{\Psi(f_i)\rho(\Omega_i)}{\phi(f_i)}. \end{aligned} \quad (7)$$

The host galaxy is more likely to be present but unobserved when a large fraction of the host-galaxy flux distribution is below the detection limit (the first term in this expression). A detected galaxy is more likely to be the host galaxy than a background galaxy if there is a higher probability for the host galaxy to be present at that location and flux than a background galaxy. Note that a large value of $\rho(\Omega_i)$ indicates a small error box. Indeed, if the localization probability density is set to a constant within a region (e.g., within the 99% contour), as is usually done, then ρ would have a value inversely proportional to the area of this region. However, by using $\rho(\Omega)$ we allow the use of more information about the burst localization.

2.2. Distribution of Host Galaxies

The galaxy distributions required to decide whether a host galaxy is present are based on the distribution of galaxies as a function of flux and redshift, $\Phi(f, z)$. This distribution is observed directly, although cosmologists are ultimately interested in the distribution as a function of luminosity and not flux. The background galaxy flux distribution used here is

$$\phi(f) = \int dz \Phi(f, z). \quad (8)$$

Although currently known imperfectly, $\Phi(f, z)$ can be established empirically by redshift surveys; $\phi(f)$ is more easily determined directly from galaxy counts.

We assume the host galaxy is not drawn from the same observed flux distribution as the background galaxies. While the background galaxies' flux distribution is observed (e.g., by galaxy count surveys), the host galaxy's flux distribution must be modeled. Here we develop the "minimal" cosmological model distribution; as discussed below (§ 4), reasonable variants have been proposed that may lead to different conclusions. First, it is likely that a galaxy's burst rate is proportional to the mass of the galaxy (Fenimore et al. 1993). Most cosmological theories attribute bursts to the mergers of two compact objects (e.g., neutron stars) within a binary system (Eichler et al. 1989; Narayan, Paczyński, & Piran 1992); the number of such objects is presumably proportional to the number of stars, and thus the mass. Assuming all galaxies are characterized by the same mass-to-light ratio, the background galaxy distribution should be weighted by the luminosity (or flux) to derive the host-galaxy distribution. Second, the host galaxy may be modeled to fall within a redshift range $\xi(z)$. Combining these two modeling factors gives

$$\Psi(f) = \frac{\int_0^\infty dz \xi(z) f \Phi(f, z)}{\int_0^\infty df \int_0^\infty dz \xi(z) f \Phi(f, z)}. \quad (9)$$

The burst distance scale is unknown but has been modeled using intensity distributions (e.g., Fenimore et al. 1993). An intensity quantity G intrinsic to the burst (e.g., peak photon luminosity or total energy emitted) is considered to be a fundamental burst property (perhaps a constant); however, the normalization of G is unknown. Cosmology modifies the Euclidean d^{-2} relationship between G and the related observed intensity quantity g (e.g., peak photon flux or energy fluence). Thus, the normalization of G and the distance scale are derived from the distribution of g (as are other modeling parameters). The resulting relationship can be inverted to calculate a range of likely distances for a given observed value of g . While most models assume that G is a standard candle and that the source density is constant per comoving volume, in the more general case the density may evolve, G may be characterized by a luminosity function, and a K -correction may be necessary to relate g and G (the energy band of the observation is redshifted). Ultimately this modeling should produce the distribution of sources dN/dz as a function of redshift and g . Finally,

$$\xi(z) = \frac{dN/dz}{\int dz (dN/dz)}. \quad (10)$$

In most cases, a simple model is used. Thus $\xi(z)$ is

assumed to be a delta function, providing a direct mapping between the peak photon flux or the energy fluence and the redshift. Given the uncertainties of modeling the host-galaxy distribution, the shape of the galaxy luminosity function at a given redshift $\Phi(f, z)$ can be approximated by a Schechter function (Peebles 1993, p. 120),

$$\psi(y) = \psi_0 y^\alpha e^{-y}, \quad (11)$$

where $y = L/L_* = f/f_*$. The intensity scale, L_* or f_* , is typically measured as the absolute magnitude in a given spectral band. Note that a K -correction should be applied for redshifts more than a few tenths (the flux observed in a given energy band was emitted by the galaxy in a different band); in addition, the luminosity function underwent evolution both in scale (i.e., the value of L_*) and normalization. Recent surveys find: for the b_j band $M_{b_j} = -19.72 \pm 0.09$ and $\alpha = -1.14 \pm 0.08$ (Ratcliffe et al. 1997); for the K band $M_K = -23.12$ and $\alpha = -0.91$ (Gardner et al. 1997); and for the R band $M_R = -20.29 \pm 0.02$ and $\alpha = -0.70 \pm 0.05$ (Lin et al. 1996).

To all these expressions for M_* must be added an additional term $5 \log h$, where $h = \dot{H}_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, resulting from the uncertainty in Hubble's constant H_0 ; however, this dependence on the value of H_0 is canceled by the H_0 dependence in the relationship between the flux and z (M_* is derived from observations of magnitude vs. redshift). When we do need H_0 , we use $h = 0.75$. Since α is of order -1 , we approximate $\Psi(f)$ as a simple exponential:

$$\Psi(f) = \exp(-f/f_*)/f_*. \quad (12)$$

Using the approximation for small values of z ,

$$m_* = M_* + 5 \log(3 \times 10^8 z)$$

and

$$f_*(z) = f_0 10^{-0.4 M_*} (3 \times 10^8 z)^{-2}, \quad (13)$$

where f_0 is the normalizing flux (i.e., the flux of a zero-magnitude object) for a given band and m_* is the apparent magnitude corresponding to f_* .

2.3. Sensitivity

We can evaluate how well our methodology discriminates between the presence or absence of a host galaxy in a given error box. Since $\phi(f)$ and $\Psi(f)\rho(\Omega)$ are the probabilities of the presence of a background galaxy and the host galaxy, respectively, in a given patch of sky Ω at a flux f , the expected value of the Bayes factor is

$$\begin{aligned} \left\langle \frac{p(D|H_{\text{hg}})}{p(D|H_{\text{bg}})} \right\rangle &= \int d\Omega \int_0^{f_{\text{lim}}(\Omega)} df \Psi(f) \rho(\Omega) \\ &\quad + \int d\Omega \int_{f_{\text{lim}}(\Omega)}^\infty df \\ &\quad \times [\phi(f) + \Psi(f)\rho(\Omega)] \frac{\Psi(f)\rho(\Omega)}{\phi(f)} \\ &= 1 + \int d\Omega \int_{f_{\text{lim}}(\Omega)}^\infty df \frac{\Psi(f)^2 \rho(\Omega)^2}{\phi(f)} \quad (14) \end{aligned}$$

if a host galaxy is present. If there is no host galaxy then both the second term in the brackets [i.e., $\Psi(f)\rho(\Omega)$] in the first equation of equation (14) and the integral in the second equation should not be included. Thus on average $\langle p(D|H_{\text{hg}})/p(D|H_{\text{bg}}) \rangle = 1$ without a host galaxy, which

TABLE 1
SENSITIVITY SIMULATIONS

α^a	β^b	N_{trial}^c	$\langle n_h \rangle^d$		$\langle n_b \rangle^e$		$\langle O_b \rangle^f$		$\langle O_h \rangle^g$	
			Simulation ^h	Calculation ⁱ	Simulation ^h	Calculation ⁱ	Simulation ^h	Calculation ⁱ	Simulation ^h	Calculation ⁱ
1	1	10^4	0.3677	0.3679	1.0065	1.0000	1.0014	1	1.1537	0.9325
0.1	1	10^4	0.9030	0.9048	31.610	31.623	0.9908	1	1.1872	1.1955
1.0	0.1	10^4	0.3611	0.3679	0.0298	0.0316	0.9977	1	5.7152	5.6079
0.1	0.1	10^4	0.8985	0.9048	1.0016	1.0000	1.0269	1	7.1515	7.1901
0.01	0.1	10^4	0.9888	0.9900	31.558	31.623	0.9982	1	7.1540	7.1917
1	0.01	10^4	0.3618	0.3679	0.0010	0.0010	0.9923	1	150.43	153.48
0.1	0.01	10^4	0.9053	0.9048	0.0334	0.0316	0.8232	1	196.68	196.78
0.01	0.01	10^4	0.9897	0.9900	1.0109	1.0000	0.8119	1	195.91	196.83
0.001	0.01	10^4	0.9987	0.9990	31.627	31.623	0.9087	1	196.88	196.83
1	0.001	10^4	0.3718	0.3679	0.0000	0.0000	0.6321	1	4901.4	4829.7
0.1	0.001	3×10^4	0.9079	0.9048	0.0011	0.0010	0.7906	1	6186.0	6192.1
0.01	0.001	3×10^4	0.9905	0.9900	0.0034	0.0316	1.1030	1	6191.3	6193.7
0.001	0.001	10^4	0.9997	0.9990	1.0064	1.0000	1.0062	1	6203.2	6193.7
10^{-4}	0.001	10^4	0.9998	0.9999	31.654	31.623	1.0161	1	6135.5	6193.7

^a The ratio f_{lim}/f_* .

^b The ratio f_b/f_* .

^c The number of trials in the simulation.

^d The fraction of the trials in which the host galaxy is detected.

^e The average number of background galaxies detected per error box.

^f The average value of the odds ratio when only background galaxies are present and the ratio of priors is set to unity.

^g The average value of the odds ratio when the host galaxy is present in addition to background galaxies. The ratio of priors is set to unity.

^h Results of simulation.

ⁱ Calculated using eq. (16).

may seem surprising, but results from background galaxies being mistaken occasionally for the host galaxy. Under the hypothesis H_{bg} that there are no host galaxies, $p(D|H_{\text{bg}})/p(D|H_{\text{bg}})$ will be less than 1 in most error boxes. However, in those cases where a background galaxy with the flux expected for the host galaxy falls in the error box, $p(D|H_{\text{bg}})/p(D|H_{\text{bg}})$ will be greater than 1. The more unlikely such an occurrence, the smaller the value of ϕ and therefore the larger the value of the second term of $p(D|H_{\text{bg}})/p(D|H_{\text{bg}})$ when a background galaxy is mistaken for the host galaxy.

The methodology's power to determine that a host galaxy is present in an error box for a given observation depends on the value of $\int d\Omega \int_{f_{\text{lim}}(\Omega)}^{\infty} df [\Psi(f)^2 \rho(\Omega)^2 / \phi(f)]$, the term added by the presence of a host galaxy. To evaluate this expression we assume that a constant probability error box (i.e., $\rho = 1/\Omega_0$ over an area Ω_0) is searched to a uniform detection threshold f_{lim} . We approximate the cumulative galaxy count distribution over the error box as $N(>f) = (f/f_b)^{-3/2}$ (the distribution expected for a constant galaxy density in three-dimensional Euclidean space), where f_b is the flux at which one background galaxy is expected in the error box. Since $N(>f) \propto \Omega_0$, f_b will vary from error box to error box as $f_b \propto \Omega_0^{2/3}$. From $N(>f)$ we derive $\phi(f) = (3/2)(f_b \Omega_0)(f/f_b)^{-5/2}$. As discussed in § 2.2, we use a weighted Schechter function for the host-galaxy distribution function, $\Psi(f) = \exp(-f/f_*)/f_*$. Consequently if a host galaxy is present

$$\begin{aligned} \left\langle \frac{p(D|H_{\text{hg}})}{p(D|H_{\text{bg}})} \right\rangle &= 1 + \frac{1}{2^{5/23}} \left(\frac{f_*}{f_b} \right)^{3/2} \int_{2f_{\text{lim}}/f_*}^{\infty} du u^{5/2} e^{-u} \\ &= 1 + \frac{1}{2^{5/23}} \left(\frac{f_*}{f_b} \right)^{3/2} \Gamma\left(\frac{7}{2}, 2f_{\text{lim}}/f_*\right) \end{aligned} \quad (15)$$

where $\Gamma(a, x)$ is the incomplete gamma function. Since most of the area under the curve $u^{5/2} e^{-u}$ is above $u = 1$, the value of $\Gamma(7/2, 2f_{\text{lim}}/f_*)$, and therefore of $\langle p(D|H_{\text{hg}})/p(D|H_{\text{bg}}) \rangle$, is

relatively insensitive to f_{lim} as long as $f_{\text{lim}}/f_* < \frac{1}{2}$, that is, when the detection threshold is less than the expected host-galaxy flux. On the other hand, $\langle p(D|H_{\text{hg}})/p(D|H_{\text{bg}}) \rangle$ is very sensitive to the value of f_b , the flux at which we expect one background galaxy in the error box. Since $\Gamma(7/2) = 3.32335$, $\langle p(D|H_{\text{hg}})/p(D|H_{\text{bg}}) \rangle \simeq 1 + 0.1958(f_*/f_b)^{3/2}$. Our methodology (and most likely any methodology) can discriminate between the presence and absence of a host galaxy in given error box when, on average, the host galaxy is expected to be much brighter than the brightest background galaxy.

These results were derived analytically by integrating over the number of galaxies detected in the error box. This integration treats the number of galaxies as a continuous quantity. We have verified the results with simulations with discrete bursts made under the same assumptions that led to equation (15). Define $\alpha = f_{\text{lim}}/f_*$ and $\beta = f_b/f_*$. Then our analytic formalism gives

$$\langle n_h \rangle = \exp(-\alpha), \quad \langle n_b \rangle = (\beta/\alpha)^{3/2}, \quad \langle O_b \rangle = 1,$$

and

$$\langle O_h \rangle = 1 + 0.1958\beta^{-3/2} \frac{\Gamma(7/2, 2\alpha)}{3.32335}, \quad (16)$$

where n_h and n_b are the numbers of host galaxies and background galaxies, respectively, detected in an error box. $\langle O_h \rangle$ ($\langle O_b \rangle$) is the odds ratio if host and background galaxies (only background galaxies) are present, assuming the prior ratio is 1. In each trial of a given simulation we generated one host galaxy with normalized flux $\epsilon_h = f_h/f_*$ and 100 background galaxies with normalized fluxes $\epsilon_i = f_i/f_*$, where all the ϵ values were drawn from the appropriate distribution functions. Then

$$\langle O_b \rangle = 1 - e^{-\alpha} + \sum_{i=1}^{100} \frac{e^{-\epsilon_i} \epsilon_i^{5/2}}{3/2\beta^{3/2}} \theta(\epsilon_i - \alpha)$$

and

$$\langle O_h \rangle = \langle O_b \rangle + \frac{e^{-\epsilon_h} \epsilon_h^{5/2}}{3/2 \beta^{3/2}} \theta(\epsilon_h - \alpha), \quad (17)$$

where $\theta(x)$ is the Heaviside function that is used here to enforce the requirement that a galaxy be detectable to be included. The results of our simulations are provided by Table 1. As can be seen, the agreement with the analytic formulae in equation (16) is very good. The largest deviations occur when $\langle n_b \rangle$ is very small, and even 10^4 trials do not provide sufficient statistics.

3. APPLICATION

In this section we apply our methodology to observations of gamma-ray burst error boxes reported in the literature. Not all details are provided in these publications, and we make a number of simplifying assumptions; for example, a constant probability density over the error box is used. We write the ratio of likelihoods (eq. [7]) as $p(D|H_{\text{bg}})/p(D|H_{\text{bg}}) = q_1 + \sum_{i=1}^{n_d} q_{2i}$, where

$$q_1 = \int d\Omega \int_0^{f_{\text{lim}}(\Omega)} df \Psi(f) \rho(\Omega) = 1 - \frac{\Omega_s}{\Omega_0} e^{-f_{\text{lim}}/f_*} \quad (18)$$

and

$$q_{2i} = \frac{\Psi(f_i) \rho(\Omega_i)}{\phi(f_i)} = \frac{e^{-f_i/f_*}}{f_* \Omega_0 \phi(f_i)}. \quad (19)$$

In these expressions Ω_0 is the size of the error box ($\rho \propto 1/\Omega_0$) and $\Omega_s \leq \Omega_0$ is the size of the searched region. Since we are testing the “minimal” cosmological model, for the host-galaxy distribution function $\Psi(f)$ we use the weighted Schechter function in equation (12). We derived the distribution of background galaxies $\phi(f)$ from the review paper by Koo & Kron (1992). In their Figure 1 they compile galaxy counts in the K , R , and b_j bands from a number of different sources. We parameterized the differential galaxy count distribution by choosing values that fell within the cluster of observed data points.

3.1. K -Band Observations of Larson & McLean

Larson & McLean (1997) observed the error boxes of nine recent bursts localized by the third Interplanetary Network, as well as a number of control fields, primarily in

the K band. Many of the error boxes were only partially observed because of corrections to the error boxes after the observations. They found many relatively bright galaxies both within and immediately outside of the error boxes. The error boxes are typically $\sim 8 \text{ arcmin}^2$, and background galaxies are expected; Larson & McLean (1997) recognize that they cannot distinguish between host and background galaxies in their data. Based on their observations of control fields and galaxy counts from the literature, they report that the overall galaxy density in and near the error boxes is about a factor of 2 larger than average, which might result from clustering around the host galaxy.

Although Larson & McLean (1997) recognize that their observations are consistent with, but do not prove, the existence of host galaxies, their earlier work (Larson et al. 1996) was interpreted as showing there was not a host-galaxy issue. Therefore we analyzed their observations to determine what statement could be made about the existence of host galaxies. In Table 2 we list for each of the nine error boxes studied by Larson & McLean (1997) the size of the box, the fraction of the box covered by the observations, and the K magnitude of the brightest galaxy in the box. We assume the probability density ρ is constant within the error boxes. Larson (1997) provides an estimated redshift for these bursts. Ideally, we would use a list of magnitudes for all detected galaxies above the limiting magnitude (here $K \sim 18.5$), but here we only have the brightest galaxy. Therefore, we use the magnitude of the brightest galaxy as both the limiting magnitude and the magnitude of the single detected galaxy for each box.

The flux at which we expect to find background galaxies is $f_b = 5.8 \times 10^{-28} \Omega_0^{2/3} \sim 2.32 \times 10^{-27} \text{ ergs s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$ for $\Omega_0 \sim 8 \text{ arcmin}^2$. Therefore f_b , f_* , and the flux of the detection threshold (which is also the flux of the brightest galaxy) are all comparable. In Table 2 we compute the various terms of the likelihood ratio. The first term, q_1 , which is also the probability that the host galaxy is fainter than the detection threshold, increases when the box is not observed completely. The product of the likelihood ratios q_{tot} for each error box is the likelihood ratio for the ensemble; here the product is 0.248, which indicates that we cannot determine whether the expected host galaxies are present or absent.

TABLE 2
ANALYSIS OF THE LARSON & MCLEAN OBSERVATIONS

Burst	z^a	Ω_0^b	Cov. ^c	K_i	$f(K_i)^d$	$f_*(z)^e$	$\phi(K_i)^f$	q_1^g	q_{2i}^h	q_{tot}^i
GRB 910122.....	0.19	19.33	0.41	15.3	4.70×10^{-27}	3.38×10^{-27}	$1.87 \times 10^{+25}$	0.898	0.203	1.102
GRB 910219.....	0.20	7.30	1.0	15.7	3.25×10^{-27}	3.05×10^{-27}	$4.49 \times 10^{+25}$	0.656	0.344	1.000
GRB 920325.....	0.22	26.5	0.87	14.0	1.56×10^{-26}	2.52×10^{-27}	$1.22 \times 10^{+24}$	0.998	0.025	1.024
GRB 920406.....	0.12	1.65	1.0	17.1	8.96×10^{-28}	8.47×10^{-27}	$8.26 \times 10^{+26}$	0.100	0.078	0.178
GRB 920501.....	0.17	2.60	0.91	14.8	7.45×10^{-27}	4.22×10^{-27}	$6.41 \times 10^{+24}$	0.844	2.429	3.274
GRB 920525.....	0.15	3.54	1.0	17.0	9.83×10^{-28}	5.42×10^{-27}	$6.87 \times 10^{+26}$	0.166	0.063	0.229
GRB 920711.....	0.23	2.50	0.93	15.5	3.91×10^{-27}	2.31×10^{-27}	$2.90 \times 10^{+25}$	0.830	1.097	1.927
GRB 920720.....	0.20	3.78	0.57	15.5	3.91×10^{-27}	3.05×10^{-27}	$2.90 \times 10^{+25}$	0.842	0.830	1.672
GRB 920723.....	0.11	4.46	0.88	16.0	2.47×10^{-27}	1.01×10^{-26}	$8.65 \times 10^{+25}$	0.311	0.201	0.512

^a Burst redshift from Larson 1997.

^b Size of the error box in arcmin^2 .

^c Fraction of the error box covered by the observations.

^d The flux corresponding to a given K magnitude, using a $K = 0$ flux of $0.62 \times 10^{-20} \text{ ergs s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$.

^e The K -band flux ($\text{ergs s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$) corresponding to an L_* galaxy at a given redshift based on Gardner et al. 1997.

^f The differential galaxy distribution (galaxies $\text{arcmin}^{-2} \text{ flux}^{-1}$) based on the cumulative distribution in Fig. 1 of Koo & Kron 1992.

^g The probability that the host galaxy is fainter than the detection limit.

^h The value of q_{2i} for the detected galaxy.

ⁱ Total likelihood ratio for the error box, the sum of q_1 and all the q_{2i} .

3.2. *HST* Observations of Schaefer et al.

Schaefer et al. (1997) observed four small error boxes with the WFPC2 on *HST* in both the *B* and UV bands (they also studied a much larger error box with the Faint Object Camera [FOC]; this error box is not considered here). Here we apply our methodology to their *B*-band observations; in our analysis we use b_j distribution functions. In each case a detection threshold is given. Sources are found in three of the four error boxes, but we consider only the small number of sources that are identified as galaxies.

Two of the error boxes were only partially observed. Schaefer et al. (1997) stated that ground-based observations of the GRB 790613 error box show there are no sources *HST* would have detected in the 10% of the error box that was not observed. Similarly, B. E. Schaefer (1997, private communication) reports there are no galaxies to the *HST* detection limit in the $\sim 15\%$ of the error box unobserved by *HST*. Thus $\Omega_s = \Omega_0$ (see eq. [18]) for these two boxes.

Note that Larson (1997) and Schaefer et al. (1997) give z values for GRB 920406 that differ by a factor of 2. Larson uses redshifts provided by E. Fenimore (1995, private communication), while Schaefer et al. (1997) give distances calculated from peak energy fluxes assuming a peak luminosity of 6×10^{50} ergs s $^{-1}$. This peak luminosity was calculated for $H_0 = 75$ km s $^{-1}$ Mpc $^{-1}$, which we used to convert the distances to redshifts.

Table 3 presents the results of applying our methodology to the *HST* observations. As Schaefer et al. (1997) concluded, the detection thresholds are sufficiently fainter than the expected f_* such that the host galaxy should have been observed. In addition, the observed galaxies are as faint as the expected background galaxies for such small error boxes; for a threshold of $B = 23$ we expect ~ 3 galaxies per arcmin 2 . The product of the likelihood ratios is 2×10^{-6} , which indicates that these observations strongly favor the hypothesis that only background galaxies and no host galaxies are found in the error boxes observed by *HST*. Note that $f_b = 7.7 \times 10^{-29} \Omega_0^{2/3}$ ergs s $^{-1}$ cm $^{-2}$ Hz $^{-1}$ and thus $f_* \gg f_b$; by § 2.3 our methodology should be able to determine that host galaxies are present in these error boxes.

4. DISCUSSION

Almost all the quantities required for analyzing an error box are observables, although they may be difficult to

derive and may be characterized by large uncertainties. However, the host-galaxy distribution function is model dependent, and any conclusions based on applying our methodology are a statement about the validity of the host-galaxy model. Here we assume that bursts occur within galaxies, and that the number of sources is proportional to the luminosity of the galaxy. This model is consistent with the scenario where bursts result from the merger of neutron star binaries. In analyzing the observations from the literature we used the “minimal” cosmological model, although in § 2.2 we outline how a distribution for the source redshift can be derived.

As discussed above, this methodology can also be applied to scenarios where the burst source is ejected from the host galaxy by expanding the error box by the angular distance the source may have traveled before bursting. For arcmin 2 scale error boxes, broadening the error box may be inconsequential, but for the well-localized bursts resulting from the newly discovered optical transients, the size of the error box may be determined primarily by the distance the source traveled before bursting.

While the minimal cosmological host-galaxy model we use is consistent with current modeling assumptions in studies of the burst database, reasonable variants can result in significantly different conclusions, and the analysis of an error box may have to be revisited as the source models evolve. For example, the observed burst distribution may be characterized by a broad luminosity function (Horack, Emslie, & Meegan 1994, Horack et al. 1996, and Hakkila et al. 1995, 1996 found that the luminosity function must be narrow, but Lored & Wasserman 1997a, 1997b and Brainerd 1997 dispute this conclusion), allowing a given burst to originate over a broad redshift band. The source density may be greater in galaxies that have undergone starbursts, which may favor small galaxies at moderate redshifts (Sahu et al. 1997).

Our methodology compares two hypotheses: (1) a host galaxy is present in addition to background galaxies; or (2) only background galaxies are present. This analysis is applied to each error box independently; the product of the likelihood ratios compares these hypotheses for the data set as a whole. This methodology does not test directly whether the candidate host galaxies are indeed distributed according to the model distribution function. Fenimore et al. (1993) introduced a test that considers the distribution of the sta-

TABLE 3
ANALYSIS OF THE SCHAEFER ET AL. OBSERVATIONS

Burst	Ω_0^a	z^b	$f_*(z)^c$	B_{lim}^d	$f(B_{\text{lim}})^e$	B_i	$f(B_i)^e$	$\phi(B_i)^f$	q_1^g	q_{2i}^h	q_{tot}^i
GRB 790325.....	2.0	0.145	1.81×10^{-27}	23.0	2.80×10^{-29}	22.53	4.32×10^{-29}	4.53×10^{28}	0.015	0.006	0.021
GRB 790406.....	0.26	0.115	2.88×10^{-27}	22.8	3.37×10^{-29}	0.012	...	0.012
GRB 790613.....	0.76	0.100	3.81×10^{-27}	23.2	2.33×10^{-29}	21.29	1.35×10^{-28}	4.61×10^{27}	0.006	0.072	...
						21.61	1.01×10^{-28}	8.32×10^{27}	...	0.040	...
						21.57	1.05×10^{-28}	7.73×10^{27}	...	0.044	0.162
GRB 920406.....	2.0	0.263	5.53×10^{-28}	23.0	2.80×10^{-29}	0.049	...	0.049

^a Size of the error box in arcmin 2 .

^b Burst redshift converted (assuming $h = 0.75$) from distance given by Schaefer et al. 1997, which in turn is based on the peak flux.

^c The *B*-band flux (ergs s $^{-1}$ cm $^{-2}$ Hz $^{-1}$) corresponding to an L_* galaxy at a given redshift based on Ratcliffe et al. 1997.

^d The limiting *B* magnitude.

^e The flux corresponding to a given *B* magnitude, using a $B = 0$ flux of 4.44×10^{-20} ergs s $^{-1}$ cm $^{-2}$ Hz $^{-1}$.

^f The differential galaxy distribution (galaxies arcmin $^{-2}$ flux $^{-1}$) based on the cumulative distribution in Fig. 1 of Koo & Kron 1992.

^g The probability that the host galaxy is fainter than the detection limit. The value is reported on the first line for a given error box.

^h The value of q_{2i} for the detected galaxy.

ⁱ Total likelihood ratio for the error box, the sum of q_1 and all the q_{2i} . The resulting value is reported on the last line for a given error box.

tistic $S = \int_0^{f_i} df \Psi(f)$, where f_i is the flux of the brightest galaxy in, or the detection threshold for, the i th error box. If the f_i indeed correspond to the host galaxies, then the statistic S should be distributed uniformly between 0 and 1, and should have an average value of $\frac{1}{2}$. However, this test does not evaluate whether f_i corresponds to the host galaxy.

5. SUMMARY

We have developed a methodology to evaluate whether the detections and nondetections of galaxies in gamma-ray burst error boxes are consistent with the presence of the burst source's host galaxy, or whether all the detections can be attributed to unrelated background galaxies. This methodology relies on the distribution of background galaxies, which is observed, and the flux distribution for the host galaxy, which must be modeled. Any conclusions are dependent on the host-galaxy model. In addition to evaluating the observations of a particular error box, our methodology also predicts its likely sensitivity for a given error box. Of course, the methodology can be used for other candidate host sources, or for similar astrophysical problems.

The methodology allows the maximal use of observational data. Thus a variable probability density for the burst's location can be used instead of assuming that this probability is constant across the error box. The detection threshold is allowed to vary across the error box; this permits the treatment of partially observed error boxes.

As examples we applied this methodology to the K -band observations of Larson & McLean (1997) and the *HST* observations of Schaefer et al. (1997). Larson & McLean (1997) found a large number of galaxies in or near nine error boxes, but our analysis shows that these detections can easily be explained as background galaxies. We find that the error boxes Larson et al. observed were too large to discriminate between the presence or absence of a host galaxy. Schaefer et al. (1997) found only faint galaxies; our analysis shows that brighter host galaxies should have been observed, but were not. Our sensitivity analysis verifies that these error boxes are small enough to determine that a host galaxy is indeed present. Therefore, within the assumptions of the "minimal" cosmological model used in our analysis (e.g., standard candle bursts; no evolution; the number of potential sources is proportional to the size of the galaxy), the expected host galaxies are absent.

In the future we will apply this methodology to other observations of small error boxes, and explore other host models.

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