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POWER SPECTRUM OF VELOCITY FLUCTUATIONS IN THE UNIVERSE

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ABSTRACT

We investigate the power spectrum of velocity fluctuations in the universe starting from four different measures of velocity: the power spectrum of velocity fluctuations from peculiar velocities of galaxies, the rms peculiar velocity of galaxy clusters, the power spectrum of velocity fluctuations from the power spectrum of density fluctuations in the galaxy distribution, and the bulk velocity from peculiar velocities of galaxies. There are various way of interpreting the observational data.

(1) The power spectrum of velocity fluctuations follows a power law $V^2(k) \sim k^2$ on large scales, achieves a maximum $V(k) \sim 500$ km s^{-1} at a wavelength of $\lambda \sim 120\ h^{-1}$ Mpc, and declines as $V^2(k) \propto k^{-0.8}$ on small scales. This type of power spectrum is predicted by a mixed dark matter model with density parameter $\Omega_0 = 1$. This model is consistent with all data observed, except the rms peculiar velocity of galaxy clusters.

(2) The shape of the power spectrum of velocity fluctuations is similar to that in (1), but the amplitude is lower (~ 300 km s^{-1} at $\lambda \sim 120\ h^{-1}$ Mpc). This power spectrum is predicted by a low-density cold dark matter model with a density parameter $\Omega_0 \simeq 0.3$.

(3) There is a peak in the power spectrum of velocity fluctuations at a wavelength of $\lambda \simeq 120\ h^{-1}$ Mpc; on larger scales the power spectrum decreases with an index $n \simeq 1.0$. The maximum value of the function $V(k)$ is ~ 420 km s^{-1}. This power spectrum is consistent with the power spectrum of the galaxy distribution in the Stromlo-APM redshift survey provided that the parameter β is in the range 0.5–0.6.

(4) There is a peak in the power spectrum as in (3), but on larger scales the amplitude of fluctuations is higher than that estimated starting from the observed power spectrum of galaxies. For the parameter β in the range 0.4–0.5, the observed rms cluster peculiar velocity is consistent with the rms amplitude of the bulk flow of ~ 340 km s^{-1} at a scale of 60 h^{-1} Mpc. In this case, the value of the function $V(k)$ at wavelength $\lambda = 120\ h^{-1}$ Mpc is ~ 350 km s^{-1}.

In the future, larger redshift surveys and more accurate observations of peculiar velocities of galaxies and clusters will help to constrain the power spectrum of velocity fluctuations in the universe.

Subject headings: cosmology: theory — dark matter — galaxies: distances and redshifts — large-scale structure of universe

1. INTRODUCTION

The velocity of matter in the universe, $\boldsymbol{u}(\boldsymbol{r})$, can be expressed as a sum of the mean Hubble expansion velocity $\boldsymbol{v}_{\rm H} = H_0 \boldsymbol{r}$ and a field of velocity fluctuations

$$\boldsymbol{v}(\boldsymbol{r}) \equiv \boldsymbol{u}(\boldsymbol{r}) - H_0 \boldsymbol{r} , \tag{1}$$

where H_0 is the Hubble constant. The peculiar velocity field $\boldsymbol{v}(\boldsymbol{r})$ in the volume V_u can be expressed in terms of its Fourier components,

$$\boldsymbol{v}(\boldsymbol{r}) = \frac{V_u^{1/2}}{(2\pi)^{3/2}} \int \boldsymbol{v}_k \exp(i\boldsymbol{k}\boldsymbol{r}) d^3k , \tag{2}$$

and quantified in terms of the power spectrum $P_V(k) \equiv \langle |v_{\boldsymbol{k}x}|^2 + |v_{\boldsymbol{k}y}|^2 + |v_{\boldsymbol{k}z}|^2 \rangle$. If the field $\boldsymbol{v}(\boldsymbol{r})$ is an isotropic Gaussian field, then the different Fourier components are uncorrelated, and the power spectrum provides a complete statistical description of the field.

Previous studies in the literature have investigated the density fluctuations and the field of velocity fluctuations in real space (for a review see Dekel 1994; Strauss & Willick 1995). This paper, however, concentrates on the power spectrum of velocity fluctuations. We will describe the velocity spectrum by

$$V^2(k) \equiv \frac{1}{2\pi^2} k^3 P_V(k) . \tag{3}$$

The function $V^2(k)$ gives the contribution to the velocity dispersion per unit interval in ln k,

$$\langle v^2 \rangle \equiv \frac{1}{V_u} \int v^2(\boldsymbol{r}) d^3r = \frac{1}{2\pi^2} \int P_V(k) k^2 dk = \int V^2(k) \frac{dk}{k} . \tag{4}$$

The rms velocity fluctuation on a given scale r can be expressed as

$$\langle v^2(r) \rangle = \int V^2(k) W^2(kr) \frac{dk}{k} , \tag{5}$$

where $W(kr)$ is the Fourier transform of the window function applied to determine the peculiar velocity field. For a Gaussian window function, the rms velocity of matter is given by

$$v_{\rm rms}^2(r) = \int V^2(k) \exp(-r^2k^2) \frac{dk}{k} . \tag{6}$$

We can study the rms velocity of matter in the universe using clusters of galaxies as tracers. Bahcall, Gramann, & Cen (1994) and Gramann et al. (1995) compared the motions of clusters of galaxies with the motion of the underlying matter distribution in different cosmological models. The rms cluster peculiar velocity is similar to the rms peculiar velocity of matter smoothed with a Gaussian window of radius $r \simeq 3\ h^{-1}$ Mpc. The observed peculiar velocity function of galaxy clusters was investigated by Bahcall & Oh (1996). They found an rms one-dimensional cluster peculiar velocity of $\langle v_{1D}^2 \rangle^{1/2} = 293 \pm 28$ km s^{-1}. This corresponds to a three-dimensional rms velocity of $\langle v^2 \rangle^{1/2} = 507 \pm 48$ km s^{-1}.

What is the origin of the velocity dispersion of galaxies and clusters of galaxies? Does the velocity dispersion of galaxy systems originate mostly from the small-scale velocity fluctuations of matter with wavelengths $\lambda < 100$ h^{-1} Mpc, or from the large-scale velocity fluctuations of matter with wavelengths $\lambda > 100$ h^{-1} Mpc? Or is there a peak in the function $V^2(k)$ at $\lambda \sim 100$ h^{-1} Mpc that contributes most to the velocity dispersion?

We will examine the power spectrum of velocity fluctuations and the rms velocity of matter starting from the power spectrum of density fluctuations derived from large galaxy surveys. In the linear approximation, the continuity equation yields a relation between the density contrast δ and the peculiar velocity,

$$\nabla \cdot \boldsymbol{v} = -f(\Omega_0) H_0 \delta \,, \tag{7}$$

where the function $f(\Omega_0)$ is the linear velocity growth factor and Ω_0 is the cosmological density parameter at the present moment. The function $f(\Omega_0) \approx \Omega_0^{0.6}$ (Peebles 1980). In Fourier space, equation (7) takes the form

$$\boldsymbol{v}_k \cdot i\boldsymbol{k} = -f(\Omega_0) H_0 \delta_{\boldsymbol{k}} \,, \tag{8}$$

where $\delta_{\boldsymbol{k}}$ is the Fourier transform of the density field. The linear growing mode is irrotational. If the velocity field has no vorticity, the function $V^2(k)$ can be determined as

$$V^2(k) = \frac{1}{2\pi^2} f^2(\Omega_0) H_0^2 k P(k) \,, \tag{9}$$

where $P(k) \equiv \langle | \delta_{\boldsymbol{k}} |^2 \rangle$ is the power spectrum of density fluctuations.

The power spectrum of density fluctuations in the mass distribution has been estimated by Kolatt & Dekel (1997) using galaxy peculiar velocities. In this paper, we will derive the power spectrum of velocity fluctuations on the basis of their results. We find that the power spectrum estimated by Kolatt & Dekel (1997) corresponds to an rms velocity $\langle v^2 \rangle^{1/2} \simeq 700$ km s^{-1} for the matter distribution smoothed on scales of ~ 3 h^{-1} Mpc. This value is larger than that observed by Bahcall & Oh (1996) for the rms cluster peculiar velocity. Therefore, either the power spectrum of density fluctuations estimated by Kolatt & Dekel (1997) is overestimated or the rms cluster peculiar velocity determined by Bahcall & Oh (1996) is underestimated. Available data are insufficient to distinguish between these scenarios, so we must consider both possibilities.

We will examine the power spectrum of velocity fluctuations starting from the power spectrum of density fluctuations derived from large redshift surveys of galaxies. The power spectrum of the galaxy distribution has been measured from a number of large galaxy surveys. In this paper, we will investigate the peculiar velocity field in the Stromlo-APM and Las Campanas redshift surveys (Tadros & Efstathiou 1996; Lin et al. 1996). The amplitude of the velocity fluctuations derived from the galaxy distribution depends on the parameter $\beta = f(\Omega_0)/b$, where b is the bias factor for the galaxies. We will estimate the parameter β on the basis of the observed rms cluster peculiar velocity.

The power spectrum of the galaxy distribution in the Stromlo-APM redshift survey peaks at a wavenumber $k = 0.052$ h Mpc^{-1} (or at a wavelength $\lambda = 120$ h^{-1} Mpc). Available data, however, are insufficient to say whether the peak in the Stromlo-APM survey reflects a real feature in the galaxy distribution. It is likely that the decline in the power spectrum at wavenumbers $k \leq 0.052$ h Mpc^{-1} is partly due to the effects of the uncertainty in the mean number density of optical galaxies (see Tadros & Efstathiou 1996 for a discussion of this effect). Einasto et al. (1997) found a well-defined peak at the same wavelength, $\lambda = 120$ h^{-1} Mpc, in the power spectrum of galaxy clusters. On the other hand, there is no well-defined peak in the three-dimensional power spectrum of the galaxy distribution in the Las Campanas redshift survey derived by Lin et al. (1996). There is a striking peak at $\lambda \approx 100$ h^{-1} Mpc in the two-dimensional power spectrum of the Las Campanas redshift survey (Landay et al. 1996). A similar peak at 128 h^{-1} Mpc in the one-dimensional power spectrum of a deep pencil-beam survey was detected by Broadhurst et al. (1990). If there is an excess of power in the universe on around a scale of 120 h^{-1} Mpc, then this scale contributes most to the velocity dispersion of galaxy systems. We will investigate the relation between the velocity dispersion on scales of ~ 3 h^{-1} Mpc and the maximum value for the power spectrum of velocity fluctuations on wavelengths of ~ 120 h^{-1} Mpc.

To characterize the large-scale peculiar velocity field, we can use the bulk velocity of galaxies. The bulk velocity averaged over spheres of radius r is determined as

$$v_b^2(r) = 9 \int V^2(k) \frac{(\sin kr - kr \cos kr)^2}{(kr)^6} \frac{dk}{k} \,. \tag{10}$$

The bulk velocity of galaxies on ~ 60 h^{-1} Mpc scales is determined by the amplitude of the density and velocity fluctuations in the universe on scales with a wavenumber $k \leq 0.05$ h Mpc^{-1} ($\lambda \geq 120$ h^{-1} Mpc). We will estimate the bulk velocity starting from the power spectrum of the galaxy distribution.

In the linear approximation, the power spectrum of velocity fluctuations is directly related to the power spectrum of density fluctuations. Formally, these power spectra are identical in their information content. Consequently, one might ask why it is necessary to investigate the velocity power spectrum at all. The properties of the peculiar velocity field, however, are best visualized and understood in terms of the velocity spectrum, just as the properties of the density field are best expressed in terms of the density spectrum. For instance, the quantities discussed in this paper (velocity dispersion and bulk velocity) are easily derived from the velocity power spectrum. It is therefore advantageous to combine both approaches to get a better understanding of the large-scale matter distribution in the universe.

This paper is organized as follows. In § 2 we estimate the power spectrum of velocity fluctuations from peculiar velocities of galaxies and analyze the rms velocity of matter in the universe in more detail. In § 3 we analyze the power spectrum of the galaxy distribution measured from various redshift surveys, and in § 4 we present a method for estimating the power spectrum of velocity fluctuations and the rms velocity of matter starting from the power spectrum of the distribution of galaxies. In § 5, we examine the power spectrum of velocity fluctuations in the Las Campanas redshift survey, and in § 6 we investigate the peculiar velocity fluctuations in the Stromlo-APM redshift survey. Discussion and summary are presented in § 7.

A Hubble constant of $H_0 = 100\, h$ km s^{-1} Mpc^{-1} is used throughout this paper.

2. PECULIAR VELOCITIES OF GALAXIES AND CLUSTERS OF GALAXIES

Kolatt & Dekel (1997, hereafter KD) derived the power spectrum of density fluctuations from the Mark III catalog of peculiar velocities (Willick et al. 1997). This catalog consists of more than 3000 galaxies from several different data sets of spiral and elliptical/S0 galaxies with distances estimated by the Tully-Fisher and D_n-σ methods. The fractional error in the distance to each galaxy is on the order of 17%–21%.

KD used the POTENT method to recover the smoothed three-dimensional velocity field from the observed radial velocities (Bertschinger et al. 1990). This method assumes that the velocity field is potential. The velocity field was smoothed with a Gaussian of radius 12 h^{-1} Mpc, and then the density field was computed using a quasi-linear solution for the continuity equation. This approximation reduces to equation (7) in the linear regime. KD applied an empirical correction procedure to recover the true power spectrum from the observed power spectrum of density fluctuations. This correction procedure was based on mock catalogs designed to mimic the observational data.

KD derived values for the function $f^2(\Omega_0)P(k)$ with 1 σ errors. Figure 1*a* shows the rms amplitude of velocity fluctuations, $V(k)$, computed on the basis of their results, using equation (9). The function $V(k)$ has been calculated for the wavenumber range $0.061 < k < 0.172\; h$ Mpc^{-1}. For the wavenumbers $k = 0.172\; h$ Mpc^{-1}, $k = 0.102\; h$ Mpc^{-1}, and $k = 0.061\; h$ Mpc^{-1}, the rms amplitude of velocity fluctuations are $V(k) = 414 \pm 52$ km s^{-1}, $V(k) = 489 \pm 66$ km s^{-1}, and $V(k) = 502 \pm 96$ km s^{-1}, respectively. To describe the power spectrum of velocity fluctuations for the peculiar velocity data, we can use the fitting function

$$V^2(k) = 2V^2(k_0)(k/k_0)^{n+1}[1 + (k/k_0)^{n+m}]^{-1}\,, \qquad (11)$$

where $k_0 = 0.052\; h$ Mpc^{-1}, $V(k_0) = 496$ km s^{-1}, $n = 1$, and $m = 1.85$. This function is consistent with the data at a confidence level of $> 99\%$ (based on a χ^2 test).

We have estimated the rms peculiar velocity of matter corresponding to the power spectrum estimated by KD. The rms peculiar velocity was computed using equation (6). Figure 1*b* shows the rms peculiar velocity for the matter distribution at radii $r = 1\; h^{-1}$ Mpc to $r = 5\; h^{-1}$ Mpc for the velocity spectrum of equation (11). At a smoothing radius $r = 3\; h^{-1}$ Mpc, the rms peculiar velocity $v_{\rm rms} = 709$ km s^{-1}.

The rms peculiar velocity calculated using equation (11) can be considered as a lower limit to the power spectrum derived by KD. We have assumed that on scales with wavenumber $k < 0.06\; h$ Mpc^{-1} ($\lambda > 100\; h^{-1}$ Mpc), the velocity spectrum decreases monotonically. If there is a peak in the velocity spectrum on scales with wavenumber $k \sim 0.05\; h$ Mpc^{-1}, or if the turnover in the spectrum occurs at larger scales, then the rms peculiar velocity would be higher than the value computed using equation (11). Therefore, the power spectrum estimated by KD corresponds to an rms peculiar velocity that is larger than 700 km s^{-1} for the matter distribution on scales of $\sim 3\; h^{-1}$ Mpc.

For comparison, we show in Figure 1*b* the rms cluster peculiar velocity found by Bahcall & Oh (1996). They determined the peculiar velocity function of galaxy clusters using an accurate sample of peculiar velocities of clusters obtained by Giovanelli et al. (1996). This sample consists of 22 clusters and groups of galaxies with peculiar velocities based on Tully-Fisher distances to Sc galaxies. Bahcall & Oh (1996) found an rms one-dimensional cluster peculiar velocity of 293 ± 28 km s^{-1}, which corresponds to a three-dimensional rms velocity of 507 ± 48 km s^{-1}.

Numerical simulations show that the velocity distribution of clusters is similar to that of the matter when the matter distribution is smoothed with a Gaussian of radius 3 h^{-1} Mpc (Bahcall et al. 1994; Gramann et al. 1995). The velocity distribution of the unsmoothed matter exhibits higher velocities than the clusters, especially for $\Omega = 1$ models, because of the high velocity dispersion of matter within clusters. The 3 h^{-1} Mpc smoothing of the matter distribution eliminates this nonlinear effect. The rms cluster peculiar velocity is similar to, or somewhat higher than, the rms peculiar velocity of the smoothed matter. Therefore, the rms cluster velocity determines an upper limit to the rms velocity of matter on scales of $\sim 3\; h^{-1}$ Mpc.

The power spectrum of velocity fluctuations estimated from the mass power spectrum of KD corresponds to an rms peculiar velocity that is larger than expected on the

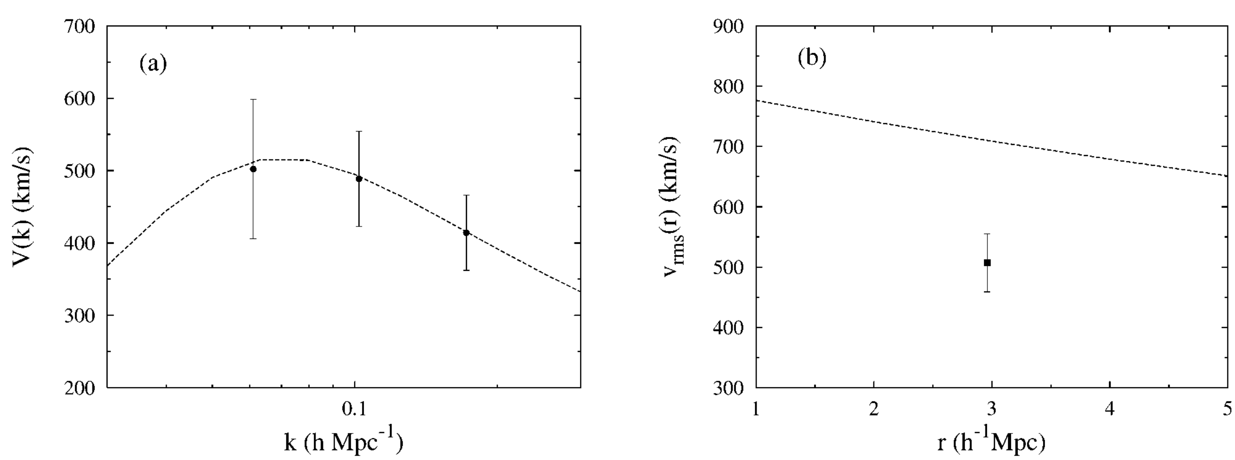

FIG. 1.—(*a*) The rms amplitude of velocity fluctuations derived from peculiar velocities of galaxies (*filled circles*). The dashed line is the fitting function given by eq. (11). (*b*) The rms peculiar velocity of matter for the velocity spectrum of eq. (11) (*dashed line*). The filled square shows the observed rms peculiar velocity of galaxy clusters.

basis of the observed rms cluster peculiar velocity determined by Bahcall & Oh (1996). Given the large errors associated with peculiar velocity measurements of galaxies, this discrepancy is not very large. The sample used by Bahcall & Oh (1996) consisted of only 22 clusters; a small sample can introduce significant statistical uncertainties. On the other hand, the peculiar velocities of galaxies used to estimate the power spectrum are contaminated by distance errors, as well as being sparsely and nonuniformly sampled. The systematic errors may be more complicated than envisaged by KD.

The three-dimensional rms cluster peculiar velocity, ~ 500 km s^{-1}, is in reasonable agreement with the results of Marzke et al. (1995), who studied the rms relative peculiar velocity between galaxy pairs separated by $\sim 1\ h^{-1}$ Mpc. They found an rms one-dimensional velocity of $\sigma_{12} = 540 \pm 180$ km s^{-1} from an analysis of the CfA-2 and Southern Sky redshift surveys. Assuming that the velocities of the galaxies are isotropic and independent, a three-dimensional rms velocity $v_{\rm rms} = 500$ km s^{-1} corresponds to a pairwise rms velocity $\sigma_{12} = (2/3)^{1/2} v_{\rm rms} = 408$ km s^{-1}. However, the rms velocity of galaxies is probably higher than the rms velocity of clusters because of the high velocity dispersion within the clusters. Also, the velocities of galaxy pairs with separations of $\sim 1\ h^{-1}$ Mpc are correlated. Together, these effects can easily explain the difference between the value of 408 km s^{-1}, predicted on the basis of the cluster rms velocity, and the measured value of 540 km s^{-1}.

3. POWER SPECTRUM OF THE GALAXY DISTRIBUTION

Let us now consider the power spectrum of the galaxy distribution determined from different redshift surveys. The power spectrum of the galaxy distribution in the Stromlo-APM redshift survey has been computed by Tadros & Efstathiou (1996, hereafter TE). The Stromlo-APM redshift survey is a 1 in 20 sparsely sampled subset of 1787 galaxies selected from the APM galaxy survey (Maddox et al. 1990). The survey is described in detail by Loveday et al. (1992). The median redshift of the Stromlo-APM survey is $z = 0.05$. TE estimated the power spectrum of density fluctuations using different volume-limited and flux-limited samples. They found that galaxy density power spectra are insensitive to the volume limit and to the weights applied in the analysis of flux-limited samples. They also tested the power spectrum estimator against simulated galaxy catalogues constructed from N-body simulations and showed that the methods applied provide nearly unbiased estimates of the power spectrum at wavenumbers $k > 0.04\ h$ Mpc^{-1}. At smaller wavenumbers, the power spectrum may be underestimated.

Figure 2 shows the power spectrum of galaxy clustering, $P^s_{\rm gal}(k)$, in the Stromlo-APM redshift survey with the 1 σ errors derived by TE. We present estimates for the flux-limited sample with $P(k) = 8000\ h^{-3}$ Mpc^3 in the weighting function (see TE for details). The power spectrum of the galaxy distribution peaks at wavenumber $k_0 = 0.052\ h$ Mpc^{-1} ($\lambda = 120\ h^{-1}$ Mpc). To describe the power spectrum in the Stromlo-APM survey, we can use the fitting function

$$P(k) = \begin{cases} P(k_0)\left(\dfrac{k}{k_0}\right)^n , & \text{if } k < k_0 , \\ P(k_0)\left(\dfrac{k}{k_0}\right)^m , & \text{if } k > k_0 , \end{cases} \qquad (12)$$

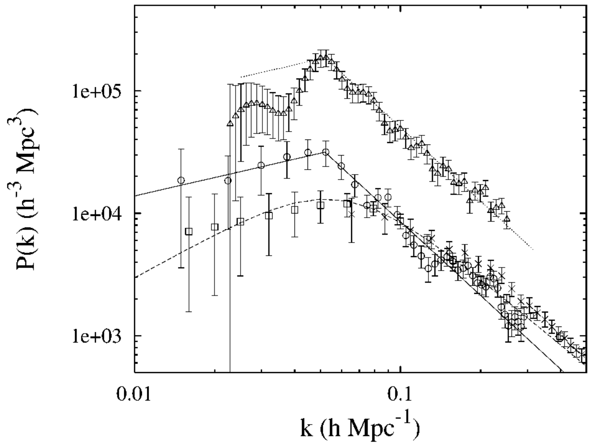

FIG. 2.—The power spectrum of the galaxy distribution. Open circles and open squares show the power spectrum of the galaxy distribution in the Stromlo-APM and Las Campanas redshift surveys, respectively. To describe the power spectrum in these surveys, we use eq. (12) (*solid line*) and eq. (13) (*dashed line*). Crosses show the power spectrum of the galaxy distribution in the SSRS2 + CfA2 redshift survey. For comparison, the open triangles represent the power spectrum of the distribution of galaxy clusters. The dotted line is the function given in eq. (12) multiplied by a factor of ~ 6.

where $k_0 = 0.052\ h$ Mpc^{-1}, $P(k_0) = 3.16 \times 10^4\ h^{-3}$ Mpc^3, $n = 0.5$, and $m = -2$. This function is consistent with the power spectrum in the Stromlo-APM survey at a confidence level of $\sim 70\%$.

The Las Campanas redshift survey contains 23,697 galaxies, with an average redshift of $z = 0.1$, distributed over six slices in the north and south Galactic caps (Shectman et al. 1996). Figure 2 shows the three-dimensional power spectrum of galaxy clustering computed by Lin et al. (1996). The observed power spectrum of galaxy clustering in the Las Campanas survey can be fitted by

$$P(k) = 2P(k_0)\left(\frac{k}{k_0}\right)^n \left[1 + \left(\frac{k}{k_0}\right)^{m+n}\right]^{-1} , \qquad (13)$$

where $k_0 = 0.06\ h^{-1}$ Mpc, $P(k_0) = 1.28 \times 10^4\ h^{-3}$ Mpc^3, $n = 1.2$, and $m = 1.8$. The function given in equation (13) is consistent with the power spectrum in the Las Campanas survey at a confidence level of $> 99\%$. Figure 2 also shows the power spectrum of the galaxy distribution in the SSRS2 + CfA2 redshift survey determined by da Costa et al. (1994). The power spectrum is presented for a volume-limited sample with a distance limit $r = 101\ h^{-1}$ Mpc.

At wavenumbers $k \geq 0.06\ h$ Mpc^{-1} ($\lambda < 100\ h^{-1}$ Mpc), the power spectra from the different redshift surveys are consistent. On larger scales, the power spectrum of the galaxy distribution is relatively poorly constrained by observations. At wavenumbers $k \simeq 0.04$–$0.06\ h$ Mpc^{-1}, the power spectrum of the galaxy distribution in the Stromlo-APM survey is a factor of 2 higher than the power spectrum derived from the Las Campanas survey. There is no well-defined peak in the three-dimensional power spectrum derived by Lin et al. (1996). Landay et al. (1996) measured the two-dimensional power spectrum of the Las Campanas survey and found a strong peak in the power spectrum at $\sim 100\ h^{-1}$ Mpc. The signal was detected in two independent directions on the sky and identified with numerous structures visible in the survey as walls and voids. Given the geometry of the Las Campanas survey, the three-dimensional analysis is not as sensitive as the two-dimensional analysis to structures on scales of $>50\ h^{-1}$ Mpc. The comparison with the power spectrum of the galaxy distribution in the Stromlo-APM survey shows that at wavenumbers $k < 0.06\ h\ \mathrm{Mpc}^{-1}$, the three-dimensional power spectrum computed by Lin et al. (1996) is probably underestimated.

As discussed by TE, the peak in the power spectrum of the Stromlo-APM survey may be caused by the effects of uncertainty in the mean number density of galaxies and may not reflect a real feature of the galaxy distribution. However, independent evidence for the presence of a preferred scale in the universe at around 120 h^{-1} Mpc comes from an analysis of the distribution of galaxy clusters. Figure 2 shows the power spectrum of the distribution of galaxy clusters as determined by Einasto et al. (1997). The power spectrum is calculated for 869 Abell clusters with measured redshifts. The power spectrum of the distribution of galaxy clusters has a well-defined peak at the same wavenumber, $k_0 = 0.052\ h^{-1}$ Mpc, as the power spectrum of galaxies in the Stromlo-APM survey. For wavenumbers $k > k_0$, the shape of the clusters' power spectrum is similar to the shape of the power spectrum for galaxies in the Stromlo-APM survey. This comparison suggests that the peak observed in the power spectrum of the Stromlo-APM survey is a real feature in the distribution of galaxies.

4. METHOD FOR DETERMINING THE VELOCITY POWER SPECTRUM FROM REDSHIFT DATA

To estimate the power spectrum of velocity fluctuations from a given power spectrum of galaxy clustering in redshift space, we assume that (1) the power spectrum of galaxy clustering in real space is $P_{\mathrm{gal}}(k) = b^2 P(k)$, where b is the bias factor, and (2) the relation between the power spectra of density and velocity fluctuations is given by the linear theory relation (eq. [9]). We assume that these assumptions hold for wavenumbers $k < 0.15\ h\ \mathrm{Mpc}^{-1}$ ($\lambda > 42\ h^{-1}$ Mpc) and examine the power spectrum of velocity fluctuations in this range.

Galaxy peculiar velocities cause a distortion of the clustering pattern measured in redshift space compared to the true pattern in real space (see, e.g., Kaiser 1987; Gramann, Cen, & Bahcall 1993). To take into account the redshift-space distortions, we use the following analytic model:

$$P^s_{\mathrm{gal}}(k) = \left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right) G^2(\beta, k\sigma_v) P_{\mathrm{gal}}(k) , \tag{14}$$

where the parameter $\beta = f(\Omega_0)/\mathrm{b}$ and the function G is given by

$$\begin{aligned} G^2(\beta, k\sigma_v)\left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right) &= \left[\frac{\sqrt{\pi}\ \mathrm{erf}\ (y)}{8y^5}\right](3\beta^2 + 4\beta y^2 + 4y^4) \\ &\quad - \left[\frac{\exp\ (-y^2)}{4y^4}\right](3\beta^2 + 2\beta^2 y^2 + 4\beta y^2) , \end{aligned} \tag{15}$$

where $y = k\sigma_v/H_0$. The first factor in equation (14) is expected from linear theory (Kaiser 1987). The function $G(\beta, k\sigma_v)$ describes the suppression of the power spectrum on small scales, as given by Peacock & Dodds (1994). For $k \to 0$ (linear regime), the function $G(\beta, k\sigma_v) \to 1$. The small-scale peculiar velocities are assumed to be uncorrelated in position and are drawn from a Gaussian distribution with one-dimensional dispersion σ_v. Numerical simulations have shown that the analytic model given by equation (14) provides a good match to the peculiar velocity distortion in redshift space (Gramann et al. 1993; TE). In the mixed dark matter model, the redshift-space distortion can be fitted with equation (14) using the parameter $\sigma_v \simeq 500\ \mathrm{km\ s^{-1}}$; in the low-density cold dark matter models, we can describe the velocity distortion in redshift space using σ_v in the range 200–350 km s^{-1}, depending on the amplitude of the power spectrum. The velocity dispersion, σ_v, depends on the power spectrum of density and velocity fluctuations in the universe, but this relation is not linear. It can be determined using numerical simulations for a given model. In this paper, we will examine the redshift-space distortion for various assumed values of σ_v.

Using equation (14), the power spectrum of velocity fluctuations is determined as

$$\begin{aligned} V^2(k) &= \frac{1}{2\pi^2} H_0^2 F^2(\beta) G^{-2}(\beta, k\sigma_v) k P^s_{\mathrm{gal}}(k) , \\ F^2(\beta) &= \frac{\beta^2}{1 + 2\beta/3 + \beta^2/5} . \end{aligned} \tag{16}$$

We use equation (16) to calculate the power spectrum of velocity fluctuations from a power spectrum of the galaxy distribution.

The amplitude of the velocity fluctuations derived from the galaxy distribution depends on the parameter β. This parameter can be estimated on the basis of the observed mass function of galaxy clusters. The present data for the cluster mass function indicate that β is less than 1, the preferred range being $\beta \simeq 0.4$–0.7 (e.g., Bahcall & Cen 1993; White, Efstathiou, & Frenk 1993; Eke, Cole, & Frenk 1996). However, the value of β determined starting from the cluster mass function depends on the bias factor for galaxies on scales of $r < 10\ h^{-1}$ Mpc. This is not necessarily equal to the bias factor for the larger scales ($k < 0.15\ h^{-1}$ Mpc) examined in this paper.

We have investigated the redshift-space distortion at the maximum wavenumber, $k = 0.15\ h\ \mathrm{Mpc}^{-1}$, used to estimate the power spectrum of velocity fluctuations. At this wavenumber, the nonlinear effect on the power spectrum in redshift space can be quite large, especially if the galaxy velocity dispersion is high. For a velocity dispersion $\sigma_v = 600\ \mathrm{km\ s^{-1}}$, we find that the function $V(k)$ is enhanced by $\sim 17\%$ compared to the linear theory prediction. For high-velocity dispersions, the nonlinear correction to the velocity spectrum at wavenumbers $k \leq 0.15\ h\ \mathrm{Mpc}^{-1}$ is therefore important. However, the true one-dimensional rms velocity of galaxies is probably significantly less than 600 km s^{-1}. This value of σ_v corresponds to an rms pairwise velocity of $\sigma_{12} \simeq 850$–$900\ \mathrm{km\ s^{-1}}$. For a velocity dispersion of $\sigma_v = 400\ \mathrm{km\ s^{-1}}$, the function $V(k)$ is only $\sim 8\%$ larger than expected from the linear approximation at a wavenumber $k = 0.15\ h\ \mathrm{Mpc}^{-1}$. In this case, we can use the linear approximation to estimate the power spectrum of velocity fluctuations at wavenumbers $k < 0.15\ h\ \mathrm{Mpc}^{-1}$.

To determine the rms velocity of matter in the universe, we use the equation

$$v^2_{\mathrm{rms}}(r) = v^2_P(r) + v^2_L = \int V^2(k) \exp\ (-r^2 k^2) \frac{dk}{k} + v^2_L , \tag{17}$$

where the function $v_P(r)$ describes the contribution of fluctuations derived from the galaxy power spectrum in a given redshift survey, and the parameter v_L is the contribution from large-scale fluctuations in the universe that may exist on scales that are comparable to or greater than the size of the redshift survey.

To determine the function $v_P(r)$ in the Las Campanas survey, we use the velocity spectrum derived directly from the redshift data in the range $0.013 < k < 0.15\ h\ \mathrm{Mpc}^{-1}$; on scales larger and smaller than this range, we use an approximation (see eq. [18] below). Using this approximation, the contributions to the velocity dispersion v_{rms}^2 at a radius of 3 h^{-1} Mpc are $\sim$2.5% and $\sim$23% from fluctuations with wavenumbers $k < 0.013\ h\ \mathrm{Mpc}^{-1}$ and $k > 0.15\ h\ \mathrm{Mpc}^{-1}$, respectively. To determine the function $v_P(r)$ in the Stromlo-APM survey, we use the velocity spectrum derived directly from the galaxy power spectrum in the range $0.007 < k < 0.15\ h^{-1}$ Mpc; outside this range we again use an approximation (see eq. [22] below). Using this approximation, the contribution from fluctuations with wavenumbers $k < 0.007\ h^{-1}$ Mpc is $\sim$2%, while fluctuations at wavenumbers $k > 0.15\ h^{-1}$ Mpc contribute $\sim$10% to the velocity dispersion at a radius $r = 3\ h^{-1}$ Mpc.

Let us now investigate the power spectrum of velocity fluctuations and the rms velocity of matter starting from the power spectrum of the galaxy distribution in the Las Campanas and Stromlo-APM redshift surveys.

5. VELOCITY FLUCTUATIONS IN THE LAS CAMPANAS GALAXY SURVEY

Figure 3*a* shows the function $V(k)$, computed from equation (16), for the power spectrum of galaxy clustering in the Las Campanas redshift survey. The rms amplitude of velocity fluctuations is presented for the parameter $\beta = 0.7$. To see the effect of redshift-space distortion in more detail, we investigated the function $V(k)$ for different values of the velocity dispersion σ_v. Figure 3*a* shows the function $V(k)$ for three values of the velocity dispersion, $\sigma_v = 0$, 400, and 600 km s^{-1}. The power spectrum of velocity fluctuations derived from the Las Campanas survey increases up to the wavenumber $k \simeq 0.06\ h\ \mathrm{Mpc}^{-1}$ ($\lambda \simeq 100\ h^{-1}$ Mpc) and flattens out for larger wavenumbers. The maximum value for the function $V(k)$ is shifted to smaller scales compared to the power spectrum of the galaxy distribution and occurs in the range $0.08 < k < 0.1\ h\ \mathrm{Mpc}^{-1}$. To describe the power spectrum of velocity fluctuations derived from the Las Campanas survey we can use the fitting function

$$V^2(k) = 2V^2(k_0)(k/k_0)^{n+1}[1 + (k/k_0)^{n+m}]^{-1}\ , \quad (18)$$

where $k_0 = 0.06\ h\ \mathrm{Mpc}^{-1}$, $n = 1.2$, $m = 1.7$, and the value of the velocity power spectrum at a wavenumber k_0 is given by

$$V(k_0) = 625F(\beta)\ \mathrm{km\ s^{-1}}\ . \quad (19)$$

For the parameter $\beta = 0.7$ ($F(\beta) = 0.56$), the rms amplitude of velocity fluctuations $V(k_0) = 350\ \mathrm{km\ s^{-1}}$. At a wavenumber $k_0 = 0.06\ h\ \mathrm{Mpc}^{-1}$, the nonlinear correction to the function $V(k)$ is small and can be neglected ($\sim$1.3% if $\sigma_v =$ 400 km s^{-1}, $\sim$3% if $\sigma_v = 600$ km s^{-1}). At smaller scales, the function $V(k)$ is better fitted with index $m = 1.7$, rather than the $m = 1.8$ used in equation (13). The fitting function of equation (18) is consistent with the power spectrum in the Las Campanas survey at a confidence level of $>$85% (if $\sigma_v \leq 600$ km s^{-1}).

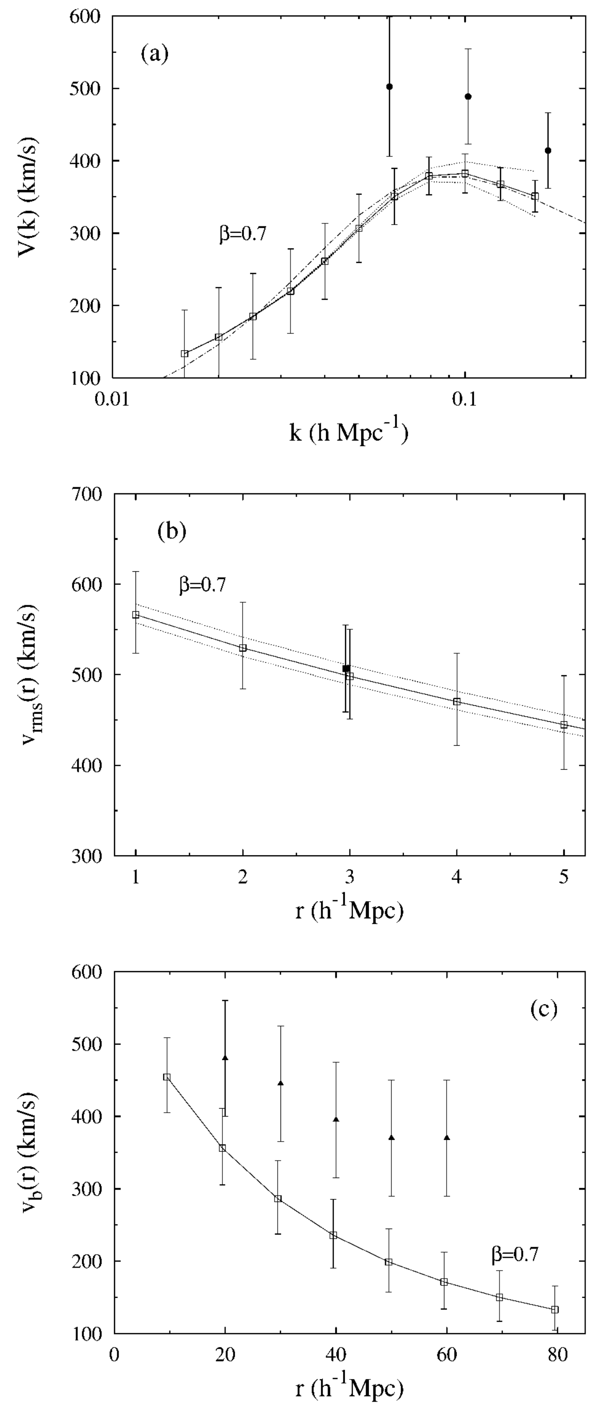

FIG. 3.—Velocity fluctuations in the Las Campanas redshift survey for the parameters $\beta = 0.7$ and $\sigma_v = 400$ km s^{-1} (*open squares with solid line*). (*a*) The rms amplitude of velocity fluctuations. Dotted lines show the function $V(k)$ for the velocity dispersion $\sigma_v = 0$ (*lower curve*) and $\sigma_v = 600$ km s^{-1} (*upper curve*). The dot-dashed line represents the approximation given in eq. (18). Filled circles demonstrate the velocity rms amplitude derived from peculiar velocities. (*b*) The rms peculiar velocity of matter for the velocity spectra presented in panel (*a*). Filled square shows the observed rms cluster peculiar velocity. (*c*) The rms amplitude of the bulk flow. Filled triangles represent the observed bulk velocities derived from peculiar velocities of galaxies.

For comparison, we show in Figure 3*a* the rms amplitude of velocity fluctuations derived from peculiar velocities of galaxies. KD compared the power spectra of density fluctuations derived from peculiar velocities with galaxy power spectra determined from various large galaxy surveys and derived best-fitting values for the parameter β in the range 0.77–1.21. The power spectrum of the galaxy distribution in the Las Campanas redshift survey is consistent with the power spectrum estimated from peculiar velocities when the parameter $\beta \approx 1.0$.

Let us now consider the rms peculiar velocity of matter, starting from the power spectrum of the galaxy distribution. Figure 3*b* shows the rms velocity computed from equation (17) for the velocity spectra presented in Figure 3*a*, assuming that the parameter $v_L = 0$. The fluctuations at wavenumbers $k < k_0$ contribute $\sim 33\%$ to the velocity dispersion of galaxy systems, and $\sim 67\%$ of this velocity dispersion is generated on smaller scales. The rms peculiar velocity of matter at the smoothing radius $r = 3\ h^{-1}$ Mpc can be written as

$$v_{\rm rms}(r = 3\ h^{-1}\ {\rm Mpc}) = (870 \pm 90) G_{\rm int}(\sigma_v) F(\beta)\ {\rm km\ s^{-1}}\ . \quad (20)$$

The function $G_{\rm int}(\sigma_v)$ describes the correction due to the nonlinear redshift-space distortions. For the velocity dispersions $\sigma_v = 400$ km s^{-1} and $\sigma_v = 600$ km s^{-1}, the function $G_{\rm int} = 1.02$ and 1.05, respectively. (The nonlinear correction also depends on the parameter β, but this dependence is very weak and is neglected here). For $\beta = 0.7$ and $\sigma_v = 400$ km s^{-1}, the rms peculiar velocity $v_{\rm rms}(r = 3\ h^{-1}$ Mpc) = (498 ± 50) km s^{-1}. The small-scale velocity dispersion of the matter is consistent with the observed dispersion of galaxy clusters when β is in the range 0.6–0.7.

Figure 3*c* shows the bulk velocities that correspond to the power spectrum of the galaxy distribution in the Las Campanas survey for $\beta = 0.7$. The bulk velocities were computed using equation (10). The bulk velocity at a radius $r = 60\ h^{-1}$ Mpc can be written as

$$v_b(r = 60\ h^{-1}\ {\rm Mpc}) = (305 \pm 75) F(\beta)\ {\rm km\ s^{-1}}\ . \quad (21)$$

For the parameter $\beta = 0.7$, the rms amplitude of the bulk flow averaged on a scale of $r = 60\ h^{-1}$ Mpc is (170 ± 40) km s^{-1}.

For comparison, we plot in Figure 3*c* the observed bulk velocities derived from the Mark III catalog of peculiar velocities for radii of 30, 40, 50 and 60 h^{-1} Mpc (Dekel 1994). The observed velocities are determined in a sphere centered on the Local Group and represent a single measurement of the bulk flow on large scales. The average velocity of galaxies in the sphere of radius $r = 60\ h^{-1}$ Mpc around us is estimated as 370 ± 80 km s^{-1}. Assuming the distribution of bulk velocities is a Maxwellian distribution with rms velocity $\simeq 170$ km s^{-1}, the probability of measuring a bulk velocity ≥ 300 km s^{-1} is only 2.5%.

The difference between the small-scale velocity dispersion of galaxy systems and the large-scale velocity dispersion at radius $r = 60\ h^{-1}$ Mpc is determined by the amplitude of velocity fluctuations at intermediate wavenumbers $k \sim 0.05$–$0.1\ h$ Mpc^{-1} ($\lambda \sim 120$–$60\ h^{-1}$ Mpc). If the rms velocity of galaxy systems is ~ 500 km s^{-1} and the rms amplitude of the bulk flow, averaged over a scale of $r = 60\ h^{-1}$ Mpc, is ≥ 300 km s^{-1}, then the contribution from velocity fluctuations at intermediate wavenumbers must be ≤ 400 km s^{-1}. This situation is consistent with the amplitude of velocity fluctuations derived from the Las Campanas survey only when $\beta \leq 0.6$ and $v_L \geq 0$. If the amplitude of the large-scale velocity fluctuations at wavenumbers $k < 0.06\ h$ Mpc^{-1} is higher than that estimated starting from the power spectrum of the galaxy distribution in the Las Campanas survey, the observed rms peculiar velocity of clusters is consistent with a lower amplitude for the velocity fluctuations at smaller wavelengths and with a lower value of the parameter β.

6. VELOCITY FLUCTUATIONS IN THE STROMLO-APM GALAXY SURVEY

Figure 4*a* shows the function $V(k)$, computed from equation (16), for the power spectrum of galaxy clustering in the Stromlo-APM redshift survey. The rms amplitude of velocity fluctuations is presented for $\beta = 0.55$. As for the Las Campanas survey, we use the parameter β that is consistent with the observed dispersion of galaxy clusters (see below). Figure 4*a* shows the results for velocity dispersions $\sigma_v = 0, 400$, and 600 km s^{-1}.

The power spectrum of velocity fluctuations, like the power spectrum of the galaxy distribution in the Stromlo-APM redshift survey, peaks at wavenumber $k_0 = 0.052\ h$ Mpc^{-1} (or at wavelength $\lambda = 120\ h^{-1}$ Mpc). At smaller wavelengths, the function $V(k)$ declines, reaching a minimum value at $k = 0.127\ h$ Mpc^{-1}. To describe the power spectrum of velocity fluctuations in the Stromlo-APM survey, we use the function

$$V^2(k) = \begin{cases} V^2(k_0)(k/k_0)^{n+1}\ , & \text{if } k < k_0\ , \\ V^2(k_0)(k/k_0)^{m+1}\ , & \text{if } k > k_0\ , \end{cases} \quad (22)$$

where $n = 0.5$ and $m = -2$, as in equation (12), and the value of the velocity spectrum at its maximum is given by

$$V(k_0) = 915 F(\beta)\ {\rm km\ s^{-1}}\ . \quad (23)$$

For $\beta = 0.55$ [$F(\beta) = 0.46$], the maximum value for the velocity rms amplitude is $V(k_0) = 420$ km s^{-1}. The function in equation (22) is consistent with the power spectrum in the Stromlo-APM survey at a confidence level of $\geq 70\%$ (assuming that $\sigma_v \leq 600$ km s^{-1}).

The power spectrum of the galaxy distribution in the Stromlo-APM redshift survey is consistent with the power spectrum estimated from peculiar velocities of galaxies by KD when $\beta \approx 0.8$–0.9. For $\beta = 0.55$, these two power spectra are only consistent at wavenumbers $k \sim 0.06\ h$ Mpc^{-1}. At smaller scales, the function $V(k)$ derived from the distribution of galaxies is smaller (by a factor of ~ 1.6 at $k = 0.1\ h$ Mpc^{-1}).

Figure 4*b* shows the rms velocity computed from equation (17) for the velocity spectra presented in Figure 4*a*, if $v_L = 0$. By substituting equation (22) into equation (17), we find that for the index $m = -2$, the velocity dispersion is given by

$$v_{\rm rms}^2(r) = V^2(k_0)\left[\frac{1}{n+1} + g(rk_0)\right]\ ,$$
$$g(rk_0) = \exp(-r^2k_0^2) - \sqrt{\pi} rk_0[1 - {\rm erf}(rk_0)]\ . \quad (24)$$

Figure 4*b* shows that equation (24) provides a good match to the velocity dispersion in the Stromlo-APM survey. The first factor in equation (24) gives the contribution from large-scale velocity fluctuations at wavenumbers $k < k_0$. To compute this, we assumed that $\exp(-r^2k^2) = 1$ for $k < k_0$. For an index of $n = 0.5$, the contribution of large-scale fluctuations is $\sim 1/(n+1) = 2/3$. The function $g(rk_0)$ gives the

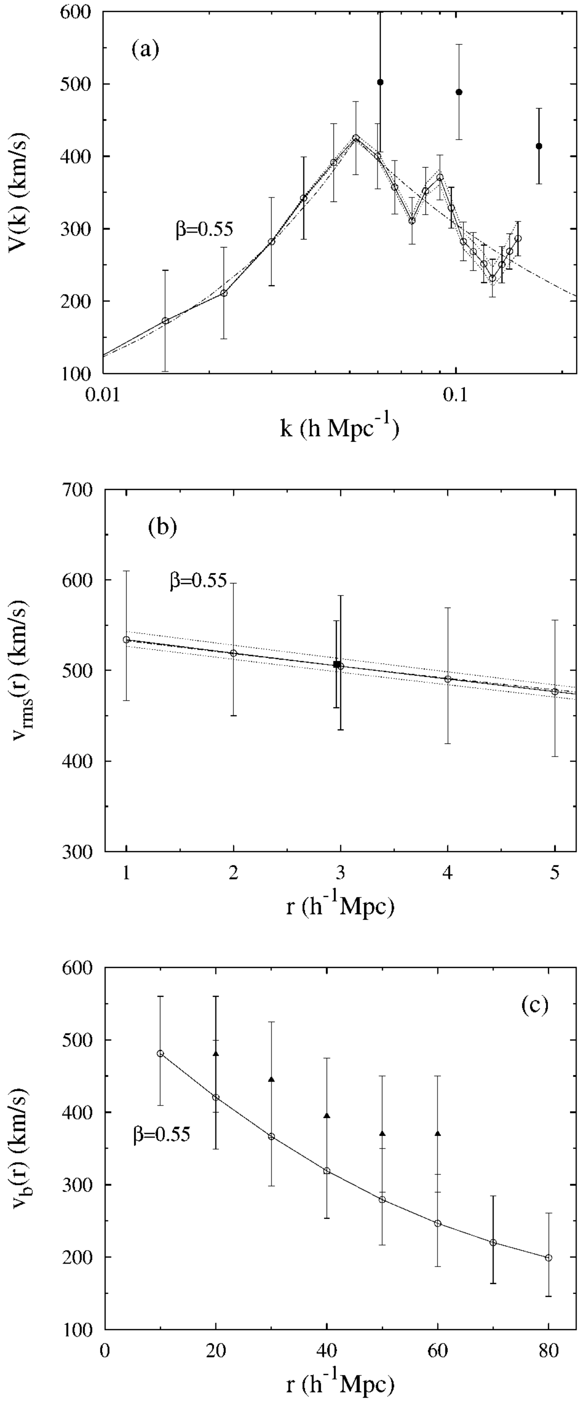

FIG. 4.—Velocity fluctuations in the Stromlo-APM redshift survey for the parameters $\beta = 0.55$ and $\sigma_v = 400$ km s^{-1} (*open circles with solid line*). (*a*) The rms amplitude of velocity fluctuations. Dotted lines show the function $V(k)$ for the velocity dispersion $\sigma_v = 0$ and $\sigma_v = 600$ km s^{-1}. The dot-dashed line shows the approximation given by eq. (22). Filled circles represent the velocity rms amplitude from peculiar velocities. (*b*) The rms peculiar velocity of matter for the velocity spectra presented in panel (*a*). The dot-dashed line shows the approximation given by eq. (24). Filled square represents the observed rms cluster peculiar velocity. (*c*) The rms amplitude of the bulk flow. Filled triangles represent the observed bulk velocities derived from the peculiar velocities.

contribution from small-scale velocity fluctuations at wavenumbers $k > k_0$. For the parameters $k_0 = 0.052$ h Mpc^{-1} and $r = 3$ h^{-1} Mpc, the function $g(rk_0) \approx 3/4$. Therefore, the large-scale fluctuations at wavenumbers $k < k_0$ and small-scale fluctuations at wavenumbers $k > k_0$ give similar contributions (47% and 53%, respectively) to the velocity dispersion of matter smoothed at radius $r = 3$ h^{-1} Mpc, and the rms velocity of matter is given by

$$v_{\rm rms}^2(r = 3\ h^{-1}\ {\rm Mpc}) \approx 1.4V^2(k_0)\ . \qquad (25)$$

The fluctuations at wavenumbers $0.04 < k < 0.07$ h Mpc^{-1}, around the maximum at $k_0 = 0.052$ h Mpc^{-1}, contribute $\sim$33% to the velocity dispersion of galaxy systems.

For the Stromlo-APM survey, the rms peculiar velocity of matter at radius $r = 3$ h^{-1} Mpc depends on the parameter β as

$$v_{\rm rms}(r = 3\ h^{-1}\ {\rm Mpc}) = (1080 \pm 160) G_{\rm int}(\sigma_v) F(\beta)\ {\rm km\ s^{-1}}\ . \qquad (26)$$

Since the contribution from small-scale fluctuations is less important in the Stromlo-APM survey than in the Las Campanas survey, the function $G_{\rm int}(\sigma_v)$ for the Stromlo-APM survey is also somewhat smaller. For velocity dispersions of $\sigma_v = 400$ km s^{-1} and $\sigma_v = 600$ km s^{-1}, the function $G_{\rm int} = 1.015$ and 1.035, respectively. For $\beta = 0.55$ and $\sigma_v = 400$ km s^{-1}, the rms peculiar velocity $v_{\rm rms}(r = 3$ h^{-1} Mpc$) = (505 \pm 75)$ km s^{-1}. The small-scale velocity dispersion of matter is consistent with the observed dispersion of galaxy clusters when the parameter β is in the range 0.5–0.6.

Figure 4*c* shows the bulk velocities that correspond to the power spectrum of the galaxy distribution in the Stromlo-APM survey for $\beta = 0.55$. The bulk velocity at radius $r = 60$ h^{-1} Mpc can be written in the form

$$v_b(r = 60\ h^{-1}\ {\rm Mpc}) = (535 \pm 145) F(\beta)\ {\rm km\ s^{-1}}\ . \qquad (27)$$

For $\beta = 0.55$, the bulk velocity $v_b(r = 60$ h^{-1} Mpc$) = (245 \pm 70)$ km s^{-1}. For an rms velocity of $\simeq$250 km s^{-1}, the probability of measuring a bulk velocity larger than 300 km s^{-1} is about 23%. The probability of measuring a velocity larger than 350 km s^{-1} is 12%.

Up to this point, we have assumed that the power spectrum of the galaxy distribution decreases monotonically for wavelengths $\lambda > 120$ h^{-1} Mpc. There may, however, be a significant contribution to the power from fluctuations on scales comparable to, or greater than, the size of the Stromlo-APM survey. In this case, the observed rms cluster peculiar velocity would be consistent with a smaller amplitude for the velocity fluctuations at smaller wavelengths, and thus with a lower value of β.

Figure 5 shows the properties of the peculiar velocity field for $\beta = 0.45$ and $v_L \geq 0$. For $\beta = 0.45$ [$F(\beta) = 0.39$], the maximum value for the velocity rms amplitude is $V(k_0) = 350$ km s^{-1}. The rms velocity of matter at the smoothing radius $r = 3$ h^{-1} Mpc is $v_{\rm rms} \simeq 420$ km s^{-1}, if $v_L = 0$ and $v_{\rm rms}(r = 3$ h^{-1} Mpc$) \simeq 500$ km s^{-1}, if there is an additional contribution from the large-scale fluctuations in the universe characterized by $v_L = 270$ km s^{-1}. For $\beta = 0.4$, we obtain a similar estimate for the velocity dispersion of galaxy systems (500 km s^{-1}) by assuming a value of $v_L \simeq 330$ km s^{-1}. In the latter case, the fluctuations determined from the galaxy distribution in the Stromlo-APM redshift survey contribute only $\sim$58% to the velocity dispersion of galaxy systems; the rest comes from scales greater than the size of the redshift survey.

Figure 5*c* shows the bulk velocities that correspond to the power spectrum of the galaxy distribution in the Stromlo-APM survey for $\beta = 0.45$. The bulk velocity at a

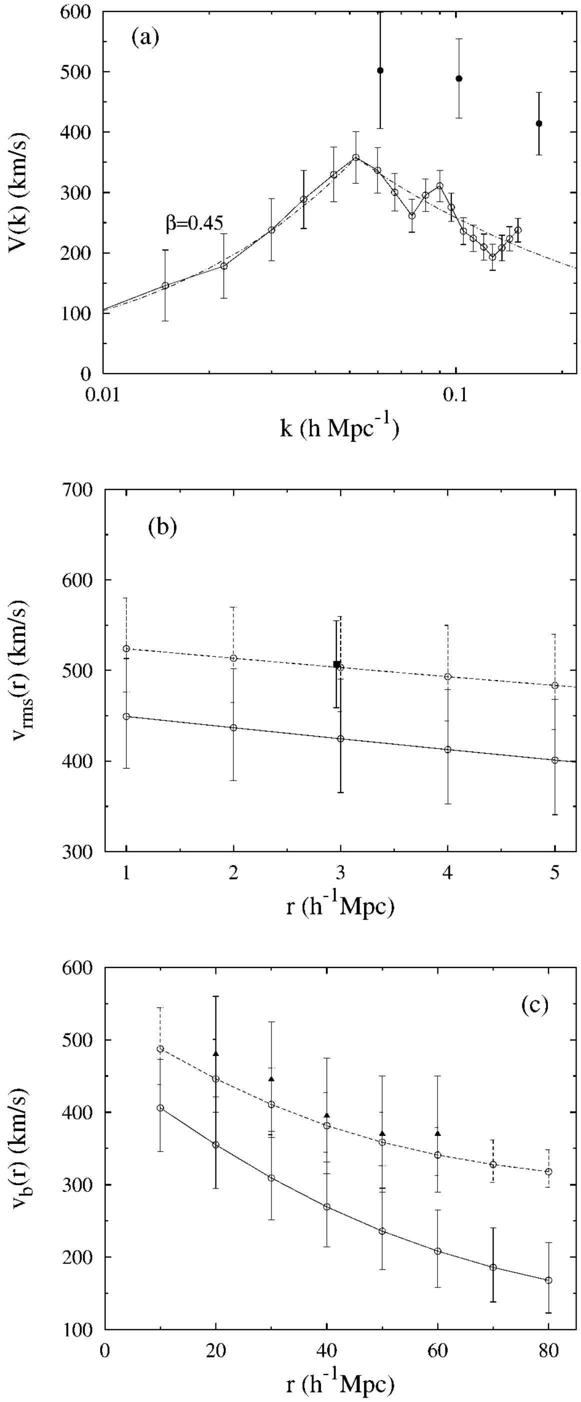

FIG. 5.—Velocity fluctuations in the Stromlo-APM redshift survey for the parameter $\beta = 0.45$. (*a*) The rms amplitude of velocity fluctuations (*open circles with solid line*). The dot-dashed line shows the approximation given by eq. (22). Filled circles represent the velocity rms amplitude from peculiar velocities. (*b*) The rms peculiar velocity of matter for $v_L = 0$ (*solid line*) and for $v_L = 270$ km s^{-1} (*dashed line*). Filled square represents the observed rms cluster peculiar velocity. (*c*) The rms amplitude of the bulk flow for $v_L = 0$ (*solid line*) and for $v_L = 270$ km s^{-1} (*dashed line*). Filled triangles represent the observed bulk velocities derived from the peculiar velocities.

radius $r = 60\ h^{-1}$ Mpc is $\simeq 210$ km s^{-1} if $v_L = 0$, and the rms amplitude of the bulk flow increases to $\simeq 340$ km s^{-1} if $v_L = 270$ km s^{-1}. (Here we assume that the contribution from large-scale fluctuations is similar at radii $r = 3\ h^{-1}$ Mpc and $r = 60\ h^{-1}$ Mpc. The contribution at larger radii can be somewhat smaller, depending how the large-scale power on wavenumbers of $\lambda \geq 120\ h^{-1}$ Mpc is distributed.)

7. DISCUSSION AND SUMMARY

In this paper, we have examined the power spectrum of velocity fluctuations in the universe starting from the peculiar velocities of galaxies and clusters of galaxies and from the power spectrum of the galaxy distribution in redshift surveys. There are various possible models for interpreting this data:

1. The power spectrum of velocity fluctuations follows a power law, $V^2(k) \sim k^2$, on large scales, achieves a maximum at wavenumbers $k_0 \sim 0.05$–$0.06\ h$ Mpc^{-1}, and declines as a power law, $V^2(k) \propto k^{-0.8}$, on smaller scales. The value of the function $V(k)$ at its maximum is ~ 500 km s^{-1}, and the rms velocity of matter smoothed with a Gaussian of radius $3\ h^{-1}$ Mpc is ~ 700 km s^{-1}. This power spectrum of velocity fluctuations is consistent with the power spectrum of density fluctuations derived by KD from peculiar velocities of galaxies, and with the power spectrum of the galaxy distribution in redshift surveys, provided that the parameter β is in the range 0.8–1.0. Data for peculiar velocities of galaxies yield the rms amplitude of velocity fluctuations $V(k) = 414 \pm 52$ km s^{-1} at wavenumber $k = 0.17\ h$ Mpc^{-1}; the velocity rms amplitude increases to $V(k) = 502 \pm 96$ km s^{-1} at $k = 0.06\ h$ Mpc^{-1} (see Fig. 1).

This power spectrum of velocity fluctuations is predicted in a mixed cold + hot dark matter model (CHDM) with density $\Omega_0 = 1.0$. Figure 6*a* shows the rms amplitude of velocity fluctuations predicted in CHDM models with neutrino densities of $\Omega_\nu = 0.2$ and $\Omega_\nu = 0.3$. We have used the transfer function computed by Pogosyan & Starobinsky (1995) and the *COBE* normalization derived by Bunn & White (1997). The initial fluctuations are assumed to be adiabatic and scale-invariant with $n = 1$. The baryonic density $\Omega_B = 0.05$ and $h = 0.5$. The power spectrum of velocity fluctuations predicted in these models is in good agreement with fluctuations derived from peculiar velocities of galaxies. This model is not consistent with the observed rms peculiar velocity of galaxy clusters determined by Bahcall & Oh (1996).

2. The shape of the power spectrum of velocity fluctuations is similar to that in model (1), but the amplitude of the power spectrum is lower. The transition between positive and negative spectral indices is smooth, without the peak at wavelength $\lambda \sim 120\ h^{-1}$ Mpc. The rms velocity of matter on scales of $\sim 3\ h^{-1}$ Mpc is in the range 450–500 km s^{-1}. This rms velocity is consistent with the power spectrum of the galaxy distribution in the Las Campanas redshift survey when the parameter β is in the range 0.6–0.7. The rms amplitude of velocity fluctuations is $\simeq 350$ km s^{-1} at a wavelength of $\lambda \simeq 100\ h^{-1}$ Mpc, and the rms amplitude of the bulk flow on a scale of $\sim 60\ h^{-1}$ Mpc is $\simeq 170$ km s^{-1}. This value is not consistent with the observed bulk velocity of galaxies.

A smooth power spectrum of velocity fluctuations is predicted in low-density cold dark matter (CDM) models. Figure 6*b* shows the rms amplitude of velocity fluctuations predicted in a flat CDM model with density parameter of $\Omega_0 = 0.3$, baryonic density $\Omega_B = 0.0125\ h^{-2}$, and a normalized Hubble constant $h = 0.65$. In this model, the rms peculiar velocity of matter $v_{\rm rms}(r = 3\ h^{-1}$ Mpc$) = 480$ km s^{-1}, and the bulk velocity is $v_b(r = 60\ h^{-1}$ Mpc$) = 265$ km s^{-1}.

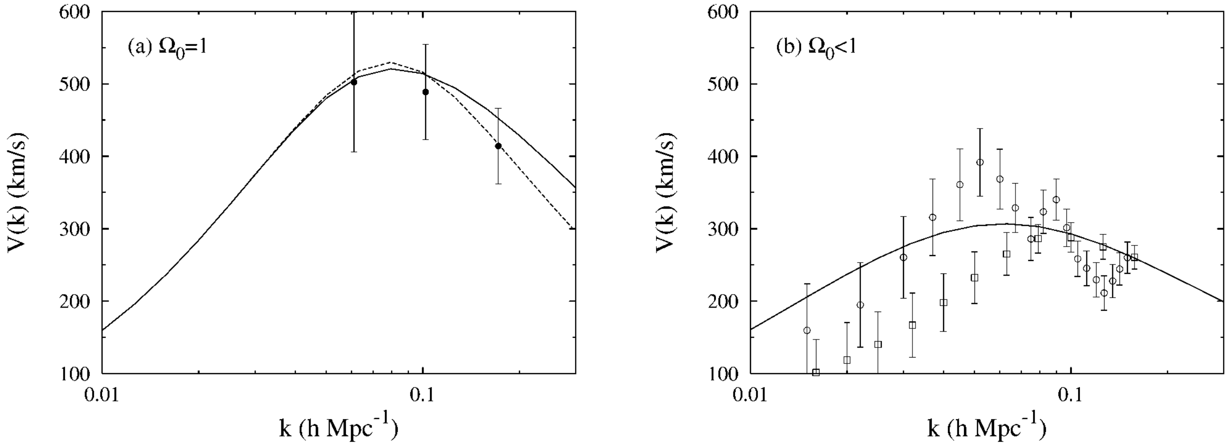

FIG. 6.—(*a*) The rms amplitude of velocity fluctuations in the CHDM models with neutrino densities $\Omega_\nu = 0.2$ (*solid line*) and $\Omega_\nu = 0.3$ (*dashed line*). Filled circles represent the velocity rms amplitude derived from peculiar velocities of galaxies. (*b*) The rms amplitude of velocity fluctuations in the flat CDM model with density parameter $\Omega_0 = 0.3$. Open circles and squares show the velocity rms amplitude for $\beta = 0.5$ derived from galaxy distribution in the Stromlo-APM and Las Campanas redshift surveys, respectively.

For comparison, we show in Figure 6*b* the function $V(k)$ derived from the Las Campanas and Stromlo-APM redshift surveys for $\beta = 0.5$. In the $\Omega_0 = 0.3$ model, this value of β gives a bias parameter $b \approx 1.0$. The amplitude of velocity fluctuations predicted in the low-density CDM model is consistent with the power spectrum of the galaxy distribution for wavenumbers $k > 0.06\ h^{-1}$ Mpc. This model is not consistent with the power spectrum of density fluctuations derived by Kolatt & Dekel (1997) from peculiar velocities of galaxies.

3. There is a peak in the power spectrum of velocity fluctuations in the universe at a wavelength of $\lambda_0 \simeq 120\ h^{-1}$ Mpc (or wavenumber $k_0 \simeq 0.05\ h$ Mpc^{-1}), and on larger scales, the power spectrum decreases with an index $n \simeq 0.5$–1.0. The maximum value of the function $V(k)$ is $\sim$420 km s^{-1} at a wavelength $\lambda = 120\ h^{-1}$ Mpc. The bulk velocity in this model is $v_b(r = 60\ h^{-1}\ \mathrm{Mpc}) \simeq 250$ km s^{-1}. The power spectrum of density fluctuations derived from peculiar velocities of galaxies by Kolatt & Dekel (1997) is correct on large scales, $\lambda \sim 100\ h^{-1}$ Mpc, but overestimated on smaller scales.

This power spectrum of velocity fluctuations is consistent with the power spectrum of the galaxy distribution in the Stromlo-APM redshift survey provided that β is in the range 0.5–0.6 (see Fig. 4). If the bias parameter $b \approx 1.0$, this value of β corresponds to a density parameter $\Omega_0 \approx 0.4$. This power spectrum of velocity fluctuations is also consistent with the observed rms peculiar velocity of galaxy clusters. The small-scale fluctuations at wavelengths $\lambda < \lambda_0$ and large-scale fluctuations at wavelengths $\lambda > \lambda_0$ give similar contributions to the velocity dispersion of galaxy systems.

The power spectrum of density and velocity fluctuations in the universe depends on the physical processes in the early universe. The peak in the power spectrum of the galaxy distribution at wavelength $\lambda \simeq 120\ h^{-1}$ Mpc may be generated during the era of radiation domination or earlier. One possible explanation for the presence of such a peak in the power spectrum is an inflationary scenario with a scalar field whose potential has a localized feature around some value of the field (Starobinsky 1992). In this scenario, the value of the corresponding characteristic scale in the universe is a free parameter, but the form of the power spectrum around this scale serves as a discriminating characteristic.

4. There is a peak in the power spectrum of velocity fluctuations in the universe at wavelengths $\lambda \simeq 120\ h^{-1}$ Mpc, as in model (3) above, but on larger scales the amplitude of fluctuations is higher than that estimated starting from the power spectrum of the galaxy distribution in the Stromlo-APM redshift survey and equation (22). For example, there could be another peak in the power spectrum of velocity fluctuations at wavelengths of $\lambda > 200\ h^{-1}$ Mpc. In this case, the fluctuations on large scales would contribute significantly to the velocity dispersion of galaxy systems. The observed rms cluster peculiar velocity is consistent with a smaller amplitude for the velocity fluctuations at intermediate wavelengths, $\lambda \sim 60$–$120\ h^{-1}$ Mpc, and thus with a lower value of β. For values of β in the range 0.4–0.5, the observed rms cluster peculiar velocity is consistent with the rms amplitude of the bulk flow, $\simeq$340 km s^{-1} at the scale of 60 h^{-1} Mpc. In this case, the value of the function $V(k)$ at wavelength $\lambda = 120\ h^{-1}$ Mpc is $\simeq$350 km s^{-1}. The power spectrum of velocity fluctuations in this model is not consistent with the power spectrum derived from peculiar velocities of galaxies.

Available data are insufficient to rule out any of the possibilities listed here. Direct measurements of the density parameter indicate that the mean density in the universe is lower than critical, the preferred range being $\Omega_0 \simeq 0.3$–0.5 (e.g., Dekel, Burstein, & White 1996). If the density parameter $\Omega_0 \simeq 0.4$, we can exclude the first model. This model predicts that clusters of galaxies move with high peculiar velocities and that the rms velocity of clusters is $\sim$750 km s^{-1}. Accurate peculiar velocities of galaxy clusters can serve as a discriminating test for this model. Larger redshift surveys, such as the Sloan Digital Survey (Gunn & Weinberg 1995), are required to accurately determine the power spectrum of the galaxy distribution on scales of $\lambda > 100\ h^{-1}$ Mpc and so to distinguish between the models listed here.

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