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GALACTIC BINARY GRAVITATIONAL WAVE NOISE WITHIN THE LISA FREQUENCY BAND

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ABSTRACT

Gravitation wave noise associated with unresolved binary stars in the Galaxy is studied with the special aim of determining the upper frequency at which it stops contributing to the rms noise level of the proposed space-borne interferometer (LISA). The upper limit to this background is derived from the statistics of SN Ia explosions, some of which can be triggered by binary white dwarf coalescences. The upper limiting frequency at which binary stochastic noise crosses LISA rms sensitivity is found to lie within the range $\approx$0.03–0.07 Hz, depending on the Galactic binary white dwarf coalescence rate. To be reliably detectable by LISA, the energy density of the relic cosmological background per logarithmic frequency interval should be $\Omega_{\rm GW} h_{100}^2 > 10^{-8}$ at $f > 0.03$ Hz.

Subject headings: binaries: close — gravitation — waves — white dwarfs

1. INTRODUCTION

Binary systems constitute at least half the stellar Galactic population ($\sim 10^{11}$ stars) and are reliable sources of gravitational waves (GWs). For a binary system consisting of two solar-mass stars, the characteristic time of orbital decay due to angular momentum removal by GW becomes shorter than the Hubble time ($\approx 15 \times 10^9$ yr) if its orbital period is less than about 14 hr. Observations of binary pulsars with a neutron star (NS) as secondary component (Taylor 1992) provide us with the strongest observational evidence for this fundamental process, and the most compact NS of black hole (BH) binaries merging as a result of GW emission are considered primary real targets for the initial GW interferometric LIGO-type detectors (Abramovici et al. 1992). An enormous energy is released during the coalescence of a compact NS + NS binary (typically, of the order of 10^{53} ergs), but the Galactic merging rate of NS + NS binaries is fairly small, 10^{-4} to a few 10^{-5} yr^{-1} (see Lipunov, Postnov, & Prokhorov 1997 for more details). To detect an acceptable number of such mergers per year (for example, three events per year with signal-to-noise ratio 3–5, see Thorne (1987)), the GW detectors should be sensitive to distances up to 200 Mpc. Typical frequencies of GWs emitted during compact binary NS coalescence are about 100–1000 Hz, depending on the mass of the stars involved.

Orbital frequencies at which a binary system may be observable are limited by the size of the components; the evolution of the orbital separation strongly changes when one of the stars fills its critical Roche lobe. Typically, mass exchange between the components has a much larger effect on the binary separation than the orbital angular momentum removal as the result of GWs, so the orbital frequency of a given binary system may fall within a wide range from a fraction of an hour to several years, depending on the initial parameters and details of evolution. The evolution of any star, however, results in the formation of a compact remnant (BH, NS, or white dwarf [WD]), so when two such compact stars remain in a binary, its orbital evolution is totally controlled by the removal of orbital angular momentum by GWs (the possible rare exceptions are a very hot young WD with a strong stellar wind or a low-mass WD companion being evaporated by a strong pulsar emission; we will not consider them here). When only GW emission drives the orbital evolution a simple analytical treatment is sufficient to fully describe it.

Since the number of binary stars in the Galaxy is very large, the GWs emitted by them (at strictly twice the orbital frequency if the orbit eccentricity is zero) form a stochastic background in the frequency range 10^{-7} to ~ 1 Hz (Mironovskij 1965; Rosi & Zimmerman 1976; Lipunov & Postnov 1987; Lipunov, Postnov, & Prokhorov 1987; Hills, Bender, & Webbink 1990; Lipunov et al. 1995). This background is interesting by itself; however, it is viewed as a noise burying a possible cosmological gravitational wave background (CGWB) that bears the unique imprint of physical processes occurring at the very early (near-Plankian) age of the universe (see, e.g., Grishchuk 1988 for a review).

A stochastic GW background is commonly measured in terms of the energy density per logarithmic frequency interval related to the critical energy density to close the universe, $\Omega_{\rm GW} = dE_{\rm GW}/d\ln f/\rho_{\rm cr} c^2$ ($\rho_{\rm cr} \approx 1.9 \times 10^{-29} h_{100}^2$ g cm^{-3}, where $h_{100} = H_0/100$ km s^{-1} Mpc^{-1} is the present value of the Hubble constant and c is the speed of light). Cosmic microwave background fluctuations experimentally detected in the last few years (Smoot et al. 1992; Strukov et al. 1993) put upper bounds on CGWB of order $\Omega_{\rm GW} = 10^{-14}$ at both LISA and LIGO frequencies assuming equation of state $p = -\epsilon$ at the inflationary stage of the universe. In this model, the CGWB spectrum is inversely proportional to the frequency keeping $\Omega_{\rm GW}(f) =$ constant at $f > 10^{-14}$ Hz (Rubakov, Sazhin, & Veryaskin 1982). If this is real, this CGWB has no chance of being detected with ongoing GW detectors (see Schutz 1997; Allen 1997). We should note, however, that the measurements of cosmic microwave background temperature fluctuations put constraints on the CGWB only at very low frequencies $f_H \sim H_0 \sim 10^{-18}$ Hz, so the dependence of CGWB energy density upon frequency is very crucial.

The situation, however, starts to change after a recent processing of the *COBE* data has provided us with some fresh information about the power-law spectral index of

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primordial perturbations (Bennett et al. 1996; Brukhanov et al. 1996). The point is that the spectrum of the primordial perturbations may be directly recalculated into the spectrum of relic GW background (see Grishchuk 1997 and references therein). According to Grishchuk (1997), the fact that the *COBE* data point to some deviations from Harrison-Zeldovich spectrum of the initial perturbations translates into the deviations from $p = -\epsilon$ equation of state at the epoch of inflation which would lead to serious changes in the CGWB as a whole. Now its energy density may increase with frequency reaching $\Omega_{\rm GW} = 10^{-8}$ at $f = 10^{-2}$ Hz. This conclusion holds regardless of the real cause of the observed CMB temperature fluctuations (either they are due to density fluctuations or relic gravitational waves). If Grishchuk's calculations are correct, this opens a novel possibility of detecting CGWB with the proposed LISA space interferometer (Schutz 1997). The conventional technique for detection of the stochastic GW backgrounds assumes the cross-correlation of outputs of at least two independent interferometers (Grishchuk 1976; Compton & Schutz 1996, and reference therein). The LISA project does not plan to have two independent interferometers; however, the strength of the signal ($\Omega_{\rm GW} = 10^{-8}$) is predicted to be so high that probably the signal could be directly detected.

This important finding raises the question: what frequency does the GW confusion limit from binary systems become lower than the detectability threshold dictated by the signal-to-noise ratio equal one? In other words, beginning at what frequency can we be sure that no known noise sources of astrophysical origin exist and therefore only cosmological background can contribute to the noise at these frequencies?

The aim of the present paper is to answer this question using new calculations of binary stellar evolution (see Lipunov et al. 1996a, 1996b, 1997 for more details and full references). The structure of the paper is as follows. In § 2 we derive the upper limit on the binary stochastic background in our Galaxy at frequencies 1 mHz–0.1 Hz (LISA diapason). At these frequencies, the background is mostly due to binary WD mergers. We use the SN Ia statistics to constrain galactic binary WD merging rate $\mathscr{R} < 1/300\ {\rm yr}^{-1}$. The upper limit on the background at these frequencies is $h_{\rm lim}(f) \simeq 4 \times 10^{-20}\,(f/10^{-3}\ {\rm Hz})^{-2/3}$ for the inverse-average distance to typical binary WD 10 kpc. In § 3, we calculate the binary Galactic background using Monte Carlo modeling of binary star evolution in the Galaxy. The comparison of the calculated backgrounds with the proposed LISA sensitivity is given in § 4. In the Appendix we give an alternative derivation of equation (8) for the part of the background formed by coalescing binary stars only.

2. UPPER BOUNDS ON THE BINARY STOCHASTIC GW BACKGROUND

It is widely recognized that merging compact binary stars (WD + WD, NS + NS ...) determine the high-frequency part of binary stochastic background (e.g., Lipunov et al. 1987; Hils et al. 1990). In this section we wish to show that this portion is mostly significant in the LISA frequency band and provides an upper limit to this background in general.

2.1. *GW Backgrounds Formed by Merging Binary Stars*

Consider a model galaxy consisting entirely of binary systems. Let us assume a stationary star formation rate (which describes well the situation in our Galaxy). We will be interested in only LISA frequency range, 10^{-4}–10^{-1} Hz, inside which only coalescing binary white dwarfs and binary neutron stars contribute. Even if binary neutron stars coalesce at a rate of 1/10,000 yr in the Galaxy (Lipunov et al. 1987, 1995; Tutukov & Yungelson 1993), their number still should be much smaller than the white dwarf binaries, and in this section we restrict ourselves to considering only binary WD.

As mentioned in the introduction, the stochastic GWB may be fully characterized by the energy emitted per logarithmic frequency interval. In the case of a binary system this energy is exactly equal to the change in the orbital energy of two stars during the time period needed for the GW frequency (which is twice the orbital frequency for circular orbits) to pass the frequency interval $\Delta f \approx f$. Therefore, in the stationary situation, the net energy emitted in GWs will be determined by the rate $\mathscr{R}$ at which the systems enter the specific frequency interval and the rate of their orbital frequency change $\dot{f}$. Clearly, if the binary orbit evolves only due to GW emission, the result will depend on the rate $\mathscr{R}$ only.

Indeed, the stationary implies that the number of binary WDs per unit logarithmic frequency interval may be determined from the continuity equation

$$dN/d\ln f \equiv N(f) = \mathscr{R}(f/\dot{f})\ . \quad (1)$$

The total energy emitted per second per unit logarithmic frequency interval at f by all such binaries in the galaxy is

$$dE/(dt\, d\ln f) \equiv L(f) = \sum_i L_i(f) = \tilde{L}(f)N(f) = \tilde{L}(f)\mathscr{R}(f/\dot{f})\ , \quad (2)$$

where $\tilde{L}(f) \propto (\tilde{\mathscr{M}}f)^{10/3}$ is the characteristic GW luminosity at frequency f which is dependent on the so-called "chirp mass" $\mathscr{M}$ of the binary system:

$$\mathscr{M} = M(\mu/M)^{3/5} \quad (3)$$

(M is the total and μ reduced masses). We use $\mathscr{M}$ for some average mass of the typical binary (see discussion followed eq. [6] below). For the orbital frequency change due to GWs we find

$$(f/\dot{f})_{\rm GW} = (2/3)(E_{\rm orb}/\dot{E}_{\rm orb})_{\rm GW} \quad (4)$$

(we have used the fact that $E_{\rm orb} \propto M_1 M_2/a$ and the third Kepler's law for binary semimajor axis a), so if $\dot{E}_{\rm orb} = (dE/dt)_{\rm GW}$ we arrive at

$$L(f) = (2/3)\mathscr{R}E_{\rm orb}(\tilde{\mathscr{M}}, f)\ . \quad (5)$$

For an isotropic background we would have

$$\Omega_{\rm GW}(f)\rho_{\rm cr}c^2 = L(f)/(4\pi c\langle r\rangle^2) = \mathscr{R}E_{\rm orb}(\tilde{\mathscr{M}}, f)/(6\pi c\langle r\rangle^2)\ , \quad (6)$$

where $\langle r\rangle$ is the inverse-square average distance to the typical source. Strictly speaking, this distance (as well as the chirp mass) may be a function of frequency since the binaries characterized by different chirp masses $\mathscr{M}$ may be differently distributed in the galaxy. We are highly ignorant about the real distribution of binaries in the Galaxy, but taking the mean photometric distance for a spheroidal distribution in the form

$$dN \propto \exp(-r/r_0)\exp[-(z/z_0)^2]$$

r is the radial distance to the Galactic center and z the height above the Galactic plane) with $r_0 = 5$ kpc and $z_0 = 4.2$ kpc with $\langle r \rangle \approx 7.89$ kpc is sufficient for our purposes.

Substituting $E_{\rm orb} \sim \mathscr{M} c^2 (\mathscr{M} f)^{2/3}$ into equation (6) we obtain

$$\Omega_{\rm GW}(f) \approx 2 \times 10^{-8} \mathscr{R}_{100} (f/10^{-3}\ {\rm Hz})^{2/3} \times (\tilde{\mathscr{M}}/M_\odot)^{5/3} (\langle r \rangle/10\ {\rm kpc})^{-2} h_{100}^{-2}\ , \quad (7)$$

where $\mathscr{R}_{100} = \mathscr{R}/(0.01\ {\rm yr}^{-1})$ is the Galactic rate of binary WD mergers.

In terms of the characteristic dimensionless amplitude of the noise background that determines the signal-to-noise ratio when cross-correlating outputs of two independent interferometers (cf. Thorne 1987, eq. [65]) we have

$$\begin{aligned} h_c(f) &= (1/2\pi)(H_0/f)\Omega_{\rm GW}^{1/2}\ , \\ &\approx 7.5 \times 10^{-20} \mathscr{R}_{100}^{1/2} (f/10^{-3}\ {\rm Hz})^{-2/3} \\ &\quad (\tilde{\mathscr{M}}/M_\odot)^{5/6} (\langle r \rangle/10\ {\rm kpc})^{-1}\ , \end{aligned} \quad (8)$$

irrespective of H_0 (naturally). Explicit derivation of equation (8) is given in the Appendix. Equation (8) shows that at high frequencies of interest here the GW background is fully determined by the galactic rate of binary WD mergers and is independent of (complicated) details of binary evolution at lower frequencies (for examples of calculated spectra at lower frequencies see Lipunov & Postnov 1987; Lipunov et al. 1987; Hils et al. 1990, and below).

All above considerations are valid for frequencies at which more than one binary system fall within the logarithmic frequency interval $\Delta f \approx f$. Formally this limiting frequency is specified by the requirement $N(f) \geq 1$, i.e.

$$f < f_{\rm cr} = 0.1\ {\rm Hz} (\tilde{\mathscr{M}}/M_\odot)^{-5/8} \mathscr{R}_{100}^{3/8}\ , \quad (9)$$

which is, however, already close to the limiting orbital frequency for a double WD binary. As the number of systems per logarithmic frequency interval rapidly decreases with frequency, $N(f) \propto f^{-8/3}$, the stochastic background starts forming shortly below this limiting frequency. Note that combining equations (8) and (9) we can find the relationship $h(f_{\rm cr}) \geq 8 \times 10^{-22} (f_{\rm cr}/0.01\ \ {\rm Hz})^{2/3} (\mathscr{M}/M_\odot)^{5/3} (r/10\ \ {\rm kpc})^{-1}$ above which binary-merger-formed GW background appears; this limit depends on the mean galactic distance r, which is known much more reliably than binary WD merger rate $\mathscr{R}$. For $\mathscr{M} < 1\ M_\odot$, however, this boundary lies below the rms LISA sensitivity at all frequencies.

2.2. *Notes on the Extragalactic Binary GW Background*

The derivation of the binary background given above (see also the Appendix) becomes more accurate for extragalactic binaries. The contribution from extragalactic binaries may be shown to be smaller than from galactic ones at all frequencies (see, e.g., Hils et al. 1990; Lipunov et al. 1995), but for completeness we give here the final result in terms of $\Omega_{\rm GW}$ which bears a clear physical meaning.

At the detector's frequency $f = f'/(1+z)$, galaxies lying inside the proper volume $dV(z)$ at redshift z contribute

$$\rho_{\rm cr} c^2\, d\Omega_{\rm GW}(f) = L(f')/(4\pi c D(z)^2) n(z) dV(z)\ , \quad (10)$$

where $L(f)$ is the GW luminosity at the source frequency f within each galaxy, $D(z)$ is the luminosity distance, n_G is the present density of galaxies, and $n(z) = n_G(1+z)^3$ is the density of galaxies at the redshift z without evolutionary effects. In the universe with zero curvature $dV(z) = 4\pi d^2(z) d[d(z)]$, where $d(z) = D(z)/(1+z)$ is the proper motion distance. Taking into account that the present density of galaxies can be rewritten through the fraction of luminous matter baryons,

$$n_G \approx 0.013 ({\rm Mpc}^{-3})(\Omega_b/0.005) h_{100}^2\ . \quad (11)$$

Equation (10) can be recasted in the form

$$d\Omega_{\rm GW}(f) = (2/3)\Omega_b \mathscr{R} \times \{E_{\rm orb}[\mathscr{M}, f(1+z)]/M_G c^2\}(1+z) d[d(z)]/c\ . \quad (12)$$

where M_G is the baryon mass of a typical galaxy per which the merging rate $\mathscr{R}$ is calculated.

For flat ($\Omega = 1$) standard cosmological model without Λ-term $d(z) = (2c/H_0)[1 - 1/(1+z)^{1/2}]$. Noticing that $E_{\rm orb}[f(1+z)]$ scales as $(1+z)^{2/3}$ (eq. [A1] from the Appendix), we may integrate equation (12) up to z_*, the redshift at which first WD mergers had started, to obtain

$$\begin{aligned} \Omega_{\rm GW}(<z_*) &= (6/7)\Omega_b t_{\rm H} \mathscr{R} (E_{\rm orb}/M_G c^2)[(1+z_*)^{7/6} - 1]\ , \\ &= (3/7)\Omega_b t_{\rm H} \mathscr{R} (\mathscr{M}/M_G)(x/2)^{2/3}[(1+z_*)^{7/6} - 1]\ , \\ &\approx 10^{-9} (\Omega_b/0.005) \mathscr{R}_{100} (10^{11} \mathscr{M}/M_G)(\mathscr{M}/M_\odot)^{2/3} \\ &\quad \times (f/1\ {\rm Hz})^{2/3} h_{100}^{-1} [(1+z_*)^{7/6} - 1]\ . \end{aligned} \quad (13)$$

This expression bears very clear physical meaning simply as the fraction of energy emitted in GW over the Hubble time ($t_{\rm H} = 2/3H_0$) by WD mergers with respect to the rest-mass energy of baryons in stars in the universe. The ratio between the cosmological and galactic backgrounds is

$$\begin{aligned} \Omega_{\rm GW}(z_*)/\Omega_{\rm GW} &= \langle r \rangle^2 \int^{z_*} (1+z)^{2/3} n(z) dV(z)/D(z)^2\ , \\ &\approx 0.05 (\Omega_b/0.005)(\langle r \rangle/10\ {\rm kpc})^2 h_{100}^{-1} [(1+z_*)^{7/6} - 1]\ , \end{aligned} \quad (14)$$

regardless of the poorly known merger rates, which shows that cosmological binary background is generally a few times smaller than the galactic one. Strong source evolution with redshift would increase this ratio. We study these effects elsewhere (Kosenko & Postnov 1997, in preparation).

2.3. *Sn Ia Rate as the Upper Limit of Binary WD Merger Rate*

The galactic merger rate of close binary WDs is unknown. One possible way to recover it is searching for close white dwarf binaries. A recent study (Marsh, Dhillon, & Duck 1995) revealed a larger fraction of such systems than had previously been thought. Still, the statistics of such binaries in the Galaxy remains very poor.

If coalescing binary WDs are associated with SN Ia explosions, as proposed by Iben & Tutukov (1984) and further investigated by many authors (for a recent review of SN Ia progenitors see Branch et al. 1995), their coalescence rate can be constrained using much more representative SN Ia statistics. Branch et al. (1995) concluded that coalescing CO-CO binary WDs remain the most plausible candidates, contributing most to the SN Ia explosions. The galactic rate of SN Ia is estimated $4 \times 10^{-3}\ {\rm yr}^{-1}$ (Tamman, Löffler, & Schröder 1994; van den Bergh & McClure 1994), which is close to the calculated rate of CO-CO coalescences [$\sim$(1–3) $\times 10^{-3}$]. The coalescence rate for He-CO WDs and

He-He WDs (other possible progenitors of SN Ia) falls 10 times short of that for CO-CO WD (Branch et al. 1995).

We note further that the idea of binary WD mergers as the main progenitors of SN Ia explosions started falling out of favor in the last few years (see especially critical studies by Nomoto et al. 1997). Evolutionary, double WDs are formed through the common envelope stage (Iben & Tutukov 1984; Webbink 1984). In the 1990s, with the calculation of new opacities, the possibility appeared for a WD, accreting even at a very high rate from the secondary companion in a binary system, to avoid the common envelope formation because of efficient stellar wind (Hachisu, Kato, & Nomoto 1996). This mechanism decreases the double WD formation rate and explains why the observed space density of binary degenerate dwarfs is smaller than that derived from Iben & Tutukov's scenario. Therefore, we may conclude that the observed SN Ia rate provides a secure *upper limit* to the double WD merger rate regardless of the evolutionary considerations.

3. A MODEL OF THE GALACTIC BINARY GW BACKGROUND

In this section we discuss the galactic GW background calculated with the scenario machine code for binary population synthesis in our Galaxy (see Lipunov et al. 1996a, 1996b for full description of the code). Briefly, we use a model galaxy with the total stellar mass of 10^{11} $M_\odot$ and half mass in binaries. We assume a constant star formation rate of 1 $M_\odot$ yr^{-1} which is a reasonable approximation for our Galaxy. Assuming the Salpeter mass distribution function $dN/dM \propto (M/M_\odot)^{-2.35}$, this star formation rate requires the minimal mass of the star to be $M_{\rm min} = 0.1\ M_\odot$ in order to produce the total stellar mass of the Galaxy during the Hubble time of 15 billion years. The initial mass ratios of the modeled binaries $q = M_2/M_1$ are assumed to be uniformly distributed from 0 to 1 (see, however, discussion in Lipunov et al. 1996a). The important evolutionary parameter, the efficiency of the common envelope stage $\alpha_{\rm CE}$, was fixed at 1. Another crucial evolutionary parameter, the distribution of the kick velocity imparted to a neutron star at birth, is not very important here as we are interested mostly in low-mass stars (with initial masses $M_1 \leq 10\ M_\odot$) which evolve ultimately into WDs.

The position of the Sun in the outskirts of the Galaxy makes the binary stochastic background anisotropic (see Lipunov et al. 1995). The exact calculations of this background thus would require the knowledge of the spatial distribution of binary stars in the Galaxy, which is uncertain, so we restrict ourselves to consider the average amplitude of the stochastic background using the mean photometric distance in the Galaxy 7.9 kpc as explained in the previous section.

The calculated background is shown in Figure 1 by the thick solid line. For comparison we plotted Bender & Hils's (1997) data (*filled quadrangles*).

4. DISCUSSION AND CONCLUSIONS

The upper frequency at which the stochastic GW background formed by galactic merging binaries becomes smaller than the detector's noise limit $h_{\rm rms}(f)$ may be derived from equation (8):

$$f_{\rm lim} = (0.02\ {\rm Hz})(h_{\rm rms}/10^{-20})^{-3/2}\mathscr{R}_{100}^{3/4}(\mathscr{M}/M_\odot)^{5/4} \times (r/10\ {\rm kpc})^{-3/2}\ .$$

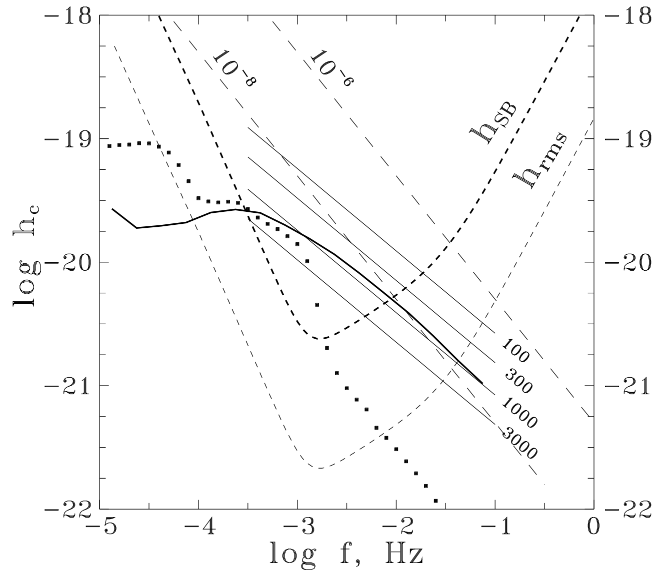

FIG. 1.—Galactic binary GW background h_c as given by Bender & Hils 1997; (*filled quadrangles*) and calculated for a model spiral galaxy with the total stellar mass 10^{11} $M_\odot$ (*solid curve*). Average photometric distance 7.9 kpc is assumed. Thin straight lines marked with 100, 300, 1000, 3000 are the analytical upper limit (eq. [8]) for binary WD merger rates 1/100, 1/300. 1/1000, and 1/3000 yr^{-1} in a model spiral galaxy, respectively, assuming $\mathscr{M} = 0.52\ M_\odot$. Straight dashed lines labeled 10^{-8} and 10^{-6} show GW backgrounds corresponding to constant $\Omega_{\rm GW}$. The proposed LISA rms noise level ($h_{\rm rms}$) and sensitivity to bursts $h_{\rm SB} = 5(5)^{1/2}h_{\rm rms}$ are also reproduced (see Thorne 1995; Fig. 14).

The upper limit (8) is plotted in Figure 1 for different rates of binary WD mergers $\mathscr{R}_{100} = 1$, 1/3, 1/10, 1/30 assuming the chirp mass $\mathscr{M} \approx 0.52\ M_\odot$ (as for two CO white dwarfs with equal masses $M_1 = M_2 = 0.6\ M_\odot$). These lines intersect the proposed LISA rms sensitivity at

$$f > f_{\rm lim} \approx 0.03\text{–}0.07\ {\rm Hz}\ . \quad (15)$$

This means that at frequencies higher than 0.07 Hz no continuous GW backgrounds of galactic origin are presently known to contribute above the rms level of LISA space laser interferometer. The contribution from extragalactic binaries is still lower regardless of the poorly known binary WD merging rate (at least in the limit of no strong source evolution with z). Other possible sources could be extragalactic massive BH binary systems (e.g., Hils & Bender 1995). Their number in the universe can be fairly high (e.g., Rees 1997), but no reliable estimates of their contribution are available at present. The lower limit (15) is already close to the LISA sensitivity limit at 0.1 Hz, but we stress that the assumptions used in its derivation are upper limits, so the actual frequency beyond which no binary stochastic backgrounds contribute may be three times lower. This precise limit depends on the details of binary WD formation and evolution which are still poorly known.

Figure 1 demonstrates that the calculated GW background intersects LISA sensitivity curve at frequencies $\sim$0.05 Hz, and Bender & Hils's curve at even lower frequencies $\sim$0.01 Hz. The latter is probably due to Bender & Hils's curve being derived from observational estimate of double WD galactic density in the solar neighborhoods; we stress once more than once formed, the binary WD will evolve until the less massive companion fills its Roche lobe;

unless the mass ratio is sufficiently far from one (see Webbink 1984), the merger should occur. Therefore, Bender & Hils's curve provides a secure *lower limit* to the galactic binary stochastic GW background. We also note that if the coalescence of two WD with inequal masses is prevented by mass transfer process (then the orbital period of the system begins increasing), some features on the shape of the background may emerge (see discussion of the minimum orbital period for cataclysmic variables in Lipunov et al. 1987).

Presently, we cannot rule out the high galactic double WD merger rate (1/300–1/1000 yr^{-1}) and therefore can consider f_{lim} to lie within the frequency range 0.01–0.07 Hz. We conclude that no GW background of galactic origin above these frequencies should contribute at the rms-noise level of LISA interferometer, and hence the detection of an isotropic stochastic signal at frequencies 0.03–0.1 Hz with an appreciable signal-to-noise level (which possibly may be done using one interferometer) would strongly indicate its cosmological origin. To be detectable by LISA, the power of relic GW background should be $\Omega_{\mathrm{GW}} h_{100}^2 > 10^{-8}$ in this frequency range.

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APPENDIX

EXPLICIT DERIVATION OF EQUATIONS

Equation (8) can be derived explicitly, without using the notion of Ω_{GW} (instead, one can use h_c to determine Ω_{GW}; both means are, of course, fully equivalent). We reproduce here this equation keeping proportionalities

$$h_c(f) \propto (1/r)\mathscr{R}^{1/2}\mathscr{M}^{5/6}f^{-2/3} .$$

We find it convenient to introduce new variables:

$$R_g = 2G\mathscr{M}/c^2 , \quad \text{gravitational radius of the chirp mass,}$$

$$R_l = c/(\pi f) , \quad \text{light cylinder radius for orbital frequency, } \pi f/2$$

and their dimensionless ratio

$$x(\mathscr{M}, f) \equiv x = (R_g/R_l) \propto \mathscr{M}f .$$

This unit allows us to write all relevant quantities in a compact form with clear physical meaning keeping G and c and facilitating numerical estimates. In these units, the orbital energy of a binary system with chirp mass $\mathscr{M}$ is

$$E_{\mathrm{orb}} = -(1/2)GM_1M_2/a = -(1/2)\mathscr{M}c^2(x/2)^{2/3} . \tag{A1}$$

The energy flux per unit time per unit area carried by a GW is

$$dE/(dA\,dt) = c^3/(16\pi G)\overline{(\dot{h}_+(t)^2 + \dot{h}_\times(t)^2)} , \tag{A2}$$

where h_+ and $h_\times$ are wave polarizations, the overbar means averaging over several cycles of the wave. Switching to the frequency domain in usual way (see, e.g., Thorne 1987), we obtain

$$dE/(r^2\,d\Omega\,df) = (c^3/G)(\pi f^2/2)(|\tilde{h}_+(f)|^2 + |\tilde{h}_\times(f)|^2) , \tag{A3}$$

where r is the distance to the source, $d\Omega = dA/r^2$ is the elementary solid angle.

Let us describe the stochastic backgrounds formed by many independent sources within unit logarithmic frequency interval at f by the characteristic strain amplitude defined as (see eq. [A2])

$$h_c^2(f) = 4G/(\pi c^3)(1/f^2)\sum_i^{N(f)} dE_i/(r_i^2\,d\Omega\,dt) , \tag{A4}$$

where $N(f) = \mathscr{R}f/\dot{f}$ is the number of sources assuming the stationary source birth rate R (see eq. [1]). We wish to show that this equation gives exactly the same result as equation (8).

Substituting equations (A2) and (A3) into equation (A4), using equation (1), and introducing, as before, some effective distance and the chirp mass of a typical source (now we will not mark them by special symbols), we arrive at

$$h_c^2(f) = \mathscr{R}f\langle|\tilde{h}(f)|^2\rangle , \tag{A5}$$

where $\langle|\tilde{h}(f)|^2\rangle = \langle\sum_{i=1}^2 |\tilde{h}_i(f)|^2\rangle$ stands for angle-averaged Fourier-components (with dimension [Hz^{-2}]) of $h(t)$:

$$\langle|\tilde{h}(f)|^2\rangle = (1/4\pi)(1/f^2)\int 2G/(\pi c^3)(dE/r^2\,df\,d\Omega)d\Omega ,$$

$$= (1/4\pi r^2)2G/(\pi c^3)(dE/df)/f^2 . \tag{A6}$$

(Note that eq. [A5] can also be derived directly from Parseval's theorem and the expression for the variance per logarithmic frequency interval of a stochastic noise associated with unresolved independent sources.)

Now notice that for circular orbits (our initial assumption) $dE/df = (dE_{\rm orb}/df)$ if far from coalescence (which is the case under consideration), so

$$dE/df = -(1/3)(\mathscr{M}c^2/f)(x/2)^{2/3} \propto \mathscr{M}^{5/3}f^{-1/3}\ , \tag{A7}$$

and from equation (A6)

$$\langle|\tilde{h}(f)|^2\rangle = (1/12\pi r^2)R_l^2/f^2(x/2)^{5/3} \propto \mathscr{M}^{5/3}f^{-7/3} \tag{A8}$$

(see eq [44] in Thorne 1987). Finally, from equation (A5) we obtain

$$h_c^2(f) = (\mathscr{R}/6\pi)(R_g/r)^2(x/2)^{-1/3}/f \propto \mathscr{M}^{5/3}f^{-4/3}\ , \tag{A9}$$

which yields exactly equation (8).

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