UPPER LIMIT TO THE DISTANCE TO THE LARGE MAGELLANIC CLOUD

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ABSTRACT

We remeasure the "light echo" times to the near and far side of the ring around SN 1987A using the method of Gould and the new reductions of the original UV spectra by Sonneborn et al. Under the assumption that the ring is circular, we obtain an upper limit to the distance to the Large Magellanic Cloud $\mu_{LMC} < 18.37 \pm 0.04$. This result is very close to Gould's original value and contradicts claims that the new spectra measurements lead to a significant increase in the distance estimate and a substantial increase in the statistical errors. We present new evidence (partly provided by A. P. S. Crotts) that the ring is intrinsically elliptical with axis ratio $b/a \sim 0.95$. For an elliptical ring, $\mu_{LMC} < 18.44 \pm 0.05$. We elaborate on Gould's original argument that the upper limit is robust, i.e., that it is only weakly sensitive to unmodeled aspects of the ring such as its hydrodynamic properties and ionization history.

Subject headings: galaxies: distances and redshifts — Magellanic Clouds — supernovae: individual (SN 1987A)

1. INTRODUCTION

The distance to the Large Magellanic Cloud (LMC) has been controversial. A low value ($\mu_{LMC} = 18.28 \pm 0.13$) is obtained from RR Lyraes calibrated by statistical parallax (Layden et al. 1996; Popowski & Gould 1997), while much higher values have recently been obtained from Hipparcos-based calibrations of RR Lyraes by Reid (1997) ($\mu_{LMC} = 18.65 \pm 0.1$) and by Gratton et al. (1997) ($\mu_{LMC} = 18.63 \pm 0.06$) and of Cepheids by Feast & Catchpole (1997) ($\mu_{LMC} = 18.70 \pm 0.10$).

The light curves of fluorescent UV line emission from the ring around SN 1987A permit an independent determination of the distance to the LMC (Panagia et al. 1991; Gould 1994, 1995; Sonneborn et al. 1997). Stripped to its essence, the method can be understood as follows. From simple geometric considerations the light curve should have two cusps, the first corresponding to the excess time for light to travel from the supernova to the near side of the ring to Earth (relative to direct travel from the supernova to Earth), and the second corresponding to the excess light travel time to the far side of the ring. The cusps arise because the parabola of constant delay time is tangent to the ring at these two times. The cusps are clearly observable in the N III] and N IV] lines. Assuming that the ring is coplanar with the supernova, the light travel time across the apparent minor diameter of the ring is then ($t_+ + t_-$) where $t_\pm$ are the times of the two cusps. The apparent major and minor diameters of the ring ($\theta_\pm$) have been measured by Plait et al. (1995) who find

$$\theta_+ = 1\overset{\prime\prime}{.}716 \pm 0\overset{\prime\prime}{.}022 , \quad \theta_- = 1\overset{\prime\prime}{.}242 \pm 0\overset{\prime\prime}{.}022 . \quad (1.1)$$

If the ring is assumed to be circular, one can therefore immediately derive a distance

$$D_{SN} = \frac{c(t_+ + t_-)}{\theta_+} . \quad (1.2)$$

Since the error in θ_+ is small ($\sim 1\%$), equation (1.2) has the potential to yield a remarkably precise distance provided that $t_\pm$ can be measured equally well.

If the ring is assumed to be circular, then there are two independent methods of estimating its inclination, i, one from the ratio of the axes, $\eta_\theta \equiv \theta_-/\theta_+$, and the other from the ratio of the delay times, $\eta_t \equiv t_-/t_+$,

$$i_\theta = \cos^{-1} \eta_\theta , \quad i_t = \frac{\pi}{2} - 2 \tan^{-1} \eta_t^{1/2} . \quad (1.3)$$

In practice therefore, rather than directly applying equation (1.2), one should appropriately weight both of these estimates of i to determine the angular size and inclination of the ring. A more serious potential problem with equation (1.2) is that it ignores various physical effects that could affect the measured values of $t_\pm$ other than simple light travel times. We address these in §§ 4 and 5. Nevertheless, equation (1.2) gives a good representation of the underlying simplicity and geometric nature of the method.

2. PREVIOUS WORK

To use the supernova ring to measure the distance to the LMC, one must both measure the distance to the ring and also determine the distance of the center of mass of the LMC relative to the ring. Since these are logically distinct, we review them separately.

2.1. *LMC Distance Relative to the Ring*

Jacoby et al. (1992) were the first to point out that at the position of SN 1987A, ~ 1 kpc from the center of the LMC bar, the plane of the LMC is ~ 500 pc in front of the LMC center of mass, assuming a standard LMC inclination angle of 27° (Bessel, Freeman, & Wood 1986). In principle, the supernova could be anywhere along the line of sight, but Gould (1995) argued that since SN 1987A is a Population I object, its most likely position is in the plane of the LMC. While fairly compelling, this argument is nevertheless wrong. Xu, Crotts, & Kunkel (1995) mapped the three-dimensional structure of the dust in front of SN 1987A using light echos. If the supernova were in the plane, one would expect to find large quantities of dust within ~ 100 pc of the supernova and progressively lower densities farther away. In fact, Xu et al. (1995) find almost no material within 100 pc, while the largest concentration of dust, the

huge N157C cloud, lies 490 pc in front of SN 1987A, implying that the plane of the LMC is ~ 500 pc in front of the center of mass. Since the supernova is ~ 500 pc behind the plane, we conclude that the supernova and the LMC are at the same distance, $\mu_{LMC} = \mu_{SN}$.

2.2. *Distance to the Ring*

Panagia et al. (1991) measured the light curves from four ions (N III], N IV], N V, and C III]). By comparing these with models, they found $t_+ = 413 \pm 24$ days and $t_- = 83 \pm 6$ days. They combined these with early measurements of $\theta_\pm$ by Jakobsen et al. (1991; which are $\sim 3\%$ smaller than eq. [1.1]) to obtain a distance of $\mu_{SN} = 18.55 \pm 0.13$. Dwek & Felten (1992) argued that the model adopted by Panagia et al. (1991) was inappropriate for a ring geometry. Gould (1995) developed the suggestion of Dwek & Felten (1992) into a systematic mathematical treatment. He then re-analyzed the data as presented in figure form by Panagia et al. (1991) and derived new delay times $t_+ = 390.0 \pm 1.8$ days, $t_- = 75.0 \pm 2.6$ days. Using these and the new measurements of the ring size by Plait et al. (1995; see eq. [1.1]), he obtained a distance estimate $\mu_{SN} = 18.350 \pm 0.035$. Gould (1995) restricted his analysis to the nitrogen ions, arguing that the carbon ion could not be put on a common basis with the other three. He also argued that since N V is a permitted line, it could well be optically thick which would vitiate the basic analysis. Nevertheless, Gould (1995) included N V in the analysis for completeness since, as he showed, it changed neither the best fit nor the error bars significantly.

Sonneborn et al. (1997) have now re-reduced the data originally used by Panagia et al. (1991). They obtain a new estimate for t_+ and t_- (or rather t_{max} and t_{rise} which are not exactly the same things, see § 6) based on a high-order polynomial fit to the N III] light curve,

$$t_{max} = 399 \pm 15 \text{ days},$$

$$t_{rise} = 83 \pm 4 \text{ days} \quad (\text{Sonneborn N III}]). \quad (2.1)$$

Based on these values including their "more realistic errors," they scale the results of Gould (1995) to obtain

$$\mu_{SN} = 18.43 \pm 0.10 \quad (\text{Sonneborn N III}]). \quad (2.2)$$

Although Sonneborn et al. (1997) do not regard this as a definitive result (they defer a more detailed treatment to future work), it has encouraged some to believe that the conflict between the supernova-ring distance and other estimates of the distance to the LMC is substantially less severe than originally claimed by Gould (1995 see, for example, Feast & Catchpole 1997). We therefore investigate the effect of applying Gould's (1995) method to the new reductions by Sonneborn et al. (1997).

3. REDETERMINATION OF THE RING DISTANCE

Except where otherwise noted, we follow the analysis given (and justified in some detail) by Gould (1995). We model the light curves according to the prescription given by his equation (2.3). This assumes that each point on the ring responds promptly (1 day rise time) to the EUV blast and then exponentially decays. (We relax the assumption of prompt response below.) We restrict attention to N III] and N IV] (see § 2.2) and to data from the first 700 days (see Gould 1995). We slightly modify Gould's (1995) procedure for establishing error bars. Gould assumed equal error bars

for all points and normalized these to make $\chi^2/dof \equiv 1$. However, Sonneborn et al. (1997) give exposure times for each data point. For N III] (but not N IV]), these fall into two classes: 48 with exposure times of ~ 300 minutes and 15 with exposure times of ~ 80 minutes. We find that the latter have about twice the scatter about the best fit curve as the former. We therefore set the errors on the shorter exposures at twice the value for the longer exposures before normalizing them by $\chi^2_{min} = 59$ (corresponding to 63 points less 4 fitting parameters, i.e., $t_\pm$, amplitude, and decay time). Note that some of raw N III] lines are saturated and are excluded from the analysis.

Figures 1–3 show best fits for the light curves together with the N III] and N IV] data upon which they are based. Figure 1 shows the data from Panagia et al. (1991) for N III while Figures 2 and 3 show the Sonneborn et al. (1997) data for N III] and N IV]. The fit to the data in Figure 1 is from Gould (1995), and the solid curves in Figures 2 and 3 are from the present work using the formalism of Gould (1995). (Fig. 2 also shows a 9th order polynomial fit which is discussed in § 6.)

Comparison of Figures 1 and 2 shows that there are many more points in the former (143 vs. 63). Part of the reason is that the circles and crosses represent separate reductions of the same data by the GSFC and VILSPA stations, respectively. Gould (1995) did not realize that the 45 VILSPA points were redundant and so incorrectly included them in his analysis. However, that still leaves the problem of why Panagia et al. (1991) show 98 points (*circles*) while Sonneborn et al. (1997) show only 63. Since Panagia et al. (1991) have not published their data in tabular form, we cannot resolve this issue. We assume that the new reductions by Sonneborn et al. (1997) are correct.
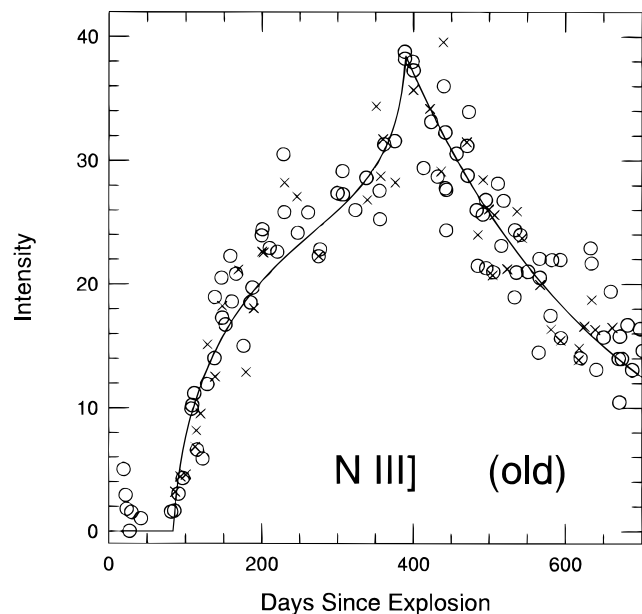

FIG. 1.—Previous data and light-curve fit for N III] emission from SN 1987A (based on Fig. 3 of Gould 1995) with data taken from Panagia et al. (1991). Intensity is in units of 10^{-14} ergs cm^{-2} s^{-1}. Shown are data points from GSFC (*circles*) and VILSPA (*crosses*). The curve is the best fit to the data using eq. (2.3) of Gould (1995) with $t_- = 83$ days, $t_+ = 390$ days and assuming a rise time of $t_* = 1$ day. The remaining parameters are the amplitude $A = 67$ and the decay time $\tau = 276$ days. VILSPA data have been multiplied by an amplitude correction factor $Q = 0.88$ which minimizes scatter of the fit.
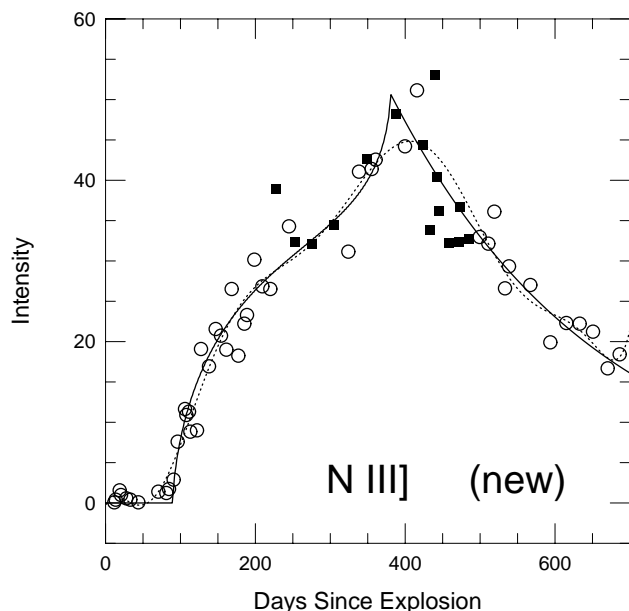
FIG. 2.—New data and light-curve fit for N III] emission from SN 1987A with re-reduced data taken from Sonneborn et al. (1997). Intensity is in units of 10^{-14} ergs cm^{-2} s^{-1}. Shown are data points from short (~ 80 minute; *solid squares*) and long (~ 250 minute; *open circles*) exposures. The solid curve is the best fit to the data using eq. (2.3) of Gould (1995) with $t_- = 88$ days, $t_+ = 381$ days and assuming a rise time of $t_* = 1$ day. The remaining parameters are the amplitude $A = 86$ and the decay time $\tau = 279$ days. The dashed curve is an $n = 9$ order polynomial fit to the data (following Sonneborn et al. 1997).

Figure 2 also shows a modest shift in the second cusp from 390 to 381 days for N III]. N IV] shows an almost identical shift. See below.

Figure 4 shows $\Delta\chi^2(t_-) \equiv \chi^2(t_-) - \chi^2_{\min}$, where $\chi^2(t_-)$ is the value of χ^2 for the best fit subject to constraining t_- to

FIG. 4.—Goodness of fit (χ^2 relative to its minimum value) for the light curves of N III] (*solid*) and N IV] (*dashed*), as a function of the time of the first caustic, t_-. The sum of the two curves is shown as a bold line. Value t_+ is held fixed at 381 and 378 days for N III] and N IV], respectively. To be compared with Fig. 1 of Gould (1995).

that value and t_+ to 381 and 378 days for N III] and N IV], respectively. The minimum value (defined above to be equal to the number of data points minus the number of parameters) is subtracted out to allow easy comparison of different curves. Figure 4 shows N III] and N IV] separately, as well as their sum. Figure 5 shows $\Delta\chi^2(t_+)$, which is defined similarly, with t_- held fixed at 66 and 88 days, i.e., at its best-fit values as shown in Figure 4.
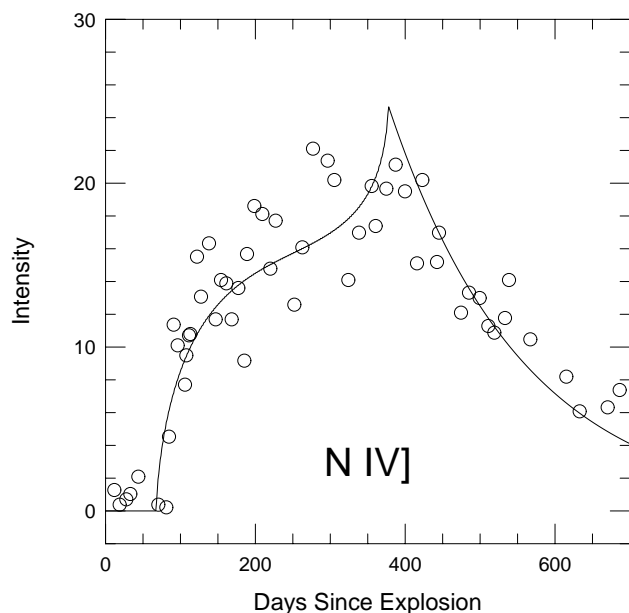

FIG. 3.—New data and light-curve fit for N IV] emission from SN 1987A with re-reduced data taken from Sonneborn et al. (1997). Similar to Fig. 2, except $t_- = 66$ days, $t_+ = 378$ days, $A = 53$, and $\tau = 180$ days. Only one type of data point is shown because all exposures are of similar duration.
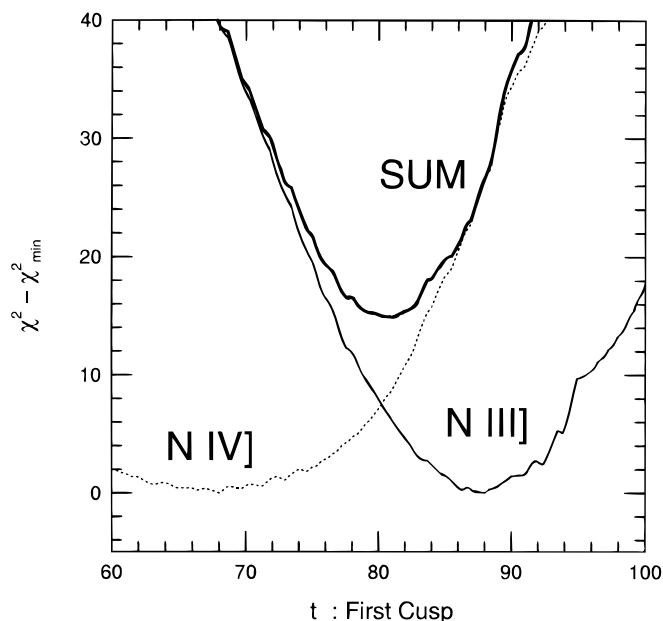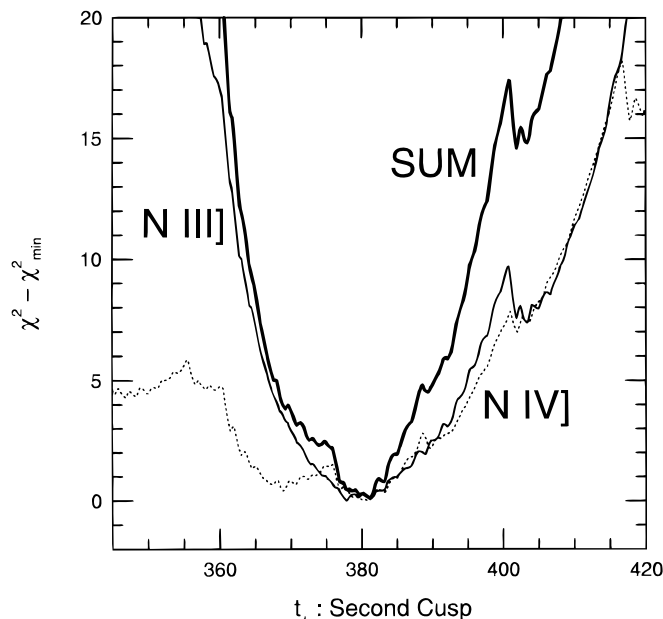

FIG. 5.—Goodness of fit (χ^2 relative to its minimum value) for the light curves of N III] (*solid*) and N IV] (*dashed*), as a function of the time of the second caustic, t_+. The sum of the two curves is shown as a bold line. Value t_- is held fixed at 88 days for N III] and 66 days for N IV. To be compared with Fig. 2 of Gould (1995).

When we use these measurements of $t_\pm$ to evaluate μ_{SN}, we will directly employ the χ^2 values shown in Figures 4 and 5. However, for purposes of discussion, it is useful to state the results in terms of best fits. To this end, we follow Gould (1995) and estimate the best fit as the center of the "2 σ interval" ($\Delta\chi^2 < 4$) and the error as one-fourth of the width of this interval. We then find $t_- = 87.8 \pm 2.7$ days and $t_+ = 380.7 \pm 6.3$ days for N III], and $t_- = 65.6 \pm 5.6$ days and $t_+ = 377.8 \pm 8.6$ days for N IV]. For the combined fit, we find

$$t_- = 80.5 \pm 1.7 \text{ days}, \quad t_+ = 378.3 \pm 4.8 \text{ days}. \quad (3.1)$$

Before using these results to measure the distance to the ring, we briefly comment on the nature of the changes relative to Gould's (1995) determination. The most striking difference is the increase in the error in t_+ based on N III] from 3.2 to 5.6 days, a factor of 1.75. A factor ~ 1.5 of this is due to the reduced number of points (see above). Most of the rest is due to the fact that the lower quality points (*squares*) are concentrated near the peak which adversely affects the accuracy of its determination. The best fit values for t_+ are consistent for the two ions, but those for t_- are not. This is basically the same situation found by Gould (1995), although the inconsistency for t_- is now less severe. The overall best-fit values rose by ~ 5 days for t_- and fell by ~ 12 days for t_+. This opposing motion will imply that the estimate of the distance changes very little, but that the consistency between i_t and i_θ is decreased.

Our primary method of estimating the distance to the ring is to assume that the ring is circular with unknown distance D, radius r, and angle of inclination i. For each distance, we then sum over all combinations of r and i and weight by the probability of obtaining the observed values of $\theta_\pm$ and $t_\pm$ given their model values. The model values are $\theta_+(D, r, i) = 2r/D$, $\theta_-(D, r, i) = 2r \cos i/D$, and $t_\pm(D, r, i) = (r/c)(1 \pm \sin i)$. The relative probability of a given distance is then

$$P(D) = \int dr \int di \, \exp\left[-\frac{1}{2} \left\{ \chi^2[t_+(D, r, i)] + \chi^2[(t_-(D, r, i)] \right. \right.$$

$$\left. \left. + \left[\frac{\theta_+(D,r,i) - \theta_{+,\text{obs}}}{\sigma_+} \right]^2 + \left[\frac{\theta_-(D,r,i) - \theta_{-,\text{obs}}}{\sigma_-} \right]^2 \right\} \right],$$

$$(3.2)$$

where $\theta_{\pm,\text{obs}}$ and $\sigma_\pm$ are given by equation (1.1), and $\chi^2[t_\pm(D, r, i)]$ are given by Figures 4 and 5. We then find,

$$\mu_{SN} = 18.372 \pm 0.035 \quad \text{(all data)}. \quad (3.3)$$

4. ALTERNATIVE INTERPRETATIONS

There are, however, several alternative viewpoints on how to treat the data. First, the two determinations of t_+ are quite consistent, but the two determinations of t_- are discrepant at the 4 σ level. Gould (1995) argued that this probably arose from the fact that the ring almost certainly deviates from the simple model that forms the basis of the light curve analysis. He showed that the determinations of t_+ were likely to be robust in the face of these deviations but those of t_- were not. Thus, one might assume that the ring is circular but ignore all information about t_-. The resulting distance is then $D_{SN} = ct_+/[\theta_+(1 + \sin i_\theta)]$, or

$$\mu_{SN} = 18.29 \pm 0.05 \quad \text{(excluding } t_-). \quad (4.1)$$

Second, it is not immediately obvious that the optical emission lines used by Plait et al. (1995) to measure the size of the ring arise from the same gas that generated the UV fluorescent emission used to measure $t_\pm$. Plait et al. (1995) find an upper limit for the full width half maximum of the ring of 0″.121, i.e., 7% of its major diameter. If the UV emission actually arose from the inner edge of the ring, but the optical emission used to measure the size of the ring came from the ring as a whole, then the UV light curve would yield an underestimate of the light-travel time across the optical ring diameter by up to 7%, and so (from eq. [1.2]) *underestimate* the distance to the ring by the same amount. In this case,

$$\mu_{SN} < 18.53 \pm 0.04 \quad \text{(optically thick N III/thin O III)}, $$

$$(4.2)$$

where the inequality reflects the fact that Plait et al. (1995) find only an upper limit for the ring thickness.

We consider this scenario to be highly implausible, fundamentally because O III and N III have similar ionization potentials so it is difficult to see how the mean radii of their emission distributions could be substantially different. We note the following specific observational evidence against separate emission. First, Plait et al. (1995) find that the best fit model for the O III emission is a crescent ring such as would be produced if the ring were optically thick to the EUV blast and thus only the inner face were illuminated. If this model is correct, then only a thin film of gas would contain either N III or O III, so the emission from the two ions would be cospatial. Plait et al. (1995) cannot actually rule out a toroidal distribution for the [O III] emission, such as would be produced if the ring were optically thin to the EUV blast.

However, even if the [O III] emission is toroidal, it should still be cospatial with N III]. The [O III] emission was first observed 1278 days after the supernova core collapse and, thus, 2.4 yr after the peak of fluorescent emission at $t_+ = 380$ days. If the ring were in fact optically thick to the EUV blast, so that only the inner face fluoresced in N III] and N IV], then how did the O III in the rest of the ring become ionized? There are only two possibilities: either it was ionized by a shock wave propagating through the ring from the inner face, or it was ionized by other wavelengths of UV radiation to which the ring was optically thin. There are two arguments against the shock-wave hypothesis. First, if such a turbulent process had proceeded across the ring in only 2.4 yr, then one would expect that in the next 2.4 yr, the ring would show some evidence of gas motions on the scale of its thickness. To the contrary, however, Plait et al. (1995) report that the late time behavior of the ring over the following 2.7 yr is simple fading. Second, from the Plait et al. (1995) measurement of the ring thickness and the 2.4 yr maximum time for the shock propagation, we can infer a minimum shock speed of 10^4 km s^{-1}. One would then expect a substantial imprint on the bulk expansion of the ring. However, Crotts & Heathcote (1991) measure an expansion velocity smaller than this by a factor 10^{-3}. If the O III was ionized by other wavelengths of UV radiation to which the ring was optically thin, then this radiation should have also generated N III (which has a similar ionization potential) throughout the ring. Since there is vastly more nitrogen in the ring as a whole than there is on the inner face, the centroid of the N III] emission should then have

been close to the center of the optical ring (as was assumed in deriving eq. [3.3]).

In brief, the fact that N III (which Fig. 2 shows to have a very well defined light curve) and O III have similar ionization potentials implies that their physical distributions should be similar. Equation (4.2) therefore represents an extreme upper limit for a physically implausible scenario.

Another possibility that warrants consideration is that the N III peak could be due to emission from relatively dense regions but the O III emission seen 2.4 yr later could be dominated by surrounding regions at lower density. This idea has been suggested at recent conferences but not published. However, in contrast to the scenario just discussed, such a process would not lead to a shift in the mean distance of the emitting gas from the center of the ring and, therefore, would not affect the estimate of the distance to the ring.

Finally, the assumption that the ring is circular may not be valid. Gould (1994) showed that if the ring is elliptical, but if $i_t \sim i_\theta$ to within statistical errors, then the inferred distance is overestimated by a factor $(1 + 0.4e^4)$, where e is the eccentricity. Using equations (1.1), (1.3), and (3.3), we find

$$i_\theta = 43°.6 \pm 1°.3 , \quad i_t = 40°.5 \pm 0°.5 . \quad (4.3)$$

These values are discrepant at the 2 σ level, implying that Gould's (1994) theorem does not strictly apply. For the particular set of measurements (1.1) and (3.3), the maximum distance occurs if the apparent minor axis of the ellipse is aligned with the true minor axis, in which case equation (1.2) is modified to become $D_{SN} = c(t_+ + t_-) \cos i_\theta / \theta_-$. This yields

$$\mu_{SN} = 18.44 \pm 0.05 \quad \text{(aligned ellipse)} . \quad (4.4)$$

The axis ratio in this case would be

$$\frac{b}{a} = \frac{\cos i_\theta}{\cos i_t} = 0.95 \pm 0.02 \quad \text{(ring)} , \quad (4.5)$$

and the eccentricity would be $e \sim 0.3$. Crotts, Kunkel, & Heathcote (1995) showed that the three-dimensional structure of the double-lobed nebula (of which the ring forms a "waist") is close to axisymmetric but did not attempt to quantify the possible deviations from axisymmetry. However, A. P. S. Crotts (1997, private communication) has now conducted a new analysis of the Crotts et al. (1995) data and has generously made these available in advance of publication. He finds that the nebula is intrinsically flattened in the sense that the shorter diameter is approximately aligned with the apparent minor axis of the ring. If the axis of the three-dimensional structure is assumed to be inclined at $i = 40°.5$, then Crotts finds that the axis ratio of the nebula is almost independent of distance from the ring and has a mean value of

$$\frac{b}{a} = 0.95 \pm 0.02 \quad \text{(nebula)} . \quad (4.6)$$

The striking agreement between equations (4.5) and (4.6) (which were derived independently) suggests that the ring may well be elliptical.

5. UPPER LIMITS

Equation (3.3) and its various modifications in § 4 implicitly assume that the delay times are exactly equal to the excess distance divided by the speed of light. That is, it is assumed that the fluorescent emission begins immediately when the EUV blast hits the ring gas. It is possible that the emission is delayed while the gas recombines from highly ionized states or by some other unrecognized process. However, such delays can only cause one to *overestimate* the physical size of the ring and thus, by equation (1.2), overestimate the distance to the ring. Thus, all distance estimates should be regarded as upper limits rather than measurements. Since, the ring could plausibly be elliptical, we adopt the overall upper limit from equation (4.4), and find

$$\mu_{LMC} < 18.44 \pm 0.05 \quad \text{(upper limit)} . \quad (5.1)$$

Instead of settling for an upper limit, one might attempt to estimate the response time of the gas and include this effect in the determination of μ_{LMC}. This approach is obviously less conservative, and it is not the primary one that we use in this paper. Nevertheless, for some purposes a best estimate is to be preferred to an upper limit, even if it is less reliable.

We proceed cautiously. We note from the detailed light curves shown in Figure 9 of Sonneborn et al. (1997) that the initial response of the gas is clearly a function ionization state, with the lower states responding later. In particular, N III] seems to be responding about 14 days later than N IV]. N IV] seems to be responding later than N v, but since the latter may be optically thick in emission, this difference may be due to emission arising from different patches of gas. We therefore assume that the N IV] responds instaneously (after the 1 day rise time of the EUV blast) and that the N III] responds 14 days later. This is equivalent to moving the N III] (*solid*) curves by 14 days to the left in Figures 4 and 5. For the t_- graph shown in Figure 4, this translation improves the agreement between the ions substantially. For t_+, the previous good agreement between the ions is not affected because the N IV] minimum is rather broad and the translation moves the N III] minimum from one end of this trough to the other.

The resulting parameter estimates are $t_+ = 370.8 \pm 5.0$ days, $t_- = 72.3 \pm 2.4$ days, and $i_t = 42°.3 \pm 0°.8$. Comparing the last with equation (4.3), we see that ring is now consistent with being circular. If it is assumed to be circular as in § 3, then $\mu_{LMC} = 18.26 \pm 0.04$. If it is assumed to be elliptical with an orientation as described above equation (4.4), then $\mu_{LMC} = 18.30 \pm 0.05$. The two results are similar because the ring is consistent with being circular.

These estimates lie significantly below the upper limit (5.1). We caution that they are derived with the aid of assumptions that are not as secure as those used to derive the basic results of this paper. Nevertheless, they serve to underscore the fact that the upper limit (5.1) is conservative and that the true value is probably lower because of the delayed response of the gas, especially N III].

6. CRITIQUE OF DISTANCE ESTIMATE BY SONNEBORN ET AL. (1997)

As we discussed in § 2, Sonneborn et al. (1997) used the same data that we have analyzed here to derive a longer time to peak light ($t_{max} = 399$ days vs. $t_+ = 379$ days) with larger errors (15 days vs. 4.8 days), implying a longer distance to the supernova ring, also with larger errors ($\mu = 18.43 \pm 0.10$ vs. $\mu = 18.37 \pm 0.04$). While Sonneborn et al. (1997) regarded their analysis as only "preliminary,"

others have taken it as superceeding that given by Gould (1995) who derived results very similar to those reported here. We therefore address four questions related to these conflicting distance estimates. Why did Sonneborn et al. (1997) obtain a longer time to peak light? What significance does this longer time to maximum have for the distance to the ring? Which curve gives a better fit to the data? Why are the Sonneborn et al. (1997) error estimates so much larger than ours? The answers to these questions are directly related to the nature of the method proposed by Gould (1995) for obtaining an *upper limit* to the ring distance.

Why did Sonneborn et al. (1997) obtain a longer time to peak light? Sonneborn et al. (1997) fit the data to a polynomial of order n, where n is a "high," but otherwise unspecified number. In order to make a concrete comparison of our work with theirs, we choose $n = 9$ and fit to the first 700 days of data. This choice is made based on the local minimum in $\chi^2_{\mathrm{eff}} = \chi^2 - (n + 1)$ at $n = 9$. That is, there is no statistical justification for incremental increases in the number of parameters beyond $n = 9$. (For $n \geq 13$, χ^2_{eff} begins to decline again, but the resulting curves have the clear appearance of "following the scatter.") It is immediately clear from Figure 2 that the prompt-response curve has an earlier maximum than the polynomial because it has an *asymmetric cusp* which rises much more steeply than it falls. The polynomial, by contrast, is approximately symmetric in the neighborhood of the peak.

What significance does this longer time to maximum have for the distance to the ring? In a word, "none." It is certainly possible to draw curves that come close to most data points and that have peaks at times $t_{\mathrm{max}} > t_+$. However, the t_{max} from such curves are not related in any simple way to the light-travel time to the far side of the ring and therefore cannot be used to estimate the size of the ring (and hence its distance). The polynomial in Figure 2 is an excellent example of such a curve. It shows a characteristic half-width at maximum

$$\Delta t = \left[\frac{F(t_{\mathrm{max}})}{F''(t_{\mathrm{max}})} \right]^{1/2} = 125 \text{ days} , \qquad (6.1)$$

where $F(t)$ is the polynomial. Since the light curve from prompt-response fluorescence is cuspy, the only *physical* way to produce such a broad peak is for the response function of the gas to have a rise time $\sim 2 \, \Delta t \sim 250$ days. If this were the case, the excess light travel time would be a time $\sim t_{\mathrm{max}} - \Delta t \sim 275$ days, not 400 days.

Which curve gives a better fit to the data? The prompt-response curve is favored over the polynomial by more than 3 σ. To determine this, we normalize the error bars (as above) to make $\chi^2 = 59$ (the number of degrees of freedom) for the best-fit prompt-response curve. We then find $\chi^2 = 63$ for the polynomial. Since the polynomial fit has six more free parameters, this implies $\Delta\chi^2 = 10$, corresponding to 3.1 σ. However, we stress that this worse fit of the polynomial curve has no bearing on the problem of establishing an upper limit to the distance. If the polynomial gave a better fit, it would not imply a larger estimate for the ring size. On the contrary, it would tend to indicate a slow rise time for the fluorescence and hence an even smaller (and closer) ring.

Why are the Sonneborn et al. (1997) error estimates three times larger than ours (15 vs. 5 days)? First, we derived our estimate from N III] and N IV] together while they used only

N III]. If we restrict our measurement to N III], the error in t_+ is 6.3 days. Second, as we now show, Sonneborn et al. (1997) significantly overestimated their errors. Consider a general linear function, $F(t; a_1, \ldots, a_n) = \Sigma_i a_i f_i(t)$, of which a polynomial is one example. Let a_i^* be the best-fit parameters for the data. The time of maximum of this best-fit curve is a solution of the equation $F'(t_{\mathrm{max}}; a_i^*) = 0$, where the prime indicates differentiation with respect to time. If the parameters of the fit deviate by δa_i, then the solution of this equation will change by $\delta t_{\mathrm{max}} \simeq -\Sigma_i \delta a_i f_i'(t_{\mathrm{max}})/F''$. Thus, the error in t_{max} is,

$$[\mathrm{var} \, (t_{\mathrm{max}})]^{1/2} = \langle (\delta t_{\mathrm{max}})^2 \rangle^{1/2}$$

$$= \frac{[\Sigma^n_{i,j=1} c_{ij} f_i'(t_{\mathrm{max}}) f_j'(t_{\mathrm{max}})]^{1/2}}{F''(t_{\mathrm{max}})} , \quad (6.2)$$

where $c_{ij} \equiv \langle \delta a_i \delta a_j \rangle$ is the covariance matrix of the a_i as derived by standard methods (e.g., Press et al. 1989) and as returned by linear fitting programs. That is, the error in t_{max} is the ratio of the error in the first derivative to the second derivative, both evaluated at t_{max}. Using equation (6.2), we find $[\mathrm{var} \, (t_{\mathrm{max}})]^{1/2} = 6.3$ days for the polynomial curve, i.e., the same as for the cuspy prompt-response curve.

7. CONCLUSIONS

The new reductions of the UV fluorescent emission-line data for SN 1987A do not result in any major change in Gould's (1995) upper limit for the distance modulus to the LMC under the assumption that the ring is circular, $\mu_{\mathrm{LMC}} < 18.37 \pm 0.04$, although the precision of the agreement between this and equation (3.3) is the result of an accidental cancellation of two effects, each 0.02 mag. However, new observational evidence for an elliptical ring (previously considered to be an implausible hypothesis by most workers in the field) raises the upper limit to $\mu_{\mathrm{LMC}} < 18.44 \pm 0.05$. The result is still in strong conflict with the recent Hipparcos-calibrated estimates of $\mu_{\mathrm{LMC}} \sim 18.65 \pm 0.1$ based on RR Lyraes and Cepheids (Reid 1997, Gratton et al. 1997; Feast & Catchpole 1997).

The robustness of the upper limit derives from the fact that the delay times $t_\pm$ measure the light-travel time across the diameter of the ring (eq. [1.2]). There are possible physical mechanisms that could retard these times and so cause one to overestimate the distance. However, there are none (except superluminal motion) that could accelerate them.

We have shown that the larger value of the time of maximum derived by Sonneborn et al. (1997) is the result of choosing a nonphysical parameterization of the light curve. Exactly the same criticism could be made of the original determination by Panagia et al. (1991), and, in fact, just such a criticism was made by Dwek & Felten (1992) and elaborated upon by Gould (1995). We note that the simple four-parameter physically based light curve of Gould (1995) fits the current data significantly better than a 10 parameter polynomial.

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