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# THE QUANTUM RADIATION FORMULAE OF A NEW RADIATION MECHANISM IN CURVED MAGNETIC FIELDS

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## ABSTRACT

A new radiation mechanism for relativistic charged particles moving in curved magnetic fields has been put forward recently by Zhang & Cheng. This radiation mechanism generalizes all the classical results of ordinary synchrotron and curvature radiation and reveals the inherent linkage and unification between them. Since the magnitude of the pulsar magnetic field is generally $\sim 10^{11}$–$10^{13}$ G, the quantum effects must be taken into account. Applying the generalized method of equivalent photon scattering developed recently by Lieu & Axford, we present the quantum-limited synchrocurvature radiation formulae in this paper, which elaborates the essence of this new radiation mechanism in a deeper sense. The formulae we present could be extensively applied to various kinds of practical curved magnetic fields, including those of pulsars.

*Subject headings:* MHD — polarization — radiation mechanisms: nonthermal — relativity

## 1. INTRODUCTION

Of the radiation from a charged particle, synchrotron and curvature radiation are the most important. They are extensively applied in astrophysics, especially to account for the radiation mechanisms in pulsar magnetospheres (Michel 1991; Mészáros 1992). In the previous discussions, they have been treated as two independent mechanisms and expressed as two different formulae, although they are both radiation from accelerated particles moving in magnetic fields. The radiation features of synchrotron radiation are independent of the radius of curvature of magnetic fields, while curvature radiation is caused by particles moving along the curved field lines; they do not exist at the same time. Magnetic field lines, however, are often curved, as, e.g., in pulsar magnetospheres; therefore, it is unreasonable to apply these two radiation mechanisms in different sections of pulsar magnetospheres as had been done in all previous models.

During the last decade, some researchers have not been satisfied with the idea that synchrotron radiation formulae, which are suitable only for particles moving in uniform and straight magnetic fields, have to be applied in curved magnetic fields. Some of them thought that there is also an ingredient of curvature radiation, in addition to synchrotron radiation, in the total power radiated, but they did not know what the ratio is, so they attempted to replace it with an adjustable parameter; however, they did not succeed.

The influence of the curvature of a magnetic field on the radiation from relativistic charges has been considered in several recent works by Zhang & Cheng (Zhang & Cheng 1995a, 1995b; Cheng & Zhang 1996). They proposed a new and more general radiation mechanism, instead of simple synchrotron and curvature radiation mechanisms, to describe the radiation emitted by charged particles moving in curved magnetic fields; this is consequently referred to as synchrocurvature radiation. It has also been demonstrated that the spectrum, power, characteristic frequency, and polarization properties of this radiation could be expressed by the same formulae that generalize their common characteristics and discrepancies. Moreover, these formulae are identical to those of either synchrotron radiation or curvature radiation in certain parametric regions. This implies that these two known radiation mechanisms are united; i.e., that they are only two special cases of a universal radiation mechanism. Therefore, we solved the above problem of total power by deriving formulae for synchrocurvature radiation that provide its radiation spectrum, characteristic frequency, and polarization properties.

The universal radiation formulae we derived indicate that the radiation properties of both synchrotron and curvature radiation models in curved magnetic fields differ obviously from realistic cases. For instance, curvature radiation will not exist if the pitch angle $\alpha \neq 0$, but the power calculated with the synchrotron radiation formula differs by several orders from that calculated with the synchrocurvature radiation formula that takes the curvature of the magnetic field lines into account, as shown in Figure 1. Furthermore, our results also give a more accurate definition of curvature radiation. Curvature radiation had been considered as a radiation mechanism only for charges moving along the magnetic field lines (i.e., at pitch angle $\alpha = 0$). The synchrotron radiation mechanism is the case for $\alpha \neq 0$, even for very small $\alpha$. The power of synchrotron radiation decreases with $\sin^2 \alpha$, and synchrotron radiation becomes curvature radiation suddenly when $\alpha = 0$. The criterion we give is that the radiation spectrum of a charged particle will become the curvature radiation spectrum when $\sin^2 \alpha \ll r_B/\rho$ (where $r_B$ is the cyclotron radius and $\rho$ is the radius of curvature of the magnetic field); thus the curvature of magnetic fields plays a remarkable role even when $\alpha \neq 0$.

In fact, the magnetic field of a pulsar is not only curved but also very strong ($\sim 10^{12}$ G). The characteristic frequency $\omega_c$ calculated on the basis of classical electrodynamics will increase with the magnitude of the magnetic field. When $\hbar\omega_c$ becomes comparable to the energy of the relativistic particle, the quantum effects become important. Therefore, it is necessary to generalize the synchrocurvature radiation mechanism to the quantum limit. The quantum effects are characterized by the parameter $\xi = \hbar\omega_c/\gamma mc^2$. For synchrotron radiation, $\gamma B/B_c$ is often used instead, where $B_c = m^2c^3/\hbar e \simeq 4.414 \times 10^{13}$ G is the Schwinger magnetic field and $\gamma$ is the Lorentz factor. Theoretically, the complete description of all the radiation processes in very strong magnetic fields can be obtained within the framework of relativistic quantum electrodynamics (QED). However, it

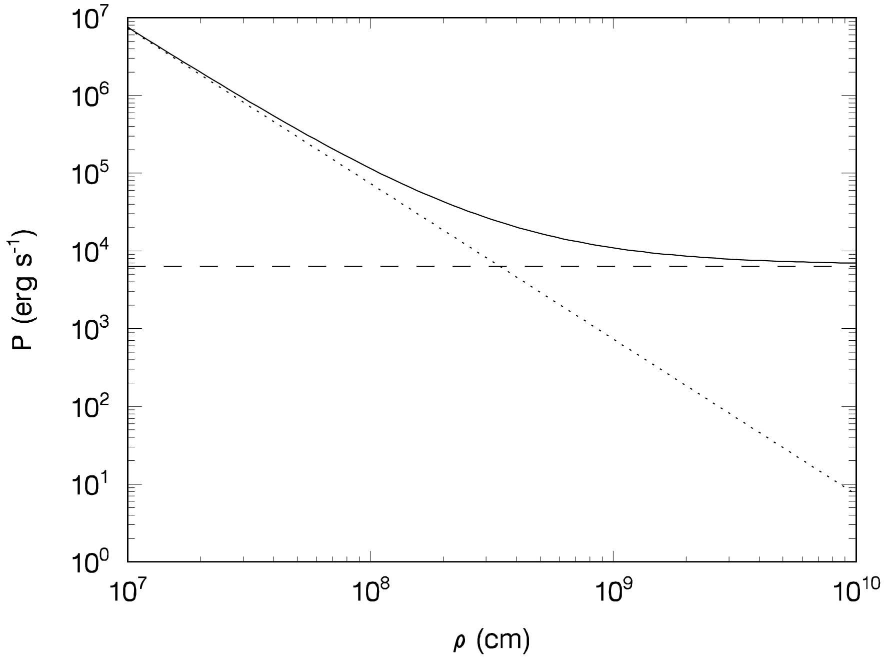


FIG. 1.—Radiation power $P$ versus $\rho$ for the synchrocurvature radiation mechanism (*solid curve*), the synchrotron radiation mechanism (*dashed line*), and the curvature radiation mechanism (*dotted line*). (Parameters are $\sin\alpha = 10^{-3}$, $\gamma = 2 \times 10^7$, and $B = 10^5$ G.)

is almost impossible to calculate quantized synchrocurvature radiation by applying QED as we have done with synchrotron radiation (Harding & Preece 1987; Sokolov & Ternov 1986; Herold, Ruder, & Wunner 1982; Bussard 1984; Latal 1986; Barring 1988; Mészáros 1992) because the symmetry of the particles' motion in a curved magnetic field is less than in a uniform and straight one and because it is ambiguous to define a spin operator in a curved magnetic field. Fortunately, we derive the radiation spectra, power, and polarization properties of synchrocurvature radiation in quantum limits, adopting the generalized method of equivalent photon scattering developed by Lieu & Axford (1993, 1995). The main idea of this elegant method is that all the classical and quantum radiation properties can be obtained simply from the viewpoint of the inverse Compton scattering of equivalent photons in a generalization of the Fermi-Weizsäcker-Williams method (GFWW). The distinguishing advantage of this method is that it is easy to generalize the emissivity formula to fit into the quantum limit and to include the case of three-dimensional orbits in a nonuniform field. Consequently, this is a very practical method (Lewin & Barber 1995).

In this paper, the synchrocurvature radiation mechanism of Zhang & Cheng is generalized to suit superstrong magnetic fields and charges with spin $\frac{1}{2}$. The quantum-limited synchrocurvature radiation formulae for a spinless particle (a Klein-Gordon particle) and for a spin $\frac{1}{2}$ particle (a Dirac particle) are presented in § 2 and § 3, respectively. In § 4, the discrepancy between classical and quantum radiation properties are also discussed. Because they take quantum effects into account, these formulae are suitable for describing the radiation from the magnetosphere of a pulsar, which provides a favorable opportunity to develop perfect models.

## 2. QUANTUM-LIMITED SYNCHROCURVATURE RADIATION FOR KLEIN-GORDON PARTICLES

For simplicity, let us consider a charged particle with mass $m$ and charge $e$ that is moving around a circular magnetic field that is uniform in the $z$- direction. Using the approximations mentioned, the position vector of the particle at a particular instant can be expressed as

$$\boldsymbol{r} = -\left\{\rho \sin(\Omega_0 t) + \frac{\gamma_B}{2}[\sin(\Omega_0 + \omega_B)t + \sin(\Omega_0 - \omega_B)t]\right\}\boldsymbol{i}^0 + \left\{\rho \cos(\Omega_0 t) + \frac{r_B}{2}[\cos(\Omega_0 - \omega_b)t + \cos(\Omega_0 + \omega_B)t]\right\}\boldsymbol{j}^0 + r_B \sin(\omega_B t)\boldsymbol{k}^0 , \quad (1)$$

where $\rho$ is the curvature radius of the field lines, $\Omega_0$ is the angular velocity of the guiding center along the magnetic field lines, $\omega_B$ is the cyclotron frequency, and $r_B$ is the cyclotron radius, respectively. Without loss of generality, we can assume that the observer is located on the $(x, z)$ plane and let $\theta_0$ be the angle between the position vector of the observer and $-\boldsymbol{x}^0$. Then $\boldsymbol{n} = -\cos\theta_0 \boldsymbol{i}^0 + \sin\theta_0 \boldsymbol{k}^0$. The spectral energy density in the range of the solid angle between $\Omega_0$ and $\Omega_0 + d\Omega_0$ and in the frequency range between $\omega$ and $\omega + d\omega$ for a Klein-Gordon particle in arbitrary relativistic motion can be obtained according to the work of Lieu & Axford (1995); that is,

$$\frac{d^2E}{d\omega\, d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\frac{1}{\eta}\left|\int_{-T/2}^{T/2} e^{i\omega(t-\boldsymbol{nr}/c)/\eta}[\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{v})]dt\right|^2 , \quad (2)$$

where the recoil factor is defined as

$$\eta = 1 - \frac{\hbar\omega_c}{\gamma mc^2} . \tag{3}$$

In order to obtain the observed spectrum, we need to calculate the above integral, square it, and integrate it with the solid angle. After a series of calculations similar to the previous ones (Zhang & Cheng 1995a, 1995b; Cheng & Zhang 1996), the radiation spectra at the two polarization directions are given by

$$\frac{dP_{\parallel}}{d\omega} = \frac{\sqrt{3}e^2\gamma}{4\pi r_c}\frac{\omega}{\omega_c}\left[\int_{\omega/\eta\omega_c}^{\infty} K_{5/3}(y)dy - K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right)\right] \tag{4}$$

and

$$\frac{dP_{\perp}}{d\omega} = \frac{\sqrt{3}e^2\gamma}{4\pi r_c}\frac{\omega}{\omega_c}\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2}\left[\int_{\omega/\eta\omega_c}^{\infty} K_{5/3}(y)dy + K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right)\right], \tag{5}$$

where the characteristic frequency of the radiation $\omega_c$ and the curvature radius of the particle trajectory $r_c$ can be expressed as

$$\omega_c = \frac{3}{2}\gamma^3 c\frac{1}{\rho}\left(\frac{r_B^3 + \rho r_B^2 - 3r_B\rho^2}{\rho r_B^2}\cos^4\alpha + \frac{3\rho}{r_B}\cos^2\alpha + \frac{\rho^2}{r_B^2}\sin^4\alpha\right)^{1/2} \tag{6}$$

and

$$r_c = \frac{c^2}{(r_B+\rho)\Omega_0^2 + r_B\omega_B^2} , \tag{7}$$

respectively, while

$$Q_2^2 \equiv \left(\frac{r_B^2 + \rho r_B - 3\rho^2}{\rho^3}\cos^3\alpha\cos\theta_0 + \frac{3}{\rho}\cos\alpha\cos\theta_0 + \frac{1}{r_B}\sin^3\alpha\sin\theta_0\right)\frac{1}{r_B} , \tag{8}$$

where $K_{5/3}(x)$ and $K_{2/3}(x)$ are Bessel functions modified by 5/3 are 2/3 orders, respectively. Hence, the total radiation spectrum is given by

$$\begin{aligned}\frac{dP}{d\omega} &= \frac{dP_{\parallel}}{d\omega} + \frac{dP_{\perp}}{d\omega}\\ &= \frac{\sqrt{3}e^2\gamma}{4\pi r_c}\frac{\omega}{\omega_c}\left\{\left[\int_{\omega/\eta\omega_c}^{\infty} K_{5/3}(y)dy - K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right)\right] + \frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2}\left[\int_{\omega/\eta\omega_c}^{\infty} K_{5/3}(y)dy + K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right)\right]\right\} .\end{aligned} \tag{9}$$

The total power radiated can be obtained by integration as

$$P = \int_0^{\omega_{\max}} \frac{dP}{d\omega}d\omega , \tag{10}$$

where $\omega_{\max}$ is the cut-off frequency that is given by

$$\omega_{\max} = \frac{\gamma mc^2}{\hbar} . \tag{11}$$

Now, by transferring the argument from $\omega/\omega_c$ to $x \equiv \omega/\eta\omega_c$, we find

$$\frac{\omega}{\omega_c} = \frac{x}{1+\xi x} , \tag{12}$$

where $\xi$ is the characteristic parameter defined above. Therefore, the total power radiated is given by

$$\begin{aligned}P = \frac{3\sqrt{3}e^2\gamma^4 cQ_2}{8\pi r_c}\Bigg(&\left\{1 + \frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2}\right\}\int_0^{\infty}\frac{x\,dx}{(1+\xi x)^3}\int_x^{\infty}K_{5/3}(y)dy\\ &+ \left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - 1\right\}\int_0^{\infty}\frac{x}{(1+\xi x)^3}K_{2/3}(x)dx\Bigg) .\end{aligned} \tag{13}$$

When $\xi \ll 1$, we discard the terms of higher order than $\xi^2$ and use the integration formula

$$\int_0^{\infty} y^{q-1}K_p(y)dy = 2^{q-2}\Gamma\left(\frac{q-p}{2}\right)\Gamma\left(\frac{q+p}{2}\right) . \tag{14}$$

Thence, we have

$$P = \frac{3\sqrt{3}e^2\gamma^4 cQ_2}{8\pi r_c}\left(\left\{1 + \frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2}\right\}\Gamma\left(\frac{7}{3}\right)\Gamma\left(\frac{2}{3}\right)\left(1 - \xi\,\frac{55\sqrt{3}}{24} + \xi^2\,\frac{56}{3}\right)\right.$$
$$\left. + \left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - 1\right\}\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{2}{3}\right)\left(1 - \xi\,\frac{5\sqrt{3}}{2} + \xi^2\,\frac{64}{3}\right)\right). \tag{15}$$

Equation (15) here will become equation (14) of Zhang & Cheng (1995a), which is the result of the classical case, if $\xi \to 0$. For the ultra–quantum mechanical case ($\xi \gg 1$), $y = \omega/\omega_c \ll 1$ for all $\omega$ will be true because the spectrum cuts off at the energy of the relativistic particle $\gamma mc^2$, which is less than $\hbar\omega_c$. Then, using the asymptotic formula

$$\lim_{y\to 0} K_p(y) = 2^{p-1}\Gamma(p)y^{-p} \tag{16}$$

and the integral formula

$$\int_0^\infty \frac{x^n}{(1+\xi x)^m}\,dx = \frac{1}{\xi^{n+1}} \times \frac{\Gamma(n+1)\Gamma(m-n-1)}{\Gamma(m)}, \tag{17}$$

we have

$$P = \frac{e^2\gamma^4 cQ_2\,\Gamma(2/3)}{12\sqrt[3]{2}\,r_c}\left\{1 + \frac{3[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2}\right\}\xi^{-4/3}. \tag{18}$$

## 3. QUANTUM-LIMITED SYNCHROCURVATURE RADIATION FOR DIRAC PARTICLES

For a Dirac particle in arbitrary relativistic motion, the spectral energy density is given by (Lieu & Axford 1995)

$$\frac{d^2E}{d\omega\,d\Omega} = \frac{e^2\omega^2}{4\pi^2c^3}\,\frac{1}{\eta}\left\{\left[1 + \frac{1}{2}\left(\eta + \frac{1}{\eta} - 2\right)\right]\left|\int v_y\,e^{i\omega(t-\mathbf{n}\mathbf{r}/c)/\eta}\,dt\right|^2 + \left|\int v_x\,\frac{\sin(\zeta-\theta_0)}{\cos\zeta}\,e^{i\omega(t-\mathbf{n}\mathbf{r}/c)\eta}\,dt\right|^2\right.$$
$$\left. + \frac{\gamma^2}{2}\left(\eta + \frac{1}{\eta} - 2\right)\left|\int\left[1 - \frac{v}{c}\cos(\zeta-\theta_0)\right]\frac{v_x}{\cos\zeta}\,e^{i\omega(t-\mathbf{n}\mathbf{r}/c)/\eta}\right|^2\right\}, \tag{19}$$

where $\zeta$ is defined as

$$\tan\zeta = -\frac{v_z}{v_x}. \tag{20}$$

We notice that the $\chi \equiv (\zeta - \theta_0)$ is approximately the angle between the direction of the velocity of motion and the direction to the observer $\boldsymbol{n}$ and that $\chi \sim 1/\gamma \ll 1$. Expanding $\chi$ in a Taylor series, we obtain

$$\frac{dP}{d\omega} = \frac{dP_\parallel}{d\omega} + \frac{dP_\perp}{d\omega}$$
$$= \frac{\sqrt{3}e^2\gamma}{4\pi r_c}\,\frac{\omega}{\omega_c}\left\{\left[\int_{\omega/\eta\omega_c}^\infty K_{5/3}(y)dy - K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right)\right.\right.$$
$$+ \frac{1}{2}\left(\eta + \frac{1}{\eta} - 2\right)\left[-\frac{3}{16}\int_{\omega/\eta\omega_c}^\infty K_{5/3}(y)dy + \frac{11}{16}K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right) + \frac{\eta\omega_c}{8\omega}K_{1/3}\left(\frac{\omega}{\eta\omega_c}\right)\right]$$
$$\left. + \left[1 + \frac{1}{2}\left(\eta + \frac{1}{\eta} - 2\right)\right]\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2}\left[\int_{\omega/\eta\omega_c}^\infty K_{5/3}(y)dy + K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right)\right]\right\}. \tag{21}$$

From the above equations, the total power is given by

$$P = \frac{3\sqrt{3}e^2\gamma^4 cQ_2}{8\pi r_c}\left(\left\{1 + \frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2}\right\}\int_0^\infty \frac{x\,dx}{(1+\xi x)^3}\int_x^\infty K_{5/3}(y)dy\right.$$
$$\left. + \left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - 1\right\}\int_0^\infty \frac{x}{(1+\xi x)^3}\,K_{2/3}(x)dx\right)$$
$$+ \left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - \frac{3}{16}\right\}\frac{1}{2}\int_0^\infty \frac{x\,dx}{(1+\xi x)^3}\left(\frac{1}{1+\xi x} + 1 + \xi x - 2\right)\int_x^\infty K_{5/3}(y)dy$$
$$+ \left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} + \frac{11}{16}\right\}\frac{1}{2}\int_0^\infty \frac{x\,dx}{(1+\xi x)^3}\left(\frac{1}{1+\xi x} + 1 + \xi x - 2\right)K_{2/3}\left(\frac{\omega}{\eta\omega_c}\right)$$
$$+ \frac{1}{16}\int_0^\infty \frac{x\,dx}{(1+\xi x)^3}\left(\frac{1}{1+\xi x} + 1 + \xi x - 2\right)K_{1/3}(x). \tag{22}$$

It is easy to obtain the total power radiated for two limit cases, $\xi \ll 1$ and $\xi \gg 1$, in the same way as we derive that for a Klein-Gordon particle. For the case $\xi \ll 1$, we have

$$P = \frac{e^2\gamma^4 cQ_2}{4\pi r_c}\left(\left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} + 1\right\}\frac{4}{3}\left(1 - \xi\,\frac{55\sqrt{3}}{24} + \xi^2\,\frac{56}{3}\right)\right.$$
$$+\left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - 1\right\}\left.\left(1 - \xi\,\frac{5\sqrt{3}}{2} + \xi^2\,\frac{64}{3}\right)\right)$$
$$+\left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - \frac{3}{16}\right\}\frac{56}{27}\,\xi^2 + \left\{\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} + \frac{11}{16}\right\}\frac{16}{9}\,\xi^2 + \frac{1}{12}\,\xi^2 . \tag{23}$$

In the above discussions, it is shown that the term $(1/2)[\eta + (1/\eta) - 2]$ guarantees that the radiation due to the spin-flip radiation is proportional to $\xi^2$, i.e., to $\hbar^2$. This conclusion is also drawn within the framework of QED. For the ultra–quantum mechanism case ($\xi \gg 1$), we get

$$P = \frac{e^2\gamma^4 cQ_2}{12\sqrt[3]{2}\pi r_c}\left[\left(\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} + 1\right)2\Gamma\left(\frac{2}{3}\right)\xi^{-4/3} + \left(\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - 1\right)\Gamma\left(\frac{2}{3}\right)\xi^{-4/3}\right.$$
$$\left.+\left(\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} - \frac{3}{16}\right)\frac{14}{9}\,\Gamma\left(\frac{2}{3}\right)\xi^{-4/3} + \left(\frac{[(r_B+\rho)\Omega_0^2 + r_B\omega_B^2]^2}{c^4Q_2^2} + \frac{11}{16}\right)\frac{7}{9}\,\Gamma\left(\frac{2}{3}\right)\xi^{-4/3} + \frac{5\sqrt[3]{2}}{144}\,\Gamma\left(\frac{1}{3}\right)\xi^{-2/3}\right]. \tag{24}$$

## 4. DISCUSSION

The quantum-modified synchrocurvature radiation formulae for Klein-Gordon and Dirac particles, applying the GFWW method, are presented in this paper. These formulae are universal because they are suitable both in classical and quantum limits. In Figure 2, we show the calculated radiation spectra of synchrotron, curvature, and synchrocurvature radiation based on classical electrodynamics, the GFWW method for Klein-Gordon particles, and the GFWW method for Dirac particles, respectively. It is evident that the calculation according to the ordinary synchrotron radiation formula, even with the quantum modification, is not the real case in curved magnetic fields. Our results provide the quantum modification not only of the synchrotron radiation mechanism but also of the curvature radiation mechanism. It is difficult to obtain all these results within the framework of QED. The classical radiation spectrum of synchrocurvature radiation and its corresponding radiation spectra in two quantum cases are redrawn in Figure 3. The difference between the classical case and quantum cases is small at the low-frequency region; i.e., the quantum modification can be ignored. However, when the radiation frequency approaches the cut off-frequency $\omega_{\max}$, the quantum mechanical effects become important in that the energy-momentum of the radiated photon can no longer be neglected, as it is in classical electrodynamics. The important disparity between the quantum spectrum and the classical spectrum is that the former is cut off at the energy of the relativistic charge $\gamma mc^2$, which is

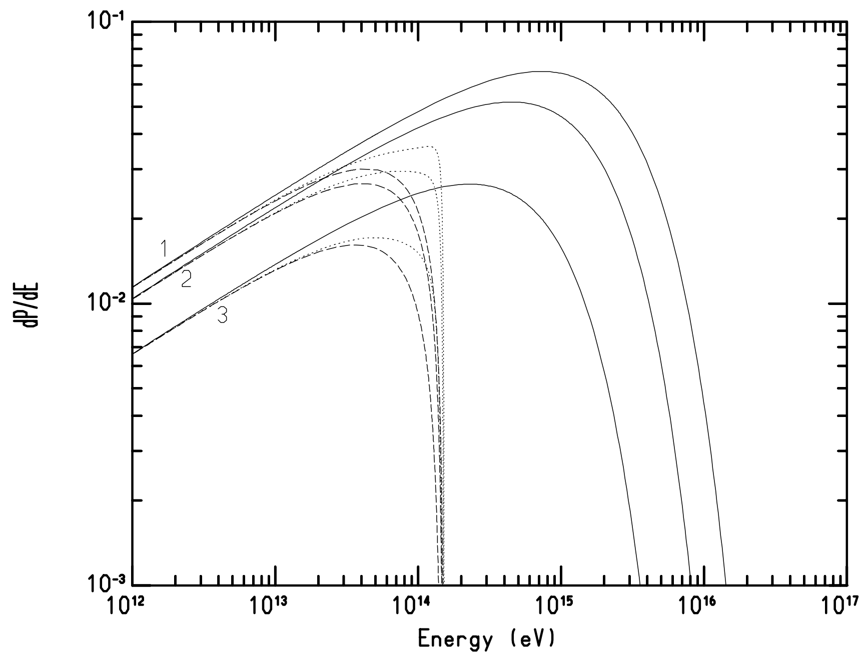


FIG. 2.—Radiation spectra, $dP/d\omega$, for the synchrocurvature radiation mechanism (curves labeled 1), the synchrotron radiation mechanism (curves labeled 2), and the curvature radiation mechanism (curves labeled 3) calculated on the basis of classical electrodynamics (*solid lines*), the GFWW method for Klein-Gordon particles (*dashed lines*), and the GFWW method for Dirac particles (*dotted lines*). (Parameters are $\sin\alpha = 10^{-6}$, $\gamma = 3 \times 10^8$, $B = 10^{12}$ G, and $\rho = 1 \times 10^6$ cm.)

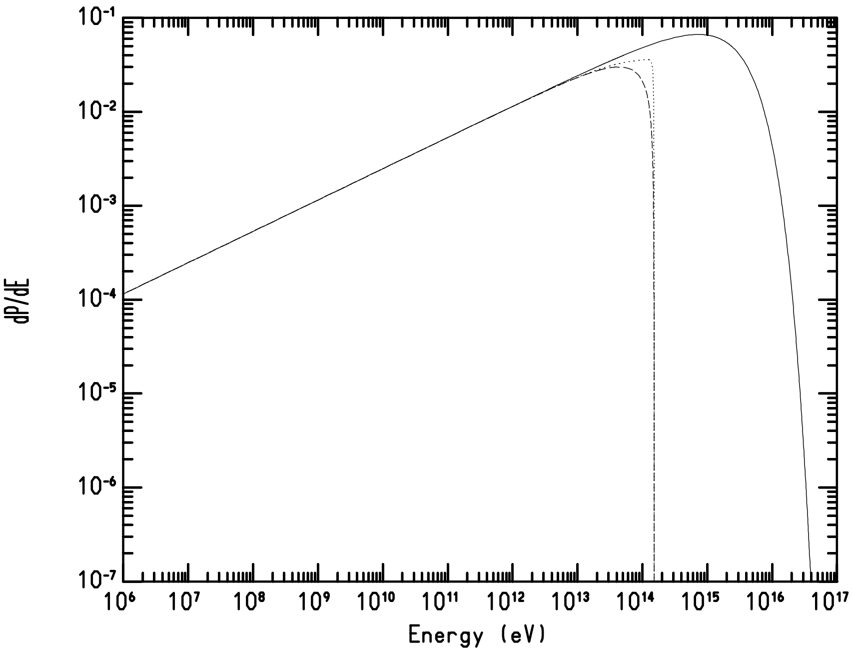


FIG. 3.—Radiation spectra for the classical (*solid line*) and quantum-limited (*dashed* and *dotted lines*) synchrocurvature radiation mechanisms. The parameters are identical to those in Fig. 2.

the common characteristic of all quantum radiation spectra. Figure 4 shows the differences between the total power radiated according to the quantum-limited synchrocurvature radiation mechanism for Klein-Gordon and Dirac particles and the well-known quantum results of Sokolov & Ternov (1986) as functions of the Lorentz factor. These differences are similar to those between classical synchrocurvature radiation and synchrotron radiation that are emphasized in previous papers (Zhang & Cheng 1995a, 1995b; Cheng & Zhang 1996). It can be seen easily from Figure 4 that synchrocurvature radiation reaches the ultra–quantum limit more easily than does synchrotron radiation because of the contribution from the curvature of the magnetic field. The total power of classical and quantum-limited synchrocurvature radiation in magnetic fields with different radii of curvature are shown in Figure 5.

Figure 6 shows the influence of quantum effects on polarization. As previously pointed out (Zhang & Cheng 1995a, 1995b; Cheng & Zhang 1996), one can differentiate classical synchrocurvature radiation from synchrotron and curvature radiation by measuring the degree of polarization. The degree of polarization of quantum-limited synchrocurvature radiation is

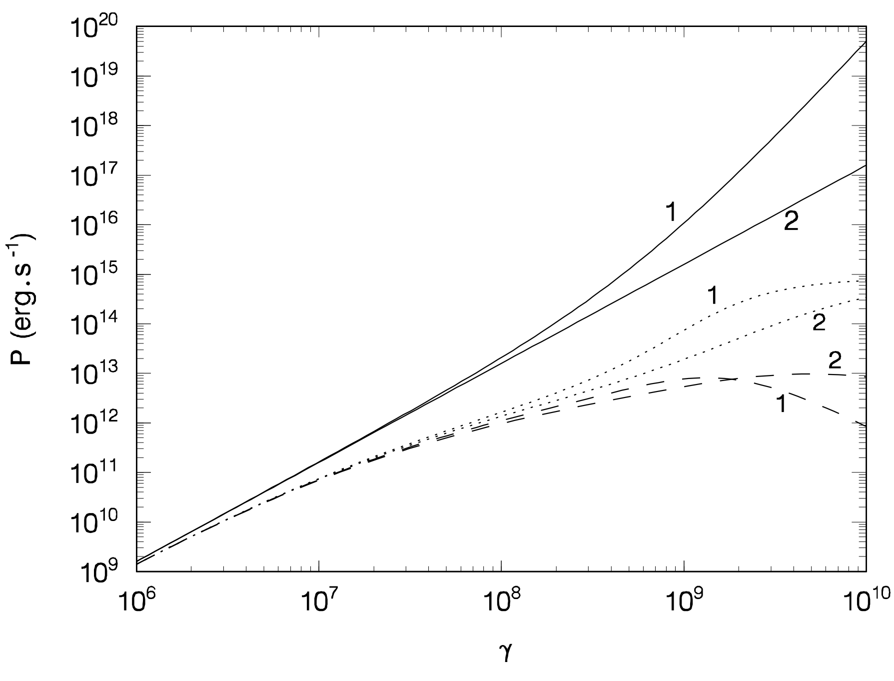


FIG. 4.—Radiation power of quantized synchrotron radiation (curves labeled 2) and quantum-limited synchrocurvature radiation (curves labeled 1) as a function of $\gamma$. The parameters are identical to those in Fig. 2.

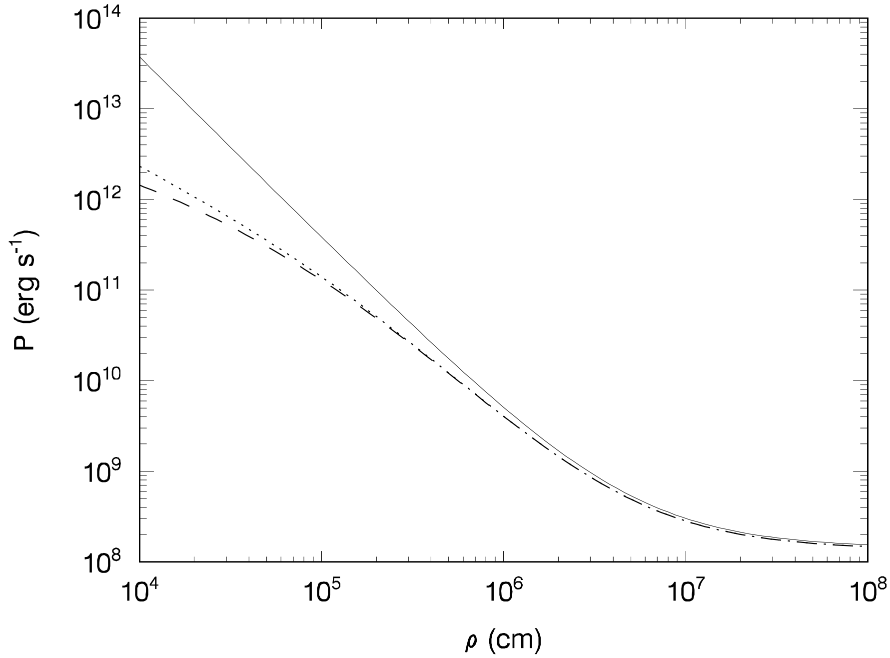


FIG. 5.—Radiation power of classical (*solid line*) and quantum-limited (*dashed* and *dotted lines*) synchrocurvature radiation as functions of $\rho$ ($\sin \alpha = 10^{-6}$, $\gamma = 3 \times 10^7$, and $B = 10^{10}$ G).

evidently lower than 0.5. Meanwhile, there is a region of decreasing degree of polarization near $\omega_{\max}$ for Dirac particles. This is the genuine quantum effect that might be beneficial in differentiating synchrocurvature radiation mechanisms from the other two conventional mechanisms.

Although Lieu & Axford's method is not a rigorous one within the framework of QED, it is effective in dealing with the radiation from charges moving in magnetic fields and in obtaining results that are impossible to calculate on the basis of the QED. Our confidence in this method comes from the fact that it can reproduce exactly almost all the radiation properties of synchrotron radiation in quantum limits. It is safe to say that the differences between the results we derive here and the results that will be obtained on the basis of QED (if possible) will be very small in a wide range of parameters. It must be emphasized again as in previous papers (Zhang & Cheng 1995a; Cheng & Zhang 1996) that we consider it reasonable to neglect the nonuniformity of the magnetic field at small scales because it influences the shape of only a single pulse. The observation time, as we know, is about $1/\gamma^3$ times less than the time interval between two pulses, so its influence on the total radiation spectrum

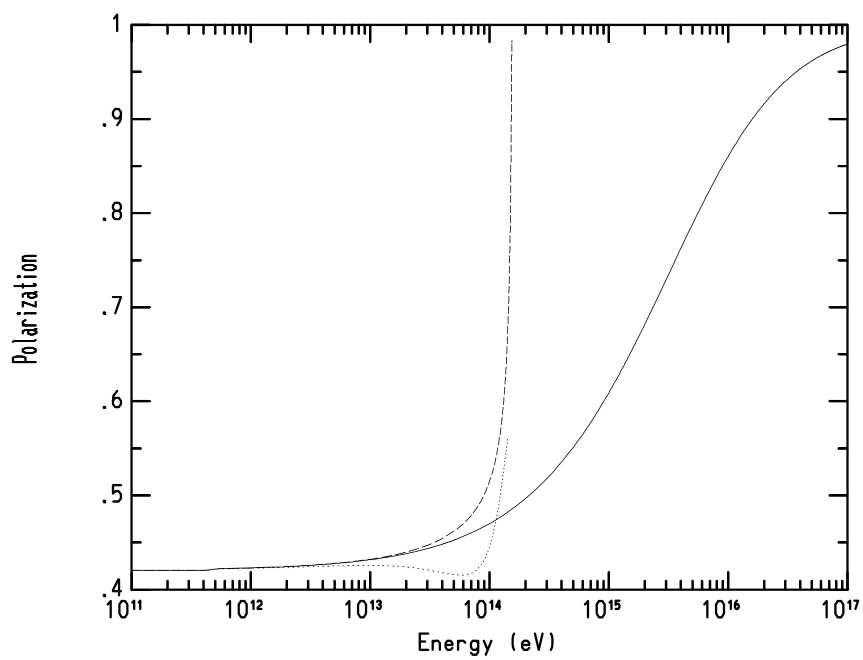


FIG. 6.—Degree of the polarization for the classical (*solid line*) and quantum-limited (*dashed* and *dotted lines*) synchrocurvature radiation mechanisms. The parameters are identical to those in Fig. 2.

is small. Therefore, Zhang & Cheng can give the results in simple expressions of the $K$ Bessel function without considering the variation of a single pulse form, and thus they can connect curvature radiation with synchrotron radiation. Because the magnetic field we considered here does not vary obviously in the coherent length scale, this assumption is reasonable based on the views of Lieu, Axford, & McKenzie (1997). Starting from the classical formula, we generalize this formula to the quantum limit, applying the Lieu & Axford method completely in the same approximation. Our purpose is not to improve the accuracy of synchrotron radiation estimates but to provide the main quantum modification to curvature radiation and synchrocurvature radiation mechanisms. Following the work of Lieu et al. (1997), more detailed considerations will be taken into account in the future.

It is noticed that the quantum mechanical effect will become observable when $\xi \sim 1$. This condition is easy to satisfy in the magnetosphere of pulsar, especially for high-energy radiation from the particles with high energy. Therefore, the quantum mechanical effects of synchrocurvature radiation must be taken into account. The formulae we present here would be useful in doing so.

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