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DETECTION OF THE HALO COMPONENT OF THE FIELD sdB POPULATION

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ABSTRACT

A new, complete sample of 20 faint sdB stars isolated from the US survey is reported. Because the sample contains stars with magnitudes as faint as $B = 18$, it is uniquely situated to probe the field sdB spatial distribution at relatively large distances out of the Galactic plane. A V/V_{max} analysis provides strong evidence that the majority of these stars are not part of a disklike stellar density distribution. Using a statistical treatment for population membership, a further V/V_{max} analysis shows that the spatial distribution of the US sdB stars is consistent with a combined disk-plus-halo stellar density law. This analysis also provides a preliminary value for the sdB halo-to-disk density ratio in the solar neighborhood that is larger by a factor of $\gtrsim 7$ than that for the field stars in general. It is likely that even relatively bright samples of sdB stars, such as those isolated from the PG survey, contain significant numbers of halo stars.

Subject headings: Galaxy: halo — Galaxy: structure — stars: early-type — stars: horizontal-branch — stars: statistics — subdwarfs

1. INTRODUCTION

The hot subdwarf B (sdB) stars are thought to be the field counterparts to the blue extended horizontal-branch (EHB) stars that are observed in globular clusters (Greenstein & Sargent 1974; Heber et al. 1986). They are thus identified with the helium-core burning stage of stellar evolution. The favored scenario for creating such stars is through evolutionary processes on the red giant branch. According to this picture, the sdB progenitors on the red giant branch would have had to suffer enough mass loss to leave the stars with only a very thin hydrogen shell. The thin hydrogen shells of sdB stars are probably inadequate to support the typical post–horizontal-branch evolution that eventually leads to the onset of thermal pulsing on the asymptotic giant branch. Instead, post-EHB evolutionary tracks appear to provide a more direct route toward the white dwarf cooling sequences (e.g., Caloi 1989; Dorman, Rood, & O'Connell 1993).

This and other, competing hypotheses regarding the details of the origins and evolution of sdB stars are in the process of being tested through observations of both field and globular cluster EHB stars. For field sdB stars, the determination of spatial distributions is one type of observation that can contribute important evidence for evolutionary scenarios. Such data can provide information on stellar-population membership and can test for evolutionary links with other types of stars through comparison of spatial distributions, space densities, and birthrates.

Studies of spatial distributions are best undertaken with complete samples, and color excess surveys provide the most efficient way of isolating samples of hot stars such as sdB stars. Indeed, the most common type of object found in the Palomar-Green (PG) survey (Green, Schmidt, & Liebert 1986) was the sdB star. Two recent analyses using large samples of ~ 200 sdB stars from the PG survey have shown that these stars belong to a disk population with a characteristic scale height of ~ 500 pc (Villeneuve et al. 1995, hereafter V95; Saffer & Liebert 1995, hereafter SL95). These results generally confirm earlier studies of the spatial distribution and kinematics of sdB stars (see V95 for a comprehensive review of past studies). However, the derived scale height of ~ 500 pc is at least a factor of ~ 2 larger than that obtained by previous studies that used smaller and brighter sdB samples. At the same time, there is evidence (Thejll et al. 1994) that the hotter, more luminous sdO stars might have an even larger scale height than this new value for the sdB stars, which is inconsistent with the hypothesis that the sdB and sdO stars are related through evolution.

One possible reason for the apparent inconsistencies between the newer and older sdB scale height determinations and between the sdB and sdO scale height determinations is that the analyses of fainter and/or more distant samples could probe stellar populations that are distributed differently than the stars contained in brighter samples. Specifically, it is possible that sdB (and sdO) stars could be distributed in multiple disk systems, each with their own scale height (e.g., Gilmore & Reid 1983), and there is already some kinematic evidence that sdB stars extend into the halo (e.g., Colin et al. 1994; Thejll et al. 1997). The existence of such multiple components to the spatial distribution of sdB stars could contribute to the type of trends that have been seen in the derived scale heights.

The purpose of this paper is to explore the nature of the field sdB star spatial distribution with the help of a new, complete sample of faint sdB stars. Section 2 of this paper presents the new sample of 20 sdB stars and describes how they were isolated from the US survey. What the sample might lack in size, it makes up for in its faint-magnitude coverage; the faint-magnitude completeness limits of this sample reach $B = 18.35$, at least ~ 2.5 mag fainter than previously studied sdB samples. In § 3 the new US sdB sample is used in a V/V_{max} analysis that takes advantage of this unique faint-magnitude leverage to show that most of these stars belong to a halo population of sdB stars. Section 4 provides a summary and discussion.

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2. ISOLATION OF THE US sdB SAMPLE

2.1. *The US Survey*

The US survey for ultraviolet excess starlike objects (Usher, Mitchell, & Warnock 1988 and references therein; Usher & Mitchell 1990) was conducted by means of three-color (u, b, v) Palomar 1.2 m Schmidt photographic plates taken according to the Tonantzintla prescription (Haro & Luyten 1962). The US survey covers seven high-latitude ($|b| > 30°$) fields, each covering $\sim$40 square degrees, centered on Selected Areas 28, 29, 55, 57, 71, 82, and 94. Candidate color excess objects were first selected by eye on the three-color plates, and then quantitatively isolated and color classified in relative, two-color $(u-b)$, $(b-v)$ diagrams constructed from uncalibrated iris photographic photometry of the three-color images (Usher 1981). The goal of the US survey is to be complete in the selection of starlike objects with colors $(U-V) < 0$ or with ultraviolet excess relative to the blackbody line in the $(U-B)$, $(B-V)$ two-color diagram.

Calibrated photographic B magnitudes with rms errors of 0.1–0.2 mag were measured for all selected objects using single-color plates. Magnitudes in the US catalogs range from $B \simeq 10$ to $B \simeq 20$. The claimed faint-magnitude completeness limits for the US survey fields range from $B = 16.9$ to $B = 18.7$. The bright-magnitude completeness limit is estimated to be $B = 10.0$, after the recovery of a number of bright, color excess candidates missed in the original US selection (Mitchell & Usher 1998).

The US survey covers a total of $\sim$206 square degrees of sky within the central regions of the seven fields. Within this area, and within the field-dependent magnitude completeness limits, the US catalogs provide complete samples of starlike objects with colors bluer and/or more ultraviolet than halo F/G subdwarfs and A stars. With temperatures exceeding 23,000 K, sdB stars should be cataloged completely by the US survey.

2.2. *Candidate Spectrophotometry*

A spectrophotometric survey of the blue and ultraviolet excess objects in the US catalogs has been conducted as a first step in isolating complete samples of hot stars and QSOs. These observations reach area-dependent, faint-magnitude completeness limits ranging from $B = 16.5$ to $B = 18.3$ within the US lists. Since these limits are typically brighter than the US catalog limits, they define the faint-magnitude completeness limits of the samples that are isolated. The observations were conducted with the use of the KPNO 2.1 m telescope and the Intensified Image Dissector Scanner and GoldCam instruments during a number of runs between 1984 January and 1993 February under a wide range of weather and photometric conditions. Wide entrance apertures of $\sim 8''$ were used to obtain low-resolution spectra with low-to-moderate signal-to-noise ratios. Most of the spectra were obtained with a 300 lines mm^{-1} grating resulting in a spectral resolution of $\sim$14 Å FWHM and a spectral coverage of 3500 Å $< \lambda <$ 7000 Å. A few spectra have also been obtained at $\sim$10, $\sim$7, and $\sim$4 Å FWHM resolutions.

The data were reduced using a combination of KPNO mountain reduction and IRAF software. The quality of the resultant flux- and wavelength-calibrated spectra has allowed the observed US objects to be reliably classified as QSOs/active galactic nuclei (AGNs) or stars. The stars were further classified into the major subgroups as given by Greenstein & Sargent (1974) and Green et al. (1986) through visual inspection and comparison with bright flux standards that spanned a wide range of hot-star types. Preliminary samples of both QSOs and hot stars from the US survey have been previously reported (Mitchell, Warnock, & Usher 1984; Mitchell, Howell, & Usher 1987).

2.3. *The US sdB Sample*

The sdB stars were isolated from within the larger US sample of hot stars through the visual appearance of the Balmer and helium absorption lines, Balmer jump, and continuum shape. Spectra for five of the sdB stars are shown in Figure 1. The reliability of the sdB classifications was enhanced through the ability to compare spectra of nearly uniform quality over a wide range of stellar types, including A and B stars, white dwarfs, and the sdO and sdB stars. A further check on the reliability of the sdB classifications was accomplished at the cooler temperatures in the course of separating the sdB stars from a sample of halo B stars (Mitchell et al. 1998). The goal was to create the present sdB sample guided by previously established classification criteria for sdB and sdOB stars (e.g., Greenstein & Sargent 1974; Vauclair & Liebert 1987). The sdB stars were deemed to meet the following criteria: $T_{\rm eff} > 23{,}000$ K and $4.5 \lesssim \log g \lesssim 6.0$ (i.e., effective temperatures hotter than the second Newell gap [Newell & Graham 1976] along the horizontal branch/EHB, and surface gravities significantly larger than those of main-sequence stars but less than those of white dwarfs). Furthermore, at the hotter temperatures, the only requirement used here to separate sdB stars from sdO stars was the absence of significant He II λ4686 absorption in the sdB spectra. This presumably limits $T_{\rm eff} \lesssim 40{,}000$ K for the sdB stars. Thus, if there are any sdO stars in the US sdB sample, they are either helium weak or their He II λ4686 line went undetected in the moderate-S/N spectra. Estimates for the atmospheric parameters of the individual US sdB stars will be reported elsewhere.

The presence or lack of He I absorption lines was not used as part of the present classification criteria. The US sdB sample is therefore similar in nature to the sample of 68 EHB stars from the PG survey studied by Saffer et al. (1994) that contained sdB, sdOB, and helium-enhanced subdwarf

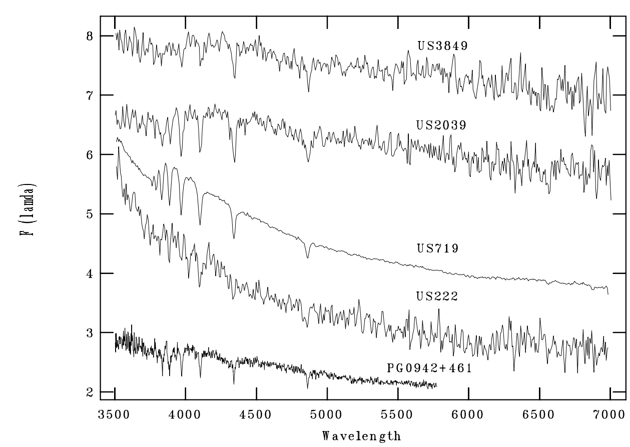


FIG. 1.—Examples of spectra for five sdB stars obtained during the spectrophotometric survey of the US catalogs. The sdB stars cover a wide range of B magnitudes. The spectra of the four US objects have spectral resolutions of $\sim$14 Å FWHM. The spectrum of the PG object has a spectral resolution of $\sim$4 Å FWHM.

stars. The only difference is the possibility that somewhat lower gravity post-EHB stars could be lurking in the US sdB sample due to the smaller surface gravity lower limit of the US sdB sample. Because of their short evolutionary timescales, post-EHB stars are rare relative to the EHB stars, and thus their number in the US sdB sample is expected to be quite low.

The complete sample of US sdB stars is presented in Table 1. The stars are listed by their US numbers in Column (1). Equatorial coordinates and finder charts can be found in the original US survey papers. PG 0942+461 is included as the only sdB star found during the effort to recover bright color excess objects that might have been missed in the US survey, and is discussed below. The second column contains the B magnitudes. These are taken directly from the US catalogs, except for (1) extinction/reddening corrections in the SA71 and SA94 US survey fields; (2) vignetting corrections of -0.1 mag for those objects appearing close to the edge of the single-color B plates; and (3) corrections for a known photometric offset in the SA29 survey field (Usher et al. 1983). Column (3) lists the faint-magnitude completeness limit of the subregion containing the star (the bright-magnitude completeness limit is taken to be $B = 10$ uniformly across the sample), and column (4) contains the total area within the complete sample over which the star, at its B magnitude, could have been discovered. Galactic latitudes and longitudes are provided in the fifth and sixth columns. Columns (7)–(9) contain the distance from the Sun, perpendicular distance from the Galactic plane, and radial distance from the Galactic center, respectively. These distances depend on the assumptions discussed in § 3.1.

Because the intrinsically blue colors of the sdB stars lie far from the $(U-V) \simeq 0$ color limit of the US survey, the completeness of the US sdB sample should be high. However, there is an increased potential for incompleteness at the brighter magnitudes due to the nature of object selection in the US survey (Warnock et al. 1986; Usher et al. 1988). Efforts to recover color excess objects missed in the original US selection so that this type of incompleteness could be evaluated and reduced are described in Mitchell & Usher (1998). Only one missed sdB star was found: PG 0942+461. This object was given the generic sd spectral classification in the PG catalog; but it was observed to have color excess on one of the US three-color plates, and was subsequently confirmed as an sdB star during the spectroscopic survey (see Fig. 1). The addition of this star reduces the incompleteness at bright magnitudes in the US sdB sample. Although it is possible that another undiscovered, bright ($10 < B < 15$) sdB star could exist within the US survey fields, this is unlikely given the efforts already made to check for such objects. The results obtained in § 3.2 bear this out.

Another potential source of incompleteness and unreliability in the US sdB sample is classification confusion between the various hot-star types due to the moderate quality of the spectra available. The most likely source of classification confusion is between the hot sdB and sdO stars. This is because the Balmer and helium absorption lines are relatively weak at temperatures $T_{\rm eff} \sim 40{,}000$ K. Also at such temperatures optical colors are relatively insensitive to temperature differences. Only higher quality spectra would mitigate such errors. Higher quality spectra would also provide the ability to distinguish between sdB stars on or near the EHB and post-EHB stars. Proposals to obtain this kind of follow-up spectroscopy of the US sdB and sdO stars at KPNO have been unsuccessful thus far.

3. THE SPATIAL DISTRIBUTION OF THE US sdB STARS

3.1. *sdB Star Distances*

Stellar distances are clearly of central importance to the study of spatial distributions. For the present study, the distances to the US sdB stars have been estimated by assuming that all of the sdB stars have an absolute B magnitude (M_B) that is equal to the average M_B of the sdB population. The decision to calculate distances using a single (average) M_B for all of the US sdB stars, as opposed to calculating distances individually for each star from derived

TABLE 1

THE US sdB SAMPLE

Name (1)	B (2)	$B_{\rm faint}$ (3)	Area (deg)^2 (4)	b (deg) (5)	l deg (6)	r (pc) (7)	z (pc) (8)	R (pc) (9)
US 95	13.8	17.85	206	88.19	47.44	1200	1200	8560
US 177	13.8	17.85	206	87.44	46.01	1200	1200	8550
US 222	16.7	17.85	181	84.78	90.22	4570	4550	9650
US 229	15.2	17.85	206	86.87	52.31	2290	2290	8730
US 719	12.3	18.05	206	46.90	173.37	600	440	8920
US 909	16.8	18.05	179	48.67	172.13	4790	3590	12180
US 1027	17.7	18.05	119	49.82	172.58	7240	5540	14270
US 1796	16.7	18.25	181	39.57	176.94	4570	2910	12370
US 2039	18.0	18.05	88	40.86	176.62	8320	5440	15750
US 2474	15.2	18.05	206	71.62	201.44	2290	2170	9430
US 2600	17.4	17.55	148	72.63	208.44	6310	6020	11840
US 3273	17.6	18.35	119	−49.97	174.84	6920	5300	13980
US 3289	16.4	18.35	206	−50.45	175.84	3980	3070	11450
US 3447	15.5	17.05	206	−46.89	173.25	2630	1920	10470
US 3470	17.3	18.15	157	−49.83	178.23	6030	4600	13210
US 3783	10.6	17.25	206	−35.40	167.01	280	160	8720
US 3802	15.6	17.15	206	−35.47	167.59	2750	1600	10820
US 3849	17.3	17.35	157	−33.31	166.69	6030	3310	13850
US 3979	15.3	16.55	206	65.28	7.68	2400	2180	7820
PG 0942+461	14.3	18.05	206	48.88	173.11	1510	1140	9560

NOTE.—Table columns are explained in § 2.3. All distances in the table assume $M_B = +3.4$.

atmospheric parameters, was guided by the following considerations. Recent studies that used distances derived from accurate spectroscopic and photometric data (Moehler, Heber, & de Boer 1990; Saffer 1991; V95) have indicated that the rms scatter of individually determined sdB absolute V magnitudes about the mean is only ~ 0.5 mag. Some of this scatter is due to a real trend of decreasing M_V with $T_{\rm eff}$ in the H-R diagram. By assuming a similar scatter of 0.5 mag in M_B and combining it with the US survey B magnitude errors of ~ 0.15 mag, an rms distance error of $\sim 25\%$ is obtained. This value compares favorably to the level of random error that is induced in individually determined distances by the typical uncertainties estimated for spectroscopically derived log g and $T_{\rm eff}$ (Moehler et al. 1990; Saffer et al. 1994; V95). Given that the quality of the available spectroscopic data for the US sdB stars is likely to be somewhat poorer than that of the data used in the cited studies, there would not appear to be any advantage in deriving distances individually for the US sdB stars.

The other major contributor to distance error for the US sdB stars is the *systematic* error caused by the uncertainty in the average M_B itself. Recent studies have produced significantly different determinations for the average sdB M_V, ranging from $+3.5$ (Moehler et al. 1990) and $+3.7$ (V95) to $\sim +4.5$ (Saffer 1991; Liebert, Saffer, & Green 1994; Thejll et al. 1997). An error in the accepted average M_B corresponds to a systematic error in the derived distances to the US sdB stars, and this in turn would lead to scale errors in any distance parameters derived for the spatial distribution. To establish a basis for comparing results with the V95 study in the following sections, the average M_B for the US sdB stars is taken to be $+3.4$, a value that corresponds (after applying an average color correction) to the average $M_V = +3.7$ derived by V95 from their reference sequence of effective temperatures and surface gravities. All of the distances shown in Table 1 were calculated using this value for M_B.

3.2. *Unweighted $V/V_{\rm max}$: Sample Completeness and Initial Spatial-Distribution Analysis*

Basic information on sample completeness and spatial distributions can be obtained from a $V/V_{\rm max}$ analysis (Schmidt 1968). In particular, the methodology used throughout § 3 is based on an incoherent $V/V_{\rm max}$ analysis of region-independent samples (Avni & Bahcall 1980). The region-independent subsamples of the US sdB sample are identified by their unique $B_{\rm faint}$ values in Table 1. For each US sdB star, i, the following volume ratio has been formed

$$\frac{V_i}{V_{{\rm max},i}} = \frac{\int_{r_{{\rm min},i}}^{r_i} r^2\,dr}{\int_{r_{{\rm min},i}}^{r_{{\rm max},i}} r^2\,dr}\,, \qquad (1)$$

where r_i is the distance of the sdB star from the Sun, $r_{{\rm min},i}$ is the minimum distance at which the sdB star could have been found without its B magnitude becoming brighter than the bright-magnitude completeness limit of the sample, and $r_{{\rm max},i}$ is the maximum distance at which the sdB star could have been found without its B magnitude becoming fainter than the faint-magnitude limit of the subregion containing the sdB star. These distances are given by

$$r_i = 10^{1+0.2(B_i - M_B)}\,,$$

$$r_{{\rm min},i} = 10^{1+0.2(B_{\rm bright} - M_B)}\,,$$

and

$$r_{{\rm max},i} = 10^{1+0.2(B_{{\rm faint},i} - M_B)}\,, \qquad (2)$$

where B_i is the object's apparent magnitude, $B_{\rm bright}$ is the sample bright-magnitude limit, $B_{{\rm faint},i}$ is the faint-magnitude limit of the subregion containing the star (given in Table 1), and M_B is the average absolute magnitude of the sdB stars.

The volume ratios for the 20 sdB stars have been averaged to obtain $\langle V/V_{\rm max}\rangle = 0.25 \pm 0.07$, where the error is equal to the measured standard deviation of the individual values about the mean divided by $20^{1/2}$. This average is significantly below the value of 0.5 expected for a uniformly distributed sample, and it indicates a decreasing space density away from the Sun (as would be expected from previous results on the sdB stars) and/or sample incompleteness at the faint magnitudes.

To investigate this result further, a number of brighter and fainter subsamples have been formed from the full US sdB sample, and for each of these $\langle V/V_{\rm max}\rangle$ has been calculated as described above. The brighter subsamples are formed by imposing systematically brighter faint-magnitude upper limits that force the elimination of the faintest sdB stars one or two stars at a time. In the process of imposing this faint-magnitude upper limit (= faint $B_{\rm lim}$), only those $B_{{\rm faint},i}$ that are greater than faint $B_{\rm lim}$ are modified to be equal to faint $B_{\rm lim}$; the other $B_{{\rm faint},i}$ are left unchanged, as is the sample bright limit. Likewise, the fainter subsamples are created by imposing systematically fainter bright-magnitude lower limits that force the elimination of the brightest sdB stars, one or two stars at a time. In this case, the bright-magnitude lower limit (= bright $B_{\rm lim}$) directly replaces the sample bright limit, $B_{\rm bright}$, throughout, while the $B_{{\rm faint},i}$ are left unchanged.

The resulting values of $\langle V/V_{\rm max}\rangle$ are plotted in Figure 2 versus the faint-magnitude upper limits and bright-magnitude lower limits that were imposed to create the subsamples. First, the relative stability of the $\langle V/V_{\rm max}\rangle$ values, especially at both ends of the plot (bright $B_{\rm lim} < 12$ and faint $B_{\rm lim} > 17$), indicates that there is no significant sample incompleteness at either bright or faint magnitudes.

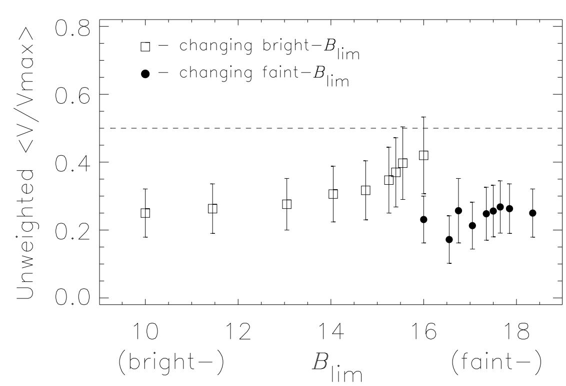


FIG. 2.—Values of unweighted $\langle V/V_{\rm max}\rangle$ are plotted for various subsamples of the US sdB sample vs. the bright and faint-magnitude limits that were imposed to create the subsamples. Error bars represent the measured errors of the means. Open squares denote subsamples that become systematically fainter (and smaller) as the imposed bright $B_{\rm lim}$ become fainter (the fainter subsamples shown contain 20, 19, 18, 16, 15, 13, 12, 11, and 10 sdB stars, as plotted from left to right). Filled circles denote subsamples that become systematically brighter (and smaller) as the imposed faint $B_{\rm lim}$ become brighter (the brighter subsamples shown contain 20, 19, 18, 17, 16, 14, 13, 11, and 10 sdB stars, as plotted from right to left). The rightmost and leftmost data points plotted are the same, representing the full 20 member US sdB sample with bright $B_{\rm lim} = 10$ and faint $B_{\rm lim} = 18.35$.

Second, the $\langle V/V_{max}\rangle$ values themselves are all less than 0.5, which indicates that none of the subsamples are uniformly distributed. However, for the fainter subsamples, where bright $B_{lim} > 14$, the trend of increasing $\langle V/V_{max}\rangle$ values is a first indication that the fainter US sdB stars might be distributed more uniformly than the brighter ones. This can also be seen from a direct comparison of the open and filled symbols plotted at $B_{lim} = 16$ in Figure 2, which shows that the 10 faintest stars in the US sdB sample are distributed more uniformly than the 10 brightest stars.

3.3. *Weighted V/V_{max}: An Attempt to Constrain the Disk Scale Height of the Field sdB Stars*

As discussed in § 1, two recent studies have found the disk scale height of the field sdB stars to be ~ 500 pc, a factor of ~ 2 larger than values obtained from previous studies. These two studies used samples of sdB stars from the PG survey that were both larger (~ 200 stars) and fainter ($B_{faint} \simeq 14.6$, V95; $B_{faint} \simeq 15.6$, SL95) than the samples used in previous analyses. The apparent trend for fainter sdB samples to give larger scale heights could be due to vagaries in the different samples and/or analysis methods used. However, it is also possible that the fainter (i.e., more distant) sdB samples actually probe different stellar distributions. To explore this possibility is the central purpose of this paper.

With a faint-magnitude completeness limit extending at least 2.5 magnitudes fainter than the limits of the PG sdB samples recently analyzed, and with an average perpendicular height above the plane of ~ 2.9 kpc, the US sdB sample should have the leverage to constrain the disk scale height of the field sdB population. A density-weighted V/V_{max} analysis (Schmidt 1968; Avni & Bahcall 1980) can be used to derive the disk scale height of the US sdB stars. For each US sdB star, *i*, the following density-weighted volume ratio has been formed

$$\frac{V'_i}{V'_{max,i}} = \frac{\int_{r_{min,i}}^{r_i} r^2 \rho_D \, dr}{\int_{r_{min,i}}^{r_{max,i}} r^2 \rho_D \, dr} , \quad (3)$$

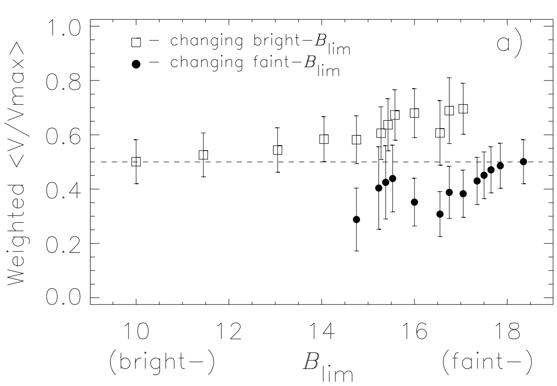
where r_i, $r_{min,i}$, and $r_{max,i}$ are defined in equation (2). The weighting factor, ρ_D, is the disk density law (e.g., Bahcall & Soneira 1980) given by

$$\rho_D = \rho_D(z, d) = \rho_{D0} \exp\left[-\left(\frac{z}{z_0}\right) - \left(\frac{d}{d_0}\right)\right] , \quad (4)$$

where z is the perpendicular distance above the Galactic plane, d is the in-plane distance from the Galactic center, z_0 is the scale height of the Galactic disk, and d_0 is the disk scale length. The in-plane distance is calculated using a value of 8.5 kpc for the distance of the Sun from the Galactic center. When calculating weighted volumes, the density weights are rendered unitless by setting ρ_{D0} to a unitless value of 1.0. The solution for the disk scale height is obtained by holding d_0 fixed at a value of 3.5 kpc, while z_0 is iteratively solved for by requiring $\langle V'/V'_{max}\rangle = 0.5$.

The solution obtained for the 20 US sdB stars is $z_0 = 1430$ pc (estimated 1 σ range: 810–1780 pc). This is almost a factor of 3 larger than the value of $z_0 = 500$ pc obtained from the PG sdB samples! The present result is not very sensitive to the value of d_0; even in the limit $d_0 \to \infty$, corresponding to no radial disk density gradient, the solution is $z_0 = 1325$ pc (estimated 1 σ range: 830–1630 pc). The dependence of the derived z_0 for the US sdB stars on the values chosen for d_0 and the average sdB M_B is shown in Mitchell (1997). The 1 σ error range on z_0 is calculated from the changes in z_0 that are required to return $\langle V'/V'_{max}\rangle$ back to a value of 0.5 after adding to and subtracting from $\langle V'/V'_{max}\rangle = 0.5$ an amount equal to the expected error on the mean weighted volume ratio $[=(12 \times n)^{1/2}$, where n is the number of stars in the sample]. The size of the derived 1 σ error range is relatively large; its size is driven both by the size of the sample *and how well the assumed density law matches the data.*

This result is investigated further by calculating $\langle V'/V'_{max}\rangle$ for the same systematically brighter and fainter subsamples of the US sdB sample that are described in § 3.2. For these calculations the disk density model is held fixed, with $d_0 = 3.5$ kpc and $z_0 = 1.43$ kpc (the z_0 solution found for the full US sdB sample). The $\langle V'/V'_{max}\rangle$ values and their measured errors are plotted in Figure 3*a*. This figure shows



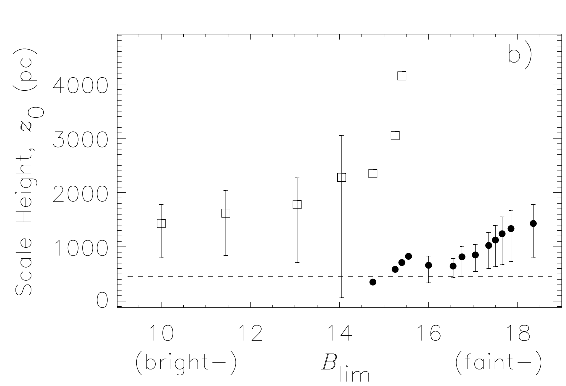


FIG. 3.—Values of (*a*) weighted $\langle V'/V'_{max}\rangle$ and (*b*) derived scale heights are plotted for various subsamples of the US sdB sample vs. the bright and faint-magnitude limits that were imposed to create the subsamples. Symbols are defined as in Fig. 2. The systematic trends in both of these plots work in the opposite direction from the effects of incompleteness and provide strong evidence that a halo component to the field sdB population has been detected. (*a*) The weighting factor used in the volume integrals is a disk density law with a scale height fixed at the value of 1.43 kpc that was derived for the full US sdB sample. Error bars represent the measured errors of the means. The fainter subsamples contain 20, 19, 18, 16, 15, 13, 12, 11, 10, 9, 7, and 6 sdB stars, as plotted from left to right. The brighter subsamples contain 20, 19, 18, 17, 16, 14, 13, 11, 10, 9, 8, 7, and 5 sdB stars, as plotted from right to left. (*b*) The scale height for each subsample is derived by requiring $\langle V'/V'_{max}\rangle = 0.5$. Error bars represent 1 σ ranges based on the expected error of $\langle V'/V'_{max}\rangle = 0.5$; their size is driven both by the size of the subsample and the ability of the assumed disk density law to match the data. Error bars are not plotted if they intersect $z_0 = 0$. The fainter subsamples in this figure contain 20, 19, 18, 16, 15, 13, and 12 sdB stars, as plotted from left to right. The brighter subsamples in this figure are the same as in Fig. 3*a*.

that the $\langle V'/V'_{max} \rangle$ values for the subsamples trend significantly away from the result obtained for the full US sdB sample. The trends for the fainter subsamples (*open squares*) indicate an sdB space density that decreases *less* rapidly with distance above the Galactic plane than predicted by the assumed disk density model. On the other hand, the brighter subsamples (*filled circles*) show that the space density of the closer US sdB stars falls off *more* rapidly with height above the plane than the assumed disk density model!

In order to see what this evidence means for the disk scale height of the field sdB stars, a solution for z_0 has been obtained for each of the US sdB subsamples using the same technique that was employed for the full US sdB sample (i.e., by requiring $\langle V'/V'_{max} \rangle = 0.5$ for each subsample). The derived values for z_0 for each of the subsamples are plotted in Figure 3*b*. The error bars represent the estimated 1 σ range in z_0 as discussed above. No error bars are shown for those data points where the error bars are large enough to intersect $z_0 = 0$. The large error bars are themselves evidence that the assumed disk density law is not a very good match to the data, especially for the fainter subsamples. For this reason, the derived scale heights are not well constrained.

Despite the relatively large errors on any one scale height solution, there are striking, systematic trends in Figure 3*b* that are important for the clues they provide. First, the trends exhibited in this figure (and Fig. 3*a*) are *opposite* those expected if sample incompleteness were to exist near the bright- and/or faint-magnitude completeness limits (e.g., see Table 3 of V95 for the kind of trends caused by sample incompleteness). There is no reasonable distribution of incompleteness within the US sdB sample that could explain the trends seen in Figure 3*b*. Second, there is no evidence for an upper limit to the derived scale heights of the fainter US sdB subsamples (the derived scale heights for even fainter subsamples are off the scale of this figure). Third, the scale heights of the brighter subsamples do show some sign of converging to a level that is close to the value $z_0 = 500$ pc derived by V95 and SL95. These results are inconsistent with the hypothesis that all of the US sdB stars belong to a disk density distribution. Instead, these results suggest that the US sdB sample contains stars from both a disk *and* halo population of field sdB stars.

3.4. *Weighted V/V_{max}: A Test for a Combined Disk-plus-Halo Density Distribution for the Field sdB Stars*

The US sdB sample is too small to attempt to derive an accurate density model for the halo component of the field sdB population. However, a test can be performed for the *consistency* between the US sdB sample and reasonable, predetermined models for the density distributions of the disk and halo populations. Unfortunately, it is not known a priori to which of the two populations any of the individual US sdB stars belong. Thus it is not possible to unambiguously assign to each US sdB star the proper density weighting function for the canonical V'/V'_{max} analysis. Instead, for the purpose of this consistency test, the density weighting is assigned through a statistical treatment of the population membership.

The sdB disk density model used for this analysis is given in equation (4). The sdB disk scale height and density normalization constant are taken from the V95 study: $z_0 = 450$ pc, and ρ_{D0} is determined from the space density of disk sdB stars in the solar neighborhood, $\rho_{D\odot} = 3 \times 10^{-7}$ pc^{-3}. The stellar density model used for the halo component of the sdB population is one that is expected to be valid for the general halo population at distances larger than $\sim$0.5 kpc from the Galactic center (Young 1976; Bahcall & Soneira 1980):

$$\rho_H(R) = \rho_{H0} \exp \frac{-b(R/R_0)^{0.25}}{(R/R_0)^{7/8}} , \quad (5)$$

where R is the distance from the Galactic center, $R_0 = 2.83$ kpc is the halo radial scale length, and $b = 7.669$. The halo density normalization constant, ρ_{H0}, is determined from the constraint

$$\rho_H(R_\odot) = \Gamma \times \rho_{D\odot} , \quad (6)$$

where $\rho_H(R_\odot)$ is the space density of halo sdB stars in the solar neighborhood, and Γ is therefore the halo-to-disk sdB space density ratio in the solar neighborhood (to be determined as part of the analysis).

The membership of the individual US sdB stars in the disk and halo populations is handled statistically. Each star, *i*, is assigned normalized probabilities of being a member of the disk, $P_{D,i}$, *and* halo, $P_{H,i}$, based on its location within the stellar density models given in equations (4)–(6)

$$P_{D,i} = \frac{\rho_D(z_i, d_i)}{\rho_{TOTAL,i}} ,$$

$$P_{H,i} = \frac{\rho_H(R_i)}{\rho_{TOTAL,i}} ,$$

and

$$\rho_{TOTAL,i} = \rho_D(z_i, d_i) + \rho_H(R_i) , \quad (7)$$

where $P_{D,i} + P_{H,i} = 1.0$. Note that these normalized probabilities contain no dependence on any density normalization constants other than the factor Γ. The probabilities are used to calculate the following density weight for object *i*

$$W_i = P_{D,i} \frac{\rho_D(z_i, d_i)}{\rho_{D0}} + P_{H,i} \frac{\rho_H(R_i)}{\rho_{H0}} . \quad (8)$$

This density weight is then used to calculate the following density-weighted volume ratio

$$\frac{V'_i}{V'_{max,i}} = \frac{\int_{r_{min,i}}^{r_i} W_i r^2 \, dr}{\int_{r_{min,i}}^{r_{max,i}} W_i r^2 \, dr} . \quad (9)$$

In this manner each object contributes both a disk and halo component to $V'_i/V'_{max,i}$ according to its location within the assumed density distributions. In calculating the sample average, $\langle V'/V'_{max} \rangle$, this treatment should statistically approximate the situation where the correct population membership is known for each star, as long as the assumed density distributions correctly describe the data. On the other hand, to the extent that the stars do not follow the assumed density distributions, both the statistical approach to assigning the weights and the density weights themselves would be in error, and the analysis would not be expected to produce sample averages close to 0.5.

The only free parameter in the above disk-plus-halo density model is Γ. Thus, the first step in the consistency test is to obtain a solution for the value of Γ by requiring $\langle V'/V'_{max} \rangle = 0.5$ for the full, 20 member US sdB sample. The solution obtained for the halo-to-disk sdB density ratio in the solar neighborhood is $\Gamma = 0.3$ (estimated 1 σ range: 0.1–1.0), with a 2 σ lower limit of $\Gamma > 0.03$. Even the value of the lower limit is surprisingly large, given that previous studies have established that this ratio is 0.0015 for the field star population as a whole (Bahcall & Soneira 1980; Reid & Majewski 1993).

The solution for Γ alone does not test the consistency between the combined disk-plus-halo density model and the US sdB sample. One test for consistency *is* provided by monitoring the values of $\langle V'/V'_{\rm max} \rangle$ that are obtained through equation (9) for the various brighter and fainter subsamples of the US sdB sample that were created as described in § 3.2. In these calculations the value of Γ is held fixed at 0.3. As can be seen in Figure 4*a*, all of the derived values of $\langle V'/V'_{\rm max} \rangle$ are consistent with 0.5, to within the expected errors, over a wide range of systematically brighter (closer) and fainter (more distant) subsamples. These relatively stable results stand in contrast to the unstable trends exhibited in Figure 3, and provide strong evidence that the combined disk-plus-halo density model is a better representation of the US sdB sample than a disk density distribution alone.

The above results indicate that the assumed models for the halo and disk stellar density distributions are an *adequate* representation of the US sdB data. However, the search for *optimal* models and their parameters must await analysis of a larger sample of both bright and faint sdB stars. Such a study will be important for finding a more accurate value of Γ. As one example of the dependency of Γ on the values assigned to the model parameters, the above analysis is repeated with a different set of assumed values for the average sdB M_B and the sdB disk scale height. These particular parameters deserve special scrutiny because there is some evidence that the values used in the above analysis might not be the best choices. Recent spectroscopic and kinematic studies have obtained relatively consistent values for the average sdB M_V of $\sim +4.5$ (Saffer 1991; Liebert et al. 1994; Thejll et al. 1997), a value that is 0.8 mag fainter than the V95 reference M_V that has been used so far in the present analysis. In addition, if the PG sdB samples analyzed by V95 and SL95 are "contaminated" by some number of halo stars, then it is likely that the disk scale heights derived from those samples are too large.

Thus, in repeating the test for consistency between the combined disk-plus-halo density distribution and the US sdB data, the distances of equation (2) are calculated using $M_B = +4.2$ (color corrected from $M_V = +4.5$) and the disk scale height of equation (4) is set to $z_0 = 300$ pc (a value less than the V95 and SL95 result, but at the high end of the older determinations). The switch to a fainter M_B produces a systematic decrease in the calculated distances of the sdB stars (e.g., $\langle z \rangle$ is now approximately 2.0 kpc for the full sample, compared to $\langle z \rangle \simeq 2.9$ kpc previously). With these changes, a value of $\Gamma = 0.12$ (estimated 1 σ range: 0.04–0.4, and 2 σ lower limit of $\Gamma > 0.01$) is derived by requiring the $\langle V'/V'_{\rm max} \rangle$ of equation (9) to equal 0.50 for the full, 20 member US sdB sample. These values are a factor of ~ 3 smaller than the values obtained above in the initial analysis, but they are still significantly larger than the value of 0.0015 established for the field star population as a whole. As before, values of $\langle V'/V'_{\rm max} \rangle$ are obtained for the various brighter and fainter subsamples of the US sdB sample, while the value of Γ is held fixed at 0.12. These results are plotted in Figure 4*b*, and they are essentially the same as those plotted in Figure 4*a*. Although close inspection reveals that the results of Figure 4*b* indicate a slightly poorer fit, the differences between the two sets of results are insignificant. Thus, the disk-plus-halo model remains a superior representation of the US sdB sample, even under significant, but reasonable, variation of some of the model parameters.

4. SUMMARY, DISCUSSION, AND FUTURE WORK

The new sample of 20 sdB stars that has been isolated from the US survey is the first complete sample of sdB stars to reach significantly beyond the magnitude limits of the PG survey. The distance leverage afforded by this faint-magnitude coverage has allowed a $V'/V'_{\rm max}$ analysis to reveal that these stars are inconsistent with a disk-only stellar density distribution. Instead their distribution is more consistent with a combined disk-plus-halo density law. Although a few individual sdB stars have previously been found to have space motions that are consistent with halo membership (e.g., Saffer 1991; Colin et al. 1994), this is the first detection of the halo component of the sdB population through a spatial-distribution analysis. The existence

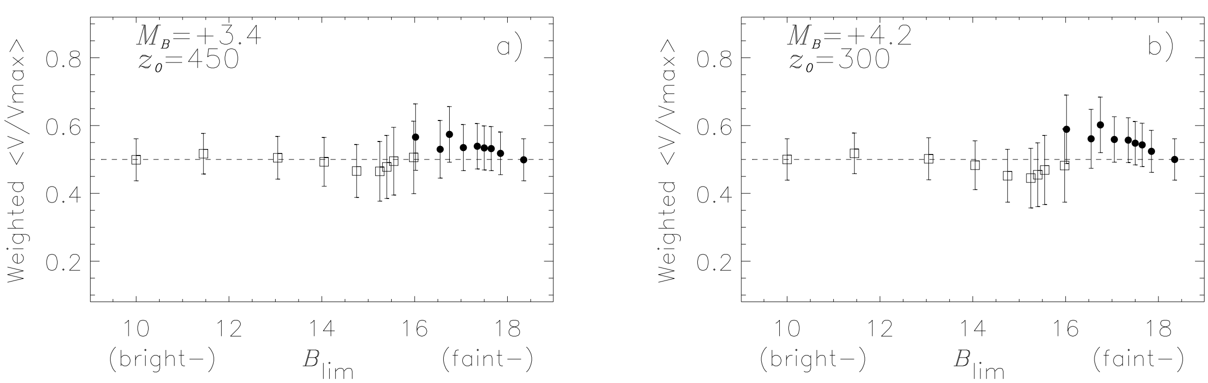


FIG. 4.—Values of weighted $\langle V'/V'_{\rm max} \rangle$ are plotted for various subsamples of the US sdB sample vs. the bright and faint-magnitude limits that were imposed to create the subsamples. The density weighting consists of the combined disk-plus-halo density distribution discussed in § 3.4. Symbols are defined as in Fig. 2. The fainter subsamples shown contain 20, 19, 18, 16, 15, 13, 12, 11, and 10 sdB stars; the brighter subsamples contain 20, 19, 18, 17, 16, 14, 13, 11, and 10 sdB stars. The relative stability of these results stands in contrast to the striking trends in Fig. 3, and provides further evidence that there is a significant component of the halo sdB population in the US sdB sample. (*a*) The results obtained when the distances to the sdB stars are based on an average sdB $M_B = +3.4$ and the disk scale height is fixed at 450 pc. (*b*) The results obtained for distances based on an average sdB $M_B = +4.2$ and a disk scale height of 300 pc.

of a thick disk ($z_0 \simeq 1$ kpc) component to the sdB population is not ruled out by the present analysis; but the analysis does show that such a component is not a dominant presence in the US sdB sample (e.g., see Fig. 3*b*). There is some evidence in Figure 4 for the existence of excess power at intermediate distances (~ 1–2 kpc) that causes the $\langle V'/V'_{\rm max}\rangle$ values of both the moderately faint and moderately bright subsamples to wander from 0.5. However, these deviations of the $\langle V'/V'_{\rm max}\rangle$ values are not significant relative to the errors.

The $V'/V'_{\rm max}$ analysis[2] of the US sdB sample has also provided a loosely constrained initial value of $\Gamma = 0.12$ for the halo-to-disk sdB space density ratio in the solar neighborhood. Although the errors on this initial determination are large, even the 2 σ lower limit value of ~ 0.01 is a factor of ~ 7 larger than the accepted ratio for the field star population as a whole. Such numbers would imply that the metal-poor halo population can produce a horizontal-branch morphology that is, by a factor of $\gtrsim 7$, more heavily weighted toward the "extreme" blue end than the horizontal branch produced by the relatively metal-rich disk population. This is in qualitative agreement with the trend generally observed for the horizontal-branch morphologies of globular and open clusters of different metallicities.

What do these numbers imply for the relative mix of disk and halo stars to be found in samples of field sdB stars? With $\Gamma = 0.12$, the disk and halo models discussed in § 3.4 indicate that only about $\sim 15\%$ of the US sdB sample (i.e., three stars) belong to the disk population. These three stars are most likely to be among the brightest in the sample. Although the US sdB sample is dominated by halo stars, it is important to correctly account for these three disk stars; a $V'/V'_{\rm max}$ analysis shows that the full 20 member US sdB sample is *not* consistent with a halo-only density distribution. That same analysis does show that the faint half of the US sdB sample *is* consistent with a halo-only density distribution. These results imply, perhaps not too surprisingly, that high-latitude sdB stars fainter than $B \simeq 15.5$ (i.e., with $z \gtrsim 1.5$ kpc) will predominantly come from the halo population. But what about the brighter sdB stars? The present results indicate that complete samples of high-latitude sdB stars with $B_{\rm faint} \simeq 14.5$ (approximately the faint limit of the V95 sample) should contain roughly 60% disk stars and 40% halo stars; at $B_{\rm faint} \simeq 13$ the percentages are roughly 80% disk and 20% halo stars. These are perhaps surprisingly large percentages of halo stars to expect in such relatively bright samples. However, the existence of halo sdB stars in such numbers could account for at least some of the trouble previous analyses have had in arriving at consistent values for the sdB disk scale height. Furthermore, these numbers are in rough agreement with recent kinematic results. Colin et al. (1994) have found one, and perhaps two, out of the five bright ($V \lesssim 14.5$) PG sdB stars they studied to have a Galactic orbit that is inconsistent with membership in a disk distribution with a scale height of only ~ 500 pc. In addition, the Thejll et al. (1997) analysis of a larger, but similarly bright, sample of 31 PG sdB stars (from which two sdB stars had already been excluded for clearly having halo kinematics) shows that the sample has a larger velocity dispersion than expected for an old disk population.

The sdB stars sit just coolward of the sdO stars in the H-R diagram, and they are among the candidates for the evolutionary progenitors of the sdO stars. One means of testing for a possible evolutionary relationship is to compare the spatial distributions of the two types of hot subdwarfs. Thejll et al. (1994) found that 21 He-rich PG sdO stars tended to have large distances above the Galactic plane, which they interpret as evidence for membership of these stars in an older population such as the halo or thick disk. Such a spatial distribution for the sdO stars was inconsistent with the disk distribution accepted for the sdB stars at that time, and this was seen as one piece of evidence against an evolutionary connection between the sdO and sdB stars. The results of the present paper show that the sdB stars also have a significant halo component, and that this component could begin to contribute $\sim 50\%$ of the stars in sdB samples with faint-magnitude completeness limits as bright as $B_{\rm faint} \simeq 15$. Because the sdO stars are more luminous than the sdB stars by an average of ~ 1 magnitude (e.g., Thejll et al. 1997), a population of sdO stars that is spatially coincident with the sdB population could be expected to provide halo stars that would begin to dominate sdO samples with faint-magnitude completeness limits as bright as $B_{\rm faint} \simeq 14$. This is approximately the average magnitude of the PG sdO stars studied by Thejll et al. (1994). Thus, it appears that the data presently available are consistent with the hypothesis that the sdO and sdB stars have similar spatial distributions. The possibility that sdO and sdB stars are related through evolutionary processes cannot yet be ruled out on the basis of spatial-distribution arguments.

Further improvements to the US sdB sample and its analysis are possible. Better quality spectroscopy coupled with model atmosphere fitting (e.g., as in Saffer et al. 1994) for all of the US hot subdwarf candidates would provide accurate atmospheric parameters for better classifications and distance estimates. Such data would allow the complete samples of faint US sdB and sdO stars to be established more accurately. In addition, extending the spectroscopic survey to the faint-magnitude completeness limits of the US catalogs would provide important new members to the faint end of the samples. Both of these steps to improve the observational basis of the US samples would increase the accuracy of the spatial-distribution analysis.

An accurate picture of the various components of the field sdB star spatial distribution is a prerequisite for a better understanding of the evolutionary relationships between the sdB stars and the other types of hot, evolved stars. Space densities and birthrates will need to be obtained for the sdB stars of each spatial component. Thus, it is clearly important to confirm the results reported in this paper through the use of larger samples of faint field sdB stars that cover a wider range of Galactic coordinates. Such analyses would be able to better constrain the various parameters of the halo and disk density distributions. In particular, larger samples would be able to test for the existence of a thick disk component. The information on the sdB halo density distribution that can be obtained at relatively faint magnitudes will in turn provide important support for a cleaner determination of the sdB disk density parameters from the PG samples at the brighter magnitudes where there appears to be a significant overlap of the disk and halo components. Even now, a reexamination of the spatial distribution of the PG sdB sample might prove fruitful in light of the present findings.

[2] The numbers quoted throughout this section are based on the results obtained with the models of § 3.4, where $M_B = +4.2$ and $z_0 = 300$ pc.

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