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SN 1987A: EVOLUTION OF THE ENVELOPE TEMPERATURE AS DEDUCED FROM Hα EMISSION

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ABSTRACT

In this paper, we present a new method for determining the temperature and ionization evolution of the hydrogen envelope of SN 1987A by comparing the observed Hα light curve and one which is derived from the recombination theory for hydrogen lines. This is a simple and convenient method for determining the temperature and ionization evolution which could be applicable to the late time of all Type II supernovae.

Subject headings: line: formation — supernovae: individual (SN 1987A)

1. INTRODUCTION

SN 1987A exploded 10 years ago but remains a topic of great astrophysical interest. Continuous monitorings in all bands of the electromagnetic spectrum have been made and a wealth of data and information has been obtained which has greatly promoted our understanding of supernova physics, both in theory and in observation. Undoubtedly, enthusiasm for studies of SN 1987A will continue into the future. Except for the persistent monitoring, it may be most important to process the available observational data and give an insightful analysis.

In this paper, we study the temperature and ionization evolution of the expanding hydrogen envelope. In particular, we analyze the observed light curves of the hydrogen emission line in order to find the temperature-variation curve. Obviously, temperature is an important parameter in supernova physics. But so far the study of this subject has been unsatisfactory compared with that given to other problems, e.g., the density, the chemical composition and spatial distribution, etc. (e.g., Pinto & Woosley 1988). Some discussions of this topic can be found in the literature, and several methods have been used to estimate the temperature, e.g., by solving the energy equilibrium equation (which expresses the local balance of heating by fast electrons and cooling by atomic and molecular radiation), by considering the condition of dust formation, or by using the observed line ratios of certain elements, etc. All these methods have defects and are difficult to use in calculating the temperature with confidence (McCray 1993). However, most work has centered on estimates of the temperature of clumps which are composed of heavy elements (e.g., the Fe/Co/Ni clumps) and are distributed throughout the envelope with small filling factor (Li, McCray, & Sunyaev 1993; Li & McCray 1992; Moseley et al. 1989; Liu, Dalgarno, & Lepp 1992; Roche, Aitken, & Smith 1991, etc.). Such clump temperatures are probably not representative of the whole envelope where the dominant elements are hydrogen and helium.

The uncertainty in temperature estimates is troublesome for studies of atomic and molecular processes in the envelope of SN 1987A, for example, in the calculations of hydrogen recombination lines at high optical depth (Xu et al. 1992). These authors calculate the time evolution of the hydrogen lines under the assumption of constant temperature ($T = 3000$ K). They state, however, that their model results are insensitive to the assumed temperature.

Recently, following Xu et al., we also assumed a constant temperature to derive the Hα light curve using recombination theory (You et al. 1994). The resultant curve is markedly different from observations. We recognized the need to check the assumption of constant temperature and that temperature evolution has to be taken into consideration.

Another important parameter in supernovae physics is the ionization fraction χ_e. Kozma & Fransson (1992) have given a simple expression which describes the time evolution of χ_e. The formula shows that the ionization fraction of hydrogen gas in the envelope is temperature-dependent, $\chi_e = \chi_e(T, t)$ (see our eq. [12]). In order to obtain the evolution curve $\chi_e \sim t$, they also assumed a constant T.

In this paper, we give a quantitative calculation for time evolution of the temperature as well as the ionization fraction of the gas in the expanding envelope of SN 1987A using a new method. The basic idea is that information on the evolution of either $T(t)$ or $\chi_e(t)$ must be involved in the line variations because the change of T and χ_e in envelope are the very two factors which control the intensity-variation behavior of recombination lines. In the following sections, we will illustrate how to extract information about the evolution of $T(t)$ and $\chi_e(t)$ from the observed Hα light curves.

2. CALCULATION OF Hα LIGHT CURVE

Hydrogen is the most abundant element in the envelope of SN 1987A. A representative temperature for the entire envelope can be found from the analysis of the light curve of hydrogen recombination lines, among which Hα is the strongest. After $t \gtrsim 120$ days, the envelope turns to the nebula phase. Figures 1, 2, and 3 present the observed Hα light curves given by Phillips & Williams (1991), Terndrup et al. (1988), and McCray (1993), respectively. Figure 3 shows the fractional luminosity of $L_{\mathrm{H}\alpha}/L_{\mathrm{B}}$, i.e., the ratio of Hα luminosity to the bolometric luminosity. The common feature of these light curves is that they all show a turning point at $t \simeq 300$ days, by which the light curves can be divided into two parts which have different variation behavior. In the early stage (120 days $\lesssim t \lesssim$ 300 days), the Hα luminosity $L_{\mathrm{H}\alpha}$ decreases with t rather slowly, much slower than the decreasing of bolometric luminosity. (This is

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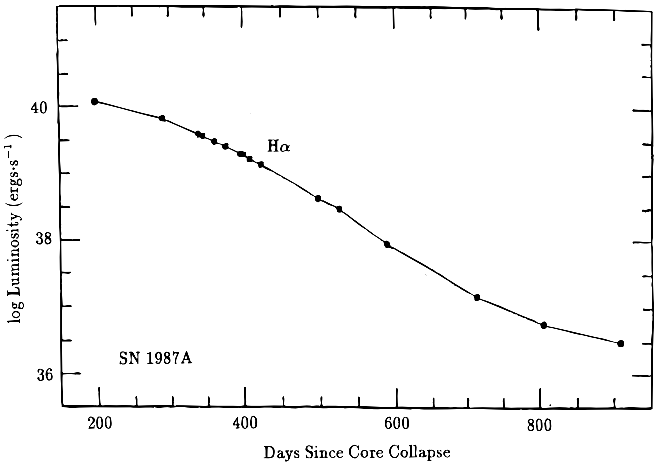

FIG. 1.—Hα light curve (taken from Phillips & Williams 1991)

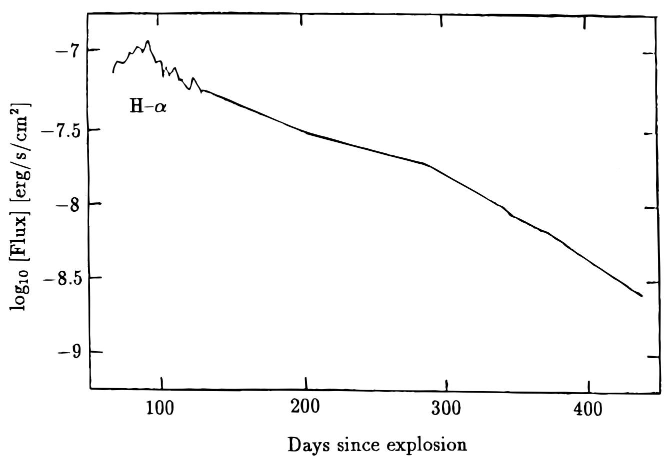

FIG. 2.—Hα light curve (taken from Terndrup et al. 1988)

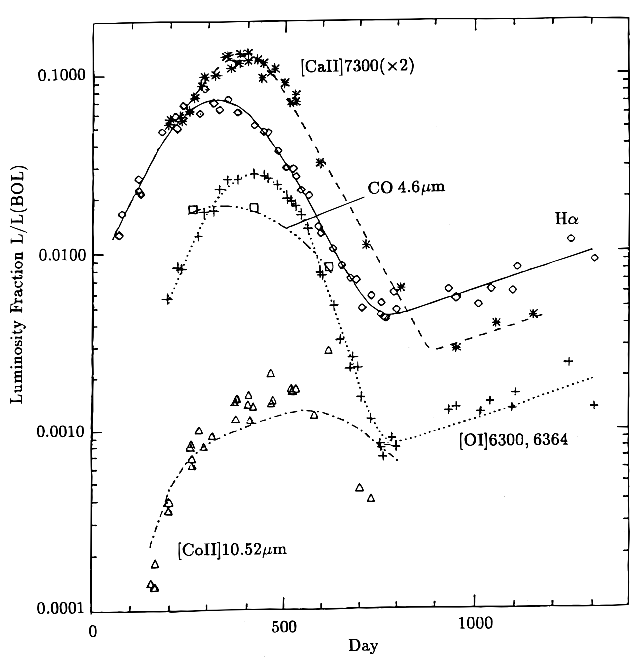

FIG. 3.—Fractional luminosities of various emission lines to the bolometric luminosity of SN 1987A, $L/L_{\rm B}$, from McCray (1993).

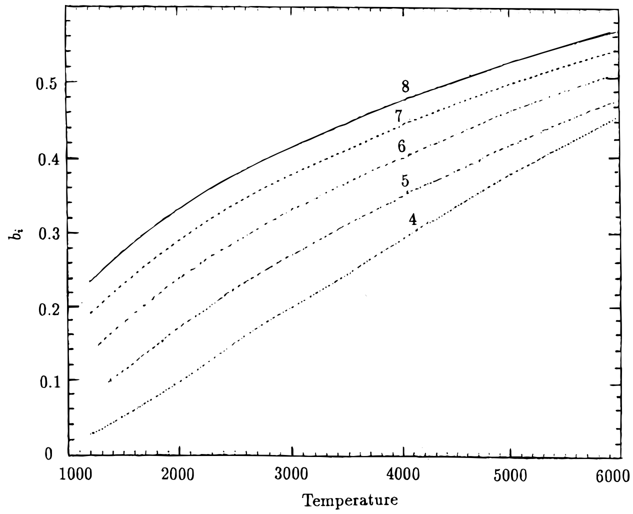

FIG. 4.—Dependence of b_i ($i = 4$–8) on the temperature T, where b_i represents the deviation of the population of level i from its thermal equilibrium.

shown most clearly by the fractional luminosity $L_{\rm H\alpha}/L_{\rm B}$, as in Fig. 3, where the ratio $L_{\rm H\alpha}/L_{\rm B}$ increases with t for $t \lesssim 300$ days). In the late stage (300 days $\lesssim t \lesssim 500$ days), $L_{\rm H\alpha}$ decreases relatively faster, and after $t \gtrsim 500$ days, it decreases even more steeply. We can fit the Hα light curve (e.g., in Fig. 1) by an empirical power-law formula and get $L_{\rm H\alpha} \propto t^{-1.98}$ for the early stage (120 days $\lesssim t \lesssim 300$ days), $L_{\rm H\alpha} \propto t^{-3.52}$ for 300 days $\lesssim t \lesssim 500$ days. After 500 days, the power law index is much steeper.

In order to derive the theoretical Hα light curve, it is necessary to consider the radiative transfer of emission lines, at least for the early stage while the envelope gas is denser and hotter and $\tau_{\rm H\alpha} \gg 1$ (which constitutes recombination case C). In this case, Hα photons are scattered many times before escaping from the envelope, and this will affect the Hα luminosity. In the late stage, the envelope becomes optically thin in Hα ($\tau_{\rm H\alpha} \ll 1$; as can be seen below, $\tau_{\rm H\alpha} \propto t^{-2}$), and the multiple scatterings of Hα photons is a negligibly small effect. In the following calculations, we will derive a formula for the Hα luminosity for $\tau_{\rm H\alpha} \gg 1$ and for $\tau_{\rm H\alpha} \ll 1$, respectively.

For a homologously expanding envelope, the treatment of the radiative transfer of emission lines becomes quite simple. From the observed line profile of Hα one can infer that the hydrogen is distributed approximately uniformly throughout the major part of the envelope (within $v_{\rm max} \lesssim 2500$ km s^{-1}), with no apparent cavity at the center of envelope. At $t \geq$ several weeks, the envelope is in free expansion and there exists a radial gradient of velocity [$v = v(r)$]. The expanding velocity increases outward from $v_{\rm min} \simeq 0$ to $v_{\rm max} \simeq 2500$ km s^{-1}. Therefore, relative to any given absorbing atom located in a certain layer, there is a Doppler redshift for any photon emerging from the inner layers. So the resonant line absorption only occurs in a small region around the emitting atom. The optical depth of this layer is (Sobolev 1960; Castor 1970; Xu et al. 1992)

$$\tau_{lu} = \frac{\pi e^2}{m_e c} f_{lu} \lambda_{ul} t N_l \left(1 - \frac{g_l N_u}{g_u N_l}\right) = \frac{\lambda_{lu}^3 t g_u A_{ul} N_l}{8\pi g_l} \left(1 - \frac{g_l N_u}{g_u N_l}\right), \quad (1)$$

where t is the age, f_{lu} is the absorption oscillation strength, A_{ul} is the radiative decay rate, λ_{lu} is the line wavelength, and N_u, $g_u(N_l, g_l)$ are the population density and statistical weight of the upper (lower) state, respectively. For the Hα line, equation (1) becomes $\tau_{\mathrm{H}\alpha} \simeq (\pi e^2/m_e c^2)\, f_{23}\, \lambda_{\mathrm{H}\alpha}\, t N_2 \simeq (\lambda_{\mathrm{H}\alpha}^3 g_3 A_{32} N_2 t/8\pi g_2)$, where $\lambda_{\mathrm{H}\alpha} = 6560$ Å. For homologous expansion, $N_l \propto t^{-3}$, and therefore $\tau_{lu} \propto t^{-2}$, decreases very rapidly with t.

According to the Sobolev approximation (Sobolev 1960; Castor 1970; Kirshner & Kwan 1975), the probability that a line photon will escape the envelope is given by $P_{\mathrm{esc}} = \tau_{lu}^{-1}[1 - \exp(-\tau_{lu})] \approx (1/\tau_{lu})$. Thus the local line emissivity is $j_{\nu ul}^{\mathrm{eff}} = j_{\nu_{ul}} P_{\mathrm{esc}} \approx (N_u A_{ul} h\nu_{ul}/\tau_{lu})$. In other words, the effect of absorption can be expressed by a replacement $j_\nu \to j_\nu^{\mathrm{eff}} \approx (j_\nu/\tau_{lu})$, equivalently, by the replacement $A_{ul} \to A_{ul}^{\mathrm{eff}} \equiv (A_{ul}/\tau_{lu})$. For a freely expanding envelope, the volume $V_{\mathrm{env}} \simeq (4\pi/3) v_{\mathrm{max}}^3 t^3$, and the total luminosity of Hα is

$$L_{\mathrm{H}\alpha} \simeq j_{\mathrm{H}\alpha}^{\mathrm{eff}} V_{\mathrm{env}} \simeq \frac{32\pi^2 h\nu_{\mathrm{H}\alpha}^4}{3c^3} \frac{g_2}{g_3} v_{\mathrm{max}}^3 \frac{N_3}{N_2} t^2 \quad (\tau_{\mathrm{H}\alpha} \gg 1)\,, \tag{2}$$

and

$$L_{\mathrm{H}\alpha} \simeq \frac{4\pi}{3} A_{32} h\nu_{\mathrm{H}\alpha} v_{\mathrm{max}}^3 N_3 t^3 \quad (\tau_{\mathrm{H}\alpha} \ll 1)\,. \tag{3}$$

For both the optically thick and optically thin cases, in order to derive the theoretical Hα light curve $L_{\mathrm{H}\alpha} \sim t$, it is necessary to find the populations N_2 and N_3 and their dependence on t. The conventional method of calculating the population N_i is to solve the statistical equilibrium equations in the steady states $[(dN_i/dt) = 0]$:

$$\begin{aligned}
&\Gamma f_i^{\mathrm{ion}} N_i + N_i \\
&\quad \times \left[\sum_{j<i} (N_e C_{ij} + A_{ij}^{\mathrm{eff}} + \zeta_{ij}) + \sum_{j>i} (N_e C_{ij} + \zeta_{ij}) + \zeta_{ic}\right] \\
&= \alpha_i N_e N_{\mathrm{H}^+} + \sum_{j>i} N_j (N_e C_{ji} + A_{ji}^{\mathrm{eff}} + \zeta_{ji}) \\
&\quad + \sum_{j<i} N_j (N_e C_{ji} + \zeta_{ji}) + \Gamma f_i N_{\mathrm{H}}\,,
\end{aligned} \tag{4}$$

where N_e is the electron number density, C_{ij} is the collision coefficient, α_i is the recombination rate to level i, ζ_{ji} is the stimulated transition rate due to the external radiation field (the effect of photoexcitation due to the local line emission has been included in A_{ji}^{eff}), and ζ_{ic} accounts for the photoionization of level i. The term $\Gamma f_i^{\mathrm{ion}} N_i$ represents the nonthermal excitation-ionization from level i by the fast electrons, where Γ is a parameter to characterize the rate (per hydrogen atom) of ionization and excitation by nonthermal electrons, f_i^{ion} is the fraction of Γ contributing to the ionization and excitation of level i, and N_i is the number density of hydrogen atoms in level i. On the right side of equation (4), $\Gamma f_i N_{\mathrm{H}}$ represents the nonthermal excitation of a hydrogen atom from the ground level to level i by the fast electrons, where f_i is the fraction of Γ contributing to level i. N_{H} is the number density of hydrogen atoms.

Γ can be written as $\Gamma \sim [L_{\mathrm{B}}/N(\mathrm{H})E]\ \mathrm{s}^{-1}$, where $E(\sim 20$ eV; see McCray 1991) characterizes the average energy needed for each excitation or ionization and $N(\mathrm{H})$ is the total number of hydrogen atoms in the envelope. After 120 days, the observed bolometric luminosity $L_{\mathrm{B}} \lesssim 10^{41}$ ergs s^{-1}. Taking the total mass of hydrogen as 10 $M_\odot$, we get $\Gamma \leq 2.5 \times 10^{-7}\ \mathrm{s}^{-1}$. For our purpose, only the populations of levels with $i \geq 3$ are needed (see eq. [6]). Xu & McCray (1991) give $f_i \lesssim 6.20 \times 10^{-2}$ for the ionization fraction $\chi_e = 0.01$ and $i \geq 3$. From Pinto & Woosley's (1988) model, $N_{\mathrm{H}} \sim 10^{10}\ \mathrm{cm}^{-3}$ at $t_y = 1$. So $\Gamma f_i N_{\mathrm{H}} < 1.5 \times 10^2$. On the other hand, we have $\alpha_i N_e N_{\mathrm{H}^+} \gtrsim 10^4$. Therefore, the nonthermal term $\Gamma f_i N_{\mathrm{H}}$ is sufficiently small and can be neglected from equation (4). Likewise, ζ_{ji} and ζ_{ic} is much smaller than effective radiative transition rate (for $i \geq 3$) and can also be neglected. Finally, we neglect the term $\Gamma f_i^{\mathrm{ion}} N_i$ on the left-hand side because of the inequality $\Gamma f_i^{\mathrm{ion}} N_i \ll \Gamma f_i N_{\mathrm{H}} (N_i \ll N_{\mathrm{H}})$.

For SN 1987A, the density of hydrogen becomes $N_{\mathrm{H}} \sim 10^{10}\ \mathrm{cm}^{-3}$ at $t = 1$ yr. Setting the ionization fraction $\chi_e \simeq 10^{-2}$, we get for the electron density $N_e \sim 10^8\ \mathrm{cm}^{-3}$, which ensures the condition $N_e \ll N_{ij}^{\mathrm{cr}} \equiv A_{ij}^{\mathrm{eff}}/c_{ij}$ for $t_y \gtrsim 1$ (in fact, the condition $N_e \ll N_{ij}^{\mathrm{cr}}$ is valid even at $t \simeq 120$ days). Thus all the collision terms can be neglected after $t \gtrsim 120$ days. Finally equation (4) simplifies to

$$N_i \sum_{i>k} A_{ik}^{\mathrm{eff}} = \alpha_i N_e N_{\mathrm{H}^+} + \sum_{k>i} N_k A_{ki}^{\mathrm{eff}}\,. \tag{5}$$

We solve equation (5) for the populations $N_i (i = 4, 5, 6, \ldots)$. Usually, the population N_j is written as

$$N_j = b_j N_j^* = b_j N_e N_{\mathrm{H}^+} \frac{h^3}{(2\pi mkT)^{3/2}} \frac{g_j}{2} \exp\left(\frac{I_{\mathrm{H}}}{j^2 kT}\right),$$

where N_j^* is the thermalized population of level j given by Saha equation and b_j is the deviation factor of level j from the thermal equilibrium. Inserting $N_j = b_j N_j^*$ into equation (5) and taking T as a parameter, we get the b_i equations which are N_e-independent because the product $N_e N_{\mathrm{H}^+}$ cancels on both sides. Therefore, equation (5) becomes

$$\begin{aligned}
b_i\left[g_i \exp\left(\frac{I_{\mathrm{H}}}{i^2 kT}\right) \sum_{j<i} A_{ij}^{\mathrm{eff}}\right] &= \alpha_i \left[\frac{h^3}{2(2\pi mkT)^{3/2}}\right]^{-1} \\
&\quad + \sum_{j>i} \left[g_j \exp\left(\frac{I_{\mathrm{H}}}{j^2 kT}\right) A_{ji}^{\mathrm{eff}}\right] b_j\,; \quad i = 4, 5, 6, \ldots\,.
\end{aligned}$$

Taking $i = 8$, from equation (5) we get b_8 under the approximation that $b_k = 1$ for $k \geq 9$. Then, taking $i = 7$ and using the solved b_8, we can get b_7 (still set $b_k = 1$ for $k \geq 9$), and so forth. Hence we obtain the solutions $b_j (j = 4, 5, 6, 7, 8$; see Fig. 4).

For $i = 3$, equation (5) simplifies to

$$N_3 \frac{A_{32}}{\tau_{32}} \simeq \alpha_3 N_e N_{\mathrm{H}^+} + \sum_{k>3} N_k A_{k3}\,. \tag{6}$$

On the left side of equation (6), we have neglected $A_{31}^{\mathrm{eff}} = (A_{31}/\tau_{31})$ because of $\tau_{31} \gg \tau_{32}$ and $(A_{31}/\tau_{31}) \ll (A_{32}/\tau_{32})$. On the right-hand side, all A_{k3}^{eff} values are approximately replaced by A_{k3}, i.e., $A_{k3}^{\mathrm{eff}} \simeq A_{k3}$. We have assumed that gas is optically thin for Paschen lines ($\tau_{3k} \ll 1$). This is a good approximation for the envelope of SN 1987A. Therefore from equation (6) we get

$$N_3 = \frac{(\alpha_3 + \beta_3)\tau_{32} N_e N_{\mathrm{H}^+}}{A_{32}}\,,$$

where

$$\beta_3 \equiv \sum_{k>3}^{\infty} b_k \frac{h^3 A_{k3}}{(2\pi mkT)^{3/2}} \frac{g_k}{2} \exp\left(\frac{I_{\mathrm{H}}}{K^2 kT}\right),$$

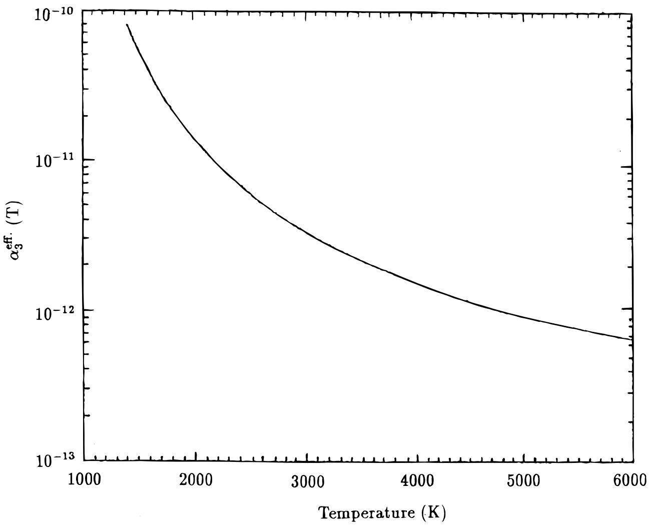

FIG. 5.—Variation of the effective recombination coefficient $\alpha_3^{\rm eff}(T)$ with temperature T, $\alpha_3^{\rm eff} \sim T$ curve.

is called the rate coefficient of indirect recombination. The direct recombination coefficient $\alpha_n(T)$ can be calculated by use of the following formula (Rybicki & Lightman 1979):

$$\alpha_n(T) = 3.26 \times 10^{-6} T^{-3/2} n^{-3} \exp\left(\frac{I_{\rm H}}{n^2 kT}\right) K\left(\frac{I_{\rm H}}{n^2 kT}\right),$$

where $K(t) = \int_t^\infty (e^{-x}/x)dx = \int_1^\infty (e^{-t\xi}/\xi)d\xi$ is an exponential integral. Defining $\alpha_3^{\rm eff}(T) \equiv \alpha_3(T) + \beta_3(T)$ as the total effective recombination rate coefficient, we get

$$N_3 = \frac{\alpha_3^{\rm eff}(T) N_{\rm H^+} \tau_{32}}{A_{32}} . \qquad (7)$$

It is easy to see that both the $\alpha_3(T)$ and $\beta_3(T)$ are monotonically decreasing functions of T because of the exponential factor $(I_{\rm H}/eK^2kT)$, and so is $\alpha_3^{\rm eff}(T)$. Inserting the N_2-dependent $\tau_{32} \equiv \tau_{\rm H\alpha} = (\lambda_{\rm H\alpha}^3/8\pi)(g_3/g_2)A_{32} N_2 t$ into equation (7), we obtain the ratio (N_3/N_2):

$$\frac{N_3}{N_2} = \frac{\lambda_{\rm H\alpha}^3}{8\pi} \frac{g_3}{g_2} \alpha_3^{\rm eff}(T) t N_{\rm e} N_{\rm H^+} . \qquad (8)$$

Inserting (N_3/N_2) into equation (2), we get the Hα luminosity in the case $\tau_{\rm H\alpha} \gg 1$,

$$L_{\rm H\alpha} = \frac{4\pi h\nu_{\rm H\alpha}}{3} v_{\rm max}^3 \alpha_3^{\rm eff}(T) N_e N_{\rm H^+} t^3 \quad (\tau_{\rm H\alpha} \gg 1) . \qquad (9)$$

In the optically thin case ($\tau_{\rm H\alpha} \ll 1$; note, however, $\tau_{31} \equiv \tau_{\rm Ly\alpha} \gg 1$), the equilibrium equation (5) for the level 3 becomes

$$N_3 A_{32} = \alpha_3 N_e N_{\rm H^+} + \sum_{k>3}^{\infty} N_k A_{k3} ,$$

where we continue to neglect the term $A_{31}^{\rm eff} = (A_{32}/\tau_{31})$ on the left-hand side, this gives an expression for N_3 (similar to eq.[7])

$$N_3 \frac{\alpha_3^{\rm eff}(T) N_e N_{\rm H^+}}{A_{32}} \qquad (10)$$

where $\alpha_3^{\rm eff}(T)$ is the same as in equation (7). Notice that equation (10) differs from equation (7) only by a factor $\tau_{\rm H\alpha}$. Substituting equation (10) into equation (3), we get the Hα luminosity for the optically thin case ($\tau_{\rm H\alpha} \ll 1$),

$$L_{\rm H\alpha} = \frac{4\pi h\nu_{\rm H\alpha}}{3} v_{\rm max}^3 \alpha_3^{\rm eff}(T) N_e N_{\rm H^+} t^3 \quad (\tau_{\rm H\alpha} \ll 1) .$$

It is surprising at first glance that the formula of $L_{\rm H\alpha}$ for the optically thin case is exactly the same as the optically thick case (eq. [9]). The physics is easy to understand. In the optically thick case, the radiative transition rate decreases from A_{32} to (A_{32}/τ_{32}), which decreases the emergent luminosity of Hα line. Nevertheless, the decrease of escape probability markedly increases the population of level 3, N_3 (see eq. [7]), which enhances the Hα emission. Hereafter we will use the same equation equation (9) to calculate the light curve both for $\tau_{\rm H\alpha} \gg 1$ and $\tau_{\rm H\alpha} \ll 1$. We rewrite it as

$$L_{\rm H\alpha} = K\alpha_3^{\rm eff}(T) N_e N_{\rm H^+} t^3 , \qquad (11)$$

where K is a constant.

The time evolution of density $N_{\rm H^+} \simeq N_e \sim t$ can be inferred simply from the observations of the infrared continuum. Observations in the 1.5–12 μm band between 1987 November and 1988 April (McCray 1990) show that the emission flux $F_\nu^{\rm IR} \propto t^{-4}$. Observations also show that the infrared continuum is quite flat ($F_\nu^{\rm IR} \propto \nu^0$), suggesting optically thin free-free radiation (bremsstrahlung). In that time, there is no evidence for thermal infrared emission by dust, which should have a very different spectrum (Dwek 1988). So the observed infrared flux $F_\nu^{\rm IR}$ should be proportional to the emission measure $\mathrm{EM}(+) = \int N_e N_+ dV \simeq N_e^2 V$. For free expansion, the emitting volume $V \propto t^3$. From these we get $N_{\rm H^+} \simeq N_e \propto t^{-3.5}$, which is correct and valid up to $t \lesssim$ 410 days (1988 April). Substituting this relation into equation (11), we get $L_{\rm H\alpha} = K\alpha_3^{\rm eff}(T) t^{-4}$. Unfortunately, such a theoretical expression of the Hα light curve is only valid for $t \lesssim 410$ days.

In order to improve the calculation of the $N_e(t)$ evolution, we have to have a knowledge of the evolution of the ionization fraction $\chi_e(t)$ because of the relation $N_{\rm H^+} \simeq N_e \simeq \chi_e N_{\rm H}$, where $N_{\rm H} \propto t^{-3}$ for free expansion. According to Kozma & Fransson (1992), $\chi_e(t)$ can be derived as

$$X_e(t) = CT^{0.44} t^{1/2} \exp\left(\frac{-t}{222\ \rm days}\right) . \qquad (12)$$

From equation (12) we see χ_e is weakly dependent on temperature T as mentioned in the introduction of this paper, $\chi_e = \chi_e(T, t)$. equation (11) can be rewritten as

$$L_{\rm H\alpha}(t) = K\alpha_3^{\rm eff}(T) \chi_e^2(T, t) N_{\rm H}^2 t^3 . \qquad (13)$$

Substituting equation (12) into equation (13), we have

$$L_{\rm H\alpha}^{\rm theor}(t) = \tilde{K}\alpha_3^{\rm eff}(T) T^{0.88} \exp\left(\frac{-t}{111\ \rm days}\right) t^{-2} . \qquad (14)$$

$\tilde{K}$ is a constant. Equation (14) is the final form of the theoretical Hα light curve in this paper which will be compared with the observed Hα light curve. We next show that $T(t)$ and $\chi_e(t)$ curves can be obtained from the comparison between the observations and equation (14).

3. TIME-EVOLUTION CURVES OF TEMPERATURES AND IONIZATION FRACTION

In order to deduce the temperature evolution, it is necessary to calculate the coefficient $\alpha_3^{\rm eff}(T) \equiv \alpha_3(T) + \beta_3(T)$. The

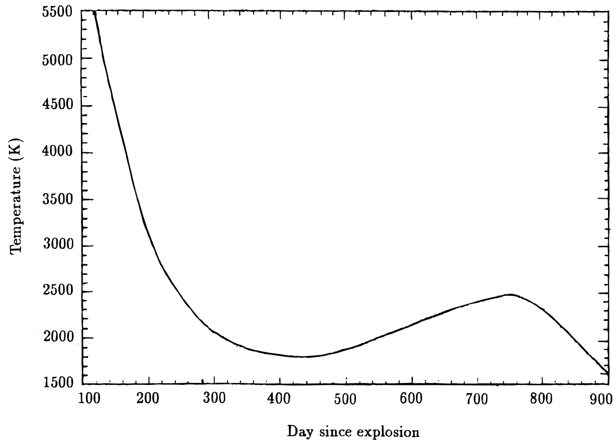

FIG. 6.—Calculated temperature evolution $T \sim t$ curve

calculated $\alpha_3^{\rm eff}(T)$ versus T curve is shown in Figure 5, where $\alpha_3^{\rm eff}$ is increasing with the decrease of temperature as we expected (the capture of electrons by the positive ions will be much easier in the low temperature). The calculated coefficients b_i for case C are shown in Figure 4.

Setting the expression for $L_{\rm H\alpha}^{\rm theor}(t)$ given by equation (14) equal to the observed value $L_{\rm H\alpha}^{\rm Obs}(t)$ (e.g., that given by Phillips & Williams 1991 in Fig. 1) gives

$$L_{\rm H\alpha}^{\rm theor}(t) = \tilde{K}\alpha_3^{\rm eff}(T)T^{0.88}\exp\left(\frac{-t}{111\ \rm days}\right)t^{-2} = L_{\rm H\alpha}^{\rm obs}(t)^* \ . \quad (15)$$

Taking the temperature of photosphere at $t \simeq 120$ days ($T_0 \simeq 5500$ K) as the initial value of temperature of the hydrogen gas in the nebula phase (the photosphere vanished at $t \simeq 120$ days and the nebula phase occurred), we derive the curve $\alpha_3^{\rm eff}(T)T^{0.88} \sim t$. Comparing this curve with curve $\alpha_3^{\rm eff}(T) \sim T$ (Fig. 5), the time evolution of temperature of hydrogen gas $T \sim t$ is obtained, as shown in Figure 6. Using the curve $T \sim t$, the evolution curve of ionization fraction $\chi_e \sim t$ is obtained by use of equation (12), as shown in Figure 7 [the constant C in eq. (12) is obtained from the observed value given by McCray 1993, i.e., $\chi_e = n({\rm H}^+)/n({\rm H}) \simeq 0.05$ at $t = 260$ days. Taking this value as an initial value of $\chi_e(t)$, C is determined].[5]

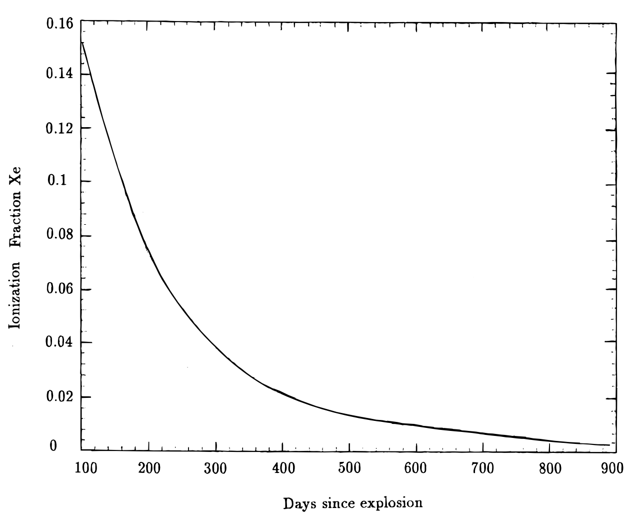

FIG. 7.—Evolution curve of ionization fraction, $\chi_e \sim t$ curve, calculated by use of equation (12) and the $T \sim t$ curve (Fig. 6).

4. DISCUSSION

In this paper we deduced the temperature evolution of SN 1987A's hydrogen envelope by comparing the observed Hα light curve to one derived from the recombination theory for hydrogen lines. This is a simple and convenient method for determining the temperature and its dependence on time. We expect that this method could be applied to other astrophysical problems.

The temperature dependence of $L_{\rm H\alpha}^{\rm theor}$ comes from the temperature dependence of $\alpha_3^{\rm eff}(T)$ (eqs. [11] and [14]). The physics is easy to understand: the recombination is higher at low temperature, which favors Hα emission.

The basic equation $L_{\rm H\alpha}^{\rm theor} = K\alpha_3^{\rm eff}(T)N_e N_{\rm H^+} t^3$ is correct both for $\tau_{\rm H\alpha} \gg 1$ and $\tau_{\rm H\alpha} \ll 1$, as mentioned in § 2. This leads us to conclude that the turning point at $t \simeq 290$ days in the observed Hα light curve is caused by a discontinuity in $T(t)$. It is a temperature effect, rather than a effect of optical depth.

From Figure 6 we see that in the early nebula phase ($120 \lesssim t \lesssim 300$ days), the temperature drops drastically, from $T_0 \simeq 5500$ K (temperature of photosphere) down to $T \lesssim 2000$ K. This rapid temperature decrease implies a rapidly increasing of $\alpha_3^{\rm eff}(T)$ in equations (11) and (14). The efficient recombination to level 3 favors Hα emission which causes the flattening of the observed Hα light curve. During the period $300 \lesssim t \lesssim 550$ days, the temperature is in the range $1800 \lesssim T \lesssim 2000$ K, and nearly constant. Therefore, the approximation of constant temperature suggested by Xu & McCray (1991) is acceptable for $300 \lesssim t \lesssim 500$ days. But their estimated value, $T = 3000$ K, seems to be higher. Figure 6 shows the average constant is $T \simeq 1900$ K, which favors dust formation. Such a calculated T is confirmed by observations of the far-infrared continuum (4–13 μm), which shows that dust was beginning to form at $t \simeq 350$ days (Meikle et al. 1993), and dust features became prominent at $t \simeq 550$ days (Roche et al. 1993; Moseley et al. 1989).

A surprising result is that the envelope temperature increases again from $T \simeq 1900$ to $\simeq 2500$ K at $500 \lesssim t \lesssim 750$ days. This is due to the lowered efficiency of recombination to cool the gas: χ_e is so small that recombination is slow, and the recombination timescale $t_{\rm rec}$ becomes so large that the steady state models (i.e., the emitted recombination luminosity equals the instantaneous radioactive energy input) is no longer correct (Fransson & Kozma 1993). The integrated deposition of the radiative energy will dominate over recombination cooling, and the temperature increases after $t \gtrsim 800$ days. The recombination rate is so low that χ_e is nearly a constant, which Fransson & Kozma (1993) have described as "freeze out." However, the temperature decreases again after $t \gtrsim 750$ days, from $T \simeq 2500$ K down to $T \simeq 1600$ K in ages $750 \lesssim t \lesssim 900$ days. A possible explanation is that there are some new cooling mechanisms operating at late times. For example, adiabatic cooling will dominate over the recombination cooling. Another possi-

[5] The possible absorption of Hα photons by dust has not been taken into theoretical calculation (eqs. [11] or [14]). The influence of dust-absorption in waveband 6000–7000 Å is small and can be expressed simply by a small modification of the value of $\tilde{K}$ in eq. (14), which only causes little changes in the behavior of time evolution.

bility is the cooling by bremsstrahlung radiation at radio wavelength. In late stages, the expanding envelope becomes transparent to free-free radiation in radio waveband. Radio observations show a positive detection at 1.3 mm (Biermann et al. 1990, 1992) after $t \gtrsim 800$ days, which confirms that free-free radiation is a cooling mechanism.

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REFERENCES

Biermann, P. L., Chini, R., Haslam, G., Kreysa, E. & Lemke, R. 1990, A&A, 236, L17
Biermann, P. L., Chini, R., Haslam, G., Kreysa, E., Lemke, R., & Sievers, A., 1992, A&A, 255, L5
Castor, J. I. 1970, MNRAS, 149, 111
Dwek, E. 1988, ApJ, 329, 814
Fransson, C., & Kozma, C. 1993, ApJ, 408, L25
Kirshner, R. P., & Kwan, J. 1975, ApJ, 197, 415
Kozma, C., & Fransson, C. 1992, ApJ, 390, 602
Li, H., & McCray, R. A. 1992, ApJ, 387, 309
Li, H., McCray, R. A., & Sunyaev, R. A. 1993, ApJ, 419, 824
Liu, W., Dalgarno, A., & Lepp, S. 1992, ApJ, 396, 69
McCray, R. A. 1993, ARA&A, 31, 175
———. 1991, in Supernovae and Stellar Evolution, ed. A. Ray & T. Velusamy (Singapore: World Scientific), 1
———. 1990, in Molecular Astrophysics, ed. T. H. Hartquist (Cambridge: Cambridge Univ. Press), 439
Meikle, W. P. S., Spyromilio, J., Allen, D. A., Varani, G. F., & Cumming, R. J. 1993, MNRAS, 261, 535
Moseley, S. H., et al. 1989, ApJ, 347, 1119
Phillips, M. M., & Williams, R. E. 1991, in Supernovae, ed. S. E. Woosley (New York: Springer), 36
Pinto, P. A., & Woosley, S. E. 1988, ApJ, 329, 820
Roche, P. F., Aitken, D. K., & Smith, C. H. 1991, MNRAS, 252, 39
Rybicky, G. B., & Lightman, A. P. 1979, Radiative Processes in Astrophysics (New York: Wiley), 167
Soboleve, V. V. 1960, Moving Envelopes of Stars (Cambridge: Harvard Univ. Press)
Terndrup, D. M., et al. 1988, in Proc. Elizabeth & Fredrick White Meeting on SN 1987A (Canberra, Australia)
Xu, Y., & McCray, R. A. 1991, ApJ, 375, 190
Xu, Y., McCray, R. A., Oliva, E., & Randich, S. 1992, ApJ, 386, 181
You, J. H., Wei, C. Y., Fu, C. Q., & Lin, X. B. 1994, Sci. China, 24(2), 186