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AMBIPOLAR DIFFUSION–DRIVEN TEARING INSTABILITY IN A STEEPENING BACKGROUND MAGNETIC FIELD

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ABSTRACT

Tearing instability is shown to be greatly enhanced by ambipolar diffusion near the locations where two-dimensional magnetic fields reverse signs. The growth time is found to be on the order of the ambipolar diffusion timescale, independent of electrical resistivity. Consistent with the earlier works, the enhancement in the reconnection of field lines indeed arises from the development of a singular current perturbation. Such magnetic reconnection is driven by the steepening effect of ambipolar diffusion on the magnetic fields near the field-reversal lines. Although the background magnetic fields are evolving on a similar timescale as the perturbations, we show that magnetic reconnection proceeds substantially faster than the steepening of the background fields.

In contrast to the conventional tearing instabilities in three dimensions, where the magnetic reconnection occurs only in the flux surfaces that contain the free energy, a positive Δ', the three-dimensional extension of this new tearing instability can occur only in regions where the Lorentz force reverses the sign. This report outlines how the three-dimensional reconnection may proceed in a similar fashion as the two-dimensional counterpart does.

Subject headings: diffusion — galaxies: magnetic fields — ISM: magnetic fields — MHD

1. INTRODUCTION

Magnetic diffusion is the mechanism for unlocking magnetic fields from an otherwise perfectly conducting astrophysical fluid, whereby the pressure exerted on the fluid by the fields can be reduced. Among all types of magnetic diffusion, ambipolar diffusion is the most important kind in unlocking the magnetic fields in the weakly cosmic-ray–ionized molecular clouds (Shu, Adams, & Lizano 1987; McKee et al. 1993). Although magnetic fields can slip from the neutral fluid under the action of ambipolar diffusion, the fields are still frozen onto a small amount of ionized gases in the fluid. Since the fluid trajectories can never cross at any given space and time, the magnetic field topology can, as a result, never change without unlocking the field lines from the ionized gases by the electrical resistivity. On the other hand, magnetic reconnection is, so far, known to be the most effective means in releasing the field energy into intense heat and fluid motions. Since magnetic reconnection requires changes in the field-line topology, the ambipolar diffusion *per second* can never produce the violent events of field-line reconnection. However, in molecular clouds the resistive diffusion is often much smaller than the ambipolar diffusion, and it is therefore a natural question to ask whether the presence of ambipolar diffusion can somehow assist the resistive diffusion. Moreover, it has been suggested a few years ago that the α-dynamo model can work only in the presence of much more efficient resistive dissipation than what was conventionally thought to exist for the linear dynamo model. This new finding results from nonlinear feedbacks of the Lorentz force on the fluid motions (Vainshtein & Cattaneo 1992; Cattaneo 1994; Gruzinov & Diamond 1994). One therefore hopes that if the ambipolar diffusion can indeed help accelerate resistive dissipation, it may perhaps help resolve this difficulty.

Recently, Brandenburg & Zweibel (1994) have shown that ambipolar diffusion can lead to sharpening of a smooth current channel into a singular current sheet, in a model where the field lines are stretched by an underlying fluid that rotates differentially. The current sheet has been found to develop at the field-reversal surface. The results lead to an important conclusion that the combined force of ambipolar diffusion and differential rotation can yield a configuration prone to rapid magnetic reconnection. Since resistivity was not included in their calculations, no magnetic reconnection was observed, in accordance with the above physical picture.

An important issue for magnetic reconnection is its timescale in the limit of vanishing resistivity. This timescale usually scales with some inverse fractional power of the resistive diffusivity η. Well-known examples include the Sweet-Parker reconnection scaled as $\eta^{-1/2}$ (see review by Parker 1979), the Petschek reconnection as $\ln(\eta)$ (Petschek 1964) and the tearing instability as $\eta^{-3/5}$ (Furth, Killeen, & Rosenbluth 1963). In the model of Brandenburg & Zweibel (1994), the development of the current sheet can be accelerated by differential rotation on the dynamical timescale, even in the limit of small ambipolar diffusion, although the final formation of the current sheet must require the presence of ambipolar diffusion. Once a singular current sheet is formed, magnetic reconnection can proceed rapidly as demonstrated by earlier works (Chiueh & Zweibel 1987; Liewer & Payne 1990).

In view of these results, one may question what intrinsic reconnection timescale it would be when no differential rotation is present, and whether the large-scale current sheet can ever have time to develop at all in the sole presence of ambipolar diffusion. The above questions are only meaningful when the relevant timescales are well separated, in other words, the resistive timescale is much longer than the ambipolar diffusion timescale which is, in turn, much longer than the ideal magnetohydrodynamic (MHD) timescale. This timescale ordering is generally valid in cool molecular clouds. In the present work, we intend to answer the above questions for a *time-dependent* background field.

Our problem is formulated as follows. Given a smooth and evolving magnetic-field reversal configuration, can there be any resistive instability such as the tearing instability, and if so, what should the instability growth rate be? Brandenburg & Zweibel (1995) have studied this problem for a current sheet equilibrium where the ion dynamics is included. They have numerically found that the tearing mode grows at a rate substantially larger than that expected for the conventional tearing mode, as a result of the singular current channel.

The basic configuration under consideration is an ideal MHD, two-dimensional equilibrium, where all forces on the neutral fluids are balanced. In addition, we let the equilibrium magnetic field near the field-reversal line be $\boldsymbol{B}_0 = B_0' x\hat{y}$, where the prime stands for the spatial derivative. If ambipolar diffusion is a small effect, the timescale for the equilibrium to evolve is long and we may regard this equilibrium as always remaining force-balanced as the magnetic fields diffuse. (This timescale will be estimated later.) The following stability analysis considers perturbations around such a force-balance equilibrium. Since the force-balance equilibrium evolves as well, the temporal Fourier expansion can no longer be valid. We will adopt a power-law temporal expansion to code with this situation instead.

Strategy for solving the present problem goes as follows. We divide the problem into an inner problem and an outer problem. For the former, the length scale is small, on the order of the ambipolar diffusion layer width, and for the latter the length scale is of the global scale on the order of $\langle B_0 \rangle / B_0'$, where $\langle B_0 \rangle$ is the globally averaged field strength. This problem is an eigenvalue problem, with the instability growth rate being the eigenvalue. The effect of actual resistive diffusivity η, which is the primary drive for field-line reconnection, is contained in the inner problem. On the other hand, since we consider the equilibrium to be ideal MHD stable, the outer solution must satisfy this requirement accordingly. We will solve the inner and outer problems separately and then match the two solutions together in an asymptotic sense.

Section 2 contains a new formulation of stability analysis for an evolving background. In § 3, unstable tearing modes are identified. It is shown that although the perturbations grow on the ambipolar diffusion timescale, they evolve substantially faster than the background fields. We conclude this work and discuss possible extension to three-dimensional reconnection in § 4.

2. FORMULATION IN A COMOVING COORDINATE

The induction equation for a two-dimensional field obeys

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times (\boldsymbol{V} \times \boldsymbol{B}) = \nabla \times \left[\left(\frac{\epsilon B^2}{\rho^{3/2}} + \eta \right) \nabla \times \boldsymbol{B} \right] , \quad (1)$$

where the right-hand side contains both the ambipolar diffusion and resistive diffusion, and the coefficient ϵ is a small dimensional constant to be explicitly given at the end of this analysis. Since the force-balance equilibrium has a reversed-field configuration, the ambipolar diffusion alone can produce steepening of the magnetic field at the reversal line (Zweibel 1994; Brandenburg & Zweibel 1994). The timescale for the steepening can be estimated by setting $\boldsymbol{V} = 0$ and $\eta = 0$ in equation (1). Substituting the field configuration $\boldsymbol{B}_0 = B_0' x\hat{y}$ to equation (1), one can straightforwardly show that equation (1) becomes an evolution equation for $B_0'(t)$:

$$\frac{dB_0'}{dt} = (2\epsilon\rho_0^{-3/2})(B_0')^3 . \quad (2)$$

The solution can be easily obtained, $B_0'(\tau) = (\rho_0^{3/4}/2\epsilon^{1/2})\tau^{-1/2}$, where $\tau \equiv t_\infty - t$ and t_∞ is the time at which the singular current sheet forms. It is clear that the evolution timescales with ϵ^{-1}.

In Brandenburg & Zweibel's (1994) work, they have adopted the boundary conditions that *constant* diffusive flux of magnetic fields is injected from both boundaries toward the field-reversal layer, and the magnetic fields reach a stationary state at the expense of flux annihilation at the field-reversal layer. It yields a different evolutionary path for $B_0(x, t)$ from the one described by equation (2). Our background solution $B_0(x, t)$ blows up within some finite time at a given location x. However, an infinitely large magnetic field strength is unphysical, and hence one should interpret such a solution with care. In fact, this solution is valid only in the comoving frame of reference, where the coordinate shrinks with time as $\tau^{1/2}$. One may define the comoving coordinate as $\xi \equiv x/\tau^{1/2}$, such that the background field $B_0(x, t)[=(\rho_0^{3/4}/2\epsilon^{1/2})\xi]$ becomes stationary in the ξ coordinate. This self-similar solution only holds within some finite range of ξ, but outside this range of ξ the solution should make a transition and matches to the boundaries. Since the present solution produces a current sheet in a much more vigorous fashion than the one obtained by Brandenburg & Zweibel, one expects that such a background field must possess a stronger tendency in steepening itself. Although its detailed mechanism remains to be investigated, one possible cause for the strong field steepening can be the fact that the present background solution contains *no* sink of magnetic flux, which warants *no* flux annihilation to take place at the field-reversal layer throughout the evolution. In this model, flux annihilation can only occur through the tearing instability to be discussed below.

Having addressed the background fields, we now turn to the perturbations. Since the instability has a much longer timescale than that of the ideal MHD, one expects that the perturbed motions should avoid coupling to the sound waves and hence they are incompressible. Thus, the two-dimensional perturbed motion can be expressed as $\delta \boldsymbol{V} = \hat{z} \times \nabla\phi$, where ϕ is the stream function. On the other hand, the two-dimensional perturbed field can also be expressed in terms of a flux function A: $\delta \boldsymbol{B} = \hat{z} \times \nabla A$. For operational convenience, one can uncurl equation (1) to take advantage of the flux function and obtain an evolution equation for the flux function. Let the perturbations be proportional to e^{iky}, we find the perturbed flux equation:

$$\frac{\partial A}{dt} - ikB_0' x\phi = \frac{2\epsilon[B_0'(t)]^2 x}{\rho_0^{3/2}} \frac{\partial A}{\partial x} + \left\{ \eta + \frac{\epsilon[B_0'(t)x]^2}{\rho_0^{3/2}} \right\} \left(\frac{\partial^2}{\partial x^2} - k^2 \right) A . \quad (3)$$

The outer problem considers ideal-MHD perturbations. As the timescale of interest is much longer than that of the ideal MHD, we shall seek the ideal-MHD static solution. Since the nonideal effects on the right-hand side can be ignored, it follows that the outer solution $\phi_{\text{out}} = 0$. The requirement of *static solutions* has already incorporated the fact that the perturbations under consideration are ideal-MHD *stable*. On the other hand, one must include all terms in equation (2), except for the k^2 term on the right (since the inner layer width is assumed much smaller than the k^{-1}), to describe the inner solutions $A_{\rm in}$ and $\phi_{\rm in}$. To match to $\phi_{\rm out}$ we must demand that $\phi_{\rm in} \to 0$ at infinity. Upon ignoring k^2 and inserting $B_0 = B_0'(t)x$ in equation (3), we find that the inner problem can be greatly simplified in separating the time dependence.

Again, we may scale x with $\tau^{1/2}$ and define the comoving coordinate $\xi \equiv x/\tau^{1/2}$ in place of x. Thus, the partial differential $\partial/\partial x = \tau^{-1/2}(\partial/\partial\xi)$ and $\partial/\partial t = -\partial/\partial\tau + (\xi/2\tau)(\partial/\partial\xi)$. Substitute the new coordinate into equation (3) and let A and ϕ be proportional to $\tau^{-\gamma}$. The induction equation becomes

$$\gamma A - i\left(\frac{k\xi\rho_0^{3/4}}{2\epsilon^{1/2}}\right)\phi = \left(\eta + \frac{\xi^2}{4}\right)\frac{d^2A}{d\xi^2} . \tag{4}$$

The other equation to be considered is that of ϕ. For an incompressible perturbation, the momentum equation becomes a vorticity equation after one takes a curl operated on the momentum equation:

$$\frac{\partial}{\partial t}\nabla^2\phi = ik\left[B_0\left(\frac{\partial^2}{\partial x^2} - k^2\right)A + \frac{d^2B_0}{dx^2}A\right] . \tag{5}$$

Note that the fluid has been assumed to be invicid. For the outer problem, we let $\phi_{\rm out} = 0$ and only the left-hand side survives. It is the balance equation for the perturbed magnetic forces. On the other hand, the relevant terms in equation (3) are the highest derivative terms for the inner problem, and they satisfy

$$\gamma\frac{d^2\phi}{d\xi^2} = i\left(\frac{k\xi\rho_0^{3/4}}{2\epsilon^{1/2}}\right)\frac{d^2A}{d\xi^2} , \tag{6}$$

in the comoving coordinate.

3. UNSTABLE TEARING MODES

We are now in a position to analyze the stability problem. Consider the inner problem. Equations (4) and (6) are combined to yield

$$-\chi\frac{d^2A_{\rm in}}{d\chi^2} = \gamma\frac{d^2(A_{\rm in}/\chi)}{d\chi^2} - \frac{d^2}{d\chi^2} \times\left\{\frac{1}{\chi}\frac{d}{d\chi}\left[\left(\eta D^2 + \frac{\chi^2}{4}\right)\frac{dA_{\rm in}}{d\chi}\right]\right\} , \tag{7}$$

where we have normalized the length to the ambipolar length scale $D \equiv (4\epsilon\gamma/k^2\rho_0^{3/2})^{1/2}$ and $\chi \equiv \xi/D$.

There is a trivial solution to this equation. That is, $A_{\rm in} = \chi$. This solution has no change in field line topology since $\delta B_x(\propto A) = 0$ at the original field-reversal location. Any change of field line topology into magnetic islands must yield a *finite* $A_{\rm in}$ at $\chi = 0$.

To ensure a finite $A_{\rm in}$ at $\chi = 0$, we begin the construction of solution near $\chi = 0$ inside the resistive layer. In this region, the left-hand side of equation (7) can be ignored since it describes the effect of fluid inertia and becomes important only in the ambipolar diffusive layer. Having dropped the inertia term, we recover equation (4) with the second term on the left-hand side ignored:

$$\frac{d}{d\chi}\left\{\left[\eta D^2 + \left(\frac{\chi}{2}\right)^2\right]\frac{dA_{\rm in}}{d\chi}\right\} - \gamma A_{\rm in} = 0 . \tag{8}$$

Near $\chi = 0$ the solution behaves as $A_{\rm in} = 1 + (\gamma/2\eta D^2)\chi^2 + O(\chi^4)$. At large distances, the solution keeps increasing outward as $A_{\rm in} \to \chi^\beta$ where $\beta = [-1 + (1 + 16\gamma)^{1/2}]/2$. This asymptotic behavior continues until the solution extends into the ambipolar diffusive layer where the inertia term on the left-hand side of equation (7) becomes important.

In the ambipolar diffusive layer we can ignore the resistive term on the right-hand side of equation (7). Let $f \equiv \chi^2 d(A_{\rm in}/\chi)/d\chi$, and equation (9) becomes

$$\frac{d^3f}{d\chi^3} + \frac{2}{\chi}\frac{d^2f}{d\chi^2} - \left(4 + \frac{2 + 4\gamma}{\chi^2}\right)\frac{df}{d\chi} + \frac{8\gamma}{\chi^3}f = 0 . \tag{9}$$

At a small distance, equation (9) permits power-law solutions, and one of them is the extension of the asymptotic power-law solution from the resistive layer. It approaches $f \sim \chi^{[-1+(1+16\gamma)^{1/2}]/2}$ as $\chi \to 0$. Due to the third-order derivative in equation (9), there should be three independent solutions. At large distances, one of the solutions diverges and another converges exponentially as $f \sim e^{\pm 2\chi}$, which are obtained by balancing the first and third terms of equation (9). Another asymptotic solution behaves as $f \sim$ const., obtained by balancing the third term itself in equation (9).

The solution matching demands $\Delta'_{\rm in}$ of the inner solution to be matched to $\Delta'_{\rm out}$ of the outer solution, where $\Delta'_{\rm out} \equiv (d[\ln(A_{\rm out})]/d\xi)|_{0-}^{0+}$ for the outer solution and $\Delta'_{\rm in} \equiv (d[\ln(A_{\rm in})]/d\xi)|_{-\infty}^{\infty}$ for the inner solution (Furth et al. 1963). A positive $\Delta'_{\rm out}$ means that the ideal MHD outer solution $A_{\rm out}$ must contain a concave kink across the diffusive layer, and $\Delta'_{\rm out}$ measures the amount of kink in $A_{\rm out}$. The inner solution must adjust itself in such a way that it provides the proper amount of kink that $A_{\rm out}$ needs. In some sense, Δ' can be viewed as the amount of free energy stored in the outer ideal-MHD region and released by the resistive dissipation so as to make it possible for a perturbation to grow. In the conventional tearing mode analysis, it is the matching condition for Δ' that in turn determines the instability growth rate, which serves as the eigenvalue of the problem.

The $f =$ const. asymptotic solution yields an asymptotic $A_{\rm in} = 1 + c_0\chi - 4\gamma/\chi^2$, where c_0 is the integration constant. The exponential decay f also yields an asymptotic exponential decay component plus a term linearly proportional to χ, also resulting from the constant of integration. Finally the exponentially divergent f yields an asymptotic $A_{\rm in} \sim \chi e^{2\chi}$.

Combination of the two finite asymptotic solutions can yield a finite $\Delta'_{\rm in}$ by adjusting the integration constants to proper values. On the other hand, the exponentially divergent branch is definitely unphysical since it yields a large $\Delta'_{\rm in} \propto \epsilon^{-1/2}$ for a small ambipolar diffusion parameter ϵ. An outer solution that has a correspondingly large $\Delta'_{\rm out}$ must be the one that entails an $A_{\rm out} \to 0$ with a finite $dA_{\rm out}/dx$ near $x = 0$. Such an outer solution is only possible when the system is on the verge of an ideal MHD instability (Newcomb 1960), which is not the situation of concern here. However, among the three branches of solutions, the exponentially divergent branch can generally dominate at large distances, and the eigenvalue problem thus consists of choosing the proper γ such that the exponentially divergent branch can be entirely eliminated. The proper solution must therefore approach the $f =$ const. branch asymptotically.

Before the numerical search for the proper γ, it is illuminating to analyze the solution trajectory in a qualitative way in order to show that the proper γ can exist. This approximate treatment is, in a sense, similar to the WKB method where we express the solution as $f = \chi^{\alpha(\chi)}$. In view of the fact that $\alpha = [-1 + (1 + 16\gamma)^{1/2}]/2 > 0$ at the origin is to decrease to another value $\alpha = 0$ at $\chi \to \infty$, if $\gamma \sim O(1)$, the quantity $|d\alpha/d\xi|$ must be small. Thus, we may substitute the power-law expression of f into equation (9) and ignore the derivatives of α to find the lowest order approximation to the solution. The result is an algebraic equation,

$$4\chi^2 = \frac{(\alpha - 2)(\alpha^2 + \alpha - 4\gamma)}{\alpha} . \tag{10}$$

Near $\chi = 0$, we find that $\delta\alpha/\delta\chi < 0$, and the solution trajectory points to the correct direction to approach the desired $\alpha = 0$ afar. In fact, a careful scrutiny of equation (10) shows that as the positive α decreases, the numerator increases and the denominator decreases, resulting in an increasing right-hand side. On the other hand, as the solution extends to an ever-larger χ, the left-hand side also increases, consistent with the qualitative behavior of the right-hand side. These facts strongly suggest the possibility for an asymptotically constant f to exist.

To pin down this possibility, one needs to conduct numerical integration. Except for the origin, equation (9) is a rather "soft" differential equation. Near the origin, the analytical power-law solution can be used for extrapolation of solutions to a small but finite χ so that direct integration at the singular origin is avoided. A second-order Runge-Kutta scheme is adopted for numerical integration. The most unstable mode is found to have a $\gamma \approx 1.25$, within a numerical error less than 0.3%. The eigenfunction f contains no node.

Thus, the power index of the most unstable perturbation turns out to assume a finite value, independent of either the resistive diffusivity η or the small ambipolar diffusion parameter ϵ. The dynamics of the unstable perturbations, and hence the magnetic reconnection, can therefore proceed substantially faster than the steepening of the background fields, which remains stationary in the comoving coordinate.

4. DISCUSSION AND CONCLUSION

To sum up the above analysis, we conclude that magnetic reconnection or tearing instability is accelerated by the ambipolar diffusion in regions where the magnetic fields locally vanish. The timescale in question is on the order of the ambipolar diffusion time across the field-reversal layer, independent of the resistive diffusivity η. Although the background fields are steepening on the same timescale, magnetic reconnection can always proceed at a substantially faster pace before a large-scale current sheet forms. In the comoving reference frame where the background magnetic fields appear static, the unstable perturbations grow as $(t_\infty - t)^{-1.25}$. Our result also implies that the rapid formation of magnetic islands in the original field-reversal layer should immediately fill the region with reconnected fields of finite strength. In addition, the large-scale current sheet, similar to the one obtained by Brandenburg & Zweibel (1994), cannot, therefore, form in the presence of this nonstationary background field. As long as the overall magnetic field reverses the sign on the large scale, the steepening of small-scale magnetic fields should continue in the nonlinear stage and the new reconnection sites shift to the X-points and O-points of the magnetic islands, where the field strengths vanish.

We now estimate the values of relevant quantities for molecular cloud parameters and examine the applicability of the present study. The magnetic fields are estimated 30 μG, the neutral mass density 10^{-21} cm^{-3} g, the ion mass density 10^{-26} cm^{-3} g, the ion-neutral collision time about 3×10^7 s, and therefore the quantity $\epsilon \sim 6$ cm$^{-3/2}$ g$^{1/2}$ s and the ambipolar diffusivity $\epsilon B_0^2/\rho^{3/2} \sim 10^{23}$ cm^2 s^{-1} (Shu et al. 1987). In addition, one may also estimate the resistive diffusivity η for the cloud ions of about 50 K, and it turns out that $\eta \sim 10^{11}$ cm^{-2} s^{-1}, more than 10 orders of magnitude smaller than the above ambipolar diffusivity. Separation of an ambipolar diffusive layer and a resistive layer is thus justified. If we take the typical length scale of the magnetic fields to be 1 pc, it follows that the ambipolar diffusion timescale is about 3×10^6 yr. This is the timescale for the development of the current sheet and is also the timescale for magnetic reconnection at the field-reversal layer. We may compare this timescale with the Alfvén transit time 5×10^5 yr and find that one may indeed separate the ideal MHD timescale from the ambipolar diffusion timescale.

Moreover, this ambipolar diffusion timescale is more than 6 orders of magnitude greater than the ion-neutral collision time, justifying the diffusion approximation in the strong ion-neutral coupling limit. This diffusion approximation also holds even in the small resistive layer which has a width about the square root of η times the ambipolar diffusion timescale, yielding the resistive layer width of about 2×10^{12} cm. By comparison, the mean-free path of the ion-neutral collision can be estimated to be about 2×10^{12} cm as well, for ions of a reasonable thermal speed about 0.7 km s^{-1}. In sum, these estimates justify the assumptions involved in the analysis for parameters in molecular clouds.

As pointed out previously (Brandenburg & Zweibel 1995), the ion inertia and pressure can become important near the null-field region of two-dimensional magnetic field, thereby breaking the one-fluid approximation. However, in the general three-dimensional case, the rapid reconnection may occur in regions where the Lorentz force vanishes but the magnetic field remains finite. Hence the one-fluid limit can generally be restored in the three-dimensional reconnection layer. A brief discussion is given below concerning how the present results can be extended to three-dimensional fields.

In three dimensions, the ambipolar effects are more complicated and contain not just diffusion. Despite the additional complications, the driving mechanism of ambipolar diffusion still persists, and we will now show that the ambipolar diffusion-driven reconnection can occur in regions where the Lorentz force of the three-dimensional magnetic fields acts in the opposite directions in squeezing the fields locally. The ambipolar effects take the form (Shu et al. 1987; Zweibel 1989)

$$\epsilon \nabla \times \left[\frac{(\boldsymbol{J} \cdot \boldsymbol{B})\boldsymbol{B} - B^2 \boldsymbol{J}}{\rho^{3/2}} \right] , \tag{11}$$

added to the ideal induction equation for the magnetic fields. The first term is the three-dimensional effect, and the second term is the usual ambipolar diffusion. The three-dimensional effect can cancel the diffusion when magnetic fields are force free.

We will now consider a simple three-dimensional model, for which we add a uniform field directed in the third dimension z and the equilibrium field components in the x and y directions remain the same. Again, we let the magnetic fields depend only on the x and y coordinates. Since the background fields are not force free despite the addition of B_z, the background fields must evolve according to the following equations:

$$\frac{\partial B_{y0}}{\partial t} = -\epsilon \frac{\partial}{\partial x}\left[\left(\frac{B_{y0}^2}{\rho_0^{3/2}}\right)\frac{\partial B_{y0}}{\partial x}\right], \tag{12}$$

$$\frac{\partial B_{z0}}{\partial t} = \left(\frac{\epsilon}{2}\right)\frac{\partial}{\partial x}\left[\left(\frac{B_{z0}}{\rho_0^{3/2}}\right)\frac{\partial B_{y0}^2}{\partial x}\right]. \tag{13}$$

Equation (12) is identical to that for the evolution of two-dimensional fields and can be cast into equation (2); on the other hand, equation (13) describes how B_{0z} is enhanced by the compression of ionized gases. If $B_{y0}(x) = B'_{y0}\,x$ as assumed in the earlier analysis, one may easily find from equation (13) that B_{z0} can remain uniform and its magnitude increases with time, scaled as $\tau^{-1/4}$.

The perturbations developed in such an evolving background become much more complicated than what have been analyzed in the previous sections. We will not work out the details in the present work but point out the key physics that underlie the rapid magnetic reconnection in three dimensions. Since rapid magnetic reconnection entails the appearance of singular current perturbations, we will now focus on the current perturbations in equation (11). It is straightforward to find that any current perturbation parallel to $\boldsymbol{B}_0$ must cancel in equation (11). Since only the perpendicular component of the current perturbation survives, one can again show that the first term in equation (11) vanishes and only the second term survives, which has exactly the form of ambipolar diffusion with a finite diffusion coefficient proportional to $B_{z0}^2 + B_{y0}^2$. However, this projected form of ambipolar diffusion is deceptive. When explicitly writing out the surviving current perturbations, we have

$$\left(\frac{\epsilon}{\rho^{3/2}}\right)\{\hat{z}[B_{z0}\,B_{y0}\,\delta j_y - B_{y0}^2\,\delta j_z] + [\hat{y}B_{y0}\,B_{z0}\,\delta j_z - B_{z0}^2(\delta j_x\,\hat{x} + \delta j_y\,\hat{y})]\}\ , \tag{14}$$

upon uncurling equation (11) as was conducted previously for obtaining equation (3). Note that the coefficients of δj_x and δj_y are finite everywhere, whereas that of δj_z vanishes when B_{y0} vanishes at $x = 0$. These suggest that δj_z should be singular at $x = 0$ whereas δj_x and δj_y can remain smooth.

In order to show that the three-dimensional reconnection resembles that in two dimensions, we also note that equation (14) possesses parity symmetry with respect to $x = 0$ when the perturbations depend only on the x and y coordinates. We have found from the two-dimensional analysis that δA_z has an even parity, and so does δj_z. If the z-component of equation (14) also has an even parity, the δj_y must be an odd function, implying an even δB_z and an even δj_x. Indeed, the parities of these quantities are consistent with that of the remaining two components in equation (14). We now focus on the z-component of equation (14). Since δj_y is a smooth function at $x = 0$ and it has an odd parity, we immediately find that the first term of the z-component in equation (14), small near $x = 0$, can be unimportant in the resistive region where reconnection takes place, and the remaining term contributing to magnetic reconnection indeed becomes identical to that for two dimensions (see eq. [3]). Hence, the three-dimensional magnetic reconnection driven by ambipolar diffusion, at least within the framework of the above simple model, has some resemblance to the two-dimensional counterpart.

The natural application of the ambipolar–diffusion-driven magnetic reconnection can be the galactic dynamo problem. One hopes that the magnetic flux can be generated in dense molecular clouds, where the ambipolar effects are pronounced, on scales much larger than the individual clouds, as a result of the much enhanced resistive dissipation described in this report. If so, the large-scale dynamo magnetic fields must thread through many individual clouds around the galaxy, and they should follow the spatial distribution of the clouds. However, in the conventional $\alpha - \omega$ dynamo model, it requires, first, the injection of helicity into the magnetic fields and, second, turbulence. Both are not favorable to the operation of dynamo in molecular clouds. First, the helicity is believed to be given by the cyclonic motion arising from the combination of Coriolis force and gravity in each galactic hemisphere. As the molecular clouds are mostly distributed within a narrow layer, about few hundred parsec width, on the equatorial plane of the galaxy, the cyclonic motion should be small. Second, the clouds are required, and actually observed, to be turbulent, instead of being in hydromagnetic equilibrium as assumed in the present tearing-mode analyses. It is unclear whether the magnetic reconnection modeled in the present work can persist in a turbulent environment.

To remedy the first difficulty, one may argue that the sites of helicity injection do not need to coincide with the sites of magnetic dissipation. If the field lines in between molecular clouds puff out of the galactic plane, they can receive sufficient helicity from the cyclonic motion at high altitudes. Since field lines are connected and magnetic helicity is a large-scale quantity, the helicity can always be transferred into the clouds for the dynamo to operate.

As to the second issue, there may not be a simple resolution. To appreciate the level of difficulty for this problem, we recall that the magnetic field steepening is originated from inbalance of the Lorentz force on either side of a fluid parcel. The sign of the relative force inbalance around this fluid parcel must persist for, at least, one ambipolar diffusion time for the singular current perturbations to develop. However, fluid turbulence evolves on a short dynamical timescale, and it is difficult to maintain the sign of relative force inbalance for so long that the singular current perturbations can have a sufficient time to develop. Of course, if turbulence is intermittent, then magnetic reconnection driven by ambipolar diffusion may have a chance to occur in the quiescent regions. Unfortunately, research on intermittency in turbulence, especially for compressible gases, is still in its infancy, and at present there is hardly any concrete solution to this problem.

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