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REVISIT TO SCHLÜTER-TEMESVARY SELF-SIMILAR SUNSPOT MODELS

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ABSTRACT

Sunspot models based on self-similarity have been reassessed by computing them with the use of a shape function incorporating the Wilson effect. It is found that the newly computed models not only remove the inconsistency spotted by Osherovich in 1982 from Yun's models but also account fairly well for the observed relation between field strength and effective temperature. By virtue of empirical relations found by recent high-resolution observations, the computed models can be characterized by the effective temperature at the sunspot center if an appropriate field gradient is assumed. The vertical gradient of the angle of inclination in the outer part of the model spot is found to be 0°.01–0°.03 km^{-1}, in agreement with the finding of Solanki, Walther, & Livingston in 1993. This demonstrates the importance of accounting for the Wilson effect in constructing self-similar sunspot models.

Subject headings: Sun: magnetic fields — sunspots

1. INTRODUCTION

In constructing magnetostatic models of sunspots, the concept of self-similarity proposed by Schlüter & Temesvary (1958, hereafter ST58) has been extensively exploited (e.g., Deinzer 1965; Yun 1970; Osherovich 1982; Flå et al. 1982; Osherovich & Garcia 1989), simply because the self-similarity simplifies the mathematical treatment of the equation of magnetohydrodynamic equilibrium. Yun (1968) improved Deinzer's model of sunspots (Deinzer 1965) by introducing an "effective surface monopole" that controls the inclination of the field lines over a sunspot at the surface. He demonstrated that the computed models fairly well represent average observed sunspots (Yun 1970). However, Osherovich (1982) argued that for any field distribution characterized by the similarity condition, the angle ψ_p at the outer edge of a model sunspot should not exceed $\psi_{\rm pmax} = 66°.6$. Yun's (1970) models, however, exceed this limit, which violates the constraint.

To overcome these shortcomings, Osherovich introduced a return-flux term into the ST field line function, allowing the field lines to bend over and return to the neighboring photosphere. However, recent elaborate high-resolution observations (e.g., Lites et al. 1993; Solanki, Ruedi, & Livingston 1992; Solanki & Schmidt 1993) have shown that there is no significant return flux outside sunspots.

Recently, Skumanich (1992a, 1992b) made a comprehensive study of the lateral structure of sunspots and found that the exponential flux distribution representing the vertical self-similarity holds approximately for regions inside $r = 0.7R_p$ at the surface of a sunspot. Adam (1990) made magnetic field measurements of single, isolated, circular spots to determine their field direction by accurately locating the fringe positions of Zeeman components of the line Fe I λ6302.5. She found that the tangent of field line inclination is a linear function of radial distance from sunspot center, consistent with the *Hubble Space Telescope* similarity field configuration. These elaborate studies lead us to reconsider the earlier shortcomings raised by Osherovich (1982) against Yun's model of sunspots.

The purpose of the present work is to demonstrate that the shortcomings revealed by Osherovich (1982) are not inherent in the ST similarity but can be attributed to the fact that the Wilson effect has not been properly taken into account for the surface boundary condition in Yun's models. In § 2 we describe how to incorporate the Wilson effect into Yun's models. In § 3 the physical characteristics of the computed models are discussed and reassessed by observations. Finally, a brief summary and conclusion will be presented in § 4.

2. MODEL AND CALCULATION

2.1. *Basic Formulation*

The basic formulation for sunspot models based on the similarity assumption and the computational procedure are well described by Yun (1970) and Yun & Park (1993). So here we make a brief presentation of their theoretical formulation.

For an axially symmetric and untwisted circular sunspot, the horizontal component of the magnetostatic equation in cylindrical coordinates is

$$\frac{\partial P}{\partial R} = \frac{B_Z}{4\pi}\left(\frac{\partial B_R}{\partial Z} - \frac{\partial B_Z}{\partial R}\right), \tag{1}$$

where P is the gas pressure and B_R and B_Z refer to the radial and vertical components of the field. The Z-axis is taken to be perpendicular to the solar surface and directed toward the center of the Sun. The depth at which $\tau = \frac{2}{3}$ in the quiet atmosphere is taken as $Z = 0$.

The similarity assumption (ST58) leads us to express the vertical component of the field $B_Z(R, Z)$ at a depth Z in the form of

$$B_Z(R, Z) = B_Z(0, Z)\,\frac{D(\alpha)}{D(0)}, \tag{2}$$

with $\alpha = R\xi(Z)$, where $\xi(Z)$ is a scale factor and $D(\alpha)$ is the shape function. Here $D(\alpha)$ is assumed to decrease monotonically with α, and the normalization factor $D(0)$ is determined by

$$\Phi = 2\pi \int_0^\infty D(\alpha)\alpha\, d\alpha\,, \tag{3}$$

where Φ is the total flux of the sunspot. By virtue of the similarity assumption and $\nabla \cdot \boldsymbol{B} = 0$, equation (1) reduces

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to a simple ordinary differential equation,

$$fyy'' - y^4 + 8\pi \Delta P = 0 , \quad (4)$$

where $y = [B_Z(0, Z)]^{1/2}$, $\Delta P = P(\infty, Z) - P(0, Z)$, and f is given by

$$f = \frac{2}{D(0)} \int_0^{\infty} D^2(\alpha)\alpha \, d\alpha . \quad (5)$$

The temperature and pressure distributions are determined by using the diffusion approximation in the radiative region and the mixing-length theory in the convective region (Yun 1970; Yun & Park 1993). The extent of the suppression of convection due to the presence of the magnetic field in the spots is determined by the value of the ratio of the mixing length to the pressure scale height. For the normal photosphere where there is no inhibition of the convective energy transport, the value of the ratio is assumed to be unity; for the spot, it is determined as an eigenvalue by requiring the solution of equation (4) to obey certain boundary conditions.

To solve the differential equation given by equation (4), two input parameters, namely, the magnetic flux Φ and the effective temperature $T_{\rm spot}$ at the sunspot axis of symmetry, should be specified. The two input parameters can be combined into one as can be seen below. According to the recent IR spectroscopic study made by Kopp & Rabin (1992) on simple sunspots, the square of the measured maximum field strength of the spots is found to be linearly proportional to the effective temperature at the sunspot center normalized to that of the quiet region (see Fig. 7 in Kopp & Rabin 1992). In estimating the field strength, they directly measured the Zeeman splittings of the line Fe I λ15648. Their measurements are thought to be more reliable than any other visible observations, since in IR the stray light is considerably reduced and the Zeeman splittings are increased by nearly a factor of 9. The relation they found is approximated as

$$B_0^2[\mathrm{kG}] = -50.8(T_{\rm spot}/T_{\rm ph}) + 46.2 \quad (6)$$

with 4000 K $\leq T_{\rm spot} \leq$ 4400 K when $T_{\rm ph}$ is set to be 5800 K. On the other hand, Kopp & Rabin (1992) carefully examined the dependence of the maximum field strength B_0 on the umbral size. A similar study was also made by Martinez Pillet & Vázquez (1993) with high-resolution Stokes V profiles taken from simple sunspots. This relation can be summarized as

$$B_0[\mathrm{kG}] = 0.1R_u[\mathrm{arcsec}] + 2 , \quad (7)$$

with $R_u > 5''$. According to the recent estimate made by Solanki & Schmidt (1993), the ratio of the umbra to the entire spot, R_u/R_p, is found to be about 0.42. Accordingly, once $T_{\rm spot}$ is known, the umbral size R_u is uniquely determined from equations (6) and (7). A unique relation between $T_{\rm spot}$ and Φ can be approximated as

$$\log \Phi = 28.7 - 9.1(T_{\rm spot}/T_{\rm ph}) , \quad (8)$$

when the values $R_u/R_p = 0.42$ and $\Phi = 0.38\pi R_p^2 B_0$ (Yun 1968) are taken. This unique relation allows us to combine the two input parameters Φ and $T_{\rm spot}$ into one for sunspots with $R_u > 7''$.

2.2. *New Shape Functions*

It is generally accepted that the observed photospheric magnetic field distribution is well represented by Skumanich's simple dipole model (Skumanich 1992a). The radial and the vertical components of the Skumanich field configuration are given by

$$B_Z = \frac{1}{2} B_0 \left[\frac{2 - \alpha^2}{(1 + \alpha^2)^{5/2}}\right] \quad (9)$$

and

$$B_R = \frac{3}{2} B_0 \left[\frac{\alpha}{(1 + \alpha^2)^{5/2}}\right] , \quad (10)$$

where B_0 refers to the photospheric magnetic field strength at the sunspot center and $\alpha = R(\tau)/Z_0$. Z_0 is the location of the dipole source. In the present study B_0 in equations (9) and (10) is set to be 3000 G, and the value of α_p at the outer edge of the penumbra is assumed to be unity (see Skumanich 1992a). In this case the inclination angle at the sunspot boundary comes out to be about 72° (e.g., Lites et al. 1993). Figure 1 shows the observed vertical and radial field distributions specified by the Skumanich dipole model. It should be noted here that these field distributions are referenced to a given optical depth (e.g., $\tau_\lambda = 1$). The observed vertical field configuration (eq. [9]) is used as a model constraint for deriving new shape functions.

According to Solanki, Walther, & Livingston (1993) and Martinez Pillet & Vázquez (1993), the Wilson depression ranges from 400 to 800 km in accordance with Gokhale & Zwaan (1972). In the present work, the Wilson depression is set to be 600 km as a mean value. To express the Wilson depression $\Delta Z(\alpha)$ as a function of α, we assumed that its radial dependence follows the following sine function;

$$\Delta Z(\alpha) = 600\{0.5 \sin [\pi(\alpha - 0.5)] + 0.5\} \text{ km} . \quad (11)$$

Equation (11) is found to represent quite well the radial dependence, which has been determined semiempirically by

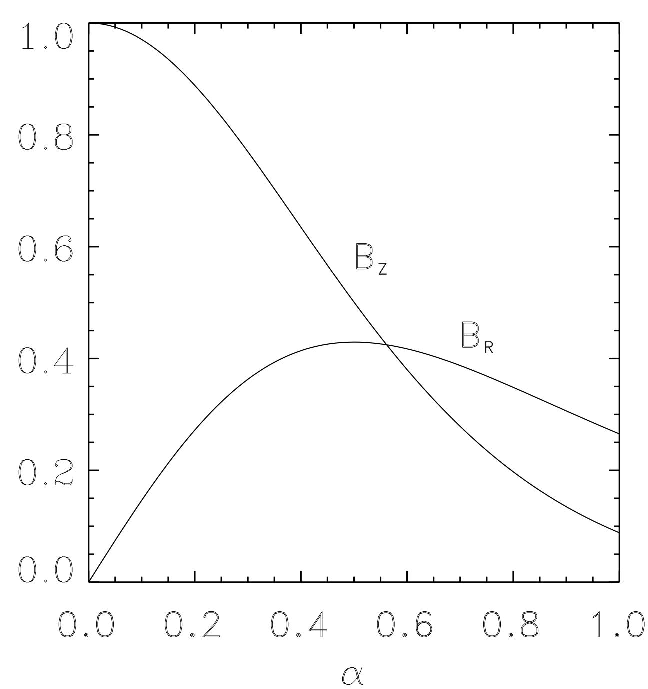

FIG. 1.—Magnetic field distribution at a given optical depth, adopted from the dipole model by Skumanich (1992a). The longitudinal configuration is used as a model constraint to derive shape functions after taking into account the Wilson effect.

Solanki et al. (1993). According to Hagyard et al. (1983) and others, the observed vertical field gradient ranges from 0.2 G km^{-1} to several G km^{-1} (for reference see Balthasar & Schmidt 1993). Hagyard et al. (1983) suggest 0.2 G km^{-1}, while Balthasar & Schmidt (1993) suggest 2–3 G km^{-1} and Pahlke & Wiehr (1990), 2 G km^{-1}. Balthasar & Schmidt (1993) obtained the nearly independent vertical field gradient on location within a sunspot. In the present study, we have considered four different cases: $dB_Z/dZ = 0$ G km^{-1}, 0.5 G km^{-1}, 1 G km^{-1}, and 2 G km^{-1}. By virtue of equations (9) and (11), the longitudinal field configuration referenced to a common geometrical depth ($Z = Z_D$) can be obtained by

$$B_Z(R, Z_D) = B_Z(\alpha) + \Delta Z(\alpha)(dB_Z/dZ) . \quad (12)$$

Here we express our new shape function as

$$D(\alpha) = D(0) \exp(-\alpha^n) = D(0) \exp[-(\alpha_p R/R_p)^n] , \quad (13)$$

where n and α_p are free parameters that will be chosen to fit $D(\alpha)/D(0)$ to the derived $B_Z(R, Z_D)/B_Z(0, Z_D)$ through equation (2). The old shape function is a Gaussian ($n = 2$) with $\alpha_p = 1.63$, which was inferred from the longitudinal field configuration referenced to optical depth. Since we have converted the magnetic field distribution referenced to optical depth to one referenced to geometrical depth, we have to derive new shape functions that will conform to the new prescription. For this work we generalize the shape function to that given by equation (13). This corresponds to an improvement of the upper boundary condition of Yun's sunspot models.

A set of the two numerical parameters n and α_p has been determined by means of nonlinear least-squares fitting for each of the four different cases considered above. The resulting values are tabulated in the third and fourth columns of Table 1. In the table we may note that case 1 (zero field gradient) is very similar to the old one chosen by Yun (1968). The total flux Φ and the parameter f conforming to the newly derived shape function have been estimated by numerically solving equations (3) and (5). We define A and C as $A = f/D(0)$ and $C = \Phi/\pi D(0)$ and list the resulting values of A and C for the four cases in Table 1. Finally, the radial component of the field $B_R(R, Z_D)$ is computed from

$$B_R(R, Z_D) = \alpha D(\alpha)(d\xi/dZ)_{Z_D}$$
$$= \exp[-(\alpha R/R_p)^n](\tan \psi_p)(\alpha/\alpha_p) .$$

2.3. *Boundary Condition*

The upper boundary condition is a free parameter, which can be specified by the inclination angle ψ_p at the outer edge of the penumbra at a given geometrical depth Z_D. The upper boundary condition is given by

$$y'(Z_D) = \frac{y^2(Z_D)}{[D(0)]^{1/2}} \frac{\tan \psi_p}{\alpha_p} , \quad (15)$$

where Z_D is taken as the depth of $\tau = \frac{2}{3}$ at the sunspot center and ψ_p is the angle of inclination of a line of force passing through the outer edge of the penumbra at the geometric depth Z_D. The value of ψ_p here should be smaller than the observed angle of inclination at the outer edge of the penumbra. However, Yun (1970) neglected the Wilson effect and made use of values based on the observations. The $\psi_{\rm pmax}$ in Table 1 is the upper limit of ψ_p, below which no off-centered local maximum of the total field strength is found. For all cases, the values of $\psi_{\rm pmax}$ are smaller than the observed inclination angle $\psi_{\rm obs}(0)$.

3. RESULT AND DISCUSSION

With the new shape functions derived in the previous section (see Table 1), four sets of magnetostatic sunspot models with $T_{\rm spot} = 4000$ K and 4400 K have been constructed, in each of which ψ_p is varied to fit the observations. In Figure 2, the field strengths at the spot axis computed for three values of ψ_p are compared with the observed ones (*solid line*) determined by the empirical relation (eq. [6]). As can be seen from the figure, the computed field strengths match well the observed one for a particular ψ_p, which is named as the most desirable one. We tabulate the most desirable ψ_p in Table 1 for comparison with the upper limit angle $\psi_{\rm pmax}$. As can be seen in Table 1, the most desirable ones of all cases, except for case 1 (zero field gradient), do not exceed the upper limit $\psi_{\rm pmax}$, beyond which the off-centered maximum field strength begins to show up. It implies that the shortcomings of Yun's model do not appear in the computed models since the upper boundary condition is different from that of Yun (1970) by taking into account the Wilson effect. In addition, the determination of the most desirable value of ψ_p for a assumed field gradient results in the models that are essentially characterized only by the effective temperature $T_{\rm spot}$ at the spot axis. This makes it possible to investigate the dependence of the physical characteristics of model spots on $T_{\rm spot}$ and/or on the spot size. The physical characteristics of the computed model sunspots are summarized in Table 2, which are specified by the most desirable $\psi_p = 50°$, 63°, 67°, 71°, and $T_{\rm spot} = 4000$ K, 4400 K, respectively. As can be seen in Table 2, thermodynamic parameters P_g and ρ depend only on the thermal structures of model spots.

So far, the vertical field gradient has been assumed to be independent of the position within a sunspot. In order to examine the radial dependence effect of the magnetic field

TABLE 1

MODEL PARAMETERS ESTIMATED FOR DIFFERENT VERTICAL FIELD GRADIENTS

Model	dB_Z/dZ (G km^{-1})	n	α_p	A	C	$\psi_{\rm pmax}$ (deg)	ψ_p (Z_D) (deg)	$\psi_{\rm obs}$ (0) (deg)
Yun	0	2.00	1.63	0.50	1.00	66.5	...	...
Case 1......	0	1.86	1.63	0.49	1.03	69.1	71	72
Case 2......	0.50	1.65	1.43	0.48	1.11	68.7	67	72
Case 3......	1.00	1.49	1.22	0.47	1.19	66.8	63	72
Case 4......	2.00	1.24	0.81	0.47	1.44	58.6	50	72
Case 5......	$-0.5\alpha + 1$	1.70	1.39	0.48	1.09	67.6	67	72

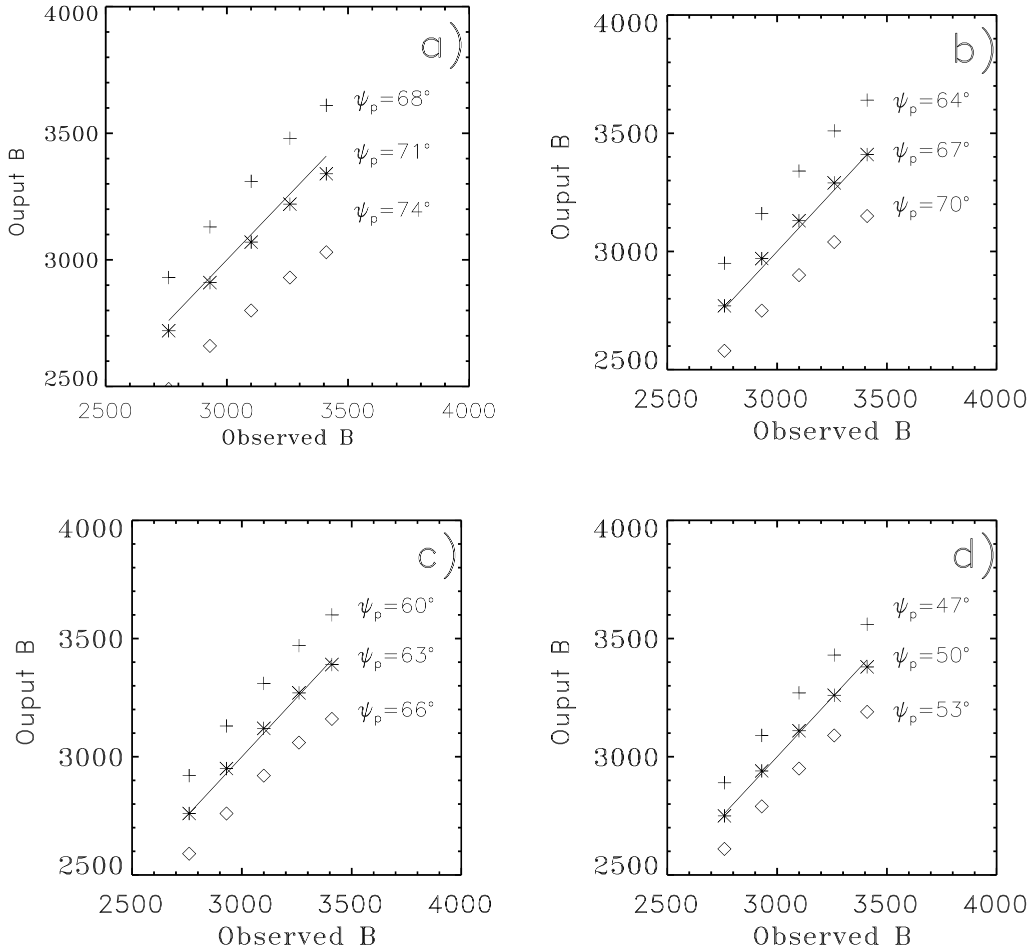

FIG. 2.—Computed maximum field strengths (*points*) at the photosphere of sunspots vs. the observed ones (*solid line*) given by eq. (6). dB_Z/dZ = (*a*) 0 G km^{-1}, (*b*) 0.5 G km^{-1}, (*c*) 1 G km^{-1}, and (*d*) 2 G km^{-1}. The two field strengths are consistent with each other for a particular ψ_p. Three ψ_p for each case are adopted for comparison.

gradient, we have taken a case characterized by

$$dB_Z/dZ = -0.5\alpha + 1 \text{ G km}^{-1} , \tag{16}$$

where we assumed that the vertical field gradient decreases with the radial distance from the spot center. With a new shape function obtained from the field configuration characterized by equation (16), we have calculated the sunspot models by the same method as in the above four cases and included the results in the last rows of Tables 1 and 2. As seen from the tables, the physical characteristics of the computed models in case 5 are nearly similar to those of case 2, where the vertical field gradient was taken as 0.5 G km^{-1}. This implies that ψ_p depends critically on the vertical gradient near the outer part of the penumbra.

Figure 3 shows the field distribution of the radial, vertical, and total magnetic field strengths at the depth of Z_d calculated by using new shape functions and equation (14) for four different field gradients (0.5, 1, 2, $-0.5\alpha + 1$ G km^{-1}). Since the inclination angle of case 4 (Fig. 3*c*) changes too much from 50° at Z_D to 72° at $Z = 0$, it could not be compatible with the observation of deep sunspot penumbra (Solanki & Schmidt 1993). Among the con-

TABLE 2

PHYSICAL CHARACTERISTICS AT THE CENTER OF MODEL SUNSPOTS AT THE SURFACE Z_D

Model	T (K)	ψ_p (deg)	Φ (Mx)	B (G)	P_g (dynes cm^{-2})	ρ (g cm^{-3})	$(dB/dZ)_c$ (G km^{-1})	R_p (Z_D) (arcsec)
Case 1……	4000	71	2.4E22	3337	1.99E5	7.86E−7	0.79	33.5
	4400	71	5.6E21	2720	1.73E5	6.22E−7	1.20	17.9
Case 2……	4000	67	2.4E22	3406	1.99E5	7.86E−7	0.79	28.0
	4400	67	5.6E21	2774	1.73E5	6.22E−7	1.20	15.0
Case 3……	4000	63	2.4E22	3387	1.99E5	7.86E−7	0.79	23.2
	4400	63	5.6E21	2759	1.73E5	6.22E−7	1.20	12.4
Case 4……	4000	50	2.4E22	3377	1.99E5	7.86E−7	0.79	14.0
	4400	50	5.6E21	2759	1.73E5	6.22E−7	1.20	7.5
Case 5……	4000	67	2.4E22	3374	1.99E5	7.86E−7	0.79	27.6
	4400	67	5.6E21	2748	1.73E5	6.22E−7	1.20	14.8

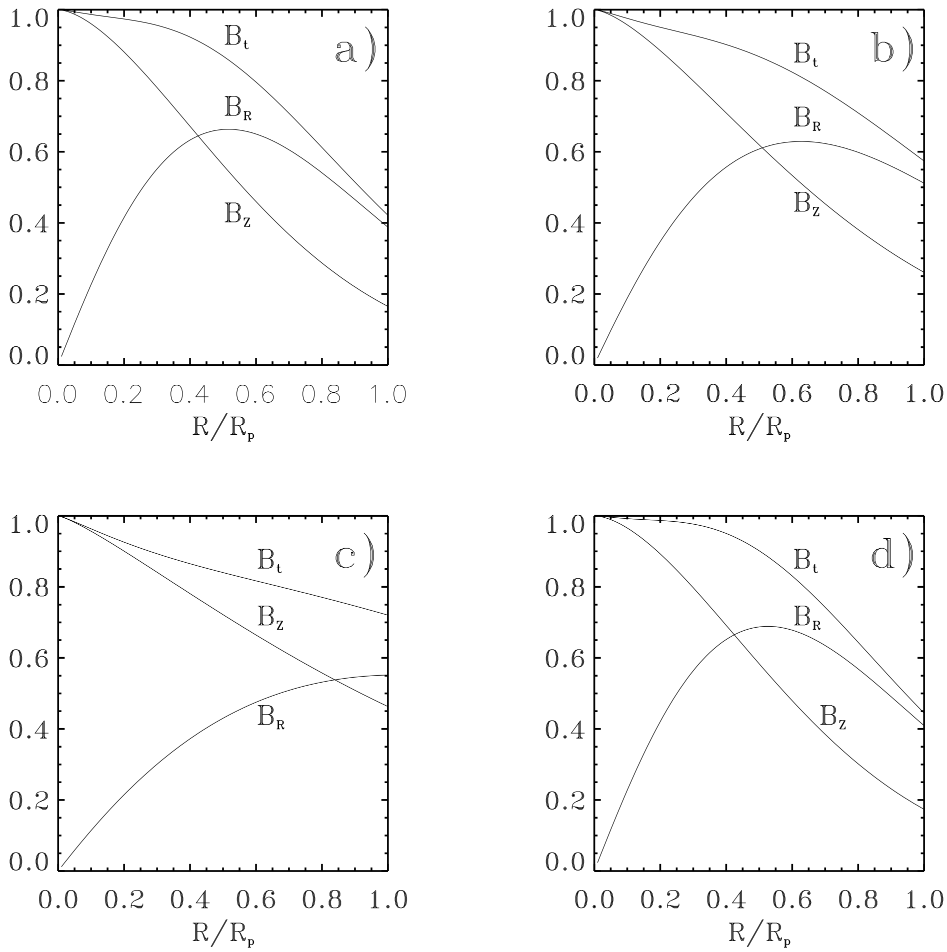

FIG. 3.—Magnetic field configuration characterized by new shape functions and the most desirable ψ_p (see Table 1). The most desirable ψ_p was determined by comparing the computed field strengths with the observed ones. dB_Z/dZ = (*a*) 0.5 G km^{-1}, (*b*) 1 G km^{-1}, (*c*) 2 G km^{-1}, and (*d*) $-0.5\alpha + 1$ G km^{-1}.

sidered models, cases 3 and 5 seem to be the most self-consistent model in the sense that the resulting $(dB/dZ)_c$ (0.8–1.2 G km^{-1}) comes out closer to the assumed vertical field gradient of 1 G km^{-1}. Considering the recent observation by Bruls et al. (1995), who found that the vertical gradient of magnetic fields in the penumbra slowly declines as a function of radial distance, we suggest that case 5 should be a most reasonable one. It is noted that the computed $d\psi_p/dZ$ ranges from 0°.01 km^{-1} to 0°.03 km^{-1} at the outer edge of the penumbra, which falls within the range suggested by Solanki et al. (1993).

Finally, we note that in our model the amount of the Wilson depression appears to have a linear relationship with $T_{\rm spot}/T_{\rm ph}$ and $T_{\rm spot}/T_{\rm ph}$ can be related to R_u, a directly observable quantity with the use of equations (6) and (7). In Figure 4 we plot the Wilson depression calculated from models as a function of R_u to get their linear relationship as

$$\mathrm{WD(km)} = 26.4 R_u[\mathrm{arcsec}] + 356.2 \qquad (17)$$

for $R_u > 7''$. We suggest this model-predicted relation could be tested by high-resolution observation in the future. The amount of the Wilson depression we presented in Figure 4 lies within the range of those suggested by Solanki et al. (1993) and Gokhale & Zwaan (1972).

The self-similarity assumption by ST58 is adopted in our study to make the problem mathematically tractable. There

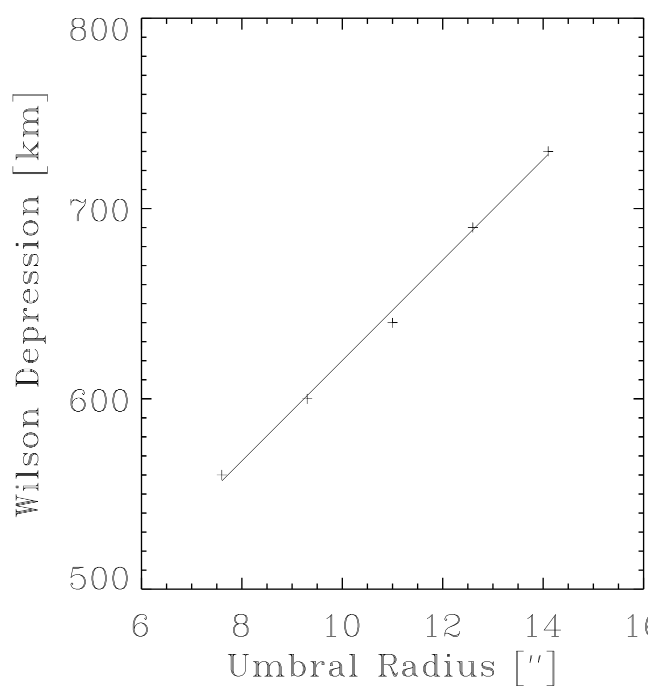

FIG. 4.—Computed Wilson depression as a function of the umbral size. Crosses are the computed values, and the solid line is their linear regression.

is no known physical reason for the sunspot field configuration to be self-similar. However, we can at least assert that the self-similarity, if any, can hold only in the region of high plasma β, where the plasma pressure can confine the magnetic field. So far no direct measurement of the subphotospheric field configuration has been made. Lee et al. (1993) investigated the radial magnetic field structure of a single isolated sunspot at the coronal base using microwave spectroscopy. They showed that the radial field distribution, at least inside the inner penumbra, has a Gaussian form and is very similar to that in the photosphere. If the field configuration is more or less self-similar even above the photosphere, we may expect that it would be more so in the subphotospheric region.

In the present study, we have not made a closer examination of the optical properties of the models. A continuum radiative transfer analysis of the emergent flux at various center-to-limb angles would give us a more definitive understanding of the Wilson effect. However, it is beyond the scope of this paper and is reserved for future studies.

The sunspot models in this study are constructed under the assumption of the azimuthal symmetry. Title et al. (1993) found from their high-resolution observations at the Swedish Solar Observatory in La Palma that the inclination angle of the magnetic field in the penumbra of sunspots oscillates rapidly with azimuth. In relation to this, Martens et al. (1996) presented a constant-α force-free model for the magnetic field in fluted sunspots. Our symmetric models do not accommodate any azimuthal variations, but are mainly concerned with the mean-field structure of isolated sunspots, especially in the high-β region in and below the photosphere.

4. SUMMARY AND CONCLUSION

In the present study, we have revisited Yun's sunspot models by taking into account the effect of the Wilson depression. For this purpose, we represented the observed field distribution of sunspots with the Skumanich dipole model and converted it to one referenced to a common geometric depth. In converting the Skumanich dipole model to the geometrically referenced one, we have made use of the radial dependence of the Wilson depression suggested by Solanki et al. (1993). With the use of the Wilson depression of Solanki et al. (1993) along with five different vertical gradients (0, 0.5, 1.0, 2.0, and $-0.5\alpha + 1$ G km^{-1}), a set of new shape functions has been derived by fitting them to the geometrically referenced longitudinal field distributions. The shape functions are assumed to be represented by $D(\alpha) = D(0) \exp(-\alpha^n)$. To reduce the number of free parameters in the model computations, we have utilized a recent empirical relation between maximum field strength and effective temperature of spots given by Kopp & Rabin (1992), together with the relation between the field strength and the umbral size suggested by Kopp & Rabin (1992) and Martinez Pillet & Vázquez (1993). With the new shape functions conforming to geometrically referenced field configurations characterized by the five different vertical field gradients, five sets of magnetostatic sunspot models have been computed by varying ψ_p.

The main result in this study is summarized as follows:

1. The computed models for a given field gradient can be characterized only by $T_{\rm spot}$, the effective temperature at the center of spots. The most desirable values of ψ_p, the upper boundary condition, for the five different cases were determined by comparing the computed maximum surface field strengths with the observed ones.
2. Especially, the computed models do not have the shortcoming raised by Osherovich (1982), demonstrating that the shortcoming is not inherent to the self-similar models. The similarity assumption would break down in the higher solar atmosphere due to the surrounding weak gas pressure as commented by Steiner, Pneuman, & Stenflo (1986) and Pneuman, Solanki, & Stenflo (1986).
3. The resulting sunspot models support the observed empirical relations given by Kopp & Rabin (1992) and Martinez Pillet & Vázquez (1993).
4. The computed $d\psi_p/dZ$ ranges from $0\overset{\circ}{.}01$ to $0\overset{\circ}{.}03$ km^{-1} at the outer edge of the penumbra, which falls within the range suggested by Solanki et al. (1993).
5. The present study supports the self-similarity for the lateral magnetic vector structure of fairly isolated sunspots found by Keppens & Martinez Pillet (1996).

We presented in Table 2 the physical characteristics of T, ψ_p, Φ, B, P_g, ρ, $(dB/dZ)_c$, and $R_p(Z_D)$ expected in each model. It is concluded that when the Wilson depression is properly taken into account, the similarity field configuration represents quite well the structure of subphotospheric sunspots.

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