

FAST AND SLOW DENSITY WAVES IN MAGNETIZED SPIRAL GALAXIES

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ABSTRACT

Nonthermal synchrotron radio emissions from gas-rich disk spiral galaxies reveal the presence of spiral magnetic field structures over galactic scales, besides the luminous optical spiral arms. From polarized radio observations, large-scale magnetic fields are inferred to lie within galactic planes and to orient roughly in the azimuthal direction. Using the magnetohydrodynamic (MHD) approximation, we develop a theory for galactic MHD density waves in a thin magnetized rotating gaseous disk, present analytical calculations for both fast and slow MHD density waves sustained by self-gravity in the tight-winding approximation, and study the phase relationships among velocity, magnetic field, and density perturbations of these MHD density waves. In particular, we relate fast MHD density waves to the global pattern of roughly coincident optical and magnetic spiral structures seen, for example, in the “Whirlpool galaxy” M51 (NGC 5194) and apply slow MHD density waves to the spiral structure in which magnetic spiral arms are interlaced with optical spiral arms discovered recently in the nearby galaxy NGC 6946. Slow MHD density waves might also manifest in other late-type gas-rich spiral galaxies such as the galaxies IC 342 and M83 (NGC 5236).

Subject headings: galaxies: magnetic fields — galaxies: spiral — MHD — polarization — waves

1. INTRODUCTION

Nonthermal synchrotron radio emissions from gas-rich disk spiral galaxies, including our own Galaxy (e.g., Heiles 1996), are primarily caused by the presence of galactic magnetic fields and relativistic cosmic-ray electrons (or positrons) confined therein and have been observed and studied for several decades (e.g., Pawsey 1965; Woltjer 1965). Typical energy densities of galactic magnetic fields, cosmic rays, and thermal gas are roughly comparable and are in the order of $\sim 1 \text{ eV cm}^{-3}$ or so (Morrison 1957; Lin 1967a; Parker 1967, 1969, 1979, 1992). Ultimately, a comprehensive theory for large-scale magnetic field structures as revealed by synchrotron radio emissions should take these relevant aspects into account. To set the stage for theoretical development, we first briefly describe some observational facts in the following.

Complementary to high-resolution optical observations of spiral galaxies (e.g., Arp 1966; Kormendy & Norman 1979; Elmegreen, Elmegreen, & Seiden 1989), radio observations with sufficiently high angular resolutions have also revealed global spiral patterns which usually correlate well with the observed optical spiral arms (Sofue, Fujimoto, & Wielebinski 1986; Beck et al. 1996). The most impressive example is offered by the magnificent “Whirlpool galaxy” M51 (NGC 5194) (Mathewson, van der Kruit, & Brouw 1972; Tully 1974a, 1974b; Segalovitz, Shane, & de Bruyn 1976; Tosa & Fujimoto 1978; Beck 1983; Sofue et al. 1986; Beck, Klein, & Wielebinski 1987; Lo et al. 1987; Rand & Kulkarni 1990; Neininger 1992; García-Burillo, Guélin, & Cernicharo 1993; Neininger & Horellou 1996; Berkhuysen et al. 1997; Beck et al. 1996) in which optical and magnetic spiral structures more or less overlap (Roberts & Yuan 1970; Tosa 1973), although the ridge lines of strongest radio synchrotron emission and neutral hydrogen H I emission at 21 cm wavelength were found to lie along the dark narrow dust lanes slightly *inside* the optical spiral arms outlined by brilliant OB stars and giant bright H II complexes. These features have also been found in other spiral galaxies (e.g., the Andromeda nebula; see Beck, Berkhuysen, & Wielebinski 1980 and Koper 1993), and the slight shift between optical and magnetic spiral structures is interpreted in terms of the time delay of $\sim 10^7$ yr in star formation as the peaks of galactic density waves sweep by (Lin & Shu 1964, 1966; Lin 1967a, 1967b, 1987; Lin, Yuan, & Shu 1969; Toomre 1977; Athanassoula 1984).

Another distinct type of large-scale spiral structure for galactic magnetic fields was discovered recently in the nearby galaxy NGC 6946 by high-resolution polarized radio observations at 6.2 cm wavelength (Beck & Hoernes 1996), in which the magnetic spiral arms lie *between* the optical spiral arms and the radial interval from a magnetic arm to its outer neighboring optical arm appears somewhat shorter. The estimated magnetic field strength in NGC 6946 falls in the range of $\sim 3\text{--}13 \mu\text{G}$, the patterns of both optical and magnetic spiral arms roughly occupy the almost rigidly rotating portion (with an angular radius of $\lesssim 4^\circ$)¹ of the galactic disk (Rogstad, Shostak, & Rots 1973; Kormendy & Norman 1979; Young & Scoville 1982; Tacconi & Young 1986; Carignan et al. 1990), and the distance to NGC 6946 was estimated to range from 5.1 Mpc (de Vaucouleurs 1979) to 10.5 Mpc (Sandage & Tamman 1975). Several earlier observations (Ball et al. 1985; Zaritsky & Lo 1986; Ishizuki et al. 1990) indicate that NGC 6946, which has no apparent companion galaxy, might have a weak central molecular bar or bulge, whereas the recent CO and infrared observations toward the central region of NGC 6946 (Regan & Vogel 1995) may be interpreted as multiple (i.e., $m = 4$) molecular spiral arms “masquerading” as a bar. In addition, there are a few other late-type gas-rich spiral galaxies, for example, IC 342 (Krause 1993; Krause, Hummel, & Beck 1989) and M83 (NGC 5236; see Sukumar & Allen 1989 and Neininger et al. 1993), which appear to bear somewhat similar global structural features of NGC 6946 (Beck 1991; Ehle & Beck 1993; Beck & Hoernes 1996). In this sense, NGC 6946 may represent a separate class of late-type gas-rich spiral galaxies.

¹ In contrast to observations of neutral hydrogen H I at 21 cm wavelength and of CO ($J = 1 \rightarrow 0$) at 2.6 mm wavelength, the H α observations by Bonnarel et al. (1988) indicate a much smaller angular size of only a few arcseconds for the rigidly rotating disk portion of the spiral galaxy NGC 6946.

Given these optical and radio observations of gas-rich spiral galaxies, it appears necessary to incorporate the effect of large-scale galactic magnetic field into the well-known galactic density wave scenario (Lin & Shu 1964, 1966; Lin 1987) in order to account for several structural aspects of radio observations of spiral galaxies. A luminous disk spiral galaxy is only a part of a typical galactic system which contains a relatively old stellar halo and a massive dark matter halo. The disk galaxy's rotation curve is determined by the total mass present in the entire system. The spiral galaxy itself contains a stellar disk component and a gaseous disk component with the former being much more massive than the latter. Young stars continuously form out of high-density regions in the gaseous disk and collectively give rise to relatively narrow extended spiral arms in blue light. The magnetic field embedded in the gaseous disk directly interacts with gas flows and with a "fat" or "inflated" disk (or an oblate spheroid) of relativistically hot cosmic rays. A complete treatment of such a comprehensive yet complex problem is certainly desirable but is beyond the limited scope of the present paper. Recently, we have taken the necessary first step (Fan & Lou 1996) to investigate possible types of tight-winding spiral magnetohydrodynamic (MHD) density waves in a magnetized gaseous disk.² In this paper, we wish to present in some detail theoretical calculations underlying the scenario outlined by Fan & Lou (1996) and discuss more thoroughly the basic physics involved.

The simplest background model that bears some realistic galactic features (Sofue et al. 1986; Beck et al. 1996) and that can be studied analytically and/or numerically is a rotating thin gaseous disk embedded with a mean ringlike azimuthal magnetic field, be it primordial (e.g., Piddington 1969, 1972, 1978) or galactic dynamo-generated (e.g., Parker 1979, 1992; Ruzmaikin, Shukurov, & Sokoloff 1988). In our perspective, galactic MHD density waves still involve important gravitational interactions. Meanwhile, in order to explain large-scale structural patterns of galactic radio observations and their spatial phase relationships with optical spiral arms, it is necessary and important to systematically study the behavior of large-scale magnetic field perturbations in association with galactic density waves in the thermal gaseous disk.

In view of the historical development of the density wave theory for spiral galaxies (Lin & Shu 1964, 1966; Lin 1987; Bertin & Lin 1996) over more than 30 years and the existing vast literature on the subject (Oort 1962; Woltjer 1962; Lin 1967a, 1967b; Becker & Contopoulos 1970; Weliachew 1975; Toomre 1977; Berkhuijsen & Wielebinski 1978; Athanassoula 1984; Binney & Tremaine 1987), our present undertaking toward an MHD density wave theory for magnetized spiral galaxies is mainly aimed at several major features of typical spiral galaxies revealed by radio and optical observations, and we anticipate that various theoretical issues will arise analogous to those associated with the standard hydrodynamic density wave theory because the energy of galactic magnetic field is much smaller than the kinetic energy of the galactic rotation.³

There are several theoretical development in the past that are generally relevant to our present investigation.

First, Lynden-Bell (1966) presented a seminal calculation of local MHD density waves in a magnetized rotating sheet in the context of galactic dynamics. Two possible MHD wave modes were found, with the stability criterion for one of the modes being the same as if the rotation and magnetic field were absent. Elmegreen (1987) obtains the Lynden-Bell result extended to the case with disk shear, and Elmegreen (1991) further studied similar problems with shear, finite-disk thickness, and thermal effects included; he performed extensive numerical explorations for various instabilities in the contexts of cloud and supercloud formation in spiral galaxies. Recently, Gammie (1996) investigated the influence of an effective viscosity of the interstellar medium on the stability of a magnetized gaseous disk. Second, Roberts & Yuan (1970) and Tosa (1973) studied stationary nonlinear MHD density wave structure in the presence of an azimuthal magnetic field and in a reference frame corotating with the pattern speed (see also Roberts 1969 for a model of hydrodynamic spiral shock wave structure using a similar approach). Their pioneer work was closely relevant to the spiral patterns seen, for example, in M51 (Mathewson, van der Kruit & Brouw 1972; Tully 1974a, 1974b; Segalovitz et al. 1976; Neininger 1992; Neininger & Horellou 1996; Berkhuijsen et al. 1997). Third, kinematic dynamo theory has been used for many years to explain the presence and overall structure of galactic magnetic fields via the amplification of a weak seed field by turbulence and differential rotation, although the origin of galactic magnetic fields and the processes that maintain them in their currently observed states still remain uncertain (Zweibel & Heiles 1997). Research publications along this general direction are very extensive (e.g., Parker 1979; Sofue et al. 1986; Ruzmaikin et al. 1988; Berkhuijsen et al. 1997; Beck et al. 1996; Zweibel & Heiles 1997); nevertheless, the magnetic spiral structure seen in NGC 6946 (Beck & Hoernes 1996) challenges the kinematic dynamo theory in its current form. Fourth, there is a theoretical effort in the past few years (Chiba & Tosa 1990; Sawa & Fujimoto 1990; Mestel & Subramanian 1991; Subramanian & Mestel 1993; Hanasz, Lesch, & Krause 1991; Otmianowska-Mazur & Chiba 1995) to incorporate basic ingredients of turbulent dynamo theory with those of density wave theory. In such an approach, the differential rotation of a disk and the velocity field associated with hydrodynamic density waves are prescribed in order to solve for the magnetic induction equation, whereas the dynamic feedback of the Lorentz force in the momentum equation is ignored. Strictly speaking, the information of both fast and slow MHD density waves (Fan & Lou 1996) is lost in such a kinematic treatment. Finally, Foglizzo & Tagger (1994, 1995) considered various wavenumber regimes for the Parker-shearing instability in a vertically stratified disk with an azimuthal magnetic field but without the self-gravity effect; their shearing instability is of the Chandrasekhar-Balbus-Hawley type (Chandrasekhar 1961; Balbus & Hawley 1991, 1992; Hawley 1995) caused by the dominant differential force against the magnetic tension in the Boussinesq approximation. Such Chandrasekhar-Balbus-Hawley type instabilities have been vigorously pursued in the theory of magnetized accretion disks (e.g., Hawley 1995; Terquem & Papaloizou 1996). Tagger et al. (1990) studied the swing amplification of spiral density waves in a disk with a magnetic field perpendicular to the disk plane.

² The problem of MHD density waves in a composite disk system consisting of stellar and magnetized gaseous disks coupled gravitationally is studied in a separate paper (Lou & Fan 1997a).

³ It was suggested a few years ago by Battaner et al. (1992) that the flat rotation curves of spiral galaxies might be caused by the presence of magnetic fields, rather than dark matter (Binney 1992). However, recent observations of Vallée (1994) indicated magnetic field strengths in M31 and our Galaxy were ~ 10 times weaker than the theoretical estimates. See also theoretical arguments given by Jokipii & Levy (1993) and Cuddeford & Binney (1993) on this issue.

The plan of this paper is as follows. In § 2, we outline the general formulation of the problem. Calculations for MHD fast and slow density waves in the tight-winding approximation are presented in § 3. In § 4, we apply our theoretical results for fast and slow MHD density waves to the “Whirlpool galaxy” M51 (NGC 5194) and the spiral galaxy NGC 6946, respectively. Some mathematical details are contained in Appendices A, B, and C for the convenience of reference.

2. FORMULATION OF THE PROBLEM

The basic ingredients of a galaxy are stars, gas and dust, magnetic fields, cosmic rays, and, perhaps, a massive halo of “dark matter.” The stars usually take up most of the visible mass of a galaxy in the later evolutionary phases. Although the gas (with the dust) contributes only several percent to the total mass of a spiral galaxy, it nevertheless plays an important role in shaping the appearance of a spiral galaxy because the most luminous young stars are borne out of the gas. The large-scale interaction between the stellar and gaseous disk components of a spiral galaxy is primarily gravitational, and the mass ratio of stellar to gaseous disk components evolve with time. The younger a galaxy, the smaller this mass ratio. The magnetic field interacts directly with the ionized gases and exerts a dynamic influence via sufficiently frequent collisions between ionized and neutral particles, on the neutral gases. Simultaneously, large-scale galactic gas motions will back react on the magnetic field to affect its overall structure and strength. Cosmic rays, that is, charged relativistic particles, are largely confined by the galactic magnetic field, and their collective motions also influence the structure and strength of the magnetic field (Parker 1969, 1979, 1992) because the energy densities of cosmic rays and magnetic field are comparable (Morrison 1957; Lin 1967a; Parker 1969, 1979). The massive halo of “dark matter,” which makes itself felt solely by virtue of gravitational effects, may play a decisive role in globally stabilizing spiral galaxies (Ostriker & Peebles 1973).

In general, we are mainly concerned with large-scale low-frequency interactions among the thermal gas, the magnetic field, and the tenuous cosmic-ray gas under the control of the gravitational force in a spiral galaxy. However, in order to limit the scope of the present analysis, we specifically focus on large-scale MHD interactions in the magnetized gaseous disk alone. For applications to galactic spiral structures (see § 4), theoretical results derived from such a limited treatment should be complemented and backed by further considerations of several physical aspects, namely, (1) the gravitational coupling of density waves in a composite system consisting of a stellar disk and a magnetized thermal gaseous disk (Lou & Fan 1997a); (2) the magnetic coupling of density waves in a composite system consisting of a magnetized thermal gaseous disk and a tenuous cosmic-ray gas (Lou & Fan 1997b); (3) a further combination of the preceding two aspects in a coherent treatment which remains a problem of considerable challenge. Even without completing a thorough investigation of aspect (3), a sensible physical scenario emerges from the separate studies of the first two aspects and the present problem. For aspect (1), one may treat the mutual gravitational coupling between density wave perturbations in stellar and magnetized gaseous disks on a completely equal footing (Lou & Fan 1997a). The basic features of MHD density waves in the magnetized thermal gaseous disk remain to a very good extent in the presence of density waves in a more massive stellar disk which was treated in a fluid approximation (e.g., Solomon 1984a, 1984b; Bertin & Romeo 1988; Elmegreen 1995; Jog 1996). The main new feature is that the enhancement of thermal gas density tracks the enhancement of stellar mass density within the Lindblad resonances, and this can be readily understood on physical ground, namely, thermal gas density can fall into the gravitational potential trough created by the enhancement of stellar mass density.⁴ Another interesting feature is that, in a certain parameter regime, the composite disk system can be unstable, as a result of gravitational coupling, while the subsystems are separately stable (cf. Jog & Solomon 1984a, 1984b on this point for the case without magnetic field; see also Vandervoort 1991 for spheroids of stars and gas). In our separate analysis (Lou & Fan 1997b) of aspect (2), cosmic rays are treated as relativistically hot gas fluid (Parker 1965); for low-frequency large-scale perturbations, cosmic-ray gas and thermal gas move together with the same transverse bulk speed but are allowed to move with different bulk speeds along the magnetic field. It has been known that, in addition to the presence of a suprathermal mode in the cosmic-ray gas (Parker 1965, 1969), density fluctuations of cosmic-ray gas associated with low-frequency large-scale MHD density waves in the magnetized thermal gaseous disk are extremely weak (Parker 1965, 1969) such that resulting enhancements of synchrotron radio emission are mainly caused by the enhancement of parallel magnetic field. Of course, there are various concurrent processes associated with active star formation along spiral arms of relatively high thermal gas density; these processes contribute significantly to unpolarized synchrotron radio emissions from spiral galaxies as well as to infrared emissions.

With these considerations in mind, it is justifiable and necessary to first work out the present partial problem for possible MHD density waves in a magnetized thermal gaseous disk alone. This partial problem forms an integral part of our overall research scheme. Together with other relevant problems that we have been studying in parallel (Fan & Lou 1997; Lou & Fan 1997a, 1997b, 1997c), it is possible to relate the results derived from this partial problem to galactic applications with appropriate qualifications.

For our present purpose of investigating large-scale magnetic field structures in the magnetized gaseous disk of a spiral galaxy, it suffices to adopt the one-fluid ideal MHD equations and thus ignore resistivity, viscosity, and ambipolar diffusion. The general nonlinear MHD equations as applied to galactic dynamics (e.g., Woltjer 1965) consist of the mass conservation equation for the gas mass density ρ

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (2.1)$$

⁴ In such a composite disk system, it also possible to have an MHD density wave mode with enhancements of thermal gas density and stellar mass density being out of phase (Lou & Fan 1997a). However, this type of MHD density wave modes appear outside the Lindblad resonances (cf. Bertin & Romeo 1988 for the hydrodynamic version of these modes).

the momentum equation for the bulk thermal gas flow velocity \mathbf{v}

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho(\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} + \rho \nabla \phi, \quad (2.2)$$

the induction equation for the magnetic field \mathbf{B}

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \quad (2.3)$$

the divergence-free condition for the magnetic field \mathbf{B}

$$\nabla \cdot \mathbf{B} = 0, \quad (2.4)$$

the Poisson equation for the negative gravitational potential ϕ

$$\nabla^2 \phi = -4\pi G \rho, \quad (2.5)$$

and the polytropic relation assumed for the gas pressure p and the gas mass density ρ

$$p = K \rho^\gamma, \quad (2.6)$$

where G is the gravitational constant, γ is the polytropic index, and K is a proportional constant. The thermal gas temperature T can be determined from the ideal gas law, once the gas pressure p and gas mass density ρ are known.

In our mathematical formulation for MHD density waves in a thin rotating gaseous disk embedded with an azimuthal ringlike (circular) magnetic field B_θ , the cylindrical coordinate system (r, θ, z) will be adopted. If one denotes the background gas flow velocity by $\mathbf{V}_0 \equiv (V_r, V_\theta, V_z)$ and the embedded background magnetic field by $\mathbf{B}_0 \equiv (B_r, B_\theta, B_z)$, our specific model for the magnetized thin gaseous disk in equilibrium is characterized by $V_r = 0$, $V_\theta = \Omega r$, with Ω being the angular rotation rate, $V_z = 0$, $B_r = 0$, $rB_\theta = F_B$, with F_B being a constant and $B_z = 0$; the polytropic relation (2.6) becomes $p_0 = K \rho_0^\gamma$, where the subscript “0” denotes the background steady state; the radial momentum equation (2.2) becomes

$$-\frac{\rho_0 V_\theta^2}{r} = -\frac{\partial p_0}{\partial r} - \frac{B_\theta}{4\pi r} \frac{\partial(rB_\theta)}{\partial r} + \rho_0 \frac{\partial \phi_T}{\partial r}, \quad (2.7)$$

and the Poisson equation (2.5) becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi_T}{\partial r} \right) + \frac{\partial^2 \phi_T}{\partial z^2} = -4\pi G \rho_T, \quad (2.8)$$

where ϕ_T is the *total* negative gravitational potential felt in the magnetized gaseous disk and ρ_T is the *total* mass density distribution present in the entire galactic system. The thin galactic disk is assumed to be of a negligible thickness. The vertical quasi-static balance involves the confinement of thermal gas and magnetic field under the actions of gas self-gravity, stellar gravity (disk plus halo), and dark matter gravity. In our background model, the radial Lorentz force in equation (2.7) vanishes because rB_θ is taken to be a constant F_B ; by this choice of B_θ which scales as r^{-1} , one must invoke other processes in the vicinity of the galactic center to avoid the singularity of B_θ there. Since the massive dark matter halo contributes to ρ_T and thus to ϕ_T , the rotation velocity $V_\theta^S(r)$ of the thin stellar disk is actually used to infer ϕ_T from observations. Since the thermal energy density of the gas is much smaller than the kinetic energy density of the galactic rotation and the radial Lorentz force is negligible, the difference between the gas rotation velocity $V_\theta(r)$ and the stellar rotation velocity $V_\theta^S(r)$ should be fairly small. In some sense, one can prescribe a $V_\theta(r)$ based on observational input in order to construct an approximate background galactic profile for the magnetized thermal gaseous disk.

Given a background profile for the thin rotating gaseous disk, it is straightforward to write out MHD perturbation equations from equations (2.1)–(2.6). For a two-dimensional propagation of MHD perturbations within the magnetized gaseous disk, there are two possible types of MHD fluctuations in general, namely, (1) the compressible MHD fluctuations with velocity and magnetic field perturbations tangential to the disk plane (e.g., Lou 1994a) and (2) the incompressible Alfvénic fluctuations with velocity and magnetic field perturbations perpendicular to the disk plane (e.g., Lou 1994b). Since only compressible MHD fluctuations that involve the gas mass density perturbation and thus couple to the gravitational potential perturbation over the galactic scale, they are the main subject of investigation in this paper. At this stage, as mentioned earlier, we shall not consider the effects of fluctuations in the stellar disk and in cosmic-ray gas solely for the sake of simplicity; our present priority is to analyze the behavior of MHD density waves in the thin rotating gaseous disk alone. The compressible MHD perturbation equations for MHD density waves are now given below, namely, the radial component of the induction equation for the radial magnetic field perturbation b_r ,

$$\frac{\partial b_r}{\partial t} = -\frac{1}{r} \frac{\partial(V_\theta b_r)}{\partial \theta} + \frac{1}{r} \frac{\partial(B_\theta v_r)}{\partial \theta}, \quad (2.9)$$

the azimuthal component of the induction equation for the azimuthal magnetic field perturbation b_θ ,

$$\frac{\partial b_\theta}{\partial t} = \frac{\partial(V_\theta b_r)}{\partial r} - \frac{\partial(B_\theta v_r)}{\partial r}, \quad (2.10)$$

the divergence-free condition for the magnetic field perturbation $\mathbf{b} \equiv (b_r, b_\theta, 0)$,

$$\frac{1}{r} \frac{\partial(r b_r)}{\partial r} + \frac{1}{r} \frac{\partial b_\theta}{\partial \theta} = 0, \quad (2.11)$$

the radial component of the momentum equation for the radial velocity perturbation v_r ,

$$\frac{\partial v_r}{\partial t} - \frac{2V_\theta v_\theta}{r} + \frac{V_\theta}{r} \frac{\partial v_r}{\partial \theta} = -\frac{1}{\rho_0} \frac{\partial p}{\partial r} + \frac{\rho}{\rho_0^2} \frac{dp_0}{dr} - \frac{B_\theta}{4\pi\rho_0 r} \left[\frac{\partial(r b_\theta)}{\partial r} - \frac{\partial b_r}{\partial \theta} \right] + \frac{\partial \phi}{\partial r}, \quad (2.12)$$

the azimuthal component of the momentum equation for the azimuthal velocity perturbation v_θ ,

$$\frac{\partial v_\theta}{\partial t} + \frac{V_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{1}{r} \frac{d(r V_\theta)}{dr} v_r = -\frac{1}{\rho_0 r} \frac{\partial p}{\partial \theta} + \frac{1}{r} \frac{\partial \phi}{\partial \theta}, \quad (2.13)$$

the mass conservation equation for the surface mass density perturbation μ ,

$$\frac{\partial \mu}{\partial t} + \frac{1}{r} \frac{\partial(\mu_0 r v_r)}{\partial r} + \frac{\mu_0}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{V_\theta}{r} \frac{\partial \mu}{\partial \theta} = 0, \quad (2.14)$$

the three-dimensional Poisson equation for the perturbed negative gravitational potential ϕ ,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = -4\pi G \mu(r, \theta) \delta(z), \quad (2.15)$$

and the perturbed polytropic relation for the gas pressure perturbation p and the gas mass density perturbation ρ ,

$$p = K \gamma \rho_0^{\gamma-1} \rho \equiv C_s^2 \rho, \quad (2.16)$$

where $\delta(z)$ is the Dirac delta function with argument z , C_s is the polytropic sound speed, and μ_0 is the surface mass density of the gaseous disk related to ρ_0 by $\rho_0(r, z) \equiv \mu_0(r) \delta(z)$. Note that for the magnetic field perturbation \mathbf{b} , one of the equations (2.9) and (2.10) may be spared, if equation (2.11) is used instead.

One can introduce the enthalpy $H \equiv \gamma p / [(\gamma - 1)\rho]$ into the present formulation, where p and ρ are nonlinear gas pressure and gas mass density in general. It follows from the perturbed polytropic relation (2.16) that the perturbation of enthalpy H , denoted by h , is given by p/ρ_0 , where p is the gas pressure perturbation and ρ_0 is the equilibrium mass density. The right-hand sides of equations (2.12) and (2.13) can then be simplified by using the enthalpy perturbation h .

By assuming the $\exp(i\omega t - im\theta)$ dependence for all MHD perturbation variables, where ω is the angular frequency in an inertial frame of reference and m is a nonnegative integer characterizing the azimuthal variation, we have from equations (2.9)–(2.14),

$$r(\omega - m\Omega)b_r = -mB_\theta v_r, \quad (2.17)$$

$$i\omega b_\theta = \frac{\partial(V_\theta b_r)}{\partial r} - \frac{\partial(B_\theta v_r)}{\partial r}, \quad (2.18)$$

$$b_\theta = -\frac{i}{m} \frac{\partial(r b_r)}{\partial r}, \quad (2.19)$$

$$i(\omega - m\Omega)v_r - 2\Omega v_\theta = -\frac{B_\theta}{4\pi\rho_0 r} \left[\frac{\partial(r b_\theta)}{\partial r} + im b_r \right] + \frac{\partial(\phi - h)}{\partial r}, \quad (2.20)$$

$$i(\omega - m\Omega)v_\theta + \frac{1}{r} \frac{d(r V_\theta)}{dr} v_r = -\frac{im}{r} (\phi - h), \quad (2.21)$$

$$i(\omega - m\Omega)\mu + \frac{1}{r} \frac{\partial(r \mu_0 v_r)}{\partial r} - \frac{im\mu_0}{r} v_\theta = 0. \quad (2.22)$$

In general, the three-dimensional Poisson equation (2.15) implies an integral representation of ϕ in terms of μ (Shu 1970a; Lin & Lau 1979), which would be difficult to pursue analytically for obtaining explicit solutions for all perturbation variables. Nevertheless, in the situation where the radial wavelength of MHD density waves is sufficiently short, it is possible to establish a local differential relation between ϕ and μ within the gaseous disk (Lin & Shu 1964, 1966; Shu 1968, 1970b; see also Appendices A and B). This approximation is referred to as the WKB or tight-winding approximation, and it applies to some, although not all, spiral galaxies reasonably well. For the two spiral galaxies M51 and NGC 6946 under consideration here, the tight-winding approximation is roughly valid. We shall adopt this tight-winding approximation in our analytical analysis because the solutions thus derived contain valuable information regarding local properties of MHD density waves in the magnetized thermal gaseous disk.

3. DISPERSION RELATIONS OF MHD DENSITY WAVES

What do we expect from MHD perturbation equations (2.15)–(2.22) in the qualitative sense? To answer this question, it is perhaps instructive to consider the simplest case in which compressible MHD perturbations propagate in a uniformly

magnetized medium in an infinite space without self-gravity. It is well known that this system supports compressible fast and slow MHD waves, in addition to incompressible transverse Alfvén waves, and the explicit dispersion relations can be readily derived (see Appendix C). Fast MHD waves propagate faster because magnetic and thermal pressure perturbations are in phase. Slow MHD waves propagate slower because magnetic and thermal pressure perturbations are out of phase. Since density and gas pressure perturbations are in phase for adiabatic or polytropic processes, density and magnetic field perturbations are in phase for fast MHD waves, while density and magnetic field perturbations are out of phase for slow MHD waves.

In the present context of MHD density waves in a thin self-gravitating gaseous disk, we consider compressible MHD perturbations because density fluctuation is coupled to gravitational potential perturbation via Poisson's equation (2.15). In other words, a spiral gravitational field will exert its dynamical influence on compressible MHD perturbations. With a cylindrical geometry and in the presence of a thin rotating disk, the analysis of MHD density waves becomes fairly complicated, but the analogs of fast and slow MHD waves described above are expected to exist. We set out to perform the relevant analysis to confirm our expectation by demonstrating the existence of fast and slow MHD density waves. It turns out that for fast MHD density waves, density and magnetic field perturbations are more or less in phase in the tight-winding approximation, while density and magnetic field perturbation enhancements are significantly phase shifted (i.e., a phase difference $\gtrsim \pi/2$ but not exactly π due to the presence of disk rotation) for slow MHD density waves.

In order to derive the local dispersion relations from equations (2.15)–(2.22) for fast and slow MHD density waves, we begin by writing

$$\phi = \Phi(r) \exp \left[i \int^r k(s) ds \right], \quad (3.1)$$

where $k(r)$ is the radial wavenumber and $\Phi(r)$ is the slowly varying amplitude of the negative gravitational potential perturbation ϕ (Goldreich & Tremaine 1979). It follows that

$$\frac{d\phi}{dr} = ik(r)\phi + \frac{\Phi'(r)}{\Phi(r)} \phi, \quad (3.2)$$

where the prime over $\Phi(r)$ denotes the radial derivative. In the tight-winding approximation with a large radial wavenumber k which is a function of r in general, Poisson's equation (2.15) can be solved approximately to give rise to the following relation

$$\frac{1}{r^{1/2}} \frac{\partial(r^{1/2}\phi)}{\partial r} \Big|_{z=0} \cong 2\pi G i \operatorname{sgn}(k)\mu + \mathcal{O}\left[\frac{\mu}{(kr)^2}\right], \quad (3.3)$$

which contains a fractional error in the order of $\mathcal{O}(kr)^{-2}$ (Shu 1970b; Goldreich & Tremaine 1979; see also Appendices A and B), where $\operatorname{sgn}(k) = +1$ for $k > 0$ and $\operatorname{sgn}(k) = -1$ for $k < 0$. By this sign convention, $k > 0$ and $k < 0$ correspond to leading and trailing spiral arms, respectively, and the value of m gives the number of spiral arms. From equations (3.2) and (3.3), one has

$$\phi = \frac{4\pi i G \operatorname{sgn}(k)\mu}{2ikr + 2r\Phi'/\Phi + 1}. \quad (3.4)$$

Substituting (3.4) into (3.2), one obtains

$$\frac{d\phi}{dr} = \left(2ikr + \frac{2r\Phi'}{\Phi} \right) \frac{2\pi i G \operatorname{sgn}(k)\mu}{2ikr + 2r\Phi'/\Phi + 1} \equiv \mathcal{F}_1 \mu, \quad (3.5)$$

which defines the complex coefficient \mathcal{F}_1 .

From equations (2.21), (2.22), and (3.4), we derive an equation relating μ , v_r , and dv_r/dr , namely,

$$\mu = \frac{i m \kappa^2 \mu_0 v_r / (2\Omega r) + i(\omega - m\Omega) r^{-1} d(r\mu_0 v_r)/dr}{(\omega - m\Omega)^2 - m^2 C_S^2 / r^2 + (m^2 / r^2) 4\pi i G \operatorname{sgn}(k)\mu_0 / (2ikr + 2r\Phi'/\Phi + 1)} \equiv \mathcal{A} v_r + \mathcal{B} \frac{dv_r}{dr}, \quad (3.6)$$

which defines the complex coefficients \mathcal{A} and \mathcal{B} , where $k^2 = (2\Omega/r) d(rV_\theta)/dr$ defines the epicyclic frequency k .

By combining perturbation equations (2.17)–(2.22), it is straightforward to derive the following differential equation

$$\frac{d^2 v_r}{dr^2} + \mathcal{D}_1 \frac{dv_r}{dr} + \mathcal{D}_2 v_r + \mathcal{D}_3 \frac{d\phi}{dr} + \mathcal{D}_4 \frac{d\mu}{dr} + \mathcal{D}_5 \mu = 0, \quad (3.7)$$

where the coefficients \mathcal{D}_i , with $i = 1, \dots, 5$, are explicitly given by

$$\mathcal{D}_1 = \frac{2(\omega - m\Omega)}{mB_\theta} \frac{d}{dr} \left(\frac{mB_\theta}{\omega - m\Omega} \right) + \frac{1}{r} + \frac{2\Omega r}{m} \frac{(\omega - m\Omega)}{C_A^2}, \quad (3.8)$$

$$\mathcal{D}_2 = \frac{(\omega - m\Omega)^2}{C_A^2} - \frac{m^2}{r^2} + \frac{(\omega - m\Omega)}{mB_\theta r} \frac{d}{dr} \left(\frac{mB_\theta}{\omega - m\Omega} \right) + \frac{(\omega - m\Omega)}{mB_\theta} \frac{d^2}{dr^2} \left(\frac{mB_\theta}{\omega - m\Omega} \right) + \frac{2\Omega(\omega - m\Omega)}{m\mu_0 C_A^2} \frac{d(r\mu_0)}{dr}, \quad (3.9)$$

$$\mathcal{D}_3 = \frac{i(\omega - m\Omega)}{C_A^2}, \quad (3.10)$$

$$\mathcal{D}_4 = -\frac{i(\omega - m\Omega)C_S^2}{\mu_0 C_A^2}, \quad (3.11)$$

$$\mathcal{D}_5 = \frac{2i\Omega r(\omega - m\Omega)^2}{m\mu_0 C_A^2} - \frac{i(\omega - m\Omega)}{C_A^2} \frac{d}{dr} \left(\frac{C_S^2}{\mu_0} \right), \quad (3.12)$$

and $C_A^2 \equiv B_\theta^2/(4\pi\rho_0)$ defines the Alfvén wave speed C_A in the magnetized thermal gaseous disk.

Substituting equations (3.5) and (3.6) into equation (3.7), one obtains a second-order ordinary differential equation for v_r ,

$$(1 + \mathcal{B}\mathcal{D}_4) \frac{d^2 v_r}{dr^2} + \left[\mathcal{D}_1 + \mathcal{B}(\mathcal{D}_3 \mathcal{F}_1 + \mathcal{D}_5) + \mathcal{A}\mathcal{D}_4 + \frac{d\mathcal{B}}{dr} \mathcal{D}_4 \right] \frac{dv_r}{dr} + \left[\mathcal{D}_2 + \mathcal{A}(\mathcal{D}_3 \mathcal{F}_1 + \mathcal{D}_5) + \frac{d\mathcal{A}}{dr} \mathcal{D}_4 \right] v_r = 0. \quad (3.13)$$

Equation (3.13) contains the information for fast and slow MHD density waves. As in equation (3.1) for $\phi(r)$, we write

$$v_r = \tilde{v}_r(r) \exp \left[i \int^r k(s) ds \right], \quad (3.14)$$

where $\tilde{v}_r(r)$ stands for the real amplitude. It follows that

$$\frac{dv_r}{dr} = (ik\tilde{v}_r + \tilde{v}_r') \exp \left[i \int^r k(s) ds \right], \quad (3.15)$$

$$\frac{d^2 v_r}{dr^2} = (ik'\tilde{v}_r - k^2\tilde{v}_r + 2ik\tilde{v}_r' + \tilde{v}_r'') \exp \left[i \int^r k(s) ds \right], \quad (3.16)$$

where the prime denotes the radial derivative. We further assume that

$$\frac{\tilde{v}_r'}{\tilde{v}_r} \sim \mathcal{O}(r^{-1}) \quad \text{and} \quad \frac{k'}{k} \sim \mathcal{O}(r^{-1})$$

in equations (3.15) and (3.16). By comparing the magnitudes of the terms on the right-hand side of equation (3.16), it follows that

$$\frac{k'\tilde{v}_r}{k^2\tilde{v}_r} \sim \mathcal{O}\left(\frac{1}{kr}\right), \quad \frac{k\tilde{v}_r'}{k^2\tilde{v}_r} \sim \mathcal{O}\left(\frac{1}{kr}\right), \quad \frac{\tilde{v}_r''}{k^2\tilde{v}_r} \sim \mathcal{O}\left(\frac{1}{kr}\right)^2.$$

In the parameter regime of large kr , the term $k^2\tilde{v}_r$ dominates in equation (3.16). In the same spirit, one can expand, term by term, the complex coefficients of equation (3.13) by assuming

$$\left| \frac{\Phi'}{ik\Phi} + \frac{1}{2ikr} \right| \ll 1;$$

these expressions are lengthy and are thus omitted here. By keeping the leading order terms, one can then derive local dispersion relations for MHD density waves. Several specific cases of interest are treated in the following four subsections.

3.1. Slow MHD Density Waves in a Rigidly Rotating Gaseous Disk

Optical spiral arms can appear in the almost rigidly rotating portion⁵ of a galactic disk (Kormendy & Norman 1979; Tacconi & Young 1986; Carignan et al. 1990). We first consider here slow MHD density waves in an almost rigidly rotating disk with the angular rotation speed $\Omega \equiv V_\theta/r \sim \text{constant}$ and with the azimuthal magnetic field $B_\theta = F_B/r$. In order to specifically determine leading order terms in equation (3.13) for slow MHD density waves in the regime of large kr , we note that $C_S \sim C_A$, $\Omega/C_A \sim k$ and, in particular, $\omega - m\Omega \sim mC_A/r$, which we shall verify when the final result comes out. With these specifications, we carefully examine the relative magnitude of each term in equation (3.13) and derive the simplified version of equation (3.13) to the leading order of large kr . This equation now reads

$$\left[1 + \frac{\tilde{\omega}^2 C_S^2}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)} \right] \frac{d^2 v_r}{dr^2} + \frac{2m\Omega r \tilde{\omega} 2\pi G \mu_0 - i\tilde{\omega}^2 r^2 2\pi G \mu_0 k}{C_A^2 r^2 |k|(\tilde{\omega}^2 - m^2 \Delta/r^2)} \frac{dv_r}{dr} - \frac{\tilde{\omega}^2 \kappa^2 + i\tilde{\omega} m \kappa^2 2\pi G \mu_0 \text{sgn}(k)/(2\Omega r)}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)} v_r \cong 0, \quad (3.17)$$

where $\tilde{\omega} \equiv \omega - m\Omega$, $\Delta \equiv C_S^2 - 2\pi G \mu_0 / |k|$. Meanwhile, by retaining only the leading terms ikv_r in equation (3.15) and $-k^2 v_r$ in equation (3.16) in the regime of large kr , we derive the local dispersion relation for slow MHD density waves (Fan & Lou 1996), namely,

$$(\omega - m\Omega)^2 \cong \frac{k^2 C_A^2 (C_S^2 - 2\pi G \mu_0 / |k|) m^2 / r^2}{\kappa^2 + k^2 (C_A^2 + C_S^2 - 2\pi G \mu_0 / |k|)}. \quad (3.18)$$

⁵ Within uncertainties of error bars, the rotation speed $V_\theta(r)$ in central regions of some disk galaxies first increases more or less linearly and then slowly bends to approach an approximately constant value with increasing r ; this situation shall be referred to as almost or nearly rigid rotation.

We see therefore that, by taking $\Omega/C_A \sim k$, $|\omega - m\Omega|$ is indeed in the order of $\mathcal{O}(mC_A/r)$ for slow MHD density waves. Note that $\kappa = 2\Omega$ for an almost rigidly rotating disk. According to dispersion relation (3.18), slow MHD density waves are locally stable when $C_S^2 - 2\pi G\mu_0/|k| > 0$, as if the disk rotation and the background azimuthal magnetic field were absent. This result is not surprising; similar results for the onset of the Jeans instability in the presence of magnetic field and/or rotation have been derived earlier (Chandrasekhar & Fermi 1953; Chandrasekhar 1954, 1961; Lynden-Bell 1966; Elmegreen 1987; Lou 1996). This stability criterion for a rigidly rotating gas disk is more stringent than Toomre's stability criterion (Safronov 1960; Toomre 1964; Julian & Toomre 1966). We emphasize that, in the absence of magnetic field and azimuthal variation, this kind of Jeans instability would not occur; although the magnetic field does not enter the stability criterion, the growth rate of such instability is anisotropic, depending upon the magnetic field direction and strength (Lynden-Bell 1966).

For locally stable slow MHD density waves with $C_S^2 > 2\pi G\mu_0/|k|$, one can readily express the magnitude of the radial wavenumber $|k|$ in terms of the background variables and other parameters by solving dispersion relation (3.18), namely,

$$\frac{1}{|k|} = \pm \frac{\{(2\pi G\mu_0)^2(\tilde{\omega}^2 - C_A^2 m^2/r^2)^2 - 4\tilde{\omega}^2 \kappa^2 [(C_S^2 + C_A^2)\tilde{\omega}^2 - C_A^2 C_S^2 m^2/r^2]\}^{1/2}}{2\tilde{\omega}^2 \kappa^2} + \frac{2\pi G\mu_0(\tilde{\omega}^2 - C_A^2 m^2/r^2)}{2\tilde{\omega}^2 \kappa^2}, \quad (3.19)$$

where $\tilde{\omega}^2 \equiv (\omega - m\Omega)^2$. One can infer from dispersion relation (3.18) that $\tilde{\omega}^2 \lesssim C_A^2 m^2/r^2$ and $\tilde{\omega}^2 \lesssim C_A^2 C_S^2 m^2/[(C_S^2 + C_A^2)r^2]$. Therefore, the minus-sign root of $|k|^{-1}$ in solution (3.19) is negative and should be discarded, and the plus-sign root of $|k|^{-1}$ in solution (3.19) is positive and corresponds to a short radial wavelength. In contrast to hydrodynamic density waves which can have both short- and long-wave branches for the same local dispersion relation (e.g., Binney & Tremaine 1987), slow MHD density waves here have only one wavelength branch.

For a packet of slow MHD density waves with a certain amount of frequency spread, one can also derive the radial group velocity S_G from dispersion relation (3.18), namely,

$$S_G = \frac{\partial \omega}{\partial k} \cong \frac{kC_A^2[\kappa^2(C_S^2 - \pi G\mu_0/|k|) + C_A^2 \pi G\mu_0 |k|]m^2/r^2}{(\omega - m\Omega)[\kappa^2 + k^2(C_A^2 + C_S^2 - 2\pi G\mu_0/|k|)]^2}, \quad (3.20)$$

where $S_G > 0$ and $S_G < 0$ correspond to inward and outward propagations, respectively. For locally stable slow MHD density waves, the sign of S_G depends on the signs of k and $\omega - m\Omega$. For example, for trailing slow MHD density waves with $k < 0$ and with the angular pattern speed $\omega_p \equiv \omega/m < \Omega$, one has $S_G > 0$, and thus the packet of slow MHD density waves moves radially inward.

We now examine the phase relationship between the surface gas mass density perturbation μ and the magnetic field perturbation $\mathbf{b} = (b_r, b_\theta, 0)$. From equations (2.17) and (2.19), we obtain

$$b_\theta = \frac{i}{\omega - m\Omega} \frac{d(B_\theta v_r)}{dr} \cong \frac{irB_\theta}{\omega - m\Omega} \left(-\frac{1}{r^2} + \frac{ik}{r} \right) v_r, \quad (3.21)$$

by keeping the leading order term in equation (3.15). Invoking the same approximation in equation (3.6) and using equations (3.18) and (3.21), we derive

$$\mu \cong \left[\frac{im\kappa^2}{2\Omega kr} + \frac{i(\omega - m\Omega)}{kr\mu_0} \frac{d(r\mu_0)}{dr} - (\omega - m\Omega) \right] \frac{(\omega - m\Omega)\mu_0 b_\theta}{B_\theta} \frac{\kappa^2 + k^2(C_A^2 + C_S^2 - 2\pi G\mu_0/|k|)}{[1 - 1/(ikr)][\kappa^2 + k^2(C_S^2 - 2\pi G\mu_0/|k|)](C_S^2 - 2\pi G\mu_0/|k|)m^2/r^2}. \quad (3.22)$$

If we assume $d[\ln(r\mu_0)]/dr \sim \mathcal{O}(r^{-1})$ and further ignore terms in the order of $\mathcal{O}[(kr)^{-1}]$ in equation (3.22), equation (3.22) reduces to

$$\mu \cong \left[\frac{im\kappa^2(\omega - m\Omega)}{2\Omega kr} - (\omega - m\Omega)^2 \right] \frac{\mu_0 b_\theta}{B_\theta} \frac{\kappa^2 + k^2(C_A^2 + C_S^2 - 2\pi G\mu_0/|k|)}{[\kappa^2 + k^2(C_S^2 - 2\pi G\mu_0/|k|)](C_S^2 - 2\pi G\mu_0/|k|)m^2/r^2}, \quad (3.23)$$

approximately. For neutral slow MHD density waves with $(\omega - m\Omega)/k$ positive or negative, μ will lead or lag b_θ by more than $\pi/2$ in phase correspondingly. Note that in the limit of a slow disk rotation (i.e., small Ω), μ and b_θ would nearly anticorrelate with a phase difference of $\sim \pi$. By retaining the leading order term in the large kr regime, equation (2.19) gives

$$b_\theta \cong \frac{kr}{m} b_r, \quad (3.24)$$

which implies that the direction of local magnetic field perturbation $\mathbf{b} \equiv (b_r, b_\theta, 0)$ is closely, but not exactly, aligned with the magnetic spiral arm.

From the mass conservation (2.22), equation (3.6), and the approximation $d[\ln(r\mu_0)]/dr \sim \mathcal{O}(r^{-1})$, one can derive the relation between v_θ and v_r to the leading order of large kr , namely,

$$v_\theta \cong \frac{[i(\omega - m\Omega)\kappa^2/(2\Omega) - kr(m/r^2)(C_S^2 - 2\pi G\mu_0/|k|)]v_r}{(\omega - m\Omega)^2 - (C_S^2 - 2\pi G\mu_0/|k|)m^2/r^2}. \quad (3.25)$$

For neutral slow MHD density waves with $C_S^2 - 2\pi G\mu_0/|k| > 0$, the denominator of equation (3.25), i.e., $(\omega - m\Omega)^2 - (C_S^2 - 2\pi G\mu_0/|k|)m^2/r^2$, is negative by dispersion relation (3.18). It follows that the phase difference between v_θ and v_r is less than $\pi/2$ and the magnitude ratio $|v_\theta/v_r|$ is $\sim \mathcal{O}(kr/m)$, that is, $|v_\theta|$ is much larger than $|v_r|$.

For small-amplitude neutral slow MHD density waves considered here, one can further estimate the upper limit for the perturbation streaming speed $|v_\theta|$ along the magnetic field line. From equations (2.17) and (3.18), we obtain $|v_r| \sim U|b_r|/|B_\theta|$ where U is less than both C_A and $(C_S^2 - 2\pi G\mu_0/|k|)^{1/2}$. It then follows from equations (3.24), (3.25), and (3.18) that $|v_\theta| \sim U|b_\theta|/|B_\theta| \lesssim U$. Therefore, in the linear regime, gas motions sliding along magnetic field lines associated with slow MHD density waves would not cause streaming instabilities or shocks. However, for slow MHD density waves with sufficiently large amplitudes (e.g., $|b_\theta| \gtrsim |B_\theta|$), both streaming instabilities (when $|v_\theta|$ exceeds C_A) and shocks (when $|v_\theta|$ exceeds C_S) may occur; these are important nonlinear effects to prevent arbitrarily large gas streaming motions along the magnetic field.

3.2. Fast MHD Density Waves in a Rigidly Rotating Gaseous Disk

Complementary to slow MHD density waves, we now proceed to analyze fast MHD density waves in a rigidly rotating disk embedded with an azimuthal magnetic field $B_\theta = F_B/r$. It is expected that the thermal gas pressure and magnetic pressure perturbations are more or less in phase in this case. Similar to analyzing slow MHD density waves, we need to determine leading order terms in equation (3.13) for fast MHD density waves in the regime of large kr . Again, we consider $C_S \sim C_A$ and $\Omega/C_A \sim k$, but take $\omega - m\Omega \sim (C_S^2 + C_A^2)^{1/2}k$ to estimate the magnitudes of all relevant terms in the coefficients of equation (3.13). After straightforward but tedious manipulations, equation (3.13) reduces to

$$\left[1 + \frac{\tilde{\omega}^2 C_S^2}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)}\right] \frac{d^2 v_r}{dr^2} - \frac{2\pi i G\mu_0 \operatorname{sgn}(k) \tilde{\omega}^2}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)} \frac{dv_r}{dr} + \left[\frac{\tilde{\omega}^2}{C_A^2} + \frac{2\Omega \tilde{\omega}}{m C_A^2 \mu_0} \frac{d(r\mu_0)}{dr} - \frac{\tilde{\omega}^2 \kappa^2 + 2\Omega r m^{-1} \tilde{\omega}^3 d[\ln(r\mu_0)]/dr}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)}\right] v_r \cong 0, \quad (3.26)$$

approximately, where $\tilde{\omega} \equiv \omega - m\Omega$, $\Delta \equiv C_S^2 - 2\pi G\mu_0/|k|$ and $\kappa^2 \equiv (2\Omega/r)d(rV_\theta)/dr$. By retaining the leading order terms in equations (3.15) and (3.16) and by ignoring m^2/r^2 terms which are in the next magnitude order, we derive the nontrivial local dispersion relation for fast MHD density waves as

$$(\omega - m\Omega)^2 \cong \kappa^2 + k^2(C_A^2 + C_S^2 - 2\pi G\mu_0/|k|). \quad (3.27)$$

Comparing to the local dispersion relation for hydrodynamic density waves (Safronov 1960; Toomre 1964; Binney & Tremaine 1987), the effective wave propagation speed C_F here in equation (3.27) becomes $C_F \equiv (C_A^2 + C_S^2)^{1/2}$ which is the fast magnetosonic wave speed. Based on dispersion relation (3.27), fast MHD density waves are locally stable for $C_A^2 + C_S^2 - 2\pi G\mu_0/|k| > 0$. Apparently, the presence of the background azimuthal magnetic field enhances the local stability of fast MHD density waves.⁶ The line of neutral stability is thus given by

$$\kappa^2 + k^2(C_A^2 + C_S^2 - 2\pi G\mu_0/|k|) = 0. \quad (3.28)$$

Since this equation is quadratic in $|k|$ and can be readily solved, the local stability for all $|k|$ requires that

$$Q_M \equiv \frac{\kappa(C_A^2 + C_S^2)^{1/2}}{\pi G\mu_0} > 1, \quad (3.29)$$

where Q_M is an MHD generalization of Toomre's Q parameter by including the Alfvén speed C_A . Denoting the radial wavelength by $\lambda \equiv 2\pi/|k|$ and defining a length scale⁷ $L \equiv 4\pi^2 G\mu_0/\kappa^2$ intrinsic to the problem, we derive from equation (3.28) the following expression

$$Q_M^2 = 1 - (2\lambda/L - 1)^2, \quad (3.30)$$

for the line of neutral stability when $Q_M \leq 1$. For $\lambda/L \rightarrow 0$, $Q_M^2 \cong 4\lambda/L$, for $\lambda/L = 0.5$, $Q_M^2 = 1$, and for $\lambda/L \rightarrow 1$, $Q_M^2 \cong (1 - \lambda/L)4\lambda/L$. These results parallel those of hydrodynamic density waves (Safronov 1960; Toomre 1964). For an equivalent plot of expression (3.30), the reader is referred to Figure 6–13 of Binney & Tremaine (1987) for hydrodynamic density waves; the only difference here is that Q_M as defined by equation (3.29) involves the Alfvén speed C_A .

For locally stable fast MHD density waves with $Q_M > 1$, dispersion relation (3.27) gives rise to two radial wavelength branches, namely,

$$\lambda = 2\pi \frac{\pi G\mu_0 \mp \{\pi^2 G^2 \mu_0^2 - (C_A^2 + C_S^2)[\kappa^2 - (\omega - m\Omega)^2]\}^{1/2}}{[\kappa^2 - (\omega - m\Omega)^2]} = \frac{L(1 \mp \{1 - Q_M^2[1 - (\omega - m\Omega)^2/\kappa^2]\}^{1/2})}{2[1 - (\omega - m\Omega)^2/\kappa^2]}, \quad (3.31)$$

where the minus- and plus-sign solutions correspond to short and long wavelength branches, respectively. Naturally, the short-wavelength branch is consistent with the WKB approximation, the starting point of our analysis. While the long-wavelength branch may violate the WKB approximation, it can nonetheless provide heuristic information needed for further numerical investigation (Toomre 1981; Binney & Tremaine 1987; Bertin & Lin 1996).

For a packet of fast MHD density waves with a certain amount of frequency spread, the radial group velocity F_G can be readily derived from dispersion relation (3.27), namely

$$F_G = \frac{\partial \omega}{\partial k} \cong \frac{\operatorname{sgn}(k)[(C_A^2 + C_S^2)|k| - \pi G\mu_0]}{(\omega - m\Omega)}, \quad (3.32)$$

⁶ Nonaxisymmetric global instabilities such as bar instabilities (Miller, Prendergast, & Quirk 1970; Hohl 1971; Ostriker & Peebles 1973; Bardeen 1975; Aoki, Noguchi, & Iye 1979) that occur in gaseous and stellar disks are not expected to be suppressed by the presence of an azimuthal magnetic field because the magnetic energy is much smaller than the kinetic energy of disk rotation.

⁷ Parameter L is the longest unstable wavelength in a disk without thermal gas and magnetic pressures; see dispersion relation (3.27).

with $F_G > 0$ and $F_G < 0$ for radially inward and outward propagations, respectively. For locally stable fast MHD density waves, the sign of F_G depends on the sign of the ratio $k/(\omega - m\Omega)$; for $k/(\omega - m\Omega) > 0$, the wave packet moves radially inward and for $k/(\omega - m\Omega) < 0$, the wave packet moves radially outward. These results parallel those of hydrodynamic density waves (Toomre 1969), except that the sound speed C_S is now replaced by the fast magnetosonic wave speed $C_F = (C_A^2 + C_S^2)^{1/2}$.

In order to determine the phase relationship between μ and $\mathbf{b} = (b_r, b_\theta, 0)$ for fast MHD density waves, we can still use equations (3.21) and (3.24) in the regime of large kr but with $\omega - m\Omega$ given by dispersion relation (3.27). Substituting equations (3.21) and (3.27) into equation (3.6) and retaining leading order terms, we obtain

$$\mu \cong - \frac{\{i(\tilde{\omega}/k)d[\ln(r\mu_0)]/dr + im\kappa^2/(2\Omega kr) - \tilde{\omega}\}\tilde{\omega}\mu_0 b_\theta}{B_\theta[1 - 1/(ikr)][\kappa^2 + k^2(C_A^2 + \Delta) - \Delta m^2/r^2]}, \quad (3.33)$$

where $\tilde{\omega} \equiv \omega - m\Omega$ and $\Delta \equiv C_S^2 - 2\pi G\mu_0/|k|$. By further ignoring next order terms of $\mathcal{O}(kr)^{-1}$ in equation (3.33) and using dispersion relation (3.27), we derive the simple relation

$$\mu \cong \frac{\mu_0 b_\theta}{B_\theta}. \quad (3.34)$$

Therefore, the gas surface mass density perturbation μ is roughly in phase with the magnetic field perturbation $\mathbf{b} = (b_r, b_\theta, 0)$, that is, optical and magnetic spiral arms nearly coincide for fast MHD density waves, the relative mass density fluctuation and the relative parallel magnetic field fluctuation are comparable in magnitude, and the direction of the magnetic field perturbation is approximately aligned with the arm to the leading order of large kr .

The relation (3.25) between v_θ and v_r remains valid for fast MHD density waves. By using dispersion relation (3.27) for fast MHD density waves and the approximation $m/(kr) \ll 1$, relation (3.25) can be further reduced to

$$v_\theta \cong \frac{ik^2 v_r}{(2\Omega)(\omega - m\Omega)}, \quad (3.35)$$

approximately. Apparently, the phase difference between v_θ and v_r is $\sim \pm \pi/2$ and the magnitudes of v_θ and v_r are more or less comparable for neutral fast MHD density waves in the regime of large kr .

Intuitively, fast MHD density waves can be readily perceived, and the nearly in-phase correlation between optical and magnetic spiral arms seen in many spiral galaxies such as the “Whirlpool galaxy” M51 (Mathewson et al. 1972; Tully 1974a, 1974b; Segalovitz et al. 1976; Neininger 1992; Neininger & Horellou 1996) and the Andromeda Nebula (also referred to as M31 and NGC 224; see Beck et al. 1980 and Koper 1993) is a strong evidence for the existence of such waves. Roberts & Yuan (1970) studied nonlinear stationary MHD spiral shock structures in a reference frame corotating with the pattern speed; what they had explored was essentially the shock manifestation of fast MHD density waves, although this terminology was not used. Similarly, the analysis of Tosa (1973), which takes into account of the effect of the magnetic pressure, also dealt with the nonlinear evolution and shock development associated with fast MHD density waves.

3.3. Fast MHD Density Waves in a Gaseous Disk with a Constant Rotation Speed

For the “Whirlpool galaxy” M51 (NGC 5194), the optical and radio spiral patterns extend far beyond the almost rigidly rotating portion of the galaxy (e.g., Burbidge & Burbidge 1964; Carranza, Crillon, & Monnet 1969; Tully 1974a, 1974b; Kormendy & Norman 1979; Mathewson et al. 1972; Neininger 1992; Neininger & Horellou 1996). For angular radius larger than the almost rigid rotation angular radius (~ 0.5), the galactic rotation speed $V_\theta = \Omega r$ remains more or less constant. It is of interest to further examine the local dispersion relation for fast MHD density waves in a magnetized gaseous disk with a roughly constant V_θ .

Again, it is assumed that the background ringlike circular magnetic field B_θ in the azimuthal direction is given by $B_\theta = F_B/r$ away from the central galactic bulge, $C_S \sim C_A$ and $\Omega/C_A \sim k$. For a nearly constant V_θ over an extended radial range, it follows from equations (2.7) and (2.8) for a thin disk galaxy that

$$\phi_T(r, z=0) = -V_\theta^2 \ln r + \text{constant}, \quad (3.36)$$

and

$$\mu_T(r) = \frac{V_\theta^2}{2\pi G r}, \quad (3.37)$$

where the background thermal gas pressure p_0 is ignored as compared to the dominant gravitational force term in equation (2.7) (Mestel 1963; Clutton-Brock 1972; Shu 1982; Binney & Tremaine 1987). For a specific background model of the gaseous disk component, one needs to input a relevant surface gas mass density profile $\mu_0(r)$. For example, one might take $\mu_0(r) \propto \mu_T(r)$ with an approximately constant proportionality coefficient.

Following the same procedure of analysis as carried out in §§ 3.1 and 3.2 for slow and fast MHD density waves in a rigidly rotating disk, we now examine leading order terms in equation (3.13) for the case of a flat rotation curve. By comparing systematically the magnitudes of all relevant terms in the regime of large kr , we derive

$$\left[1 + \frac{\tilde{\omega}^2 C_S^2}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)}\right] \frac{d^2 v_r}{dr^2} - \frac{2\pi i G \mu_0 \operatorname{sgn}(k) \tilde{\omega}^2}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)} \frac{dv_r}{dr} + \left[\frac{\tilde{\omega}^2}{C_A^2} - \frac{\tilde{\omega}^2 \kappa^2}{C_A^2(\tilde{\omega}^2 - m^2 \Delta/r^2)}\right] v_r \cong 0, \quad (3.38)$$

where the notations are the same as before. Taking the leading order terms ikv_r in equation (3.15) and $-k^2v_r$ in equation (3.16) in the WKB approximation and further dropping higher order terms, the local dispersion relation for fast MHD density waves bears the same form of equation (3.27), except that $\kappa^2 \equiv (2\Omega/r)d(rV_\theta)/dr = 2\Omega^2$ here instead of $\kappa^2 = 4\Omega^2$ as in the case of an almost rigidly rotating disk. The analysis and discussion following equation (3.27) can be carried out in the similar manner; in particular, equations (3.28)–(3.34) are valid in the present case of a flat rotation curve. Therefore, fast MHD density waves can manifest over an extended thin gaseous disk with a differential rotation profile given by $\Omega(r) \sim V_\theta/r$ with V_θ being roughly constant. In particular, magnetic spiral arms revealed by synchrotron radio continuum and optical spiral arms should roughly overlap; the time delay of $\sim 10^7$ yr in star formation leads to the observed slight shift between optical and magnetic spiral arms.

If we denote $s \equiv (\omega - m\Omega)/\kappa$, then the corotation resonance occurs when $s = 0$ and the Lindblad resonances occur when $s = \pm 1$. In the present case, the corotation radius r_{CR} is given by $r_{\text{CR}} = mV_\theta/\omega$ from the condition $\omega - m\Omega = 0$, and thus, $s = (r/r_{\text{CR}} - 1)m/2^{1/2}$ and the outer and inner Lindblad resonances occur at $r/r_{\text{CR}} = 1 \pm 2^{1/2}/m$. By solving the local dispersion relation (3.27), we derive

$$\frac{r}{r_{\text{CR}}} = 1 \pm \left(\frac{2}{m^2}\right)^{1/2} \left\{ 1 + \left[\left(\frac{Q_M^2}{2} \frac{|k|}{|k_L|} - 1 \right)^2 - 1 \right] / Q_M^2 \right\}^{1/2}, \quad (3.39)$$

where $k \equiv 2\pi/\lambda$, $k_L \equiv 2\pi/L$, with $L \equiv 4\pi^2 G\mu_0/\kappa^2$ (see footnote 7), and Q_M is defined by equation (3.29). The plus- and minus-sign solutions in equation (3.39) correspond to fast MHD density waves outside and inside the corotation, respectively. This is analogous to hydrodynamic density waves. An example of relation (3.39) with the minus sign and in the absence of magnetic field was shown in Figure 6–14b of Binney & Tremaine (1987) where r/r_{CR} versus k/k_L was displayed with $m = 2$. Although both Q_M and k_L vary with r , the qualitative behavior of fast MHD density waves should be similar. Therefore, one expects that the mechanisms of swing amplification and loop feedback (Goldreich & Lynden-Bell 1965; Mark 1976; Zang 1976; Toomre 1981; Binney & Tremaine 1987; Fan & Lou 1996) should operate to enhance fast MHD density waves in spiral galaxies with differential rotations (Fan & Lou 1997). Similar swing processes in a magnetized gaseous disk with shear can also play important roles in the formation of clouds and superclouds along galactic spiral arms (Elmegreen 1987, 1991; Gammie 1996).

3.4. Slow MHD Density Waves in a Gaseous Disk with a Flat Rotation Curve

Up to this point, it is natural to further consider the possibility of slow MHD density waves in a thin magnetized disk with a nearly constant rotation speed V_θ . As it turns out, the local analysis of equation (3.13) becomes fairly complicated if one maintains, in a consistent manner, that $C_S \sim C_A$, $\Omega/C_A \sim k$, and $\omega - m\Omega \sim \mathcal{O}(mC_A/r)$, and the resulting complex dispersion relation involves $\omega - m\Omega$ to sixth order, with its physical meaning not immediately transparent. The main source of complications in the above derivation results from the fact that many extra terms must be retained in equation (3.13) as a result of nonnegligible $d(\omega - m\Omega)/dr = m\Omega/r$ in relevant terms (cf. eqs. [3.6], [3.8], and [3.9]).

In order to avoid these unnecessary complications and to demonstrate the basic point, we adopt a different approximation by assuming $\Omega/C_A \ll k$ in the following analysis. This assumption is somewhat drastic, since it applies to either the extreme WKB regime with very short radial wavelengths or the case of a relatively low rotation speed. Nevertheless, the outcome is simple, and for our purpose, it is sufficient.

It is straightforward to show that the local dispersion relation for slow MHD density waves under such an approximation is given by

$$(\omega - m\Omega)^2 \cong \frac{C_A^2(C_S^2 - 2\pi G\mu_0/|k|)m^2/r^2}{(C_A^2 + C_S^2 - 2\pi G\mu_0/|k|)}. \quad (3.40)$$

This is very similar to the local dispersion relation (3.18) for slow MHD density waves in the case of a nearly rigid rotation, except that the κ^2 term in the denominator of (3.18) is dropped here. For a packet of slow MHD density wave with a frequency spread, the radial group velocity S_G is given by

$$S_G = \frac{\partial\omega}{\partial k} \cong \frac{\pi G\mu_0 \operatorname{sgn}(k)C_A^4 m^2/r^2}{(\omega - m\Omega)k^2(C_A^2 + C_S^2 - 2\pi G\mu_0/|k|)^2}, \quad (3.41)$$

with $S_G > 0$ and $S_G < 0$ for inward and outward propagations, respectively.

A few comments can be made with regard to dispersion relation (3.40). First, slow MHD density waves are locally stable when $C_S^2 - 2\pi G\mu_0/|k| > 0$, which can be readily satisfied in the regime of very large k . Second, $(\omega - m\Omega)^2$ is less than either $C_A^2 m^2/r^2$ or $(C_S^2 - 2\pi G\mu_0/|k|)m^2/r^2$, and hence the adjective “slow.” Third, the radial extent of slow MHD density waves tends to concentrate within a relatively narrow strip about the corotation radius r_{CR} . Finally, it is therefore not expected to observe large-scale, coherent manifestation of slow MHD density waves in the extended galactic disk portion with a nearly constant rotation speed V_θ . Nevertheless, ringlike transient features of slow MHD density waves may be observable around the corotation radius r_{CR} .

4. APPLICATIONS TO SPIRAL GALAXIES M51 AND NGC 6946

Having discussed extensively mathematical aspects of both fast and slow MHD density waves in the preceding section, we now establish the utility of these theoretical results from the perspective of observations. Before specific galactic applications,

it is important to first clarify the correspondence between large-scale MHD density waves and optical/radio spiral arms in a disk galaxy.

As mentioned earlier, a mature spiral galaxy contains a massive stellar disk which typically shows broad and smooth stellar spiral arms (in red light). Meanwhile, a spiral arm of relatively high stellar mass density creates a gravitational potential trough which tends to concentrate a high-density gaseous arm more or less locked in phase with the stellar spiral arm. Young massive OB stars and giant H II cloud complexes continuously formed out of the high-density gaseous arms then produce luminous narrow optical spiral arms (in blue light). The radiative heating of dust by profuse ultraviolet photons from OB stars in active star-forming regions also leads to the appearance of spiral arms in far-infrared emissions. In the absence of magnetic fields, the problem of gravitational coupling between density waves in stellar and gaseous disks has been studied previously for various purposes (Lin & Shu 1966; Jog & Solomon 1984a, 1984b; Bertin & Romeo 1988; Elmegreen 1995; Jog 1996; Lou & Fan 1997c). With a magnetic field embedded in the gaseous disk (Lou & Fan 1997a), the thermal gas concentration tracks the stellar mass concentration, as in the hydrodynamic case, through the gravitational interaction within the Lindblad resonances. Simultaneously, the large-scale MHD interaction in the magnetized thermal gaseous disk gives rise to the possible MHD density wave modes described in § 3. Therefore, in applications to galactic spiral structures, stellar and gaseous arms are roughly in phase, while enhancements of large-scale parallel magnetic field are either in phase (for fast MHD density waves) or significantly phase shifted (for slow MHD density waves) relative to the enhancement of thermal gas density.

Another closely relevant aspect, as already indicated earlier, is that synchrotron radio emissions come from interactions of cosmic-ray electrons and magnetic field in spiral galaxies. Since both magnetic field and cosmic-ray gas are involved in synchrotron radio emissions, the key question of pertinence here is the relative fluctuation magnitude of cosmic-ray gas density as compared to that of parallel magnetic field in association with MHD density waves in the magnetized thermal gaseous disk. It was pointed out by Parker (1965) that for large-scale low-frequency perturbations in a composite system of thermal gas, magnetic field, and cosmic rays, one can effectively treat cosmic-ray gas as a tenuous, relativistically hot “fluid.” Both thermal gas and cosmic-ray gas are “attached” to the magnetic field in transverse bulk motions, but they are allowed to move with different bulk speeds along the magnetic field. In the example worked out by Parker, the background medium was assumed to be uniform and the self-gravity was absent. For small perturbations, the system supports several wave modes, namely, incompressible Alfvén waves, suprathermal waves (i.e., sound waves in the cosmic-ray gas), slow MHD waves, and fast MHD waves (significantly modified only around the direction perpendicular to the background magnetic field). It was shown that fluctuations in cosmic-ray gas density associated with slow and fast MHD waves are extremely weak as a result of very fast sound speed (close to the speed of light c) in the cosmic-ray gas relative to thermal sound speed and Alfvén speed (both in the order of $10\text{--}20\text{ km s}^{-1}$ for an interstellar medium); the same is true for slow and fast MHD density waves (Lou & Fan 1997b) in the presence of a tenuous cosmic-ray gas. That is, spiral structures in synchrotron radio emissions *would* be mainly determined by enhancements of large-scale parallel magnetic field *if* cosmic-ray (electron) gas is distributed more or less uniformly in the azimuthal direction of a rotating disk.

There is, however, an important caveat to this. Spiral arms with high thermal gas concentrations are sites of more frequent supernovae and star formation activities which are thought to be sources of cosmic rays and random magnetic fields. Cosmic rays will propagate and diffuse to reach all accessible places in a galaxy; it remains an open question whether cosmic-ray gas density is relatively high within spiral gas arms or whether cosmic-ray gas distributes more or less evenly in the azimuthal direction (radial variations are allowed as a result of radial variations in the strength of ringlike circular magnetic fields). Furthermore, star formation activities may tend to disrupt large-scale organized galactic magnetic field to some extent and may also generate intense small-scale magnetic fields through dynamo processes. As a result, unpolarized synchrotron radio emissions, present almost everywhere in the disk of a spiral galaxy, are expected to be particularly strong along spiral gas arms. Therefore, one should be mindful to distinguish the total and polarized synchrotron radio emissions from spiral galaxies.

For applications to spiral galaxies, one should be clear about possible large-scale galactic magnetic field configurations. In particular, magnetic field directions inferred from radio polarization vectors usually appear to roughly align with spiral arms. Since galactic spiral arms rarely appear as tight as perfect azimuthal rings, it would seem paradoxical if globally *connected* magnetic field lines align with somewhat open spiral arms because the disk differential rotation would distort magnetic field lines in a few turns of galactic rotation. One way of avoiding this dilemma is to invoke a ringlike magnetic field configuration in the background disk (see Roberts & Yuan 1970; Tosa 1973). In the MHD density wave scenario, it is the collective behavior of phase-organized, shape-distorted rings of magnetic field lines that gives rise to globally coherent, somewhat open magnetic spiral structures. This situation is very much like the case in which the collective behavior of phase-organized, shape-distorted rings of gas streamlines gives rise to globally coherent, somewhat open velocity spiral structures [see, e.g., Visser 1980 for a theoretical model of the spiral galaxy M81 (NGC 3031)]. Another similar situation is the collective behavior of phase-organized shape-distorted stellar orbits (see Figs. 3 and 4 of Kalnajs 1973; or see Fig. 6–11 of Binney & Tremaine 1987) that gives rise to global spiral arms associated with density waves in a thin stellar disk. In our MHD density wave scenario, the perturbed magnetic field is locally (i.e., a segment of a distorted ring of magnetic field line) aligned with the orientation of a spiral arm, but magnetic field segments along a spiral arm are not actually connected throughout. This situation can be visualized by regarding the closed distorted gas streamlines of Visser (1980) as magnetic field lines. In this manner, magnetic and optical spiral structures can be roughly aligned or parallel in orientation, and the winding problem does not actually arise.

For fast MHD density waves, distorted gas streamlines, magnetic field lines, and gas mass density perturbations are orchestrated as such that the enhancements of gas density and magnetic field strength are roughly in phase. Whereas for slow MHD density waves, distorted gas streamlines, magnetic field lines, and gas mass density perturbations are organized such that the enhancements of gas density and magnetic field strength are significantly phase shifted (i.e., a phase difference $\gtrsim \pi/2$).

In order to get oneself used to perturbation structures of slow MHD density waves, we recall that gas materials can slide along magnetic field lines and azimuthal variations are necessary in addition to radial variations. It is then possible for gas density enhancements in spatial regions with rarefied magnetic field lines. In contrast, fast MHD density waves can appear even in the absence of azimuthal variations. The fact that polarized radio continuum observations tend to give the impression that magnetic field lines appear “connected” and aligned along the orientation of spiral arms may result from several effects. First, in observing a nearly face-on spiral galaxy, the plane of sky is discretized and the magnetic field direction is inferred for each local patch. Second, by ignoring Faraday rotation (this is a better approximation for shorter radio wavelengths), magnetic field orientation is inferred from the polarized vector by turning $\sim 90^\circ$. What is sometimes plotted at each pixel is a bar with its length proportional to the degree of polarization. Third, nonlinear MHD processes are expected to have happened in a spiral galaxy, that is, magnetic field lines distorted at various places may not be proportional to a linear scale globally. Finally, uncertainties in determining the amount of Faraday rotation and various peculiarities always exist. All of these factors together make it difficult to reconstruct actual magnetic field lines from a discretized polarized radio continuum map. Such a situation notwithstanding, one sure thing is that our scenario can only account for axisymmetric spiral structures (ASS). The possibility of bisymmetric spiral structures (BSS) as indicated by some observations (Sofue et al. 1986; Beck et al. 1996; Berkhuijsen et al. 1996) remains as an open issue at this point.

We now apply the theoretical results obtained for fast and slow MHD density waves in a thin rotating magnetized gaseous disk to the optical and magnetic spiral structures of two galaxies, namely the “Whirlpool galaxy” M51 and the galaxy NGC 6946, that prompted our investigation in the first place. From the perspective of a coherent theory for galactic MHD density waves, we propose that the spiral structure of M51 generally fits into the scenario of fast MHD density waves, while the spiral structure of NGC 6946 fits into the scenario of slow MHD density waves. More elaborations on these two cases are given below separately.

4.1. Fast MHD Density Waves in the Spiral Galaxy M51

The “Whirlpool galaxy” M51 is a well-known double galaxy consisting of NGC 5194, a magnificent spiral galaxy, and NGC 5195, a smaller companion galaxy conspicuously located near the tip of one of the two main spiral arms of NGC 5194. The apparent closeness of the two galaxies, their similar radial velocities, the extensive luminous material spread around the companion (van den Bergh 1969; Schweizer 1977), and the narrow extended connection seen in nonthermal radio emissions (Mathewson et al. 1972) provide strong observational evidence that the two galaxies are tidally interacting. It is possible that the spiral structure in the outer regions of NGC 5194 results from this tidal interaction (Toomre & Toomre 1972; Tully 1974b), although N -body simulation results (e.g., Hernquist 1990) indicate tidal interactions from the companion NGC 5195 alone cannot explain interior spiral structures of NGC 5194. There is also infrared data evidence (Rix & Rieke 1993) implying the presence of interference of a preexisting spiral pattern with the tidally induced spiral arms in NGC 5194. Optical observations (Elmegreen et al. 1989) suggest two types spiral structures in M51: density wave structures in the inner part and tidal interaction structures in the outer part. Recent synchrotron radio observations (Berkhuijsen et al. 1997) are consistent with this picture. There is a further suggestion based on simulations and dynamical analysis that instead of a transient passing, NGC 5195 may orbit around NGC 5194 as a bound system very much like the peculiar system Arp 86 (NGC 7753/7752) (Salo & Laurikainen 1993; Lin & Bertin 1995; Lin 1996). These proposals for density wave excitation mechanism in M51 aside, we are mainly interested in the physical nature of the correlation between the optical and synchrotron radio spiral structures seen in NGC 5194.

According to Table 2 and Figure 5 (Pl. 10) of Kormendy & Norman (1979), NGC 5194 has a typical galactic rotation curve with the angular radius of the optical spiral pattern being ~ 4.4 , the angular radius of its nearly rigid rotating disk portion being only ~ 0.5 , and its maximum angular radius being ~ 5.0 . The optical spirals of NGC 5194 appears mostly in the differentially rotating portion of its galactic disk. Since the radio spiral pattern with an enhanced polarization intensity nearly coincides with the optical spiral pattern in NGC 5194 (Mathewson et al. 1972; Tully 1974a, 1974b; Neininger 1992; Neininger & Horellou 1996), it is natural to interpret this phenomenon in terms of fast MHD density waves studied in §§ 3.2 and 3.3. The fact that both total and polarized radio intensities are strong within spiral arms indicates that star formation activities do not seem to disrupt completely large-scale galactic magnetic field and that whether cosmic-ray electrons are more abundant or not within spiral gas arms, enhancements of organized and random magnetic fields must be at least partially responsible for the increase of polarized and unpolarized radio intensities. We emphasize that spiral patterns associated with fast MHD density waves can manifest in both almost rigidly and differentially rotating portions of a gaseous disk, and the presence of magnetic field tends to enhance the local stability of hydrodynamic density waves (see eq. [3.27]). Given other conditions the same in a typical gaseous spiral galaxy, the inclusion of an azimuthal magnetic field tends to increase the radial group velocity of a tightly wound free spiral MHD density wave (see eq. [3.32]) and, thus, fresh waves must be continually generated to maintain the spiral pattern (Toomre 1969; Kalnajs 1972). In the case of NGC 5194, these waves might be partially excited and sustained by the external gravitational potential due to the companion NGC 5195 or these waves may be self-maintained consistent with the basic state (Bertin et al. 1989a, 1989b; Lin 1996). In the scenario of fast MHD density waves, various small-scale activities in high-density spiral arms tend to locally reduce the coherence of large-scale magnetic field spiral pattern. In particular, the tidal interaction from the companion NGC 5195 might be responsible for spiral arm distortions, for example, the intersection of one dust lane and thus magnetic field arm across the eastern optical spiral arm (Mathewson et al. 1972; Neininger & Horellou 1996; Beck 1996). Nevertheless, the overall spiral pattern of enhanced galactic magnetic field (plus cosmic-ray electrons) and a certain degree of polarized radio emissions persist (Mathewson et al. 1972; Segalovitz et al. 1976; Sofue et al. 1986; Neininger & Horellou 1996; Berkhuijsen et al. 1997).

In fact, the earlier MHD model analyses of Roberts & Yuan (1970) and of Tosa (1973) were essentially on nonlinear shock manifestations of fast MHD galactic density waves in a reference frame corotating with the constant pattern speed $\omega_p \equiv \omega/m$.

Our local linear analysis here complements their nonlinear analyses, reveals the similarity between fast MHD density waves and hydrodynamic density waves, and naturally raises the possibilities of slow MHD density waves as well as large-scale slow MHD shocks in spiral galaxies.

At present, we are unable to account for the bisymmetric spiral structure or mixed spiral structure (Beck 1996; Beck et al. 1996) in NGC 5194 because we have assumed in our model a ringlike background magnetic field in the first place. If one allows a mean galactic magnetic field to be dominantly azimuthal sufficiently far away from the center and to gradually meander through the galactic center, then a global spiral pattern of fast MHD density waves could appear bisymmetric. This is not a simple task and there are more hurdles to overcome in order to explicitly construct such a model. Before ending this subsection, we note in passing that the overall coincident ring structures of M31 in many electromagnetic wavelengths (Beck et al. 1980; Koper 1993 and references therein) can be best explained in terms of fast MHD density waves either in the tight-winding regime or in the axisymmetric approximation. The basic reason for such an identification is that enhancements of ring magnetic field, gas concentration, and all traces of Population I stars are largely in phase. This situation is expected even in the nonlinear regime.

4.2. Slow MHD Density Waves in the Nearby Spiral Galaxy NGC 6946

Barring certain irregularities and fuzziness in some regions, NGC 6946 is a multi-armed (e.g., $m = 4$) spiral galaxy with the angular radius of its optical spiral pattern being ~ 3.3 , the angular radius of its almost rigidly rotating disk portion⁸ being ~ 4.0 and its maximum angular radius shown by neutral hydrogen H I emission being ~ 11.0 (see, e.g., Table 3 and Fig. 3 [Pl. 8] of Kormendy & Norman 1979). In a near-infrared atlas of spiral galaxies (Elmegreen 1981), NGC 6946 was subsumed into the “grand-design” category as opposed to the flocculent category, although this classification is somewhat subjective. NGC 6946 has no apparent companion and may (Ball et al. 1985; Zaritsky & Lo 1986; Ishizuki et al. 1990) or may not (Regan & Vogel 1995) possess a weak central molecular bar. Its density waves are only of moderate strength and roughly occupy the disk portion with a nearly rigid rotation (Rogstad et al. 1973; Kormendy & Norman 1979). Radio continuum observations in the past decade (Klein et al. 1982; Harnett, Beck, & Buczylowski 1989; Beck 1991; Ehle & Beck 1993) revealed an overall spiral pattern of the interstellar magnetic field in NGC 6946, but the resolution was insufficient to specifically relate optical and magnetic spiral structures.

Recent high-resolution observations of polarized radio emissions at 6.2 cm wavelength from NGC 6946 revealed surprisingly the existence of coherent magnetic spiral arms lying between optical spiral arms (see Fig. 2 of Beck & Hoernes 1996; Beck et al. 1996). The two main magnetic spiral arms, containing aligned magnetic fields of strength $\sim 3\text{--}13\ \mu\text{G}$, are parallel to the optical spiral arms which are also seen in the total radio emission. The peak position of the magnetic arm appears, however, not exactly in the middle between the total-power arms, but shifts toward the outer arm (see Fig. 3 of Beck & Hoernes 1996). On the whole, since the global spiral pattern seen in polarized radio emissions corresponds so well with that of optical/total-power emissions, it is clear that large-scale magnetic fields participate in the global galactic dynamics in a nontrivial manner. As indicated earlier, it is not surprising that unpolarized radio emission should be strong within spiral gas arms where star formation activities are also strong; the fact that polarized radio intensities are weak within gaseous arms and are strong in interarm regions calls for a new global mechanism. We propose the presence of slow MHD density waves in NGC 6946 as a plausible interpretation (Fan & Lou 1996).

In order to apply the theoretical results of slow MHD density waves, we first estimate the relevant physical parameters of NGC 6946. The background gas surface mass density is taken to be $\mu_0 \sim 5 \times 10^{-3}\ \text{g cm}^{-2}$; the background azimuthal magnetic field $B_\theta \sim 1.3 \times 10^{-5}\ \text{G}$; the sound speed $C_s \sim 2 \times 10^6\ \text{cm s}^{-1}$; the radial wavenumber $k \sim 10m/r_{\text{RG}}$, where $m = 4$ and r_{RG} is the maximum rigid rotation radius taken to be $\sim 8\ \text{kpc}$ (this implies that NGC 6946 is about 7 or 8 Mpc away); the disk rotation speed at r_{RG} is taken to be $\Omega r_{\text{RG}} \sim 2 \times 10^7\ \text{cm s}^{-1}$, and the thickness D_z of the gaseous disk is taken to be $\sim 500\ \text{pc}$. Given these estimates, the Alfvén wave speed is $C_A \sim 2 \times 10^6\ \text{cm s}^{-1}$; the almost rigidly rotating disk portion of NGC 6946 completes one turn in $2\pi/\Omega \sim 2.5 \times 10^8\ \text{yr}$; and the radial group speed for a packet of slow MHD density waves at r_{RG} is $S_G \sim 4.3 \times 10^4\ \text{cm s}^{-1}$ (see eqs. [3.18] and [3.20]). For comparison, we note that the radial group speed for a packet of fast MHD density waves at r_{RG} would be $F_G \sim 2.7 \times 10^6\ \text{cm s}^{-1}$ for the same set of parameters (see eqs. [3.27] and [3.32]).

As noted earlier, for a manifestation of slow MHD density waves in an extended radial range of a gaseous disk, dispersion relation (3.18) requires an almost rigid disk rotation. We now discuss how well the available rotation curve data of NGC 6946 (Carignan et al. 1990) fits into this scenario, especially for the inner disk portion within $\sim 4'$. If one simply adopts a straight line passing through the galaxy's center to fit the data (in the least-square sense) within $r \sim 4'$ shown in Figures 4 and 6b of Carignan et al. (1990), the velocity deviation away from a rigid rotation can be as large as $\sim 30\text{--}40\ \text{km s}^{-1}$. This magnitude of velocity deviation would exceed the tolerance implied by dispersion relation (3.18), unless the sound speed C_s and the Alfvén speed C_A are several times higher than the oft-quoted value of $\sim 10\ \text{km s}^{-1}$ or the equivalent gas turbulence velocity is as high as $\sim 30\text{--}40\ \text{km s}^{-1}$. We note that C_s may not be a constant throughout the disk. In the case of our Galaxy, C_s tends to increase toward the disk center. On the observational side, Table 3 of Carignan et al. (1990) indicates that, independent of systematic uncertainties on inclination angle i and position angle P.A. of the galaxy NGC 6946, errors in determining rotation velocity alone in the inner disk can be as large as $\sim \pm 10\text{--}20\ \text{km s}^{-1}$. From their separate estimate for uncertainties of inclination angle i and position angle P.A. (see Table 2 and Fig. 6 of Carignan et al. 1990), the resulting systematic error in velocity determination can be as large as $\sim 35\ \text{km s}^{-1}$. All these uncertainties taken together, dispersion relation (3.18) for slow MHD density waves may be marginally satisfied within the angular radius of $\sim 4'$ in NGC 6946. More accurate

⁸ The disk rotation speed V_θ first increases and then slowly bends toward the horizontal with increasing radius; we refer to this situation as *almost* or *nearly rigid rotation*.

determinations of the rotation curve and effective wave propagation speeds in NGC 6946 are needed to further test of our scenario.

It is also of some interest to note that noncircular gas motions in NGC 6946 are seen by several observations (Rogstad et al. 1973; Tacconi & Young 1986; Carignan et al. 1990), and most of the velocity deviations coincide with the optical spiral arms. These velocity deviations are of the order of $\sim 10 \text{ km s}^{-1}$ and $\sim 20\text{--}30 \text{ km s}^{-1}$ in the inner and outer parts NGC 6946, respectively.

In brief summary, our proposal that a global pattern of slow MHD density waves (see § 3.1) manifests in NGC 6946 follows several clues. First, according to our analyses in §§ 3.1 and 3.4, slow MHD density waves can manifest globally only within the almost rigidly rotating portion of a gaseous disk, and the optical/magnetic spiral patterns do appear in NGC 6946 more or less within its angular radius of almost rigid rotation. Second, magnetic and density spiral arms should be largely parallel to each other with a radial phase shift more than $\pi/2$ (see eqs. [3.23] and [3.24]) for slow MHD density waves. This appears roughly consistent with the observations of spiral arms of NGC 6946 seen in polarized radio emissions and optical bands. Third, in order to account for the fact that the magnetic arm shifts toward the outer optical/total-power arm, we assume a trailing spiral (i.e., $k < 0$) in NGC 6946 following the conventional wisdom and infer the angular pattern speed $\omega_p \equiv \omega/m < \Omega$ according to equations (3.18) and (3.23) for locally stable slow MHD density waves. This choice of signs for k and $\omega - m\Omega$ assures that the peak of density perturbation radially leads the peak of magnetic field perturbation by more than $\pi/2$ in phase.⁹ Fourth, a packet of slow MHD density waves propagates radially inward (see eq. [3.20] with $k < 0$ and $\omega - m\Omega < 0$) with a group speed S_G ($\sim 4.3 \times 10^4 \text{ cm s}^{-1}$) much slower than the azimuthal rotation speed at r_{RG} . A global pattern resulting from a packet of slow MHD density waves could survive many turns of disk rotation in NGC 6946 once excited. Finally, our recent analysis (Fan & Lou 1997) indicates that slow MHD density waves are preferentially swing amplified in a disk with an almost rigid rotation.

5. DISCUSSION

Except for the inclusion of a magnetic field, our formulation of fast and slow MHD density waves in a thin rotating gaseous disk with self-gravity is very similar to that of hydrodynamic density waves (Lin & Shu 1964; 1966; Binney & Tremaine 1987; Bertin & Lin 1996). Besides the specific properties pertaining to the local stability criteria of fast and slow MHD density waves in the tight-winding approximation, the important question of global stability for such a magnetized rotating disk remains. In the absence of magnetic field, it is well known that a rotating gaseous or stellar disk under self-gravity can be disrupted by global barlike instabilities (Miller et al. 1970; Hohl 1971; Ostriker & Peebles 1973; Bardeen 1975; Aoki et al. 1979; Toomre 1981; Binney & Tremaine 1987). These global instabilities may be stabilized by increasing the stellar velocity dispersion and the sound speed in the disk or in the central bulge (Vandervoort 1970) or by introducing a sufficiently massive halo of dark matter surrounding the thin rotating disk (Ostriker, Peebles, & Yahil 1974). Since the galactic magnetic energy is typically comparable to the gas thermal energy and is considerably smaller than the kinetic energy of disk rotation, the presence of galactic magnetic field is not expected to fundamentally alter the situation of global instabilities although the effective increase of fast MHD wave speed may have a stabilizing tendency in very young spiral galaxies.¹⁰ Therefore, the apparent stability of many magnetized spiral galaxies may still require, among other mechanisms, the presence of a massive dark matter halo.

It should be noted that in the past decade or so, reliable solutions of hydrodynamic density waves have been obtained numerically (Pannatoni 1983; Bertin et al. 1989a, 1989b; Bertin & Lin 1996) by solving the relevant linear integro-differential equations (Lin & Lau 1979) without the restriction of the WKB approximation. Here the integral equation involves the formal integral solution (Shu 1970a; Lin & Lau 1979) relating surface mass density and gravitational potential perturbations from Poisson's equation (2.15). Such a modal approach has revealed that the global stability of hydrodynamic density waves depends on several parameters characterizing the background basic state (Bertin et al. 1989a, 1989b). Furthermore, by including the long-range gravitational effect, open or bar modes seen in various galaxies can be modeled in the modal formulation. In light of this important theoretical development, it is apparent that the modal formulation can also be generalized to include magnetic field effects (see eqs. [2.15]–[2.22]). Among others, it is then of considerable interest to study magnetic field configurations associated with open or bar modes of galaxies.

Another important aspect of the present problem is the development of Parker instability (or magnetic Rayleigh-Taylor instability; see Parker 1966; 1971a, 1971b, 1979) perpendicular to the disk if one takes into account finite thickness of a spiral galaxy and allows vertical motions. Such instability inevitably leads to vertical undulations along azimuthal magnetic field lines with upward portions of magnetic field lines inflated with cosmic rays. Eventually, magnetic field lines remain anchored to the galactic disk by heavier gas clouds accumulated around valleys of magnetic field lines (see, e.g., the recent review of Mouschovias 1996, and references therein). In the present context, it is important to understand the dynamical interplay of shear, swing amplification and Parker instability (e.g., Shu 1974; Elmegreen 1987, 1991; Fan & Lou 1997). When observing a nearly face-on spiral galaxy, the effect of magnetic field undulation may be averaged out. However, in order to compute quantitatively the contrast of large-scale synchrotron continuum emissions, one should keep in mind complications caused by the presence of Parker instability (Parker 1992; Mouschovias 1996).

For fast MHD density waves, the presence of magnetic field effectively increases wave propagation speed, that is, C_S^2 is now replaced by $C_S^2 + C_A^2$. There is no question that fast MHD density waves should be relevant to large-scale spiral structures in disk galaxies. As to slow MHD density waves, the presence of the background azimuthal magnetic field leads to a local

⁹ In principle, a leading spiral with $k > 0$ and $\omega - m\Omega > 0$ can give rise to a same radial phase relation between optical and magnetic spiral arms for slow MHD density waves described by eqs. (3.18) and (3.23).

¹⁰ This may be so, for example, in magnetized, extremely gas-rich, young, bluer spiral galaxies, detected by the *Hubble Space Telescope* Deep Field, in the early universe (e.g., Fukugita, Hogan, & Peebles 1996).

stability criterion $C_s^2 > 2\pi G\mu_0/|k|$ (see eq. [3.18]) as if the disk rotation and magnetic field were absent (Chandrasekhar & Fermi 1953; Lynden-Bell 1966; Lou 1996). For lower sound speed C_s , higher surface gas mass density μ_0 , and longer wavelengths, slow MHD density waves become unstable with anisotropic growth rates (Lynden-Bell 1966). By this criterion, the radial wavelength λ should be less than $C_s^2/(G\mu_0)$ for neutral slow MHD density waves. For a gas surface mass density $\mu_0 \lesssim 5 \times 10^{-3} \text{ g cm}^{-2}$, $\lambda < 10 \text{ kpc}$ for $C_s \sim 30 \text{ km s}^{-1}$ and $\lambda < 1 \text{ kpc}$ for $C_s \sim 10 \text{ km s}^{-1}$. Since the radial wavelength of spiral MHD density waves should be in the order of $\sim 1 \text{ kpc}$, the scale for slow MHD density waves estimated here should be relevant to large-scale spiral structures. It is worthwhile to point out that our theoretical results are certainly adaptable to smaller scale phenomena in magnetized gas medium with relatively high mass density and low sound speed (e.g., in a magnetized accretion disk). For nonlinear development, the fate of locally unstable slow MHD density waves and the coupling between fast and slow MHD density waves in the presence of a strong differential rotation (Fan & Lou 1997) remain to be investigated.

Fast and slow MHD density waves can be clearly distinguished for MHD perturbations with sufficiently small amplitudes. In reality, the spiral structure associated with fast MHD density waves in M51 and the spiral structure associated with slow MHD density waves in NGC 6946 may well carry nonlinear signatures (for fast MHD shocks, see Roberts & Yuan 1970 and Tosa 1973). On the theoretical side, many questions can be raised regarding the development of fast and slow spiral MHD shocks in the presence of finite-amplitude MHD density waves as well as the specific properties of these spiral MHD shocks. While much remains to be learned, several generic properties are expected. For example, the gas density, the thermal pressure and the total pressure (i.e., the sum of thermal and magnetic pressures) all increase across an MHD shock. The magnetic field strength and the angle between the shock normal and the magnetic field decrease across a slow MHD shock and increase across a fast MHD shock. The entropy must increase across an MHD shock, and, in the comoving frame of the shock, a conversion of kinetic energy to thermal energy takes place across the shock. With these in mind, it might be possible to identify galactic fast and slow spiral MHD shocks or to distinguish one from another.

Beck & Hoernes (1996) pointed out a certain similarity among several nearby spiral galaxies NGC 6946, IC 342 (van der Kruit 1973; Gräve & Beck 1988; Sofue et al. 1985; Krause et al. 1989; Krause 1993) and M83 (Ondrechen 1985; Sukumar & Allen 1989; Neininger et al. 1993), all of which are late-type gas-rich spiral galaxies without companions and have shown signatures of magnetic arms in polarized radio continuum. It is noted here that all these spiral galaxies have extended disk portions with differential rotations seen in neutral hydrogen H I emissions, but their radii of optical spiral patterns are all comparable to the maximum radii of almost rigidly rotating disk portions (see Table 1 for M83 and Table 3 for IC 342 in Kormendy & Norman 1979). Judging the radio observation history of NGC 6946 with successive improvement of angular resolution and signal-to-noise ratio, polarized radio emissions from IC 342 and M83 may reveal more candidates carrying signatures of global spiral patterns of slow MHD density waves.

Finally, in order to address the question of how MHD density waves are excited, Fan & Lou (1997) studied the local swing amplification process of fast and slow MHD density waves following the basic formalism of Goldreich & Lynden-Bell (1965; see also Goldreich & Tremaine 1978, 1979; Toomre 1981; Elmegreen 1987). We found that, after the swing amplification, fast MHD density waves manifest more readily in a disk with strong differential rotation, whereas slow MHD density waves manifest more easily in an almost rigidly rotating disk via transient Jeans instability. These theoretical results lend further support to our interpretations for optical and polarized radio observations of spiral structures in M51 and NGC 6946.

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APPENDIX A

Consider solutions of Poisson's equation (2.15) which relates surface mass density and gravitational potential perturbations for $z \neq 0$. Since both ϕ and μ contain the explicit $\exp(i\omega t - im\theta)$ dependence with $m \geq 0$ being an integer, we have immediately

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) - \frac{m^2 \phi}{r^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (\text{A1})$$

outside the thin disk at $z = 0$. The general solution of equation (A1) appears in the form of

$$\phi(r, z) = \sum_k [C_k H_m^{(1)}(kr) + D_k H_m^{(2)}(kr)] \exp(-k|z|), \quad (\text{A2})$$

where $H_m^{(1)}(x)$ and $H_m^{(2)}(x)$ are the first and second kind Hankel functions of order m with argument x , respectively, C_k and D_k represent two sets of complex coefficients, respectively, and $k > 0$. If one requires the regularity condition at $r = 0$, it is then more convenient to use the representation of Bessel functions in equation (A2) (see Shu 1968). On the other hand, if one allows a central bulge to avoid the singularity at $r = 0$, then the present representation by Hankel functions is more general. By integrating Poisson's equation (2.15) along the z direction, the jump condition for $\partial\phi/\partial z$ across $z = 0$ is simply

$$\left[\frac{\partial \phi}{\partial z} \right] = -4\pi G\mu(r), \quad (\text{A3})$$

where the brackets on the left-hand side denote the difference of $\partial\phi/\partial z$ across $z = 0$. It follows immediately that

$$\mu(r) = (2\pi G)^{-1} \sum_k k [C_k H_m^{(1)}(kr) + D_k H_m^{(2)}(kr)] . \quad (\text{A4})$$

Taking the radial and azimuthal derivatives of ϕ at $z = 0$, respectively, one derives from equation (A2)

$$\left. \frac{d\phi}{dr} \right|_{z=0} = \sum_k k [C_k H_m^{(1)}(kr) + D_k H_m^{(2)}(kr)] , \quad (\text{A5})$$

$$\left. \frac{1}{r} \frac{d\phi}{d\theta} \right|_{z=0} = -\frac{im}{r} \sum_k [C_k H_m^{(1)}(kr) + D_k H_m^{(2)}(kr)] , \quad (\text{A6})$$

where the prime denotes the derivative with respect to the argument. Note that both $H_m^{(1)}(kr)$ and $H_m^{(2)}(kr)$ diverge in the limit of $kr \rightarrow 0$. Since we are mainly interested in MHD density waves in the thin disk portion outside the central galactic bulge, this divergence causes no essential difficulty.

In the limit of very large kr , we have the leading asymptotic expressions for the Hankel functions, namely,

$$H_m^{(1)}(kr) \sim \left(\frac{2}{\pi kr}\right)^{1/2} \exp \left[i \left(kr - \frac{m\pi}{2} - \frac{\pi}{4} \right) \right] , \quad (\text{A7})$$

$$H_m^{(2)}(kr) \sim \left(\frac{2}{\pi kr}\right)^{1/2} \exp \left[-i \left(kr - \frac{m\pi}{2} - \frac{\pi}{4} \right) \right] . \quad (\text{A8})$$

By assuming $\exp(i\omega t)$ time dependence for all the perturbation variables, $H_m^{(1)}(kr)$ and $H_m^{(2)}(kr)$ correspond to radially inward and outward propagations of MHD density waves, respectively. It follows approximately that $H_m^{(1)}(kr) \sim iH_m^{(1)}(kr)$ and $H_m^{(2)}(kr) \sim -iH_m^{(2)}(kr)$ for large kr .

Therefore, for radially inward propagation of MHD density waves with $D_k = 0$ for all $k > 0$ and for radially outward propagation of MHD density waves with $C_k = 0$ for all $k > 0$, we have approximately

$$\left. \frac{d\phi}{dr} \right|_{z=0} \cong 2\pi G i \epsilon \mu + \mathcal{O} \left[\frac{\mu(r)}{kr} \right] , \quad (\text{A9})$$

in the regime of large kr , respectively, where $\epsilon = +1$ for $D_k = 0$ and $\epsilon = -1$ for $C_k = 0$. For both cases,

$$\left. \frac{1}{r} \frac{d\phi}{d\theta} \right|_{z=0} \sim -i \mathcal{O} \left[\frac{2\pi G m \mu(r)}{kr} \right] , \quad (\text{A10})$$

for very large kr . We thus arrive at the same result (11) of Lin & Shu (1964) using an alternative solution procedure. For comparison, the reader is referred to the detailed asymptotic analysis contained in the Appendix of Lin & Shu (1964; see also Binney & Tremaine 1987).

Similarly, by setting either $D_k = 0$ or $C_k = 0$ in equations (A2) and (A4) and using equations (A7) and (A8), one can readily demonstrate further that in the regime of large kr

$$\left. \frac{1}{r^{1/2}} \frac{d(r^{1/2}\phi)}{dr} \right|_{z=0} \cong 2\pi G i \epsilon \mu + \mathcal{O} \left[\frac{\mu}{(kr)^2} \right] , \quad (\text{A11})$$

which was first derived by Shu (1970b) using a different approach.

APPENDIX B

In the absence of gas pressure and magnetic field, hydrodynamic perturbation equations (2.12)–(2.15) were originally proposed by Lin & Shu (1964) to describe galactic density waves. Using the tight-winding or WKB approximation (A9) and (A10), a first-order differential equation can be derived, namely,

$$\frac{i}{r} \left[\frac{2\pi G \epsilon r \mu_0 (\omega - m\Omega)}{\kappa^2 - (\omega - m\Omega)^2} \right] \frac{d\mu}{dr} + \left\{ (\omega - m\Omega) + \frac{i}{r} \frac{d}{dr} \left[\frac{2\pi G \epsilon r \mu_0 (\omega - m\Omega)}{\kappa^2 - (\omega - m\Omega)^2} \right] + \frac{im\mu_0 \kappa^2}{2\Omega r} \frac{2\pi G \epsilon}{[\kappa^2 - (\omega - m\Omega)^2]} \right\} \mu \cong 0 , \quad (\text{B1})$$

where the epicyclic frequency κ is defined by

$$\kappa^2 \equiv \frac{2\Omega}{r} \frac{d(rV_\theta)}{dr} . \quad (\text{B2})$$

The explicit solution for surface density perturbation μ of equation (B1) is

$$\mu = \frac{\kappa^2 - (\omega - m\Omega)^2}{2\pi G \epsilon r \mu_0 (\omega - m\Omega)} \exp \left[- \int \frac{m\kappa^2 dr}{2\Omega(\omega - m\Omega)r} + i \int \frac{\kappa^2 - (\omega - m\Omega)^2}{2\pi G \epsilon \mu_0} dr \right] . \quad (\text{B3})$$

Correspondingly, the radial velocity perturbation v_r is

$$v_r = -\frac{2\pi G \epsilon \mu (\omega - m\Omega)}{\kappa^2 - (\omega - m\Omega)^2} = -\frac{1}{r\mu_0} \exp \left[- \int \frac{m\kappa^2 dr}{2\Omega(\omega - m\Omega)r} + i \int \frac{\kappa^2 - (\omega - m\Omega)^2}{2\pi G \epsilon \mu_0} dr \right] , \quad (\text{B4})$$

and the azimuthal velocity perturbation v_θ is

$$v_\theta = -\frac{ik^2}{2\Omega} \frac{2\pi G\epsilon\mu}{[\kappa^2 - (\omega - m\Omega)^2]} = -\frac{ik^2}{2\Omega r\mu_0(\omega - m\Omega)} \exp\left[-\int \frac{m\kappa^2 dr}{2\Omega(\omega - m\Omega)r} + i \int \frac{\kappa^2 - (\omega - m\Omega)^2}{2\pi G\epsilon\mu_0} dr\right]. \quad (B5)$$

For neutral galactic density waves with an almost real ω , the phase factor in equations (B3)–(B5) is precisely that obtained by equation (12) of Lin & Shu (1964).

Although near the Lindblad resonances

$$\kappa^2 - (\omega - m\Omega)^2 = 0, \quad (B6)$$

the whole analysis under the tight-winding approximation leading to the present results breaks down, perturbation variables as formally expressed by equations (B3)–(B5) do not diverge when (B6) is satisfied. Therefore, perturbation behaviors around the Lindblad resonances must be carefully treated (Goldreich & Tremaine 1979). In contrast, around the corotation singularity with

$$\omega - m\Omega = 0, \quad (B7)$$

perturbation variables do diverge as indicated by solutions (B3)–(B5).

APPENDIX C

In this appendix, we recount basic properties of adiabatic compressible MHD waves in a uniformly magnetized medium without gravity. Besides the transverse incompressible Alfvén waves, the dispersion relation for compressible MHD perturbations in such a system is simply

$$\omega^4 - (C_S^2 + C_A^2)(k_\perp^2 + k_z^2)\omega^2 + k_z^2(k_\perp^2 + k_z^2)C_A^2 C_S^2 = 0, \quad (C1)$$

where ω is the angular frequency, C_S and C_A are sound speed and Alfvén speed, respectively, and k_z and k_\perp are wavenumbers parallel and perpendicular to the background uniform magnetic field B_0 in \hat{z} -direction. Two solutions of dispersion relation (C1) are

$$\frac{\omega^2}{k^2} = \frac{(C_S^2 + C_A^2)}{2} \pm \frac{1}{2} \left[(C_S^2 + C_A^2)^2 - \frac{4k_z^2 C_S^2 C_A^2}{(k_\perp^2 + k_z^2)} \right]^{1/2}, \quad (C2)$$

with plus- and minus-signs correspond to fast and slow MHD waves, where $k^2 \equiv k_\perp^2 + k_z^2$. From the basic MHD perturbation equations, one also has

$$\frac{\rho}{\rho_0} = \frac{\omega^2}{\omega^2 - k_z^2 C_S^2} \frac{b_z}{B_0}, \quad (C3)$$

where ρ_0 is the background mass density and ρ and b_z are perturbations of mass density and parallel magnetic field, respectively. For fast MHD waves (i.e., the plus-sign solution of eq. [C2]), $\omega^2 > k_z^2 C_S^2$, and thus perturbation enhancements of mass density and parallel field are in phase. For slow MHD waves (i.e., the minus-sign solution of eq. [C2]), equation (C3) can be rearranged to the following form,

$$\frac{\rho}{\rho_0} = -\frac{b_z}{B_0} \frac{C_A^2 - C_S^2 + [(C_A^2 + C_S^2)^2 - 4k_z^2 C_A^2 C_S^2 / (k_\perp^2 + k_z^2)]^{1/2}}{2C_S^2 [1 - k_z^2 / (k_\perp^2 + k_z^2)]}, \quad (C4)$$

which shows the explicit anticorrelation of mass density and parallel magnetic field perturbation enhancements. In the limit of $k_z^2 / k_\perp^2 \ll 1$ which is relevant to our case, equation (C4) reduces to

$$\frac{\rho}{\rho_0} \approx -\frac{C_A^2}{C_S^2} \frac{b_z}{B_0}, \quad (C5)$$

approximately. Therefore, the relative mass density perturbation is much smaller than the relative compression of the magnetic field in the regime of $C_A^2 \ll C_S^2$. For $C_A^2 \gg C_S^2$, the converse holds; and for $C_A^2 \sim C_S^2$, the relative density perturbation is in the same order of the relative compression of the magnetic field.

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