THE ASTROPHYSICAL JOURNAL, 493:52–53, 1998 January 20

CAN INTERPLANETARY MEASURES BOUND THE COSMOLOGICAL CONSTANT?

JAVIER F. CARDONA AND JUAN M. TEJEIRO
Departamento de Física, Universidad Nacional de Colombia, Santafé de Bogotá, Colombia
Received 1996 June 13; accepted 1997 August 11

ABSTRACT

Introducing the cosmological constant into Einstein's field equations, we determine a bound for this constant using the values observed from Mercury's perihelion shift. The bound that we find is 10^{-45} km^{-2}, 1 order of magnitude above the accepted value. By improving by one or two decimal digits the precision of the measurements of Mercury's perihelion shift, one would have an alternative way to bound the value of the cosmological constant.

Subject headings: celestial mechanics, stellar dynamics — cosmology: theory — planets and satellites: individual (Mercury) — relativity

1. INTRODUCTION

The cosmological constant was first introduced in 1917, when Einstein attempted to construct a static model of the universe; however, the discovery of the redshift of the stars, made by Hubble, compelled Einstein to eliminate this constant from his field equations. The cosmological constant appeared again in the inflationary model of the universe (Brandenberger 1985). In this scenario, the density of positive isotropic energy of the scalar field (inflation) that dominates the first stages of the universe behaves like a cosmological constant, leading the universe to a rapid cosmological expansion (the de Sitter phase). After the transitional phase, the energy of the vacuum is transformed into radiation, and the standard model of cosmology is recovered with a vacuum energy density or cosmological constant that is practically nil. Taking as a starting point the diverse problems related to the cosmological constant (Weinberg 1989), astronomers have worked to obtain a bound for this constant from the observed data. Carroll, Press, & Turner (1992) make a complete review of the current status of these efforts to bound the cosmological constant. They conclude that the most likely bounds on the cosmological constant are $-7 < \Lambda/(3H_0^2) < 2$, which are obtained from the following methods: (1) the existence of high-redshift objects, (2) the ages of globular clusters and cosmic nuclear data, (3) galaxy counts as a function of redshift, (4) clustering and structure formation, (5) quasar absorption line statistics, (6) statistics of gravitational lensing counts, and (7) astrophysics of distant objects. The most immediately promising test for the cosmological constant is gravitational lens statistics. Kochanek has found a formal limit of $\Lambda/(3H_0^2) < 0.66$ at 95% confidence in flat cosmologies using this method in recent works (Kochanek 1996). At interplanetary distances, the effect of the cosmological constant could be imperceptible; however, one of the classical tests of the general theory of relativity (Mercury's perihelion shift) gives rise to results so sensitive to this constant that it could be used to bound it.

2. SHIFT CALCULATION WITH THE COSMOLOGICAL CONSTANT

This problem involves a spherically symmetrical mass distribution (the Sun) whose metric, taking into account the cosmological constant, receives the name of the Schwarzschild–de Sitter metric according to Hawking & Gibbons (1977). This metric takes the form

$$ds^2 = B(r, \Lambda)dt^2 - A(r, \Lambda)dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) , \quad (1)$$

where

$$B(r, \Lambda) = 1 - \frac{2GM}{c^2 r} - \frac{\Lambda r^2}{3} , \quad A(r, \Lambda) = \frac{1}{B(r, \Lambda)} ,$$

where G is the gravitational constant, M is the solar mass, and c is the light velocity. The timelike geodesics that determine the orbit of the planet are found from this metric. Thus, the perihelion shift of a planet is (Weinberg 1972)

$$\Delta\varphi = 2[\varphi(r_+) - \varphi(r_-)] - 2\pi , \quad (2)$$

where φ is the angle swept by the radiovector that goes from the Sun to the planet. From the timelike geodesics, the following relationship is determined between φ and the radial coordinate r:

$$\varphi(r) = \pm \int \left\{ A^{1/2}(r, \Lambda)dr \Bigg/ \left[r^2 \left(\frac{1}{J^2 B(r, \Lambda)} - \frac{E(\Lambda)}{J^2(\Lambda)} - \frac{1}{r^2} \right)^{1/2} \right] \right\} , \quad (3)$$

where E and J are constants of motion. Then

$$\varphi(r_+) - \varphi(r_-) = \int_{r_-}^{r_+} I(\Lambda) , \quad (4)$$

where

$$I(\Lambda) = A^{1/2}(r, \Lambda)dr \Bigg/ \left[r^2 \left(\frac{1}{J^2 B(r, \Lambda)} - \frac{E(\lambda)}{J^2(\Lambda)} - \frac{1}{r^2} \right)^{1/2} \right] .$$

Now, given that Λ is very small and since r is restricted to the dimensions of the solar system, we have that

$$\frac{2GM}{c^2 r} \gg \frac{\Lambda r^2}{3} . \quad (5)$$

The integral of the equation (4) can expand itself in a Taylor series around $\Lambda = 0$:

$$\varphi(r_+) - \varphi(r_-) \cong \int_{r_-}^{r_+} I(0) + \Lambda \int_{r_-}^{r_+} \left(\frac{dI}{d\Lambda} \right)_{\Lambda = 0} .$$

At order zero, one could use the approach

$$\left(1 - \frac{2GM}{c^2 r} \right)^{-1} \simeq 1 + \frac{2GM}{c^2 r} .$$

This approximation leads to an analytical solution of the first integral:

$$\Delta\varphi = \frac{6\pi MG}{c^2 L} + 2\Lambda \int_{r_-}^{r_+} \frac{d}{d\Lambda} \times \left\{ A^{1/2} \Big/ \left[r^2 \left(\frac{1}{J^2 B} - \frac{E}{J^2} \right)^{1/2} \right] \right\}_{\Lambda=0} dr \,, \quad (6)$$

where L is the semilatus rectum, which is defined as follows:

$$\frac{1}{L} \equiv \frac{1}{2}\left(\frac{1}{r_+} + \frac{1}{r_-}\right) .$$

Numerically solving the second integral of equation (6) and using recent measurements of Mercury's perihelion shift ($42''.98 \pm 0''.1$ century^{-1}; Will 1993), one could estimate based on equation (6) that

$$|\Lambda| < 10^{-45} \ \mathrm{km}^{-2} .$$

If we take into account the solar gravitational quadrupole moment (J_2; Will 1993) in equation (6), then

$$\Delta\varphi = 42.95(1 + 2.9 \times 10^3 J_2) \ \text{arcsec century}^{-1} .$$

The solar gravitational quadrupole moment, J_2, does not change the bound significantly since $J_2 \sim 10^{-7}$, and the contribution of this term to the whole perihelion shift will be only one part in 10^4.

3. CONCLUSIONS

The accepted bound for the cosmological constant according to Kochanek (1996) is about 10^{-46} km^{-2}. Thus, by improving in one or two decimal digits the precision of the measurements of Mercury's perihelion shift, one would have an alternative way to bound the value of the cosmological constant that, unlike the conventional bounds, would be based on mensurations at interplanetary scales. There are astronomical phenomena in which the relativistic effects are much more evident. This is the case of the well-known binary pulsar PSR 1913+16, which was discovered during the summer of 1974 in Arecibo by Hulse & Taylor (1975). It has been determined that this pulsar forms a binary set with another star whose nature is still speculated on. On determining the orbit of this pulsar, you also find that a periastron shift exists near to it at 4° year^{-1} (compare it with Mercury's perihelion shift, which is only 43″ century^{-1}) (Taylor 1975). If a good percentage of this shift is the result of relativistic effects, and if it could be determined with enough precision, one would have an alternative form of bounding Λ with greater precision.

REFERENCES

Brandenberger, R. 1985, Rev. Mod. Phys., 57, 1
Carroll, S. M., Press, W. H., & Turner, E. L. 1992, ARA&A, 30, 499
Hawking, S., & Gibbons, G. 1977, Phys. Rev. D, 15, 2738
Hulse, R., & Taylor, J. 1975, ApJ, 195, L51
Kochanek, C. 1996, ApJ, 466, 638
Taylor, J. 1975, in Ann. NY Acad. Sci., 262, 7th Texas Symp. Relativistic Astrophysics, ed. P. R. Bergmann, E. J. Fenyves, & L. Motz, 490
Weinberg, S. 1972, Gravitation and Cosmology (New York: Wiley)
———. 1989, Rev. Mod. Phys., 61, 1
Will, C. 1993, Theory and Experiment in Gravitational Physics (Cambridge: Cambridge Univ. Press)