THE DURATION–PHOTON ENERGY RELATION OF GAMMA-RAY BURSTS AND ITS INTERPRETATIONS

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ABSTRACT

Fenimore et al. have recently presented a very tight correlation between the spectral and the temporal structure in gamma-ray bursts (GRBs). In particular, they discovered that the durations of the constituent subpulses which make up the time profile of a given GRB have a well-defined power-law dependence of index $\simeq 0.45$ on the energy E of the observed photons. In this paper we present two simple models which can account in a straightforward fashion for the observed correlation. These models involve (1) the impulsive injection of a population of relativistic electrons and their subsequent cooling by synchrotron radiation, and (2) The impulsive injection of monoenergetic high-energy photons in a medium of Thompson depth $\tau_T \sim 5$ and their subsequent downgrading in energy due to electron scattering. We present arguments for distinguishing between these two models from the existing data.

Subject headings: gamma rays: bursts

1. INTRODUCTION

The issue of the nature of gamma-ray bursts, (GRBs), a contentious one since their discovery, has only deepened with the more recent observations by BATSE which confirmed their isotropic distribution in the sky while indicating a significant lack of faint events, inconsistent with a uniform distribution in space (Meegan et al. 1992). These facts have provided support to models invoking the cosmological origin of the GRBs, thus increasing further the uncertainty associated with the location, nature, and origin of the GRB phenomenon. This uncertainty is due largely to the lack of associaton of any class of interesting astrophysical objects within the positional error boxes of GRBs, which would set the distance scale and hence the luminosity associated with these events. In the absence of such positional association, attempts to gain understanding on the nature of GRBs have relied on searches for systematics in the time and/or energy domain of the observed bursts. Part of our continuing ignorance with respect to the nature of GRBs is due to the absence of any significant correlations between the GRB spectral and/or temporal properties (Paczyński 1992).

This situation seems to have recently changed: Fenimore et al. (1995) have presented a very well defined correlation between the average duration of the subpulses usually present in the time profile of a given burst, or the duration of the entire burst in the absence of subpulses, and the energy channel of observation. They have found that the average pulse width, $\Delta\tau$, as determined either by the autocorrelation function of the entire GRB or by specific fits of the time profiles of individual subpulses in a given profile, is well fitted by a power law in the energy E at which the observation is made; more specifically they have found that $\Delta\tau \simeq AE^{-0.45 \pm 0.05}$, where A is a proportionality constant. The above reference thus made quantitative an effect that had been noticed earlier by Fishman et al. (1992) and Link, Epstein, & Priedhorsky (1993), who indicated that, in

general, the various peaks in the GRB time profiles are shorter and better defined at higher energies. This unique piece of information may, therefore, serve as a stepping stone in a probing the physical processes associated with the radiation emission in GRBs and maybe the entire GRB phenomenon. More recently, Norris et al. (1996) have further elaborated on the time profiles of GRBs as a function of the energy channel of observation. These authors have found that the relation reported by Fenimore et al. (1995) indeed holds, but it may have slightly different slopes depending on the overall position of the observed pulse within the entire burst.

In this paper we present two generic models which can provide an account of the particular functional form of the observed correlation between $\Delta\tau$ and E. It is of interest to note that the physical processes associated with these two specific models are fundamentally different, despite the fact they both can produce the observed pulse duration–energy correlation. However, these models are quite simple in their concept but specific enough to consider possible that further observations and detail fits would yield systematics which could discriminate between them or will lead to further insights into the radiation mechanism of GRBs. The first model involves the impulsive injection of a power law of relativistic electrons in a given volume and their subsequent cooling by synchrotron or inverse Compton radiation, while the second one the impulsive injection of high-energy photons in an cold medium of Thompson depth $\tau_T \sim 10$ and their subsequent downgrading by electron scattering.

In § 2 the mathematical formulation of both models is presented along with sample time-dependent spectra and model pulse durations at various frequencies. In § 3 the results are discussed and conclusions are drawn concerning the impact of these results on GRBs models.

2. THE MODELS

To begin with, it is worth noting that the relation discussed by Fenimore et al. (1995) precludes from the outset the possibility that the observed spectra are the result of up-Comptonization of soft photons by repeated scatterings in a steady-state, hot electron population (see, e.g., Sunyaev

& Titarchuk 1980); in this case the higher energy photons are those which would have spent longer time within the hot, scattering medium, leading to a correlation between the burst duration $\Delta\tau$ and the photon enegy E of opposite sense to that observed.

Before we discuss the models for the energy dependence of GRB duration in detail, we present some simple, heuristic arguments which qualitatively indicate that a well defined spectro-temporal relation similar to that observed is not indeed at all surprising. These considerations are motivated by the recent cosmological scenario for GRBs suggested by Mészáros & Rees (1992) and also by some arguments concerning photon scattering in a thick media and should generally be as applicable to galactic GRB models. It is worth noting here that, despite their apparent dissimilarity, both models outlined below describe essentially the same process, namely, the energy degradation of a population of relativistic particles with one of the models referring to relativistic electrons while the other refers to photons. What is more important to bear in mind is that the observed correlation appears to require the presence of some sort of "cooling" in GRBs.

Consider the impulsive injection of relativistic particles at a given volume and assume the emission to be optically thin. If synchrotron or inverse Compton losses dominate the evolution of the electron distribution, then the loss rate per electron is $\dot{\gamma} \propto B^2\gamma^2$, where γ is the Lorentz factor of a given electron and B the magnetic field (if the losses are dominated by inverse Compton, then B^2 should be replaced by the ambient photon energy density). Therefore, the characteristic electron life time is $\Delta\tau \sim \gamma/\dot{\gamma} \propto \gamma^{-1}$. Since the characteristic energy of synchrotron (or inverse Compton) emission is $E \propto \gamma^2\epsilon_0$ (ϵ_0 is a characteristic energy), expressed in terms of E rather than γ, the time duration is $\Delta\tau \propto E^{-1/2}$.

Consider, alternatively, the impulsive injection of mono-energetic high energy photons of energy E_0 in a medium of Thomson depth $\tau_T \gg$ and of electron temperature $kT \ll E_0$. Photons of energy E suffer fractional energy loss of order $\Delta E/E = -(1 - \mu)E/m_e c^2$ per collision, where μ is the cosine of the photon scattering angle, or in terms of the photon wavelength λ, $\Delta\lambda = (1 - \mu)$. Averaged over solid angle, $\langle\Delta\lambda\rangle = 1$. So after n scatterings, $\langle\lambda_n\rangle = n + \lambda_0$ where λ_0 is the original wavelength (omitted for $n > 1$). In the diffusion regime, the dispersion of photon wavelengths is also proportional to the number of scatterings. Hence the following relation between the number of scatterings or time (in units of scattering time): $\Delta\tau \propto \lambda^2 \propto E^{-2}$. For a finite medium, this diffusion process lasts for a number of scatterings $n \propto \tau_T^2$, thus the maximum width (in wavelength) of the escaping photons would be $\Delta\lambda \propto \tau_T$. In this case, however, the precise law between the pulse width and the energy depends on the photon source distribution within the scattering medium, as it will be exhibited in more detail in the next section. Nonetheless, the qualitative correlation between the energy and duration of pulses will always be as that outlined above.

In the following, we make the above heuristic arguments more concrete by presenting well formulated solutions of the problems outlined above.

2.1. Relativistic Electron Cooling

Consider the simplest case of time-dependent synchrotron radiation by a population of nonthermal electrons.

Relativistic electrons of Lorentz factor γ with a power-law distribution of index Γ, i.e., $Q_e(\gamma) = q\gamma^{-\Gamma}$, are injected in a volume of tangled magnetic field B at an instant $t = t_0$ and are left to cool by emission of synchrotron (or inverse Compton radiation). The equation which governs the evolution of the electron distribution function $N(\gamma, t)$ in time is

$$\frac{\partial N}{\partial t} = -\frac{\partial}{\partial\gamma}\left[\langle\dot{\gamma}\rangle N(\gamma, t)\right] + Q_e(\gamma)\delta(t - t_0),$$

$$= \frac{\partial}{\partial\gamma}\left[\beta\gamma^2 N(\gamma, t)\right] + Q_e(\gamma)\delta(t - t_0), \quad (1)$$

where $\langle\dot{\gamma}\rangle = -\beta\gamma^2$ is the mean electron loss rate and $\beta = (4/3)(B^2/8\pi)(\sigma_T/m_e c^2)c$, while $Q_e(\gamma)$, as given above, represents the injection of relativistic electron population. This equation can be solved by the standard technique of converting it into an ordinary differential equation in t and integrating it along the characteristic curves $\gamma = \gamma(t)$. Thus, multiplying through by γ^2 one obtains

$$\frac{\partial(\gamma^2 N)}{\partial t} - \beta\gamma^2\frac{\partial}{\partial\gamma}\left[\gamma^2 N(\gamma, t)\right] = \gamma^2 Q_e(\gamma)\delta(t - t_0),$$

$$\frac{d(\gamma^2 N)}{dt} = \gamma^2 Q_e(\gamma)\delta(t - t_0), \quad (1a)$$

with the last step becoming obvious on identifying $\partial\gamma/\partial t$ with $-\beta\gamma^2$. The total differential of equation (1a) can be integrated along the integral curves of the electron energy γ; these are given by the solution of $\partial\gamma/\partial t = -\beta\gamma^2$, namely, $\gamma = \gamma_0/(1 + \gamma_0\beta t)$, where γ_0 is the initial electron energy. Assuming $Q_e(\gamma) = q\gamma^{-\Gamma}$, the resulting form of the electron distribution function is

$$N(\gamma, t) = \begin{cases} q\gamma^{-\Gamma}(1 - \beta\gamma t)^{\Gamma - 2} & \text{for } \gamma < 1/\beta t, \\ 0 & \text{for } \gamma > 1/\beta t \end{cases} \quad (2)$$

Given the electron distribution function $N(\gamma, t)$, the emitted spectrum can be calculated by convolving $N(\gamma, t)$ with the synchrotron emissivity $\epsilon(\gamma, \nu)$ (see, e.g., Rybicki & Lightman 1979). However, the form of the emerging spectrum can be easily calculated using the δ-function approximation to the single electron emissivity, i.e., $\epsilon(\gamma, \nu) \propto \delta(\nu - \gamma^2\nu_c)$, where $\nu_c \sim 4 \times 10^6 B(G)$ Hz is the cyclotron frequency. Assuming, further, that the injected distribution function has a minimum Lorentz factor $\gamma_m(t)$ the resulting spectrum has the form

$$F_\nu \propto \begin{cases} \nu^{1/3}, & \text{for } \nu < \nu_m(t) = \gamma_m(t)^2\nu_c, \\ \frac{1}{2}(\nu/\nu_c)^{-(\Gamma-1)/2}[1 - (\nu/\nu_c)^{1/2}\beta t]^{\Gamma-2}, & \\ & \text{for } \nu > \nu_m(t), \\ 0, & \text{for } \nu > \nu_c/\beta^2 t^2, \end{cases} \quad (3)$$

where $\nu_m(t)$ is the synchrotron frequency corresponding to the lowest energy electrons of the injected electron Lorentz factor γ_m (which is also a function of time), and ν_c is the cyclotron frequency corresponding to magnetic field B.

Figure 1 shows the time-dependent spectra obtained by integrating the distribution function of equation (2) over the exact expression of the synchrotron emissivity with an injection spectrum $\propto \gamma^{-2}$ between Lorentz factors $\gamma_m = 10^6$ and $\gamma_M = 10^9$. These specific values (especially that of γ_m) were chosen to assure that the low-energy turnover of the spectrum occurs at $\nu_m \sim 10^{20}$ Hz ($=414$ keV), as indicated by the GRB spectra (Schaeffer et al. 1994), and that the dura-
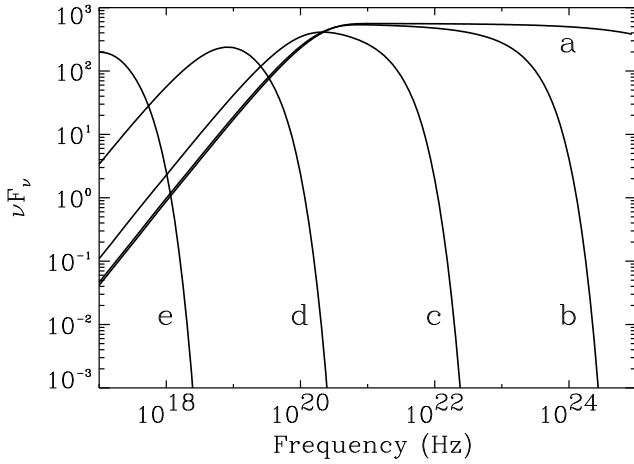
Fig. 1.—Evolution of the spectrum with time for the cooling synchrotron emitting relativistic electron model. The times are corresponding to each curve are: (a) 10^{-3} s, (b) 10^{-2} s, (c) 10^{-1} s, (d) 1 s, (e) 10 s. The B-field is assumed to be 30 G, the maximum and minimum electron Lorentz factors at injection were taken to be 10^9 and 10^6, respectively.

tion of the burst be $\tau_b \sim$ a few seconds, also in agreement with observations. These considerations then serve to also determine the value of the magnetic field associated with this emission to be $B \sim 30$ G.

If the observed durations are indeed only apparent rather than intrinsic, the result of emission from a plasma in relativistic motion toward the observer, as suggested, e.g., in the currently popular relativistic blast wave GRB models (see, e.g., Mészáros & Rees 1992), then both the break frequency v_m and the characteristic time scale $\Delta\tau$ should be shifted by the appropriate powers of the bulk Lorentz factor Γ of the flow to reflect the values at the rest frame of the fluid. In this case, the intrinsic frequency of emission, v'_m, will be shorter from the observed one $v_{m,o}$ by a factor Γ, i.e., $v'_m \simeq v_{m,o}/\Gamma$ while the intrinsic duration $\Delta\tau'$ would be longer than the observed one $\Delta\tau \simeq$ a few seconds, by a factor $\delta^{-1} = \Gamma(1 - \beta_L) \simeq \Gamma$, i.e., $\Delta\tau' \simeq \Delta\tau \cdot \Gamma$ (β_L is the velocity of the fluid, i.e., $\Gamma = (1 - \beta_L)^{-1/2}$). Using the primed (rest frame) values of the duration and break frequency to determine our values of B and γ_m leads to the following scalings for the magnetic field B and the value of γ_m on the rest frame of the fluid in terms of the observed duration $\Delta\tau$(s) and $v_{m,0} = 10^{20} v_{20}$ Hz

$$B \simeq 10^{5/3} \Gamma^{-1/3} v_{20}^{-1/3} \Delta\tau^{-2/3} \text{ s G} , \tag{4a}$$

and

$$\gamma_m \simeq 10^{17/3} \Gamma^{-1/3} v_{20}^{1/3} \Delta\tau^{1/3} \text{ s} . \tag{4b}$$

So while our calculations were performed with a field value of $B \sim 30$ G and $\gamma_m \simeq 10^6$ the values of these parameters in the fluid rest frame would depend on the value of the bulk Lorentz factor Γ by the relations given above.

Figure 2 shows the synchrotron emission as a function of time at a set of frequencies, spaced logarithmically, with each curve corresponding to a frequency 10 times smaller than the previous one. It is apparent in this figure that the burst duration is inversely proportional to the square root of the radiation frequency, in accordance with the relation reported by Fenimore et al. (1995). It is also apparent in this figure that, for the particular form of electron injection considered in this example (i.e., impulsive injection), there is no time lag in peak emission between frequencies in the range
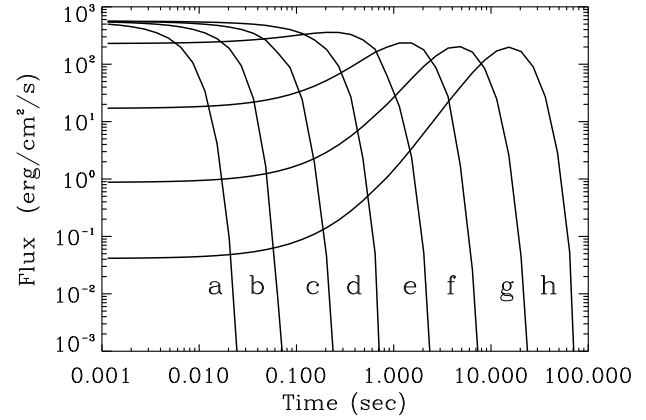

Fig. 2.—Synchrotron emission at individual frequencies as a function of time for the relativistic cooling electron model. The frequencies corresponding to the various curves are: (a) 10^{24} Hz, (b) 10^{23} Hz, (c) 10^{22} Hz, (d) 10^{21} Hz, (e) 10^{20} Hz, (f) 10^{19} Hz, (g) 10^{18} Hz, (h) 10^{17} Hz.

$v > \gamma_m(t_0)^2 v_c$, i.e., for frequencies higher than the synchrotron frequency of the lowest energy electrons at the time of injection. Time lags in peak emission develop only for frequencies smaller than that above, resulting from the "cooling" of the lowest energy electrons to energies $\gamma_m(t) < \gamma_m(t_0)$.

2.2. Energy Downgrading by Electron Scattering

Consider alternatively the injection of photons of energy $E \sim m_e c^2$ near the center of a cold ($T_e \ll E$), spherical (for simplicity) electron cloud of Thomson depth $\tau_T \sim 5$. As the photons random walk out of the cloud, they also lose energy in collisions with the ambient electrons. A monoenergetic photon pulse at the center of the cloud spreads, therefore, both in energy and in time. The duration of the pulse for photons of a given energy can be computed from the following considerations (Illarionov et al. 1979):

At a given scattering event the fractional change of the photon energy is

$$\frac{\Delta v}{v} = -\frac{(1 - \mu)hv}{m_e c^2} , \tag{5}$$

or, expressed in terms of the photon wavelength $\lambda = m_e c^2/hv$,

$$\Delta\lambda = 1 - \mu , \tag{6}$$

where μ is the cosine of the angle between the incident and scattered photon momenta.

Let $\phi(\mu) = (\frac{3}{16}\pi)(1 + \mu^2)$ denote the phase function of the Thompson cross section, such that $\int \phi(\mu)d\Omega = 1$, then the average change in the photon wavelength $\langle \Delta\lambda \rangle$ and the dispersion in the photon energies σ_λ^2 per scattering are

$$m_\lambda = \langle \Delta\lambda \rangle = \int \phi(\mu)(1 - \mu)d\Omega = 1 , \tag{7}$$

and

$$\sigma_\lambda^2 = \langle (\Delta\lambda)^2 \rangle - \langle \Delta\lambda \rangle^2 = \langle (\Delta\lambda)^2 \rangle - 1 ,$$

$$= \int \phi(\mu)(1 - \mu)^2 d\Omega - 1$$

$$= \frac{3}{8} \int_{-1}^{1} (1 + \mu^2)(1 - \mu)^2 d\mu = \frac{2}{5} . \tag{8}$$

The distribution (probability) of photons over number of scatterings n with a mean number of scatterings u is given by a Poisson distribution, $P_u(n)$, which is the limit of binomial distribution for the case of rare events. For the Poisson distribution the values of the mean m_n and dispersion σ_n^2 are equal, i.e., $\sigma_n^2 = m_n = u$.

Now, the photon distribution in wavelength λ for a given mean number of scatterings u (or alternatively after time u measured in units of the scattering time $\tau = 1/\sigma_T n_e c$) is given by the convolution

$$J_u(\lambda) = \sum_{n=0}^{\infty} \Psi_n(\lambda) P_u(n) , \qquad (9)$$

where $\Psi_n(\lambda)$ is the photon distribution after n scatterings, with mean wavelength $\langle \lambda_n \rangle = n + \lambda_0$ (λ_0 is the wavelength of the initial photon) and dispersion $\sigma_\lambda^2(n) = n \cdot \sigma_\lambda^2 = (2/5)n$ and $P_u(n)$ is the probability of n scatterings within time u.

The mean of this distribution is

$$m_\lambda^u = \int_{\lambda_0}^{\infty} J_u(\lambda) d\lambda = \sum_{n=0}^{\infty} \left[\int_{\lambda_0}^{\infty} \lambda \Psi_n(\lambda) d\lambda \right] P_u(n) ,$$

$$= \sum_{n=0}^{\infty} (n + \lambda_0) P_u(n) = u + \lambda_0 , \qquad (10)$$

and the dispersion

$$\sigma_\lambda^2(u) = \int_{\lambda_0}^{\infty} (\lambda - m_\lambda^u)^2 J_n(\lambda) d\lambda ,$$

$$= \int_{\lambda_0}^{\infty} [(\lambda - m_\lambda^n)^2 + 2(\lambda - m_\lambda^n)(m_\lambda^n - m_\lambda^u)$$

$$+ (m_\lambda^n - m_\lambda^u)^2] d\lambda P_u(n) ,$$

$$= \sum_{n=0}^{\infty} \left[\tfrac{2}{5} n + 0 + (n - u)^2 \right] P_u(n) ,$$

$$= \frac{2}{5} u + u = \frac{7}{5} u . \qquad (11)$$

Therefore the photon spectrum after time u will have the form

$$J_u(\lambda) = \frac{1}{[2\pi\sigma_\lambda^2(u)]^{1/2}} \exp\left[-\frac{(\lambda - m_\lambda^u)^2}{2\sigma_\lambda^2(u)} \right], \qquad (12)$$

which represents a Gaussian of width u about $\lambda - \lambda_0$. In fact, the above relation can be viewed both as the photon spectrum escaping after time u or alternatively as the time history for photons of a given wavelength λ. One can use the above expression to obtain the dispersion in escaping time for photons of a given energy. This is simply given by the width of the Gaussian in equation (12), which considering the definitions of $\sigma_\lambda^2(u)$ and m_λ^u, yields $\lambda - \lambda_0 \simeq 2(2u)^{1/2}$, i.e., that $\Delta\tau \propto E^{-2}$, as suggested by our heuristic arguments.

The emergent spectrum of the photons escaping over the duration of the burst is given by the convolution

$$I(\lambda) = \int_0^{\infty} J_u(\lambda) P(u) du , \qquad (13)$$

where $P(u)$ is the probability of a photon to escape after u scatterings. As a concrete example, we consider a photon source distribution according to the first eigenfunction of the diffusion operator; with this assumption, the function

$P(u)$ has the form

$$P(u) = \beta e^{-\beta u} , \qquad (15)$$

with $\beta = \pi^2/(3(\tau_0 + 2/3)^2$ being the eigenvalue of the same operator. Then the integral of equation (13) above is analytic yielding

$$I(\lambda) = \frac{\beta\lambda^2}{(1 + 7\beta/5)^{1/4}(1 + 7\beta/10)^{1/10}} \exp\left[-(\lambda - \lambda_0)\beta \right]$$

$$(16)$$

which indicates that the spectrum is a power law in energy of index -2 with an exponential turn-over at $\lambda \sim 1/\beta$.

The analytic results of the above discussion should be viewed only as an illustrative example to indicate the relationship between the photon energy E and the duration of a photon pulse $\Delta\tau$. It is also apparent from the above discussion that the precise form of this relation depends on the distribution of photon sources within the scattering cloud, the relation of equation (12), corresponding to a homogeneous, infinite medium. Our analytic calculations are also more appropriate for the case in which the high-energy photon source at the center of the electron cloud. This ensures that the escaping photons will undergo quite a few scatterings before escape and that they will lose a significant fraction of their energy in the first few scatterings. These scatterings reduce their energy to the point where the treatment of radiative transfer can be replaced by diffusion in energy and space (i.e., small energy change per scattering at the Thomson cross section). Clearly, the random walk of photons situated close to the cloud boundary cannot be described by diffusion with small energy change at each scattering event. A large fraction of the high-energy photons (assuming monoenergetic high-energy injection) undergoes only a few scatterings before escape but with large energy change per scattering (Hua & Titarchuk 1995). Thus the accurate consideration of the photon transport distributed over a finite plasma cloud could, conceivably, lead to an energy–duration relation in closer agreement to the results of Fenimore et al. (1995).

Because our analytic model is appropriate only for scattering in the Thomson regime, we extend our treatment by introducing a more realistic model. The extended model consists of two concentric spheres; it is assumed that cold electrons are uniformly distributed over the entire volume of the larger sphere of radius R_0. The smaller sphere, of radius R_s, represents the volume within which the high-energy photon sources are located and the photon injection takes place, uniformly over its entire volume. This model has an additional parameter namely the ratio of the radii of these two spheres $R_s/R_0 = \tau_s/\tau_0$. It is also assumed that the photon injection takes place impulsively at $t = 0$, at energies ~ 1 MeV. For the treatment of this more detailed model we have used a Monte-Carlo code developed by one of us (X. M. H.). The details of this Monte Carlo code have been discussed elsewhere and will not be repeated here. The interested reader is referred to the relevant publications (Hua & Titarchuk 1996).

In Figure 3 we present the widths in time associated with photons escaping from the scattering cloud at several energy bands, as obtained by our Monte Carlo calculation; the energy bands were chosen in the same fashion as in the analysis of Fenimore et al. (1995). The widths of the pulses
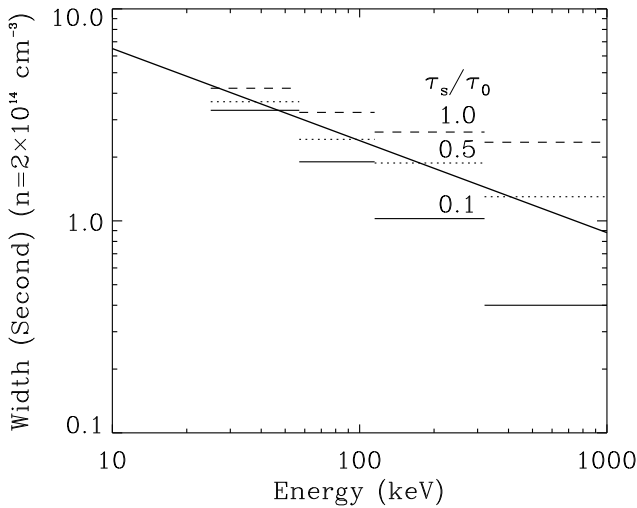
FIG. 3.—Pulse width–energy relation for the downscattering model for different values of the τ_s/τ_0 ratio.

as a function of the energy are given for three values of the ratio τ_s/τ_0, namely, 1.0, 0.5, and 0.1. The photon injection was assumed to be monoenergetic at $E_0 = 1$ MeV and the total Thomson depth of the source was taken to be $\tau_0 = 10$.
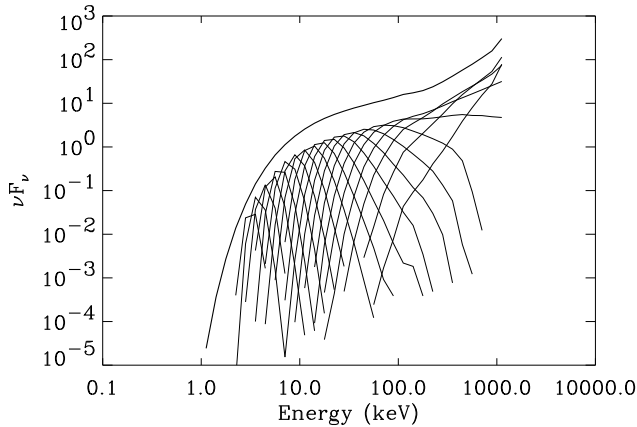

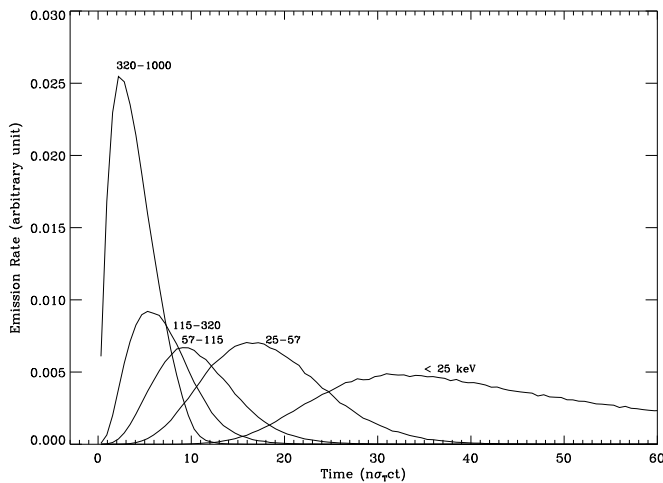FIG. 4.—Time-accumulated spectrum for the downscattering model described in the text.

FIG. 5.—Time evolution of emission for various energy channels, for the downscattering model.

One can see that the value $\tau_s/\tau_0 = 0.5$ provides good fit to the relation found by Fenimore et al. (1995), indicated by the straight solid line.

The resulting photon spectrum, in νF_ν units, accumulated over the entire burst is given in Figure 4, with the spectra escaping at specific times given by the individual curves. The times are spaced equally in logarithmic intervals. The time-accumulated spectrum is consistent with that of GRBs in that it peaks at roughly 1 MeV. Clearly, injection of photons at higher energies and/or at larger Thomson depths could produce spectra which would simulate the observed ones more accurately. Finally, in Figure 5 we present the rates of escaping photons as a function of time, for photons of a given energy range. The energy ranges were chosen to be identical to those used by Fenimore et al. (1995) to derive the energy–duration relation.

3. CONCLUSIONS AND DISCUSSION

We have presented above two models which can account for the recently discovered correlation of the GRB duration (or the duration of subpulses within a given burst) as a function of the photon energy. Both models appear to be able to reproduce the observed power-law dependence of $\Delta\tau$ on E given in Fenimore et al. (1995), as well as the general form of the observed GRB spectra. Considering in greater detail the comparison of our model spectra with those of GRBs (Schaeffer et al. 1994), the model spectra are generally consistent with those observed in that their luminosity (i.e., their νF_ν distribution) peaks at $\sim 10^{20}$ Hz, while exhibiting a power-law frequency dependence, in agreement with observations.

The slope of this low-energy power law is very well determined for the synchrotron cooling models, being proportional to $E^{4/3}$, the functional form of the single electron synchrotron emissivity. Interestingly, there exist several GRBs (e.g., GRB 910503 and GRB 910601) the low-frequency spectra of which are in agreement with this interpretation. In particular, the spectrum of GRB 910601 (Share et al. 1994) seems to be fitted very well with the single electron synchrotron emissivity over its entire range, suggesting the synchrotron origin of the radiation associated with this specific burst. However, most of the spectra in Schaeffer et al. (1994) require low-frequency power-law fits of significantly different slopes, typically $\nu F_\nu \propto E^{1/2}$. Clearly, the simple interpretation of synchrotron emission by a uniform, optically thin, impulsively injected electron population, such as described above, would be inappropriate for this type of bursts. However, more complicated models which relax the above assumptions could produce spectra agreement with observation. For example, continuous (rather than impulsive) injection of the electron population over timescales long compared to the loss timescale of the electrons with Lorentz factor γ_m would lead to spectra in agreement with this form.

On the other hand, the downscattering model could accommodate spectra of the form $\nu F_\nu \propto E^{1/2}$ since the precise slope of the low-energy power law depends on the distribution of the high-energy photon sources within the cold cloud considered. These slopes are consistent with those of the time averaged spectrum of the downscattering model given in Figure 4. In further support of this point, one should bear in mind that the spectrum of Figure 4 represents the result of a δ-function photon injection at the highest energy bin, hence the upward turn near $\simeq 1$ MeV in

Figure 4. Injection of a broader type of photon spectra or larger value for τ_0 would result in smoother spectra, conforming more to the observation.

In assessing the utility of the above models, one could claim that they both may be of relevance in modeling the spectro-temporal evolution of GRBs. The synchrotron cooling models fares a little better in reproducing the duration-energy correlation in that it requires fewer parameters to do so than the downscattering model. On the other hand, the downscattering model, because of its additional freedom, appears to be able to provide more detailed fits to a larger number of GRB spectra. Discriminating between these models requires a more detailed combination of spectral and temporal analysis of individual bursts rather than the use of their average properties. Nonetheless, the form of the observed correlations along with their present model interpretation clearly indicate that, whatever their precise mechanism, the available energy of GRBs is originally released into the highest energy particles and their evolution is determined by its subsequent cascade to the lower energy ones.

Despite the general agreement of the computed spectra of both models to those observed and their overall agreement to the duration–photon energy relation, it is hard at the present level of analysis, to draw any further conclusions concerning the physical mechanism underlying the emission of radiation in GRBs. This is due to the absence of an independent estimate of the parameters which set the scales of the time and the energy of peak emission in the νF_ν spectra, in either model. In the nonthermal model, the time is set by the magnitude of the magnetic field or rather the combination $B^2\gamma$ (or by $\rho_{rad}\gamma$ if inverse Compton is the dominant loss mechanism and and the bulk motion Lorentz factor Γ if relativistic expansion is of relevance), while in the downscattering model it is set by the density of the electron cloud through which the γ-rays propagate, both of which are indeterminate. The energy at which the νF_ν spectra peak could be used to provide some additional information on the radiative processes associated with GRBs. However, such information is independent, at this level of analysis, from that associated with the GRB timescales. In the nonthermal models, the energy of peak emission is set by the combination $\gamma_m^2 B$, while in the downscattering models it is an independent parameter, defined by the injection of primary photons.

Figures 2 and 5 exhibit an additional aspect of the spectro-temporal correlations of the GRB time profiles which has been touched upon only briefly so far, namely, that of the time lags in peak emission among the various energy channels. This issue, only a peripheral one in this note, could be further developed both theoretically and observationally and might provide additional information on the nature of the GRB emission process, or at least a sufficiently strong criterion to reject one or both of the above models.

Figure 5, indicates that the model of impulsive high-energy photon injection leads invariably to lags in the peak (highest intensity) emission between the various energy channels, in the sense that the lower energy emission always lags behind the high energy one. Due to the nondimensional character of the time axis of this figure, the lags have to be compared to the dispersion (FWHM) of the appropriate energy photons. It is apparent in the Figure 5 the lag is smaller than the dispersion (FWHM) of the burst, in qualit-

ative agreement with the results of Figure 12 of Norris et al. (1996). While the same data indicate generally lags significantly smaller than the observed FWHM of the pulses, we believe that more detailed study is required to decide the quantitative agreement of the model with the data and possibly exclude this model on the basis of this issue.

In the relativistic electron cooling model, on the other hand, the presence of such lags is not ubiquitous, even for an impulsive injection of the electron population. As shown in Figure 2, lags develop only for frequencies emitted by electrons with Lorentz factors lower than $\gamma_m(t_0)$, as discussed earlier. Indeed, as seen in Figure 2 there is little lag in the peak emission between frequencies $\gtrsim 10^{20.5}$ Hz, while there are progressively increasing lags between the peak emission of lower frequencies. Since the time axis of Figure 2 is absolute (i.e., in s) one can compare these lags directly to those discussed in Norris et al. (1996). In doing so, one should bear in mind that the lowest channel of the above reference corresponds to curve f of this figure so that no lags longer than 0.5 s should be expected for the chosen values of the parameters, in rough agreement with Norris et al. (1996).

The issue of lags in peak emission within both models can be further complicated by the relation of the injection to the loss timescales in cases in which the latter is not impulsive. A further indication of the complex nature of this issue is the fact that, as noted by Norris et al. (1996), the lags in peak emission depend not only on the energy band of observation but also the asymmetry in the burst time profiles, an issue not discussed here. We prefer not to delve into these issues in this note but concentrate instead on the relation between the observed photon energy and the burst duration since that is, in both our models, a property of the Green's function of the associated problem and hence independent of the details of injection. Some of the aspects of their findings could be accounted by the models presented above. However, since the entire burst time profile seems to be of importance in this issue, we will defer addressing it to future more detailed models of the spectro-temporal evolution of GRBs. They systematics provided by Norris et al. (1996) indicate that this issue is an important one and in need of further theoretical exploration.

Finally, we would like to point out that neither model provides an answer to why GRBs emit most of their luminosity in the γ-rays rather than at other wavelengths. The downscattering model treats the injection of primary photons as an initial condition and it is hence beyond the scope of any further discussion. The synchrotron emission model actually does fare a little better in this respect since γ-ray emission is the natural outcome of electron acceleration in shocks. It can be easily shown that for shock acceleration of electrons limited by synchrotron losses, the maximum Lorentz factor is of order $\gamma^2 \sim (e/fB)(1/\sigma_T)$ and the frequency of the emitted radiation $E_\gamma \sim (\frac{1}{8}f)(c/r_0) \simeq 2 \times 10^{22}$ Hz $\sim (24/f)$ MeV (r_0 is the classical electron radius and f is a measure of the electron free path in units of the Larmor radius), which has the interesting property of being independent of the magnetic field B. If this is indeed the reason that GRBs emit mainly in the γ-ray band, it would not leave much room for the relativistic blast wave models (Mészáros & Rees 1992) since the latter postulate an additional relativistic boost of the produced photons with bulk Lorentz factors $\Gamma \sim 10^3$ in order that they are detected in the γ-ray band.

The models presented above, despite their ability to provide good fits to the energy-duration relation of GRBs, cannot, at the present level of analysis, provide an immediate, decisive answer to the question of the nature GRBs. Nonetheless, the energy-duration relation reported by Fenimore et al. (1995) which represents the tightest correlation associated with GRBs to date, warrants their use in more detailed spectro-temporal analyses of GRB time profiles, which may set the groundwork for uncovering the nature of GRBs.

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Note added in proof.—Following the submission of our paper, the detection of X-ray and optical afterglows with absorption features attributed to adsorbing clouds at $z \simeq 0.8$ have convincingly argued for the cosmological origin of GRBs, thus obviating parts of our introduction and discussion sections; however, the basic processes and conclusions of our paper still hold true.