THE ASTROPHYSICAL JOURNAL, 492:286–297, 1998 January 1

ON THE POSSIBILITY OF THE NONEXPLOSIVE CORE CONTRACTION OF MASSIVE STARS: NEW EVOLUTIONARY PATHS FROM ROTATING WHITE DWARFS TO ROTATING NEUTRON STARS

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Received 1996 December 2; accepted 1997 August 13

ABSTRACT

We have found that a new and *nonexplosive evolutionary path* from stars in the white dwarf region to stars in the neutron star region may be possible. Such a process can be realized if we incorporate both a large amount of rotation and the temperature effect on the equation of state. The large value of angular momentum is required to make stars *secularly unstable* because of gravitational radiation emission. For high temperature matter, the contribution of the temperature and the electron fraction to the pressure becomes large enough to make rotating stars *dynamically stable* against axisymmetric perturbations. Thus, equilibrium states may exist for rotating compact stars that are *dynamically stable* against axisymmetric collapse but *secularly unstable.* The evolution of dynamically stable and secularly unstable rotating stars can proceed as follows. The secular instability is caused by the emission of gravitational waves that carry away the angular momentum of the star. Stars with less angular momentum will contract to higher density states. This process occurs rather slowly, i.e., not on a dynamical timescale but on a secular timescale of gravitational radiation emission. Consequently, compact configurations such as white dwarfs in this category may undergo nonexplosive and slow contraction. This contraction leads some configurations to neutron stars and others to black holes, depending on the mass and the angular momentum. If the final outcomes are neutron stars, they are both dynamically and secularly stable because some of the angular momentum is lost. Therefore, we have succeeded in showing that "fizzlers" can exist, although Newtonian gravity is used.

This evolution is likely to occur within the central region of massive stars. Since the central region of a massive star is hot, high-temperature effects become important. Concerning high angular momentum, massive stars in the main sequence stage usually rotate rather rapidly. It implies that the angular momentum of the core can also be large enough to lead to secular instability. Thus, cores of massive stars may contract on a long timescale without being accompanied by supernova explosions.

Subject headings: instabilities — stars: evolution — stars: interiors — stars: neutron — stars: rotation — white dwarfs

1. INTRODUCTION

Many researchers have considered that rotation would change stellar evolutionary paths considerably. Since the rotational effect acts as a counterforce against gravity, it could be possible that the contracting or collapsing stages of stars might be affected significantly, thus resulting in less violent phenomena compared with those of spherical evolution (see, e.g., Hoyle 1946).

Among the many attempts to treat the rotational evolution of compact stars (e.g., Salpeter & Wagoner 1971; Wiita & Press 1976), Shapiro & Lightman (1976) investigated the rotational effect on evolution quantitatively by constructing models of compact stars in the density region very close to that of neutron stars. Although they used very simplified models, they found that if rotation is rapid enough, some stars with lower density than neutron stars can be dynamically stable but secularly unstable. Thus, very compact stars near the neutron star density can evolve to secularly stable neutron stars on a very *long* timescale because the interaction that causes the emission of gravitational radiation is very weak. Stars of this kind have been named "fizzlers" by Gold (1974) because of their slow evolution.

In order to apply the concept of fizzlers to astrophysically important situations, we need to go beyond the results of Shapiro & Lightman. There are three reasons for that. First, Shapiro & Lightman treated a rather narrow range of the central density of rotating stars, i.e., the density region 10^{13}–10^{15} g cm^{-3}. Second, they did not solve rotating equilibrium exactly. They applied the energy variational principle to uniformly rotating spheroidal configurations. Moreover, the total energy was approximated by using simplified models. Thus, their equilibrium solutions were not exact. Third, as for the equation of state, they adopted a modified nonrelativistic Fermi gas instead of the realistic equation of state.

About 10 years ago, Müller & Eriguchi (1985) developed a new computational scheme for rapidly rotating stars and studied the existence of fizzlers in a wide range of the maximum density (10^{7}–$10^{14.4}$ g cm^{-3}). They used the equation of state of Baym, Pethick, & Sutherland (1971b) and that of Baym, Bethe, & Pethick (1971a). These equations of state represent completely catalyzed matter of zero temperature. Thus, their investigation was free from the three shortcomings mentioned before. They found that rotation stabilizes equilibrium states for the density range 10^8–$10^{12.5}$ g cm^{-3}. However, for densities larger than $10^{12.5}$ g cm^{-3}, the equation of state is too soft to make equilibrium states stable. Consequently, the secular evolution of white dwarfs or compact cores of massive stars due to gravitational emission will turn into a dynamically unstable collapse and lead to a supernova explosion or black hole formation depending on the mass and the angular momentum of the collapsing white dwarfs or stellar cores.

According to the results of Shapiro & Lightman (1976) and those of Müller & Eriguchi (1985), fizzlers seem to exist only for a part of the process of evolution from the white dwarf region to the neutron star region. However, there remains one important factor that is not taken into account in their investigations. When we consider a white dwarf state, not an isolated white dwarf, for the core of a massive star, we need to consider the effect of the high temperature of the core. Wiita & Press (1976) discussed the effect of the temperature when they considered rotational states of compact stars. However, they only stated that the temperature effect would be negligible or very small without pursuing quantitative treatments.

The temperature effect was treated quantitatively by Glatzel, Fricke, & El Eid (1981). They were interested in the evolution of rotating massive stars and constructed equilibrium states of rotating cores with high temperature. In order to take the rotational effect into account, they assumed that mechanical equilibrium states could be approximated by Maclaurin spheroids. On the other hand, concerning the matter of the core, high-temperature cores were assumed to be in a nuclear statistical equilibrium state. Therefore, the thermodynamical quantities could be determined from three quantities, i.e., the temperature, the density, and the electron fraction. Since the results of realistic computations of spherical stars show that cores consist of iron-group nuclei, they took ^{56}Fe into consideration and obtained the equation of state for constant entropy per baryon. They investigated the density range $10^{8.4}$–10^{13} g cm^{-3}. For this density range, they assumed that the values for the entropy and the electron fraction could be taken from the knowledge of spherical stars. Thus, they used rather high entropies, $1 \leq s/k \leq 2$, and rather large values of the electron fraction, $0.41 < Y_e \leq 0.4643$. Here s, k, and Y_e are the entropy per baryon, the Boltzmann constant, and the electron fraction, respectively. By applying the stability criterion for axisymmetric collapse, they found that the rotational effect combined with the temperature effect causes stars to be dynamically stable, if appropriate conditions for the entropy per baryon, the electron fraction, and the total angular momentum are satisfied. Roughly speaking, as is expected, larger values for these three quantities help stars to be dynamically stable.

Their result implies that the rotational contribution to stabilization is more effective at high temperature than at low temperature. However, Glatzel et al.'s results are obtained by using *approximate equilibrium configurations* in a rather narrow density range at lower density. Therefore, it would be interesting to investigate rotational stabilization for high-temperature stars by employing *exact equilibrium states* of rotating stars for a wider density range, say, 10^8–10^{15} g cm^{-3}.

In this paper, we develop a scheme to compute numerically exact equilibrium configurations of *rapidly rotating hot* stars. The temperature effect can be included through the equation of state that is determined by three thermodynamic variables: the entropy per baryon, the electron fraction, and the baryon number density. For fixed values of the entropy per baryon and the electron fraction, we compute sequences of equilibrium models with different angular momenta. By applying stability criteria for axisymmetric collapse and gravitational radiation emission to the obtained equilibrium configurations, we will find out whether or not dynamically stable but secularly unstable configurations can exist from the low-density region to the high-density region.

2. EQUILIBRIUM CONFIGURATIONS OF RAPIDLY ROTATING AND HOT COMPACT STARS

2.1. *Assumptions and Basic Equations*

It is assumed that the rotating stars treated in this paper satisfy the following conditions. Configurations are (1) equatorially symmetric, (2) axisymmetric, and (3) in stationary states. (4) There are no internal flows, except motion due to rotation. (5) Viscosity is neglected. We will take the temperature effect into consideration by assuming that (6) the entropy per baryon is constant throughout the star. This isentropic assumption allows us to treat equilibrium configurations as barotropes. Furthermore, (7) Newtonian gravity is used instead of general relativity even for neutron stars. Therefore, values obtained for strong gravity regions will be different from those obtained from general relativity. However, the purpose of this paper is to show that secularly unstable but dynamically stable equilibrium configurations can exist. Thus, if we are able to show that such configurations can exist in a very wide parameter range, it will be very likely that such configurations do in fact exist, even if general relativity is employed.

Under these assumptions, the hydrostatic equation can be expressed as

$$\frac{1}{\rho} \nabla p = -\nabla \phi_g + r \sin \theta \Omega^2 e_R , \tag{1}$$

where ρ, p, ϕ_g, Ω, and e_R are the density, the pressure, the gravitational potential, the angular velocity, and the unit vector in the R-direction, respectively. Here spherical coordinates (r, θ, φ) are used and $R = r \sin \theta$. The gravitational potential satisfies Poisson's equation:

$$\Delta \phi_g = 4\pi G \rho . \tag{2}$$

As for the boundary conditions, the physical quantities are regular all through space, and the surface is defined by a set of points where the pressure vanishes:

$$p = 0 \quad \text{(on the surface)} . \tag{3}$$

Concerning the gravitational potential, it is regular everywhere and tends to zero at infinity:

$$\phi_g \to 0 \quad (r \to \infty) . \tag{4}$$

To compute equilibrium configurations, we need to specify the rotation law. In this paper, we choose the following two representative laws:

$$\Omega^2 = \frac{v_0^2}{R^2 + A^2} \tag{5a}$$

and

$$\Omega^2 = \frac{j_0^2}{(R^2 + A^2)^2} , \quad (5b)$$

where R is the distance from the rotation axis and v_0, j_0, and A are all constants. We will call the first law the v-constant law (the velocity is constant for a small value of A), and we will call the second law the j-constant law (the specific angular momentum is constant for a small value of A).

As mentioned before, these stars can be treated as barotropes. Thus, the hydrostatic equation (1) can be integrated to give

$$H = -\phi_g + h_0^2 \phi_{\rm rot} + C , \quad (6)$$

where C is an integral constant and H is the enthalpy defined by

$$H = \int \frac{dp}{\rho} . \quad (7)$$

The quantity $\phi_{\rm rot}$ is the centrifugal potential, and h_0 is a constant. They can be expressed as

$$\phi_{\rm rot} = \frac{1}{2} \ln (R^2 + A^2),\ h_0^2 = v_0^2,\ v\text{-constant} \quad (8a)$$

and

$$\phi_{\rm rot} = -\frac{1}{2} \frac{1}{R^2 + A^2} , \quad h_0^2 = j_0^2,\ j\text{-constant} . \quad (8b)$$

The gravitational potential is transformed into an integral form because the boundary condition can easily be taken into account:

$$\phi_g = -G \int \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} d^3 r' . \quad (9)$$

This potential for axisymmetric configurations can be expanded in terms of the Legendre polynomials P_n as

$$\phi_g(r, \theta) = -4\pi G \sum_{n=0}^{\infty} P_{2n}(\cos \theta) \int_0^{\infty} r'^2 dr' \times \int_0^{\pi/2} \sin \theta' d\theta' f_n(r, r') P_{2n}(\cos \theta') \rho(r', \theta') , \quad (10)$$

where

$$f_n(r, r') = \begin{cases} \frac{1}{r} \left(\frac{r'}{r}\right)^n & \text{for } r' < r , \\ \frac{1}{r'} \left(\frac{r}{r'}\right)^n & \text{for } r' > r . \end{cases} \quad (11)$$

2.2. *Equation of State*

In this paper, we want to investigate rotating and hot compact stars in the range from the density of white dwarfs to that of neutron stars. In such a wide range of density, equations of state are not firmly determined because of the lack of experimental data for basic interactions of nucleons and other elementary particles. For zero temperature matter with a lower density than the nuclear matter density $\rho_n \sim 2 \times 10^{14}$ g cm^{-3}, since nuclear interactions determined by experiments are used, uncertainties of the equation of state are small. The typical equation of state below neutron drip density is that of Baym et al. (1971b) (hereafter, we will call this equation of state BPS). However, even if the temperature is zero, interactions above ρ_n are poorly understood so that many different equations of state have been proposed by considering different kinds of interactions.

Since it is very hard to determine the equation of state above the nuclear matter density from nuclear physics and/or elementary particle physics at present, other methods in which astrophysical phenomena are utilized have been considered. The mass of some neutron stars can be determined quite precisely, e.g., $M = 1.44\ M_\odot$ for the neutron star in the binary pulsar PSR 1931+16 (Taylor & Weisberg 1989). This implies that equations of state that give a maximum mass less than 1.44 $M_\odot$ have to be abandoned (see, e.g., Friedman, Ipser, & Parker 1984, 1986, 1989; Eriguchi, Hachisu, & Nomoto 1994).

Although the situation is much worse for hot dense matter, we need to treat an important astrophysical situation, i.e., the hot and dense state of a supernova explosion. Thus, many kinds of equations of state that represent the states of high temperature and high density have been proposed (e.g., Lattimer & Swesty 1991; Takatsuka, Nishizaki, & Hiura 1994; Sumiyoshi & Toki 1994). Therefore, it should be noted that conclusions obtained by using these equations of state are uncertain to some extent.

In this paper, we employ the equation of state of Lattimer & Swesty (1991). Their equation of state has been computed by using the compressible liquid drop model at finite temperature and contains one free parameter, i.e., incompressibility, whose value has not been determined definitely yet. The pressure can be computed by having a very high degree of the Helmholtz free energy of the matter, which consists of one kind of nuclei, α-particles, neutrons that drop out from neutron-rich nuclei, electrons, positrons, and photons. In their equations of state, there are three input thermodynamic variables: the baryon density, the entropy per baryon or the temperature, and the electron fraction. If the temperature and the density are specified in addition to the condition of β-equilibrium, the state of matter can be uniquely determined. However, since weak interactions with neutrino change the electron fraction through nonequilibrium processes such as gravitational collapse and/or supernova explosion for hot and high-density matter, the three variables are treated as independent in the equations of state by Lattimer & Swesty (1991).

Consequently, we will use the equation of state expressed by the following form for the high-temperature region:

$$p = p(\rho; s, Y_e) , \quad (12)$$

where s and Y_e are considered to be parameters that are constant throughout the high-temperature region of the star. Since the temperature near the surface is very low, we employ the BPS equation of state for the region where the density becomes less than 10^7 g cm^{-3}.

We need to comment on our assumption of the constancy of the electron fraction through the high-temperature region. As mentioned before, the precise equation of state has not been determined for hot matter with high densities. Furthermore, the state of hot matter would depend on neutrino interactions with matter. Nevertheless, there have been several attempts to constrain the equation of state for collapsing stars from analytical considerations (e.g., Bethe et al. 1979) and from hydrodynamical computations of the collapse of massive stars (e.g., Mezzacappa & Bruenn 1993; they started their calculations from evolutionary models of massive stars [e.g., Weaver, Zimmerman, & Woosley 1978; Hashimoto 1995]). In these investigations, the values and distributions of entropy and the electron fraction have been obtained, although they depend on the models employed in their treatments.

In particular, Mezzacappa & Bruenn (1993) computed the infall phase of massive stars up to the density region $\rho_{max} \sim 10^{14}$ g cm^{-3}. When the central density ρ_{max} reaches $\sim 10^{12}$ g cm^{-3}, owing to the neutrino trapping, the distribution of the lepton fraction becomes flat inside the central region that contains two-thirds of the total mass of the core. On the other hand, the electron fraction behaves somewhat differently. The distribution of the electron fraction becomes flat in the region mentioned above when the central density is about 10^{12} g cm^{-3}. Then the slope of the distribution begins to grow as the central density becomes higher because of the electron capture. However, a plateau in the electron fraction forms in the central region with two-thirds of the core mass when the central density reaches 10^{14} g cm^{-3}. A similar trend for Y_e was found by Myra (1988). Although Y_e decreases during contraction, after neutrino trapping, the lepton (neutrino) pressure compensates for the decrease of the electron pressure because the lepton fraction remains nearly constant. In effect, our assumption of constant Y_e can be considered the constancy of the lepton fraction. Therefore, as the first approximation, it makes the problem tractable to assume that the distribution of the electron fraction is constant throughout space. By using results obtained from this uniform distribution of the electron fraction, we will discuss the *necessary conditions* for fizzlers to appear.

It should be noted that, in the hot matter of collapsing cores, the deleptonization does not always work effectively. Therefore, contrary to the monotonically decreasing electron fraction for cold catalyzed matter due to the β-equilibrium condition (see the BPS equation of state in Fig. 1), the value of the electron fraction remains large even up to the nuclear density ρ_n. The value of Y_e may be from ~ 0.3 to ~ 0.5. At the same time, the values of entropy range from

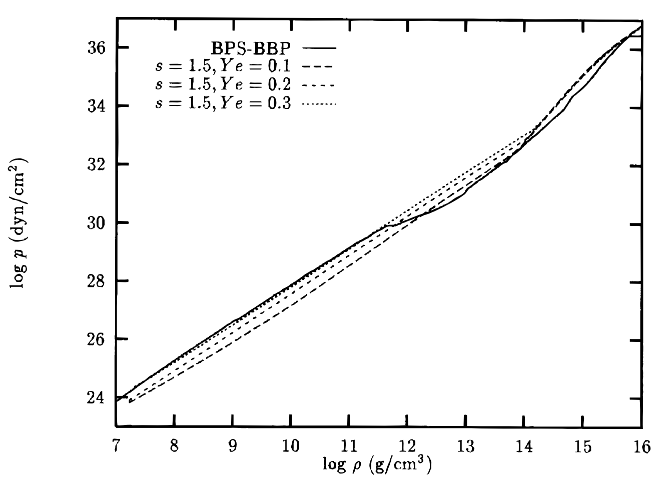

FIG. 1.—The equations of state are shown in the density-pressure plane. The parameters are $K_{com} = 180$ MeV, $s/k = 1.5$, and $Y_e = 0.1$, 0.2, and 0.3 for equations of state by Lattimer & Swesty (1991). For comparison, we also plot the equation of state of Baym et al. (1971b) and of Baym et al. (1971a).

$s \sim 0.7$ to ~ 2.0. These ranges cover the hot and dense supernova matter (e.g., Bethe 1990).

The parameters chosen in our computations are summarized in Table 1. In this table, the quantity K_{com} is the parameter that expresses the incompressibility of nuclear matter. The larger it becomes, the stiffer the equation of state above ρ_n becomes. The values of the entropy per baryon are taken so as to cover the values that appear in both the presupernova stage (see, e.g., Hashimoto 1995) and the core-collapse stage (see, e.g., Bethe et al. 1979; Mezzacappa & Bruenn 1993). Since the exact electron fraction (or lepton fraction) cannot be determined without time-dependent calculations, and since the values from various investigations vary (e.g., Bethe et al. 1979; Weaver et al. 1978; Mezzacappa & Bruenn 1993; Hashimoto 1995), we select three values ($Y_e = 0.15$, 0.2, and 0.3) that are rather small in order not to overestimate the high-temperature effect. The softness of the equation of state of Lattimer & Swesty can be estimated from the maximum mass for a spherical neutron star. If we choose $K_{com} = 180$ MeV, $s/k = 1.0$, and $Y_e = 0.3$, the maximum mass of a spherical neutron star is $M_{max} = 1.77 M_{\odot}$.

In Figure 1, the equations of state are shown for parameters $K_{com} = 180$ MeV, $s/k = 1.5$, and $Y_e = 0.1$–0.3. In this figure, the BPS-BBP equation of state is also drawn for comparison. Here BBP denotes the equation of state by Baym et al. (1971a). It should be noted that BPS-BBP is the case of zero temperature with the β-equilibrium condition. As seen from this figure, the BPS-BBP equation of state is stiff in the white dwarf region because of the higher value of Y_e but becomes softer in the density region 10^{12}–10^{14} g cm^{-3} because of the lower value of Y_e that is the result of neutronization. For the BPS equation of state, $Y_e \sim 0.3$ at $\rho = 4.3 \times 10^{11}$ g cm^{-3} where neutrons begin to drip. During the stage of neutron drip, the pressure becomes very soft. The smaller value of Y_e implies that the contribution to the total pressure of the electrons becomes weak. In Figure 2, the temperature of the equations of state is expressed as a function of the density. Above 10^{14} g cm^{-3}, all equations of state with different Y_e are bunched because the contribution of the interacting neutron gas becomes dominant. On the other hand, the BBP equation of state above ρ_n is softer than that of Lattimer & Swesty because BBP used the Reid soft core potential as the nuclear force.

2.3. *Solving Scheme*

A rotating star can be uniquely determined by two parameters, if the equation of state and the rotation law are prescribed. These two parameters should represent the

TABLE 1

MODEL PARAMETERS

	$K_{com} = 180$ MeV			
FIGURE	s/k	Y_e	Rotation Law	A/r_e
3	1.5	0.3	j-constant	0.25
4	0.5	0.3	j-constant	0.25
5	0.05	0.3	j-constant	0.25
6	1.5	0.3	j-constant	0.4
7	1.5	0.3	v-constant	0.1
8	1.5	0.2	j-constant	0.25
9	0.5	0.2	j-constant	0.25
10	1.5	0.15	j-constant	0.25

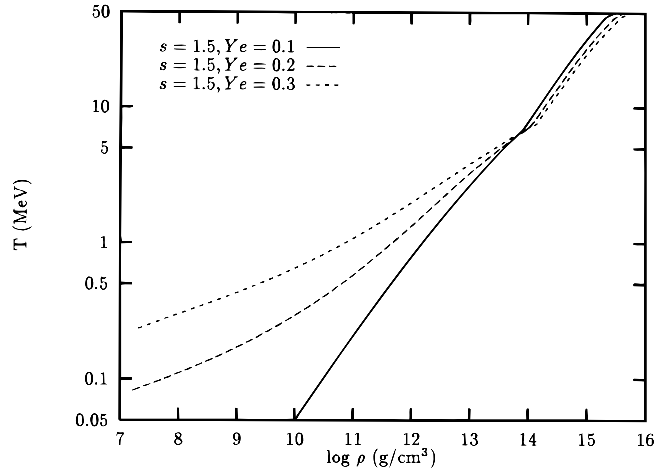

FIG. 2.—The same equations of state in Fig. 1 are plotted in the density-temperature plane.

strength of gravity and the amount of rotation. In this paper, we choose the ratio of the polar radius r_p to the equatorial radius r_e and the maximum density $\rho_{\max}$ as two model parameters. Once these parameters are chosen, the basic equations (6) and (10) are solved for the prescribed rotation law and for the equation of state by using a combination of the Eriguchi & Müller scheme (Eriguchi & Müller 1985, 1991) and the Hachisu self-consistent field (HSCF) scheme (Hachisu 1986) explained below. These two schemes are essentially iterative schemes to solve nonlinear equations, and they have proved to be very powerful. Concerning the equations of state, we employ the piecewise-polytropic approximation devised by Müller & Eriguchi (1985).

We will briefly explain the modified part of the scheme from the original schemes (Eriguchi & Müller 1985; Hachisu 1986). In this paper, we use the surface-fitted coordinate system first devised by Eriguchi & Müller (1991) and extensively used by Uryu & Eriguchi (1994, 1995). The primitive type of surface-fitted coordinate was devised originally by Eriguchi & Müller (1985). In their paper, the gravitational potential is computed by using a kind of surface-fitted coordinate to avoid the vacuum region. Since we are going to compute significantly deformed configurations because of very rapid rotation, and since the vacuum region has no meaning in Newtonian gravity, it would be helpful to increase the accuracy of the results in order to avoid the vacuum regions in the numerical computations. However, the original scheme requires more computational time because it is based on the Newton-Raphson iteration scheme. On the other hand, the HSCF scheme is the extended version of the self-consistent field method in which the density and the gravitational potential are calculated iteratively. Since the computations can be done very rapidly for the HSCF scheme, we combine the two schemes in order to speed up the computations and not to lose accuracy. One more revision is related to the mesh spacing in the r-direction. For some regions, we need to treat a very soft equation of state that requires us to use very fine mesh sizes near the central region, as was done by Müller & Eriguchi (1985).

After we obtain the equilibrium configurations, we will compute some physical quantities such as the total mass M, the total angular momentum J, the rotational energy T, and the gravitational energy W defined by

$$M = \int \rho \, d^3\boldsymbol{r} \,, \tag{13}$$

$$J = \int \rho R \Omega^2 \, d^3\boldsymbol{r} \,, \tag{14}$$

$$T = \frac{1}{2} \int \rho R^2 \Omega^2 \, d^3\boldsymbol{r} \,, \tag{15}$$

and

$$W = \frac{1}{2} \int \rho \phi_g \, d^3\boldsymbol{r} \,. \tag{16}$$

3. STABILITY CRITERIA AND FIZZLERS

The equilibrium configurations that will be obtained by the procedure explained in the previous section have to be checked for instabilities against axisymmetric and/or nonaxisymmetric perturbations.

3.1. *Dynamical Instability*

Concerning rotating configurations, we need to consider two kinds of dynamical instabilities: axisymmetric collapse and nonaxisymmetric bar mode instability. Although it is not exactly shown, the bar mode instability occurs when the ratio of the rotational energy to the absolute value of the gravitational energy exceeds a critical value that seems universal, irrespective of the rotation law and the equation of state (Ostriker & Bodenheimer 1973; Durisen & Tohline 1985), i.e.,

$$\frac{T}{|W|} > 0.26\text{–}0.27 \,. \tag{17}$$

Thus, equilibrium models whose value of $T/|W|$ exceeds this critical one will evolve into two-armed spiral-type configurations on a dynamical timescale (see, e.g., Pickett, Durisen, & Davis 1996; Smith, Houser, & Centrella 1996). However, since only very rapidly rotating configurations satisfy this condition, most of configurations treated in this paper will be free from this instability.

The other dynamical instability we need to consider is the axisymmetric " radial " instability, i.e., the axisymmetric collapse of configurations. This instability can be investigated by analyzing the oscillation modes of linearized equations and finding the zero-frequency modes. However, there is a much simpler method for analyzing this instability by using a sequence of equilibrium models. For Newtonian rotating stars, this scheme was developed by Bisnovatsyi-Kogan & Blinnikov (1974). For isentropic configurations, the dynamical instability sets in at the extreme point of the mass with respect to the central density if the specific angular momentum distribution is kept fixed, i.e.,

$$\frac{dM}{d\rho_{\max}}\Big|_{j(q)} = 0 \,, \tag{18}$$

where $j(q)$ is the specific angular momentum distribution at the point where this criterion is applied, and q is the ratio of the mass inside the distance from the rotation axis to the total mass.

In actual computations, it is not easy to find the model that satisfies condition (18) by constructing a sequence of models whose distribution of the specific angular momentum is the same. However, Bisnovatsyi-Kogan & Blinnikov show that for ordinary situations, the critical point is located very near the point where the following condition is satisfied:

$$\frac{dM}{d\rho_{\max}}|_J = 0 . \tag{19}$$

To apply this condition, we need to have an equilibrium sequence of the same total angular momentum. In this paper, we will use condition (19) to get an approximate point for the onset of the axisymmetric dynamical instability. Since, as we discussed before, there are uncertainties about the equations of state, the exact values of the instability points for these equations of state will not apply to the real equation of state. Moreover, as will be shown later, most configurations are obtained by the *j-constant rotation law* equation (5b) so that if the total angular momentum is the same, the distribution of the specific angular momentum becomes almost the same except in the very central region. Thus, condition (19) will be a good approximation of condition (18) in this paper.

3.2. *Secular Instability Due to Gravitational Radiation Emission*

Concerning rotating stars, there are two different mechanisms that will excite the secular instability of nonaxisymmetric modes. The first mechanism is related to viscosity. This instability will lead axisymmetric rotating stars to become nonaxisymmetric rotating configurations because of the existence of viscosity. The well-know bifurcation from the Maclaurin spheroids to the Jacobi ellipsoids is caused by this instability (see, e.g., Chandrasekhar 1969; Press & Teukolsky 1973). It should be noted that for higher modes (i.e., $m \geq 3$, where m is the azimuthal quantum number), only much more rapidly rotating stars will suffer from this instability (see, e.g., Chandrasekhar 1969; Tassoul 1978).

The situation mentioned above is valid for *uniformly* rotating objects. For *differentially* rotating object, only little is known. It is very likely that friction in the shear flow will cause stars to evolve secularly. However, the timescales of such evolutions have not been clarified yet. Even if the timescales might be short, the final outcomes of the evolution might not be significantly nonaxisymmetric. Therefore, in this paper, we will not take this instability into consideration, although its effect may be important.

The second instability is excited by the emission of gravitational radiation. This mechanism was found by Chandrasekhar (1970). This instability for the $m = 2$ mode occurs at the same point as the secular instability point because of the viscosity for uniformly rotating stars (Ipser & Managan 1985). Since we will investigate compact stars such as white dwarfs and neutron stars, the effect of gravitational radiation is the main mechanism that excites the instability.

In the early stages of the investigations, this instability was found to occur if $T/|W| \sim 0.14$, irrespective of the rotation law and the equation of state by Ostriker & Bodenheimer (1973). However, as precise and recent treatments by Managan (1985) and Imamura et al. (1995) have shown, the critical values of $T/|W|$ tend to decrease for compressible stars with significantly differential rotation. Thus, for differentially rotating stars, the above value should be considered an overestimated value for the onset of the secular instability.

Moreover, as Lindblom & Detweiler (1977) have shown, the two instabilities for the $m = 2$ mode mentioned above tend to cancel each other out if the strengths of the viscous and the radiative forces are of the same order. However, the viscosities for compact objects such as white dwarfs or neutron stars have not been firmly determined yet. Therefore, in this paper, we will pursue the possibility that viscosities for compact objects are small enough in comparison with the critical ones above which the viscous effect becomes dominant.

Therefore, we will employ the following condition for secular instability due to gravitational radiation emission:

$$\frac{T}{|W|} > 0.14 . \tag{20}$$

For other modes with larger m, it has been shown that rotating compressible stars become unstable for more slowly rotating configurations because of gravitational radiation emission compared with the case for the $m = 2$ mode (Managan 1985; Imamura, Friedman, & Durisen 1985; Imamura et al. 1995). Therefore, the value of equation (20) may be considered an upper limit, even for other m modes as the critical value for the onset of the secular instability, although we need to consider the cancellations due to the viscosity in order to be more precise.

The timescale $\tau_{\rm sec}$ of the growth of this instability due to gravitational radiation is given as follows (Friedman & Schutz 1975):

$$\tau_{\rm sec} \sim 10^{-7}\left(\frac{M}{M_\odot}\right)^{-3}\left(\frac{R}{100\ \rm km}\right)^{4}\left(\frac{T}{|W|}\right)^{-3} \times \left[\frac{T}{|W|} - \left(\frac{T}{|W|}\right)_c\right]^{-5} \rm s , \tag{21}$$

where $(T/|W|)_c$ is the critical value of the ratio of the rotational energy to the gravitational energy where the instability sets in. From this expression, we can see that the more slowly a star rotates, and consequently the smaller the value of $T/|W|$, the longer the timescale becomes. Therefore, the evolution due to this instability for smaller values of m is faster so that the $m = 2$ mode is the most important, as long as viscosity can be neglected (see, e.g., Ipser & Lindblom 1991; Lindblom 1995; Yoshida & Eriguchi 1995).

3.3. *Fizzlers*

Since rotating stars suffer from the instabilities discussed in the previous subsections, equilibrium configurations are divided into several categories according to their stability nature. In this paper, we will not treat very rapidly rotating configurations that are unstable to the bar mode dynamical instability. Therefore, we will consider only two instabilities: the dynamical instability for axisymmetric collapse and the secular instability due to gravitational radiation emission. Considering these two instabilities, we can assume the following three categories for equilibrium configurations:

Category 1.—Stars that are dynamically stable against axisymmetric collapse and secularly stable against gravitational radiation emission.

Category 2.—Stars that are dynamically stable against axisymmetric collapse but secularly unstable due to gravitational radiation emission.

Category 3a.—Stars that are dynamically unstable for axisymmetric collapse but secularly stable against gravitational radiation emission.

Category 3b.—Stars that are dynamically unstable for axisymmetric collapse and secularly unstable due to gravitational radiation emission.

Although the definitions of categories 3a and 3b are different, stars in both categories will collapse on a dynamical timescale. In this sense, these two categories are essentially the same. Therefore, we will use category 3 for both types of stars.

Among these three categories, the second one is the most interesting. Equilibrium configurations in this category can evolve on a long timescale compared with the dynamical timescale. Since the gravitational waves carry angular momentum away, stars in this category will contract to higher densities because of angular momentum loss. Although energy is also carried away from the stars, its amount is so small that the mass is considered to be conserved.

Therefore, we will propose the following *hypothetical* path for one of stellar evolutions. Stars in category 2 will evolve to a higher density region while keeping their masses constant. As angular momentum is lost in this process because of gravitational emission, configurations necessarily contract to higher density regions. In other words, the star will move to states in either category 1 or category 3. A star leaving category 2 can be characterized by its mass M and angular momentum J_f. For each equilibrium sequence along which the total angular momentum is constant, there exists a model whose mass is the maximum. Let this maximum mass be $M_{\max}(J)$. If $M < M_{\max}(J_f)$, the star settles down to category 1. If $M \geq M_{\max}(J_f)$, the star will continue to contract and finally collapse into a black hole. It should be noted that, as long as stars remain in the category 2 region, the evolution proceeds on a secular timescale. These are just the stars that Gold (1974) called "fizzlers."

4. COMPUTATIONAL RESULTS AND DISCUSSIONS

As was discussed before, although in this paper the configurations of strong gravity such as neutron stars are treated, we have used Newtonian gravity instead of general relativity. Thus, the values obtained for the region beyond the density $\rho_n \sim 2 \times 10^{14}$ g cm^{-3} should not be treated as exact ones, because for neutron stars with $\rho_{\max} \sim \rho_n$ the following relation holds:

$$\frac{2GM}{c^2R} \sim 0.1 , \tag{22}$$

where M and R are the mass and the radius of the neutron star, respectively. Here G and c are the gravitational constant and the velocity of light, respectively. This value implies that the strength of gravity for neutron stars up to ρ_n can be roughly treated by Newtonian gravity.

4.1. *Model Parameters*

In order to obtain an equilibrium configuration, we need to choose the equation of state and the rotation law. For the equations of state by Lattimer & Swesty (1991), we fix the parameter of incompressibility $K_{\text{com}} = 180$ MeV and adopt the two quantities s/k and Y_e as parameters. We believe that our results, in particular their qualitative behaviors, are independent of K_{com}. Concerning the rotation law, we choose one from equations (5a) and (5b) and specify the value of the parameter A. As mentioned before, besides these choices, we need to specify two more parameters: (1) the maximum density $\rho_{\max}$, which represents the strength of gravity, and (2) the ratio of the polar radius to the equatorial radius, which represents the amount of rotation. By changing these two parameters, $\rho_{\max}$ and r_p/r_e, we can have sequences of equilibrium configurations.

In actual computations, when the equation of state and the rotation law are prescribed and the parameters are specified, we start computing one equilibrium sequence by changing the ratio r_p/r_e and keeping $\rho_{\max}$ fixed. After one sequence has been completed, we change the value of the maximum density and follow the same procedure to obtain other sequences. In this way, we can have equilibrium configurations with different masses, different angular momenta, and different maximum densities. Once equilibrium configurations in a wide range of parameters have been obtained, we can select equilibrium configurations with the same total angular momentum in order to study their dynamical stability according to the criterion mentioned before, i.e., condition (19).

The equations of state and the rotation laws used in this paper are shown in Table 1. In this table, the values of the entropy per baryon, the electron fraction, the type of the rotation law, and the value of the rotation parameter A are displayed. The density range we have investigated covers $\rho_{\max} = 10^8$–$10^{14.5}$ g cm^{-3}.

In numerical computations, we used the mesh number of $(r \times \theta) = (48 \times 11)$. The number of Legendre polynomials in the expansion of the gravitational potential is 10 for most models. The accuracy of the obtained equilibrium models can be estimated by the value of the following quantity that is derived from the virial relation (see, e.g., Tassoul 1978):

$$VT = \frac{|2T + W + 3\int p\,dV|}{|W|} . \tag{23}$$

The value of VT for our models ranges from several times 10^{-3} for the worst cases to several times 10^{-4} for most models. This value is roughly considered to be the accuracy of equilibrium configurations.

4.2. *Results*

It is important to know the slope of the mass curve as a function of the maximum density for a sequence along which the total angular momentum is constant. Equilibrium configurations with positive slopes are stable against axisymmetric collapse, as discussed before.

In Figures 3, 4, and 5, the mass of equilibrium configurations is plotted against the maximum density for several sequences with constant angular momenta. All the configurations in these figures are computed by using the *j*-constant rotation law, i.e., equation (5b) with the rotation parameter $A/r_e = 0.25$. As for the equation of state, the same electron fraction is chosen, i.e., $Y_e = 0.3$, but the values of the entropy per baryon are different: $s/k = 1.5$ for Figure 3, $s/k = 0.5$ for Figure 4, and $s/k = 0.05$ for Figure 5.

In each figure, different curves represent sequences with different angular momenta. It should be noted that for spherical sequences denoted by solid curves, slopes are negative from $\rho_{\max} \sim 10^8$ g cm^{-3} to $\rho_{\max} \sim 2 \times 10^{14}$ g cm^{-3}. This implies that spherical stars in this region are dynamically unstable to axisymmetric collapse. For lower

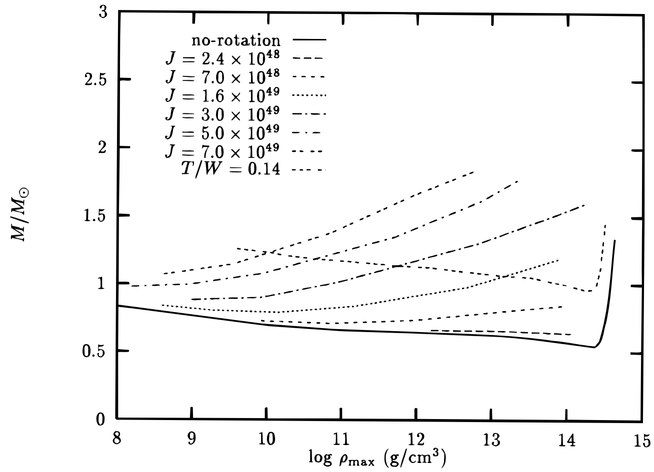

FIG. 3.—The equilibrium sequences along which the total angular momentum is the same are plotted in the maximum density–mass plane. The equation of state is that of Lattimer & Swesty (1991), with parameters $K_{\rm com} = 180$ MeV, $s/k = 1.5$, and $Y_e = 0.3$. The j-constant rotation law, equation (5b) with $A/r_e = 0.25$, is used. The lowest curve corresponds to spherical models. The secular stability limit, equation (20), is also drawn. Above this curve, all equilibrium configurations are unstable because of gravitational radiation emission.

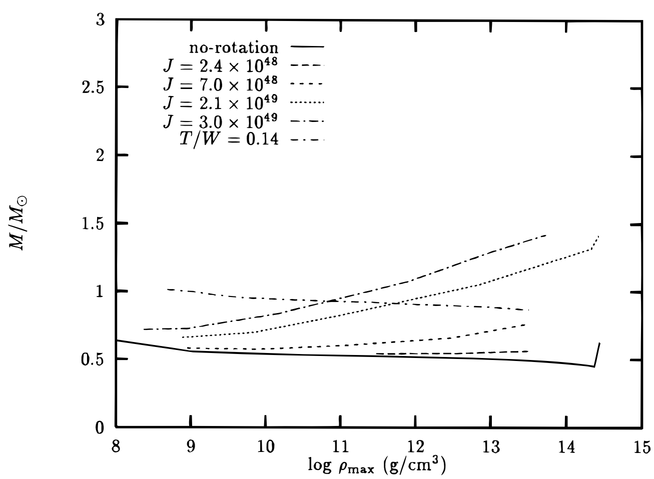

FIG. 4.—Same as Fig. 3, but for $s/k = 0.5$

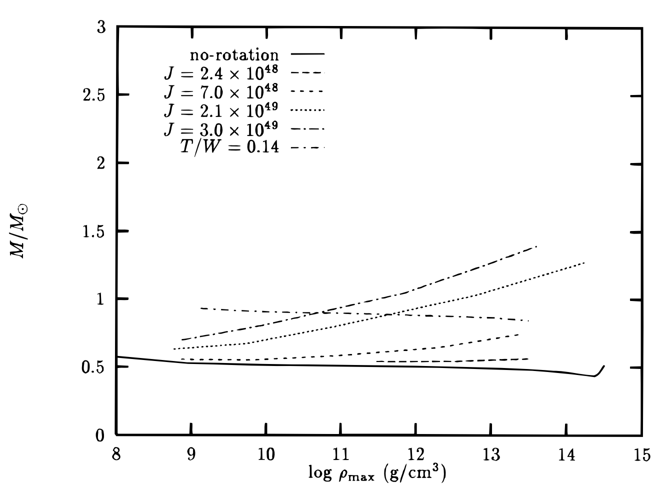

FIG. 5.—Same as Fig. 3, but for $s/k = 0.05$

angular momenta, slopes in this region are negative, but the absolute values are smaller than those of spherical sequences. If the angular momentum is larger than a certain critical value that varies depending on the value of the entropy per baryon as seen from figures, the slopes become positive. Therefore, rotation makes the equilibrium configurations stable as long as the constancy of the electron fraction and the entropy per baryon is assumed. This tendency is common to all three cases. The difference in the entropy per baryon results in different values of the mass for the same maximum density. Larger values of s allow larger values of the mass.

In these figures, the curve that marks the secular instability limit, i.e., models with $T/|W| \sim 0.14$, is also drawn. In the region above this curve, the slopes of the constant angular momentum curves are positive, i.e., dynamically stable against axisymmetric collapse. Therefore, stars in this region belong to category 2 in the previous section. It is also important to note that the slope of this critical curve is negative up to $\rho_{\max} \sim 2 \times 10^{14}$ g cm^{-3}. Since fizzlers evolve horizontally to the right direction in this plane, it implies that they always stay in this category 2 region and remain as fizzlers until they reach the neutron star region.

In order to see the influence of the rotation parameter A and the rotation law, we have computed different equilibrium sequences both for different values of A and for the different rotation law by using the same parameters for the equation of state. In Figures 6 and 7, equilibrium sequences with constant angular momenta are shown in the same maximum density–mass plane as in Figures 3–5. In Figure 6, models for the j-constant rotation law with $A/r_e = 0.4$ are shown. In Figure 7, the rotation law is that of the v-constant rotation law, i.e., equation (5a) with $A/r_e = 0.1$. As seen from these figures, the tendency is the same as that of Figures 3–5. The most important thing is that the constant angular momentum curves in these planes almost coincide if the value of the angular momentum is the same, in spite of the different angular momentum or different angular velocity distributions (see also Müller & Eriguchi 1985). This implies that once the equation of state is chosen, the structure and the stability nature of the equilibrium configurations are determined solely by the amount of the total angular momentum.

In Figures 8, 9, and 10, equilibrium sequences in the maximum density–mass plane are plotted for different

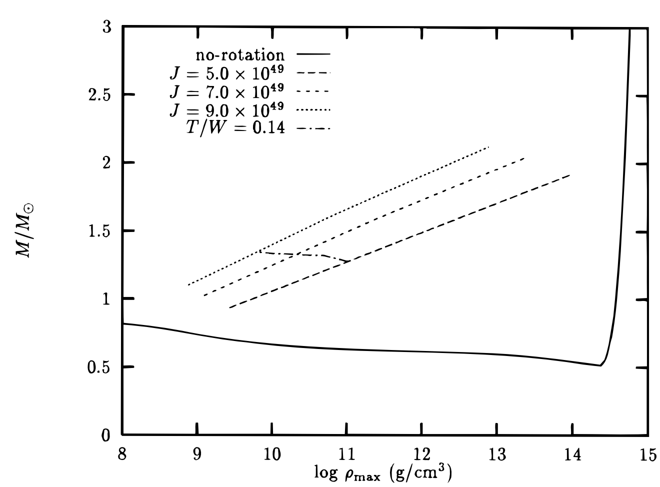

FIG. 6.—Same as Fig. 3, but for $A/r_e = 0.4$

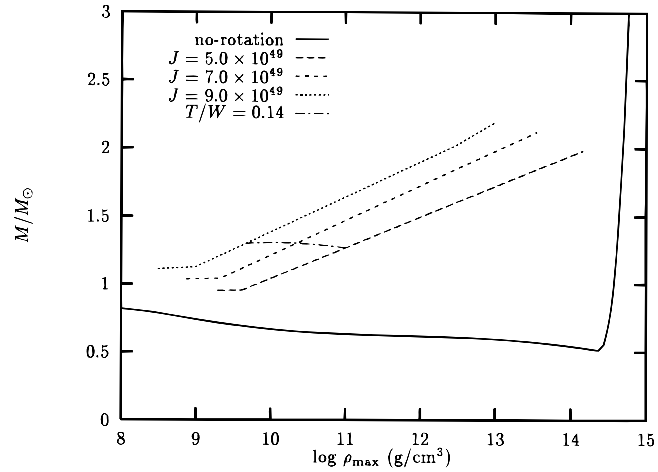

FIG. 7.—Same as Fig. 3, but for the v-constant rotation law with $A/r_e = 0.1$.

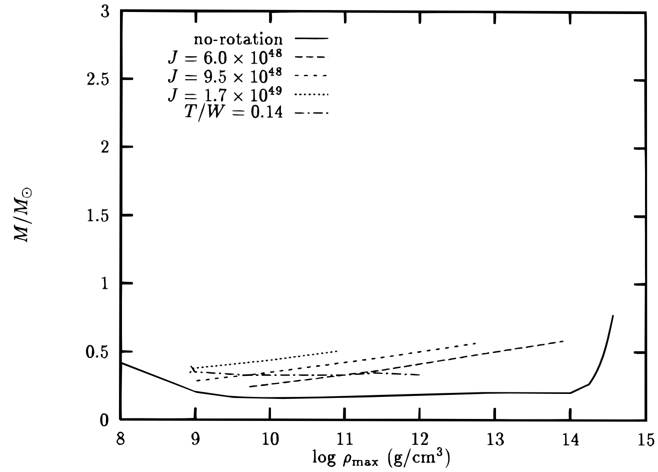

FIG. 10.—Same as Fig. 3, but for $Y_e = 0.15$

parameters of the equation of state when the j-constant rotation law with $A/r_e = 0.25$ is chosen. Comparing sequences in Figures 3, 8, and 10 for $s/k = 1.5$ and different Y_e, or sequences in Figures 4 and 9 for $s/k = 0.5$ and different Y_e, we can see the effect of the electron fraction on the structure and the stability of the equilibrium configurations.

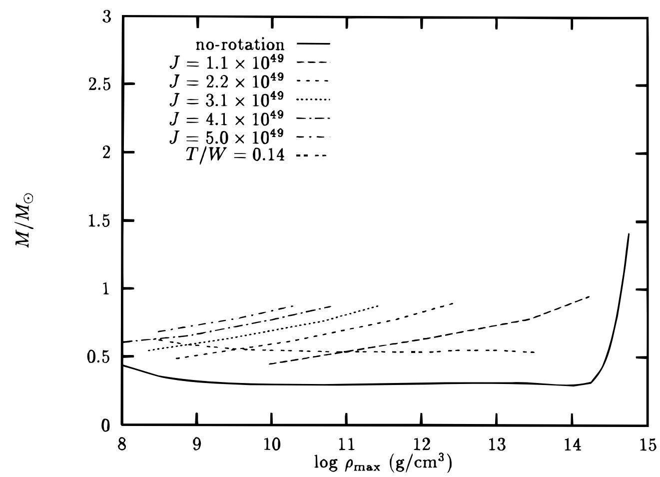

FIG. 8.—Same as Fig. 3, but for $Y_e = 0.2$

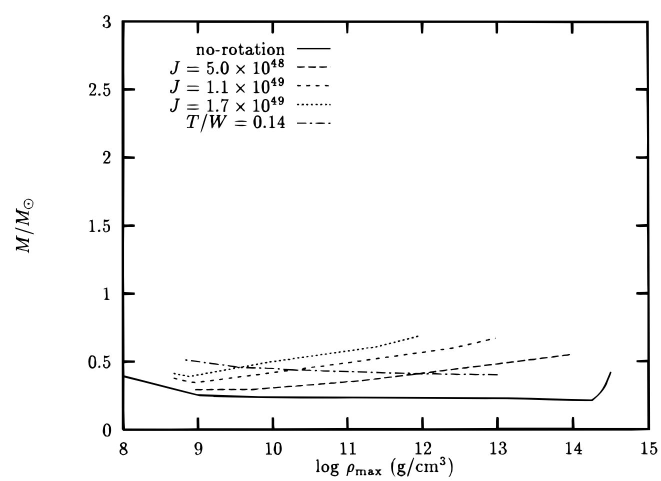

FIG. 9.—Same as Fig. 8, but for $s = 0.5$

As can be seen clearly, the mass of spherical stars depends strongly on the value of the electron fraction. Concerning the rotational effect, apart from the low-density region ($\rho_{max} \lesssim 10^9$ g cm^{-3}), if the total angular momentum exceeds some critical value that depends on the equation of state, rotation can make equilibrium sequences stable against axisymmetric collapse, although for smaller values of Y_e this stabilization mechanism becomes weaker. This is due to the fact that the contribution from the electron to the pressure is an important factor in making equilibrium configurations stable.

As explained before, for all the models we have computed, the ratio of the rotational energy to the absolute value of the gravitational energy satisfies the stability condition against the bar mode dynamical instability, i.e., $T/|W| < 0.26$.

4.3. *Discussions*

We have shown that when we take the high-temperature effect into account, dynamically stable but secularly unstable states can exist for some rotating and hot compact stars as long as the entropy per baryon and the electron fraction are assumed to be constant in space.

However, the constancy of the entropy per baryon and the electron fraction throughout the star is not guaranteed in real stars. Thus, real equilibrium sequences with constant angular momenta would be different from those obtained in this paper. As shown by Mezzacappa & Bruenn (1993), the electron fraction profiles are not only varying in space but also changing during evolution. The value of Y_e in the central region continues to decrease by keeping the value of Y_e fixed near the surface as evolution proceeds. Although their results are obtained by computing the dynamical collapse of massive stars, a similar tendency may be expected to appear for the contraction stage of hot cores of massive stars, if contraction occurs. Therefore, it seems that our equilibrium sequences obtained in this paper cannot be applied to such situations directly.

However, our purpose in this paper is not to show the existence of fizzlers exactly, but to show that the existence of fizzlers cannot always be excluded because the situation around rotating hot compact stars may be different from that of previous investigations in which the BPS-BBP equation of state for cold matter was used (Müller & Eriguchi 1985). In practice, the following discussion can be considered favorable for fizzlers, or at least we will be able to show that the necessary conditions for fizzlers can be fulfilled.

In order to apply our equilibrium sequences with the constant electron fraction, we consider the following situation for evolution. Let us choose two real stars with different maximum densities but with the same total angular momentum. We will call the real star with the lower maximum density the star at the "initial state," and the real star with the higher density the star at the "final state."

In real stars, the electron fraction is high near the surface and low in the core. Let the highest value of the electron fraction of the star at the initial state be $Y_{e,i}$ and the lowest value of Y_e of the star at the final state be $Y_{e,f}$, i.e., $Y_{e,i} \equiv \max_{\rm initial}(Y_e)$ and $Y_{e,f} \equiv \min_{\rm final}(Y_e)$. Then we can consider a *model configuration* in which the electron fraction is assumed to be constant and its value is $Y_{e,i}$, and the maximum density is the same as that of the initial state. Similarly, another *model configuration* is assumed to have a constant value of $Y_e = Y_{e,f}$ and the same maximum density as that of the final state. We will call the former model the "initial model" and the latter the "final model." It should be noted that the mass of the initial model, M_i, is *larger* than that of the real star at the initial state, and that the mass of the final model, M_f, is *smaller* than that of the real star at the final state.

The following condition is necessary for fizzlers to exist:

$$M_f \geq M_i\,, \tag{24}$$

because there arises the possibility of having equilibrium curves with positive slopes in the mass–maximum density plane. Of course, the mass of real stars may be lower than this range because steep negative slopes may be there. However, for the cold equation of state, such as the BPS-BBP equation of state, the effective change of the electron fraction amounts to 0.3 or more, i.e., from 0.4 for lower densities to 0.05 or less for higher densities as seen from Figure 1. On the other hand, for realistic collapse, according to the results of Mezzacappa & Bruenn (1993), the decrease of the electron fraction amounts only to 0.15 or so, starting from 0.45 and decreasing to 0.3 for the infall phase of massive stars. Therefore, even if the slope might be negative for some range of densities, it would not be tremendously steep. As seen from the tendency of the curves, slopes of sequences with higher angular momenta are larger. Thus, if enough angular momentum is considered, the above condition, equation (24), may be treated as a necessary condition for the existence of fizzlers.

In Figure 11, we have shown two examples for which the necessary condition, equation (24), is satisfied. The open boxes correspond to initial models for which $Y_e = 0.3$ and final models for which $Y_e = 0.2$. We connect the initial model and the final model with the same value of the angular momentum. As seen from this figure, if the angular momentum exceeds some critical value, the slopes of the connecting curves become positive. This implies that, contrary to the cold equation of state, there is a possibility that fizzlers may exist. At least we may not exclude the possibility of fizzlers from investigations in realistic situations.

Another thing to be considered is the combination of the values of the entropy and the electron fraction. The main purpose of this paper is to investigate the stabilization effect of hot matter in rather high density regions. Thus, we choose even rather low values of the electron fraction such as $Y_e = 0.2$ or 0.15 because these values may be realized in the real evolution of fizzler states. However, those low values are totally unreasonable for lower density regions where the main contribution to the pressure is the electrons. Consequently, for equilibrium configurations with lower values of the electron fraction, only the parts of the sequences in the high-density regions should be considered valid.

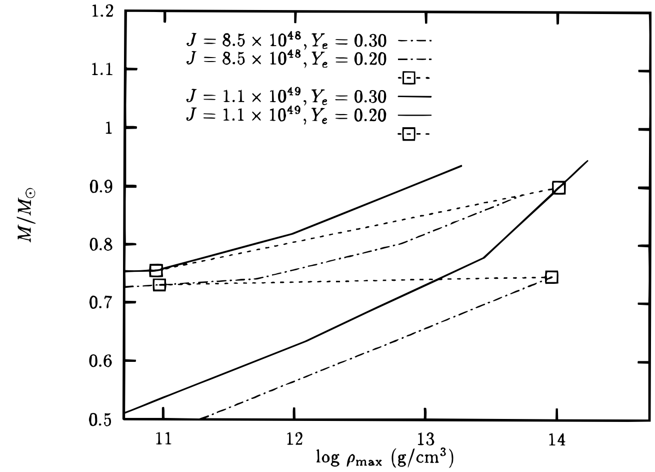

FIG. 11.—$J = 8.500 \times 10^{48}/J = 1.100 \times 10^{49}$. The idealized equilibrium sequences of constant angular momentum are shown by the dotted curves. These curves connect two states that are called the initial model and the final model in the text. The electron fraction of the initial (final) model is the largest (smallest) value of the electron fraction that is realized in the real star with the same angular momentum and the same maximum density. The open boxes correspond to initial models for which $Y_e = 0.3$ and final models for which $Y_e = 0.2$. The solid curves and dash-dotted curves correspond to equilibrium sequences with constant total angular momenta whose values are shown in the figure.

Apart from these remarks, we need to check several things further before we come to a definite conclusion about evolution on a secular timescale or the existence of fizzlers.

In order to have fizzlers, the angular momentum of stars must be larger than a critical value, say, $J \gtrsim 5 \times 10^{49}$ g cm^2 s^{-1}. If we suppose that the object is the core of a massive rotating star that consists of iron, the angular momentum of the core must be less than the total angular momentum of the whole massive star. In general, the angular momentum J can be expressed as

$$J \sim \alpha M R^2 \Omega = 7 \times 10^{51}\left(\frac{\alpha}{0.1}\right)\left(\frac{M}{10\,M_\odot}\right)\left(\frac{R}{5R_\odot}\right) \times \left(\frac{V_{\rm surf}}{100\ {\rm km\ s^{-1}}}\right)\ {\rm g\ cm^2\ s^{-1}}\,, \tag{25}$$

where $V_{\rm surf}$ is the surface velocity of the massive star and α is a constant that depends on the internal structure of the star. Typically, the value of α is 0.01–0.1 for massive stars and ~ 1 for compact stars (see, e.g., Eriguchi et al. 1992). For example, if we take a main-sequence star with $M = 20 M_\odot$, the radius is roughly $R = 6\,R_\odot$. The surface velocity $V_{\rm surf} =$ 100 km s^{-1} results in $J \sim 10^{52}$ g cm^2 s^{-1}. For such a massive star, the mass of the core is considered to be $M = 1.5 M_\odot$, and the radius $R = 0.5 R_\odot$. If we choose $\alpha \sim 1$ and assume uniform rotation for this core region, the angular momentum of the core region is roughly $J \sim 10^{50}$g cm^2 s^{-1}. Thus, there is no problem about the angular momentum.

Moreover, we need to think about the change of angular momentum distribution during contraction. In a real contraction stage, the angular momentum distribution changes from that of the initial state because of angular momentum transfer. Therefore, we need to consider evolutionary sequences in which the angular momentum distribution is changed. In general, this results in the change of the value of $T/|W|$. However, as discussed before, the critical value of our criterion for secular instability is chosen to be rather large. Consequently, even if the value of $T/|W|$ changes during contraction, evolution on a secular timescale will be expected once fizzler states are formed.

Concerning the chosen values of the entropy per baryon and the electron fraction, they are consistent with the results of spherical computations of stellar evolution and/or dynamical computations of collapsing stages (see, e.g., Hashimoto 1995) as discussed in § 2.2. According to evolutionary calculations, the values of the entropy per baryon are $s/k = 0.66$–1.19, and those of the electron fraction are $Y_e = 0.41$–0.49 in a core whose maximum density is $\rho_{\max} = (2\text{–}3) \times 10^{10}$ g cm^{-3} at the beginning of the collapse of the core (Hashimoto 1995). Similarly, Weaver et al. (1978) found that the density of the iron core is $\rho_{\max} = (3.5\text{–}6) \times 10^9$ g cm^{-3}, and that the electron fraction and the entropy per baryon are $Y_e \sim 0.43$ and $s/k \sim 1.5$, respectively, just before the core collapse. From the theoretical consideration of the dynamical collapse of spherical stars, Bethe et al. (1979) concluded that, for an evolution from $\rho_{\max} \sim 10^9$ g cm^{-3} to $\rho_{\max} \sim 10^{12}$ g cm^{-3}, the entropy and the electron fraction can be changed from $s/k \sim 1.0$ and $Y_e \sim 0.4$ to $s/k \sim 1.5$ and $Y_e \sim 0.3$ through electron capture before neutrino trapping occurs.

Since the fizzlers are slowly evolving objects, we have to consider some mechanisms that may affect evolution on a rather longer timescale compared with the dynamical timescale. The most important mechanism to be considered is energy loss by neutrino emission. Indeed, although the dynamical timescale is of an order of a millisecond, the timescale for neutrino cooling is estimated to be about 100 ms at $\rho \sim 6 \times 10^{12}$ g cm^{-3} (Bethe et al. 1979). This timescale should be compared with the evolutionary timescale of the fizzler contraction. Its evolutionary timescale can be estimated from equation (21) by assuming that $M \sim 1\ M_\odot$, $R \sim (r_p r_e)^{1/2} = 35$ km, and $T/|W| \sim 0.21$, as typical values at $\rho_{\max} \sim 10^{13}$ g cm^{-3}:

$$\tau_{\rm sec} \sim 0.1 \text{ s} . \tag{26}$$

Since this timescale is of the same order as that of neutrino cooling estimated by Bethe et al. (1979), it is likely that the decrease of entropy does not amount to a large value. In fact, for the core collapse, the nonequilibrium process of electron captures increases entropy. Therefore, our choice of the entropy and the electron fraction may be reasonable. Moreover, for the high-density region ($\rho \gtrsim 3 \times 10^{12}$ g cm^{-3}), neutrino trapping occurs that leads to the formation of the adiabatic core and also works to prevent the electron fraction from decreasing.

The timescale of fizzler evolution or the timescale of gravitational radiation emission is essentially a general relativistic effect so that it works effectively in the formation region of neutron stars. On the other hand, its effect is so weak in the lower density region, $\sim 10^9$ g cm^{-3}, that the timescale for this evolution can be roughly from several weeks to 10^3 yr (Friedman & Schutz 1975). Therefore, in this density region, cooling works effectively, and it is difficult to keep a high temperature for the star. However, according to the results of Müller & Eriguchi (1985), even stars with zero temperature can be stabilized by the rotational effect alone. In other words, once a star begins to contract because of gravitational radiation emission from the white dwarf stage, it can evolve dynamically stably up to $\rho \sim 10^{12.5}$ g cm^{-3}, even if cooling due to neutrino losses keeps the temperature low. When the star contracts to a state around $\rho \sim 10^{12.5}$ g cm^{-3}, the opacity to neutrinos may increase because of neutrino trapping so that high-temperature states can occur. The temperature effect, including the electron fraction, will act as a stabilizing factor even for the otherwise very unstable region around $\rho_{\max} \sim 10^{12}$–10^{14} g cm^{-3} as shown in this paper.

It is known that the decrease of the angular momentum ΔJ, and that of the energy ΔE because of gravitational radiation emission, are related to each other by the following relation:

$$\Delta E = \tilde{\Omega} \Delta J . \tag{27}$$

Here $\tilde{\Omega}$ is the mean angular velocity. If the angular momentum that is taken away by gravitational waves is $\Delta J \sim 10^{49}$ g cm^2 s^{-1}, and if we choose $\tilde{\Omega} \sim 10^3$ s^{-1}, which is the angular velocity of millisecond pulsars, we have $\Delta E \sim 10^{52}$ ergs. However, since, in actual situations, the angular velocity is much smaller for most stages of the evolution, this value should be a very rough upper limit for the energy loss.

Furthermore, we should consider the initial states of massive stars and their consistency with the statistics of pulsars. As our discussions show, a high entropy and large electron fraction are required to stabilize cores of massive stars. However, if the temperature becomes high, nuclear reactions begin to happen, and they could lead to an explosion if there is nuclear fuel. Therefore, in order to realize a fizzler state, we need to consider cores that consist of iron, or rather heavy white dwarfs that consist of oxygen-neon-magnesium, because there is proof that these cores without the angular momentum have collapsed (Hashimoto, Iwamoto, & Nomoto 1993). It implies that the progenitors of the fizzlers may be massive stars of $M \gtrsim 8 M_\odot$ with rather high angular momenta. Neutron stars formed in this process may be surrounded by massive envelopes so that accretion may cause the neutron stars to become black holes unless some mechanism blows off the envelope, such as strong mass loss or significant mass transfer to the companion star. Thus, the number of neutron stars born in this process may not be large. According to Lyne, Manchester, & Taylor (1985), the birthrate of pulsars is one per 30–120 yr (see also Lyne & Graham-Smith 1990 and Strom 1995). This is consistent with the statistics of supernova counts, i.e., one supernova per 12–20 yr (Tammann 1982; van den Bergh & Tammann 1991; Strom 1995). If neutron stars produced through fizzlers come from massive stars and some of them collapse into black holes, this path to neutron stars may not disturb the statistics of pulsars and supernovae.

Finally, we should discuss the stability criterion against axisymmetric dynamical collapse. As mentioned before, our criterion, equation (19), is not an exact one. In order to make the analysis exact, we need to use equation (18) or undertake a precise linear stability analysis. However, as shown in our study, the equilibrium sequences with the same total angular momentum are almost uniquely determined irrespective of the angular velocity distributions if the equation of state is the same. Therefore, although the criterion that we use for axisymmetric dynamical instability is not exact, the results obtained can be treated as reliable ones within the ambiguity of the equation of state for high temperature.

4.4. *Conclusions*

We summarize our results as follows:

1. For hot compact stars, if the angular momentum of rotating stars becomes larger than a critical value that depends on the equation of state, rotation makes stars dynamically stable against axisymmetric collapse in the density range of $\rho_{\max} \sim 10^8$ g cm^{-3} to $\rho_{\max} \sim 10^{14.5}$ g cm^{-3}, as long as the electron fraction and the entropy per baryon are assumed constant.

2. The stabilization of compact stars occurs mainly because of the effect of the high electron fraction on the pressure if rotation exists. The existence of the entropy per baryon does not affect stabilization directly but is required to guarantee the high value of Y_e in real hot dense matter.

3. Fizzlers can exist for a rather wide parameter range of equations of state and rotation because there can be equilibrium sequences that satisfy the necessary condition for fizzlers.

4. Considering the realistic evolution of real stars, the number of neutron stars produced through this fizzler path may not be large.

We would like to thank the referee, Professor R. Durisen, for his very useful comments and criticism to the original paper. This work was supported in part by the Grant-in Aid for Scientific Research of the Ministry of Education, Science, and Culture of Japan. One of the authors (M. H.) would like to thank M. Arnould and M. Rayet for their hospitality at the Institute d'Astronomie et Astrophysique in Université Libre de Bruxelles where a part of this work was done.

REFERENCES

Baym, G., Bethe, H. A., & Pethick, C. J. 1971a, Nucl. Phys. A, 175, 225
Baym, G., Pethick, C. J., & Sutherland, P. 1971b, Nucl. Phys. A, 170, 299
Bethe, H. A. 1990, Rev. Mod. Phys., 62, 801
Bethe, H. A., Brown, G. E., Applegate, J. H., & Lattimer, J. M. 1979, Nucl. Phys. A, 324, 487
Bisnovatsyi-Kogan, G. S., & Blinnikov, S. I. 1974, A&A, 31, 391
Chandrasekhar, S. 1969, Ellipsoidal Figure of Equilibrium (New Haven: Yale Univ. Press)
———. 1970, Phys. Rev. Lett., 24, 611
Durisen, R. H., & Tohline, J. E. 1985, in Protostars and Planets II, ed. D. C. Black & M. S. Matthews (Tucson: Univ. Arizona Press), 534
Eriguchi, Y., Hachisu, I., & Nomoto, K. 1994, MNRAS, 266, 179
Eriguchi, Y., & Müller, E. 1985, A&A, 146, 260
———. 1991, A&A, 248, 435
Eriguchi, Y., Yamaoka, H., Nomoto, K., & Hashimoto, M. 1992, ApJ, 392, 243
Friedman, J. L., Ipser, J. R., & Parker, L. 1984, Nature, 312, 255
———. 1986, ApJ, 304, 115
———. 1989, Phys. Rev. Lett., 62, 3015
Friedman, J. L., & Schutz, B. F. 1975, ApJ, 199, L157
Glatzel, W., Fricke, K. J., & El Eid, M. 1981, A&A, 93, 395
Gold, T. 1974, in Seventh Texas Symp. on Relativistic Astrophysics
Hachisu, I. 1986, ApJS, 61, 479
Hashimoto, M. 1995, Prog. Theor. Phys., 94, 663
Hashimoto, M., Iwamoto, K., & Nomoto, K. 1993, ApJ, 414, L105
Hoyle, F. 1946, MNRAS, 106, 343
Imamura, J. N., Friedman, J. L., & Durisen, R. H. 1985, ApJ, 294, 474
Imamura, J. N., Toman, J., Durisen, R. H., Pickett, B. K., & Yang, S. 1995, ApJ, 444, 363
Ipser, J. R., & Lindblom, L. 1991, ApJ, 373, 213
Ipser, J. R., & Managan, R. A. 1985, ApJ, 292, 517
Lattimer, J. M., & Swesty, F. D. 1991, Nucl. Phys. A, 535, 331
Lindblom, L. 1995, ApJ, 438, 265
Lindblom, L., & Detweiler, S. L. 1977, ApJ, 211, 565
Lyne, A. G., & Graham-Smith, F. 1990, Pulsar Astronomy (Cambridge: Cambridge Univ. Press)
Lyne, A. G., Manchester, R. N., & Taylor, J. H. 1985, MNRAS, 213, 613
Managan, R. A. 1985, ApJ, 294, 463
Mezzacappa, A., & Bruenn, S. W. 1993, ApJ, 405, 637
Müller, E., & Eriguchi, Y. 1985, A&A, 152, 325
Myra, E. S. 1988, Phys. Rep., 163, 127
Ostriker, J. P., & Bodenheimer, P. 1973, ApJ, 180, 171
Pickett, B. K., Durisen, R. H., & Davis, G. A. 1996, ApJ, 458, 714
Press, W. H., & Teukolsky, S. A. 1973, ApJ, 181, 513
Salpeter, E. E., & Wagoner, R. V. 1971, ApJ, 164, 557
Shapiro, S. L., & Lightman, A. P. 1976, ApJ, 207, 263
Smith, S. C., Houser, J. L., & Centrella, J. M. 1996, ApJ, 458, 236
Strom, R. G. 1995, in The Lives of the Neutron Stars, ed. M. A. Alphar, Ü. Kiziloğlu, & J. van Paradijs (Dordrecht: Kulwer), 23
Sumiyoshi, K., & Toki, H. 1994, ApJ, 422, 700
Takatsuka, T., Nishizaki, S., & Hiura, J. 1994, Prog. Theor. Phys., 92, 779
Tammann, G. A. 1982, in Supernovae: A Survey of Current Research, ed. M. J. Rees & R. J. Stoneham (Dordrecht: Reidel), 371
Tassoul, J.-L. 1978, Theory of Rotating Stars (Princeton: Princeton Univ. Press)
Taylor, J. H., & Weisberg, J. M. 1989, ApJ, 345, 434
Uryu, K., & Eriguchi, Y. 1994, MNRAS, 269, 14
———. 1995, MNRAS, 277, 1411
van den Bergh, S., & Tammann, G. A. 1991, ARA&A, 29, 363
Weaver, T. A., Zimmerman, G. B., & Woosley, S. E. 1978, ApJ, 225, 1021
Wiita, P. J., & Press, W. M. 1976, ApJ, 208, 525
Yoshida, S., & Eriguchi, Y. 1995, ApJ, 438, 830