THE ASTROPHYSICAL JOURNAL, 492:677–682, 1998 January 10

THE EXPECTED DURATION OF GAMMA-RAY BURSTS IN THE IMPULSIVE HYDRODYNAMIC MODELS

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Received 1997 March 26; accepted 1997 August 22

ABSTRACT

Depending upon the various models and assumptions, the existing literature on gamma-ray bursts (GRBs) mentions that the gross theoretical value of the duration of the burst in the hydrodynamical models is $\tau \sim r_\gamma/2\eta^2 c$, where r_γ is the radius at which the blast wave associated with the fireball (FB) becomes radiative and sufficiently strong. Here $\eta \equiv E/Mc^2$, c is the speed of light, E is the initial laboratory frame energy of the FB, and M is the baryonic mass of the same (Rees & Mészáros). However, within the same basic framework, some authors (e.g., Katz and Piran) have given $\tau \sim r_\gamma/2\eta c$. We intend to remove this confusion by considering this problem at a deeper level than has been considered so far. Our analysis shows that none of the previously quoted expressions are exactly correct, and in case the FB is produced impulsively and the radiative processes responsible for the generation of the GRB are sufficiently fast, its expected duration would be $\tau \sim a r_\gamma/2\eta^2 c$, where $a \sim O(10)$. We further discuss the probable change, if any, of this expression, in case the FB propagates in an anisotropic fashion. We also discuss some associated points in the context of the Mészáros & Rees scenario.

Subject headings: gamma rays: bursts — hydrodynamics — relativity

1. INTRODUCTION

Our present understanding of the phenomenon of gamma-ray bursts (GRBs) is based on the foundations laid by Cavallo & Rees (1978), Goodman (1986), Paczyński (1986), Eichler et al. (1989), Shemi & Piran (1990), and several other works. However, as far as the origin of actually observed nonthermal and highly complex spectra are concerned, we are indebted to another important idea (Rees & Mészáros 1992; Mészáros & Rees 1993) that cosmic fireballs (FBs) with appreciable pollution of baryonic mass, i.e., with a value of $\eta \sim 10^2$–10^4, where $\eta \equiv E/Mc^2$, could be a virtue rather than a problem. Most of our current efforts to understand the phenomenology of GRBs largely hinge on the above-mentioned framework. Mészáros & Rees (1993) suggested that the duration of the burst should be $\tau \sim r_d/2\eta^2 c$, where r_d is the so-called deceleration radius measured in the laboratory frame. At $r = r_d$, the FB is supposed to transmit half of its original momentum to the medium. The baryon-polluted FB is expected to drive a strong forward shock (or a blast wave) in the ambient medium, presumably the interstellar medium (ISM), and at $r = r_d$, the blast wave is assumed to be sufficiently strong as the FB transfers half of its momentum to it. Technically, we can define another distance, namely, r_γ, where either the blast wave or the reverse shock becomes sufficiently radiative as far as hard X-ray and gamma-ray (i.e., the main component of the observed GRBs) production is concerned. The value of r_γ will be highly model dependent and contains all the microphysics of the process, and, strictly speaking, there may not be any simple correlation between the values of r_d (largely a simpler hydrodynamic definition) and r_γ (a highly model-dependent definition). We want to emphasize here that the basic definition of τ should naturally involve r_γ and not r_d. Unfortunately, the present status of the studies on GRBs is quite preliminary, and it is not possible to unambiguously define the value of r_γ even for a relatively simple model. In this situation, practically, all the authors tacitly assume that $r_\gamma \sim r_d$. Since the present paper endeavors to analyze the question of the gross timescale in the context of the existing framework of the hydrodynamical model of GRBs, we will also use the condition $r_\gamma \sim r_d$, although we will try to retain the physical distinction between r_γ and r_d as far as possible.

However, following the same basic framework, Katz (1994; see his eq. [23]) finds that for a FB propagating in a dense ambient medium (which could be a molecular cloud), we should have $\tau \sim r_d/\eta c$. Similarly, Piran (1994) also concludes that $\tau \sim r_d/\eta c$ (see his eq. [25]). Although, in a subsequent paper (Sari & Piran 1995), the question of hydrodynamical timescales and temporal structure of GRBs has been discussed in considerable detail, we feel that the aspect of the gross overall duration has not been answered in an unambiguous manner. The foregoing work, in particular, laid specific emphasis on the temporal structure of the GRBs associated with the occurrence of the Newtonian, or, subsequently, relativistic, reverse shock that is supposed to propagate inside the FB. On the other hand, it is worth discussing that, if the fireball (FB) actually becomes radiative at the deceleration radius (where it loses half of its initial momentum and kinetic energy to the ambient medium), the likely evolution of the reverse shock from a Newtonian one to a fully relativistic one (at $r \gg r_d$) could be only of somewhat academic interest. However, in case the blast wave fails to become radiative at $r \sim r_d$ for one reason or another, the considerations due to Sari & Piran (1995) may become applicable to the actual cases. Here again, one has to address the question of the overall gross duration of the bursts.

To fully appreciate the origin of this dilemma and its resolution, we would revisit the concept of duration of pulses emitted by a FB propagating in a vacuum. For the sake of clarity, we would use the following nomenclature: γ the instantaneous bulk Lorentz factor (LF) of *any* fluid emitting the radiation and γ_F the instantaneous bulk LF of the FB. Thus γ is general nomenclature applicable to both the original FB (in a vacuum) or any section of the shocked

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fluid (in a medium) which might be emitting radiation, whereas γ_F specifically refers to the FB.

Note that the shock cannot be sufficiently strong unless $r \gtrsim r_d$. We will see that, in case the GRB eventually results from a shock that becomes sufficiently strong as well as radiative at $r = r_d = r_\gamma$, as envisaged by Mészáros & Rees (1993), we should have $\tau \sim a r_d/2\eta^2 c$, where $a \sim 10$, which means that in the scheme of Mészáros & Rees (1993) the actual duration of the bursts should be at least 1 order higher than has been contemplated so far (for a fixed η). We also try to point out that as long as the blast wave can be considered sufficiently radiative, for estimating the eventual duration of the burst, the reverse shock plays an insignificant role.

2. FIREBALL IN A VACUUM

When a FB propagates in a vacuum, we have $\gamma \equiv \gamma_F$. It is interesting to compare the radiation emitted by the spherically expanding FB with that of a relativistically moving point source showing apparent superluminal (transverse) motion (Rees 1966):

$$v_\perp = \frac{v \sin\theta}{1 - \beta\cos\theta}\,, \tag{1}$$

where $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$, and θ is the angle between the line of sight and the direction of motion. We may obtain $v_\perp > c$ in some circumstances essentially because the source of radiation moves with a speed $v \to c$, and tends to catch up with the radiation emitted by itself. This phenomenon occurs because the velocity of propagation of the radiation from the fast-moving source remains fixed at c and does not increase in a Galilean fashion. The same relativistic phenomenon is actually responsible for offering a value of τ much shorter than the expected nonrelativistic value $\sim r_\gamma/c$. Nevertheless, for considering the (apparent) time seen by the observer, we have to consider the line-of-sight velocity of the FB rather than the transverse velocity:

$$v_\parallel = \frac{v\cos\theta}{1-\beta\cos\theta}\,. \tag{2}$$

For the exact point of the FB intersected by the line of sight, i.e., for $\theta = 0$, one can see that $v_\parallel = v/(1-\beta) \approx 2v\gamma_F^2$ for $\gamma_F \gg 1$. Hence $\tau \sim r/v_\parallel \sim r/2c\gamma_F^2$. However, the observer receives light not only from a given point but also from other parts of the FB. Because of relativistic aberration, the angular extent of the region is limited to $\theta \sim 1/\gamma_F$ in the laboratory frame. Then we have

$$\tau \sim \frac{r}{c}\,\frac{1-\beta\cos\theta}{\cos\theta} \sim \frac{r}{c\gamma_F^2}\,. \tag{3}$$

Here it is assumed that $r = r_\gamma \gg r_0$, the initial radius of the FB. Initially the FB could be nonrelativistic, and in case it could manage to send radiation outside (actually it would be extremely optically thick), the duration of the burst would have been $\tau_0 \sim r_0/c$. If the explosive energy is liberated not instantaneously but over a tiny but finite timescale, the impulsive approximation is correct as long as the expected $\tau \gg \tau_0$. If the FB really becomes optically thin, i.e., the Thomson scattering optical depth of the FB fluid becomes ≤ 1 at $r = r_{\rm T}$, most of the radiation escapes over a distance $r \sim r_{\rm T}$, and we will have $\tau \sim r/c\gamma_{\rm T}^2$, where $\gamma_{\rm T}$ is the bulk Lorentz factor of the FB at this point. Dynamics of the FB shows that from the initial nonrelativistic phase, the FB becomes quickly relativistic and $\gamma_F \propto r$, and $\gamma_F \to (\eta + 1) \approx \eta$ when $r_\eta = 2r_0\eta$; then the FB coasts freely with $\gamma_F \approx \eta$ (Piran, Shemi, & Narayan 1993; Mészáros, Laguna, & Rees 1993). For a wide range of parameters of cosmic FBs, it also follows that $r_{\rm T} \gg r_\eta$, so that $\gamma_{\rm T} \approx \eta$. At the same time, the observed duration of the burst cannot be smaller than τ_0. Therefore, for the sake of consistency, we will have $\tau = r_{\rm T}/c\eta^2$ if $r_{\rm T} \geq r_0\eta^2$, and $\tau = r_0/c$ otherwise.

This exercise suggests that, in the laboratory frame, the FB appears as a narrow shell of width $\Delta r \sim r_{\rm T}/\gamma_F^2$ (if $r_{\rm T} > r_0\eta^2$). The Doppler factor associated with the superluminal motion is $\delta = [\gamma(1-\beta\cos\theta)]^{-1}$ and the comoving duration of the pulse is (see eq. [2])

$$\tau_c \approx \tau\delta \approx \frac{r}{v}\,\frac{1}{\gamma_F\cos\theta} \approx \frac{r}{c\gamma_F}\,. \tag{4}$$

Of course, this result could have been obtained directly by considering the Lorentz contraction of the length r in the fluid frame. For the baryon-polluted FBs with $\eta < 10^5 E_{51} r_{0,F}^{-2/3}$, the escape of radiation at $r \approx r_{\rm T}$ is only of pedagogic importance because only an insignificant fraction of the FB energy $\sim E/\eta$ is available in the form of FB radiation at $r \approx r_\eta < r_{\rm T}$. Almost the entire energy of the FB gets channelized into the bulk kinetic energy of the baryons and the FB cannot radiate efficiently in vacuum (Shemi & Piran 1990).

3. FIREBALL IN A MEDIUM

Mészáros & Rees (1993) pointed out that the baryonic FB should sweep the ambient interstellar medium (ISM) and drive a strong forward shock. In principle, this blast wave may be radiating all the time, but the rate of radiation cannot be substantial unless appreciable amounts of the FB momentum and energy have been transmitted to the medium and the shock becomes radiative at $r = r_\gamma$. It is also likely that the FB radiates a considerable amount of energy in nonthermal gamma rays even before $r = r_\gamma$, owing to the existence of internal shocks (Rees & Mészáros 1994). However, we implicitly assume here that the fraction of initial energy lost in this way would be less than 50%.

We can simplify the FB-shock configuration by a one-dimensional sketch following Katz (1994) and Piran (1994) (see Fig. 1). Region 4 in the figure represents the unperturbed FB whose original edge S is the contact discontinuity between the piston driving the shock and the unperturbed ISM (region 1). Region 2 is the perturbed and squeezed-shocked ISM, whereas region 3 is the perturbed and reverse-shocked FB. Let $w = e + p$ denote the proper enthalpy density for each region (with appropriate

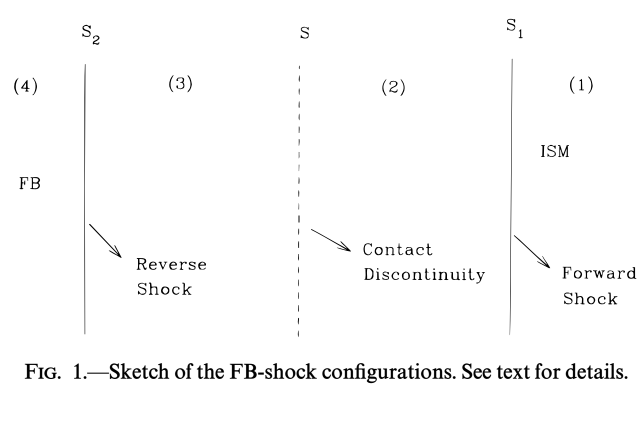

FIG. 1.—Sketch of the FB-shock configurations. See text for details.

subscript), where e denotes the proper internal energy density and p represents the pressure. Again, some clarification of the nomenclature would be in order here. A subscript to quantities such as e, p, and w will represent respective proper values in a given region, i.e., e_1 will represent the internal energy density of region 1. In contrast, since LFs are always meaningful with respect to a certain inertial frame, γ_{12} will represent the value of the LF of region 2 with respect to region 1, and vice versa. Since now the radiation is emitted not by the FB but by the shocked fluid in regions 2 and 3 with a laboratory frame bulk LF of $\gamma_{12} = \gamma_{31} = \gamma$ (the laboratory frame being the ISM at rest [region 1]), *it is the value of* γ_{12} rather than γ_F that now determines the value of τ. And it is here that the value of τ bifurcates between $r_\gamma/c\eta$ and $r_\gamma/c\eta^2$ in the literature. Therefore, we must unambiguously find the value of γ_{12} in terms of η to settle this issue. Here it may also be emphasized that, at least for the one-dimensional simplification employed by us, the laboratory frame LF of the reverse-shocked fluid in region 3 is also γ_{12}, although its LF with respect to region 4, γ_{34}, could be very much different from γ_{31}.

Now we can recall the strong shock jump conditions from Taub (1949) and Blandford & McKee (1976) and apply the same at S_1, the forward shock:

$$\frac{e_2}{n_2} = \gamma_{12}\,\frac{w_1}{n_1}\,, \tag{5}$$

$$\frac{n_2}{n_1} = \frac{\gamma_{12}\Gamma_2 + 1}{\Gamma_2 - 1}\,, \tag{6}$$

and,

$$\gamma_{S_1}^2 = \frac{(\gamma_{12}+1)[\Gamma_2(\gamma_{12}-1)+1]^2}{\Gamma_2(2-\Gamma_2)(\gamma_{12}-1)+2}\,, \tag{7}$$

where γ_{S_1} is the LF of the interface S_1 between 1 and 2. Here Γ is defined by the relation

$$p \equiv (\Gamma - 1)(e - \rho)\,, \tag{8}$$

where ρ is the rest-mass density in the respective regions (Blandford & McKee 1976). For a simple one-component fluid, physically Γ is just the ratio of specific heats, and has a value lying between 4/3 and 5/3. We expect the (forward) shocked fluid to be highly compressed and heated, and to be highly relativistic (with respect to internal energy), so that $\Gamma_2 = 4/3$ and $e_2 = p_2/3$. On the other hand, region 1, i.e., the unperturbed ambient medium, is assumed to be cold, so that $p_1 \approx 0$ and $w_1 \approx e_1 \approx mn_1c^2$, where m stands for proton mass. Then it promptly follows that

$$e_2 \approx mn_2c^2\gamma_{12}\,, \tag{9}$$

$$n_2 \approx 4\gamma_{12}n_1\,, \tag{10}$$

and

$$\gamma_{S_1} \approx \sqrt{2}\gamma_{12}\,. \tag{11}$$

If we assume the surface of contact discontinuity to be at perfect pressure equilibrium, then we will have $p_3 = p_2$, and, further, assuming the reverse-shocked fluid also to be highly relativistic (as far as internal energy is concerned), i.e., $p_3 = e_3/3$, we find $e_3 \approx e_2$. As to region 4, i.e., the unshocked part of the FB, the baryons are assumed to be coasting freely after $r > r_\eta \ll r_\gamma$ with a LF $\gamma_F \to \eta + 1 \approx \eta$ until they sacrifice a considerable portion (half at $r = r_d$) of their bulk energy to the shocked material. Therefore, at $r > r_\eta$, the FB material is also nonrelativistic in its rest frame, enabling us to write $p_4 \approx 0$, $w_4 \approx e_4 \approx mn_4c^2$. This allows us to form another set of simplified jump conditions at S_2:

$$\frac{e_3}{n_3} \approx \gamma_{34}\,\frac{w_4}{n_4} = \gamma_{24}mc^2 \tag{12}$$

and

$$\frac{n_3}{n_4} \approx 4\gamma_{34} + 3 \approx 4\alpha\gamma_{24}\,, \tag{13}$$

where

$$\alpha \equiv \left(1 + \frac{3}{\gamma_{34}}\right) \tag{14}$$

and the LF of the interface between regions 3 and 4 is

$$\gamma_{S_2} \approx \sqrt{2}\gamma_{24}\,. \tag{15}$$

The value of α lies between 1 (ultrarelativistic reverse flow, $\gamma_{34} \gg 1$) and 4 (mildly relativistic reverse flow, $\gamma_{34} \approx 1$). Following Piran (1994), if we define $f \equiv n_4/n_1$, and utilize the fact that $\gamma_F = \gamma_{12}\gamma_{24}$, we can eliminate γ_{24} from the foregoing equations to obtain

$$\gamma_{12} \approx f^{1/4}\gamma_F^{1/2}\alpha^{1/2}\,, \tag{16}$$

$$\gamma_{24} = \gamma_{34} \approx f^{-1/4}\gamma_F^{1/2}\alpha^{-1/2}\,, \tag{17}$$

and

$$n_3 \approx 4f^{-1/4}\gamma_F^{1/2}\alpha^{1/2}\,. \tag{18}$$

Katz (1994) has considered a case with $n_4 = n_1$, i.e., $f = 1$, and $\alpha = 1$ to obtain $\gamma_{12} \approx \gamma_F^{1/2}$. Thus he obtained $\tau \approx r_\gamma/c\gamma_{12}^2 \approx r_\gamma/c\gamma_F$. Let us now try to see whether this consideration of $f = 1$ is justified or not. As is well understood, each observer sees the FB within a solid angle of γ_F^{-2}, and a spherical FB will appear as a collection of γ_F^2 incoherent beamed FBs to as many observers distributed over a4π solid angle. Thus, even for a FB with angular extent 4π, we can actually take care of a considerable amount of anisotropic development. As long as a given ambient medium, which could be the background ISM ($n_1 \approx 1$) or a molecular cloud ($n_1 \sim 10^2$–10^4), has a linear width much larger than the length scales associated with the development and completion of the radiative processes associated with a GRB event, we can crudely consider the ambient medium to be uniform over a certain scale. Since the value of r_γ is expected to be less than 10^{17} cm, and the associated solid angle, γ_F^{-2}, is expected to be 10^{-4} to 10^{-8} sr corresponding to $\gamma_F \sim 10^2$–10^4, the linear scales in question are indeed much smaller than the typical cloud dimensions of the order of a few parsecs, and thus we should not be concerned with the probable different values of n_1. Given a certain fixed value of n_1, we can now uniquely find the value of n_4:

$$n_4 = \frac{M}{4\pi r^2(\Delta r)_{\rm com}m}\,, \tag{19}$$

where the comoving width of the FB is $(\Delta r)_{\rm com} = r/\gamma_F$. And since $M \approx E/\eta c^2$, we obtain

$$n_4 = \frac{E(\gamma_F/\eta)}{4\pi r^3c^2m} \approx 5\times 10^7 E_{51}r_{15}^{-3}\left(\frac{\gamma_F}{\eta}\right)\ {\rm cm}^{-3}\,. \tag{20}$$

Now we will have to confront this question: physically, which is the most appropriate definition for r_γ? If the blast wave becomes radiative at $r = r_d$, obviously, following Mészáros & Rees (1993), the radiative radius r_γ should be the deceleration radius, r_d. Simple energy and momentum conservation considerations show that at the deceleration radius, i.e., the radius when half of the initial momentum is transmitted, we have $\gamma_F = \gamma_{Fd} \approx \eta/2$, and the swept-up mass is $\approx M/\gamma_F$. This yields

$$r_d = \left(\frac{3E}{4\pi c^2 \gamma_{Fd} \eta m n_1}\right)^{1/3} \approx \left(\frac{3E}{2\pi c^2 \eta^2 m n_1}\right)^{1/3}$$
$$\approx 7 \times 10^{15} E_{51}^{1/3} \eta_3^{-2/3} n_1^{-2/3} \text{ cm} , \quad (21)$$

where $\eta_3 = \eta/10^3$ and $n_1 \to n_1/1 \text{ cm}^{-3}$. We can also contemplate the question of which is the fundamental scale length (apart from r_0) in this problem. Could it be the "Sedov length" $l \equiv E/n_1 mc^2$, where the FB sweeps a mass equal to E/mc^2 (Sari & Piran 1995)? From a dynamical point of view the concept of momentum exchange is more meaningful than the concept of swept-up mass, and in a nonrelativistic supernova remnant (SNR) case, the equality of swept-up mass implies equal momentum sharing. Therefore, in a relativistic dynamical problem (GRB), it is the "deceleration length" which is the physical equivalent to the idea of Sedov length appearing in the SNR theory. As we allow γ_F to approach 1, we find $r_d \to l$ within a small numerical factor! If the Sedov length were indeed a basic length scale in this problem, there would be a basic timescale $l/c \sim 1$ yr—a timescale that is actually appropriate for the SNR case and which may also be appropriate for low-wavelength afterglow following the main GRB.

Of course it is probable that the shock actually may not be sufficiently radiative at $r = r_d$, but it becomes so at a much later distance where the value of f would be much lower than f_d (in a spherically symmetric three-dimensional geometry), and the FB has swept an amount of mass much larger than M/γ_F for a variety of reasons (Sari & Piran 1995). Unless we have a specific prescription to describe the extent to which the FB is radiative, the problem becomes rather poorly defined in this case. The most important parameter describing the radiative maturity of the shock could be the in situ magnetic field near the blast wave, and, for almost any model of enhanced magnetic field generation, the value of the magnetic field decreases at least linearly until the background ISM value is achieved (Mészáros, Rees, & Papathanassiou 1994). Therefore, *it is highly unlikely that if the blast wave fails to become radiative at $r = r_d$, it will be radiative at a much larger radius in the framework of a purely hydrodynamic model.* Nevertheless, we would like to point out here that this whole discussion explores the question of duration of the GRB in the idealistic Mészáros & Rees (1993) framework, which assumes that even in a sparse ISM ($n_1 \leq 1$), the hydrodynamic limit is achieved at any radius. We have discussed elsewhere (Mitra 1996) that actually this framework may not be valid at all when applied to a sparse ambient medium because the mean free path of the leading particles of the blast wave (λ) is unlikely to satisfy the condition $\lambda \ll \Delta r$, where $\Delta r \sim r/\gamma^2 c$ is the laboratory frame width of the FB. This would mean that the hydrodynamical limit may not at all be achieved at the expected value of $r \sim r_d$ and the FB may not transfer any appreciable amount of energy and momentum to the ambient medium. Naturally, there may be no strong shock at all at $r = r_d$ (Mitra 1996). In such a case, the FB may interact with the ambient medium in the fluid limit at a much larger distance $r \gg r_d$, and part of the discussion by Sari & Piran (1995) may be applicable in a surprisingly unexpected way.

For the time being we ignore such a disturbing possibility and note that, in any case, it is really not necessary for the emission of the gamma rays that the reverse shock crosses the FB; the blast wave may be radiative enough on its own, although it may look for seed photons originating from any source including the reverse shock. Then at $r = r_\gamma = r_d$, the foregoing equations lead to

$$f = \frac{n_4}{n_1} = \frac{1}{6}\gamma_{Fd}\eta \approx \frac{1}{3}\gamma_{Fd}^2 \approx \frac{1}{12}\eta^2 . \quad (22)$$

Therefore, we must have $f \gg 1$ at the deceleration radius in the Mészáros & Rees (1993) scenario. Now we can go back to equation (22) to find that for $r = r_d$, we have

$$\gamma_{12} \approx (1/3)^{1/4} \alpha^{1/2} \gamma_{Fd} \quad (23)$$

and

$$\gamma_{24} \approx 3^{1/4} \alpha^{-1/2} \approx 1 . \quad (24)$$

Because of the uncertainty in the value of the bulk LF of the reverse-shock material (which we clearly find to be ~ 1) we are still not able to fix exactly the value of γ_{12}. However, we can do so by appealing to some simple physical facts. The first condition is a trivial one that we cannot have the value of any LF, in particular γ_{24}, less than 1. And in order for there to be a forward shock at all, we must have γ_{S_1} ($\approx 2^{1/2}\gamma_{12}$) $\geq \gamma_F$. Finally, there will be no reverse shock if $\gamma_{12} \geq \gamma_F$. These physical conditions are actually so powerful that we could have set the value of γ_{12} to lie between the narrow range of $\gamma_F/(2)^{1/2}$ and γ_F without carrying out much of the exercise done before. And the same conditions show that for $\gamma_F \gg 1$, irrespective of the nature of the ambient medium, we would never have a solution which admits $\gamma_{12} \ll \gamma_F$ (for instance $\gamma_{12} \approx \gamma_F^{1/2}$ discussed earlier). It is interesting to note that the value of γ_{12} should lie within such a narrow range. The physical constraints also imply that the maximum value of the bulk LF of the shocked material in the FB is $2^{1/2}$, and thus the reverse shock in a GRB problem is bound to be nonrelativistic. From such considerations, we can write the approximations

$$\frac{\gamma_F}{\sqrt{2}} \leq \gamma_{12} \leq \gamma_F \quad (25)$$

and

$$1 \leq \gamma_{24} \leq \sqrt{2} . \quad (26)$$

Now we can go back to the work of Sari & Piran (1995) to see whether the condition for occurrence of an ultrarelativistic reverse shock is valid or not. They have considered a case with $f \ll \gamma_F^2$ (see eq. [5] of Sari & Piran 1995) for which $\gamma_{34} \gg 1$ (see our eq. [16]). But, if we use our equation (16), we immediately find that, for such a value of f, we would have $\gamma_{12} \ll \gamma_F$, which is unphysical in view of the constraint (25). On the other hand, note that equations (22) and (23) are consistent with such physical constraints.

So, as long as we can assume that the radiative processes in both the forward-shocked fluid and the reverse-shocked fluid are sufficiently fast, we have $\tau \sim r_d/c\gamma_{12}^2$. But since the reverse shock is actually at best mildly relativistic, particles are not expected to be accelerated to very high Lorentz factors within the limited observed duration of GRBs. And since for radiative processes like synchrotron and inverse Compton the timescales are inversely proportional to the LF of the particles, there is a possibility that the reverse-shocked fluid might significantly stretch the expected theoretical GRB timescales. How justified is this apprehension? The strength of the signal appearing from regions 2 and 3 should depend on the ratio of the power dissipated in the two regions. And the latter should depend on the ratio of the amount of work done by S_1 on the ambient medium (1), and by S_2 on (4). Because the value of the fluid pressure is the same at S_1 and S_2 ($p_2 = p_3 = p$), the rate of compression or the rate of $p\ dV$ work done by the two shocks is $\gamma_{S1}:\gamma_{S2} \approx \gamma_F:1 \approx \eta/2:1$. Since this rate is Lorentz invariant, and the expected value of $\eta_2 \sim 10^2$–10^4, we find that the amount of available power that goes into the compression of the FB is negligible. Accordingly, the rate of work done by the reverse shock in any form, whether it is the heating of the FB, or producing enhanced magnetic field in it or producing low-energy photons to facilitate inverse Compton–boosted gamma-ray production in region 2 is actually negligible.

Thus we come to the very important conclusion that we can practically ignore the reverse shock in studying the gross timescale of GRBs within the Mészáros & Rees (1993) framework! This understanding enables us to write

$$\tau \approx \frac{ar_d}{2c\eta^2}\,, \qquad (27)$$

where $8 < a < 16$. This value of τ is obviously 1 order of magnitude higher than the usually quoted value of $\tau \approx r_d/2c\eta^2$. By recalling the value of r_d from equation (21), we can rewrite

$$\tau \approx 0.1aE_{51}^{1/3}\eta_3^{-8/3}n_1^{-1/3}\ \mathrm{s}\,. \qquad (28)$$

For a given value of η, the above-derived value of τ is at least 1 order larger than similar values used in the literature (for instance, see eq. [5.2] of Mochkovitch et al. 1995).

4. DISCUSSION

We have been able to remove a basic qualitative deficiency in the theoretical description of GRBs in the Mészáros & Rees (1993) scenario, which has become the "standard model" for understanding the phenomenology of these events, i.e., whether $\tau \sim r_d/2c\eta^2$ or $\tau \sim r_d/2c\eta$. Although none of these expressions appeared to be quite correct, our final result is obviously tilted in favor of the former relation, which is, however, to be modified by a numerical factor $8 < a < 16$. We have also emphasized an important aspect of this problem: the minimum value of the ratio of the Lorentz-invariant power liberated in the forward blast wave and the reverse shock goes as $\eta/2 \gg 1$. This point should considerably simplify the evaluation of the emitted spectrum in the problem. Although we have assumed a spherical FB, this discussion remains unchanged even for a case when the actual FB has a funnel-type structure in the proper frame. To appreciate this subtle point, consider the creation of jet-type FBs following a neutron star–neutron star collision event (Rees & Mészáros 1992). Suppose the $e^+ + e^-$ FB is due to collisions of neutrinos and antineutrinos emanating from the superhot and about-to-merge neutron stars. Then, despite the asymmetry in the binary geometry, $e^+ + e^-$ pairs would be produced over the full 4π solid angle, although there would be excess production along the symmetry axis because of the larger value of the collision angle there. Unless we are sure about the exact configuration of the neutrinosphere associated with each neutron star, or the configuration of the thick disk that may be formed as the result of merger, and also the temporal evolution of the whole pattern, it can only be a matter of conjecture as to how much excess production of pairs is achieved at a particular point with given values of r and θ. The canonical value of $E \sim 10^{50}$ or 10^{51} ergs is actually obtained by assuming a broad isotropic picture of the events, and one should actually consider the use of a fiducial value of E per unit solid angle for dealing with the anisotropic cases. In case there is some excess production in a certain direction, it does not mean that the pair flux produced in other directions somehow gets focused in that direction to result in the same canonical value of E assumed for the spherical 4π case. And the basic reason that there might be a funnel-type geometry of the FB is not the somewhat higher production of pairs in a given direction. The basic reason, instead, is that the mass flux of the debris of a neutron star–neutron star merger event is assumed to be appreciably less in those preferred directions, yielding a value of $\eta \gg 1$. If the merger event spews off a baryonic mass $\sim 10^{-3}\ M_\odot$, then even for an assumed (spherical) value of $E \sim 10^{51}$ ergs, in the isotropic case we would have $\eta \leq 1$, and there would be no GRB! It is primarily on this account that the assumption of a relatively baryon-free funnel along the symmetry axis gives rise to jetlike FBs. It only means that there are other jetlike FBs too, but they have unacceptably low values of η. Thus, in our view, once we keep a canonical value of E and tacitly assume an intrinsically spherical FB (though having various values of η in various directions), for a given observer, we must not additionally plug in the solid-angle factor in associated quantities like r_d or τ, as has been done by Mészáros, Rees, & Papathanassiou (1994). To further appreciate this point, we can specifically consider two typical equations considered by them (eqs. [2.4] and [2.16]),

$$\tau \sim 1E_{51}^{1/3}\eta_3^{-8/3}n_1^{-1/3}\theta^{-2/3}\ \mathrm{s}\,, \qquad (29)$$

and the total burst fluence,

$$S_0 = 10^{-6}E_{51}\theta^{-2}D_{28}^{-2}\ \mathrm{ergs\ cm^{-2}}\,, \qquad (30)$$

where θ is the semiangle of the funnel, and S_0 is the total bolometric fluence of the burst occurring at a distance of $D = 10^{28}$ cm. Note that, although both mathematically and physically $\theta = 0$ should correspond to the absence of any burst, the two foregoing equations would suggest $\tau \to \infty$ and $S_0 \to \infty$ for a nonexistent burst! This happens because the value of E was not scaled down in keeping with the value of θ in these equations, virtually presuming that the canonical value for an isotropic FB energy somehow gets channelized along the narrow funnels. Therefore, we reinterpret our basic result that in case the FB is conical with a solid angle $\Omega \sim \theta^2$ in its rest frame, then E_{51} is to be replaced by $(\Omega/4\pi)E_{51}$ to take care of the fractional energy channelization. It follows then that if we apply equations similar to equation (29) or equation (30), we eventually get back the original equation (28). In other words, the basic temporal properties of the FB (along a given direction) should be more or less unaltered even for an anisotropic case as long as we are able to define $\eta(\theta)$ in a meaningful way. It may be recalled here that although the generation of a relatively baryon-free, high-η jetlike FB is required for understanding the GRBs, the understanding about their genesis is largely a matter of conjecture and is something like having "an artist's conception." This is true in view of (1) the uncertainty about the physical mechanism triggering GRBs, (2) the fact that even for an assumed mechanism like a neutron star–neutron star merger, the merger geometry is unknown and evolving faster (at the moment of maximum energy liberation) on timescales shorter than the observed GRB timescales, (3) unknown effects like probable new general relativistic instabilities and the unknown nature and evolution of the coalesced object, (4) unknown extreme parameters such as temperature, density, and their profiles in the dynamic and unknown merger geometry (accretion disk, torus, etc.), (5) the unknown microphysics at such extreme conditions (e.g., the equation of state and viscosity profiles of the dynamic merged object), and also (6) the basic uncertainty about the relative importance of energy loss by gravitational radiation and neutrino emission.

The actual burst duration, even when we consider the emission of hard X-rays and gamma rays only, can obviously be longer than is suggested by equation (29) if the FB fails to radiate the available energy (50% at $r = r_d$). In fact, when the burst fails to be radiative at $r \sim r_d$, there may be a weak and prolonged burst corresponding to a larger value of r_γ. Also, if we redefine the burst duration as the one during which the FB radiates 75% of its energy, we will have a value of τ larger by a factor of a few ($\gamma_F \to \gamma_{F/2}$).

We remind the reader again that this whole discussion explored the question of duration of the GRB in the idealistic Mészáros & Rees (1993) framework, which assumes that even in a sparse ISM ($n_1 \leq 1$), the hydrodynamic limit is achieved at any radius. Since the leading protons of the FB interact with the sparse ISM extremely weakly because the background ISM magnetic field is very weak, and we are not aware of any cooperative phenomenon by which these protons may collectively interact much more strongly by self-generating strong magnetic fields on their way up, it is quite uncertain whether GRBs can be triggered in ordinary sparse ISM (Mitra 1996). Naturally, there may be no strong shock at all at $r = r_d$ unless GRBs are hatched in special dense regions of ISM, although subsequently the blast wave may propagate in the ordinary ISM and generate various low-energy afterglows. The eventual mechanism of GRBs could be considerably different from what our present understanding admits, and then part of the present discussion may be invalidated. For instance, all the existing hydrodynamical models assume that protons/ions accelerated in the shock can transfer their energy to the associated electrons sufficiently fast and that the radiative timescale of the shock is determined by the much shorter radiative timescale of the electrons. This assumption need not be correct, and though this was pointed out privately by the present author to several other authors working on the problem, this aspect has been glossed over for the sake of simplicity. We will discuss in a separate paper under what conditions the hydrodynamical description may be valid even at small values of r and how it may be possible to have a blast wave with a higher efficiency for gamma-ray production (Mitra 1998).

It may be also possible that the fundamental mechanism for the GRBs is not a one-shot collision process (involving compact objects) but, on the other hand, a process whereby energy is released erratically and in a jerky manner in the form of an "unsteady wind" over a duration of tens of seconds or even longer. In such a case the basic GRB duration will obviously be determined by the central source, although afterglow timescales may be determined by the hydrodynamic model discussed in this work.

Finally, let us point out that one can also separately define r_{optical}, r_{infrared}, and r_{radio} in the context of the hydrodynamical model for the post-GRB phase of the event, provided that one has a model for the generation of the respective radiations. After the main GRB phase, the value of γ will decrease rapidly and will approach unity, i.e., the blast wave will become Newtonian like a SNR. At each stage the afterglow will be characterized by a timescale $\tau_{\text{radiation}} \sim r_{\text{radiation}}/\gamma^2 c$, with no simple and general relationship between $\tau_{\text{radiation}}$ and η. Such afterglow timescales can obviously be arbitrarily long and exceed even hundreds of years if the GRB event occurs in the Local Group. Thus, if we are fortunate, we may be able to identify some of the presently known SNRs that have no signature of harboring a compact object at their centers as GRB remnants.

The author is thankful to the anonymous referee for several constructive suggestions that greatly improved the presentation of this work.

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