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Why is it so easy to underestimate systematic errors when measuring G?

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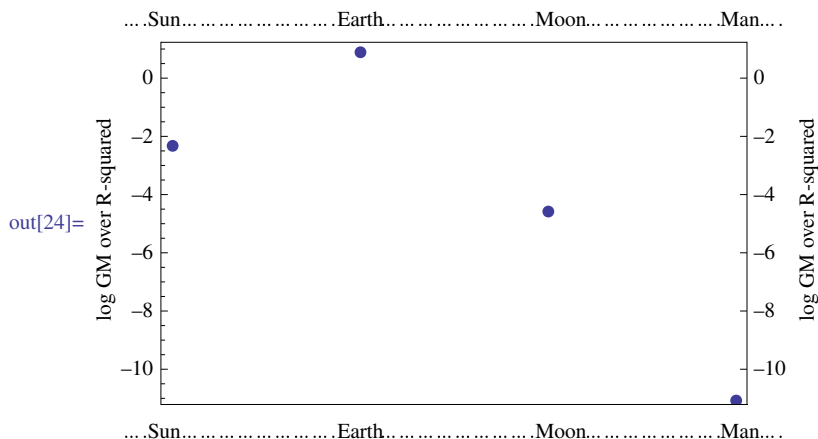
Before this Theo Murphy Meeting, my working hypothesis was that human activity during the measurement of G significantly affects the measurement itself. Noise caused by the gravity gradient of humans was indeed the reason why in one experiment the apparatus was raised 3 m above the floor. The meeting convinced me that all experimenters took adequate precautions against gravity gradients caused by human activity. During the meeting, another concern arose: was the cycle time between two states of the experiment so long that the environment changed significantly in the meantime? Once again, it appears that the experimenters were appropriately cautious. After the meeting, it became clear that ageing effects in thin, stressed wires could be an issue. I conclude with a speculation about a future 'atomic' value of G.

1. Introduction

I have been asked to speculate on why the measurement of G has had its ups and downs with no convergence in sight. My only qualification for doing so is that in 1989 there were two positive measurements of the putative fifth force, $V = -(GM/r)(1 + \alpha\,e^{-r/\lambda})$. Experiments up a television tower in North Carolina, USA, and down a mine shaft in Australia agreed that α was not zero and λ was a few hundred metres. They disagreed, however, on the sign of α. Wes Tew and I, then both at the University of Colorado, Boulder, CO, USA, helped both experimental groups to recognize that they had not treated the local topography correctly.

2. Seven measurements of G

At this Theo Murphy Meeting, Gillies & Unnikrishnan [1] has observed that the claimed value for $\Delta G/G$

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Figure 1. The Newtonian force (per kg) at the location of the apparatus from Sun, Moon, Earth and an SGman. Note that, for the Earth, $g = 9.8$ N kg^{-1} and that the common logarithm of 9.8 is $+1.0$. (Online version in colour.)
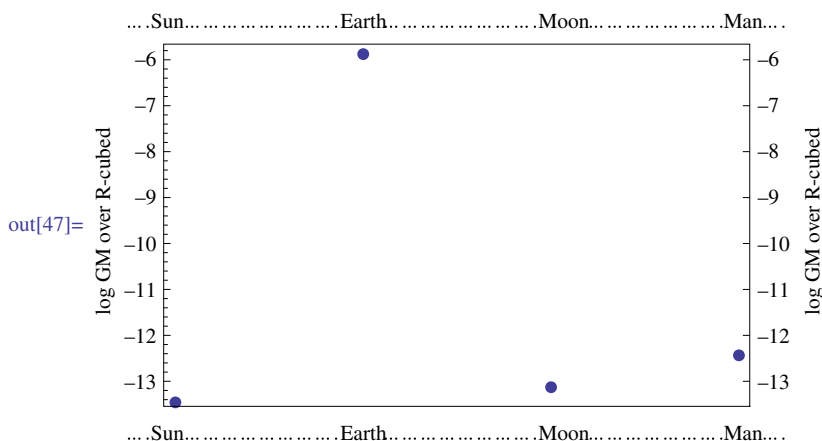
**Figure 2.** The tidal force (per kg m^{-1}) at any apparatus from Sun, Moon, Earth and an SGman. (Online version in colour.)

systematically decreases as the source mass increases. This observation is easy to understand, because the signal varies directly as the product of the source mass and the test mass.

Two approaches have been taken for the test mass. Most chose a test mass in the form a very thin flat sheet suspended by a light torsion fibre. Newman & Bantel [2] have shown that in this geometry the measured torque is proportional to $Y_{lm} = Y_{22}$ of the source. In their experiment, higher-order terms only begin to contribute at $l = 6$. Others also had clever ways to negate terms such as Y_{42}. When using a thin, flat sheet, the gravitational effects of distant objects diminish as their tidal force, or R^{-3}.

Quinn *et al.* [3] used instead a 6 kg test mass with fourfold symmetry. With such symmetry, the gravitational effect of distant objects decreases as R^{-5}. Additionally, the supporting torsion strip has a restoring torque which is almost entirely gravitational and hence lossless.

Figures 1–3 contrast these two approaches. The y-axis on all is $\log_{10} GM_x r^n$ where SI units are used, $n = -2, -3$ or -5 and the x-axis shows three astronomical sources and a terrestrial one: a 'Speake–Gillies man' (SGman), i.e. one who weighs 100 kg and is 25 m away from the apparatus [4].
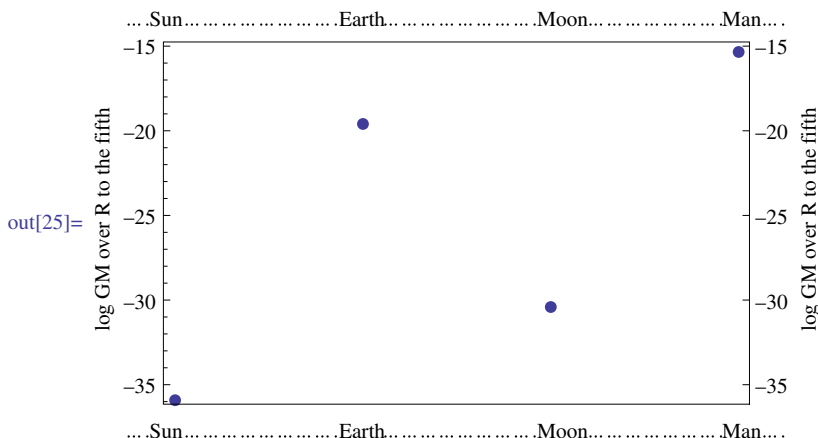
Figure 3. The 'fifth' force (per kg m^{-3}) at the site of any apparatus from Sun, Moon, Earth and an SGman. (Online version in colour.)

In the first plot, $n = -2$. This plot simply shows that the Earth dominates, but the Sun is not far behind. Baron von Eötvös made good use of the Sun's force on masses of differing composition.

The next plot is relevant to all experiments except for Quinn *et al*. [3]. Here, we see that the tidal force of the Earth at its surface is seven decades greater than that of a typical experimenter. The scale for this tidal force is simply related to the Earth's average density and is independent of size. The usual parameter is the free air gravity anomaly, $G\rho/2 = 0.30$ mgal m^{-1}, where 1 gal is 1 cm s^{-2}, and the average density of the Earth is 5.5 g cm^{-3}. In principle, one could measure G by measuring the gravitational gradient near an object of known density, say the high-density tungsten spheres used by Jesse Beams [5] and Luther *et al*. [6]. Unfortunately, gravimeters are not yet sensitive enough for the task.

Fortunately, all seven experiments are insensitive to gravity gradients from the Earth or any other stationary object. Generally, they are invulnerable because their *test* masses are stationary. For instance, the pendulum masses in the experiment of Parks and Faller [7] move less than half a wavelength of light. If the test masses hardly move but the source masses do, then an extraneous object in the environment has no effect on the measured value of G.

In one case, the test mass does move. In the experiment of Gundlach & Merkowitz [8], the fibre-supported test mass and the source masses are mounted on concentric turntables. Here, feedback ensures that the angular acceleration of the test mass and the source mass are essentially the same even though their angular frequencies differ. Gundlach & Merkowitz [8] use a method that was employed first by Jesse Beams and collaborators 45 years ago [5]. The Beams feedback mechanism 'ignores all surrounding stationary sources'. Such 'ignorance' is particularly helpful in the hands of Gundlach & Merkowitz [8]. Their laboratory is on the side of a steep slope [9].

In the third plot, $n = -5$. This plot is relevant to the experiment of Quinn *et al*. [3] that has fourfold symmetry. In this case, the extraneous forces from the SGman finally dominate over those of the Earth. But, the forces are inconsequential.

Gravity gradients do not appear to be an issue in any of the experiments presented at this Theo Murphy Meeting. Nonetheless, we form our first figure of merit by using the seven values of M_{source} and R_{source}. For each measurement, we have

$$\text{FM}_1 = \log\left[\frac{M_{\text{source}}}{25\,\text{kg}}\right] - 5\log\left[\frac{R}{25\,\text{m}}\right], \tag{2.1}$$

for Quinn *et al*. [3], and

$$\text{FM}_1 = \log\left[\frac{M_{\text{source}}}{M_{\text{earth}}}\right] - 3\log\left[\frac{2R_{\text{source}}}{R_{\text{earth}}}\right], \tag{2.2}$$

for all others.

Table 1. Two figures of merit.

	M_{source} (kg)	R_{source} (mm)	ABA_{cycle} (s)	FM_1	FM_2	$FM_1 + FM_2$
Gundlach & Merkowitz [8]	33	250	125	−1.0	−2.1	−3.1
Schlamminger [10]	13 520	780	160	0.1	−2.2	−2.1
Luo [11]	1.56	78	56 000	−0.8	−4.7	−5.5
Parks [7]	480	300	250	−0.1	−2.4	−2.5
Quinn et al. [3]	45	214	1800	10.0	−3.3	6.7
Quinn et al. [3]	45	214	61 000	10.0	−4.8	5.2
Newman [12]	118	324	14 400	−0.8	−4.1	−4.9
Tino and co-workers [13]	516	400	3600	−0.5	−3.6	−4.1

The first entry for Quinn et al. reflects their use of the original Cavendish deflection method; the second for the modern servo method.

The second figure of merit quantifies uncontrolled changes in the exterior environment. Newman & Bantel [2] has mentioned unusual rainfall even at their remote site at Hanford, WA, USA. Their protocol handles linear increases in the moisture content of the soil with time. It does not reflect quadratic and higher-order terms. Generally, the source masses are moved periodically from position A to position B. Making this transition as quickly as possible should minimize vulnerability to a changing environment. Thus, we define a second FM:

$$FM_2 = -\log \left[\frac{ABA_{cycle\ time}}{s} \right]. \tag{2.3}$$

FM_1 and FM_2 are shown in table 1.

The figure of merits clearly favour the two measurements of Quinn et al. [3] There is some virtue in having done two experiments with the same apparatus and completely different techniques. I would add the work of Newman et al. [14], which has been spread out over 15 years in two locations—most recently in an isolated spot free from 'cultural influences' such as cars and trucks. With 13 tons of mercury as a source, Schlamminger et al. [10] need not worry about highly stressed fibres. The atomic fountain of Tino and co-workers [13] promises a new technique that dispenses with fibres altogether.

(a) Fibre drift (torsion)

One asset of any experiment is PhD students, who must write a detailed account of the work. Yue Su [15] was a student working on the EOTWASH experiment in Seattle, WA, USA [9]. This experiment used a rotating torsion balance with a composition dipole. The experiment found no evidence for a fifth force [9]. Yue Su's thesis section on fibre drift is very frank: 'the main problem with the fibre used in this thesis is that it spontaneously untwists over a period of many months. Each time after we load and unload the pendulum, the fibre untwists at a relatively high drift rate ($>40\,\mu\mathrm{rads\,h^{-1}}$) after it is stretched again. This drift rate decreases slowly ($3–5\,\mu\mathrm{rad\,h^{-1}\,day^{-1}}$) at our normal running temperature (about 23.5°C). To speed up the process of untwisting, we heat the apparatus ($\simeq$30°C) above the normal running temperature for about 1 day, and then let it cool down. The heating turns out to be fairly effective: the drift rate after heating is usually below $4\,\mu\mathrm{rads\,h^{-1}}$ and remains constant ($\pm0.5\,\mu\mathrm{rads}$) throughout the data-taking time (usually about a month). We did not observe any clear correlation between the drift rate and the absolute position of the fibre, as one might expect. It seems that the fibre is aiming at a fixed drift rate (which may be a function of temperature) rather than at a fixed position. It appears that heating up and cooling down helped the fibre to reach the final drift rate'.

The fibre used in the EOTWASH experiment was a 0.8 mil tungsten fibre with a breaking strength of 100 g, a length of 80 cm and a torsion constant of 33 dyn cm rad^{-1}. This fibre is very similar to the one used by Gundlach & Merkowitz [8]. The essential difference is that now the loaded apparatus is heat treated at 100 °C above ambient. These authors correct for the twist of the fibre after their run is completed. Conceivably, there is substantial fibre drift during their individual runs. If this drift is to reduce the torsion constant, then it would also depress the value of G, which Gundlach & Merkowitz [8] report.

Heyl and Chrzanowski also reported fibre drift with a similar filament in their classic 1942 measurement of G [16]. Their description is revealing: 'two different tungsten filaments were used as a suspension for the pendulum. One of these was an ordinary commercial lamp filament, hard-drawn of diameter 0.0012 in, and tensile strength 288 g. This filament had been coiled up in the drawing process and consequently exhibited considerable drift when first set up. The measurements were not begun until 3 months after setting up by which time the drift of the resting point had been reduced to 0.04 cm in 15 h, the photographic plate being 490 cm from the filament. The total load on the filament was about 182 gm'. (Because an optical lever was used, the fibre drift rate was $(1/2)(0.04/490)(1/15) = 3\,\mu$rad h^{-1}.)

Heyl and Chrzanowski continue, 'After the completion of measurements with this filament, we obtained, through the cooperation of W. E. Forsythe, of the General Electric Lamp Works at Cleveland several pieces of specially annealed filament which had been kept straight in drawing and shipping. This filament on account of being annealed had a lower tensile strength than the hard-drawn filament and a diameter of 0.0014 in was necessary to give a tensile strength of 284 g. This filament showed a slight drift, which after 2 months' time was reduced to about half that of the hard-drawn filament'.

The disparity in the results was surprising both to the authors and to Riley Newman who reviewed the status of 'Big G' 14 years ago [17]. G as determined by its statistical error and its deviance from the CODATA 2010 value was better for the hard-drawn than for the annealed fibre. The Heyl and Chrzanowski results were $G = 6.6685 \pm 0.0016$ (annealed) and $G = 6.6755 \pm 0.0008$ (hard drawn). The latter result compares favourably with CODATA 2010 6.67384 ± 0.00080. From these limited data, I conclude that heat-treating or annealing a tungsten fibre makes its properties less, not more, reliable.

(b) Fibre drift (extension)

Parks and Faller [7] supported the mirrors of a horizontal Fabry–Pérot interferometer with two simple four-wire pendulums. The Fabry-Pérot cavity was shortened by the gravitational interaction of large source masses when these masses were moved to a compact 'inner position'. Conversely, the cavity was lengthened when the masses were moved to the 'outer position'. The authors took care to isolate the top-heavy support of the pendulums from the platform supporting the source masses.

The measured value of G is directly proportional to a spring constant k. As the authors state, 'the pendulum spring constant, up to some small corrections, is given by $k = m\omega^2$ with m the bob mass and ω the angular frequency of the pendulum when it is set into free oscillation in a separate experiment'.

My concern is with the timing of this separate experiment. Let us hope it was done sometime in May 2004 when the authors made their main measurements. As a wild speculation, suppose this separate experiment was made in May 2003. In the intervening year, microseisms from a nearby power plant could have extended the wires of the pendulum, thereby leading the experimenters to underestimate the value of G.

The apparatus of Parks and Faller [7] is located in the spectrometer laboratory on the University of Colorado Boulder campus in the USA. The apparatus, the power plant and the Colorado Centennial Foucault Pendulum form a triangle about 80 m on a side. The Foucault pendulum was commissioned in 1976. It still has the original 130-foot steel piano wire. This wire is one-sixteenth of an inch in diameter and supports a pendulum bob of 600 pounds [18]. At present,

the wire stretches about one-quarter of an inch every year. Perhaps, another wild speculation, the Parks–Faller pendulum has a similar increment in strain of 0.5 ppm per day. In an interval of 1 year, the effect on their measured $\Delta G/G$ is 180 ppm.

Schlamminger ([19], especially table 3.6) measured strains in a pit at the Paul Scherrer Institut in Villigen, Switzerland. He measured the length of the 2.3 m and 3.7 m wires leading to their top and bottom masses 13 times in 5 years. His data-taking run lasted 44 days in August and September of 2001. From measurements of wire length in May and November 2001, he found an average strain increment of 0.2 ppm per day. This increment is reduced to 0.1 ppm per day if the entire 5 year span of measurements is used. Stephan Schlamminger (2014, personal communication) concludes that such a small drift does not compromise his claimed accuracy.

3. Kuroda effect

Up to this point, this review has been written without reference to Kazuaki Kuroda. Kuroda observed that 'the measurement by the time of swing method gives a value of G that is too high by a fraction ϕ/π. If the wire is the principle source of damping, then the bias is equal to $1/(\pi Q)$, where Q is the quality factor for the main torsional mode' [20]. Reported values for Q are Luo *et al*. [11] 336 000, Quinn *et al*. [3] 100 000, and Newman & Bantel [2] 300 000. Presumably, the quality factor is considerably lower in the experiments of Park *et al*. and Gundlach *et al*. These authors specifically designed their experiments so that their fibres are hardly stressed. At present, Kuroda is the leader of a large project to complement the Laser Interferometer Gravitational Wave Observatory. The large-scale gravitational wave telescope will use sapphire fibres ($Q = 5\,000\,000$) cooled to 16 K [21].

4. BICEP2

On 17 March 2014, the Harvard–Smithsonian Center for Astrophysics in Cambridge, MA, USA, held a press conference. It announced that an observatory at the South Pole had found evidence for primordial gravitational waves emanating from the inflationary period just after the Big Bang. This stunning result caused me to search the literature to see what impact the discovery, if true, might have on Big G. Five years ago Galli *et al*. [22] speculated that ultimately a cosmic variance-limited, cosmic microwave background experiment could determine G to a 'precision of about 0.4%', a value which they deemed to be 'competitive with current laboratory measurements'. Unfortunately, these authors do not give a simple equation with G on the LHS. Such an equation is given in §5.

5. Cosinusoidal potential

Many of the experimenters at the Theo Murphy Meeting owe a debt to Fischbach *et al*. [23]. Their putative fifth force brought an excitement to a field that, until then, had given Newton and Einstein a 100% success rate. The EOTWASH group at the University of Washington, USA, began their work in 1985 when Fischbach had a sabbatical there. Jens Gundlach was a post-doc working with that group. Yue Su was a graduate student [15].

Su and I had a mission: how to invert the source problem? The EOTWASH group had found $\alpha = 0$, but they had no direct way of determining a subterranean mass anomaly that was, say, 20 km diagonally down from Seattle, WA, USA. Could one use measurements of g at the surface to infer $\Delta\rho$ as a function of depth, latitude and longitude. After all, both the Newtonian and the Yukawa potentials separately satisfy von Neumann uniqueness.

We studied Bougeur gravitational anomaly maps from land measurements of g and even gathered ships' logs of gravity in Puget Sound, WA, USA. Then, Su had an inspiration. The fifth force does not satisfy uniqueness. This even though its two constituents do so [24].

While writing our paper, it became obvious that the Yukawa potential itself was irrelevant to a continuing problem of galactic astronomers. The kinematics of a disc galaxy such as the

Milky Way differs fundamentally from that of the Solar System. In the latter, the orbital speed of a planet varies inversely as the square-root of the distance from the Sun. In Andromeda, a typical disc galaxy like our own, the velocity of stars far from the galactic centre is independent of the distance from the centre of the galaxy [25]. Rubin [26] accepts the conventional explanation, 'dark matter is responsible', but she has always hoped that something more fundamental was indicated.

The Yukawa potential does not help. Imagine a potential with a range λ_o, intermediate between that of the Solar System and the distance from the Sun to the centre of the Milky Way, R_o. The shielding factor $\exp[-R_o/\lambda_o]$ provides *less* centrifugal force than Newton does to keep the Sun in a circular orbit about the galactic centre.

Alternatively, the Yukawa potential with an imaginary value for λ_o can work. Then, we have for the potential at a distance r from a point mass

$$\phi = \left(\frac{-GM}{r}\right) \Re\left[\exp\left[\frac{i2\pi r}{\lambda_o}\right]\right] = \left(\frac{-GM}{r}\right)\cos\left[\frac{2\pi r}{\lambda_o}\right]. \tag{5.1}$$

A wide variety of astronomical data, including a striking set of wiggles in a plot of neutral hydrogen in the Milky Way ([27], fig. 7.23), leads to a nominal value of 400 pc for λ_o. Now, consider how the centripetal force from this potential balances the usual centrifugal force at the location of the Sun, $r = R_o = 8000$ pc.

$$\frac{mv^2}{r} = \frac{d}{dr}\left[\left(\frac{-GmM_{\text{eff}}}{r}\right)\left(\cos\left[\frac{2\pi}{r}\right]\right)\right] = \left(\frac{2\pi}{\lambda_o}\right)\left(\frac{GmM_{\text{eff}}}{r}\right)\left\langle\sin\left[\frac{2\pi r}{\lambda_o}\right]\right\rangle. \tag{5.2}$$

Just cancel the r's in the denominator to find $v^2 = GM_{\text{eff}}\langle\sin[2\pi r/\lambda_o]\rangle$.

Here, for the disc of the Milky Way, $M_{\text{eff}} \simeq M_{\text{enc}}$, and the average over the circular groove in which the Sun is trapped, $\langle\sin[2\pi r/\lambda_o]\rangle \simeq 0.1$.

Ten years after my collaboration with Su, George Gillies invited me to publish in a special issue of *Metrologia* on electrical charge. (All but two of the seven articles in this issue are represented by authors at this Theo Murphy Meeting). Here, I presented the case for a graviton of imaginary mass and a photon of the same, but real, mass [28]. Su's influence persisted. The article credited Vladamir Fock for having the good sense to tame Einstein's general covariance by insisting that calculations in relativistic gravitation be done in a unique gauge, the harmonic or Lorentz gauge [29].

A few years later, Bartlett [30] presented a poster that relates Newton's constant of gravitation G, the new length λ_o, the Rydberg, and the Bohr radius,

$$\frac{c^4(2\pi/\lambda_o)^2}{(2\pi G)} = \frac{13.6\,\text{eV}}{(137a_0)^3}. \tag{5.3}$$

Please note that the dimensions of both sides of the equation are those of energy density. The right-hand side of equation (5.3) can be evaluated to an accuracy of a part per million. G is known to better than 100 ppm. Plugging in values for a_o, c, R_∞ and G gives the 'atomic' value of $\lambda_o = 374$ pc. This atomic value is compatible with our nominal value of 400 pc. In the very distant future, G may be measured most accurately by an atomic fountain. Astronomers may measure λ_o with high accuracy. Then, by using the first half of the ansatz above, one can fulfil the metrologists' dream of replacing all artefacts with atoms.

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