New technique for measuring Newton's constant G

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We discuss a new technique for measuring Newton's constant G using a rotating torsion balance operated in a feedback mode. The method has several conceptually new features that reduce sensitivity to the dominant systematic uncertainties of previous experiments. We have successfuly conducted exploratory measurements that establish the feasibility of the new technique. [S0556-2821(96)50116-5]

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The Newtonian gravitational constant G is the least precisely determined fundamental constant. The accepted Committee on Data for Science and Technology (CODATA) [1] value, $G=(6.67259\pm 0.00085)\times 10^{-11}$ $\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$, is heavily dominated by the 1982 measurement of Luther and Towler [2] and was assigned an uncertainty of 128 ppm. Recently this value has been brought into question by several groups. The German Physikalisch Technische Bundesanstalt [3] obtained a value 0.6% (~40 standard deviations) higher, a New Zealand group [4] reported a value 0.1% (~7 standard deviations) lower, while a Wuppertal group [5] obtained a value 0.06% lower than the CODATA value. In addition a Russian group [6] claimed to observe a temporal and length-scale variation of G at the 0.7% level. Except for Ref. [5], which used a new double-pendulum technique, these experiments employed the classical strategy of measuring the torque on a torsion pendulum or relied on the constancy of the restoring torque of a torsion fiber undergoing large-amplitude oscillations. Kuroda [7] recently pointed out that G measurements based on detecting the change in torsional oscillation frequency may have a systematic bias due to torsion fiber inelasticity. A decisive measurement, preferably using a new technique, is needed to resolve the discrepancies in the value of this natural constant.

We have developed a new method for measuring G that is based on measuring the *angular acceleration* of a "two-dimensional" torsion pendulum. Our method, which differs from the acceleration method of Rose *et al.* [8], overcomes important sources of systematic error in previous measurements. In particular, the pendulum dimensions, mass, and density distribution need not be known precisely, and many torsion fiber properties need not be known precisely or even remain constant. We have run numerical simulations and have conducted exploratory measurements using an existing apparatus to demonstrate the feasibility of this new method.

The gravitational angular acceleration, α, of a torsion pendulum in the field of a nearby attractor can be expressed in a multipole formalism [9,10]:

$$\alpha(\phi)=\sum_{l,m}\alpha_{l,m}=-\frac{4\pi G}{I}\sum_{l=2}^{\infty}\frac{1}{2l+1}\sum_{m=-l}^{+l}mq_{lm}Q_{lm}e^{im\phi}, \tag{1}$$

where q_{lm} and Q_{lm} are the spherical multipole moments of the pendulum and multipole fields of the attractor, respectively, ϕ is the azimuthal angle between the pendulum and the attractor, and I is the pendulum moment of inertia. Equation (1) assumes that the restoring torque from the suspension fiber is negligible; we justify this approximation below. With the choice of pendulum and attractor geometries discussed below, α will be dominated by the $q_{22}Q_{22}$ term in Eq. 1. In this case

$$\alpha(\phi)=\alpha_{2,2}=-\frac{16\pi}{5}G\frac{q_{22}}{I}Q_{22}\sin 2\phi. \tag{2}$$

The quotient

$$\frac{q_{22}}{I}=\frac{\int\rho(\vec{r}_p)Y_{22}(\theta_p,\phi_p)r_p^2d^3r_p}{\int\rho(\vec{r}_p)\sin^2\theta_p r_p^2 d^3r_p}\to\sqrt{\frac{15}{32\pi}}, \tag{3}$$

where $\rho(\vec{r}_p)$ is the pendulum density, becomes a constant for a pendulum that lies entirely in a plane that includes the torsion fiber axis, so that

$$\alpha(\phi)=\alpha_{2,2}=-\sqrt{\frac{24\pi}{5}}GQ_{22}\sin 2\phi. \tag{4}$$

This allows a measurement of α to yield a precise value of G that is independent of the pendulum mass, dimensions, or density distribution. It is worth noting that uncertainties in these quantities formed the dominant contributions to the error in the value of Ref. [2].

A pendulum and attractor geometry that incorporates these ideas is shown in Fig. 1. A complete torsion pendulum apparatus mounted on a turntable is initially started rotating at a slow rate ω_t, for example 2π rad/h. Eight massive spheres centered about the pendulum exert a time-varying gravitational torque on the pendulum that, with the geometries discussed below, will be dominated by a $q_{22}Q_{22}$ coupling. The angular acceleration of the pendulum is measured by activating a feedback loop that continually adjusts the rotation rate of the turntable to follow the pendulum so that the torsion fiber never twists from its equilibrium angle. In this way the "free" angular acceleration of the pendulum is directly transferred to the turntable, and the restoring torque from the suspension fiber is driven to zero. Because the fiber

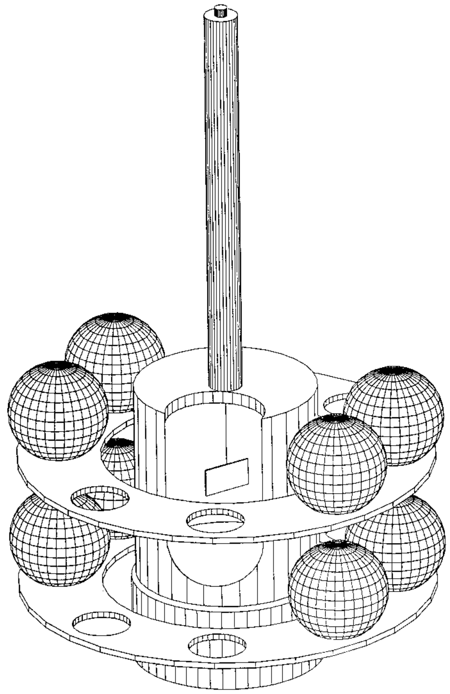

FIG. 1. Diagram of an apparatus for measuring G. The drawing shows only the essential geometrical features. The torsion balance (shown cut away to display the pendulum) sits on a turntable; the rings supporting the spheres are mounted on a second turntable (not shown). The pendulum and attractor spheres are drawn to scale. The rings have a vanishing Q_{22} moment and do not affect the result; the leading torque from the rings has $m=8$. Asymmetries in the attractor turntable can be canceled by shifting the spheres to the unoccupied holes in the rings.

never twists, the torsion constant, κ, of the fiber need not be known nor will its inelastic properties directly affect the measurement [13].

The turntable angular acceleration is determined from the change in the pulse rate of a high-resolution angle encoder, a highly linear device that does not require external calibration (because $0°=360°$). One determines G by fitting $\alpha(\phi)$ with a harmonic series in ϕ and extracting the coefficent of $\sin 2\phi$ to select angular accelerations, $\alpha_{l,m}$, with $m=2$. To eliminate accelerations caused by other objects in the lab, the attractors are placed on a second turntable that rotates at a rate ω_a whose magnitude and sign differ from ω_t. This rotation also averages out any local nonlinearities of the pendulum shaft encoder, reduces any effects from vibrations associated with either of the turntables, and puts the signal at a relatively high frequency which reduces noise, in particular low-frequency gravitational noise.

We now discuss techniques for minimizing $l>2$ gravitational torques so that α is dominated by the $q_{22}Q_{22}$ coupling. The magnitudes of $l>2$ torques are naturally reduced by factors $(R_p/R_a)^{l-2}$, where R_p is a typical dimension of the pendulum and R_a is the radius to the attractors. The leading higher-l accelerations can be made to vanish with proper pendulum and attractor design. The q_{l2} and Q_{l2} moments with odd l vanish due to symmetry about the horizontal mid-plane. A rectangular pendulum with a width w, height h, and thickness t will have a vanishing q_{42} moment if $10h^2=3(w^2+t^2)$. For such a pendulum

$$\frac{q_{22}}{I}=\frac{w^2-t^2}{w^2+t^2}\sqrt{\frac{15}{32\pi}}. \quad (5)$$

For example, a pendulum with a width $w=76.00$ mm, a height $h=41.65$ mm, and a thickness $t=2.50$ mm, has a q_{22}/I that deviates from the "two-dimensional" value by only 0.2%, while q_{42} still vanishes. By making the attractors from pairs of spheres of mass M, with vertical separation z, at a radial distance from the pendulum axis $\rho=\sqrt{3/2}z$, we eliminate the Q_{42} field; by employing two pairs of spheres on either side of the pendulum, separated by 45° of azimuth, we eliminate all odd m couplings as well as those with $m=4$. With this design, shown in Fig. 1, we have

$$Q_{22}=\sqrt{\frac{10}{7\pi}}\frac{108}{49}\frac{M}{\rho^3}, \quad (6)$$

while the leading non-$q_{22}Q_{22}$ torque, which occurs in $l=6$ order, is calculable and small:

$$\frac{\alpha_{6,2}}{\alpha_{2,2}}=\frac{99}{7683200}\frac{213(w^4+t^4)+626w^2t^2}{\rho^4}. \quad (7)$$

For ρ=25 cm, this ratio is 2.35×10^{-5}. Note that all lower-order torques, except for the $q_{22}Q_{22}$ torque of interest, are the products of two small (nominally zero) values.

We have, so far, described an ideal scenario. We now show that corrections for imperfections are small and tractable. One can determine G to 1 part in 10^5 if w is uncertain by 0.20 mm, or if density variations are as large as 0.46% [11], or if the absolute thickness and overall flatness of the pendulum are uncertain by 5 μm. Similarly, the tip of the pendulum about a horizontal axis can be as large as 2 mrad, or a rotational misalignment about an axis perpendicular to the plane can be as large as $\leqslant 10$ mrad. If the pendulum is fabricated from a quartz glass plate, one can use optical methods to measure its thickness, flatness, and density uniformity. The quartz plate faces can then be Au-coated to reflect the light beam that monitors the angular deflection (and to electrically ground the pendulum). The light beam can also be used to measure the pendulum tip by rotating the pendulum so that the light hits the opposite face of the pendulum. By constructing the attractor from spheres we eliminate problems from density nonuniformities in the attractor that otherwise would limit the precision (spherically symmetric density variations do not affect the fields of the spheres, while nonsymmetric variations can be measured and averaged out by changing the orientations of the individual spheres).

A feedback circuit with finite gain requires a small twist of the torsion fiber to derive its feedback signal. Hence a small torque is tied up in twisting the torsion fiber. As the twist angle is recorded, it is straightforward to account for this small extra torque when extracting G from $\alpha(\phi)$. If the open-loop gain (the factor by which the feedback reduces the pendulum deflection) is sufficiently high ($>10^3$), and the damping time of the free pendulum sufficiently long ($\tau_d \geqslant 10^4$ s), a 10^{-5} measurement of G can be made even if the free oscillation frequency of the torsion oscillator ω_0 and the angular-deflection calibration are known only to 1%. The angular-deflection calibration, ω_0, and τ_d are easily found by

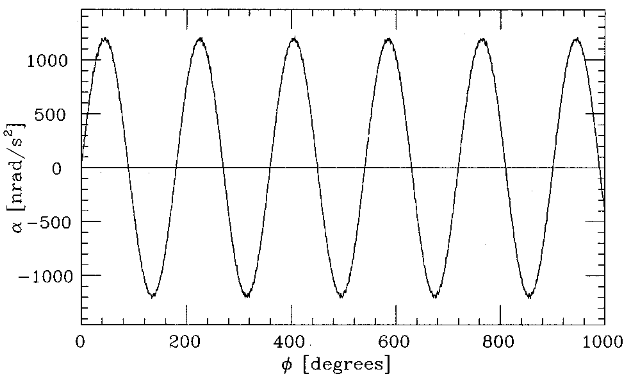

FIG. 2. Numerical simulation of a G measurement, showing the turntable angular acceleration for the pendulum and attractor geometries discussed in the text. A feedback update time of $\Delta=2$ s and a 6 db/octave electronic time constant of 3.0 s were used. Realistic values for ω_0, τ_d, and electronic noise were employed. A torsion-fiber drift rate of 1 μrad/h (see Ref. [10]) was assumed. The average turntable angular velocity was $\overline{\omega_t}=4.87$ mrad/s, which gave a 4.9% gravitational speed modulation.

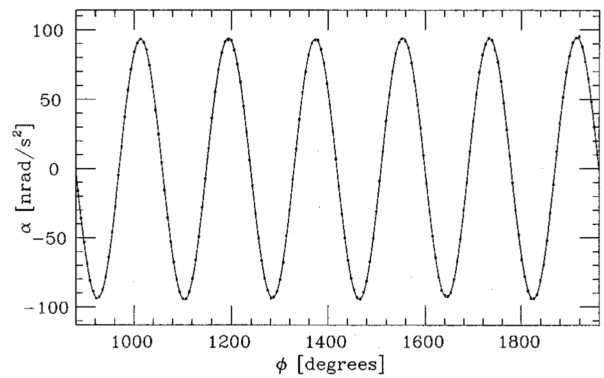

FIG. 3. Proof-of-principle demonstration using the Eöt-Wash torsion balance and a stationary attractor. We show the angular acceleration (averaged over 100 s) of the feedback turntable arising from the predominant $q_{22}Q_{22}$ gravitational coupling. The smooth curve is a harmonic fit to the data. Gravitational fluctuations from human activity in the vicinity of the test setup were the dominant noise source.

turning off the feedback and observing the pendulum response to a programmed step-change in the turntable angular velocity [9].

The performance of the feedback loop is of central importance in our technique. We have tested a feedback algorithm, first with numerical simulations and then with a torsion balance normally used for equivalence principle tests [9,10]. The feedback loop is digital and executed in software. It senses the torsion fiber twist angle θ as measured by an autocollimator and adjusts the frequency ν of the oscillator controlling the turntable rotation rate so that the autocollimator signal remains unchanged. This frequency, directly proportional to the turntable rotational velocity $\omega_t=\dot{\phi}$, is updated at regular intervals, Δ, and recorded along with the values of ϕ and θ. The feedback loop uses differential, direct, and integral terms to compute the frequency change

$$\nu_{i+1}-\nu_i \propto c_1\frac{\theta_i-\theta_{i-1}}{\Delta}+c_2\frac{\theta_i+\theta_{i-1}}{2}+c_3\Delta\sum_{m=0}^{i}\theta_m . \quad (8)$$

Stable performance is obtained with $c_3=\Delta/\overline{\tau^3}$, $c_2=3\Delta/\overline{\tau^2}-\omega_0^2\Delta-1.5c_3\overline{\tau}$, and $c_1=3\Delta/\overline{\tau}-c_2\Delta-c_3\Delta^2$, where $\overline{\tau}$ is a characteristic time which should be several times Δ [12].

Figure 2 shows a numerical simulation of a G measurement using the pendulum dimensions given above, an attractor with $Q_{22}=4.6$ g/cm^3, realistic values for ω_0, and τ_d. The feedback loop had an update time of $\Delta=2.0$ s. A 3.0 s, low-pass, 6 db/octave analog filter was placed on the autocollimator output, and $1/f^2$ noise in θ, consistent with that observed in previous experiments [10], was included. The open-loop gain could be made to exceed several times 10^3. The simulation gave a G value accurate to 10^{-5} in the equivalent of one day of operation. We then successfully implemented the same feedback scheme in an existing equivalence principle apparatus using similar parameters.

Figure 3 shows one result from a series of tests of the feedback algorithm using our Eöt-Wash rotating torsion balance [9,10]. For this test, we used a lab-fixed Pb attractor with $Q_{22}=0.52$ g/cm^3 (roughly a factor of 10 smaller than would be used in an actual G measurement), and two of the four test bodies of our normal pendulum were removed to create a sizable q_{22} moment. The average turntable speed was $\overline{\omega_t}=0.0011$ rad/s, with a 3.7% gravitational $\sin 2\phi$ speed variation, and the feedback gain was ≈ 2000. The extracted value of G agreed with the standard value to within the 2% uncertainty in Q_{22}. We found that gravity gradient fluctuations caused by human activity were the biggest source of noise. These would be considerably reduced by operating in a more favorable location.

We have presented a new method for determining G that we believe could provide a substantially improved value good to 10 ppm. Our method overcomes the most significant sources of systematic uncertainty encountered in other techniques. Initial tests demonstrate the practicality of the method. Finally, we point out that the pendulum and attractor geometry discussed here offers substantial advantages for torsion-balance G measurements based on the conventional frequency-change technique used, for example, in Ref. [2]. With our geometry the change in the squared small-oscillation frequency of the pendulum is

$$|\Delta\omega_o^2|=8G\sqrt{\frac{6\pi}{5}}\frac{w^2-t^2}{w^2+t^2}\,Q_{22}, \quad (9)$$

where Q_{22} is given by Eq. (6); we have ignored possible problems with variations in κ as discussed in Ref. [7]. Newman [14] has independently discovered the advantages of a flat pendulum in this context.

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[11] This value occurs in a worst-case scenario where the density of the central half of the pendulum is 0.46% greater than the density of the outer half.
[12] The small correction for the finite open-loop gain must account for the phase delay and attenuation of the autocollimator readout.
[13] An alternative to the feedback scheme discussed here may offer advantages in signal-to-noise. In this scheme the turntable is driven in a smooth, preprogrammed way that closely anticipates the pendulum acceleration.
[14] R.D. Newman (private communication).