Thermoelastic Correction in the Torsion Pendulum Experiment *

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The thermoelastic effect of the suspension fibre in the torsion pendulum experiment with magnetic damping was studied. The disagreement in the oscillation periods was reduced by one order of magnitude through monitoring the ambient temperature and thermoelastic correction. We also found that the period on uncertainty due to noise increases with the amplitude attenuation after thermoelastic correction.

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As a scientific instrument, the torsion balance has been used for detecting the weak force, especially in gravitational experiments, such as the measurements of the gravitational constant,[1,2] verifications of the Newtonian inverse square law,[3] searches for non-Newtonian effects or new forces in nature,[4,5] tests of the equivalence principle,[6] study of a composition dependence or a spin dependence of the gravitational interaction,[7] etc. The torsion balance could be used in either a static mode or a dynamic mode. In the static mode, a gravitational couple acts on the beam to produce an angular deflection. Absolute measurement of the deflection with high precision is very difficult in practice due to the presence of background disturbances. A better way is to operate the torsion balance in the dynamic mode and the quantity of interest is the frequency excited by the attracting masses, in effect for averaging out the low-frequency external noise. A significant advantage of this type of experiment is that it is differential in nature, having a frequency shift as its signature. This approach is quite welcome from the metrologist's point of view, since time and frequency are relatively easy to measure with accuracies well beyond those needed for most kinds of torsion pendulum experiments. Determination of the gravitational constant G with the time of swing method is the typical application of the dynamical mode. However, there are still some shortcomings in the dynamic mode, such as the nonlinear effect due to the finite amplitude, and the anelasticity of the suspension fibre. We have studied the nonlinearity of the suspension fibre, and the amplitude dependence of the quality factor of the torsion pendulum.[8-11] In Ref. 9, we found that the torsion pendulum period decreased with the amplitude attenuation, and the periods were different at the same amplitude for different sets. In our recent research on the physical properties of a torsion fibre, we found that the torsion spring constant of the fibre is temperature dependent. The physical reason is that the shear and Young's moduli of the fibre are related to the interatomic force which in turn depends on the distance between atoms in the crystal lattice. So the elasticity of the fibre is temperature dependent and this is the so-called thermoelasticity.[12] This means that the different periods of different sets in Ref. 9 may be caused by the different ambient temperature, because the electrical coil which provided the magnetic damping for the pendulum may change the temperature near the fibre. To verify this and reduce thermoelastic effect in the torsion pendulum experiment, the ambient temperature was monitored during the experiment of the torsion pendulum period versus the amplitude.

The typical nonlinear dynamical equation of the torsion balance can be written as

$$I\ddot{\theta}+\gamma\dot{\theta}+K_1\theta+K_3\theta^3=0, \tag{1}$$

where I, θ and γ are the moment of inertia, the angle displacement and the damping factor of the pendulum, respectively. The coefficient K_1 is the total equivalent torsion constant of the pendulum system, and K_3 represents the nonlinearity of the system. We can obtain the approximate solution of Eq. (1) in the small amplitude oscillation as follows[10]

$$\theta(t)\approx\theta_0\mathrm{e}^{-\beta t}\cos(\omega t)+\frac{K_3}{32K_1}\theta_0^3\mathrm{e}^{-3\beta t}\cos(3\omega t), \tag{2}$$

where $\beta=\gamma/2I$, and θ_0 is the initial amplitude of the oscillation. The amplitude dependent period is

$$T(A)=T_0\left(1-\frac{3K_3}{8K_1}A^2\right), \tag{3}$$

where

$$T_0=\frac{2\pi}{\omega_0}\left(1-\frac{\beta^2}{\omega_0^2}\right),\quad \omega_0=\frac{K_1}{I},$$

and $A=\theta_0e^{-\beta t}$ is the amplitude of the oscillation. On the other hand, the temperature dependent period of the pendulum can be written as follows

$$T(\Theta)=T(\Theta_0)[1+\alpha_{\mathrm{T}}(\Theta-\Theta_0)], \tag{4}$$

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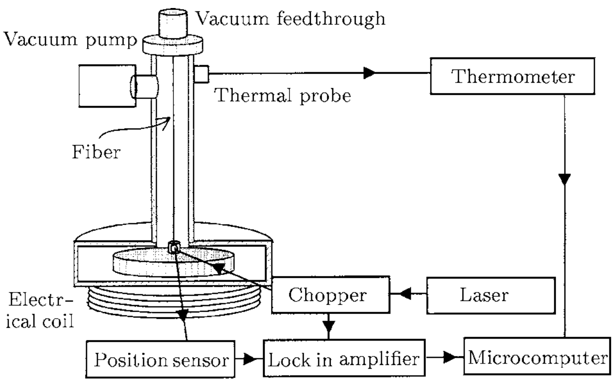

Fig. 1. Schematic diagram of the apparatus in the thermoelastic correction experiment.

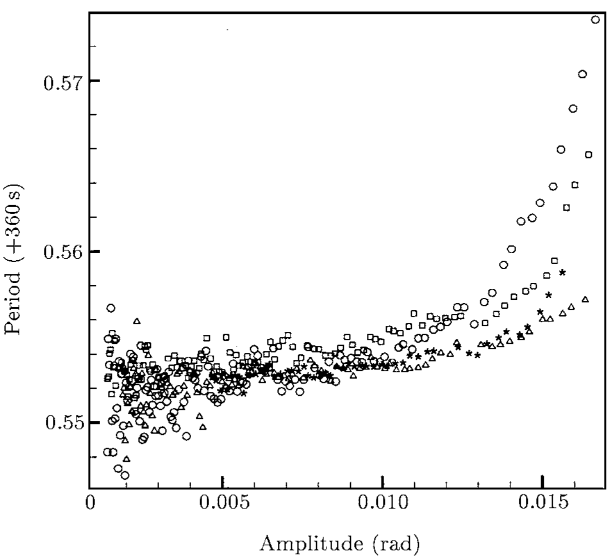

Fig. 2. Uncorrected periods for four data sets. The squares, stars, uptriangles and circles represent the data started on Nov. 3, Nov. 10, Nov. 14 and Nov. 26 1999, respectively.

where α_{T} is the temperature coefficient of the oscillation period, $T(\Theta)$ and $T(\Theta_0)$ are the periods at temperatures Θ and Θ_0, respectively. So the period at temperature Θ and amplitude A can be expressed as

$$T(\Theta, A) = T(\Theta_0, A)[1 + \alpha_{\mathrm{T}}(\Theta - \Theta_0)], \qquad (5)$$

and

$$T(\Theta_0, A) = T(\Theta_0, 0)\left[1 - \frac{3k_3}{8k_1}A^2\right]. \qquad (6)$$

According to the above analysis, we can correct the period $T(\Theta, A)$ to be $T(\Theta_0, A)$ at a certain temperature Θ_0, then we fit $T(\Theta_0, A)$ with Eq. (6). Fig. 1 shows the schematic diagram of the apparatus, composed of the torsion pendulum system which is the same as in our experiment of Ref. 9, the temperature sensor system and the data acquisition system. The disc pendulum was used in our experiment in order to eliminate the background field effects. To monitor the ambient temperature, a quartz thermometer was used in this experiment, with a resolving power of 0.001°C. The probe of the thermometer was attached onto the vacuum vessel near the suspension position. The angle displacement of the oscillation and the ambient temperature was recorded by the data acquisition system synchronously.

The pendulum oscillation was started by rotating the vacuum feedthrough. The initial amplitude of the oscillation was usually in excess of the effective maximal amplitude 0.016 rad, then it was attenuated through damping. The simple mode effect of the pendulum was damped by means of a magnetic field produced by an electrical coil under the vacuum vessel (as shown in Fig. 1). This duration usually lasted a few hours. After that, the recording system began to sample the oscillation signal of the pendulum continuously every half second until the amplitude attenuated from 0.016 to 0.001 rad. This duration lasted about four or five days, and then we adjusted the vacuum feedthrough again for the next set of experimental data. Four sets of the experimental data were obtained from Nov. 3 to Dec. 1, 1999.

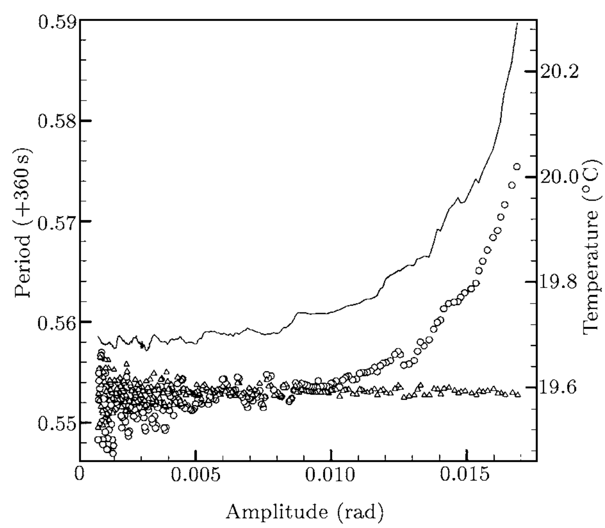

Fig. 3. Thermoelastic effect in the oscillation period of Nov. 26, 1999. The circles represent the uncorrected values and the uptriangles represent the corrected values. The ambient temperature recorded synchronously is shown as the solid line.

We fitted the period of the time-angle data by means of the period fitting method[13] and obtained the period data $T(\Theta, A)$ at temperature Θ and amplitude A. Fig. 2 shows the raw period of four data sets, from which we find there is a difference of about 0.01 s between them at amplitude 0.015 rad. Then we corrected $T(\Theta, A)$ to be $T(\Theta_0, A)$ using the temperature data with Eq. (5), where $\alpha_T = (106 \pm 3) \times 10^{-6}/°\mathrm{C}$ as was determined by our recent experiment[12] and $\Theta_0 = 19.73°\mathrm{C}$. Fig. 3 shows a typical thermoelastic correction of the pendulum period: the uptriangles represent the uncorrected value $T(\Theta, A)$ and the circles represent the corrected value $T(\Theta_0, A)$. The ambient temperature recorded synchronously is also shown in Fig. 3 as the solid line. It is noted that the period shift due to the fibre thermoelasticity is about 0.02 s at the beginning of the experiment and then it reduces with time. This is because the ambient temperature increased due to the heating of the electrical coil under the vacuum vessel and then attenuated to room temperature after the current of the coil was turned off. The four sets of the corrected periods varying with the amplitude are shown in Fig. 4. After thermoelastic correction, we fit the period data $T(\Theta_0, A)$ with Eq. (6) by means of the least-squares fitting method. The fitting results $T(\Theta_0, 0)$ are listed in Table 1.

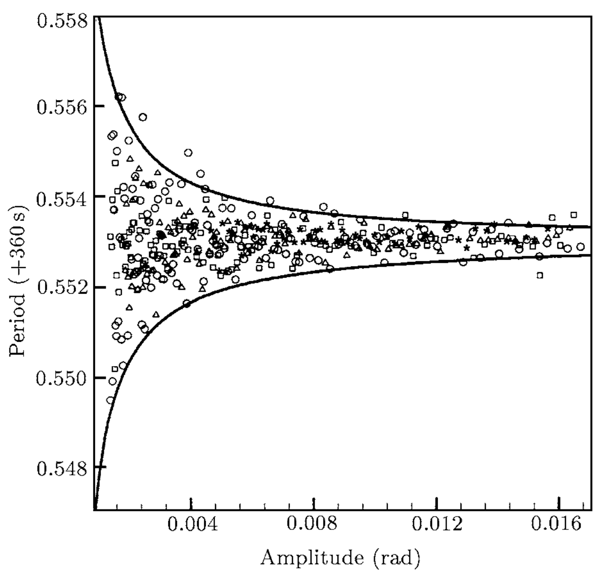

Fig. 4. Thermoelastic corrected periods for four data sets. The squares, stars, uptriangles and circles represent the data started on Nov. 3, Nov. 10, Nov. 14, Nov. 26, 1999, respectively. The period uncertainty fitted with empirical formula $\Delta T(A) = C/A$ is shown as the solid line.

From Fig. 4 and Table 1, we know that the oscillation periods for different sets of experiments agree with each other within the experimental error. After the thermoelastic correction, the nonlinear term $|k_3/k_1|$ is less than $0.030\,\mathrm{rad}^{-2}$, which is consistent with the results of the experiment without magnetic damping.[10,12] It is clear that the uncertainty of the period increases with the amplitude attenuation, which can be expressed, by a simple relation between the period deviation and the amplitude, as empirical formulae $\Delta T(A) = C/A$ and $C = 5.3 \times 10^{-6}\,\mathrm{s{\cdot}rad}$ approximately, as shown in Fig. 4 by the solid lines; this proves the hypothesis of some authors.[14]

Table 1. Fitting results of the pendulum periods of four data sets at temperature $\Theta_0 = 19.73°\mathrm{C}$.

Set No.	Duration of run (day/month, 1999)	T_0(s)	k_3/k_1 (rad^{-2})
1	03/11–08/11	360.5529	−0.029
2	10/11–12/11	360.5530	0.032
3	14/11–17/11	360.5530	0.004
4	26/11–01/12	360.5529	0.001
Average		360.5530	0.002
Uncertainty		0.0001	0.030

In conclusion, the thermoelastic effect of torsion fibre can be, and should be, corrected by monitoring the ambient temperature in the precision torsion pendulum experiment. In our experiment, the disagreement in the oscillation periods is reduced from 0.02 s to the noise level of 0.001s at the amplitude of 0.015 rad. This means that the uncertainty of determination G by means of the time-of-swing method with the same temperature variation can be reduced by one order of magnitude through the thermoelastic correction.

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