The 1986 adjustment of the fundamental physical constants*[†]

prepared by

E. Richard Cohen

Rockwell International Science Center, Thousand Oaks, California 91360

Barry N. Taylor

National Bureau of Standards, Gaithersburg, Maryland 20899

One of the early projects established by CODATA was intended to provide a recommended set of the fundamental physical constants, which are so important to the analysis and interpretation of experimental data in many scientific disciplines. The first "Recommended Consistent Values of the Fundamental Physical Constants" appeared in 1973 and was subsequently adopted by most international and national bodies in their own recommendations. This process has contributed to improved compatibility of scientific and technical data in all fields of science. CODATA presents here a revised version of these recommendations which takes into account the significant advances in metrology that have occurred since the 1973 analysis. "The 1986 adjustment of the fundamental physical constants" represents a 5-year effort involving experts from the major metrological laboratories of the world. It is hoped that this recommended data set will receive the acceptance of the scientific community which was achieved by its predecessor.

CONTENTS

*Originally appearing as CODATA Bulletin No. 63, November, 1986 (Pergamon, Oxford/New York). In view of the universal importance of the fundamental constants, the editors of *Reviews of Modern Physics* believed that it would be of value to the physics community to reprint this official report of the Committee on Data for Science and Technology (CODATA) of the International Council of Scientific Unions, which was previously available only to those with access to the CODATA Bulletins. The reprint contains a few minor modifications to the original that were made for clarity. RMP wishes to thank David R. Lide, President of CODATA, for making the article available to us.

[†]A report of the CODATA Task Group on Fundamental Constants; members of the Task Group:
T. J. Quinn, Bureau International des Poids et Mesures (Chairman, 1985—)
E. R. Cohen, Rockwell International Science Center (United States of America, Chairman, 1969—1985)
B. Kramer, Physikalisch-Technische Bundesanstalt (Federal Republic of Germany)
B. A. Mamyrin, A. F. Joffe Physical-Technical Institute (Union of Soviet Socialist Republics)
M. Morimura, National Research Laboratory of Metrology (Japan)
B. N. Oleinik, D. I. Mendeleev All Union Research Institute of Metrology (Union of Soviet Socialist Republics)
B. W. Petley, National Physical Laboratory (United Kingdom)
H. Preston-Thomas, National Research Council (Canada)
B. N. Taylor, National Bureau of Standards (United States of America)

GLOSSARY

A_{BIPM}	The time-dependent BIPM representation of the ampere prior to January 1, 1969: $A_{BIPM}=V_{BIPM}/\Omega_{BIPM}$
A_{69-BI}	The time-dependent BIPM representation of the ampere from January 1, 1969 to January 1, 1976: $A_{69-BI}=V_{69-BI}/\Omega_{69-BI}$
A_{76-BI}	The time-dependent BIPM representation of the ampere since January 1, 1976: $A_{76-BI}=V_{76-BI}/\Omega_{69-BI}$
A_{BI85}	The value of A_{76-BI} on January 1, 1985
ASMW	Amt für Standardisierung, Messwesen and Warenprüfung der DDR (Office for Standardization, Metrology and Quality Control, German Democratic Republic)
BIPM	Bureau International des Poids et Mesures (International Bureau of Weights and Measures), Sèvres, France
C_{BI85}	The BIPM representation of the coulomb on January 1, 1985: $C_{BI85}=A_{BI85}\cdot s$
CCE	Comité Consultatif d'Électricité (Consultative Committee on Electricity of the CIPM)
CERN	Organization Européenne pour la Recherche Nucléaire (European Organization for Nuclear Research), Switzerland
CGPM	Conférence Générale des Poids et Mesures (General Conference of Weights and Measures)
CIPM	Comité International des Poids et Mesures (International Committee of Weights and Measures of the CGPM)
CODATA	Committee on Data for Science and Technology of the International Council of Scientific Unions (ICSU)
CSIRO	Commonwealth Scientific and Industrial Research Organization, Australia
d_{220}	crystallographic (220) lattice spacing of silicon at 22.5 °C in vacuum
E	483 594.0 GHz/V
ELS1	extended least-squares algorithm 1 [Eq. (1.3)]
ELS2	extended least-squares algorithm 2 [Eq. (1.4)]
ETL	Electrotechnical Laboratory, Japan
g_e	$2\mu_e/\mu_B$, g factor of the free electron
$g_j(\mathrm{H})$	effective g factor of the electron in the ground (1S) state of hydrogen
g_p	$2\mu_p/\mu_B$, g factor of the free proton referred to the Bohr magneton
g'_p	$2\mu'_p/\mu_B$, g factor of the proton in a spherical sample of pure water at 25 °C
$g_p(\mathrm{H})$	effective g factor of the proton in the ground (1S) state of hydrogen
g_μ	$2\mu_\mu/(e\hbar/2m_\mu)$, g factor of the free muon
IGSN71	1971 International Gravity Standardization Net
K_A	The ratio A_{BI85}/A
K_V	The ratio $V_{76-BI}/V=(483\,594.0\ \mathrm{GHz/V})/(2e/h)$; $K_V=K_A/K_\Omega$
K_Ω	The ratio Ω_{BI85}/Ω
KhGIMIP	Kharkov State Scientific Research Institute of Metrology, USSR
LCIE	Laboratoire Central des Industries Électrique (Central Laboratory for the Electrical Industries), France
M	number of variables in a least-squares adjustment
M_p	molar mass of the proton
N	number of items of stochastic input data in a least-squares adjustment
NBS	National Bureau of Standards, US
NIM	National Institute of Metrology, Peoples' Republic of China
NML	National Measurement Laboratory (CSIRO Division of Applied Physics) [formerly, National Standards Laboratory (NSL)], Australia
NPL	National Physical Laboratory, UK
NPLI	National Physical Laboratory, India
ppm	parts per million
PTB	Physikalisch-Technische Bundesantalt (Federal Physical-Technical Institute), Federal Republic of Germany
r_i	Residual of an input datum in a least-squares adjustment: $r_i=y_i-\hat{y}_i$ where y_i is the input datum and $\hat{y}_i$ is the adjusted value
R_B	Birge ratio: $R_B=(\chi^2/\nu)^{1/2}$
R_H	Quantized Hall resistance: $R_H=h/e^2=\mu_0 c/2\alpha\approx 25\,812.8\ \Omega$; $\mu_0 c=Z_0=$impedance of free space $\approx 377\ \Omega$
s_i^2	*a priori* assigned estimate of the variance of the ith stochastic datum
SI	Système International d'Unités (International System of Units), the official name of the system of units based on the meter, kilogram, second, ampere, kelvin, mole, and candela
V_{BIPM}	The time-dependent BIPM representation of the volt prior to January 1, 1969
V_{69-BI}	The time-dependent BIPM representation of the volt from January 1, 1969

to January 1, 1976

V_{76-BI} The BIPM representation of the volt on the basis of the defined Josephson frequency-voltage quotient: $2e/h = 483\,594.0$ GHz/V_{76-BI}

VNIIM All-Union Scientific Research Institute of Metrology (Mendeleev Institute), USSR

VSL Van Swinden Laboratory, The Netherlands

w_i weight of the ith stochastic datum: $w_i = 1/\sigma_i^2$

α fine-structure constant: $\mu_0 ce^2/2h$

γ_p gyromagnetic ratio of the free proton: $\gamma_p = \omega_p/B = 2\mu_p/\hbar$

γ'_p effective gyromagnetic ratio of the proton in pure water (spherical sample at 25 °C): $\gamma'_p = \omega'_p/B$

μ_B Bohr magneton: $e\hbar/2m_e$

μ_N nuclear magneton: $e\hbar/2m_p$

μ'_p effective magnetic moment of the proton in pure water (spherical sample at 25 °C): $\gamma'_p = 2\mu'_p/\hbar$

ν degrees of freedom of a least-squares adjustment: $\nu = N - M$

ν_i effective number of degrees of freedom of a stochastic datum

ν_{Mhfs} ground-state hyperfine splitting interval in muonium

σ_i^2 Variance of the ith stochastic datum

ω'_p nuclear magnetic resonance (spin-flip) angular frequency of a proton in pure H_2O (spherical sample at 25 °C) in a magnetic field B: $\hbar\omega'_p = 2\mu'_p B$

Ω_{BIPM} The time-dependent BIPM representation of the ohm prior to January 1, 1969

Ω_{69-BI} The time-dependent BIPM representation of the ohm since January 1, 1969 Ω_{BIPM} and Ω_{69-BI} are based on the mean resistance of the same set of six resistors. There was no change in the definition of the BIPM as-maintained ohm on January 1, 1969, only in its symbol

Ω_{BI85} BIPM representation of the ohm on January 1, 1985: $\Omega_{BI85} = \Omega_{69-BI}$(January 1, 1985)

I. INTRODUCTION

A. Background

The CODATA Task Group on Fundamental Constants was established in 1969 to provide a set of accurate values of the basic constants and conversion factors that have become increasingly important for the interpretation of the numerical data of science and technology. Its 1973 report (CODATA, 1973) provided the first internationally adopted set of values for the fundamental constants of physics and chemistry. Those values were almost immediately challenged by several new measurements, and it has been clear for at least the past decade that a new adjustment was required. However, data that affect our knowledge of the physical constants are constantly appearing and it is always difficult to establish an optimal time at which to make a change in the recommended values. January 1, 1986 was established as a cutoff date for this review, recognizing that any additional changes to the 1973 values that might be introduced by newer data that are thereby excluded would almost certainly be much less than the changes already implied by the data that were available prior to that cutoff date.

During the past five years the present authors, with the guidance of the other members of the Task Group, have reviewed and analyzed a wealth of material. This report summarizes that effort and gives the new set of "best" values derived from the 1986 adjustment. This review is similar in concept to its predecessors (Taylor, Parker, and Langenberg, 1969; Cohen and Taylor, 1973). The present analysis, however, makes no distinction between data which are dependent on quantum electrodynamics (QED) for their analysis and those which are not, because there is no evidence, within the present limits of experimental precision, that the concepts of QED (as opposed to its implementation) are not valid.

An extraordinary amount of experimental and theoretical work relating to the fundamental constants has been published in the past dozen years. The measurement of frequency at infrared and visible wavelengths reached a level of development that has resulted in the redefinition of the meter in terms of the distance traveled by light in a given time. Another advance was the direct linking of atomic lattice spacings to optical wavelengths, making possible a significant improvement in the determination of the Avogadro constant. Impressive progress has been made in the precision of numerical evaluations of the quantum electrodynamics of the electron anomalous moment as well as in its experimental determination. The most striking metrological advance occurred when von Klitzing (von Klitzing, Dorda, and Pepper, 1980) observed the quantization of electrical conductance and achieved not only a direct macroscopic measurement of the fine-structure constant but a Nobel Prize as well (von Klitzing, 1986).

B. Data selection and evaluation procedures

Because of problems associated with previous analyses of such a diverse set of experimental and theoretical data, increased attention has been directed in the 1986 analysis to questions of statistical validity.

It is obviously necessary that the data, even though they may represent different physical quantities, must all be expressed in a manner that allows a coherent comparison of their uncertainties. Therefore, as part of the review, the originally assigned uncertainty of each measurement or numerical calculation was examined, and modified if necessary, to ensure that all uncertainties were expressed consistently in terms of a variance. This is in general consonance with the recommendations of the Bureau International des Poids et Mesures (BIPM) Working Group on the Statement of Uncertainties (BIPM, 1981; Giacomo, 1981). These recommendations make no distinction in principle between "random" and "systematic" uncertainties and classify uncertainties only as Type A or Type B, "according to the way in which their numerical value is estimated: (1) those which are evaluated by applying statistical methods, (2) those which are evaluated by other means."

Type-A uncertainties are characterized by the estimated variances s_i^2 and Type-B uncertainties by quantities u_j^2 that are considered to be equivalent to variances, the existence of which are assumed. Since the two types of uncertainty are to be treated mathematically in the same fashion, the distinction of notation is omitted in this report. In some cases, in order to estimate the variance of a Type-B uncertainty, it is assumed that the conceptual underlying error distribution can be represented by a rectangle of width $2a$, and that the stated error limit of $\pm a$ in such cases represents the halfwidth of that distribution. The equivalent variance is then taken to be $a^2/3$.

Two basic criteria were used in selecting the data to be included in the present adjustment.

(i) The result had to be available prior to January 1, 1986. However, this does not necessarily mean full publication in final form; an item was considered to be available if it was sufficiently complete that a meaningful uncertainty could be assigned to it.

(ii) Each datum had to have an uncertainty sufficiently small that it carried a nontrivial weight in comparison to other values of the same quantity. A datum whose total weight is less than the uncertainty in the weight of another cannot significantly affect the result when both are present. The relative uncertainty of an estimate of a variance (or of a statistical weight) is $\sqrt{2/\nu_i}$ where ν_i is the effective number of statistically independent variables that form the basis for the estimate, i.e., the effective number of degrees of freedom (Cohen, 1984). Since few experimental results will be based on as many as 50 effective degrees of freedom, the assigned uncertainties are rarely reliable to better than 10%. Furthermore, it is difficult to detect the presence of systematic errors that are less than approximately one tenth of the final uncertainty, and it is hardly significant to introduce into the analysis data that will shift the result by an amount smaller than the uncertainty due to the effect of possible undetected systematic errors. Therefore, one ought not include a datum whose weight is less than a few percent of the weight of other data measuring the same quantity or of the weight of an indirect value deduced from other data in the input set. The general rule has therefore been adopted that a measurement will not be included in the analysis if its assigned uncertainty is more than four times the uncertainty (and whose weight is therefore less than approximately 0.06 times the weight) of some other measurement or indirect evaluation of the same quantity.

The least-squares approach to the analysis of the fundamental constants has been described in greater detail in previous reviews (Cohen, Crowe, and DuMond, 1957; Taylor, Parker, and Langenberg, 1969; Cohen and Taylor, 1973). In brief, each experimental result represents a constraint on the values of a set of physical quantities, expressed as an algebraic relationship among the auxiliary constants and the unknowns. The set of M unknowns chosen for the analysis is not unique, but it must be complete and independent. Completeness of the set means that there are enough unknowns to express all of the experimental data; independence implies that no unknown can be expressed as a combination of the other unknowns, and hence that the N observations can be expressed in terms of the unknowns in essentially only one way.

The usual least squares procedure uses the so-called Birge ratio, $R_B=(\chi^2/\nu)^{1/2}$, as a factor to rescale the uncertainties of the results of an adjustment, so as to yield a value of χ^2 equal to its expectation value, $\nu=N-M$. This procedure is an *a posteriori* evaluation of the "error associated with unit weight." It is valid to consider a uniform rescaling of the weights if the assigned uncertainties have only relative significance and there is no *a priori* estimate of absolute weights, or if the data are such that the systematic errors of all input data are roughly similar. However, when the data come from different and unrelated sources with broadly different physical content, a uniform expansion of all uncertainties can hardly be justified; any rescaling of the assigned weights should consider any *a priori* information that may be available concerning the uncertainty assignment of each individual datum.

Therefore, in analyzing the input data we have considered not only the usual least-squares algorithm, but also the algorithm proposed by Tuninskii and Kholin (1975) of the Mendeleev Institute of Metrology in Leningrad (VNIIM) (Tarbeyev, 1984), as well as a modification of it suggested by Taylor (1982), and the extended least-squares algorithms described by Cohen (1976, 1978, 1980, 1984). The weight w_i associated with each experimental datum is $1/\sigma_i^2$. The new algorithms may be categorized as procedures that recognize that the true values of the variances σ_i^2 (and hence the weights) needed in the least-squares analysis are not known, but are only available as *a priori* estimates s_i^2 summed from the Type-A and Type-B components. They then use the consistency of the data to provide additional, *a posteriori*, information with which to improve these estimates.

Tuninskii and Kholin use the formalism of a cost function to modify the weight of each observational equation in such a way that χ^2 is set equal to its expectation value ν with minimum cost. The cost function they propose is

$$C=\sum\left[\frac{1}{w_is_i^2}-1\right]^2 . \tag{1.1}$$

This shall be referred to as the VNIIM algorithm.

There is an infinite cost penalty in the VNIIM algorithm if the weight of an observation is decreased to zero, but only a finite penalty if the weight becomes very large; Taylor (1982) considered, *inter alia*, a symmetrized form of the cost function that shall be used here as an example of functions that assign large cost both to very small and to very large weights:

$$C=\sum\left[\frac{1}{w_is_i^2}-2+w_is_i^2\right] , \tag{1.2}$$

which we shall refer to as the symmetric VNIIM algorithm. He has also demonstrated that the results obtained using different algorithms are not particularly sensitive to the precise form of the cost function used, since the primary influence of the cost function is defined by the quadratic dependence near $w_is_i^2=1$.

The two extended least-squares algorithms consider the information on the reliability of the variance estimates when combining the *a priori* estimate s_i^2, based on ν_i effective degrees of freedom, with the *a posteriori* estimate provided by the residuals $r_i=y_i-\hat{y}_i$, where y_i is the experimental input datum and $\hat{y}_i$ is the adjusted value in the least-squares fit, and the value of χ^2.

The first algorithm, ELS1, uses both the *a priori* standard deviation s_i and the residual to evaluate a best estimate of the variance σ_i^2. The coefficients a_i and b_i in the ansatz $\hat{\sigma}_i^2=a_is_i^2+b_ir_i^2$ are determined by requiring that this expression be an unbiased, minimum variance estimator of σ_i^2.

The expectation value of the variance of the residual is $(1/w_i)-t_{ii}$, where $1/w_i$ is the variance of y_i and t_{ii} is the variance of $\hat{y}_i$ (Cohen, 1953; Cohen, Crowe, and DuMond, 1957). The self-consistent estimate of the variance of the input datum is the mean of the *a priori* estimate s_i^2 with weight ν_i and the *a posteriori* estimate $r_i^2/(1-w_it_{ii})$ with weight 1, or

$$\frac{1}{w_i}=\frac{\nu_is_i^2+r_i^2/(1-w_it_{ii})}{\nu_i+1} . \tag{1.3}$$

Each output quantity of a least-squares adjustment is influenced, in general, by all of the input quantities. Therefore, if there are no identifiably discrepant data, it is appropriate to use all of the output to evaluate the statistically most efficient estimate for the weight of each input. The second algorithm, ELS2, replaces the ansatz given above by the more complete sum $\hat{\sigma}_i^2=a_is_i^2+\sum b_{ijk}r_jr_k$, and from this obtains the surprisingly simple result that the best estimate is a mean of the *a priori* value s_i^2 with weight ν_i and the *a posteriori* estimate $(\chi^2/\nu w_i)$ with weight ν:

$$\frac{1}{w_i}=\frac{\nu_is_i^2+\nu[(\chi^2/\nu)/w_i]}{\nu_i+\nu}$$

or, with the assumption that the estimate of ν_i is independent of the estimate of $w_i=1/\sigma_i^2$,

$$w_i=(\nu_i+\nu-\chi^2)/\nu_is_i^2 . \tag{1.4}$$

Equation (1.4) is statistically stronger than Eq. (1.3) in the ratio $(\nu_i+\nu)/(\nu_i+1)$. However, Eq. (1.4) may not always have a solution with positive weights; for the weight w_i to be positive it is necessary that $\nu_i+\nu$ be greater than χ^2. If there is more than one discrepant item in the data set, it may be impossible for the algorithm to reduce the weights of the discrepant data enough to yield a sufficiently small value of χ^2. The lack of convergence is an indication that the data are discrepant in the sense that the observed residuals are larger than can be reasonably ascribed to random fluctuations in the data. This is a contradiction of the assumptions underlying Eq. (1.4), and indicates that the algorithm cannot be used for that data set.

The two algorithms have different functions. ELS1, Eq. (1.3), is intended to identify data whose deviation from the consensus of the whole set indicates that discrepancies are present, and to correct those discrepancies by reassigning the statistical weights of the data. ELS2, Eq. (1.4), applies if the data are sufficiently "cleansed" of undetected systematic errors that the assigned weights properly represent the variances of the assumed underlying probability distributions. It replaces the constant scaling factor of the Birge ratio by a nonuniform scaling lying between 1 (if the effective degrees of freedom ν_i is high, $\nu_i\gg1$) and $R_{\rm B}$ (if ν_i is low, $\nu_i\to0$). Algorithm ELS2 is valid even if the value of χ^2 for the standard least-squares process is less than ν. Then the Birge ratio is less than unity and algorithm ELS2 reduces the uncertainties; in either case, whether the Birge ratio is larger or smaller than unity, the factor by which the standard deviation is changed by the algorithm always lies between 1 and $R_{\rm B}$.

II. REVIEW OF THE DATA

As in previous adjustments, the data are divided into two categories: the more precise data (auxiliary constants) that are not subject to adjustment because of their relatively low uncertainties, and the less precise or stochastic data that are subject to adjustment.

There is no formal basis for separation into these two categories except that a variable with an uncertainty much smaller than that of any other variable to which it is connected will not be altered by the adjustment and can hence be treated as a constant. For example, it was originally intended that the proton-electron mass ratio would be one of the unknowns of the analysis. However, as data collection proceeded, the precision with which $m_{\rm p}/m_{\rm e}$ was known increased to the point that the weight of the direct observational equation was much larger than any weight contributed by any other relationship (such as the proton magnetic moment measurements that were of such importance in previous adjustments).

If there is more than one determination of an auxiliary constant, as is the case for the Rydberg constant, the data need merely be represented by a simple weighted mean (which is itself only a one-dimensional version of least squares) and that mean used subsequently as a constant no longer subject to adjustment.

A. Auxiliary constants

All the auxiliary constants in the present adjustment have uncertainties not greater than 0.02 parts per million (ppm); in no case is the uncertainty of an auxiliary constant larger than one-tenth the uncertainty of the stochastic datum with which it appears, and one-twentieth is typical. The principal auxiliary constants used in the adjustment are given in Table I. In those few instances in which the values are themselves influenced by the adjustment, so that they must be iteratively determined, the numerical values listed in Table I correspond to the 1986 recommended values.

1. The speed of light and the definition of the meter

The new definition of the meter in terms of the speed of light (BIPM, 1983) establishes c as an exact auxiliary constant in the analysis, but it does not make wavelength metrology obsolete, nor does it assure that a given laser will provide a valid length standard. The definition of the meter is conceptual rather than operational; the practical working standards for its representation are contained in a *mise en pratique* that defines the operating conditions under which specified hyperfine-structure features of selected absorption-stabilized lasers will constitute a consistent set of frequencies and wavelengths that reproduce the meter to within the specified uncertainties (Hudson, 1984).

2. Proton-electron mass ratio

The value $m_p/m_e = 1836.152\,701(37)$ is the most recent result reported by van Dyck, Moore, and collaborators (1986) at the University of Washington from measurements of the cyclotron frequency, $\omega_c = eB/m$, of electrons and protons in the same magnetic field in a Penning trap. An almost identical, but less precise value has been reported by Gräff *et al.* (1980), $m_p/m_e = 1836.1527(11)$. A reevaluation of the data of Wineland *et al.* (1983) gives $m_p/m_e = 1836.152\,34(36)$.

In previous adjustments this ratio, or the equivalent ratio

$$\mu_p'/\mu_N = (m_p/m_e)(\mu_p'/\mu_B)\ ,$$

has been an adjustable stochastic variable. The data of Mamyrin *et al.* (1983) give a value for μ_p'/μ_N equivalent to $m_p/m_e = 1836.150\,90(79)$, while the measurements of Petley and Morris (1974) give 1836.1521(13). These earlier measurements are not in gross disagreement with the University of Washington data but they are clearly of such lower precision (and hence, lower weight) that they need not be considered in our analysis.

3. Relative atomic masses and mass ratios

The relative atomic masses of the nuclides are taken from the 1983 Atomic Mass Table of Wapstra and Audi (1985), the most recent compilation available. The values of M_p (the molar mass of the proton), and the deuteron-electron and alpha particle-electron mass ratios, m_d/m_e and m_α/m_e, are calculated from the appropriate nuclidic masses and the value of m_p/m_e adopted above.

The mass ratio m_μ/m_e is required in the evaluation of the reduced mass factor and QED terms for muonium (μ^+e^- atom) and for the QED calculation of the electron g factor. The magnetic moment ratio $\mu_\mu/\mu_p = (\mu_e/$

TABLE I. Summary of the principal auxiliary constants used in the 1986 least-squares adjustment.

Quantity	Value	Relative uncertainty (parts in 10^9)
c	299 792 458 m/s	(exact)
V_{76-BI}	(483 594.0 GHz)($h/2e$)	(exact)
m_p/m_e	1 836.152 701(37)	20
M_p	0.001 007 276 470(12) kg/mol	12
$1+m_e/m_p$	1.000 544 617 013(11)	0.011
$1+m_e/m_d$	1.000 272 443 707(6)	0.006
$1+m_e/m_\alpha$	1.000 137 093 354(3)	0.003
$1+m_e/m_\mu$	1.004 836 332 18(71)	0.71
R_∞	10 973 731.534(13) m^{-1}	1.2
$g_e/2=\mu_e/\mu_B$	1.001 159 652 193(10)	0.010
$g_\mu/2=1+a_\mu$	1.001 165 923 0(84)	8.4
μ_e/μ_p	658.210 688 1(66)	10
μ_p/μ_B	0.001 521 032 202(15)	10
μ_p'/μ_B	0.001 520 993 129(17)	11
$d\Omega_{69-BI}/dt$	−0.056 6(15) $\mu\Omega$/a	

$\mu_p)(g_\mu/g_e)(m_e/m_\mu)$ is a variable in the adjustment, but the uncertainty of $1+m_e/m_\mu$ is approximately $\frac{1}{200}$ the uncertainty of m_μ/m_e and therefore may be taken as an auxiliary constant.

The QED calculation of the electron g factor also requires a value for the τ-lepton-electron mass ratio; we use $m_\tau/m_e=3492(6)$ from the 1984 Review of Particle Properties (Wohl *et al.*, 1984).

4. Rydberg constant

The techniques of Doppler-free spectroscopy, with an increase in precision of one to two orders of magnitude compared to the data available in 1973, have rendered all earlier measurements of the Rydberg constant R_∞ obsolete. The present value is based on the 1979 measurements at Stanford (Hänsch *et al.*, 1974; Goldsmith *et al.*, 1979) and those at Yale (Amin *et al.*, 1981, 1984; Lichten, 1985) reevaluated on the basis of the new definition of the meter (including the specifications for the operating conditions of the lasers that provide its realization) and a revision of Erickson's spectroscopic energy-level calculations (Erickson, 1977, 1983) that incorporates the newer values of m_p/m_e and α. A determination of R_∞ carried out at NPL (Petley and Morris, 1979; Petley, Morris, and Shawyer, 1980), giving a result that is in agreement with the Yale and Stanford results was not included in determining the recommended value because it is quoted as having a precision poorer by a factor of 6 than the Yale datum. Thus, if it were to be included it would make no significant contribution to the final result.

5. g factor for the free electron and muon

The g factor for the free electron, $g_e=2\mu_e/\mu_B=2(1+a_e)$ where μ_e is the magnetic moment of the electron, μ_B is the Bohr magneton, and a_e is the electron magnetic moment anomaly, contributes to the adjustment in two ways: as an input variable, a_e, from which a value of the fine-structure constant may be derived; and as a fixed auxiliary constant, g_e. The value in Table I for g_e is the one most recently reported by van Dyck, Schwinberg, and Dehmelt (1984) at the University of Washington from measurements on a single electron stored in a Penning trap cooled to 4.2 K. The uncertainty assigned to this value has been increased from the value 4×10^{-12} of the experiment itself to take account of possible theoretically estimated shifts in the cyclotron orbits arising from the finite geometry of the apparatus (Brown *et al.*, 1985; van Dyck, 1985).

The g factor for the free muon, $g_\mu=2\mu_\mu/(e\hbar/2m_\mu)=2(1+a_\mu)$, enters the adjustment only as an auxiliary constant. In contrast to the electron anomaly, the muon anomaly a_μ is not known with sufficient precision either experimentally or theoretically to allow the calculation of a competitive value for the fine-structure constant. The value used here is obtained from the latest of a long series of muon measurements that have been carried out over the past twenty years at CERN (Bailey *et al.*, 1979).

6. Electron and nuclear magnetic moment ratios

The ratio μ_e/μ_p is obtained from the g factor measurements in hydrogen of Winkler *et al.* (1972) which are by far the most accurate available. These workers report the value

$$g_j(\mathrm{H})/g_p(\mathrm{H})=658.210\,706\,3(66)\quad(0.010\ \mathrm{ppm})\,. \tag{2.1}$$

The g factors must be corrected to the free-particle values using the theory of the bound-state corrections due to Faustov (1970), Grotch and Hegstrom (1971), and Close and Osborne (1971):

$$\mu_p(\mathrm{H})=\mu_p\left[1-\alpha^2\left[\frac{1}{3}-\frac{m_e m_p}{2(m_p+m_e)^2}\right]+\frac{\alpha^2(m_p+3m_e)m_e a_p}{6(m_p+m_e)^2(1+a_p)}+\cdots\right], \tag{2.2a}$$

where $a_p=\mu_p/\mu_N-1$ is the proton magnetic moment anomaly, and $\mu_e(\mathrm{H})$ is found by simply interchanging the roles of the electron and the proton. One then obtains

$$\frac{g_j(\mathrm{H})}{g_p(\mathrm{H})}=\frac{g_e}{g_p}(1+27.7\times10^{-9})\,. \tag{2.2b}$$

When this is combined with Eq. (2.1) we find

$$g_e/g_p=\mu_e/\mu_p=658.210\,688\,1(66)\quad(0.010\ \mathrm{ppm})\,. \tag{2.3}$$

The proton moment in Bohr magnetons μ_p/μ_B is obtained from this result and μ_e/μ_B given in Table I.

The ratio μ_p'/μ_B is obtained similarly from the measurements of Phillips, Cooke, and Kleppner (1977) who give the value

$$g_j(\mathrm{H})/g_p(\mathrm{H_2O},34.7\,^\circ\mathrm{C})=658.216\,009\,1(69)$$

for a spherical, pure H_2O nuclear magnetic resonance (NMR) sample at 34.7 °C. The 0.10-ppm correction required to convert this result to the 25 °C reference temperature for proton NMR measurements (indicated by a prime in this report) is derived from the work of Petley and Donaldson (1984), and hence

$$g_e/g_p'=\mu_e/\mu_p'=658.227\,597\,0(72)\quad(0.011\ \mathrm{ppm})\,. \tag{2.4}$$

The Lambe-Dicke result (Lambe, 1959, 1968) used in the 1973 adjustment is six times less precise than the value given in Eq. (2.4) and differs from it by 0.8 standard deviations. However, temperature was not recorded for this earlier measurement and possible sources of systematic effects not considered by Lambe have been pointed out (Phillips, Cooke, and Kleppner, 1977), so that a meaningful comparison with the newer measurement cannot be made.

Properties of the neutron and deuteron, while not required for the multivariate analysis of the input data, are required for the output table of recommended values.

The neutron magnetic moment is based on the measurement by Greene *et al.* (1979), of the ratio of the NMR frequency for free neutrons to that of protons in a cylindrical sample of pure water at 22 °C. Their result, corrected to standard conditions (spherical sample at 25 °C), becomes

$$\omega_n/\omega'_p = \mu_n/\mu'_p = 0.684\,996\,93(16) \quad (0.24 \text{ ppm}) .$$

The ratio of the g factors for the deuteron and electron in deuterium has been measured by Phillips, Kleppner, and Walther (1984):

$$g_d(\mathrm{D})/g_j(\mathrm{D}) = 2.332\,172\,696(25)\times 10^{-4} \quad (0.011 \text{ ppm}) .$$

This is corrected to give the g-factor ratio for the free particles using Eq. (2.2a) with the appropriate replacement of the proton by the deuteron, using $a_d = [\mu_d/(e\hbar/m_d)] - 1 = -0.143$ for the deuteron magnetic moment anomaly. The result is $\mu_d/\mu_e = 0.000\,466\,434\,553\,8(50)$, and hence

$$(\mu_d/\mu_p)_{\mathrm{PKW}} = 0.307\,012\,208\,6(45) .$$

Another determination of μ_d/μ_p has been reported by Neronov and Barzakh (1977) who give

$$(\mu_d/\mu_p)_{\mathrm{NB}} = 0.307\,012\,198\,3(8)$$

from a measurement of the ratio of NMR frequencies in HD and an improved calculation of the isotope dependence of the diamagnetic shielding and molecular bound-state corrections. The indicated uncertainty is statistical only; the total error is stated to be less than 0.010 ppm (Neronov, Barzakh, and Mukhamadiev, 1975). Since the two determinations differ by 0.033 ppm, it is more appropriate to take an unweighted average than to use the quoted uncertainties as the basis for the weights. We therefore adopt

$$\mu_d/\mu_p = 0.307\,012\,203\,5(51) \quad (0.017\text{ppm}) . \tag{2.5}$$

7. "As-maintained" volt and ohm standards

Since 1973 the major national standards laboratories have been utilizing the Josephson effect to provide a time-independent, reproducible representation of the SI unit of electrical potential difference, replacing the banks of standard cells that served previously. Therefore, all measurements of electrical potential can now be unambiguously related to the BIPM "as-maintained" unit characterized by the Josephson frequency-voltage quotient, 483 594.0 GHz/$\mathrm{V}_{76-\mathrm{BI}}$ (but of course still limited by the precision of the transfer of the Josephson frequency to the reference standard cells used in the actual experiment). The noncoherent unit $\mathrm{V}_{76-\mathrm{BI}}$ is related to the coherent SI unit through the relation

$$\mathrm{V}_{76-\mathrm{BI}} = K_\mathrm{V}\ \mathrm{V} \tag{2.6}$$

and the quantity K_V is a variable in the present adjustment. It is related to the physical constants by the definition

$$(2e/h)K_\mathrm{V} \equiv 483\,594.0 \text{ GHz/V} . \tag{2.7}$$

Measurements of electrical potential difference not directly referred to $\mathrm{V}_{76-\mathrm{BI}}$ by Josephson effect measurements were related to the BIPM as-maintained unit using the results of the triennial standard cell comparisons carried out by BIPM from 1954 to 1973, and of other bilateral interlaboratory comparisons, and interpolated to the mean date of the measurements assuming a linear drift for the as-maintained laboratory unit.

National units of resistance (based mainly on precision wire-wound resistors) have also been compared in bilateral comparisons as well as in the BIPM triennial intercomparisons. The Australian CSIRO National Measurement Laboratory has used the Thompson-Lampard calculable capacitor to maintain its unit of resistance consistent with the SI ohm since 1964, so that there is a 20-year data base tracing the time dependence of the BIPM unit, $\Omega_{\mathrm{BIPM}} = \Omega_{69-\mathrm{BI}}$. These data indicate a surprisingly constant linear drift, $d\Omega_{69-\mathrm{BI}}/dt = -0.0566(15)\ \mu\Omega/\mathrm{a}$. Because of this drift, all data are expressed in terms of Ω_{BI85}, the value of $\Omega_{69-\mathrm{BI}}$ on January 1, 1985:

$$\Omega_{\mathrm{BI85}} \equiv \Omega_{69-\mathrm{BI}}(\text{January 1, 1985})$$

and in analogy with Eq. (2.6),

$$\Omega_{\mathrm{BI85}} = K_\Omega\ \Omega . \tag{2.8}$$

The drift rate is an auxiliary constant in the 1986 adjustment but K_Ω, the value of the BIPM as-maintained ohm on January 1, 1985 expressed in SI units, is considered to be an unknown because of Type-B uncertainties associated with the transfer of capacitor measurements to resistance standards. (A typical transfer chain required to achieve a realization of the ohm from a calculable capacitor measurement might involve the series: 0.5 pF $\rightarrow$10 pF$\rightarrow$500 pF$\rightarrow$10 k$\Omega$$\rightarrow$1$\Omega$.)

The bilateral comparisons and the BIPM 1-Ω triennial intercomparisons, with the assumption that each national resistance unit has its own linear drift rate, provide the means for determining the time-dependent offset between each national standard and the BIPM as-maintained ohm, $\Omega_{\mathrm{BIPM}} = \Omega_{69-\mathrm{BI}}$.

Although the ampere is defined as a base unit in SI, its representation $\mathrm{A}_{\mathrm{BIPM}}$ is maintained as a derived unit using Ohm's law, and hence is obtained from

$$\begin{aligned} \mathrm{A}_{\mathrm{BIPM}} &= \mathrm{V}_{\mathrm{BIPM}}/\Omega_{\mathrm{BIPM}} = \mathrm{V}_{76-\mathrm{BI}}/\Omega_{69-\mathrm{BI}} , \\ \mathrm{A}_{\mathrm{BI85}} &= \mathrm{V}_{\mathrm{BI85}}/\Omega_{\mathrm{BI85}} = (K_\mathrm{V}/K_\Omega)\mathrm{A} = K_\mathrm{A}\ \mathrm{A} . \end{aligned} \tag{2.9}$$

8. Acceleration due to gravity

The value of the acceleration due to gravity at the location of the measurement is required for the determination of the ampere and the volt and in the high-field determination of the proton gyromagnetic ratio. The development of the transportable absolute laser gravimeter (Faller, 1967; Hammond and Faller, 1967) has released experimenters from the necessity of relying on a series of relative gravity differences to relate the laboratory site to a national or international Fundamental Gravity Station. The transportable gravimeter yields a measurement precision of 0.01—0.03 ppm. The International Gravity Standardization Net, IGSN71, based on 10 absolute gravity determinations and approximately 25 000 gravity differences provides a reference network of 1854 gravity stations with overall uncertainty of 0.1—0.2 ppm for the evaluation of gravity at a laboratory site (Morelli *et al.*, 1974). For those few measurements for which a modern gravity value is unavailable, the older value based on the Potsdam System has been used with a correction of -14.0×10^{-5} m s^{-2}.

B. Primary stochastic input data

The 38 items of stochastic data considered in the adjustment and an estimated value of the effective degrees of freedom (Cohen, 1984) of each determination are listed in Table II. The uncertainties of the stochastic data lie in the range 0.05—10 ppm. Of the 12 data types represented in Table II, only the Faraday, the molar mass of Si, and the muonium hyperfine interval are represented by a single measurement. Unfortunately, this redundancy is not as useful as one might hope because not all of the data are of comparable precision.

1. Type 1. Direct ohm determinations

The Thompson-Lampard calculable capacitor (Thompson and Lampard, 1956; Lampard, 1957; van der Pauw, 1958) has been used not only at CSIRO where it was developed, but in several other national laboratories as well. The measurement constitutes a determination of the permittivity of vacuum ε_0 in units that are related to the laboratory-maintained ohm. Since $\varepsilon_0=1/\mu_0 c^2$ is exactly defined in terms of the SI ohm, this allows a direct calibration of the laboratory unit in terms of SI.

The five measurements of the ohm (item 1.1: Thompson, 1968; item 1.2: Cutkosky, 1974; item 1.3: Igarashi *et al.*, 1968, 1978; Igarashi, 1983, 1984; item 1.4: Dahake *et al.*, 1983; item 1.5: Jones and Kibble, 1985) are reasonably consistent; the mean value is

$$\Omega_{\mathrm{BI85}}=\Omega-1.533(69)\ \mu\Omega\ , \qquad (2.10)$$

with a Birge Ratio $R_{\mathrm{B}}=1.13$ and $P_{\chi^2}(5.09\mid 4)=0.28$. Although the NPLI measurement is consistent with the other four it carries from 2.5 to 11 times less weight. If this low-weight datum were omitted it would decrease the mean by less than 0.0012 ppm and increase the Birge ratio to $R_{\mathrm{B}}=1.30$ with $P_{\chi^2}(5.08\mid 3)=0.17$.

2. Type 2. Direct ampere determinations

The SI ampere is defined by assigning an exact value to μ_0, the permeability of vacuum, in Ampère's law; the ampere is maintained as a unit in terms of an as-maintained or laboratory volt and ohm. The determination of the as-maintained ampere in terms of SI is accomplished by comparing the measured and computed forces or torques exerted by a current-carrying coil on a second coil. The force depends on the intensity and the geometry of the currents; the difficulties in the measurement lie in defining the precise geometry of the current paths and in measuring the relatively small forces generated because of the single layer coils that must be used in order to be able to define the geometry precisely.

The direct ampere determinations (item 2.1: Driscoll and Cutkosky, 1958; item 2.2: Vigoureux, 1965; item 2.3: Gorbatsevitch, 1973; item 2.4: Driscoll and Olsen, 1968; item 2.5: Bender and Schlesok, 1974; item 2.6: Vigoureux and Dupuy, 1980) yield a mean value,

$$\mathrm{A}_{\mathrm{BI85}}=\mathrm{A}+(2.07\pm 2.53)\ \mu\mathrm{A}\ , \qquad (2.11)$$

with $R_{\mathrm{B}}=0.65$ and $P_{\chi^2}(2.11\mid 5)=0.84$. If the relatively high NPL value (item 2.6) is deleted, the mean is decreased to $\mathrm{A}_{\mathrm{BI85}}=\mathrm{A}-(0.48\pm 3.22)\ \mu\mathrm{A}$, with $R_{\mathrm{B}}=0.33$ and $P_{\chi^2}(0.45\mid 4)=0.98$.

3. Type 3. Direct volt determinations

The volt can be realized by determining the force on the plates of a parallel plate capacitor when a potential difference (known in terms of the laboratory unit) is applied. The energy stored in a capacitor per unit area of the plates (neglecting end effects) is given by $W=\frac{1}{2}CV^2$, where $C=\varepsilon/z$ is the capacity per unit area, z is the separation, and ε is the permittivity of the medium between the plates. The force per unit area on either plate is

$$F=dW/dz=\tfrac{1}{2}\varepsilon V^2/z^2\ .$$

At LCIE, Elnékavé and Fau (1984) (item 3.1) have determined K_{V} using a Kelvin electrometer. The major uncertainties are related to the accuracy of the calculation of the end corrections and the precision with which the geometry can be defined and measured.

At CSIRO, Clothier (1965) and Sloggett *et al.* (1984, 1985) (item 3.2) have developed a novel variant of the Kelvin electrometer in which the lower electrode is a pool of mercury and the upper electrode is a semitransparent optical flat that is also one face of a laser interferometer. The level of the mercury under the electrode rises when a potential difference is applied until the gravitational forces on the mercury balance the electrical forces. The position of the mercury surface is measured interferometrically. Measurements are made at a succession of voltages and spacings chosen so that the height of the mercury is approximately constant.

The mean of the French and Australian measurements is

$$V_{76-BI}=V-7.86(58)\ \mu V\ , \tag{2.12}$$

TABLE II. Stochastic input data for the 1986 least-squares adjustment.

Item	Measurement date	Identification	Value	Relative uncertainty (ppm)	Degrees of freedom ν_i
1. Ohm, Ω_{BI85}			Ω		
1.1	1964–1985	NML (Australia)	0.999 998 54(14)	0.14	3.2
1.2	1973	NBS (US)	0.999 998 30(11)	0.11	1.5
1.3	1977–1984	ETL (Japan)	0.999 998 40(23)	0.23	5.3
1.4	1981	NPLI (India)	0.999 998 50(36)	0.36	2.4
1.5	1983	NPL (UK)	0.999 998 68(14)	0.14	3.2
2. Ampere, A_{BI85}			A		
2.1	1956	NBS (US)	0.999 997 4(84)	8.4	7.2
2.2	1963	NPL (UK)	0.999 998 2(59)	5.9	6.5
2.3	1966–1969	VNIIM (USSR)	0.999 998 6(61)	6.1	6.0
2.4	1967	NBS (US)	1.000 002 7(97)	9.7	2.6
2.5	1972	ASMW (GDR)	1.000 003 2(79)	7.9	2.8
2.6	1976	NPL (UK)	1.000 006 2(41)	4.1	2.4
3. Volt, V_{76-BI}			V		
3.1	1982–1983	LCIE (France)	0.999 996 7(24)	2.4	4.5
3.2	1985	NML (Australia)	0.999 991 86(60)	0.60	2.8
4. Faraday F			C_{BI85}/mol		
4.1	1975–1984	NBS (US)	96 486.00(13)	1.33	10.7
5. Proton gyromagnetic ratio, γ'_p, low field			$10^4\ s^{-1}/T_{BI85}$		
5.1	1968	ETL (Japan)	26 751.180(87)	3.25	4.9
5.2	1971–1976	NPL (UK)	26 751.177(15)	0.54	3.2
5.3	1977	NIM (PRC)	26 751.399(22)	0.82	9.3
5.4	1978	NBS (US)	26 751.371 9(64)	0.24	8.4
5.5	1980	VNIIM (USSR)	26 751.241(17)	0.62	7.4
5.6	1981	ASMW (GDR)	26 751.412(57)	2.13	16.6
6. Proton gyromagnetic ratio, γ'_p, high field			$10^4\ C_{BI85}$/kg		
6.1	1961–1964	KhGIMIP (USSR)	26 751.30(15)	5.4	5.1
6.2	1974	NPL (UK)	26 751.676(27)	1.0	2.4
6.3	1981	NIM (PRC)	26 751.564(96)	3.6	3.9
6.4	1983	ASMW (GDR)	26 751.466(86)	3.2	1.5
7. Si lattice spacing, d_{220}			pm		
7.1	1973–1976	NBS (US)	192.015 904(19)	0.10	2.4
7.2	1981	PTB (FRG)	192.015 560(45)	0.23	1.1
8. Si molar volume, M/ρ			$10^{-6}\ m^3$/mol		
8.1	1973	NBS (US)	12.058 808(14)	1.15	3.7
9. Quantized Hall resistance, R_H			Ω_{BI185}		
9.1	1985	PTB (FRG)	25 812.846 9(48)	0.18	4.8
9.2	1983–1984	NBS (US)	25 812.849 5(31)	0.12	1.9
9.3	1984	ETL (Japan)	25 812.843 2(40)	0.16	2.9
9.4	1984	NPL (UK)	25 812.842 7(34)	0.13	2.9
9.5	1984	VSL (Netherlands)	25 812.839 7(57)	0.22	3.4
9.6	1984–1985	LCIE (France)	25 812.850 2(39)	0.15	3.7
10. Inverse fine-structure constant, α^{-1}					
10.1	1981–1984	U Wash/Cornell (US)	137.035 994 2(89)	0.065	1.2
10.2	1984	Yale Univ. (US)	137.036 041(82)	0.60	1.4
11. Muon magnetic moment, μ_μ/μ_p					
11.1	1982	Los Alamos/Yale (US)	3.183 346 1(11)	0.36	6.3
11.2	1982	SIN (Switzerland)	3.183 344 1(17)	0.53	10.1
12. Muon hyperfine interval, ν_{Mhfs}			kHz		
12.1	1982	Los Alamos/Yale (US)	4 463 302.88(62)	0.14	1.4

but there is a difference of a factor of 4 in the uncertainty of the two measurements, which translates to a factor of 16 in their relative statistical weights. The difference between the two values is (4.80 ± 2.46) ppm, or approximately two standard deviations.

4. Type 4. Faraday constant

The Faraday constant was an important input to the 1973 adjustment, but was deleted in the final analysis because of discrepancies with other data and the possibility of unsuspected systematic errors in the determination of the molar mass of the silver sample (Cohen and Taylor, 1973). The 1960 NBS experiments were repeated in a series of measurements extending over a nine-year period (1975—1984). The electrochemical equivalent of Ag was measured by Bower and Davis (1980) while the isotopic analysis and the determination of the molar mass of the sample of Ag used in the electrolysis is due to Powell, Murphy, and Gramlich (1982). More recently, motivated by the present adjustment of the constants, additional measurements of the impurity content of the Ag used in the experiment were carried out (Taylor, 1985).

Because $\mu_p'/\mu_N=(\mu_p'/\mu_B)(m_p/m_e)$ and M_p are auxiliary constants in the present analysis, a measurement of the Faraday is equivalent to a measurement of γ_p',

$$\gamma_p'=(\mu_p'/\mu_N)F/M_p\ . \tag{2.13}$$

Since in the NBS experiments F was measured in $C_{NBS}=A_{NBS}\cdot s$, and transferred to $C_{BI85}=A_{BI85}\cdot s$, this Faraday measurement is equivalent to a high-field γ_p' determination (see below).

The physical chemist who uses F to relate electrochemical and thermodynamic measurements requires the Faraday in units $\mathrm{J\,mol^{-1}}/V_{LAB}$. In a system of units for which electrical energy is not equal to mechanical energy, the distinction between "coulomb" and "joule/volt" is significant. The Faraday measured in terms of J/V is a determination of $\gamma_p K_V$, which is neither a low-field nor a high-field determination.

5. Type 5. Gyromagnetic ratio (low field)

The gyromagnetic ratio of the proton γ_p' is given by ω_p'/B, where ω_p' is the NMR angular frequency of protons in H_2O (spherical sample, 25 °C) in an external magnetic flux density B. It may be determined by two different methods that in fact yield two different quantities because of the way the unit of electric current enters into the determination of B. When the magnetic flux density is calculated from the current flowing through the precision solenoid used to generate the field and its measured geometry, one has the so-called "low-field" measurement ($B\approx1$ mT): the solenoid is a single-layer coil so that the geometry of the current paths may be accurately determined, and carries low current in order that the thermal effects will be small. The appropriate unit for γ_p' is then $\mathrm{s^{-1}T^{-1}}$.

The low-field γ_p' data are in disagreement, with $\chi^2=198.0$ for five degrees of freedom, giving a Birge ratio $R_B=6.29$ and $P_{\chi^2}(198.0\mid5)<10^{-40}$. The NPL measurement (item 5.2) (Vigoureux and Dupuy, 1973, 1980) and the VNIIM measurement (item 5.5) (Studentsov, Khorev, and Shifrin, 1981) are significantly lower than the NIM measurement (item 5.3) (Chiao, Liu, and Shen, 1980), the NBS measurement (item 5.4) (Williams and Olsen, 1979) and the ASMW measurement (item 5.6) (Schlesok and Forkert, 1985). The low value of the ETL measurement (item 5.1) (Hara *et al.*, 1968) is not inconsistent only because of its low precision. (It contributes only 3.1 to the value of χ^2, although it differs by only 0.13 ppm from the highly discrepant NPL value that contributes 116.6.)

Of all the gyromagnetic ratio data, the most glaring discord comes from the NPL low-field value. The measurements of the proton resonance frequency were completed in December 1975 after which the coil dimensions were measured, but no verification was made (by repeating the frequency measurements), that the measurement process did not affect the coils. Because the measurements were forced to terminate prematurely, we consider it to be an incomplete effort which should not be included in the final adjustment. While deleting the NPL value reduces the value of χ^2 to 63.5 with four degrees of freedom, this is still highly discrepant: $R_B=3.98$ and $P_{\chi^2}(63.5\mid4)<10^{-10}$.

6. Type 6. Gyromagnetic ratio (high field)

When B is generated by an electromagnet and determined by measuring the dimensions of and the mechanical force exerted on a current-carrying conductor, one has the so-called "high-field" measurement ($B\approx0.5$T): the appropriate unit for γ_p' is $\mathrm{s^{-1}/(N\,m^{-1}/A)=A\,s/kg}$.

The high-field γ_p' data are less discrepant than the low-field data, but they are also less precise; the four values give $\chi^2=12.00$ with three degrees of freedom, $R_B=2.00$, and $P_{\chi^2}(12.00\mid3)=0.0074$. The Kharkov (KhGIMIP) measurement (item 6.1) (Yagola, Zingerman, and Sepetyi, 1962, 1966) would appear to be discrepant; it differs from the mean of the other three values (item 6.2: Kibble and Hunt, 1979; item 6.3: Chiao, Liu, and Shen, 1980; Wang, 1984; item 6.4: Schlesok and Forkert, 1985) by 2.4 standard deviations of that difference.

When the NBS Faraday measurement is expressed as a determination of the gyromagnetic ratio through Eq. (2.13), it becomes $26\,751.716(36)\times10^4\ C_{BI85}/\mathrm{kg}$; this is (2.8 ± 1.6) ppm higher than the mean of the direct measurements and (15.5 ± 5.6) ppm higher than the Kharkov value.

7. Type 7. Silicon lattice spacing

The first direct measurement of an atomic lattice spacing in terms of a known optical wavelength was carried out at NBS (Deslattes *et al.*, 1974, 1976) using the x-ray—optical interferometer (XROI) (item 7.1). The NBS value for silicon d_{220} is, however, inconsistent with the later measurement at PTB (item 7.2) using a somewhat different realization of the same concept (Becker *et al.*, 1981; Becker and Siegert, 1984; Siegert and Becker, 1984; Seyfried, 1985). The two results differ by more than 7 times the uncertainty of their difference, so that it is unjustified to keep both of them in the same adjustment. Additional measurements at PTB (Becker, Seyfried, and Siegert, 1982; Siegert, Becker, and Seyfried, 1984) have confirmed the quoted PTB precision, while work at NBS has verified that the Si crystals used by PTB and by NBS have the same lattice spacing to within 0.4 ppm (Deslattes and Henins, 1984). The 0.23 ppm uncertainty implied by this limit of error is the major contributor to the uncertainty assigned to the PTB determination, although it actually applies not to the PTB measurement itself but to the precision with which the result can be related to the NBS silicon molar volume determination (see below). Deslattes has indicated (Deslattes, 1985) that significant corrections to his early pioneering measurements with the NBS XROI have been identified, but that it is premature to give a new value.

8. Type 8. Molar volume of silicon

The molar volume of Si has been measured only at NBS (Deslattes, 1980a, 1980b). The NBS value is based on determinations of the isotopic composition (Barnes *et al.*, 1975; Ku, 1983) and density of pure Si single crystals (Bowman, Schoonover, and Carroll, 1974a, 1974b, 1975), corrected for known impurities and expressed in terms of the density at 22.5 °C in vacuum.

9. Type 9. Quantized Hall resistance

Quantum Hall effect measurements using heterostructures and MOSFET's have demonstrated over the past few years that the quantized Hall resistance relation $R_{\rm H}=h/e^2=\mu_0 c/2\alpha$ (von Klitzing, 1986) is valid at a precision of 1 in 10^7 or better in the limit that there is zero dissipation in the direction of current flow. (The measurements only confirm that $R_{\rm H}$ is the same for different materials, and that to this precision it is independent of specific operating conditions. However, it is difficult to imagine a nonzero correction term that would be independent of the solid-state environment or of the operating conditions to this level.) The six measurements (item 9.1: Bliek *et al.*, 1985; item 9.2: Cage, Dziuba, and Field, 1985; item 9.3: Wada *et al.*, 1985; item 9.4: Hartland, Davies, and Wood, 1985; item 9.5: van der Wel *et al.*, 1985; item 9.6: Delahaye *et al.*, 1986) give a mean value,

$$R_{\rm H}=25\,812.846\,1(16)\ \Omega_{\rm BI85} \tag{2.14}$$

in terms of the BIPM as-maintained ohm, with a Birge ratio of 1.007. From this, one finds

$$\alpha^{-1}(\Omega/\Omega_{\rm BI85})=137.036\,204\,4(85)\ .$$

The precision of 0.062 ppm is surprising for a physical measurement in a many-body system.

10. Type 10. Fine-structure constant

The fine-structure constant α (item 10.1) can be calculated from the measurement of the electron magnetic moment anomaly carried out at the University of Washington by van Dyck, Schwinberg, and Dehmelt (1984) combined with the extensive quantum electrodynamic calculations of the theory, which culminate in Kinoshita's numerical evaluation of the eighth-order terms (Kinoshita and Lindquist, 1981a, 1981b, 1983):

$$\tfrac{1}{2}g_e-1=a_e=\tfrac{1}{2}\alpha/\pi+C_2(\alpha/\pi)^2+C_3(\alpha/\pi)^3 + C_4(\alpha/\pi)^4+\cdots+\delta a\ ,$$

where $\delta a=1.69(4)\times10^{-12}$ is the sum of the non-QED contributions for hadronic vacuum polarization and weak interactions, and

$$C_2=-0.328\,478\,444,\quad C_3=1.1763(13)\ ,$$

$$C_4=-0.8\pm2.5\ .$$

A measurement of the fine structure in atomic He (Lewis, Pichanick, and Hughes, 1970; Frieze *et al.*, 1981; Kponou *et al.*, 1981) can yield a value for α (item 10.2) only if the calculation of the energy levels (Lewis and Serafino, 1978) can be carried out with sufficient precision. Unfortunately, the computational uncertainty is relatively large, and uncalculated terms in the perturbation expansion of the wave-functions of the two-electron system may contribute at the part in 10^7 level.

The two measurements of the fine-structure constant differ in *a priori* assigned weights by a factor of 85. Although the He fine-structure result is not in disagreement (within its stated uncertainty) with the anomalous electron moment value, the latter carries so much more weight that little is gained by including the former, and it is therefore justified to give this datum no further consideration in the analysis.

11. Type 11. Muon magnetic moment

The ratio of the magnetic moment of the muon to that of the proton $\mu_\mu/\mu_{\rm p}$ has been determined from the hyperfine structure of muonium (μ^+e^-) in a magnetic field by Mariam (1981; Mariam *et al.*, 1982) (item 11.1) and from the precession frequency measurements of muons stopped in liquid bromine targets by Klempt *et al.* (1982) (item 11.2). These measurements are similar to the corresponding hyperfine measurements for hydrogen (Sec. II.A.6), and to the anomalous muon precession (Sec. II.A.5), respectively. The mean of the two values is

3.183 345 47(95), with $R_B = 0.99$; there is no indication of any discrepancy or systematic errors.

12. Type 12. Muonium hyperfine splitting

The measurement of the hyperfine splitting interval ν_{Mhfs} in muonium (Mariam, 1981; Mariam *et al.*, 1982) will yield a value of $\alpha^2\mu_\mu/\mu_p$ if it is combined with the QED corrections to the original Fermi theory. This has been provided in large part by Sapirstein, Terray, and Yennie (1984) and Bodwin, Yennie, and Gregorio (1985), but still-uncalculated higher-order terms may be significant at the level of experimental precision that is now achievable. Because of this, a 0.13-ppm Type-B uncertainty, corresponding to ± 1 kHz limit of error, has been included in the uncertainty of the theory of the experiment.

C. Secondary stochastic data

In addition to the data of Table II, which form the basis for the least-squares analysis, there are three other stochastic quantities that must be considered in a general survey of the fundamental physical constants, but which have uncertainties so large that they appear in a separate category. In essence, the results of the multivariate adjustment become auxiliary constants with respect to these data.

1. Molar gas constant

The 1973 recommended value for the gas constant R was based on measurements of the molar volume of oxygen and nitrogen. Measurements of volume are beset with problems of sorption of gas on the walls of the vessel, an effect that was not fully appreciated when the original experiments were carried out (1924—1952). In contrast to such extensive measurements, the speed of sound is an intensive measurement that avoids the necessity of an absolute volume determination. Quinn, Colclough, and Chandler (1976) have used an acoustic interferometer to find the speed of sound in argon at the temperature of the triple point of water (Colclough, 1979, 1984a; Colclough, Quinn, and Chandler, 1979):

$$c_0^2 = 94\,756.75(58)\ \mathrm{m^2\,s^{-2}}\quad (6.1\ \mathrm{ppm})\ , \tag{2.15}$$

where the uncertainty is Type A only.

The gas constant is given by

$$R = M(\mathrm{Ar})c_0^2/\gamma T\ , \tag{2.16}$$

where $M(\mathrm{Ar})$ is the molar mass of argon and $\gamma = \frac{5}{3}$ is the specific heat ratio for an ideal monatomic gas.

The isotopic abundances of atmospheric argon measured by Nier (1950) [which have been adopted by the International Commission on Atomic Weights and Isotopic Abundances of IUPAC (de Bièvre *et al.*, 1984; Holden and Martin, 1984; Holden, Martin, and Barnes, 1984; Peiser *et al.*, 1984) as representative of the composition of atmospheric argon], the relative isotopic abundance measurements of the NPL argon, and the nuclidic masses of Wapstra and Audi (1985) lead to a molar mass for the argon used in the NPL measurements,

$$M(\mathrm{Ar_{NPL}}) = 0.039\,947\,753(75)\ \mathrm{kg/mol}\quad (1.9\ \mathrm{ppm})\ , \tag{2.17}$$

and hence to

$$R = 8.314\,510(70)\ \mathrm{J\,mol^{-1}\,K^{-1}}\quad (8.4\ \mathrm{ppm})\ , \tag{2.18}$$

where the assigned uncertainty is compounded from 6.1 ppm Type-A uncertainty and 5.8 ppm Type-B uncertainty. The value of R given in Eq. (2.18) is 3.6 ppm higher than that given by Colclough, Quinn, and Chandler (1979), who used $M(\mathrm{Ar_{NPL}}) = 0.039\,947\,6$ kg/mol based on the results of Melton *et al.* (1971) for the isotopic composition of argon. Although the change we have introduced into the Colclough, Quinn, and Chandler result is within the range of its stated experimental uncertainty, Eq. (2.18) has the advantage of being based on values related directly to the present IUPAC adopted reference abundances and its recommendation, $A_r = 39.9478$, for the relative atomic weight of atmospheric argon.

All the earlier data on the gas constant, including the 1973 recommendation based on Batuecas's evaluation of his own measurements as well as those of other workers, are omitted in determining the present recommended value because of the uncertainties to be assigned to them [particularly in view of the analysis of the systematic errors associated with such measurements by Quinn, Colclough, and Chandler (1976) and Colclough (1984b)]. The uncertainty of this older work (given as 31 ppm in 1973) must now be considered to be at least five or six times larger than that of the new determination.

2. Stefan-Boltzmann constant

By far the most accurate determination of the Stefan-Boltzmann constant available today is the one obtained by Quinn and Martin (1985) from the electrical calorimetric measurements of the radiation emitted by a black body at the temperature of the triple point of water. They give

$$\begin{aligned}\sigma &= 5.669\,67(76)\times 10^{-8}\ \mathrm{W_{NPL}\,m^{-2}\,K^{-4}}\\ &= 5.669\,59(76)\times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}\\ &\qquad (134\ \mathrm{ppm})\ ,\end{aligned} \tag{2.19}$$

where $\mathrm{W_{NPL}}$ is the watt as maintained in electrical units at NPL at the time of the measurement:

$$1\ \mathrm{W_{NPL}} = 1\ \mathrm{V_{NPL}^2/\Omega_{NPL}} = 0.999\,985\,92(62)\ \mathrm{W}$$

based on the 1986 adjusted values for $\mathrm{V_{76-BI}}$ and Ω_{BI85}.

This represents a significant increase in precision over the Blevin and Brown (1971) determination with an uncertainty of approximately 500 ppm discussed in the 1973 adjustment. However, since the theoretical expression for σ is

$$\sigma = \pi^2 (R/N_A)^4 / 60\hbar^3 c^2 , \qquad (2.20)$$

a more accurate value may be calculated from the measured molar gas constant, Eq. (2.18), and the 1986 recommended values of N_A and $\hbar$:

$$\sigma = 5.670\,51(19) \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}\ \ (34\ \ \mathrm{ppm}) .$$

Thus, the direct measurement is 4 times less precise than the indirect value. Since the difference between the two is 1.1 times the standard deviation of that difference, the two values are not in disagreement. However, there is presently insufficient precision in the direct measurement for it to significantly influence a determination of the gas constant. [A *pro forma* weighted mean of R from acoustic interferometry, Eq. (2.18), and $R = 8.314\,175(280)$ $\mathrm{J\,mol^{-1}\,K^{-1}}$ from the Quinn-Martin measurement of σ, yields $R = 8.314\,490(68)\ \mathrm{J\,mol^{-1}\,K^{-1}}$ (8.2 ppm).]

The situation could change with an increase in precision of this measurement by a factor of 2 or 3; then it would be quite appropriate to compute a value of the gas constant from the Stefan-Boltzmann constant and to combine that with the direct gas constant data. With the present status of the measurements, such a treatment is premature.

3. Newtonian constant of gravitation

There is no established relationship between the gravitational constant G and other physical quantities; it stands completely uncoupled from the remainder of the adjustment. Measurements of G can at present have no effect on our knowledge of the values of any other constants.

The 1973 recommended value was based on the measurements of Heyl (1930) and of Heyl and Chrzanowski (1942) (see also Cohen and Taylor, 1973):

$$G = 6.6720(41) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}\ \ (615\ \mathrm{ppm}) .$$

The results of a Heyl-type oscillating torsion balance experiment that was an outgrowth of a University of Virginia program (Rose *et al.*, 1969; Towler *et al.*, 1971) transferred to NBS in 1973 were reported by Luther and Towler (1982, 1984):

$$G = 6.672\,59(43) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}\ \ (64\ \mathrm{ppm}) , \qquad (2.21)$$

where the uncertainty is composed of a 40-ppm statistical (Type A) component and a 50-ppm nonstatistical (Type B) component.

Other measurements of G have been reported by Facy and Pontikis (1970, 1971), Pontikis (1972), Sagitov *et al.* (1979) and Karagioz *et al.* (1976, 1981). Both Pontikis's and Karagioz's data are internally inconsistent, indicating the presence of systematic effects that had not been adequately evaluated. Only the Sagitov data appear to be statistically valid. This experiment is similar in principle to the Heyl torsion balance and yields

$$G = 6.674\,49(81) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}\ (121\ \mathrm{ppm}) ,$$

where the uncertainty is the statistical standard deviation of the mean of 20 values. No information is given on the evaluation of the systematic uncertainties, so that it is impossible to combine properly these measurements with those of Luther and Towler.

We therefore adopt the Luther and Towler result, Eq. (2.21), as the recommended value, but in view of the difficulties in properly evaluating the systematic uncertainties of this type of measurement and the limited number of runs carried out, we shall arbitrarily double the uncertainty; it therefore appears below in Tables VI and VII as 128 ppm.

III. DATA ANALYSIS

Section II reviewed the data available for consideration in determining the values of the fundamental constants, and indicated in a cursory way some of the consistencies and inconsistencies in each type of data in order to establish a basis for a more complete analysis. In this section we consider their consistency by looking at relationships among data of different types. The data will then be more systematically analyzed using least-squares and the other algorithms summarized in Sec. II.

A. Relationships among data of different types

The most obvious of the relationships among the variable is $\mathrm{V} = \mathrm{A}\,\Omega$; from Eq. (2.12) and Eq. (2.10) we find

$$\begin{aligned} \mathrm{A_{BI85}} &= 0.999\,992\,14(58)\,\mathrm{V}/0.999\,998\,467(69)\ \Omega \\ &= \mathrm{A} - 6.32(59)\ \mu\mathrm{A} , \end{aligned} \qquad (3.1)$$

which shows a discrepancy of (8.4 ± 2.6) ppm with respect to the direct determination of the ampere, Eq. (2.11). Equation (3.1) represents an improvement in precision of the determination by a factor of 4.3, or a larger statistical weight by a factor of 19. The six direct ampere determinations together have less than 6% the statistical weight of Eq. (3.1).

Another indirect evaluation of the ampere may be obtained from a comparison of the results of low-field and high-field γ'_p determination. Since the two methods determine the same physical quantity, one may write

$$\gamma'_p = \gamma_{lo}\,\mathrm{s^{-1}}/\mathrm{T_{BI85}} = \gamma_{hi}\mathrm{A_{BI85}}\,\mathrm{s/kg} ,$$

where γ_{lo} and γ_{hi} are the dimensionless numerical magnitudes. The laboratory units are given by $\mathrm{A_{BI85}} = K_A \mathrm{A}$ and hence $\mathrm{T_{BI85}} = K_A \mathrm{T}$ since in the low-field determination the magnetic field is calculated from the geometry of, and the current in, the precision solenoid and is therefore proportional to the magnitude of the local unit of current.

TABLE III. Comparison of high- and low-field measurements of γ'_p.

Lab	$A_{BI85}-A$ (μA)	Uncertainty (ppm)
NPL	-9.34	0.57
NIM	-3.10	1.83
ASMW	-1.03	1.92
NBS	-6.43	0.68

The SI tesla is, by definition $T=kg\,s^{-2}\,A^{-1}$ and hence

$$K_A^2=\gamma_{lo}/\gamma_{hi} \; . \tag{3.2}$$

The data from those laboratories that have carried out both low-field and high-field determinations (including the NBS Faraday as a high-field determination of the gyromagnetic ratio) yield the results listed in Table III.

The weighted mean of these four determinations is $-7.54(41)$ μA based on the *a priori* estimates of the variances. The data are discordant; they yield $\chi^2=30.0$ with 3 degrees of freedom. The Birge ratio is $R_B=3.16$ and $P_{\chi^2}(30.0 \mid 3)<10^{-5}$. However, the NIM and ASMW measurements are not precise enough to carry significant weight in comparison with the NBS and NPL values, and at least one of those two purportedly high precision results must be in error since they disagree by 3.3 times the standard deviation of their difference. The probability of a discrepancy as large as this, with the standard deviation being based on 13.2 effective degrees of freedom, is approximately 0.25%.

We look at values of the Avogadro constant in order to have an independent appraisal of the discordant Si lattice-spacing data. From the fundamental definition of N_A one has

$$N_A=nM(\mathrm{Si})/\rho(\mathrm{Si})v \; , \tag{3.3}$$

where $n=8$ is the number of Si atoms in a unit cell, M(Si) is the mean molar mass of the Si crystal, ρ(Si) is the density, and $v=a^3=(\sqrt{8}d_{220})^3$ is the volume of a unit cell. Using the NBS data for the molar volume and for d_{220} gives $N_A=6.022\,097\,3(72)\times10^{23}$/mol; if the PTB value for d_{220} is used, the result is 5.4 ppm larger, $6.022\,129\,7(81)\times10^{23}$/mol.

In comparison, an indirect value of N_A may be found from

$$N_A=\tfrac{1}{2}R_H F(2e/h) \; . \tag{3.4}$$

The significant feature of this expression is that it is independent of the actual values of the laboratory electrical units as long as they are consistent (i.e., the resistance unit in terms of which R_H is expressed and the unit of voltage defined by the assigned value of $2e/h$ are consistent with the current or charge unit in terms of which the Faraday is expressed). Then, with $2e/h=483\,594.0$ GHz/V_{76-BI}, F expressed in terms of C_{BI85}, and using the mean value of the quantized Hall resistance given in Eq. (2.14), Eq. (3.4) gives $N_A=6.022\,143\,3(80)\times10^{23}$/mol, a value that is (2.3 ± 1.9) ppm larger than the value based on the PTB lattice spacing and (7.7 ± 1.8) ppm larger than that based on the NBS value. This further supports the conclusion that the NBS lattice-spacing measurement (item 7.1) should be rejected.

Another relationship that is independent of the electrical units used is

$$\alpha^3=\frac{2\mu_0 R_\infty}{(\mu'_p/\mu_B)}\frac{\gamma'_p}{(2e/h)R_H} \tag{3.5}$$

if γ'_p is a low-field determination (and hence with units s^{-1}/T_{LAB}). Then if $2e/h$ is expressed in LAB volts and R_H in LAB ohms, the magnitude of the laboratory units in terms of SI cancels. It is most useful here to use this expression to find a value of the low-field gyromagnetic ratio from a precise value of the fine-structure constant, since it is the gyromagnetic ratio data, not the fine-structure data, that are discordant. With α^{-1} from item (10.1) and the quantized Hall resistance from Eq. (2.14), Eq. (3.5) gives

$$\gamma'_p=26\,751.3617(55)\times10^4\ s^{-1}/T_{BI85} \; ,$$

a result that is more precise than any of the direct measurements, and that only differs from the most precise of those data (item 5.4) by $-(0.38\pm0.31)$ ppm.

A value for the fine-structure constant with a precision comparable to that obtained from the anomalous electron moment can be deduced from the quantized Hall resistance combined with the value of the BIPM as-maintained ohm given in Eq. (2.10). This leads to $\alpha^{-1}=137.035\,994\,3(127)$, in remarkable agreement with the anomalous moment result (item 10.1), $137.035\,994\,2(89)$. An independent value for α may also be found from the Los Alamos/Yale muonium hyperfine structure measurements (items 11.1 and 12.1). These data give $\alpha^{-1}=137.036\,003(26)$, which agrees within its uncertainty with item 10.1, but with only $\frac{1}{9}$ the weight.

One can calculate a value of the muon moment from the muonium hyperfine splitting, $\Delta\nu_{Mhfs}$, given a value for the fine-structure constant, α. This indirect value of μ_μ/μ_p is statistically independent of item (11.1), even though it comes from the same measurements, because the primary error contribution is the Type-B uncertainty in the theoretical expression for the hyperfine interval. Using the value 137.035 994 2 from the electron anomalous moment implies $\mu_\mu/\mu_p=3.183\,345\,68(61)$, a value that is consistent with the direct determinations.

There are still other relationships that may be written down by combining those given above, but in general they add no essential new information to that which is already available in the relationships given here. The purpose of this survey is only to indicate the extent to which the data are in agreement or disagreement. The full treatment of the data requires a complete multivariate analysis.

B. Multivariate analysis of the data

The total number of observational equations in this analysis is $N=38$, but because of the redundancy of the measurements there are only 12 distinct types. These can be expressed in terms of $M=5$ adjustable unknowns which, in turn, may be used to calculate all other constants of interest. The twelve types of observational equations are given in Table IV. In this table the quantity E is an abbreviation for 483 594.0 GHz/V; this allow us to write

$$V_{76-\mathrm{BI}}=483\,594.0\ \mathrm{GHz}(h/2e)\ , \tag{3.6a}$$

$$V_{76-\mathrm{BI}}=K_{\mathrm{V}}\ \mathrm{V}\ ,$$

$$2e/h=E/K_{\mathrm{V}}\ , \tag{3.6b}$$

for the definition of the BIPM as-maintained volt. The numerical quantity K_{V} defined by Eq. (3.6a) is used as a basic variable of the least-squares analysis, as indicated in Sec. II.A.7.

The five adjustable unknowns are taken to be α, K_{Ω}, K_{V}, d_{220}, and $\mu_{\mu}/\mu_{\mathrm{p}}$. (For convenience in numerical output, the calculations are actually performed with the variable α^{-1}). In Table IV the quantities to the left of the centered dot are auxiliary constants, those to the right are the stochastic variables of the adjustment.

It is clear that some of the data discussed in Sec. II will have very little statistical weight in any least-squares analysis, and there is ample evidence, even from the cursory review given above, that others should be removed from the set of retained data because of discrepancies. An adjustment of all 38 data by standard least squares gives $\chi^2=324.9$ with $\nu=38$, or a Birge ratio $R_{\mathrm{B}}=3.14$. Since the expectation value of χ^2 is 33 ± 8.1 and the expectation value of the Birge ratio is 1 ± 0.12, the discrepancy has been given clear quantitative expression. The *pro forma* probability that χ^2 would by chance be this large is $P_{\chi^2}(324.9\mid 33)\approx10^{-29}$.

TABLE IV. Observational equations for the 1986 least-squares adjustment.

1.	$\Omega_{\mathrm{BI85}}=K_{\Omega}\ \Omega$
2.	$A_{\mathrm{BI85}}=K_{\mathrm{V}}K_{\Omega}^{-1}\ \mathrm{A}$
3.	$V_{76-\mathrm{BI}}=K_{\mathrm{V}}\ \mathrm{V}$
4.	$F_{\mathrm{BI85}}=\dfrac{M_{\mathrm{p}}cE}{4R_{\infty}(m_{\mathrm{p}}/m_{\mathrm{e}})}\cdot\alpha^2K_{\mathrm{V}}^{-2}K_{\Omega}$
5.	$\gamma'_{\mathrm{p}}(\mathrm{lo})_{\mathrm{BI85}}=\dfrac{c(\mu'_{\mathrm{p}}/\mu_{\mathrm{B}})E}{4R_{\infty}}\cdot\alpha^2K_{\Omega}^{-1}$
6.	$\gamma'_{\mathrm{p}}(\mathrm{hi})_{\mathrm{BI85}}=\dfrac{c(\mu'_{\mathrm{p}}/\mu_{\mathrm{B}})E}{4R_{\infty}}\cdot\alpha^2K_{\mathrm{V}}^{-2}K_{\Omega}$
7.	$d_{220}(\mathrm{Si})=d_{220}(\mathrm{Si})$
8.	$V_{\mathrm{m}}(\mathrm{Si})=\dfrac{M_{\mathrm{p}}\mu_0c^2E^2}{2\sqrt{8}(m_{\mathrm{p}}/m_{\mathrm{e}})R_{\infty}}\cdot\alpha K_{\mathrm{V}}^{-2}d_{220}^3$
9.	$(R_{\mathrm{H}})_{\mathrm{BI85}}=\frac{1}{2}\mu_0c\cdot\alpha^{-1}K_{\Omega}^{-1}$
10.	$\alpha=\alpha$
11.	$\mu_{\mu}/\mu_{\mathrm{p}}=\mu_{\mu}/\mu_{\mathrm{p}}$
12.	$\nu_{\mathrm{Mhfs}}=\dfrac{16}{3}\dfrac{R_{\infty}c(\mu_{\mathrm{p}}/\mu_{\mathrm{B}})}{(1+m_{\mathrm{e}}/m_{\mu})^3}q\cdot\alpha^2(\mu_{\mu}/\mu_{\mathrm{p}})$
	$q=1.000\,957\,61(14)$

A simple listing of the differences between two sets of adjusted values can give a distorted image of the essential difference because it cannot show the correlations that exist among them. In order to obtain a proper representation of these differences, the variance matrix of the data may be interpreted as the metric tensor of the space spanned by the variables of the least-squares adjustment. The metric distance of the adjusted point from a fixed point in this space (i.e., the distance measured in units of the standard deviation in the specified direction) is given by

$$d^2=\sum_{r,s}(x_r-x_r^0)w_{rs}(x_s-x_s^0)\ , \tag{3.7a}$$

where w_{rs} is the weight matrix of the least-squares adjustment. It thus provides a measure of distance that is independent of any choice of correlated output values that might be used to demonstrate the difference. Equation (3.7a) is expressed in terms of the output variables of the adjustment, but the solution point $\{x_r\}$ can equally well be expressed in terms of the input data in the least-squares adjustment and the distance can equivalently be written as

$$d^2=\sum_i w_i(y_i-y_i^0)^2\ , \tag{3.7b}$$

where y_i is the ith stochastic input datum and y_i^0 is its value calculated at the fixed point defined by the origin values $\{x_r^0\}$.

Table V summarizes the differences among the solutions produced by the various algorithms applied to different data sets, showing the distance d of each solution from a fixed point. This point corresponds to the ELS2 adjustment for the set of 22 input data [data set (e)]. It is evident from the data presented in this table that once items 5.2 and 7.1 are deleted from consideration, all of the algorithms considered here [including standard least squares with the uncertainties reexpressed using "external error," i.e., expanded uniformly by the multiplicative factor $R_{\mathrm{B}}=(\chi^2/\nu)^{1/2}$] yield results that are quite similar; none of these results differ by more than one standard deviation from the results of ELS2 applied to data set (e) that form the basis for the 1986 recommended values.

The VNIIM-based algorithms modify the uncertainties assigned to the input data so as to give χ^2 equal to the number of degrees of freedom, making the largest changes to those data that are most discrepant. Whereas the standard least-squares procedure [Table V, data set (a), Least squares (external error)] achieves consistency by expanding all uncertainties by a factor of 3.1, the original VNIIM algorithm applies factors that range from 1.001 for the muonium hfs (item 12.1) to 4.1 for the NPL low-field γ'_{p} measurement (item 5.2). The modified VNIIM algorithm applies factors of 1.000 and 5.0 to these data.

The extended least-squares algorithm ESL1 is even

TABLE V. Comparison of adjustments using different data sets and different algorithms. In this table d is the "distance" from a fixed origin, measured in standard deviations, of the point in variable space representing the solution given by the specified algorithm for various data sets. The origin is the output of algorithm ELS2 applied to set (e), corresponding to the 1986 CODATA Recommended Values. Only the adjusted values of α and K_V are given since these are the most sensitive to the different data sets, and are the variables that are most significant for defining the derived output data.

Algorithm	Data set	(a)	(b)	(c)	(d)	(e)
	Description of data set	All data	set (a), omit items 5.2, 7.1, and 10.2	set (b), omit items 2.1–2.6, 5.1, and 5.6	set (c), omit items 3.1, 5.5, 6.1, and 6.4	set (d), omit item 5.3
		$N=38$	$N=35$	$N=27$	$N=23$	$N=22$
		$\nu=33$	$\nu=30$	$\nu=22$	$\nu=18$	$\nu=17$
Least squares	χ^2	324.9	106.6	89.8	19.5	17.09
(internal error)	α^{-1}	137.036 010 2(59)	137.035 995 9(60)	137.035 996 1(60)	137.035 988 3(60)	137.035 989 6(61)
	$(K_V-1)\times 10^6$	−6.77(28)	−7.24(29)	−7.34(29)	−7.59(30)	−7.59(30)
	d	17.688	1.738	1.518	0.222	0.0052
Least squares	χ^2	33	30	22	18	17
(external error)	α^{-1}	137.036 010(18)	137.035 996(11)	137.035 996(12)	137.035 988 3(63)	137.035 989 6(61)
	$(K_V-1)\times 10^6$	−6.77(87)	−7.24(54)	−7.34(58)	−7.59(31)	−7.59(30)
	d	5.637	0.922	0.751	0.213	0.0052
VNIIM	χ^2	33	30	22	18	17
	α^{-1}	137.035 995 0(81)	137.035 992 2(72)	137.035 992 5(73)	137.035 988 4(62)	137.035 989 6(61)
	$(K_V-1)\times 10^6$	−6.88(61)	−7.29(36)	−7.38(36)	−7.59(30)	−7.59(30)
	d	7.888	0.965	0.775	0.219	0.0066
VNIIM	χ^2	33	30	22	18	17
(symmetric)	α^{-1}	137.035 993 3(72)	137.035 990 7(69)	137.035 991 2(69)	137.035 988 3(62)	137.035 989 6(61)
	$(K_V-1)\times 10^6$	−7.17(44)	−7.31(33)	−7.39(33)	−7.59(30)	−7.59(30)
	d	15.999	0.899	0.667	0.224	0.0037
ELS1	χ^2	50.7	42.0	32.5	17.3	15.16
	α^{-1}	137.035 990 9(55)	137.035 989 7(58)	137.035 990 1(57)	137.035 988 3(60)	137.035 990 2(57)
	$(K_V-1)\times 10^6$	−7.32(28)	−7.34(28)	−7.41(28)	−7.57(28)	−7.57(28)
	d	1.169	0.925	0.681	0.233	0.177
ELS2	χ^2				18.2	17.01
	α^{-1}				137.035 987 8(64)	137.035 989 5(61)
	$(K_V-1)\times 10^6$				−7.59(31)	−7.59(30)
	d				0.272	**0.00**

more severe in its expansion of the uncertainties of the data for set (a). For item 5.2 s_i is increased by a factor of 6.35, and for item 7.1, s_i is increased by a factor of 9.91. Except for item 5.5, whose uncertainty is increased by a factor 2.73, no other uncertainty is increased by a factor larger than 1.8. Since the muonium hfs result is consistent with the other data, the algorithm actually reduces its uncertainty. This datum has a relatively low input weight because of the introduction of an allowance for uncalculated terms in the theoretical expression used in its evaluation. The good agreement may be taken as evidence that the estimate of uncalculated terms was realistic but slightly pessimistic, and that the total uncertainty could be reduced from 0.14 ppm to 0.11 ppm. This 0.11 ppm is still more than three times the experimental uncertainty, and corresponds to assigning 0.8 kHz instead of 1 kHz to the theoretical estimated limit of error. Since the choice of 1 kHz was to some extent an accident of our number system, the "corrected" estimate cannot be considered to be any less realistic than the original estimate.

Algorithm ELS1 reduces the value of χ^2 for data set (a) to 50.7 with a Birge ratio $R_B=1.24$. This still is indicative of discrepant data; the probability that χ^2 would be as large as this is only 0.025. The discrepancies in set (a) were anticipated and three data were identified as inappropriate for inclusion in the final adjustment: item 5.2, NPL low-field γ'_p, because it represents an uncompleted measurement; item 7.1, NBS d_{220}, because it is in serious disagreement with other data and recently shown to be in error; and item 10.2, He fine structure, because it is based on an inadequately developed theoretical expression.

When these three data are deleted the result is data set (b) with 35 observations and 30 degrees of freedom; the standard least-squares adjustment gives $\chi^2=106.6$ with

$R_B = 1.89$. This set still has discrepancies; the probability of values as large as these is $P_{\chi^2}(106.6 \mid 30) < 2\times 10^{-11}$.

As a point of departure, the values resulting from this least-squares adjustment are

$$\alpha^{-1} = 137.035\,996(11)\,, \tag{3.8a}$$

$$V_{76-\mathrm{BI}} = [1 - 7.24(54)\times 10^{-6}]\ \mathrm{V}\,, \tag{3.8b}$$

$$\Omega_{\mathrm{BI85}} = [1 - 1.524(92)\times 10^{-6}]\ \Omega\,, \tag{3.8c}$$

$$d_{220} = 192.015\,553(74)\ \mathrm{pm}\,, \tag{3.8d}$$

$$\mu_\mu/\mu_p = 3.183\,345\,71(87)\,, \tag{3.8e}$$

where the uncertainties have been computed from external consistency. These uncertainties (which are expanded by a factor of $R_B = 1.9$ from the internally computed values) are all significantly smaller than the corresponding uncertainties of the 1973 adjustment. Although data set (b) contains discrepant data, none of the five adjusted quantities differs from its value in the final recommended set by more then 0.67 standard deviations of that difference, and only K_V differs from the final recommended value by more than one standard deviation of that value.

The largest remaining discrepancy is produced by item 5.5, the low-field γ_p' determination of Studentsov *et al.*. It contributes by itself almost half of the total χ^2 (i.e., 52.6) and its difference from the value of γ_p' deduced from the remainder of the data is 7.4 times the standard deviation of that difference. There are several other significant discrepancies: item 2.6, the newer NPL direct ampere determination ($\Delta\chi^2 = 8.5$, $\delta x = 2.9\sigma$); item 6.1, the Kharkov high-field γ_p' determination ($\Delta\chi^2 = 6.3$, $\delta x = 2.5\sigma$); item 6.4, the ASMW high-field γ_p' determination ($\Delta\chi^2 = 5.5$, $\delta x = 2.4\sigma$). All of these were identified as discrepant in the preliminary examination.

In order to reduce χ^2 to $\nu = 30$ the VNIIM algorithm expands the *a priori* uncertainties by factors that range from 1.00 for the consistent muonium hfs (item 12.1) to 2.46 for the discrepant determination of Studentsov *et al.* of γ_p' (item 5.5). In comparison with this, the symmetric VNIIM algorithm introduces an even larger factor, 2.97. For both algorithms the weight assigned to this datum is reduced to the point that it contributes less than 1% to the determination of the output value of γ_p'. The datum has thus been effectively deleted from the analysis, yet it contributes 9.0 to the total χ^2 of 30 for the original VNIIM algorithm and 6.3 to χ^2 for the symmetric VNIIM algorithm.

The extended least-squares algorithm ELS1 does not force the value of χ^2 to be equal to ν, and gives $\chi^2 = 42.0$, a value which has a probability of being exceeded by chance, $P_{\chi^2}(42.0 \mid 30) = 0.071$. In reaching this value the algorithm modifies the uncertainties by factors ranging from 0.76 to 2.74. The determination of Studentsov *et al.* is treated similarly to the way it is treated in the previously discussed algorithms. The expansion factor, 2.74, reduces the relative contribution of this datum to χ^2 from 49% to 18% while its contribution to the output value of γ_p' is reduced to 0.5%.

Because of these discrepancies, the second extended least-squares algorithm, ELS2, does not give a solution with positive weights.

If the extremely discrepant γ_p' result of Studentsov *et al.* (item 5.5) is removed from data set (b), the value of χ^2 is reduced to 52.1; this is still an uncomfortably large value with a probability $P_{\chi^2}(52.1 \mid 29) = 0.005$, and the other discrepant quantities identified above still contribute to the disagreement to essentially the same extent. The extended least-squares algorithm ELS1 reduces χ^2 to 34.5, with a Birge ratio $R_B = 1.09$ and $P_{\chi^2}(34.5 \mid 29) = 0.22$, while ELS2 still produces negative (and therefore, unreal) weights.

Data set (b) contains several items that contribute very little to the determination of the adjusted values of the unknowns. None of the direct ampere determinations (items 2.1—2.6) contributes more than 0.5% (and all six, less than 1.3%) of the weight of the adjusted value of the ampere, a value defined more precisely from the ratio of the low-field to the high-field γ_p' measurements, or from ohm and volt realizations. The ohm, in turn, is only partially defined by the calculable capacitor data; equally as much weight is attached to the "indirect" value of the ohm resulting from the combination of the quantized Hall resistance and the implied value of the fine-structure constant—both the direct value of item 10.1 and the indirectly deduced value from such relationships as Eq. (3.5).

The low-weight data—items 2.1—2.6, item 3.1; items 5.1, 5.3, and 5.6, and item 6.1—none of which contribute more than $\frac{1}{15}$ as much as another direct or indirect determination of the same quantity, may be omitted without greatly affecting the results [Table V, data sets (c), (d), and (e)]. In addition to these eleven deletions, the relatively low weight and somewhat discrepant ASMW high-field γ_p' determination (item 6.4) is also eliminated. The remaining 22 data items (58% of the original inventory) make up data set (e). The standard least-squares procedure gives $\chi^2 = 17.09$ for these data, with 17 degrees of freedom. Since the value of χ^2 is only slightly greater than ν, there is very little difference among the results produced using different algorithms. This can be seen quantitatively in Table V, where there are no significant differences except for ELS1. Because the weight of each input datum is adjusted in ELS1 based on its own deviation from the consensus value, the total reduction of χ^2 is larger than using algorithm ELS2, and χ^2 is reduced to an adjusted value of 15.16.

IV. RECOMMENDED VALUES AND DISCUSSION

In this section we present the 1986 CODATA Recommended Set of numerical values of the physical constants and discuss the results with respect to other choices of the data and alternative algorithms. Also given is a comparison of the 1986 Set with the 1973 Set. The primary source of the differences between the 1973 and 1986 numbers is identified and the differences in reliability of the two adjustments are discussed.

A. 1986 recommended values of the fundamental physical constants

Based on the results of applying the various algorithms to the data and on a consideration of the effects on consistency produced by eliminating data that are either obviously discrepant relative to their claimed precision or of such low weight that they contribute little to the determination of the final result (or both), data set (e) consisting of 22 items of stochastic data is taken to be the appropriate data set to define the 1986 recommended values of the fundamental constants.

Because algorithm ELS2 appears to be best grounded statistically, we use it as the basis for the recommended set. The adjusted value of χ^2 for this algorithm is 17.01 with 17 degrees of freedom. The results of the application of algorithm ELS2 to set (e) are presented in three tables: Table VI gives a short list, and Table VII gives a more extensive list, of the fundamental constants of physics and chemistry; Table VIII presents a set of related values, such as the quantities $V_{\mathrm{BI-76}}$, Ω_{BI85}, and d_{220} that are a necessary part of the data of the adjustment, but that cannot be considered as fundamental in the same sense as the quantities of Table VII. In addition, Table IX gives some technologically and metrologically useful energy conversion factors.

The equations relating the quantities in Tables VI—IX to the variables of the least-squares adjustment and to the auxiliary constants of Table I are fairly direct. (In many cases when the relation is not obvious from the definition of the quantity itself, an expression is given in the tables.) More complete discussions have been given previously (Taylor, Parker, and Langenberg, 1969; Cohen and Taylor, 1973) and will not be repeated here. The x-ray data in Table VIII are based on the measurements of Deslattes and Henins (1973) and Kessler, Deslattes, and Henins (1979) who determined the ratios of the wavelengths to the d_{220} spacing in Si.

Because the variables in an analysis such as this are statistically correlated, it must be remembered that the uncertainties associated with a computed value can in general only be found with the use of the full variance ma-

TABLE VI. Summary of the 1986 recommended values of the fundamental physical constants. An abbreviated list of the fundamental constants of physics and chemistry based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Quantity	Symbol	Value	Units	Relative uncertainty (ppm)
speed of light in vacuum	c	299 792 458	$\mathrm{m\,s^{-1}}$	(exact)
permeability of vacuum	μ_0	$4\pi\times10^{-7}$	$\mathrm{N\,A^{-2}}$	
		$=12.566\,370\,614\ldots$	$10^{-7}\ \mathrm{N\,A^{-2}}$	(exact)
permittivity of vacuum, $1/\mu_0 c^2$	ε_0	$8.854\,187\,817\ldots$	$10^{-12}\ \mathrm{F\,m^{-1}}$	(exact)
Newtonian constant of gravitation	G	6.672 59(85)	$10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$	128
Planck constant	h	6.626 075 5(40)	$10^{-34}\ \mathrm{J\,s}$	0.60
$h/2\pi$	$\hbar$	1.054 572 66(63)	$10^{-34}\ \mathrm{J\,s}$	0.60
elementary charge	e	1.602 177 33(49)	$10^{-19}\ \mathrm{C}$	0.30
magnetic flux quantum, $h/2e$	Φ_0	2.067 834 61(61)	$10^{-15}\ \mathrm{Wb}$	0.30
electron mass	m_e	9.109 389 7(54)	$10^{-31}\ \mathrm{kg}$	0.59
proton mass	m_p	1.672 623 1(10)	$10^{-27}\ \mathrm{kg}$	0.59
proton-electron mass ratio	m_p/m_e	1 836.152 701(37)		0.020
fine-structure constant, $\mu_0 ce^2/2h$	α	7.297 353 08(33)	10^{-3}	0.045
inverse fine-structure constant	α^{-1}	137.035 989 5(61)		0.045
Rydberg constant, $m_e c\alpha^2/2h$	R_∞	10 973 731.534(13)	$\mathrm{m^{-1}}$	0.0012
Avogadro constant	N_A, L	6.022 136 7(36)	$10^{23}\ \mathrm{mol^{-1}}$	0.59
Faraday constant, $N_A e$	F	96 485.309(29)	$\mathrm{C\,mol^{-1}}$	0.30
molar gas constant	R	8.314 510(70)	$\mathrm{J\,mol^{-1}\,K^{-1}}$	8.4
Boltzmann constant, R/N_A	k	1.380 658(12)	$10^{-23}\ \mathrm{J\,K^{-1}}$	8.5
Stefan-Boltzmann constant, $(\pi^2/60)k^4/\hbar^3c^2$	σ	5.670 51(19)	$10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$	34
		Non-SI units used with SI		
electron volt, $(e/\mathrm{C})\mathrm{J}=\{e\}\mathrm{J}$	eV	1.602 177 33(49)	$10^{-19}\ \mathrm{J}$	0.30
(unified) atomic mass unit, $1\ \mathrm{u}=m_u=\frac{1}{12}m(^{12}\mathrm{C})$	u	1.660 540 2(10)	$10^{-27}\ \mathrm{kg}$	0.59

TABLE VII. 1986 recommended values of the fundamental physical constants. This list of the fundamental constants of physics and chemistry is based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full variance matrix must be used in evaluating the uncertainties of quantities computed from them.

Quantity	Symbol	Value	Units	Relative uncertainty (ppm)
		GENERAL CONSTANTS		
		Universal constants		
speed of light in vacuum	c	299 792 458	$\mathrm{m\,s^{-1}}$	(exact)
permeability of vacuum	μ_0	$4\pi\times10^{-7}$	$\mathrm{N\,A^{-2}}$	
		$=12.566\,370\,614\ldots$	$10^{-7}\ \mathrm{N\,A^{-2}}$	(exact)
permittivity of vacuum, $1/\mu_0 c^2$	ε_0	$8.854\,187\,817\ldots$	$10^{-12}\ \mathrm{F\,m^{-1}}$	(exact)
Newtonian constant of gravitation	G	6.672 59(85)	$10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$	128
Planck constant	h	6.626 075 5(40)	$10^{-34}\ \mathrm{J\,s}$	0.60
in electron volts, $h/\{e\}$		4.135 669 2(12)	$10^{-15}\ \mathrm{eV\,s}$	0.30
$h/2\pi$	$\hbar$	1.054 572 66(63)	$10^{-34}\ \mathrm{J\,s}$	0.60
in electron volts, $\hbar/\{e\}$		6.582 122 0(20)	$10^{-16}\ \mathrm{eV\,s}$	0.30
Planck mass, $(\hbar c/G)^{1/2}$	m_{P}	2.176 71(14)	$10^{-8}\ \mathrm{kg}$	64
Planck length, $\hbar/m_{\mathrm{P}}c=(\hbar G/c^3)^{1/2}$	l_{P}	1.616 05(10)	$10^{-35}\ \mathrm{m}$	64
Planck time, $l_{\mathrm{P}}/c=(\hbar G/c^5)^{1/2}$	t_{P}	5.390 56(34)	$10^{-44}\ \mathrm{s}$	64
		Electromagnetic constants		
elementary charge	e	1.602 177 33(49)	$10^{-19}\ \mathrm{C}$	0.30
	e/h	2.417 988 36(72)	$10^{14}\ \mathrm{A\,J^{-1}}$	0.30
magnetic flux quantum, $h/2e$	Φ_0	2.067 834 61(61)	$10^{-15}\ \mathrm{Wb}$	0.30
Josephson frequency-voltage quotient	$2e/h$	4.835 976 7(14)	$10^{14}\ \mathrm{Hz\,V^{-1}}$	0.30
quantized Hall conductance	e^2/h	3.874 046 14(17)	$10^{-5}\ \mathrm{S}$	0.045
quantized Hall resistance, $h/e^2=\mu_0 c/2\alpha$	R_{H}	25 812.805 6(12)	Ω	0.045
Bohr magneton, $e\hbar/2m_{\mathrm{e}}$	μ_{B}	9.274 015 4(31)	$10^{-24}\ \mathrm{J\,T^{-1}}$	0.34
in electron volts, $\mu_{\mathrm{B}}/\{e\}$		5.788 382 63(52)	$10^{-5}\ \mathrm{eV\,T^{-1}}$	0.089
in hertz, μ_{B}/h		1.399 624 18(42)	$10^{10}\ \mathrm{Hz\,T^{-1}}$	0.30
in wavenumbers, μ_{B}/hc		46.686 437(14)	$\mathrm{m^{-1}\ T^{-1}}$	0.30
in kelvins, μ_{B}/k		0.671 709 9(57)	$\mathrm{K\,T^{-1}}$	8.5
nuclear magneton, $e\hbar/2m_{\mathrm{p}}$	μ_{N}	5.050 786 6(17)	$10^{-27}\ \mathrm{J\,T^{-1}}$	0.34
in electron volts, $\mu_{\mathrm{N}}/\{e\}$		3.152 451 66(28)	$10^{-8}\ \mathrm{eV\,T^{-1}}$	0.089
in hertz, μ_{N}/h		7.622 591 4(23)	$\mathrm{MHz\,T^{-1}}$	0.30
in wavenumbers, μ_{N}/hc		2.542 622 81(77)	$10^{-2}\ \mathrm{m^{-1}\,T^{-1}}$	0.30
in kelvins, μ_{N}/k		3.658 246(31)	$10^{-4}\ \mathrm{K\,T^{-1}}$	8.5
		ATOMIC CONSTANTS		
fine-structure constant, $\mu_0 ce^2/2h$	α	7.297 353 08(33)	10^{-3}	0.045
inverse fine-structure constant	α^{-1}	137.035 989 5(61)		0.045
Rydberg constant, $m_{\mathrm{e}}c\alpha^2/2h$	R_∞	10 973 731.534(13)	$\mathrm{m^{-1}}$	0.0012
in hertz, $R_\infty c$		3.289 841 949 9(39)	$10^{15}\ \mathrm{Hz}$	0.0012
in joules, $R_\infty hc$		2.179 874 1(13)	$10^{-18}\ \mathrm{J}$	0.60
in eV, $R_\infty hc/\{e\}$		13.605 698 1(40)	eV	0.30
Bohr radius, $\alpha/4\pi R_\infty$	a_0	0.529 177 249(24)	$10^{-10}\ \mathrm{m}$	0.045
Hartree energy, $e^2/4\pi\varepsilon_0 a_0=2R_\infty hc$	E_{h}	4.359 748 2(26)	$10^{-18}\ \mathrm{J}$	0.60
in eV, $E_{\mathrm{h}}/\{e\}$		27.211 396 1(81)	eV	0.30
quantum of circulation	$h/2m_{\mathrm{e}}$	3.636 948 07(33)	$10^{-4}\ \mathrm{m^2\,s^{-1}}$	0.089
	h/m_{e}	7.273 896 14(65)	$10^{-4}\ \mathrm{m^2\,s^{-1}}$	0.089
		Electron		
electron mass	m_{e}	9.109 389 7(54)	$10^{-31}\ \mathrm{kg}$	0.59
		5.485 799 03(13)	$10^{-4}\ \mathrm{u}$	0.023
in electron volts, $m_{\mathrm{e}}c^2/\{e\}$		0.510 999 06(15)	MeV	0.30
electron-muon mass ratio	m_{e}/m_μ	4.836 332 18(71)	10^{-3}	0.15
electron-proton mass ratio	$m_{\mathrm{e}}/m_{\mathrm{p}}$	5.446 170 13(11)	10^{-4}	0.020
electron-deuteron mass ratio	$m_{\mathrm{e}}/m_{\mathrm{d}}$	2.724 437 07(6)	10^{-4}	0.020
electron—α-particle mass ratio	m_{e}/m_α	1.370 933 54(3)	10^{-4}	0.021
electron specific charge	$-e/m_{\mathrm{e}}$	$-1.758\,819\,62(53)$	$10^{11}\ \mathrm{C\,kg^{-1}}$	0.30

TABLE VII. (*Continued*).

Quantity	Symbol	Value	Units	Relative uncertainty (ppm)
		Electron		
electron molar mass	$M(\mathrm{e}), M_{\mathrm{e}}$	5.485 799 03(13)	10^{-7} kg/mol	0.023
Compton wavelength, $h/m_{\mathrm{e}}c$	λ_{C}	2.426 310 58(22)	10^{-12} m	0.089
$\lambda_{\mathrm{C}}/2\pi=\alpha a_0=\alpha^2/4\pi R_\infty$	$\bar{\lambda}_{\mathrm{C}}$	3.861 593 23(35)	10^{-13} m	0.089
classical electron radius, $\alpha^2 a_0$	r_{e}	2.817 940 92(38)	10^{-15} m	0.13
Thomson cross section, $(8\pi/3)r_{\mathrm{e}}^2$	σ_{e}	0.665 246 16(18)	10^{-28} m^2	0.27
electron magnetic moment	μ_{e}	928.477 01(31)	10^{-26} $\mathrm{J\,T^{-1}}$	0.34
in Bohr magnetons	$\mu_{\mathrm{e}}/\mu_{\mathrm{B}}$	1.001 159 652 193(10)		1×10^{-5}
in nuclear magnetons	$\mu_{\mathrm{e}}/\mu_{\mathrm{N}}$	1 838.282 000(37)		0.020
electron magnetic moment anomaly, $\mu_{\mathrm{e}}/\mu_{\mathrm{B}}-1$	a_{e}	1.159 652 193(10)	10^{-3}	0.0086
electron g factor, $2(1+a_{\mathrm{e}})$	g_{e}	2.002 319 304 386(20)		1×10^{-5}
electron-muon magnetic moment ratio	$\mu_{\mathrm{e}}/\mu_{\mu}$	206.766 967(30)		0.15
electron-proton magnetic moment ratio	$\mu_{\mathrm{e}}/\mu_{\mathrm{p}}$	658.210 688 1(66)		0.010
		Muon		
muon mass	m_{μ}	1.883 532 7(11)	10^{-28} kg	0.61
		0.113 428 913(17)	u	0.15
in electron volts, $m_{\mu}c^2/\{e\}$		105.658 389(34)	MeV	0.32
muon-electron mass ratio	m_{μ}/m_{e}	206.768 262(30)		0.15
muon molar mass	$M(\mu), M_{\mu}$	1.134 289 13(17)	10^{-4} kg/mol	0.15
muon magnetic moment	μ_{μ}	4.490 451 4(15)	10^{-26} $\mathrm{J\,T^{-1}}$	0.33
in Bohr magnetons,	$\mu_{\mu}/\mu_{\mathrm{B}}$	4.841 970 97(71)	10^{-3}	0.15
in nuclear magnetons,	$\mu_{\mu}/\mu_{\mathrm{N}}$	8.890 598 1(13)		0.15
muon magnetic moment anomaly, $[\mu_{\mu}/(e\hbar/2m_{\mu})]-1$	a_{μ}	1.165 923 0(84)	10^{-3}	7.2
muon g factor, $2(1+a_{\mu})$	g_{μ}	2.002 331 846(17)		0.0084
muon-proton magnetic moment ratio	$\mu_{\mu}/\mu_{\mathrm{p}}$	3.183 345 47(47)		0.15
		Proton		
proton mass	m_{p}	1.672 623 1(10)	10^{-27} kg	0.59
		1.007 276 470(12)	u	0.012
in electron volts, $m_{\mathrm{p}}c^2/\{e\}$		938.272 31(28)	MeV	0.30
proton-electron mass ratio	$m_{\mathrm{p}}/m_{\mathrm{e}}$	1 836.152 701(37)		0.020
proton-muon mass ratio	m_{p}/m_{μ}	8.880 244 4(13)		0.15
proton specific charge	e/m_{p}	9.578 830 9(29)	10^{7} $\mathrm{C\,kg^{-1}}$	0.30
proton molar mass	$M(\mathrm{p}), M_{\mathrm{p}}$	1.007 276 470(12)	10^{-3} kg/mol	0.012
proton Compton wavelength, $h/m_{\mathrm{p}}c$	$\lambda_{\mathrm{C,p}}$	1.321 410 02(12)	10^{-15} m	0.089
$\lambda_{\mathrm{C,p}}/2\pi$	$\bar{\lambda}_{\mathrm{C,p}}$	2.103 089 37(19)	10^{-16} m	0.089
proton magnetic moment	μ_{p}	1.410 607 61(47)	10^{-26} $\mathrm{J\,T^{-1}}$	0.34
in Bohr magnetons	$\mu_{\mathrm{p}}/\mu_{\mathrm{B}}$	1.521 032 202(15)	10^{-3}	0.010
in nuclear magnetons	$\mu_{\mathrm{p}}/\mu_{\mathrm{N}}$	2.792 847 386(63)		0.023
diamagnetic shielding correction for protons in pure water, spherical sample, 25 °C, $1-\mu_{\mathrm{p}}'/\mu_{\mathrm{p}}$	$\sigma_{\mathrm{H_2O}}$	25.689(15)	10^{-6}	
shielded proton moment (H_2O, sph., 25 °C)	μ_{p}'	1.410 571 38(47)	10^{-26} $\mathrm{J\,T^{-1}}$	0.34
in Bohr magnetons	$\mu_{\mathrm{p}}'/\mu_{\mathrm{B}}$	1.520 993 129(17)	10^{-3}	0.011
in nuclear magnetons	$\mu_{\mathrm{p}}'/\mu_{\mathrm{N}}$	2.792 775 642(64)		0.023
proton gyromagnetic ratio	γ_{p}	26 752.212 8(81)	10^{4} $\mathrm{s^{-1}\,T^{-1}}$	0.30
	$\gamma_{\mathrm{p}}/2\pi$	42.577 469(13)	$\mathrm{MHz\,T^{-1}}$	0.30
uncorrected (H_2O, sph., 25 °C)	γ_{p}'	26 751.525 5(81)	10^{4} $\mathrm{s^{-1}\,T^{-1}}$	0.30
	$\gamma_{\mathrm{p}}'/2\pi$	42.576 375(13)	$\mathrm{MHz\,T^{-1}}$	0.30

TABLE VII. (*Continued*).

Quantity	Symbol	Value	Units	Relative uncertainty (ppm)
		Neutron		
neutron mass	m_n	1.674 928 6(10)	10^{-27} kg	0.59
		1.008 664 904(14)	u	0.014
in electron volts, $m_nc^2/\{e\}$		939.565 63(28)	MeV	0.30
neutron-electron mass ratio	m_n/m_e	1 838.683 662(40)		0.022
neutron-proton mass ratio	m_n/m_p	1.001 378 404(9)		0.009
neutron molar mass	$M(\mathrm{n}),M_n$	1.008 664 904(14)	10^{-3} kg/mol	0.014
neutron Compton wavelength, h/m_nc	$\lambda_{C,n}$	1.319 591 10(12)	10^{-15} m	0.089
$\lambda_{C,n}/2\pi$	$\bar{\lambda}_{C,n}$	2.100 194 45(19)	10^{-16} m	0.089
neutron magnetic moment[a]	μ_n	0.966 237 07(40)	10^{-26} J T^{-1}	0.41
in Bohr magnetons	μ_n/μ_B	1.041 875 63(25)	10^{-3}	0.24
in nuclear magnetons	μ_n/μ_N	1.913 042 75(45)		0.24
neutron-electron magnetic moment ratio	μ_n/μ_e	1.040 668 82(25)	10^{-3}	0.24
neutron-proton magnetic moment ratio	μ_n/μ_p	0.684 979 34(16)		0.24
		Deuteron		
deuteron mass	m_d	3.343 586 0(20)	10^{-27} kg	0.59
		2.013 553 214(24)	u	0.012
in electron volts, $m_dc^2/\{e\}$		1 875.613 39(57)	MeV	0.30
deuteron-electron mass ratio	m_d/m_e	3 670.483 014(75)		0.020
deuteron-proton mass ratio	m_d/m_p	1.999 007 496(6)		0.003
deuteron molar mass	$M(\mathrm{d}),M_d$	2.013 553 214(24)	10^{-3} kg/mol	0.012
deuteron magnetic moment[a]	μ_d	0.433 073 75(15)	10^{-26} J T^{-1}	0.34
in Bohr magnetons,	μ_d/μ_B	0.466 975 447 9(91)	10^{-3}	0.019
in nuclear magnetons,	μ_d/μ_N	0.857 438 230(24)		0.028
deuteron-electron magnetic moment ratio	μ_d/μ_e	0.466 434 546 0(91)	10^{-3}	0.019
deuteron-proton magnetic moment ratio	μ_d/μ_p	0.307 012 203 5(51)		0.017
		PHYSICO-CHEMICAL CONSTANTS		
Avogadro constant	N_A, L	6.022 136 7(36)	10^{23} mol^{-1}	0.59
atomic mass constant $m_u=\frac{1}{12}m(^{12}\mathrm{C})$	m_u	1.660 540 2(10)	10^{-27} kg	0.59
in electron volts, $m_uc^2/\{e\}$		931.494 32(28)	MeV	0.30
Faraday constant, N_Ae	F	96 485.309(29)	C mol^{-1}	0.30
molar Planck constant	N_Ah	3.990 313 23(36)	10^{-10} J s mol^{-1}	0.089
	N_Ahc	0.119 626 58(11)	J m mol^{-1}	0.089
molar gas constant	R	8.314 510(70)	J mol^{-1} K^{-1}	8.4
Boltzmann constant, R/N_A	k	1.380 658(12)	10^{-23} J K^{-1}	8.5
in electron volts, $k/\{e\}$		8.617 385(73)	10^{-5} eV K^{-1}	8.4
in hertz, k/h		2.083 674(18)	10^{10} Hz K^{-1}	8.4
in wavenumbers, k/hc		69.503 87(59)	m^{-1} K^{-1}	8.4
molar volume (ideal gas), RT/p $T=273.15$ K, $p=101\,325$ Pa	V_m	0.022 414 10(19)	m^3 mol^{-1}	8.4
Loschmidt constant, N_A/V_m	n_0	2.686 763(23)	10^{25} m^{-3}	8.5
$T=273.15$ K, $p=100$ kPa	V_m	0.022 711 08(19)	m^3 mol^{-1}	8.4
Sackur-Tetrode constant (absolute entropy constant),[b] $\frac{5}{2}+\ln[(2\pi m_ukT_1/h^2)^{3/2}kT_1/p_0]$ $T_1=1$ K, $p_0=100$ kPa	S_0/R	$-1.151\,693(21)$		18
$p=101\,325$ Pa		$-1.164\,856(21)$		18
Stefan-Boltzmann constant, $(\pi^2/60)k^4/\hbar^3c^2$	σ	5.670 51(19)	10^{-8} W m^{-2} K^{-4}	34
first radiation constant, $2\pi hc^2$	c_1	3.741 774 9(22)	10^{-16} W m^2	0.60

TABLE VII. (*Continued*).

Quantity	Symbol	Value	Units	Relative uncertainty (ppm)
		PHYSICO-CHEMICAL CONSTANTS		
second radiation constant, hc/k	c_2	0.014 387 69(12)	m K	8.4
Wien displacement law constant, $b=\lambda_{\max}T=c_2/4.965\,114\,23\ldots$	b	2.897 756(24)	10^{-3} m K	8.4

[a]The scalar magnitude of the neutron moment is listed here. The neutron magnetic dipole is directed oppositely to that of the proton, and corresponds to the dipole associated with a spinning negative charge distribution. The vector sum, $\boldsymbol{\mu}_d=\boldsymbol{\mu}_p+\boldsymbol{\mu}_n$, is approximately satisfied.
[b]The entropy of an ideal monatomic gas of relative atomic weight A_r is given by

$$S=S_0+\tfrac{3}{2}R\ln A_r-R\ln(p/p_0)+\tfrac{5}{2}R\ln(T/\mathrm{K})\ .$$

trix. An extended variance matrix, containing the variances, covariances, and correlation coefficients of the output variables, is given in Table X. The information in this variance matrix is redundant, inasmuch as its rank is 5. The variable d_{220} is omitted from this table because there is little need for the correlations of this output value with other data. Since the basically more significant quantity N_A, the Avogadro constant, does appear in the table, there is no loss of information by omitting d_{220}.

To use Table X, note that the covariance between two quantities Q_k and Q_s which are functions of a common set of variables $x_i (i=1,\ldots,N)$ is given by

$$v_{ks}=\sum_{i,j=1}^{N}\frac{\partial Q_k}{\partial x_i}\frac{\partial Q_s}{\partial x_j}v_{ij}\ , \tag{4.1}$$

where v_{ij} is the covariance of x_i and x_j. In this general

TABLE VIII. Maintained units and standard values. A summary of "maintained" units and "standard" values and their relationship to SI units, based on a least-squares adjustment with 17 degrees of freedom. The digits in parentheses are the one-standard-deviation uncertainty in the last digits of the given value. Since the uncertainties of many of these entries are correlated, the full covariance matrix must be used in evaluating the uncertainties of quantities computed from them.

Quantity	Symbol	Value	Units	Relative uncertainty (ppm)
electron volt, $(e/\mathrm{C})\mathrm{J}=\{e\}\mathrm{J}$	eV	1.602 177 33(49)	10^{-19} J	0.30
(unified) atomic mass unit, $1\ \mathrm{u}=m_u=\frac{1}{12}m(^{12}\mathrm{C})$	u	1.660 540 2(10)	10^{-27} kg	0.59
standard atmosphere	atm	101 325	Pa	(exact)
standard acceleration of gravity	g_n	9.806 65	m s^{-2}	(exact)
		"As-maintained" electrical units		
BIPM maintained ohm, $\Omega_{69-\mathrm{BI}}$, $\Omega_{\mathrm{BI85}}\equiv\Omega_{69-\mathrm{BI}}$(January 1, 1985)	Ω_{BI85}	$1-1.563(50)\times10^{-6}=0.999\,998\,437(50)$	Ω	0.050
Drift rate of $\Omega_{69-\mathrm{BI}}$	$\dfrac{d\Omega_{69-\mathrm{BI}}}{dt}$	$-0.056\,6(15)$	μΩ/a	
BIPM maintained volt, $\mathrm{V}_{76-\mathrm{BI}}=483\,594.0$ GHz$(h/2e)$	$\mathrm{V}_{76-\mathrm{BI}}$	$1-7.59(30)\times10^{-6}=0.999\,992\,41(30)$	V	0.30
BIPM maintained ampere, $\mathrm{A}_{\mathrm{BIPM}}=\mathrm{V}_{76-\mathrm{BI}}/\Omega_{69-\mathrm{BI}}$	$\mathrm{A}_{\mathrm{BI85}}$	$1-6.03(30)\times10^{-6}=0.999\,993\,97(30)$	A	0.30
		X-ray standards		
Cu x unit: $\lambda(\mathrm{CuK}\alpha_1)\equiv1537.400$ xu	xu(CuKα_1)	1.002 077 89(70)	10^{-13} m	0.70
Mo x unit: $\lambda(\mathrm{MoK}\alpha_1)\equiv707.831$ xu	xu(MoKα_1)	1.002 099 38(45)	10^{-13} m	0.45
Å*: $\lambda(\mathrm{WK}\alpha_1)\equiv0.209\,100$ Å*	Å*	1.000 014 81(92)	10^{-10} m	0.92
lattice spacing of Si (in vacuum, 22.5 °C),[a]	a	0.543 101 96(11)	nm	0.21
$d_{220}=a/\sqrt{8}$	d_{220}	0.192 015 540(40)	nm	0.21
molar volume of Si, $M(\mathrm{Si})/\rho(\mathrm{Si})=N_A a^3/8$	V_m(Si)	12.058 817 9(89)	cm^3/mol	0.74

[a]The lattice spacing of single-crystal Si can vary by parts in 10^7 depending on the preparation process. Measurements at PTB indicate also the possibility of distortions from exact cubic symmetry of the order of 0.2 ppm.

TABLE IX. Energy conversion factors. To use this table note that all entries on the same line are equal; the unit at the top of a column applies to all of the values beneath it. Example: 1 eV = 806 544.10 m^{-1} = 11 604.45 K.

	J	kg	m^{-1}	Hz
1 J=	1	$1/\{c^2\}$ $1.112\,650\,06\times10^{-17}$	$1/\{hc\}$ $5.034\,112\,5(30)\times10^{24}$	$1/\{h\}$ $1.509\,188\,97(90)\times10^{33}$
1 kg=	$\{c^2\}$ $8.987\,551\,787\times10^{16}$	1	$\{c/h\}$ $4.524\,434\,7(27)\times10^{41}$	$\{c^2/h\}$ $1.356\,391\,40(81)\times10^{50}$
1 m^{-1}=	$\{hc\}$ $1.986\,447\,5(12)\times10^{-25}$	$\{h/c\}$ $2.210\,220\,9(13)\times10^{-42}$	1	$\{c\}$ 299 792 458
1 Hz=	$\{h\}$ $6.626\,075\,5(40)\times10^{-34}$	$\{h/c^2\}$ $7.372\,503\,2(44)\times10^{-51}$	$1/\{c\}$ $3.335\,640\,952\times10^{-9}$	1
1 K=	$\{k\}$ $1.380\,658(12)\times10^{-23}$	$\{k/c^2\}$ $1.536\,189(13)\times10^{-40}$	$\{k/hc\}$ 69.503 87(59)	$\{k/h\}$ $2.083\,674(18)\times10^{10}$
1 eV=	$\{e\}$ $1.602\,177\,33(49)\times10^{-19}$	$\{e/c^2\}$ $1.782\,662\,70(54)\times10^{-36}$	$\{e/hc\}$ 806 554.10(24)	$\{e/h\}$ $2.417\,988\,36(72)\times10^{14}$
1 u=	$\{m_uc^2\}$ $1.492\,419\,09(88)\times10^{-10}$	$\{m_u\}$ $1.660\,540\,2(10)\times10^{-27}$	$\{m_uc/h\}$ $7.513\,005\,63(67)\times10^{14}$	$\{m_uc^2/h\}$ $2.252\,342\,42(20)\times10^{23}$
1 hartree=	$\{2R_\infty hc\}$ $4.359\,748\,2(26)\times10^{-18}$	$\{2R_\infty h/c\}$ $4.850\,874\,1(29)\times10^{-35}$	$\{2R_\infty\}$ 21 947 463.067(26)	$\{2R_\infty c\}$ $6.579\,683\,899\,9(78)\times10^{15}$

form, the units of v_{ij} are the product of the units of x_i and x_j and the units of v_{ks} are the product of the units of Q_k and Q_s. For most cases involving the fundamental constants, the variables x_i may be taken to be the fractional change in the physical quantity from some fiducial value, and the quantities Q can be expressed as powers of physical constants Z_j according to

$$Q_k=q_k\prod_{j=1}^{N}Z_j^{Y_{kj}}=q_k\prod_{j=1}^{N}Z_{0j}^{Y_{kj}}(1+x_j)^{Y_{kj}}\ , \tag{4.2}$$

where q_k is a constant. If the variances and covariances are then expressed in relative units Eq. (4.1) becomes

$$v_{ks}=\sum_{i,j=1}^{N}Y_{ki}Y_{sj}v_{ij}\ , \tag{4.3}$$

where the v_{ij} are to be expressed, for example, in (parts in $10^9)^2$. Equation (4.3) is the basis for the expansion of the variance matrix to include e, h, m_e, N_A, and F. In terms of correlation coefficients r_{ij} defined by $v_{ij}=r_{ij}(v_{ii}v_{jj})^{1/2}=r_{ij}\varepsilon_i\varepsilon_j$, where ε_i is the standard deviation ($\varepsilon_i^2=v_{ii}$), we may write

$$\varepsilon_k^2=\sum_{i=1}^{N}Y_{ki}^2\varepsilon_i^2+2\sum_{j<i}^{N}Y_{ki}Y_{kj}r_{ij}\varepsilon_i\varepsilon_j\ . \tag{4.4}$$

As an example of the use of Table X, consider the calculation of the uncertainty of the Bohr magneton $\mu_B=eh/4\pi m_e$. In terms of the variables of the 1986 adjustment this ratio is given by

$$\mu_B=(2\pi\mu_0R_\infty E)^{-1}\cdot(\alpha^{-1})^{-3}K_V\ , \tag{4.5}$$

where the quantities in the parentheses to the left of the centered dot are taken to be exact. Using Eq. (4.3) with $i=1$ corresponding to α^{-1} and $i=2$ corresponding to K_V, and dropping the subscript k because there is only a single quantity, $Q=\mu_B$, gives

$$\varepsilon^2=Y_1^2v_{11}+2Y_1Y_2v_{12}+Y_2^2v_{22}\ , \tag{4.6}$$

where $Y_1=-3$ and $Y_2=1$. Thus taking the appropriate entries from Table X leads to

$$\varepsilon^2=[9(1997)-6(-1062)+87\,988]\times(10^{-9})^2 \tag{4.7}$$

or $\varepsilon=0.335$ ppm. Alternatively, one may evaluate eh/m_e directly from Table X, using $i=5$ corresponding to e, $i=6$ to h, and $i=7$ to m_e with $Y_5=1$, $Y_6=1$, and $Y_7=-1$. Then

$$\varepsilon^2=Y_5^2v_{55}+2Y_5Y_6v_{56}+2Y_5Y_7v_{57}+Y_6^2v_{66}+2Y_6Y_7v_{67}+Y_7^2v_{77} \tag{4.8a}$$

$$=[92\,109+2(181\,159)-2(175\,042)+358\,197-2(349\,956)+349\,702]\times(10^{-9})^2 \tag{4.8b}$$

which also yields $\varepsilon=0.335$ ppm.

The 1986 analysis does not make a distinction between the full data set and a data set from which all input quantities depending on QED theory are deleted. A major objective of the 1969 adjustment (and to a lesser extent, of the 1973 adjustment also) was the examination of the difference between the WQED (without QED) results and the results of an adjustment containing all of the not-otherwise-deleted data in order to test the validity of QED theory. If the data dependent on QED information in the present adjustment, item (10.1), the electron magnetic moment anomaly and item (12.1), the muonium

TABLE IX. (*Continued*).

K	eV	u	hartree
$1/\{k\}$ 7.242 924(61)$\times 10^{22}$	$1/\{e\}$ 6.241 506 4(19)$\times 10^{18}$	$1/\{m_{\mathrm{u}}c^2\}$ 6.700 530 8(40)$\times 10^{9}$	$1/\{2R_\infty hc\}$ 2.293 710 4(14)$\times 10^{17}$
$\{c^2/k\}$ 6.509 616(55)$\times 10^{39}$	$\{c^2/e\}$ 5.609 586 2(17)$\times 10^{35}$	$1/\{m_{\mathrm{u}}\}$ 6.022 136 7(36)$\times 10^{26}$	$\{c/2R_\infty h\}$ 2.061 484 1(12)$\times 10^{34}$
$\{hc/k\}$ 0.014 387 69(12)	$\{hc/e\}$ 1.239 842 44(37)$\times 10^{-6}$	$\{h/m_{\mathrm{u}}c\}$ 1.331 025 22(12)$\times 10^{-15}$	$1/\{2R_\infty\}$ 4.556 335 267 2(54)$\times 10^{-8}$
$\{h/k\}$ 4.799 216(41)$\times 10^{-11}$	$\{h/e\}$ 4.135 669 2(12)$\times 10^{-15}$	$\{h/m_{\mathrm{u}}c^2\}$ 4.439 822 24(40)$\times 10^{-24}$	$1/\{2R_\infty c\}$ 1.519 829 850 8(18)$\times 10^{-16}$
1	$\{k/e\}$ 8.617 385(73)$\times 10^{-5}$	$\{k/m_{\mathrm{u}}c^2\}$ 9.251 140(78)$\times 10^{-14}$	$\{k/2R_\infty hc\}$ 3.166 829(27)$\times 10^{-6}$
$\{e/k\}$ 11 604.45(10)	1	$\{e/m_{\mathrm{u}}c^2\}$ 1.073 543 85(33)$\times 10^{-9}$	$\{e/2R_\infty hc\}$ 0.036 749 309(11)
$\{m_{\mathrm{u}}c^2/k\}$ 1.080 947 8(91)$\times 10^{13}$	$\{m_{\mathrm{u}}c^2/e\}$ 931.494 32(28)$\times 10^{6}$	1	$\{m_{\mathrm{u}}c/2R_\infty h\}$ 3.423 177 25(31)$\times 10^{7}$
$\{2R_\infty hc/k\}$ 3.157 733(27)$\times 10^{5}$	$\{2R_\infty hc/e\}$ 27.211 396 1(81)	$\{2R_\infty h/m_{\mathrm{u}}c\}$ 2.921 262 69(26)$\times 10^{-8}$	1

hyperfine-splitting interval, are deleted, the remaining 20 items (15 degrees of freedom) have $\chi^2=16.53$ for standard least squares, which is reduced to 15.24 by algorithm ELS2. The distance of the ELS2 solution from the recommended set is $d=0.29$. The WQED value of α^{-1} is 137.035 984 6(94). This differs from the recommended value by (-0.036 ± 0.059) ppm. The WQED value of K_{V} differs from the recommended value by (0.01 ± 1.03) ppm. There is no clear basis for any distinction between QED and WQED data.

The validity of the theory of the quantum Hall effect (QHE) can also be investigated. If the QHE data are deleted from the recommended set, the solution is displaced by a distance $d=0.038$ and the value of α^{-1} is 137.035 998 4(79); the difference from the recommended value is (-0.009 ± 0.071) ppm. A value of α^{-1} from the Hall resistance data and the ohm determinations, 137.035 994 3(127), was mentioned in Sec. III.A. This differs by only (0.043 ± 0.085) ppm from the value above, where the uncertainty is evaluated taking into account the correlations from the direct ohm (Type 1) determinations. Thus, based on the presently available observational data, there is no evidence of any discrepancy in the QHE theory at current levels of precision.

B. Comparison with the 1973 adjustment

There are significant changes in the recommended values of the physical constants from 1973 to 1986. The auxiliary constants $m_{\mathrm{p}}/m_{\mathrm{e}}$ and R_∞ show improvements by factors of 19 and 63, respectively. The uncertainties of the 1986 recommended values are typically a factor of 10 smaller than their 1973 counterparts and the uncertainty of α has decreased by a factor of 18. It will be interesting to see if this rate of improvement can be sustained for another thirteen years; if so, the turn of the 21st century should be an exciting period for metrology.

The distance d of the 1973 adjustment from the present adjustment, using Eq. (3.7a) and the 1973 variance matrix

TABLE X. Expanded matrix of variances, covariances, and correlation coefficients for the 1986 recommended set of fundamental physical constants. The elements of the variance matrix appear on and above the major diagonal in (parts in $10^9)^2$; correlation coefficients appear in *italics* below the diagonal. The values are given to as many as six digits only as a matter of consistency. The correlation coefficient between m_{e} and N_{A} appears as -1.000 in this table because the auxiliary constants were considered to be exact in carrying out the least-squares adjustment. When the uncertainties of $m_{\mathrm{p}}/m_{\mathrm{e}}$ and M_{p} are properly taken into account, the correlation coefficient is -0.999 and the variances of m_{e} and N_{A} are slightly increased.

	α^{-1}	K_{V}	K_Ω	μ_μ/μ_{p}	e	h	m_{e}	N_{A}	F
α^{-1}	1 997	−1 062	925	3 267	−3 059	−4 121	−127	127	−2 932
K_{V}	*−0.080*	87 988	90	−1 737	89 050	177 038	174 914	−174 914	−85 864
K_Ω	*0.416*	*0.006*	2 477	1 513	−835	−744	1 105	−1 105	−1 939
μ_μ/μ_{p}	*0.498*	*−0.040*	*0.207*	21 523	−5 004	−6 742	−208	208	−4 796
e	*−0.226*	*0.989*	*−0.055*	*−0.112*	92 109	181 159	175 042	−175 042	−82 933
h	*−0.154*	*0.997*	*−0.025*	*−0.077*	*0.997*	358 197	349 956	−349 956	−168 797
m_{e}	*−0.005*	*0.997*	*0.038*	*−0.002*	*0.975*	*0.989*	349 702	−349 702	−174 660
N_{A}	*0.005*	*−0.997*	*−0.038*	*0.002*	*−0.975*	*−0.989*	*−1.000*	349 702	174 660
F	*−0.217*	*−0.956*	*−0.129*	*−0.108*	*−0.902*	*−0.931*	*−0.975*	*0.975*	91 727

(Cohen and Taylor, 1973), is $d=3.41$. Since d^2 is a statistic with a χ^2 distribution, the probability that this is a chance occurrence is $P_{\chi^2}(11.62 \mid 5)=0.04$. The most significant revision in the recommended values of the fundamental constants is the change in K_V, and the increase of 7.75 ppm that it implies for the recommended value of $2e/h$. The 1973 value is lower than the 1986 value by 3.0 standard deviations. A related change appears in N_A (the 1973 correlation coefficient between e/h and N_A is 0.985, the 1986 coefficient is 0.997); the 1973 value is (15.2 ± 5.1) ppm lower than the new recommendation, also a change of three standard deviations.

Since the fine-structure constant α, which is proportional to $e(e/h)$, has changed only by $+0.37$ ppm, the increase in $2e/h$ is strongly correlated to a decrease in e. If e/h increases and e decreases, then h must decrease twice as much. Furthermore, the quantity $N_A h$ is proportional to the Compton wavelength and hence to α^2/R_∞; a decrease in h is coupled with an increase in N_A and with an increase (approximately half as large) in F. The changes from the 1973 values of many of the entries in Table VII are thus strongly correlated and all of the large changes can be directly linked to the change in K_V. This is seen quantitatively in Table X; the magnitudes of the correlation coefficients between the variables K_V, e, h, m_e, and N_A are all greater than 0.975, and the correlation coefficients between these variables and F are all greater than 0.90.

The source of a major part of the difference between 1973 and 1986 is the deletion, in 1973, of two Faraday determinations which seemed to be discrepant with the remaining data (Cohen and Taylor, 1973); in hindsight this "discrepancy" was not that severe. In fact, adjustment No. 40 in that analysis, which differs from the 1973 recommended set (No. 41) only in its retention of the two Faraday determinations, gives a value of $2e/h$ that is 5.3 ppm higher than the 1973 recommendation and (2.5 ± 2.0) ppm lower than the present value. The normalized residuals of the two Faraday measurements in adjustment No. 40 were 1.76 and 1.50.

In view of this experience it is important to point out that there are no similar data discrepancies in the present analysis; the deleted data have been either extremely discrepant or of very low weight. It should not be forgotten that even data set (b) of Table V has d less than 1 for all of the adjustments except the standard least squares with *a priori* (internal) error assignments, and that the level of precision is uniformly no more than a factor of 2 poorer than the recommended values. Thus, it is improbable that any future reassessment of the current data could change the recommendations of the present analysis by as much as two standard deviations.

ACKNOWLEDGMENTS

We gratefully acknowledge helpful conversations and correspondence with a host of colleagues, many of whom provided us with information and results prior to publication. One of us (B.N.T.) warmly thanks (Mrs.) Sheila A. Taylor for writing and executing the many computer programs he used in the course of this work, and T. E. Keiss for assistance with some of the numerical computations.

REFERENCES

Amin, S. R., C. D. Caldwell, and W. Lichten, 1981, Phys. Rev. Lett. **47**, 1234.

Amin, S. R., C. D. Caldwell, and W. Lichten, 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 117.

Bailey, J., *et al.*, 1979, Nucl. Phys. **B 150**, 1.

Barnes, I. L., *et al.*, 1975, J. Res. Natl. Bur. Stand. **79A**, 727.

Becker, P., *et al.*, 1981, Phys. Rev. Lett. **46**, 1540.

Becker, P., P. Seyfried, and H. Siegert, 1982, Z. Phys. B **48**, 17.

Becker, P., and H. Siegert, 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 317.

Bender, D., and W. Schlesok, 1974, Metrologia **10**, 1.

deBièvre, P., M. Gallet, N. E. Holden, and I. L. Barnes, 1984, J. Phys. Chem. Ref. Data **13**, 809.

BIPM, 1981, Procès Verbaux des séances CIPM **49**, 70e session, 1981 (BIPM, Sèvres, France).

BIPM, 1983, Comptes Rendus des séances de la 17e CGPM, 1983 (BIPM, Sèvres, France).

Blevin, W. R., and W. J. Brown, 1971, Metrologia **7**, 15.

Bliek, L., *et al.*, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 304; and private communication.

Bodwin, G. T., D. R. Yennie, and M. A. Gregorio, 1985, Rev. Mod. Phys. **57**, 723.

Bower, V. E., and R. S. Davis, 1980, J. Res. Natl. Bur. Stand. **85**, 175.

Bowman, H. A., R. M. Schoonover, and C. L. Carroll, 1974a, J. Res. Natl. Bur. Stand. **78A**, 13.

Bowman, H. A., R. M. Schoonover, and C. L. Carroll, 1974b, Metrologia **10**, 117.

Bowman, H. A., R. M. Schoonover, and C. L. Carroll, 1975, "Reevaluation of the densities of the four NBS silicon crystal standards," Natl. Bur. Stand. Report No. NBSIR 75-768.

Brown, L. S., G. Gabrielse, K. Helmerson, and J. Tan, 1985, Phys. Rev. A **32**, 3204.

Cage, M. E., R. F. Dziuba, and B. F. Field, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 301; and private communication.

Chiao, W. C., R. M. Liu, and P. Shen, 1980, IEEE Trans. Instrum. Meas. **IM-29**, 238.

Close, F. E., and H. Osborne, 1971, Phys. Lett. **34B**, 400.

Clothier, W. K., 1965, Metrologia **1**, 181.

CODATA, 1973, "Recommended consistent values of the fundamental physical constants," CODATA Bulletin No. 11 (International Council of Scientific Unions—Committee on Data for Science and Technology, 51 Blvd de Montmorency, 75016 Paris, France, Dec. 1973).

Cohen, E. R., 1953, Rev. Mod. Phys. **25**, 706.

Cohen, E. R., 1976, "'Extended' least squares," Rockwell International Science Center Report No. SCTR-76-1.

Cohen, E. R., 1978, "An extended least-squares algorithm for treating inconsistent data," Rockwell International Science Center Report No. SCTR-78-11.

Cohen, E. R., 1980, in *Metrology and Fundamental Constants,* proceedings of the International School of Physics Enrico Fermi, Course LXVIII, edited by A. Ferro-Milone, P. Giacomo, and S. Leschiutta (North-Holland, Amsterdam), p. 581.
Cohen, E. R., 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 391.
Cohen, E. R., K. M. Crowe, and J. W. M. DuMond, 1957, *Fundamental Constants of Physics* (Interscience, New York).
Cohen, E. R., and B. N. Taylor, 1973, J. Phys. Chem. Ref. Data **2**, 663.
Colclough, A. R., 1979, Acustica **42**, 28.
Colclough, A. R., 1984a, private communication.
Colclough, A. R., 1984b in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 263.
Colclough, A. R., T. J. Quinn, and T. R. D. Chandler, 1979, Proc. R. Soc. London Ser. A **368**, 125.
Cutkosky, R. D., 1974, IEEE Trans. Instrum. Meas. **IM-23**, 305.
Dahake, S. L., *et al.*, 1983, IEEE Trans. Instrum. Meas. **IM-32**, 5; and private communication.
Delahaye, F., *et al.*, 1986, Metrologia **22**, 103.
Deslattes, R. D., 1980a, in *Metrology and Fundamental Constants,* proceedings of the International School of Physics Enrico Fermi, Course LXVIII, edited by A. Ferro-Milone, P. Giacomo, and S. Leschiutta (North-Holland, Amsterdam), p. 38.
Deslattes. R. D., 1980b, Ann. Rev. Phys. Chem. **31**, 435.
Deslattes, R. D., 1985, private communication.
Deslattes, R. D., *et al.*, 1974, Phys. Rev. Lett. **33**, 463.
Deslattes, R. D., *et al.*, 1976, Phys. Rev. Lett. **36**, 898.
Deslattes, R. D., and A. Henins, 1973, Phys. Rev. Lett. **31**, 972.
Deslattes, R. D., and A. Henins, 1984, private communication.
Driscoll, R. L., and R. D. Cutkosky, 1958, J. Res. Natl. Bur. Stand. **60**, 297.
Driscoll, R. L., and P. T. Olsen, 1968, Document CCE/68-7, submitted to CCE, CIPM, 12[e] session (October 1968).
van Dyck, R. S., Jr., 1985, private communication.
van Dyck, R. S., Jr., F. L. Moore, D. L. Farnum, and P. B. Schwinberg, 1986, Bull. Am. Phys. Soc. **31**, 244; and private communication.
van Dyck, R. S., Jr., P. B. Schwinberg, and H. G. Dehmelt, 1984, in *Atomic Physics 9,* edited by R. S. van Dyck, Jr. and E. N. Fortson (World Scientific, Singapore), p. 53.
Elnékavé, N., and A. Fau, 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 465; Document CCE/83-2, submitted to CCE, CIPM, 16[e] session, March, 1983; and private communication.
Erickson, G. W., 1977, J. Phys. Chem. Ref. Data **6**, 831.
Erickson, G. W., 1983, private communication.
Facy, L., and C. Pontikis, 1970, C. R. Acad. Sci. Ser. B **270**, 15.
Facy, L., and C. Pontikis, 1971, C. R. Acad. Sci. Ser. B **272**, 1397.
Faller, J. E., 1967, Science **158**, 60.
Faustov, R., 1970, Phys. Lett. B **33**, 422.
Frieze, W., E. A. Hinds, V. W. Hughes, and F. M. J. Pichanick, 1981, Phys. Rev. A **24**, 279.
Giacomo, P., 1981, Metrologia **17**, 69.
Goldsmith, J. E., E. W. Weber, and T. W. Hänsch, 1979, Phys. Rev. Lett. **41**, 1525.
Gorbatsevich, S. V., 1973, private communication.
Gräff, G., H. Kalinowsky, and J. Traut, 1980, Z. Phys. A **297**, 35.
Greene, G. L., *et al.*, 1979, Phys. Rev. D **20**, 2139.
Grotch, H., and R. Hegstrom, 1971, Phys. Rev. A **4**, 59.
Hammond, J. A., and J. E. Faller, 1967, IEEE J. Quantum Electron. **QE-3**, 597.
Hänsch, T. W., *et al.*, 1974, Phys. Rev. Lett. **32**, 1336.
Hara, K., H. Nakamura, T. Sakai, and N. Koizumi, 1968, unpublished.
Hartland, A., G. J. Davies, and D. R. Wood, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 309.
Heyl, P. R., 1930, J. Res. Natl. Bur. Stand. **5**, 1243.
Heyl, P. R., and P. Chrzanowski, 1942, J. Res. Natl. Bur. Stand. **29**, 1.
Holden, N. E., and R. L. Martin, 1984, Pure Appl. Chem. **56**, 653.
Holden, N. E., R. L. Martin, and I. L. Barnes, 1984, Pure Appl. Chem. **56**, 675.
Hudson, R. P., 1984, Metrologia **19**, 163.
Igarashi, T., 1983, private communication.
Igarashi, T., 1984, private communication.
Igarashi, T., Y. Koizumi, and M. Kanno, 1968, IEEE Trans. Instrum. Meas. **IM-17**, 226.
Igarashi, T., H. Nishinaka, and K. Murakami, 1978, Document CCE/78-12, submitted to CCE, CIPM, 15[e] session.
Jones, R. G., and B. P. Kibble, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 181.
Karagioz, O. V., *et al.*, 1976, Izv. Acad. Nauk SSSR, Fiz. Zemli **12**, 106 [Izv. Acad. Sci. USSR, Phys. Solid Earth **12**, 351 (1976)].
Karagioz, O. V., A. A. Silin and V. P. Izmaylov, 1981, Izv. Acad. Nauk SSSR, Fiz. Zemli **17**, 92 [Izv. Acad. Sci. USSR, Phys. Solid Earth **17**, 66 (1976)].
Kessler, E. G., Jr., R. D. Deslattes, and A. Henins, 1979, Phys. Rev. A **19**, 215.
Kibble, B. P., and G. J. Hunt, 1979, Metrologia **15**, 5.
Kinoshita T., and W. B. Lindquist, 1981a, Phys. Rev. Lett. **47**, 1573.
Kinoshita T., and W. B. Lundquist, 1981b, "Eighth order magnetic moment of the electron, IV," and "V," Cornell University Reports Nos. CLNS-81/509 and CLNS-81/510 (Newman Laboratory of Nuclear Studies, 1981).
Kinoshita T., and W. B. Lindquist, 1983, Phys. Rev. D **27**, 867, 877, 886.
Klempt, E., *et al.*, 1982, Phys. Rev. D **25**, 652.
von Klitzing, K., 1986, Rev. Mod. Phys. **58**, 519.
von Klitzing, K., G. Dorda, and M. Pepper, 1980, Phys. Rev. Lett. **45**, 494.
Kponou, A., *et al.*, 1981, Phys. Rev. A **24**, 264.
Ku, H., 1983, private communication.
Lambe, E. B. D., 1959, "A measurement of the g-value of the electron in the ground state of the Hydrogen atom," Ph.D. thesis (Princeton University).
Lambe, E. B. D., 1968, in *Polarisation, Matière et Rayonnement* (Societé Française de Physique, Paris), p. 441.
Lampard, D. G., 1957, Proc. Inst. Elec. Eng. (London) Part C **104**, 2701.
Lewis, M. L., and P. H. Serafino, 1978, Phys. Rev. A **18**, 867.
Lewis, S. A., F. M. J. Pichanick, and V. W. Hughes, 1970, Phys. Rev. A **2**, 86.
Lichten, W., 1985, private communication.
Luther, G., and W. R. Towler, 1982, Phys. Rev. Lett. **48**, 121;

also in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C., 1984), p. 573.

Mamyrin, B. A., N. N. Aruyev, and S. A. Alekseenko, 1983, Zh. Eksp. Teor. Fiz. **84**, 1978 [Sov. Phys. —JETP **57**, 1152 (1983)].

Mariam, F. G., 1981, "High precision measurement of the muonium ground-state hyperfine interval and the muon magnetic moment," Ph.D. thesis (Yale University).

Mariam, F. G., *et al.*, 1982, Phys. Rev. Lett. **49**, 993.

Melton, C. E., W. Massey, and B. N. Abels, 1971, Z. Naturforsch. **26a**, 1241.

Morelli, C., *et al.*, 1974, "The International Gravity Standardization Net 1971 (IGSN71)," Publ. Spéc. No. 4 (Bur. central de l'assoc. intl. de géodesie, Paris).

Neronov, Yu. I., and A. E. Barzakh, 1977, Zh. Eksp. Teor. Fiz. **72**, 1695 [Sov. Phys.—JETP **45**, 871 (1977)].

Neronov, Yu. I., A. E. Barzakh, and Kh. Mukhamadiev, 1975, Zh. Eksp. Teor. Fiz. **69**, 1872 [Sov. Phys.—JETP **42**, 950 (1975)].

Nier, A. O., 1950, Phys. Rev. **77**, 789.

van der Pauw, L. J., 1958, Philips Res. Rep. **13**, 1.

Peiser, H. S., *et al.*, 1984, Pure Appl. Chem. **56**, 695.

Petley, B. W., and R. W. Donaldson, 1984, Metrologia **20**, 81.

Petley, B. W., and K. Morris, 1974, J. Phys. A **7**, 167.

Petley, B. W., and K. Morris, 1979, Nature (London) **279**, 141.

Petley, B. W., K. Morris, and R. E. Shawyer, 1980, J. Phys. B **13**, 3099.

Phillips, W. D., W. E. Cooke, and D. Kleppner, 1977, Metrologia **13**, 179.

Phillips, W. D., D. Kleppner, and F. G. Walther, 1984, private communication.

Pontikis, C., 1972, C. R. Acad. Sci. Ser. B **274**, 437.

Powell, L. J., T. J. Murphy, and J. W. Gramlich, 1982, J. Res. Natl. Bur. Stand. **87**, 9.

Quinn, T. J., A. R. Colclough, and T. R. D. Chandler, 1976, Philos. Trans. R. Soc. London Ser. A **283**, 367.

Quinn, T. J., and J. E. Martin, 1985, Philos. Trans. R. Soc. London Ser. A **316**, 85.

Rose, R. D., *et al.*, 1969, Phys. Rev. Lett. **23**, 655.

Sagitov, M. U., *et al.*, 1979, Dok. Acad. Nauk SSSR **245**, 567 [Sov. Phys.—Dokl. **245**, 20 (1981)].

Sapirstein, J. R., E. A. Terray, and D. R. Yennie, 1984, Phys. Rev. D **29**, 2290.

Schlesok W., and J. Forkert, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 173.

Seyfried, P., 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 313.

Siegert, H., and P. Becker, 1984, in *Precision Measurement and Fundmental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 321.

Siegert, H., P. Becker, and P. Seyfried, 1984, Z. Phys. B **56**, 273.

Sloggett, G. J., *et al.*, 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 469.

Sloggett, G. J., *et al.*, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 187; and private communication.

Studentsov, N. V., V. N. Khorev, and V. Ya. Shifrin, 1981, Izmeritel, Tekh. **1981**, (No. 6), 56 [Meas. Tech. **24**, 501 (1981)].

Tarbeyev, Yu. V. 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 483.

Taylor, B. N., 1982, "Numerical comparisons of several algorithms for treating inconsistent data in a least-squares adjustment of the fundamental constants," Natl. Bur. Stand. Report No. NBSIR 81-2426.

Taylor, B. N., 1985, J. Res. Natl. Bur. Stand. **90**, 91.

Taylor, B. N., W. H. Parker, and D. N. Langenberg, 1969, Rev. Mod. Phys. **41**, 375; also published as *The Fundamental Constants and Quantum Electrodynamics* (Academic, New York, 1969).

Thompson, A. M., 1968, Metrologia **4**, 1; Procès Verbaux des séances CIPM **51**, p. 57, 72^e session, 1983 (BIPM, Sèvres, France, 1983); *ibid.* **53**, p. 83, 74^e session, 1985.

Thompson, A. M., and D. G. Lampard, 1956, Nature (London) **177**, 888.

Towler, W. R., *et al.*, 1971, in *Precision Measurement and Fundamental Constants,* edited by D. N. Langenberg and B. N. Taylor (Natl. Bur. Stand. US Spec. Publ. 343) (US GPO, Washington, D.C.), p. 485.

Tuninskii, V. S., and S. V. Kholin, 1975 "Concerning changes in the methods for adjusting the physical constants," Mendeleev Research Institute of Metrology (VNIIM), Leningrad, internal report; Metrologiya **1975**, (No. 8), 3 (1975) (in Russian).

Vigoureux, P., 1965, Metrologia **1**, 3.

Vigoureux, P., and N. Dupuy, 1973, NPL Quantum Metrology Division of Electrical Science Report No. Qu-26.

Vigoureux, P., and N. Dupuy, 1980, NPL Division of Electrical Science Report No. DES-59.

Wada, T., K. Shida, H. Nishinaka, and T. Igarashi, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 306.

Wang, Z. X., 1984, in *Precision Measurement and Fundamental Constants II,* edited by B. N. Taylor and W. D. Phillips (Natl. Bur. Stand. US Spec. Publ. 617) (US GPO, Washington, D.C.), p. 505.

Wapstra, A. H., and G. Audi, 1985, Nucl. Phys. A **432**, 1.

van der Wel, W., K. J. P. M. Harmans, R. Kaarls, and J. E. Mooij, 1985, IEEE Trans. Instrum. Meas. **IM-34**, 314.

Williams, E. R., and P. T. Olsen, 1979, Phys. Rev. Lett. **42**, 1575.

Wineland, D. J., J. J. Bollinger, and W. M. Itano, 1983, Phys. Rev. Lett. **50**, 628; **50**, 1333(E).

Winkler, P. F., D. Kleppner, T. Myint, and F. G. Walther, 1972, Phys. Rev. A **5**, 83.

Wohl, C. G., *et al.*, 1984, Rev. Mod. Phys. **56**, S1.

Yagola, G. K., V. I. Zingerman, and V. N. Sepetyi, 1962, Izmeritel, Tekh. **1962**, (No. 5), 24 [Meas. Tech. **1962**, 387 (1962)].

Yagola, G. K., V. I. Zingerman, and V. N. Sepetyi, 1966, Izmeritel, Tekh. **1966**, (No. 7), 44 [Meas. Tech. **1966**, 914 (1966)].