Determination of the Gravitational Constant at an Effective Interaction Distance of 112 m

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The Newtonian gravitational constant has been determined at an effective interaction distance of 112 m. A high-precision balance was used to compare the weights of two 1-kg stainless steel masses located above and below the variable water level of a pumped-storage lake. Water-level changes up to 43 m produced a maximum weight difference of 1290 μg, which could be measured with a resolution of < 1 μg. The data yield a value for the gravitational constant G of $(6.6700 \pm 0.0054) \times 10^{-11}$ $\mathrm{m^3\,kg^{-1}\,s^{-2}}$ (1σ), in agreement with laboratory determinations. New limits for the strength of possible new intermediate-range forces are placed.

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Since 1986 [1] several experiments have been performed to search for deviations from Newtonian gravity. The deviations are interpreted in terms of a new fundamental interaction called "fifth" force. Composition-dependent violation of Newtonian gravity is well excluded by torsion-balance experiments [2]. This paper describes an experiment to search for a composition-independent violation of the inverse-square law in the range 10 cm to 100 m [2,3]. The potential energy describing the interaction between two masses, including a Yukawa-type term, may be written as

$$V(r) = -G_\infty \frac{m_1 m_2}{r}(1 + \alpha e^{-r/\lambda}),$$

where α is the strength of the new force relative to gravity and λ the Compton wavelength of the exchange particle. The resulting force is

$$F(r) = G(r)\frac{m_1 m_2}{r^2},$$
$$G(r) = G_\infty[1 + \alpha(1 + r/\lambda)e^{-r/\lambda}],$$

and the gravitational "constant" becomes distance dependent.

At geophysical distances (10 m to 10 km) experiments of different kinds have been carried out. In "Airy"-type experiments the gravity gradient towards the center of the Earth is measured by using gravimeters. High tower experiments [4] have put limits on the variation of G with distance, but provide no direct estimate of the gravitational constant. In mines [5], in boreholes [6], and in the ocean [7] the value of G could be determined with the density of the traversed material. These experiments mainly suffer from the insufficient knowledge of local topography and density anomalies in the Earth's crust. Additionally, the measurements are disturbed by moving the gravimeters between readings.

Most of these problems can be solved by using movable sources of known mass and fixed instruments. In "lake" experiments the gravity as a function of a variable water mass is measured by using gravimeters or balances. With gravimeters [8,9] problems result from calibration of instruments and from individual instrumental drift. With a single balance, also these problems can be removed. Moore *et al.* [10] employed an electrostatic beam balance with a separation of test masses of 12 m in a tower of the Splityard Creek reservoir in Australia. At an effective interaction distance of 22 m they obtained for G a deviation from the laboratory value of $(0.2 \pm 0.8)\%$. The uncertainty was mainly due to vibration of the supporting tower.

The experiment reported here belongs to the lake experiments using a balance. The experimental site is at the Gigerwald lake, a pumped-storage reservoir for peak-power production in eastern Switzerland (46°55′ N, 9°24′ E at 1335 m above sea level). At one side the 2.5-km-long lake is confined by a 147-m-high and 430-m-wide concrete dam of parabolic shape allowing maximum water-level changes of 90 m.

The basic idea of the Gigerwald experiment was to measure the weight difference of two test masses located above and below the variable water level with a single balance (see Fig. 1).

Since the weight difference is measured in a short time, balance drifts are negligible. Time-variable gravity effects originating from distances much larger than the separation of test masses completely vanish (e.g., tides). By

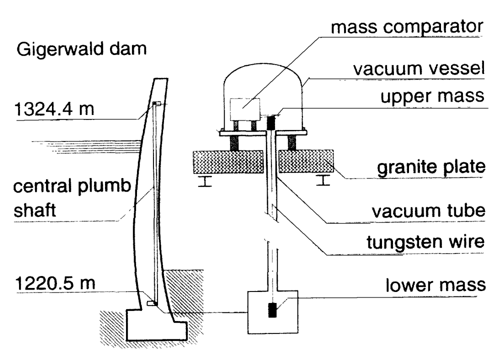

FIG. 1. Scheme of the Gigerwald experiment.

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comparing the weight differences at several water levels even the static local gravity from the surroundings cancels. Finally, the recorded gravity signal is just due to the interaction between the locally moved mass (water and air) and the test masses.

The high-precision balance, a modified Mettler-Toledo mass comparator, is mounted at the top of the central plumb shaft of the dam. It is a single-pan flexure-strip balance and works on the substitution principle. The two masses to be compared are attached one after the other below the pan suspended on one side of the beam. The weight of the attached 1-kg (upper or lower) mass is compensated by a fixed counterweight placed on the other side of the beam and, up to 2 g, by a servo-controlled current through a coil immersed in the flux of a permanent magnet. The applied current is calibrated with standard masses. The mass-to-force conversion is established by the local gravity at the balance which was found to be $9.804\,208\,1(3)\ \mathrm{m\,s^{-2}}$ [11].

The two test masses are made of polished stainless steel (316L). The upper mass of 1.114 45 kg is hanging just below the balance at 1324.359 m and the lower mass of 1.099 87 kg is suspended by a 100-μm diam tungsten wire of 15.72 g at the bottom of the shaft. Their center of mass separation is 103.822(2) m. To avoid systematic weighing errors they are suspended concentrically which required the upper mass to be a hollow cylinder, whereas the lower mass is of a cylindrical shape.

The suspension devices, crucial to the success of this experiment, are of 4.24 g and of 3.04 g for the upper and the lower mass, respectively. They consist of two chain-link-like saddle surfaces made of polished stainless steel coated with tungsten carbide. Because of its low friction coefficient, the displacement of the bearing point out of the centering position is $< 50\ \mu$m. This diminishes the transmission of torques to the balance. Additionally the hardness of tungsten carbide avoids surface deformation. Between the suspension devices and the balance double-crossed knife edges are placed to further reduce remaining torques. To avoid air convection and variable buoyancy the balance and the test masses are at 5×10^{-3} mbar. With a three-stage temperature control system the temperature of the balance is held constant with a precision of 0.3 mK over several weeks.

The average weight of each mass is taken over a time of 3 min. Afterwards the mass is detached from the balance lifting it by 1 mm, simultaneously the other mass is lowered and suspended on the balance. During this exchange the load on the balance is held constant within 1 g. This avoids relaxation effects in the flexure strips. The weight difference is then calculated by linear interpolation between two successive measurements of the upper mass at the time the lower mass was measured. Every 12 min the weight difference is determined. After 60 mass exchanges a calibration of the scale sensitivity is carried out by adding a small mass of 0.999 993 g to the balance. No significant change of scale sensitivity was observed.

Since the gravity effect of a infinite horizontal sheet of thickness h and density ρ is proportional to ρh and approximately remains true in our case of a finite lake, it is advantageous to determine the water level via the water pressure instead of via floats or similar methods. We used a high-precision pressure transducer (Rittmeyer W1Q) with a resolution of 1 mm and an absolute accuracy of 1 cm. The instrument automatically corrects for the ambient air pressure.

Figure 2 shows the digitally recorded raw data of water level and weight difference during a period of 120 days. Water-level changes up to 43 m produced a maximum weight signal of 1290 μg. The weight difference was measured with a short-time resolution of 0.5 μg. As long as water-level changes are well between the test masses, the weight difference varies almost linearly by 33 μg/m. Whenever water level approaches the height of the upper mass, the weight signal diminishes and becomes even negative for higher levels.

The shore and dam contour was determined by air photogrammetry and conventional surveying, respectively. Data processing yielded 2-m-equidistant contour lines. The dam shape is known to better than 1 cm. The shoreline coordinates have a random uncertainty of 30 cm. Their systematic uncertainty is much less than 5 cm as a result of a well surveyed control network. The coordinates of test masses are uncertain by less than 3 mm. Not the whole shore is bounded by rocks; there are also layers of scree, where water is seeping in. From geological surveys based on numerous drill holes the solid rock boundary is known. The porosity of the scree is estimated by geologists to be 0.30 ± 0.03, which enabled us to calculate the effect of water seepage.

Water density was measured in the laboratory to be slightly denser than pure water by $1.2(2) \times 10^{-4}\ \mathrm{g\,cm^{-3}}$. Temperature profile measurements in the lake revealed no significant change of temperature with depth except

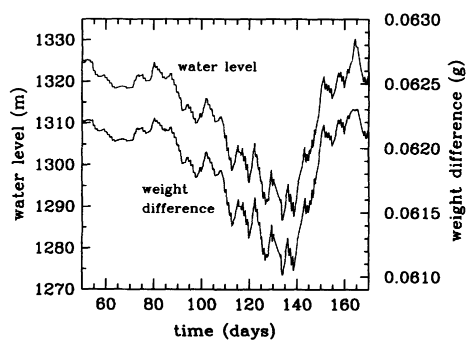

FIG. 2. Raw data of the water level and the weight difference from March to June 1993 (the local gravity at the balance is $9.804\,208\ \mathrm{m\,s^{-2}}$). The peaks are caused by daily, weekly, and seasonal water-level changes depending on power consumption.

for a thin boundary layer below the surface. During the measuring period water temperature varied from 3°C to 6°C. In this range water-density variations are negligible. The density which enters the calculation of the gravity effect is the density difference between water and air. The mean density of air is determined to be 0.001 08 $\mathrm{g\,cm^{-3}}$.

To calculate the Newtonian gravity signal the method of Talwani and Ewing [12] was used. They give an exact formula for the vertical gravity of a flat horizontal lamina whose contour is represented by an arbitrary n-sided polygon. The effect of the Earth's curvature is negligible. The effective interaction distance r_{eff} is

$$r_{\mathrm{eff}} = \left(\int r_1\, dF_1^z - \int r_2\, dF_2^z\right) \Big/ \left(\int dF_1^z - \int dF_2^z\right),$$

where the r_i's and the dF_i^z's are the distances, respectively, the vertical gravitational forces between a water element and the two test masses. The integration over the Gigerwald lake gives $r_{\mathrm{eff}} = (112 \pm 2)$ m depending on the water level.

The measured and calculated weight difference as a function of water level is shown in Fig. 3. The weight differences at same heights must be equal for all times. Any discrepancies are interpreted as change of mass distribution in the environment, e.g., change of soil moisture. Variable atmospheric pressure has an effect of $< 0.1\ \mu$g on the weight signal. Observed lake oscillations ("seiches") with amplitudes of 2 mm also have no effect. Dam movements originating from temperature and water pressure changes do not produce a significant uncertainty, since both lake water and test masses are moved the same way. Effects of balance tilts due to dam movements cancel by measuring the weight difference and by repeated scale calibration. Ground vibrations were not observed.

TABLE I. Error budget.

Source	Uncertainty (parts per 1000)
Dam contour	0.30
Shore contour	0.37
Porosity of scree	0.61
Test mass positions	0.04
Water level	0.12
Water density	0.05
Random experimental error	0.18
Total RSS (1σ)	0.81

In order to minimize the effect of soil moisture in the weight signal, the following model equation for the measured weight difference is taken:

$$\Delta F(t) = a \Delta F_{\mathrm{Newton}}(t) + b_i t + c_i.$$

The parameter a represents the ratio G/G_{lab} over the whole measuring period, where G_{lab} denotes the currently accepted laboratory value of the gravitational constant [13]. b_i and c_i are the coefficients of linear drifts in time periods of about 5 days. The drifts are $< 0.5\ \mu$g per day and vary in sign. The value for a determined from data is 0.999 61(18), which is the weighted mean of the 1992 measurement [1.000 17(76)] and 1993 measurement [0.999 58(19)]. For a consistency check the 1993 data were subdivided into two data sets of measurements with water level above and below 1305 m. The results are 0.999 29(26) and 0.999 81(33), respectively, in reasonable agreement with each other. The final uncertainty in this determination of G is obtained by taking the root-sum-square (RSS) of the uncertainties listed in Table I. The

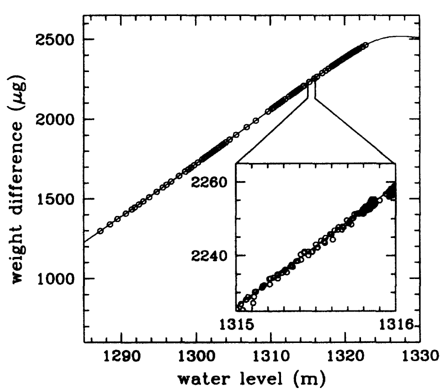

FIG. 3. The solid curve is the calculated weight difference of the two test masses as a function of the water level following pure Newtonian gravity (the origin is set at 1240 m for an empty lake). The measured gravity data (open circles), corrected only with a linear drift, fit well the predicted gravity effect. Each circle represents the average of 100 measurements. The inset shows a typical region where all gravity data are presented.

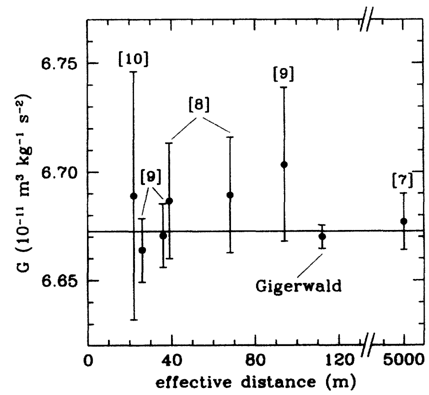

FIG. 4. Results of G experiments at geophysical distances as a function of the effective interaction distance. The quoted precision of each experiment is given as 1σ-error bars. The solid line represents the laboratory value G_{lab}. Numbers in brackets refer to the citations in the text. The results from Ref. [8] and from Ref. [9], respectively, are not independent of each other.

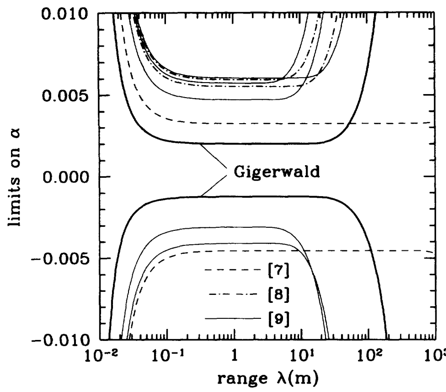

FIG. 5. Excluded strengths α and ranges λ for a single Yukawa model at the 2σ level arising from experiments measuring directly the gravitational constant at geophysical distances. Constraints from other experiments are not shown. Numbers in brackets refer to the references in the text.

largest component of the error budget is the uncertainty of water seepage in the scree. It turns out to be the limiting element of this experiment. The resulting value of the gravitational constant is

$$G = (6.6700 \pm 0.0054) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}},$$

which is in agreement with laboratory values. In Fig. 4 the result of this experiment is presented with other G experiments at geophysical distances.

Together with the laboratory value G_{lab} the Gigerwald experiment sets useful limits on the strength $\alpha(\lambda)$ of a fifth force for values of λ from more than hundred meters down to few centimeters. Figure 5 illustrates the constraints placed by G experiments at geophysical distances on such an interaction with 95% confidence. A more detailed report on this experiment is in preparation.

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[1] E. Fischbach *et al.*, Phys. Rev. Lett. **36**, 3 (1986).
[2] For a review, see E. G. Adelberger *et al.*, Annu. Rev. Nucl. Part. Sci. **41**, 269 (1991), or E. Fischbach and C. Talmadge, Nature (London) **356**, 207 (1992); for an index of measurements and theory see E. Fischbach *et al.*, Metrologia **29**, 213 (1992).
[3] M. V. Moody and H. J. Paik, Phys. Rev. Lett. **70**, 1195 (1993).
[4] D. H. Eckhardt *et al.*, Phys. Rev. Lett. **60**, 2567 (1988); J. Thomas *et al.*, Phys. Rev. Lett. **63**, 1902 (1989); C. Jekeli *et al.*, Phys. Rev. Lett. **64**, 1204 (1990); C. C. Speake *et al.*, Phys. Rev. Lett. **65**, 1967 (1990); Y.-C. Liu *et al.*, Phys. Lett. A **169**, 131 (1992).
[5] F. D. Stacey *et al.*, Rev. Mod. Phys. **59**, 157 (1987).
[6] M. Ander *et al.*, Phys. Rev. Lett. **62**, 985 (1989).
[7] M. A. Zumberge *et al.*, Phys. Rev. Lett. **67**, 3051 (1991).
[8] G. Müller *et al.*, Phys. Rev. Lett. **63**, 2621 (1989); G. Müller *et al.*, Geophys. J. Int. **101**, 329 (1990).
[9] M. Oldham *et al.*, Geophys. J. Int. **113**, 83 (1993).
[10] G. I. Moore *et al.*, Phys. Rev. D **38**, 1023 (1988); G. I. Moore *et al.*, J. Phys. E **21**, 534 (1988).
[11] Measurement made by E. Klingele, ETH Zürich.
[12] M. Talwani and M. Ewing, Geophysics **25**, 203 (1960).
[13] G. G. Luther and W. R. Towler, Phys. Rev. Lett. **48**, 121 (1982).

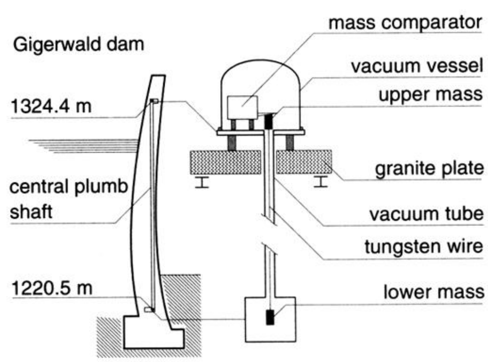

FIG. 1. Scheme of the Gigerwald experiment.