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Project SEE (Satellite Energy Exchange): proposal for space-based gravitational measurements

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Abstract. Project SEE (Satellite Energy Exchange) is an international effort to organize a new space mission for fundamental measurements in gravitation, including tests of the equivalence principle (EP) by composition dependence (CD) and inverse-square-law (ISL) violations, determination of G, and a test for non-zero G-dot. The CD tests will be both at intermediate distances (a few metres) and at long distances (radius of the Earth, R_E). Thus, a SEE mission would obtain accurate information self-consistently on a number of distinct gravitational effects. The EP test by CD at distances of a few metres would provide confirmation of earlier, more precise experiments. All other tests would significantly improve our knowledge of gravity. In particular, the error in G is projected to be less than 1 ppm. Project SEE entails launching a dedicated satellite and making detailed observations of free-floating test bodies within its experimental chamber.

Keywords: gravity, gravitational constant, Cavendish, equivalence principle, inverse-square law

1. Introduction

The concept of SEE (Satellite Energy Exchange), a new approach to space-based determination of G and other gravitational parameters, was proposed in considerable detail in 1992 [1–4]. The SEE concept was subsequently analysed by a team from the Russian Gravitational Society based at Gosstandard [5–11]. A number of proposals for determining G in space, including SEE, have been compared and analysed [12, 13].

Further information may be obtained at the Project-SEE web site: `http://gravity.phys.utk.edu/see`

A SEE mission would essentially entail observing the mutual orbital perturbation of test bodies in a restricted three-body situation. The fundamental interaction is an *exchange of energy* between two co-orbiting satellites, as first envisaged and described by Sir George Darwin a century ago [14]. Experimental observations of a SEE-type interaction between co-orbiting satellites of Saturn have led to a number of recent papers [15–17]. Very recently, a Canadian team discovered that the Earth itself has a companion asteroid with which it engages in a SEE-type interaction [18].

2. Objectives of a SEE mission

Since data from SEE observations reflect the net result of all forces acting on the test bodies, they are naturally adapted to detecting and measuring deviations from Newtonian gravity, especially equivalence principle (EP) violations, by both inverse-square-law (ISL) violations and composition dependence (CD), and possibly also deviations from General Relativity. Specifically, a SEE mission can make the following measurements and tests.

(1) Test of EP by ISL at test-body separations of a few metres.

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Table 1. Expected accuracy of SEE tests and measurements.

Test/measurement	Expected accuracy
EP/ISL at a few metres	2×10^{-7}
EP/CD at a few metres	$<10^{-7}$ ($\therefore \alpha < 10^{-4}$)
EP/ISL at $\sim R_E$	$<10^{-10}$
EP/CD at $\sim R_E$	$<10^{-16}$ ($\therefore \alpha < 10^{-13}$)
G	0.33 ppm
(G-dot)/G	$<10^{-13}$/yr in one year
GR violations	Undetermined

(2) Test of EP by CD at test-body separations of a few metres.
(3) Test of EP by ISL at test-body separations of $\sim R_E$ (radius of the Earth).
(4) Test of EP by CD at test-body separations of $\sim R_E$.
(5) Realization of the gravitational constant G.
(6) Search for non-zero G-dot (time variation of G).
(7) Possible search for violations of General Relativity (GR).

The expected accuracies of the various tests and measurements are indicated in table 1.

The results of precise tests of gravitational parameters have considerable implications for unification theories in at least three respects, as follows.

(A) Gravity is now the 'missing link' in grand unification.
(B) Nearly all modified theories of gravity and unified theories predict some violations of EP, either by deviations from the Newtonian law (ISL) or by the CD of gravity accelerations, due to the appearance of new possible massive particles (partners) [19–21].
(C) Most such theories also predict non-zero values of G-dot. These areas are very attractive for tests by orbiting satellites such as SEE.

In a SEE mission, we create initial conditions necessary to observe the encounter phase of the orbits first described by Darwin and later observed in the Solar System. The SEE satellite is basically a long cylinder, and inside it are two free-floating test bodies, a large 'Shepherd' having a mass of a few hundred kilograms and a small 'Particle' having a mass of 100 g (see figure 1). The capsule is manoeuvered by a 'disturbance compensation system' (DISCOS) to avoid collision with the Shepherd, and the Particle is launched into a very similar orbit, so that it is *approaching* the Shepherd. If the approach is sufficiently slow, the exchange of gravitational energy between the two test bodies will essentially reverse the initial difference in their orbital radii, thus causing them to recede from each other. The encounter is nearly one-dimensional, since the change in orbital radii is of the order of 10 cm, while the variation in the horizontal separation of the test bodies during an encounter is several metres. The attitude of the SEE satellite is essentially horizontal (like an arrow in flight), so the long cylindrical shape of the SEE capsule is well suited to enclosing the narrow trajectories of the encounters.

The SEE concept is rooted in the tradition of orbital-perturbation analysis. That is, we will seek to make very precise measurements of small effects, by allowing time to magnify them naturally. As with all such analyses—from the discovery of Uranus to the explanation of the perihelion precession of Mercury—our analysis methods will disentangle the sought-after effects from each other and from various background effects (such as the influence of the Moon and of the Earth's harmonics). Although in some cases the background effects may be large, they will generally be calculable and, since SEE provides for *controlled* experiments, we will often have the added luxury of being able to choose the phases of the effects under investigation both relative to each other and relative to the unwanted background effects.

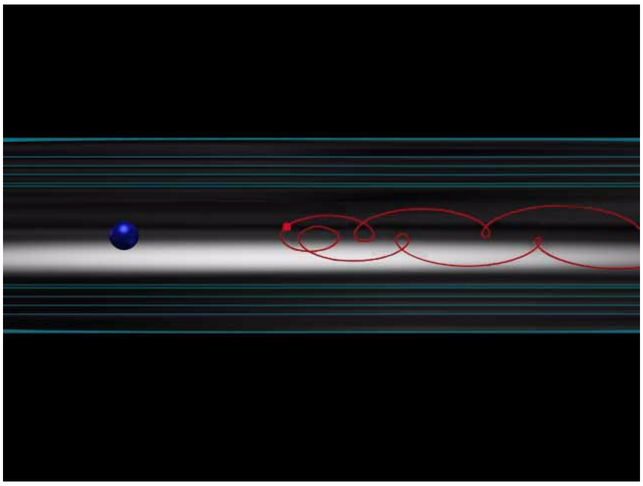

Figure 1. Cutaway view of SEE satellite showing Shepherd and Particle in SEE encounter.

The Russian team of the SEE collaboration, the Moscow research group of the Russian Gravitational Society (RGS), has been working for the past decade in the field of multidimensional gravity and cosmology, and, in particular, for the past two years, on gravitational effects of p-branes [22].

In recent decades there has been significant progress in unifying weak and electromagnetic interactions, and there exist some more modest achievements in grand unification theory (GUT), supersymmetric, string and superstring theories. Considerable theoretical progress has also been made concerning the implications of various models for possible variations of the 'constants' of nature, although most experimental and observational data have been restricted chiefly to the gravitational constant G—and with disappointingly inconclusive results. In addition, we still need more precise measurements of temporal and spatial variations of G at ranges of a few metres and we need adequate theoretical models. Significant progress has also been achieved in tests of gravitational effects in strong fields (i.e. binary pulsars) and in observational cosmology (CMB anisotropy), but we are still lacking an adequate and well established model of the very early Universe. At present, new theories with membranes, p-branes and more vague M- and F-theories are being created and studied, but constraints imposed by existing experimental results are generally weak as criteria for selecting the appropriate version of these theories. It is important to develop a thorough understanding of various multidimensional models of gravitation and cosmology because such models provide very promising approaches to manifestations of unified fundamental interaction theories, possible temporal and range variations of fundamental physical 'constants', and a number of other basic and long-standing problems of modern cosmology and black hole physics.

We must emphasize that the SEE mission is capable of probing for violations in the interactions of the test bodies with *each other* (rather than only with the Earth or the Sun). Thus, SEE is able to increase the existing ISL precision in the range of 1 to 100 m by several orders of magnitude. This capability is very important in view of recent result in the range of 20 to 500 m, which was obtained by Achilli and colleagues [23]. They found a positive result for the deviation from the Newtonian law (ISL) and interpreted the result in terms of a Yukawa-type potential. The Achilli result needs to be verified in other independent experiments.

3. Why space?

Despite the remarkable ingenuity which has long characterized gravity research, it is probably inevitable that accuracy attainable from terrestrial experiments will plateau, and that only in space can a quantum leap in accuracy be achieved. There are some signs that this may already have happened. To wit, the uncertainty of the gravitational constant G has been accepted as 128 ppm for nearly two decades. Nevertheless, most of the stated errors in the new determinations presented at this conference are still of order 100 ppm, and the scatter (1-sigma) of their G values about the mean is about 140 ppm (this excludes the two experiments which claim somewhat larger errors of 1700 and 1400 ppm). Thus, notwithstanding the unprecedented increase both quantitatively and qualitatively in G research during this remarkable decade, these new results suggest that it may be very difficult indeed to reduce the uncertainty in G much below 100 ppm by terrestrial methods. Some investigators now project errors as low as 10 ppm in the near future, based on ongoing improvements in their experiments. Within two or three years it will be known (1) whether it will in fact be possible to report such small error estimates and, more importantly, (2) whether the scatter of the various revised values for G will also support error estimates of 10 ppm. Finally, we must note that even this view may be somewhat optimistic, since it takes no account of the long-standing puzzle of the value obtained the Physikalisch-Technische Bundesanstalt (PTB), which is more than 1/2% above the CODATA value for G [24]!

Furthermore, a potential advantage of using space for gravity research is the potential for simultaneous determination of a number of different gravitational parameters. SEE has this capability. Even under the most favourable scenarios for ground-based determinations of G, it is doubtful that there will be enough sensitivity 'to spare' to permit multiple-parameter determinations.

It might seem that the problems of terrestrial apparatus must inexorably yield to new technologies—that the promise of ever increasing *sensitivities* would also lead to ever improving *accuracy*. However, this may not be true, since it is various systematic errors which limit the ultimate attainable accuracy in terrestrial experiments [25].

Nevertheless, advocacy of space for gravitational research does not mean that the environment of space is inherently friendly to the researcher. On the contrary, a number of problems are much more difficult in space than on Earth, and these problems must be understood thoroughly in order to solve them and take advantage of the fact that space is an inherently quiet environment. SEE is among about 25 known proposals for space-based realization of G. However, only three of these proposals go beyond a general outline of the intended gravitational interaction and also explore in some detail the many other physical effects which are present in the space environment and which would seriously vitiate accuracy if not identified and minimized or mitigated [12].

Here we note a common misconception about Project SEE: the gravity gradient is not a problem for Project SEE. On the contrary, SEE *must* have the gravity gradient, because this is an orbital-perturbation experiment. We want the full gravitational field of the Earth. Explicit account must be taken of the non-spherical character of the Earth's gravitational field, as noted in the original SEE proposal [1], especially as regards the largest terms [5, 7, 9].

4. Evolution of SEE mission concepts

Analysis by the SEE teams during the past seven years has led to a number of developments in the SEE concept. These are outlined briefly in this section.

Although the original SEE proposal [1] envisaged three-axis stabilization of the capsule, several advantages of rotating the capsule slowly about the cylinder axis were later realized. A counter-rotating reaction wheel will cancel the axial component of angular momentum of the remainder of the satellite. This cancellation allows the capsule to tumble once per orbit without any applied torques (except for very small torques required for 'trimming' the orientation).

Considerable analysis has been carried out on the temperature of and thermal effects in the SEE satellite, focusing first on the equilibrium temperature of its various concentric cylinders based on the somewhat idealized model of heat flow by radiation only [26] and subsequently turning to questions of temperature distribution. These studies include estimates of the allowable deviations from perfect temperature uniformity [12] and estimates of the actual expected nature and extent of such deviations, including both the axial distribution [27, 28] and the azimuthal distribution [29], and estimates of the size of conductive (non-radiative) heat flows [29]. The results from references [26] and [27] are largely incorporated into reference [28].

We briefly summarize here the results of the analyses in reference [29]. (1) For capsule rotation periods of the order of a few minutes, we find the azimuthal temperature variation in the outermost cylindrical shell of the satellite will be of the order of a few hundred millidegrees. The variation in all other shells will of course be many orders of magnitude smaller. (2) For several reasonable assumptions about the nature of the physical connection between successive cylinders in the SEE satellite, we find that the heat flow is strongly radiation-dominated except between the two innermost cylinders, and even here the radiation flow exceeds the conductive flow. A remaining issue is how to keep the instrumentation thermally isolated from the experimental chamber.

Finally, we have also shown that it is practicable to measure the temperatures with sufficient accuracy [28]. Collectively, these analyses indicate that our thermal-management concepts for the SEE satellite are sound and there will be no unpleasant surprises when the analyses are repeated in more detail using somewhat revised satellite designs.

We have developed a completely novel method of distance measurement for the purpose of determining the positions of the test bodies within the experimental chamber. Known as the Micron Accuracy Absolute Ranging System (MAARS), it is based on Fresnel diffraction and is capable of sub-micron precision at stand-off distances exceeding one metre [30]. MAARS is suitable for continually determining the positions of the test bodies at any desired time interval, but, because MAARS obtains absolute distances (rather than relative distances, as with traditional interferometry) it is not necessary to keep the beam on the test bodies continuously. Otherwise, as pointed out in [1], it would be necessary to operate with sub-nanowatt laser power in order to prevent unacceptably large forces due to the radiation pressure from the laser beam. MAARS meets the bulk of our distance-measuring requirements.

Capsule rotation with a period of the order of a few minutes opens up the possibility of a test of EP by CD, which is several orders of magnitude more accurate than by the method described in [1]. The rotation of the capsule this facilitates nanometre-level comparisons of the Earth radii of the orbits of two test bodies which are floating near each other within the experimental chamber. These could be two Particles which have been launched simultaneously into slightly different orbits. Such a measurement will require traditional interferometry and thus will require continuous (vis à vis *continual*) contact of the laser beam with both test bodies for a few minutes (long enough for the capsule to rotate once or twice about its axis) at each of several different places in the orbits of the two bodies. Thus, the duty cycle may be fairly low; for example, the required observation time might be 5 to 10 min per orbital revolution.

Concepts for a test of the time variation of G have evolved significantly since the original proposal [1]. In principle G-dot may be measured by using the rotation period of the Shepherd as a clock, since the Shepherd is in a drag-free orbit. The availability of long-term centimetre-level tracking means that the constancy of G can be tested at a cosmologically significant level [13]. This also requires knowledge of the harmonics of the Earth's field on a time-varying basis† [31]. Knowledge to the required accuracy has been technologically possible for some time, as indicated by analysis carried out for the Geopotential Research Mission (GRM) some 15 years ago [32]. A recent proposal now under consideration by NASA would provide monthly determinations of the geopotential with sensitivity equivalent to micrometre-level changes in sea level over regions several hundred kilometres in extent. This is more than sufficient to allow a determination of G-dot at a cosmologically significant level.

† One of the authors (AJS) is grateful to Rogers Ritter for pointing out the hitherto oversight of the need for precision time-varying geopotential for a G-dot experiment based on using the orbital period as a clock.

We note that any drag-free satellite could in principle be used to determine G-dot. However, the SEE Shepherd is drag-free in a very different sense from most drag-free satellites: the mass of the SEE satellite is configured to make it 'gravitationally invisible' to anything inside the experimental chamber. Thus, it is not necessary to continually fire small jets to counteract drag, as is the case with a traditional satellite. Thus, the SEE approach is favourable in limiting the unwanted perturbative effects on the would-be drag-free body.

Perhaps the single most important change in the SEE concept which is now under consideration is a suggestion by Nordtvedt to give the orbit an eccentricity of the order of 0.01 [33]. The rationale for such a change is that the best tests of the ISL at distances $\sim R_E$ are now those obtained from observing LAGEOS 2. *A fortiori*, observation of the SEE shepherd should be able to yield substantially better results, since it is drag-free and has active communication capability. The extent of this potential improvement is under investigation now.

5. Error budget for realization of the gravitational constant *G*

The various disturbing effects which are thought to have the potential to contribute to errors in the realization of the gravitational constant G on a SEE mission are being evaluated. The current status of this evaluation is shown in table 2. We find that the value of G can be determined with an error of about 0.33 ppm by a SEE mission. Note that the total error is dominated by the error in the mass of the Shepherd. The Appendix provides a discussion of the analysis of each source of error.

6. Conclusion

Space offers the prospect of a quantum leap in the accuracy of gravitational experiments. Although space is a challenging environment for research, the inherent quiet of space can be exploited to make very accurate determinations of G and other gravitational parameters, providing that care is taken to understand the many physical phenomena in space which have the potential to vitiate accuracy. A SEE mission will have the capability to make such determinations of multiple parameters simultaneously. The implications of the results for grand unification are substantial.

Appendix: discussion of *G* error budget

The methods of estimating the various effects tabulated in the error budget (table 2) are now described.

A.1. Distance resolution

The estimates of the effects of distance resolution on $\Delta G/G$ are conservative in two respects. (1) In laboratory tests our distance-measuring system MAARS has substantially surpassed the value of the resolution assumed for these calculations, namely 1 micron. (2) Although the analysis described below showed that accuracy varies significantly

Table 2. Error budget for realization of G.

Error source	Average or rms force	$\delta G/G$ ($\times 10^{-9}$)	Comments
Distance resolution I	NA	49	Assuming ISL
Distance resolution II	NA	54	Testing ISL
Time resolution	NA	Negligible	
Blackbody radiation	$<0.6 \times 10^{-18}$ N	<4	$\Delta T = 0.0001$ K
Electrostatic, Shepherd part	2.9×10^{-18} N@$s = 3$ m	<50	$q_s = 0.24$ pC, $V_s = 12$ mV $q_p = 0.24$ pC, $V_p = 103$ mV
Electrostatic, Particle image	$<2.4 \times 10^{-17}$ N ($\perp$)	$\ll 170$	$\Delta z = 1$ m from end wall
Lorentz forces	$\ll 5 \times 10^{-14}$ N ($\perp$)	Negligible	Shielded/compensated
Earth's gravity gradient	NA	Zero	SEE is an orbital-perturbation experiment
Earth's non-spherical field	Calculable	Negligible	
Fields of Moon, Jupiter, etc	Calculable	Negligible	
Shepherd mass uncertainty	Shepherd's gravity $\times 300 \times 10^{-9}$	300	Ref. [34]
Shepherd's mass defects	Small	<50	
Capsule mass defects	$<13 \times 10^{-18}$ N	<90	Many defects $\sim$10 mg
Outgassing	Small	Small	Obviate by baking
Total I[a], assuming ISL		<330	
Total II, testing ISL		<331	

[a] For computing the total error, the individual errors are added in quadrature. The bounded errors are evaluated at the upper bound, while the gross overestimate for the Particle image is evaluated—very conservatively—as one-third of the given overestimate. To wit,

$$[49^2 + 4^2 + 50^2 + (170/3)^2 + 300^2 + 50^2 + 90^2]^{1/2} = 330.$$

with the trajectory geometry, and that the optimum geometry leads to a smaller value of $\Delta G/G$, we have elected to report a mean value of $\Delta G/G$ for the various geometries in this error budget. After more careful analyses of these two effects, we expect to be able to report a far smaller value.

The effect of distance resolution on $\Delta G/G$ was calculated by two of the authors (Kolosnitsyn and Alexeev) by analysing simulated trajectories using several different functionals and several different methods of minimization. As an example, one functional was

$$S = \sum_{i=1}^{N}[(x_i - x_i^{(\mathrm{theor})})^2 + (y_i - y_i^{(\mathrm{theor})})^2] \qquad (A.1)$$

where the data set $(x_i^{(\mathrm{theor})},\ y_i^{(\mathrm{theor})})$ describes an error-free trajectory of the particle and the set $(x_i,\ y_i)$ describes the corresponding 'experimental' trajectory, which includes gaussian noise of the assumed value $\sigma = 1$ micron. The object is to minimize this functional in the space of the five variables (G, x_0, y_0, v_{x0}, and v_{y0}). The location of the minimum point of course gives the 'experimental' value of G, while the curvature along the G axis at this point in this five-dimensional space gives the uncertainty $\Delta G/G$. If the distribution of the coordinate errors is gaussian and the functional S reaches its minimum, then it follows that the ratio S/σ is distributed according to the χ^2 distribution with $2N$ degrees of freedom. Consequently, a single trajectory may be used to estimate the error ΔG in the realization of the gravitational constant. Multiple trajectories of course provide substantially tighter errors.

The minimum of S was located by both the gradient-descent (gd) and the sequence-descent (sd) methods. The results were in close agreement. The results for the uncertainty, using 11 trajectories, were

$$\Delta G/G_{gd} = 4.69 \times 10^{-8} \qquad (A.2)$$

and

$$\Delta G/G_{sd} = 5.24 \times 10^{-8}. \qquad (A.3)$$

Again we note that these values are based on an average over all trajectory geometries rather than on optimum geometries, and that the value of the coordinate error σ is conservative.

Another functional which was tried for the analysis, denoted S_x, employed only the x (horizontal) coordinates. It gave results almost as good as S itself, which indicates that nearly all the information is in the horizontal coordinates. This is to be expected, since the trajectories are very narrow—almost one-dimensional. A third functional, S_y, used only the y (vertical) coordinates and, predictably, gave much poorer results.

Preliminary analysis has also been carried out with a fourth functional, which uses the absolute values rather than the squares of the coordinate differences:

$$S^* = \sum_{i=1}^{N}[|x_i - x_i^{(\mathrm{theor})}| + |y_i - y_i^{(\mathrm{theor})}|]. \qquad (A.4)$$

An x-only version S_x^* of the absolute-value functional S^* and a y-only version S_y^* were also tried.

A.2. Time resolution

Time resolution is not an issue in G realization. Moreover, time resolution issues are negligible in all SEE experiments except G-dot.

A.3. Blackbody radiation pressure

Control of the temperature of the SEE experimental chamber is achieved by enclosing it within a series of concentric open cylinders which act as thermal radiation barriers. With this construction, cryogenic temperatures within the chamber are achieved and controlled to less than 0.1 mK, while fine-tuning is achieved by liquid-crystal windows in the outermost cylinder [28].

Radiation pressure within the capsule may put unbalanced forces on the Particle or the Shepherd, or both, unless the temperature of the capsule walls is uniform and equal to that of the Shepherd. However, there is no natural mechanism for maintaining thermal equilibrium between the capsule walls and the Shepherd, since heat transfer by radiation is very slow at low temperatures. Therefore the general strategy must be to adjust the temperature of the capsule walls slightly so that it follows that of the Shepherd. The temperature of the Shepherd is essentially stable [28], with the exception that Van Allen protons (see below) may cause a secular temperature increase on the order of 0.01 K/yr. This slow rate of change can easily be tracked by the liquid crystals.

Because of its low mass/area ratio, the Particle is of more concern than the Shepherd. Here we examine a worst-case scenario for the Particle, namely that there is a temperature difference of $\Delta\Theta = 10^{-4}$ K between the two ends of the capsule. For exposition purposes we also assume that the Shepherd's temperature matches that of the end of the chamber in which the Shepherd is located.

The net axial force on the Particle due to this effect would be

$$f = 4\Theta^3 \Delta\Theta \varepsilon\sigma/c \times \pi R_p^2 = 2.49 \times 10^{-18}\ \mathrm{N} \qquad \text{(A.5)}$$

if there were no internal reflections and if all emitted photons travelled parallel to the chamber axis (i.e. normally out from the end walls rather than Lambertian). Here ε is the emissivity, σ is the Stefan–Boltzmann constant, c is the speed of light and R_p is the radius of the Particle. We take $\varepsilon = 0.05$ and $R_p = 2.1$ cm (consistent with assuming that Particle has the density of aluminium and a mass of 100 g). In fact there are multiple internal reflections, with the result that the excess radiation from the 'hot' end of the capsule ultimately strikes the test bodies almost isotropically. Equation (A.5) implicitly incorporates the effects of reflections on the side walls of the capsule by treating the end walls as infinite sheets. We note that equation (A.5) overstates the momentum impacts on the Particle by assuming that all radiation is directed parallel to the chamber axis. Taking account of these effects reduces the force by somewhat more than a factor of four compared to the result in equation (A.5).

Thus, the net radiation-pressure force on the Particle due to a temperature difference of 0.1 mK between the two ends of the capsule (worst-case scenario) is

$$f < 0.6 \times 10^{-18}\ \mathrm{N}. \qquad \text{(A.6)}$$

This is more than eight orders of magnitude below the gravitational force of the Shepherd at a distance of 3 m.

The remainder of this section is a demonstration that a factor of about four does indeed result from taking account of multiple reflections and treating the photon directions more realistically.

The effects of multiple reflections can be illustrated by a simple model. For purposes of analysis we consider only the *excess* photons which originate from the warmer end of the capsule, which we call the 'left' end. Because of multiple reflections, the stream of photons which are in flight and travelling left to right may be expressed as an infinite series, assuming that a fraction β is lost to absorption during each one-way trip. The result is clearly

$$p = \sum_{n=0}^{\infty} (1-\beta)^{2n} = [1-(1-\beta)^2]^{-1}. \qquad \text{(A.7)}$$

The stream of original photons in flight travelling in the opposite direction is $p \times (1-\beta)$. So the *excess* proportion of photons which are travelling to left to right is

$$p_{net} = p - p \times (1-\beta) = \beta p = \beta[1-(1-\beta)^2]^{-1}. \qquad \text{(A.8)}$$

Retaining the assumption that all photons travel parallel to the chamber axis, the fraction β of photons which is lost during each one-way trip is only the reflection loss, ε, so we have

$$p_{net} = \varepsilon[1-(1-\varepsilon)^2]^{-1} = 0.51. \qquad \text{(A.9)}$$

That is, the effect of multiple reflections from the end walls is to reduce the net radiation-pressure force to 1.27×10^{-18} N (51% of the expression in equation (A.5)).

The assumption that all photons travel parallel to the chamber axis clearly results in an overstatement of the blackbody-radiation force, since the directions of the photon are not actually parallel to the chamber axis. This fact decreases the number of initially radiated photons that are available for collisions, and it also means that the photons which do eventually strike the Particle are nearly isotropic rather than aligned parallel to the axis, thus reducing the average axial component of the impulse from each collision. Although detailed modelling has not been done, it seems evident that these directional effects reduce the force by more than a factor of two. QED.

A.4. Electrostatic effects

Charging of the SEE test bodies by protons in the Van Allen belts is a significant problem which must be solved. Van Allen electrons are not a problem because they are not penetrating. We had previously shown that charging of the test bodies by very high energy cosmic rays was not a problem because these events are comparatively rare [1]. It also turns out that the South Atlantic Anomaly (SAA)—a region of intense Van Allen activity which results from the low altitude of the Earth's magnetic field lines over the South Atlantic Ocean—is not a problem for SEE because the SAA is overwhelmingly a low-energy phenomenon.

We have set limits on the allowable charge on the test bodies, namely 0.24 pC (1.5×10^6 electronic charges), as shown in table 2. These limits correspond to potentials of about 103 mV and 12 mV at the respective surfaces of the Particle and the Shepherd. The limits are based on the interaction of each test body with its own image charge when it is near the end of the SEE experimental chamber. The limits are proportionately stricter for the Shepherd because of the very demanding requirements of the G-dot determination. Thus, the Shepherd–Particle interaction *per se* is not the determining factor with respect to charge limits on the test bodies.

Proton fluxes experienced by the SEE satellite in various orbits have been calculated using software created at the Nuclear Physics Institute (NPI) of Moscow State University, namely the packages Space Environment Effects 2 (SEE2) and Space Radiation Effects Information System (SEREIS). These packages were developed from the NASA models AP8-max and AP8-min for calculating proton fluxes. However, the AP8 models use only the Solar maximum of 1970 and the Solar minimum of 1964, while the NPI packages use a more updated version of the Earth's magnetosphere, which takes account of its evolution on a scale of decades.

The results of this study may be summarized as follows.

(1) Charging of the SEE satellite and test bodies occurs almost exclusively during the crossing of the neighbourhood of the magnetic equator. The duration of the charging periods (FWHM) is about 12 min. Maximum charging rates occur in the central Atlantic (not the south Atlantic), and the smallest peaks in charging rates occur at the equatorial crossing in the Indian Ocean. The latter peaks are roughly an order of magnitude smaller than the central Atlantic peaks.
(2) When the SEE satellite is in a charging peak (crossing the magnetic equator), the time required for the charge on the test bodies to reach the maximum allowable values, as listed above, is a matter of seconds, not minutes. Thus, we must have the capabilities to detect and remove the charge as it builds up, on a time scale of seconds.
(3) We believe that detection and measurement of the charge on the test bodies can probably be achieved relatively easily by an array of minute microvoltmeters attached to the inner wall of the experimental chamber.

Several methods for removing positive charge are now being evaluated. Two candidate methods are as follows.

(1) *Passive*. Fill the experimental chamber with a polarized gas, such as a singly halogenated alkane (freon family) or an amine. The gas pressure must be below 10^{-6} Torr to avoid unacceptable Brownian motion of the Particle. We must determine whether the charge-removal rate will be adequate at such low pressure. A further issue is whether the adsorption of gas on the Shepherd would be stable or fluctuating; a fluctuation of a few hundred monolayers would cause unacceptable variation in the mass of the Shepherd.
(2) *Active*. Shoot electron beams directly at test bodies. The number of electrons needed is of the order of 10^8 s^{-1}. Although this approach has the inherent drawback of requiring that an active system must perform correctly for many years, it is simple in principle and will accomplish the goal.

A number of potentially deleterious side effects of the use of electron beams have been evaluated and discounted, e.g. heating effects in the Shepherd are negligible (recall that even the Van Allen belt protons produced negligible heating); space-charge effects are minimal, both in terms of electrostatic interaction with the test bodies and in terms of potential instability of the electron beam; momentum-transfer effects are marginally significant but can easily be averted simply by using two electron beams firing from approximately opposite directions.

Since it has become virtual folk wisdom in recent years that the South Atlantic Anomaly (SAA) is *the* problem for satellites with regard to charge accumulation, we must reiterate that our Van Allen peaks for the SEE orbit are at the magnetic equators, not at the SAA. Thus, the SAA is no problem for SEE: the SAA is a low-energy phenomenon, and the SEE satellite is naturally shielded against low-energy protons because we have elected to use radiation barriers rather than liquefied gas to achieve and control cryogenic temperatures. These radiation barriers are several centimetres thick *in toto*, so they stop all but the high-energy protons and also the secondaries. In graphs displaying time histories of test body charging calculated with the NPI models for SEE, the SAA does not even show up as a bump or even a shoulder at proton energies above 13 MeV, which is the energy necessary to penetrate 1 mm (see figure 2). In short, the SAA has *nothing* to do with SEE. We hope that this matter is now put to rest.

The issue of solar flares is also of concern, since the fluxes in Van Allen belts may increase more than an order of magnitude during flares. Therefore the SEE measurements cannot be carried out during the peak year of solar maximum. Furthermore, flares can occur even during solar minimum years. One strategy is to simply quit taking data during flares. However, for G-dot, we need long data runs (several years) so we need the capability to remove charge at a sufficient rate to keep ahead of all but the largest flares which might occur during non-solar-maximum periods. We need a protocol which can keep the charge on the Shepherd below its threshold during all but the three or four worst flares which typically occur during the non-solar-maximum period. We note, however, that it might be acceptable for the charge on the Shepherd to exceed its threshold value for brief periods (a few minutes). Thus, a discharging technique which cannot match the charging rate continuously at all times might nevertheless be acceptable.

A.5. Lorentz forces

The magnetic part of the Lorentz force, $q\boldsymbol{v} \times \boldsymbol{B}$, arises from the Earth's magnetic field and the orbital velocity of the respective test body. The field of the Earth is about 1/3 G at the SEE altitude, and the orbital velocities are $\sim$7.1 km s^{-1}. In the absence of shielding and compensation, the Lorentz force on the Particle would vary between 6×10^{-15} N and 6×10^{-14} N, depending mainly on the orientation of the Earth's field, when the charge on the Particle is $\sim$0.24 pC. The force is of course perpendicular to the Particle's velocity and therefore nearly perpendicular to the gravitational force due to the Shepherd. The vector $\boldsymbol{v} \times \boldsymbol{B}$ is largest and roughly

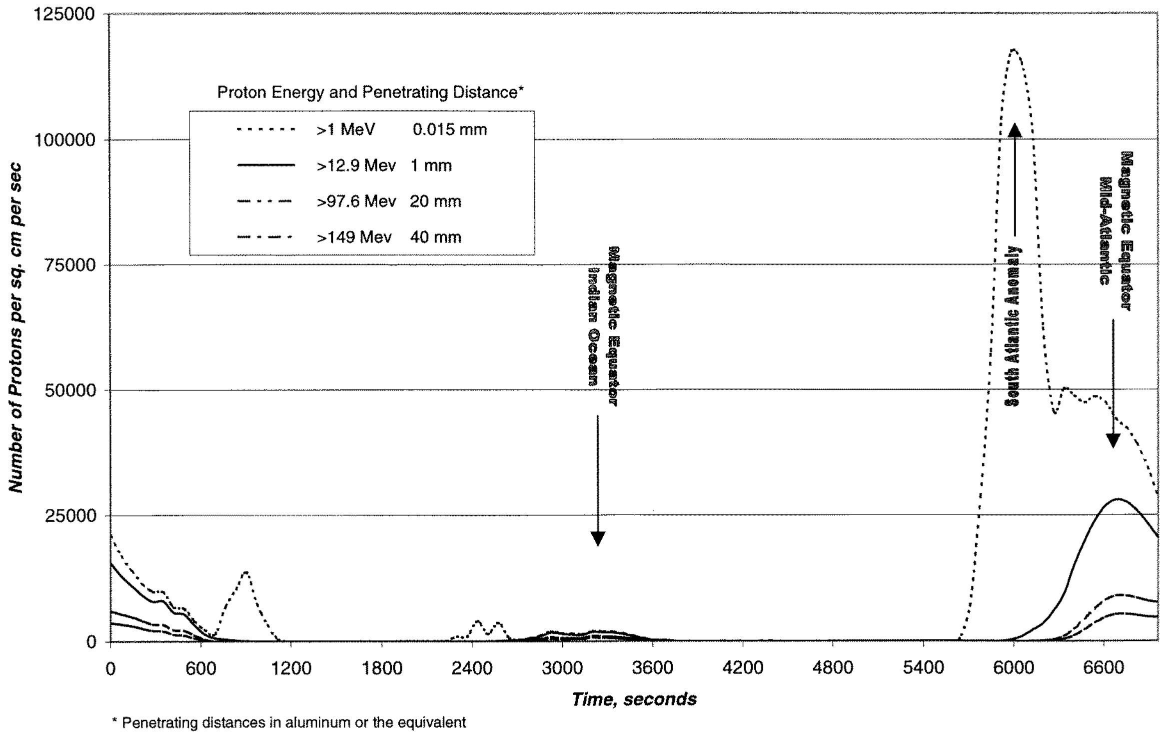

Figure 2. The Van Allen proton flux in a SEE orbit peaks at the magnetic equators, except for very low energy protons, which can penetrate less than 1 mm (aluminium or equivalent). The very high flux at the South Atlantic Anomaly (SAA) consists entirely of low-energy protons and therefore does not affect the SEE test bodies.

horizontal in the polar regions, and it is smallest and nearly vertical (usually downward, since the SEE satellite is in a retrograde orbit) near the magnetic equator. The component of the force which is parallel to the gravitational force of the Shepherd can exceed 2×10^{-14} N only in the form of a brief (<15 min) impulse when passing near a magnetic pole, and only if the Particle is near—but not at—its closest approach to the Shepherd.

The above paragraph assumes no magnetic shielding. Clearly, a capability to reduce the magnetic part of the Lorentz force by one or two orders of magnitude is desirable.

Passive shielding, by a thin μ-metal layer around the experimental chamber, could reduce the Lorentz force by two orders of magnitude. If active compensation is also needed, large Helmholtz-like (but necessarily odd-shaped) coils on the outer shell of the SEE satellite could create a magnetic field which is uniform to better than 1%. Thus, such coils could also reduce the field of the Earth to a residual which is more than two orders of magnitude below the uncompensated field.

If methods are found to measure the charge on the Particle and Shepherd with high accuracy (better than 1%), then it would be possible to simply calculate the Lorentz forces, thus obviating the need for shielding and compensation.

Finally, we note that the magnetic part of the Lorentz force on the Shepherd is generally in the same direction as the perturbing force which arises from the quadrupole moment of the Earth's gravitational field, especially in the polar regions. Thus, the Lorentz force will slightly augment the rate of precession of the Shepherd's orbital plane. This fact may slightly influence the choice of orbit inclination required to achieve a Sun-synchronous orbit.

A.6. Effects of the Earth's non-spherical field, the Moon, Jupiter, etc

Gravitational effects of the Earth's harmonics and of other bodies in the Solar System are calculable. For example, the gradient of the Moon's gravity will introduce an oscillation of a few microns into the Shepherd–Particle separation [1]. These effects can readily be accounted for in the overall fits.

A.7. Shepherd mass uncertainty

This is by far the dominant error in G realization for SEE. Experiments at the Physikalish-Technische Bundesanstalt by Debler have established that large masses can be measured within approximately 0.33 ppm [34]. In fact, the total G error, taking all other disturbances into account, is less than 10% above the error due to the Shepherd mass uncertainty alone. We note that SEE is the only existing or proposed G experiment which is not limited by other disturbances.

A.8. Shepherd's mass defects

The original SEE proposal indicated that mass defects in the Shepherd would have negligible impacts on G realization [1]. This was based on the reasoning that mass defects large enough to be detectable on Particle motion at close range could be modelled, and then any residual would have minimal impact at the larger ranges which would be used for all SEE measurements.

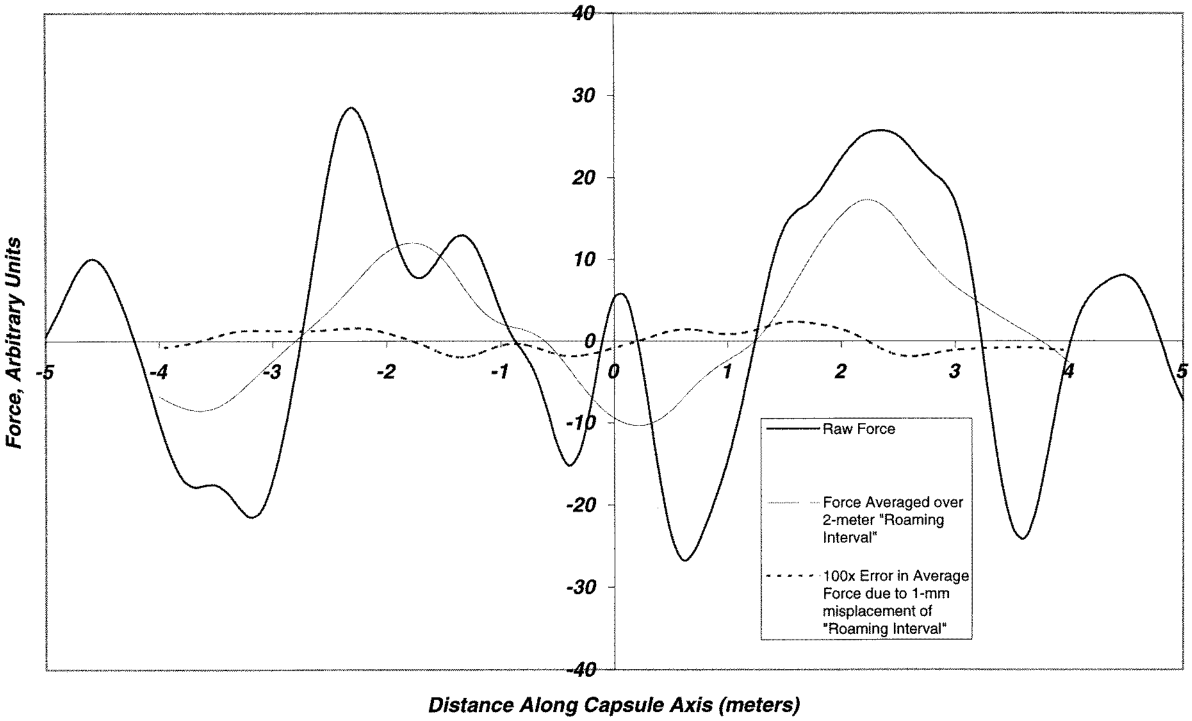

Figure 3. The force due to 1000 simulated mass defects in the experimental chamber walls is shown as the bold full curve. When the Shepherd is moved back and forth in a 'roaming interval' the mean force on it is reduced (full curve). The average force on the Shepherd in principle vanishes when it is placed for equal amounts of time in two roaming intervals and when the capsule is flown both 'forwards' and 'backwards'; the error due to misplacement of the Shepherd (failure to exactly replicate the location of the roaming interval) by 1 mm is more than two orders of magnitude lower than the force *per se* (broken curve). The *unmodelled* force is less than these values by more than an order of magnitude. Statistical considerations reduce the effective force further.

Recent studies by one of the authors (Konstantinov) assumed a distorted Shepherd and solved for gravitational parameters using to the above-described methods of Kolosnitsyn. These investigations are couched in terms of the impacts of errors in the moments of the Shepherd, especially the error in the quadrupole moment ΔJ_2, upon the position of the Particle. Konstantinov found that 1 μm changes would result from $\Delta J_2 = 10^{-5}$ if the trajectory were smooth or from $\Delta J_2 = 10^{-4}$ if the trajectory were prolately cycloidal. These uncertainties in J_2 are easily within modern machining capability. Therefore we have no reason to fear that mass defects in the Shepherd will degrade the accuracy of G.

A.9. Capsule mass defects

Although the mass of the SEE satellite will be configured so that its internal gravitational force field is in principle zero, the effect of defects (excesses and deficits) must be considered.

The force on a test body due to any point-mass defect will in principle have time-varying components along each coordinate axis. Because of the rotation of the capsule about its long axis, the resulting movement of a point-mass defect will of course cause an oscillation in its gravitational force on a test body. However, we have shown that, if the rotation period of the capsule is a few minutes, then in practice all point-mass defects may be regarded as uniform rings of mass [29]. To wit, the amplitude of the resulting periodic part of the test body motion is of the order of 1 nm or less for a 1 g defect in the inner wall of the experimental chamber, unless the test body is within a few centimetres of the wall. Thus, we may confidently treat all mass defects as rings (which of course cause no oscillation).

To study the effects of mass defects on the various SEE measurements, we have simulated large numbers ($\sim$1000) of ring defects with a random-number generator. Fourier analysis of spatial variation of the resulting gravitational forces along the axis of the rings shows that the spectrum peaks at a wavelength about 2.4 times the diameter of the ring. Thus, if all the defects were on the inner wall of the experimental chamber, the most prominent wavelengths would be about 2.4 m. We note that the (discrete) Fourier spectrum closely resembles the Fourier transform of the force of a single ring, as would be expected.

To date we have assumed that all the mass defects are on the inner wall of the experimental chamber and we have studied the force only on the cylinder axis. Clearly, defects in the outer cylinders will be weaker and will have longer wavelengths than defects in the inner wall, while the off-axis force spectrum will be stronger and have shorter wavelengths than the on-axis spectrum. Thus, our current simple models are probably representative of reality.

The force field within the experimental chamber may be modelled by observing the variation of test body velocities with position while in orbit—a process which we call 'self-calibration'. We have previously shown that a single mass defect of 10 mg or perhaps 1 mg could be seen by the large bump in the time trace of the Particle which occurs when the Particle passes by the defect. We may now interpret the peak amplitude of the Fourier spectrum as the peak force due to an equivalent (10 or 1 mg) defect. The Fourier spectrum falls a factor of ten from its peak (at 2.4 m) to wavelengths of ~83 cm, which is about the $n = 24$ component. It follows that about 23 terms are required to fit the on-axis force due to any ensemble of mass defects. Moreover, the problem is not completely arbitrary, since we know that all defects are rings of diameters between 1 and 2 m. We expect that the constraints imposed by this fact will allow a satisfactory fit of the force throughout the experimental chamber with no more than 23 additional terms.

After the fits, residual forces due to mass defects will remain. Strategies exist to effect maximum cancellation of these unmodelled forces. The key element of these strategies, which was recognized even before the original SEE proposal was published, is to vary the positions of the test bodies and to fly the capsule both 'forward' and 'backwards'. Recent analysis allows us to quantify the benefits of this general approach. The most critical need is for the G-dot experiment, since the on-axis force on the Shepherd must average to zero with high precision over periods of months or years. We have shown that if we (1) establish a 'roaming interval' of 2 to 3 m length in each end of the capsule, meaning that the position of the Shepherd varies back and forth in this interval, (2) fly the capsule both 'forwards' and 'backwards' for equal amounts of time over a period of several months and (3) are able to replicate the location of a given roaming interval within 3 mm when we bring the Shepherd back to it, then the rms net force is approximately three orders of magnitude below the peak unmodelled force (see figure 3). That is, the long-term average force on the Shepherd is three orders of magnitude smaller than the smallest detectable force. Thus, the expected perturbation due to mass defects is about four orders of magnitude lower than the peak force due to defects. The importance of these strategies cannot be overestimated.

A.10. Outgassing

There are two issues entailing outgassing: possible forces on the test bodies due to small outgassing jets, either from the bodies themselves or in the inner wall of the experimental chamber, and possible mass loss of the Shepherd due to outgassing. Outgassing from any other part of the satellite would cause no problem, because the drag-free concept being employed by SEE means that the process of compensation for external forces acting on the satellite does not affect the test bodies. Thus, it is not necessary to bake any of the electronics or other controls of the SEE satellite. The critical parts for baking are the test bodies themselves and the inner shell of the experimental chamber. All these components can withstand fairly high temperatures, so it will be possible to obviate outgassing by baking these components.

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