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The theoretical significance of *G*

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Abstract. The quantization of gravity and its unification with the other interactions present one of the greatest challenges of theoretical physics. Current ideas suggest that the value of G might be related to the other fundamental constants of physics, and that gravity might be richer than the standard Newton–Einstein description. This gives added significance to measurements of G and to Cavendish-type experiments.

Keywords: gravitational constant

Cavendish's famous experiment [1], carried out in 1798 using an apparatus conceived by the Reverend John Michell, gave the first accurate determination of the strength of gravitational coupling. The reader is referred to the excellent review of Everitt [2] for an authorative discussion of this classic experiment. According to Everitt (who took a 95% confidence limit on the mean of Cavendish's 23 measurements) the value obtained by Cavendish for the mean density of the Earth was 5.48 ± 0.10 (the modern value being 5.57). This corresponds to a fractional precision of 1.8%. As is discussed in detail in the other contributions to this Cavendish bicentennial conference, our present knowledge of the value of Newton's constant G seems to be uncertain at the $\sim 10^{-3}$ level. This contrasts starkly with our knowledge of other fundamental constants of physics (e.g. $\alpha_{\mathrm{e.m.}} = e^2/(4\pi\hbar c)$ and particle masses) which are known with a part in a million precision, or better. (Note, however, that the strong coupling constant, at the Z-boson mass scale, $\alpha_s(m_Z)$ is known only with a fractional precision of 1.7% [3].) The purpose of this work is to discuss briefly the significance of the value of G, and more generally of Cavendish experiments, within the current framework of theoretical physics.

Let us immediately note that, as with any other fundamental constant of physics, G should be measured with state-of-the-art precision, even if the significance of its value within the framework of physics is unknown (or small). The main point I wish to make here is that many theoretical developments of twentieth century physics suggest that there is an especially deep significance attached to the value of G. This gives all the more importance to measurements of G. (Though, as we shall see, our current theoretical understanding is incomplete and cannot yet make full use of any precise value of G.)

As a starting point, let us recall that the strength of gravity is strikingly smaller than that of the three other known interactions. Indeed, in quantum theory, the strengths of the electromagnetic, weak and strong interactions are measured by three dimensionless numbers $\alpha_1, \alpha_2, \alpha_3$ which are smaller, but not much smaller, than unity. Here, $\alpha_i \equiv g_i^2/(4\pi\hbar c)$, with $i = 1, 2, 3$, where g_1, g_2, and g_3 are the coupling constants of the gauge groups $U(1)$, $SU(2)$ and $SU(3)$ respectively†. The values of the α_is depend on the energy scale at which they are measured, i.e. they depend on the distance scale‡ over which the interaction is being probed. (For instance, the strong coupling constant α_3, measuring the strength of Quantum Chromodynamics (QCD), is of order unity at the energy scale $\Lambda_{\mathrm{QCD}} \sim 200$ MeV, and becomes small at high energy scales, i.e. at very short distances.) The numerical values of the α_is at the energy scale defined by the mass of the Z-boson, $m_Z \approx 91$ GeV, are [4]

$$\alpha_1(m_Z) = \frac{1}{58.97 \pm 0.08}$$

$$\alpha_2(m_Z) = \frac{1}{29.61 \pm 0.13} \qquad \alpha_3(m_Z) = \frac{1}{8.3 \pm 0.5}. \tag{1}$$

When the energy scale μ increases, $\alpha_1(\mu)$ and $\alpha_2(\mu)$ increase, while $\alpha_3(\mu)$ decreases. It seems (if one makes extra assumptions about the existence and spectrum of new (supersymmetric) particles at higher energies) that the three gauge couplings unify to a common numerical value

$$\alpha_1(m_U) \simeq \alpha_2(m_U) \simeq \alpha_3(m_U) \equiv \alpha_U \simeq \tfrac{1}{25} \tag{2}$$

at a very high energy scale

$$m_U \simeq 2 \times 10^{16}\ \mathrm{GeV}. \tag{3}$$

In contrast to the numerical values (1) or (2), the corresponding 'gravitational fine-structure constant', $\alpha_g(m) \equiv Gm^2/\hbar c$, obtained by noting that the gravitational interaction energy Gm^2/r is analogous to the electric one $e^2/(4\pi r)$, is strikingly small ($\alpha_g(m) \sim 10^{-40}$) when m is taken to be a typical particle mass. Indeed

$$\alpha_g(m) \equiv \frac{Gm^2}{\hbar c} \simeq 6.707 \times 10^{-39} \left(\frac{m}{\mathrm{GeV}}\right)^2. \tag{4}$$

† Here, $\alpha_1 \equiv (5/3)\alpha_Y$ with Y being the weak hypercharge ($Y(e_R) = 1$). The usual fine-structure constant $\alpha = e^2/(4\pi\hbar c) \simeq 1/137$ corresponds to a combination of α_Y and α_2.

‡ Recall that in relativistic quantum mechanics (using $c = 1$) an energy scale E corresponds to a distance scale $L_E = \hbar/E \simeq 0.2$ Fermi $(E/\mathrm{GeV})^{-1}$.

For a long time, this enormous numerical difference was viewed as a challenge. At face value it seems to imply that gravity has nothing to do with the three other interactions. However, some authors tried to find a natural origin for numbers as small as (4). In particular, Landau [5] conjectured that the very small value of α_g might be connected with the value of the fine-structure constant $\alpha = (137.035\,9895(61))^{-1}$ by a formula of the type $\alpha_g \simeq A\exp(-B/\alpha)$, with A and B being numbers of order unity. More recently, 't Hooft [6] resurrected this idea in the context of instanton physics, where such exponentially small factors appear naturally. He suggested that the value $B = \pi/4$ was natural, and he considered the case where $m = m_e$, the electron mass. It was noted (for fun) in [7] that the simple-looking value $A = (7\pi)^2/5$ happens to give excellent agreement with the experimental value of G. If we define (for fun) a simple-looking 'theoretical' value of G by

$$\alpha_g^{\text{theory}}(m_e) \equiv \frac{G^{\text{theory}} m_e^2}{\hbar c} \equiv \frac{(7\pi)^2}{5}\exp\left(-\frac{\pi}{4\alpha}\right) \tag{5}$$

one finds that it corresponds to $G^{\text{theory}} = 6.672\,3458 \times 10^{-8}$ cm^3 g^{-1} s^{-2}, in excellent agreement with the CODATA value: $G^{\text{CODATA}}/G^{\text{theory}} = 1.000\,04 \pm 0.000\,13$!

The first aim of this exercise is to exhibit one explicit example of a possible theoretical prediction for G. The second aim is to serve as an introduction to the currently existing 'predictions' for the value of G which are numerically inadequate, but which are conceptually important. Indeed, the main message of this work is that the gravitational interaction is currently believed to play a central role in physics, and to unify with the other interactions at a very high energy scale. The main argument is that gravity, like the other interactions, should be described by quantum theory. However, quantizing the gravitational field has turned out to be a much more difficult task than quantizing the other interactions. Let us recall that the electromagnetic interaction was quantized in the years 1930–1950 (QED), and that the weak and strong interactions were quantized in the 1970s and 1980s (Standard Model of weak interactions and QCD). The methods used to quantize the electroweak and strong interactions are strongly connected with the fact that the (quantum) coupling constants of these interactions, $\alpha_i = g_i^2/(4\pi\hbar c)$, are dimensionless. In contrast, we see from equation (4) that the quantum gravitational coupling constant $G/\hbar c$ has dimensions of an inverse mass squared or (using the correspondence $L_E = \hbar/E$) of a distance squared. This simple fact has significant consequences on the quantization of gravity—it means that gravity becomes very strong at high energies, i.e. at short distance scales. This is directly apparent in equation (4). If we consider a quantum process involving the mass-energy scale μ, the associated dimensionless analogue of the fine-structure constant will be $\alpha_g(\mu) = G\mu^2/\hbar c$ and will grow quadratically with μ. This catastrophic growth renders inefficient the (renormalizable quantum field theory) methods used in the quantization of the other interactions. It suggests that gravity defines a maximum mass scale, or a minimum distance. There is, at present, only one theory that indeed contains such a fundamental length scale and that succeeds (at least in the perturbative sense) in making sense of quantum gravity, namely String Theory. This theory (which is not yet constructed as a well defined, all encompassing framework) contains no dimensionless parameter and only one dimensional parameter $\alpha' = \ell_s^2 = m_s^{-2}$ where ℓ_s is a length and m_s a mass (we henceforth often use units where $\hbar = 1 = c$). In the simplest case (where the theory is perturbative and no large dimensionless numbers are present) String Theory makes a conceptually very elegant prediction: it predicts that the 'fine-structure constants' of *all* the interactions, including the gravitational one, must become equal at an energy scale of the order of the fundamental string mass m_s. In other words, it predicts (in the simplest case) that

$$\alpha_g(m_U) \simeq \alpha_1(m_U) \simeq \alpha_2(m_U) \simeq \alpha_3(m_U) \text{ at } m_U \sim m_S. \tag{6}$$

This yields $G \sim \alpha_U/m_U^2$. Taking into account some numerical factors [8] yields something like (γ denoting Euler's constant)

$$\frac{G}{\hbar c} \simeq \frac{\mathrm{e}^{1-\gamma}}{3^{3/2}4\pi}\frac{\alpha_U}{m_U^2}. \tag{7}$$

When one inserts the 'experimental' values (2) and (3) for α_U and m_U, one finds that the right-hand side of (7) is about 100 times larger than the actual value of G. Many attempts have been made to remedy this discrepancy [9]. However, the main message I wish to convey here is that modern physics tries to unify gravity with the other interactions and suggests the existence of conceptually important links, such as equation (7), between G and the other coupling constants of physics. It is quite possible that, in the near future, a better prediction for G will exist.

I wish to mention that the exponential-type relations (5) between G, α and the particle mass scales are also (roughly) compatible with the type of unification predicted by String Theory. Indeed, the hadronic mass scale (Λ_{QCD}) determining the mass of the proton, the neutron and the other strongly interacting particles is (via the Renormalization Group) predicted to be exponentially related to the string mass m_s. Roughly

$$m_p \sim m_s \exp(-b/\alpha_U) \tag{8}$$

where b is a (known) number of order unity. Combining (8) with (6) leads to

$$\alpha_g(m_p) = Gm_p^2/\hbar c \sim \alpha_U\,\mathrm{e}^{-2b/\alpha_U} \tag{9}$$

where α_U is the common value of the gauge coupling constants at unification.

Finally, String Theory (and other attempts at quantizing gravity, and/or unifying it with the other interactions) makes other generic predictions that might be testable in Cavendish-type experiments. Indeed, a generic prediction of such theories is that there are more gravitational-strength interactions than the usual (tensor) one described by Einstein's general relativity. In particular, the usual tensor gravitational field $g_{\mu\nu}(x)$ is typically accompanied by one or several scalar fields $\varphi(x)$. As many high-precision tests of relativistic gravity have put stringent limits on any long-range scalar gravitational fields (see e.g. [10]), there are two possibilities (assuming that such scalar partners of $g_{\mu\nu}$ do exist in nature).

(i) The scalar gravitational field $\varphi(x)$ is (like $g_{\mu\nu}$) long-ranged, but its strength has been reduced much below the usual gravitational strength G by some mechanism. (A natural cosmological mechanism for the reduction of any scalar coupling strength has been discussed in [11] and [12].)

(ii) The initially long-ranged field $\varphi(x)$ has acquired a mass term m_φ, i.e. it has become short-ranged (decreasing with distance like $e^{-m_\varphi r}/r$) but its strength is still comparable to (or larger than) G [13, 14].

In the first case, the best hope of detecting such a deviation from standard gravity is to perform ultrahigh-precision tests of the equivalence principle [12]. In the second case, deviations from standard (Newtonian) gravity might appear in short-distance Cavendish-type experiments [13, 14]. Indeed, it is possible (but by no means certain) that the mass (and therefore the range m_φ^{-1}) of such gravitational-strength fields be related to the supersymmetry breaking scale m_{SUSY} by a relation of the type $m_\varphi \sim G^{1/2} m_{\mathrm{SUSY}}^2 \sim 10^{-3}\ \mathrm{eV}(m_{\mathrm{SUSY}}/\mathrm{TeV})^2$. Therefore, if $m_{\mathrm{SUSY}} \sim 1$ TeV, the observable strength of gravity would increase by a factor of order unity at distances below $m_\varphi^{-1} \sim 1$ mm [14, 15]. More recently, another line of thought has suggested that gravity could be even more dramatically modified below some distance r_0 [16]. In principle, Cavendish-type experiments performed for separations smaller than r_0 might see a change of the $1/r^2$ law: the exponent 2 being replaced by an exponent larger than or equal to 4 [16]! (Note, however, that in these models r_0 is already constrained to be smaller than $\sim 1\ \mu$m.) I also wish to mention here a general argument put forward by Weinberg [17], suggesting the existence of a new gravitational-related interaction with range *larger* than 0.1 mm. (The recent announcement of the measurement of a non-zero cosmological constant goes some way to confirm the importance of such a submillimetre scale.)

In conclusion, I hope to have shown that G measurements and Cavendish-type experiments have now reached a new significance as possible windows on the physics of unification between gravity and the other interactions.

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