ISSN 0202-2893, Gravitation and Cosmology, 2009, Vol. 15, No. 1, pp. 65–68. © Pleiades Publishing, Ltd., 2009.

Problems of Measurement of the Newtonian Gravitational Constant[1]

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Received October 19, 2008

Abstract—Due to weakness of gravity, the accuracy of the Newtonian gravitational constant G is essentially below the accuracy of other fundamental constants. The current value of G, recommended by CODATA in 2006, based on all results available at the end of 2006, is $G = (6.67428 \pm 0.00067) \times 10^{-11}$ m^3 kg^{-1} s^{-2} with a relative error of 100 ppm. New experiments at a level of accuracy of 10 to 30 ppm are now in progress in some world gravitational laboratories. One of the problems of improving the accuracy of G is a precision measurement of the period of eigen oscillations of the torsion balance. In this work, numerical modelling of torsion and swing oscillations is under the condition that the torsion system is subject to random noise. A new method of high-precision estimation of the period of torsion oscillations has been developed. The dependence of the torsion period value and the error of its estimation on the level of the acting seismic noise has been studied. To measure the Newtonian gravitational constant at the accuracy level of 10 ppm, the environmental seismic noise must be below 10^{-2} mGal.

PACS numbers: 04.80.Cc, 02.60.Cb

DOI: 10.1134/S0202289309010162

1. INTRODUCTION

The Newtonian gravitational constant G, together with Planck's constant $\hbar$ and the speed of light c are the fundamental constants of nature which represent the fundamental limits: c is the maximum speed, $\hbar$ is the minimum angular momentum and G is the gravitational radius of a unit mass (the maximum radius of a sphere for relativistic gravitational collapse). Due to weakness of gravity, the accuracy of G is essentially below the accuracy of other fundamental constants. The modern value of the Newtonian gravitational constant, which was recommended by CODATA in 2007, based on the data, which were available at the end of 2006, is equal to $G = (6.67428 \pm 00067) \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The accuracy of the best experimental results is 15 to 40 ppm, although the scatter of the results is large enough. Therefore new experiments at a level of accuracy of 10 to 30 ppm are rather topical [1].

The traditional instrument for carrying out high-precision gravitational experiments, above all, measurement of the Newtonian gravitational constant, is the torsion balance. New technological approaches and optimization of the configuration of an experimental setup show that the gravitational constant can be measured at an accuracy level of 10 to 30 ppm. Nevertheless, there are a number of problems which should be solved to achieve this goal. One of such problems is the precision of measurement of the period of eigen oscillations of the torsion pendulum. The goal of this paper is to propose a method of high-precision estimation of the period of torsion oscillations and to study a dependence of the value of the torsion period and the error of its estimation on the acting seismic noise level.

2. TORSION BALANCE

The detailed structure of the torsion balance which is used in the Huazhong University of Science and Technology (HUST) experiment for measuring G, is shown in Fig. 1. The test mass, a rectangular bar, is suspended from point O by a tungsten fiber of length l. To describe the dynamics of the torsion balance, two coordinate systems are defined. One of them is a stationary Cartesian coordinate system $OXYZ$, with the origin at O located at the suspension point of the torsion balance. The other coordinate system, $O_1X_1Y_1Z_1$, is fixed rigidly with the test body, the torsion bar. Its origin is at O_1 which is the point of attachment of the fiber to the torsion bar. The center of mass of the torsion balance is located at the point O_2, at a distance l_0 from the point O_1 in the vertical direction. The suspension point is driven

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[1] Talk given at the International Conference RUSGRAV-13, June 23–28, 2008, PFUR, Moscow.

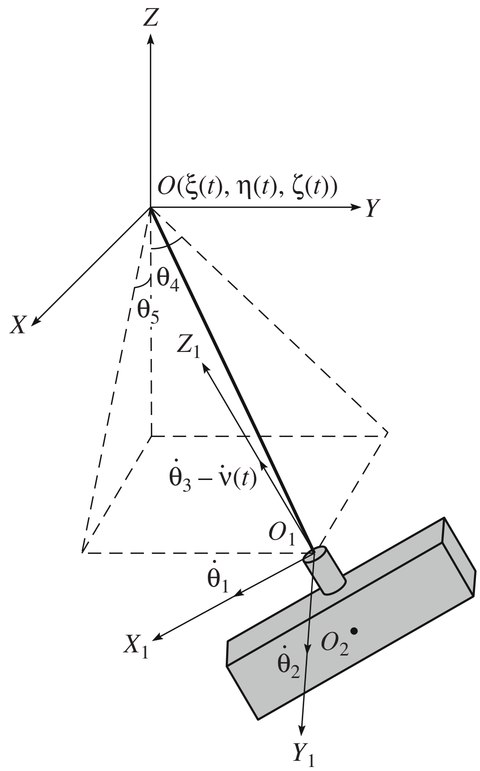

Fig. 1. The torsion balance and the coordinate systems chosen to describe its motion.

by fluctuation forces (seismic noise) which cause its random shifts $\xi(t)$, $\eta(t)$ and $\zeta(t)$ along the X, Y and Z directions, respectively. Other fluctuation forces, acting directly on the torsion bar (e.g., random fluctuations of a residual gas in the vacuum chamber, temperature variations etc), can provide a torsion rotation $\nu(t)$.

In such problem setting, the system has five degrees of freedom, which are marked as Θ_1, Θ_2, Θ_3, Θ_4, and Θ_5. Here, the parameters Θ_1 and Θ_4 represent the rotation angles around the axes X_1 and X and describe swing oscillations in the YZ plane. The parameters Θ_2 and Θ_5 represent rotation angles around the axes Y_1 and Y and describe swing oscillations in the XZ plane. The angle Θ_3 represents rotation about the Z axis and describes the principal torsion oscillations.

We have studied the behavior of oscillations of the torsion balance [1]. A numerical experiment has shown that oscillations in the swing degrees of freedom are excited by a random noise of seismic origin and occur with an amplitude varying in time. A spectral analysis of swing oscillations has shown that oscillations in each swing degree of freedom are beating of all quasi-harmonic swing modes with a random amplitude changing in time. The swing frequencies are defined by the geometrical parameters of the torsion balance. The random character of the swing oscillations is determined by the seismic noise acting on the suspension point. It was also shown that even with damping, due to the action of the seismic noise, the swing oscillations are a steady process.

The torsion balance is a complex nonlinear system, and due to nonlinear relations between different degrees of freedom, there emerge new oscillations, which are torsional mode couplings, or coupled modes [2]. The frequencies of coupled modes are simply linear combinations of swing-mode frequencies. If the coupled-mode frequencies are close to the frequency of the torsion oscillations, the latter can be disturbed. The principles of suppression of low-frequency torsional mode couplings are considered in [1, 2].

3. INFLUENCE OF ENVIRONMENTAL NOISE ON THE TORSIONAL OSCILLATIONS

The motion of the torsion balance is strongly influenced by various external disturbances, especially by the seismic noise. For a torsion balance with extremely high sensitivity, even small vibrations of the ground and changes of the environmental temperature could significantly change the period of torsion oscillations. Since the seismic noise is stochastic, variations of the period of torsional oscillations are also stochastic. A general approach to reducing the stochastic noise to a certain extent is to use the statistical mean methods (e.g., the nonlinear least-square fitting method). A theoretical analysis shows that for such methods the relative precision is proportional to $(1/m)^{1/2}$ (m is the total number of periods). For a reasonable time of measurement, these methods still cannot give a satisfactory result. Meanwhile, it is known that one of the most efficient methods, which is used for statistical analysis in the fields of seismology and global positioning systems [3], is the correlation, or phase, method. Therefore, to effectively suppress the influence of the stochastic noise on the torsion balance, our colleagues from HUST adopted and developed the correlation method for determination of the period of torsional oscillations [4]. Estimation of the effectiveness of the correlation method shows that the relative precision of period determination is improved proportionally to $(1/m)^{3/2}$. The idea of this method is as follows. Let a time series consist of m torsion oscillations. The phase angle φ_j at the j-th period can be determined by a cross-correlation function of a reference signal with the frequency ω_0 and the real data. To keep the phase unambiguous, the frequency of the reference signal ω_0 should be close to the frequency of the experimental

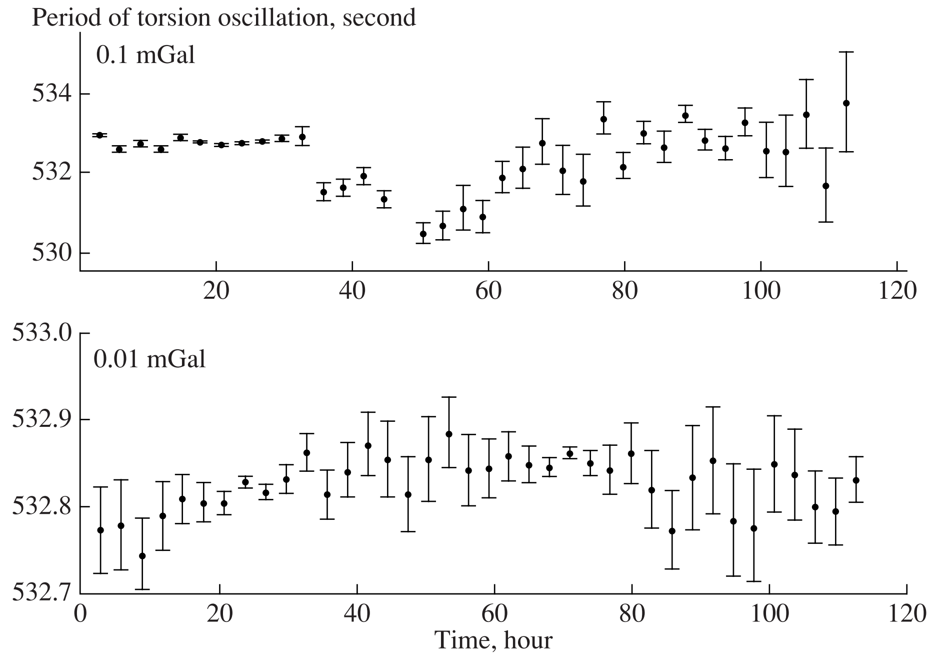

Fig. 2. Estimation of the period of torsional oscillations for different seismic noise levels. Each value was calculated for the "linear-slope" part equal to 20 oscillations. The vertical bars are standard deviations.

data. The frequency ω_0 can be easily obtained by FFT analysis. In some approximation, the phase φ_j can be expressed as

$$\varphi_j = \varphi_0 + 2\pi \frac{\omega_0 - \omega_{\text{ex}}}{\omega_0} j + \Delta\varphi_j, \quad (1)$$

where φ_0 is the initial value for linear fitting and $\Delta\varphi_j$ is a random variable determined by the environmental noise. Estimation of the frequency ω_{ex} on the basis of j oscillations can be done by linear fitting of Eq. (1),

$$\hat{\omega}_{\text{ex}} = \omega_0 - \frac{\omega_0 K}{2\pi}, \quad (2)$$

where K is the slope of Eq. (1). The estimator $\hat{\omega}_{\text{ex}}$ in Eq. (2) can be shown to be unbiased.

If the dependence of the phase on the period number has a linear character ($K = \text{const}$), the frequency ω_{ex} can be estimated using all data (m oscillations) simultaneously. If the torsion oscillations are noticeably disturbed by the environmental noise, the phase behavior can be nonlinear ($K \neq \text{const}$). In this case the time series is divided into parts with $K \approx \text{const}$, and ω_{ex} is estimated for each such "linear-slope" part. Then the value of ω_{ex} is calculated as a mean value of all "linear-slope" parts.

Numerical simulation of the motion of a torsion balance subject to seismic noise and estimation of the frequency of torsional oscillations have been done for the torsion balance used in the HUST experiment. The main parameters of the torsion balance are presented in Table 1. The time series of data have been obtain as a solution to the set of equations describing the motion of a torsion balance with five degrees of freedom [1]. The length of the time series was 118 h (it means 800 oscillations for the typical period of torsional oscillations of 532 s). The period of torsion oscillations was estimated by the phase method. The level of seismic noise acting on the torsion balance (in terms of acceleration) varied from 10^{-3} mGal to 10^{-1} mGal. The results of a numerical simulation show that, for the seismic noise of 10^{-3} mGal, the dependence of the phase on the number of period has a linear character and the torsion period can be estimated using all data simultaneously. For the seismic noise of 10^{-2} mGal, the phase behavior becomes

Table 1. Main parameters of the torsion balance

Mass	75 g
Size:	
length	91.5 mm
height	27.6 mm
width	12 mm
Fiber:	
length	889 mm
diameter	25 μm
Torsion constant	6.47×10^{-9} Nm

Table 2. Dependence of the period of torsional oscillations on the environmental seismic noise level

Seismic noise, mGal	Period of torsion oscillation, s (mean for 800 periods)	Period of torsion oscillation, s (mean for 20 $\times$ 40 periods)
0.001	532.80882 ± 0.00016 (0.3 ppm)	532.80788 ± 0.00036 (0.6 ppm)
0.005	532.80851 ± 0.00030 (0.6 ppm)	532.80463 ± 0.00068 (1 ppm)
0.01		532.8258 ± 0.0010 (2 ppm)
0.05		532.6541 ± 0.004 (8 ppm)
0.1		532.390 ± 0.062 (100 ppm)

nonlinear, and the period can be estimated by dividing the data into "linear-slope" parts. Finally, for the seismic noise of 10^{-2} mGal, the phase dependence is a complicated process having nonlinear parts and steps. In this case, the period estimation accuracy is essentially lower than it is required in the experiment (100 ppm against 10–30 ppm). The results of the analysis are summarized in Table 2. Fig. 2 shows estimates of the torsion period obtained for the length of "linear-slope" parts equal to 20 periods.

4. CONCLUSION

One of the most important problems which restricts the accuracy of measurement of the Newtonian gravitation constant is measurement of the period of eigen oscillations of the torsion balance. Due to a very high sensitivity, the torsion balance responses to tiny external noises, which accordingly results in changes of the torsional oscillation period. The general approach in reducing the stochastic noise is averaging the fluctuations over a period. The phase method of period estimation can improve the uncertainty by a factor of $1/m$ (m is the total number of periods).

The accuracy of period estimations depends on the level of seismic noise which affects the torsion balance. Estimation of the period on the accuracy level of a few ppm is provided by the level of environmental seismic noise with amplitudes smaller than 10^{-2} mGal. The value of the torsional period depends on the seismic noise level. Changing the seismic noise level by one order leads to a change in the torsion period value of a few tens of ppm. To provide the absence of systematic errors in the period values caused by the noise, the noise conditions should not change during the experiment.

ACKNOWLEDGMENTS

This work was supported by the Russian foundation for Basic Research, Grant No. 05-02-39014.

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