RIVISTA DEL NUOVO CIMENTO VOL. 19, N. 2 1996

A comparative survey of proposals for space-based determination of the gravitational constant G

A. J. SANDERS (1) and G. T. GILLIES (2)

(1) *Department of Physics and Astronomy, University of Tennessee Knoxville, TN 37996-1200, USA*

(2) *Department of Mechanical, Aerospace, and Nuclear Engineering University of Virginia - Charlottesville, VA 22901, USA*

(ricevuto il 27 Novembre 1995)

1. – Introduction

Since virtually the beginning of the space age, researchers have been attracted to the prospect of utilizing the relatively quiet environment of space for making an accurate determination of the Newtonian gravitational constant G, which has proved to be exceptionally elusive in terrestrial experiments (Working Group (1965)). The need for an improved measurement of G is well known. It is the least precisely determined of all the fundamental constants of nature, having the presently accepted value (Cohen and Taylor (1986 and 1987)) of $G = (6.67259 \pm 0.00085) \cdot 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$. The relative uncertainty in this value, 128 parts per million (ppm), is 10^2 to 10^5 times larger than that of most of the constants which arise in atomic and nuclear physics (1).

(1) Until recently the gas constant R and some 10 physicochemical constants derived from it were also conspicuous for their large errors. Now a recent reduction in the uncertainty in R, which will correspondingly reduce those of the derivative physicochemical constants in the next CODATA least-squares adjustment (Cohen and Taylor (1994)), means that G and its derivative constants, *viz.*, the Planck mass, time, and length, will soon stand almost completely alone in having large uncertainties: Only two other constants will still have uncertainties over 1.8 ppm: the Stefan-Boltzmann constant and the anomalous magnetic moment of the muon, both of which will have uncertainties of about 7 ppm.

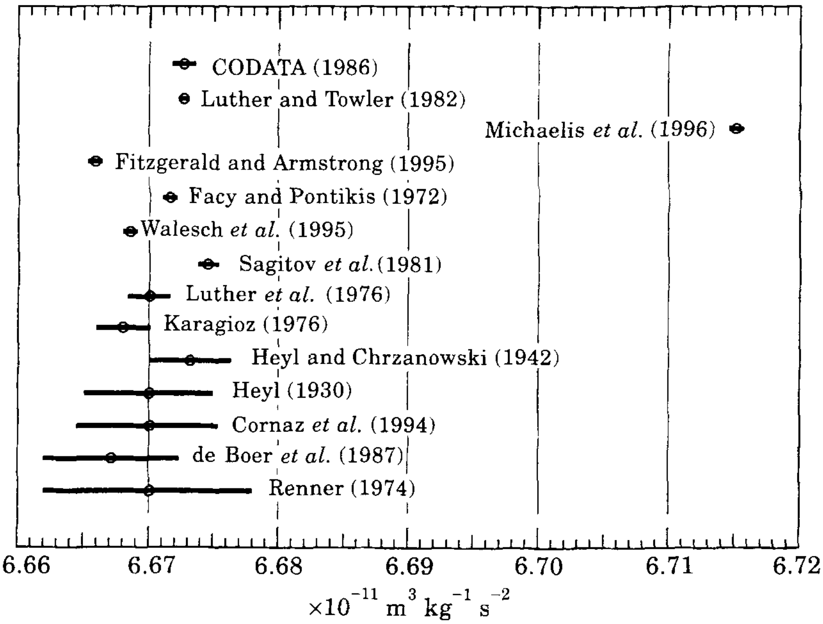

Fig. 1. – Some of the recent high-precision determinations of the Newtonian gravitation constant G. Since 1930 a number of investigators have determined G to three significant figures or better. Although six determinations now claim errors of about 100 parts per million (ppm), we note that all of these results exclude each other's values for G within their quoted errors. The CODATA value is based on Luther and Towler's result, with the exception that the error was doubled by the CODATA committee.

Moreover, the six highest-precision measurements of G (Luther and Towler (1982); Michaelis *et al.* (1996); Fitzgerald and Armstrong (1995); and Walesch *et al.* (1995); Sagitov *et al.* (1981); and Pontikis (1972)), each claiming an uncertainty of ≈ 100 ppm, all exclude each other within the limits of their quoted errors, as shown in fig. 1. These measurements generally differ from the CODATA value by multiple standard deviations. The largest departure, some 0.6% (6000 ppm), is exhibited by the result of the PTB in Braunschweig (Michaelis *et al.* (1996)), while the one-standard-deviation scatter among the remaining five values which claim ≈ 100 ppm is in fact over 500 ppm. Another new measurement claiming somewhat lower accuracy is consistent with the CODATA value (Cornaz *et al.* (1994)). We further note that still other new determinations, which are also expected to provide very precise results, are now in progress (*e.g.*, Luther (1995)). Some might argue that the size and nature of these discrepancies may be such as to reopen the possibility that there is new physics to be explored in this regime of interaction strength. Whether or not this is so, these results would seem to indicate that we may now be reaching the limit of performance with the paradigm of terrestrially-based mechanical instrumentation as used to determine the absolute value of G.

The difficulty in determining G is attributable in large part to the extreme weakness and the simultaneous ubiquity of the gravitational force. The Newtonian attraction between fundamental particles is $\approx 10^{40}$ times weaker than the electromagnetic or nuclear forces between them. Thus, only ponderable, electrically neutral test bodies have been of use in making absolute measurements of G. Moreover, the stray gravitational fields due to other bodies, particularly the Earth, cannot be screened out, since the gravitational force cannot be shielded. Signal-to-noise ratio is thus an important issue in any laboratory gravitational experiment, and therefore significant efforts are made to isolate such experiments from cultural and natural sources of gravity gradients and time-varying fields.

For two centuries now the torsion pendulum has been the instrument of choice for laboratory determinations of G, because of its ability to largely nullify the effects of the Earth's field. However, it is subject to a number of well-known limitations, which to date have contributed to the uncertainty of G. A number of these, including thermal noise, seismic disturbances, and mechanical-load restrictions have been evaluated by Gillies and Ritter (Gillies and Ritter (1993)). Despite many ingenious approaches for circumventing these problems (*e.g.*, supporting the test masses by magnetic suspension or mercury bearings), the uncertainty in G has proven to be stubbornly resilient, having decreased by only about a factor of 10 per century over the past 200 years (Speake and Gillies (1987), Melnikov (1994)).

The difficulties inherent in terrestrial determinations of G have in no small measure fueled the hope that somehow the relatively quiet environment of space could be exploited to achieve a great leap in the accuracy of G. However, the availability of a so-called «zero-g» environment and very high vacuum in space does not concomitantly endow the orbiting laboratory with immunity from other perturbations. Indeed, potential error sources related to low-frequency vibrations and to thermal effects, especially disequilibrium and fluctuations, may be more difficult to control in a spacecraft than in a well-isolated terrestrial laboratory. Moreover, the necessary physical separation of the spacecraft from the ground-based experimenter raises a number of issues pertaining to control, data acquisition, and, more seriously, repair of unforeseen problems. Even after very careful planning, a mission may still be susceptible to simple mistakes and to problems due to failure to anticipate some previously unobserved behavior (these are illustrated in the case of the Hubble telescope by, respectively, the focusing problem and the thermally-induced vibration of solar-cell panels). Finally, there is no escape from the fact that it is very, very costly to mount and complete any space-based scientific mission.

In this article, we review the various proposals for experiments for measuring G in space and discuss the prospects for achieving a low-uncertainty determination. Our scope in this paper is the *physics* underlying the design of these experiments. We examine a number of interesting previous proposals and find that, despite their attractiveness, the fundamental physical principles were often not sufficiently well-developed to reveal serious and potentially limiting design problems (however, in some cases substantial modification of the designs might allow retention of the general concept). The situation with respect to the terrestrial determination of G is discussed in detail elsewhere (Gillies (1987, 1988, 1990), Gillies and Sanders (1993)).

After even the most exhaustive physical assessment is completed, of course, a host of practical issues must still be carefully addressed during the various phases of designing an actual spacecraft for construction. Although the issues which arise during this stage are equally critical to mission success, they are chiefly in the domain of engineering and therefore beyond the scope of this paper. We note that Walker (1993) has summarized and presented some very sensible guidelines about what works and what does not work in addressing such issues.

TABLE I. – *Comparison of EPIC proposals.*

Author(s)	Berman and Forward 1969	Wilk 1971	Vinti 1970, 1972
Basic design	Artificial orbital system Tunnel-in-sphere oscillator Resonance with optical flats (terrestrial)	Variants of tunnel in sphere Gravitational clock 3 orthogonal tunnels in sphere One tunnel in sphere	Tunnel-in sphere oscillators (Artificial orbital system also briefly discussed)
Special focus	Equations of motion	Equations of motion	Elaboration on Wilk's proposal
Restraints on test bodies	None in artificial orbital system or in orbital version of tunnel-in-sphere oscillator	Yes; various	Tethers and gimbals on sphere «Suspension system, perhaps electrostatic,» on test mass
Treatment of Earth's field	Viewed as unwanted perturbation	Viewed as unwanted perturbation	«The first-order gravity-gradient term causes trouble»
Treatment of precision-measurement issues	Qualitative. Estimates effects of non-sphericity and density inhomogeneity of large test body and of errors in tunnel shape and position Force due to mass of mother satellite mentioned	Wide range of issues treated, including thermal noise, temperature stability, electromagnetic effects, gas pressure, and density variation	Largely absent
Comments	Also describes terrestrial resonance method using sphere with two tunnels No mention that the natural orbital motion of test body results in a pseudo-restoring force, which confounds force due to sphere	Proposed accuracy and precision of measurement seem optimistic	Spherical-symmetry assumption open to question

Smalley 1975	Farinella and colleagues 1979, 1980	Ritter and Gillies 1981, 1984	Steenbeck and Treder 1982
Chiefly a review article	Scattering simulations near Lagrange points of a source mass (1980)	Resonant excitation of test body in soft magnetic trap	Artificial binary and tunneled-sphere oscillator
Gravitational clocks Test-body dynamics as idealized by the authors	Astrodynamics of test bodies	Precision rotating field sensor	Creation of a multipurpose experiment
Axles	None	Source mass on axle Test mass in magnetic trap	None
Various, according to view of each author	Viewed as perturbation	None	Viewed as perturbation
Focused on selected ultimate limits assuming ideal conditions	Qualitative	Qualitative	Mainly limited to questions of resolution
Proposed a gravitational clock consisting of a small mass oscillating back and forth through a hole traversing a massive disk	Exploratory No usable trajectories identified	Potentially a very high-sensitivity technique	Also intended to test for gravitational shielding and the existence of gravitational waves

TABLE I. – *Continued.*

Author(s)	Hills 1986	Avron and Livio 1986	Baker and Falk 1987*a*, 1987*b*
Basic design	Artificial binary inside a spherical shell or ballon (*)	Restricted 3-body problem	Chiefly a critical analysis of conditions for stability of orbits
Special focus	Use of large-radius (100 m) orbit to test for 5th force	Attempt to simplify form of equations of motion	Artificial binary both in conventional orbit and in cycloidal motion
Restraints on test bodies	None	None	None
Treatment of Earth's field	Viewed as unwanted perturbation	Viewed as unwanted perturbation Coriolis effect	Investigated whether the large gradients in near-Earth orbits preclude stable binary orbits
Treatment of precision-measurement issues	Only resolution issues considered	Qualitative	N.A.
Comments	Spherical symmetry of shell/balloon would be spoiled by imperfections and instrumentation Solar radiation pressure would significantly displace orbit	Perspective of article rests largely on the idealization of a location «far from any gravitating body»	The finding of cycloidal motion (similar to later findings of Project SEE) was omitted in subsequent published reports

(*) Hills also suggests placing several satellites in orbit around an Earth-crossing asteroid at different radii to test the inverse-square law as a means of probing for a 5th force of intermediate range.

2. – Categorization criteria

The various published proposals for measuring G in space fall naturally into two groups: The first group, which we dub «Exploration of Principles under Idealized Conditions» (EPIC), concentrated on describing the basic physical principles of the proposed experiments. The authors typically chose to forgo assessments of the precision-measurement aspects of their experiments, and instead concentrated on the underlying physical mechanisms. Nevertheless, the early EPIC proposals were generally conceptually elegant and, in some cases, impressively detailed. Wilk's work was especially wide-ranging (Wilk (1971)). It analyzed a number of different schemes which had been proposed, including several informal and previously unpublished suggestions, and also considered a number of generic questions, such as (in appendix G of that document) the suitability of various metals for construction of the test masses. An overview of the EPIC proposals is presented in table I. According to our

Roxburgh *et al.* 1989	Hall 1990	Prasanna 1993	Antonyuk *et al.* 1993
Excitation of ultra-sensitive gradiometers by rotating dumbbell or rod	Chiefly a critical analysis of conditions for stability of orbits	Artificial binary located at the Earth-Moon Lagrange points L_4 or L_5	Scattering simulations near Lagrange points of a source mass
Adaptation of special accelerometers to space-based gravity studies	Artificial binary	Equations of motion of two-, three-, and four-body problems	N.A.
Axles	None	None	None
Geodesy of high-order harmonics of Earth's field is major goal	Tidal force viewed as perturbation	Intrinsic in multi-body treatment (However, effects of Earth's and Moon's J_2 are negligible)	Perturbation
Preliminary	Brief	Brief discussion in Conclusion section	Perturbations are presumed to be the same as in the SEE capsule
Published proposal is brief description of a general concept	Abstract only Evidently a continuation of 1987 proposal of Baker and Falk	M. S. Thesis under J. L. Junkins	Similar to work of Farinella (1980)

categorization scheme, some of the more recent proposals are also placed in the EPIC category if they are chiefly investigations of selected analytical issues, such as orbital stability or simplifying the equations of motion, rather than comprehensive proposals.

Twenty years ago Smalley reviewed the then-existing proposals for measuring G in space (Smalley (1975)). His review was very much in the spirit of the EPIC proposals themselves—a comparative survey of the physical principles as presented in the proposals. His treatment of errors focused on the ultimate limiting accuracy under the idealized conditions posited by the authors, and it did not entail attempts to uncover overlooked sources of error or basic flaws in experimental design. More recently, Spallicci has reviewed a number of proposals in a similar spirit (Spallicci (1989)). The present article tries to extend these efforts in a more critical and analytical direction. Specifically, we attempt to examine important potential sources of experimental error, many of which are of particular concern in precision-measurement experiments.

The hallmark of the second group of experiments is that they not only treat the conceptual essence of the proposed experiment but also attempt to identify and deal with the many other physical interactions which occur in an actual experiment. Accounting for the effects of such interactions is essential to achieving the sought-for accuracy. An overview of these proposals is presented in table II.

Given this set of criteria, the second group is comprised, then, of only three proposals, namely the NEWTON proposal of the University of Pisa (Nobili *et al.* (1989, 1990 and 1993)), the Satellite Energy Exchange (SEE) proposal of the University of Tennessee (Sanders and Deeds (1992*a* and 1992*b*) and Sanders *et al.* (1993)), and the proposed G/ISL (G/Inverse-Square-Law) experiment of the STEP project (Blaser *et al.* (1993 and 1994), Paik and Blaser (1993), and Paik (1994)). The NEWTON and SEE proposals incorporate extensive discussions of the experimental errors which the authors anticipate, including specific ways to either mitigate or compensate for the expected errors. For example, to minimize the gravitational effects of unmodeled mass, the SEE proposal calls for specially designed radiation barriers to eliminate the occasional warping and buckling which may occur in conventional foil barriers, while the NEWTON proposal calls for locating liquid fuel tanks at the ends of the spacecraft. Similarly, the G/ISL proposal calls for operating a pair of matched accelerometers in a differential-signal mode in order to reject common-mode noise. However, we note that the discussion of experimental uncertainties in the G/ISL proposal *per se* is less exhaustive than those in SEE and NEWTON, although perturbation analysis for the overall STEP project is very advanced and incorporates a number of elegant methods for obviating possible errors.

None of these proposals has yet reached the level of specificity entailed in engineering feasibility studies (although planning for many aspects of the overall STEP mission is in a very advanced stage). Moreover, given the minute size of the gravitational force and the high-accuracy goal of the proposed experiments (~ 1 part in 10^6), it is possible that important effects still remain to be identified. Thus, all three proposals warrant continued, detailed scrutiny at the present stage by physicists and space scientists prior to the necessary engineering-feasibility assessments (see for example Keyser (1992 and 1993)). A synopsis of each of the proposals listed in table II follows, and is in turn followed by some comments on various EPIC proposals.

Figure 2 indicates the time sequence of the various proposals to measure G in space. Note that this has become a lively field of inquiry during the past ten years, with new proposals now appearing at the rate of about one per year. Some of the recent proposals have exhibited great sensitivity to the difficulties of mounting a space mission and to the scarce resource of flight opportunities. This has significantly elevated the quality and detail of the proposals.

Project NEWTON

The NEWTON proposal (Nobili *et al.* (1989, 1990 and 1993)) entails analyzing the motion of a small (2 kg) test-body in orbit around a large (75 kg, 10 cm radius) attracting mass. The planned orbital radius is 20 cm, so the orbital period would be about 2 hours. The test bodies would be shielded by a cylindrical capsule about 3.5 m

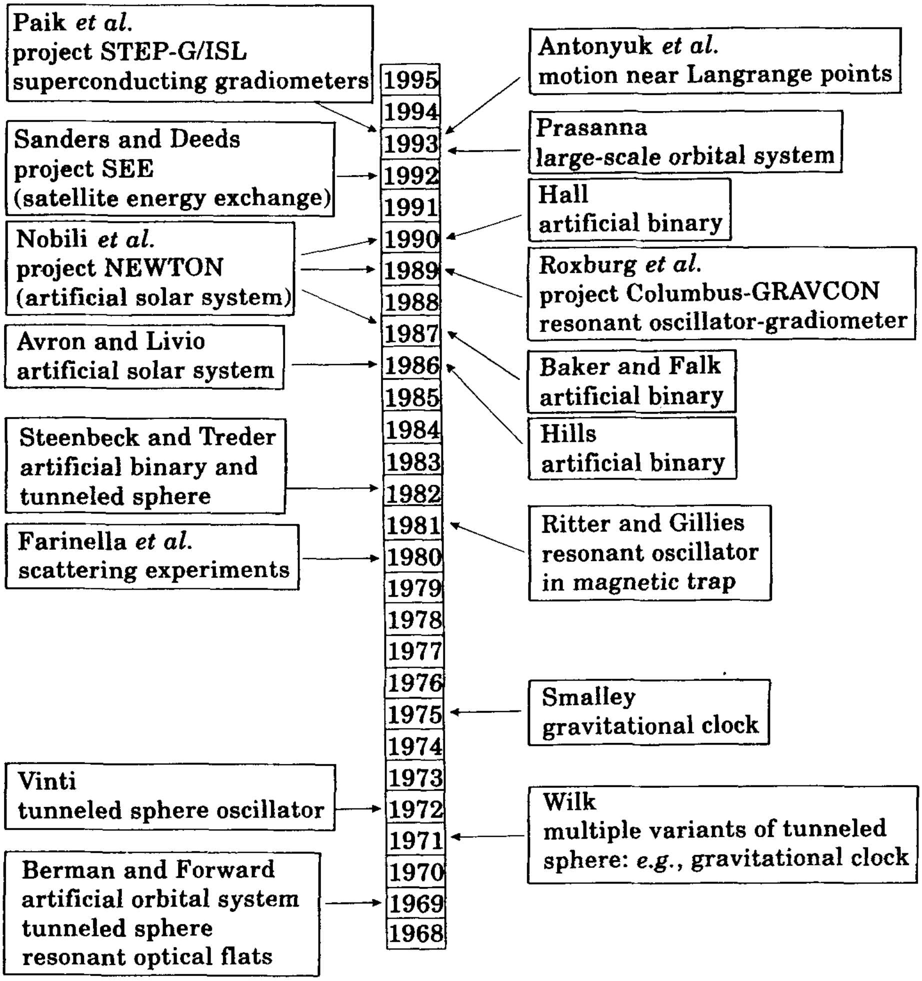

Fig. 2. – A timetable of several of the proposals for the determination of G in space.

long and 3 m in diameter, as illustrated in fig. 3. The entire satellite would be placed in geosynchronous orbit ($R = 42\,200$ km) in order to minimize tidal effects. The capsule is to be spin-stabilized, rotating roughly once per minute, with its axis parallel to the Earth's axis of rotation. A disturbance compensation system (DISCOS) would control thrusters to counteract the solar-radiation pressure. The test bodies would be located near the center of mass of the capsule. The orbital plane of the test-body motion would be nearly perpendicular to the cylinder axis, so that the expected anisotropy of blackbody radiation between the two ends of the capsule would be roughly normal to the mutual gravitational attraction of the two test bodies. The capsule would be operated at an ambient temperature of ~ 300 K. A preliminary form of this proposal was published earlier (Nobili *et al.* (1987)). In addition to a determination of G, a NEWTON mission is expected to achieve several other objectives, as discussed below in sect. 9.

The NEWTON apparatus is essentially a conventional artificial solar system, and therefore its dynamics are generally familiar and in principle well understood: during

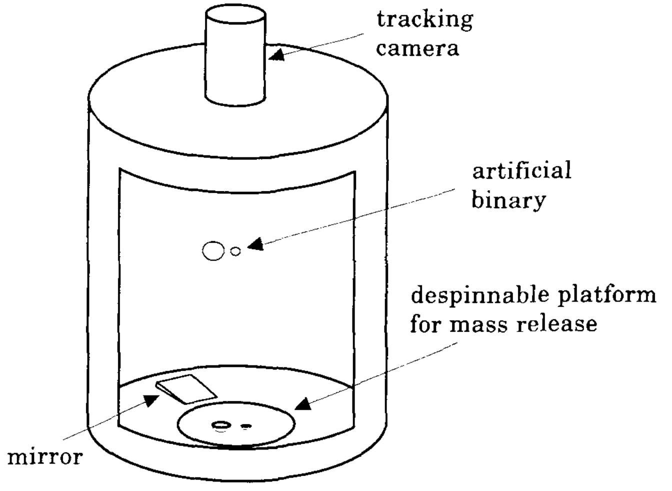

Fig. 3. – Schematic diagram of the NEWTON capsule showing the enclosed artificial orbital system. The artificial planet and satellite (the large and small test bodies, respectively) are located near the center of mass of the capsule. The relative motion of the test bodies would be tracked by the camera and lights shown at the top. The cylindrical capsule would be spin-stabilized about its symmetry axis, which would be parallel to the Earth's axis. The spin axis of the artificial planet and the orbital angular momentum of the artificial solar system would also be in this direction. Small hydrazine-powered thrusters (not shown) on the sides of the cylinder would fire about every 30 minutes to offset the drag due to solar-radiation pressure (atmospheric drag is insignificant at the altitude of the planned geosynchronous orbit). The hydrazine fuel would be stored as far away from the test bodies as possible to reduce perturbations, especially those due to sloshing. Reproduced from Nobili *et al.* (1990) with the permission of the authors.

the past three and a half centuries, few problems in physics have received more scrutiny than the problem of an orbital system with a hierarchy of satellites. Therefore, orbital systems are naturally among the most-often proposed means for space-based determinations of G. (EPIC proposals using orbital systems include Berman and Forward (1969), Steenbeck and Treder (1982), Hills (1986), Avron and Livio (1986), Baker (1987) and Hall (1990).) Two of these proposals (Steenbeck and Treder (1982) and Hills (1986)) also entailed tests for non-Newtonian effects.

Project SEE

The SEE proposal (Sanders and Deeds (1992*a* and 1992*b*) and Sanders *et al.* (1993)) employs a novel test-body configuration, namely the encounter phase of George Darwin's horseshoe orbit (Darwin (1897)). In the SEE method, a large mass and a small test-body «float» freely inside the SEE capsule as independent, co-orbiting satellites of Earth. The small mass is repeatedly «launched» toward the large mass, and then the mutual perturbation of the two bodies during the encounter is analyzed. As a consequence of the virial theorem, they appear to repel each other, a seemingly paradoxical result and a novel feature of this arrangement. The duration of a single encounter is typically one to three days, and the relative motion of the two bodies during an encounter is very nearly one-dimensional and in the «along-track» direction. The SEE type of interaction is illustrated in nature by Janus and

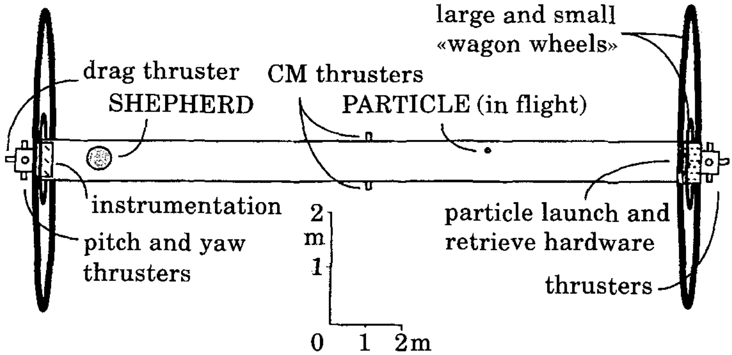

Fig. 4. – Schematic diagram of the SEE capsule showing the enclosed large and small test bodies (the shepherd and particle, respectively). The particle would be launched toward the shepherd every few days, and the relative motion of the test bodies during the encounter would be tracked interferometrically as the particle first approached and then receded from the shepherd. The masses and radii of the large rings shown at each end of the capsule are chosen so that their gravitational field within the cylinder nearly cancels the field due to the cylinder itself, thus making the entire capsule «gravitationally invisible» to the test bodies. The same purpose can be achieved, or alternatively the field can be fine-tuned, by careful design of the distribution of mass along the cylinder walls. The capsule would be oriented with its long axis approximately parallel to its orbital velocity (like an arrow in flight). Station keeping would normally be accomplished by jerk-free thrusts and torques obtained from solar-radiation pressure and interactions with the Earth's magnetic field. (The thrusters shown are only for auxiliary purposes.)

Ephimetheus, the co-orbiting satellites of Saturn (Dermott and Murray (1981*a* and 1981*b*), Yoder *et al.* (1983)).

The SEE capsule is to be a long thin cylinder (~ 20 m long, 1 m diameter), shown in fig. 4. Its mass will be distributed so that the internal local gravitational field is essentially zero. A low Earth orbit (altitude 1500 km) is planned, with its parameters chosen to make it Sun-synchronous and also to keep the nodal line nearly perpendicular to the Earth-Sun line. The capsule is to be oriented with its axis approximately parallel to its velocity vector, like an arrow in flight, which also makes the cylinder axis roughly perpendicular to the Earth-Sun line. Thus, this axis must «tumble» once per orbit. A «forward-thinking» (predictive) DISCOS system would maintain attitude control and counteract atmospheric drag by means of jerk-free torques and thrusts. The SEE capsule would be operated at moderate cryogenic temperatures (*e.g.*, ~ 78 K) to minimize blackbody radiation. Novel radiation barriers would provide extremely precise thermal stability, in order to minimize the anisotropy of thermal radiation and capsule warping due to thermal expansion (Sanders and Deeds (1992*a* and 1992*b*)). The capsule may be oriented with the same side always toward the Sun, or, alternatively, it may be spun about the cylinder axis at about 0.5 Hz. (If the latter case, the axial component of angular momentum will be canceled by a counter-rotating reaction wheel, so that virtually no applied torques will be needed to achieve the once-per-orbit tumble.)

As discussed in sect. 8 and 9 below, a SEE mission is also intended to establish new upper bounds on violations of the inverse-square law, the equivalence principle (EP), and possibly $|\mathrm{d}G/\mathrm{d}t|$.

Several years prior to the SEE proposal by Sanders and Deeds, the principle of the SEE-type interaction and its relation to the co-orbiting satellites of Saturn had been pointed out in an unpublished manuscript by Baker and Falk (Baker and Falk (1987*a*)). However, this aspect of their manuscript was not mentioned in subsequent materials that ultimately were published. These included a brief NASA technical report (Baker (1987)) and two abstracts (Baker and Falk (1987*b*) and Hall (1990)) (2).

STEP G/ISL

A determination of G is planned as a co-experiment on the STEP (Satellite Test of the Equivalence Principle) mission rather than as the primary objective, as in the case of NEWTON and SEE. This co-experiment design, known as G/ISL, is described in the 1993 STEP Phase A Study Report (Blaser *et al.* (1993)). The previous STEP Assessment Study Report had included a brief discussion of preliminary plans for a determination of G at a modest accuracy level (10 to 1000 ppm) employing the same accelerometers used for the equivalence-principle test. The present G/ISL proposal is very sophisticated, not to say virtuoso, in its approach to the experiment's design (Blaser *et al.* (1993 and 1994), Paik and Blaser (1993), Reinhard *et al.* (1994), and Paik (1994)).

The STEP G/ISL apparatus consists of a matched pair of accelerometers located at opposite ends of the STEP quartz reference block. Each accelerometer, in turn, consists essentially of a small cylindrical source mass moving back and forth inside of two hollow cylindrical (3) test masses. The source mass is approximately 2 cm long and 2 cm in diameter, and its mass is 93.6 g. The mean diameters of the «inner» and «outer» test masses are about 4 and 6.5 cm, respectively. Both are about 9 cm long and have masses of about 300 g. All the masses are magnetically levitated. They are mounted coaxially, with the common axis being essentially that of the dewars which contain the apparatus for all the STEP experiments. Minute axial movements of the test masses are detected by SQUIDs. The common axis of the STEP dewar and the G/ISL masses is to be oriented perpendicularly to the orbital plane of the STEP satellite. A sketch of one of the G/ISL accelerometers, consisting of the source mass inside the «inner» and «outer» test mass, is shown in fig. 5. The G/ISL masses are isolated electromagnetically from each other by superconducting shields. The operating temperature of all STEP experiments, including G/ISL, is to be 2 K, with a view for keeping Brownian-motion disturbances of the test bodies to a minimum. A recent revision in plans now calls for keeping the helium in the outer dewar supercritical (and thus at a somewhat higher temperature) in order to eliminate the liquid-gas interface, which is intended to minimize any potential problem due to «sloshing» of the helium (Paik (1995)).

The two source masses are driven back and forth along the axis at about 0.003 Hz (period ~ 5 min). They are driven synchronously and in opposition so that the pair of accelerometers can operate as a *differential* accelerometer. Differential acceleration analysis is expected to provide a very effective filter against common-mode

(2) The SEE proposal was conceived without knowledge of Baker and Falk's earlier unpublished manuscript. We are pleased to set the record straight at this point by acknowledging here and drawing attention to the prior independent work by Baker and colleagues.

(3) Recent plans call for adding flat surfaces to the test masses of the other (non-G/ISL) STEP experiments, making their cross-sections nearly square, in order to avert coupling of rotation to axial movement via the patch effect (Lockerbie (1996)).

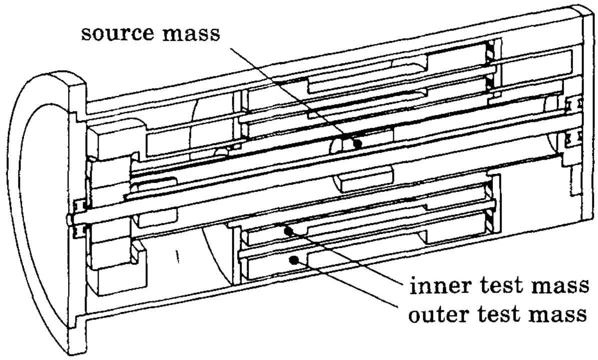

Fig. 5. – Cutaway view of one half of the STEP-G/ISL apparatus. The other half of the apparatus, 75 cm away along the symmetry axis (not shown), contains a mirror image of the instruments shown here. The inner test masses on the two ends form a gradiometer. The outer test masses form a separate gradiometer. The source masses are constrained to move along the axis by magnetic bearings. The gravity signals from the moving masses are detected by the two gradiometers to perform the G and ISL (Inverse-Square-Law) experiments. Reproduced from Blaser *et al.* (1993) with the permission of the author. Figure 10 below is a detailed schematic view of a single G/ISL test mass.

accelerations. Moreover, the source masses are driven with a wave form that allows them to spend most of their time near the points where the coupling to the test masses is at a maximum. The presence by design of this local maximum in the gravitational force is exploited to reduce accuracy requirements in distance measurement. This technique has been used by Hulett, Koldewyn and others (Hulett (1969) and Koldewyn (1976)) in terrestrial determinations of G. Finally, of course, the analysis will be phase-sensitive at the driving frequency of the source masses, thus filtering out much of the background at other phases or frequencies.

The STEP satellite is to be placed in a low ($h \approx 550$ km), nearly polar ($i \approx 97.6°$), nearly circular ($e \approx 0.001$), and Sun-synchronous orbit. With proper choice of the right ascension of the ascending node, the STEP satellite will be in continuous sunlight for about eight months, which is somewhat more than the design mission life. The sensitivity of the STEP EP experiment to the effects of the South Atlantic Anomaly in the Van Allen belts is the principal motivation for selecting a low-altitude orbit, and the launch year was also chosen with an eye to minimizing these effects. To wit, the average rate of charging of the lower Van Allen belts is paradoxically *lowest* during periods of high solar activity, since high activity heats and expands the atmosphere, thus increasing the shielding againt solar protons. However, the years of the solar maximum itself must be avoided because of the likelihood of solar flares.

Note that the SEE and STEP orbits are both Sun-synchronous with planes approximately perpendicular to the Earth-Sun line. SEE is just above the minimum altitude for year-round continuous sunlight; STEP is lower for shielding from solar photons, and therefore continuously sunlit only part of the year, but long enough for an 8-month emission.

The G/ISL experimental design is an interesting offshoot of the STEP design philosophy, as previously conceived for the EP experiment. Obvious similarities include accelerometer construction and the strong emphasis placed on methods of obviating sources of errors rather than correcting for them. For example, cylindrical geometry is used for the test mass to minimize metrological problems, and the common low temperature within the dewar is expected to minimize a number of potential thermal errors.

Alternative test-body dynamics and geometries: the EPIC proposals

In the remainder of this section we briefly recount a number of less well-known test-body configurations which have been proposed for determining G in space. Discussion of most of the systems described in this section can be found in the previous reviews by Smalley and Spallicci (Smalley (1975), Spallicci (1989)).

An unusual approach to test-body dynamics was the «scattering-experiment» encounters which were considered briefly by the Pisa group (Farinella, Nobili *et al.* (1980)). These entailed observing the motion of a small particle in the vicinity of the Lagrange point L_2 of a larger body in Earth orbit. Several manifolds of trajectories were identified theoretically and explored by numerical modeling. Figure 6 shows two such trajectories. These authors have subsequently turned their efforts to the artificial-solar-system approach of Project NEWTON. A similar but somewhat more detailed investigation, which also entails studying the motion

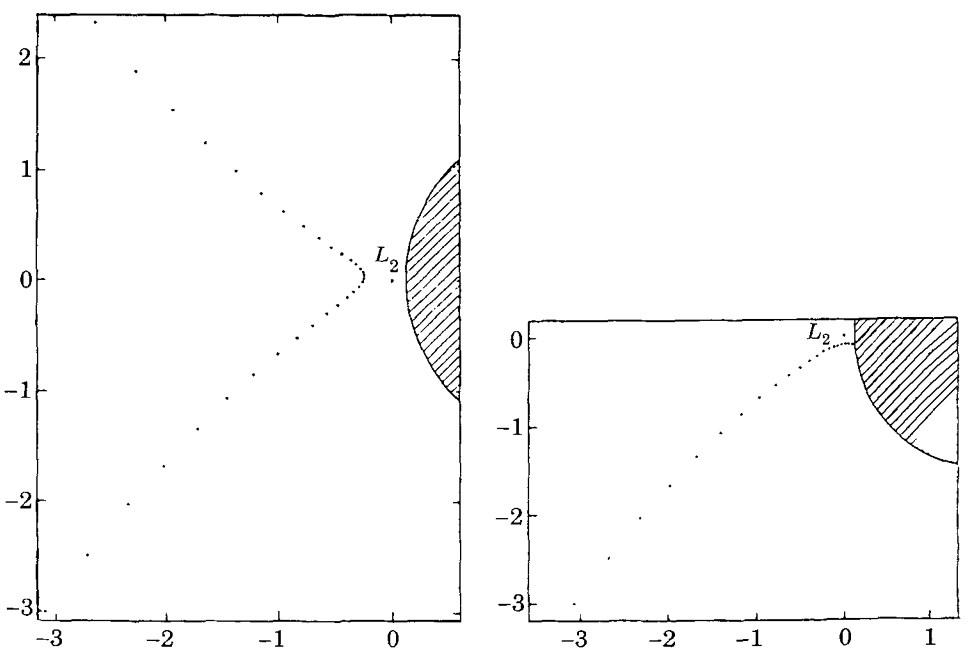

Fig. 6. – An early exploration of the dynamics of non-gravitationally-bound test bodies. Numerical simulations of the motion of a small test body in the vicinity of a large test body were carried out by Farinella, Milani, and Nobili using a wide range of experimental parameters. Trajectories were classified according to whether the parameter values caused the small body to veer away from the large body (fig. 9*a*)) or to collide with it (fig. 9*b*)). The interval between dots is 86.4 s, so the durations of the simulations were about 50 min and 30 min, respectively. No quasi-stable trajectories suitable for determining G were identified. Reproduced by permission of the authors and the publisher.

of a particle in the vicinity of Lagrange points of a larger body, has recently been carried out by a Russian group (Antonyuk *et al.* (1993 and 1994)).

Several oscillator systems have been proposed in which a small test body would be placed in a tunnel passing through a large mass. The value of G would be inferred from the natural frequency of oscillation of the small body as it passed back and forth through the large mass under the influence of its gravity. A great deal of attention has been given to such systems in which the large mass is a sphere with a tunnel passing through its center (Berman (1967*b*), Berman and Forward (1969), Chapman (1971), Wilk (1971), Vinti (1970 and 1972), and Steenbeck and Treder (1982 and 1984)). In this instance the oscillator would in principle be harmonic, since the restoring force is simply proportional to the displacement of the small body from the center of the sphere, neglecting the mass lost by drilling the tunnel.

The proposal of Steenbeck and Treder (1982) also anticipated tests for two non-Newtonian effects, namely partial gravitational shielding, in the framework of the Majorana equation, and the existence of gravitational waves. Further, they point out that their proposed apparatus is unique among gravitational-wave detectors, in that it entails no mechanical restraints of the test bodies (Steenbeck and Treder (1982), p. 280).

Other oscillator systems have been proposed employing principles similar to that of the tunneled sphere, in that the value of G would be inferred from the natural oscillation frequency of the small body as it passed back and forth through the large mass. These include systems in which the hole is drilled through a long cylinder, as proposed by Worden and Everitt (Smalley (1975), p. 11 and p. 16). This system was actually designed as an Eötvös-experiment apparatus [4]. Smalley, in turn, proposed using a very short cylinder—virtually a disk—as the attracting body with the hole through it (Smalley (1975)). He called the arrangement the «flat-plate spherical mass oscillator».

An oscillator proposed by Wilk entails a rigid three-armed cross on an axle for mounting three small oscillating masses, which are attached at the ends of the arms (Blood (1971)). This apparatus, called the «rotor», is surrounded by a set of three identical tunneled spheres, called the «stator», which are positioned so that the small masses are free to move back and forth through the tunnels as the arm rotates.

Finally, in a terrestrial version of the tunneled-sphere oscillator, two small masses are suspended from the arms of an ordinary beam balance in such a way that they are located inside two vertical tunnels drilled into the *same* sphere (Berman and Forward (1969)).

The term «gravitational clock» is sometimes applied to any of these systems, signifying that they are all approximately harmonic oscillators and that G would be inferred from the natural frequency of oscillation. They all would enjoy the advantage in principle that the raw data are chiefly time measurements, which have negligible resolution problems. However, these proposals suffer from the generic problems faced by tunnel-in-sphere arrangements, as discussed below in sect. 4 and 5.

Several proposed systems are based on a resonance principle, wherein the excitor masses are moved at a constant rate and G is inferred chiefly from the amplitude of

---

[4] From a historical point of view, it is interesting to observe the evolution of the concepts of the STEP and G/ISL accelerometers from early papers and another early report by Everitt (1977).

oscillation of the small masses, rather than from their natural frequency, since their motion is phase-locked to the pre-determined driving frequency. To our knowledge all proposed resonant systems have called for an axle to mount and move the excitor mass(es). One such system employs optically flat rectangular plates which are parallel and nearly touching (Berman (1967*a*)). The excitor plate is mounted on the axle, which is turned at a constant rate, and the other plate is on a resonant mount. By choosing the dimensions of the plates carefully, it is possible to eliminate the quartic term in the potential, thus making the oscillator very nearly harmonic. Another system involves an excitor consisting of a sphere moving along a circular path at a constant rate. The oscillating mass in this case is another sphere located at the middle of the circular path and confined to small displacements by a very soft magnetic trap (Ritter and Gillies (1981 and 1984)).

A number of apparatuses, including most rotating resonance systems, could in principle be operated in the Beams mode—that is, the attracting mass(es) could be continuously accelerated so that a constant angular orientation is maintained relative to the small-mass rotor. Blood discusses a variant of the three-armed gravitational clock designed for operation in the Beams mode. This device, which has four-fold symmetry, has no tunnels in the spheres, since there is no longer any need for the small masses to move back and forth through the attracting spheres (Blood (1971)).

A unique variant on the tunneled-sphere scheme was the MIT group's complex three-tunnel centrifugal balance (Wilk (1971)), which relied neither on natural frequency nor on resonance to infer G. Rather, this device was designed to measure the gravitational attraction of the sphere on three small test masses within its three mutually orthogonal tunnels by observing their positions as a function of the sphere's angular velocity. Curiously, the orientation of the rotation vector was chosen arbitrarily, rather than perpendicular to one or two of the tunnel axes. Consequently, the centrifugal forces would not be even approximately parallel to the tunnel axes, thus requiring strong constraints to hold the test bodies on the axes.

3. – Background forces in the experimental arrangements

As indicated above, the EPIC proposals are generally overviews which forgo discussion of the extreme strictures of a precision-measurement experiment. Although this approach is useful for introducing measurement concepts, it often leaves unexplored various systematic effects which would ultimately limit experimental accuracy. For example, some authors (Vinti (1970 and 1972) and Avron and Livio (1986)) proposed G experiments on board-manned spacecraft. However, the reasons that terrestrial determinations of G typically require the experimental apparatus to be isolated from human activity also generally apply in space. In fact, the proximity of the astronauts and the activity associated with their work would pose a virtually insuperable problem. This is, of course, because the mass distribution in the vicinity of the test bodies would be continually varying, and also because the vibrations induced in structural supports by impulses due to astronauts might vitiate measurements of test-body accelerations (probably to an even greater extent than in terrestrial experiments, which are typically situated in a quiet location and/or mounted rigidly on a large inertial mass, such as a block of stone or some other type of stable platform). Effects of this kind are likely to spoil the measurement of G at the level of one part in 10^2 or 10^3, and thus be three or four orders of magnitude too large to allow a measurement of G to one part in 10^6. Most of the EPIC proposals did not investigate the significance of such problems. Those solutions which were proposed, including one to ask the astronauts to keep as motionless as possible for certain periods of time (Vinti (1972), p. 224), would likely be either unworkable or insufficient.

Although a comprehensive, quantitative assessment of experimental errors was outside the scope of the EPIC proposals, some did attempt to identify the most relevant sources of uncertainty. For example, Berman and Forward pointed out the issues of the gravitational force due to the mother ship and of non-sphericity and density inhomogeneities in test bodies (Berman and Forward (1969)). Moreover, regarding the former problem, and in a separate paper, Forward provided a detailed theoretical description of a method for flattening space-time (Forward (1982)). The obvious need to offset atmospheric drag is discussed in most of the EPIC proposals; see for example Vinti's comments on a «drag free laboratory» (Vinti (1972), p. 209). However, these proposals seldom took account of the accelerations due to solar-radiation pressure, even though they are typically larger at altitudes over 500 km than accelerations due to atmospheric drag (see for example Braginski and Manukin (1977)).

As indicated above, Wilk identified a wide range of effects which would enter his error budgets (Wilk (1971)). Moreover, for example, in the case of that document's description of the space adaptation of the Beams experiment, the appendices therein provided a comprehensive and quantitative treatment of such effects (Blood (1971) and Lee (1970)). (A thorough analysis of the experimental errors foreseen in the originally conceived terrestrial version of the Beams experiment was undertaken at the University of Virginia in the 1960s (Kramer (1967)).) Unfortunately, however, Wilk's analyses of other potential methods for measuring G were narrowly focused on the problems arising from only a few idealized effects, despite the fact that these analyses were extremely detailed. The resulting lack of comprehensiveness left many potentially important effects unaccounted for and, in some instances, entailed unfeasible suggestions. For example, although Wilk recognized the need for drag-free capability to offset atmospheric drag, he suggested a solar orbit as a second-choice alternative. This would in fact be much worse, unfortunately, since a capsule in solar orbit would be displaced several hundred meters radially by solar-radiation pressure, thus entirely preventing the test bodies from floating freely within the capsule as intended.

The NEWTON, SEE, and STEP proposals devote considerable attention to the effects of solar-radiation pressure. The SEE and STEP proposals, in particular, examine these topics in detail. The acceleration due to solar-radiation pressure may vary greatly, and, moreover, acceleration discontinuities would actually occur upon entering and leaving the Earth's shadow (Sanders and Deeds (1992*a*)). Eclipses can also lead to difficult thermal-control problems (Barlier *et al.* (1991)). The SEE and STEP satellites would largely obviate these problems by using Sun-synchronous orbits that are in continuous sunlight (non-eclipsing) and in planes roughly normal to the Earth-Sun line.

4. – Gravitational kinematics of the test masses

All proposals for gravitational measurements in space with free-floating test bodies entail the restricted three-body problem. The mathematical tasks of finding equations which represent the forces and then manipulating these expressions to derive the resulting equations of motion have proven to be surprisingly difficult. In this section we discuss the approaches taken by various investigators and the kinds of difficulties which have arisen.

Most proposals for measuring G in space, including all artificial-binary or artificial-solar-system and tunnel-in-sphere arrangements, share a treacherous conceptual starting point: a postulated far-away location where there are no other gravitational fields or sources that would complicate the pure two-body interaction. A typical statement is: «The "cleanest" possible measurement of G would be to place two balls of known mass in orbit [around each other] far from any gravitating body» (Avron and Livio (1986)). Unfortunately, the inexorable next step away from this idealization is to note that the experiment will be in Earth orbit and then to introduce the Earth's actual field as a *perturbation*. This is a problem because the Earth's field is so dominant: To a very good approximation, each test body is independently in orbit around the Earth, and it is instead the interaction between the two test bodies which constitutes the small perturbation. The inevitable result of treating such a large force as a perturbation is a quagmire of mathematical difficulties in deriving and interpreting the equations of motion, particularly for experiments in low Earth orbit. In short, it is as if the mind of the physicist yearns for a corollary of Newton's first law which would hold that, in the absence of any mutual interaction, two bodies in very similar orbits will maintain a constant relative position in some sense. This cannot be, of course. As a tacit assumption, however it seems to have made its way into most G-in-space proposals.

In point of fact, if the two test bodies were completely non-interacting, then each would travel in an essentially Keplerian orbit, and therefore their actual relative motion would be that each body would execute cycloids with respect to the other. (The size and phasing of the cycloids would depend on how the orbital elements differed.) Thus, unless the two bodies were extremely close, the result of their very weak mutual gravitational attraction would be to slightly perturb the shape and precession rate of these cycloids. As an example of the kinematics of such a situation,

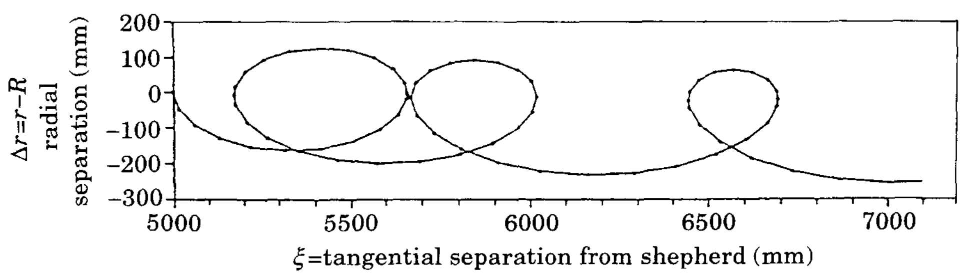

Fig. 7. – Cycloidal motion in a SEE encounter. The particle (not shown) travels in a series of cycloidal loops as it approaches from the right and moves toward the shepherd (not shown), which is located 5 m to the left at the coordinates $(\xi, \eta) = (0, 0)$. (Note offsets of the origin on both axes.) In this illustration the position of the particle at its point of closest approach to the shepherd is at the extreme left of the graph, *viz.*$(\xi, \eta) = (5\,\mathrm{m}, 0)$. The particle then recedes along a very similar cycloidal path, which for clarity is not shown. Each loop in the path corresponds to one orbital period of the spacecraft (about two hours). In the special case that the particle and the shepherd both have zero eccentricity asymptotically, the path is basically U-shaped and lacks the cycloids (see fig. 4 of Sanders and Deeds (1992a)).

see fig. 7, which shows a numerical simulation of the trajectory of the particle in the SEE experiment. This simulation assumes that the particle's mass is negligible and that the shepherd is in a circular orbit (Sanders and Deeds (1992*a* and 1992*b*)); this is an extension of the work of Darwin (1897). Quite similar results were found previously by Baker and Falk (1987*a*). More recently, very extensive simulations by Melkinov and colleagues (Melkinov *et al.* (1993), Bronnikov *et al.* (1993*b*) and Alekseev *et al.* (1994)) have also confirmed these findings ([5]). Their simulations also showed that a non-circular orbit for the shepherd does not vitiate the stability of a SEE encounter or its utility for determining G.

The non-zero *gradient* of the Earth's gravitational force is generally identified as the central «problem» by those authors who try to represent the Earth's field as a perturbation on an idealized force-free region. The tunnel-in-sphere proposals are especially hobbled by the approach of treating the Earth's field as the perturbation. The proponents envisioned the particle trajectories as simple oscillations along a straight line through the center of the sphere. Although in an ideal force-free location this motion would indeed occur, in Earth orbit the body within the sphere will not travel along a straight line through the tunnel in the sphere; rather it will execute a cycloid almost as if the sphere were absent, and this would lead to a collision with the tunnel walls. However, if the tunnel were oriented perpendicular to the satellite's orbital plane, then the oscillation of the body would merely vary its orbital plane, not its energy, which would essentially obviate the cycloidal motion and, hence, any collisions with the tunnel walls. (Ironically, Berman and Forward recognized the desirability of this tunnel orientation, but only because it would minimize the dependence of the oscillation period on the Earth's field gradient. They made no mention of the issue of collisions (Berman and Forward (1969)).) G/ISL is essentially a sophisticated tunnel-in-sphere scheme, except now it is the «sphere» which moves, and the system is non-resonant. G/ISL also circumvents the cycloid/collision problem by orienting the axis of motion perpendicular to the orbital plane. This experiment relies very heavily, of course, on restraining forces.

A further problem is that the orbital plane of the satellite will precess (see, for example, Kaula (1966)), except for polar and equatorial orbits. The EPIC proposals do not mention the need either to choose such a special orbit or to otherwise make the sphere rotate in order to keep the tunnel perpendicular to the orbital plane. Moreover, they presented no analysis concerning how the direction of oscillation of the test body in such arrangements might also precess.

This instance of the neglect of a kinematical effect illustrates a widespread, but understandable, weakness in proposals for determining G in space. Since the three-body problem poses difficult challenges in even the simplest cases, and since a number of germane results of celestial mechanics are not widely known outside the

---

([5]) The original SEE proposal had found that the trajectory of the particle with respect to the shepherd would be simply U-shaped (without cycloids) if the shepherd were in a circular Earth orbit and if the particle had also originally been in a zero-eccentricity Earth orbit (at asymptotic distances from the shepherd). However, simulations by Melnikov *et al.* (1993) of the SEE interaction found that a SEE encounter invariably changes the eccentricity of the particle's Earth orbit slightly, thus inducing small oscillations in its trajectory relative to the shepherd as it recedes from the shepherd. These oscillations are pronounced if the encounter distance is 2 m or less.

astrodynamics community, orbital-kinematic effects have often been overlooked or oversimplified. Neglecting precession effects is especially likely to cause problems, since these effects accumulate over a long period of time. Thus, even if the conceptual model of an interaction is accurate in the short term, precessions or other subtle long-term effects which escape scrutiny might make the actual system unstable over the long periods of time needed to attain the desired accuracy.

For one variant of the tunneled-sphere experiment, *viz.*, Wilk's three-tunnel centrifugal-balance method, it was proposed that auxiliary forces be applied to keep the small test body on the tunnel axis. For this purpose restraints were proposed, including those involving gimbals and electrostatic forces (Vinti (1972)). However, the authors made no mention that collisions with the tunnel walls would occur even in the basic single-tunnel version in the absence of restraints. Unfortunately, the auxiliary forces applied by such restraints, in any variant of the tunneled-sphere method, would potentially be comparable to the mutual gravitational attraction of the test bodies.

A simple way of estimating the required size of these auxiliary forces is to note that unrestrained bodies of mass m at orbital radius $\sim R$ would, as mentioned above, execute relative cycloids that consist of essentially harmonic motion in two dimensions at the orbital frequency $\omega = \sqrt{(M_{\mathrm{E}} G/R^3)}$, which, therefore, entails apparent forces of roughly $m\omega^2 x = (mM_{\mathrm{E}} G/R^3)x$, where x is an oscillation amplitude and M_{E} is the mass of the Earth. The auxiliary force F_{A} supplied by the restraints must clearly be of similar size in order to alter this motion (*e.g.*, to restrict motion to a straight line). Comparing F_{A} to the maximum gravitational attraction F_{M} which the tunneled sphere can supply, we find that the minimum ratio of the forces is

$$F_{\mathrm{A}}/F_{\mathrm{M}} \sim (mM_{\mathrm{E}} G/R^3)\,a/(mMG/a^2) = (\varrho_{\mathrm{E}}/\varrho_{\mathrm{M}}) \times (R_{\mathrm{E}}/R)^3\,, \tag{1a}$$

where M is the mass of the tunneled sphere, a is its radius, and ϱ_{E} and ϱ_{M} are the mean densities of the Earth and the tunneled sphere, respectively (we have set the oscillation amplitude equal to a). For a heavy metal, the density ratio can be slightly less than 0.3 and, hence, the minimum ratio of the forces becomes

$$\begin{cases} F_{\mathrm{A}}/F_{\mathrm{M}} \sim 0.001 \text{ in geosynchronous orbit } (R \sim 6.6\,R_{\mathrm{E}})\,, \\ F_{\mathrm{A}}/F_{\mathrm{M}} \sim 0.2 \text{ in low Earth orbit } (R \sim 1.2\,R_{\mathrm{E}})\,. \end{cases} \tag{1b}$$

Auxiliary forces of this size are unacceptable unless they can be made very accurately perpendicular to the gravitational attraction. To achieve 1 ppm accuracy in G, the perpendicularity error would therefore have to be below 1 milliradian in geosynchronous orbit or 5 microradians in low Earth orbit. Although precise perpendicularity is very naturally achieved by a Cavendish balance in terrestrial experiments (see, for example, Gillies and Ritter (1993)), no such guarantee of perpendicularity exists in the restraints proposed by the MIT group.

An important orbital-dynamics consideration is that none of the G/ISL test masses is traveling quite in the plane of an Earth orbit, but, rather, all are slightly displaced from the plane in which the c.m. of the STEP satellite would travel if it were fully drag compensated. Therefore the test masses must be held back from this plane by some small restraining force, which must be stable (albeit not *known*) to great accuracy. To this end, the force is supplied by means of a persistent current in a type-I superconductor, which is therefore extremely stable (Blaser *et al.* (1993), Paik (1994), and Paik (1995)). We note, however, that the force *per se* may vary slightly due to displacements of the mass. Since the force must be parallel to the gravitational force which is being measured, the absolute error in determining the restraining force translates almost directly into an absolute error in the force which is the object of the G determination.

For the artificial-solar-system proposals, the problem of the Earth's gradient has also proved to be particularly vexing in two respects. First, it makes the equations of motion complicated, in contrast to those which would apply in an ideal force-free location. Second, and more importantly, it raises disturbing questions about orbital stability. The mutual attraction of the test bodies may actually be less than the difference between the Earth's force on them at different points in their orbits, unless either the large test mass is on the order of several tons or the orbits are so small that the test bodies are almost touching.

However, both of these difficulties may be more perceptual than actual (Hénon (1974)). First, there is no intrinsic reason why complicated equations of motion should preclude an effective experiment. Nevertheless, the resulting barrier to insight may diminish the chance for optimal trajectory design and, more seriously, increase the chance of overlooking sources of error. As to the orbital-stability question, the review by Baker finds that this concern is largely unfounded, at least for retrograde motion when there are fewer than 25 orbital revolutions (Baker (1987)). Actually, the Earth-moon system is also weakly bound by the same standard, *viz.*, the strength of the Earth's field at 380 000 km *vs.* the gradient of the Sun's field multiplied by the Earth-moon separation; the ratio is only 7 : 3. In short, the net effect of these two «problems» has probably been only to create a distraction to the space-gravity community.

On the other hand, in the event that non-retrograde orbits of test bodies are generally unstable when the satellite is in low Earth orbit, then it would seem that orbital-precession effects would limit observation times to a few months, unless strategies can be devised to cause the orbital plane of the artificial solar system to precess synchronously with the satellite's orbit. NEWTON has minimized this problem by making most of the relevant angular momenta parallel. The orbital angular momentum of the artificial solar system is to be parallel to the rotational angular momenta of the capsule, the large test body, and the Earth, which will essentially obviate precession. Nevertheless, occasional orbit corrections will still be necessary because of solar and lunar perturbations, since the relevant angular momenta are not parallel to those listed above (Nobili *et al.* (1990)).

The SEE method (Sanders and Deeds (1992*a* and 1992*b*) and Sanders *et al.* (1993)) does not encounter these orbital-kinematics problems because it is based intrinsically on orbital-perturbation techniques. Its starting point is, very naturally, the Earth's actual field, not an idealized force-free region. The SEE method recognizes that the test bodies are predominantly *satellites of the Earth*, not of each other, and it then infers the gravitational interaction between the test bodies from the perturbations of their Earth orbits. This point of view also results in a very straightforward formulation of the equations of motion. The Lagrangian for the co-planar bodies, if one body has negligible mass, is

$$L = (1/2)\, m(\dot{r}^2 + r^2 \dot{\theta}^2) + M_{\mathrm{E}} mG/r + MmG/s \tag{2}$$

and therefore the equations of motion of the particle are immediately seen to be

$$\ddot{r} - r\dot{\theta}^2 = -M_{\mathrm{E}}G/r^2 - (MG/s^3)\eta \tag{3}$$

and

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -(MG/s^3)\xi\,, \tag{4}$$

where s is the separation of the test bodies, r and θ are the polar coordinates of the particle, and m, M and M_{E} are the masses of the particle, the shepherd, and the Earth. The rectangular coordinates (ξ, η), which represent essentially the along-track and radial directions at the location of the particle, are defined by

$$\xi = R\sin(\theta - \Theta)\,, \tag{5a}$$

$$\eta = r - R\cos(\theta - \Theta) \tag{5b}$$

and

$$s^2 = \xi^2 + \eta^2 = |\boldsymbol{r} - \boldsymbol{R}|^2\,, \tag{5c}$$

where R is the orbital radius of the shepherd. Note that eqs. (3) and (4) reduce to the classical two-body problem if $M = 0$ or $s \sim \infty$. The equations of motion of the shepherd are simply $R =$ constant and $\Theta = \omega_0 t$.

The perturbation inferences are actually drawn from the *relative* motion of the test bodies (essentially, the trajectory of the smaller body relative to the larger). In short, the SEE method observes the natural orbital motion of the test bodies, perturbed only by each other and not by any artificial restraints. What is measured in SEE is, of course, the cycloidal motion of the test bodies. The small gravitational interaction between the test bodies results in a perturbation on those cycloids which would be followed by non-interacting bodies traveling in slightly different Keplerian orbits. Analyzing the relative motion yields the value of G and any dependence of G on the separation of the test bodies from each other or the Earth.

5. – Spacecraft and orbital-dynamics considerations

The success of any space-based gravity experiment depends on the design of the spacecraft (shielding capsule) and on the choice of orbit. Performance requirements of the spacecraft/capsule center on the need to:

1) shield the test masses from drag and external radiation pressure (especially solar);

2) provide a stable platform for all reference coordinate systems;

3) minimize gravitational forces due to the capsule itself and ensure that this force field has essentially no time variation.

The second requirement implies that the capsule should be jerk-free and nearly rigid, while rigidity, in turn, requires thermal stability. This and the jerk-free stipulation require careful choice of orbit. The EPIC proposals generally did not focus on these critical issues.

Special problems occur for an artificial binary or solar system with large test-body separations. These are mostly due to the requirement for a very large shielding capsule. Hills has made the most expansive proposal of this nature, suggesting test masses of several tons and a metal balloon several hundred meters in radius (Hills (1986)). The Pisa group also contemplated the use of a balloon (Farinella *et al.* (1979), p. 297) but subsequently pointed out that, with a flexible balloon, the necessary extreme precision with regard to the mass distribution of the balloon is unlikely to be attained (Nobili *et al.* (1988)). Certainly it cannot be known *a priori*, since such a large shield would have to be folded at launch and then assembled in orbit. However, it is now possible to measure with sufficient accuracy large masses such as that proposed by Hills (Debler (1991)).

The further suggestion to use an *inflated* balloon as the shielding capsule (Hills (1986)) would exacerbate these problems. The chief concern is for transient effects which would occur when forces were put on the balloon by the thrusters. Quasi-steady-state effects, in contrast, tend to be very small, according to our calculations. To wit, we estimate that both the centrifugal equatorial bulge due to rotation and the one-sided flattening due to solar-radiation pressure would be micron-size effects, assuming reasonable values for the gas pressure and for the elastic-constant properties of the balloon material.

Solar-radiation pressure would displace a large cross-section shield substantially from the orbits of the test bodies unless it were either very massive or else constantly pushed by compensating thrusters. For example, when the orbital plane is perpendicular to the Earth-Sun line, the displacement Δ of an uncompensated spherical shell is simply

$$\Delta = P_S R^3 A/(GM_E m) = kR^3 A/m, \tag{6}$$

where P_S is the solar-radiation pressure of $4.7 \cdot 10^{-6}$ Pa (assuming total absorption), R is the satellite's orbital radius, A is the cross-sectional area of the balloon (πr^2), and M_E and m are the masses of the Earth and the balloon. An obvious requirement for a practical working space is $\Delta \ll r$. In order to achieve $\Delta = r/10$ passively with a balloon of 350 m radius in low Earth orbit (altitude 1000 km), a balloon mass of 50 000 kg would be required, while in geosynchronous orbit ($R = 42\,200$ km) the balloon would need the impractically large mass of 10^7 kg.

Alternatively, if a low-mass balloon with an active compensation system were used, the system would have to supply a thrust of about 1.8 N almost continuously. Although the size of this thrust is modest, the fuel requirement for supplying it continuously would be enormous. A useful perspective is that the total impulse required during a one-year mission ($5.7 \cdot 10^7$ N·s) is enough to provide escape velocity to a 5-ton object.

Artificial-solar-system proposals employing small capsules would encounter relatively minor radiation-displacement problems if in low Earth orbit, but the problem is substantial at the geosynchronous radius. For example, the force on the NEWTON capsule, with its cross-sectional area facing the Sun of 11 m^2, would be about $5 \cdot 10^{-5}$ N, assuming total absorption. Equation (6) predicts that, without compensation, the capsule would be displaced a little over 23 meters at the geosynchronous radius, but only about 11 cm in low Earth orbit (1000 km altitude). Thus, at the high altitude planned for NEWTON, a compensation system is essential and must be in continual operation, providing the equivalent of a steady thrust of $\sim 5 \cdot 10^{-5}$ N. This will require 2 kg of hydrazine fuel per year, assuming a nozzle velocity of 0.8 km/s (Nobili *et al.* (1990)). However, there is some reason for concern about the jerking which may result from operating restrictions on the NEWTON DISCOS, as discussed below in sect. 6.

The forces on the test bodies due to convection currents within a gas-filled experimental chamber, as proposed by Hills, would be very difficult to model accurately, yet their effects and those due to viscous drag and Brownian motion might be too large to ignore. We note that, since the turn of the century, terrestrial determinations of G have very often been done in vacuum to minimize both viscous and free-molecular-flow gas couplings. The first to do so was Braun (Braun (1897)). The chief effect of Brownian motion is a stochastic wandering of the orbital velocity v of the small test body. The r.m.s. change during time τ at a pressure P and temperature T is given by the expression

$$\delta v = (r_b/m_b) \times (\beta\pi P\tau)^{1/2} \times (m_g kT)^{1/4}\,, \tag{7}$$

where r_b and m_b are the radius and mass of the test body, β is a shape parameter (on the order of unity for spherical bodies, and less than 2 for arbitrary shapes), m_g is the mass of a gas molecule of the residual gas in the capsule (assumed monatomic), and k is Boltzmann's constant.

As a practical matter, the threshold pressure, above which drag and Brownian-motion effects cannot be neglected, is about 0.02 Pa ($1.5 \cdot 10^{-4}$ torr). Note that eq. (7) indicates that a pressure higher than 0.06 Pa ($4.5 \cdot 10^{-4}$ torr) would be necessary to cause the orbital velocity of the NEWTON small test body to wander more than 1 ppm during a one-year observation period ($\beta = 2$; worst-case assumption). Very similar threshold pressures obtain for SEE and G/ISL. For example, although SEE's much shorter observation time (~ 3 days, *vs.* one year for NEWTON) is favorable, this effect is offset by the much lower mass of the small test body proposed for SEE (100 g, *vs.* 2 kg for NEWTON). Moreover, at the threshold pressure the residual gas would be close to the free-molecular-flow regime, with a mean free path on the order of 25 centimeters [6]. Similarly, the very small value of the product $m_g T$ in the last term of eq. (7) is favorable for G/ISL, but this is approximately offset by the extremely low velocity of the test masses ($\sim 10^{-5}$ m/s), so that the threshold pressure resulting in $\delta v/v \approx 10^{-6}$ is about the same for G/ISL as for NEWTON and SEE.

The actual pressure in the NEWTON, SEE and STEP capsules will of course be far lower, and therefore Brownian-motion effects should be negligible. However, outgassing may initially lead to a much higher effective pressure (Keyser (1993)), although this effect would die out within a year (Sanders and Deeds (1993)).

On the other hand, a large balloon which depends on inflation to maintain its shape, as proposed by Hills, would clearly require a relatively higher pressure, say 100 Pa (0.75 torr). At this pressure, the mass of the *small* test body would have to be over 700 kg in order to keep $\delta v/v$ in one year below 1 ppm, assuming $v \approx 10^{-4}$ m/s. Moreover, viscous drag clearly cannot be neglected at 100 Pa.

Finally, the inherent appeal of a spherical *shape* for the shielding capsule (*e.g.*, Hills (1986) and Antonyuk *et al.* (1993)), as if to endow it with a spherically symmetric *mass distribution*, is only fleeting, since the instrumentation mass cannot be

---

[6] $\lambda \approx 25$ cm for neon (a likely atmosphere for the capsule), at $P = 0.06$ Pa and $T = 190$ K, as calculated from expressions such as those given by Reif (1965).

distributed with spherical symmetry. Thus, the apparent advantage of starting with a spherical shape for the capsule is essentially nullified.

The choice of scale for an artificial binary or solar system poses an inherent dilemma: 1) If the separation of the test bodies as they orbit around each other is very small and nearly constant, and especially if the bodies are nearly in contact, then the determination of G is particularly sensitive to errors due to density inhomogeneities, spurious interactions between the test bodies, and other difficulties of the type that plagues terrestrial experiments. Moreover, the dependence of G on separation cannot be investigated. 2) Alternatively, large separations of the test bodies require an enormous mass for the larger test body. This, in turn, entails high lift-off costs, significant fabrication difficulties and, possibly, unacceptable uncertainties in mass. The shielding capsules would also likely be very cumbersome.

The SEE method avoids these pitfalls because the separation of the test bodies is large, it varies substantially during the encounter, and it is in the «along-track» direction(7). Moreover, the fact that the separation in a SEE encounter is chiefly along-track means that the orbits of the test bodies can be contained in a high-aspect-ratio cylindrical capsule and, hence, disturbances of the capsule due to atmospheric drag and solar-radiation pressure should be relatively unimportant. Despite the test-body separation of over ten meters, the along-track cross-section can be on the order of $1\,\mathrm{m}^2$. A conventional orbiting system generally requires that the cross-sectional area be at least an order of magnitude larger, since all capsule dimensions must be more than twice the separation of the test bodies (orbital radius).

The price which must be paid for SEE's approach to test-body and capsule geometry is twofold: First, the large test-body separation means that the gravitational interaction between the test bodies is relatively small, which in turn means that perturbations must be very closely controlled. Second, and perhaps more seriously, the long aspect ratio of the capsule means that its structural vibration modes will be at very low frequencies, which in turn means that the modes may be relatively easily excited by a number of environmental effects, such as slight thermal fluctuations in the outer skin. A preliminary analysis of vibration modes of the SEE capsule, which treats the limited case of a solid cylinder, finds that the fundamental frequency for transverse vibration (flexing) is about 2 Hz (Alekseev *et al.* (1993*b*)). A more detailed analysis, reflecting the fact that the cylinder is hollow and thin-walled, has yet to be done and is expected to show that this fundamental is somewhat below

(7) The large separation essentially obviates the density-inhomogeneity problem, while high variability of separation allows both investigation of G as a function of separation and accurate determination of the multipole moments of the gravitational potential of the larger test mass (the «shepherd») and the capsule. That is, «satellite geodesy» within the capsule becomes very practicable. This is a crucial advantage; it means that the harmonics of the gravitational fields of the capsule and the shepherd can be determined in orbit, as described further below in sect. 7. This process may be described as a «self-calibration» capability and is probably unique to SEE. In contrast, all terrestrial and most other proposed space-based experiments must rely largely on *a priori* modeling of the mass distribution to compute gravitational fields of the capsule and the test bodies. SEE also proposes to employ a Cook-Marussi stack of cylinders (Cook (1968), p. 235) rather than a sphere as the shepherd mass. This significantly alleviates the not-inconsiderable problem of determining sphericity, which further reduces the problem of errors in the shepherd mass distribution.

1 Hz. Techniques of active vibration control are very promising for obviating potential problems of this nature in spacecraft. For example, more than a decade ago, the application of such methods to a long thin cylinder with proportions similar to that of the SEE capsule was successfully modeled and demonstrated (Swigert and Forward (1981) and Forward (1981)). The various «smart materials» techniques which are now available should further increase the range of effective options for vibration control. Such materials could be incorporated in the capsule walls *per se* and also possibly in external struts and guy wires.

6. – Apparatus-related uncertainties

In addition to the various potential forces on the test bodies due to external effects, which the shielding capsule is designed to exclude, investigators must also pay close attention to the situation inside the capsule, lest the effects of self-induced forces vitiate accuracy. Great care is necessary to understand and minimize these forces, since the gravitational interaction between the test bodies is so weak. In this section we discuss the major effects of this sort and the ways that various investigators have addressed them.

Terrestrial investigations generally go to great lengths to limit the temperature variation within the experimental chamber, since convection effects and anisotropy in blackbody radiation can produce unacceptably large forces. Dicke's classic equivalence-principle experiment is a fine example (Roll *et al.* (1964), especially sect. IV.G). Similarly, in a space-based determination of G by any method, radiation pressure within the capsule, as distinct from that impinging on its exterior, is a critical issue. Careful attention must be given to the radiation-pressure forces due both to lasers or other light sources and to blackbody-radiation anisotropy, which would result from thermal non-equilibrium. For example, the force on a test body due to an imbalance of 1 milliwatt is $3.3 \cdot 10^{-12}$ N (assuming complete absorption). For comparison, in the SEE experiment the gravitational force of the shepherd on the small test body is $3.3 \cdot 10^{-9}$ N at 1 m separation and $3.3 \cdot 10^{-11}$ N at 10 m separation. To cope with this difficulty, the authors of the SEE proposal specified that the tracking-laser power should be ~ 10 nW. The SEE group is continuing to actively investigate the problems of managing radiation pressure, especially blackbody anisotropy. STEP carries the suppression of blackbody radiation much further by operating at ~ 2 K. This has been a key concept underlying STEP since its earliest roots (see, for example, Everitt (1977)).

Most space-based proposals, unfortunately, have not explored the radiation-pressure issue. Moreover, several of them propose to use television cameras for observing the test bodies, which implies bathing the capsule interior with some level of luminous intensity. The resulting net force on the test bodies due to radiation pressure might thus be comparable to their mutual gravitational attraction. Although remedies for this are admittedly possible in principle, any suggested solutions would require careful evaluation.

The NEWTON proposal provides an estimate of the perturbation due to blackbody radiation anisotropy arising from a thermal gradient within the capsule (Nobili *et al.* (1990)). This represents essentially the temperature difference between capsule end-walls. Specifically, if $T \approx 300$ K and $\Delta T = 1$ K, then the resulting force on the small test body is $2.2 \cdot 10^{-4}$ (220 ppm) in relation to the gravitational force due to the large test body. This effect is very sensitive to temperature, varying as $T^3 \Delta T$. However, the NEWTON authors expect the effect on the actual uncertainty in G will be much less than 220 ppm, because the plane of the orbit will be roughly perpendicular to the capsule axis, and therefore the radiation-pressure force will be roughly perpendicular to the gravitational force.

A much more serious problem, which is not mentioned in the NEWTON proposal, will arise from any temperature difference between the large test mass and the average of the capsule walls. This is potentially quite serious because the direction of the resulting force on the small test body, due to radiation anisotropy, is radial with respect to the large test body and therefore masquerades as a change in its gravity. At 300 K, a temperature difference of 0.1 K between the large test mass and the walls would introduce an error of nearly 2 ppm in G. Moreover, the thermal-relaxation time constant of the large test body will be extremely long (probably a matter of days), which means that the resulting error in G may repeat itself. That is, the bias in G might be so nearly stable that it would not reveal itself through inconsistencies in the data (a masking effect).

The large amount of short wavelength, essentially visible incident radiation needed for video-based detection and (in NEWTON) for trajectory-correction maneuvers further exacerbates this circumstance. Blackbody emission will all be in the infrared ($\lambda \geqslant 10\ \mu m$), and, therefore, for any given surface the effective absorptivity $(1 - R)$ may not be equal to the emissivity ε (Planck (1991)). Consequently, the steady state approached by the system would not be a thermal-equilibrium state. Thus, the large test mass will probably tend toward a different temperature from that of the walls.

SEE also confronts blackbody-anisotropy problems, although they are greatly attenuated by operation at moderately cryogenic temperatures (~ 78 K). However, the price which must be paid for the small radiative transfers is that the time constants for approach to thermal equilibrium are extremely long—a matter of weeks or months. This means that the temperature of the shepherd will be sensibly constant throughout an encounter. The principal blackbody-anisotropy problems of SEE are those arising from temperature differences, both between opposing capsule walls and between the two test bodies. For example, a potential temperature difference between the two ends of the capsule is of concern, since the resulting force would be nearly parallel to the gravitational force.

It appears that these effects are amenable to correction, however. Fluorescence-based remote thermometry capable of detecting temperature differences on the order of 0.001 K is now feasible, provided the actual temperatures can be made fairly close to the nominal design temperature (Allison (1994))([8]). Adjustments of the capsule-wall temperature by various active-control mechanisms could then bring the walls into equilibrium with the shepherd and each other. Nevertheless, an issue of concern for SEE is that the decay time for approaching thermal equilibrium naturally through radiation becomes very long as the temperature decreases. (It eventually reaches an asymptotic value, since both

([8]) Incident laser power is not expected to disturb the thermal equilibrium significantly, since fluorescence thermometry requires only microsecond pulses every few minutes, and, therefore, the time-averaged power will be many orders of magnitude below peak power.

the heat capacity and the rate of radiation vary as T^3.) However substantial blackbody radiation still exists at ~ 78 K.

The geometry of the SEE capsule and the test-body positions suggest that an alternative method may be feasible for corrections of those effects which arise from a temperature deviation of the shepherd. Specifically, the interior side walls of the capsule will behave optically like mirrors in the infrared, and one of the authors has shown that this leads to significant focusing effects, which vary substantially with the positions of the test bodies in the capsule. The excess blackbody radiation due to any temperature difference between the shepherd and the capsule walls can therefore be substantially magnified (by at least a factor of 5) by varying the optical locations of the test bodies with respect to the capsule walls. This might be accomplished by simply slewing the capsule slightly. Examination of the experimental data according to whether the small test mass is in or out of the focal region with respect to the shepherd will either 1) show that the shepherd is in good thermal equilibrium with the walls or 2) provide a basis for subtracting the effects due to non-equilibrium. A similar procedure is unlikely to work for NEWTON because the two test bodies must remain so close together that focusing effects cannot significantly alter the total radiation pressure of the large test body on the small test body.

The issue of possible electrostatic interaction between the test bodies is addressed in the SEE and NEWTON proposals but not in most EPIC proposals. The SEE proposal found that this is not an important issue (Sanders and Deeds (1992*a*), p. 500), and thus the silence of most authors on this point may indicate that they had determined that the effect is also negligible in their systems. However, the NEWTON proposal finds that this will be a major perturbation, reasoning that the plasma must be the same inside and outside the capsule (Nobili *et al.* (1990), p. 398). It suggests dealing with electrostatic interaction by installing electrometers in the capsule and carrying out a Fourier analysis of the output. SEE, on the other hand, will use a closed rather than open experimental chamber, and thus eliminate plasma inside the capsule, except for that produced by cosmic rays. The use of a closed capsule might also be practicable for a NEWTON mission. This would reduce the perturbation due to electrostatic interaction between the two bodies many orders of magnitude (perhaps a factor of 10^8). However, interaction with induced charges in the walls, which is the major electrostatic effect in SEE, is not discussed in either the NEWTON or the STEP proposal.

Superconducting barriers in the proposed STEP G/ISL experiment would completely eliminate electrostatic interactions between the test masses (although it might leave open the possibility of induced charges on the chamber walls). In any case, the short-period (~ 5 min) phase-sensitive analysis would go a long way toward reducing any residual effects.

The Pisa group has suggested attaching a conducting wire «of negligible mass» to one of the test bodies in a preliminary version of the NEWTON proposal (Nobili *et al.* (1987), p. 440). It is not clear how the attachment of any wire to an otherwise free-floating test body could be done so as to avoid exerting unacceptable forces. We note that various proposals have previously called for tethering a satellite to the Shuttle to provide a low-vibration platform for gravity experiments (see, for example, Lorenzi and Gullahorn (1988)). It must be emphasized that the rationale for such «tethered satellites» is to avoid the large residual accelerations and vibrations which are inherent in manned spacecraft. Although this is achieved whenever the line is not taut, a slack condition does not conversely imply the absence of forces of the tether line upon the tethered satellite, but rather only a vast reduction in residual accelerations compared to those on-board the Shuttle. In contrast, a much higher standard is essential for the large test-body in a G experiment: the goal is to shield both test bodies from all extraneous forces, so it would be unacceptable to introduce any significant hard-to-model force, such as that due to a slack tether.

Admittedly, it is necessary to know the forces only on the *small* test body, but this generally entails observing its position primarily in relation to the larger test body, which precludes adulteration of its position by tugging on it with a «negligible» wire. One might assume that the shielding capsule *per se* could serve as the reference platform for measuring the motion of the test bodies, since it is «drag free». However, the capsule is subject to buffeting and is therefore drag free only in an approximate and time-averaged sense. It cannot be a sufficiently stable platform to be used as a standard against which to compare the position of an individual test mass. Therefore the large body is essential as a reference for determining the motion of the small test body. This is true for any experimental arrangement involving two free-floating test bodies (SEE, artificial solar system, scattering experiments, etc.). This also reveals another potential fundamental shortcoming in most of the tunnel-in-sphere experiments: Since they probably would not be able to avoid the incorporation of some form of restraints, they would likely be forced to rely on the shielding capsule as the reference body. We note that STEP largely obviates this problem by the use of differential accelerometers, thus greatly suppressing the common-mode acceleration.

Nevertheless, in any G-in-space experiment it is important that the capsule be as free as possible of vibrations, since the orientation of the capsule and the relative positions of selected points in its interior must be known to considerable accuracy. This calls for a strategy to coordinate a number of methods for minimizing jerks due to station-keeping and the experimental procedures *per se*. The EPIC proposals generally made no comment on this matter. The SEE and STEP proposals address it in some detail (Sanders and Deeds (1992*a*); Blaser *et al.* (1993)), using a variety of approaches. Both would use magnetic torque bars (Walker (1993), p. 521) to eliminate jerks in capsule-orientation maneuvers and continuous-sunlight (eclipse-free) orbits to improve thermal stability and to eliminate solar-radiation-pressure jerks. SEE will obtain jerk-free thrust for precision station-keeping by a novel application of solar-sailing principles. It will use the sides of the capsule itself as sails in order to «tack» across the solar radiation, thus providing the minute along-track thrusts needed to maintain a high-altitude orbit. STEP will use jerk-free proportional thrusters fueled by boil-off from the helium dewars for both attitute control and drag compensation.

The NEWTON proposal did not address these problems, except for a brief reference to the possibility of a «continuous-thrust orbital-control system» (Nobili *et al.* (1990), p. 406); in other portions of the report, including the conclusion, all discussion is in terms of jets. Moreover, the NEWTON capsule's orientation and spin rate ($\sim$ once per minute) would evidently require the DISCOS thrusts to be supplied in very short bursts, which might exacerbate the jerk problem. Because the sunlight strikes the capsule chiefly on the sides rather than the ends of the cylinder, the required thrust must be directed chiefly radially rather than along the cylinder axis, and therefore it is reasonable to assume that the longest allowable burst for any individual thruster is the time in which the capsule turns about 30 degrees, *viz.* 5 seconds. Moreover, the NEWTON proposal called for very infrequent (every 2000 s) bursts, which would require the individual impulses to be relatively large ($\sim 10^{-2}$ N). The jerks from such intermittent impulses could excite the very low-frequency vibration modes of the capsule, which might compromise the rigidity of the optical system, with serious effects on the accuracy of the experiment. Such vibrations are a major concern in any satellite requiring precision orientation (see for example Reasenberg *et al.* (1988)). In the case of the NEWTON capsule, vibration problems might be substantially alleviated by the use of Field Emission Electric Propulsion (FEEP) thrusters, which are being considered by the Pisa group for another experiment. FEEP thrusters are capable of supplying micro-newton forces either continuously or in pulses with rise time ~ 1 ms (Nobili *et al.* (1995), Marcuccio and Andrenucci (1996) and Vannaroni *et al.* (1996)). The fuel requirement of FEEP thrusters is also extremely small, which in a gravity experiment is a substantial advantage.

As a general principle, there is much to be said for *avoiding* spacecraft vibrations rather than isolating against them. Recent proposals for astronomical satellites, such as POINTS and HIPPARCOS, exhibit this principle superbly (Reasenberg *et al.* (1988) and Froeschle and Mignard (1981)). We believe there are valuable lessons here for gravity-measurement satellites as well.

A major problem faced in the manipulation of any object in near-zero gravity and in a vacuum is that it is hard to «let go». So-called «stiction» effects, which normally are barely noticeable in the terrestrial environment, will impart unwanted impulses to a body upon release (Herndon (1993) and Nobili *et al.* (1990), p. 403). This effect is perhaps of minimal concern in artificial-binary and artificial-solar-system arrangements, which typically require only one launch for periods of months or years (provided, of course, no actual collisions result from launch errors). However, it is a significant concern for SEE, since a new launch is required every few days. Extensive computer modeling has confirmed that the SEE-encounter trajectories are very robust, in the sense that completely operable trajectories are obtained even if the launch error is substantial (Melnikov *et al.* (1993), Alekseev *et al.* (1993*a*), Bronnikov *et al.* (1993*b*)). For some purposes, however, it is desirable to be able to replicate a previous trajectory as closely as possible. The original SEE proposal suggests the possible use of eddy currents to stop the particle after an encounter and to re-insert it into a desired trajectory (Sanders and Deeds (1992*a*)). However, the authors are concerned that the fields intended to steer the particle might produce unacceptable heating in the shepherd (Deeds (1994)). Hence, stiction remains a major concern in the SEE experiment. A potential solution is to coat the test bodies with a thin film of a high-temperature superconductor (Mashburn (1994)).

Stiction should be of concern in the STEP G/ISL experiments only when masses are being uncaged. This happens infrequently, and moreover the magnetic bearings would prevent collisions until any transients which might result from a «rough» uncaging procedure could dissipate.

A potential problem for G/ISL is that the STEP capsule contains a substantial amount of mass in the immediate proximity of the G/ISL test masses. Some of this is expected to be in motion, including other test masses and the liquid helium in the dewar. Uncertainties and variations of the mass distributions surrounding terrestrial G experiments have always been a difficult problem to overcome and, in fact, place a limit on torsion-balance determinations of G. This holds even for rotated Cavendish balances, which are able to partially average out the forces due to the mass distributions surrounding the experimental apparatus (see the discussion of gravity gradients in Gillies and Ritter (1993)).

The motion of all test masses in STEP is in principle very well accounted for. However, the uncertainties intrinsic to identifying the time-varying distribution of the mass of the (superfluid) liquid helium in a partially filled dewar may potentially result in serious mass-accounting problems. At the time when the Phase-A Study Report was published, the G/ISL authors indicated that the motion of the liquid helium could probably be dealt with satisfactorily, but that, if this proved not to be true, then an alternative approach could be used. In it, the entire G/ISL calibration and data collection processes would be carried out during the brief period when there is no liquid-gas interface, *i.e.* when the helium in the outer dewar has been exhausted and the helium from the inner dewar either is still in place or has just been transferred to the outer dewar, thus filling it completely. In this scenario, the complete G/ISL protocol, including calibrations, would be carried out in a few days. We note that this compressed timetable would leave very little margin for a «shake-down» period, when any unanticipated problems might be discovered and corrected. Subsequently, the STEP team has devised a method to further reduce helium «sloshing», which involves keeping the outer dewar in supercritical phase so that there is no liquid-gas interface, while keeping the inner dewar completely full of superfluid helium (Paik (1995)). This approach is intended to obviate the possibility of being forced to conduct the G/ISL experiment under unreasonably tight time requirements. However, we note that, since the density of supercritical helium varies strongly with temperature, significant density gradients may still exist within the outer dewar. Thus, although this approach can eliminate the liquid-gas interface *per se*, it is not clear whether unacceptable sloshing might nevertheless occur in the form of movements of pockets of different spatial density.

7. – Self-calibration

A common reaction to proposals involving the determination of G within a capsule is to suppose that the relative accuracy can be no better than that with which the mass distribution of the capsule is known. This is a misconception. Moreover, the gravitational field due to the capsule itself can be determined very accurately, at least in the case of SEE.

Regarding the first point, there is no particular relation between the total mass of the capsule and the amount of unknown mass at various locations which would cause a given relative error in a G determination. Rather, the critical variables are the absolute amount and the distribution of the unknown mass, not its ratio to the total capsule mass. Other significant variables are the geometry of the trajectories, including the proximity of the test bodies, and whether the capsule is spinning.

The gravitational field due to the capsule can be mapped in detail by observing the motion of the enclosed test bodies, using analytical methods that are in principle no different from those used to map the gravitational field of the Earth by observing the orbits of artificial satellites (see, for example, Kaula (1966)).

At the most fundamental level, the significance of the self-calibration principle is that it exploits the space environment to achieve something impossible in the terrestrial environment, where investigators typically have no choice but to rely on *a priori* mass accounting to estimate extraneous gravitational forces. As pointed out in

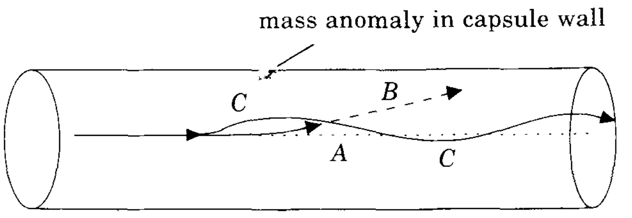

Fig. 8. – Passage of the particle (small test body) past a mass anomaly in the capsule wall. If the mass of the capsule is distributed so that it has zero internal field, then, in the absence of other gravitational fields, the trajectory of the particle would be a straight line through the capsule (path A; dotted line). Any anomaly in the capsule wall, as shown above, would of course cause the particle to execute a hyperbola with the anomaly at its focus (path B; dashed line), *if* the bodies were not in orbit around the Earth. The particle's energy would also be temporarily changed in the vicinity of the anomaly. However, since all the bodies will in fact be in Earth orbits, the result of such an encounter would be a perturbation of the particle's orbit. Any changes in semi-major axis (*i.e.* energy) would be only temporary, but the changes in eccentricity and/or orbital plane would persist after the encounter, resulting in an oscillatory motion relative to the capsule (path C; solid line). Thus, the residual gravitational field of the capsule can be determined by systematic analysis of the particle's motion. This is self-calibration.

the NEWTON proposal (Nobili *et al.* (1990), p. 393), space-borne experiments should be designed with the unique opportunities of space consciously in mind, since it is unlikely that the potential advantages of space can ever be fully exploited by a revised version of any terrestrial experiment. Self-calibration is a particularly appropriate example of this philosophy.

As an illustration of the effectiveness of such methods, we consider the ability to detect a small mass anomaly in the walls of the SEE capsule: Because of the geometry of the SEE trajectory, entailing great distances from the shepherd, the particle essentially drifts for long periods in a straight line, with its velocity changing very slowly and smoothly. When it passes a point-mass anomaly Δm at an impact parameter b while it is in such a «drift» mode, as illustrated in fig. 8, then the associated potential energy at closest approach is

$$\Delta U = -\Delta m\, m\, G/b\,, \tag{8}$$

where m is the mass of the particle. Equating ΔU to the corresponding change in kinetic energy and solving for the relative increment to velocity yields

$$\Delta v/v = \Delta m\, G/(bv^2)\,. \tag{9}$$

This result is exact only in an otherwise gravity-free region; since the bodies are all in orbit around the Earth, an accurate description is possible only in terms of changes in their Earth orbits. Nevertheless, this result demonstrates considerable sensitivity to small mass anomalies. For example, if $\Delta m = 1$ g, $b = 50$ cm (the maximum possible), and $v = 10^{-4}$ m/s (typical), then $\Delta v/v = 1.3 \cdot 10^{-5}$, so the particle will require 0.01 s less to traverse the 10 cm segment of its trajectory which is nearest the anomaly. Thus, masses of ~ 0.1 g or perhaps even 0.01 g can be detected.

An enormous advantage of the large separation between the SEE test bodies is that detailed knowledge of the gravitational field of the shepherd is not required for this procedure. In fact, very simple curve-fitting techniques should be sufficient to reveal a «bump» in the velocity, indicating a Δm in the wall. For example, if there are no anomalies and the test-body separation is 10 m, then a simple quadratic fit over a 40 cm path has residuals of only $\Delta v/v = 0.7 \cdot 10^{-7}$, which is more than an order of magnitude smaller than the actual velocity bump due to a 1 g anomaly, *viz.*, $\Delta v/v \sim 10^{-5}$. That is, an empirical plot of particle velocity *vs.* time (or distance) will exhibit a bump roughly ten times as large as the residuals of a quadratic fit to the velocity in the absence of the anomaly. Thus, this procedure is almost independent of the force due to the shepherd when the separation is large, say, $\geqslant 10$ m. *A fortiori*, the effects of any mass anomalies in the shepherd should decouple almost completely from those due to anomalies in the capsule walls, no matter what the fitting procedure. Of course, an actual SEE mission would employ a sophisticated general fitting procedure and take explicit account of the gravity of the shepherd, resulting in much improved accuracy.

In visualizing this process, it may be helpful to initially imagine that the SEE capsule were rotating about its axis, which would effectively «spread» any mass anomalies as rings around the cylinder. That is, this would physically integrate out the ϕ-dependence of any mass anomaly, leaving only the z (axial) dependence of the anomaly. (The required rotation rate to do this is about once every 100 seconds, so that the particle would move only ~ 1 cm during one rotation.) This process would capture the most important effects of the mass anomalies in the capsule wall, since the forces on the particle due to the ϕ-dependence are roughly perpendicular to the gravitational force due to the shepherd and, therefore, are much less important than those arising from the z-dependence. Note that the results would be analogous to a representation of the Earth's field in terms of zonal harmonics, which reflects only the θ-dependence and neglects the ϕ (spherical coordinate) dependence.

If the SEE capsule is three-axis stabilized rather than rotating about its axis, the ϕ (cylindrical coordinate) dependence of its field is also needed, which requires launching the particle into trajectories lying close to the wall (say, within 15 or 20 cm) along a number of different «strips» of the wall without rotating the capsule. After 20 or 30 SEE encounters, requiring less than 3 months, the whole capsule wall should be mapped well enough to go to at least the 10th or 12th order in the ϕ-dependence of the Fourier-Bessel series.

A gradual quasi-monotonic variation in mass density in a direction parallel to the axis of the SEE capsule would, if undetected, cause a more serious perturbation, since it would masquerade as a change in G or a violation of the inverse-square law. Fortunately, such an effect can be detected by reversing the positions of the test bodies and by varying the location of the shepherd in the capsule.

Because the test-particle separation in SEE can be varied so greatly, it should be possible to map the shepherd's field without confounding it with the field of the capsule. The capability to vary the test-body separation can also be exploited to «leverage» the precision of the shepherd's field. The shepherd's field can be mapped by SEE encounters having very small test-body separations at closest approach (say, ~ 1 m), which is much less than the typical distance at closest approach in the SEE encounters that are used for determining G (say, $\geqslant 3$ m). Thus, since the contributions of the harmonics enter as powers of the separation, the absolute precision available in a G-determination encounter will be at least an order of magnitude better than in the encounters used to determine the harmonics.

Finally, a «capsule geodesy» program obviously cannot be successful unless either the gravitational field which is being mapped remains stable or the changes therein can be accounted for accurately. The SEE proposal describes detailed measures to keep the mass distribution of the capsule constant, such as maintaining a high degree of thermal stability to avoid capsule distortion, and even the avoidance of conventional foil insulation because of the buckling and warping problem (Sanders and Deeds (1992*a*), p. 500).

The NEWTON proposal also indicates plans to use the principles of «capsule geodesy» as described above to calibrate the fields of the capsule and large test-body in orbit. In principle this is a major improvement over the preliminary proposal, which called for relying on *a priori* mass accounting (Nobili *et al.* (1987)). Moreover, NEWTON would obviate the need for measuring the ϕ-dependence of the capsule field by rapidly (once per minute) rotating it. Unfortunately, however, there may be several significant obstacles to achieving effective capsule geodesy in the case of NEWTON ([9]). The process of determining multiple parameters for the geodesy equations requires multiple runs with high-precision data under varying geometries (see, for example, Kaula (1966)). However, since the NEWTON experimental protocol typically entails experimental runs of at least several months, rather than several days or hours, to obtain high-precision data, there may be insufficient time during the life of the mission to employ a wide variety of different geometries. Moreover, the NEWTON proposal presents compelling reasons for essentially fixing the geometry of the experiment, making the orbital radius of the artificial solar system roughly constant at 20 cm. (The radius apparently cannot be varied more than about a factor of two.) It especially calls for making the plane of the orbit roughly perpendicular to both the capsule axis and the Earth's axis of rotation. The rationales are, respectively, to keep blackbody radiation pressure normal to the gravitational attraction between the test bodies and to prevent unwanted perturbation of the plane of inclination of the test-body orbit.

Finally, the NEWTON proposal does not include any discussion of means to assure capsule rigidity, which is necessary to keep its mass distribution constant ([10]), and which in turn requires a high degree of thermal stability to prevent warping and the avoidance of jerks to prevent low-frequency vibrations. The thermal-stability requirement is complicated by the thermal shocks which will occur upon entering and leaving the Earth's shadow, which we estimate will happen approximately 91 days per year (45 or 46 days centered on each equinox) for the orbit of the NEWTON capsule.

Unless the present NEWTON protocol can be changed to allow for 1) substantial variations in experimental geometry, 2) a sharp upgrade in precision and resolution, and 3) a high degree of capsule rigidity, it may be difficult for NEWTON to achieve full self-calibration. The problems of precision and resolution and capsule rigidity could largely be solved through redesign. For example, interferometry could be introduced in the tracking system, while reassessment of the thermal-management issues might yield ways to insure that the capsule would keep its shape. In fact, very

([9]) The *ad hoc* committee, Fundamental Physics Assessment Group (FPAG), may also have expressed doubts, in its review of the NEWTON proposal, about NEWTON's capability for capsule geodesy, as indicated by the collaboration's reply to FPAG (Nobili (1993*a*)).
([10]) The possibility of thermal distortion of the SEE capsule also remains of concern.

recent work by members of the NEWTON team on an equivalence-principle proposal («GG-FEEP^2») seems to hold promise for solving the mechanical and thermal problems of NEWTON (Genta *et al.* (1996), Catastini and Nobili (1996), Arduini *et al.* (1996)). However, we note that this work entails the use of conventional insulation blankets, consisting of foil interleaved with bridal cloth. This raises concerns that buckling of the foil may cause unacceptable fluctuations in the mass distribution of the capsule (Sanders and Deeds (1992*a*), p. 500). NEWTON's eclipse seasons warrant particular attention and analysis, due to the thermal shocks which will occur upon entering or leaving the Earth's shadow.

Although a number of barriers to self-calibration by NEWTON can be overcome by redesign, the narrow range of experimental geometries as currently planned seems intrinsic to the NEWTON concept, thus limiting the possibilities for full self-calibration. To illustrate the situation, suppose that the data-reduction problem anticipated in the NEWTON proposal (Nobili (1990), p. 401) is expressed in terms of simultaneous equations. The resulting associated matrix would be very ill-conditioned, if not singular, because so little variation in experimental geometry is allowed. The use of alternative techniques, such as iterative methods, would avoid mathematical difficulties but still leave the harmonic coefficients poorly resolved.

For STEP G/ISL, *in situ* calibration of mass distribution and gravitational field would not seem to be possible. However, the approach of STEP is intended to obviate any requirement for calibrating the capsule field, in that all measurements are short-period and oscillatory, thus filtering out extraneous forces, provided they are either stable or at frequencies greatly different from that of the signal. (Although we examine below in sect. 9 the possibility that the inverse-square law test of G/ISL may be limited by its reliance upon *a priori* measurements, this problem need not affect the G-determination aspect of G/ISL.)

8. – Error assessments

Although formal error budgets have not yet been prepared for either SEE, NEWTON, or STEP-G/ISL, all three proposals contain very extensive discussions of known errors and order-of-magnitude estimates of their effects on accuracy. This allows some preliminary statements to be made about their likely error budgets.

The uncertainty in the shepherd mass is expected to be the largest item in the SEE error budget. It will be about 0.3 ppm, based on recent results in the accurate measurement of large masses (Debler (1991)). Other items expected to contribute more than 0.1 ppm to the SEE error budget are capsule mass anomalies, optical system biases, capsule distortion, optical system variations, capsule orientation, test-body separation, thermal gradients in the capsule, and the shepherd-capsule temperature difference. The last two sources of error are currently being evaluated, and it is expected that strategies now being developed to control and minimize these sources of error will also put them in the range of 0.1 to 0.3 ppm. The total error is expected to be about 1 ppm. A detailed discussion of error sources in SEE is provided in sect. V and VI of Sanders and Deeds (1992*a*). Detailed analysis of a number of specific sources has been carried out by Melnikov and colleagues (Osipova (1993), Lidov and Vashkov'yak (1994), and Bronnikov *et al.* (1993*a* and 1993*b*)). Finally, we note that the uncertainty in the distance between the test bodies at closest approach is not a significant source of error. This is a fortuitous result of the SEE kinematics, as discussed in sect. V of Sanders and Deeds (1992*a*), p. 498.

The NEWTON proposal provides a detailed «perturbation analysis» (subsect. 3.3 of Nobili *et al.* (1990)) which lists a wide range of perturbations by cause. A number of these can be very accurately calculated and, therefore, substantially corrected, as noted in the text of the proposal. The items which, in our opinion, will still be significant after appropriate corrections have been made are (the numbers in parenthesis are the sizes of the perturbations *without corrections*, stated first absolutely, in units of $m/s^2 \cdot 10^{-9}$, and then stated, after the semicolon, in relation to the average gravitational force due to the large test body, expressed in ppm: uncertainty in the radius of the Earth orbit of the spacecraft (2.3; 1.8), internal radiation of illuminating lamps on the small test body (3.7; 2.8), uncertainty in the distance between centers of mass of the spacecraft and the large test body (6.3; 4.8), liquid-fuel uncertainty (6.7; 5.2), spacecraft gravity gradient (7.9; 6.1), uncertainty in the mass of the large test body (13; 10), electrical interaction (49; 38), and spacecraft temperature gradients (290; 220). In a separate section the difficulty of measuring the separation between the test bodies is emphasized (subsect. 4.2 of Nobili *et al.* (1990)). (We also call attention to a general treatment of satellite perturbations by three of the NEWTON authors (Milani *et al.* (1987)).)

Note that we have listed the items in ascending order of the perturbation. The contributions to the eventual error budget would of course be much less than the numbers indicated above in most cases, due to corrections which are now known to be possible but are still incompletely evaluated. Further, it is likely that most of the errors listed are conservatively estimated. In particular, the uncertainty in the mass of the large test body is now most likely overstated, in view of the above-mentioned recent advances in large-mass metrology. Hence, the NEWTON mass error may be on the order of only ~ 0.1 to ~ 0.3 ppm.

Three other items in the above list can be virtually eliminated using methods discussed in the SEE proposal. These are the uncertainty in the distance between the centers of mass of the spacecraft and the large test body, the spacecraft gravity gradient, and electrostatic interactions. Although most proposals for determining G in space require that the test bodies be located very close to the capsule center of mass, this consideration is virtually inconsequential in the SEE experiment, because the mass of the SEE capsule is distributed so that in principle it creates no gravitational force within its interior. Thus, the only gravitational force of the capsule on the test bodies will be that due to anomalies in the capsule walls, which will be detected by «capsule geodesy» as described above. The electrostatic perturbation in NEWTON could be reduced many orders of magnitude (perhaps a factor of 10^8) by using a closed capsule, as also noted above.

Another uncertainty—that in the separation of the test bodies (*i.e.* the orbital radius)—can perhaps be finessed. This is a common worry in artificial-solar-system proposals (see, for example, Berman and Forward (1969), p. 100, and Hills (1986)). However, if the nature of the orbit were sufficiently well understood, then its dimensions could be accurately inferred from the period and the velocities at various phases in the orbit. (For example, the circumference of a circular orbit is obviously $2\pi a = vT$, so its radius a is known from the velocity v and the period T.) However, this would require very accurate measurement of the test bodies' relative velocity and the satellite attitude, which in turn would probably require interferometric methods and star sightings, respectively. In any case, a direct measurement of the orbital radius may be unnecessary [11].

Internal radiation due to the NEWTON illumination lamps could perhaps be reduced at least an order of magnitude by using chopping or strobing techniques. For example, 10 or 100 watt flashes of a few milliseconds duration at intervals of 1 s might be sufficient. Finally, if FEEP thrusters, rather than a hydrazine-based system, could be used for the NEWTON DISCOS, then the uncertainty due to fuel mass would be virtually eliminated.

Thus, of the various effects listed above which are expected to contribute significantly to the error in a determination of G by NEWTON, all except the spacecraft temperature gradient can probably be reduced below 1 ppm by straightforward revisions of the experimental design.

However, more serious problems are expected from two sources of uncertainty which were not identified in the NEWTON proposal, namely

1) Thermal disequilibrium between the large test body and the capsule walls.

2) Capsule distortion, especially thermally induced warping.

These have already been discussed above in sect. **6** and **7**. We would only emphasize that the significance of the second of these is that, if the capsule is subject to time-varying distortions, then it is unlikely that «capsule geodesy» can be employed to map the fields of the capsule and the large test mass. This is of concern, since the NEWTON proposal does not discuss methods for assuring that the capsule shape will remain constant, thus minimizing capsule distortion.

Table II in the STEP Phase-A Study Report, «Summary of Disturbances», outlines the major potential disturbances anticipated by STEP, the levels to which they must be controlled to achieve the accuracy goals, and the means for doing so. The table reflects the STEP philosophy that all errors should be made negligible. We note, however, that a formal error assessment as applied to G/ISL has not yet been presented.

The small size of the STEP G/ISL source mass (a cylinder approximately 2 cm long and 2 cm in diameter with a mass of 93.6 g) means that it has a fairly large surface/mass ratio. This may make it relatively more vulnerable to surface disturbances than the large masses in either SEE or NEWTON. For example, a scratch only a few monolayers deep incurred during caging or uncaging might be significant in the G/ISL error budget.

The potential for contaminant-adhesion errors is a facet of the more general problem that any G determination which claims 1 ppm will strain the definition of the kilogram, as pointed out in the SEE proposal (Sanders and Deeds (1992*a*), pp. 498-499). It is potentially more serious for G/ISL because of the large surface/mass ratio. The variation among the three experiments in the surface/mass ratio of their source masses is due chiefly to differences in the mass value itself rather

---

([11]) It appears that NEWTON should have ample resolution for this approach: Some 330 orbital revolutions will occur every month. Assuming conservatively that 10-arc-second accuracy is available in measuring the orientation of the capsule and in defining and measuring the phase angle of the small test body in its orbit, then in a one-month data run the period could be measured to less than 0.1 ppm.

TABLE II. – *Comparison of recent detailed proposals.*

Author(s)	Nobili and colleagues 1987, 1990	Sanders and Deeds 1992*a*	Paik/STEP 1993
Basic design	Artificial solar system. NEWTON mission	Satellite Energy Exchange (SEE)	Source mass driven back and forth inside coaxial hollow cylindrical test masses
Special focus	Exploration of mechanics of test bodies	New test-body dynamics, namely the encounter phase of Darwin's horse-shoe orbit	Near-null force region at center designed to make any Yukawa force stand out clearly
Source mass	75 kg sphere, heavy metal	500 kg Cook-Marussi stack, heavy metal	93.6 g cylinder, platinum-iridium alloy
«Small» mass	2 kg sphere	100 g ~ 10 different materials	~ 300 g hollow cylinders, 2 optically clear single crystals with heavy and light nuclei, respectively
Mass ratio	37.5	5000	~ 0.3
Separation of centers of mass	20 cm	~ 1 m to ~ 15 m	$\sim$ zero (source mass is coaxial with test masses at c.m.)
Temperature	Ambient (~ 300 K)	Moderately cryogenic (~ 78 K)	Cryogenic (~ 2 K)
Detection method	TV camera, using angle information only (low resolution)	Optical interferometry (high resolution)	SQUIDs (ultra-high resolution)
Measurement period	1 month to 1 year	1 hour to 3 days	5 minutes to few hours
Restraints on test bodies	Wire of «negligible mass» to bleed charge from large test body (1987)	None	All masses magnetically levitated on a common axis; only axial components of acceleration are treated
Treatment of Earth's field	Viewed as perturbation Averts problem by high (geosynchronous) orbit.	Earth's field viewed as natural; test-body interaction viewed as the perturbation (not *vice versa*)	High-order geodesy is a major co-experiment in STEP
Capsule control	Hydrazine-fueled jets (not jerk-free)	Solar sailing and magnetic torque bars (jerk-free)	Helium boil-off and magnetic torque bars (jerk free)
Significant unresolved precision-measurement issues	Blackbody radiation anisotropy, especially radially from large test body Capsule rigidity Geodesy capability	Thermal effects, especially internal blackbody radiation Stiction	Test-mass metrology
Comments	Much related work done by these authors on STEP and other equivalence-principle tests in space	Substantial background work on microgravity and nanogravity related to SEE has been undertaken	Elegant experimental design in keeping with those of the other STEP experiments

than in density or shape. Thus we may say simply that the surface/mass ratio varies as $m^{-1/3}$, in which case the relative surface/mass ratios for SEE, NEWTON, and STEP, respectively, are 1 : 2 : 18. Thus, any effects of this nature would be about an order of magnitude higher in STEP than in SEE or NEWTON.

In summary, the cumulative effect of the perturbations enumerated by the NEWTON, SEE, and G/ISL proposals should be compatible with the accuracy goals of the proposals. Thermal problems will warrant further attention, particularly the two problems in NEWTON which are discussed above but not mentioned in the NEWTON proposal.

9. – Other experimental objectives

Most proposed experiments for gravitational measurement have one primary objective, but may also be capable of making related measurements at interesting levels of precision. This capability is concomitant with the expectations of both NEWTON and SEE and is intrinsic to the entire STEP mission. Testing the equivalence principle is of course the primary purpose of STEP, and a test of the

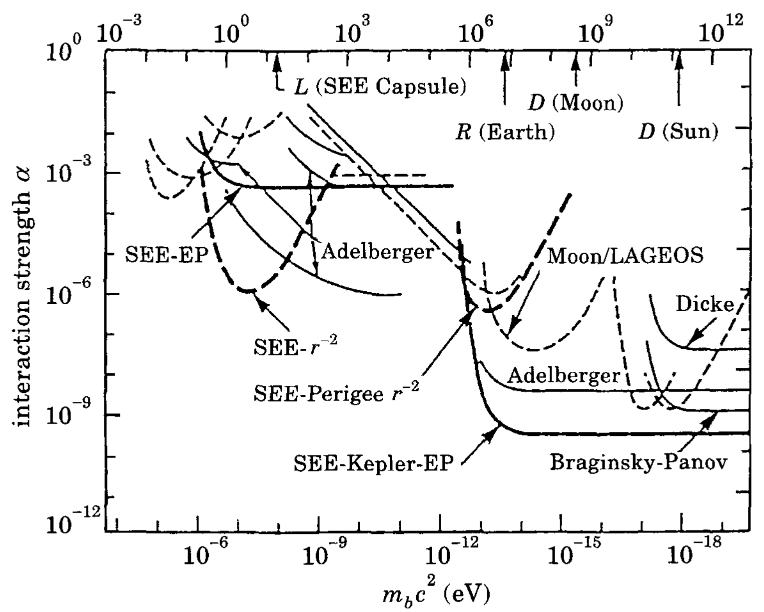

Fig. 9. – Tests of limits placed on the Yukawa parameter α by previous experiments, and projected possibilities for a SEE mission. The limits which have been placed by various experiments on the magnitude of the field-strength parameter α, which appears in various non-Newtonian theories, depend on the assumed value of the characteristic distance Λ according to the experimental parameters, especially the interaction range. The figure suggests that a SEE mission may be capable of yielding very interesting short-range results by testing the inverse-square law (see curve SEE-r^{-2}), while also yielding very interesting long-range results via an Eötvös-type experiment. For the latter, SEE searches for violations of Kepler's third law (see curve SEE-Kepler-EP). This figure is adapted from de Rújula (1986). Figures 1.1 and 5.4 in Blaser *et al.* (1993) give the limits expected from a STEP mission, including G/ISL.

inverse-square law is a part of STEP's proposed G/ISL (G/Inverse-Square-Law) experiment in this mission.

In addition to determining G, a SEE mission is intended to establish new upper bounds on violations of the inverse-square law at distances of the order of meters, by using measurements at different distances. It might possibly also provide the best test to date of violations of the equivalence principle (EP) at distances exceeding the radius of the Earth, by testing Kepler's third law with test bodies of different compositions. Of course a violation of the inverse-square law would be interpreted by present-day theory as an EP violation. The composition-dependent EP test is expected to be especially sensitive, *viz.* a few parts in either 10^{13} or 10^{15}, depending on which experimental configuration is used (see Sanders and Deeds (1992*a*), p. 502). If the gravitational interaction between two bodies of mass m_1 and m_2, including a possible non-Newtonian term, is written as

$$U_{12}(r) = -(m_1 m_2 G/r) \times (1 + \alpha \exp[-r/\Lambda]), \tag{10}$$

where α and Λ are the field strength and characteristic interaction length of a fifth force, as parameterized in the standard Yukawa fashion, then these EP tests may be expressed in terms of bounds on the field-strength parameter α, as shown in fig. 9.

The inverse-square-law test of G/ISL is also expected to provide similar improvement in the bound on α at short range (in fact, at $\Lambda \sim 2$ cm), as shown in fig. 5.4 of the Phase-A Report, while the main EP tests are expected to bound α at about 10^{-14} at long range ($\Lambda > R_E$), as shown in their fig. 1.1. In addition, the STEP geodesy experiment is expected to improve the existing bound on α at $\Lambda \sim 100$ km by an inverse-square test (fig. 5.4 of Blaser *et al.* (1993)).

The data from a SEE mission may also yield a useful bound on $|\mathrm{d}G/\mathrm{d}t|$, which is quite significant in itself because this would be the only extant data from a controlled, non-astronomical experiment. For a review of the literature on $\mathrm{d}G/\mathrm{d}t$, see the papers of Gillies and Melnikov (Gillies (1987, 1988 and 1990); Melnikov (1994)).

G/ISL employs an elegant approach to distinguish a possible non-Newtonian force from the Newtonian. Each G/ISL test mass is essentially a hollow cylinder with thick walls at the ends, and may therefore be regarded as a combination of a simple hollow cylinder having uniform walls and a pair of flush-fitting cylindrical sleeves on the ends, as shown in fig. 10. For certain critical ratios of the masses and dimensions of the sleeves relative to those of the main cylinder, the gradient of the Newtonian gravitational field vanishes in the middle of the cylinder. By making the sleeves

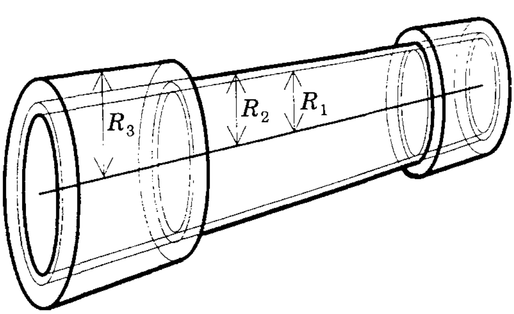

Fig. 10. – Schematic drawing of a G/ISL test mass.

slightly larger than these critical values, the force near the center can be shaped as shown in fig. 5.3*b* of Blaser *et al.* (1993). From that figure it is clear that small regions $[0, z_3]$ and $[0, -z_3]$ exist on the axis on both sides of the origin along which the Newtonian force averages to zero. The crux of the ISL (Inverse-Square-Law) test in STEP is to measure the average force in these predetermined regions, and thence to interpret any non-zero average force as non-Newtonian.

One result of this arrangement is that extremely small errors in the relative sizes of the main cylinder and the sleeves would produce a significant non-zero average of the Newtonian force over the supposed null-average regions $[0, z_3]$ and $[0, -z_3]$. If so, then the metrological accuracy presumed for STEP (1 ppm) may be insufficient to yield STEP's accuracy goal for α (also 1 ppm). One should also consider the possibility that the assumptions underpinning the metrology goals themselves may inadvertently be too optimistic. The potential difficulties are outlined in the following paragraphs.

We denote the inner and outer radii of the main cylinder R_1 and R_2, and we denote the inner and outer radii of the sleeves R_2 and R_3. Note that R_2 is both the outer radius of the actual main cylinder and the inner radius of the actual sleeves, as shown in fig. 10.

Very small changes in the radius R_2 can be used to fine-tune the relative masses of the main cylinder and the sleeves (with only slight effect on the total mass). By this means, the shape of the Newtonian force F_z in the central region (cf. fig. 5.3*b* of Blaser *et al.* (1993)), including the locations of the crossover points and the null-average intervals $[0, z_3]$ and $[0, -z_3]$, can also be fine-tuned.

The edges $\pm z_3$ of the null-average region must be known with extreme accuracy. Unfortunately, however, they must be determined *a priori* from the dimensions of the masses. That is, there appears to be no information available which would allow *in situ* calibration. Again, from fig. 5.3*b* of Blaser *et al.* (1993) it is evident that the edge of the null-average regions is at about $z_3 \approx 0.264$ cm and that the average non-Newtonian acceleration in this region would be $\langle a_{\text{Yuk}} \rangle \approx 2.1 \cdot 10^{-17}\ \text{m/s}^2$ if $\alpha = 1 \cdot 10^{-6}$. (If the Newtonian force in the central region were exactly cubic, then z_3 would be simply the crossover point times $\sqrt{2}$.)

The excess Newtonian force due to an error ΔR_2 is calculated straightforwardly as the force due to a differential cylindrical volume element which extends between the two sleeves:

$$\Delta F_z = (GM\,\Delta m/L) \times \{[(L/2 - z)^2 + R^2]^{-1/2} - [(L/2 + z)^2 + R^2]^{-1/2}\}, \tag{11}$$

where M is the source mass, and Δm, R, and L are the mass, radius, and length of the differential cylinder. Clearly the differential mass is $\Delta m = 2\pi R\,\Delta R\,L\varrho$. From fig. 5.3*a* of Blaser *et al.* (1993), we estimate the dimensions as $L \approx 4.5$ cm; $R = R_2 \approx 2.04$ cm.

The reason for concern about whether metrological accuracy can be sufficient to achieve the G/ISL accuracy goal for α is that the shape of the force in the central region is so critically sensitive to the parameters, such as R_2. G/ISL assumes 1 ppm metrology accuracy. In applying this assumption to R_2, it is unclear whether the radius R_2 itself or the wall thickness $(R_2 - R_1)$ is assumed known to 1 ppm. With the former assumption, $\Delta R_2 = 2.0 \cdot 10^{-8}$ m; with the latter, $\Delta R_2 = 2.6 \cdot 10^{-9}$ m (based on $R_2 - R_1$). In either case, it is straightforward to calculate the mean Newtonian force over the supposedly null-average interval. The results are that the average

Newtonian accelerations, calculated from eq. (11), are

$$\begin{cases} \langle a_\mathrm{N} \rangle = 38 \times \langle a_\mathrm{Yuk} \rangle, & \text{if } \Delta R_2 = 2.0 \cdot 10^{-8}\,\mathrm{m}, \text{ and} \\ \langle a_\mathrm{N} \rangle = 4.8 \times \langle a_\mathrm{Yuk} \rangle, & \text{if } \Delta R_2 = 2.6 \cdot 10^{-9}\,\mathrm{m}, \end{cases} \tag{12}$$

for $\alpha = 1 \cdot 10^{-6}$. That is, an error in R_2 which is either 1 ppm in R_2 *per se* or 1 ppm in the wall thickness $(R_2 - R_1)$ would cause an error of either 38 ppm or 4.8 ppm, respectively, in α, values much larger than STEP's accuracy goal for α of 1 ppm. Effects on α of similar magnitude must also result from errors in the other parameters. Furthermore, any errors which violate the presumed symmetry (such as a taper in radii or wall thickness, unequal lengths of the sleeves, or an overall warp) would cause effects which would be difficult to interpret. Thus, it may be that, when account is taken of these effects, the metrology contribution to the error budget for α will be about 10 ppm if the *wall thicknesses* can be measured to 1 ppm, or about 100 ppm if only the *radii* can be measured to 1 ppm. Other issues, such as alignment errors of the test mass and expansion due to any sources of heating, may also warrant investigation. Moreover, the goal of measuring and maintaining the thickness of a cylinder wall to 1 ppm, or 2.6 nm, may be difficult to achieve. We note that this corresponds to only a few monolayers of atoms on each side.

The measurement of G would be by far the principal gravitational objective of a NEWTON mission. Although the same authors are currently involved in several related proposals for tests of the equivalence principle (Barlier *et al.* (1991), Bramanti *et al.* (1992), Nobili (1993), Spallicci (1993), and Nobili *et al.* (1995)), the possibility of using the NEWTON vehicle for this purpose is mentioned only obliquely in the NEWTON proposal (Nobili *et al.* (1990), p. 400, and Nobili *et al.* (1987), p. 438).

As a result of the necessary «drag-free» operation of the satellite, either a NEWTON, SEE, or STEP mission would provide at very little extra effort several important benefits (after Nobili *et al.* (1990)), including:

1) A very reliable gauge of solar-radiation pressure at all times.

2) A platform for accurate calibration of the next generation of accelerometers.

3) A sensitive determination of the tesseral harmonics of the Earth's field which are resonant with the period of the satellite.

A potentially much more important and interesting resonance effect has recently been pointed out by Damour and Esposito-Farèse (1994*a* and 1994*b*):

4) Resonant orbit changes associated with relativistic effects described by the various non-Einsteinian PPN parameters.

Differences among the three satellites in their orbits and principles of control would result in somewhat different methods for inferring solar-radiation pressure and atmospheric drag. In either a SEE or a NEWTON mission the enclosed, free-floating test bodies would serve as the proof mass, which allows the capsule *per se* to fluctuate substantially about the ideal geodetic trajectory. In contrast, the STEP vehicle would be maintained continuously under extremely precise three-axis compensation. The SEE satellite would be in a 1500 km altitude orbit, and the solar-radiation pressure would be inferred from the displacement of the capsule orbit. The NEWTON satellite would be in a geosynchronous orbit, and the solar-radiation pressure could be inferred either from the cumulative impulse required to *prevent* a secular change of the orbit or from the drift of the capsule with respect to the center of mass of the test bodies during the interval between thruster bursts. The STEP satellite would be in a very low altitude orbit, so solar-radiation pressure could be inferred from the thrust required to maintain the cross-track position. In addition, a STEP or SEE mission would also provide data on atmospheric drag at the altitude of 550 or 1500 km.

Concerning resonances with tesseral harmonics in the Earth's ordinary geopotential field, this technique, which was discovered by Yionoulis (Yionoulis (1965 and 1966)), accentuates the effect of those harmonic coefficients of a given order m if the rotation period of the Earth is approximately equal to m times the period of the satellite. Thus, King-Hele has emphasized the value of resonances for detecting inaccuracies in individual coefficients in an expansion of the geopotential, even in cases when the expansion gives an overall satisfactory fit (see, for example, King-Hele (1980)). Unfortunately, however, the result most readily obtained from resonances is a lumped sum of the coefficients of all *degrees* of the order m unless data from multiple satellites are available.

A SEE mission could best provide data on the terms of order 12, although this would require a somewhat higher altitude (~ 1680 km) in order to make its period roughly one-twelfth of the Earth's rotation period. An inconvenient side effect of any long-period resonance in the orbital period is that it would complicate the interpretation of a $\mathrm{d}G/\mathrm{d}t$ experiment, perhaps even to the point that its credibility would be compromised by the required corrections. A STEP mission could provide resonance data on terms of order 15. Increasing the altitude by only 15.5 km would put STEP exactly on the $m = 15$ resonance (assuming a sun-synchronous orbit). A NEWTON mission, because of its geosynchronous orbit, would be sensitive only to $m = 1$ terms. However, even resonance effects would be severely attenuated by such a large orbital radius.

The possibility of finding resonances due to non-Einsteinian terms is of course much more intriguing. The motivation to search for such terms at this time is based chiefly on the work of Damour and colleagues concerning the convergence of recent developments in theory and in experimental capability. On the theory side, despite strong experimental evidence that any scalar term (such as arises in Brans-Dicke theory) must be extremely small, Damour and colleagues advocate a reappraisal of the possibility that gravity may entail scalar, vector, or non-Einsteinian-tensor terms, on the grounds that any scalar term would now quite naturally be very small as a consequence of cosmological expansion (Damour and Esposito-Farèse (1993), Damour and Nordtvedt (1993*a* and 1993*b*) and Damour and Polyakov (1994)). On the experimental side, they argue that the capability for centimeter-level tracking, as demonstrated with the LAGEOS satellites, means that several non-Einsteinian effects might now be observable with drag-free satellites in low Earth orbits through orbital-resonance techniques (Damour and Esposito-Farèse (1994*a* and 1994*b*)). The possible resonances arise from near-vanishing denominators, which are sums and differences of various precession rates.

Fortunately, the planned orbits of both NEWTON and SEE are very close to resonances for the PPN parameter α_1. A non-zero value of this parameter would indicate that a preferred reference frame exists. To wit, any geosynchronous orbit is very useful, such as that planned for NEWTON, although the ideal period is actually about 30 hours. However, the duration of the mission must be long enough that the departure angle between the orbit's perigee and the reference direction can be accurately measured. Damour and Esposito-Farèse estimate that this would require 12 to 15 years. Such a long mission duration raises questions of satellite longevity. In particular, 12 years is a very long time to expect a disturbance-compensation system to perform with no failures. Moreover, for a DISCOS system which relies on consumable fuel, the fuel requirements would also be significant. The determination of G might be compromised by the perturbation due to a large fuel mass which changes substantially over the life of the mission. This raises concerns in the case of NEWTON, since its DISCOS uses about 2 kg/y of hydrazine fuel. However, these concerns would be greatly alleviated if the NEWTON DISCOS system can be altered to use FEEP thrusters.

The planned STEP orbit is also fairly close to several potential α_1 resonances, but the short mission duration (8 months) is not compatible with the very long times needed to observe such a resonance, and drag-free control would be lost after the helium had been exhausted. Moreover, the use of solar sailing to extend the mission would not be feasible, since atmospheric drag at the STEP altitude is comparable to the force of solar-radiation pressure.

The low Earth Sun-synchronous orbit previously planned for SEE is close to several of the eccentricity resonances identified by Damour and Esposito-Farèse which are suitable for measuring α_1. The resonances result from near-zero values of various combinations of the mean motion of the Earth around the Sun, $n_E = 2\pi/T_E$, and the rates of precession of perigee and the orbital plane, which are given to lowest order by the expressions

$$\dot{\omega} = (3/4)J_2\sqrt{(GM_E/R_E^3)} \times (a/R_E)^{-7/2} \times (4 - 5\sin^2 I), \tag{13}$$

$$\dot{\Omega} = -(3/2)J_2\sqrt{(GM_E/R_E^3)} \times (a/R_E)^{-7/2} \times (\cos I), \tag{14}$$

where J_2 represents the quadrupole moment of the Earth, M_E and R_E are the mass and radius of the Earth, and a and I are the radius and inclination of the satellite's orbit. The expression of Damour and Esposito-Farèse (1994*a*) for the eccentricity (eqs. (38) and (54)) contains non-Einsteinian terms whose denominators are sums and differences of these rates, *viz.* $\dot{\omega} \pm \dot{\Omega} \pm n_E$ and, simply, $\dot{\omega} \pm n_E$. It is convenient to express the resonant condition in terms of the values of the perigee precession rate $\dot{\omega}$. Since $n_E = 2\pi/\mathrm{y}$, and since a Sun-synchronous orbit by definition has orbital precession rate $\dot{\Omega} = 2\pi/\mathrm{y}$, it follows that the resonance requirement is met by a SEE orbit when the perigee precession rate is a multiple of $-2\pi/\mathrm{y}$. Figure 11 shows that this occurs for three different values of the inclination angle. The resonance conditions and the Sun-synchronous requirement are met simultaneously for three different values of $\dot{\omega}$:

i) at $\dot{\omega} = -\dot{\Omega} - n_E = -4\pi/\mathrm{y}$, which occurs at $I = 101.54°$,

ii) at the double coincidence at $\dot{\omega} = -\dot{\Omega} = -n_E = -2\pi/\mathrm{y}$, which occurs at $I = 106.85°$, and

iii) at the triple coincidence at $\dot{\omega} = 0 = -\dot{\Omega} + n_E = +\dot{\Omega} - n_E = 0 \times (-2\pi/\mathrm{y})$, which occurs at $I = 116.57°$.

The first resonance above, at $I = 101.54°$, occurs almost exactly at the nominal orbit originally proposed for SEE (102° inclination). The double coincidence, at

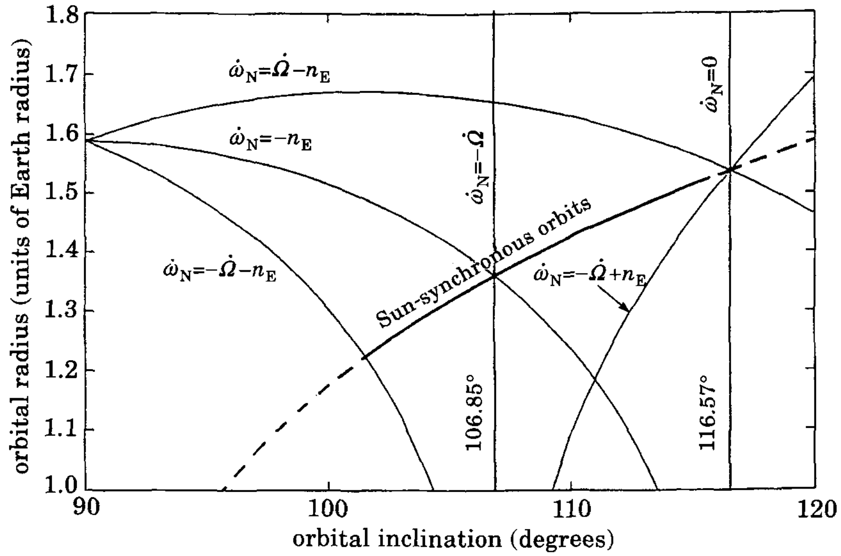

Fig. 11. – Relationships between orbital radius and inclination which satisfy the Sun-synchronous condition and various non-Einsteinian resonances. The heavy curve traversing the figure diagonally indicates the Sun-synchronous condition, while the solid portion thereof indicates the range of parameters which causes the satellite to be in continuous sunlight year-round. The two intersections of the Sun-synchronous curves with resonances, at $I = 101.54°$ and $106.85°$, would allow tests for these resonances on a SEE mission. The intersection at $I = 116.57°$ is slightly outside the continuous-sunlight range.

106.85°, is significantly stronger than the others. However, it would require a substantially higher altitude (2300 km rather than 1500 km).

Note that only a *Sun-synchronous* orbit can satisfy the coincidence requirements, thus allowing two or more resonance effects to contribute simultaneously. However, if the coincidence is exact, then mutual cancellation among the various terms in the resonance equations given by Damour and Esposito-Farèse (Damour and Esposito-Farèse (1994*a*), eqs. (38) and (54)) may occur, depending on the signs of the numerators. This difficulty can be averted by offsetting the nodal precession rate $\dot\Omega$ slightly from the exact Sun-synchronous value ($2\pi/\mathrm{y}$). The resulting orbit curve will therefore lie slightly above or below the Sun-synchronous curve shown in fig. 11. Thus, the signs of the individual terms in the resonance equations can be predetermined by careful choice of the inclination angle I and the offset of the nodal precession rate.

Even as resonances, any non-Einsteinian effects will be very small. Given the existing limits on the PPN parameters, none of the effects would cumulate at the rate of more than a few tens of centimeters per year, and for some effects the maximum possible rate is much less. This implies long observation periods and very stringent demands for long-term tracking accuracy. The latter, in turn, may require a more accurate knowledge of the Newtonian field of the Earth than is available at present. Damour and Esposito-Farèse point out the need for numerical simulations to clarify this point (Damour and Esposito-Farèse (1994*a*), p. 1698).

Many applications for drag-free satellites are as yet unrealized, even though this technology has existed for nearly 30 years (Lange (1964), Staffs (1974)). To date only one fully drag-free satellite has ever actually flown, namely TRIAD-I of the Applied Physics Laboratory (APL). Several other APL satellites in the NOVA series and the ESA satellite CASTOR, which used the CACTUS compensation system, were drag-free in the along-track direction but lacked a full three-dimensional DISCOS system (however, CASTOR accelerations were *measured* in all three axes (Gay (1995))). Moreover, although TRIAD-I operated in a fully drag-free mode, a telemetry failure resulted in the loss of most of the aeronomy data concerning the operation of the DISCOS system, which are necessary to infer the drag (Black (1990 and 1994)).

10. – Conclusion

A determination of G is a precision-measurement experiment. Therefore it calls for stringent metrology and a thorough knowledge of competing effects. These issues were generally sidestepped in the EPIC proposals, in favor of examining the idealized physical mechanisms of the experimental arrangement employed [12]. In general, very careful measures such as were taken by Turneaure *et al.* (1986) are needed to assure that small effects can be observed under the actual experimental conditions in space. Great care will also be required with regard to the NEWTON, SEE, and STEP-G/ISL experiments, lest their accuracy be vitiated by overlooked or insufficiently evaluated interactions.

The common practice of approaching the kinematics of the test-body interaction from the point of view of an idealized force-free region has caused considerable difficulty in conceptualizing the test-body interactions and in deriving and interpreting the equations of motion. This approach requires viewing the Earth's gravity gradient as a «problem». At best this has resulted in the expenditure of a great deal of effort, and it has in some instances led investigators to the problematic step of proposing artificial restraints in order to force the motion of the test bodies to imitate that which would result in an idealized force-free region. In contrast, the point of view that both of the test bodies are in orbit around the Earth and are only slightly perturbed by each other is much more natural, both mathematically and conceptually.

In any G-in-space experiment, it is important to minimize vibrations of the capsule and to maintain its thermal stability. The motivation for thermal stability is to minimize both the changes in the size and shape of the capsule and the net force on the test bodies due to blackbody radiation anisotropy. A reasonable and achievable stipulation is that thermal stability be sufficient to maintain rigidity (all intra-capsule distances) to within about one optical wavelength. Vibration avoidance also requires essentially jerk-free thrust and torque.

The satellite housing a G experiment must be drag free, which entails counteracting atmospheric drag itself and also compensating for micrometeorite strikes, solar-radiation pressure, and Earth-radiation pressure. Except at very low altitudes, solar-radiation pressure is the dominant effect, and at extremely

(12) The authors of several of the EPIC proposals were experienced in spacecraft-based design and undoubtedly recognized the need for much further study.

high (geosynchronous) orbits, it blows an uncompensated satellite out of its orbital plane by ten meters or more.

Careful choice of orbit is a critical element in a strategy for achieving these ends. Sun-synchronous orbits are well suited to these purposes. Such an orbit was prescribed in the SEE and STEP proposals. The problems accompanying geosynchronous orbits raise serious questions about their practicability.

The issue of mass distribution errors in the large test mass, due to both density inhomogeneities and shape errors (such as non-sphericity), warrant close scrutiny, especially in experiments which entail small separations between the test bodies. The use of cylindrical geometry for the test masses, as in a Cook-Marussi stack, may substantially alleviate errors in the latter regard.

It is very important that substantial capacity for self-calibration be incorporated to the greatest possible extent in the design of any experiment for making measurements of Newtonian gravity in space. In particular, *in situ* analysis of the mass distribution of the capsule and large test body is essential, since it is unlikely that this distribution can be modeled *a priori* to sufficient accuracy. SEE has this capability intrinsically, as a consequence of the large variation both in the separation of the test bodies and in their positions within the capsule.

Finally, the instrument-design requirements for measuring G in space overlap substantially with those for accomplishing several related objectives which also entail stringent precision-measurement capability. These include measurements of drag and radiation pressure, trials of next-generation accelerometers, exploration of selected harmonics in the Earth's geopotential field through resonances, and tests for various effects bearing on fundamental aspects of gravitation theory, including possible violations of the equivalence principle, time variation of G, and non-Einsteinian values of the PPN parameters. NEWTON, SEE and STEP are all capable of pursuing a number of these objectives.

Note added in proofs.

Fujii (1972) has also discussed the possibility of space-based gravitational experiments, within the context of searching for a gravity-like force of intermediate range.

* * *

The authors are pleased to acknowledge the interest and assistance of many colleagues, including: Steve Allison, Carlo Arduini, Harold Black, George Bush, Giuseppe Catastini, Mike Cates, Thibault Damour, Dan DeBra, Ed Deeds, Francis Everitt, Gilles Esposito-Farèse, Paolo Farinella, Glen Fountain, Michel Gay, Nick Locherbie, John Junkins, Doug Mashburn, Vitaly Melnikov, Riley Newman, Anna Nobili, Ho Jung Paik, Lee Pryor, Rogers Ritter, Matt Scudiere, Larry Smalley, A. D. A. M. Spallicci, Jim Thompson, Paul Worden and Steve Yionoulis. We are also grateful to *Astrophysics and Space Science* and *ESA Journal* for permission to reproduce figures. We thank the following institutions for their assistance, and where appropriate, sponsorship: The University of Tennessee, Knoxville; Oak Ridge National Laboratory; the UTK/ORNL Science Alliance; The University of Virginia; Gosstandard, Moscow (Russian Bureau of Standards); and the Scientific Research Center for Surface and Vacuum Studies, Moscow.

REFERENCES

ALEKSEEV A. D., BRONNIKOV K. A., KOLOSNITSYN N. I., KONSTANTINOV M. YU., MEL'NIKOV V. N. and RADYNOV A. G., *Analysis of the project for a space experiment to measure the gravitational constant and possibly the ones of non-Newtonian-type interactions*, paper presented to *Eighth Russian Gravitational Conference* (*Moscow, May, 1993a*) (presented as *Анализ проекта космического Зксперимента по измерению констант гравитационного и возможныощ неньютоновскио взаимодействий*).

ALEKSEEV A. D., BRONNIKOV K. A., KOLOSNITSYN N. I., MEL'NIKOV V. N. and RADYNOV A. G., *Error sources in Earth satellite measurements of gravitational interaction parameters*, *Meas. Tech.*, **36** (1993*b*) 1070-1077 (translation of *Источники ошибок в измерениях параметров гравитационного взаимодействия на спутниках Земли*, *Izmer. Tekh.*, **10** (1993*b*) 6-9).

ALEKSEEV A. D., BRONNIKOV K. A., KOLOSNITSYN N. I., MEL'NIKOV V. N. and RADYNOV A. G., *Simulation of the procedure for measuring the gravitational constant on an Earth satellite*, *Meas. Tech.*, 37 (1994) 1-5 (translation of *Моделирование процедуры измерния гравитационной постоянной на спутнкие Земли*, *Izmer. Tekh.*, **1** (1994) 3-5).

ALLISON S. W., private communication (1994).

ANTONYUK P. N., BRONNIKOV K. A. and MEL'NIKOV V. N., *Determination of the gravitational constant from moving bodies in the vicinity of a libration point*, *Meas. Tech.*, **36** (1993) 837-844 (translation of *Определение гравитационной постоянной при движении частицы в окрестности точки цибрации*, *Izmer. Tekh.*, 8 (1993) 3-6).

ANTONYUK P. N., BRONNIKOV K. A. and MEL'NIKOV V. N., *Determination of the gravitational constant G by particle motion inside a satellite in the vicinity of a libration point*, *Astron. Lett.*, **20** (1994) 59-61 (translation of *Определение гравитационной постоянной при движении частицы в окрестности точки либрации*, *Pis'ma Astron. Ž.*, **20** (1994) 72-75).

ARDUINI C., GENOVESE L. and LANEVE G., *On thermal models of satellites for gravitational experiments*, in *Summary Book, Second National Meeting on Galileo Galilei* (*GG-FEEP*^{2}), *Pisa, May 3-5, 1995* (Università di Pisa) 1996, pp. 68-69.

AVRON Y. and LIVIO M., *Considerations regarding a space-shuttle measurement of the gravitational constant*, *Astrophys. J.*, **304** (1986) L61-64.

BAKER S. D., *Some considerations on measuring the Newtonian gravitational constant G in an orbiting laboratory*, NASA memorandum, Contract No. NGT-01-008-021 (Marshall Space Flight Center and The University of Alabama in Huntsville, August 11, 1987), pp. 7.

BAKER S. D. and FALK A. F., *Orbits inside a spacecraft and measuring the gravitational constant G*, preprint, Rice University and University of North Carolina (1987*a*).

BAKER S. D. and FALK A. F., *Measuring the Newtonian gravitational constant G in an orbiting laboratory*, *Bull. Am. Phys. Soc.*, **32** (1987*b*) 91.

BARLIER F., BLASER J.-P., CAVALLO G., DAMOUR T., DECHER R., EVERITT C. W. F., FULIGNI F., LEE M., NOBILI A. M., NORDTVEDT K., PACE O., REINHARD P. and WORDEN P. W., *STEP* (*Satellite Test of Equivalence Principle*): *Assessment Study*, ESA/NASA report SCI (91) 4 (January, 1991), pp. 67.

BERMAN D., *Discussion and analysis of the rotating flat plate Newtonian gravitational constant experiment*, Appendix F in FORWARD R. L., *Research toward feasibility of an instrument for measuring vertical gradients of gravity*, Report AFCRL-67-0631, Air Force contract No. AF 19(628)-6134 (Air Force Cambridge Research Laboratories, Bedford, Mass.) 1967*a*.

BERMAN D., *Discussion and analysis of the vertically tunneled sphere-Newtonian gravitational constant experiment*, Appendix G in FORWARD R. L., *Research toward feasibility of an instrument for measuring vertical gradients of gravity*, Report AFCRL-67-0631, Air Force contract No. AF 19(628)-6134 (Air Force Cambridge Research Laboratories, Bedford, Mass.) 1967*b*.

BERMAN D. and FORWARD R. L., *Free-fall experiments to determine the Newtonian gravitation constant* (G), in *Advances in the Astronautical Sciences*, edited by H. JACOBS, *Exploitation of Space for Experimental Research*, Vol. **24** (American Astronomical Society, Tarzana, Cal.) 1969, pp. 95-115.

BLACK H. D., *Early development of Transit, the Navy navigation satellite system, J. Guidance, Contr., Dyn.*, **13** (1990) 577-585.

BLACK H. D., private communication (1994).

BLASER J.-P., BYE M., CAVALLO G., DAMOUR T., EVERITT C. W. F., HEDIN A., HELLINGS R. W., JAFRY Y., LAURENCE R., LEE M., NOBILI A. M., PAIK H. J., REINHARD R., RUMMEL R., SANFORD M. C. W., SPEAKE C., SPENCER L., SWANSON P. and WORDEN P. W., *STEP* (*Satellite Test of Equivalence Principle*): *Report on the Phase A Study*, ESA/NASA report SCI (93) 4 (March, 1993).

BLASER J.-P., CRUISE A. M., DAMOUR T., JAFRY Y., LAURENCE R., LEÓN J., PAIK H. J., REINHARD R., RUMMEL R. and WORDEN P. W., *STEP* (*Satellite Test of Equivalence Principle*): *Assessment Study*, ESA/NASA report SCI (94) 5 (May, 1994).

BLOOD B. E., *Error study of the Beams experiment concept used in a $\Delta G/G$ detector*, Report RN-63 (January, 1971), Appendix B in WILK L. S. (Editor), Progress Report PR-8, *Studies of space experiments to measure gravitational constant variations and the Eötvös ratio*, NASA contract No. NAS 9-8328 (MIT Measurement Systems Laboratory, Cambridge, Mass.) 1971.

BRAGINSKY V. B. and MANUKIN A. B., *Measurement of Weak Forces in Physics Experiments*, edited by D. H. DOUGLASS (University of Chicago Press, Chicago) 1977.

BRAMANTI D., NOBILI A. M. and CATASTINI G., *Test of the equivalence principle in a non-drag-free spacecraft, Phys. Lett. A*, **164** (1992) 243-254.

BRAUN C., *Die Gravitations-Konstante, die Masse und mittelere Dichte der Erde nach einer neuen experimentellen Bestimmung, Denkschr. Akad. Wiss. Wien, Math. Naturwissen. Klasse*, **64** (1897) 187-258.

BRONNIKOV K. A., KOLOSNITSYN N. I., KONSTANTINOV M. YU., MEL'NIKOV V. N. and RADYNOV A. G., *Measurement of the gravitational interaction parameters of an Earth-orbiting satellite, Meas. Tech.*, **36** (1993*a*) 845-852 (translation of *Измерение параметров гравитационного взаимодействия на спутнике Земли, Izmer. Tekh.*, 8 (1993*a*) 6-10).

BRONNIKOV K. A., KOLOSNITSYN N. I., KONSTANTINOV M. YU., MEL'NIKOV V. N. and RADYNOV A. G., *Numerical modeling of the trajectories of particles for measuring the gravitational constant on an artificial satellite, Meas. Tech.*, **36** (1993*b*) 951-957 (translation of *Численное моделирование траекторий частиц для измерения гравитационной постоянной на ИСЗ, Izmer. Tekh.*, **9** (1993*b*) 3-6).

CATASTINI G. and NOBILI A., *The GG-FEEP*^{2} *thermal stability*, in *Summary Book, Second National Meeting on Galileo Galilei* (*GG-FEEP*^{2}), *Pisa, May 3-5, 1995* (Università di Pisa) 1996, pp. 30-31.

CATASTINI G., BRAMANTI D., NOBILI A. M., FULIGNI F. and IAFOLLA V., *The pico gravity box: An efficient passive noise attenuator in space, ESA J.*, **16** (1992) 401-417.

CHAPMAN P. K., *Synchronous orbit sphere*, in WILK L. S. (Editor), Progress Report PR-8, *Studies of space experiments to measure gravitational constant variations and the Eötvös ratio*, NASA contract No. NAS 9-8328 (MIT Measurement Systems Laboratory, Cambridge, Mass.) 1971.

COHEN E. R. and TAYLOR B. N., *The 1986 adjustment of the fundamental physical constants, CODATA Bulletin*, **63** (1986) 12.

COHEN E. R. and TAYLOR B. N., *The fundamental physical constants, Phys. Today*, **40** (1987) BG11-BG15.

COHEN E. R. and TAYLOR B. N., *The fundamental physical constants, Phys. Today*, **47** (1994) BG9-BG16.

COOK A. H., *A new determination of the constant of gravitation, Contemp. Phys.*, **9** (1968) 227-238.

CORNAZ A., HUBLER B. and KÜNDIG W., *Determination of the gravitational constant at an effective interaction distance of* 112 m, *Phys. Rev. Lett.*, **72** (1994) 1152-1155.

DAMOUR T. and ESPOSITO-FARÈSE G., *Nonperturbative strong-field effects in tensor-scalar theories*, *Phys. Rev. Lett.*, **70** (1993) 2220-2223.

DAMOUR T. and ESPOSITO-FARÈSE G., *Testing for preferred-frame effects in gravity with artificial Earth satellites*, *Phys. Rev. D*, **49** (1994*a*) 1693-1706.

DAMOUR T. and ESPOSITO-FARÈSE G., *Orbital tests of relativistic gravity using artificial satellites*, *Phys. Rev. D*, **50** (1994*b*) 2381-2389.

DAMOUR T. and NORDTVEDT K., *General relativity as a cosmological attractor of tensor-scalar theories*, *Phys. Rev. Lett.*, **70** (1993*a*) 2217-2219.

DAMOUR T. and NORDTVEDT K., *Tensor-scalar cosmological models and their relaxation toward general relativity*, *Phys. Rev. D*, **48** (1993*b*) 3436-3450.

DAMOUR T. and POLYAKOV A. M., *The string dilaton and a least coupling principle*, *Nucl. Phys. B*, **423** (1994) 532-558.

DARWIN G. H., *Periodic orbits*, *Acta Math.*, **21** (1897) 99-242.

DEBLER E., *Set-up of mass scales above* 1 kg *illustrated by the example of a* 5 t *mass scale*, *Metrologia*, 28 (1991) 85-94.

DEEDS W. E., private communication (1994).

DERMOTT S. F. and MURRAY C. D., *The dynamics of tadpole and horseshoe orbits: Theory*, *Icarus*, 48 (1981*a*) 1-11.

DERMOTT S. F. and MURRAY C. D., *The dynamics of tadpole and horseshoe orbits II: Experiment*, *Icarus*, 48 (1981*b*) 12-22.

DE VENUTO F., DOBROWOLNY M., FULIGNI F., IAFOLLA V., NOZZOLI S. and VANNARONI G., *Direct measurement of the FEEP thruster force in the micronewton range*, in *Summary Book, Second National Meeting on Galileo Galilei (GG-FEEP*^{2}*), Pisa, May 3-5, 1995* (Università di Pisa) 1996, p. 14.

EVERITT C. W. F., *Feasibility analysis of gravitational experiments in space*, Final Report on Contract No. 954524, W. W. Hansen Laboratories of Physics, Stanford University (September, 1977), pp. 32.

FARINELLA P., MILANI A. and NOBILI A. M., *Measurement of the gravitational constant in space*, in *Instabilities in Dynamic Systems*, edited by V. G. SZEBEHELY, *NATO ASI Ser. C*, **47** (Reidel, Dordrecht) 1979, pp. 296-297.

FARINELLA P., MILANI A. and NOBILI A. M., *The measurement of the gravitational constant in an orbiting laboratory*, *Astrophys. Space Sci.*, **73** (1980) 417-433.

FITZGERALD M. P. and ARMSTRONG T. R., *Newton's gravitational constant with uncertainty less than* 100 ppm, *IEEE Trans. Instrum. Meas.*, 44 (1995*a*) 494-497.

FORWARD R. L., *Research toward feasibility of an instrument for measuring vertical gradients of gravity*, Report AFCRL-67-0631, Air Force contract No. AF 19(628)-6134 (Air Force Cambridge Research Laboratories, Bedford, Mass.) 1967.

FORWARD R. L., *Electronic damping of orthogonal bending modes in a cylindrical mast—Experiment*, *J. Spacecraft and Rockets*, **18** (1981) 11-17.

FORWARD R. L., *Flattening space-time near the Earth*, *Phys. Rev. D*, **26** (1982) 735-744.

FROESCHLE M. and MIGNARD F., *Numerical simulation of the signal and data processing of the HIPPARCOS satellite*, *Appl. Opt.*, **20** (1981) 3251-3258.

FUJII Y., *Scale invariance and gravity of hadrons*, *Ann. Phys. (N.Y.)*, **69** (1972) 494-521.

GAY M., private communication (1995).

GENTA G., BRUSA E. and ROSINI S., *Numerical stability analysis of the GG and GG-FEEP*^{2} *rotating systems*, in *Summary Book, Second National Meeting on Galileo Galilei (GG-FEEP*^{2}*), Pisa, May 3-5, 1995* (Università di Pisa) 1996, pp. 24-26.

GILLIES G. T., *The Newtonian gravitational constant: An index of measurements*, *Metrologia*, **24** (Suppl.) (1987) 1-56.

GILLIES G. T., *Status of the Newtonian gravitational constant*, in *Gravitational Measurements,* *Fundamental Metrology, and Constants*, edited by V. DE SABBATA and V. N. MELNIKOV (Kluwer Academic Publishers, Dordrecht) 1988, pp. 191-214.

GILLIES G. T., *Reseource letter MNG-1: Measurements of Newtonian gravitation*, *Am. J. Phys.*, 58 (1990) 525-534.

GILLIES G. T. and RITTER R. C., *Torsion balances, torsion pendulums, and related devices*, *Rev. Sci. Instrum.*, 64 (1993) 283-309.

GILLIES G. T. and SANDERS A. J., *Getting the measure of gravity*, *Sky and Telescope*, 85 (1993) 28-32.

HALL C., *Alternative method to measure the gravitational constant*, *Bull. Am. Phys. Soc.*, **35** (1990) 1328.

HÉNON M., *Vertical stability of periodic orbits in the restricted problem*, *Astron. Astrophys.*, **30** (1974) 317-321.

HERNDON J. N., private communication (1993).

HILLS J. G., *Space measurement of the gravitational constant using an artificial binary*, *Astron. J.*, **92** (1986) 986-988.

HULETT M. J., *On the Newtonian Gravitational Constant*, B.A. Thesis, Wesleyan University (1969), pp. 97.

KAULA W. M., *Theory of Satellite Geodesy* (Blaidsdell, Waltham, Mass.) 1966.

KEYSER P. T., *Perturbative forces on an artificial binary for measuring G*, *Phys. Lett. A*, **167** (1992) 29-31.

KEYSER P. T., *Perturbative forces in the proposed Satellite Energy Exchange experiment*, *Phys. Rev. D*, 47 (1993) 3658-3659.

KING-HELE D. G., *The gravity field of the Earth*, *Philos. Trans. R. Soc. London, Ser. A*, **294** (1980) 317-328.

KOLDEWYN W. A., *A new method for measuring the Newtonian gravitational constant G*, Ph.D. Dissertation, Wesleyan University (1976), pp. 171.

KRAMER JOANN S., *Determination of Newton's gravitational constant G with increased precision; a theoretical analysis*, M.Sc. Thesis, University of Virginia (1967), pp. 160.

LANGE B., *The drag-free satellite*, *Am. Institute Aeron. Astron. J.*, 2 (1964) 1590-1606.

LEE W. N., *A preliminary analysis of the effects of using non-spherical masses in a Beams-type measurement of G or $\Delta G/G$*, Report RE-74 (October, 1970), Appendix D in WILK L. S. (Editor), Progress Report PR-8, *Studies of space experiments to measure gravitational constant variations and the Eötvös ratio*, NASA contract No. NAS 9-8328 (MIT Measurement Systems Laboratory, Cambridge, Mass.) 1971.

LIDOV M. L. and VASHKOV'YAK M. A., *On quasi-satellite orbits for experiments on refinement of the gravitation constant*, *Astron. Lett.*, **20** (1994) 188-198 (translation of *О квазиспутниковых орбитах для Зксперимента по уточнению гравитационной постоянной*, *Pis'ma Astron. Ž.*, **20** (1994) 229-240).

LOCKERBIE N., *Gravitational balancing of cylindrical bodies with flat surfaces and its application to the design of test masses for the STEP experiment*, in *Summary Book, Second National Meeting on Galileo Galilei (GG-FEEP^2), Pisa, May 3-5, 1995* (Università di Pisa) 1996, pp. 32-33.

LORENZI E. C. and GULLAHORN G. E., *Recent developments in gravity gradiometry from the Space–Shuttle-borne tethered satellite system*, *J. Appl. Phys.*, **63** (1988) 216-223.

LUTHER G. G., *A redetermination of the Newtonian gravitational constant currently being undertaken at the Los Alamos National Laboratory*, *Bull. Am. Phys. Soc.*, **40** (1995) 976.

LUTHER G. G. and TOWLER W. R., *Redetermination of the Newtonian gravitational constant, G*, *Phys. Rev. Lett.*, 48 (1982) 121-123.

MARCUCCIO S. and ANDRENUCCI M., *FEEP thrusters: state of the art*, in *Summary Book, Second National Meeting on Galileo Galilei (GG-FEEP^2), Pisa, May 3-5, 1995* (Università di Pisa) 1996, pp. 10-13.

MASHBURN D. N., private communication (1994).

MELNIKOV V. N., *Fundamental physical constants and their stability: A review, Internat. J. Theor. Phys.*, 33 (1994) 1569-1579.

MELNIKOV V. N., KONSTANTINOV M. YU., KOLOSNITSIN N. I., BRONNIKOV K. A., RADYNOV A. G., ALEKSEEV A. D. and ANTONYUK P. N., *Report for SEE Project* (1993).

MICHAELIS W., HAARS H. and AUGUSTIN R., *A new determination of the gravitational constant G, Metrologia*, 32 (1996) 267-276.

MILANI A., NOBILI A. M. and FARINELLA P., *Non-Gravitational Perturbations and Satellite Geodesy* (Adam Hilger, Bristol) 1987.

NOBILI A. M., *FPAG scientific assessment of NEWTON proposal*, communication to Fundamental Physics Assessment Group (FPAG), J.-P. BLASER (Chairman) (October 21, 1993*a*).

NOBILI A. M., *Test of the equivalence principle in space, Nuovo Cimento C*, **16** (1993*b*) 789-793.

NOBILI A. M., BRAMANTI D., POLACCO E., CATASTINI G., GENTA G., BRUSA E., MITROFANOV V. P., BERNARD A., TOUBOUL P., COOK A. J., HOUGH J., ROXBURGH I. W., POLNAREV A., FLURY W., BARLIER F. and MARCHAL C., *Proposed new test of the equivalence test in space*, Università di Pisa, preprint (February 1995), p. 41.

NOBILI A. M., MILANI A., POLACCO E., ROXBURGH I. W., BARLIER F., AKSNES K., EVERITT C. W. F., FARINELLA P., ANSELMO L. and BOUDON Y., *NEWTON: A manmade planetary system in space to measure the constant of gravity G: Proposal for the new medium-size mission of the European Space Agency (ESA)* (1989), p. 28.

NOBILI A. M., MILANI A., POLACCO E., ROXBURGH I. W., BARLIER F., AKSNES K., EVERITT C. W. F., FARINELLA P., ANSELMO L. and BOUDON Y., *The NEWTON mission—A proposed manmade planetary system in space to measure the gravitational constant, ESA J.*, **14** (1990) 389-408.

NOBILI A. M., MILANI A., POLACCO E., BRAMANTI D., CATASTINI G., ROXBURGH I. W., BARLIER F., AKSNES K., EVERITT C. W. F., FARINELLA P., ANSELMO L. and BOUDON Y., *NEWTON: A manmade planetary system in space to measure the constant of gravity G: Proposal for the M3 medium-zise mission of ESA* (May, 1993).

NOBILI A. M., MILANI A. and FARINELLA P., *Testing Newtonian gravity in space, Phys. Lett. A*, **120** (1987) 437-441.

NOBILI A. M., MILANI A. and FARINELLA P., *The orbit of a space laboratory for the measurement of G, Astron. J.*, **95** (1988) 576-578.

OSIPOVA A. V., *Parameter constraints in a theoretical model for trial-body motion in a satellite experiment designed to refine the gravitational constant, Meas. Tech.*, **36** (1993) 1305-1310 (translation of *Об ограничениях на параметры теоретической модели движения пробных тел в спутниковом зксперименте по уточнению гравитационной постоянной, Izmer. Tekh.*, 12 (1993) 3-6).

PAIK H. J., *Superconducting accelerometry: its principles and applications, Class. Quantum Grav.*, **11** (1994) A133-A144.

PAIK H. J., private communication (1995).

PAIK H. J. and BLASER J.-P., *Constant of gravity and inverse-square law experiments on STEP*, in *Proceedings of the STEP Symposium, University of Pisa, April, 1993* (in press).

PLANCK M., *The Theory of Heat Radiation* (Dover, New York, N.Y.) 1991 (translation of *Vorlesungen über die Theorie der Wärmestrahlung*).

PONTIKIS C., *Détermination de la constante de gravitation par la méthode de résonance, C. R. Acad. Sci. Paris*, **274** (1972) 437-440.

REASENBERG R. D., BABCOCK R. W., CHANDLER J. F., GORENSTEIN M. V., HUCHRA J. P., PEARLMAN M. R., SHAPIRO I. I., TAYLOR R. S., BENDER P., BUFFINGTON A., CARNEY B., HUGHES J. A., JOHNSTON K. J., JONES B. F. and MATSON L. E., *Microarcsecond astronomy: An instrument and its astrophysical applications, Astron. J.*, **96** (1988) 1731-1745.

REIF F., *Fundamentals of Statistical and Thermal Physics* (McGraw-Hill, New York, N.Y.) 1965, pp. 465-471.

REINHARD R., JAFRY Y. and LAURENCE R., *STEP—A fundamental-physics laboratory in space*, *ESA J.*, **18** (1994) 229-235.

RITTER R. C. and GILLIES G. T., *A satellite measurement of G to a part per million*, University of Virginia Preprint (unpublished) (1981), pp. 5.

RITTER R. C. and GILLIES G. T., *A satellite measurement of G to a part per million*, *Bull. Am. Phys. Soc.*, **29** (1984) 703.

ROLL P. G., KROTKOV R. and DICKE R. H., *The equivalence of inertial and passive gravitational mass*, *Ann. Phys.*, **26** (1964) 442-517.

ROXBURGH I. W., ASKNES K., BARLIER F., BOUDON Y., EVERITT C. W. F., MILANI A., NOBILI A. M. and TRAVKOL R., *GRAVCON: A proposal to measure the constant of gravity G using the man-tended free flyer of space station*, submitted in response to the *Call for New Mission Proposals for the Next Medium Size Project (M2) Based on the Use of Free Fliers and Space-Station-Related Facilities* (November 1989) pp. 13.

DE RÚJULA A., *On forces weaker than gravity*, *Phys. Lett. B*, **180** (1986) 213-220.

SAGITOV M. U., MILYUKOV V. K., MONAKHOV YE. A., NAZARENKO V. S. and TADZHIDINOV KH. G., *A new determination of the Cavendish gravitational constant*, *Dokl. Akad. Nauk SSSR*, **245** (1981) 20-22 (translation of *Новое определение кавендишевой гравитационной постоянной*, *Dokl. Akad. Nauk CCCP*, **245** (1979) 567-569).

SANDERS A. J. and DEEDS W. E., *Proposed new determination of the gravitational constant G and tests of Newtonian gravitation*, *Phys. Rev. D*, **46** (1992*a*) 489-504.

SANDERS A. J. and DEEDS W. E., *Project SEE: Proposed new determination of the gravitational constant G and tests of Newtonian gravitation*, *Bull. Am. Phys. Soc.*, **37** (1992*b*) 1675.

SANDERS A. J. and DEEDS W. E., *Reply to «Perturbative forces in the proposed Satellite Energy Exchange experiment»*, *Phys. Rev. D*, **47** (1993) 3660-3661.

SANDERS A. J., DEEDS W. E. and GILLIES G. T., *Proposed new space-based method for more accurate gravitational measurements*, in *The Earth and the Universe: Festschrift in honour of Hans-Jürgens Treder*, edited by W. SCHRÖDER (International Association of Geomagnetism and Aeronomy, Bremen-Rönnebeck, Germany) 1993, pp. 360-365.

SMALLEY L. L., *Gravitational clock: A proposed experiment for the measurement of the gravitational constant G*, NASA Technical Memorandum NASA TM X-64920 (Space Sciences Lab., Georg C. Marshall Space Flight Center, Marshall Space Flight Center, Ala., January 24, 1975), pp. 31.

SPALLICCI A. D. A. M., *The equivalence principle and the gravitational constant in experimental relativity*, in *Proceedings of the VIII Italian Conference on General Relativity and Gravitational Physics, Cavalese (Trento), August 30-September 3, 1988*, edited by M. CERDONIO, R. CIANCI, M. FRANCAVIGLIA and M. TOLLER (World Scientific, Singapore) 1989, pp. 520-536.

SPALLICCI A. D. A. M., *The Columbus space program and gravity science*, in *Advanced Series in Astrophysics and Cosmology*, FANG LI ZHI and R. RUFFINI (Series Editors), *The First William Fairbank Meeting on Relativistic Gravitational Experiments in Space, Proceedings of the Meeting held at the University of Rome, September 10-14, 1990*, Vol. 7 (World Scientific, Singapore) 1993, pp. 505-508.

SPEAKE C. C. and GILLIES G. T., *Why is G the least precisely known physical constant?*, *Z. Naturforsch. A*, **42** (1987) 663-669.

STAFF OF THE SPACE DEPARTMENT, Johns Hopkins University, Applied Physics Lab., and STAFF OF THE GUIDANCE AND CONTROL LABORATORY, Stanford University, *A satellite freed of all but gravitational forces: TRIAD-I*, *J. Spacecraft and Rockets*, **11** (1974) 637-644.

STEENBECK M. and TREDER H.-J., *Zur Bestimmung der Gravitationskonstanten und zum Nachweis von Gravitationsstrahlung in einem kosmischen Laboratorium*, *Astron. Nachr.*, **303** (1982) 277-282.

STEENBECK M. and TREDER H.-J., *Möglichkeiten der experimentellen Schwerkraftforschung*, in *Veröffentlichungen des Forschungsbereichs Kosmische Physik*, edited by H. STILLER and H.-J. TREDER, Vol. **11** (Akademie-Verlag, Berlin) 1984.

SWIGERT C. J. and FORWARD R. L., *Electronic damping of orthogonal bending modes in a cylindrical mast—Theory*, *J. Spacecraft and Rockets*, 18 (1981) 5-10.
TURNEAURE J. P., EVERITT C. W. F., PARKINSON B. W. *et al.*, *The Gravity-Probe-B relativity gyroscope experiment: Approach to a flight mission*, in *Proceedings of the Fourth Marcel Grossman Meeting on General Relativity*, edited by R. RUFFINI (Elsevier, Amsterdam) 1986, pp. 411-464.
VINTI J. P., *Analysis of an experiment to determine the gravitational constant in an orbiting space laboratory*, Report No. RE-72 (September, 1970), Appendix E in WILK L. S. (Editor), Progress Report PR-8, *Studies of space experiments to measure gravitational constant variations and the Eötvös ratio*, NASA contract No. NAS9-8328 (MIT Measurement Systems Laboratory, Cambridge, Mass.) 1971.
VINTI J. P., *Theory of an experiment in an orbiting space laboratory to determine the gravitational constant*, *Celestial Mech.*, 5 (1972) 204-254.
WALESCH H., MEYER H., PIEL H. and SCHURR J., *The gravitational force at mass separation from* 0.6 m *to* 2.1 m *and the precise measurement of G*, *IEEE Trans. Instrum. Meas.*, **44** (1995) 491-493.
WALKER J. A., *Approaches to spacecraft design*, in *Advanced Series in Astrophysics and Cosmology*, FANG LI ZHI and R. RUFFINI (Series Editors), *The First William Fairbank Meeting on Relativistic Gravitational Experiments in Space, Proceedings of the Meeting held at the University of Rome, September 10-14, 1990*, Vol. 7 (World Scientific, Singapore) 1993, pp. 515-526.
WILK L. S. (Editor), Progress Report PR-8, *Studies of space experiments to measure gravitational constant variations and the Eötvös ratio*, NASA contract No. NAS9-8328 (MIT Measurement Systems Laboratory, Cambridge, Mass.) 1971.
WORKING GROUP ON THE PHYSICAL SCIENCE, G. P. WOLLARD (Chairman), Chapter VI, *Physics and Geophysics*, in *Space Research: Directions for the Future: Report of the Study by the Science Space Board*, H. H. HESS, Chaiman (National Academy of Sciences - National Research Council, Washington, D.C.) 1966, pp. 315-338.
YIONOULIS S. M., *A study of the resonance effects due to the Earth's potential function*, *J. Geophys. Res.*, **70** (1965) 5991-5996.
YIONOULIS S. M., *J. Geophys. Res.*, **71** (1966) 1289-1291 (Erratum).
YODER C. F., COLOMBO G., SYNNOTT S. P. and YODER K. A., *Theory of motion of Saturn's coorbiting satellites*, *Icarus*, **53** (1983) 431-443.

Direttore responsabile: RENATO ANGELO RICCI

Questo periodico è iscritto all'Unione Stampa Periodica Italiana

Questo fascicolo è stato realizzato in fotocomposizione dalla Monograf, Bologna
e stampato dalla tipografia Compositori, Bologna
nel mese di settembre 1996
su carta patinata ecologica chlorine-free
prodotta dalle *Cartiere del Garda S.p.A.*, Riva del Garda (TN)