GRAVITY AS A KEY PROBLEM OF THE MILLENIUM

Vitaly N. Melnikov

Institute of Gravitation and Cosmology, Peoples' Friendship University of Russia, 6 Miklukho-Maklaya St., Moscow 117198, and
Center for Gravitation and Fundamental Metrology, VNIIMS, 3-1 M. Ulyanovoy St., Moscow 117313, Russia
and Depto.de Fisica, CINVESTAV, Apartado Postal 14-740, Mexico 07360, D.F.[1]

Gravitation as a fundamental interaction that governs all phenomena at large and very small scales, but still not well understood at a quantum level, is a missing cardinal link to unification of all physical interactions. Problems of the absolute G measurements and its possible time and range variations are reflections of the unification problem. Integrable multidimensional models of gravitation and cosmology make up one of the proper approaches to study basic issues and strong field objects, the Early Universe and black hole physics in particular. The choice, nature, classification and precision of determination of fundamental physical constants are described. The problem of their temporal variations is also discussed, temporal and range variations of G in particular. A need for further absolute measurements of G, its possible range and time variations is pointed out. The novel multipurpose space project SEE, aimed for measuring G and its stability in space and time 3-4 orders better than at present, may answer many important questions posed by gravitation, cosmology and unified theories.

1.   Introduction

The second half of the 20th century in the field of gravitation was devoted mainly to theoretical study and experimental verification of general relativity and alternative theories of gravitation with a strong stress on relations between macro and microworld fenomena or, in other words, between classical gravitation and quantum physics. Very intensive investigations in these fields were done in Russia by M.A.Markov, K.P.Staniukovich, Ya.B.Zeldovich, A.D.Sakharov and their colleagues starting from mid 60's. As a motivation there were: singularities in cosmology and black hole physics, role of gravity at large and very small (planckian) scales, attempts to create a quantum theory of gravity as for other physical fields, problem of possible variations of fundamental physical constants etc. A lot of work was done along such topics as [3]:
    - particle-like solutions with a gravitational field,
    - quantum theory of fields in a classical gravitational background,
    - quantum cosmology with fields like a scalar one,
    - self-consistent treatment of quantum effects in cosmology,
    - development of alternative theories of gravitation: scalar-tensor, gauge, with torsion, bimetric etc.
    As all attempts to quantize general relativity in a usual manner failed and it was proved that it is not renormalizable, it bacame clear that the promising trend is along the lines of unification of all physical interactions which started in the 70's. About this time the experimental investigation of gravity in strong fields and gravitational waves started giving a powerful speed up in theoretical

---

[1]e-mail: melnikov@rgs.phys.msu.su, melnikov@fis.cinvestav.mx

studies of such objects as pulsars, black holes, QSO's, AGN's, Early Universe etc., which continues now.

But nowadays, when we think about the most important lines of future developments in physics, we may forsee that gravity will be essential not only by itself, but as a missing cardinal link of some theory, unifying all existing physical interactions: week, strong and electromagnetic ones. Even in experimental activities some crucial next generation experiments verifiing predictions of unified schemes will be important. Among them are: STEP - testing the corner stone Equivalence Principle, SEE - testing the inverse square law (or new nonnewtonian interactions), EP, possible variations of the newtonian constant G with time, absolute value of G with unprecedented accuracy [39]. Of course, gravitational waves problem, verification of torsional, rotational (GPB), 2nd order and strong field effects remain important also.

We may predict as well that thorough study of gravity itself and within the unified models will give in the next century and millenium even more applications for our everyday life as electromagnetic theory gave us in the 20th century after very abstract fundamental investigations of Faraday, Maxwell, Poincare, Einstein and others, which never dreamed about such enormous applications of their works.

Other very important feature, which may be envisaged, is an increasing role of fundamental physics studies, gravitation, cosmology and astrophysics in particular, in space experiments. Unique microgravity environments and modern technology outbreak give nearly ideal place for gravitational experiments which suffer a lot on Earth from its relatively strong gravitational field and gravitational fields of nearby objects due to the fact that there is no ways of screening gravity.

In the developement of relativistic gravitation and dynamical cosmology after A. Einstein and A. Friedmann, we may notice three distinct stages: first, investigation of models with matter sources in the form of a perfect fluid, as was originally done by Einstein and Friedmann. Second, studies of models with sources as different physical fields, starting from electromagnetic and scalar ones, both in classical and quantum cases (see [3]). And third, which is really topical now, application of ideas and results of unified models for treating fundamental problems of cosmology and black hole physics, especially in high energy regimes. Multidimensional gravitational models play an essential role in the latter approach.

The necessity of studying multidimensional models of gravitation and cosmology [1, 2] is motivated by several reasons. First, the main trend of modern physics is the unification of all known fundamental physical interactions: electromagnetic, weak, strong and gravitational ones. During the recent decades there has been a significant progress in unifying weak and electromagnetic interactions, some more modest achievements in GUT, supersymmetric, string and superstring theories.

Now, theories with membranes, p-branes and more vague M- and F-theories are being created and studied. Having no definite successful theory of unification now, it is desirable to study the common features of these theories and their applications to solving basic problems of modern gravity and cosmology. Moreover, if we really believe in unified theories, the early stages of the Universe evolution and black hole physics, as unique superhigh energy regions, are the most proper and natural arena for them.

Second, multidimensional gravitational models, as well as scalar-tensor theories of gravity, are theoretical frameworks for describing possible temporal and range variations of fundamental physical constants [3, 4, 5, 6]. These ideas have originated from the earlier papers of E. Milne (1935) and P. Dirac (1937) on relations between the phenomena of micro- and macro-worlds, and up till now they are under thorough study both theoretically and experimentally.

Lastly, applying multidimensional gravitational models to basic problems of modern cosmology

and black hole physics, we hope to find answers to such long-standing problems as singular or nonsingular initial states, creation of the Universe, creation of matter and its entropy, acceleration, cosmological constant, origin of inflation and specific scalar fields which may be necessary for its realization, isotropization and graceful exit problems, stability and nature of fundamental constants [4], possible number of extra dimensions, their stable compactification etc.

Bearing in mind that multidimensional gravitational models are certain generalizations of general relativity which is tested reliably for weak fields up to 0.001 and partially in strong fields (binary pulsars), it is quite natural to inquire about their possible observational or experimental windows. From what we already know, among these windows are:

– possible deviations from the Newton and Coulomb laws, or new interactions,

– possible variations of the effective gravitational constant with a time rate smaller than the Hubble one,

– possible existence of monopole modes in gravitational waves,

– different behaviour of strong field objects, such as multidimensional black holes, wormholes and p-branes,

– standard cosmological tests etc.

Since modern cosmology has already become a unique laboratory for testing standard unified models of physical interactions at energies that are far beyond the level of the existing and future man-made accelerators and other installations on Earth, there exists a possibility of using cosmological and astrophysical data for discriminating between future unified schemes.

As no accepted unified model exists, in our approach we adopt simple, but general from the point of view of number of dimensions, models based on multidimensional Einstein equations with or without sources of different nature:

– cosmological constant,

– perfect and viscous fluids,

– scalar and electromagnetic fields,

– their possible interactions,

– dilaton and moduli fields,

– fields of antisymmetric forms (related to p-branes) etc.

Our program's main objective was and is to obtain exact self-consistent solutions (integrable models) for these models and then to analyze them in cosmological, spherically and axially symmetric cases. In our view this is a natural and most reliable way to study highly nonlinear systems. It is done mainly within Riemannian geometry. Some simple models in integrable Weyl geometry and with torsion were studied as well.

Here we dwell mainly upon some problems of fundamental physical constants, the gravitational constant in particular, upon the SEE project shortly (see A.Sanders' paper in this volume) and exact solutions in the spherically symmetric case, black hole and PPN parameters for these solutions in particular, within a multidimensional gravity.

2. Fundamental physical constants

2.1. In any physical theory we meet constants which characterize the stability properties of different types of matter: of objects, processes, classes of processes and so on. These constants are important because they arise independently in different situations and have the same value, at any rate within accuracies we have gained nowadays. That is why they are called fundamental physical constants (FPC) [3]. It is impossible to define strictly this notion. It is because the constants, mainly dimensional, are present in definite physical theories. In the process of scientific

progress some theories are replaced by more general ones with their own constants, some relations between old and new constants arise. So, we may talk not about an absolute choice of FPC, but only about a choice corresponding to the present state of the physical sciences.

Really, before the creation of the electroweak interaction theory and some Grand Unification Models, it was considered that this *choice* is as follows:

$$c, \ \hbar, \ \alpha, \ G_F, \ g_s, \ m_p \ (\text{or } m_e), \ G, \ H, \ \rho, \ \Lambda, \ k, \ I, \tag{1}$$

where α, G_F, g_s and G are constants of electromagnetic, weak, strong and gravitational interactions, H, ρ and Λ are cosmological parameters (the Hubble constant, mean density of the Universe and cosmological constant), k and I are the Boltzmann constant and the mechanical equivalent of heat which play the role of conversion factors between temperature on the one hand, energy and mechanical units on the other. After adoption in 1983 of a new definition of the meter ($\lambda = ct$ or $\ell = ct$) this role is partially played also by the speed of light c. It is now also a conversion factor between units of time (frequency) and length, it is defined with the absolute (null) accuracy.

Now, when the theory of electroweak interactions has a firm experimental basis and we have some good models of strong interactions, a more preferable choice is as follows:

$$\hbar, \ (c), \ e, \ m_e, \ \theta_w, \ G_F, \ \theta_c, \ \Lambda_{QCD}, \ G, \ H, \ \rho, \ \Lambda, \ k, \ I \tag{2}$$

and, possibly, three angles of Kobayashi-Maskawa — θ_2, θ_3 and δ. Here θ_w is the Weinberg angle, θ_c is the Cabibbo angle and Λ_{QCD} is a cut-off parameter of quantum chromodynamics. Of course, if a theory of four known now interactions will be created (M-, F-or other), then we will probably have another choice. As we see, the macro constants remain the same, though in some unified models, i.e. in multidimensional ones, they may be related in some manner (see below). From the point of view of these unified models the above mentioned ones are low energy constants.

All these constants are known with different *accuracies*. The most precisely defined constant was and remain the speed of light c: its accuracy was 10^{-10} and now it is defined with the null accuracy. Atomic constants, e, $\hbar$, m and others are determined with errors $10^{-6} \div 10^{-8}$, G up to 10^{-4} or even worse, θ_w — up to 10%; the accuracy of H is also about 10%. An even worse situation is now with other cosmological parameters (FPC): mean density estimations vary within an order of magnitude; for Λ we have now data that its corresponding density exceeds the matter density (0.7 of the total mass).

As to the *nature* of the FPC, we may mention several approaches. One of the first hypotheses belongs to J.A. Wheeler: in each cycle of the Universe evolution the FPC arise anew along with physical laws which govern this evolution. Thus, the nature of the FPC and physical laws are connected with the origin and evolution of our Universe.

A less global approach to the nature of dimensional constants suggests that they are needed to make physical relations dimensionless or they are measures of asymptotic states. Really, the speed of light appears in relativistic theories in factors like v/c, at the same time velocities of usual bodies are smaller than c, so it plays also the role of an asymptotic limit. The same sense have some other FPC: $\hbar$ is the minimal quantum of action, e is the minimal observable charge (if we do not take into account quarks which are not observable in a free state) etc.

Finally, FPC or their combinations may be considered as natural scales determining the basic units. While the earlier basic units were chosen more or less arbitrarily, i.e., the second, meter and kilogram, now the first two are based on stable (quantum) phenomena. Their stability is believed to be ensured by the physical laws which include FPC.

Another interesting problem, which is under discussion, is why the FPC have values in a very narrow range necessary for supporting life (stability of atoms, stars lifetime etc.). There exist several possible but far from being convincing explanations [40]. First, that it is a good luck, no matter how improbable is the set of FPC. Second, that life may exist in other forms and for another FPC set, of which we do not know. Third, that all possibilities for FPC sets exist in some universe. And the last but not the least: that there is some cosmic fine tuning of FPC: some unknown physical processes bringing FPC to their present values in a long-time evolution, cycles etc.

An exact knowledge of FPC and precision measurements are necessary for testing main physical theories, extention of our knowledge of nature and, in the long run, for practical applications of fundamental theories. Within this, such theoretical problems arise:

1) development of models for confrontation of theory with experiment in critical situations (i.e. for verification of GR, QED, QCD, GUT or other unified models);

2) setting limits for spacial and temporal variations of FPC.

As to a *classification* of FPC, we may set them now into four groups according to their generality:

1) Universal constants such as $\hbar$, which divides all phenomena into quantum and nonquantum ones (micro- and macro-worlds) and to a certain extent c, which divides all motions into relativistic and non-relativistic ones;

2) constants of interactions like α, θ_w, Λ_{QCD} and G;

3) constants of elementary constituencies of matter like m_e, m_w, m_x, etc., and

4) transformation multipliers such as k, I and partially c.

Of course, this division into classes is not absolute. Many constants move from one class to another. For example, e was a charge of a particular object – electron, class 3, then it became a characteristic of class 2 (electromagnetic interaction, $\alpha = \frac{e^2}{\hbar c}$ in combination with $\hbar$ and c); the speed of light c has been in nearly all classes: from 3 it moved into 1, then also into 4. Some of the constants ceased to be fundamental (i.e. densities, magnetic moments, etc.) as they are calculated via other FPC.

As to the *number* of FPC, there are two opposite tendencies: the number of "old" FPC is usually diminishing when a new, more general theory is created, but at the same time new fields of science arise, new processes are discovered in which new constants appear. So, in the long run we may come to some minimal choice which is characterized by one or several FPC, maybe connected with the so-called Planck parameters — combinations of c, $\hbar$ and G:

$$L = \left(\frac{\hbar G}{c^3}\right)^{1/2} \sim 10^{-33} \text{ cm},$$
$$m_L = (c\hbar/2G)^{1/2} \sim 10^{-5} \text{ g},$$
$$\tau_L = L/c \sim 10^{-43} \text{ s}. \tag{3}$$

The role of these parameters is important since m_L characterizes the energy of unification of four known fundamental interactions: strong, weak, electromagnetic and gravitational ones, and L is a scale where the classical notions of space-time loose their meaning.

2.2. The problem of the gravitational constant G measurement and its stability is a part of a rapidly developing field, called gravitational-relativistic metrology (GRM). It has appeared due to the growth of measurement technology precision, spread of measurements over large scales and a tendency to the unification of fundamental physical interaction [6], where main problems arise and are concentrated on the gravitational interaction.

The main subjects of GRM are:

- general relativistic models for different astronomical scales: Earth, Solar System, galaxes, cluster of galaxies, cosmology - for time transfer, VLBI, space dynamics, relativistic astrometry etc.(pioneering works were done in Russia by Arifov and Kadyev, Brumberg in 60's);

- development of generalized gravitational theories and unified models for testing their effects in experiments;

- fundamental physical constants, G in particular, and their stability in space and time;

- fundamental cosmological parameters as fundamental constants: cosmological models studies, measurements and observations;

- gravitational waves (detectors, sources...);

- basic standards (clocks) and other modern precision devices (atomic and neutron interferometry, atomic force spectroscopy etc.) in fundamental gravitational experiments, especially in space...

There are three problems related to G, which origin lies mainly in unified models predictions:

1) absolute G measurements,

2) possible time variations of G,

3) possible range variations of G – non-Newtonian, or new interactions.

Absolute measurements of G. There are many laboratory determinations of G with errors of the order 10^{-3} and only 4 on the level of 10^{-4}. They are (in 10^{-11} m^3 kg^{-1} s^{-2}):

  1. Facy and Pontikis, France   1972 —   6,6714 ± 0.0006
  2. Sagitov et al., Russia   1979 —   6,6745 ± 0.0008
  3. Luther and Towler, USA   1982 —   6,6726 ± 0.0005
  4. Karagioz, Russia   1988 —   6,6731 ± 0.0004

¿From this table it is evident that the first three experiments contradict each other (the results do not overlap within their accuracies). And only the fourth experiment is in accord with the third one.

The official CODATA value of 1986

$$G = (6,67259 \pm 0.00085) \cdot 10^{-11} \cdot m^3 \cdot kg^{-1} \cdot s^{-2} \tag{4}$$

is based on the Luther and Towler determination. But after very precise measurements of G in Germany and New Zealand the situation became more vague. Their results deviate from the official CODATA value by more than 600 ppm.

As it may be seen from the Cavendish conference data [53], the results of 7 groups may agree with each other only on the level 10^{-3}. The most recent and precise G-measurement [56] diverge also from the CODATA value of 1986.

This means that either the limit of terrestrial accuracies has been reached or we have some new physics entering the measurement proceedure [6]. The first means that, maybe we should turn to space experiments to measure G [39], and and second means that a more thorough study of theories generalizing Einstein's general relativity or unified theories is necessary.

There exist also some satellite determinations of G (namely $G \cdot M_{\text{Earth}}$) on the level of 10^{-9} and several less precise geophysical determinations in mines.

The precise knowledge of G is necessary, first of all, as it is a FPC; next, for the evaluation of mass of the Earth, of planets, their mean density and, finally, for construction of Earth models; for transition from mechanical to electromagnetic units and back; for evaluation of other constants through relations between them given by unified theories; for finding new possible types of interactions and geophysical effects; for some practical applications like increasing of modern

gradiometers precision, as they demand a calibration by a gravitational field of a standard body depending on G: high accuracy of their calibration (10^{-5} - 10^{-6}) requires the same accuracy of G. (I am indebted to Dr.N.Kolosnitsyn for this last remark.)

The knowledge of constants values has not only a fundamental meaning but also a metrological one. The modern system of standards is based mainly on stable physical phenomena. So, the stability of constants plays a crucial role. As all physical laws were established and tested during the last 2-3 centuries in experiments on the Earth and in the near space, i.e. at a rather short space and time intervals in comparison with the radius and age of the Universe, the possibility of slow *variations* of constants (i.e. with the rate of the evolution of the Universe or slower) cannot be excluded a priori.

So, the assumption of absolute stability of constants is an extrapolation and each time we must test it.

2.3. *Time Variations of G.* The problem of variations of FPC arose with the attempts to explain the relations between micro- and macro-world phenomena. Dirac was the first to introduce (1937) the so-called "Large Numbers Hypothesis" which relates some known very big (or very small) numbers with the dimensionless age of the Universe $T \sim 10^{40}$ (age of the Universe in seconds 10^{17}, divided by the characteristic elementary particle time 10^{-23} seconds). He suggested (after Milne in 1935) that the ratio of the gravitational to strong interaction strengths, $Gm_p^2/\hbar c \sim 10^{-40}$, is inversely proportional to the age of the Universe: $Gm_p^2/\hbar c \sim T^{-1}$. Then, as the age varies, some constants or their combinations must vary as well. Atomic constants seemed to Dirac to be more stable, so he chose the variation of G as T^{-1}.

After the original *Dirac hypothesis* some new ones appeared and also some generalized *theories* of gravitation admitting the variations of an effective gravitational coupling. We may single out three stages in the development of this field:

1. Study of theories and hypotheses with variations of FPC, their predictions and confrontation with experiments (1937-1977).

2. Creation of theories admitting variations of an effective gravitational constant in a particular system of units, analyses of experimental and observational data within these theories [30, 3] (1977-present).

3. Analyses of FPC variations within unified models [6, 4, 1] (present).

Within the development of the first stage from the analysis of the whole set of existed astronomical, astrophysical, geophysical and laboratory data, a conclusion was made [30, 31] that variations of atomic constants are excluded, but variations of the effective gravitational constant in the atomic system of units do not contradict the available experimental data on the level $10^{-11} \div 10^{-12}$year^{-1}. Moreover, in [32, 30, 31] the conception was worked out that variations of constants are not absolute but depend on the system of measurements (choice of standards, units and devices using this or that fundamental interaction). Each fundamental interaction through dynamics, described by the corresponding theory, defines the system of units and the system of basic standards.

Earlier reviews of some hypotheses on variations of FPC and experimental tests can be found in [3, 4].

Following Dyson (1972), we can introduce dimensionless combinations of micro- and macro-

constants:

$$\alpha = e^2/\hbar c = 7,3 \cdot 10^{-3}, \qquad \gamma = Gm^2/\hbar c = 5 \cdot 10^{-39},$$
$$\beta = G_F \, m^2 c/\hbar^3 = 9 \cdot 10^6, \qquad \delta = H\hbar/mc^2 = 10^{-42},$$
$$\varepsilon = \rho G/H^2 = 2 \cdot 10^{-3}, \qquad t = T/(e^2/mc^3) \approx 10^{40}$$

We see that α, β and ε are of order 1 and γ and δ are of the order 10^{-40}. Nearly all existing hypotheses on variations of FPC may be represented as follows:

Hyposesis 1 (standard):

α, β, γ are constant, $\delta \sim t^{-1}$, $\varepsilon \sim t$.

Here we have no variations of G while δ and ε are determined by cosmological solutions.

Hyposesis 2 (Dirac):

α, β, ε are constant, $\gamma \sim t^{-1}$, $\delta \sim t^{-1}$.

Then $\dot{G}/G = 5 \cdot 10^{-11}$year^{-1} if the age of the Universe is taken to be $T = 2 \cdot 10^{10}$ years.

Hyposesis 3 (Gamow):

$\gamma/\alpha = Gm^2/e^2 \sim 10^{-37}$, so e^2 or α are varied, but not G, β, γ; $\varepsilon = $ const, $\alpha \sim t^{-1}$, $\delta \sim t^{-1}$.

Then $\dot{\alpha}/\alpha = 10^{-10}$ year^{-1}.

Hyposesis 4 (Teller):

trying to account also for deviations of α from 1, he suggested $\alpha^{-1} = \ln \gamma^{-1}$.

Then β, ε are constants, $\gamma \sim t^{-1}$, $\alpha \sim (\ln t)^{-1}$, $\delta \sim t^{-1}$

$$\dot{\alpha}/\alpha = 5 \cdot 10^{-13} year^{-1}. \tag{5}$$

The same relation for α and γ was used also by Landau, DeWitt, Staniukovich, Terasawa and others, but in approaches other than Teller's.

Some other variants may be also possible, e.g. the Brans-Dicke theory with $G \sim t^{-r}$, $\rho \sim t^{r-2}$, $r = [2 + 3\omega/2]^{-1}$, a combination of Gamow's and Brans-Dicke etc. [3].

2.4. There are different astronomical, geophysical and laboratory *data* on possible variations of FPC.

Astrophysical data:

a) from comparison of fine structure ($\sim \alpha^2$) and relativistic fine structure ($\sim \alpha^4$) shifts in spectra of radio galaxies, Bahcall and Schmidt (1967) obtained

$$|\dot{\alpha}/\alpha| \le 2 \cdot 10^{-12} \text{ year}^{-1}; \tag{6}$$

b) comparing lines in optical ($\sim Ry = me^4/\hbar^2$) and radio bands of the same sources in galaxies Baum and Florentin-Nielsen (1976) got the estimate

$$|\dot{\alpha}/\alpha| \le 10^{-13} \text{ year}^{-1}, \tag{7}$$

and for extragalactic objects

$$|\dot{\alpha}/\alpha| \le 10^{-14} \text{ year}^{-1}; \tag{8}$$

c) from observations of superfine structure in H-absorption lines of the distant radiosource Wolf et al. (1976) obtained that

$$|\alpha^2 (m_e/m_p) g_p| < 2 \cdot 10^{-14} ; \tag{9}$$

¿from these data it is seen that Hyposeses 3 and 4 are excluded. Recent data only strengthen this conclusion. Comparing the data from absorption lines of atomic and molecular transition spectra in high redshifts QSO's, Varshalovich and Potekhin, Russia, [41] obtained for $z = 2.8 - 3.1$:

$$|\dot{\alpha}/\alpha| \leq 1.6 \cdot 10^{-14} \text{ year}^{-1} \tag{10}$$

and Drinkwater et al. [43]:

$$|\dot{\alpha}/\alpha| \leq 10^{-15} \text{ year}^{-1} \text{ for } z = 0.25 \tag{11}$$

and

$$|\dot{\alpha}/\alpha| \leq 5 \cdot 10^{-16} \text{ year}^{-1} \text{ for } z = 0.68 \tag{12}$$

for a model with zero deceleration parameter and $H = 75 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$.

The same conclusion is made on the bases of *geophysical data*. Indeed,

a) α-decay of $U_{238} \rightarrow Pb_{208}$. Knowing abundancies of U_{238} and P_{238} in rocks and independently the age of these rocks, one obtains the limit

$$|\dot{\alpha}/\alpha| \leq 2 \cdot 10^{-13} \text{ year}^{-1}; \tag{13}$$

b) from spontaneous fission of U_{238} such an estimation was made:

$$|\dot{\alpha}/\alpha| \leq 2,3 \cdot 10^{-13} \text{ year}^{-1}. \tag{14}$$

c) finally, β-decay of Re_{187} to Os_{187} gave:

$$|\dot{\alpha}/\alpha| \leq 5 \cdot 10^{-15} \text{ year}^{-1} \tag{15}$$

We must point out that all astronomical and geophysical estimations are strongly model-dependent. So, of course, it is always desirable to have *laboratory tests* of variations of FPC.

a) Such a test was first done by the Russian group in the Committee for Standards (Kolosnitsyn, 1975). Comparing rates of two different types of clocks, one based on a Cs standard and another on a beam molecular generator, they found that

$$|\dot{\alpha}/\alpha| \leq 10^{-10} \text{ year}^{-1}. \tag{16}$$

b) From a similar comparison of a Cs standard and SCCG (Super Conducting Cavity Generator) clocks rates Turneaure et al. (1976) obtained the limit

$$|\dot{\alpha}/\alpha| \leq 4.1 \cdot 10^{-12} \text{ year}^{-1}. \tag{17}$$

c) More recent data were obtained by J. Prestage et al. [44] by comparing mercury and H-maser clocks. Their result is

$$|\dot{\alpha}/\alpha| \leq 3.7 \cdot 10^{-14} \text{ year}^{-1}. \tag{18}$$

All these limits were placed on the fine structure constant variations. From the analysis of decay rates of K_{40} and Re_{187}, a limit on possible variations of the weak interaction constant was obtained (see approach for variations of β, e.g. in [33])

$$|\dot{\beta}/\beta| \leq 10^{-10} \text{ year}^{-1}. \tag{19}$$

But the most strict data were obtained by A. Schlyachter in 1976 (Russia) ¿from an analysis of the ancient natural nuclear reactor data in Gabon, Oklo, because the event took place $2 \cdot 10^9$ years ago. They are the following:

$$|\dot{G}_s/G_s| < 5 \cdot 10^{-19} \text{ year}^{-1},$$
$$|\dot{\alpha}/\alpha| < 10^{-17} \text{ year}^{-1}, \tag{20}$$
$$|\dot{G}_F/G_F| < 2 \cdot 10^{-12} \text{ year}^{-1}.$$

Quite recently Damour and Dyson [42] repeated this analysis in more detail and gave more cautious results:

$$|\dot{\alpha}/\alpha| \leq 5 \cdot 10^{-17} \text{ year}-1 \tag{21}$$

and

$$|\dot{G}_F/G_F| < 10^{-11} \text{ year}^{-1}. \tag{22}$$

So, we really see that all existing hyposeses with variations of atomic constants are excluded.

2.5. Now we still have no unified theory of all four interactions. So it is possible to construct systems of measurements based on any of these four interactions. But practically it is done now on the basis of the mostly worked out theory — on electrodynamics (more precisely on QED). Of course, it may be done also on the basis of the gravitational interaction (as it was partially earlier). Then, different units of basic physical quantities arise based on dynamics of the given interaction, i.e. the atomic (electromagnetic) second, defined via frequency of atomic transitions or the gravitational second defined by the mean Earth motion around the Sun (ephemeris time).

It does not follow from anything that these two seconds are always synchronized in time and space. So, in principal they may evolve relative to each other, for example at the rate of the evolution of the Universe or at some slower rate.

That is why, in general, variations of the gravitational constant are possible in the atomic system of units (c, $\hbar$, m are constant) and masses of all particles — in the gravitational system of units (G, $\hbar$, c are constant by definition). Practically we can test only the first variant since the modern basic standards are defined in the atomic system of measurements. Possible variations of FPC must be tested experimentally but for this it is necessary to have the corresponding theories admitting such variations and their certain effects.

Mathematically these systems of measurement may be realized as conformally related metric forms. Arbitrary conformal transformations give us a transition to an arbitrary system of measurements.

We know that scalar-tensor and multidimensional theories are corresponding frameworks for these variations. So, one of the ways to describe variable gravitational coupling is the introduction of a *scalar field* as an additional variable of the gravitational interaction. It may be done by different means (e.g. Jordan, Brans-Dicke, Canuto and others). We have suggested a variant of gravitational theory with a conformal scalar field (Higgs-type field [34, 3]) where Einstein's general relativity may be considered as a result of spontaneous symmetry breaking of conformal symmetry (Domokos, 1976) [3]. In our variant spontaneous symmetry breaking of the global gauge invariance leads to a nonsingular cosmology [35]. Besides, we may get variations of the effective gravitational constant in the atomic system of units when m, c, $\hbar$ are constant and variations of all masses in the gravitational system of units (G, c, $\hbar$ are constant). It is done on the basis of approximate [36] and exact cosmological solutions with local inhomogenity [37].

The effective gravitational constant is calculated using the equations of motions. Post-Newtonian expansion is also used in order to confront the theory with existing experimental

data. Among the post-Newtonian parameters the parameter f describing variations of G is included. It is defined as

$$\frac{1}{GM}\frac{d(GM)}{dt} = fH. \tag{23}$$

According to Hellings' data [38] from the Viking mission,

$$\tilde{\gamma} - 1 = (-1.2 \pm 1.6)\cdot 10^3, \qquad f = (4 \pm 8)\cdot 10^{-2}. \tag{24}$$

In the theory with a conformal Higgs field [36, 37] we obtained the following relation between f and $\tilde{\gamma}$:

$$f = 4(\tilde{\gamma} - 1). \tag{25}$$

Using Hellings' data for $\tilde{\gamma}$, we can calculate in our variant f and compare it with f from [38]. Then we get $f = (-9, 6 \pm 12, 8)\cdot 10^{-3}$ which agrees with (24) within its accuracy.

We used here only Hellings' data on variations of G. But the situation with experiment and observations is not so simple. Along with [38], there are some other data [3, 4]:

1. ¿From the growth of corals, pulsar spin down, etc. on the level

$$|\dot{G}/G| < 10^{-10} \div 10^{-11} \text{ year}^{-1}. \tag{26}$$

2. Van Flandern's positive data (though not confirmed and critisized) from the analysis of lunar mean motion around the Earth and ancient eclipses data (1976, 1981):

$$|\dot{G}/G| = (6 \pm 2)10^{-11} \text{ year}^{-1}. \tag{27}$$

3. Reasenberg's estimates (1987) of the same Viking mission as in [38]:

$$|\dot{G}/G| < (0 \pm 2)\cdot 10^{-11} \text{ year}^{-1} \tag{28}$$

4. Hellings' result in the same form is

$$|\dot{G}/G| < (2 \pm 4)\cdot 10^{-12} \text{ year}^{-1}. \tag{29}$$

5. A result from nucleosythesis (Acceta et al., 1992):

$$|\dot{G}/G| < (\pm 0.9)\cdot 10^{-12} year^{-1}. \tag{30}$$

6. E.V.Pitjeva's result, Russia (1997), based on satellites and planets motion:

$$|\dot{G}/G| < (0 \pm 2)\cdot 10^{-12} year^{-1} \tag{31}$$

As we see, there is a vivid contradiction in these results. As to other experimental or observational data, the results are rather inconclusive. The most reliable ones are based on lunar laser ranging (Muller et al, 1993 and Williams et al, 1996). They are not better than 10^{-12} per year. Here, once more we see that there is a need for corresponding theoretical and experimental studies. Probably, future space missions like Earth SEE-satellite [39] or missions to other planets and lunar laser ranging will be a decisive step in solving the problem of temporal variations of G and determining the fates of different theories which predict them, since the greater is the time interval between successive masurements and, of course, the more precise they are, the more stringent results will be obtained.

As we saw, different theoretical schemes lead to temporal variations of the effective gravitational constant:

1. Empirical models and theories of Dirac type, where G is replaced by $G(t)$.

2. Numerous scalar-tensor theories of Jordan-Brans-Dicke type where G depending on the scalar field $\sigma(t)$ appears.

3. Gravitational theories with a conformal scalar field arising in different approaches [3, 50].

4. Multidimensional unified theories in which there are dilaton fields and effective scalar fields appearing in our 4-dimensional spacetime from additional dimensions [51, 1]. They may help also in solving the problem of a variable cosmological constant from Planckian to present values.

As was shown in [4, 51, 1] temporal variations of FPC are connected with each other in *multidimensional models* of unification of interactions. So, experimental tests on $\dot{\alpha}/\alpha$ may at the same time be used for estimation of $\dot{G}/G$ and vice versa. Moreover, variations of G are related also to the cosmological parameters ρ, Ω and q which gives opportunities of raising the precision of their determination.

As variations of FPC are closely connected with the behaviour of internal scale factors, it is a direct probe of properties of extra dimensions and the corresponding theories [7, 8, 1].

2.6. *Non-Newtonian interactions, or range variations of G.* Nearly all modified theories of gravity and unified theories predict also some deviations from the Newton law (inverse square law, ISL) or composition-dependent violations of the Equivalence Principle (EP) due to appearance of new possible massive particles (partners) [4]. Experimental data exclude the existence of these particles at nearly all ranges except less than *millimeter* and also at *meters and hundreds of meters* ranges. The most recent result in the range of 20 to 500 m was obtained by Achilli et al. using an energy storage plant experiment with gravimeters. They found a positive result for the deviation from the Newton law with the Yukawa potential strength α between 0.13 and 0.25. Of course, these results need to be verified in other independent experiments, probably in space ones [39].

In the Einstein theory G is a true constant. But, if we think that G may vary with time, then, from a relativistic point of view, it may vary with distance as well. In GR massless gravitons are mediators of the gravitational interaction, they obey second-order differential equations and interact with matter with a constant strength G. If any of these requirements is violated, we come in general to deviations from the Newton law with range (or to generalization of GR).

In [5] we analyzed several classes of such theories:

1. Theories with massive gravitons like bimetric ones or theories with a Λ-term.

2. Theories with an effective gravitational constant like the general scalar-tensor ones.

3. Theories with torsion.

4. Theories with higher derivatives (4th-order equations etc.), where massive modes appear leading to short-range additional forces.

5. More elaborated theories with other mediators besides gravitons (partners), like supergravity, superstrings, M-theory etc.

6. Theories with nonlinearities induced by any known physical interactions (Born-Infeld etc.)

7. Phenomenological models where the detailed mechanism of deviation is not known (fifth or other force).

In all these theories some effective or real masses appear leading to Yukawa-type deviation from the Newton law, characterized by strength and range.

There exist some model-dependant estimations of these forces. The most well-known one belongs to Scherk (1979) from supergravity where the graviton is accompanied by a spin-1 partner (graviphoton) leading to an additional repulsion. Other models were suggested by Moody and Wilczek (1984) – introduction of a pseudo-scalar particle – leading to an additional attraction between macro-bodies with the range $2 \cdot 10^{-4}$ cm $< \lambda < 20$ cm and strength α from 1 to 10^{-10} in this range. Another supersymmetric model was elaborated by Fayet (1986, 1990), where a spin-1 partner of a massive graviton gives an additional repulsion in the range of the order 10^3 km and α of the order 10^{-13}.

A scalar field to adjust Λ was introduced also by S. Weinberg in 1989, with a mass smaller than 10^{-3}eV$/c^2$, or a range greater than 0.1 mm. One more variant was suggested by Peccei, Sola and Wetterich (1987) leading to additional attraction with a range smaller than 10 km. Some p-brane models also predict non-Newtonian additional interactions in the mm range, what is intensively discussed nowadays. About PPN parameters for multidimensional models with p-branes see below.

2.7. SEE - Project

We saw that there are three problems connected with G. There is a promising new multi-purpose space experiment SEE - Satellite Energy Exchange [39], which addresses all these problems and may be more effective in solving them than other laboratory or space experiments.

This experiment is based on a limited 3-body problem of celestial mechanics: small and large masses in a drag-free satellite and the Earth. Unique horse-shoe orbits, which are effectively one-dimensional, are used in it.

The aims of the SEE-project are to measure: Inverse Square law (ISL) and Equivalence Principle (EP) at ranges of meters and the Earth radius, G-dot and the absolute value of G with unprecedented accuracies.

We studied some aspects of the SEE-project [57] :

1. Wide range of trajectories with the aim of finding optimal ones:
- circular in spherical field;
- the same plus Earth quadrupole modes;
- elliptic with eccentricity less than 0.05.
2. Estimations of other celestial bodies influence.
3. Estimation of relative influence of trajectories to changes in G and α.
4. Modelling measurement procedures for G and α by different methods, for different ranges and for different satellite altitudes: optimal - 1500 km, ISS free flying platform - 500 km and also for 3000 km.
5. Estimations of some sources of errors:
- radial oscillations of the shepherd's surface;
- longitudal oscillations of the capsule;
- transversal oscillations of the calsule;
- shepherd's nonsphericity;
- limits on the quadrupole moment of the shepherd;
- limits on addmissible charges and time scales of charging by high energy particles etc.
6. Error budgets for G, G-dot and $G(r)$.

The general conclusion is that the SEE-project may really improve our knowledge of these values by 3-4 orders better than we have nowadays.

3. Multidimensional Models

The history of the multidimensional approach begins with the well-known papers of T.K. Kaluza and O. Klein on 5-dimensional theories which opened an interest to investigations in multidimensional gravity. These ideas were continued by P. Jordan who suggested to consider the more general case $g_{55} \neq$ const leading to a theory with an additional scalar field. They were in some sense a source of inspiration for C. Brans and R.H. Dicke in their well-known work on a scalar-tensor gravitational theory. After their work a lot of investigations have been performed using material or fundamental scalar fields, both conformal and non-conformal (see details in [3]).

A revival of ideas of many dimensions started in the 70's and continues now. It is completely due to the development of unified theories. In the 70's an interest to multidimensional gravitational models was stimulated mainly by (i) the ideas of gauge theories leading to a non-Abelian generalization of the Kaluza-Klein approach and (ii) by supergravitational theories. In the 80's the supergravitational theories were "replaced" by superstring models. Now it is heated by expectations connected with the overall M-theory. In all these theories, 4-dimensional gravitational models with extra fields were obtained from some multidimensional model by dimensional reduction based on the decomposition of the manifold

$$M = M^4 \times M_{\text{int}}, \tag{32}$$

where M^4 is a 4-dimensional manifold and M_{int} is some internal manifold (mostly considered to be compact).

The earlier papers on multidimensional gravity and cosmology dealt with multidimensional Einstein equations and with a block-diagonal cosmological or spherically symmetric metric defined on the manifold $M = mathbbR \times M_0 \times \ldots \times M_n$ of the form

$$g = -dt \otimes dt + \sum_{r=0}^{n} a_r^2(t) g^r \tag{33}$$

where (M_r, g^r) are Einstein spaces, $r = 0, \ldots, n$. In some of them a cosmological constant and simple scalar fields were also used [15].

Such models are usually reduced to pseudo-Euclidean Toda-like systems with the Lagrangian

$$L = \frac{1}{2} G_{ij} \dot{x}^i \dot{x}^j - \sum_{k=1}^{m} A_k e^{u_i^k x^i} \tag{34}$$

and the zero-energy constraint $E = 0$.

It should be noted that pseudo-Euclidean Toda-like systems are not well-studied yet. There exists a special class of equations of state that gives rise to Euclidean Toda models [9].

Cosmological solutions are closely related to solutions with spherical symmetry [16]. Moreover, the scheme of obtaining the latter is very similar to the cosmological approach [1]. The first multidimensional generalization of such type was considered by D. Kramer and rediscovered by A.I. Legkii, D.J. Gross and M.J. Perry (and also by Davidson and Owen). In [52] the Schwarzschild solution was generalized to the case of n internal Ricci-flat spaces and it was shown that a black hole configuration takes place when the scale factors of internal spaces are constants. It was shown there also that a minimally coupled scalar field is incompatible with the existence of black holes. In [10] an analogous generalization of the Tangherlini solution was obtained, and an investigation of singularities was performed in [26]. These solutions were also generalized to the electrovacuum case with and without a scalar field [11, 13, 12]. Here, it was also proved that BHs exist only

when a scalar field is switched off. Deviations from the Newton and Coulomb laws were obtained depending on mass, charge and number of dimensions. In [12] spherically symmetric solutions were obtained for a system of scalar and electromagnetic fields with a dilaton-type interaction and also deviations from the Coulomb law were calculated depending on charge, mass, number of dimensions and dilaton coupling. Multidimensional dilatonic black holes were singled out. A theorem was proved in [12] that "cuts" all non-black-hole configurations as being unstable under even monopole perturbations. In [14] the extremely charged dilatonic black hole solution was generalized to a multicenter (Majumdar-Papapetrou) case when the cosmological constant is non-zero.

We note that for $D = 4$ the pioneering Majumdar-Papapetrou solutions with a conformal scalar field and an electromagnetic field were considered in [24].

At present there exists a special interest to the so-called M- and F-theories etc. These theories are "supermembrane" analogues of the superstring models in $D = 11, 12$ etc. The low-energy limit of these theories leads to models governed by the Lagrangian

$$\mathcal{L} = R[g] - h_{\alpha\beta}g^{MN}\partial_M\varphi^\alpha\partial_N\varphi^\beta - \sum_{a\in\Delta}\frac{\theta_a}{n_a!}\exp[2\lambda_a(\varphi)](F^a)^2, \tag{35}$$

where g is a metric, $F^a = dA^a$ are forms of rank $F^a = n_a$, and φ^α are scalar fields.

In [46] it was shown that, after dimensional reduction on the manifold $M_0 \times M_1 \times \ldots \times M_n$ and when the composite p-brane ansatz is considered, the problem is reduced to the gravitating self-interacting σ-model with certain constraints. For electric p-branes see also [17, 18, 20] (in [20] the composite electric case was considered). This representation may be considered as a powerful tool for obtaining different solutions with intersecting p-branes (analogs of membranes). In [46, 29] Majumdar-Papapetrou type solutions were obtained (for the non-composite electric case see [17, 18] and for the composite electric case see [20]). These solutions correspond to Ricci-flat (M_i, g^i), $i = 1, \ldots, n$ and were generalized to the case of Einstein internal spaces [46]. The obtained solutions take place when certain *orthogonality relations* (on couplings parameters, dimensions of "branes", total dimension) are imposed. In this situation a class of cosmological and spherically symmetric solutions was obtained [27]. Special cases were also considered in [22]. Solutions with a horizon were considered in detail in [21, 27]. In [21, 28] some propositions related to (i) interconnection between the Hawking temperature and the singularity behaviour, and (ii) to multitemporal configurations were proved.

It should be noted that multidimensional and multitemporal generalizations of the Schwarzschild and Tangherlini solutions were considered in [13, 25], where the generalized Newton formulas in a multitemporal case were obtained.

We note also that there exists a large variety of Toda solutions (open or closed) when certain intersection rules are satisfied [27].

We continued our investigations of p-brane solutions based on the sigma-model approach in [46, 18, 20]. (For the pure gravitational sector see [17, 45, 46].)

We found a family of solutions depending on one variable describing the (cosmological or spherically symmetric) "evolution" of $(n+1)$ Einstein spaces in the theory with several scalar fields and forms. When an electro-magnetic composite p-brane ansatz is adopted, the field equations are reduced to the equations for a Toda-like system.

In the case when n "internal" spaces are Ricci-flat, one space M_0 has a non-zero curvature, and all p-branes do not "live" in M_0, we found a family of solutions to the equations of motion (equivalent to equations for Toda-like Lagrangian with zero-energy constraint [27]) if certain *block-orthogonality relations* on p-brane vectors U^s are imposed. These solutions generalize the

solutions from [27] with an orthogonal set of vectors U^s. A special class of "block-orthogonal" solutions (with coinciding parameters ν_s inside blocks) was considered earlier in [28].

We considered a subclass of spherically symmetric solutions. This subclass contains non-extremal p-brane black holes for zero values of "Kasner-like" parameters. A relation for the Hawking temperature was presented (in the black hole case).

We also calculated the Post-Newtonian Parameters β and γ (Eddington parameters) for general spherically symmetric solutions and black holes in particular [54]. These parameters depending on p-brane charges, their worldvolume dimensions, dilaton couplings and number of dimensions may be useful for possible physical applications.

Some specific models in classical and quantum multidimensional cases with p-branes were analysed in [47]. Exact solutions for the system of scalar fields and fields of forms with a dilatinic type interactions for *generalized intersection rules* were studied in [48], where the PPN parameters were also calculated.

Finally, a *stability* analysis for solutions with p-branes was carried out [49]. It was shown there that for some simple p-brane systems multidimensional black branes are stable under monopole perturbations while other (non-BH) sperically symmetric solutions turned out to be unstable.

Acknowledgement

References

[1] V.N. Melnikov, "Multidimensional Classical and Quantum Cosmology and Gravitation. Exact Solutions and Variations of Constants." CBPF-NF-051/93, Rio de Janeiro, 1993;
V.N. Melnikov, in: "Cosmology and Gravitation", ed. M. Novello, Editions Frontieres, Singapore, 1994, p. 147.

[2] V.N. Melnikov, "Multidimensional Cosmology and Gravitation", CBPF-MO-002/95, Rio de Janeiro, 1995, 210 p.;
V.N. Melnikov. In: *Cosmology and Gravitation. II*, ed. M. Novello, Editions Frontieres, Singapore, 1996, p. 465.

[3] K.P. Staniukovich and V.N. Melnikov, "Hydrodynamics, Fields and Constants in the Theory of Gravitation", Energoatomizdat, Moscow, 1983, 256 pp. (in Russian).

[4] V.N. Melnikov, *Int. J. Theor. Phys.* **33**, 1569 (1994).

[5] V. de Sabbata, V.N. Melnikov and P.I. Pronin, *Prog. Theor. Phys.* **88**, 623 (1992).

[6] V.N. Melnikov. In: "Gravitational Measurements, Fundamental Metrology and Constants", eds. V. de Sabbata and V.N. Melnikov, Kluwer Academic Publ., Dordrecht, 1988, p. 283.

[7] V.D. Ivashchuk and V.N. Melnikov, *Nuovo Cimento* **B 102**, 131 (1988).

[8] K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, *Nuovo Cimento* **B 102**, 209 (1988).

[9] V.R. Gavrilov, V.D. Ivashchuk and V.N. Melnikov, "Integrable Pseudo-euclidean Toda-like Systems in Multidimensional Cosmology with Multicomponent Perfect Fluid", *J. Math. Phys.* **36**, 5829 (1995).

[10] S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov, *Phys. Lett.* **A 161**, 98 (1991).

[11] S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov, *Chinese Phys. Lett.* **8**, 439 (1991).

[12] K.A. Bronnikov and V.N. Melnikov, *Annals of Physics (N.Y.)* **239**, 40 (1995).

[13] V.D. Ivashchuk and V.N. Melnikov, *Class. Quantum Grav.*, **11**, 1793 (1994).

[14] V.D. Ivashchuk and V.N. Melnikov, "Extremal Dilatonic Black Holes in String-like Model with Cosmological Term", *Phys. Lett.* **B 384**, 58 (1996).

[15] U. Bleyer, V.D. Ivashchuk, V.N. Melnikov and A.I. Zhuk, "Multidimensional Classical and Quantum Wormholes in Models with Cosmological Constant", gr-qc/9405020; *Nucl. Phys.* **B 429**, 117 (1994).

[16] V.D. Ivashchuk and V.N. Melnikov, "Multidimensional Gravity with Einstein Internal Spaces", hep-th/9612054; *Grav. & Cosmol.* **2**, 177 (1996).

[17] V.D. Ivashchuk and V.N. Melnikov, "Intersecting p-brane Solutions in Multidimensional Gravity and M-Theory", hep-th/9612089; *Grav. & Cosmol.* **2**, No 4, 297 (1996).

[18] V.D. Ivashchuk and V.N. Melnikov, *Phys. Lett.* **B 403**, 23 (1997).

[19] V.D. Ivashchuk and V.N. Melnikov, "Sigma-Model for Generalized Composite p-branes", hep-th/9705036; *Class. Quant. Grav.* **14**, 11, 3001 (1997).

[20] V.D. Ivashchuk, M. Rainer and V.N. Melnikov, "Multidimensional Sigma-Models with Composite Electric p-branes", gr-qc/9705005; *Gravit. & Cosm.* **4**, No. 1 (13) (1998).

[21] K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, "The Reissner-Nordström Problem for Intersecting Electric and Magnetic p-branes", gr-qc/9710054; *Grav. & Cosmol.* **3**, 203 (1997).

[22] K.A. Bronnikov, U. Kasper and M. Rainer, "Intersecting Electric and Magnetic p-Branes: Spherically Symmetric Solutions", gr-qc/9708058.

[23] M.A. Grebeniuk, V.D. Ivashchuk and V.N. Melnikov, "Multidimensional Cosmology for Intersecting p-branes with Static Internal Spaces", *Grav. & Cosmol.*, **4**, No 2(14), 145 (1998).

[24] N.M. Bocharova, K.A. Bronnikov and V.N. Melnikov, *Vestnik MGU (Moscow Univ.)*, **6**, 706 (1970)(in Russian, English transl.: Moscow Univ. Phys. Bull., **25**, 6, 80 (1970)) — the first MP-type solution with a conformal scalar field;
K.A. Bronnikov, *Acta Phys. Polonica* , **B4**, 251 (1973);
K.A. Bronnikov and V.N. Melnikov, in *Problems of Theory of Gravitation and Elementary Particles*, **5**, 80 (1974) (in Russian) — the first MP-type solution with conformal scalar and electromagnetic fields.

[25] V.D. Ivashchuk and V.N. Melnikov, *Int. J. Mod. Phys. D* **4**, 167 (1995).

[26] V.D. Ivashchuk and V.N. Melnikov, "On Singular Solutions in Multidimensional Gravity", hep-th/9612089; *Grav. and Cosmol.* **1**, 204 (1996).

[27] V.D. Ivashchuk and V.N. Melnikov, "Multidimensional Classical and Quantum Cosmology with Intersecting p-branes", hep-th/9708157; *J. Math. Phys.*, **39**, 2866 (1998).

[28] K.A. Bronnikov, "Block-orthogonal Brane Systems, Black Holes and Wormholes", hep-th/9710207; *Grav. and Cosmol.* **4**, No 2 (14), (1998).

[29] V.D. Ivashchuk and V.N. Melnikov, "Mudjumdar-Papapetrou Type Solutions in Sigma-model and Intersecting p-branes", hep-th/9702121, *Class. Quant. Grav.* **16**, 849 (1999).

[30] V.N. Melnikov and K.P. Staniukovich. In: "Problems of Gravitation and Elementary Particle Theory (PGEPT)", Atomizdat, Moscow, 1978, v.9, p.3 (in Russian).

[31] N.A. Zaitsev and V.N. Melnikov. In: PGEPT, 1979, v.10, p.131.

[32] V. Canuto et al. *Phys. Rev.* **D 16**, 1643 (1977).

[33] M. Novello and P. Rotelli, *J. Phys. A* **5**, 1488 (1972).

[34] K.A. Bronnikov, V.N. Melnikov and K.P. Staniukovich. ITP-68-69, Kiev, 1968.

[35] V.N. Melnikov. *Dokl. Acad. Nauk* **246**, N 6, 1351 (1979);
V.N. Melnikov and S.V. Orlov, *Phys.Lett.* **70 A**, N4, 263 (1979).

[36] V.N. Melnikov and A.G. Radynov. In: PGEPT, 1984, v.14, p.73.

[37] V.N. Melnikov and A.G. Radynov. In: "On Relativity Theory", Singapore, WS, 1985, v.2, p.196.

[38] R. Hellings, *Phys. Rev. Lett.* **51**, 1609 (1983).

[39] A. Sanders and W. Deeds. *Phys. Rev.* **D 46**, 480 (1992).

[40] J.D. Barrow, gr-qc/9711084.

[41] D.A. Varshalovich and A.Y. Potekhin. *Space Sci. Rev.* **74**, 259 (1995); *Pis'ma Atron. Zh.* **22**, 3 (1996); *Astron. Lett.*,**22**, 6 (1996).

[42] T. Damour and F. Dyson. *Nucl. Phys.* , **B 480**, 37 (1996).

[43] M. Drinkwater et al., *MNRAS*, 1999, November, astro-ph/9711290.

[44] J.D. Prestage et al., *Phys. Rev. Lett.* **74**, 3511 (1995).

[45] V.D. Ivashchuk and V.N. Melnikov, *J. Math. Phys.* **40**, 6568 (1999), gr-qc/9901001.

[46] S. Cotsakis, V.D. Ivashchuk and V.N. Melnikov. *Grav. & Cosm.* **5**, 52 (1999).

[47] M.A. Grebeniuk, V.D. Ivashchuk, V.N. Melnikov and R. Triay, Grav. & Cosmol. **5**, 229 (1999).

[48] V.D. Ivashchuk and V.N. Melnikov, Proc. 2nd Int. Gamov Conf., Odessa, Ukraine, 1999; *Grav. & Cosmol.* **5**, 313 (1999), hep-th/9910041; *Class.Quant.Grav.*, 2000, to appear.

[49] K.A. Bronnikov and V.N. Melnikov. *Grav. & Cosmol.* **5**, 306 (1999); *Nucl.Phys.*, 2000, to appear.

[50] V.N. Melnikov et al., *GRG* **17**, 63 (1985).

[51] V.N. Melnikov. In: "Results of Science and Technology. Gravitation and Cosmology". Ed.: V.N. Melnikov, 1991, v.1, p.49 (in Russian).

[52] K.A. Bronnikov and V.D. Ivashchuk. In: Abstr. 8th Sov. Grav. Conf., Erevan, EGU, 1988, p.156.

[53] *Measurement Science and Technology*, **10**, (1999).

[54] V.D. Ivashchuk and V.N. Melnikov. In: Lecture Notes in Physics, vol.537. Mathematical and Quantum Aspects of Relativity and Cosmology. Eds.: S.Cotsakis and G.Gibbons. Proc. 2 Samos Meeting on Cosmology, Geometry and Relativity. Springer, Berlin, 2000, p.214.

[55] E.V. Pitjeva. In: Dynamics and Astrometry of Natural and Artificial Celestial Bodies. Kluwer Acad. Publ., Netherlands, 1997, p. 251.

[56] J.H. Gundlach and S.M. Merkowitz, gr-qc/0006043.

[57] A.D. Alexeev, K.A. Bronnikov, N.I. Kolosnitsyn, M.Yu. Konstantinov, V.N. Melnikov and A.G. Radynov. *Izmeritelnaya tekhnika,* 1993, N 8, p.6; N 9, p.3; N 10, p.6; 1994, N1, p.3. *Int. J. Mod. Phys.*, 1994, **D 3**, N 4, p.773.

P.N. Antonyuk, K.A. Bronnikov and V.N. Melnikov. *Astron.Lett.*, 1994, **20**, N 1, p.59. *Grav. Cosm.*, 1997, **3**, N 4(12).

K.A. Bronnikov, M.Yu. Konstantinov and V.N. Melnikov, *Grav. Cosm.*, 1996, **2**, N 4(8), p.361. *Izmerit. Tekhnika*, 1996, N 5, p.3.

A.D. Alexeev,..., V.N.Melnikov et al. Ibid., 1997, N 10, p.3. *Grav.Cosm.*, 1999, **5**, N1(17), p.67.

A.J.Sanders,..., V.N.Melnikov et al. Ibid, 1997, **3**, p.287. *Meas.Sci.Technol.*, 1999, **10**, p.514. *Class.Quant.Grav.*, 2000, **17**, N 1.

V.N. Melnikov. *Science in Russia*, 2000, N 6, November.