Testing General Relativity with Atom Interferometry

Savas Dimopoulos, Peter W. Graham, Jason M. Hogan, and Mark A. Kasevich

Department of Physics, Stanford University, Stanford, California 94305, USA
(Received 10 October 2006; published 15 March 2007)

The unprecedented precision of atom interferometry will soon lead to laboratory tests of general relativity to levels that will rival or exceed those reached by astrophysical observations. We propose such an experiment that will initially test the equivalence principle to 1 part in 10^{15} (300 times better than the current limit), and 1 part in 10^{17} in the future. It will also probe general relativistic effects—such as the nonlinear three-graviton coupling, the gravity of an atom's kinetic energy, and the falling of light—to several decimals. In contrast with astrophysical observations, laboratory tests can isolate these effects via their different functional dependence on experimental variables.

Experimental tests of general relativity (GR) have gone through two major phases. The original tests of the perihelion precession and light bending were followed by a golden era from 1960 until today (see, e.g., [1]). These tests were in part motivated by alternatives to GR, such as Brans-Dicke theory [2]. More recently, the cosmological constant problem suggests that our understanding of gravity is incomplete, motivating a number of proposals for modifying GR at large distances [3]. Also, alternatives to the dark matter hypothesis have led to theories where gravity changes at slow accelerations or galactic scales [4].

Presently, most GR parameters have been tested at the part per thousand level. Typical GR tests involve the study of planets, stars, or light moving over astronomical distances for extended periods of time. In this Letter we argue that high-precision tests of GR are possible in the lab using the motion of individual, quantum-mechanical atoms moving over short distances for brief periods of time. Specifically, atom interferometry can lead to laboratory tests of GR that are competitive with astrophysical tests and potentially superior to them in the long run. There are two reasons for this: one is the high accuracy of atomic physics methods—reaching, for example, 16-decimal clock synchronization [5]. The second is that such an experiment has several control parameters, allowing us to isolate and measure individual relativistic terms and control backgrounds by using their scalings with these parameters. In contrast, with astrophysical observations we have limited control and often cannot disentangle the relativistic effects.

Our proposed experiment relies on light-pulse atom interferometers. These have already achieved extreme accuracy as inertial sensors in a variety of configurations including gyroscopes [6], gradiometers [7], and gravimeters [8]. The first generation of atom inteferometry experiments will push the limits on the principle of equivalence (PoE) and begin measuring GR effects and placing constraints on parametrized post-Newtonian (PPN) parameters, as shown in the third column of Table I. The next three columns show further possible improvements.

To calculate the effect of GR corrections to Newtonian gravity on an atom interferometer, we need the metric governing the motion of the atoms and photons in the experiment. Consider a Schwarzschild space-time in the PPN expansion ($\hbar = c = 1$):

$$ds^2 = (1 + 2\phi + 2\beta\phi^2)dt^2 - (1 - 2\gamma\phi)dr^2 - r^2 d\Omega^2, \tag{1}$$

where $\phi = -\frac{GM}{r}$ is the Earth's gravitational potential, and β and γ are PPN parameters. The major effect neglected here is the rotation of the Earth. The Newtonian effects of the rotation are an important background, but the possible rotation related GR effects are undetectable in the interferometer configurations considered here.

Combining the geodesic equations for the spatial $\vec{x}^i$ ($i = 1, 2, 3$) and t, the coordinate acceleration of an atom in the frame of Eq. (1) (with $\vec{v} = \frac{d\vec{x}}{dt}$) is

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}[\phi + (\beta + \gamma)\phi^2] + \gamma[3(\vec{v} \cdot \hat{r})^2 - 2\vec{v}^2]\vec{\nabla}\phi$$
$$+ 2\vec{v}(\vec{v} \cdot \vec{\nabla}\phi). \tag{2}$$

This illustrates two classes of leading GR corrections to the Newtonian force law. The $\nabla\phi^2$ terms are related to the nonlinear (non-Abelian) nature of gravity indicating that gravitational energy gravitates through a three-graviton vertex. To see this, note the divergence of the gravitational field given in Eq. (2) is nonzero because of these terms. Just as for an electric field, a nonzero divergence of the gravitational field implies a local source density—in this case, a local energy density—proportional to that divergence $\nabla \cdot \nabla\phi^2 = 2(\nabla\phi)^2$. So the local energy density is proportional to the field squared. This energy gives rise to the $\nabla\phi^2$ terms. The other terms are velocity-dependent forces related to the gravitation of the atom's kinetic energy. The nonlinear GR corrections are smaller than Newtonian gravity by a factor of $\phi \sim 7 \times 10^{-10}$ while the velocity-dependent forces are smaller by $v^2 \sim 10^{-15}$ for the atom velocities we are considering. We will show that the non-

TABLE I. Experimental precision for measuring GR effects, PPN parameters, and principle of equivalence (PoE) violations. The initial atom interferometer (AI) limits assume the 10 m experiment described in the text. The potential upgrade would implement $200\hbar k$ LMT beam splitters. The future experiment assumes a 100 m interferometer. A possible improvement using Heisenberg statistics is shown in the far future column. All precision values assume 10^6 s of integration.

Tested Effect	Current limit	AI initial	AI upgrade	AI future	AI far future
PoE	3×10^{-13}	10^{-15}	10^{-16}	10^{-17}	10^{-19}
PPN (β, γ)	10^{-4}–10^{-5}	10^{-1}	10^{-2}	10^{-4}	10^{-6}

linear terms can only be measured through a gradient of the force produced and so are reduced by an additional factor of $\frac{10\text{ m}}{R_{\text{earth}}} \approx 10^{-6}$ for a 10 m long experiment. Both effects are then $\sim 10^{-15}g$.

Experimental setup.—To measure these small accelerations we consider first a one-dimensional gravimeter arranged to measure vertical accelerations with respect to the Earth. In a ground-based atom interferometer gravimeter, a dilute ensemble of cold atoms is launched upward with velocity v_L. The atoms then follow trajectories in accordance with Eq. (2). During their free fall, a sequence of laser pulses along the vertical direction serve as beam splitters and mirrors by coherently transferring photon recoil kicks of momentum $\hbar k_{\text{eff}}$ to each atom [9]. For example, the laser pulse (gray) at $t = 0$ in Fig. 1 acts as a beam splitter, putting the atom in a superposition of the initial velocity state (solid black line) and a state with higher velocity (dashed black line). The resulting spatial separation of the halves of the atom is proportional to the interferometer's acceleration sensitivity. We consider the beam splitter-mirror-beam splitter ($\frac{\pi}{2} - \pi - \frac{\pi}{2}$) sequence [10] the simplest implementation of a gravimeter and the matter-wave analog of a Mach-Zender interferometer.

To test GR, we propose a Rb interferometer with an initial precision of $\sim 10^{-15}g$. This will be achieved with an evaporatively cooled atom source, large momentum transfer (LMT) beam splitters, and an $L \approx 10$ m tall vacuum system for the atoms' flight, allowing a long interrogation time $T = 1.34$ s. To reduce technical noise, including laser phase and vibrational noise, differential strategies similar to those used in current gravity gradiometers will be employed. For example, to test the PoE, the differential acceleration between ^{85}Rb and ^{87}Rb can be measured in a simultaneous dual species fountain [11].

Evaporatively cooled, rather than laser cooled, atomic sources [12] should enable a significant performance improvement over the previous generation of sensors. First, evaporatively cooled sources enable tighter control over systematic errors related to the initial position and velocity of the atomic ensemble. Second, evaporatively cooled sources achieve temperatures ($<1~\mu$K) low enough to implement LMT beam splitters, which are highly velocity selective. Promising LMT beam splitter candidates include optical lattice manipulations [13], sequences of Raman pulses [14], and adiabatic passage methods [15]. Finally, the low temperatures available with these sources eliminate

signal losses due to expansion of the ensemble over long interferometer interrogation times.

Overall sensitivity is both a function of the effective momentum transfer of the atom optics ($\hbar k_{\text{eff}}$) and the signal-to-noise ratio (SNR) of the interference fringes. Because of technical advances in implementation of normalized detection methods for atomic clocks and sensors [16], interference fringes can now be acquired with high SNRs limited only by quantum projection noise (atom shot noise) for ensembles of up to $\sim 10^7$ atoms. Using $10\hbar k (= \hbar k_{\text{eff}})$ LMT beam splitters, a $\sim 10^7$ atom evaporatively cooled source and an interrogation time of $T = 1.34$ s results in a sensitivity of $7 \times 10^{-13}g/\text{Hz}^{1/2}$, and in a precision of $\sim 10^{-15}g$ after a day of integration. This estimate is based on realistic extrapolations from current performance levels, which are at $10^{-10}g$ [17].

Interferometer phase shifts.—The total phase shift in the interferometer is the sum of three parts: the propagation phase, the laser interaction phase, and the final wave packet separation phase [18]. The usual formulas for these must be modified in GR to be coordinate invariants. Our calculation will be discussed in greater detail in [19]. The space-time paths of the atoms and lasers are geodesics of Eq. (1). The propagation phase is

$$\phi_{\text{propagation}} = \int L dt = \int m ds, \tag{3}$$

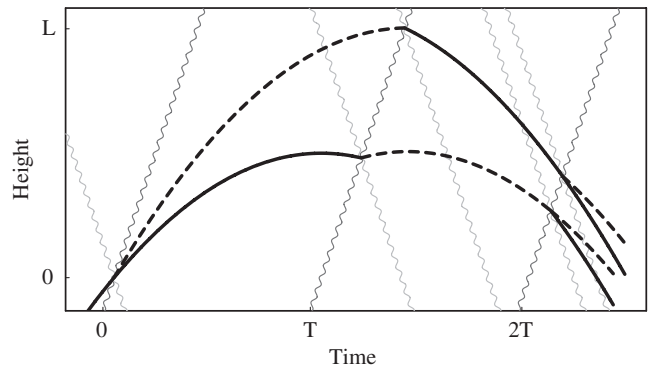where L is the Lagrangian and the integral is along the

FIG. 1. A space-time diagram of a light-pulse atom interferometer. The black curves indicate the geodesic motion of a single atom near the surface of the Earth. Laser light used to manipulate the atom is incident from above (light gray) and below (dark gray) and travels along null geodesics. The finite speed of the light has been exaggerated.

atom's geodesic. The separation phase is taken as

$$\phi_{\text{separation}} = \int \bar{p}_\mu dx^\mu \sim \bar{E}\Delta t - \vec{\bar{p}} \cdot \Delta \vec{x}, \qquad (4)$$

where, for coordinate independence, the integral is over the null geodesic connecting the classical endpoints of the two arms of the interferometer, and $\bar{p}$ is the average of the classical 4-momenta of the two arms after the third pulse. The laser phase shift due to interaction with the light is the constant phase of the light along its null geodesic, which is its phase at the time it leaves the laser. Corrections due to the atomic wave function size Δx [20] and the laser pulse time can be calculated nonrelativistically [21] but do not affect the leading order GR signal.

In Table II we list some of the phase shifts that arise from an analytic relativistic calculation. Effectively, the local gravitational acceleration is expressed as a Taylor series in the height above the Earth's surface. The first phase shift in Table II represents the effect of the leading order (constant) piece of the local acceleration while the 2nd and 5th terms are the next gradients in the Taylor series. Notice that even the second gradient of the gravitational field is relevant for this interferometer. The 3rd term arises from the second order Doppler shift of the laser's frequency as seen by the moving atom. The 4th, 6th, 7th, and 8th terms arise only from GR and are not present in a Newtonian calculation. The 4th and 7th terms arise in part from the nonlinear nature of gravity.

The 6th term receives contributions from the velocity-dependent forces in Eq. (2), but its coefficient is independent of γ. There are two canceling contributions to this term coming from the γ terms in the force law for the atom and the photon. This term thus measures the effect of gravity on light and the velocity-dependent force on the atom. If we put a different parameter, δ_{light}, in front of the ϕ in the component g_{00} of the metric governing the motion of the light, we would see this term as $(4 + \delta_{\text{light}} + \gamma_{\text{light}} - \gamma_{\text{atom}})k_{\text{eff}}gT^2v_L^2$, where the γ's are the PPN parameters in the metrics for the light and the atom. This term then tests a matter-light principle of equivalence, namely, that they both feel the same metric.

TABLE II. A partial phase shift list. The sizes of the terms assume the initial design, sensitive to accelerations $\sim 10^{-15}g$.

	Phase shift	Size (rad)	Interpretation
(1)	$-k_{\text{eff}}gT^2$	3×10^8	gravity
(2)	$-k_{\text{eff}}(\partial_r g)T^3 v_L$	-2×10^3	1st gradient
(3)	$-3k_{\text{eff}}gT^2 v_L$	4×10^1	Doppler shift
(4)	$(2 - 2\beta - \gamma)k_{\text{eff}}g\phi T^2$	2×10^{-1}	GR
(5)	$-\frac{7}{12}k_{\text{eff}}(\partial_r^2 g)T^4 v_L^2$	8×10^{-3}	2nd gradient
(6)	$-5k_{\text{eff}}gT^2 v_L^2$	3×10^{-6}	GR
(7)	$(2 - 2\beta - \gamma)k_{\text{eff}}\partial_r(g\phi)T^3 v_L$	2×10^{-6}	GR 1st grad
(8)	$-12k_{\text{eff}}g^2 T^3 v_L$	-6×10^{-7}	GR

Previous works on GR and interferometry [22] have not dealt with a specific, viable experiment or a full relativistic calculation. Important effects, such as the influence of gravity on light, the corresponding changes to the separation and laser phases, and the nonlinearity of gravity were not discussed. The typical perturbation theory calculation, which integrates the linearized GR Lagrangian over the unperturbed Newtonian trajectories, does not give the correct coefficients or even the dependence on PPN parameters of the phase shifts in Table II. For example, the aforementioned cancellation of the γ's in $k_{\text{eff}}gT^2v_L^2$ would be missed by such a calculation.

Measurement strategies.—To test GR, the relativistic terms must be experimentally isolated from the total phase shift. Many effects contribute to the total phase shift including the Newtonian gravity of the nearby environment, magnetic fields, and the Earth's rotation. Many of these effects will be much larger than the GR effects. We employ magnetic shielding, a rotation servo to null Earth rate during the interferometer interrogation time, and, if necessary, strategically placed masses to "shim" the local gravity field. The local field can also be characterized to high precision using the conventional gravity response (1st term in Table II) at shorter times T.

To pick out the relevant terms, four different control parameters can be used: k_{eff}, v_L, T, and R, the distance from the center of the Earth. Also, with a different design, the angle of the interferometer can be varied. These parameters can be varied by order 1, except for R which can only be varied by $\sim 10^{-6}$ in a ground-based experiment. The different scalings of the relativistic terms with these parameters allow many backgrounds to be rejected.

For example, the 6th term in Table II, $k_{\text{eff}}gT^2v_L^2$, scales differently with the control parameters than the backgrounds and so can be directly measured. Practically, to reduce systematics and technical noise, this would require a differential measurement where clouds of atoms are launched simultaneously with different velocities.

The largest GR term, $k_{\text{eff}}gT^2\phi$, is more difficult to measure. Its scalings with the three main control parameters, k_{eff}, v_L, and T allow most backgrounds to be ignored. However, it must still be picked out of the Newtonian background, $k_{\text{eff}}gT^2$. It originates in part from the $\nabla\phi^2$ term in Eq. (2) and is thus due to the nonlinear nature of gravity. Since it is impossible to have a force that scales as $\frac{1}{R^3}$ outside the mass distribution in the center of mass frame in Newtonian gravity, the R scaling of this term may provide a way to distinguish it from a Newtonian gravitational field. A differential measurement of the R scaling would involve two interferometers running simultaneously at different heights. The same laser should be used for both interferometers to reduce systematics and technical noise. At best, such a differential setup would measure the gradient of this term, thus reducing it to $\sim 10^{-7}$ rad or $\sim 10^{-15}g$, if the interferometers are placed 10 m apart.

In practice, a measurement of this term or of the third GR term, $k_{\text{eff}}\partial_r(g\phi)T^3v_L$, could be masked by the

Newtonian gravity of local mass inhomogeneities. One possible approach to rejecting the Newtonian background is to employ three such differential accelerometer measurements along three mutually orthogonal axes. If these three measurements are added together, all Newtonian gravity gradient contributions to the measurement must cancel, since, in the absence of a local source, the divergence of the gravitational field is zero in Newton's theory. In GR this divergence is not zero, as can be seen from the $\nabla\phi^2$ term in Eq. (2). For such a strategy to be effective, however, the Newtonian field needs to be sufficiently slowly varying over the measurement baseline, which can be tested by varying the baseline.

Table I summarizes the experimental precision possible for measuring GR effects and the PPN parameters β and γ. The initial atom interferometry limits assume the 10 m experiment described earlier, but many upgrades are possible. For example, using $200\hbar k$ beam splitters increases the sensitivity by a factor of ~ 10. Expanding to a 100 m long interferometer would increase the sensitivity by a factor of 10. For the GR terms this improvement would be even larger; for example, terms 6 and 7 scale as L^2 ($v_L \sim T$, $L \sim T^2$). With these improvements it is possible to reach limits on the PPN parameters of $\sim 10^{-4}$, competitive with present limits. Finally, the noise performance can be improved by using entangled states instead of uncorrelated atom ensembles [23]. For a suitably entangled source, the Heisenberg limit is SNR $\sim n$, a factor of $\sqrt{n}$ improvement. For $n \sim 10^6$ entangled atoms, the potential sensitivity improvement is 10^3. Progress using these techniques may soon make improvements in SNR on the order of 10 to 100 realistic [24].

Discussion.—By combining a long interrogation time, LMT beam splitters, and Heisenberg statistics, a ground-based interferometer could exceed the precision of present astrophysical tests of GR. Even at comparable precisions, laboratory tests provide an important complement to astrophysical ones. An advantage of laboratory tests is that they can isolate specific GR effects, such as the nonlinear coupling and the gravitation of kinetic energy, that are not isolated in astrophysical tests. For example, the lunar laser ranging (LLR) test of the PoE constrains the PPN parameters β and γ, but it cannot test for the existence of the nonlinear coupling. In Newton's theory the weak equivalence principle (in the sense of equal Earth and Moon accelerations towards the Sun) is also satisfied and there is no nonlinear coupling at all. Only in the subclass of deviations from GR given by the PPN expansion does LLR imply a nonlinear coupling equal to that predicted by GR to 3 parts in 10^4. This happens because PPN is a restricted class of deviations that does not include many theories. Using atom interferometers, the nonlinear coupling can be directly and unambiguously measured, allowing us to test a truly relativistic effect that does not occur in Newton's theory.

Finally we consider whether the Hubble expansion rate, H, is measurable through the force it exerts on separated atoms. Unfortunately, according to the PoE, local experiments feel the rest of the Universe through its tidal forces proportional to Riemann $\sim H^2$, which is too small. In particular, in GR there is no local effect linear in H, as is sometimes invoked to explain the Pioneer anomaly [25]. Similarly, the PoE prevents the detection of our free-fall towards the galactic dark matter in a local experiment.

[1] C. M. Will, Living Rev. Relativity **9**, 3 (2006); *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993).
[2] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).
[3] T. Damour and A. M. Polyakov, Nucl. Phys. **B423**, 532 (1994); G. R. Dvali *et al.*, Phys. Lett. B **485**, 208 (2000); N. Arkani-Hamed *et al.*, hep-th/0209227.
[4] N. Arkani-Hamed *et al.*, J. High Energy Phys. 05 (2004) 074; M. Milgrom, Astrophys. J. **270**, 365 (1983); J. D. Bekenstein, Phys. Rev. D **70**, 083509 (2004).
[5] W. H. Oskay *et al.*, Phys. Rev. Lett. **97**, 020801 (2006).
[6] T. L. Gustavson *et al.*, Phys. Rev. Lett. **78**, 2046 (1997).
[7] M. J. Snadden *et al.*, Phys. Rev. Lett. **81**, 971 (1998).
[8] A. Peters *et al.*, Metrologia **38**, 25 (2001).
[9] *Atom Interferometry*, edited by P. R. Berman (Academic, New York, 1997).
[10] M. Kasevich and S. Chu, Phys. Rev. Lett. **67**, 181 (1991).
[11] H. Marion *et al.*, Phys. Rev. Lett. **90**, 150801 (2003).
[12] C. Pethick and H. Smith, *Bose-Einstein Condensation in Dilute Gases* (Cambridge University Press, Cambridge, England, 2002).
[13] J. H. Denschlag *et al.*, J. Phys. B **35**, 3095 (2002).
[14] J. M. McGuirk *et al.*, Phys. Rev. Lett. **85**, 4498 (2000).
[15] M. Weitz *et al.*, Phys. Rev. Lett. **73**, 2563 (1994).
[16] G. Santarelli *et al.*, Phys. Rev. Lett. **82**, 4619 (1999); J. McGuirk *et al.*, Opt. Lett. **26**, 364 (2001); S. Bize *et al.*, J. Phys. B **38**, S449 (2005).
[17] J. B. Fixler *et al.*, Science **315**, 74 (2007).
[18] P. Storey and C. Cohen-Tannoudji, J. Phys. II (France) **4**, 1999 (1994); K. Bongs, R. Launay, and M. Kasevich, Appl. Phys. B **84**, 599 (2006).
[19] S. Dimopoulos, P. Graham, J. Hogan, and M. Kasevich (to be published).
[20] This stationary phase approximation neglects accelerations of size $(\partial_r^2 g)\Delta x^2 \lesssim 10^{-20} g$ for $\Delta x \lesssim 1$ mm.
[21] C. J. Borde, Gen. Relativ. Gravit. **36**, 475 (2004).
[22] C. J. Borde *et al.*, gr-qc/0008033; K. Varju and L. H. Ryder, Phys. Rev. D **62**, 024016 (2000); S. Wajima *et al.*, Phys. Rev. D **55**, 1964 (1997); J. Anandan, Phys. Rev. D **30**, 1615 (1984); J. Audretsch and K.-P. Marzlin, Phys. Rev. A **50**, 2080 (1994).
[23] D. Leibfried *et al.*, Science **304**, 1476 (2004).
[24] A. K. Tuchman *et al.*, Phys. Rev. A **74**, 053821 (2006).
[25] J. D. Anderson *et al.*, Phys. Rev. Lett. **81**, 2858 (1998).