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A preliminary study of a torsion balance based on a spherical superconducting suspension

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Abstract. We present details of the design and construction of a new superconducting, magnetically suspended torsion balance in which the levitation coil and the lift surface of the float lie on the surfaces of concentric spheres. We compare results from calculations of the variation of the inductance with the levitated height and transverse motion of the float with experimental measurements and show that the levitation system is stable. Angular motion of the torsion balance is detected using superconducting pick-up coils whose inductance is modulated by float rotation. The subsequent change in current flowing in the persistent circuit containing the coils is measured using a flux-gate magnetometer. The pick-up coils exert a restoring torque on the float which can be modified by adjusting the persistent current stored. Periods down to 60 s should be obtainable for a current of 2.5 A. Preliminary results of ring-down experiments in He gas at a pressure of 53Pa show that periods of angular oscillations of 24 s with quality factor, Q, of about 200 can be obtained. The moment of inertia of the float is 2×10^{-5} kg m^2. The observed period of 24 s indicates that there is an additional restoring torque in the system which may be due to trapped flux. The observed value of Q is consistent with gas damping.

Keywords: gravity gradients, weak forces, vacuum, cryogenic, superconducting suspension, gradiometry

1. Introduction

The determination of Newton's constant of gravitation by Cavendish [1] in 1798 marked the beginning of the modern era of gravitation experiments. The measurement tool which has served experimentalists in this field of research since this time has been the torsion balance. Despite its great success as a detector of weak forces, the classical torsion balance is known to have several disadvantages: It is sensitive to ground tilts and horizontal and vertical ground vibrations. The torsion fibre exhibits anelasticity and $1/f$ noise [2–5]. The sensitivity of a torsion balance (the angular twist per unit applied torque) is optimized when the torsion fibre is as fine as possible and the test masses suspended from it are as light as possible [6]. This reduces the gravitational torques to a magnitude which can be comparable to those of competing forces due to electrostatic, magnetic and thermal effects. In response to these well known design faults and due to the continued interest in the field of experimental gravitation [7, 8] many experimenters have attempted to develop new approaches to the challenge of weak-force measurement. For a review of this activity and a summary of the characteristics of torsion balances, the reader is referred to the excellent article by Gillies and Ritter [9]. Of particular interest is the recent work of Quinn *et al* [10], who showed that heavily loaded broad flexure strips enhance the gravitational signal-to-noise ratio beyond that possible with the standard round-section fibre. However, a far more radical approach has been followed by Moody and Paik [11] and Worden [12], who have exploited the advantages associated with operation at liquid-helium temperatures. Paik has developed gravity gradiometers based on fairly stiff mass-spring systems (with resonance frequencies of around 10 Hz) and SQUID displacement transducers. Worden has investigated the possibility of combining the exquisite sensitivity of the SQUID displacement transducer with a superconducting magnetic levitation system of low stiffness (10^{-3} Hz). This system was conceived for operation in the zero-g environment of a drag-free satellite and is the basis of the proposed test of the universality of free fall in space [13]. Karen *et al* [14] showed that it was possible to construct a low-stiffness torsion balance for laboratory use by levitating a niobium-coated spherical shell in the magnetic field of a superconducting loop. The shell had a moment of inertia

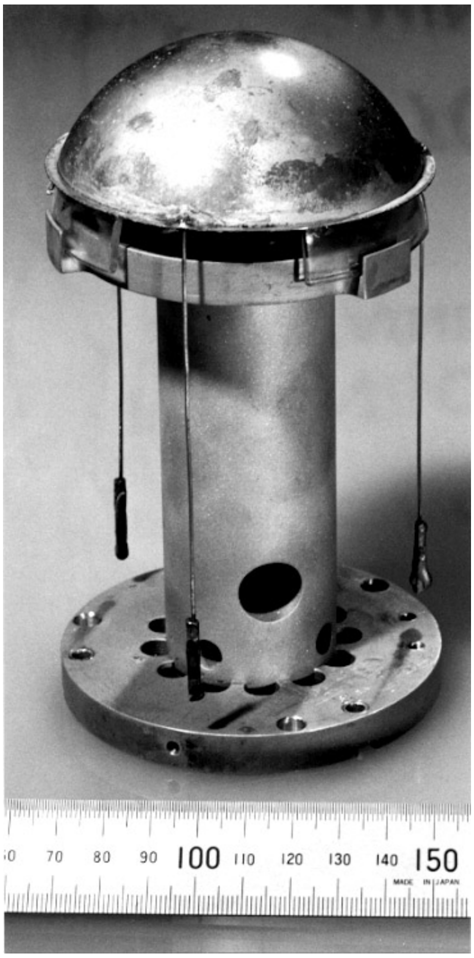


Figure 1. The prototype spherical superconducting torsion balance.

of 1.4 g cm^2 and rotation periods in the range 22–63 s were observed. The anomalous behaviour of the levitated shell was attributed to the presence of trapped flux. In this paper we report on preliminary work in the construction, modelling and testing of a torsion balance which is levitated from a novel design of superconducting bearing and is designed specifically for operation in the terrestrial laboratory.

2. The spherical superconducting torsion balance

The instrument comprises a coil which is patterned onto a substrate in the form of a spherical segment of radius R_1. The magnetic field generated by the superconducting coil produces a lift pressure on the superconducting inner surface of a float which is part of a spherical shell of radius R_2, with $R_2 > R_1$. In the current design three test masses in the form of cylinders are suspended from the rim of the float by rigid wires. Rotational motion of the float about a vertical axis is detected using pick-up coils in a persistent superconducting circuit of a similar design to those developed by Worden [12] and Paik [15]; the inductances of two pairs of pick-up coils are modulated by the rotation of superconducting vertical strips which are mounted on the float close to its centre of figure. At present the read-out from the persistent circuit is achieved with a flux-gate magnetometer; however, in future experiments this will be replaced by a SQUID. The key design features are as follows.

(i) Operation of the levitation coil in persistent mode in vacuum should lead to an essentially lossless suspension exhibiting negligible anelasticity and low thermal noise.
(ii) The azimuthal symmetry of the coil substrate and float levitation surface and the near azimuthal symmetry of the coil itself should, ideally, generate a negligible restoring torque about the vertical axis. The restoring torque of the torsion balance should then be definable by its torsional coupling to the rotation detector which can be fixed by the choice of current stored in the pick-up coils.
(iii) The design of the float is such that its centre of mass lies close to its centre of figure, or buoyancy. This should minimize the coupling of the float to horizontal ground vibrations.
(iv) Because the magnetic pressure acts radially at the inner surface of the float, rotation of the coil about a horizontal axis due to ground tilt should not couple torques to the vertical axis of the float.
(v) Rotational accelerations of the coil due to ground vibrations about the vertical axis will generate noise torques in the torsion balance; however, a second generation device with two torsion balances operating in coupled differential mode would eliminate this noise in the fashion adopted by Paik and Worden in their differential accelerometers.

Figure 1 shows a photograph of the prototype of the torsion balance which operates at 4.2 K and which has been constructed to investigate the feasibility of the design. The float shell, of mass 9.7 g, is constructed from copper sheet of initial thickness 400 μm which is spun onto a mandrill and then etched to an average thickness of 110 μm. The inner radius of the shell is $R_2 = 42.9$ mm† and extends to a colatitude of 67°. The sphericity of the inner surface is approximately 1%. A lead film of average thickness 20 μm is thermally evaporated onto the inner surface of the float. Lead test masses of mass 1 g each are attached to the rim of the float shell with copper wire of diameter 0.9 mm and length 120 mm. The mass of the complete float is 14.7 g, its centre of gravity lies approximately 9 mm below the centre of figure and its radius of gyration is 34 mm.

Figure 2 shows the superconducting levitation coil. The substrate is at present manufactured from Pyrex glass with an outer radius of $R_1 = 42.4$ mm and sphericity of 0.25%. The coil is fabricated by thermal evaporation of a lead film followed by coating with a photoresist. The coil is patterned onto the resist using a computer-controlled laser beam. The coil is approximately helical and comprises segments of constant latitude extending over approximately 120° of longitude which are connected by inclined segments extending over a few degrees of longitude. The extent in latitude of the coil, which in the current design is between 15° and 60°, is chosen as a compromise between the lift force per unit area and the transverse stability. A planar geometry would maximize the lift force but would give no transverse stability to the float. We discuss this point in more detail in section 3. After developing the photoresist, unwanted lead is removed by wet etching. Details of this process can be found in Trenkel [16]. The thickness of the lead film is

† All dimensions are given at room temperature.

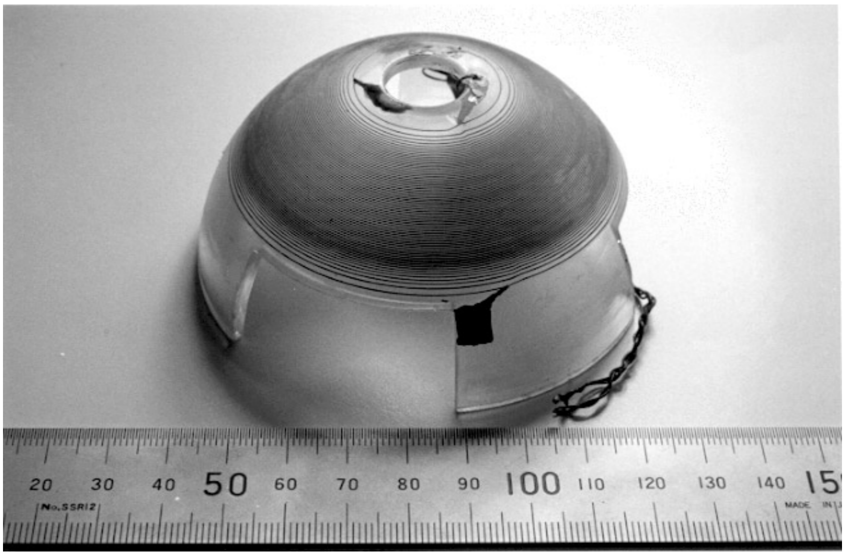


Figure 2. The levitation coil. The lead windings are 150 μm wide and 35 μm thick.

35 ± 4 μm and the width of the coil windings is 150 μm. We have studied the optimization of the coil-windings geometry in detail [17] in order to maximize the lifting capacity of the suspension. The maximum load is set by the critical current, i_c, that is sustainable by the coil. This depends on the value of the gap, $g = R_2 - R_1$, between the coil and the float inner surface, the pitch, p, and the cross-sectional dimensions of the windings. In principle, a bipolar coil would be desirable because the magnetic field due to the bearing would decay quickly as a function of distance, reducing any possible coupling to the rotation detector. However, an average lifting field of about 100 G is necessary in order to support the float and the critical field of Pb at 4.2 K is only 470 G. A bipolar coil made from Pb would require a pitch larger than the gap ($p \geqslant g$) and a levitation gap of order $g \approx 2t$, where t is the thickness of the windings. Given the limited sphericity of our float, a gap of the order of 100 μm or less was considered unfeasible and it was decided to use unipolar windings. Here, the critical current depends both on the surface field due to each individual winding (the self-field) and on the field due to all the other windings (the external field). We have concluded that an optimum winding geometry (or current distribution) can be achieved by progressively increasing the spacings between the windings towards the edges of the coil. This reduces the magnetic field component perpendicular to the long side of the windings (our windings have aspect ratios of approximately 4:1), which is enhanced through demagnetization effects and therefore limits the overall critical current.

In our present design the average current density carried by the windings is reduced from its maximum value at the centre of the coil, where $p = 350$ μm, to zero at the edges over 20% of the coil length. The gap is currently 0.5 mm. The maximum persistent current sustainable by this coil–float combination is 3.75 A. We will discuss how the lifting capacity of the coil is related to this critical current in section 3.1 below.

A circuit diagram of the rotation detector is shown in figure 3(a). It comprises the following elements:

(i) two pairs of sensing coils ($L_{11,12}$ and $L_{21,22}$) which are arranged as shown in figure 3(b) in the horizontal plane containing the centre of figure of the coil substrate,
(ii) an output coil (L_0) which is coupled to a flux-gate magnetometer,
(iii) a flux transformer which can be used to apply torques onto the float and
(iv) two heat switches (HS_1 and HS_2) used for injection of current into the rotation detector.

Two superconducting strips (S1 and S2 in figure 3(b)) on the surface of a tube, whose axis coincides with the rotation axis of the float, modulate the inductances of the coils, which in turn forces a current to flow through the output coil.

The output of the sensor has been measured and is given, to a good approximation, as

$$\Phi_{out} = \Phi_m \sin(2\phi) \qquad (1)$$

where Φ_{out} is the output flux coupled to the magnetometer pick-up coil (L_0 in figure 3(a)), ϕ is the rotational angle of the float and Φ_m has been measured to be $3 \times 10^7 \Phi_0$ rad^{-1} for a maximum stored current of 2.5 A (where Φ_0 is the fundamental flux quantum, $h/(2e)$).

The rotation detector exerts a restoring torque on the float which can be adjusted by modifying the persistent current that is stored. In the present set-up the maximum current which can be stored should result in a torsional stiffness of 2×10^{-7} N m rad^{-1} and a period of 60 s. We now discuss the characteristics of the spherical magnetic bearing and describe its performance in preliminary experiments.

3. The characteristics of the spherical magnetic suspension

3.1. Theoretical considerations

A full analysis of the characteristics of the suspension would include a study of the static stability and dynamics of the float, including couplings of all six degrees of freedom. Here we will assume perfect azimuthal symmetry and restrict our

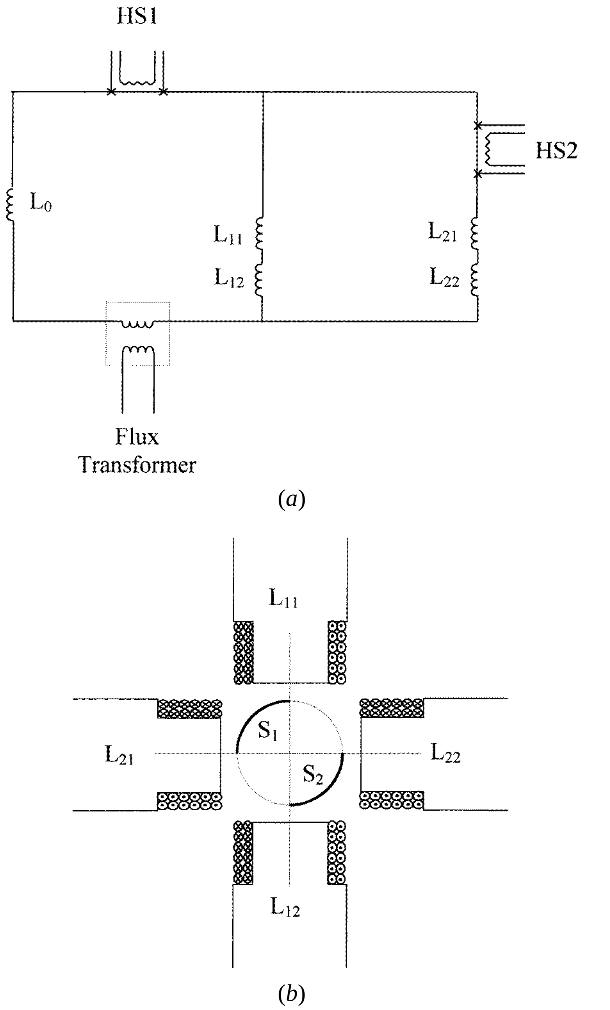


Figure 3. The circuit diagram of the rotation detector. (*a*) The circuit diagram of the rotation detector showing heat switches (HS1 and HS2) and current injection connections. The sensing coil L_0 is coupled to the flux-gate magnetometer. (*b*) A schematic view of pick-up coils (L_{11}, L_{12}, L_{21} and L_{22}) and superconducting strips (S_1, S_2) which are attached to the float.

discussion to the static stability of three degrees of freedom, namely displacement in the vertical, z, and horizontal, x, directions and rotation about the horizontal, θ. We have not, as yet, studied the θ degree of freedom in any depth and assume that the overlap of the float of 7° over the coil windings is sufficient that this stiffness can be neglected. Because experiments at 4.2 K are expensive and time consuming we have developed detailed numerical models for calculating stabilities both in the z and in the x direction and also the lifting capacity of the coil. We now discuss these models and compare their predictions with experimental measurements.

Clearly an important characteristic of the magnetic suspension system is the lifting capacity. If the total energy stored in the coil–float system is $\frac{1}{2}Li^2$ then the lifting force is given as [18]

$$F_z(z) = \frac{1}{2}\frac{\mathrm{d}L}{\mathrm{d}z}i^2 - mg \tag{2}$$

where L is the inductance of the levitation coil as a function of the height, z, of the float and g is the local gravitational acceleration of the Earth. We can define α as the mass per unit current squared which the coil will levitate. The lifting capacity can then be found as

$$M = \frac{1}{2}\left[\frac{\mathrm{d}L}{\mathrm{d}z}\right]_{z=0}\frac{{i_c}^2}{g} = \alpha i_c^2 \tag{3}$$

where $z = 0$ corresponds to the nominal levitated position of the float where the centres of figure of the float and coil substrate coincide. As mentioned in the previous section, i_c is a complex function of the geometries of the float and coil. For the present float and coil we have measured $i_c = 3.75$ A. The other crucial feature of any suspension is its stability both for vertical and for transverse motion. The restoring force due to magnetic forces depends on whether the coil–float system is in a constant-current or constant-flux mode [19]. In the constant-current mode the stiffness, K_{zi}, becomes

$$K_{zi} = -\left[\frac{\mathrm{d}F_z}{\mathrm{d}z}\right]_{i,z=0} = -\frac{1}{2}\left[\frac{\mathrm{d}^2L}{\mathrm{d}z^2}\right]_{z=0} i^2. \tag{4}$$

However, in the constant-flux mode we find

$$K_{z\Phi} = -\left[\frac{\mathrm{d}F_z}{\mathrm{d}z}\right]_{\Phi,z=0} = K_{zi} + \frac{\Phi^2}{L^3}\left[\frac{\mathrm{d}L}{\mathrm{d}z}\right]^2_{z=0}. \tag{5}$$

Provided that K_{zi} is positive the suspension is stable in this degree of freedom and $K_{z\Phi} > K_{zi}$. For motion in the transverse or x direction we expect that

$$\left[\frac{\mathrm{d}L}{\mathrm{d}x}\right]_{x=0} = 0$$

and therefore there is no distinction between the stiffnesses in constant-current and constant-flux modes:

$$K_x = -\left[\frac{\mathrm{d}F_x}{\mathrm{d}x}\right]_{x=0} = -\frac{1}{2}\left[\frac{\mathrm{d}^2L}{\mathrm{d}x^2}\right]_{x=0} i^2. \tag{6}$$

We have developed numerical models which predict the characteristics of the spherical magnetic suspension. The basic technique which we employ is the solution of the boundary-value problem by solving the integral Fredholm equation of the second kind. In our simplest model (model 1) we follow the method employed by Bourke [20], in which ficticious surface ‘charges’ coat the lift surface of the float. The lift surface is divided into bands at different latitudes and charge densities for each band are found such that the field due to all surface charge elements cancels the normal component of the magnetic field generated by the coil. The total magnetic field anywhere can then be calculated from the sum of the contributions from the coils and the charges. The vertical lifting force can be calculated by summing the magnetic pressures exerted on the float. Bourke further showed that it was possible to extend this method to calculate the transverse forces exerted by the magnetic field when the float was translated away from the axis of symmetry. Model 1 is reasonably simple insofar as we neglect the cross-section of the coil windings and ignore the field on the upper surface of the float. We have developed a much more comprehensive model of the magnetic levitation system (model 2) which seeks current-density distributions both on the coil windings and on the float such that the *tangential* component of the magnetic field at the interior of all superconducting surfaces is annulled. The flux and inductance can then be calculated from a line integral of the vector potential around the coil [16, 21]. We have not yet extended this model to the case in which the float is not axially symmetrical and we will be able, ultimately, to use it to predict values of the critical current. However, the inclusion of current elements on the coil windings makes the programme somewhat machine intensive.

3.2. Comparisons of model predictions with experimental results

The inductance as a function of the vertical displacement of the float was measured under dc at 4.2 K with currents comparable to those required for levitation using a superconducting transformer. From measurements of the ratio of the current induced in the secondary to that flowing in the primary we could directly compare the inductance value of the coil and associated float with a calibrated inductance. The known inductor was wound from lead wire and its value was determined at 4.2 K and at 1 kHz using a commercial component bridge with an accuracy of 1.3%.

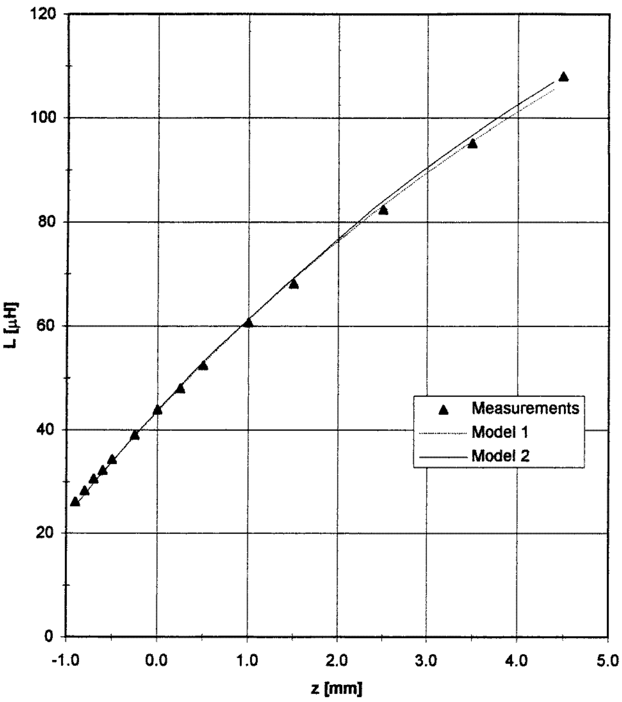
Measurements of L are plotted as a function of the levitation height together with the predictions of the models in figure 4(*a*). A float of $R = 43.4$ mm (which was slightly larger than that of the float used in the levitation experiments) was used for these measurements. The inductance values plotted for model 1 were obtained by integration of the lifting force to within an arbitrary constant, which was chosen to ensure a best fit with the measurements. The agreement between predictions and measurements is very good. It can be seen that, in this case, the two models are also in good agreement. As shown in figure 4(*a*), the slope of the inductance versus height decreases with height and this ensures, given the discussion in section 3.1, that the suspension is stable against perturbations in z.

Figure 4(*b*) shows the variation of the inductance versus x measured using an ac bridge at 1 kHz and at a float height of $z = 1$ mm, together with the predictions of model 1. The inductance has a maximum value when the float is centred and thus the suspension is stable against perturbations in the x direction.

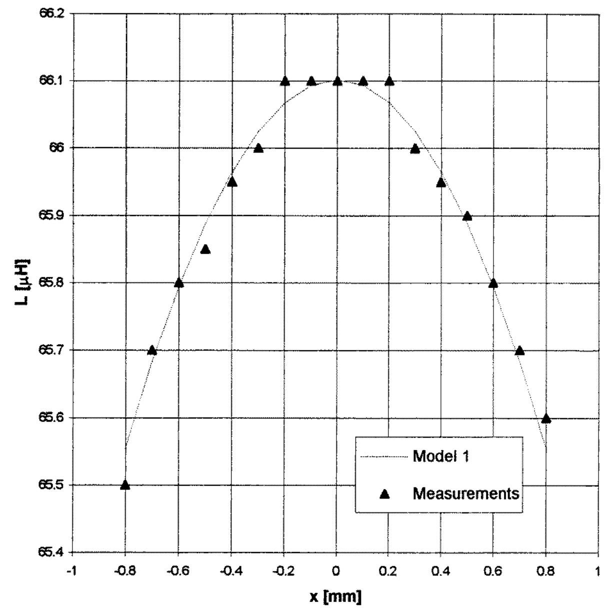
From the inductance measurements the values for α, K_{zi} and K_{xi} have been determined to be 1.1 g A^{-2}, 24 N m^{-1} and 16 N m^{-1} respectively. This corresponds to resonance frequencies of 6 and 5 Hz for the vertical and horizontal modes.

4. The performance of the torsion balance

We have performed preliminary levitation experiments in a partial vacuum to prove the viability of the suspension. The experiments were run in a Dewar flask providing magnetic shielding at the level of a milligauss. Helium gas at a pressure of 53 Pa was used as an exchange gas. A current of 1 A was stored in the rotation detector and



(*a*)



(*b*)

Figure 4. A plot of measured and calculated values of the inductance: (*a*) as a function of the vertical height of the float and (*b*) as a function of the horizontal translation of the float.

a persistent current of 3.7 A was used to levitate the float. Rotational oscillations of the float were excited by coupling magnetic flux into the detection circuit as described above.

With the exchange gas present a quality factor, Q, of about 200 at a period of 25 s was observed. This level of damping corresponds to a thermal noise torque of 3×10^{-15} N m $Hz^{-1/2}$. Subsequently the gas pressure was increased and the change in Q was measured. In the range 53–118 Pa Q was, to a good appromixation, inversely proportional to the pressure. Above 118 Pa no significant variation in Q was measured. This pressure corresponds to a mean free path of about 0.1 mm, which is comparable to the gap between the float and the levitation bearing.

For a stored current of 1 A we would expect a period of 120 s if all the restoring torque were generated by the detector coils. An oscillation period of 25 s indicates therefore that the majority of the restoring torque is being provided by a parasitic stiffness of about 10^{-6} N m rad^{-1}. Furthermore, adjustment of the persistent currents stored in the detector circuit did not change the period in a consistent manner. We suspect that the restoring torque that is observed is due to flux being trapped in the levitation system.

The noise in our magnetometer, about 10^{-8} T Hz$^{-1/2}$, limits the angular resolution of our torsion balance to around 10^{-4} rad Hz$^{-1/2}$ (which, together with the current parasitic stiffness, corresponds to a torque sensitivity of about 10^{-10} N m Hz$^{-1/2}$). It is interesting to note that the use of a dc SQUID would increase the angular sensitivity by some nine orders of magnitude [16].

5. Conclusions

We have shown the feasibility of the design and manufacture of a new torsion-balance suspension. Our preliminary experiments appear promising. We have achieved stable levitation of a 14.7 g float using a persistent current. If we calculate the level of torque noise due to thermal noise from damping processes we find an amplitude spectral density of 3×10^{-15} N m Hz$^{-1/2}$. This potential sensitivity should be realizable by coupling a dc SQUID to the rotation detector. The torque noise is comparable to that achieved recently in an electrostatically suspended torsion balance [22]. By varying the residual gas pressure we have established that the dominant damping mechanism is due to the viscosity of the gas.

We have observed rotational stiffnesses which are significantly larger than those that could be expected from the rotation detector. We are currently improving the quality of the coil and float and are investigating whether the parasitic stiffnesses are due to trapped flux.

Since the experiments described here, we have made significant improvements. The floats are now electroformed and have a higher sphericity (0.01%) and, with a gap of 0.3 mm, we have achieved critical currents of 4.5 A. We are currently investigating the damping as a function of the pressure of helium gas.

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