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FUNDAMENTAL PROBLEMS IN METROLOGY

THE DESIGN OF A LABORATORY EXPERIMENT TO DETECT NEW FORCES

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An experiment to detect new interactions, described by a Yukawa-type potential, on equipment with a uniform gravitational field is considered. It is shown that in the Yukawa α–λ potential parameter space a sensitivity can be achieved at a level of $\alpha \approx 10^{-10}$ in a range of values of $\lambda \approx 0.1$–10^7 m.

***Key words**: "dark" matter, new interactions, inverse-square law, experiment in a uniform gravitational field, spectrum of non-Newtonian forces.*

Observations of supernova, the anisotropy of the background microwave radiation and the curves of rotation of the galaxies show that the invisible (hidden, "dark") mass of the Universe, according to different estimates, exceeds the visible (luminous) mass (25% against 5%). In addition to this "dark" matter, there should be unusual matter with negative pressure, which has come to be called "dark energy" and which makes up about 70% of the total energy in the Universe. Its equation of state is $\varepsilon = \omega p$, where ε is the energy density, p is the pressure, and $\omega \leq -1$. According to modern ideas on primary nucleosynthesis and the formation of the galaxies, hidden mass has a nonbarion nature. Microrefraction experiments also agree with the nonbarion origin of the hidden mass, although their present accuracy is still insufficient to answer this question uniquely. The importance of the problem of hidden ("dark") mass, and also of "dark" energy, which at the present time is modeled by the cosmological constant or by special scalar fields with interaction (quintessence) potentials or by "phantom" fields with negative energy, etc., is due to the fact that the average density of the Universe and, consequently, the choice of an adequate cosmological model, depend on its solution. An attempt to explain "dark" mass by massive electron (muon) neutrinos does not agree with experimental data. One of the widespread hypotheses on the nature of hidden mass proposes the existence of different massless or light elementary particles, which lead to an additional Yukawa-type interaction between massive bodies. This additional interaction leads to an apparent breakdown of the inverse-square law, which acts in Newtonian gravitation [1–6]. New particles are forecast, in particular, by unified gauge theories, supersymmetry and supergravitation (arion, scalar axion, dilaton, antigraviton with spin equal to unity, etc. [3, 6]). It is an extremely complex problem to detect these particles with modern accelerators (see, for example, [6, 7]) due to their small mass (of the order of 10^{-6}–10^{-4} eV for an axion) or large mass (10 GeV – 2 TeV for a heavy neutralino). One other new problem is worth mentioning, connected, obviously, with the modification of Newton's law at considerable distances (of an order greater than solar distances). This is the anomalous acceleration of the Pioneer-10 and Pioneer-11 spacecraft directed towards the Sun (see [8]), which has not yet received a satisfactory explanation.

A Yukawa-type interaction (when it is normalized to Newtonian interaction) is described by the potential $V(r) = \alpha G m e^{-r/\lambda}/r$, which is determined by the dimensionless value of the interaction α and the spatial scale λ. At the present time, the search for such an interaction is being carried out over a wide range of values of λ, beginning with atomic scales of

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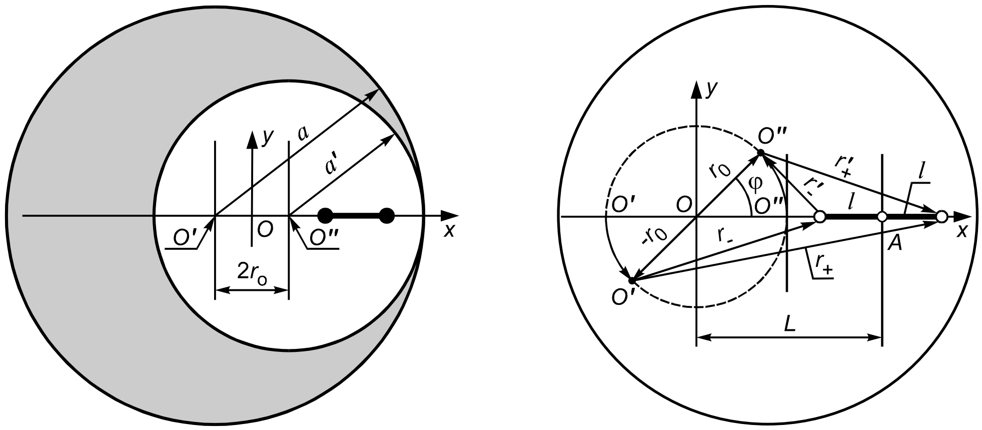

a *b*

Fig. 1. Conventional sketch of a section of the equipment (*a*) and the spherical cavity (*b*) in a horizontal plane.

the order of 10^{-10} m and up to astronomical orders of 10^{14} m. Measurements at atomic distances are being carried out using atomic power microscopes. For scales of the order centimeters to meters the most effective instruments are gravimetric instruments. At considerable distances, data on the fall of bodies, the motion of satellites (LAGEOS), the Moon, planets, and pulsars are used. New results in this range were obtained in [9].

In this paper, we consider a possible experiment to detect new forces with a finite interaction radius using unique equipment constructed at the All-Russia Research Institute of Metrology to calibrate seismometers and accelerometers, and estimates were obtained over a wide range of the parameters of Newtonian interaction, namely, the advance with respect to the spatial parameter λ is 9 orders of magnitude, from several centimeters to 10^7 m, and with respect to the parameter α, characterizing the new interaction, it is 6–7 orders of magnitude.

The equipment consists of a sphere with spherical cavities of radius a and a', respectively, displaced a distance of $2r_0$ with respect to the center of the sphere. The sphere is placed on a turntable, which is suspended in air using air bearings and is rotated slowly ($\omega/2\pi \approx 10^{-2}$–$10^{-4}$ Hz) with high stability ($\delta\omega/\omega \approx 10^{-6}$). A uniform gravitational field is produced in the spherical cavity, which, when the platform is rotated, leads to the occurrence of an alternating signal, by which the instrument, which is in the spherical cavity, is calibrated. In Fig. 1, we conventionally represent a section of the equipment in the horizontal plane (for clarity this is not shown in the correct proportions). The centers of the sphere O' and of the spherical cavity O'' move in a horizontal plane in a circle of radius r_0 with center at the point O (see Fig. 1*a*). A torsion balance in the form of a dumbell suspended on an elastic filament, the axis of rotation of which is denoted by the letter A (see Fig. 1*b*) is used as the sensitive element. The sensitive element is rigidly fastened to the equipment using a special suspension system via a channel along the axis of rotation. Since the centers of the sphere and its spherical cavity are shifted with respect to one another, then, due to the absence of symmetry, the torsion balance experiences a moment of forces.

New forces, as mentioned above, are described by a Yukawa-type potential (see [3]). The moment of the forces acting on the sensitive element (the dumbell on the elastic filament), placed in the spherical cavity, was calculated in [4]. The calculation reduced to finding the potential energy of the torsion balance and its dependence on the angle of rotation of the sphere.

The potential energy of the sensitive element in the spherical cavity, according to [4], depends on the distances between the centers of the sphere O, the spherical cavity O'' and the centers of both spheres of the dumbell r_-, r_+, r'_-, and r'_+:

$$U(P) = \alpha m \sigma G \frac{2\pi}{\mu^2} \beta(\mu s_0) \left(A' \left[\frac{\sinh \mu r'_+}{\mu r'_+} + \frac{\sinh \mu r'_-}{\mu r'_-} \right] - A \left[\frac{\sinh \mu r_-}{\mu r_-} + \frac{\sinh \mu r_+}{\mu r_+} \right] \right). \quad (1)$$

According to the drawing in Fig. 1,

$$r_{\pm} = r_0 + \mathbf{i}(L \pm l); \quad r'_{\pm} = -r_0 + \mathbf{i}(L \pm l),$$

where $\mathbf{i}$ is the unit vector along the OX axis, whence it follows that

$$r_{\pm} = (r_0^2 + (L \pm l)^2 + 2r_0(L \pm l)\cos\varphi)^{1/2}; \tag{2}$$

$$r'_{\pm} = (r_0^2 + (L \pm l)^2 - 2r_0(L \pm l)\cos\varphi)^{1/2}. \tag{3}$$

In spherical coordinates, the potential energy of the dumbell in a field of Yukawa forces, as follows from relations (1)–(3), is a function of the angle φ:

$$U(\varphi) = \alpha m\sigma G\frac{2\pi}{\mu^2}\beta(\mu s_0)F(\varphi), \tag{4}$$

where

$$F(\varphi) = A'\left(\frac{\sinh\mu r'_+}{\mu r'_+} + \frac{\sinh\mu r'_-}{\mu r'_-}\right) - A\left(\frac{\sinh\mu r_+}{\mu r_+} + \frac{\sinh\mu r_-}{\mu r_-}\right). \tag{5}$$

Here α and μ are parameters characterizing the value and spatial scale $\lambda = 1/\mu$ of the interaction, s_0 and m are the radius and mass of the small weights (of spherical form) at the ends of the balance beam of the torsion balance of length $2l$, L is the distance between the center of the dumbell (the point A in Fig. 1b) and the axis of rotation of the sphere O', a is the radius of the sphere, a' is the radius of the spherical cavity, $A = (1 + \mu a)e^{-\mu a}$, $A' = (1 + \mu a')e^{-\mu a'}$, $r_{\pm}$ are the distances from the center of the sphere (O') to the small spheres of the balance beam, closest to the center (–) and furthest from it (+) (see Fig. 1), $r'_{\pm}$ are the distances from the center of the spherical cavity (O'') to the small spheres of the balance beam – closest to the center (–) and furthest from it (+), σ is the density of the material of the sphere, G is the gravitational constant, r_0 is the radius of the circle along which the centers of the sphere and of the spherical cavity move (the distance between them is $2r_0$), and $\beta(x)$ is the scale parameter, defined by the formula

$$\beta(x) = \frac{3}{x^3}(x\cosh x - \sinh x). \tag{6}$$

Inside the spherical cavity, a uniform gravitational field is produced, the strength of which g (the acceleration of the force of attraction) is everywhere constant inside the cavity $g = (4\pi/3)G\sigma 2r_0$ [2]. The uniform Newtonian gravitational field does not produce a moment of the forces. Forces which differ from Newtonian produce it. As a result, for the ideal arrangement of the configuration of the sphere, in the form of the experiment considered, unlike all other experiments, new forces can be detected on a zero background of Newtonian gravitation.

The potential energy $U(\varphi)$ according to (1)–(3) is a periodic function and, consequently, can be expanded in a Fourier series. The dependence on the angle φ is contained in the function $F(\varphi)$. According to [4], we have

$$F(\varphi) = (A' - A)\sum_{m=0}^{\infty} D_{2m}^{(3)}\varepsilon_m \cos 2m\varphi - (A' + A)\sum_{m=0}^{\infty} D_{2m+1}^{(3)}\cos(2m+1)\varphi. \tag{7}$$

The coefficients of even and odd harmonics are

$$D_{2m}^{(3)} = \sum_{s=m}^{\infty}\frac{4s+1}{2^{4s-1}}\binom{2(s-m)}{s-m}\binom{2(s+m)}{s+m} i_{2s}(\mu r_0)\{i_{2s}(\mu(L+l)) + i_{2s}(\mu|L-l|)\}; \tag{8}$$

$$D_{2m+1}^{(3)} = \sum_{s=m}^{\infty} \frac{4s+3}{2^{4s+1}} \binom{2(s-m)}{s-m} \binom{2(s+m+1)}{s+m+1} i_{2s+1}(\mu r_0) \times$$

$$\times \{ i_{2s+1}(\mu(L+l)) - i_{2s+1}(\mu |L-l|) \}. \tag{9}$$

For brevity, we have introduced the notation $i_n(x)$ for the modified spherical Bessel functions $\sqrt{\pi / 2x} I_{n+1/2}(x)$ [5].

The correctness of the expansion of the function $F(\varphi)$ in Fourier series (7)–(9) is confirmed by a parallel calculation of this function from formula (5). The modified spherical Bessel functions $i_n(x)$, which determine the amplitudes of the harmonics (8) and (9), for small values of the argument, when $x < 1$, can be expanded in power series (see, for example, [5]):

$$i_0(x) = 1 + \frac{x^2}{3!} \left(1 + \frac{x^2}{20} \left(1 + \frac{x^2}{42} \left(1 + \frac{x^2}{72} \right) \right) \right); \tag{10}$$

$$i_2(x) = \frac{x^2}{15} \left(1 + \frac{x^2}{14} \left(1 + \frac{x^2}{36} \left(1 + \frac{x^2}{66} \right) \right) \right); \tag{11}$$

$$i_4(x) = \frac{x^4}{945} \left(1 + \frac{x^2}{22} \left(1 + \frac{x^2}{52} \right) \right); \tag{12}$$

$$i_6(x) = \frac{x^6}{135135} \left(1 + \frac{x^2}{30} \right); \tag{13}$$

$$i_1(x) = \frac{x}{6} \left(2 + \frac{x^2}{20} \left(4 + \frac{x^2}{42} \left(6 + \frac{x^2}{9} \right) \right) \right); \tag{14}$$

$$i_3(x) = \frac{x^3}{105} \left(1 + \frac{x^2}{18} \left(1 + \frac{x^2}{44} \right) \right); \tag{15}$$

$$i_5(x) = \frac{x^5}{10395} \left(1 + \frac{x^2}{26} \right). \tag{16}$$

The accuracy of formulas (10)–(16) is characterized by an error of the order of $x^{10}/10!$ for "even" functions $i_{2s}(x)$ and $x^9/9!$ for "odd" functions $i_{2s+1}(x)$. The amplitudes of the harmonics D_{2m} and D_{2m+1} are proportional to the product of the leading arguments, $(xy)^{2m}/((m+2)!)^2$, $(xy)^{2m+1}/((m+3)!)^2$, respectively. The arguments x and y for the gravitational measure [1], which it is proposed to used for the experiment, differ by not more than a factor of 6, and for large λ they all become less than unity. The function $F(\varphi)$ can then be calculated, leaving only the first three – four harmonics in it.

The function $F(\varphi)$ was calculated by two methods using formulas (5) and (7)–(9). The calculations were carried out for the case when $L < l$ and two "sets" of arguments:

$$x = r_0\mu = 0.06, \quad y = \mu(L + l) = 0.32, \quad z = \mu(L - l) = 0.04;$$

$$x = r_0\mu = 0.006, \quad y = \mu(L + l) = 0.032, \quad z = \mu(L - l) = 0.004,$$

TABLE 1. Results of a Calculation of the Function $F(\varphi)$ for Different Values of the Arguments

φ, °	$F(\varphi)\cdot 10^2$ for $x = 0.06$; $y = 0.32$; $z = 0.04$; $\lambda = 50$ cm		$F(\varphi)\cdot 10^3$ for $x = 0.006$; $y = 0.032$; $z = 0.004$; $\lambda = 500$ cm	
	(5)	(7)–(9)	(5)	(7)–(9)
0	7.18832328684	7.18832328680	1.206464854314	1.206464854314
10	7.20379359666	7.20379359662	1.208164208318	1.208164208318
20	7.24973459152	7.24973459150	1.213210636321	1.213210636321
30	7.32475073395	7.32475073393	1.221450805167	1.221450805167
40	7.42656324978	7.42656324976	1.232634341505	1.232634341505
50	7.5520793097	7.5520793096	1.246421439254	1.246421439254
60	7.69748592847	7.69748592847	1.262393184438	1.262393184438
70	7.85836573618	7.85836573618	1.280064283673	1.280064283673
80	8.02983111090	8.02983111090	1.298897809561	1.298897809561
90	8.20667260850	8.20667260850	1.328321514956	1.328321514956

corresponding to two values of the spatial parameter λ (50 cm and 500 cm) and the parameters of the experimental equipment $r_0 = 3$ cm, $L = 7$ cm, and $l = 9$ cm. The results are presented in the table.

As follows from the table, for the first set of arguments ($\lambda = 50$ cm) of the spherical functions, the greatest of these is the value $y = 0.32$, and the function $F(\varphi)$, calculated from (5), is identical with the results of a calculation using (7) to twelve and eleven significant digits. For the second set of arguments ($\lambda = 500$ cm), when all the arguments are an order of magnitude less, they coincide to thirteen significant digits. This good agreement between the results obtained by two different methods confirms, in particular, the correctness of the expansion of the function $F(\varphi)$ in a Fourier series.

Replacing the function $F(\varphi)$ in the potential energy of the beam balance (4) by its representation in a Fourier series (7) and differentiating with respect to the angle φ, we obtain the moment of the forces produced by the new interaction:

$$M = \frac{dU(\varphi)}{d\varphi} = \alpha m_0 \beta(\mu s_0) G\rho \frac{4\pi}{\mu^2}\left\{-(A' - A)\sum_{m=1}^{\infty} D_{2m}^{(3)} 2m \sin 2m\varphi + \right.$$

$$\left. + (A' + A)\sum_{m=0}^{\infty} D_{2m+1}^{(3)}(2m+1)\sin(2m+1)\varphi\right\}. \quad (17)$$

Fourier series (17) for the rotation of the sphere, when $\varphi = \omega t$, converts into the spectrum of the expected signal. We have for the first and second harmonics of this signal

$$M_{1\omega} = \alpha m_0 \beta(\mu s_0) G\rho 4\pi\lambda^2 (A' + A) D_1 \sin\omega t;$$

$$M_{2\omega} = -\alpha m_0 \beta(\mu s_0) G\rho 4\pi\lambda^2 (A' - A) 2D_2 \sin 2\omega t.$$

The coefficients of the expansion D_1 and D_2 when $L > l$ are found from the relations

$$D_1^{(1)} = \sum_{s=0}^{\infty} \frac{4s+3}{2^{4s+1}} \binom{2s}{s} \binom{2(s+1)}{s+1} i_{2s+1}(\mu r_0)\{i_{2s+1}(\mu(L+l)) + i_{2s+1}(\mu(L-l))\};$$

$$D_2^{(1)} = \sum_{s=1}^{\infty} \frac{4s+1}{2^{4s-1}} \binom{2(s-1)}{s-1} \binom{2(s+1)}{s+1} i_{2s}(\mu r_0)\{i_{2s}(\mu(L+l)) + i_{2s}(\mu(L-l))\}.$$

When $L = l$, the argument in the spherical Bessel function $z = \mu|L-l| = 0$, and for all $n \neq 0$ the spherical functions $i_n(0) = 0$ (see (6)). As a result,

$$D_1^{(2)} = \sum_{s=0}^{\infty} \frac{4s+3}{2^{4s+1}} \binom{2s}{s} \binom{2(s+1)}{s+1} i_{2s+1}(\mu r_0) i_{2s+1}(\mu 2l);$$

$$D_2^{(2)} = \sum_{s=1}^{\infty} \frac{4s+1}{2^{4s-1}} \binom{2(s-1)}{s-1} \binom{2(s+1)}{s+1} i_{2s}(\mu r_0) i_{2s}(2l).$$

For the case when $L < l$, it follows from (8) and (9) that

$$D_1^{(3)} = \sum_{s=0}^{\infty} \frac{4s+3}{2^{4s+1}} \binom{2s}{s} \binom{2(s+1)}{s+1} i_{2s+1}(\mu r_0)\{i_{2s+1}(\mu(L+l)) - i_{2s+1}(\mu|L-l|)\};$$

$$D_2^{(3)} = \sum_{s=1}^{\infty} \frac{4s+1}{2^{4s-1}} \binom{2(s-1)}{s-1} \binom{2(s+1)}{s+1} i_{2s}(\mu r_0)\{i_{2s}(\mu(L+l)) + i_{2s}(\mu|L-l|)\}. \qquad (18)$$

The first harmonic is of particular interest. Its amplitude is proportional to the product of three factors, which are functions of the scale factor λ: $M_{1\omega} \approx \beta(s_0/\lambda)\lambda^2 D_1(1/\lambda)$. For large values of λ, the arguments in the factor $\beta(x)$ and of the modernized spherical Bessel functions are proportional to $1/\lambda$ and become small. In this case, the factor $\beta(x) \to 1$ (as $x \to 0$), and in the coefficient D_1, according to (18), the first term, proportional to the product $i_1(x)i_1(y)$ becomes the leading term. The Bessel functions for small values of the argument $i_1(z)$ in the first approximation are proportional to its argument ($i_1(x) \sim x$, (see (14)), so that when $\lambda \to \infty$, the argument $x \to 0$, and the amplitude of the first harmonic approaches a constant value, i.e., $M_{1\omega} \approx \alpha m \beta(\mu s_0) G\rho 4\pi\lambda^2 D_1 \approx \text{const}$. When $\lambda \to \infty$, we have $\beta \to 1$ and $A \to 1$,

$$A' \to 1, \quad (A + A') \to 2, \quad x \ll 1, \quad y \ll 1, \quad i_1(x) \approx x/3,$$

$$D_1 \approx \frac{3}{2} 2 \frac{r_0}{3\lambda} \frac{L+l}{3\lambda} = \frac{r_0(L+l)}{3\lambda^2}.$$

Finally, for large λ we obtain the estimate

$$M_{1\omega} \approx \alpha m 8\pi G\rho \frac{r_0(L+l)}{3}. \qquad (19)$$

The amplitudes of the remaining harmonics decrease as $\lambda \to 0$ and $\lambda \to \infty$, and when the spatial scale of the new interaction has a value of the order of the radius of the sphere ($\lambda \approx a$), they reach a maximum.

The amplitude of the first harmonic depends very much on the arrangement of the torsion balance in the cavity of the gravitational measure. It is a maximum when it is at the greatest distance from the axis of rotation. When the axis of the torsion balance (the point A in Fig. 1b) coincides with the axis of rotation (the point O), then $L = 0$ and, as follows from (18), in this case the first harmonic (and all odd harmonics) disappears, and the second harmonic becomes the greatest (this result was obtained previously in [6, 7]).

We will use the usual condition as the criterion of the detection of a new force, namely, the equality of the signal/noise ratio to unity. To estimate the limit resolution, we will consider as the noise the fundamentally irremovable thermodynamic fluctuations, which, in a torsion balance with a long relaxation time, are given by the formula [8]:

$$M_{fl} = \xi\sqrt{\frac{8JkT}{\tau\tau^*}}.$$

Here ξ is a coefficient of the order of unity, J is the moment of inertia of the torsion balance, k is Boltzmann's constant, T is the temperature, τ is the measurement time, and τ^* is the relaxation time. Equating this noise moment to the signal at the first harmonic, we obtain the following relation for estimating the limit resolution of the parameter α:

$$\alpha m_0 \beta(\mu s_0) G\rho 4\pi\lambda^2 (A' + A) D_1 = \xi\sqrt{\frac{8JkT}{\tau\tau^*}}. \tag{20}$$

When $\lambda \to \infty$, the limit resolution of the parameter of the new interaction is

$$\alpha \approx \frac{3}{8\pi}\frac{\xi}{mG\rho r_0(L+l)}\sqrt{\frac{8JkT}{\tau\tau^*}}. \tag{21}$$

We will consider equipment (see Fig. 1) with the following parameters: $L = l = 9$ cm, $m = 20$ g, $J = 2\,m_0 l^2 = 3240$ g·cm^2, $\rho = 7.9$ g·cm^{-3}, $a = 30$ cm, $a' = 23.5$ cm, and $T = 300$ K. We will choose the measurement time τ and the relaxation time τ^* of the torsion balance to be $2.59\cdot10^6$ sec (one month) and $3.72\cdot10^7$ sec (one year), respectively. The relaxation time τ^* corresponds to an oscillation period $\tau_0 = 900$ sec and a Q-factor of the torsion balance $Q = 130{,}000$, which is observed when using an elastic filament of rhenium [9]. Substituting the gravitational constant G and Boltzmann's constant k into (21), we arrive at the following estimate for the maximum sensitivity, limited by thermodynamic noise:

$$\alpha_T \approx 7.04\cdot10^{-10}. \tag{22}$$

Rotation of the gravitational measure generates interference in synchronism with the useful signal. This occurs due to inaccurate construction of the gravitational measure and of the torsion balance. This means that the gravitational field of the measure becomes nonuniform and, consequently, second derivatives of the gravitational potential arise. Inaccuracy in constructing the torsion balance leads to the occurrence of nondiagonal components of the torsion balance, in particular, J_{xz} and J_{yz}. All this implies that the following synchronous interference will occur:

$$M_t = U_{xz}J_{yz} - U_{yz}J_{xz}.$$

Since the tensor of the second derivatives $\hat{U}$ (like the inertia tensor $\hat{J}$) is symmetrical, rotations of the system of coordinates around all three of its axes, in principle, may lead to a diagonal form and thereby eliminate the nondiagonal components U_{xz}, U_{yz} (J_{xz}, J_{yz}). However, the conditions of the experiment do not allow rotation around all the axes, and hence it is impossible to eliminate the components U_{xz}, U_{yz} (and also J_{xz}, J_{yz}). Their value is determined by the errors in constructing the torsion balance.

The components of the tensor of the second derivative of U_{xz} and U_{yz} have constant values in a system of coordinates rigidly connected to the rotating gravitational measure. In a fixed system of coordinates, the components of the tensor are converted according to the rule

$$\hat{U}' = \hat{g}\,\hat{U}\,\hat{g}^{-1}.$$

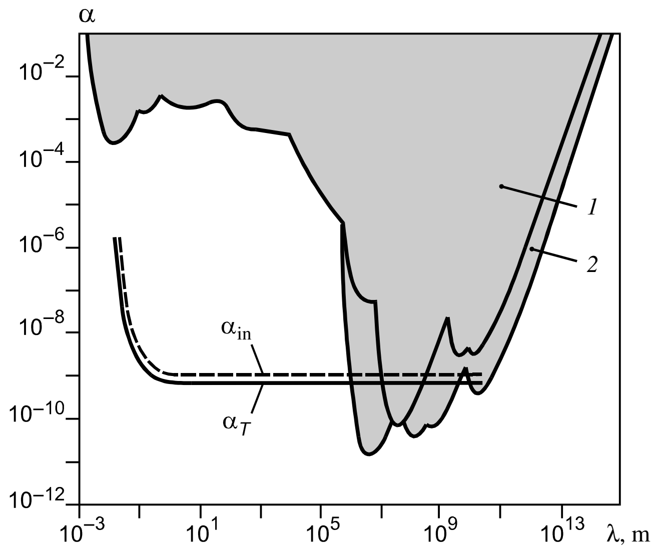

Fig. 2. State of the experimental and observed data for checking the inverse square law (*1*, *2* are the regions in which there are no deviations from the 1998 and 2004 data, respectively) and the expected results of an experiment to detect new forces (curves of α_{in} and α_T).

Here the sign "∧" relates to the tensor and the matrix of rotation around the z axis by an angle φ. In accordance with this rule,

$$U'_{xz} = U_{xz}\cos\varphi - U_{yz}\sin\varphi; \quad U'_{yz} = U_{xz}\sin\varphi + U_{yz}\cos\varphi.$$

Replacing the components of the tensor of the second derivatives in the general formula (20) by the values with primes and substituting (22) into it, we obtain

$$M_t = (U_{xz}J_{yz} - U_{yz}J_{xz})\cos\varphi - (U_{yz}J_{yz} + U_{xz}J_{xz})\sin\varphi.$$

We will assume that the errors in constructing the torsion balance are approximately the same for all the axes: $\delta x \approx \approx \delta y \approx \delta z \approx \delta l$. Then, starting from the definition of the inertia tensor, its components J_{xz} and J_{yz}, due to the errors, can be estimated in order of magnitude as

$$J_{xz} \approx J(\delta l/l); \quad J_{yz} \approx J(\delta l/l)^2.$$

The second derivatives of the gravitational potential, which arise due to errors in constructing the torsion balance, were considered in [4]. Using the results obtained in this paper, we can estimate the components of the second derivatives:

$$\delta V_{xz} = \sqrt{\frac{4\pi}{5}}G\rho\frac{1}{a'}\sqrt{\frac{3}{2}}\left(u_{21} + u_{2-1}\right) \approx G\rho\frac{\delta a'}{a'};$$

$$\left|\delta V_{yz}\right| = \sqrt{\frac{4\pi}{5}}G\rho\frac{1}{a'}\sqrt{\frac{3}{2}}\left|u_{2-1} - u_{21}\right| \approx G\rho\frac{\delta a'}{a'}.$$

Finally, for synchronous interference, which can be called "tidal," since it is connected with the second derivatives of the gravitational potential, we obtain from (20) the estimate

$$M_t \approx JG\rho \frac{\delta l}{l} \frac{\delta a'}{a'} \cos(\varphi + \varphi_0). \tag{23}$$

It follows from (23) that there is a limitation on the sensitivity due to inaccurate construction of the torsion balance and of the sensitive element

$$\alpha m_0 \beta(\mu s_0) 4\pi\lambda^2 (A' + A) D_1 = J \frac{\delta l}{l} \frac{\delta a}{a}.$$

Hence, using the limit value of the moment of the forces (19), we can write the instrumental limitation on the sensitivity of the equipment:

$$\alpha_{\text{in}} \approx \frac{3}{16\pi} \frac{J}{m r_0 l} \frac{\delta l}{l} \frac{\delta a}{a}.$$

The error in constructing the VNIIM equipment $\delta a/a$ is of the order of $3 \cdot 10^{-4}$, while the sensitive element of the torsion balance can be constructed with an error of $\delta l/l \approx 1 \cdot 10^{-5}$.

Taking into account the parameters of the torsion balance, given above, we obtain an instrumental estimate of the limit sensitivity, which can be achieved in the experiment described:

$$\alpha_{\text{in}} \approx 1.08 \cdot 10^{-9}.$$

In Fig. 2, we show curves of α_T and α_{in} for thermodynamic and instrumental interference, which limit the possibilities of the experiment, and also the limits on the parameters α and λ, obtained from the results of experiments and astronomical observations (and astrophysical observations with binary pulsars) (regions *1* and *2*). The proposed experiment using unique equipment will enable a large lacuna at the beginning of the graph to be filled – to progress by 9 orders of magnitude with respect to the spatial parameter λ from several centimeters to 10^7 m. As regards the parameter α, characterizing the new interaction, the progress will be by 6–7 orders of magnitude. One of the experiments proposed can replace a set of planned experiments, but also considerably limit the possibility of the existence of carriers of new probable interactions, predicted by unification theories.

Note also that the proposed experiment allows of a different interpretation and can be regarded as an experiment to check the inverse square law, i.e., Newton's law of gravitation: the Yukawa potential may be taken as a deviation from Newton's law. The absence of such deviations is represented in Fig. 2 by regions *1* and *2*.

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