A value for *G* from beam-balance experiments

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Abstract. A high-precision measurement of the Newtonian gravitational constant G by means of a beam balance is being carried out at the University of Zürich. We have made test measurements in which the gravitational force of 10^3 kg of water was used and G was determined with a relative standard uncertainty of 220×10^{-6}. Currently, measurements with 13.5×10^3 kg of mercury are in progress. These measurements are not completed yet, but a preliminary result is presented.

Keywords: gravitational constant, beam balance, mercury

1. Introduction

We have designed an experiment using a beam balance to measure the Newtonian gravitational constant G. The basic idea is very simple. The weight difference of two test masses is measured with a modified Mettler–Toledo balance, which is one of the most precise balances in the world. The weight difference is changed by the gravitational force of two movable tanks filled with a liquid of known density. G can be calculated from the measured change of the weight difference. The idea for this experiment arose from our previous experiment at the Gigerwald storage lake [1].

Up to now, only a few attempts to measure G by means of a beam balance have been made [2], none of which attained a precision comparable to that of a torsion balance. Owing to recent developments in beam-balance technology, the beam balance is becoming quite competitive [3, 4]. Further features of the experiment are an optimized mass arrangement and a favourable measuring procedure. In the following sections a brief description of the general principles and the set-up is given. A detailed description can be found elsewhere [5, 6].

2. General principles

As shown in figure 1, two test masses are suspended on separate wires such that they hang on the same axis but at different levels. The test masses are alternately connected to the beam balance and their weight difference is determined. Effects that are equal for both test masses such as tidal forces and zero-point drifts of the balance cancel out. The weight difference is modulated by the gravitational force of two movable field masses, which are moved between two positions. In the first position, the field masses are located between the test masses, whereas in the second, the test masses are between the field masses. This is repeated many times and the resulting change of the weight difference is the gravitational signal from which G is calculated. As long as disturbing effects are independent of the field-mass position they cancel out, can be eliminated or cancel out on the average.

The field masses are cylinders with a central hole. The gravitational force of each field mass has extrema close to both ends of the hole. The field masses are moved such that the test masses are placed at an extremum. Therefore, the gravitational force acting on a test mass is, to first order, independent of the test-mass position. This reduces the required accuracy of the distances between the field and test masses. Furthermore, only a moderate homogeneity is needed for the density of the test masses.

The field masses are tanks, which can be filled with a liquid. A liquid is used, because the density of liquids is more homogeneous than that of typical solids. Mercury is used because of its high and well-known density. Measurements made with water allow an important check of consistency. The effect of the tanks can be measured with empty tanks.

3. The experimental set-up

The experiment is located in a 4.5 m deep pit inside an experimental building at the Paul-Scherrer-Institut (PSI). This pit has thick concrete walls and good mechanical and thermal stability. A schematic view of the experiment is shown in figure 2. The experiment extends over two rooms, separated by an insulating working platform. The upper room contains the beam balance, the electronics and the auxiliary equipment. The lower part houses the field masses. Both parts are thermally and mechanically insulated from each other. Special care was taken to obtain temperature stability and each room has its own temperature-stabilization system.

The balance is placed on a granite plate, which is supported by two steel girders fixed to the walls of the pit. In order to avoid errors due to convection, buoyancy and other gas-pressure forces, the balance is operated in vacuum. The vacuum system consists of a 0.5 m diameter chamber housing

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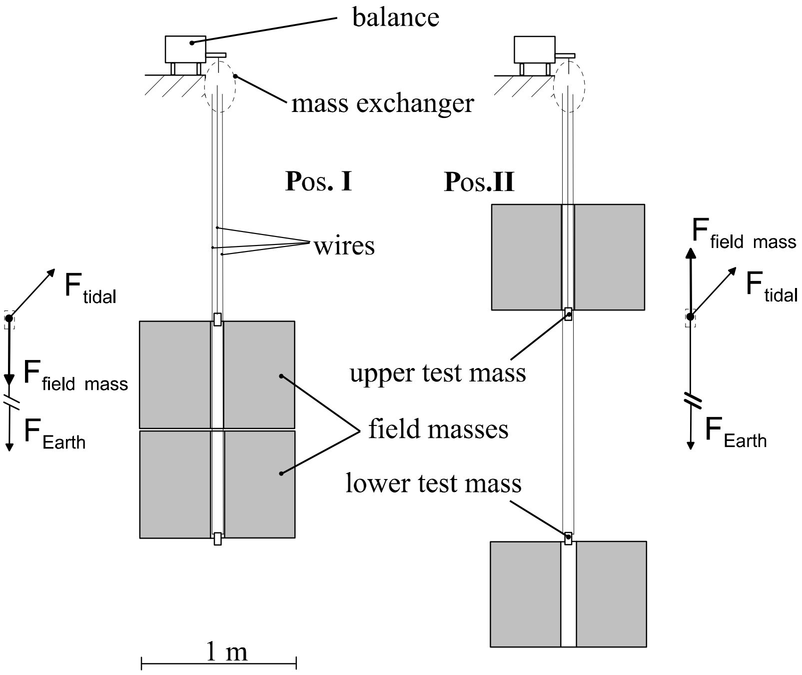

Figure 1. A schematic view of the mass arrangement. The test masses are alternately connected to the balance by means of the 'mass exchanger' and their weight difference is determined. The field masses are alternately rested in two positions. The corresponding forces on the upper test mass due to the Earth, the field masses and the tidal forces are plotted on the left- and right-hand sides, respectively.

the balance and a long tube enclosing the test masses. The pressure is about 10^{-4} Pa. The test masses are 1 kg weights with a cylindrical shape, 45 mm in diameter and 77 mm high.

The tanks employed for the field masses are made of stainless steel, having an outer diameter of 1046 mm and an inner diameter of 100 mm. They are 700 mm high. Each tank has a volume of 500 l. The corresponding 6.8×10^3 kg of mercury results in an amplitude of the gravitational signal of about 0.8 mg. To get the same signal with water instead of mercury, more than 2000×10^3 kg of water would be necessary. For the movements of the tanks a robust tower was constructed. The tanks are suspended by means of ball-bearing nuts on three spindles with left-and right-hand threads, so that they can be moved vertically in opposite directions. The spindles are coupled by a gearbox and driven by a synchronous motor.

The balance is a modified version of a commercially available Mettler–Toledo balance (AT1006 comparator). This balance was constructed for a high-precision comparison of 1 kg standard masses. The resolution of the balance used in this experiment was increased from 1 μg to 0.1 μg. It is a single-pan, flexure-strip balance with a servo-controlled beam. It works on the substitution principle, which means that the masses to be compared are placed one after the other on the same pan. The exchange of the test masses is a critical part of the experiment because the reproducibility of the weighing depends strongly on the performance of the exchange mechanism [6, 7].

In our experiment a reproducibility of 300 ng is achieved. The reproducibility is the standard deviation of ten weight comparisons, each taking about 11 min. We found no difference in the reproducibility of the balance correlated to day–night, week–weekend and vacuum pumps on–off measurements.

For the calibration a 1 g standard mass is placed on the balance in addition to a test mass. The statistical uncertainty of the calibration is 0.5×10^{-6}. The calibration has been tested with two 0.5 g standard masses and no discrepancy was found. To convert the balance reading into force, the local gravity at the balance, $g = 9.807\,233\,5(6)$ ms^{-2} [8] is used.

4. Measurements and results

The whole measuring procedure is fully automated and computer controlled. First, the weight difference of the two test masses is determined while the field masses are in position I. The test masses are alternately connected to the balance and the readings for each test mass are averaged over 120 s. The weight difference is calculated by linear interpolation between two successive measurements. One complete cycle takes 11 min. After approximately 4 h the field masses are moved to the second position, which takes about 5 min. Then the weight difference is determined again. After another 4 h the tanks are moved back to the first position. This procedure is repeated many times. The balance is calibrated in each field mass position. No variation of the calibration factor was found.

As a test, each tank was filled with 500 kg water, this being taken as an important test of the whole experiment. Since we used pure water the temperature-dependent density of water was taken from the literature [9]. The temperature of the water was measured to within an accuracy of ± 0.1 K, resulting in a density uncertainty of 21 ppm. With a measuring time of 20 days the gravitational signal of about 110 μg has been measured with a statistical uncertainty of 9 ng (one standard deviation). To determine G, the gravitational force from the field masses on the test masses

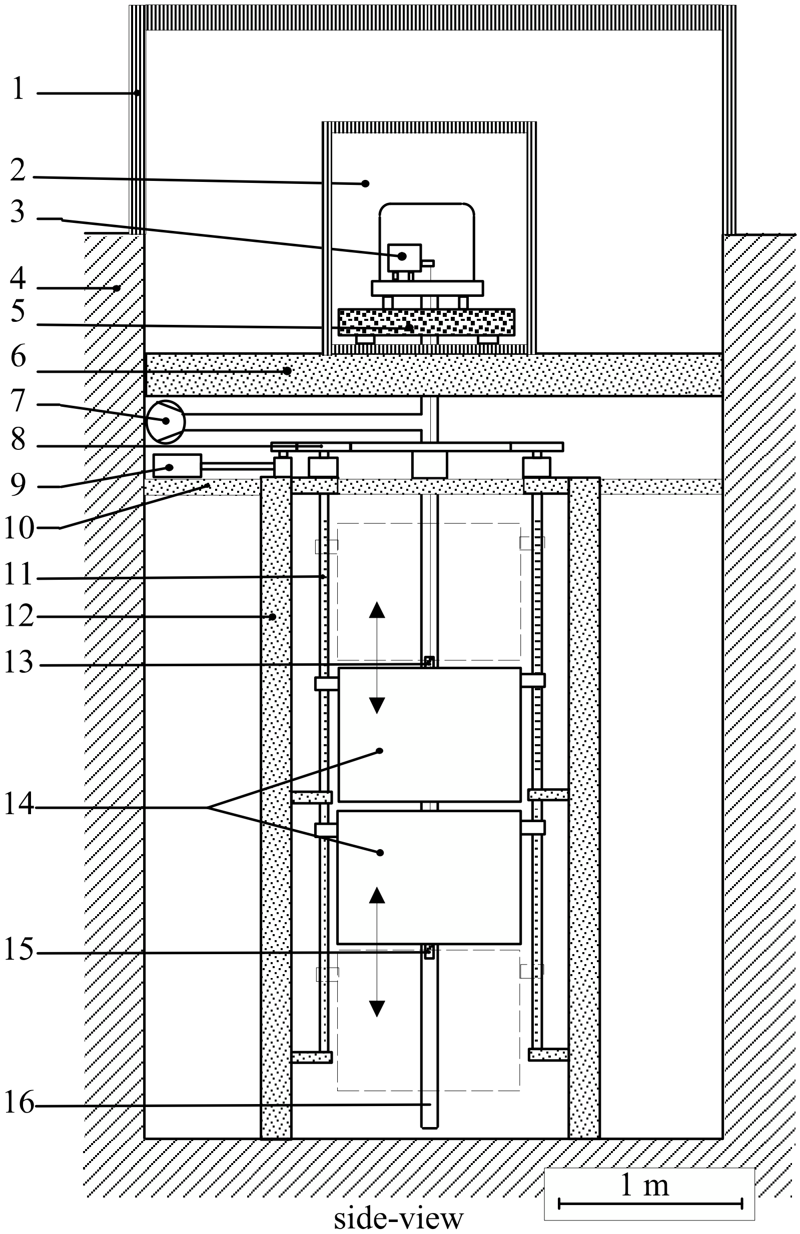

Figure 2. A side view of the experimental set-up: 1, enclosure; 2, thermally insulated chamber; 3, balance inside the vacuum chamber; 4, concrete walls of the pit; 5, granite plate; 6, steel girders supporting the balance; 7, vacuum pumps; 8, gearbox; 9, motor; 10, working platform; 11, spindle; 12, steel pillar; 13, upper test mass; 14, field masses; 15, lower test mass; and 16, vacuum tube.

was computed by numerical integration. The results from measurements made with full and empty tanks were found to be consistent within the statistical uncertainty. The weighted average is

$$G_{water} = (6.6754 \pm 0.0005_{stat} \pm 0.0014_{sys}) \\ \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}.$$

The first error is the statistical uncertainty and the second error is the systematic uncertainty. The compilation of the systematic uncertainties is given in table 1. These measurements were first published in [10] with a relative uncertainty of 240×10^{-6}. Since then, further investigations have been performed. The resulting value for G did not change.

During the winter of 1997/1998 the tanks were filled with mercury. The mercury was purchased in 400 flasks, each with a volume of 2.5 l. The filling procedure was similar to the procedure used for filling the tanks with water [5]. To prevent air bubbles from being locked into the tanks, the tanks were evacuated before the filling. Then, the liquid from one flask was pressed through a pipe into a small intermediate vessel. This vessel served as an airlock and the liquid was transferred via low pressure into the tanks. This was repeated for every flask and the tanks were evacuated again, before the last flask was emptied. In order to determine the mass of the mercury, each flask was weighed before and after the filling. For this purpose, the mass of each flask was balanced against a standard mass. The amount of mercury remaining in the filling system was found in a similar manner. In this way the total mass of mercury in the tanks was determined to be 13 520.635(27) kg [6]. The temperature-dependent density is taken from [11]. Since the values for the volume of the tanks determined from the measurement of dimensions on

Table 1. Systematic uncertainties of the gravitational constant, measured with water-filled and with mercury-filled tanks. The uncertainties are grouped according to the contributions of the test masses, the tanks, the liquid and further effects. They are estimated at a confidence level of 68.3% and the total uncertainty is the root-mean-square value. This table is slightly different from the one published in [5].

	$\Delta G/G$ (10^{-6})	
Source of uncertainty	Water	Mercury
Test-mass position	11	10
Test-mass dimension	2.7	2.0
Density inhomogeneity	$\leqslant 1$	$\leqslant 2.2$
Masses of test masses	0.27	0.27
Joints and spindle drive	16	2.2
Shape and volume	14	2
Dimensions of the tanks	11	1.6
Density inhomogeneity	$\leqslant 5$	$\leqslant 0.06$
Masses of the tanks	4	0.5
Air displaced by the tanks	7	2
Density of the liquid	15	18
Mass of the liquid	8	0.7
Nonlinearity of the balance	$\leqslant 200$	$\leqslant 200$
Systematic variation	—	80
Sorption effect	45	6.4
Integration	$\leqslant 13$	$\leqslant 5$
Tilting effect	$\leqslant 18$	$\leqslant 4$
Calibration	5	8
Local gravity	0.06	0.06
Total	209	217

the measuring table and from the water and the mercury employed are in agreement to within their uncertainties of 40 ppm, the density difference between our mercury and high-purity mercury must be below 40 ppm [6]. Many samples were taken during the filling so that later the density could be measured accurately.

Measurements with mercury-filled tanks have been in progress since the beginning of 1998. Some of the first data for the modulated weight difference are plotted in figure 3. The amplitude of the gravitational signal from February to July is plotted in figure 4. Unfortunately, the data are clearly not consistent. Two groups of data points can be distinguished.

The first group is from the measurements which were made during February 1998, about 3 weeks after the filling of the tanks had been completed. The measured gravitational signal was found to vary significantly. Obviously, it is not possible to draw any conclusions from the four data points. The raw data such as the weight difference, the calibration factor and the temperature also do not exhibit variations which could account for the variation of the gravitational signal.

During March 1998, the vacuum system was vented in order to measure the relative distances of field and test masses, after which the system was evacuated again. The following measurements exhibited no systematic variations and the results are consistent to within the statistical uncertainties.

The relative difference between the two measurements is 160×10^{-6}. Many possible reasons for this difference were discussed and are currently being investigated [6].

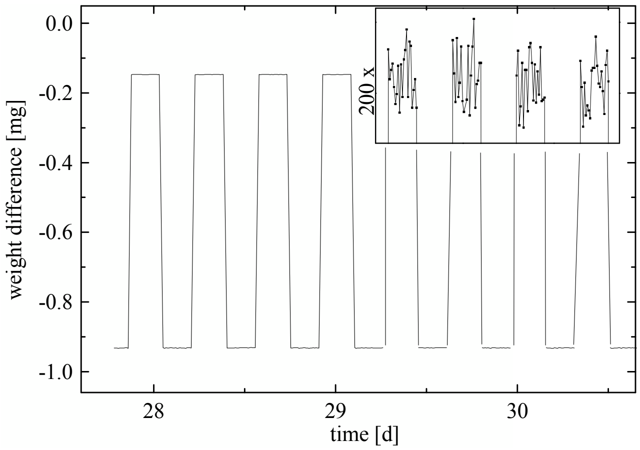

Figure 3. The modulation of the weight difference due to the mercury-filled tanks. The inset shows the weight difference enlarged 200-fold while the field masses are in position I (with the tanks together).

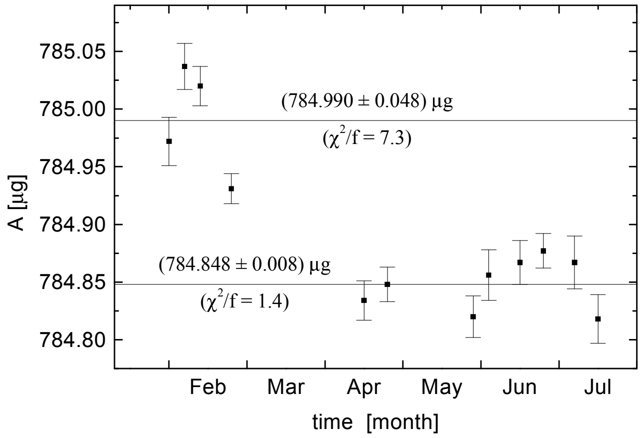

Figure 4. The measured gravitational signal of the mercury-filled tanks. The amplitudes are the average of the measurements during a week. The results can be divided into two measuring periods. The average amplitude of each period and the χ^2 per degree of freedom is given.

Up to now, no satisfactory explanation has been found. Hence, the best thing to do at this moment is to take the arithmetic average of the two measurements and a systematic uncertainty of $\pm 80 \times 10^{-6}$ equal to the discrepancy of the two determinations. Thus, we give a preliminary value for the gravitational constant of

$$G_{mercury} = (6.6749 \pm 0.0014_{sys}) \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}.$$

Since the statistical uncertainty is negligible in this case, only the systematic uncertainty is given, the compilation of which is given in table 1 [6].

The measurements made with water-filled tanks and with mercury-filled tanks are found to be consistent to within their assigned uncertainties. This uncertainty is at present dominated by a possible nonlinearity of the balance. A nonlinearity in the characteristic curve of the balance could influence the measurement since the amplitudes of the gravitational signal (0.8 mg) and the calibration mass (1 g) differ by more than a factor of 1000. So far, only an upper limit of 200 ppm has been estimated, but an experiment to investigate precisely the linearity of the balance is in preparation. It should be mentioned that a possible nonlinearity would effect both results and uncertainties in the same way. This is the reason why at present the measurement made with mercury-filled tanks did not result in a smaller uncertainty than the water measurement. This result, which differs from the presently accepted value by about two of our standard deviations, represents the current state of our experiment. Many improvements and investigations can be made and the prospects for reaching the design accuracy of the experiment of 10×10^{-6} are good.

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