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# A simple Cavendish experimental apparatus

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A simple Cavendish apparatus is described that allows measurement of the gravitational constant G and makes observable the gravitational attraction between commonplace objects. The apparatus consists of a torsion balance constructed from readily available materials, including lead bricks and fishing weights ("sinkers"). A computer program is used to determine the gravitational field at the location of the small mass due to a nearby lead brick, which allows students to gain experience with numerical methods. Experimental results obtained are compatible with the accepted value of G. © 2016 American Association of Physics Teachers.

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#### I. INTRODUCTION

The gravitational constant G is the least accurately known of the fundamental constants of physics. It was estimated by Newton<sup>1</sup> who assumed the surface acceleration due to gravity, the average density, and radius of the Earth. His estimate of the density being between 5 and 6 g cm<sup>-3</sup> corresponds to G = 6 to  $7 \times 10^{-11}$  m<sup>3</sup> kg<sup>-1</sup> s<sup>-2</sup>. The first relatively accurate direct measurement of G was made by Cavendish<sup>2</sup> in 1798, who viewed his measurement as a means to determine the average density. He found an Earth density of 5.448 g cm<sup>-</sup> corresponding to his measurement of  $G = 6.74 \times 10^{-11}$  m<sup>3</sup> kg<sup>-1</sup> s<sup>-2</sup>, which is within 1% of the currently accepted value. For an historical perspective on the measurement of G, see Clotfelder<sup>3</sup> and Falconer.<sup>4</sup>

As recently noted by Speake and Quinn, "Had Cavendish walked into any of the modern torsion balance labs, he would have immediately known what was going on....Cavendish would have been surprised, however, to find that after so many years, measurement accuracy has improved only modestly-not nearly as much as it has for almost every other physical quantity." Further, Speake and Quinn, among others, point out that the values of G recently measured by various groups using a variety of techniques are not consistent with each other and vary by much more than would be expected from the precisions assigned by these groups. For example, Parks and Faller<sup>6</sup> measured  $G = 6.67234(14) \times 10^{-11}$  m<sup>3</sup>  $kg^{-1} s^{-2}$ , while Quinn *et al.*<sup>7,8</sup> measured G = 6.67554(16)

 $\times 10^{-11}$  m<sup>3</sup> kg<sup>-1</sup> s<sup>-2</sup>. These values differ by more than 20 standard deviations. This inconsistency might arise from misunderstandings of the uncertainties, but could also be an indication of new fundamental physics.<sup>5</sup> This situation shows that the measurement of G is not a closed field of physics and future researchers may wish to participate in the resolution of this puzzle.

While gravity is perhaps the most evident of the fundamental forces, attracting commonplace objects toward the center of Earth and on a grander scale keeping planets, stars, and galaxies in their orbits, the attraction of commonplace object to each other is so small that it goes completely unnoticed. To allow students to observe the effect of the gravitational attraction between fishing weights and lead bricks, a simple and inexpensive torsion balance was designed. Measurement of G with this torsion pendulum allows students to appreciate its sensitivity. The apparatus is simple enough that it can be constructed by students, and it offers opportunities for modification and improvement. The design is intended to measure G to within about 10%. (Corrections, which are relatively small on that scale, will be discussed in Sec. V, as will the effect of errors associated with uncertainties of lengths and masses.)

The general approach for the Cavendish experiment is to use a torsion pendulum so that large masses produce a gravitational torque

$$\tau_g = m(g_l + g_r)\ell = mG(k_l + k_r)\ell \tag{1}$$

on the system. This equation gives the torque generated by the attraction of smaller masses, each of mass m, to larger masses, each of mass M. The smaller masses are attached to the ends of two rods, each of length  $\ell$ , that are suspended by a thin wire as part of a torsion pendulum (see Fig. 1). The quantities  $g_l$  and  $g_r$  are, respectively, the gravitational field strengths of the left and right large masses at the locations of the (left and right) small masses. These fields are proportional to G and can thus be written as  $g_l = Gk_l$  and  $g_r = Gk_r$ , which defines the quantities  $k_l$  and  $k_r$ .

If all the masses were spherical, the quantities  $k_l$  and  $k_r$  would be given by  $M/s^2$ , where s is the separation distance between the centers of the small and large masses. For the apparatus described here, we have found that the small masses can be assumed to be spherical while the larger masses—standard  $2 \times 4 \times 8$  in. lead bricks—cannot. The students must therefore carry out a numerical calculation of  $k_l$  and  $k_r$ . (A similar numerical procedure is used to verify the assumption that treating the small masses as spherical does not introduce a significant error.)

In addition to the gravitational torque, there is a restoring torque  $\tau_w = \kappa \theta$  produced by the twist (through angle  $\theta$ ) of the supporting wire. The angle  $\theta$  is determined from the deflection of a laser beam reflected from a mirror attached to the center of the torsion pendulum. In static equilibrium, the gravitational and restoring torques will balance, so

$$G = \frac{\kappa \theta_s}{m(k_l + k_r)\ell},\tag{2}$$

where  $\theta_s$  is the (static) angular displacement from the true equilibrium position (with no lead bricks present). A measurement of G thus requires that we measure all of the quantities on the right-hand side of Eq. (2). While it is reasonably straightforward to measure m and  $\ell$  directly, determining the other quantities is more difficult.

To place the present apparatus and approach in context, we list here a few other apparatuses that are intended for student use.

- Block *et al.* describe an apparatus in which metal pipes are used to enclose the torsion pendulum, and a mirror is attached to one end of the bar holding the small masses.
- Dousse and Rhême<sup>10</sup> describe a Cavendish balance for the undergraduate laboratory that is capable of high accuracy.
- A demonstration apparatus that is permanently mounted to a wall is described by Meissner.

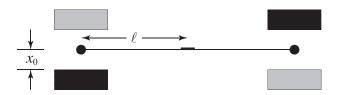


Fig. 1. A schematic diagram of the torsion pendulum as viewed from above. The small masses (circles) are attached to lever arms of length  $\ell$  and a mirror is attached to the center (small rectangle). The lead bricks are shown as large rectangles in their initial position (black) and their final position (gray). The magnitude of the gravitational torque on the pendulum is given by Eq. (1); the torque changes sign when the lead bricks are moved. The (perpendicular) distance from the brick face center to the center of the small mass is given by  $x_0$ , while  $y_0$  and  $z_0$  (not shown) give the horizontal and vertical distances from the brick face center to the small mass. The parameter values are given in Table I.

- Leybold, Inc. produces a commercial Cavendish apparatus for which Purcell<sup>12</sup> has given a positive review.
- Pasco, Inc. produces a commercial Cavendish apparatus that uses tungsten weights.

While all of these apparatuses can be used to adequately measure G, the apparatus described here has multiple advantages: it is simpler, much less expensive, and can be constructed from scratch by students.

The remainder of this paper is organized as follows. The apparatus is described more fully in Sec. II. Section III describes the experiment and explains the procedures used to determine the torsion constant  $\kappa$  and the quantities  $k_l$  and  $k_r$ . The results of the torsion pendulum experiment are provided in Sec. IV, as is our determination of  $\theta_s$  and our final result for G. Lastly, an error analysis is presented in Sec. V.

#### II. APPARATUS

A photograph of the complete experimental apparatus is shown in Fig. 2, and a separate photograph of the arms and mirror of the torsion pendulum is displayed in Fig. 3. The relevant physical parameters are listed in Table I. The torsion balance uses lead fishing weights ("sinkers") for the small masses and lead bricks for the large masses. Lead bricks can be obtained for about \$60 each; the cumulative cost of the remaining parts is well under \$100. The small masses and the swinging part of the balance are contained within a rectangular box with approximate dimensions of  $30\,\mathrm{cm}\times75\,\mathrm{cm}\times3.8\,\mathrm{cm}$ . Sheet aluminum is used for all faces, and pieces of



Fig. 2. A photograph of the apparatus. Two standard size lead bricks sit in opposing positions with respect to the rectangular box. The laser is mounted on top of a smaller lead brick and secured with duct tape. The mirror is visible in the center. The supporting wire is epoxied between two thin pieces of wood shown at the top of the vertical tube. The wood rests on top of a small piece of cardboard into which a slit had been cut for the wire. A third lead brick on the far side of the box and centered on the apparatus provides stability.



Fig. 3. The torsion pendulum with the mirror attached at the pivot. Holes have been drilled transverse to the axis of a cylindrical hub to accommodate the arms, which are hollow aluminum tubes. The mirror is attached to the flat face of the cylindrical hub with epoxy. The sinkers are attached to the arms by thin wire, which is secured to the arms with epoxy. The supporting wire is attached between the mirror and the hub. This wire passes up inside a vertical tube and the entire pendulum is housed in a rectangular box (see Fig. 2). The scale at the top right has a length of one foot (30.5 cm).

wood,  $3.8 \, \mathrm{cm} \times 1.9 \, \mathrm{cm} \times 75 \, \mathrm{cm}$ , are used at the top and bottom of the box to space the aluminum sheets  $3.8 \, \mathrm{cm}$  apart. These sheets are held in place with epoxy, and aluminum tape was used to seal all edges.

A 2.5 cm-diameter hole was drilled in the center of the upper piece of wood into which a 70 cm-long and 2.5 cm-diameter aluminum tube was epoxied. The rods that hold the small masses are made of thin-walled aluminum tubing, available at local hardware stores. These arms are inserted into holes drilled radially into a 1 cm-thick, hollow, cylindrical aluminum hub with outer and inner diameter of 2.5 cm and 1.2 cm, respectively. The holes were drilled in such a way that the two arms are angled as seen in Fig. 3, which prevents the arms from being unstable to rotation in the vertical plane. A 304 V stainless steel wire  $^{13}$  of radius 89  $\mu$ m was sandwiched between the small mirror used to observe the deflection and a flat face of the hub; this assembly was then epoxied together. The wire was brought up through to the top of the vertical aluminum tube and epoxied between two thin pieces of wood, each approximately  $3 \text{ cm} \times 3 \text{ cm}$  in size. The wire passes through a slit cut in a square piece of cardboard, mounted on top of the tube, upon which the thin wood pieces rest. This arrangement allows for manual rotation of the wire from the top.

The wire was electrically connected to the tube, which in turn was connected to the metal box. This grounding, as well as the use of the metal tape, was made to eliminate electric

Table I. Parameters of the apparatus; errors in the rightmost digit are indicated in parentheses.

Parameter	Description	Value
m	Small mass	0.110(1) kg
M	Large mass (brick)	11.9(1) kg
$m_\ell$	Lever arm mass (each)	0.004(1) kg
$\ell$	Distance of <i>m</i> from axis	0.300(5) m
$I_m$	Mom. of inertia of small masses	$0.0198(6) \text{ kg m}^2$
$I_{\ell}$	Mom. of inertia of lever arms	$0.00024 \mathrm{kg} \mathrm{m}^2$
$I_{ m hub}$	Mom. of inertia of hub	$6.03 \times 10^{-7} \text{ kg m}^2$
$l_w$	Length of wire	0.84(1) m
$r_w$	Radius of wire	89(1) μm
$x_0$	Perp. dist. to brick face center (bfc)	0.032(3) m
<i>y</i> <sub>0</sub>	Parallel horizontal distance to bfc	0.000(5) m
<i>z</i> <sub>0</sub>	Vertical distance to bfc	0.013(5) m

fields inside the box, although this may not be necessary. To visually observe the small weights, a  $2\,\mathrm{cm}\times4\,\mathrm{cm}$  opening was cut into one of the  $30\,\mathrm{cm}\times3.8$  cm ends of the box. Meanwhile, a  $2.5\,\mathrm{cm}$ -diameter hole was cut in one face of the box, allowing the laser beam to strike the mirror. The laser beam was reflected from the mirror onto a wall,  $8.5\,\mathrm{m}$  away, where a ruler was placed to measure the position of the reflected light. The laser was angled slightly upward so that as the beam reflects from the mirror, it is in a plane nearly perpendicular to the box face and to the mirror. Both holes were covered with thin transparent plastic to eliminate any motion of air, which could reach the balance and cause significant unwanted motion.

#### III. EXPERIMENT AND DATA ANALYSIS

The two large bricks are placed so that each one gives rise to a gravitational torque of the same sign on one of the small masses (see Figs. 1 and 2). The other orientation has the bricks on the opposite sides of the box so that the torque is in the opposite direction. The wire is rotated at the top of the tube until the small masses are in equilibrium as near as discernible by eye in the middle of the box. This procedure takes patience because the pendulum damping is very small. After equilibration, the laser spot on the wall corresponds to the stable position for this orientation of the bricks. At that point, the lead bricks are quickly moved to the other orientation. The pendulum begins to oscillate and the position of the reflected laser beam is recorded as a function of time.

#### A. Determination of the torsion constant $\kappa$

We determine  $\kappa$  from the (angular) oscillation frequency  $\omega$  of the torsional pendulum via

$$\kappa = \omega^2 I,\tag{3}$$

where I is the moment of inertia of the pendulum. Because the moment of inertia and the oscillation frequency can be determined accurately, this technique is preferred over obtaining  $\kappa$  from the shear modulus of stainless steel and the radius of the wire. (Since  $\kappa$  is proportional to the fourth power of the wire radius, an error of 10% in wire radius produces almost a 50% change in  $\kappa$ , not to mention any uncertainty in the shear modulus itself.)

Our measurement for  $\omega$  was made with the bricks in place, which leads to a small error that arises due to the field variation over the path of the small masses. This error is small because the range of motion is small and the bricks approximate a plane for which there would be no field variation. A cleaner technique would be to determine  $\kappa$  by measuring the oscillation frequency without the bricks in place.

## B. Determination of the gravitationally field strength

Because the amplitude of the motion of the small masses is only a fraction of a millimeter, this motion is ignored in the calculation of the gravitational fields. While it is possible to obtain analytic expressions via direct integration, a simpler approach is to perform the calculation numerically. Thus, to determine the gravitational field components  $g_l$  and  $g_r$ , the brick is divided into small cubical elements, each of edge length a and mass  $\rho a^3$ , where  $\rho$  is the density of lead. If we let  $x_i$ ,  $y_i$ , and  $z_i$  be distances from the ith element of the

brick to the center of the small (assumed spherical) mass, with  $x_i$  being the horizontal distance perpendicular to the box face,  $y_i$  being the horizontal distance parallel to the box face, and  $z_i$  being the vertical distance, then

$$g_l = Gk_l \approx G\rho a^3 \sum_i \frac{x_i}{\left(x_i^2 + y_i^2 + z_i^2\right)^{3/2}}.$$
 (4)

Carrying out these numerical calculations allows us to determine the quantities  $k_l$  and  $k_r$ .

For convenience, the position of the lead brick is specified by the distances  $x_0$ ,  $y_0$ , and  $z_0$  from the center of the small mass to the center face of the lead brick.

#### IV. EXPERIMENTAL RESULTS

When the positions of the lead bricks are reversed (see Fig. 1), the torsion pendulum will begin to oscillate. The position *s* of the laser spot as a function of time undergoes damped oscillations, as shown in Fig. 4. These data were fit to a function of the form

$$s = s_0 + Ae^{-t/T_d}\cos(\omega t + \phi), \tag{5}$$

where  $s_0$  is an arbitrary offset, A is the initial amplitude of the oscillation,  $T_d$  is the time constant for the damping,  $\omega$  is the (angular) oscillation frequency, and  $\phi$  is the initial phase of the oscillator. The exact causes of the damping are presumably air resistance and internal friction in the twisting of the wire. Equation (5) assumes the damping force is proportional (linear) to the angular velocity. Based on the quality of the fit in Fig. 4, it is apparent that the damping is linear over the time range shown. Thus, Eq. (5) provides a reasonable representation of the data.

To fit the data, we used the Levenberg-Marquardt non-linear least squares algorithm described in, for instance, *Numerical Recipes*. <sup>15</sup> The errors for the laser spot position were assumed to be time independent and were set equal to

 $\sqrt{\sum_t \left[s_{\rm ex}(t) - s_{\rm th}(t)\right]^2/\nu}$ , where  $s_{\rm ex}(t)$  is the experimental spot displacement,  $s_{\rm th}(t)$  is the theoretical spot displacement from Eq. (5), and  $\nu$  is the number of degrees of freedom. This method is commonly used for determining self-consistent errors and is what one would expect if the deviations are random and the fitting function is correct

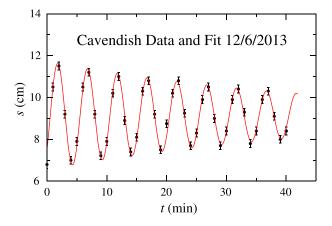


Fig. 4. Measurements of the reflected laser spot position as a function of time after reversing the positions of the lead bricks. The curve is a fit using Eq. (5).

(essentially assuming a reduced chi-squared  $\chi^2_{\nu} = \chi^2/\nu = 1$ ). The position error obtained using this procedure was 0.18 cm, which was used for the error bars in Fig. 4. This error seems reasonable given that the spot size at the wall was approximately 0.7 cm across. The fit to the data using Eq. (5) is shown as the smooth curve in Fig. 4; Table II lists the fit parameters and their errors.

So far, we have directly measured the quantities m and  $\ell$ , and have calculated  $k_l$  and  $k_r$  using a numerical procedure. In order to calculate G from Eq. (2), we still need to determine  $\kappa$  and  $\theta_s$ . Once  $\omega$  is found from fitting Eq. (5) to the experimental data,  $\kappa$  can be determined using Eq. (3). The final piece of the puzzle is thus to find  $\theta_s$ .

To determine  $\theta_s$  we note that when the bricks are in their initial position (see Fig. 1), the system is at rest with the torsion pendulum at a static angle  $\theta_s$  with respect to its true equilibrium position (if there were no bricks at all). When the bricks are moved to their new location, the new equilibrium angle will be  $-\theta_s$  with respect to the true equilibrium position. Thus, the equilibrium angle will have shifted by  $2\theta_s$ , which means the pendulum will (initially) oscillate about the new equilibrium angle with an (angular) amplitude of  $2\theta_s$ . If the laser light shines on a wall that is a distance L from the mirror, the result will be a laser spot that oscillates with an amplitude of  $A = 2L\theta_s$ .

When everything is substituted into Eq. (2) we find

$$G_{\text{exp}} = 6.8 \times 10^{-11} \,\text{N} \,\text{m}^2/\text{kg}^2.$$
 (6)

Compared to the accepted value 16 of

$$G_{\rm acc} = 6.67384 \times 10^{-11} \,\mathrm{N} \,\mathrm{m}^2/\mathrm{kg}^2,$$
 (7)

there is a relative error of  $\delta G/G$  of approximately 1.9%. While this appears to be a reasonably accurate measurement, we have yet to quantify the measurement error.

### V. ERROR ANALYSIS

As described in the introduction, the intent of the design was to provide an easily constructed apparatus that clearly demonstrates the attraction between everyday masses and provides a measurement of Newton's constant G to within about 10%. With this aim, one only had to be modestly careful in making measurements of the various parameters that are used to determine G to ensure that they are unlikely to introduce errors comparable to 10%. There are two kinds of errors that arise: corrections that could have been made and measurement inaccuracies. The latter are assumed to be random. We discuss each of these errors in this section.

Table II. Results of fitting Eq. (5) to the experimental data using the Levenberg-Marquardt algorithm. Error bars on the data were adjusted so as to yield a normalized  $\chi_{\nu}^2$  of approximately one (0.998).

Parameter	Value	Error
<i>s</i> <sub>0</sub>	9.14 cm	0.03 cm
A	2.58 cm	0.10 cm
$T_d$	47.0 min	4.6 min
ω	$1.260{\rm min}^{-1}$	$0.002{\rm min^{-1}}$
$\phi$	4.02 rad	0.04 rad

#### A. Corrections

#### 1. Moment of inertia

The moments of inertia of the small masses, the lever arms, and the hub are listed in Table I. We included only the small masses in the moment of inertia of the pendulum. While the moment of inertia of the hub is negligible, the moment of inertia of the lever arms would add a 1.2% correction to the value of G.

#### 2. Field calculation

For the cell size in the sums used to calculate the fields [see Eq. (4)], we used a = 0.001 m. It was subsequently determined that values of a smaller than 0.0003 m gave identical values for the field. Between a = 0.001 m and a = 0.0003 m, there was a reduction of the field by 0.5%.

## 3. Field due to the second brick

We only included the gravitational field due to the nearer of the two lead bricks. One can calculate the field of the second brick in the same way as for the nearer brick by using  $y_0 = 0.6$  m. The result is a fractional decrease of the torque by 0.25%.

#### 4. Gravitational torques on the lever arms and the hub

Each lever arm has a mass of 4 g. To determine the torque, the lever arms were divided into segments of 1 cm. The gravitational field from the bricks was calculated for each segment and the torque was determined using the accepted value of G. Summing over the segments of the lever arms gave a torque on the lever arms of  $3.9 \times 10^{-11}$  N m. This torque is less than 1% of the torque on the small masses  $(6.459 \times 10^{-9} \text{ N m})$ . The torque on hub is even smaller than the torque on the lever arms.

## 5. Determination of the torsion constant κ

As described in Sec. III A, our measurement of  $\omega$  is made with the bricks in place, but we neglected the variation in the field strength due to the motion of the small masses. The fractional change  $\delta \kappa / \kappa$  is proportional to the change  $\delta g / g$ where g is gravitational field calculated assuming no displacement of the small mass and  $\delta g$  is the field calculated at the static position. The result is a 0.8% underestimate in the value of  $\kappa$ .

# **B.** Measurement errors

The errors for m,  $\ell$ ,  $x_0$ ,  $y_0$ , and  $z_0$  are our best estimates for the actual measurement errors for each quantity. The errors of the remaining parameters come from the fit to the data. To determine how each error affects the value of G, we calculate how G changes as a result of each error, and use this to determine the relative error in G for each error. When these relative errors (summarized in Table III) are added in quadrature, the result is a net relative error of  $\delta G/G = 0.102$ , which meets our design goal of 10% overall error. Taking this relative error into account, our final measurement result is

$$G_{\rm exp} = 6.8(7) \times 10^{-11} \,\mathrm{N} \,\mathrm{m}^2/\mathrm{kg}^2.$$
 (8)

Table III. How parameter errors affect the final measurement.

Parameter	Portion of final error, $\delta G/G$	
$x_0$	0.046	
y <sub>0</sub>	0.0034	
$z_0$	0.00036	
A	0.04	
I	0.013	
ω	0.0016	
$T_d$	0.0006	
m	0.01	
M	0.01	

#### C. Improved precision

It is possible that the measurement precision could be improved. For example, the laser could be detected electronically and sampled over the spot to yield a positional accuracy of perhaps 0.1 mm. The positions of the bricks, if placed with great care, might also be determined to 0.1 mm. The lead fishing weights could be replaced with actual spherical masses and their lever arms measured, again with great care, to 0.1 mm. With these improvements, one might be able to obtain  $\delta G/G \approx 3 \times 10^{-3}$ .

## VI. CONCLUSION

The experimental results presented here (obtained by students) provide a reasonable measurement of the gravitational constant G. Thus, the apparatus could be used for an extended project in a high school or university due to its simplicity and low cost.

#### ACKNOWLEDGMENT

The authors would like to thank Paul Kossler for assistance in the construction of the apparatus.

a)Electronic mail: wjkoss@wm.edu

<sup>&</sup>lt;sup>1</sup>I. Newton, "Principia book III The system of the world Proposition 10, Theorem 10," 1687; see I. B. Cohen, Isaac Newton: The Principia (Univ. California Press, Berkeley (1999), p. 815.

<sup>&</sup>lt;sup>2</sup>H. Cavendish, "Experiments to determine the density of the earth," Philos. Trans. London 88, 469–526 (1798).

<sup>&</sup>lt;sup>3</sup>B. E. Clotfelder, "The Cavendish experiment as Cavendish knew it," Am. J. Phys. 55, 210–213 (1987).

<sup>&</sup>lt;sup>4</sup>I. Falconer, "Cavendish: the man and the measurement," Meas. Sci. Technol. 10, 470-485 (1999).

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gravitational constant," Phys. Rev. Lett. 105, 110801-1–110801-4 (2010).

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<sup>11</sup> Hans Meissner, "Demonstration of gravitational attraction with the Cavendish balance," Am. J. Phys. 25, 639-640 (1957).

<sup>&</sup>lt;sup>12</sup>E. M. Purcell, "Gravitational torsion balance," Am. J. Phys. 25, 393 (1957).

<sup>&</sup>lt;sup>16</sup>Peter J. Mohr, Barry N. Taylor, and David B. Newell, "CODATA recommended values of the fundamental physical constants: 2010," Rev. Mod. Phys. 84, 1527–1605 (2012).



# **String Driver**

This is the simplest possible driver for the experiment on standing waves on a string. The string, whose mass per unit length has been measured, is attached to one end of the vibrating steel blade, and the other end runs over a pulley and down to a mass-hanger. The electromagnet runs on standard 115 V, 60 cycle alternating current, so the frequency is known. Masses are carefully added to the hanger until a standing wave is produced. A graph of the square of the wavelength vs the tension is a straight line, and from this the speed of sound in the string may be obtained. The wavelength is measured directly, and the speed of the signal in the string can then be calculated. This device, in the Greenslade Collection, was sold by Cenco for \$12.00 in 1940. (Notes and picture by Thomas B. Greenslade, Jr., Kenyon College)

<sup>&</sup>lt;sup>13</sup>The wire cost ~\$8 and was obtained from Component Supply Company, 1220 New Hope Road, Fort Meade, FL 33841.

<sup>&</sup>lt;sup>14</sup>The laser cost ∼\$7 and was purchased from Parts Express, 725 Pleasant Valley Dr., Springboro, OH 45066 (model DC-LP11112, part number 073-082). The laser is powered by an Enercell voltage source, but any source of 5 V will do.

<sup>&</sup>lt;sup>15</sup>William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, *Numerical Recipes*, 3rd ed. (Cambridge U.P., New York, NY, 2007), pp. 773–839.