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Musings on the use of a bifilar suspension in the measurement of *G*

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Abstract. Torsion balances and torsion pendulums have been associated with the measurement of the Newtonian gravitational constant from Cavendish through Eötvös and Heyl. The usual suspension consists of a single fine fibre supporting the small test mass, ordinarily a dumbbell of some type. Problems with this system are discussed and some of the advantages of using a bifilar suspension are posited.

Keywords: gravitational constant, bifilar suspension

1. Introduction

Big G (the Newtonian gravitational constant) is small. Little g (the acceleration due to gravity) is big. Measurements of G are performed in the presence of disturbing influences of g. The great attraction of using torsion balances and pendulums [1–3] is that they may be employed in such a way that the forces of g and G are orthogonal. This attraction should not be an overwhelming factor, since no measurement of G has ever lived up to its initial billing.

2. Use of a bifilar suspension

In a time-of-swing measurement of G, a small mass system or bob, having a large gravitational quadrupole moment relative to its mass is supported by a fine fibre suspension system. The frequency of the oscillation squared is $\omega^2 = \kappa/I$ where κ (the torsion constant) is the second derivative of the potential with respect to the angle and I is the moment of inertia of the small oscillating mass (bob). κ may be composed of several components: κ_f (the torsion constant of the supporting fibre resulting from the shear modulus of the fibre material); κ_G (the torsion constant produced by the attraction of some very well defined large masses placed near the small system). This is the quantity, linear in G, which is used in order to determine G. The frequency of the oscillation of the bob is measured, the large masses are moved and this frequency is again measured. The measured frequencies are squared and the difference taken; if κ_f is constant during the time of the measurement, κ_f may be eliminated.

$$\omega_1^2 = (\kappa_f + \kappa_{G_1})/I \qquad \omega_2^2 = (\kappa_f + \kappa_{G_2})/I.$$

Thus $I\Delta\omega^2$ is some linear function of κ_G and $G = \kappa_G$/(functions of the masses' size and positions).

This process seems simple and straightforward. Fibres with κ as small as 10^{-4} or 10^{-5} dyne cm rad^{-1} can be obtained. Angles as small as 10^{-6} rad can be measured, so we can measure forces as small as 10^{-11} dynes (10^{-16} N) but not without a price. In order to maximize the sensitivity, the fibre is usually loaded to about half its breaking strength. It has been found that the ambient position of the bob, even after several months under load, is not stable. The position of the bob drifted something in excess of 10^{-4} rad per day with a time constant of weeks or months, as previously observed by Roll *et al* [4], probably due to the continuous loading and unloading of the fibre by microseisms.

We [5] have observed drift in the torsion constant of as much as 10^{-3} per day, typically 10^{-4}, which is in line with the observations of several previous experimenters (see Heyl and Chrzanowski [3] for example) (figure 1). This is probably temperature related; however, cooled (liquid nitrogen) fibres apparently do not suffer as badly from this problem [6]. The signal-to-noise ratio in a measurement of this type is usually proportional to the amplitude of the oscillation in the limit of small oscillations. However, the noise inherent in an amplitude of a few 10^{-2} rad (limited by the autocollimator in use) is not expected to be the limit in precision for a measurement with a hoped-for precision of 10^{-5}. In addition to the problems of drift, Kuroda [7] has pointed out that all measurements using the time-of-swing method of determining G should take the shift of the measurement due to the anelastic properties of the fibre into consideration.

Heyl and Chrzanowski in their 1942 paper [3] mention their attempt to use, without success, a bifilar system. A bifilar suspension might, we hope, ameliorate to some extent some of the problems outlined above, i.e.

(1) drift in the torsion constant due to

  (a) microseisms
  (b) temperature variations;

(2) drift in the ambient position;
(3) reduction of metrology problems in a small mass system.

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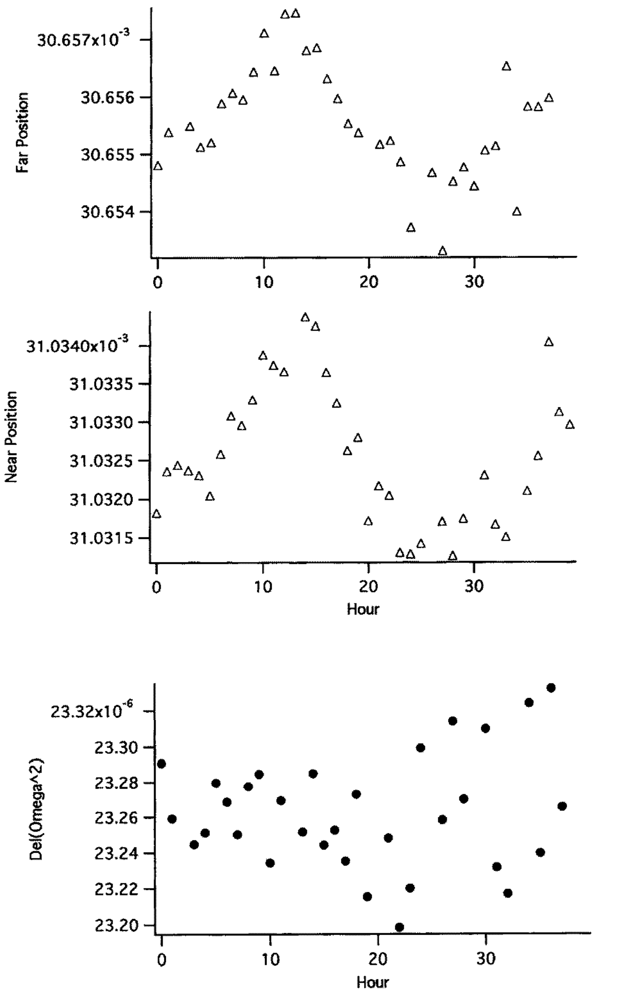

Figure 1. The upper two graphs represent a measurement of the frequency of oscillation of the torsion pendulum with large masses at two different positions with the measurements interleaved. The lower graph represents the difference of these squared frequencies. The drift in the frequencies and lack of drift in the differences indicate perhaps that the apparent drift is in the torsion constant of the suspension fibre.

Using a bifilar suspension, we now have an added torsion constant κ_g:

$$\omega^2 = (\kappa_f + \kappa_G + \kappa_g)/I.$$

This torsion constant κ_g of a bifilar suspension is created by the rising and falling of the bob as the bob oscillates and is linear in m (the supported mass) and g. The constancy of g over time is greater than we can expect for the constancy in κ_f at room temperature, however well the temperature is controlled. If we choose a large κ_g (compared to κ_f) the twist in the fibre (caused by temperature changes or by the aforementioned microseisms) is still present but the drift in the position of the bob is now reduced by the ratio κ_f/κ_g. As the bob oscillates through an angle θ, assuming the lengths of the fibres stay constant, the bob must necessarily rise and fall. This change in height h is quadratic in θ for small values of θ and proportional to the separation of the fibres at the top, $2r_t$, and the separation at the bottom, $2r_b$, and inversely proportional to the length of the fibres, l. It can easily be shown that this change in height, h is

$$h = \frac{r_t r_b \theta^2}{2l}.$$

The potential due to this change in height is

$$U = mgh = \frac{mgr_t r_b \theta^2}{2l}.$$

The torque, when the system rotates through some small angle θ, is

$$T = \frac{\partial U}{\partial \theta} = \frac{mgr_t r_b \theta}{l}$$

The torsion constant, the partial derivative of the torque with respect to θ is therefore

$$\kappa_g = (r_t r_b/l)mg.$$

This leads to the question, how should the spacing of the two fibres r_t and r_b and therefore κ_g be chosen? In order to make an intelligent choice of κ_g, we need to know the size of κ_G. In turn, what can we expect for the torsion constant due to G?

As an example let us take the values, sizes and quantities now available to me. The large masses that I am currently using are the masses that Jesse Beams used and which were loaned to us at the National Bureau of Standards (Gaithersburg), Towler and myself [8] used these masses in the 1982 redetermination of G and they are presently on loan to me at Los Alamos National Laboratory (LANL). They are 10 cm in diameter and 10.5 kg in mass. They are mounted on a 30 cm cervit table leaving at most 7 or 8 cm for the radius of the small mass system (bob). The torsion constant, the first derivative of the torque on the small mass system due to the presence of the large masses, is to a first approximation (a few per cent for these values)

$$\kappa_G = GMm(r_m^2/r_M^3)$$

where r_m is the radius of the small mass system and r_M is the radius of the large mass. κ_G is therefore of the order of a few 10^{-3} dyne cm rad^{-1}.

Remembering how G is measured:

$$\omega^2 = (\kappa_f + \kappa_g + \kappa_G)/I$$

we want κ_g to be large compared with κ_f but not so large as to overwhelm κ_G (the quantity we really want to measure). Fifty times κ_G would seem to be the maximum allowed value of κ_g (something of the order of 0.1 dyne cm rad^{-1}), not so small a value that it takes an eternity to measure ω. If, for instance, we choose the separation of the fibres, r, at the top and bottom to be the same ($m = 10$ gm, $l = 50$ cm), r turns out to be about $1/30$ cm. For a change in angle θ, the change in height of the small mass system is $(r_t r_b/2l)\theta^2$. This is astounding in that, for the dimensions appropriate for this experiment, the rise and fall of the bob is of the order of ångströms. How can the length of a tungsten fibre 8 microns in diameter and 0.5 m long be defined to within a few ångströms? However, in the test devices we have built, the measured and calculated κ_g values agreed to about 10% (very close considering my machining tolerances). If $\kappa_g + \kappa_f$ remains constant during the period of the measuring cycle, it is not necessary to know them exactly since they will have cancelled out. This is one reason for choosing κ_g to be as large as possible.

So, can we cancel out the temperature variation of κ_g? Again, $\kappa_g = (r_t r_b/l)$ times {things which have no temperature components}. If we want the variation in κ_g as a function of temperature to be zero, the sum of the coefficients of thermal expansion of the structures which define the rs must equal the coefficient of thermal expansion of the material of the supporting fibre. In the present work, the supporting fibre is tungsten with a coefficient of $4 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$. The spacing of the fibres at the bottom, $2r_b$, is defined by clamping the fibres between two molybdenum plates with a coefficient of $5 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$. Molybdenum is used because it is hard, so as to constrain the position of the fibres consistently, and has a very low magnetic susceptibility. The top defining structure must therefore have a coefficient of $-1 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$. Sapphire has a coefficient of $8 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$. This, along with molybdenum can be combined to have a coefficient of $-1 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$ as described later. This presumes that the changes in temperature in the various parts of the apparatus occur concurrently (which may or may not be true).

The problems in designing this apparatus are as follows.

(1) Keeping the tension in the two fibres the same.
(2) Providing a kinematic orientation of the top of the bifilar system without obviating (#1).
(3) Defining the separation of the fibres at the top with a temperature coefficient of $-1 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$ without destroying the properties (1) and (2).
(4) Providing a high-Q attachment at the bottom which maintains the separation of the fibres.
(5) Designing a support for the small masses which, while having negligible mass and moment of inertia, still maintains the separation of the masses to the accuracy and precision needed.

A discussion of these problems now follows.

(1) With regard to the tension of the fibres, folding a single fibre (rather than using two distinct fibres) is the current solution to this problem. The fibre is run over a small ball bearing (approximately 4 mm in diameter). The measured torque necessary to rotate the bearing is less than 100 dyne cm; how much less I do not know. Since the tension in the fibre is of the order of 10 000 dynes, the tension on the two halves of the fibre above the first two rods will be the same to better than 1%, perhaps very much less.

(2,3) After running the fibre over the ball bearing, the fibre is constrained by running it over three sapphire rods 600 microns in diameter set at a small angle (about 10°). The small angle is an attempt to reduce the force and thereby the friction between the rods and the fibre, reducing wear and keeping the tension in the two halves equal (figure 2). The first two rods, which define the spacing of the fibres, are mounted on a molybdenum plate. The fibres are then run over another sapphire rod and this defines the orientation direction of the system, somewhat like the nut or bridge of a guitar. Sapphire is chosen because it is hard (Mohs number of 9), smooth (drawn from a melt) and rods of several sizes are available (hard so that the tungsten does not eat into it (thus maintaining a constant separation between the fibres) and smooth so that it allows the fibre to come to its equilibrium position and to keep the tension in the fibres the same). As mentioned above, the two sapphire rods which define the separation of the fibres at the top of the suspension are positioned by mounting them in a groove in a molybdenum plate. Given that the coefficient of thermal expansion of sapphire is $8 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$ and that of molybdenum is $5 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$ and if the groove in the molybdenum plate is machined so that the separation between the sapphire rods is equal to the diameter of one of the rods (i.e. the groove is three times the diameter of one of the rods which we take as unit length), the expansion of the groove in the molybdenum plate for a unit change in temperature will be 15×10^{-6} units of length and the sapphire will expand so as to close the gap by 16×10^{-6} units of length, i.e. more than offsetting the increase in the gap due to the expansion of the molybdenum. This excess expansion of the sapphire will be just enough to produce an effective coefficient of thermal expansion of $-1\,^\circ\mathrm{C}^{-1}$ (figure 3).

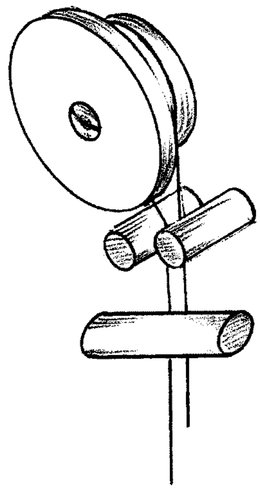

Figure 2. Schematic diagram of the upper support, showing the passage of the folded bifilar fibre over the ball bearing through the two sapphire rods which define the top spacing and over the single sapphire rod which defines the effective length and orientation of the bifilar suspension.

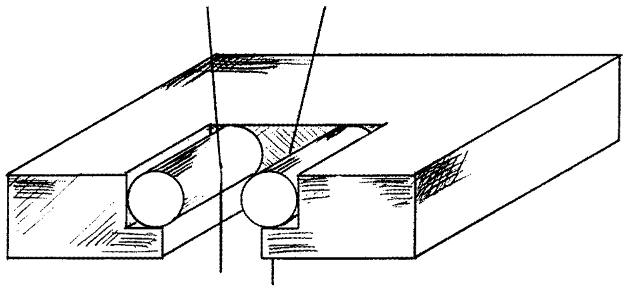

Figure 3. The molybdenum plate and nested sapphire rods which determine the separation of the fibres at the top. The coefficient of thermal expansion of the molybdenum and that of sapphire combine to produce an effective coefficient of thermal expansion of $-1 \times 10^{-6}\,^\circ\mathrm{C}^{-1}$.

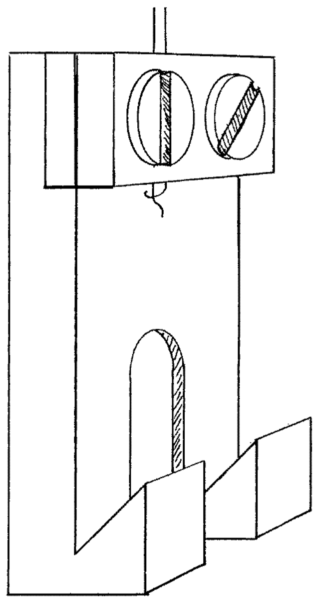

Figure 4. The bottom attachment of the two fibres consists of clamping between a pair of molybdenum plates 4.5 mm wide, 3 mm long and 0.75 mm thick.

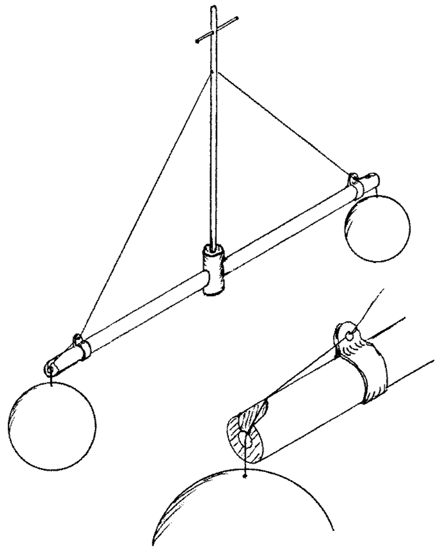

Figure 5. The current configuration of the small mass system showing details of the ends of the quartz tubing spacer.

(4) The fibres are constrained at the bottom by clamping between molybdenum plates a few millimetres square and 0.75 mm thick. The Q of the bottom constraint (figure 4) and of the system in its entirety has not yet been determined.

(5) The current configuration of the small mass system is shown in figure 5.

The use of a bifilar suspension also helps to reduce the metrology requirements of the small mass system. In this small mass system, the masses of the supporting members are for this approximation very much less than m and are therefore not taken into consideration here. Again, $\omega^2 = (\kappa_f + \kappa_g + \kappa_G)/mr_s^2$ where r_s is the radius of the small mass system and, since κ_g and κ_G are large compared to κ_f, one may neglect κ_f and therefore $\omega^2 = (\kappa_g + \kappa_G)/mr_s^2$. Now, both the numerator and the denominator on the right-hand side contain m linearly; therefore to a first approximation, the precision necessary in the determination of m is considerably reduced. Of course, when finally determining G, all masses and moments of inertia are considered. While the actual value of the mass of the small masses is now relatively unimportant, their separation is not. The small masses are suspended by fine (12 micron) tungsten fibres. The advantage of mounting the small masses in this way is that the centre of mass is necessarily beneath the ends of the defining rod (plus some correction for the diameter of the fibres), the length of the rod (quartz tubing 2.5 mm in diameter quartz having a very low coefficient of thermal expansion) having been determined beforehand. With increased bracing, a considerable decrease in the extraneous moment of inertia may be accomplished by further reducing the diameter of the quartz supporting arm. Several current attempts to improve the stability of the suspension system in the measurement of G include the works by Quinn *et al* [9], Newman and Bantel [10] and others. A bifilar suspension, while under consideration for at least half a century, has never been implemented. Here we go.

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