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The application of laser metrology and resonant optical cavity techniques to the measurement of *G*

Wei-Tou Ni†, Der-Kuang Liu†‡, Tai-Tung Liu†, Hsien-Hao Mei†, Sheau-shi Pan§, Chin-Poh Pang† and Hsien-Chi Yeh§

† Centre for Gravitation and Cosmology, Department of Physics, National Tsing Hua University, Hsinchu, Taiwan 300, Republic of China
‡ Institute of Physics, National Chiao Tung University, Hsinchu, Taiwan 300, Republic of China
§ Centre for Measurement Standards, Industrial Technology Research Institute, Hsinchu, Taiwan 300, Republic of China

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Abstract. Our goal is to measure the gravitational constant G to within 1–10 ppm using a high-finesse Fabry–Pérot optical cavity. The two high-reflectivity mirrors of the Fabry–Pérot cavity will be suspended separately by two nearly identical simple pendulums of length about 1 m. For the source of attraction, we have built a rotating mass source with 12 lead bricks (each brick is of mass 11.5 kg) on each of two opposite sides of a rotating table. The period of rotation can be adjusted. We normally use a rotation period of 200 s. For this rotating mass, the relative equilibrium displacement of the two mirrors is over 10 nm and we are working for a preliminary measurement of this distance to one part in 10^4. Auxiliary quantities such as the positions of the test mass and the positions of Fabry–Pérot cavity mirrors can be monitored using real-time laser metrology. The stability and real-time monitoring of the pendulums, building stability, and the compensation or averaging of mass inhomogeneity need special considerations.

Keywords: gravitational constant, Fabry–Pérot interferometer, laser metrology, fundamental constant, fundamental physics

1. Introduction

As part of the world wide effort to improve the accuracy of our knowledge of the gravitational constant G [1, 2] we are applying optical techniques developed for other fundamental physics experiments that enable us to detect and measure very small displacements of suspended mirrors. We have devised a prototype experiment to measure G, similar to that described by Walesch *et al* [3], who used a suspended microwave resonant cavity subject to the gravitational influence of a translational moving mass to measure G with 120 ppm uncertainty. Encouraged by their results, we adapted our optical set-up used for vacuum birefringence measurements [4] to a method of measuring G using an optical resonant cavity [5]. This new method has the potential of measuring the gravitational constant G to 1 ppm or better. We plan to use a high-finesse Fabry–Pérot optical cavity, the two high-reflectivity mirrors of which will be separately suspended by two nearly identical pendulums. Our first intention was to use X-pendulums with a period longer than 5 s to gain sensitivity. However, it was difficult to stabilize the periods of such X-pendulums so we started with simple pendulum suspensions of length 1 m (pendulum period 2 s). The optical resolution will still be better than 1 ppm.

For the source of attraction, we have built a rotating gravitational source with 12 lead bricks (each brick is of mass 11.5 kg) on each of two opposite sides of a rotating table, the period of rotation of which can be adjusted. We typically use a rotation period of 200–300 s. For this rotating mass, the relative equilibrium displacement of the two mirrors is over 10 nm.

In section 2 we discuss the basic principles of the measurement scheme and describe the set-up of the rotational gravitational source. Section 3 describes the laser metrology set-up for measuring the relative displacement of the two pendulums. Section 4 presents the beat frequency measurement scheme for the suspended Fabry–Pérot cavity and discusses strategies. Section 5 presents a brief outlook.

2. Basic principle of the measurement

The basic principle of the measurement is the same as that used in the Wuppertal experiment [3] and is shown in figure 1. As the mass M moves back and forth or rotates,

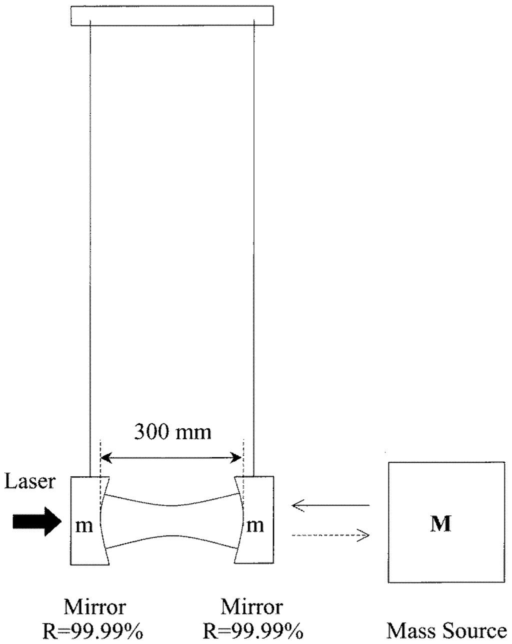

Figure 1. The basic principle of our measurement.

the equilibrium positions of the two mirrors change. This relative position change can be measured by laser metrology (section 3) or by the optical resonant-frequency change of the Fabry–Pérot cavity (section 4).

We have constructed a preliminary rotating source mass made up of two composite masses of 138 kg each at the end of a rectangular 80 cm long steel table. The table rotates about its centre, thus providing an alternating gravitational signal to the suspended test masses hung at the end of the pendulums. The period of rotation can be adjusted to be between 20 and 500 s. This test mass assembly is intended for use in preliminary tests of our method and will be replaced by a fully engineered version in due course.

3. Detection of relative displacement using laser metrology

Over the past few years we have developed one-dimensional, two-dimensional two-axis and two-dimensional three-axis subnanometre laser metrology and real-time motion control systems for various purposes [6–8]. Using a fast data acquisition and processing system, we can perform mid-point cyclic averaging in real-time to minimize nonlinearity errors in order to control the motion of an object to subnanometre or picometre precision. With a VMEbus-based data acquisition and processing system, the residual nonlinearity reached 1.5 pm rms using offline fourth-order cyclic averaging. We have built a three-axis (X, Y, θ_z) laser metrology and real-time motion control system with an rms X-axis positioning error of 0.6 nm, rms Y-axis positioning error of 0.06 nm and rms θ_z-axis positioning error of 4×10^{-9} rad for a 12 μm movement. The results of real-time control for 1 nm steps and for circular motions are shown in figure 2. We are currently extending our work to a complete six-axis motion system and are implementing a two-stage scheme for one-dimensional motion over a 5 mm range with subnanometre accuracy.

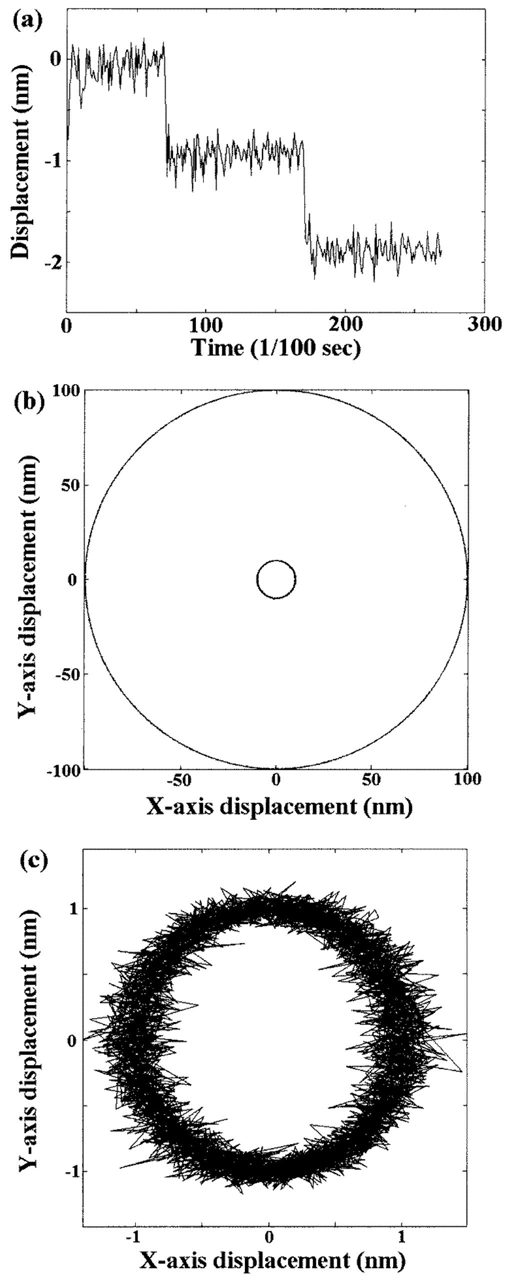

Figure 2. Examples of real-time laser metrology and motion control: (a) one nanometre steps; (b) circular motions of radii 100 nm and 10 nm; (c) circular motion of radius 1 nm.

In our development of laser metrology, measurement accuracy reaches 1 pm. For a relative displacement of 10 nm, the fractional displacement measurement could reach 10^{-4}; this is comparable to the fractional uncertainty of the CODATA G value. Since laser displacement measurement using a heterodyne interferometer is easier to implement than a Fabry–Pérot resonant cavity measurement and a fractional accuracy of 10^{-4} is good to study the improvement of systematic errors and environmental influences, we began with this method of detection. The detection scheme is shown in figure 3. The relative displacement of the two suspended mirrors is obtained by adding the displacement signals of two heterodyne interferometers. Preliminary tests have been made and data acquisition is underway.

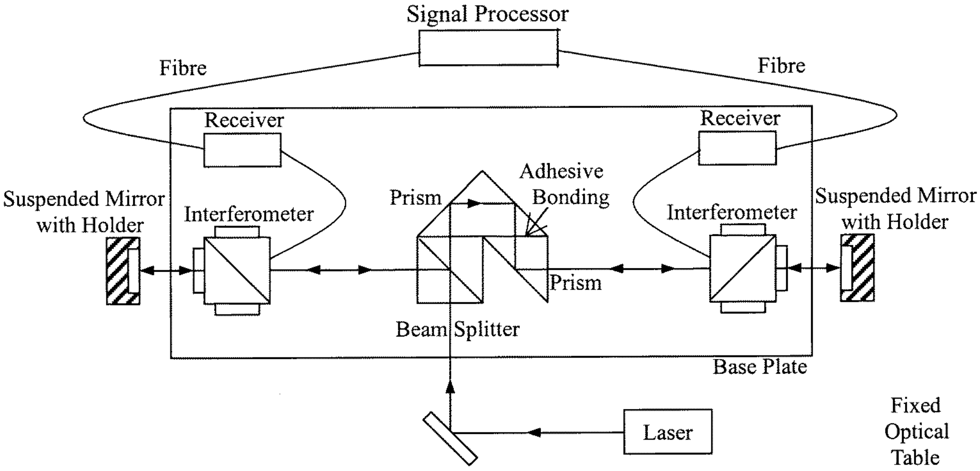

Figure 3. Schematic diagram of the laser metrology set-up.

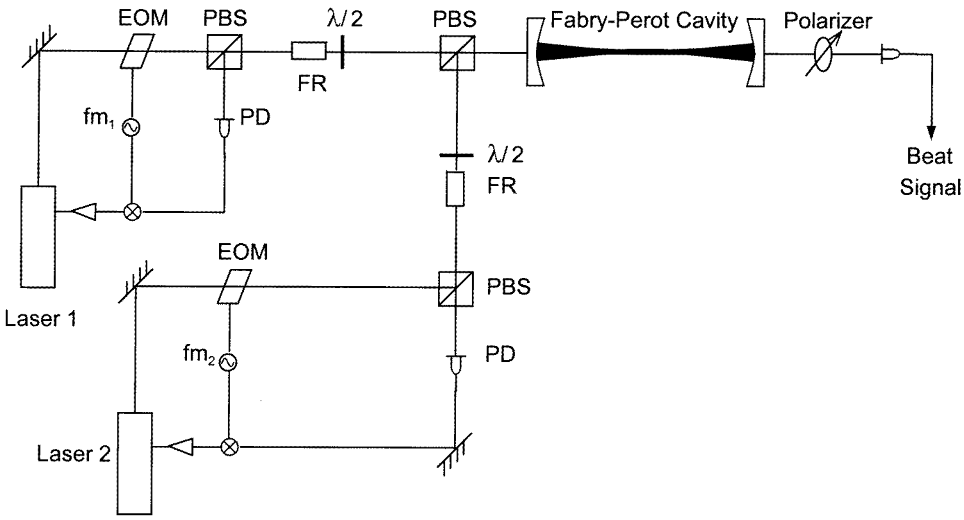

Figure 4. Schematic diagram of the Fabry–Pérot cavity beat measurement.

4. Fabry–Pérot cavity beat measurement

Our heterodyne interferometry scheme [4, 5] is shown in figure 4. Two stable diode-pumped frequency-doubled Nd:YAG lasers are independently locked to adjacent axial modes of the Fabry–Pérot interferometer, and the beat frequency between them is measured. Each laser beam is phase modulated at an RF frequency by a phase electro-optical modulator (EOM) and goes through a polarizing beam splitter (PBS), a Faraday rotator (FR) and $\lambda/2$ waveplate. The two laser beams are then combined by another PBS and mode matched to the high-finesse Fabry–Pérot interferometer. The reflected beams from the Fabry–Pérot cavity are used to lock both lasers independently to adjacent axial modes of the Fabry–Pérot interferometer using a Pound–Drever locking scheme. The beat signal between the two lasers is detected using the transmitted beam by an array of fast photodiodes with a linear polarizer in front. This arrangement reaches the shot-noise limit for locking and maximizes the signal-to-noise ratio (SNR) for the beat signal simultaneously.

For a cavity of length 300 mm, the frequency difference of two neighbouring longitudinal modes is about 500 MHz. For a cavity finesse of 30 000 and a laser of wavelength 532 nm, the cavity linewidth is 8.87 pm. If we resolve this linewidth to one part in 10 000, then the displacement resolution is 887 am (0.887 fm) and a 10 nm displacement can be resolved to 0.1 ppm. When a displacement of 887 am is converted into a frequency difference measurement, it corresponds to 1.48 μHz. The merit of this method is thus to convert the measurement of the gravitational force into a highly accurate frequency measurement. Auxiliary quantities, such as the positions of the test masses and the positions of the Fabry–Pérot cavity mirrors, can be monitored using real-time laser metrology. The stability and real-time monitoring of the pendulums, building stability, and the compensation or averaging of mass inhomogeneity need special consideration. Flexure pendulum suspensions may need to be considered.

At present we use a vacuum can designed as an end tank for our 3.5 m/7 m prototype interferometer for vacuum birefringence measurements [4]. In future G measurements we will make a dedicated vacuum tank of smaller diameter to house the rotatable Fabry–Pérot detector. The rotatable Fabry–Pérot detector will be placed on the Huber Model 440 Goniometer which we presently use to test for spatial anisotropy for polarized electrons [9]. For the source configuration, we need a system such that the gravitational effect on the detector is rather insensitive to small movements of the source position, just as R Newman, J H Gundlach, H Parks and J E Faller talked about in their presentations. In this way, the position metrology for the source configuration is less demanding. The homogeneity requirement is also less stringent. Instead of a rotating source on one side of the tank, we have been studying optimization of a rotating source surrounding the vacuum tank of the Fabry–Pérot detector. This will enable us to study the gravitational SNR at the double modulated frequency. In the study of noises for interferometric detectors, the seismic gravity-gradient noise has been analysed by Saulson [10], Hughes and Thorne [11], and Beccaria *et al* [12]. From their studies we conclude that gravitational noise around 10 Hz or above will not hinder the accuracy of our projected goal. However, gravitational noise around 1 Hz or below will need to be analysed and studied empirically. This is one of our goals of implementing a G measurement with a laser metrology detection scheme.

5. Outlook

In this Cavendish bi-centenary conference on the gravitational constant, we have listened to descriptions of measurements of G to within 10^{-4} and the progress of measurements of G to within 10^{-5} or 10^{-6}. With enduring effort, measurements of G with 1–10 ppm accuracy will be reached not too long from now.

Acknowledgment

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References

[1] Quinn T 1998 Welcome and introductory comments *The Gravitational Constant: Theory and Experiment 200 Years After Cavendish (London, 23–24 November 1998)*

[2] Gillies G 1999 The gravitational constant: theory and experiment 200 years after Cavendish *The Gravitational Constant: Theory and Experiment 200 Years After Cavendish (London, 23–24 November 1998) Meas. Sci. Technol.* **10** 421 (this issue)

[3] Walesch H, Meyer H, Piel H and Schurr J 1995 The gravitational force at mass separations from 0.6 m to 2.1 m and the precise measurement of *G IEEE Trans. Instrum. Meas.* **44** 491–3

[4] Ni W-T, King S-K, Cheng H-W, Shy J-T, Mio N, Tsubono K and Chui T C P 1996 Test of quantum electrodynamics and search for light scalar/pseudoscalar particles using ultra-high sensitive interferometers *Proc. Seventh Marcel Grossmann Meeting on General Relativity (Stanford, CA, 24–30 July 1994)* (Singapore: World Scientific) pp 1628–30

[5] Ni W-T 1995 A new method to measure the gravitational constant (in Chinese with an English abstract) *Proc. 4th Metrology Conference of the Chinese Metrology Society (Hsinchu, Taiwan, Republic of China, 6 May 1995)* (Hsinchu: CMS) pp 255–9

[6] Ni W-T, Pan S-s, Peng G-S, Shy J-T, Tseng S-M, Tsao S-L, Wang S-E, Wu J, Wu S-A and Yeh H-C 1996 Progress in laboratory R & D for fundamental physics space missions—weak light phase-locking, fibre-linked heterodyne interferometry, fibre delay line and picometre real-time motion control *Class. Quantum Grav.* **13** A311–15

[7] Yeh H-C, Ni W-T and Pan S-S 1999 Subnanometer laser metrology and subnanometer real-time motion control *Preprint* submitted

Yeh H-C 1998 Study in subnanometer laser metrology and subnanometer real-time motion control *PhD Thesis* National Tsing Hua University, Hsinchu

[8] Ni W-T *et al* 1994 A new method to measure silicon lattice spacings and possible new definition of the kilogram: scheme and progress *1994 Conference on Precision Electromagnetic Measurements Digest (Boulder, Colorado, USA, 27 June–1 July 1994)* (Piscataway, NJ: IEEE) pp 341–2

[9] Hou L-S and Ni W-T 1998 Test of spatial anisotropy for polarized electrons using a rotatable torsion balance *CD-ROM Proc. Int. Workshop on Gravitation and Astrophysics (Tokyo, 17–19 November 1997)* ed Kazuaki Kuroda (Tokyo: ICRR)

[10] Saulson P R 1983 Terrestrial gravitational noise on a gravitational wave antenna *Phys. Rev.* D **30** 732

[11] Hughes S A and Thorne K S 1998 Seismic gravity-gradient noise in interferometric gravitational-wave detectors *Phys. Rev.* D **58** 122 001–1-27

[12] Beccaria M *et al* 1998 Relevance of Newtonian seismic noise for the VIRGO interferometer sensitivity *Class. Quantum Grav.* **15** 3339–62