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Nonlinearity of tungsten fiber in the time-of-swing method

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Abstract

A symmetric disc torsion pendulum was designed to study the nonlinearity of a tungsten fiber. The result shows that the uncertainty of G due to the fiber nonlinearity would be less than 1 ppm if the oscillation amplitude of the pendulum was in the order of 0.01 rad.

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Since the first measurement by Cavendish [1], there have been about 300 laboratory measurements of the Newtonian gravitational constant G in recent 200 years [2,3]. However, the 1986 CODATA value for G was 6.67259×10^{-11} $\mathrm{m^3 kg^{-1} s^{-2}}$ with an uncertainty of 128 parts per million (ppm), much large than that of all other fundamental constants [4]. Furthermore, recent determinations of G are wildly different from each other and the 1986 CODATA value [5], which are listed in Table 1. This situation – disagreement far in excess of error estimate – suggests that a new value for G should be recommended. In 1998, the CODATA recommended value for G is 6.673×10^{-11} $\mathrm{m^3 kg^{-1} s^{-2}}$ [6]. It is noted that the new recommended CODATA value for G is essentially the same as the 1986 value but the relative uncertainty placed on this value has been expanded to 1500 ppm. This situation indicates that investigations on all kinds of the systematic errors of different methods are desirable.

In the determination of G, the torsion pendulum is a mainstay instrument, and the time-of-swing method has commonly been used [10,11,14–16]. In this method, source masses modulate the oscillation frequency of a torsion pendulum, and the gravitational constant can be determined by measuring the influence of the source masses on the pendulum's oscillation period at different configurations. The time-of-swing method has the great advantage of requiring no displacement measurement or force calibration. But as with all the other methods, it is also subject to some systematic errors associated with the dynamic properties of the pendulum, such as the anelasticity of the torsion fiber and nonlinearity of the oscillation. The anelasticity indicates that the fiber spring constant is dependent on the oscillation frequency. Kuroda pointed out that the time-of-swing method may have a systematic bias due to the changes of the spring constant at two different con-

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Table 1
Recent results of the experimental measurements of G

Source	G value ($\times 10^{-11}$ $m^3kg^{-1}s^{-2}$)	Error estimate (ppm)	Deviation from 1986 CODATA value (ppm)
1986 CODATA value [4]	6.67259	128	0
1998 CODATA value [6]	6.673	1500	+61
Michaelis et al. [7]	6.7154	83	+6420
Fitzgeral et al. [8]	6.6656	95	−1050
Walesh et al. [9]	6.6719	83	−105
Karagioz et al. [10]	6.6729	78	+45
Bagley et al. [11]	6.6740	102	+210
Schurr et al. [12]	6.6754	240	+420
Schwarz et al. [13]	6.6873	1400	+2210
J. Luo et al. [14]	6.6699	105	−405

figurations, and this upward fraction bias should be $1/\pi Q$ [17,18]. Bagley and Luther tested this hypothesis by means of a pendulum with different fibers, which have different Q factors [11]. Newman and Bantel tested the anelastic properties of the torsion fiber undergoing large amplitude oscillation at low temperature and demonstrated that the fractional error introduced by the linear anelasticity of a fiber in the measurement of G is bounded by $0 \leq \delta G/G \leq \frac{1}{2}Q^{-1}$ [19,20]. While the nonlinearity of the oscillation indicates that the period of the pendulum oscillation varies with its amplitude, and the typical equation of the oscillation can be written as,

$$I\ddot{\theta} + \gamma\dot{\theta} + K_1\theta + K_3\theta^3 = 0, \tag{1}$$

where I, θ and γ are the inertial moment, the angle displacement of the pendulum and the damping factor, respectively. The coefficient K_1 is the total equivalent torsion constant of the pendulum system, which usually includes three parts: the torsion constant of the fiber K_{1f}, the gravitational torsion constant of the attracting masses K_{1a} and the torsion constant of the background gravitational field K_{1b} [14]. The coefficient K_3 represents the nonlinearity of the pendulum system, which can also be classified into three parts K_{3f}, K_{3a} and K_{3b}, correspondingly. Using the Harmonic Balance Method [21], we can obtain the approximate solution of Eq. (1) at a small amplitude oscillation as follows,

$$\theta(t) \approx \theta_0 e^{-\beta t}\cos\omega t + \frac{K_3}{32K_1}\theta_0^3 e^{-3\beta t}\cos 3\omega t, \tag{2}$$

where $\beta = \gamma/2I$, θ_0 is the initial amplitude of the oscillation. The frequency ω^2 is shifted from its unperturbed value $\omega_0^2 = K_1/I$ to

$$\omega^2(A) = \omega_0^2 - \beta^2 + \frac{3K_3}{4I}A^2, \tag{3}$$

where $A = \theta_0 e^{-\beta t}$ is the amplitude of the oscillation at time t. Correspondingly, the period of the oscillation can be written as

$$T(A) = T_0\left[1 - \frac{I\beta^2}{K_1} + \frac{3K_3}{4K_1}A^2\right]^{-1/2}, \tag{4}$$

where $T_0 = 2\pi/\omega_0$ is the unperturbed period of the pendulum. In the time- of-swing method, the Newtonian gravitational constant G is determined by

$$G = \frac{I\Delta(\omega_0^2)}{\Delta k_1}, \tag{5}$$

where $\Delta(\omega_0^2)$ is difference of the frequency square measured at two configurations, $\Delta K_1 = G\Delta k_1$ is difference of the equivalent torsion constants at the two cases. From Eqs. (3) and (5), we can obtain the systematic error due to the nonlinearity of a system as follows

$$\frac{\delta G}{G} = \frac{\delta\left[\Delta(\omega_0^2)\right]}{\Delta(\omega_0^2)} = \frac{3A(A\delta K_3 + 2K_3\delta A)}{4I\Delta(\omega_0^2)}. \tag{6}$$

To reduce these effects, we should select small amplitude operation. However, the noise contribution to the period measurement will increase with the amplitude attenuation [22], and it sets a limitation to the precision of the experiment. In Ref. [23], we found that the nonlinear term due to the attracting masses K_{3a} can approach to zero in an optimum design. In this Letter, our research is concentrated on the nonlinear properties of a tungsten fiber, which has been used as the suspension wire in the torsion pendulum experiments [10,11,14,15].

A schematic of the apparatus used in our measurement is shown in Fig. 1. A symmetric disc aluminum torsion pendulum of 57.415 mm in diameter and 9.988 mm in thickness was designed to measure the nonlinearity of a tungsten fiber. The symmetry of the pendulum can eliminate the influence of the background gravitational field on the period of the oscillation, so the first term in Eq. (6) can be neglected. The pendulum is suspended by the fiber and located in a stainless vacuum vessel maintained at vacuum of 1.5×10^{-5} Pa by an ion pump. The thickness and length of the fiber are 25 μm and 513 mm, respectively, which has been suspended for 3 years and has been used in our experiment of determining G [14], and the drift of it was less than 2×10^{-6} rad/day. The height and the initial amplitude of the torsion pendulum can be adjusted by a vacuum feed-through. The apparatus was located in an electromagnetic shield room, which stands on a heavy vibration isolation platform in our cave laboratory [14].

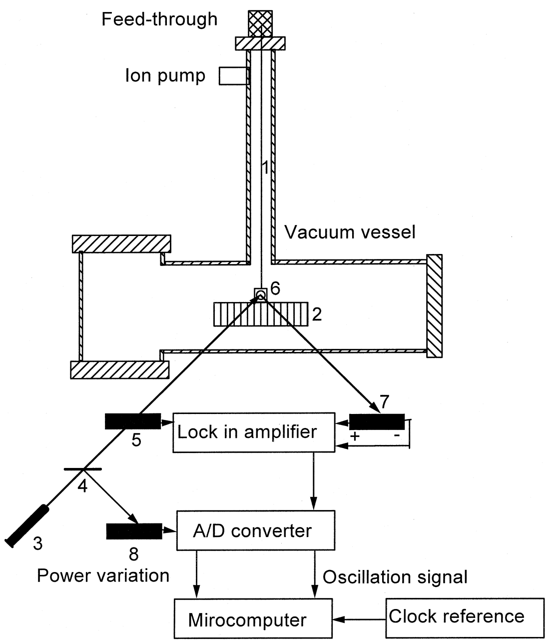

Fig. 1. A schematic of the apparatus used in measurement of of the nonlinearity of a tungsten fiber. Where 1 is the tungsten fiber, 2 is the torsion pendulum, 3 is the laser source, 4 is the beam splitter, 5 is the optical chopper, 6 is the mirror, 7 is the position sensor used to measure the pendulum oscillation and 8 is the photodiode used to monitor the power variation of the laser.

The rotation of the torsion pendulum was detected by an optical lever. The beam from a frequency-stabilizer He–Ne laser 3 ($\Delta\nu/\nu \leq 1 \times 10^{-8}$ /day, $P = 0.6$ mW) was split by a beam splitter 4. About 10% of the beam was reflected to a photodiode 8 (SD-380-23-21-051, made by Silicon Detector Corporation, USA) for monitoring the power variation of the laser. The remainder of the beam went through an optical chopper 5 (SR540, made by Stanford Research Systems Company, USA), and then was reflected by the mirror 6 attached on the center of the pendulum with a reflection angle about 5 degrees. The beam finally fell on a position sensor 7 (SD-1161-21-11-391), which was placed about 450 mm far from the mirror. The difference signal voltage from the position sensor went through a lock-in-amplifier (SR830) and then was converted into a series of time-angle data through a 16-bit A/D converter, and finally recorded by a microcomputer. The output voltage signal from the photodiode is also recorded digitally, and the stability of the laser power was approximately in the level of 0.5% during four days. A square-wave trigger signal of 2 Hz from a time interval counter (SR620, $\Delta\nu/\nu \leq 5 \times 10^{-10}$ /day) was used as the sampling frequency.

Data were taken for each set in the following procedure. The torsion pendulum was started to oscillate by adjusting a rotary vacuum feed-through. The initial amplitude of the oscillation was usually in excess of the effective span of 2.96 cm of the position sensor, and it gradually attenuated to the effective span under the damping of the system. After that, the recording system begun to sample the oscillation signal of the pendulum continuously each half-second until the amplitude attenuated from 0.015 rad to 0.002 rad. This duration lasted about four days, and then we adjusted the vacuum feed-through again for the next set of the experimental data. We obtained five sets of the experimental data from March 24 to April 27 in 1999.

To calibrate the amplitude of the oscillation, the position sensor set (450 ± 1) mm far from the mirror

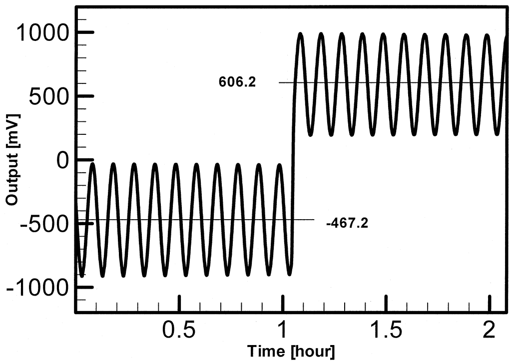

Fig. 2. The result of calibrating experiment. The equilibrium position of the output voltage was changed from -467.2 mV to 606.2 mV after the position sensor was shifted 5000 μm transversely. The amplitude per unit voltage of the pendulum oscillation was 0.518×10^{-2} rad/V.

was shifted (5000 ± 2) μm in transversely by a micrometer during the oscillation of the pendulum. The equilibrium position of the output voltage was changed from (-467.2 ± 0.1) mV to (606.2 ± 0.1) mV as shown in Fig. 2. It means that the amplitude per unit voltage of the pendulum oscillation was 0.518×10^{-2} rad/V. The period of the oscillation of the pendulum was fit by means of the Period Fitting Method [24]. Fig. 3 shows a typical four days record of the oscillation period varying with its amplitude, and each circle represents the mean-value of the periods of five consecutive oscillations of the pendulum. The uncertainty in the fit period increases from 0.15 ms to 0.45 ms with attenuation of the oscillation amplitude from 0.015 rad to 0.002 rad.

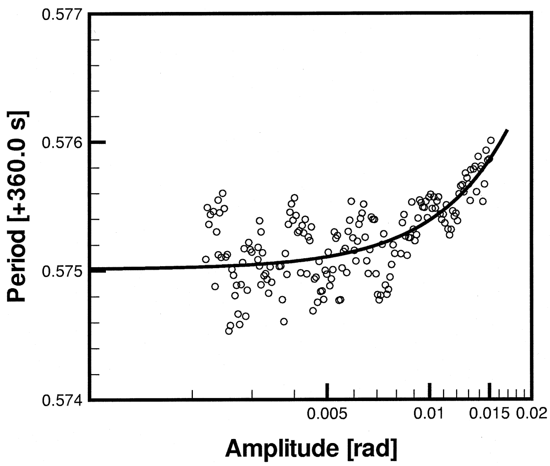

Fig. 3. Typical record of the period versus the amplitude of the pendulum oscillation during April 23 to April 27, 1999. Each circle represents the mean-value of the period of five consecutive oscillations of the pendulum, and the uncertainty in the fit period increases from 0.15 ms to 0.45 ms with attenuation of the oscillation amplitude from 0.015 rad to 0.002 rad. The solid line represents the fitting result with Eq. (4) by means of the least-square method, in which $T_0 = 360.5750$ s and $K_3/K_1 = -0.028$ rad^{-2}.

Table 2
Experimental values of T_0 and K_3/K_1

Set No.	Duration of run (day/month)	T_0 (s)	K_3/K_1 (rad^{-2})
1	19/03-23/03	360.5785	-0.027
2	06/04-10/04	360.5768	-0.015
3	11/04-15/04	360.5760	-0.038
4	17/04-21/04	360.5755	-0.020
5	23/04-27/04	360.5750	-0.028
	average	360.5764 ± 0.0012	-0.026 ± 0.008

We fit the period $T(\theta)$ with Eq. (4) by means of the least-square method. The unperturbed period T_0 and the nonlinear term of the tungsten fiber K_3/K_1 are listed in Table 2 for five sets of the experimental data. From Table 2, we can see that the mean-value of K_3/K_1 is only about -0.026 rad^{-2} with standard uncertainty of ± 0.008 rad^{-2}. For a typical experiment, the difference of the frequency squares $\Delta(\omega_0^2)$ is about 40% times of ω_0^2. If we assume the oscillation amplitude $A = 1.0 \times 10^{-2}$ rad and $\delta A = 1.0 \times 10^{-3}$ rad as well as $|K_3/K_1| = 0.026$ rad^{-2} at the same time, the uncertainty in determination of G due to the nonlinearity of the tungsten fiber would be less than 1 ppm according to Eq. (6). It means that nonlinearity of a tungsten fiber can be reasonably neglected in the small amplitude oscillation.

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