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FUNDAMENTAL PROBLEMS IN METROLOGY

MICROGRAVIMETRIC BASE SURVEY AT THE GRAVIMETRIC STATION OF THE ALL-UNION SCIENTIFIC–RESEARCH INSTITUTE OF METROLOGY

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The results are presented of a microgravimetric survey of two bases of the gravimetric station of the All-Union Scientific–Research Institute of Metrology (VNIIM) using an E-54 variometer. The measurements form part of preparations for performing comparisons of absolute ballistic gravimeters at the observatory.

The presently achieved accuracy of measuring free-fall acceleration is characterized by an error of 2-3 μGal for absolute measurements and an error of hundredths or thousandths of a microgal for relative measurements (with cryogenic gravimeters). At this level of accuracy, the gravitational field turns out to be dependent on many factors: astronomical (lunar—solar tides, motion of the Earth's pole, nonuniformity of the Earth's rotation), meteorological (movement of atmospheric masses, sagging of the surface of the Earth under their action, geophysical (displacement of the inner core of the Earth, a change in the density of the medium before an earthquake), etc. (for more detail, see [1]). A systematic investigation of these factors, necessary in particular in order to maintain the national gravimetric network at the state-of-the-art level in gravimetry is possible only within a framework of gravimetric observatories. One such observatory has been set up in the D. I. Mendeleev All-Union Scientific–Research Institute of Metrology (VNIIM) in an underground laboratory.

The underground laboratory is located at a depth of 45 m in a 150 m long 7.2 m diameter adit. Microtraverse reference points for periodic leveling were laid down in the adit and two gravimetric bases were made in its center. There are bore holes for determining the underground water level in the territory near the laboratory. The work program of the gravitational observatory will comprise regular absolute measurements of the free-fall acceleration g, systematic measurements of tidal changes in g, tilt measurements, observations of the underground water level, understanding of the correlation between meteorological factors and the gravimetric measurements, etc. It is proposed to link the gravimetric station of the observatory to the fundamental national gravimetric network and through it directly to the worldwide gravimetric network.

The most accurate absolute measurements of g are presently made by the ballistic method. Here it is appropriate to remark that the first measurements of g using this method were made in the P. N. Agaletskii All-Union Scientific–Research Institute of Metrology [2] in 1954-1956 even before the creation of laser ballistic gravimeters. The falling body was made in the form of a rod, with a glass photographic plate, located in an 80 cm long evacuated cylindrical chamber. The chamber was dropped from a height of 14 m along two guides. Observations were made of the descent of the chamber relative to the guides and of the rod relative to the chamber. The absolute value of g was determined with an accuracy of 1 mGal and a correction of 14 mGal to the Potsdam system was established. This was later confirmed in the 1960's by measurements using ballistic gravimeters. The gravimetric station at Potsdam was taken as the international zero-point after absolute measurements of g using a reversible (Kater's) pendulum had been made there in 1898-1904.

At the present time the role of an international zero-point has actually passed to the International Bureau of Weights and Measures in Sèvres in France where international measurements of the absolute value of g have been regularly made since 1979 using ballistic laser gravimeters. These measurement detected discrepancies in the values of g [3-5] resulting from local

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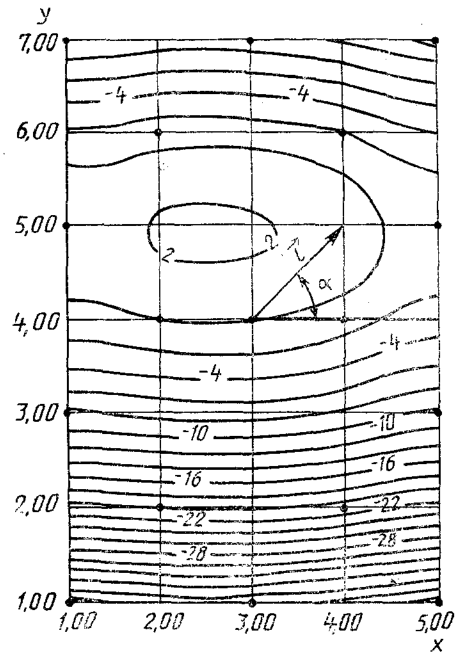

Fig. 1. Field contours of the increment $\Delta g(x, y)$.

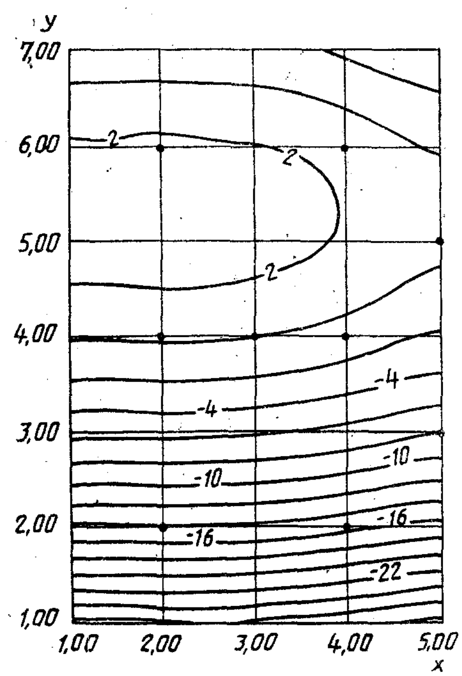

Fig. 2. Smoothed picture of the increment field contours.

nonuniformities of the gravitational field [6, 7]. Knowing these nonuniformities, the results of the measurements can be reduced to a single point, and this makes it possible considerably to reduce the overall error. The reduction must be performed not only in height but also in a horizontal plane [8] in which the nonuniformity of the gravitational field can be quite high, as indicated by the measurements reported in [9, 10].

The microgravimetric surveys of bases described in the literature [9, 11] were as a rule carried out with high-precision Sodin gravimeter having a single-reading accuracy of the order of 5 μGal. It is consequently necessary to make 20-30 measurements of one base in order to obtain an error of the order of 1 μGal (0.6-1.0 μGal in [9]). Such measurements are laborious and the Sodin instruments themselves require a high accuracy when working with them [11].

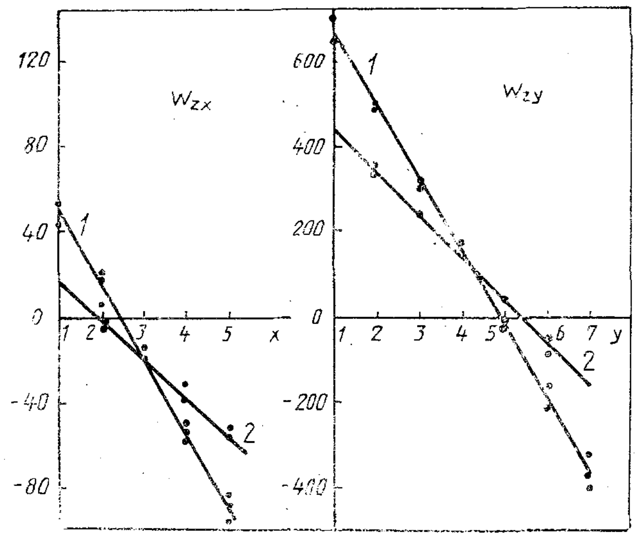

Fig. 3. Results of measurements of W_{zx} and W_{zy}.

We have carried out a microgravimetric survey of two bases at the VNIIM gravimetric station using an E-54 gravitational variometer. As is well known, this instrument actually measures the horizontal gradients W_{zx} and W_{zy}, and by integrating these one can determine the field of the increments Δg (relative to any point of the base). The results of a survey of a 1.90 × 2.50 m base are given below. The measurements were made using a mesh with a step length of 0.3 m on two levels, 50 and 106 cm, at the points shown in Figs. 1 and 2. Repeat measurements were made at several points in order to estimate the experimental error. These showed W_{zx} and W_{zy} to be determined with errors respectively of the order of 0.20 E (1%) and 0.73 E (0.5%). The field of the increments Δg was found from the formulas:

$$\Delta g=\int_{r_0}^{r} dl\,\mathrm{grad}\,g=\int_{x_0,y_0}^{x,y}(dx W_{zx}+dy W_{zy})=\int_0^l dl(W_{zx}\cos\alpha+W_{zy}\sin\alpha). \tag{1}$$

In Eq. (1) l is measured from the central point to the variable point in the direction of an angle α (see Fig. 1), $r_0 = (x_0, y_0)$ is the position of the central point ($x_0 = 3$, $y_0 = 4$) (see Fig. 1), $r(x, y)$ is the radius vector of the variable point at which the increment Δg is calculated.

The gradients W_{zx} and W_{zy} were previously found using linear interpolation for those points of the base mesh where they were not measured. The interpolation error was estimated from the difference in values of the same quantity at a given point which were obtained approaching it along different paths. Thus, the horizontal gradient at point *33* (see Fig. 1) can be determined by measuring the values at *22* and *44* and also at *24* and *42*. The deviation from the average value in this case is 0.82 E, and this is equivalent to an error of 0.025 μGal in each step (0.3 m). The maximum interpolation error was of the order of 10 E which, translated to the increment error, is equivalent to 0.3 μGal per step or the order of 1 μGal over the extent of the base.

The interpolation error which was estimated by the trapezium method is well known to be estimated from the formula:

$$\delta g=(1/2)(\Delta x)^2 y''(\xi).$$

The second derivative $y''(\xi)$ is easily estimated from the measured values. For the integration over x we obtain an error in the range 0.012-0.054 μGal and for that over y an error in the range 0.10-0.13 μGal. In summary, the principal error in determining Δg is due to that in measuring the horizontal gradients and to the interpolation error, and is within 1 μGal.

Figure 1 shows contours of equal increments $\Delta g(x, y)$. Attention is drawn to the fact that the equal-increment contours are extended along the x axis. This is due to the specific features of the gravitational environment, in that the base is located in a cylindrical horizontal adit, at its center. The x axis is directed along the axis of the cylinder. It is well known that the self-field inside a cylinder depends only on the coordinates y and z which are orthogonal to the longitudinal x axis. This property of the gravitational field of a cylinder is reflected in the results obtained, in that they are weakly dependent on the

x coordinate and strongly dependent on the y (and also the z) coordinate in the transverse direction. The maximum Δg is displaced relative to the center of the base.

Another method of determining the increments $\Delta g(x, y)$ is to represent it in the form of a power function of the two variables, of order n, with undetermined coefficients chosen from the conditions for the best approximation to the experimental data. We used this method. Since the contours near the maximum consist of almost regular ovals, the increments $\Delta g(x, y)$ (at least near the maximum) can be described by the function

$$\Delta g(x,y)=Ax+By-ax^2-by^2+cxy+C \tag{2}$$

(the negative signs before the third and fourth terms, with $a > 0$ and $b > 0$ represent the condition for a maximum). Of course, in general Eq. (2) must contain terms of the third, fourth, etc. powers. The term cxy determines the orientation of the contours. In our case the contours, and particularly the ovals near the maximum, are strictly orientated along the coordinate axes. It hence follows that $c = 0$. In this case the derivative $\partial g/\partial x = W_{zx}$ must be a linear function of x alone, while the derivative $\partial g/\partial y = W_{zy}$ must be a linear function of y alone.

Figure 3 gives the measured values of W_{zx} and W_{zy}. For both levels they lie on straight lines (*1* and *2*):

$$W_{zx}=A-2ax,\quad W_{zy}=B-2by. \tag{3}$$

The coefficients A, B, $2a$, and $2b$ were found by the least squares method. For the first (lower) level we have

$$A=2{,}437,\quad B=25{,}452,\quad a=0{,}501,\quad b=2{,}596. \tag{4}$$

The points of intersection of the straight lines in Eq. (3) with the abscissa axis determine the position of the maximum of the $\Delta g(x, y)$ field:

for the first level

$$x_{\max}=A/2a=2{,}43,\quad y_{\max}=B/2b=4{,}90,$$

and for the second level

$$x_{\max}=A/2a=1{,}86,\quad y_{\max}=B/2b=5{,}49.$$

Integrating Eq. (3) we arrive at formula (2) with c absent:

$$\Delta g(x,y)=Ax+By-ax^2-by^2+C.$$

The increments $\Delta g(x, y)$ are found relative to the central point having coordinates $x_0 = 3$, $y_0 = 4$ at which $\Delta g = 0$. This conditions gives $C = 63.078$.

All the coefficients are expressed in microgals; x and y assume dimensionless values. An equivalent representation is of the form:

$$\Delta g(x,y)=A'(x-x_0)+B'(y-y_0)-a(x-x_0)^2-b(y-y_0)^2, \tag{5}$$

where $x_0 = 3$ and $y_0 = 4$ are the coordinates of the central point of the base; A', B' are related to A, B by the expressions $A' = A - 2ax_0$, $B' = B - 2by_0$.

Hence, for the first level

$$A'=-0{,}569,\quad B'=4{,}685.$$

For the second level

$$A'=-0,588,\quad B'=4,150,\quad a=0,257,\quad b=1,556.$$

A comparison of the increments Δg obtained by integrating the field of the horizontal gradients in calculations using Eqs. (2) and (5) showed the discrepancy to be less than 0.5 μGal at all points of the base. Thus, the formulas (2) and (5) are valid not only near the maximum point but also over the entire base.

The confidence limits for the polynomial coefficients (4) were calculated for a 70% confidence level. For 32 degrees of freedom (for a number of readings 16 $\times$ 2) they were found to be:

$$\delta A=A\cdot 3,91\cdot 10^{-2},\quad \delta B=B\cdot 1,04\cdot 10^{-2},\quad \delta a=a\cdot 6,55\cdot 10^{-2},$$

The variance $D(\Delta g) = \sigma^2$ was then estimated from Eq. (5), taking into account the relationships between A', B' and A, B. The root mean square deviation σ, averaged over the base, was found to be 1 μGal. This value characterizes the accuracy of formulas (2) and (5).

Similar estimates for the second level, despite almost halving the statistics (the number of readings was $9 \times 2 = 18$) turned out to be rather better than for the lower level. This was because the measurement conditions were such that the instrument on the second level possessed a greater mechanical stability than that on the first level and produced more stable results. In summary, $\sigma \approx 0.8$ μGal for the second level. On account of the relatively small number of measurements for the second level, the increments Δg were not determined using the integration method.

Figure 2 gives a picture of the contours. They are smoother, with the reduction in the values of Δg at the edges of the base being roughly halved. The position of the maximum is also displaced. The smoothing of the gravitational field on moving away from the base can also be seen in Fig. 3 from the reduction in the values of the second derivatives of W_{zx} and W_{zy}. Attention is directed to the fact that the location of the contours is not repeated at the different levels. This confirms the remark made in [8] concerning the need to reduce the results of absolute measurements not only in a vertical plane but also in a horizontal plane.

The results obtained are in agreement with those of the 1991 survey of the same base using a Sodin gravimeter. Measurements were made with an accuracy of the order of 1 μGal at five points for one level, at a height of 5-10 cm. There is qualitative agreement between the previous results and the new results. In both cases a considerable fall off is observed in the measured values of Δg in the transverse direction of the adit (the y axis) with small changes in Δg in the longitudinal direction (the x axis), the smallest values in this and other cases being on the line $y = 0$.

The use of variometers to carry out a highly accurate microgravimetric survey can be considered to be promising both because of the accuracy achieved and also because of the time taken to make the measurements. It should be emphasized that a root mean square deviation of 1 μGal was obtained even for single readings at one point.

These measurements form part of the preparations for performing comparisons of absolute ballistic gravimeters at the gravimetric observatory. Taken together with the measured value of the vertical gradient (260 μGal/m) they solve the problem of reducing the results of measurements in the vertical and horizontal planes. They provide evidence that the underground laboratory possesses a high level of protection from interference and a stable hydrological situation. This promises extremely favorable conditions for the functioning of the gravimetric observatory.

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