The mean density of the Earth

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The historical measurements of the mean density of the Earth are reviewed and the significance of its value is briefly discussed.

Introduction

As with many things concerning the mass of planetary bodies, it all started with Isaac Newton around 1687. Turning to the *Principia*, *Book III, The System of the World*, Proposition 10, Theorem 10 (see, for example Cohen[1]) we read:

> 'If the earth were not denser than the seas, it would emerge from those seas and, according to the degree of its lightness, a part of the earth would stand out from the water, while all those seas flowed to the opposite side. By the same argument the spots on the sun are lighter than the solar shining matter on top of which they float. And in whatever way the planets were formed, at the time when the mass was fluid, all heavier matter made for the centre, away from the water. Accordingly, since the ordinary matter of our earth at its surface is about twice as heavy as water, and a little lower down, in mines, is found to be about three or four or even five times heavier than water, it is likely that the total amount of matter in the earth is about five to six times greater than it would be if the whole earth consisted of water, especially since it has already been shown above that the earth is about four times denser than Jupiter.'

The fact that the average of Newton's 'five or six' is very close to today's value of the mean relative density of the Earth shows just how prescient he was. The mean density of the Earth ρ_E was an extremely important quantity in early Renaissance science as it provided a strong clue as to planetary composition. The mass of the Earth, M_E was simply given by

$$M_E = \frac{4\pi R_E^3 \rho_E}{3},$$

where R_E is the radius of the Earth, a quantity that could easily be obtained by noting the way in which stellar altitudes varied as the observer changed latitudes.

The effect of mass is also central to Isaac Newton's second law of motion. In the planetary context, where mass is not changing, this can be summarised by equating the force acting on a body to the product of the mass of that body and the acceleration that the force produces. Consider the force exerted by the Sun on the orbiting Earth. Here

$$G\frac{M_{Sun} M_E}{d_{Sun}^2} = M_E \frac{V_E^2}{d_{Sun}} = M_E \frac{4\pi^2 d_{Sun}^2}{d_{Sun} T_E^2}, \quad [1]$$

where G is Newton's constant of gravitation, d_{Sun} is the average distance between the Earth and the Sun, V_E is the velocity of the Earth along its orbit and T_E is the time Earth takes to get around the orbit, i.e. the year. The equations in [1] can be rearranged to give

$$M_{Sun} = \frac{4\pi^2}{G}\left[\frac{d_{Sun}^3}{T_E^2}\right], \quad [2]$$

which is the basic equation behind Kepler's Third (i.e. Harmonic) Law. Considering the force exerted by the Earth on the orbiting Moon we can also write

$$M_E = \frac{4\pi^2}{G}\left[\frac{d_{Moon}^3}{T_M^2}\right], \quad [3]$$

where d_{Moon} is the average distance between the Earth and the Moon and T_M is the time it takes the Moon to orbit the Earth, i.e. the month. In Newton's time G was not known accurately so the masses of solar system objects with satellites were usually quoted in terms of ratios. Taking the Sun as an example equation [2] would be divided by equation [3], leading to

$$\frac{M_{Sun}}{M_E} = \left(\frac{d_{Sun}}{d_{Moon}}\right)^3 \times \left(\frac{T_M}{T_E}\right)^2.$$

Applying a formula similar to this to the orbital parameters of our Moon and the moons of Jupiter led Newton to the conclusion that 'the earth is four times denser than Jupiter', as given in the quotation above.

Newton's constant of gravitation, G, is given by

$$G = \frac{gR_E^2}{M_E} = \frac{3g}{4\pi R_E \rho_E}.$$

where g is the acceleration of gravity at the Earth's surface, a quantity that can easily be obtained by timing a swinging pendulum.

The radius of the Earth

Greek science indicates that the sphericity of the Earth was accepted by many from about the time of Anaximander (BC 610–547). Aristotle (BC 384–322) said that the circumference of our planet was 400,000 stadia. (see *De Caelo, Book II*, Chapter 14). This estimate possibly came from noting

Table 1. Historical 'best estimate' values of *L*

If the radius of the Earth is R_E, then its circumference is $2\pi R_E$ and the distance L one has to travel along a polar great circle in order to change one's latitude by one degree is $2\pi R_E/360$. The table below shows how the 'best estimate' value of L has changed over history.

Author	*Date*	*L (km)*
Aristotle	360 BC	200
Eratosthenes	250 BC	128.4
Yi-Hsing	AD 723	132.3
Abdallah al-Mamum	AD 814	90
Fernel (Paris)	AD 1527	101.4
Norwood	AD 1637	120.5
Picard (Paris)	AD 1669	111.2
Today	AD 2000	110.946

how ships gradually disappeared from view as they moved away from port, across the sea.

The first recorded actual measurement[2] of the Earth's radius was by Eratosthenes of Alexandria (BC circa 276–194), the librarian of the great museum in that city. At noon on the day of the summer solstice he noted that the difference between the altitude of the Sun at Syene (latitude 24° 6′N, longitude 32° 51′E) and Alexandria (latitude 31° 9′N, longitude 29° 53′E) was one fiftieth of a complete circle, *i.e.* about 7.2°. The difference in longitude was ignored and the distance between the two places was found by professional pacers to be 5000 stadia, giving the circumference of the Earth as 250,000 stadia. The singular of the word stadia was stadion (in Greek) and stadium (in Latin). But what is the length of a stadion?[3] The stadion at Olympus, where the original Olympic Games were held, is recorded as being 192.3m long; the Philetaerian (Babylonian–Persian) stadion as 198.4m, the Italian stadion 186m and the Ptolemaic stadion 212.6m.

Unfortunately we have little idea as to which stadion was used by Eratosthenes. The most commonly quoted modern definition of the stadion equates it to a Greek furlong, this being 125 Roman paces, the pace being a typical double stride of a marching Roman soldier (*i.e.* the distance between two successive falls of the same foot). This makes the Roman pace equal to about 1.4795m and the stadion equal to about 184.9m. A Roman mile is, as you would expect, exactly 1000 paces. If this is the case the Eratosthenian value for the Earth's circumference is just over 15% too large.

Let us convert all the measurements of the size of planet Earth to a common unit, this being the average distance one has to travel along a polar great circle in order to change one's latitude by one degree. As the polar radius of Earth is 6356.755km, this distance, L, is about 110.946km. Eratosthenes found it to be 128.4km and Aristotle 200km. Yi-Hsing (683–727 AD) a Chinese astronomer/monk, measured the length of the shadows cast by the solsticial and equinoctial noon Sun, and the altitude of the pole star in thirteen places throughout China and concluded that L was 132.3km. In Mesopotamia the Caliph Abdallah al-Mamum (AD 814) supervised observations of the pole star along a north–south line and found $L = 90$km.

Modern European observations are given in Table 1. The last figure in this table is fairly close to the value expected by the definition of the metric unit of length. A committee of the French Academy of Science sat at the time of the French Revolution and decided that the 'metre' would be exactly one ten-millionth part of a quarter of the Earth's circumference measured through the poles. In 1792 the French physicist and geometrist Jean–Charles Borda (1733–1799) defined the unit metre by measuring the distance between Dunkirk and Barcelona (which are on the same meridian), the metric system becoming official in 1792. In the ideal metric system $L = 4\times10^4 / 360 = 111.111111$km.

Table 2. Values suggested for the mean density of the Earth, as a function of the date of publication

Author	*Date*	*Mean density of Earth (kg m⁻³)*	*Method*
Newton	1687	5000–6000	guesswork
Maskelyne	1776	4500	Mount Schiehallion
Cavendish	1798	5448 ± 33	torsion balance
Playfair	1811	4720 ± 150	Mount Schiehallion
Hutton	1821	4950	Schiehallion results
Bouguer	1821	4390	deviation of pendulum, Milan
Sabine	1827	4770	pendulum near Milan
Reich	1837	5490	torsion balance
Giulo	1841	4950	pendulum near Milan
Baily	1842	5670	torsion balance
Reich	1852	5580	torsion balance
Airy	1854	6600	*g* variation, Harton mine
James & Clerke	1855	5300	pendulum, Edinburgh
Cornu & Baille	1873	5530 ± 30	torsion balance
Mendenhall	1880	5770	pendulum
Von Jolly	1881	5690	*g* variation over 21m
von Sterneck	1883	5650 ± 650	*g* variation, silver mine
Poynting	1892	5490	torsion balance
Boys	1895	5527	torsion balance
Richarz & Krigar–Menzel	1898	5505	*g* variation
Heyl	1930	5517	torsion balance
Zahradnicek	1933	5528	torsion balance
Jeffreys	1939	5517 ± 4	torsion balance
Heyl & Chrzanowski	1942	5514	torsion balance

Once L is known it is a trivial step to calculate the radius of the Earth, R_E , using

$$R_E = 360L / 2\pi.$$

The density of the Earth and other planetary objects

The volume of the Earth ($4\pi R_E^3 / 3$) can be divided into the mass of the Earth to give the mean density of the Earth (see Table 2). These data are plotted in Figure 1. All the methods of estimating the mass of Earth depend on the measurement of Newton's constant of gravitation G. In one set of experiments the gravitational influences of 'natural' masses were considered. A typical example would be the Mount Schiehallion experiment. The effect that this Perthshire, conical, isolated mountain of hopefully well-known density had on a nearby suspended pendulum was estimated by Neville Maskelyne (1732–1811), our fifth Astronomer Royal. Similar measurements were made near Arthur's Seat in Edinburgh, on Mount Chimborazo in the Andes and on mountains near Milan.

Other 'natural' mass experiments used the flat Earth's crust. Here the acceleration of gravity would be measured at the

Earth's surface and then at the bottom of a nearby mine of known depth. The variation in g was then easily related to the value of G, assuming that is, that the density of the Earth's crust is well-known. George Biddell Airy (1801–1892) carried out a 'g variation' experiment at the Harton coalpit in Sunderland in 1854 and similar experiments were made in silver mines in Saxony and Bohemia in the 1880s. Unfortunately, as can be seen from Figure 2, these 'natural' mass experiments did not yield very satisfactory results. There is an inevitable uncertainty in the estimated density of mountains and landforms. Also there is ignorance about the density of underlying and nearby strata. These resulted in large systematic errors.

A better approach to G measurements used 'prepared' masses. Here the gravitational attraction between two spheres of known mass, size and separation was measured using delicate torsion balances. The first experiment of this kind was carried out by the great English natural philosopher and chemist Henry Cavendish (1731–1810). Significant improvements were made in this approach when quartz fibres replaced the original metal wire fibres supporting the deviating test mass. It can be seen from Figure 2 that the estimation of G using the torsion balance[4] consistently gave the best results.

The Earth's mean density of around 5520kg m^{-3} hovers between the density of laboratory iron (7000kg m^{-3}) and the density of crustal surface rock[5] (2700kg m^{-3}) and this immediately suggested to early researchers that Earth was a mixture of these two substances, with the rock in the surface regions and the iron at the centre. The proposed iron core[6] was also extremely helpful when William Gilbert, in 1600, and Edmund Halley, in 1692, attempted to explain the source of the Earth's magnetic field.

The density of the Earth is compared with the densities of other solid solar system bodies in Table 3. Two densities are quoted for each object. The mean density is obtained by simply dividing the mass by the volume. The 'uncompressed' density $\rho\,(P = 0)$ is model-dependent and has been obtained[7] by removing the effect of gravitational compression, and thus assumes that the pressure is zero throughout the whole volume. This makes a considerable difference for large massive bodies like Earth, where the central pressure[9] of

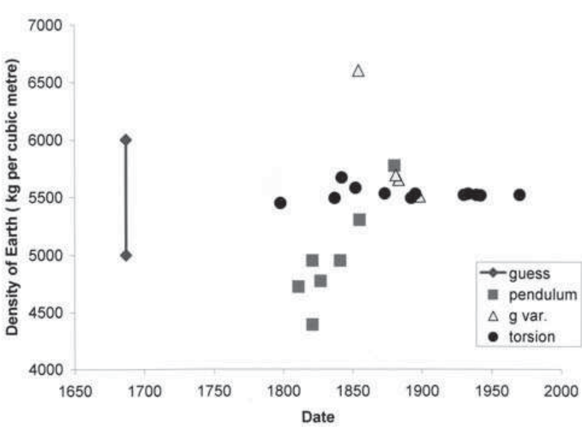

Figure 1. The historical estimates of the density of the Earth are plotted as a function of date. The actual values are given in Table 2.

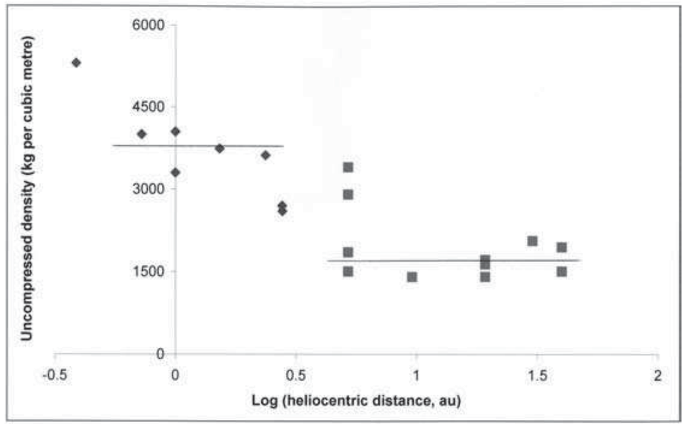

Figure 2. The uncompressed (zero pressure throughout) densities of large solid bodies in the solar system (see Table 3) are plotted as a function of the logarithm of the heliocentric distance (in AU) over the Mercury–Pluto range. The two horizontal lines indicate the mean density values on the sunward and anti-sun sides of the original snow-line.

about 360 GPa causes considerable compression and a large increase in the material density. The effect of the pressure variation with depth becomes negligible for bodies with radii less than about 1500km.

The uncompressed density is plotted as a function of the logarithm of the heliocentric distance in Figure 2. The terrestrial planets have an uncompressed density of around 3740 ± 150kg m^{-3}, this value underlining their general rocky/metallic composition (the relative abundance by weight of elements in the whole Earth is approximately 35% iron, 29.5% oxygen, 15% silicon, 12.5% magnesium, 3% nickel, 1.5% sulphur, 1% calcium, 1% aluminium and 1.5% the rest[10]). Mercury has an exceptionally large uncompressed density indicating that it is metal rich and has probably lost a considerable amount of its outer rocky mantle during the massive asteroidal bombardment that it suffered in the early days of the solar system. The Moon, on the other hand, has a lower than expected density.

Table 3. The densities[7,8] of large solid bodies in the solar system

Name	*Radius (km)*	*Distance (AU)*	*Mean density (kg m^{-3})*	*Uncompressed density (kg m^{-3})*
Mercury	2439	0.3871	5427	5300
Venus	6051	0.7233	5204	4000
Earth	6378	1.0000	5515	4050
Moon	1738	1.0000	3304	3300
Mars	3396	1.5237	3933	3740
Ceres	457	2.769	2700 ± 140	(2700)
Pallas	262	2.770	2600 ± 500	(2600)
Vesta	251	2.361	3620 ± 350	(3620)
Ganymede	2634±10	5.2042	1940 ± 20	1500
Callisto	2503±5	5.2042	1850 ± 4	1400
Io	1821.3±0.2	5.2042	3530 ± 6	3400
Europa	1565±8	5.2042	3020 ± 40	2900
Titan	2575±2	9.5751	1881 ± 5	1400
Titania	788.9±1.8	19.19	1710 ± 50	(1710)
Oberon	761.4±2.6	19.19	1630 ± 50	(1630)
Umbriel	584.7±2.8	19.19	1400 ± 160	(1400)
Triton	1352.6±2.4	30.2	2054 ± 32	(2054)
Pluto	1137±8	39.91	1940 ± 120	(1940)
Charon	586±13	39.91	1500 ± 200	(1500)

This is thought to indicate its unusual origin mechanism. It has been suggested that a Mars-sized asteroid hit the differentiated Earth in the early days of the planetary system and the Moon was formed[11] from the Earth's mantle ejecta. The densities of the bodies in the terrestrial planet region of the solar system indicate that these formed at temperatures above the freezing point of water.

The solid satellites of the outer gas-giant planets have an uncompressed density of around 1660 ± 90 kg m^{-3}, this indicating that most of them probably have high-density rocky/metallic interiors with large lower-density water ice mantles. Interestingly the large Jovian satellites seem to fall into both density camps. The dominant variable causing this heliocentric density variation would have been the temperature of the condensing pre-planetary nebula. Other things (such as albedo and emissivity) being equal, this temperature is expected to vary as the inverse of the square root of the heliocentric distance. For the expected solar luminosity of the time the Jovian region would be very close to the 'snow line'. At the dawn of the solar system this line was about 4 to 5 AU from the Sun, in a region where the temperature was about 250K, the temperature at which water vapour converts into snowflakes in the near-vacuum of space. Sunward of this line it is too hot for water-snow to condense. Beyond this region water-ice becomes a very important constituent of planetary bodies. The existence of the snow line explains the positioning of the two horizontal lines in Figure 2.

It has been estimated[12] that the typical ice/rocky-metallic mass ratio in the early outer solar system was about 2.2. The scatter of the data points in Figure 2 has been taken to indicate that the inner and outer solar system solid bodies have densities that fluctuate randomly about the mean values given above. Apart from the differentiation either side of the initial snow-line there seems to be no clear density trend with heliocentric distance.

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