Novel torsion balance for the measurement of the Newtonian gravitational constant

T. J. Quinn, C. C. Speake and R. S. Davis

Abstract. We report on the design and preliminary performance of a novel torsion balance for the measurement of the Newtonian gravitational constant. The design takes advantage of the properties of a wide, heavily loaded torsion strip to enhance the ratio of gravitational signal-to-noise torques beyond that achievable by the traditional Cavendish torsion balance. The results of preliminary experiments suggest that an accuracy of 1 part in 10^4 should be possible.

1. Introduction

From the time of Cavendish, the torsion balance using a wire suspension has been the principal device used to measure the Newtonian gravitational constant G. It has long been considered that the principal advantage of the torsion balance over other methods is that the forces to be measured are orthogonal to those resulting from the gravitational attraction of the Earth. It was pointed out in 1895 by Boys [1] that, since the stiffness of a torsion wire increases as the fourth power of the radius and the load capacity increases only as the square of the radius, the most advantageous configuration for high sensitivity is a very fine wire carrying a very small mass. Almost all measurements of the Newtonian gravitational constant made since then have employed a torsion balance with the common design features that the fibre was a fine circular wire and the test masses were small. Minimizing fibre stiffness was an essential requirement before the days of opto-electronic or electronic position sensing. Most of the test masses were spheres at the ends of a rod, i.e. a quadrupole mass distribution, and the total mass of the test-mass assembly, even in modern torsion-balance measurements of G, was only a few grams. In these experiments, while the sensitivity was adequate, the magnitude of the gravitational signal was always very small and great efforts had to be made to reduce perturbing forces.

There is now a considerable revival in interest in the measurement of G because, despite nearly two hundred years of effort, there is no consensus on its value even to an uncertainty of 0,1 %. Indeed, a result produced by Michaelis et al. at the Physikalisch-Technische Bundesanstalt [2], after long and careful experimentation, is nearly 0,6 % higher than the 1986 value given by the CODATA Task Group on Fundamental Constants [3]. The 1986 CODATA value, assigned an uncertainty of 1,28 parts in 10^4, was based on the work of Luther and Towler [4] who had given their result an uncertainty one half of that subsequently assigned by the CODATA group. A recent repeat of the measurement by Bagley [5] gives a result that differs from the 1981 value by 8 parts in 10^4.

It has been suggested by Kuroda [6] that the origin of differences between values of G obtained using torsion balances lies in the anelastic behaviour of the material of the torsion suspension [7]. This has focused attention on the importance of the properties of the suspension in a torsion balance and led him to predict that earlier measurements of G made using a wire suspension should be corrected downwards. However, the value obtained by Michaelis et al. using a torsion balance with a liquid mercury suspension is higher than all others. There is thus still no clear explanation of outstanding discrepancies.

In recent experiments on the behaviour of torsion strips [8-9] it was found that for wide strips under heavy loads there exists an additional gravitational restoring torque that can be much larger than the elastic component. There may be considerable advantages for certain laboratory experiments in operating a torsion strip in such a way that its restoring torque is mostly gravitational and, therefore, lossless. We have therefore designed a torsion balance for the measurement of the Newtonian gravitational constant that uses a wide, heavily loaded torsion strip.

2. Properties of heavily loaded torsion strips

As the ratio of width to thickness of a torsion strip increases, its behaviour departs progressively from that of a torsion wire. The torsion constant c_1 of a strip of

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width b, thickness t and length L, where $L \gg b \gg t$, is given by

$$c_1 = c_0 + c_\mathrm{g}, \tag{1}$$

where

$$c_0 = bt^3F/3L \tag{1a}$$

and

$$c_\mathrm{g} = Mgb^2/12L. \tag{1b}$$

Here, F is the modulus of rigidity of the material and g is the local acceleration due to gravity. The first term on the right of (1), c_0, is that due to Saint Venant and is generally taken to describe the torsion constant of a wide strip but, in fact, this is true only at zero load; the second term, c_g, is the gravitational component of the torsion constant. For a wide enough strip under heavy load, $c_\mathrm{g} \gg c_0$. Since it does not contain the modulus of elasticity, the term c_g should be independent of material properties such as anelasticity and, therefore, should be lossless. We have confirmed that c_g is indeed gravitational, and apparently lossless, using Cu-Be strips of thickness 50 µm and widths 5 mm and 10 mm [9]. It is interesting to note that Heyl and Chrzanowski [10] attempted to exploit the desirable properties of a gravitationally restored torsion element in the form of a bifilar suspension. They found, however, that instabilities at the ends of the wires were too great.

The equivalent expression for a round torsion wire of radius r is

$$c_\mathrm{w} = \frac{\pi r^4}{2L}\left(F + \frac{Mg}{\pi r^2}\right), \tag{2}$$

where $Mg/\pi r^2$ is the stress, σ, in the wire. Since σ is always much smaller than F, the restoring torque of a wire is always dominated by the elastic component.

The ratio of the restoring torques of a strip and wire having the same length and same shear modulus, and under the condition that the stress in each is the same, i.e. $\pi r^2 = bt$, is thus given by

$$\frac{c_1}{c_\mathrm{w}} = \frac{2\pi t}{3b} + \frac{\pi b}{6t} \cdot \frac{\sigma}{F}. \tag{3}$$

For the torsion strip used in the experiments described here, in which b/t = 42, the ratio c_1/c_w = 0,31. Note, however, that the second term is the dominant one so that most of the restoring torque of the strip is gravitational and not subject to the anelastic effects and corresponding noise of the wire whose restoring torque is wholly elastic.

3. Design of torsion balance

The new design of a torsion balance for the measurement of G is shown in Figure 1. Its basic design innovations are the following:

(a) The suspension is in the form of a wide, heavily loaded torsion strip so that its restoring torque is mainly gravitational in origin and thus lossless.

(b) The suspended test mass assembly of four 0,47 kg masses resting on a 16 cm diameter disk has a mass of more than 2 kg which, with the four source masses of 10 kg each, results in a gravitational torque, τ, of some 2×10^{-9} N m, nearly three orders of magnitude larger than that generally achieved in such experiments [4].

(c) The four test masses have a hexadecupole mass configuration so that the torsion pendulum is very insensitive to distant moving masses since the sensitivity of this system to such masses falls off as the fifth power of the distance compared with the third power for the quadrupole distribution of a dumbbell arrangement. In addition, the symmetry of the system greatly reduces the coupling of swing to rotation about the vertical [11].

The device described here is a prototype designed to test the principles and to provide a preliminary determination of G to an uncertainty of 1 % or perhaps even of 0,01 %. For a measurement of G with such an uncertainty the question of the uniformity of density of the test and source masses poses no particular problem, nor do manufacture and geometric measurement.

The torsion balance is made from a disk some 160 mm in diameter hanging from the 30 µm thick, 1,25 mm wide and 80 mm long torsion strip of Cu-Be. The Cu-Be strip is made from a fully dispersion-hardened rolled sheet cut to shape electrochemically. Equally spaced and resting on the disk are the four cylindrical Cu-Te test masses each of about 0,47 kg. Four 10 kg source masses are arranged on a carousel that can be rotated about the axis of the torsion strip. Such a configuration of test and source masses gives a hexadecupole mass distribution. It remains to be demonstrated experimentally that any residual torque resulting from a small quadrupole mass distribution, caused for example by engineering or density inhomogeneities in the disk, are negligible.

To facilitate the geometric measurements, the torsion balance and source mass assembly are mounted on the work-table of a small coordinate measuring machine. In this way it is easy to ensure that all dimensional measurements have uncertainties of only a few micrometres. Movements of the source masses, for example, can be measured directly in terms of the basic coordinate system and the previously determined positions of the source and test masses. It should be noted that for the hexadecupole disposition of source and test masses shown in Figure 1, the position of the ensemble of test masses with respect to that of the source masses is relatively uncritical. For example, a displacement of the set of test masses of 300 µm with respect to the source masses results in an error in G of only 3 parts in 10^5. For a device of the

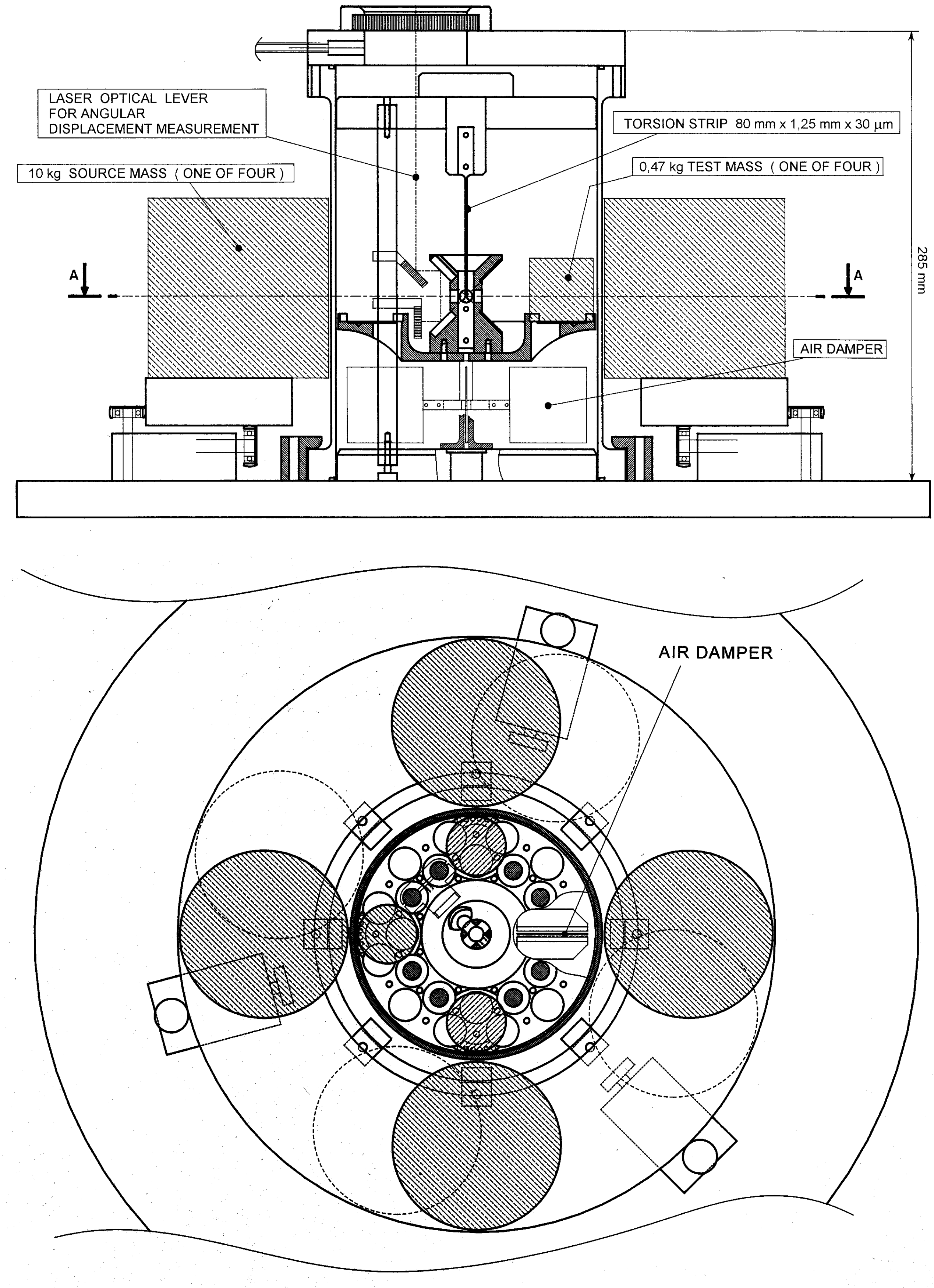

Figure 1. Schematic drawings of the torsion pendulum showing configuration of test and source masses (a) side view; (b) top view (only one of four air dampers is shown).

dimensions shown, uncertainties in shape, separation and orientation of the source and test masses of 5 μm lead to uncertainties in G of 5 parts in 10^5. With more care and a high-performance coordinate measuring machine, these uncertainties could be reduced to 1 part in 10^5.

In the preliminary arrangement shown in Figure 1, the rotation is detected by an optical lever. The reflector attached to the torsion disk is designed to be almost completely insensitive to rocking of the pendulum. The sensitivity to rotation, on the other hand, is increased by multiple reflections so that the reflected laser beam is deflected by up to sixteen times the angle of rotation of the pendulum. This has the advantage that angular movements of the incoming laser beam, which are not multiplied by sixteen, are correspondingly reduced with respect to the gravitational signal.

In designing an experiment such as this, one parameter that is almost impossible to estimate a priori is the effect of seismic noise, including tilt, on the position of the torsion balance. Another parameter is the magnitude of the drift in zero position resulting from relaxation in the suspension and from temperature changes. For this reason we made a preliminary montage of the balance complete with test and source masses, air damping and optical readout of angular position of the balance. The balance was mounted in an aluminium case but no provision was made to reduce the air pressure.

4. Results of preliminary tests

The results of preliminary experiments are satisfactory. For a gravitational torque provided by rotation of the four 10 kg source masses through angles of 22,5° either side of the neutral position, the signal-to-noise ratio is encouraging, see Figure 2.

The period of the pendulum is about eighty seconds. Zero drift over some 72 hours is a few percent of the gravitational signal, despite the absence of temperature control in the laboratory. Most of the noise in the signal results from fluctuations in the direction of the laser beam used in the optical lever.

Experiments were made to measure the effects of ground tilt on the gravitational signal. The marble slab on which the prototype device is mounted was tilted by about 120 μrad in each of two orthogonal directions at angles of 45° to the major axis of the torsion strip. In one direction the resulting rotation of the pendulum corresponded to 0,4 % of τ/c_1 and in the other direction no change was measurable. The uncertainty of these measurements was about 0,2 % of τ/c_1. Since ground tilts are generally two orders of magnitude below these artificially induced tilts, the sensitivity of the device to such tilts is not a problem, at least at the level of 0,01 %. To eliminate this problem completely, it is proposed to suspend the torsion strip from a crossed-knife gimbal of the design used in the BIPM flexure strip balance [12].

For preliminary measurements of the gravitational constant, we have already noted that the deflection of the torsion balance is measured by the deflection of a laser beam multiply reflected in a mirror system mounted on the torsion disk. The torsion constant of the strip is then deduced from the period of the torsion balance using the calculated value of the moment of inertia of the system. Operation at atmospheric pressure limits the accuracy of this method.

On the basis of the results of the preliminary experiments we plan to measure the gravitational constant with an uncertainty of 1 % in the next few months and then to seek an uncertainty of 0,1 %. Observations of statistical uncertainties in the data obtained so far indicate that a determination at a much higher accuracy is feasible. To achieve this we plan to

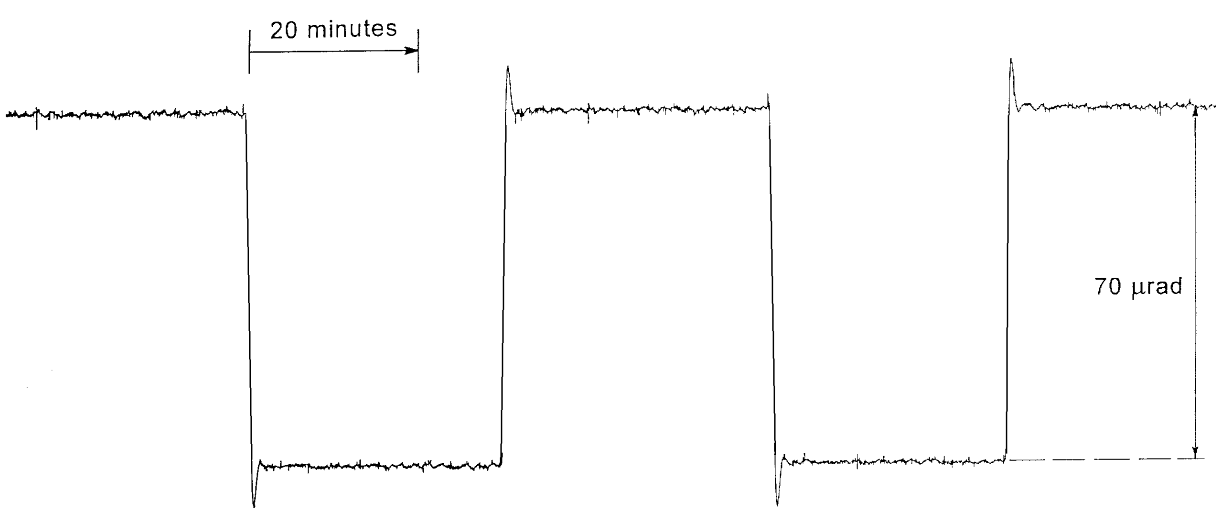

Figure 2. Example of chart recording showing deflection of laser beam indicating a 70 μrad rotation of the torsion pendulum on turning the source-mass carousel through an angle of 45°.

operate in vacuum using a capacitive angular sensor and an electrostatic force transducer [13].

Finally, we note that the heavily loaded torsion strip has a fundamental advantage over the traditional round-section fibre (wire) in that it has a greater ratio of gravitational signal to thermal-noise torque. The amplitude spectral density of thermal-noise torques acting on a torsion balance due to anelastic losses in the suspension is

$$n = \left(4k_{\mathrm{B}}T\,\frac{c_0}{\omega}\,\frac{\Delta F}{F}\right)^{1/2}, \qquad (4)$$

where c_0 is the elastic component of the restoring torque. For the torsion strip, C_0 is given by (1a); for the torsion wire, $C_0 = \frac{\pi r^4 F}{2L}$. The modulus defect is denoted as $\Delta F/F$ and ω is approximately the cadence of the experiment. This result follows from Speake and Quinn [14] and Quinn et al. [8]. An idealized torque signal τ, due to a gravitational field a, acting on a mass m, with balance armlength l, may be written

$$\tau = mla. \qquad (5)$$

We can now compare the performance of a strip with that of a wire, both having the same length L, and being loaded to the same tensile stress σ, for experimental designs having equal values of m, ω and l. Denoting the ratios of signal-to-noise for torsion strip and torsion wire experiments as sn_{s} and sn_{w} respectively, we can calculate the ratio of the signal-to-noise ratios

$$\frac{sn_{\mathrm{s}}}{sn_{\mathrm{w}}} = \left(\frac{3}{2\pi}\,\frac{b}{t}\right)^{1/2}. \qquad (6)$$

Although not yet used in our G apparatus, we have shown that a strip having b/t =100 is perfectly feasible [9], a value which gives significant advantage to the torsion strip. In this case, the ratio of the oscillation periods of the two balances would be

$$\frac{T_{\mathrm{s}}}{T_{\mathrm{w}}} = \left(\frac{6F}{\pi\sigma}\,\frac{t}{b}\right)^{1/2}. \qquad (7)$$

For the values of tensile stress achieved in our experiments (σ/F = 0,01) the periods of the two balances and therefore their response times would be comparable (see (3)). The result given in (7) will be important for experiments such as those which seek a breakdown in the universality of free fall where the signal-to-noise ratio is limited by the thermal noise in the wire suspensions. A considerable improvement in the precision of such tests should be possible by employing the strategy described above, i.e. high-resolution angular detector, large test masses and short-period torsion pendulum (relative to small mass systems). To take full advantage of the short period of the torsion pendulum, we envisage using a mass-exchanger system similar to that developed at the BIPM for beam balances [15] to permute the test masses on the pendulum disk rather than rotating the whole apparatus.

Note added in proof: Results of recent experiments by Bagley and Luther tend to confirm the Kuroda hypothesis (Bagley C. H., Luther G. G., Preliminary results of a determination of the Newtonian constant of gravitation: A test of the Kuroda hypothesis, *Physics Review Letters*, 1997, **78**, 3047-3050).

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