St. Petersburg Paper (TGSMM-2013)

PRECISION MEASUREMENT OF THE GRAVITATIONAL QUANTITIES g and G

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Key words: gravity, little g, Newtonian constant of gravitation, big G, precision measurement, scientific thinking

Summary: The precision measurement of g and G has a measurement history that dates from the beginnings of scientific thought. And though the measurement accuracy with which we can measure g, the free-fall acceleration due to our Earth's gravity, has been improved on by nearly eight orders of magnitude during the past 400 years, the accuracy with which G, the Newtonian Constant of Gravitation, is known has barely increased by three orders of magnitude during its nearly 300 year measurement history. In this paper, I will discuss what has driven (and impeded) this progress and I will also point out how various different ideas for measuring these two quantities have come about. Finally, I will point out how some of these ideas and/or technologies have directly benefited other areas of scientific research. Throughout this paper, an important and underlying theme will be the interconnectivity and commonality of all precision measurement experiments.

In this paper I will show that measurement capability has implemented much of scientific progress by extending the reach of our hands and quickening the response of our eyes. I will address the question, "What is precision measurement science?" and answer the question, "Should it be precision measurement or precision measurements?" In either case, this is an area of science that for one reason or other has sadly never received much recognition. In a paper published in 1983, Pipkin and Ritter [Ref. 1] begin their article on ***Precision Measurements and Fundamental Constants*** with the sentence, "An important but largely unrecognized area of the physical sciences consists of metrology, the science of measurement, and the determination of the fundamental constants..." Again in 1999, during the centenary of the American Physical Society, the APS created a slide entitled "Throughout the Year we are Celebrating ALL (capitalization theirs) Areas of Physics" [Fig. 1]. In that neither gravitational physics nor precision measurement made the list of "ALL" areas of physics, I inquired as to why these two areas got lost, and was told that there wasn't room for them despite the glaringly empty space following the last (important but rather arbitrary) entry, "Education."

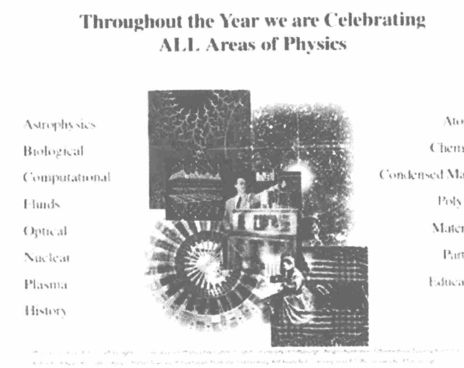


Fig. 1

With this brief introduction, let me turn first to the measurement of little g—a quantity that, incidentally, is not all that "little". Its measurement has most likely been performed since primitive man picked up a stone and dropped it…or threw it at a menacing animal. I would guess that in either case, early man not only noticed that the stone fell but also noticed that it fell faster and faster. A clever one—a future mathematician—might have first dropped one stone and then dropped a second and then "subtracted" the two fall times and noted the very small difference. Another very clever one (a prototype physicist) might have even performed an early "null" experiment by dropping two stones simultaneously and noting that they both hit at (nearly) the same time.

Today, little g can be measured with exquisite precision, and this measurement finds use in a variety of important applications. Nevertheless, though the process of measuring this free-fall acceleration is familiar to all, the process of trying to measure little g ***precisely*** in one's laboratory can at times be rather frustrating [Fig. 2]. Nature, while never dealing off the bottom of the deck, oftentimes seems to hold all the aces.

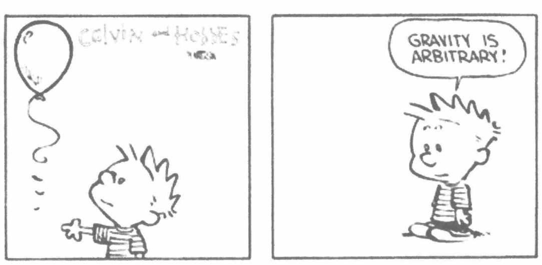
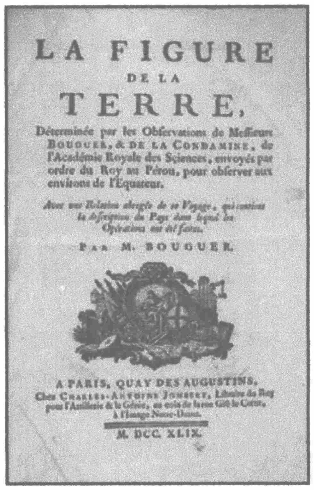


Fig. 2

Galileo is credited with having introduced the scientific method. It has been said that, "Science came down from Heaven to Earth on the inclined plane of Galileo." In his studies of motion, he used an inclined plane to dilute gravity, thus slowing the motion of "falling" (or in his case sliding or rolling) so that this motion could be studied using the modest "clocks" of his day (e.g., his pulse). Galileo also discovered and recognized the importance of pendulums—suspended masses whose time-of-swing can be accurately measured by timing a large number of swings.

A contemporary of Galileo's, Giovanni Battista Riccioli, who was born roughly 30 years after Galileo and died some 30 years after he did, questioned some of the results attributed to Galileo's leaning-tower-of-Pisa experiments, and accordingly carried out his own free fall experiments. He is now credited with being the first person to actually measure the value of g. To do this, he used a local 100-meter-tall tower from which he dropped masses and, aided by his fellow clergy, used rapidly swinging (i.e., short) pendulums as clocks; in this way he determined the value of little g. This early free fall determination produced a number that is within 5% of today's value.

The next 350 years of little g determinations were essentially all made with pendulums of various types because they yielded higher accuracies than could be obtained using the method of free fall. During this period, little g measurements made at different latitudes were used to determine the figure (i.e., the shape) of the earth [Fig. 3]. Kater, in this period, used his "reversible pendulum" [Fig. 4], to carry out the great trigonometric survey of Britain. What the use of a reversible pendulum did was to better answer the question, "What is the 'length' that should be used to determine g knowing a pendulum's period?" Kater believed that his little g measurement accuracy was 1 part in 10^6, but, in fact, the "accuracy" of his measurements was closer to 1 part in 10^5. Nevertheless, the precision of his difference measurements proved good to 1 part in 10^6! [In spite of his reducing the length measurement to only needing to measure the distance between the two knife edges on which the pendulum was swung, other factors, such as knife edge deformation, still reared their ugly heads at this level of accuracy.]


LA FIGURE
DE LA
TERRE,
Déterminée par les Observations de Messieurs BOUGUER, & DE LA CONDAMINE, de l'Académie Royale des Sciences, envoyés par ordre du Roy au Pérou, pour observer aux environs de l'Equateur.

PAR M. BOUGUER.

A PARIS, QUAY DES AUGUSTINS,

M. DCC. XLIX.

Fig. 3

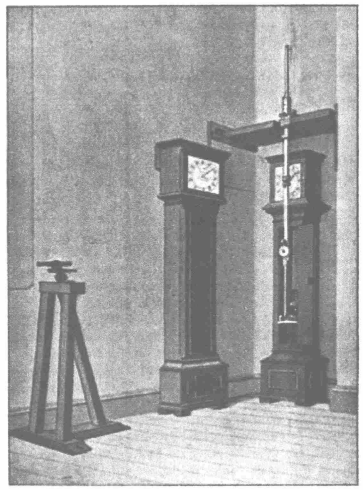


Fig. 4

Following what was roughly a 300-year period of using pendulums to determine both absolute and relative gravity, relative gravimeters (masses on springs used in a clever "geometry" that results in a large change in the masses position for a small change in gravity) began to be used. The adjacent figure [Fig. 5] shows the internal structure of such a device. Perhaps the most famous of these devices was the LaCoste and Romberg relative gravimeter, an instrument…and a company, both of which grew out of a cleverly solved problem that had been assigned to Lucien LaCoste when he was a student in one of Professor Romberg's classes. [Moral: all students should carefully do their assigned homework.]

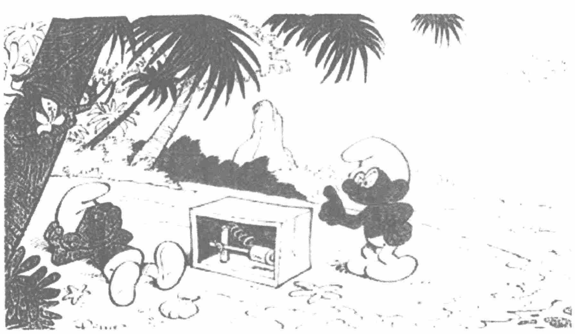


Fig. 5

Following the end of World War II and the concurrent dramatic progress in timing accuracies, scientists returned, 350 years after Riccioli's early measurements, to the method of free fall to determine g. The first of these determinations used the methods of geometrical optics to locate the position of a dropped body as it fell. When I began looking for an experiment that I could do for my Ph.D. thesis, my adviser (Professor R. H. Dicke) suggested that I look into using an interferometer to define these free-fall positions. The scientific interest in measuring little g at that time was that changes in its measured value would reflect (could be used to look for) variations in big G with time.

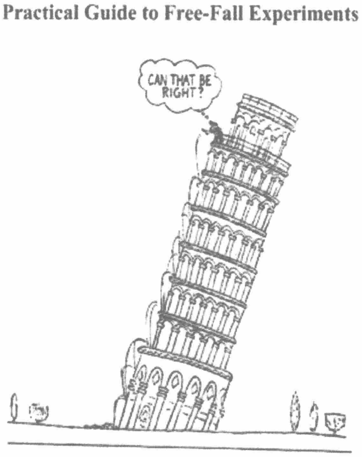


Fig. 6

As all students are taught to do, I immediately rushed to the library to see what I could learn; however the one book I found [Fig. 6] was not particularly helpful. The problems of using interferometry to measure g was made particularly difficult, because the mega-Hertz fringe frequencies that would result after a few centimeters of free fall required a bright source bright so that the fringes would be well defined. When I began as a student, there were no lasers so I ended up designing an interferometer that used white light fringes to define three positions of the object as it fell; I used the sun (by far the brightest light source available to me at that time) to illuminate this interferometer. A separate measurement, involving classical interferometer methods using monochromatic light sources, was carried out to determine interferometrically the spacing of the three white light fringes that occurred during each drop.

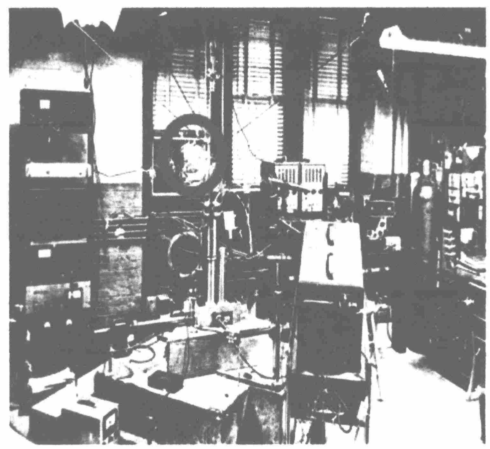


Fig. 7

The next figure [Fig. 7] shows my laboratory as it was in the late 1950s. The circled object sitting outside of the window was a Silvermann heliostat that was used to direct an image of the sun to the entrance pinhole of my white light interferometer. Shown [Fig. 8] are three examples of the three sets of mega-Hz frequencies white light fringes.

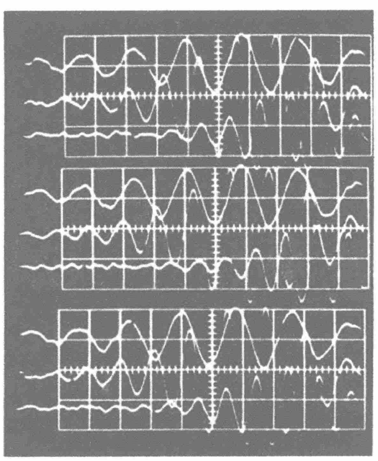


Fig. 8

The other problem at that time was that there were no commercially available corner cubes. [When one drops things to measure g, one needs to use rotation-insensitive mirrors.] As a result, I ended up using a lens-mirror cat's-eye assembly [Fig. 9] for my retro-reflecting "mirror". Finally, after 5 years of laboratory work, this interferometric method resulted in a 5-cm-measurement-length free-fall determination of little g with an accuracy of 7 parts in 10^7.

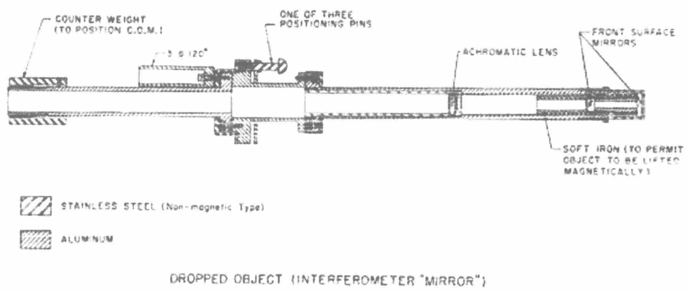


Fig. 9

In 1966, I attended the IUGG General Assembly that was held in Berkeley that year, and gave a talk on my thesis work. During the break that followed my talk, while standing in the lobby outside of the lecture hall, I saw Lucien LaCoste (I had earlier asked another person to point him out to me.) so I walked up to him and said, "Hello, I'm Jim Faller" and he said "Yes I know, I just heard your talk." I then asked him if he thought that someday absolute instruments might replace (his) relative instruments in making field measurements of g. His response was, "Yes, but I'm not worried." [Sixty years later, absolute instruments are now (finally) being

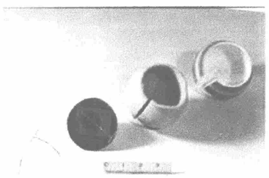


Fig. 10

used to do this; however, the most effective (but not always used) way to carry out a gravitational survey is to create a course grid of absolute measurements and intersperse it with more closely spaced relative measurements that are tied to the absolute points.]

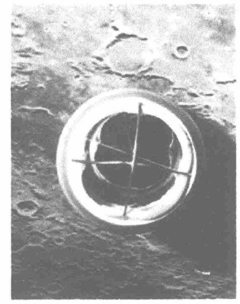


Fig. 11

By the end of my thesis work, new and useful measurement tools had become available. Corner cubes were commercially available. As it turned out, fabrication methods to make them had been developed during the Second World War for a number of military applications. As a result, their availability made it much easier to design a dropped object for use in free-fall gravimeters. A second and even more important development was the invention of various types of lasers. Both giant-pulse and continuous lasers were developed. In addition, the soon-developed stabilized versions of these continuous lasers became an implementing technology for building absolute gravimeters.

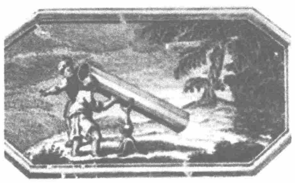


Fig. 12

Occasionally, throughout this paper, I will introduce brief diversions from the ongoing story of g and G to point out the beneficial "fallout" of ideas and/or technologies that are transferable to other areas of science. A reader could skip these and simply continue with the story. I hope, however, that some readers will find these diversions helpful to their understanding of how science comes about. As the first example of how little g "knowledge" resulted in a "new experiment", my knowledge of corner cubes and their retro-reflecting properties together with the by then availability of giant pulse lasers led me to suggest what has became known as the lunar laser ranging experiment.

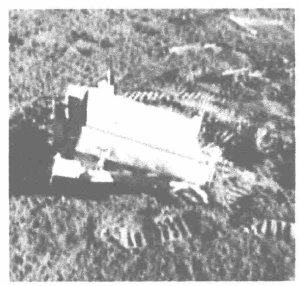


Fig. 13

A model of a corner-cube package was made that could be "rolled" onto the lunar surface by a surveyor-type soft-landed mission. Figure 10 show a possible construction of this bottom-heavy corner cube package and Figure 11 shows it "deployed" on the lunar surface. Lunar ranging [Fig. 12] would then involve sending a pulse of laser light towards a retro-reflecting "mirror" placed on the lunar surface, detecting the returned pulse of light, and precisely measuring its (roughly) 2.5 second round-trip travel time. Variations in this travel time contain scientific information relevant to gravitational physics as well as lunar and earth-based science. The good news is that NASA "listened" to this idea, and arrays of retroreflectors were subsequently placed on the moon by the Apollo 11, 14, and 15 astronauts. The adjacent figure [Fig. 13] shows the retro-reflector array as placed on the moon by the Apollo 14 astronauts. [The Apollo 14 array shown here is identical to the (earlier-placed) Apollo 11 array. However, since the Apollo 14's face is not in shadow, you can see the 100 individual cubes that make up this lunar "mirror".]

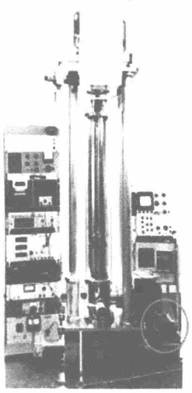


Fig. 14

Returning now—hopefully not too many of you will say "finally"—to the story of gravimeter development, the availability of stabilized lasers made possible the Hammond-Faller absolute gravimeter. [Fig. 14] This instrument used (circled near the bottom of the picture) a by-then-available Spectra Physics 119 Lamb-dip stabilized laser with an accuracy of 5 parts in 10^8 as its light source. Also shown in [Fig. 15] are the frequency-swept free-fall fringes that result when the dropped "mirror" (a corner cube) of the interferometer is released. Having electronic access to this swept-frequency signal that goes from DC to 10 MHz during the drop makes it straight-forward to determine a value for g. This instrument also made the first trans-Atlantic absolute gravity transfer measurement between Wesleyan University in Middletown, Connecticut (my academic home at that time), the NPL (Bushy House), and the BIPM in Sevres France.

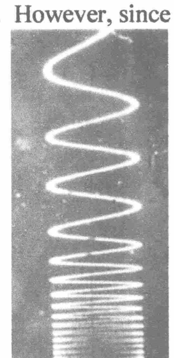


Fig. 15

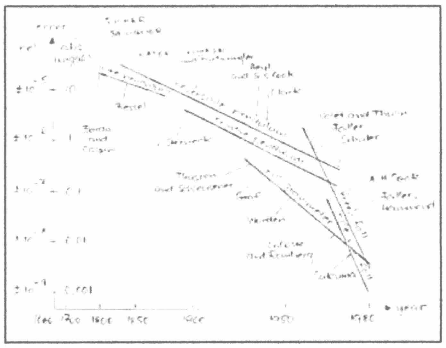


Fig. 16

The summary graph [Fig. 16] covering the first 300 hundred years of gravity measurements is due to Wolfgang Torge. Though the rate of accuracy improvement is seen here to be ever increasing with time, this trend plateaued out in subsequent years because of the appearance of various systematic (and hard to deal with) errors that came into play with further increases in measurement accuracy.

Here, let me make a very brief aside to comment on the question "why does progress always take so long?" [Fig. 17] is from a "physics of sports" talk that I have given. The picture shows that the basket (the year is 1892) did not have a hole in its bottom. As a result the game was unnecessarily slowed down because, after every score, someone had to shinny up the pole and extract the ball from the basket. In spite of the obvious-to-us-today solution of simply cutting a hole in the bottom of the basket, it took 10 years before this idea was implemented. Ideas, that in retrospect seem obvious to us, require someone to first think of them before any "progress" can happen.

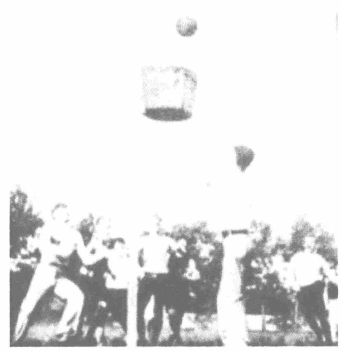


Fig. 17

The next gravimeter shown here diagrammatically [Fig 18] was built by Mark Zumberge as a part of his Ph.D. thesis. It utilized two novel and important ideas. The first was the use of a drag-free dropping chamber to surround the dropped object, an idea that occurred to me because I knew about the drag-free satellite that Stanford had developed. [Incidentally, the use of a drag-free chamber as applied to gravimetry had appeared earlier in the Russian literature; however, I was unaware of this development at that time.] The second important idea was to make use of a "super spring" to isolate the reference mirror in this gravimeter. Bob Rinker, another student of mine, had for his Ph.D. thesis created a very long-period spring that could be used for isolation by electronically feeding back on the top of a 30-centimeter-long spring to cause it to (nearly) track the motional amplitude of the mass hanging at its bottom. This idea resulted in an effective "kilometer-long" spring with a period of roughly 30 seconds that far better isolated the reference mirror of the free-fall-measuring interferometer. The adjacent figure [Fig. 19] shows (on the right) the scatter with this smaller and therefore more portable gravimeter without using the spring. On the left it shows the greatly reduced scatter when a super spring is used to isolate the reference corner cube. [Not surprisingly, this "super spring" idea has found other applications. For example, it is being used by LIGO to isolate their suspended masses from ground motion by electronically lowering the resonant frequency of their isolating tables.]

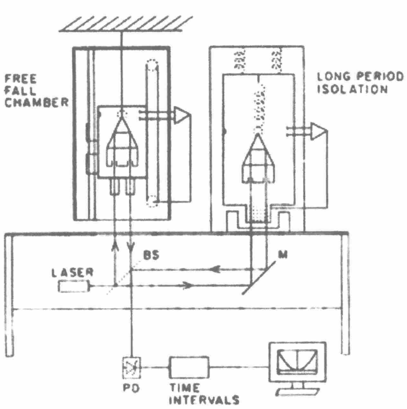


Fig. 18

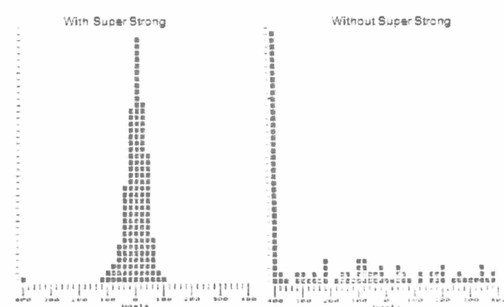
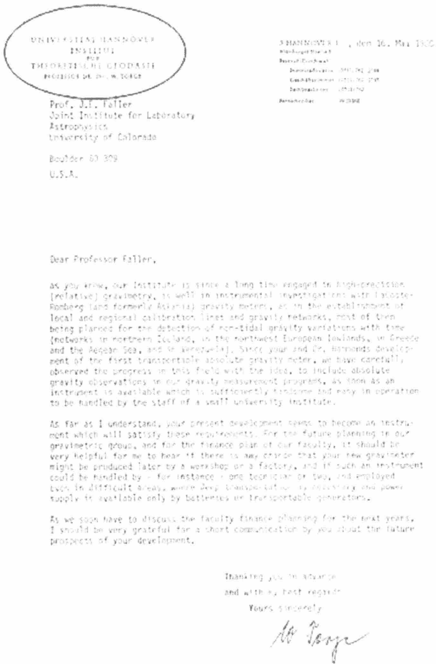


Fig. 19

At this point I would like you to look at two letters. The first letter [Fig. 20], written by Wolfgang Torge in May of 1980, asked if his institute could obtain from us an absolute gravimeter that they could use in their research programs. The answer Tim Niebauer (who was my student then) and I gave was, "Yes". Very importantly, this request pointed out to us the considerable interest that existed in having absolute gravimeters that they could use by the precision measurement, geodetic, and geophysical communities. [We ended up building 6 JILAg absolute gravimeters at that time; and, of course, the story goes on from there.]

The second letter [Fig. 21], again from Torge, was written two years later in 1982. The interesting point to me is that during this time period, the name of the institute was changed from the Institute for Theoretical Geodesy (the name of his institute that appears at the top left of his 1980 letter) to the Institute for Earth Measurement (i.e., *Erdmessung*). This was a recognition that "geophysics had come of age"; it was now possible to measure and study as they were happening quantities of geophysical and geodetic importance using the newly



UNIVERSITÄT HANNOVER
INSTITUT
FÜR
THEORETISCHE GEODÄSIE
PROFESSOR DR.-ING. W. TORGE

3 HANNOVER 1, den 16. Mai 1980

Prof. J.E. Faller
Joint Institute for Laboratory Astrophysics
University of Colorado

Boulder 80 309
U.S.A.

Dear Professor Faller,

as you know, our Institute is since a long time engaged in high-precision (relative) gravimetry, as well in instrumental investigations with LaCoste-Romberg (and formerly Askania) gravity meters, as in the establishment of local and regional calibration lines and gravity networks, most of them being planned for the detection of non-tidal gravity variations with time (networks in northern Iceland, in the northwest European lowlands, in Greece and the Aegean Sea, and in Venezuela). Since your and Dr. Hammonds development of the first transportable absolute gravity meter, we have carefully observed the progress in this field with the idea, to include absolute gravity observations in our gravity measurement programs, as soon as an instrument is available which is sufficiently handsome and easy in operation to be handled by the staff of a small university institute.

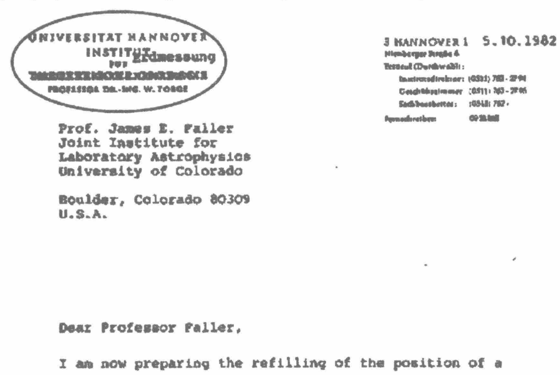
As far as I understand, your present development seems to become an instrument which will satisfy these requirements. For the future planning in our gravimetric group, and for the finance plan of our faculty, it should be very helpful for me to hear if there is any chance that your new gravimeter might be produced later by a workshop or a factory, and if such an instrument could be handled by - for instance - one technician or two, and employed even in difficult areas, where [illegible] transportation is [illegible] and power supply is available only by batteries or transportable generators.

As we soon have to discuss the faculty finance planning for the next years, I should be very grateful for a short communication by you about the future prospects of your development.

Thanking you in advance
and with my best regards
Yours sincerely

W. Torge

Fig. 20



UNIVERSITÄT HANNOVER
INSTITUT für Erdmessung
PROFESSOR DR.-ING. W. TORGE

3 HANNOVER 1 5.10.1982

Prof. James E. Faller
Joint Institute for
Laboratory Astrophysics
University of Colorado

Boulder, Colorado 80309
U.S.A.

Dear Professor Faller,

I am now preparing the refilling of the position of a

Fig. 21

developed high-accuracy measurement technique of absolute gravimetry as well as using other techniques such as GPS, and VLBI.

The JILAg gravimeter shown here [Fig. 22] was then and still is today a remarkably good instrument. Nevertheless, it had one design flaw that made it less than perfect. But like the hole in the bottom of the basket problem that I pointed out earlier, this particular flaw needed to be recognized before it could be "fixed".

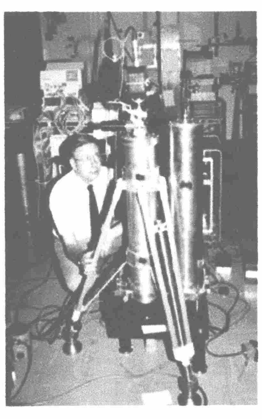


Fig. 22

As luck would have it, during the time when we were constructing the 6 JILAg gravimeters, a paper appeared (in Physical Review Letters) that suggested that suggested the possibility of a 5th force in nature; in addition to Newtonian gravity's $1/r^2$ squared force dependence, there might also be a short range Yukawa-type force which, amongst other things, would cause the equivalence principle to be violated when the attracting mass was at close range. Tim and I responded by using two of the JILAg dropping chambers, out of the six that we had, to create a modern-day version of Galileo's leaning-tower-of-Pisa experiment. We modified one JILAg dropped object so that it contained a large amount of Cu and modified another so that it contained a lot of Uranium. We then carried out [Fig. 23] a side-by-side free-fall equivalence test that showed to 5 parts in 10^{10} that both objects fell at the same rate. ***But*** (by about a factor of 10) we didn't do quite as well as we thought we should have been able to do given that this was a null experiment.

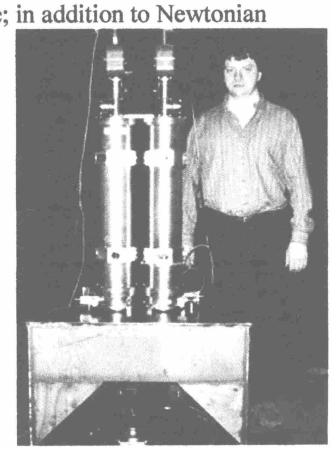


Fig. 23

We eventually realized that though the masses of the two dropped objects were the same, the response of the floor and that of the (wooden) box containing the interferometer on which the two dropping chambers sat were not uniform, and inevitably gave rise to a very small first-order tilt that the instrument would sense as it was systematically related to the moment of release. The "ahah" moment came when we realized that an "inline" instrument with the dropping chambers one above the other or an absolute gravimeter with its dropping chamber and its super spring in line would serve to put a hole in the bottom of this particular basket! Subsequently, when AXIS Instruments was formed to "provide the world with a source of absolute gravimeters" their new instrument was developed as an inline instrument giving it only a second-order measurement dependency on tilt.

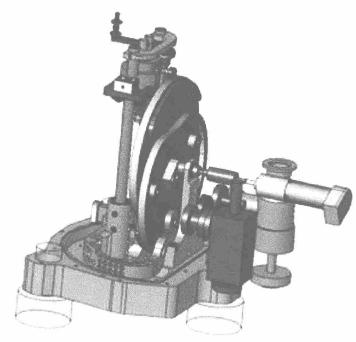


Fig. 24

During this period, another student of mine, Arthym Vitushkin, a name many of you know, developed with me a new type of cam-driven absolute gravimeter [Fig 24] that necessarily had a short drop (2 cm) and a high measurement rate, i.e., 3 drops per second. One of the instrument's simultaneously driven cams was used to drive (control) the dropping chamber, and the second was used to drive upwards an auxiliary mass to keep the center of mass of the entire apparatus fixed throughout the measurement process. With this instrument, at least in principle, we were able to remove, I think for the first time, the effects of drop-related recoil from the measurement of little g.

The evolution from the JILAg through many years of FG5 instruments and today's FG5X instrument [Figs. 25 and 26] which involved first AXIS Instruments that became MicrogSolutions and finally became MicrogLacoste, involved a difficult technical as well as a difficult funding journey where Tim and others both sacrificed financially and risked their careers by gambling that they would ultimately, as they did, succeed. MicrogLacoste's latest free-fall absolute gravimeter—the FG5X—is, by the way, a cable-driven recoil-compensated gravimeter. Circled in the second figure shown here near the top is the dropped mass and circled near the bottom is the recoil balancing mass that is pulled up as the dropping chamber is accelerated down.

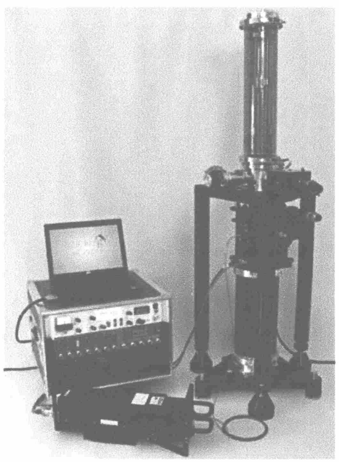


Fig. 25

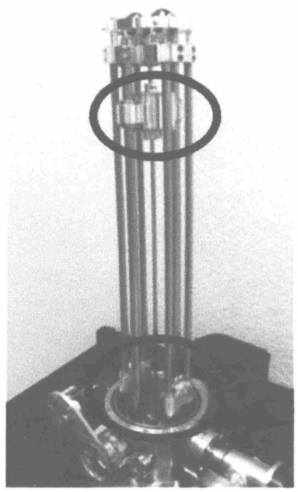


Fig. 26

Fig. 27 is a graph showing the measurement accuracies achieved using the method of interferometric free fall during the past 40 years. [The point on the far right represents the FG5X.] During this time frame, the ever-increasing rate of improvement seen in Torge's graph of the first 300 years of gravimetry is seen to plateau out as one runs into a sea of difficult-to-deal-with systematic errors that act to limit the accuracy for today's (and I would suggest tomorrow's) absolute gravimeters to approximately 1 part in 10^9.

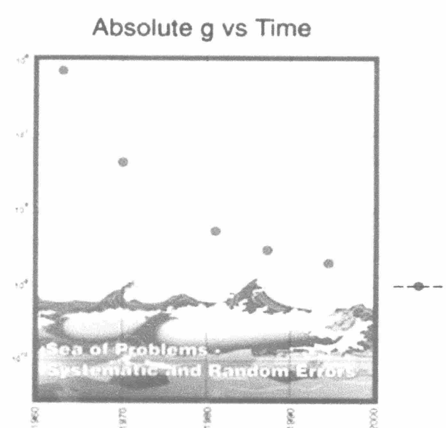


Fig. 27

The various implementing ideas and technologies that have brought absolute gravimeters to today's level of accuracy consist of the availability of quality corner cubes, stabilized lasers, super-spring isolation, drag free dropping chambers and recoil-corrected dropping mechanisms. Though I have reviewed this history using instruments in which I have direct knowledge regarding their development, similar developments have concurrently taken place at many other laboratories throughout the world.

Figure 28 shows both the stationary and "transportable" apparatus developed by Sakuma (also pictured) using the method of rise and fall that he developed at the BIPM. Rise and fall offers a considerably longer measurement time for a given apparatus size, but does so at the cost of the considerably greater mechanical complexity that is required to prevent cube rotation from being instigated at the moment of release.

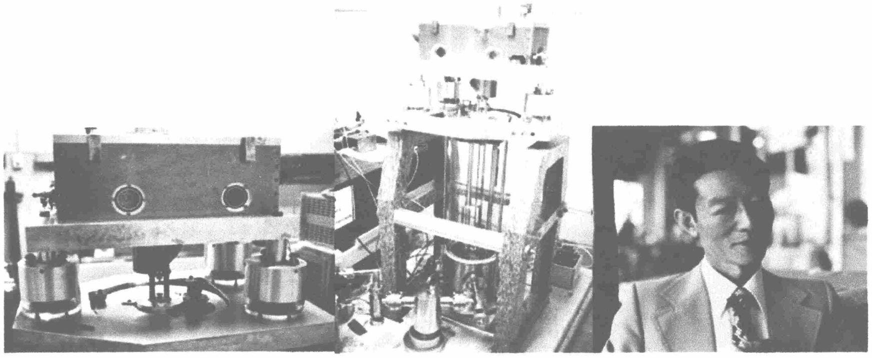


Fig. 28

[In the freefall case the dropped object is released at essentially zero velocity. In the case of rise and fall, it is released at the maximum velocity associated with the required upward throw.] Additionally, in the days when good vacuums were not that easy to obtain, the rise-and-fall method also offered immunity to the level of vacuum that was achieved. On the other hand, with today's drag free-dropping chambers and today's turbo and ion pumps, vacuum issues rarely enter into the discussion.

Finally, a second figure [Fig. 29] shows some of the other apparatuses that have been developed during this same time period, while a third figure [Fig. 30] shows 5 different instruments that participated in one of the intercomparisons of absolute gravimeters that were held for many years at the BIPM.

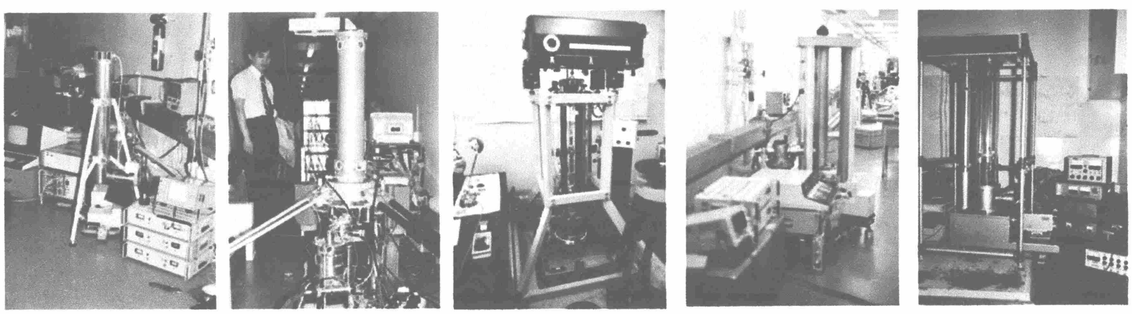


Fig. 29

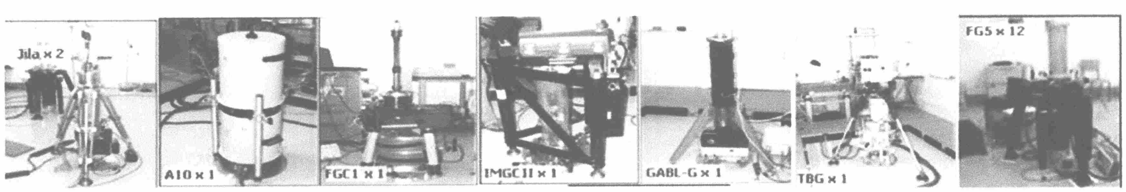


Fig. 30

An important question, one that influences the dynamics of funding, is what are the uses for little g.

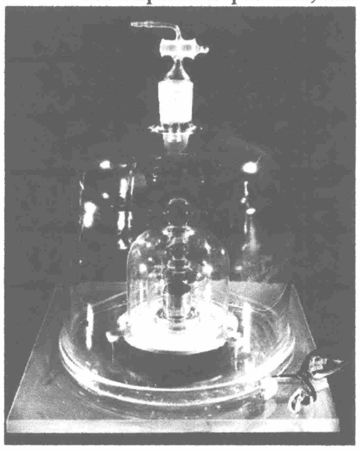


Fig. 31

And here the good news is that there are many: If one wants to replace the artifact kilogram [Fig. 31] that is used as a mass standard via the Watt balance approach, one needs to know in real time the local value for g. Absolute g field surveys are today being used to both search for and monitor subsurface densities and density variations with time. The first of the next viewgraphs [Fig. 32] shows an A10-based gravity survey on the ice in Prudhoe Bay, Alaska. The second [Fig.33] shows an A10 land survey in Indonesia. And finally the last application [Fig. 34] shows a 12-year history of absolute measurements made at Churchill in Manitoba, Canada. This figure shows a monotonic decrease in the measured value of gravity at this site because of the still-ongoing post-glacial rebound of the Earth's crust as it recovers (moves up) from previously being weighted down by the glacial cover. There are considerably more important applications: establishing calibrations lines, providing the gravitational field measurements needed for precise comparisons of atomic clocks, etc.

Fig. 32

Fig. 33

But what, I hear you ask, about the future? Will someday scientists be dropping atoms [Fig. 35] rather than dropping today's corner-cube-containing macroscopic bodies? If you were to ask me that question, my answer would be, "Maybe, but I'm not worried." The "sea of problems" affects (and limits) atom gravimeters just as it limits today's classical gravimeters. Though atoms are small and therefore give rise to no recoil effects at least two of today's absolute gravimeters have found ways to mitigate these recoil effects. And, yes atoms are small, but today's atom gravimeters are still large but, of course some day they may become much smaller. Finally, all absolute gravimeters, and this includes those dropping atoms, require similar isolation from ground vibrations for at least some of the non-falling parts of the instrument.

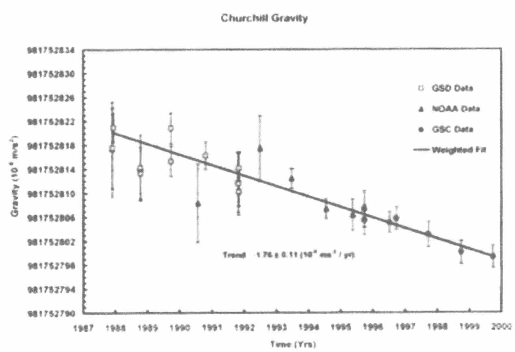


Fig. 34

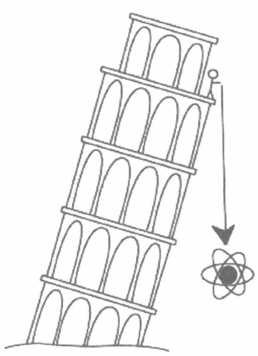


Fig. 35

At this point, let us turn to the other gravitational quantity: the Newtonian constant of gravitation, i.e., big G, a constant that strength-wise is really not very big, at least in the context of laboratory-sized experiments. At the present time, the experimental situation regarding its numerical value is not good, for there is a wide divergence in recent values of G. Several recent determinations (all with error bars of a few parts in 10^5) differ by as much as 5 parts in 10^4! But let us start at the beginning.

The story of how Newton "discovered" gravity has him sitting under an apple tree when an apple falls down and strikes him in the head, and *voila* he discovers universal gravitation! As one can see from the figure shown here [Fig. 36], there was a lot of serendipity involved in this story, even if it is true, because had the apple missed hitting him, he would probably have only reached out, grabbed it, and eaten it.

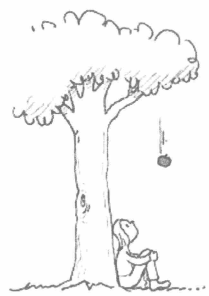


Fig. 36

If one Googles Universal Gravitation and then looks at Wikipedia's offering on this subject, the following figure [Fig. 37] appears: $F_1=F_2$ and there is indeed an inverse square dependence of the force; but look at the shown size of the two force arrows! They're huge. Any student studying this figure will come away with the (wrong) impression that the force of gravity between two (laboratory) spheres is really quite large. [On a cosmic scale, the force is indeed quite large; for were one to replace the gravitational force of the Sun on the Earth by a steel cable capable of the same force, the diameter of that cable would have to be roughly the same as the diameter of the earth!

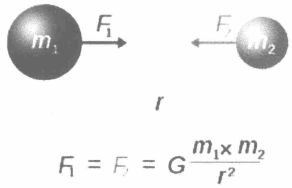


Fig. 37

If you are wondering how Newton's "thought experiment" that he did 100 years prior to these "mountain men" experiments could have been so accurate, Figure 38 is intended to offer an explanation.

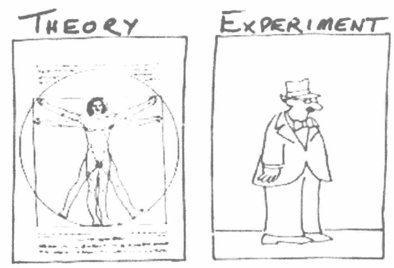


Fig. 38

Only 20 years after Maskelyne completed his experiment, Cavendish's famous paper "Experiments to Determine the Density of the Earth" was published [Ref. 2]. He begins his paper, "Many years ago, the Rev. John Michell, of this Society, contrived a method of determining the density of the earth, by rendering sensible (possible) the attraction of small quantities of matter; but as he was engaged in other pursuits, he did not complete the apparatus till a short time before his death, and did not live to make any experiments with it." [After several intermediary owners, Cavendish inherited this apparatus that he then used with great experimental skill to determine the density of the Earth.] So the first thing we learn about Cavendish is that he was fair; he properly credited Michell for the truly brilliant idea of using a torsion balance [Fig. 39] to (effectively) measure G. [A torsion balance serves to (or in practice, perhaps it just nearly does?) remove the force of little g that is some 10^{10} times larger than the big G forces, from influencing the experiment. Were I ever to teach a course on "100 Great Ideas in Physics," Michell's torsion balance would certainly be included.]

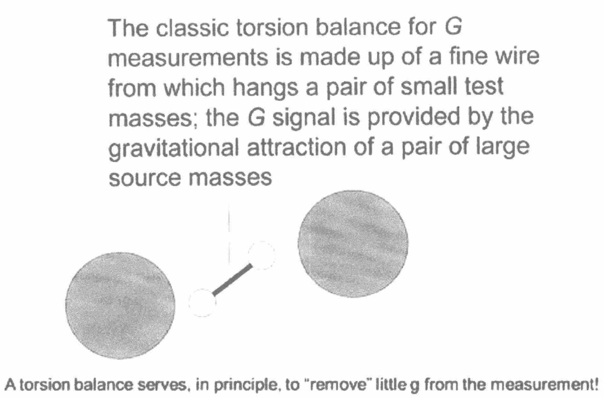


Fig. 39

Cavendish, who by this time was in his sixties, single handedly carried out this experiment. He also got a number of other things right. Not only did he arrange to be a brilliant experimentalist, but he also arranged to have wealthy parents so that with the monies he inherited, he could (and did) "retire" into a private world of science without ever having to seek funding for his work.

The opening sentence of the second paragraph of this paper is, "The apparatus is very simple"...and it was. Just like putting a hole in the bottom of the basket was a very simple idea to speed up the game—***once it was recognized***—so was the use of a torsion balance to measure G. The use of a torsion balance allowed him, using well-defined laboratory masses, to measure G, and he did this with an accuracy of 1%.

Equally interesting is Cavendish's comment that he makes later in this paper that "the mean angular position of his torsion balance kept increasing for half an hour, or an hour, after the motion of the weights." Though he didn't know a cause for what he observed, today we know that this "continuing increase" was caused by the anelasticity of his fiber.

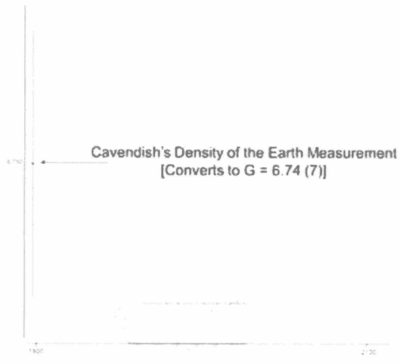


Fig. 40

The next figure [Fig. 40] shows Cavendish's Density of the Earth that converts to G = 6.74 (7), compared with some of today's measurements for G. His number is approximately 1% high. However, if one attempts to correct his number for the effects of the anelasticity of his Cu fiber, a reasonable guess for the Q of his fiber would suggest a correction of order 1%. And since the sign of the correction lowers his number, it would bring his G value into reasonable agreement with today's value!

At this point I want invest two paragraphs to briefly mention two other not as well known, but nevertheless interesting, big G experiments. The first of these was proposed by a Dr. W. Laska in 1889 [Fig. 41]. It was "an experiment that was never was." No results were ever published. It proposed to replace the torsion fiber with a knife edge that, provided it was located exactly in the middle of the vertical beam, would remove little g from the measurement. However, the most interesting feature was Laska's proposed, even though never implemented, use of optical interferometry to measure displacements. Newton's rings, formed in white light with the lens-flat combination at the top of the beam were to be used to measure the very small sideways displacement of the beam when the hollow sphere, D, was filled with mercury so as to gravitationally pull the mass, E, to the side.

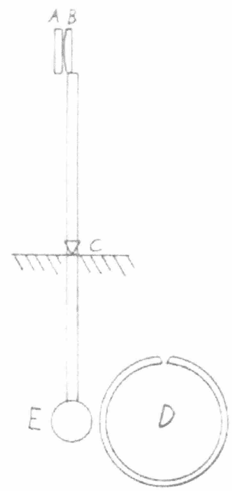


Fig. 41

The second of these experiments, which was performed by Poynting (of the vector fame) and was published in 1891, did not use a torsion fiber either; but rather [Fig. 42] it used a pan scale balance where the masses, m, were alternately attracted by a larger mass, M, located on a beam that could be rotated to alternately move this mass beneath them. The first year of this work, however, was completely wasted because the M/2 mass, (shown in this figure) was NOT initially included as a part of the experiment. As a result, the big-G-caused balance tilts were masked by the (inevitable) floor tilts that occurred when the single mass M was rotated! [Floor tilting issues that impact big G determinations also affect little g measurements.]

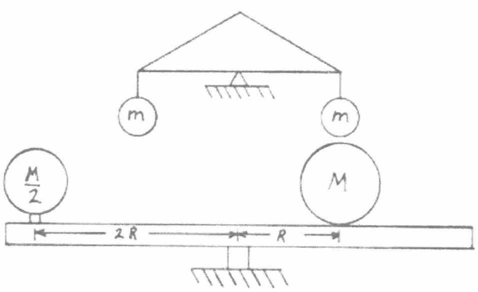
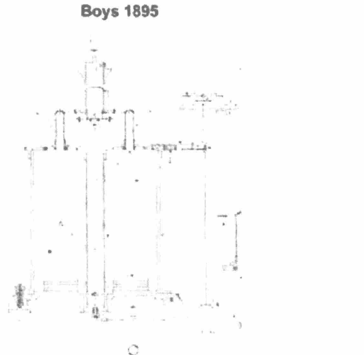


Fig. 42

The next figure [Fig. 43] shows diagrammatically the classic experiment of C.V. Boys in 1895 that improved our knowledge of the value of G to 1 part in 10^3. Boys introduced many new ideas into physics; the principal one was the use of fused-silica fibers that he learned to draw to support his torsion beam. His paper was also entitled "On the Newtonian Constant of Gravitation," officially determining G rather than the density of the Earth. Figure 44 shows his response to the Royal Society where he vigorously argues that his experiment did nothing so trivial as determining the weight of an insignificant planet...but was rather determining a constant of universal applicability.

Boys 1895



C.V. Boys, "On the Newtonian Constant of Gravitation," Phil. Trans. R. Soc. London A, **186**, 1 (1895)

Fig. 43

> C.V. Boys to the Royal Society
>
> "In spite of the courteously expressed desire of your distinguished and energetic secretary, that I should indicate in the title that, to put I vulgarly, I had been weighing the earth, I could not introduce as the object of my work anything so casual as an accidental property of an insignificant planet…The earth has no more to do with the investigation than the table has upon which the apparatus is supported."

Fig. 44

Boys [Fig. 45] clearly recognized the importance of—and was proud of—his discovery of the value of quartz fibers that made it possible for the first (and sadly the last) time "to obtain the value of Newton's Constant with a degree of accuracy ***as great as*** that with which electrical and magnetic units are known!"

1894.] *on the Newtonian Constant of Gravitation.* 377

opposition of circumstance, knowing that by my discovery of the value of the quartz fibre, and my development of the design of this apparatus, I had, for the first time, made it possible to obtain the value of Newton's Constant with a degree of accuracy as great as that with which electrical and magnetic units are known; though I have up to the present succeeded to an extent which is greater, I believe, than was expected of me, I am not yet entirely satisfied. I hope to make one more effort this autumn, but the conditions under which I have to work are too difficult; I cannot make the prolonged series of experiments in a spot remote from railways or human disturbance; I cannot escape from that perpetual command to come back to my work in London; so after this I must leave it, feeling sure that the next step can only be made by my methods, but by some one more blest in this world than myself.

[C. V. B.]

Fig. 45

Boys also wrote the book "Soap Bubbles and the Forces that Mould Them" based on the Royal Society Christmas Lecture that he gave for the children of London. He wrote in a charming and fascinating manner that made the reading of his papers a joy. If papers today were allowed to be written in a manner that would make them half as interesting as Boys' writings were, it would make reading journal articles a lot more enjoyable.

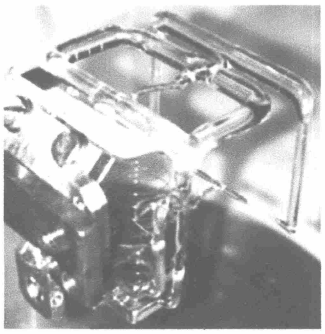


Fig. 46

At this point, I would just briefly point out that fused-silica fibers (Boys' technology) continue to be used in scientific instruments. Figure 46 shows the all fused silica golf-ball-sized "Gravilog" sensor that is used to make down-bore-hole gravity measurements at the 5 to 10 μgal level of precision. Another quartz-fiber application [Fig. 47] is seen in the lower mirror assembly used in the LIGO gravitational wave detector. Here the 30 kilogram lower interferometer mirror is suspended from the upper control mass by fused silica fibers that are used because of their high Q (and therefore low thermal noise) character.

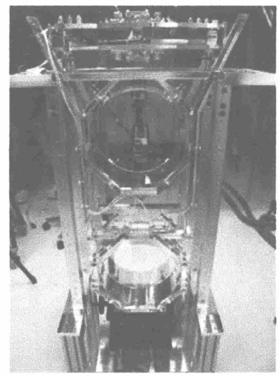


Fig. 47

The "classic" G determination of Heyl and Chrzanowski that was done in 1942 and that appears in the NBS Journal of Research is one of the many lengthy (as all of the early papers on this subject were) G papers that I read when I was a student. In this experiment [Fig. 48] cylindrical masses (a shape easy to machine with high accuracy at that time) were positioned so as to add to (while in the near position) the restoring strength of the torsion fiber and to subtract from (when in the far position) the restoring strength. Measuring the torsion periods for these two positions and (incorrectly) assuming that a fiber's torsion constant is independent of frequency plus knowing a few dimensions and mass values let one straightforwardly determine a value for G. Of interest to me, and perhaps it will also be interesting to you, was my recent discovery that Heyl's work on big G was ***not*** supported by NBS, but rather it was something he did in his spare time.

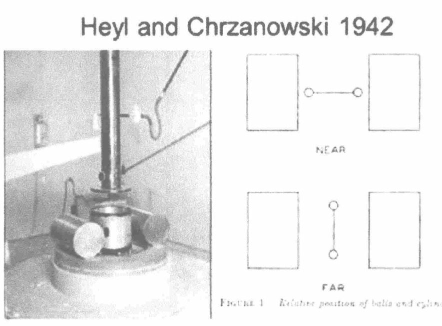


Fig. 48

Forty years later Luther and Towler published their 7.5 parts in 10^5 big G determination [Ref. 3] in which the period of a torsion pendulum was altered by the presence of two 10.5 kg tungsten balls [Fig. 49]. [By this time, the fabrication of spheres was nearly as easy as the fabrication of cylinders, and therefore spheres were employed because of their much-simpler-than-cylinders $1/r^2$ force dependence.] The difference in the squares of the frequencies with and without the balls was thought (see next paragraph) to be proportional to G. The value of G determined by this very carefully carried out experiment became the principle basis of CODA's

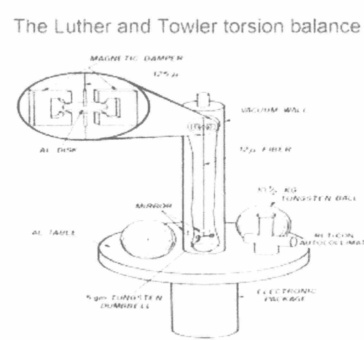


Fig. 49

1986 recommended value for this constant to which it assigned an error that was roughly double the error that Luther and Tower had assigned to their determination.

Two hundred years after Cavendish's "observation" of anelasticity, and sixteen years after the Luther-Towler experiment, Kazuaki Kuroda pointed out [Fig. 50] that all big G torsion-balance-based experiments need to make a correction for anelasticity [Ref. 4]. [The inclusion of this correction would serve to lower the Luther-Towler value for G by a few parts in 10^4; their fiber had a considerably higher Q than did Cavendish's copper fiber.]

Fig. 50

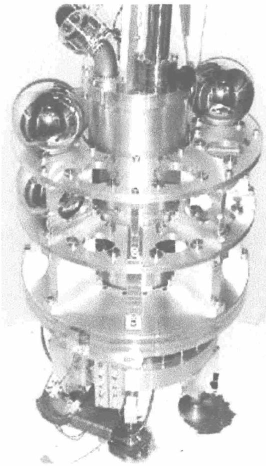


Fig. 51

I began the section of this paper that deals with the precision measurement of G by stating that today's experimental situation regarding its numerical value is not good. I will conclude this section by describing several recent determinations that are for me the source of this measurement dilemma. All of these experiments report errors of a few parts in 10^5, but their G values differ by as much as 5 parts in 10^4.

The four G determinations (two of them were done by the same group) that I want to call to your attention avoided the fiber-anelasticity problem by devising other measurement methods to determine G. Each is described in a *Physical Review Letter* of only four pages; and I recommend them to you for your "leisure" reading. In these new approaches, they did not allow their fiber to twist, or they used a heavily-loaded flat fiber, or they used a four-wire pendulum-supported mass. In both of the latter cases, "anelasticity-free" little g provided the great majority of the restoring force that balanced the gravitational (big G) pull of the reference masses. [I note here that though Michell's great contribution to science was the idea of a torsion balance that removed little g from the experimental arena, several of these "new" experiments chose to put it back in.]

The first of these experiments, the beautifully conceived and carried out experiment of Grundlach and Merkowitz, (published in 2000 [Ref. 5]) servoed the apparatus [Fig. 51] in such a way as to keep their torsion balance's fiber from twisting. They thereby avoided their experiments sensitivity to torsion fiber properties including anelasticity that could have led to a bias in their measurement. This experiment served as the principle basis for CODATA's 2002 (and also 2006) recommended values for G; and it still stands today as the determination with the lowest estimated error (1.4 parts in 10^5).

Two experiments were carried out at the BIPM. The first of these was published in 2001 [Ref. 6]. This experiment was subsequently redone 12 years later with significant instrumental improvements and published in September of this year [Ref. 7]. In both of these determinations, the basic principles of the experiment remained the same; a torsion balance was suspended from a Be-Cu strip that was 30 um thick, 2.5 mm wide and 160 mm long. For this type of fiber the torsion constant consists of two parts, an elastic part and a gravitational part. The gravitational part is identical, though the physics is not quite the same, to what you would calculate for a two wire suspension. Most importantly, given that they heavily loaded this strip, the gravitational (little g) part provided 97% of the total restoring torque and thereby greatly reduced the contribution of the troubling first part that contains the frequency-dependent shear modulus. Recent text books (for some unknown reason) tend to omit the second (and for this experiment helpful) part so that the experimental physicist's motto, "A month or two in the laboratory will save you an hour in the library" would not be true unless you happened to look up the equation for the torsion constant of a flat fiber in a fairly old text.

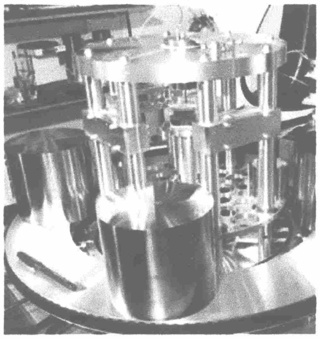


Fig. 52

In carrying out this determination, there were two modes of operation, free deflection (the Cavendish method) and electrostatic servo-control (where a electrostatic force is used to balance out the gravitational force). In both experiments (2001 and 2013) each of these two methods produced statistically the same answer, and both experiments have error bars that overlap. Finally, as you can see, the BIPM apparatus [Fig. 52] is indeed a thing of beauty.

Though this kind of beauty is lacking in the apparatus that Harold Parks and I used in our 2010 big G determination [Ref. 8] I nevertheless believe that our experiment represented the gathering of a great deal of experimental art in order to determine a value for G.

The next figure [Fig. 53] shows schematically the JILA G experiment. A Fabry-Perot interferometer measures the spacing between the two pendulum bobs with respect to a suspension-point-located reference cavity. The bobs are made of oxygen-free copper and have a mass of 780 g. The pendulum length is 72 cm, and the spacing between the bob centers is 34 cm. When the four 120 kg tungsten source masses (which are floated on air bearings) are moved from one position to another, the horizontal gravitational force on each pendulum bob changes by 480 nN, giving rise to a change in pendulum bob separation. In this experiment, little g provides most of the "spring," with only a small part coming from the bending of the wires.

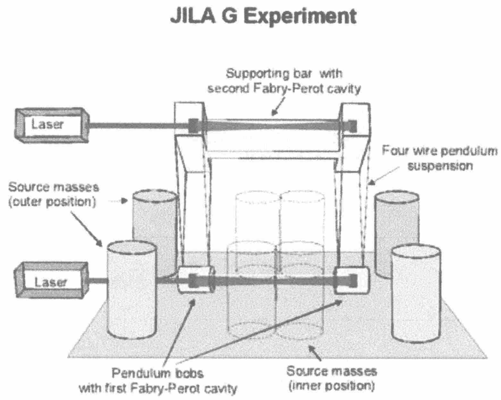


Fig. 53

The down side of using little g as the opposing spring is that the deflections are very small. However, our method of converting this small displacement (roughly 1/10 of a wavelength of light) into the frequency domain (where it produces a 120 MHz difference in the beat frequency between the two lasers) gave us sufficient sensitivity to make a measurement at the parts in 10^5 level of accuracy.

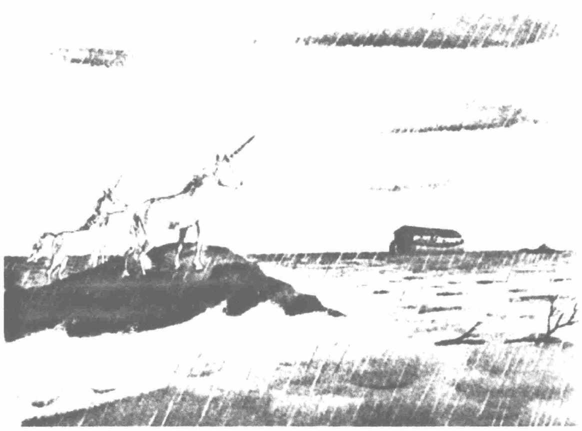


Fig. 54

To avoid a tilt issue similar to Poynting's, we needed to make the two four-wire suspensions exactly the same length, or at least nearly the same to minimize the correction that would need to be applied for systematic tilts. (Were one of our masses hanging from a longer suspension it would move more sideways when the apparatus is tilted than would the mass hanging on a shorter suspension.) In practice this adjustment is not as easy as it might seem though we were able to make the two pendulum lengths equal to within a few thousandths of a centimeter. [Many years ago at a meeting on gravitational wave detection, Vladimir Braginski, responding to a theorist's question of "why don't you just make the wire lengths the same?" answered by saying, "To change the length of the wire is almost as difficult as changing the height of my wife."]

But there was a major problem with our experiment: We got the wrong answer. Our result did not agree with Gundlach's value and hence with the CODATA-recommended value…or, for that matter with the 2001 BIPM result that was about as high as we were low from the recommended CODATA value. What to do? We worried that somehow or somewhere we had missed the boat [Fig. 54]. Accordingly, we spent the next 6 years looking under every stone and rock, expecting that some previously unrecognized systematic error source would show up; but in spite of all our searching, we were unable to find any not-already-included effects that would change our value for G. So we finally submitted for publication a result that agreed well with the 1986 CODATA recommended value, and in turn with the Luther-Towler 1982 value, but did not agree with CODATA's most recently recommended value.

The responses of our three reviewers, reproduced here in part, were all quite fair. Referee A thought that our "agreement" with Luther et al. was interesting, although, it was noted that it might be a coincidence. I, of course, would prefer to think of it as being interesting. Referee B thought the paper well written and clear for the general reader, and though he believed the reported discrepancy was due to some unidentified systematic effect, he nevertheless credited us for having "done due diligence in investigating all the major sources of error in this very challenging measurement."

Uncertainty component	$\delta G/G$ ($\times 10^{-5}$)
Six critical dimensions	1.4
All other dimensions	0.8
Source mass density inhomogeneities	0.8
Pendulum spring constants	0.7
Total mass measurement	0.6
Interferometer	0.6
Tilt due to source mass motion	0.1
Day-to-day scatter	0.4
Combined uncertainty	2.1

Fig. 55

The next figure [Fig. 55] identifies the major uncertainty components, and I simply note that to "understand" the 2 parts in 10^4 difference between our value and the one CODATA has recommended, we would have to be off in our estimate of at least one of our error sources by over a factor of 10!...unless of course we had overlooked something. And though we do make mistakes in estimating errors, I would like to believe that they are not at that level. [Lennart Robertsson (of the BIPM) once said regarding the weak point of error budgets, "You don't include what you don't know"…and I would add, "or think of." And I'm sure we didn't. The next figure [Fig. 56] points out that it is often-times very difficult to recognize the source of a problem. Some years ago, in connection with a talk I gave in 2002 at the CPEM meeting held that year in Ottawa, I wrote what appears as Figure 57 noting that physicists are human; they want their "new" number to agree with earlier measurements of the same quantity.

Fig. 56

Referee C thought that our approach was a nice alternative to the usual torsion-balance measurement, and had no recommendation on a value for G, or most importantly its ***realistic*** error bar (highlighting mine).

Because we are aware of earlier results (or predictions) we tend to look for and find systematic errors which permit us to correct our result until it stands at least in the shadow of these measurements—at this point, we stop looking, fold up our equipment, and publish our "new" result in substantial agreement with…

From the History of Precision Measurements by James Faller

Fig. 57

The lower figure [Fig. 58] shows the CODATA Recommended values for G, beginning in 1973. The vertical spacings in this figure reflect the time intervals between new recommendations, and the horizontal position reflects the numerical increases or decreases in the value that they gave as their recommended number.

In the last paragraph of Harold's and my PRL we wrote, "Having now completed our measurement, we are reminded of Cavendish's description of his 1798 experiment, "The apparatus is very simple." That statement also applies to the experiment that we report here. We would add, "The measurement is very hard."

Figure 59, a compendium of recent G measurements, appeared first in the 2013 T. J. Quinn et. al. paper [Ref. 7]) that describes their recent determination. The spread in the results of these measurements of G shows that this determination is hard for everybody. A glance at this chart shows three groups: there is the group on the left, a clustering in the middle, and two measurements on the right. The two determinations on the right, both BIPM determinations agree well with each other, but are high. The four measurements on the left also agree with each other, but are low. There

CODATA RECOMMENDED VALUES FOR G

(SI Units)

1973	**6.6720 (41) x 10^{-11} (6.1 x 10^{-4})**
1986	**6.67259 (85) x 10^{-11} (1.28 x 10^{-4}) [Luther-Towler]**
1998	**6.673 (10) x 10^{-11} (1.5 x 10^{-3}) [PTB]**
2002	**6.6742 (10) x 10^{-11} (1.5 x 10^{-4}) [G&M]**
2006	**6.67428 (67) x 10^{-11} (1 x 10^{-4})**
2010	**6.67384 (80) x 10^{-11} (1.2 x 10^{-4})**

Fig. 58

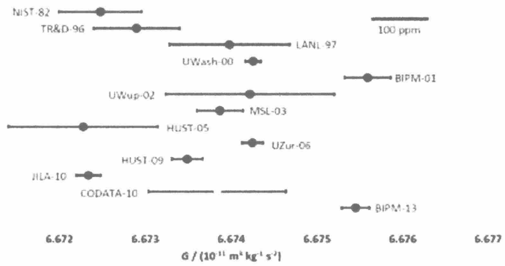


Fig. 59

are also four measurements in the middle that agree very well with each another. The remaining measurement, HUST-05, sits on the fence between the four measurements in the middle and the four on the left.

One of the very real problems with learning about some of these recent big G determinations is that the results of the ones with the smallest error bars have only been published in *Physical Review Letters*. Because of that journal's strict page limit (4 or if you're very lucky 5 pages), the descriptions of these many-years-long experiments are simply far too short to do justice to the work or to reveal to the reader just exactly was done. By contrast, Cavendish's "Measuring the Density of the Earth" paper is 57 pages long. And Cavendish didn't have a word processor! [Had Cavendish published in the Physical Review Letters, I doubt that he would have had space to include his observation of anelasticity.]

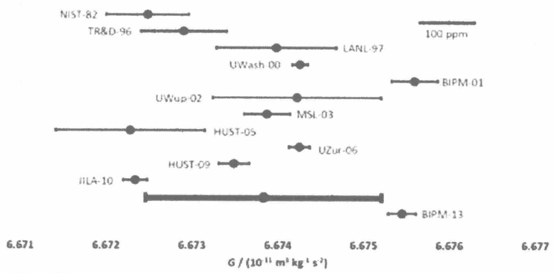


Fig. 60

What to do? I have a self-serving, but nevertheless reasonable, suggestion: I believe that a far better fit for ***all*** of these reported experiments would result if the one-sigma error bar associated with CPEM's present recommended number were increased by a factor of something less than two. [Fig. 60] This would admit that in spite of a lot of careful efforts, we probably don't know the value of G as accurately as we might like our colleagues to think we do. But it would respond to the third reviewer's (referee C's) "realistic error bar" hint as well as make some allowance for Lennart Robertsson's telling comment on the "weak point of error budgets. This would also make it plausible, within something like a 10 or 15 % chance, that either the JILA experiment or the BIPM's two experiments might be right. And of course, they might also be both wrong.

Now let us turn to some of the uses of knowing the value of big G….and here the answer is, "There are none!" Or at least there are almost none. Furthermore, none of the uses that I know of presently needs a value for G to better than 1%. Two uses familiar to the participants at this meeting are: One needs a value for G that is good to roughly 10% in order to correct absolute gravimeters for the self (attracting) mass of the instrument on the dropped object; and a second is that for borehole gravity measurements, one needs the value of G to roughly 1% in order to determine the actual density of the (assumed) horizontal mass layers.

"Ideas are like violets in the springtime, they spring up in many places at the same time."

(and I would add, in different centuries)

Gauss writing to the elder Bolyai about his son's discovery of non-Euclidean geometry…and urging him to publish!

Fig. 61

Much more important is the fact that the various error types that infect (affect?) big G determinations appear again and again in every area of physics. Accordingly, recognizing and understanding big G's measurement challenges has and is creating a knowledge base that directly transfers to "doing" all of the rest of physics. The measurement of G will always be ***the*** mountain that future generations of physicists could learn much from, and therefore society will benefit by continuing to encourage them to undertake this climb.

Fig. 62

Finally, let me conclude with a few thoughts beginning with Figure 61. Many of the ideas and technologies that grew out of the precision measurement of g and G have also sprung up in different places at the same time; but, in addition, these same ideas have appeared and will continue to appear in different centuries. The field of precision measurement is a field of multiple and time-consuming subtleties—a field where one learns to recognize the truth in the saying, "More things are known than are actually true." (J. R. Pierce). Great patience is required to do these kinds of experiments [Fig. 62] …and to get the right equation. And finally precision measurement physics is a thick board [Fig. 63] as well as an important and fascinating area of science.

"I HAVE LITTLE PATIENCE WITH SCIENTISTS WHO TAKE A BOARD OF WOOD, LOOK FOR ITS THINNEST PART, AND DRILL A GREAT NUMBER OF HOLES WHERE DRILLING IS EASY."
ALBERT EINSTEIN

Fig. 63

I show here a modified Figure 1 [Fig.64] to point out that the APS ***did*** have space, in spite of what they told me, to include both precision measurement and gravitational physics under the heading "ALL Areas of Physics." And, finally, in keeping with Bach, Figure 65 points out my belief that precision measurement, fundamental constants, and instrumental capabilities belong in the pedal line of science, for they set the tonality of and underpin the details of the works we physicists discover and play for the world. Lastly, Figure 66 answers an earlier question; it surely should be Precision Measurement…without the "***s***"; for this area of physics is ***not*** just a collection of accurate measurements but rather it is a unified field in which measurement science is used in a broad range of ***related*** experiments.

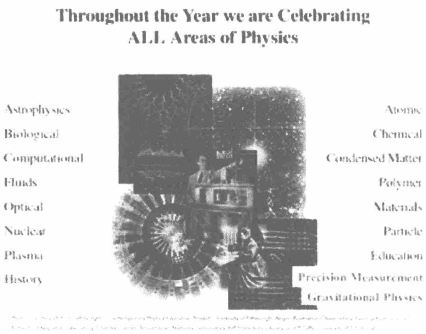


Fig. 64

Fig. 65

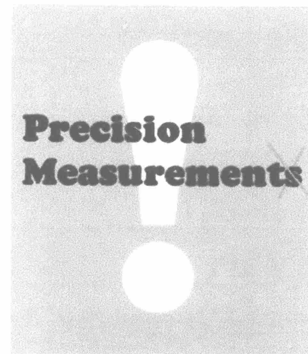


Fig. 66

Acknowledgements

I would like to thank the organizers of the IAG TG=SMM 2013 Symposium for (again) sponsoring a meeting that gives precision measurement scientists who are working in the field of terrestrial gravimetry an opportunity to hear and learn what their fellow scientists are working on, thinking about, and have accomplished. I would also like to thank Julie Phillips and Eyvon Petty of JILA's technical support staff for helping me with the writing and preparation of this paper. Finally, I would like to thank my wife, Jocelyne Bellenger/Faller, for her patience and understanding during the six hectic weeks when I was working on this paper.

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