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Large Number Hypothesis

: A Review

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Abstract Large dimensionless numbers, arising out of ratios of various physical constants, intrigued many scientists, especially Dirac. Relying on the coincidence of large numbers, Dirac arrived at the revolutionary hypothesis that the gravitational constant G should vary inversely as the cosmic time t . This hypothesis of Dirac, known as Large Number Hypothesis (LNH), sparked off many speculations, arguments and new ideas in terms of applications. Works done by several authors with LNH as their basic platform are reviewed in this work. Relationship between some of those works are pointed out here. Possibility of time-variations of physical constants other than G are also discussed.

Keywords Large numbers · variable G · anthropic principle

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1 Introduction

Physical constants are not uncommon to us. During our study of various laws of nature, we come across a number of constants entangled with those laws. But, it was Dirac who discovered an apparently unseen thread joining up

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those physical constants by a simple yet interesting law, viz., “Law of Large Numbers”. Using that law, Dirac arrived at his “Large Number Hypothesis” (LNH) which has profound influence on the world of physics. In fact, a plethora of works have been done, both at theoretical and observational level with LNH as their starting point.

These works can be divided into three broad categories:

- I. The first type of works are centered around modification of Einstein’s gravitational theory and related equations for adopting the idea of G variation as predicted by LNH [1, 2, 3, 4, 5, 6, 7, 8];
- II. Arguments and counter arguments in favor of and against LNH for justifying and refuting that hypothesis characterize the second category of works [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22];
- III. Testing and applications of LNH can be classified as the third line of investigation [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37].

Although so many years have passed after the inception of LNH, it has not lost its significance. Instead, perhaps it has gained a new momentum after the discovery of accelerating universe. The present cosmological picture emerging out of SN Ia observations [38, 39] reveals that the present universe is accelerating. Some kind of exotic repulsive force in the form of vacuum energy is supposed to be responsible for this acceleration which started about 7 Gyr. ago. This repelling force is termed as *dark energy* and is designated by Λ . On the way of investigating dark energy, many variants of Λ have been proposed including a constant Λ . But, compatibility with other areas of physics demands that Λ should be slowly time decreasing. Moreover, the currently observed [40, 41] small ($\approx 10^{-35} s^{-2}$) value of Λ suggests that it has decreased slowly from a very high value to its present nearly zero value. This type of time dependency of Λ has a similarity, so far as the main spirit of the idea is concerned, with that of the gravitational constant G as proposed by Dirac in his LNH [42]. Both Λ and G have descended from a very high initial value to its present small value because the universe is so old. It should be mentioned here that a long ago, while dealing with large dimensionless numbers, Eddington [43] proposed a large number involving the cosmological term Λ (then regarded as a constant geometric term) viz. $ch(m_n m_e / \Lambda)^{1/2} \simeq \sqrt{N}$, where m_n and m_e are the masses of nucleon and electron respectively, h is the Planck constant, c is the velocity of light and N is the total number of particles in the universe. It has also been shown that variable Λ models are generally characterized by particle creation [44] which is also a feature of LNH. All these suggest that the link between LNH and Λ can, in no way, be ignored. In fact, in recent years, a combined framework of LNH and the cosmological parameter is used to address a number of important issues such as explanation of flat galactic rotation curves [45], unification of LNH with general theory of relativity [46], possible implications of a variable fine-structure constant [34, 35] etc. Also, the problem of unifying all other forces with gravity demands an understanding of the coincidence of large numbers and hence LNH [47].

On the other hand, for unfolding the true nature of dark energy, a number of variants of the cosmological term have been suggested, one of them being kinematical Λ of phenomenological character (for an overview see [48]). Now,

since LNH predicts that the gravitational constant G should vary with time, a probable inter-connection between LNH and Λ cannot be ruled out. So an investigation about the behaviour of the scale factor and other cosmological parameters within the framework of varying G and Λ may be helpful in finding the significance of LNH in context of the present cosmological scenario. Such an attempt is made by Ray and Mukhopadhyay [37] in their work with phenomenological Λ model under varying G . In this review, most of the works mentioned in this section, will be discussed in subsequent sections in the relevant contexts. The paper is organized as follows: idea of large numbers and Dirac's LNH are given in Sec. 2 and 3 respectively. Sec. 4 deals with consequence of LNH in terms of modifications of gravitational theory and dynamical equations related to it. Arguments in favor of and against LNH are described in Sec. 5 while Sec. 6 is devoted to testing and applications of LNH. Finally, some discussions are being done in Sec. 7.

2 An Overview of the Large Numbers

Although Dirac's name is closely associated with large numbers, but it was Weyl [49, 50] who initiated the idea of large numbers. He arrived at the number 4×10^{42} through a comparison between the electron's radius r_e given by $q_e^2/4\pi\epsilon_0 m_e c^2$ with the hypothetical radius of a particle with a charge q_e and an electrostatic energy equal to that of electron's gravitational energy. It is easy to see that the above ratio is equal to that of electron's electrostatic Coulomb force (F_e) and electron's gravitational energy (F_g) as well as that of the energy of electron's electrostatic field (E_0) and electron's self-energy (E_g) due to its gravitational field. These connections prompted Weyl to speculate that the same ratio might also be held between the radius of the universe to that of an electron. Weyl's speculation was found to be nearly correct when astronomer Stewart showed [51] that the ratio of the radius of the universe and electron was only two orders of magnitude smaller (10^{40}) than that of Weyl's number. In the same year, Eddington [52] speculated that if N be the number of particles in the universe, then using the equation

$$\frac{GM_u}{R_u} = C_g^2 \quad (1)$$

where M_u is the mass of the universe, R_u is the radius of curvature of the universe, C_g is the maximum speed of recession of distant galaxies and G is the gravitational constant, one can write

$$\frac{F_e}{F_g} = \frac{E_0}{E_g} = \frac{q^2}{\pi\epsilon_0 r_e} / \frac{Gm_e^2}{r_e} = \sqrt{N} = \text{Weyl's number} \quad (2)$$

From equation (2), by substituting known values of the parameters, we can arrive at the number $N = 1.7507 \times 10^{85}$ which is known as Eddington's magic number. It is to be noted that Jordan [53] pointed that if M^* and m_e be typical stellar mass and electron mass respectively, then

$$\frac{M^*}{m_e} \approx 10^{60} \approx (10^{40})^{3/2}. \quad (3)$$

Recently Shemi-zadeh [54] has arrived at some large numbers of the order of 10^{60} by comparing cosmological parameters such as Hubble radius (R_H), mass of the universe (M_H), CMBR temperature (T_γ) with their counterparts of the macroscopic world, viz., Planck length (l_P), Planck mass (m_P), Planck energy (E_P) etc. For instance,

$$\frac{R_H}{l_P} \approx 10^{60} \approx \frac{E_P}{T_\gamma}. \quad (4)$$

Before leaving this section, special attention should be made for three large numbers involving gravitational constant G , atomic constants h , m_p and cosmological parameters like Hubble term H and ρ , the matter density of the observable universe. These three numbers are designated by N_1 , N_2 , N_3 and are given by [55]

$$N_1 \equiv \frac{\bar{h}c}{Gm_p^2} \simeq \frac{1}{6} \times 10^{39}, \quad (5)$$

$$N_2 \equiv \frac{mc^2}{hH} \simeq \frac{1}{3} \times 10^{39}, \quad (6)$$

$$N_3 \equiv \frac{\rho c^3}{m_p H^3} \simeq 10^{79}. \quad (7)$$

It is easy to see that N_1 , N_2 and N_3 can be related by

$$N_1 \simeq N_2 \simeq \sqrt{N_3}. \quad (8)$$

Without wasting more time, we conclude this section here only mentioning that a list (may not be exhaustive) of large numbers is given in Table 1.

3 Dirac's LNH and its extension

Even to a non-initiated person it seems surprising that the order of the ratios of fundamental constants of the macro and micro physical world can be so close in so many cases as demonstrated by equations (1)-(8). So, it is quite natural that a person like Dirac would be intrigued by those coincidence. Dirac thought that the coincidence of large numbers was far from being accidental. He had a firm belief that the coincidence seen among various cosmological and atomic constants was a manifestation of a hitherto unknown theory linking up the quantum mechanical origin of the universe to the various cosmological parameters. Dirac [59,60] pointed out that the ratio of electrical ($e^2/4\pi\epsilon_0 r^2$) and gravitational ($Gm_p m_e/r^2$) forces between proton and electron in a hydrogen atom is a large number of the order of 10^{40} , i.e.

$$\frac{e^2}{4\pi\epsilon_0 Gm_p m_e} \approx 10^{40} \quad (9)$$

Table 1 List of Large Numbers

Physical Constants Involved	Large Numbers	References
1. Electrostatic force F_E and gravitational force F_G between a proton and an electron	$F_E/F_G \simeq 10^{40}$	[47, 55, 56]
2. Radius of the universe R and the radius of an electron r	$R/r \simeq 10^{40}$	[55, 57]
3. Intensity of electromagnetic and gravitational interaction of elementary particles	$\frac{e^2/\hbar c}{Gm_e^2/\hbar c} \simeq 10^{40}$	[54, 57]
4. Mass of a typical star M_* and electron mass m_e	$M_*/m_e \simeq 10^{60} = (10^{40})^{3/2}$	[54]
5. Mass of the universe M_U and proton mass m_p	$M_U/m_p \simeq 10^{80} = (10^{40})^2$	[54]
6. Mass of the universe M_U and Planck mass m_P	$M_U/m_P \simeq 10^{61}$	[54]
7. Hubble radius R_H and Planck length l_P	$R_H/l_P \simeq 10^{60}$	[47, 54]
8. Planck mass density ρ_P and the observed matter density of the universe ρ	$\rho_P/\rho \simeq 10^{120} = N_1^{1/3}$	[54]
9. Planck energy E_P and CMBR temperature T_γ	$(E_P/T_\gamma)^2 \simeq 10^{60}$	[54]
10. Planck mass m_P and neutrino mass m_ν	$(m_P/m_\nu)^2 \simeq 10^{61}$	[54]
11. Planck mass m_P and electron mass m_e	$(m_P/m_e)^3 \simeq 10^{62}$	[54]
12. Planck mass m_P and pion mass m_π	$(m_P/m_\pi)^3 \simeq 10^{63}$	[54]
13. Planck mass density ρ_P and the current critical matter density of the universe ρ_c	$\rho_P/\rho_c \simeq 10^{121}$	[56]
14. Electron mass m_e and Hubble parameter H	$m_e c^2/\hbar H \simeq 10^{39}/3$	[57]
15. Matter density of the observed universe ρ , proton mass m_p and Hubble parameter H	$\rho c^3/m_p H^3 \simeq 10^{79}$	[57]
16. Number of nucleons N_4 in the universe	$\rho_0 (cH^{-1})^3/m_p = 10^{80} = (10^{40})^2$	[58]
17. Number of proton N_p and baryons N_b	$N_p/N_b \simeq 10^{10} = (10^{40})^{1/4}$	[58]

where e is the charge of an electron, m_p is the proton mass and ϵ_0 is the permittivity of space. Again, the ratio of the then age of the universe (2×10^9) and the atomic unit of time ($e^2/4\pi\epsilon_0 m_e c^3$) is also nearly of the same size, i.e.

$$\frac{4\pi\epsilon_0 m_e c^3}{e^2} \approx 10^{40}. \quad (10)$$

Dirac suggested that the two quantities in the left hand side of equations (9) and (10) are equal, i.e.

$$\frac{e^2}{4\pi\epsilon_0 G m_p m_e} \simeq \frac{4\pi\epsilon_0 m_e c^3}{e^2}. \quad (11)$$

Relying on equation (11), Dirac proposed that as a consequence of causal connections between macro and micro physical world, some of the fundamental constants cannot remain constant for ever; rather they should vary with time, however small the change might be. This is known as Dirac's LNH. According to LNH, atomic parameters cannot change with time and hence the gravitational constant should vary inversely with time, expressed in atomic units, i.e.

$$G \propto \frac{1}{t}. \quad (12)$$

Also, the hypothesis demands that creation of matter occurs continuously in the universe. This creation of matter can occur in two possible ways, viz., “additive creation” and “multiplicative creation”. According to “additive creation theory”, matter is created through the entire space and hence in intergalactic space also. In “multiplicative creation theory”, creation of matter occurs only in those places where matter already exists and this creation proceeds in proportion to the amount and type of atoms already existing there. According to general relativity, G is constant and hence we cannot readily consider G as a variable quantity in Einstein equation. To overcome this difficulty, Dirac considered two metrics. The equations of motion and classical mechanics are governed by the Einstein metric which remains unaltered while the other metric, known as atomic metric, includes atomic quantities and the measurement of distances and times by laboratory apparatus [61]. The interval $ds(A)$ separating two events as determined by apparatus in atomic system of units (a.s.u.) will be different from the interval $ds(G)$ between the same two events as measured in the gravitational system of units (g.s.u.). This implies that equations written in g.s.u. and a.s.u. cannot be used at a time until one of them is converted to the other system of units [6]. The velocity of light is unity for both metrics. Considering the case of a planet orbiting the sun, Dirac [62] showed that the relationship of Einstein and atomic metric was different for additive and multiplicative creation theory. In terms of the atomic distance scale, the solar system is contracting for the additive creation model while it is expanding in multiplicative creation.

Zeldovich [63] extended Dirac’s LNH by including the cosmological parameter in its realm and defined Λ by

$$|\Lambda| = \frac{8\pi G^2 m_p^6}{h^4}. \quad (13)$$

Then Zeldovich showed that Λ produces the same gravitational field in the vacuum as that produced by matter in space and hence the cosmological term should be included in the field equations as full fledged term in the presence of ordinary matter. The gravitational energy of the vacuum was interpreted by Zeldovich as interactions of virtual particles separated by a distance $h/m_p c$ and the amount of energy created by the gravitational interactions of these particles is given by

$$\epsilon = \frac{G m_p^6 c^4}{h^4}. \quad (14)$$

LNH was further extended by Sakharov [64] who proposed a gravitational theory based on the consideration of vacuum fluctuations. He suggested that there should be a fundamental length $\sim 10^{-33} \text{ cm}$, less than which the theory is not valid. According to Matthews [65], in the extended LNH, the mass of a proton (chosen by Dirac) should be replaced by the effective mass of the vacuum energy density of the relevant epoch because this replacement would agree with experimental results. It has already been mentioned that Eddington [43] linked up the cosmological parameter Λ with the total number of particles of the universe through the relation

$$ch(m_n m_e / \Lambda)^{1/2} \simeq \sqrt{N}. \quad (15)$$

Above relation tells us readily that if N increases with the age of the universe, then Λ is also time dependent. Berman [66] has called this whole idea as Generalized LNH.

4 Consequences of Dirac's LNH

4.1 Variable G Cosmology

Dirac's LNH has many significant consequences and as such depending on it, a plethora of works have been done. Most of these works are centered around the variation of G .

4.1.1 Scale-covariant Theory

It is well known that Einstein's equations of general relativity do not permit any variation in the gravitational constant G because of the fact that the Einstein tensor has zero divergence and by energy conservation law $T_{\mu\nu}$ is also zero. So, in the light of Dirac's LNH, some modifications of Einstein equations are necessary. This is because, if we simply allow G to be a variable in Einstein equations, then energy conservation law is violated [2, 22]. So, the study of the effect of varying G can be done only through modified field equations and modified conservation laws. For this purpose, Canuto et al. [1, 2] developed a scale-covariant (also termed as scale-invariant) theory. In this theory a gauge function β is chosen and the essence of scale-covariant theory lies in the fact that physical laws remain unaffected by the choice of the gauge function. Scale-covariant theory is developed by scale transformation and using different dynamical systems for measuring space-time distances. Let us start with the conventional Einstein equation

$$\bar{G}_{\mu\nu} = -8\pi\bar{T}_{\mu\nu} + \Lambda\bar{g}_{\mu\nu} \quad (16)$$

having the line element

$$\bar{d}s^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu. \quad (17)$$

Using the transformation

$$ds = \beta^{-1}(x)\bar{d}s \quad (18)$$

and by considering necessary modification of Ricci tensors [67] (i.e. of Einstein tensors as well) it has been shown [2] that the field equations, in general units, can be written in modified form as

$$G_{\mu\nu} + 2\frac{\beta_{\mu;\nu}}{\beta} - 4\frac{\beta_\mu\beta_\nu}{\beta^2} - g_{\mu\nu}\left(2\frac{\beta^\lambda_{;\lambda}}{\beta} - \frac{\beta^\lambda\beta_\lambda}{\beta^2}\right) = -8\pi T_{\mu\nu} + \Lambda g_{\mu\nu} \quad (19)$$

where $T_{\mu\nu}$ is the energy-momentum tensor and Λ is the so called cosmological term which is related to $\bar{\Lambda}$ by the relation

$$\Lambda = \beta^2\bar{\Lambda}. \quad (20)$$

It is also shown that energy conservation law can be written in a modified form as

$$\dot{\rho} + (\rho + p)u^\mu_{;\mu} = -\rho \left(\frac{\dot{G}}{G} + \frac{\dot{\beta}}{\beta} \right) - 3p \frac{\dot{\beta}}{\beta}. \quad (21)$$

The corresponding Friedmann equations with the conventional Robertson-Walker metric

$$ds^2 = dt^2 - R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (22)$$

reduces to the form

$$\left(\frac{\dot{R}}{R} + \frac{\dot{\beta}}{\beta} \right)^2 + \frac{k}{r^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3}, \quad (23)$$

$$\frac{\ddot{R}}{R} + \frac{\ddot{\beta}}{\beta} + \frac{\dot{\beta}}{\beta} \frac{\dot{R}}{R} - \frac{\dot{\beta}^2}{\beta^2} = -\frac{4\pi G}{3}(3p + \rho) + \frac{\Lambda}{3}. \quad (24)$$

The energy conservation laws, in this case, become

$$\dot{\rho} + 3 \frac{\dot{R}}{R}(\rho + p) = -\frac{\rho}{G\beta} \frac{d}{dt}(G\beta) - 3p \frac{\dot{\beta}}{\beta}. \quad (25)$$

However, in the scale-covariant theory [1,2], the cosmological term Λ is not a constant rather it varies like β^2 . Now, determination of $\beta(t)$ is very crucial for scale-covariant theory because determination of $\beta(t)$ enables us to compare that theory with observational results. As $\beta(t)$ gives us the liberty in choosing the system of units, it is not possible to determine it within the theory. This means that we cannot formulate any dynamical equation for determining $\beta(t)$ and hence imposition of external constraint is obligatory here. Relation with gauge fields and the cosmological parameter suggests that β is inversely proportional to t . Again, adopting LNH, it can be shown that β can alternatively be written as $\beta = t_0/t$, where t_0 is the present age of the universe [1,2,3].

Using their scale-covariant cosmology, Canuto and Hsieh [9] have shown that it is possible to reconcile $3K$ blackbody radiation with Dirac's LNH and scale factor $R(t)$ as well as curvature constant k can be determined without using m versus z relation or any other classical cosmological test. It has also been demonstrated there [9] that $k = 0$ i.e. universe is flat and the relation between G and the gauge function $\beta(t)$ is given by

$$G\beta^2 = \text{constant}. \quad (26)$$

It may be mentioned here that recent observational results provided by WMAP, COBE etc. support scale-covariant cosmology regarding the geometry of the universe.

Being intrigued by Dirac's LNH Peng [46] has modified Einstein's general relativity theory by allowing G to be a variable (as suggested by Dirac) so

that the new theory can be applied to cosmology without any inconsistency. In Peng's work, a tensor term arising from variation of G plays the role of the cosmological term Λ . Moreover, natural constants which evolve with time (viz., m_e, m_p, e, \hbar, K_B etc.) are modified there [46] systematically so that the new constants remain really constant for sufficiently long time. In another work, Peng [4] has modified Einstein's general theory of relativity by considering the gravitational constant G as a variable. This modification is achieved by including a tensor term which crops up naturally from the derivative of G and not from the cosmological term Λ . Unlike Dirac, Peng assumed that m_e, m_p and e are not constants but evolve according to the rules $Gm \propto t$ and $e^2 \propto tm_e$ where m may be any one of m_e, m_p or M , the total mass of the universe. Then, by making a fundamental assumption of the form

$$\phi^2 = (G/G_0)^{1/n} = (t/t_0)^{-1} \quad (27)$$

where ϕ^2 is a dimensionless variable, Peng [4] showed that in the modified Einstein equation the cosmological term Λ_α^β is determined by ϕ . For the usual FLRW metric, the modified field equations become

$$R^3 \frac{d\phi^2}{dt} = (3 - \omega/2)^{-1} 8\pi G_0 \tilde{\rho}_m(t') R^3(t') t, \quad (28)$$

$$\phi^2 \frac{-3(\dot{R}^2 + k)}{R^2} - 3\phi^2 \dot{R}/R - \omega \dot{\phi}^2/2 = \frac{-8\pi G_0 \tilde{\rho}(t') R^3(t')}{R^3} \quad (29)$$

where t' is an arbitrary constant argument.

It has been shown by Peng [4] that this modified theory is consistent with LNH. Moreover, Hubble's relation, derived from this new theory is compatible with observational results. Einstein's theory is shown to be a special case of the modified theory for phenomena of short duration extended to short distances.

For reconciling LNH with Einstein's theory of gravitation, Lau [5] selected the general form of Einstein's field equations, viz.,

$$R^{\mu\nu} - (1/2)g^{\mu\nu}R + \Lambda g^{\mu\nu} = -8\pi G T^{\mu\nu} \quad (30)$$

and showed that for a time-dependent G and Λ , LNH satisfies equation (30). According to Lau [5], time varying G is important only for the early period or for knowing the entire evolutionary history of the universe. But at a time sufficiently away from the Big-Bang, G can conveniently be regarded as a constant. He argues also that at the present epoch, cosmological term Λ has a very small value and hence can be approximated to zero. With a constant G and zero Λ , equation (30) reduces to the conventional form

$$R^{\mu\nu} - (1/2)g^{\mu\nu}R = -8\pi G T^{\mu\nu} \quad (31)$$

of Einstein's field equations. It is to be noted that being motivated by LNH and assuming time dependent Λ and G , Lau and Prokhovnik [68] have developed a scalar-tensor theory in terms of an action principle as a modification of Einstein's theory.

4.1.2 Dirac's theory of variable G

Another G -variable theory is Dirac's theory [59,7] which is a theory of the background structure of the universe and its geometrical properties are described by a gauge function β which determines the ratio of gravitational units and electromagnetic units [22]. Dirac's theory is related to the question of a zero or non-zero cosmological term Λ . Field equations and LNH imply that two natural gauges are associated with Dirac's theory [59,7]. One gauge, where $\Lambda = 0$, is known as additive creation (zero gauge) theory. Another gauge in which Λ is finite is known as multiplicative creation theory. So, continuous creation is an integral part of Dirac's theory.

4.1.3 Hoyle-Narlikar theory

Hoyle-Narlikar theory [8] is based on two main principles, viz., conformal invariance and the absorber theory of radiation [22]. For conformal invariance, natural laws remain invariant under changes of the gauge function β and hence possesses a technical similarity with Dirac's theory. The second principle is a theory of electromagnetism and has been shown to be an integral part of cosmology by Hoyle-Narlikar theory.

5 Status of Dirac's LNH

5.1 Arguments against Dirac's LNH

Time and again, a number of objections have been raised against Dirac's LNH [59,60]. According to Falik [17], $3K$ blackbody radiation cannot be reconciled with LNH, not even in the version of LNH that is constructed by taking into account the blackbody radiation. Because, LNH cannot explain the large amount of helium present in the universe. Again, a prediction of Dirac's LNH is that the number of nucleons in the universe should vary as t^2 where t is the cosmic age. Norman [18], citing some examples, has shown that multiplicative creation theory is in huge disagreement with observational results regarding isotropic abundance ratios of some elements. For instance, it has been found [69,70] that $^{13}\text{C}/^{12}\text{C}$ is 1.1×10^{-2} while according to LNH [18] it is as large as 4.3. Also, according to Norman's [18] calculation $^{40}\text{K}/^{39}\text{K}$ ratio obtained from Dirac's LNH is 14 while in reality, it is 1.3×10^{-14} [71]. Thus, in both the cases the values as provided by LNH are much higher than the actual values. Another point of objection against Dirac's LNH is self-consistent within the universe of Einstein de-Sitter model. If only curvature constant and the cosmological term are both zero, then Dirac's theory is compatible with the well known Friedmann-Lemaître model. Otherwise, the relation $N_1 = N_2 \simeq \sqrt{N_3}$ should be considered as accidental and hence no inference can be drawn regarding variation of G and continuous creation of matter [19]. Many cosmological models proposed by Dirac require photon creation or destruction. On the other hand, for any cosmological model that obeying Cosmological Principle and evolving in time, number of photons

in a co-moving volume must be conserved. Steigman [21] has shown that cosmological models based on Dirac's LNH are in sharp disagreement with the standard Planck spectrum. For verifying the plausibility of continuous creation of matter as predicted by Dirac relying on his LNH, Steigman [20] developed cosmological models with two modes of particle creation. It has been shown there that creation of particles is unnecessary because, as it is expected, the number of particles N varies as t^2 where t is the cosmic age.

5.2 Arguments in favor of Dirac's LNH

However, many workers have supported Dirac's LNH by raising various counter-arguments. Canuto and Hsieh [9] have provided a physical basis needed for Dirac's cosmology and have derived the relevant dynamical equations. It has also been shown by them [9] that LNH is not in disagreement with $3K$ blackbody radiation, rather it is of utmost importance for predicting the scale factor and the curvature constant. In another work, Canuto and Hsieh [10] have refuted Falik's [17] argument by showing that Falik's conclusion is based on two faulty assumptions, viz.,

$$\rho_\gamma \sim T^4, \quad (32)$$

$$RT = \text{constant} \quad (33)$$

where ρ_γ is the energy density of radiation in local thermodynamic equilibrium, T is the equilibrium temperature and R is the scale factor of RW metric. According to Canuto and Hsieh [10], correct relations should have been

$$\rho_\gamma \sim \beta^2 G^{-1} t^4, \quad (34)$$

$$\beta RT = \text{constant}. \quad (35)$$

Moreover, in the opinion of Canuto and Hsieh [10] the relation $G \sim t^{-1}$ should not be extrapolated backwards in the early epoch as done by Falik [17] because, this relation is compatible with observational results only in the present matter dominated universe [11,12]. Bishop [13] has opposed Julg's [19] argument by stating that one should not assume LNH and $G\rho R^3 \sim \text{constant}$ simultaneously because LNH cannot be discussed within FRW models. Not only that, the relation $G\rho R^3 \sim \text{constant}$ is inconsistent with two other relations proposed by Dirac [60,59,62], viz.,

$$N \sim \rho R^3 = \text{constant}, \quad (36)$$

$$GN \sim t \quad (37)$$

where N is the number of nucleons in the universe.

LNH can be used to determine the gauge function $\beta(x)$ involved in the scale covariant theory developed by Canuto and others [1,2]. It was previously found that this $\beta(x)$, when applied in computations of stellar evolution,

support multiplicative creation but not additive creation. Slothers [72] made a rough calculation for determining the effects of multiplicative creation on the luminosity of a white dwarf and showed that the lower bound of the luminosity was at least one order of magnitude higher than that which was observed physically. But, through a detailed calculation, Lodenquai [32] has refuted Slothers's conclusion regarding lower bound of white dwarf luminosity and hence viability of multiplicative creation theory is not lost. Relying on his expansion centre model [73,74] related to Milky Way, Lorenzi [16] has been able to develop a cosmological picture of the universe which is shown to agree with Dirac's LNH. Chao-wen and Slothers [75] have shown that Dirac's multiplicative creation theory do not violate any observed fact about sun, but theory of additive creation is not of that status. Genreith [76] has shown that Dirac's LNH can be explained by a fractal model of the universe. Moreover, Dirac's conjecture regarding an inter-connection between micro and macro universe and that of Einstein about the role of gravitational fields in the structure of elementary particles can be reconciled using that model [76].

However, Holographic principles can be helpful for explaining Dirac's LNH. In very simplified form, holographic principle states that the entropy S (actually, S is the entropy divided by σ , the Boltzmann constant) of a physical system subject to gravity is bounded from above by a quarter of its boundary area in Planck units i.e.

$$S \leq \frac{A}{4l_p^2}. \quad (38)$$

Also, \hbar , G and c provide a natural system of units of length and mass given by

$$l_p = \sqrt{\frac{\hbar G}{c^3}}, \quad (39)$$

$$m_p = \sqrt{\frac{\hbar c}{G}}. \quad (40)$$

According to Bousso [14] the holographic principle leads to the prediction that the number of degrees of freedom N available in the universe is related to the cosmological parameter Λ by the relation

$$N = \frac{3\pi}{\Lambda l_p^2 \ln 2}. \quad (41)$$

The N bound conjecture states that, S is bounded by $N \ln 2$. Marugan and Carneiro [15] have shown that if one assumes a homogeneous, isotropic and flat universe dominated by the cosmological term Λ , then Dirac's large number coincidence can be explained in terms of the holographic N bound conjecture.

Another objection raised against $G \propto 1/t$ relation comes from stellar astrophysics. The problem is like this: since stellar luminosity is proportional to $G^7 M_\star^5$ [22], then variation of G implies an abnormally high solar luminosity

which does not fit with observations. But, according to Wesson and Goodson [22] the relation $GM_\star = \text{constant}$ [2] leads to the result that L is nearly constant. If this consideration, along with the change of earth-sun distance due to change in G (and hence possible change in mass as well), is taken into account then this problem of variable G cosmology may be solved.

Davidson [30], in an attempt for testing Dirac cosmology in the light of observational results, has shown that if one assumes Dirac's LNH and admits the existence of two metric scales, viz., atomic scale or A scale and Einstein scale or E scale then many important observational cosmological features like Hubble's parameter, the cosmic age, the cosmic mass density follow naturally. It is to be noted here that Dirac's LNH-based cosmology differs from that of canonical general relativistic (GR) cosmology. For instance, according to Dirac cosmology, the atomic age of the universe is $T/3$ where T is the observed Hubble time, whereas in General Relativity it is $2T/3$ for $\Omega_0 = 1$.

6 Application and Testing of LNH

6.1 Cosmology and Astrophysics

Following Dirac's [59, 60] approach regarding large number coincidence, Barrow [31] has predicted proton half-life period. His prediction can be compared with experimental results for verification. Relying on the experimental results of the time variation of the fine-structure constant [77, 78], Berman and Trevisan [34] considered a model in which electric permittivity ϵ_0 and magnetic permeability μ_0 vary with time such that the speed of light remains constant. Using LNH, it has been possible to judge the time dependency of N , the number of nucleons in the universe [34]. Moreover, value of the deceleration parameter as estimated from the same investigation is compatible with the supernova data [38, 39] while the calculated value of the cosmological term falls within the acceptable range. In another work, relying on the experimental data of Webb et al. [78] regarding the time variation of the fine structure constant, Berman and Trevisan [35] have derived possible time variations of some other parameters of the universe, viz., number of nucleons in the universe, the speed of light, gravitational constant and the energy density. It has been possible to calculate [35] the value of the deceleration parameter which points towards an accelerating universe and hence supports SN Ia data [38, 39].

It should be mentioned here that Gomide [79] studied cosmological models with varying c and (or) varying ϵ_0 including LNH in his framework of study. Berman and Trevisan [34] elaborated a full model containing a Jordan-Brans-Dicke (JBD) framework with time-varying speed of light. In another work, Berman and Trevisan [35] have commented that similar conclusions could be attained by applying Dirac's LNH with $c = c(t)$.

Being intrigued by the suggestion of Dirac, regarding time variation of G , Garcia-Berrow et al. [33] have investigated about the effect of a decreasing G on the cooling rate of white dwarfs. It has been shown there that variable G strongly affects the cooling rate of white dwarfs at low luminosity. The star expands and cooling process gets accelerated due to decrease in G . For two

separate cases, two upper bounds for \dot{G}/G are derived [33] which are in good agreement with those obtained from binary pulsar.

Using the Weak Field Approximation, Whitehouse and Kraniotis [45] have determined the value of the cosmological parameter Λ from galactic rotation curves and have found that it agrees with their theoretically derived value of the same parameter using extended LNH. In the same paper, values of other cosmological parameters, viz., gravitational constant G , gravitational modification constant and the effective mass density were predicted. They have also shown that within the extended LNH, only two parameters, viz., fundamental length and the vacuum energy density are sufficient to completely specify the cosmological parameters for that epoch. This approach, according to them [45], may be helpful for finding a fundamental theory linking up the atomic and cosmological parameters and Dirac's dream may come true. Moreover, it has been shown by them [45] that the flat rotation curve of galaxies may be explained by the cosmological term Λ and presence of dark matter is not necessary if Newton's gravitational equation is modified in the form

$$F_m = -\frac{Gmm_0}{r^2} + G_\Lambda mr \quad (42)$$

where G_Λ is the gravitational force exerted by the cosmological term Λ and represents a fifth fundamental force which is directly proportional to the distance. However, the value of Λ derived in [45] is negative and hence represents a decelerating universe, which goes against the modern picture of an accelerating universe with a repulsive cosmological term.

Gilson [36] has shown that implications of Dirac's LNH can be directly derived from his quantum theory of gravitation [80] and three large numbers $\alpha\hbar/Gm_pm_e$, c/Hr_e and N can be expressed as three closed formulae with definite coefficients involving other known physical constants. Secondly, from the theory mentioned above, it has been possible to develop two quantum Friedmann cosmologies in which the cosmological parameter Λ plays a very basic and fundamental role having nice agreement with measurement. Dahlen and Honl [81] has shown that variability of G , as proposed by Dirac, suggests that QSO's are normal galaxies at their early stages and vice-versa.

Constructing two solar models for testing two types of matter creation of Dirac's cosmology, Carignan et al. [82] have shown that the first model fits very well with the theory of multiplicative creation of Dirac while the second model does not support the modified multiplication theory [83] in the sense that it shows an excess solar neutrino flux and presence of excessive hydrogen on the surface. Assuming Dirac's multiplicative theory (i.e. $G \propto t^{-1}$ and $M \propto t^2$) VandenBerg [84] has theoretically calculated isochrones and luminosity functions for old stellar systems. His results do not show any difference from normal stellar evolution regarding colour-magnitude diagram. Applying his scalar-tensor theory to a cosmological model which obeys LNH, Lau and Prokhorovnik [68] have deduced the time dependent form of the cosmological parameter. A viable explanation for the smallness of Λ is also provided by them along with the possible significance of the scalar field [68].

6.2 Impact on Planetary Science

LNH has cast its shadow in the field of planetary science also. According to the literature, LNH has tremendous influence on the rotation of the earth. Blake [23] has shown that early Dirac, additive creation and multiplicative creation theories have different predictions about variations of lengths of month and year. Comparing fossil data regarding these variations with those predicted by LNH, Blake [23] has suggested that early Dirac and additive creation versions of LNH cannot be accepted here while multiplicative creation theory fits well with observations.

LNH and hence variation of G has played a role in the calculation of earth's rate of expansion also. The average rate of expansion, calculated by several geophysicists, is 0.48 mm. per year. Such a large expansion cannot be explained in terms of geophysical processes. Jordan [24] and Egyed [25] proposed that the amount of expansion may be explained if one admits the variability of G , as suggested by Dirac. Lyttleton and Fitch [26], taking into consideration a variable G , calculated the change of radius of earth consisted of a liquid core and a mantle. Yubushita [27] constructed an earth model with a liquid core, a mantle and an outer shell and obtained the following relationships between rates of change of earth's radius and that of G for three versions of Dirac's LNH:

(i) For no creation model,

$$\frac{\dot{R}}{R} = -0.062 \times \frac{\dot{G}}{G}, \quad (43)$$

(ii) For additive creation,

$$\frac{\dot{R}}{R} = -0.33 \times \frac{\dot{G}}{G}, \quad (44)$$

(iii) For multiplicative creation,

$$\frac{\dot{R}}{R} = -0.61 \times \frac{\dot{G}}{G}. \quad (45)$$

Taking $\dot{G}/G = -1/t$ and using the value of the Hubble parameter as $6 \times 10^{-11} yr^{-1}$, he finally obtained $\dot{R} = 7 \times 10^{-3} cm yr^{-1}$ for no creation, $\dot{R} 1.9 \times 10^{-2} cm yr^{-1}$ for additive creation and $\dot{R} 2.5 \times 10^{-2} cm yr^{-1}$ for multiplicative creation models. According to Yubushita [27], both no creation and additive creation models are inadequate for explaining the calculated rate of earth's expansion. The third option, i.e. multiplicative creation theory can be consistent with the observational data if the value of the Hubble parameter H be taken to be $10^{-10} yr^{-1}$. In another paper, Yubushita [28] has also shown that if G changes in accordance with Dirac's LNH, then the radius of the primordial earth would have been 700 km. less than the present value. Of course, under some special assumptions, this change in radius is shown to be energetically feasible. Analyzing lunar occultation data since 1955, Van Flandern [29] calculated moon's mean longitude. Faulkner [61] showed that Van Flandern's result is consistent with Dirac's additive creation theory.

6.3 Amount of Variation of G

Ever since Dirac's proposition of a possible time variation of G , a volume of works has been centered around the act of calculating the amount of variation of the gravitational constant. Gaztanaga et al. [85], relying on data provided by SN Ia [38,39] have shown that the best upper bound of the variation of G at cosmological ranges is given by

$$-10^{-11} \leq \left| \frac{\dot{G}}{G} \right| \leq 0 \quad (46)$$

where z , the red-shift, assumes the value nearly equal to 0.5. Observation of spinning-down rate of pulsar PSR J2019+2425 provides the result [86,87]

$$\left| \frac{\dot{G}}{G} \right| \leq (1.4 - 3.2) \times 10^{-11} \text{yr}^{-1}. \quad (47)$$

Depending on the observations of pulsating white dwarf star G 117-B 15A, Benvenuto et al. [88] have set up the astereoseismological bound on \dot{G}/G as

$$-2.50 \times 10^{-10} \leq \left| \frac{\dot{G}}{G} \right| \leq 4 \times 10^{-10} \text{yr}^{-1} \quad (48)$$

while using the same star Biesiada and Malec [89] have shown that

$$\left| \frac{\dot{G}}{G} \right| \leq 4.1 \times 10^{-11} \text{yr}^{-1}. \quad (49)$$

Using Nordtvedt's [90] expression for \dot{G}/G in generalized Brans-Dicke theory, Sahoo and Singh [91] have shown that for some particular values of the coupling constant ω_0 , viz., -1.9 or -1 , the numerical value of \dot{G}/G at present is about 2×10^{-10} per year which lies within the observational limit. Various ranges of time variations of G , provided by both theoretical and observational results are enlisted in Table 2.

It may be mentioned that, being motivated by LNH, Ray and Mukhopadhyay [37] have confronted theoretically derived values of \dot{G}/G obtained by solving Friedmann equations (for flat model) with time dependent G and time dependent Λ of phenomenological character for testing the plausibility of experimentally determined values of variation of the gravitational constant. It has been shown by them that for certain values of the parameters of the models, theoretically and experimentally determined values agree with each other. Moreover, it has been found that G decreases with time as suggested by Dirac [59]. Starting from Raychaudhuri's equation [58] for the Brans-Dicke theory [92], Berman [93] has derived an expression for \dot{G}/G involving the deceleration parameter q and density parameter Ω . It has been shown there that for no variation of G , one must have $2q = \Omega$. Now, for an accelerating universe $q < 0$ whereas Ω is always positive. This implies that time variation of G is guaranteed by an accelerating universe.

Table 2 Values of α and β for average \dot{G}/G when $t_0 = 14$ Gyr, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $z \simeq 0.5$

Ranges of $\dot{G}/G \text{ yr}^{-1}$	Sources	α	β
$-(1.10 \pm 1.07) \times 10^{-11} < \frac{\dot{G}}{G} < 0$	PSR 1913 + 16 [94]	-0.0852	0.4074
$-1.60 \times 10^{-12} < \frac{\dot{G}}{G} < 0$	Helioseismological data [95]	-0.0115	0.0670
$(-1.30 \pm 2.70) \times 10^{-11}$	PSR B1855+09 [86,96]	-0.1023	0.4698
$-1.40 \times 10^{-11} < \frac{\dot{G}}{G} < +2.60 \times 10^{-11}$	Loren-Aquilar et al. [97]	0.0410	-0.2812
$-10^{-11} \leq \frac{\dot{G}}{G} < 0$	Supernove Type Ia [85]	-0.0769	0.3750
$(-8 \pm 5) \times 10^{-11}$	Lunar occultation [29]	-1.333	1.6
$(-6.4 \pm 2.2) \times 10^{-11}$	Lunar tidal acceleration [29]	-0.8421	1.4328
-15.30×10^{-11}	Early Dirac theory [23]	11.7692	2.0582
-5.1×10^{-11}	Additive creation theory [23]	-0.5730	1.2644
$(-16 \pm 11) \times 10^{-11}$	Multiplication creation theory [61]	8.00	2.0869
$-4.0 \times 10^{-13} < \frac{\dot{G}}{G} < +3.0 \times 10^{-13}$	Big Bang Nucleosynthesis [98]	-0.0003	-0.0021
$-2.5 \times 10^{-10} \leq \frac{\dot{G}}{G} \leq +4.0 \times 10^{-11}$	WDG 117-B15A [88]	-3.0	1.8
$ \frac{\dot{G}}{G} \leq +4.10 \times 10^{-10}$	WDG 117-B15A [99]	1.1319	3.09
$-(0.6 \pm 4.2) \times 10^{-12}$	Double-neutron star binaries [100]	-0.0043	0.0254
$(0.46 \pm 1.0) \times 10^{-12}$	Lunar Laser Ranging [101]	0.0318	-0.2110
$1 \times 10^{-11 \pm 1}$	Wu and Wang [102]	0.0666	-0.5

7 Discussions

Ever since the inception of the idea of large numbers, it has drawn attention of researchers in various fields ranging from geophysics and earth science to astrophysics and cosmology. In this article we have reviewed all these works in a systematic way to get the present status and future prospects. Here, main thrust is given on cosmology, astrophysics and planetary science in terms of LNH.

Now, success of a theory depends on its experimental verification, applications in different fields and significant predictions. From the previous sections it is clear that Dirac's LNH satisfies all these criteria. LNH related works have an interdisciplinary flavor in the sense that researchers of various fields have worked on this particular topic in a unique manner. Not only that, some of the works done on LNH from different standpoint seem to have a relation between them. Berman and Trevisan [34] investigated about a possible time variation of electric permittivity ϵ_0 and magnetic permeability μ_0 and showed that total density of the universe ρ is approximately proportional to t^{-2} . Ray and Mukhopadhyay [37] considered a flat Friedmann model with variable Λ and G . It is interesting to note that time variation of ρ in [37], for a small value of the parameter α , will be similar to that [34]. Again, in another work of Berman and Trevisan [35] regarding time variation of the gravitational constant, velocity of light etc., it is shown that $\rho \propto t^{-1}$ which may also be obtained from the work of Ray and Mukhopadhyay [37] for negative α , i.e. attractive Λ . In this regard it is to be noted here that Gilson [80] has proposed a model in which necessity of dark matter for explaining the flat galactic rotational curve is alleviated by invoking the idea of an attractive

Λ producing a decelerating universe. It is already mentioned that negative Λ can be obtained from the work of Ray and Mukhopadhyay [37] for a negative α without creating any disturbing physical feature relating the scale factor, energy density and gravitational constant. Shemi-zadeh [54], starting from various large number coincidence and making some suppositions involving the fine structure constant, the Hubble parameter, the Planck frequency etc., has derived simple expressions for the age of the universe, the deceleration parameter, the Hubble parameter etc. According to Shemi-zadeh [54], his work can be extended further with suitable physical models which may be connected to the Brane world models. It may be mentioned here that unlike most of the literature where $G \propto t^{-1}$, Milne [103] found that G is directly proportional to the cosmic age t . Recently, using Dirac's LNH, Belinchon [104] has arrived at the same relation between G and t as that of Milne [103]. In his bulk viscous model (although not directly related to or motivated by LNH) with time-varying Λ and G , Arbab [105] has shown that G increases with time. In a multi-dimensional model with an Einstein internal space and a multi-component perfect fluid, Dehnen et al. [106] have considered expressions for \dot{G} . A suggestion for a possible mechanism for small \dot{G} in the case of two non-zero curvature without matter is made there [106]. For the 3-space with negative curvature and internal space with positive curvature, an accelerating universe with small value of \dot{G}/G near t_0 is obtained which is shown to be compatible with the exact solution of Gavrilov et al. [107]. In another example, in the same work [106], with two Ricci-flat factor spaces and two matter sources (dust + 5-brane), a sufficiently small variation of G is obtained.

About seventy years ago, Dirac pointed out the possibility of time-variation of a fundamental constant in the context of a full-fledged cosmological model. It is evident from the present review that various ideas have been germinated by LNH. Moreover, in recent years, changeability of other constants related to physical laws is getting more and more attention to the researchers of various fields, viz., physics, geophysics, astrophysics and cosmology [108]. Amount of variations of some of the so called constants, viz., fine-structure constant [77, 78, 109, 110, 111, 112, 113, 114] and ratio of proton and electron mass [112, 115] are claimed to have been measured with reliable accuracy. It has already been mentioned in the introduction that the erstwhile cosmological parameter Λ , once regarded as a constant, is now largely accepted as a function of time. These experimental and observational results have already sparked off ideas regarding probable variations of other fundamental constants and much more are expected to come in near future. So, in every respect, LNH has triggered off many new ideas, some of which may be fundamental in nature. Recently, in a work of Shukurov [116] related to interstellar matter, it has been found that the number of Christmas dinner per galaxy turns out to be of the order of 10^{40} . Even the Higgs scalar-tensor theory, in which the mass of the particles appeared through gravitational interaction [117, 118, 119], is also found to be compatible with LNH [120]. On the other hand, Carter [121] is of the opinion that using anthropic principle (both weak and strong) large number coincidence can be explained within the framework of conventional physics and no exotic idea, like time variation of G , is necessary. Thus it may be in-

ferred that we are still carrying a legacy of Dirac's Large Number Hypothesis - a simple idea with profound implication.

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