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The string dilaton and a least coupling principle

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Abstract

It is pointed out that string-loop modifications of the low-energy matter couplings of the dilaton may provide a mechanism for fixing the vacuum expectation value of a massless dilaton in a way which is naturally compatible with existing experimental data. Under a certain assumption of universality of the dilaton coupling functions, the cosmological evolution of the graviton–dilaton–matter system is shown to drive the dilaton towards values where it decouples from matter ("Least Coupling Principle"). Quantitative estimates are given of the residual strength, at the present cosmological epoch, of the coupling to matter of the dilaton. The existence of a weakly coupled massless dilaton entails a large spectrum of small, but non-zero, observable deviations from general relativity. In particular, our results provide a new motivation for trying to improve by several orders of magnitude the various experimental tests of Einstein's Equivalence Principle (universality of free fall, constancy of the constants, etc.).

1. Introduction

At present we know only one theory which treats gravity in a way consistent with quantum mechanics: string theory. In the low-energy limit (low in comparison with the Planck mass) string theory gives back classical general relativity, with, however, an important difference. All versions of string theory predict the existence of a (four-dimensional) scalar partner of the tensor Einstein graviton: the dilaton. It may happen that this scalar field acquires a mass due to some yet unknown dynamical mechanism. This is the generally adopted view, and if so there will be no observable macroscopic difference between string gravity and Einstein gravity if that mass is large enough [1]. In this paper we will discuss another possibility: that the dilaton remains massless. This immediately leads to the

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dramatic conclusion that all coupling constants and masses of elementary particles, being dependent on the dilaton scalar field, should be, generally speaking, space and time dependent, and influenced by local circumstances. This conclusion is of course not new and it was precisely the reason for discarding the possibility that we are going to discuss. Indeed, it has been stated that the existence of a massless dilaton contributing to macroscopic couplings would, at once, entail the following observable consequences: (i) Jordan–Fierz–Brans–Dicke-type [2] deviations from Einstein's theory in relativistic ($O(Gm/c^2r)$) gravitational effects [3], (ii) cosmological variation of the fine-structure constant, and of the other gauge coupling constants [4], and (iii) violation of the (weak) equivalence principle [5]. As the strength of the coupling of the dilaton to matter is expected to be comparable to that of the (spin-2) graviton, and even larger in the case of hadrons [5], the above observable consequences seem to be in violent conflict with experiment. Indeed, present experimental data give upper limits of order: (i) 10^{-3} for a possible fractional admixture of a scalar component to the relativistic gravitational interaction [6], (ii) 10^{-15} yr^{-1} for the fractional variation with time of the fine-structure constant [2] [7], (iii) 10^{-11}–10^{-12} on the universality of free fall (weak equivalence principle) [8–11] [3]. (See [12,13] for reviews of the comparison between gravitational theories and experiments.)

In this paper, we point out that non-perturbative string-loop effects (associated with worldsheets of arbitrary genus in intermediate string states) can naturally reconcile the existence of a massless dilaton with existing experimental data if they exhibit the same kind of universality as the tree-level dilaton couplings. By studying the cosmological evolution of general graviton–dilaton–matter systems we show that the dilaton is cosmologically attracted toward values where it decouples from matter, a situation which we call the "Least Coupling Principle". Roughly speaking, the origin of the attraction is the following. Masses of different particles depend on the dilaton ϕ, while the source for the dilaton is the ϕ-gradient of these masses. It is therefore not surprising to have a fixed point where the ϕ-gradient of the masses is zero. (With some important differences discussed below, this mechanism is similar to the generic attractor mechanism of metrically-coupled tensor–scalar theories discussed in [14].) This cosmological attraction is so efficient that the presently existing experimental limits do not place any significant constraints on the physical existence of a massless dilaton. Most importantly, we give quantitative estimates for the level of residual deviation from Einstein's theory expected at

[1] More precisely, using the formulas (2.11) and (3.6) below with $B(\Phi)=e^{-2\Phi}$ one can check that the experimental results on the inverse-square law at small distances [1] constrain the mass of a tree-level coupled dilaton to be larger than 1.6×10^{-3} eV at the 2σ level.

[2] Note that, within the QCD framework, it does not make sense to speak of the variation of any strong-interaction coupling constant (the hadron mass scale adjusting itself such that $\alpha_{\mathrm{strong}}\simeq1$).

[3] The most recent analysis of Lunar Laser Ranging data [11] finds that the fractional difference in gravitational acceleration toward the Sun between the (silica-dominated) Moon and the (iron-dominated) Earth is $(-2.7\pm6.2)\times10^{-13}$.

the present cosmological epoch, notably for the violation of the equivalence principle.

It is to be noted that the cosmological attractor mechanism discussed here could apply as well to the other gauge-neutral massless scalar fields (moduli) present in string theory. Through one-loop threshold effects, the coupling parameters and masses of elementary particles acquire a dependence on the VEVs of the moduli fields. If that dependence were to exhibit the kind of universality discussed below, our mechanism would fix the moduli to values where they decouple from matter. To fix ideas, and not commit ourselves to specific compactification models, we shall phrase our discussion below in terms of the dilaton field only, which is a universal partner of the graviton in all string models.

2. The graviton–dilaton–matter system

One of the main features of (super) string theories is that, at critical dimension, they contain massless modes, two of which describe gravitons and dilatons (and their superpartners). There are also, in general, many other massless particles which come from the excitations of the conformal "matter" on the string's worldsheet. At the tree level in the string-loop expansion (spherical topology for intermediate worldsheets) the effective action describing the massless modes (here considered directly in four dimensions) has the general form [15–17]

$$S_{\text{tree}} = \int \mathrm{d}^4x\sqrt{\hat{g}}\; \mathrm{e}^{-2\Phi}\Big((\alpha')^{-1}\Big[\hat{R} + 4\hat{\Box}\,\Phi - 4(\hat{\nabla}\Phi)^2\Big] - \tfrac{1}{4}k\hat{F}^a_{\mu\nu}\hat{F}^{a\mu\nu} - \bar{\hat{\psi}}\hat{D}\hat{\psi}$$

$$+ \ldots + \sum_{n\geqslant 1} \mathrm{O}\Big[(\alpha'\partial^2)^n\Big]\Big). \tag{2.1}$$

Here, $\hat{g}_{\mu\nu}$ (often denoted $G_{\mu\nu}$) denotes the metric appearing in the σ-model formulation of string theory and is used for defining all the covariant constructs entering Eq. (2.1) ($\hat{\nabla}$, $(\hat{F})^2$, $\hat{D}$, ...); Φ denotes the dilaton; a summation over the various possible gauge fields ($F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$) and fermions ($\hat{D} = \hat{\gamma}^\mu(\hat{\nabla}_\mu + A^a_\mu t^a)$) is understood; the ellipsis stand in particular for the ill-understood remaining scalar sector of the theory (Higgs fields and their Yukawa couplings, and possibly other gauge-neutral scalar (moduli), or pseudo-scalar (axion, etc.), fields); and the last term symbolically denotes the infinite series of higher-derivative terms representing the low-energy effects of all the massive string modes on which one has to integrate to get the effective action for the massless modes.

The remarkable feature that, when formulating the action in terms of the "string frame" metric $\hat{g}_{\mu\nu}$, the dilaton couples, at the string tree level, in a universal, multiplicative manner to all the other fields derives from the fact that $g_s \equiv \mathrm{e}^{\Phi}$ plays the role of the string coupling constant. In the σ-model formulation, this is easily seen to follow from applying the Gauss–Bonnet theorem ($(4\pi)^{-1}\int \mathrm{d}^2\xi\sqrt{h}\;\; R^{(2)}(h) = \chi = 2(1-n)$; n = number of handles) to the Fradkin–Tseytlin σ-model dilaton term, $S_{\text{dil}} = (4\pi)^{-1}\int \mathrm{d}^2\xi\sqrt{h}\;\; \Phi(X)R^{(2)}$. In a constant (or slowly varying) dilaton background, the genus-n string-loop contribution to any string transition amplitude contains the factor $\exp(-S_{\rm dil}) = e^{2(n-1)\Phi} = g_s^{2(n-1)}$. Therefore, when taking into account the full string-loop expansion, the effective action for the massless modes will take the general form

$$S = \int d^4x\sqrt{\hat{g}}\left(\frac{B_g(\Phi)}{\alpha'}\hat{R} + \frac{B_\Phi(\Phi)}{\alpha'}\left[4\hat{\Box}\Phi - 4(\hat{\nabla}\Phi)^2\right] - B_F(\Phi)\frac{k}{4}\hat{F}^2 - B_\psi(\Phi)\bar{\hat{\psi}}\hat{D}\hat{\psi} + \dots\right) \tag{2.2}$$

At this stage of development of string theory, one does not know how to control the structure of the various dilaton coupling functions $B_i(\Phi)$ ($i = g, \Phi, F, \psi, \dots$) beyond the fact that in the limit $\Phi \to -\infty$ ($g_s \to 0$) they should admit an expansion in powers of $g_s^2 = e^{2\Phi}$ of the form

$$B_i(\Phi) = e^{-2\Phi} + c_0^{(i)} + c_1^{(i)}e^{2\Phi} + c_2^{(i)}e^{4\Phi} + \dots. \tag{2.3}$$

It will be important for the following that higher-order powers of g_s in Eq. (2.3) (or non-perturbative effects in $1/g_s^2$) can generate functions $B_i(\Phi)$ exhibiting a local maximum.

We did not include in Eq. (2.1) a cosmological constant term. This term is absent when the supersymmetry is unbroken. However, one must break supersymmetry to describe the low-energy world. One must find a dynamical mechanism which breaks gauge symmetry and supersymmetry, and gives correct masses to the massless particles. All known such mechanisms generate unacceptably large cosmological constants, essentially because the vacuum energies of bosons and fermions do not cancel anymore. We do not attempt, in the present paper, to solve this very serious problem, but simply assume the existence of some mechanism (maybe related to the massless dilaton) which screens away the cosmological constant at macroscopic distances.

Concerning the low-energy effects of all the massive string modes, we shall assume for simplicity that, like at tree level, Eq. (2.1), they are equivalent to introducing a cutoff at a *Φ-independent* string mass scale $\hat{\Lambda}_s \sim (\alpha')^{-1/2}$, when measuring distances by means of the *string frame* metric $\hat{g}_{\mu\nu}$.

It is convenient to transform the action (2.2) by introducing several Φ-dependent rescalings. One can put both the gravity and the fermion sectors into a standard form by: (i) introducing the "Einstein metric",

$$g_{\mu\nu} \equiv CB_g(\Phi)\hat{g}_{\mu\nu} \tag{2.4}$$

(with some numerical constant C [4]), (ii) replacing the original dilaton field Φ by the variable [5]

$$\phi \equiv \int d\Phi\left[\frac{3}{4}\left(\frac{B_g'}{B_g}\right)^2 + 2\frac{B_\Phi'}{B_g} + 2\frac{B_\Phi}{B_g}\right]^{1/2} \tag{2.5}$$

[4] We shall choose C such that the string units and the Einstein units coincide at the present cosmological epoch: $CB_g(\Phi_0) = 1$.

(where a prime denotes $d/d\Phi$), and (iii) rescaling the Dirac fields

$$\psi \equiv C^{-3/4} B_g^{-3/4} B_\psi^{1/2} \hat{\psi}. \tag{2.6}$$

The transformed action can be decomposed into a gravity sector ($g_{\mu\nu}$, ϕ) and a matter one (ψ, A, ...),

$$S[g, \phi, \psi, A, \cdots] = S_{g,\phi} + S_{\mathrm{m}}, \tag{2.7a}$$

$$S_{g,\phi} = \int d^4x \sqrt{g} \left(\frac{1}{4q} R - \frac{1}{2q} (\nabla\phi)^2 \right), \tag{2.7b}$$

$$S_{\mathrm{m}} = \int d^4x \sqrt{g} \left[-\bar{\psi} D \psi - \tfrac{1}{4} k B_F(\phi) F^2 + \ldots \right]. \tag{2.7c}$$

Here, $q \equiv 4\pi\bar{G} \equiv \frac{1}{4}C\alpha'$ ($\bar{G}$ denoting a *bare* gravitational coupling constant), $B_F(\phi) \equiv B_F[\Phi(\phi)]$ and the ellipsis stand for the (more complicated) Higgs sector. One should note that the string cutoff mass scale acquires a dependence upon the dilaton in Einstein units:

$$\Lambda_{\mathrm{s}}(\phi) \equiv C^{-1/2} B_g^{-1/2}(\phi) \hat{\Lambda}_{\mathrm{s}}. \tag{2.8}$$

Essential to the following will be the dilaton dependence of the matter lagrangian. One does not know at present how to relate string models to the observed particle spectrum. The basic clue that we shall follow is the dilaton dependence of the gauge coupling constants: $g^{-2} = kB_F(\phi)$ from (2.7c). To connect the (bare) effective action (2.7) (integrated over the massive string modes) to the low-energy world, one still needs to take into account the quantum effects of the light modes between the string scale $\Lambda_{\mathrm{s}}(\phi)$ and some observational scale. In the case of an asymptotically free theory the ratio of the IR confinement mass scale Λ_{conf} to the cutoff scale, is, at the one-loop level, exponentially related to the inverse of the gauge coupling constant appearing in the bare action:

$$\Lambda_{\mathrm{conf}} \sim \Lambda_{\mathrm{s}} \exp(-8\pi^2 b^{-1} g^{-2}) = C^{-1/2} B_g^{-1/2}(\phi) \exp\left[-8\pi^2 b^{-1} k B_F(\phi)\right] \hat{\Lambda}_{\mathrm{s}}, \tag{2.9}$$

where the one-loop coefficient b depends upon the considered gauge field as well as the matter content. The mass of hadrons is, for the most part, generated by QCD effects and is simply proportional to Λ_{QCD} (with some pure number as proportionality constant). The dilaton dependence of the QCD part of the mass of hadrons is therefore given by (2.9) with $b = b_3$ and $B_F = B_3$ being the appropriate QCD quantities. However, the lepton masses, and the small quark contributions to the mass of hadrons, are not related to Λ_{QCD} (at least in any known way). Their dilaton dependence is defined by specific mechanisms of spontaneous symmetry breakdown (and compactification) which depend on particular string models and are not well established at present. Let us note that in technicolor-type models, as well as in no-scale supergravity ones [18], all the particle mass scales are related to the fundamental cutoff scale by formulas of the type (2.9).

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[5] When $B_g = B_\Phi$ the quantity under the square root in Eq. (2.5) is positive definite.

As a minimal ansatz, we can assume that the mass (in Einstein units) of any type of particle, labelled A, depends in a non-trivial way on the VEV of the dilaton through some of the functions B_i appearing in (2.2):

$$m_A(\phi) = m_A\left[B_g(\phi), B_F(\phi), \dots\right]. \tag{2.10}$$

The essential new feature allowed by non-perturbative string-loop effects (i.e. arbitrary functions $B_i(\Phi)$, Eq. (2.3)) is the possibility for the function $m_A(\phi)$ to admit a minimum for some finite value of ϕ, say $\phi_{\rm m}$. Assuming this, we shall see below that the cosmological evolution naturally attracts ϕ toward $\phi_{\rm m}$. However, if the various coupling functions $B_i(\phi)$ differ from each other, the values of ϕ where $m_A(\phi)$ reaches a minimum will depend, in general, on the type of particle considered. It will be seen below that this weakens the attraction effect of the cosmological expansion, and, more importantly, leaves room for violations of the equivalence principle at a probably unacceptable level (see the footnote following Eq. (6.13)). This suggests to concentrate on the case where string-loop effects preserve the universal multiplicative coupling present at tree level, Eq. (2.1), i.e. the case where all the dilaton coupling functions coincide: $B_i(\phi) = B(\phi)$ for $i = F, g, \dots$. More generally, it would suffice that the various dilaton coupling functions be related to some common function B, say $B_i(\phi) = f_i[B(\phi)]$. In this "universal $B(\phi)$" case, the values of ϕ where the function $m_A(\phi) = m_A[B(\phi)]$ reaches a minimum will (generically) coincide with the (A-independent) values $\phi_{\rm m}$ where the function $B(\phi)$ is maximal. Even more generally, the minimal requirement for our cosmological scenario to apply is the existence of one special value $\phi_{\rm m}$ of ϕ about which all the mass functions $m_A(\phi)$ reach a minimum. As discussed below, this assumption leads very naturally (without fine-tuning, or the need to inject small parameters) to a situation where the present deviations from general relativity are so small as to have escaped detection.

This universality hypothesis is a rather strong conjecture. Since quark and lepton masses appear after supersymmetry and gauge symmetry breaking, the validity of our conjecture depends also on the concrete mechanism of symmetry breaking. It is interesting, however, to remark that it is sufficient to obtain an approximate universality. One possible scenario would be of the following type. Above the scale of supersymmetry breaking $m_{\rm SUSY}$, one may have exact universality (including non-perturbative string effects). Below $m_{\rm SUSY}$, non-universal dependence on ϕ may appear due to a combination of non-supersymmetric interactions and non-renormalizable interactions via higher string modes. Then it is not unreasonable to expect that the size of the universality-violating effects will contain the factor $\alpha' m_{\rm SUSY}^2$. As we shall indicate below, one can experimentally tolerate such non-universalities if that factor is smaller than $\sim 10^{-6}$ which corresponds to $m_{\rm SUSY} \lesssim 10^{14}$ GeV.

Let us indicate various possible scenarios which hint at ways in which the desired universality could be realized in the supersymmetric region. We leave a detailed analysis of these possibilities to the future.

First of all, there is a number of non-renormalization theorems [19], both for the dilaton and the moduli dependences. Essentially, they claim that in the low-energy region only the Kähler potential gets renormalized. (With the possibility of some (controllable) infrared anomaly in the gauge functions.) It can then be shown that by assuming certain features of the Kähler potential one can obtain the minimal universality required for cosmologically attracting the dilaton (or moduli) to a fixed point.

Target-space duality suggests itself as a natural way of satisfying our minimal universality requirement for cosmologically attracting a modulus to a self-dual value. First, we notice that in some orbifold models the mass scale at which the gauge couplings would become strong is a modular invariant function of an (untwisted) modulus $T = T_1 + iT_2$ which is $\propto [T_2 \,|\, \eta(T)\,|^4]^{-1/2}$ [20] and admits (when $T_1 = 0$) a local minimum at $T_2 = 1$ as a consequence of the symmetry $T_2 \to 1/T_2$. We notice that this symmetry (and therefore the existence of an extremum) will hold to all orders of string perturbation theory. Second, the calculation of threshold corrections to Yukawa couplings (in some orbifold models) finds that the boundary relation between some (untwisted) Yukawa couplings and some (E_6) gauge coupling at the string unification scale does not receive any moduli-dependent correction at the one-loop level [21].

A related mechanism for universality is the conjectured S duality [22]. It claims that all physical quantities depend on the dilaton–axion field $S \equiv a + i e^{\Phi}$ in a modular invariant way (here a denotes the axion partner of the dilaton Φ). This again includes the transformations $S \to -1/S$ and $\Phi \to -\Phi$, and would ensure that all physical quantities have an extremum at $\Phi = 0$.

These (disconnected) arguments are very far from being conclusive, but we feel that they may indicate useful avenues for future research.

When we shall need in the following to estimate quantitatively the dependence of particle masses on ϕ, we shall assume that the mass of any particle A is of the form suggested by Eq. (2.9):

$$m_A(\phi) = \mu_A B^{-1/2}(\phi) \exp\left[-8\pi^2 \nu_A B(\phi)\right] \hat{\Lambda}_s, \tag{2.11}$$

with μ_A and ν_A pure numbers of order unity. We believe that our main qualitative conclusions do not depend strongly on the specific form of the assumption (2.11).

For the quantitative estimates below we need to choose some specific value of the string unification scale $\hat{\Lambda}_s \propto (\alpha')^{-1/2}$. The theoretical value $\hat{\Lambda}_s = e^{(1-\gamma)/2} 3^{-3/4} g M_{\text{Planck}}/4\pi \simeq g \times 5.27 \times 10^{17}$ GeV has been suggested [23]. Here g denotes the common (modulo possible Kac–Moody level factors of order unity) value of the gauge coupling constants at the string scale, expected to be $g \sim 0.6$ [6]. To fix ideas, we shall take $\hat{\Lambda}_s = 3 \times 10^{17}$ GeV.

[6] In our scenario it would be fixed to the value $g \sim B_F(\phi_m)^{-1/2}$. Note that the corresponding value $B_F(\phi_m) \sim 3$ does not, by itself, tell us whether it is reached at perturbative or non-perturbative values of the string coupling constant $g_s = e^{\Phi}$.

3. Classical cosmology with a dilaton

The gravitational field equations derived from Eqs. (2.7) read

$$R_{\mu\nu} = 2\partial_\mu\phi\, \partial_\nu\phi + 2q\left(T_{\mu\nu} - \tfrac{1}{2}Tg_{\mu\nu}\right), \tag{3.1a}$$

$$\Box\,\phi = -q\sigma, \tag{3.1b}$$

where the source terms, defined by $T^{\mu\nu} \equiv 2g^{-1/2}\delta S_{\mathrm{m}}/\delta g_{\mu\nu}$, $\sigma \equiv g^{-1/2}\delta S_{\mathrm{m}}/\delta\phi$, are related by the energy balance equation: $\nabla_\nu T^{\mu\nu} = \sigma\nabla^\mu\phi$.

In the case of a Friedmann cosmological model, $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}l^2$ with $\mathrm{d}l^2 = (1-Kr^2)^{-1}\,\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), K = 0, +1$ or -1, the field equations give ($T^\mu_\nu = \mathrm{diag}(-\rho, P, P, P)$; $H \equiv \dot{a}/a$, the overdot denoting $\mathrm{d}/\mathrm{d}t$)

$$-3\frac{\ddot{a}}{a} = q(\rho + 3P) + 2\dot{\phi}^2, \tag{3.2a}$$

$$3H^2 + 3\frac{K}{a^2} = 2q\rho + \dot{\phi}^2, \tag{3.2b}$$

$$\ddot{\phi} + 3H\dot{\phi} = q\sigma. \tag{3.2c}$$

In the following, we concentrate on the spatially flat case ($K = 0$). Following [14], we can combine Eqs. (3.2) to write a simple equation for the cosmological evolution of the dilaton with respect to the logarithm of the cosmological scale factor: $p \equiv \ln a + \text{const.}$ (not to be confused with the pressure P). Denoting $\mathrm{d}/\mathrm{d}p$ by a prime, one gets ($K = 0$)

$$\frac{2}{3-\phi'^2}\phi'' + (1-\lambda)\phi' = \frac{\sigma}{\rho}, \tag{3.3}$$

where $\lambda \equiv P/\rho$.

Except during phase transitions, the material content of the Universe can be classically described as a superposition of several (weakly interacting) gases labelled by A, i.e. by an action of the form

$$S_m[g, \phi, x_A] = -\sum_A \int m_A[\phi(x_A)]\left[-g_{\mu\nu}(x_A^\lambda)\,\mathrm{d}x_A^\mu\,\mathrm{d}x_A^\nu\right]^{1/2} \tag{3.4}$$

(the massless particles being obtained by taking the limit $m_A \to 0$ with $m_A u_A^\mu \equiv m_A\, \mathrm{d}x_A^\mu/\mathrm{d}s_A$ fixed). In Eq. (3.4) the summation over A includes a sum over the statistical distribution of the A-type particles. The gravitational source terms corresponding to Eq. (3.4) read

$$T^{\mu\nu}(x) = \frac{1}{\sqrt{g(x)}}\sum_A \int \mathrm{d}s_A\, m_A[\phi(x_A)]u_A^\mu u_A^\nu \delta^{(4)}(x - x_A), \tag{3.5a}$$

$$\begin{aligned}\sigma(x) &= -\frac{1}{\sqrt{g(x)}}\sum_{\mathrm{A}} \int \mathrm{d}s_A\, \alpha_A[\phi(x_A)]m_A[\phi(x_A)]\delta^{(4)}(x - x_A)\\ &= \sum_A \alpha_A[\phi(x)]T_A(x),\end{aligned} \tag{3.5b}$$

where

$$\alpha_A(\phi) \equiv \frac{\partial \ln m_A(\phi)}{\partial \phi} \tag{3.6}$$

measures the strength of the coupling of the dilaton to the A-type particles. In the second Eq. (3.5b), $T_A = -\rho_A + 3P_A$ denotes the trace of the A-type contribution to the total $T^{\mu\nu} = \Sigma_A T_A^{\mu\nu}$. It is easy to see that when the different A-gases are non-interacting, their corresponding sources satisfy the separate energy balance equations: $\nabla_\nu T_A^{\mu\nu} = \sigma_A \nabla^\mu \phi = \alpha_A T_A \nabla^\mu \phi$.

In the string context, it is natural to assume that the string scale Λ_s subsumes both what is usually meant by "Planck scale" and "GUT scale", leaving essentially no room for a quasi-classical inflationary era. We leave to future work a discussion of primordial stringy cosmology, and content ourselves by describing the evolution of dilatonic cosmologies through a radiation-dominated era, followed by a matter-dominated one.

4. Evolution of the dilaton during the radiation-dominated era

During a radiation-dominated era (Universe dominated by ultra-relativistic gases) the gravitational source terms are approximately given by

$$\rho \simeq 3P \simeq g_*(T)\tfrac{1}{30}\pi^2 T^4, \tag{4.1a}$$

$$\sigma \simeq 0, \tag{4.1b}$$

where $g_*(T) = \Sigma_{\rm Bose} g_A^{\rm B}(T_A/T)^4 + \frac{7}{8}\Sigma_{\rm Fermi} g_A^{\rm F}(T_A/T)^4$ is the effective number of relativistic degrees of freedom in the cosmic soup at temperature T. (The sum defining $g_*(T)$ is taken only over particles with mass $m_A \ll T$; because of possible previous decouplings the corresponding relativistic gases may not all have the temperature T, e.g. $T_\nu = (\frac{4}{11})^{1/3} T_\gamma$ below 1 MeV.) Eq. (4.1b) suggests that the dilaton does not evolve during the radiation era. More precisely, Eq. (3.3) with $\lambda \simeq \frac{1}{3}$ shows that $\phi(p)$ behaves as a particle, with velocity-dependent mass, submitted to a constant friction. In a few p-time units, $\phi(p)$ will exponentially come to rest. (See [14] for the exact solution of the damped evolution of $\phi(p)$ when σ/ρ is negligible.) However, something interesting happens each time the Universe cools down to a temperature $T \sim m_A$ defining the threshold for the participation of the species A to the relativistic soup. When $T \sim m_A$, the term on the right-hand side of the p-time evolution of ϕ is well approximated by

$$\frac{\sigma_A}{\rho_{\rm tot}} = -\frac{15}{\pi^4}\frac{g_A}{g_*(T)}\tau_\pm(z_A)\alpha_A(\phi), \tag{4.2}$$

where $z_A \equiv m_A/T$ and

$$\tau_\pm(z) \equiv z^2 \int_z^\infty \mathrm{d}x \frac{(x^2 - z^2)^{1/2}}{\mathrm{e}^x \pm 1}, \tag{4.3}$$

where the upper (lower) sign corresponds to A being a fermion (boson). In the approximation (justified by the results to be discussed) where the dilaton contributions to the Einstein equations (3.2a), (3.2b) are negligible one has $T \propto a^{-1}$, and therefore $p = \ln z_A$ (with an adapted choice of origin for p). Then, as a function of p, the (everywhere positive) function $\tau_{\pm}$ is proportional to e^{2p} when $p \to -\infty$, rises up to a maximum $\simeq 1.16$ when $z_A \simeq 0.87$, and falls quickly to zero as $\exp(\frac{5}{2}p - e^{p})$ when $p \to +\infty$. (This maximum occurs because for $T \gg m_A$ the (ultra-relativistic) particles do not contribute to σ, while for $T \ll m_A$ there are exponentially few particles.) Remembering the definition (3.6) of $\alpha_A(\phi)$, it is easy to see that if the function $m_A(\phi)$ has a minimum, say $\phi_{\rm m}^A$, and if the initial value of ϕ, say $\phi_-^A \equiv \phi(p = -\infty) = \phi(T \gg m_A)$, is sufficiently near $\phi_{\rm m}^A$, Eq. (3.3) will describe a damped, transient non-linear attraction of $\phi(p)$ around $\phi_{\rm m}^A$. (Note that, from the above discussion, the initial velocity is zero to an exponential accuracy $\sim \exp(p_{A'} - p_A)$.) The existence of such an attraction mechanism by mass thresholds during the radiation era was noticed in [14] in a related context (generalized Jordan–Fierz–Brans–Dicke theories characterized by a universal, A-independent coupling function $\alpha_A(\phi) = \alpha(\phi) = \partial a(\phi)/\partial\phi$). In the context of [14], it seemed natural to assume that the curvature of the function $\ln m(\phi) = a(\phi) + \ln m_0$ near its minimum was of order unity. This rendered the presently discussed attraction mechanism very ineffective. An important new feature of the present, dilatonic, context is that the curvature of $\ln m_A(\phi)$ near its minimum is expected to be large compared to one. This follows from the expected exponential dependence on $B_F(\phi)$ of the mass scales of the low-energy particle spectrum, Eq. (2.9). To fix ideas and be able to make some quantitative estimates, we shall take the form (2.11) with $\mu_A = 1$. This yields

$$\alpha_A(\phi) = -\left(\ln\frac{\hat{\Lambda}_{\rm s}}{m_A} + \frac{1}{2}\right)\frac{\partial \ln B(\phi)}{\partial\phi} = +\ln\frac{\hat{\Lambda}'_{\rm s}}{m_A}\frac{\partial \ln B^{-1}(\phi)}{\partial\phi}, \tag{4.4}$$

where $\hat{\Lambda}'_{\rm s} = e^{1/2}\hat{\Lambda}_{\rm s} \simeq 5 \times 10^{17}$ GeV.

We see that a minimum $\phi_{\rm m}$ of $m_A(\phi)$ corresponds to a maximum of $B(\phi)$ (or a minimum of $B^{-1}(\phi)$). Let us denote by κ the curvature of the function $\ln B^{-1}(\phi)$ near its minimum $\phi_{\rm m}$. In the parabolic approximation

$$\ln B^{-1}(\phi) \simeq \ln B^{-1}(\phi_{\rm m}) + \tfrac{1}{2}\kappa(\phi - \phi_{\rm m})^2, \tag{4.5}$$

one gets

$$\alpha_A(\phi) = \beta_A(\phi - \phi_{\rm m}), \tag{4.6a}$$

$$\beta_{\rm A} = \kappa \ln\frac{\hat{\Lambda}'_{\rm s}}{m_A} = \kappa\left(40.75 - \ln\frac{m_A}{1\ \text{GeV}}\right). \tag{4.6b}$$

Inserting Eq. (4.6a) into Eq. (4.2) and then into Eq. (3.3) (written in the approximation $\lambda \simeq \frac{1}{3}$) yields

$$\phi''(p) + \phi'(p) = s_A(p)[\phi(p) - \phi_{\rm m}], \tag{4.7}$$

with

$$s_A(p) = -\frac{45}{2\pi^4}\beta_A \frac{g_A}{g_*(T)} \tau_{\pm}(e^p). \tag{4.8}$$

Within a good approximation one can replace the temperature-dependent quantity $g_A/g_*(T)$ by its initial value, say $f_A^{\rm in} \equiv g_A/g_*^{\rm in}$, in which $g_*{}^{\rm in} \equiv g_*(T \gg m_A)$ contains the contribution $\frac{7}{8}g_A$ (or g_A) if A is a fermion (or boson). Eq. (4.7) describes a damped motion submitted to a transient harmonic force tending to attract ϕ toward $\phi_{\rm m}$. The final outcome of this motion is to leave (when $p = +\infty$) ϕ nearer to $\phi_{\rm m}$ than it was when it started at rest at $p = -\infty$. We define the attracting factor of the A-type mass threshold as $m_{\pm}(b_A) \equiv [\phi(+\infty) - \phi_{\rm m}^A]/[\phi(-\infty) - \phi_{\rm m}^A]$, where the suffix $\pm$ in the left-hand side corresponds to the fermion/boson case and where $b_A \equiv \beta_A f_A^{\rm in} \equiv \beta_A g_A/g_*{}^{\rm in}$. There are two quite different regimes in this mass-threshold attraction mechanism: when $b_A \ll 1$ ($b_A < 0.5$ sufficing), $\phi(p)$ moves monotonically toward $\phi_{\rm m}$ by a small amount given by integrating over p the force term on the right-hand side of (4.7) evaluated at the original position of ϕ ("kick" approximation). The result is (see [14])

$$m_{\pm}(b_A) = 1 - \tfrac{1}{2}b_A\begin{pmatrix}\frac{7}{8}\\ 1\end{pmatrix} + O(b_A^2), \tag{4.9}$$

where the upper (lower) coefficient corresponds to the fermion (boson) case, respectively. In this first case the attracting power of the A threshold is rather weak (hence the conclusion of [14] that the total radiation era attraction is rather ineffective in the case of usual tensor–scalar theories with $\beta_A = O(1)$ and $\Sigma_A f_A \sim \ln \frac{100}{10} \simeq 2.3$). By contrast, in the present, dilatonic context one expects $\kappa \sim 1$, $\beta_A \sim 40$ and therefore $b_A \gg 1$ for many mass thresholds (the most efficient mass thresholds being the latest in the radiation era which tend to have the largest $f_A^{\rm in}$: notably the e^+e^- threshold with $f_e^{\rm in} = 4/10.75 \simeq 0.372$). When $b_A \gg 1$ ($b_A > 2$ sufficing in practice) one can analytically solve Eq. (4.7) by a WKB-type approach. (With some subtleties compared to the usual WKB approximation, as the matching between the damped and oscillating regions must be done via Bessel functions instead of the usual Airy ones.) Qualitatively the motion of $\phi(p)$ begins by a slow roll toward $\phi_{\rm m}$, continues by WKB oscillations around $\phi_{\rm m}$, and terminates as a damped inertial motion. The final analytical results for the attraction factor reads ($b_A \gg 1$)

$$m_{\pm}(b_A) = (C_{\pm}b_A)^{-1/4}\cos\theta_{\pm}^A, \tag{4.10a}$$

with $C_+ = \frac{15}{8}$, $C_- = \frac{15}{4}$ and

$$\theta_{\pm}^A = \int_{-\infty}^{+\infty}[-s_A(p)]^{1/2}\,dp - \tfrac{1}{4}\pi = b_A^{1/2}I_{\pm} - \tfrac{1}{4}\pi, \tag{4.10b}$$

with $I_+ \simeq 1.2743$, $I_- \simeq 1.4029$. Note that when $b_A \to \infty$, $|m_{\pm}(b_A)|$ tends to zero as $O(b_A^{-1/4})$. Fig. 1 represents the two functions $m_{\pm}(b)$, obtained by numerically integrating Eq. (4.7).

One must take into consideration the fact that mass thresholds can occur only for particles whose masses are smaller than the critical temperature of the phase

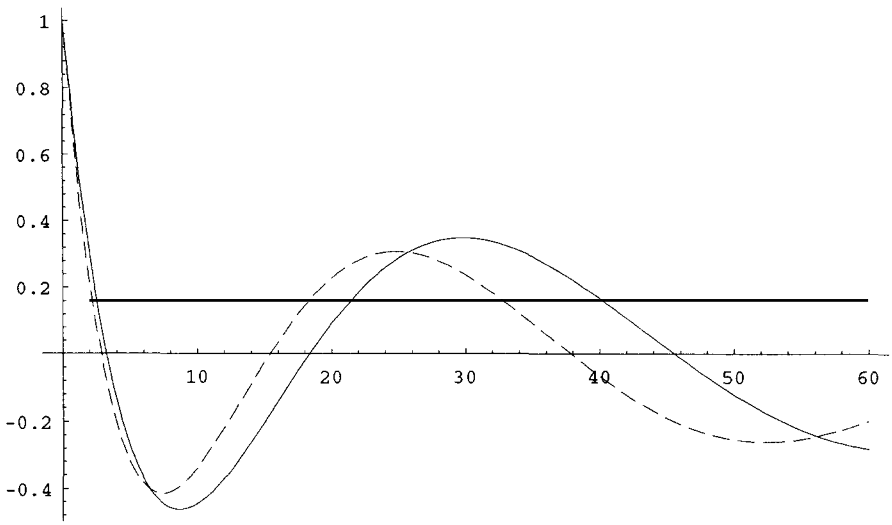

Fig. 1. The factor by which ϕ is attracted (when the early Universe cools down through $T \sim m_A$) toward a minimum ϕ_{m} of the function $m_A(\phi)$ is plotted as a function of $b_A = \beta_A f_A^{\mathrm{in}}$. The solid (dashed) line corresponds to A being a fermion (boson).

transition through which they acquired a mass (e.g. the pions are the only hadrons to take into account). The Higgs threshold is to be considered as part of the electroweak phase transition, and the strange quark threshold overlaps with the quark–hadron phase transition. This leaves nine, clearly present, mass thresholds associated (in decreasing temperature scale) with the top quark ($f_{\mathrm{t}}^{\mathrm{in}} = 12/106.75$, $\beta_{\mathrm{t}} \simeq 35.74\kappa$), the Z^0 ($f_{\mathrm{Z}}^{\mathrm{in}} = 3/95.25$, $\beta_{\mathrm{Z}} \simeq 36.24\kappa$), the $\mathrm{W}^{\pm}$ ($f_{\mathrm{W}}^{\mathrm{in}} = 6/92.25$, $\beta_{\mathrm{W}} \simeq 36.37\kappa$), the bottom quark ($f_{\mathrm{b}}^{\mathrm{in}} = 12/86.25$, $\beta_{\mathrm{b}} \simeq 39.14\kappa$), the tau ($f_{\tau}^{\mathrm{in}} = 4/75.75$, $\beta_{\tau} \simeq 40.17\kappa$), the charmed quark ($f_{\mathrm{c}}^{\mathrm{in}} = 12/72.25$, $\beta_{\mathrm{c}} \simeq 40.35\kappa$), the pions ($f_{\pi}^{\mathrm{in}} = 3/17.25$, $\beta_{\pi} \simeq 42.74\kappa$), the muon ($f_{\mu}^{\mathrm{in}} = 4/14.25$, $\beta_{\mu} \simeq 43.00\kappa$), and the electron ($f_{\mathrm{e}}^{\mathrm{in}} = 4/10.75$, $\beta_{\mathrm{e}} \simeq 48.33\kappa$). The quoted values of f_A^{in} and β_A show that $b_A \equiv \beta_A f_A^{\mathrm{in}}$ is typically a few times κ (with extreme values 1.14κ and 17.98κ for the Z and e respectively). A look at Fig. 1 shows immediately that if κ is of order unity, each mass threshold will a be rather efficient attractor. The compound effect of all those attractors is discussed below.

Besides mass thresholds, phase transitions provide another possible attractor mechanism for the dilaton during the radiation-dominated era. During a phase transition the vacuum energy density V changes from some positive value, say $V^{\mathrm{in}} = g_{\mathrm{vac}}(\frac{1}{30}\pi^2)T_{\mathrm{c}}^4$, when $T > T_{\mathrm{c}}$ to a comparatively negligible value when $T < T_{\mathrm{c}}$. For instance, in the case of the QCD (quark–hadron) phase transition one has $T_{\mathrm{c}} \simeq 200$ MeV and $g_{\mathrm{vac}} = \frac{34}{3}$ (in a simple model [24] describing the unconfined phase as a relativistic gas of gluons and u and d quarks – besides γ, e, ν, and μ – and the confined phase as a relativistic gas of pions). Besides its dependence on the temperature, the vacuum energy density is also a function of the dilaton.

Therefore the vacuum term in the matter action, $S_{\text{vac}} = -\sqrt{g}\ V(\phi, T)$, will generate a corresponding source term $\sigma_{\text{vac}} = -\partial V/\partial\phi$ in the right-hand sides of the dilaton evolution equations (3.1b), (3.2c) or (3.3). In the simple model of the QCD phase transition just described, one can estimate the source term σ_{vac} by assuming that the dilaton dependence of V is essentially contained in the ϕ-dependence of the critical temperature T_c. In turn, the latter dependence is obtained from $T_c \sim \Lambda_{\text{QCD}}$ with $\Lambda_{\text{QCD}}(\phi)$ given by Eq. (2.9) with the appropriate one-loop coefficient. This shows that ϕ will be attracted toward a minimum of $\Lambda_{\text{QCD}}(\phi)$. More precisely, if we assume, to fix ideas, that $B_g(\phi) = B_F(\phi)$ in Eq. (2.9) and that ϕ is near the maximum ϕ_m of $B(\phi)$, one gets, in the parabolic approximation (4.5) (setting as above $\lambda = P_{\text{tot}}/\rho_{\text{tot}}$ and $p = \ln(T_c/T)$),

$$\phi''(p) + \tfrac{3}{2}[1 - \lambda(p)]\phi'(p) = s_{\text{vac}}(p)[\phi(p) - \phi_m], \tag{4.11}$$

where $s_{\text{vac}}(p) \simeq -6\beta_{\text{vac}} f_{\text{vac}} e^{4p}$ when $p \to -\infty$, with $\beta_{\text{vac}} = \kappa \ln(\hat{\Lambda}_s/T_c)$ and $f_{\text{vac}} = g_{\text{vac}}/g_*(T > T_c)$. After the phase transition, when $p \to +\infty$, one expects $s_{\text{vac}}(p)$ to fall quickly to zero as $\exp(-aT_c/T) = \exp(-ae^p)$ with a of order unity. In the limit where $b_{\text{vac}} \equiv \beta_{\text{vac}} f_{\text{vac}}$ is large enough to make ϕ oscillate around ϕ_m, one can solve Eq. (4.11) by a WKB-type approach. The final result for the attraction factor due to a phase transition, $p(b_{\text{vac}}, \ldots) \equiv [\phi(+\infty) - \phi_m]/[\phi(-\infty) - \phi_m]$, reads

$$p(b_{\text{vac}}, \ldots) = 2^{3/2}\pi^{-1}\Gamma(\tfrac{5}{4})a^{1/2}(6b_{\text{vac}})^{-1/8}\, e^{-I} \cos\theta, \tag{4.12}$$

where $I = \frac{1}{4}\int_{-\infty}^{+\infty} [1 - 3\lambda(p)]\, dp$, and where the angle θ depends on the two functions $\lambda(p)$ and $s_{\text{vac}}(p)$. (In the approximation $\lambda(p) = \frac{1}{3}$, one finds $\theta = \int_{-\infty}^{+\infty}[-s_{\text{vac}}(p)]^{1/2}\, dp - \frac{1}{8}\pi$.) In the case of the QCD phase transition, one has $\beta_{\text{vac}} \simeq 42.36\kappa$ and $f_{\text{vac}} = \frac{34}{3}/51.25 \simeq 0.2211$. If κ is of order unity, $b_{\text{vac}} \simeq 9.37\kappa$ is probably large enough to render valid the WKB result (4.12). This yields an attraction factor $p_{\text{QCD}} \simeq 0.49a^{1/2}\kappa^{-1/8} \cos\theta$. In the case of the electroweak phase transition, rough estimates give $b_{\text{vac}} \simeq \frac{1}{4}\lambda\kappa$ where λ denotes the quartic self-coupling of the Higgs. Its seems therefore probable that $b_{\text{vac}}^{\text{electroweak}} \lesssim 1$, so that the electroweak transition has only a weak attracting effect on ϕ. We conclude that phase transitions seem to have only a modest effect on ϕ. It would be at present meaningless to refine the calculation of the effect on ϕ of the electroweak and QCD phase transitions (even the order of the transitions is in doubt, not to mention the precise redshift dependence of $\lambda(p)$ and $s_{\text{vac}}(p)$). In fact, until one has some understanding of the cosmological constant problem, it does not make much sense to compute any gravitational effect linked to phase transitions. In the following we shall therefore neglect the effect of the phase transitions with respect to that of the nine mass thresholds discussed above.

5. Evolution of the dilaton during the matter-dominated era

The matter content of the Universe near the end of the radiation era and during the subsequent matter era can be described as the superposition of a relativistic gas ("radiation", i.e. photons and three neutrinos in the standard picture) and of a non-relativistic one ("matter"; made of particles of mass $m_{\rm m}(\phi)$). From Eqs. (3.5) the source terms for the cosmological evolution equations (3.2) read $\rho = \rho_{\rm r} + \rho_{\rm m}$, $P = P_{\rm r} + P_{\rm m}$, $\sigma = -\alpha_{\rm m}(\phi)(\rho_{\rm m} - 3P_{\rm m})$ with $P_{\rm r} = \frac{1}{3}\rho_{\rm r}$, $P_{\rm m} \simeq 0$, and $\alpha_{\rm m}(\phi) \equiv \partial \ln m_{\rm m}(\phi)/\partial\phi$. Either from the definition (3.5a) or from the separate energy balance equations discussed below Eq. (3.6), one deduces that, during the expansion, $\rho_{\rm r} \propto a^{-4}$ while $\rho_{\rm m} \propto m_{\rm m}(\phi) a^{-3}$. Finally, the evolution of ϕ with respect to the p-time $p \equiv \ln a + \text{const.}$ is given by the equation

$$\frac{2}{3-\phi'^2}\phi'' + [1 - \lambda(p, \phi)]\phi' = -[1 - 3\lambda(p, \phi)]\alpha_{\rm m}(\phi), \tag{5.1}$$

with $3\lambda(p, \phi) = [1 + Cm_{\rm m}(\phi){\rm e}^{p}]^{-1}$, C being some constant. In the approximation where the radiation era has already attracted ϕ very near a minimum $\phi_{\rm m}$ of $m_{\rm m}(\phi)$, we can consider that $m_{\rm m}(\phi) \simeq$ const. in $\lambda(p, \phi)$. Choosing now the origin of p at the equivalence between radiation and matter ($\rho_{\rm r}(p=0) = \rho_{\rm m}(p=0)$), we get simply $\lambda(p) = 3^{-1}(1 + {\rm e}^{p})^{-1}$. Neglecting ϕ'^2 in Eq. (5.1) and using the harmonic approximation (4.6), we find that ϕ satisfies a linear differential equation which can be rewritten as a hypergeometric equation. Denoting $x \equiv {\rm e}^{p} \equiv a/a_{\rm equivalence}$ we have

$$x(x+1)\partial_x^2\phi + \left(\tfrac{5}{2}x + 2\right)\partial_x\phi + \tfrac{3}{2}\beta_{\rm m}(\phi - \phi_{\rm m}) = 0. \tag{5.2}$$

The condition of regularity of ϕ when $x \to 0$, say $\phi(x=0) = \phi_{\rm rad}$ ($\phi_{\rm rad}$ denoting the value of ϕ at the end of the radiation era, before the transition to the matter era around $p = 0$), selects uniquely the solution of (5.2) to be $\phi_{\rm m} + (\phi_{\rm rad} - \phi_{\rm m}) \times F[a, b, c; -x]$. Here $F[a, b, c; z]$ denotes the usual (Gauss) hypergeometric series. The values of the parameters are

$$a = \tfrac{3}{4} - i\omega, \quad b = \tfrac{3}{4} + i\omega, \quad c = 2, \tag{5.3}$$

with $\omega \equiv [\frac{3}{2}(\beta_{\rm m} - \frac{3}{8})]^{1/2}$. In other words, the attraction factor of the matter era up to the present time, $F_{\rm m} \equiv (\phi_{\rm now} - \phi_{\rm m})/(\phi_{\rm rad} - \phi_{\rm m})$, is given by

$$F_{\rm m} = F[a, b, c; -Z_0], \tag{5.4}$$

where $Z_0 \equiv \exp(p_0) \equiv a_{\rm now}/a_{\rm equivalence}$ denotes the (Einstein frame) redshift separating us from the moment of equivalence between matter and radiation. As Z_0 is large (see Eq. (6.5a) below), we can use the asymptotic behavior of the hypergeometric function (together with the properties of Euler's Γ function) to get more explicit forms for $F_{\rm m}$. Whatever be the sign of $\beta_{\rm m} - \frac{3}{8}$ (i.e. in the two cases where ω is real or pure imaginary) one can write

$$F_{\rm m} = 2^{1/2}\pi^{-1/2}2^{2i\omega}\Gamma_2{\rm e}^{-3p/4}{\rm e}^{i\omega p} + (i\omega \leftrightarrow -i\omega), \tag{5.5}$$

with $\Gamma_2 \equiv \Gamma(2i\omega)/\Gamma(2i\omega + \frac{3}{2})$. Actually, from the estimate (4.6b) we expect $\beta_{\rm m}$ to be (much) larger than $\frac{3}{8}$ (indeed, if $m_{\rm m} \sim 1$ GeV, one would need κ to be smaller than 9.2×10^{-3} to make $\beta_{\rm m} < \frac{3}{8}$). In that case ($\omega$ real), one can compute the modulus of the complex number Γ_2 in terms of elementary functions to get

$$F_{\rm m} = \left(\frac{\operatorname{cotanh}(2\pi\omega)}{\pi\omega\left(\omega^2 + \frac{1}{16}\right)}\right)^{1/2} \exp\left(-\tfrac{3}{4}p_0\right)\cos\theta_0, \tag{5.6}$$

with $\theta_0 \equiv \omega p_0 + 2\omega \ln 2 + \operatorname{Arg} \Gamma_2$. As in the case of attraction by mass thresholds (when $\beta_A f_A^{in} \gtrsim 1$), the attraction factor (5.6) is proportional to a cosine (when $\beta_m > \frac{3}{8}$) because Eq. (5.1) describes a damped oscillation around the minimum ϕ_m of $\ln m_m(\phi)$. When $\beta_m < \frac{3}{8}$, ϕ slowly rolls down toward ϕ_m without oscillating (overdamped oscillator). (See also [14] in which the transition between radiation domination and matter domination was approximated – in the analytical formulas – as being a sharp one.)

6. Observable consequences of a cosmological relaxed massless dilaton

Sections 4 and 5 have exhibited several efficient mechanisms for driving the VEV of the dilaton toward a value where it decouples from matter. However, none of these mechanisms is a perfect attractor. The important question remains of giving quantitative estimates of the residual coupling strength of the dilaton at various cosmological epochs and of the corresponding observable effects.

The quantitative estimates of the efficiency of the cosmological attraction of the dilaton depend very much on the universality, or lack thereof, of the dilaton couplings. If the dilaton coupling functions $B_g(\Phi)$, $B_F(\Phi)$, $B_H(\Phi)$ (the latter representing the class of couplings to the fundamental Higgs sector, if it exists as such) are unrelated functions, one expects the mass functions (2.10) to have minima (if any) at different values of ϕ, say ϕ_m^A. For instance, the lepton and quark masses will involve B_H while hadron masses will all be proportional to $B_g^{-1/2} \exp[-8\pi^2 b_3^{-1} k_3 B_3]$ ($B_3 \equiv B_{SU(3)}$). In such a non-universal case, the various mass thresholds, and phase transitions, will not attract ϕ to the same value, but will tend to reshuffle each time the value of ϕ. In that case, the only efficient fixing of the value of ϕ would arise during the matter era, ϕ being attracted toward of minimum of $m_m(\phi)$ where the label "m" represents the type of matter which dominates the present universe.

By contrast, one can consider the case where all the dilaton coupling functions coincide, $B_i(\Phi) = B(\Phi)$, or more generally where they depend on a common function: $B_i(\Phi) = f_i[B(\Phi)]$. In this universal $B(\Phi)$ case, all the mass thresholds, as well as the QCD phase transition and the matter era, tend to attract ϕ to a common value, some value ϕ_m where $B(\phi)$ reaches a maximum [7]. In the universal case, the cosmological evolution is an extremely efficient way of pinning down the value of ϕ. Moreover, as ϕ is pinned down to an extremum of $B(\phi)$, i.e. to a value where $\partial B(\phi)/\partial\phi$ and $\partial m_A(\phi)/\partial\phi$ vanish, one can say that the universal dilaton coupling case illustrates some "Principle of Least Coupling" in the sense that the universe is attracted to dilaton values extremizing the strengths of the interaction. It would be worth exploring whether imposing this universality provides a sensible

[7] Note that a primordial (inflationary type) phase transition – as well as the electroweak one, if the Higgs sector is fundamental – could instead attract ϕ to a *minimum* of $B(\phi)$ through a transient vacuum energy $\propto B(\phi)$.

way of selecting a preferred class of string models. In the following, we leave open the two possibilities, universal/non-universal, in our discussion of the observable consequences of our scenario.

The earliest observational information we have about cosmology concerns the primordial abundance of the light elements (mainly helium 4, with traces of deuterium, helium 3 and lithium 7). Let us discuss the production of helium 4 as an example. In the standard scenario of homogeneous primordial nucleosynthesis, the abundance of Helium is mainly determined by the neutron to proton ratio at the temperature where the rate of interconversion $\mathrm{n} \leftrightarrow \mathrm{p}$ due to weak interactions becomes slower than the cosmological expansion rate (freeze-out) (see [25]). Neglecting the small additional effect of free neutron decay, one can write an approximate analytical formula for the primordial helium abundance (by weight), Y, of the form, $Y = 2/(\mathrm{e}^{aX} + 1)$ where a is a pure number of order unity and where X denotes the following dimensionless combination of coupling constants and masses:

$$X \equiv g_2^{4/3}\left(1 + 3g_A^2\right)^{1/3} \frac{m_\mathrm{n} - m_\mathrm{p}}{m_\mathrm{W}} \left(\frac{m_\mathrm{Planck}}{m_\mathrm{W}}\right)^{1/3} \left(\frac{g_*}{10.75}\right)^{-1/6}. \tag{6.1}$$

Here g_2 denotes the SU(2) coupling constant, $g_A \simeq 1.26$ the axial/vector coupling of the nucleon, and g_* the effective number of relativistic degrees of freedom at freeze-out (retained here to allow easy comparisons between the effect of a change in g_* – e.g. an additional light neutrino – and the effects of changing, e.g., Newton's constant $G = m_\mathrm{Planck}^{-2}$, or Fermi's one $G_\mathrm{F} = g_2^2/8m_\mathrm{W}^2$). A remarkable fact about the combination X is that it is numerically of order unity thanks to a delicate compensation between large $((m_\mathrm{Planck}/m_\mathrm{W})^{1/3} \simeq (1.52 \times 10^{17})^{1/3})$ and, small $((m_\mathrm{n} - m_\mathrm{p})/m_\mathrm{W} \simeq 1.61 \times 10^{-5})$ factors. This fact prevents us from proposing an educated guess of the quantitative dependence of X on the bare dilaton coupling constants $B_F(\phi)$, $B_g(\phi), \dots$. Even the sign of $\partial \ln X/\partial \ln B^{-1}$ (when $B_i(\phi) = B(\phi)$) is unclear. On the other hand one can estimate that $\partial Y/\partial \ln X \simeq -0.44$ both from the rough analytical formula for $Y(X)$ and from the numerical computations of the dependence of Y on the neutron half-life or on g_*. We can therefore write the value of the helium abundance predicted by a scenario modified by the presence of a dilaton as

$$Y^\mathrm{dil}(\eta) = Y^\mathrm{GR}(\eta) - 0.22\frac{\partial \ln X}{\partial \ln B^{-1}}\kappa(\phi_\mathrm{rad} - \phi_\mathrm{m})^2, \tag{6.2}$$

where we have reestablished the slight dependence of Y upon the baryon to photon ratio, η. In the standard, general relativistic scenario the dependence of the GR-predicted abundances on η is crucially used, together with the observed values of the light-element abundances, to set upper bounds on η, and thereby upper bounds on the ratio of the present total baryon mass density to the closure density, Ω_b. The standard conclusion being that baryons fail to close the Universe by at least a factor five, $\Omega_\mathrm{b} < 0.2$ [25]. Eq. (6.2) (to be completed by the corresponding dilaton-modified predictions for the other light elements) suggests that a dilatonic Universe could naturally accomodate $\Omega_b = 1$ if the value ϕ_{rad} of ϕ at freeze-out (i.e. just after the electron mass threshold) differs by a small (but not too small) amount from the minimum ϕ_m. (For instance, in the case of the helium abundance, the dilaton correction term on the right-hand side of Eq. (6.2) should be approximately -0.03, and $\partial \ln X / \partial \ln B^{-1}$ should be positive.) It would be interesting to reexamine in full numerical detail primordial nucleosynthesis within the type of dilaton scenario considered here to assess whether it could naturally reconcile $\Omega_b = 1$ with the observed abundances of light elements. Let us only note here that the rather modest attraction toward ϕ_m which is probably needed in such a scenario seems more natural in the non-universal case. Indeed, in the universal $B(\phi)$ case, all the nine mass thresholds compound their effect to drive ϕ very near some universal minimum ϕ_m. More precisely, $\phi_{rad} - \phi_m = F_r \times (\phi_{in}\text{-}\phi_m)$ where ϕ_{in} is the "initial" value of ϕ (meaning in this work, before the electroweak phase transition) and where the total attracting power of the radiation era is given by

$$F_r(\kappa) = \left(\prod_{A=1}^{9} m_{\pm}(\beta_A f_A^{in}) \right) \left(\prod_{i=2,3} p(\beta_i^{vac} f_i^{vac}) \right). \tag{6.3}$$

The values of β_A and f_A^{in} to be used in the attraction factors of each of the nine mass thresholds have been given above (remember that the $+$ $(-)$ sign corresponds to fermions (bosons)). The second factor in Eq. (6.3) corresponds to the effect of the two known phase transitions electroweak (2) and QCD (3). In view of the uncertainty in the calculation of the effect of phase transitions, and anyway of their expected modest contribution (see above), we shall neglect the attraction power of these phase transitions in the following. The small but non-zero value of $\phi_{rad} - \phi_m = F_r(\kappa)\Delta\phi$ (with $\Delta\phi \equiv \phi_{in} - \phi_m$) implies that all the gauge coupling constants squared, $g^2 \propto B^{-1}(\phi)$, differed, at the end of radiation era, from their present values g_0^2 by a fractional amount

$$\frac{g_{rad}^2 - g_0^2}{g_0^2} \simeq \tfrac{1}{2}\kappa(\phi_{rad} - \phi_m)^2 = \tfrac{1}{2}\kappa[F_r(\kappa)\Delta\phi]^2 \tag{6.4}$$

(where we used the fact that $\phi_0 - \phi_m \ll \phi_{rad} - \phi_m$ because of the matter era attraction). As one *a priori* expects $\Delta\phi \equiv \phi_{in} - \phi_m$ to be of order unity, the function $\frac{1}{2}\kappa F_r^2(\kappa)$, which is plotted in Fig. 2, illustrates the remarkable efficiency (in the universal case) of the radiation era in pinning down the values of the physical coupling constants.

During the subsequent matter era, ϕ is (in the universal case) further driven toward ϕ_m by the factor $F_m(\kappa, Z_0)$, Eqs. (5.4)–(5.6). The numerical value of the matter era attraction factor F_m is proportional to $Z_0^{-3/4}$, where $Z_0 \equiv \exp(p_0)$ denotes the redshift separating us from the epoch of equivalence between matter and radiation. In the approximation $m_m(\phi) \simeq$ const. introduced at the beginning of Section 5, this redshift is given by [14]

$$Z_0 = \frac{\rho_0^{matter}}{\rho_0^{rad}} \simeq 13350\,\Omega_{75}, \tag{6.5a}$$

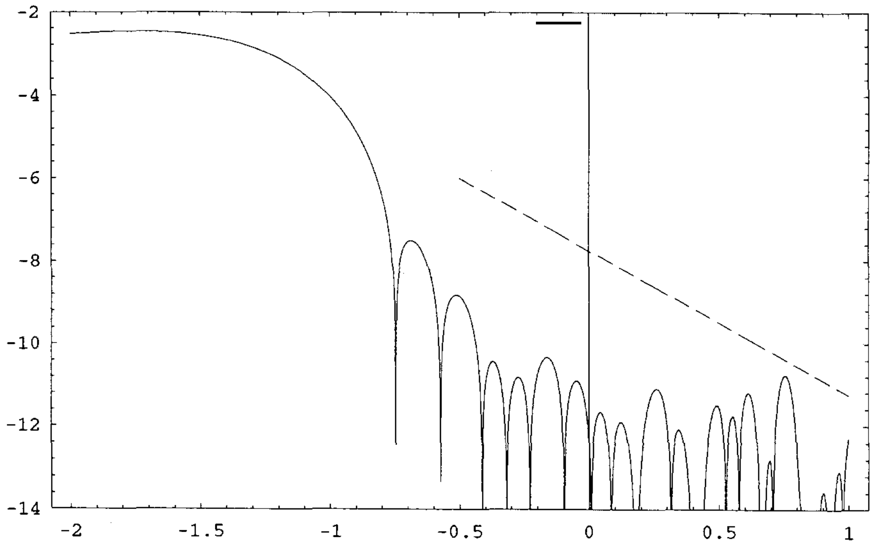

Fig. 2. The solid line represents $\log_{10}[(g^2_{\rm rad} - g^2_0)/g^2_0]$ as a function of $\log_{10}\kappa$, i.e. the fractional deviation (left over at the end of the radiation era) of the gauge coupling constants $g^2_{\rm rad} \propto B^{-1}(\phi_{\rm rad})$ from their present values g^2_0, versus the curvature κ of the function $\ln B^{-1}(\phi)$ near its minimum. The dashed line represents an analytical estimate (when $\kappa \gtrsim 1$) of that deviation, obtained by assuming that the phases θ of the WKB results Eq. (4.10a) are randomly distributed.

where

$$\Omega_{75} \equiv \frac{8\pi \overline{G} \rho_0^{\rm matter}}{3(75\ {\rm km\cdot s^{-1}\ Mpc^{-1}})^2} = \frac{\rho_0^{\rm matter}}{1.0568\times 10^{-29}\ {\rm g\cdot cm^{-3}}}. \tag{6.5b}$$

Under the hypothesis of a spatially flat Universe ($K=0$), generally assumed in this paper, Ω_{75} is linked to the present value of Hubble's "constant", H_0, by $\Omega_{75} = (H_0/75\ {\rm km\cdot s^{-1}\ Mpc^{-1}})^2$. In that case, the observational limits $50 < H_0/1$ km $\cdot$ s^{-1} Mpc^{-1} < 100 imply $0.44 < \Omega_{75} < 1.78$. On the other hand, if one assumes that the Universe is spatially hyperbolic ($K=-1$), one must modify the coefficients of the evolution equation (3.3) for ϕ by retaining the K-dependent terms. However, it was shown in [14] that as long as $\Omega_{75} > 0.05$ this modification of Eq. (3.3) has a small effect, and that the matter era attraction factor of $K=-1$ Universes is well approximated by the $K=0$ formula (5.6), with Z_0 given by Eqs. (6.5a), (6.5b). The main difference is that now Ω_{75} is not related to H_0, and can be smaller than 0.44. In fact, present observational data are compatible with $\Omega_{75} \sim 0.1$.

Finally, the scenarios considered here predict that the present value of ϕ, say ϕ_0, differs from the minimum $\phi_{\rm m}$ by $\phi_0 - \phi_{\rm m} = F_{\rm t}(\kappa, Z_0)\Delta\phi$ where $\Delta\phi \equiv \phi_{\rm in} - \phi_{\rm m}$ and where the total attraction factor is

$$F_{\rm t}(\kappa, Z_0) \equiv F_{\rm r}(\kappa) F_{\rm m}(\kappa, Z_0). \tag{6.6}$$

There are three kinds of presently observable consequences of having ϕ_0 near, but different from, $\phi_{\rm m}$: (i) violations of the (weak) equivalence principle, (ii) modifications of relativistic gravity, and (iii) slow changes of the coupling constants of physics, notably the fine-structure constant α and Newton's constant G.

To discuss the modifications of the gravitational sector, we can make use of the results of [26] on the relativistic gravitational interaction of condensed bodies in generic metrically-coupled tensor–scalar theories. Indeed, the action describing the classical interaction of massive particles of various species under the exchange of the $g_{\mu\nu}$ and ϕ fields is given by $S_{g,\phi} + S_{\mathrm{m}}[g, \phi, x]$, where $S_{g,\phi}$ is given by (2.7b) and S_{m} by Eq. (3.4). This action is identical to the one studied in Section 6 of [26]. We conclude that, at the newtonian approximation, the interaction potential between particle A and particle B is $-G_{AB} m_A m_B / r_{AB}$ where $r_{AB} \equiv |\boldsymbol{x}_A - \boldsymbol{x}_B|$ and

$$G_{AB} = \overline{G}\left(1 + \alpha_A^{(0)} \alpha_B^{(0)}\right). \tag{6.7}$$

Here $\overline{G}$ is the bare gravitational coupling constant entering the action (2.7b), and $\alpha_A^{(0)}$ is the present strength of the coupling of the dilaton to A-type particles, i.e. the value of (3.6) taken at the cosmologically determined VEV ϕ_0. (In diagrammatic language, the two terms on the right-hand side of Eq. (6.7) are, respectively, the one-graviton exchange contribution ($\overline{G}$) and the one-dilaton exchange one ($\overline{G} \alpha_A^{(0)}\ \alpha_B^{(0)}$).) Two test masses, made respectively of A- and B-type particles, will fall in the gravitational field generated by an external mass m_E with accelerations a_A and a_B differing by

$$\left(\frac{\Delta a}{a}\right)_{AB} \equiv 2\frac{a_A - a_B}{a_A + a_B} = \frac{\left(\alpha_A^{(0)} - \alpha_B^{(0)}\right)\alpha_E^{(0)}}{1 + \frac{1}{2}\left(\alpha_A^{(0)} + \alpha_B^{(0)}\right)\alpha_E^{(0)}} \simeq \left(\alpha_A^{(0)} - \alpha_B^{(0)}\right)\alpha_E^{(0)}. \tag{6.8}$$

All precision tests of the gravitational interaction used macroscopic bodies made of (neutral) atoms. Let the labels $A, B, \ldots$ denote some atoms. In the approximation where one neglects $m_{\mathrm{u}}/m_{\mathrm{N}}$, $m_{\mathrm{d}}/m_{\mathrm{N}}$, $m_{\mathrm{e}}/m_{\mathrm{N}}$, α and α_{weak}, the mass of an atom is a pure (dilaton-independent) number times a *QCD-determined* mass scale, say $u_3(\phi)$. In this approximation $\alpha_A(\phi) = \partial \ln m_A / \partial\phi$ is *independent* of the type of atom considered and is equal to $\alpha_3(\phi) = \partial \ln u_3/\partial\phi$. The dilaton dependence of u_3 is determined by Eq. (2.9). Choosing u_3 so that its present value $u_3(\phi_0)$ is numerically equal to the atomic mass unit, $u = 931.49432$ MeV, we see from Eqs. (4.6) that

$$\alpha_A(\phi_0) \simeq \alpha_3(\phi_0) = \beta_3(\phi_0 - \phi_{\mathrm{m}}) = \beta_3 F_{\mathrm{t}}(\kappa, Z_0)\Delta\phi, \tag{6.9}$$

with $\beta_3 = \kappa \partial \ln u_3 / \partial \ln B^{-1} \simeq 40.82\kappa$.

In this approximation, the dilaton mimics a usual Jordan–Fierz(–Brans–Dicke) field, i.e. a scalar field coupled exactly to T_μ^μ. The main observational consequences of the body-independent coupling (6.9) are modifications of *post-newtonian* relativistic effects, $\mathrm{O}(v^2/c^2)$ beyond the newtonian $1/R$ interaction (weak-gravitational-field case) [8]. The latter are measured by the two Eddington parame-

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[8] In view of the positiveness of β_3, the recent results of [27] show that the deviations from general relativity are further quenched in the strong-gravitational-field case of binary neutron star systems.

ters $\gamma_{\mathrm{Edd}}-1$ and $\beta_{\mathrm{Edd}}-1$ (which vanish in general relativity). From [26] we see that in the approximation (6.9)

$$1-\gamma_{\mathrm{Edd}}=2\frac{\alpha_3^2}{1+\alpha_3^2}\simeq 2(\beta_3)^2\left[F_{\mathrm{t}}(\kappa,Z_0)\Delta\phi\right]^2, \tag{6.10}$$

$$\beta_{\mathrm{Edd}}-1=\frac{1}{2}\frac{\beta_3\alpha_3^2}{\left(1+\alpha_3^2\right)^2}\simeq \tfrac{1}{2}(\beta_3)^3\left[F_{\mathrm{t}}(\kappa,Z_0)\Delta\phi\right]^2. \tag{6.11}$$

Note also that the value of Newton's gravitational constant (in Einstein units) is $G_{\mathrm{N}}=\overline{G}(1+\alpha_3^2)$.

Much more sensitive tests of the existence of dilaton couplings are obtained by looking at violations of the weak equivalence principle, i.e. at the body dependence of $\alpha_A(\phi_0)$ beyond the QCD approximation (6.9). To do this, we shall retain the leading $m_{\mathrm{u}}/m_{\mathrm{N}}$, $m_{\mathrm{d}}/m_{\mathrm{N}}$, $m_{\mathrm{e}}/m_{\mathrm{N}}$, and α corrections to the mass of an atom. First, the mass of the nucleons have the form, $m_{\mathrm{p}}=m_{\mathrm{N3}}+b_{\mathrm{u}}m_{\mathrm{u}}+b_{\mathrm{d}}m_{\mathrm{d}}+C_{\mathrm{p}}\alpha$, $m_{\mathrm{n}}=m_{\mathrm{N3}}+b_{\mathrm{d}}m_{\mathrm{u}}+b_{\mathrm{u}}m_{\mathrm{d}}+C_{\mathrm{n}}\alpha$, where m_{N3} ($\simeq u_3$) is the pure QCD approximation to the nucleon mass, and where b_{u}, b_{d}, C_{p}/u_3 and C_{n}/u_3 are pure numbers (in the approximation of negligible strange quark content one has $b_{\mathrm{u}}=\langle p\,|\,\bar{u}u\,|\,p\rangle/2m_{\mathrm{N}}$, $b_{\mathrm{d}}=\langle p\,|\,\bar{d}d\,|\,p\rangle/2m_{\mathrm{N}}$) [28]. Second, the mass of an atom can be approximately decomposed as

$$m(\mathrm{atom})=Zm_{\mathrm{p}}+Nm_{\mathrm{n}}+Zm_{\mathrm{e}}+E_3^{\mathrm{nucleus}}+E_1^{\mathrm{nucleus}},$$

where Z is the atomic number and N the number of neutrons, and where E_3^{nucleus} denotes the strong-interaction contribution to the binding energy of the nucleus, and E_1^{nucleus} the Coulomb interaction energy of the nucleus.

In terms of the baryon number $B\equiv N+Z$, the neutron excess $D\equiv N-Z$, and the Coulomb energy term $E\equiv Z(Z-1)/(N+Z)^{1/3}$, the mass of an atom can be written as

$$m(\mathrm{atom})=u_3M_3+\sigma' B+\delta' D+a_3\alpha u_3 E, \tag{6.12}$$

where M_3 is a pure number ($=B+$ strong-interaction binding contribution) and where we have defined

$$\sigma'\equiv\sigma+\tfrac{1}{2}C_{\mathrm{n}}\alpha+\tfrac{1}{2}C_{\mathrm{p}}\alpha+\tfrac{1}{2}m_{\mathrm{e}},\quad \delta'\equiv\tfrac{1}{2}\delta+\tfrac{1}{2}C_{\mathrm{n}}\alpha-\tfrac{1}{2}C_{\mathrm{p}}\alpha-\tfrac{1}{2}m_{\mathrm{e}},$$

with the usual definitions for $\sigma\equiv\frac{1}{2}(m_{\mathrm{u}}+m_{\mathrm{d}})(b_{\mathrm{u}}+b_{\mathrm{d}})$, $\delta\equiv(m_{\mathrm{d}}-m_{\mathrm{u}})(b_{\mathrm{u}}-b_{\mathrm{d}})$. (Note the factor $\frac{1}{2}$ in the first term of the definition of δ'.) Finally, by differentiating the logarithm of (6.12) we get a more precise expression than (6.9) for the dilaton coupling strength,

$$\alpha_A(\phi_0)\simeq\alpha_3(\phi_0)+\frac{\partial\hat{\sigma}}{\partial\phi_0}\left(\frac{B}{M}\right)_A+\frac{\partial\hat{\delta}}{\delta\phi_0}\left(\frac{D}{M}\right)_A+a_3\frac{\partial\alpha}{\partial\phi_0}\left(\frac{E}{M}\right)_A, \tag{6.13}$$

where we have introduced $\hat{\sigma}\equiv\sigma'/u_3$, $\hat{\delta}\equiv\delta'/u_3$ and approximated $M_3\simeq M\equiv$

$m(\text{atom})/u_3$ in the corrections terms [9]. Finally, from Eq. (6.8) we get an equivalence-principle violation of the form

$$\left(\frac{\Delta a}{a}\right)_{AB} = \left[\kappa F_{\mathrm{t}}(\kappa, Z_0)\Delta\phi\right]^2 \left[C_B \Delta\left(\frac{B}{M}\right) + C_D \Delta\left(\frac{D}{M}\right) + C_E \Delta\left(\frac{E}{M}\right)\right]_{AB}, \tag{6.14}$$

where $(\Delta X)_{AB} \equiv X_A - X_B$ and where $C_B = \lambda_{u_3} \partial\hat{\sigma}/\partial \ln B^{-1}$, $C_D = \lambda_{u_3} \partial\hat{\delta}/\partial \ln B^{-1}$, $C_E = \lambda_{u_3} \lambda_\alpha a_3 \alpha$, $\lambda_{u_3} \equiv \partial \ln u_3/\partial \ln B^{-1}$ and $\lambda_\alpha \equiv \partial \ln \alpha/\partial \ln B^{-1}$. Numerically, our usual estimate (2.9) gives $\lambda_{u_3} \simeq 40.82$, and the idea of unification of gauge couplings at the string scale gives $\lambda_\alpha \simeq 1$. (E.g. in the simplest SU(5)-type GUT the value of the fine-structure constant at the QCD-confining energy scale u_3 – such that $\alpha_{\text{strong}}(u_3) \simeq 1$ – is given by

$$\alpha(u_3)^{-1} = \tfrac{22}{7}\alpha_{\text{GUT}}^{-1} - \tfrac{10}{21}\alpha_{\text{strong}}(u_3)^{-1} \simeq \tfrac{22}{7}\alpha_{\text{GUT}}^{-1} \propto B(\phi).$$

We have also $a_3\alpha = 0.717\ \text{MeV}/u_3 = 0.770 \times 10^{-3}$ from the fit of atomic masses to the Bethe–Weizsäcker formula. We can therefore estimate the coefficient of the nuclear Coulomb energy term in Eq. (6.14) to be $C_E \simeq 3.14 \times 10^{-2}$. As for the other two coefficients, C_B and C_D, it is much less clear how to estimate them. From the experiment-derived values of $\sigma = 35 \pm 5$ MeV and $\delta = 2.05 \pm 0.30$ MeV, plus the theoretical estimates $C_{\mathrm{p}}\alpha = 0.63$ MeV, $C_{\mathrm{n}}\alpha = -0.13$ MeV [28], one can compute $\hat{\sigma} = 3.8 \times 10^{-2}$ and $\hat{\delta} = 4.2 \times 10^{-4}$. From the point of view of their dilaton dependence $\hat{\sigma}$ and $\hat{\delta}$ are the sum of four terms proportional to m_{u}/u_3, m_{d}/u_3, m_{e}/u_3 and α. It is impossible at present to reliably guess the dilaton dependence of the mass ratios $m_{\text{quark}}/m_{\text{hadron}}$ and $m_{\mathrm{e}}/m_{\text{hadron}}$. The numbers we would get for $\partial\hat{\sigma}/\partial \ln B^{-1}$ would be very different were we to assume our usual exponential link to the string scale, or some other assumption. It seems, however, reasonable to estimate that the order of magnitude of $\partial\hat{\sigma}/\partial \ln B^{-1}$ and $\partial\hat{\delta}/\partial \ln B^{-1}$ will be *at most* that given by the exponential assumption (2.11), and *at least* that obtained by differentiating only the fine-structure constant contributions to $\hat{\sigma}$ and $\hat{\delta}$. This yields corresponding rough upper and lower bounds for the coefficients of the B and D contributions: $1.1 \times 10^{-2} \lesssim |C_B| \lesssim 7.5$, $1.7 \times 10^{-2} \lesssim |C_D| \lesssim 8.2 \times 10^{-2}$. If these upper bounds are correct, one can check that the last term in Eq. (6.14) will be numerically dominant for pairs (A, B) having a large difference in atomic number. (Indeed, E/M is roughly proportional to $Z^{2/3}$.) The largest effect would arise in comparing uranium (for which $E/M \simeq 5.7$) with

[9] Note that the assumption of a universal $B(\phi)$ is crucial to ensure that all the terms in Eq. (6.13) have in common a very small factor $\phi_0 - \phi_{\mathrm{m}}$. If, e.g., the mass of leptons (and/or α) depended on a completely different function of ϕ than the mass of hadrons, Eq. (6.13) would, at best, predict that the equivalence principle is violated at the (unacceptable) level $\sim [O(\alpha) + O(m_{\text{lepton}}/m_{\mathrm{N}})]^2$, in the favourable case where the Universe is assumed to be dominated by hadronic matter. See, however, below for the case of small universality violations.

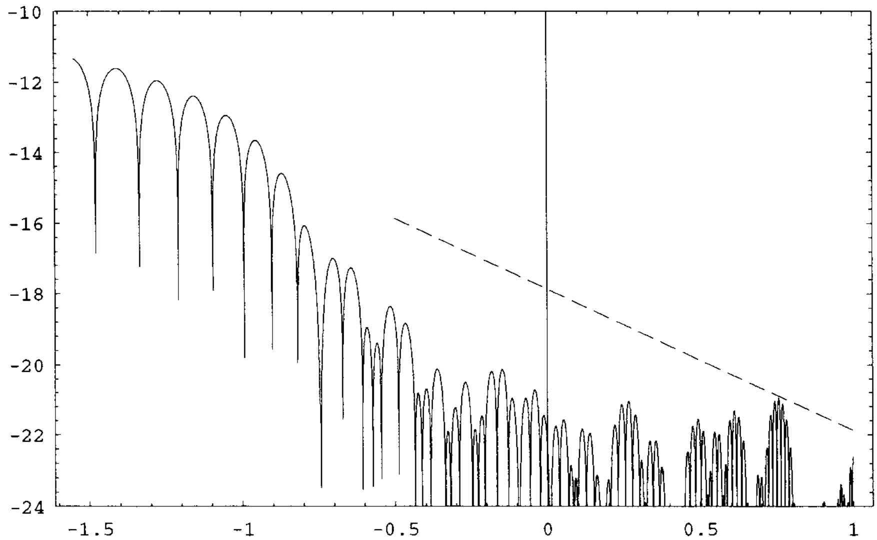

Fig. 3. The solid line represents $\log_{10}(\Delta a/a)_{\rm max}$ as a function of $\log_{10}\kappa$, i.e. the expected present level of violation of the equivalence principle (when comparing uranium with a light element) as a function of the curvature κ of the (string-loop induced) function $\ln B^{-1}(\phi)$ near a minimum $\phi_{\rm m}$. The dashed line represents an analytical estimate (when $\kappa \gtrsim 1$) of that violation obtained by assuming random phases θ in Eqs. (4.10a) and (5.6).

hydrogen (any light element would do nearly as well). For such a pair, Eq. (6.14) yields

$$\left(\frac{\Delta a}{a}\right)_{\rm max} = 0.18\left[\kappa F_{\rm t}(\kappa, Z_0)\Delta\phi\right]^2. \tag{6.15}$$

The right-hand side of Eq. (6.15) is plotted in Fig. 3 as a function of κ (assuming $\Delta\phi = 1$, and $\Omega_{75} = 1$). As one *a priori* expects κ to be of order unity, Fig. 3 shows that, within the scenario considered here (including universal dilaton couplings), the present tests of the equivalence principle (at the 10^{-11}–10^{-12} level) do not put any significant constraints on the existence of a massless dilaton. It is interesting to remark that another experimentally viable possibility would be to have only an approximate universality of the dilaton couplings. If, for instance, the masses of hadrons and leptons depended on slightly different functions, say $B(\phi)$ and $B(\phi)+\epsilon(\phi)$, the violation of the equivalence principle would (when neglecting $\phi_0-\phi_{\rm m}$) be proportional to $(\partial\epsilon/\partial\phi_0)^2$. If, as suggested above, the violation of universality were proportional to $\alpha' m^2_{\rm SUSY}$, we would have $\Delta a/a \propto (\alpha' m^2_{\rm SUSY})^2 \sim (m_{\rm SUSY}/\Lambda_{\rm s})^4$.

The situation is even worse if we consider tests of post-newtonian gravity. Indeed, from Eqs. (6.10) and (6.15) we have the link

$$\left(\frac{\Delta a}{a}\right)_{\rm max} \simeq 5.4\times10^{-5}(1-\gamma_{\rm Edd}), \tag{6.16}$$

showing that the present and planned levels of testing of post-newtonian gravity, i.e. 10^{-3} and 10^{-7} at best for $\gamma_{\rm Edd}$, correspond, respectively, to equivalence-principle tests at the levels 5×10^{-8} and 5×10^{-12}. The other link $\beta_{\rm Edd} - 1 = \frac{1}{4}\beta_3(1 - \gamma_{\rm Edd}) \simeq 10.2\kappa(1 - \gamma_{\rm Edd})$ shows that $\beta_{\rm Edd}$ tests do not fare essentially better.

The last observational consequence of our scenario to discuss is the residual present variation of the coupling constants of physics. From $\partial \ln \alpha / \partial \ln B^{-1} \simeq 1$, with $\ln B^{-1} = \text{const.} + \frac{1}{2}\kappa[\phi(t) - \phi_{\rm m}]^2$ and a present time dependence of ϕ given by Eq. (5.6) (in which the leading term is $\cos(\omega p + \text{const.})$), we deduce that

$$\left(\frac{\dot\alpha}{\alpha H}\right)_0 = -\kappa\left[\omega \tan\theta_0 + \tfrac{3}{4}\right]\left[F_{\rm t}(\kappa, Z_0)\Delta\phi\right]^2. \tag{6.17}$$

Using $\omega \equiv [\frac{3}{2}(\beta_{\rm m} - \frac{3}{8})]^{1/2}$ with $\beta_{\rm m} \sim 40.8\kappa$ (if the particles dominating the Universe have a mass not very different from the GeV scale), we have the approximate link $(\dot\alpha/\alpha H)_0 \sim -43\kappa^{-1/2} \tan\theta_0 (\Delta a/a)_{\rm max}$. This link shows again that equivalence-principle tests are the most sensitive way of searching for possible dilaton couplings. (In the foreseeable future, ultrastable cold-atom clocks might probe the level $\dot\alpha/\alpha \sim 10^{-16}\ {\rm yr}^{-1} \sim 10^{-6}H_0$ which corresponds to $\Delta a/a \sim 10^{-8}$.)

Finally, the time variation of the gravitational coupling constant (measured in Einstein units) is

$$\left(\frac{\dot G}{GH}\right)_0 = -2\left[\omega \tan\theta_0 + \tfrac{3}{4}\right]\left[\beta_3 F_{\rm t}(\kappa, Z_0)\Delta\phi\right]^2. \tag{6.18}$$

In actual $\dot G$ experiments one is comparing an orbital frequency n (e.g. let us consider that of a planet around the Sun) to an atomic frequency ν. Taking into account the adiabatic invariants of the orbital motion (angular momentum and eccentricity) and assuming an atomic clock based on the Bohr frequency $\propto m_{\rm e}\alpha^2$, the directly measured quantity will be

$$\frac{\dot n}{n} - \frac{\dot\nu}{\nu} = 2\frac{\dot G}{G} + 2\frac{\dot m_{\rm s}}{m_{\rm s}} + 3\frac{\dot m_{\rm p}}{m_{\rm p}} - \frac{\dot m_{\rm e}}{m_{\rm e}} - 2\frac{\dot\alpha}{\alpha}, \tag{6.19}$$

where $m_{\rm s}$, $m_{\rm p}$ denote the masses of the Sun and of the planet. (Contrary to Eq. (6.18), Eq. (6.19) is valid in any system of units.) Eq. (6.19) gives finally [10]

$$\frac{\dot n}{n} - \frac{\dot\nu}{\nu} = -H_0\left(\omega \tan\theta_0 + \tfrac{3}{4}\right)\left[F_{\rm t}(\kappa, Z_0)\Delta\phi\right]^2\left(4\beta_3^2 + 5\beta_3 - \beta_{\rm e} - 2\kappa\right), \tag{6.20}$$

in which the term coming from Eq. (6.18) dominates. In spite of the large factor $(\beta_3/\kappa)^2 \simeq (40.8)^2$, the scaling of the prediction (6.20) with the Hubble rate H_0 makes it pale in comparison with equivalence principle tests. (On the other hand, this large factor renders $\dot G$ experiments competitive with $\dot\alpha$ ones.)

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[10] The link between $\dot\phi_0$ and $\dot n/n$ is more involved if n is the orbital frequency of a binary neutron star system: see [29], which must be completed by taking into account the changes in the rest-masses of the stars, and the non-perturbative gravitational self-energy effects [27].

7. Conclusions

Einstein's starting point in constructing general relativity was the interpretation of the universality of free fall in terms of a universal coupling of matter to a common metric tensor $g_{\mu\nu}$. It has since been felt that such a universal metric coupling was the only theoretically natural way of explaining how the long-range fields participating in gravity [11] could satisfy the high-precision tests of the equivalence principle (now reaching the 10^{-12} level).

The present work suggests that other types of couplings of a long-range scalar field Φ, though a priori entailing strong violations of the equivalence principle, could provide theoretically natural ways of explaining why no violations have been seen at the 10^{-12} level. The technically simplest such type of couplings is a universal multiplicative coupling of Φ to all the other fields, $\mathscr{L}_{\text{tot}} = B(\Phi)\mathscr{L}_0(g_{\mu\nu}, \nabla\Phi, A_\mu, \psi, \dots)$ with $B(\Phi)$ admitting a maximum. More general types of couplings, all involving some assumption of (possibly approximate) universality of the Φ-dependence, have been discussed above.

It maybe worthwhile to summarize in qualitative terms [12] the basic reasons why a massless dilaton is rendered nearly invisible during the cosmological evolution: (i) Each time, during the radiation era, the Universe passes through a temperature $T \sim m_A$, the A-type particles and antiparticles become non-relativistic before annihilating themselves and disappearing from the cosmic soup; this provides a source term for the dilaton proportional to the ϕ-gradient of $m_A(\phi)$, which attracts ϕ toward a minimum ϕ_{m}^A of $m_A(\phi)$; Eq. (4.6b) and Fig. 1 suggest that each such attraction is moderately efficient, leaving ϕ nearer to ϕ_{m}^A by a factor $\sim \frac{1}{3}$. (ii) Under the assumption of universality of the dilaton coupling functions $B(\phi)$, the minima of all the mass functions $m_A(\phi)$ will coincide and the ~ 9 mass thresholds of the radiation era will compound their effects to attract very efficiently ϕ toward some common minimum ϕ_{m}. (iii) In the subsequent matter era, ϕ will be continuously attracted toward a minimum of the mass function $m_{\text{m}}(\phi)$ corresponding to the (non-relativistic) matter dominating the Universe. (Under the same universality condition this minimum will be again ϕ_{m}.) The attraction factor due to the matter era is inversely proportional to the $\frac{3}{4}$th power of the redshift $Z_0 \sim 1.3 \times 10^4$ separating us from the end of the radiation-dominated era. (iv) As a consequence of the very efficient total attraction toward ϕ_{m}, the present strength of the coupling of the dilaton to any type of matter $\alpha_A(\phi)$, being proportional to the ϕ-gradient of $m_A(\phi)$, is very small. The present deviations from general relativity in the interaction between two masses, m_A and m_B, are proportional to the product $\alpha_A \alpha_B$ and are therefore extremely small. (v) The equivalence-principle tests are very sensitive, but they probe only differences $(\alpha_A - \alpha_B)\alpha_C$ which,

[11] In Einstein's theory gravity is mediated by only one, spin-2, field; but in metrically-coupled tensor–scalar theories, gravity is mediated both by a spin-2 and a spin-0 field. In the latter case, the universal metric coupled to matter is a combination of the two pure-spin fields: $g_{\mu\nu}^{\text{univ}} = A^2(\phi) g_{\mu\nu}^*$.

[12] Simplified quantitative estimates are provided below.

because of the known universal features of QCD-generated masses, contain as supplementary small parameters either the ratio of the quark masses to the nucleon mass, or the fine-structure constant.

From a theoretical point of view, our work suggests a criterion for selecting a preferred class of string models: namely those exhibiting the kind of universality of the dilaton dependence required by our scenario, with a function $B(\phi)$ admitting a maximum [13]. It will take, however, an improvement in our current understanding of supersymmetry breaking in string theory to see whether the universality required by the Least Coupling Principle is a viable option, providing a reasonable selection criterion for SUSY breaking mechanisms. It is to be noted that in this paper we had always in mind the coupling of the (four-dimensional) dilaton which is such an intimate partner of the graviton that it seems reasonable to assume that it remains massless in the low-energy world [14]. However, the cosmological attractor mechanism described here could also apply to the other gauge-neutral scalar fields (moduli) present in string theory. Because of threshold effects, the gauge coupling function B_F acquires a non-trivial dependence on the moduli fields [20]. Therefore, we have here with the help of target-space duality, a possible mechanism for fixing the moduli to values where they decouple from the other fields.

From an experimental point of view, our results provide a new incentive to improving the precision of equivalence-principle tests (universality of free fall, constancy of the constants, etc.). Fig. 3 suggests, when assuming that the curvature of $B(\phi)$ near its maximum is of order unity – say $0.1 < \kappa < 10$ –, to look for a present level of violation of the universality of free fall somewhere between 10^{-14} and 10^{-23}. Actually, one should not consider the results plotted in Fig. 3 too seriously. On the one hand, even within the precise assumptions made in the text, the predicted maximal value of $\Delta a/a$ contains an unknown factor $\simeq \Omega_{75}^{-3/2}(\phi_{\text{in}} - \phi_{\text{m}})^2$ which could be $\gtrsim 10$. On the other hand, our assumption (2.11) with $\mu_A = 1$ has entailed a specific phasing of the various oscillations undergone by ϕ during the radiation era, i.e. specific choices of where on the curves of Fig. 1 $[\phi(p) - \phi_{\text{m}}]/[\phi(-\infty) - \phi_{\text{m}}]$ ends up being when $p \to +\infty$. It is possible that the assumption (2.11) has overestimated the combined attraction power of the radiation era mass thresholds. A different estimate is obtained by multiplying the WKB approximations (4.10a) (all valid as soon as $\kappa \gtrsim 1$), assuming that all the oscillation angles $\theta_{\pm}^A$ are randomly distributed on the circle. Under the latter assumption it makes sense to compute a RMS value of the radiation era attraction factor ($\langle \cos \theta_{\pm}^A \rangle_{\text{RMS}} = 1/\sqrt{2}$). Neglecting as above the effect of the phase transitions one finds $[F_{\text{r}}(\kappa)]_{\text{RMS}} = 1.87 \times 10^{-4} \times \kappa^{-9/4}$ for the radiation era attraction factor, and from

[13] This maximum can be a local one, or it can arise at infinity in ϕ-space if it corresponds to a finite value of $\ln B(\phi)$. This is not the case for the tree-level coupling function $e^{-2\Phi}$, for which Φ would tend to continuously roll toward $-\infty$ during the cosmological expansion, and worse, by Eq. (4.4), Φ would cause deviations from general relativity, including violations of the equivalence principle, of order unity or more.

[14] It is enticing to assume that the presently obscure mechanism ensuring the vanishing of the cosmological constant allows both 'gravitational' fields to remain long-ranged in the low-energy world.

Eq. (5.6) with $\omega \gg 1$, $\langle \cos \theta_0 \rangle_{\mathrm{RMS}} = 1/\sqrt{2}$ and $\Omega_{75} = 1$, $[F_{\mathrm{m}}(\kappa)]_{\mathrm{RMS}} = 1.47 \times 10^{-5} \times \kappa^{-3/4}$ for the matter era attraction factor. This leads to a total attraction factor $[F_{\mathrm{t}}(\kappa)]_{\mathrm{RMS}} = 2.75 \times 10^{-9} \times \kappa^{-3}$ and to the following analytical estimate of the RMS value of the maximum value of the equivalence principle violation:

$$\left(\frac{\Delta a}{a} \right)^{\max}_{\mathrm{RMS}} = 1.36 \times 10^{-18} \kappa^{-4} (\Delta \phi)^2. \tag{7.1}$$

The comparison of Fig. 3 with Eq. (7.1) (valid if $\kappa \gtrsim 1$ and the angles $\theta^A_{\pm}$ are randomly distributed) indicates that the phasing of the radiation era oscillations tends to be destructive. It is possible that alternative assumptions, different from (2.11), yields values of $(\Delta a/a)$ nearer to the RMS analytical estimate (7.1). This would have the consequence that presently planned satellite tests of the equivalence principle [30] which aim at the level $\Delta a/a \sim 10^{-17}$, would probe a larger domain of values of κ, $\Delta\phi$ and Ω_{75}.

In conclusion, high-precision tests of the equivalence principle can be viewed as windows on string-scale physics. Not only could they discover the dilaton, but, by fitting observed data to the expected composition dependence (6.14) of the equivalence-principle violation, they could give access to the ratios C_B/C_E, C_D/C_E which are delicate probes of some of the presently most obscure aspects of particle physics: Higgs sector and unification of coupling constants.

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References

[1] V.P. Mitrofanov and O.I. Ponomareva, Sov. Phys. JETP 67 (1988) 1963
[2] P. Jordan, Nature 164 (1949) 637; Schwerkraft und Weltall (Vieweg, Braunschweig, 1955);
M. Fierz, Helv. Phys. Acta 29 (1956) 128;
C. Brans and R.H. Dicke, Phys. Rev. 124 (1961) 925
[3] J. Scherk and J.H. Schwarz, Nucl. Phys. B81 (1974) 118
[4] E. Witten, Phys. Lett. B149 (1984) 351;
M.B. Green, J.H. Schwarz and E. Witten, Superstring theory (Cambridge U.P., Cambridge, 1987)
[5] T.R. Taylor and G. Veneziano, Phys. Lett. B213 (1988) 450
[6] R.D. Reasenberg et al., Astrophys. J. 234 (1979) L219
[7] P. Sisterna and H. Vucetich, Phys. Rev. D41 (1990) 1034
[8] P.G. Roll, R. Krotkov and R.H. Dicke, Ann. Phys. (NY) 26 (1964) 442
[9] V.B. Braginsky and V.L. Panov, Sov. Phys. JETP 34 (1972) 463
[10] E.G. Adelberger et al., Phys. Rev. D42 (1990) 3267;
Y. Su et al., Phys. Rev. D, submitted

[11] J.O. Dickey, X.X. Newhall and J.G. Williams, Adv. Space Res. 9 (9) (1989) 75;
J. Müller, M. Schneider, M. Soffel and H. Ruder, Astrophys. J. 382 (1991) L101;
J.O. Dickey et al., Science (1994), in press
[12] C.M. Will, Theory and experiment in gravitational physics (Cambridge U.P., Cambridge, 1992) (revised edition)
[13] T. Damour, *in* Gravitation and quantizations, Proc. LVIIth Les Houches Summer School, July 1992, eds. B. Julia and J. Zinn-Justin (North-Holland, Amsterdam, 1994), in press
[14] T. Damour and K. Nordtvedt, Phys. Rev. Lett. 70 (1993) 2217; Phys. Rev. D48 (1993) 3436
[15] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. B158 (1985) 316
[16] C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, Nucl. Phys. B262 (1985) 593
[17] C.G. Callan, I.R. Klebanov and M.J. Perry, Nucl. Phys. B278 (1986) 78
[18] J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241 (1984) 406; B247 (1984) 373; Phys. Lett. B143 (1984) 410;
A.B. Lahanas and D.V. Nanopoulos, Phys. Rep. 145 (1987) 1
[19] M. Dine and N. Seiberg, Phys. Rev. Lett. 57 (1986) 2625;
J. Atick, G. Moore and A. Sen, Nucl. Phys. B307 (1988) 221;
M.A. Shifman and A.I. Vainshtein, Nucl. Phys. B277 (1986) 456;
H.P. Nilles, Phys. Lett. B180 (1986) 240;
I. Antoniadis, K.S. Narain and T. Taylor, Phys. Lett. B267 (1991) 37
[20] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B355 (1991) 649
[21] I. Antoniadis et al., Nucl. Phys. B407 (1993) 706
[22] A. Font et al., Phys. Lett. B249 (1990) 35;
J.H. Schwarz and A. Sen, Phys. Lett. B312 (1993) 105; Nucl. Phys. B411 (1994) 35
[23] V.S. Kaplunovsky, Nucl. Phys. B307 (1988) 145 [Erratum: B382 (1992) 436]
[24] J.P. Blaizot, Acta Phys. Pol. B18 (1988) 661;
H. Reeves, Phys. Rep. 201 (1991) 335
[25] E.W. Kolb and M.S. Turner, The early universe (Addison-Wesley, Redwood City, CA, 1990)
[26] T. Damour and G. Esposito-Farèse, Class. Quant. Grav. 9 (1992) 2093
[27] T. Damour and G. Esposito-Farèse, Phys. Rev. Lett. 70 (1993) 2220
[28] J. Gasser and H. Leutwyler, Phys. Rep. 87 (1982) 77
[29] K. Nordtvedt, Phys. Rev. Lett. 65 (1990) 953; Astrophys. J. 407 (1993) 5
[30] J.P. Blaser et al., STEP phase A report, ESA publication SCI (93)4