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Absolut Measurement of the Newtonian Force and a determination of G

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We present the latest results for the Wuppertal gravitational experiment which is based on a microwave resonator. The gravitational force of two test masses acting on the resonator is measured as a function of distance. From new data taken we determine the gravitational constant G with a value of $G = 6.67422 \cdot 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$ with an uncertainty of 147 ppm.

INTRODUCTION

The value of the Newtonian gravitational constant G has interested physicists for more than 200 years and except for the speed of light, it has the longest history of measurements. But although G has been the target of experimentalists for such a long time, it is the least well known fundamental constant. At the end of 1998 the CODATA even decided to increase the uncertainty of the accepted value for the gravitational constant from 128 ppm to 1500 ppm. This remarkable step of increasing the uncertainty instead of decreasing was made to reflect the discrepancies between recent experiments, which span a wide range of more than 0.7 %. Motivated by this fact and the uniqueness of the Wuppertaler experiment concerning distance range and method, strong efforts were performed in order to minimize the uncertainties and to determine a realistic estimate for the Newtonian constant and the remaining errors. Here we report the final results of our experiment to determine G, which has a standard uncertainty of 147 ppm.

THE FABRY-PÉROT GRAVIMETER

The Fabry-Pérot Pendulum Gravimeter at the University of Wuppertal was developed over the years since 1988 for absolute experiments on Newtonian gravity. An account of those developments can be found in several earlier papers [1][2] and contributions to conferences [3][4]. The principle set-up of our experiment is shown in Fig. 1. The heart of the experiment is an open Fabry-Pérot resonator, which is formed by two reflectors. The pendula are placed inside a vacuum tank with a pressure of $p \leq 5 \cdot 10^{-5}$ mbar. The position change of the two cavity mirrors is determined by measuring the resonance frequency of the cavity with high precision.

On each outer side of the reflectors a 576 kg mass is placed, coinciding with the axis of the resonator which are labelled field masses (FM). The two masses consist of brass and have been measured at the PTB in Braunschweig to an accuracy of 9 g each. During the measurements they are moved symmetrically and simultaneously from a reference to a measuring position near the reflectors every fifteen minutes. This causes the distance between the reflectors of the cavity to change due to the change in gravitational force acting on them. The measured position change is than compared to the expected change, which one obtains by solving four of the following equation:

$$\Delta z = \frac{G \rho_M \rho_m}{m \omega^2} \left[\int_v dv \int_V dV \, \frac{\vec{r} \cdot \vec{e}}{r^3} - \int_v dv \int_V dV \, \frac{\vec{r}_{Ref} \cdot \vec{e}}{r^3_{Ref}} \right] \quad (1)$$

The equation describes the position change of one pendulum with masse m and the volume v resulting from a displacement of one FM with a masse M and a volume V from a reference position r_{Ref} to a measurement position r. Note, that by using a symmetrical set-up with two fieldmasses we can afford relative high tolerances in the positioning of the resonator between the masses, due to the flatness of the gravitational potential.

The resonance frequency of the cavity is typically centered in the range of 20 GHz $< \omega <$ 26 GHz. A shift of Δb=10 nm (typical for the distances of the fieldmasses) in cavity length changes the resonance frequency by about 1 kHz. Because the resulting shift in the frequency is measured as a function of the field masses position, Newton's inverse-square law can be tested between the ranges of

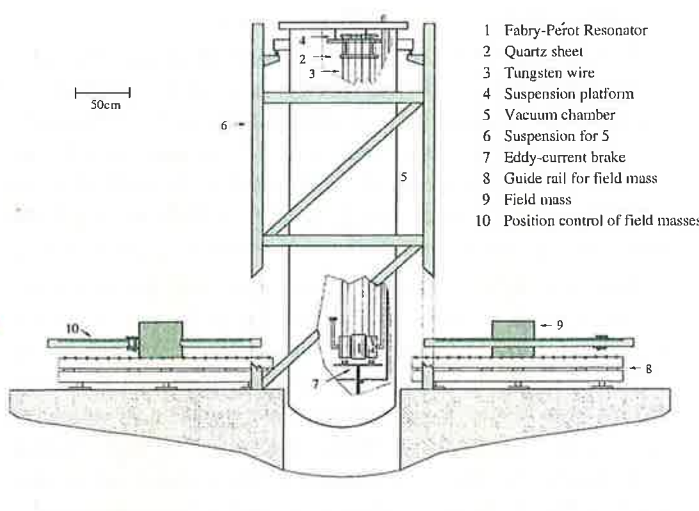

FIG. 1: Schematic view of the experimental setup with the Fabry-Pérot resonator and the two fieldmasses.

0.7 m and 2.1 m. To determine the length of the cavity a number of modes of the cavity are excited and their frequency is measured and compared with cavity theory.

The conversion factor between the change in the distance of the two pendula and the change in the resonance frequency can be deduced by cavity theory [5]

$$\beta = -\frac{b}{f}\left(1 - \frac{nc}{2\pi f}\sqrt{\frac{1}{2Rb - b^2}} - \frac{c}{8\pi^2 Rqf} + \mathcal{O}(10^{-8})\right)^{-1} . \quad (2)$$

Here b denotes the absolute length of the cavity, f the resonance frequency, c the speed of light, R the curvature radius of the mirrors, n and q are values which characterize the mode. This factor β enables us to convert the mode-dependent change in frequency Δf into the change in distance of the two pendula Δb:

$$\Delta b = \beta \cdot \Delta f . \quad (3)$$

Depending on the chosen mode, the conversion factor is in the order of $\beta \simeq 10$ nm/kHz. At least 10 modes of the cavity are easily obtained to provide a determination of b and R with sufficient precision.

As compared to our earlier measurements the data of 2000/01 should mainly benefit from the much improved position control of the field masses. The relative position of each FM is determined with two linear encoders installed parallel to the guide rails for the FMs. These linear position measuring devices possess a measuring lengths of 1.7 m with an accuracy grade of $\Delta r < \pm\ 3\ \mu$m (Nr. 10 in Fig. 1). The absolute distance of both FM is determined with an additional optical bank which was installed in front of the mounting of the vacuum chamber (Nr. 6 in Fig. 1) parallel to the guide rails for the FMs. With this system an integral accuracy of 15 μm for the absolute distance of the FM is reached.

The position of the pendula and therefore the resonance frequency of the cavity is rather sensitive to changes in the ambient temperature which is however a very smooth and slow effect and is quantitatively described by a polynomial of fourth order. Nevertheless an active heating system was installed, which reduced the temperature variations of the experiment to $\Delta t \approx 0.1$ K.

Noise on the pendula positions is mainly coming from seismic events with an amplitude range from the microseismic to large earthquakes. A set-up of eddy current brakes that can be adjusted by remote control effectively damps this noise. Furthermore the regular movement of the pendula averages the noise out to a very high degree of accuracy. So far no influence of the noise level on the final results from our measurements has been detected.

In table I the values of some further parameters for the components of the experiment are listed.

Item	Material	Dimensions (∅,l)	Mass
fieldmass	brass	ø=440mm, l=430mm	576kg
cavity	copper a. aluminum	ø=200mm, l=300mm	6.7kg
wire	tungsten	ø=200μm	3.25g

TABLE I: Some parameters for the components of the Fabry-Pérot Gravimeter.

NEW MEASUREMENTS

During the measurements it arose, that the tilting of the whole experiment is the main source of the experimental error. A tilting of the Fabry-Pérot Gravimeter leads to a change in the distance of the resonator mirrors. If the measurement itself leads to a tilt of the experiment, the measured change in the distance of the pendulums consists not only of the gravitational contribution, but also of a contribution, which is caused by the tilting:

$$\Delta b_{measurment} = \Delta b_{gravitation} \pm \Delta b_{tilt}. \quad (4)$$

The reason for the tilt of the experiment is the deformation of the floor within the area of the suspension for the vacuum chamber, caused by the high load of the flour due to the fieldmasses. Because this deformation depends on the fieldmass position, it acts as a distance depend deviation from the law of gravitation. To determine the correlation between the tilt and the position of the fieldmasses, the deflections of the mirrors had to be measured. By blocking one of the pendulums, a tilt of the experiment leads to a deflections δb of the remaining freely swinging pendulum according to $\Delta b_{fixed} = l \cdot \varphi_{fixed}$ where l defines the length of the pendulum and φ the tilt angle. So for each measuring position two measurement were performed, where one of the two pendulums was always fixed. The difference of both measurements is the pure contribution of the tilting assuming that the gravitativ contribution is equal in both series of measurements. In Fig. 2 the tilting angle as a function of the fieldmass position is plotted. The values show an approximately square dependency on the position of the masses. The determined values for the tilting enable us to calculate correction values for this contribution in a gravitation measurement. If both pendulums swing freely a tilt of the experiment varies the distance between both pendulums according to $\Delta b_{free} = \Delta l \cdot \varphi_{free}$. The scaling factor between the angle of tilting and deflection of the pendulums is the difference in the length of the two pendulums. With the assumption that the tilting of the experiment is not influenced on whether a pendulum is fixed or not, the angle of inclination φ for both measuring types is equal ($\varphi_{fixed} \equiv \varphi_{free}$). The inaccuracy in the determination of the deflection of the pendulums (δf_{tilt}) goes – scaled with the length difference – linearly into the systematic error of the gravitational constant. For

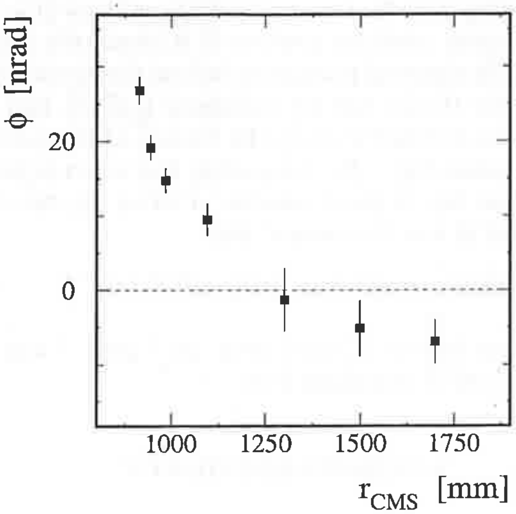

FIG. 2: The tilting angle of the experiment as a function of the measuring position of the fieldmasses.

our measurements this source of error leads to the largest contribution in the systematic error and limits the accuracy of the measurement of the gravitational constant G [6].

RESULTS

The new measurements were performed from January 2000 - May 2001. The fieldmasses were moved simultaneously and symmetrically in 15 minutes intervals between a measuring position and a reference position. The distance of the centre of gravity of the cavity and the fieldmasses for the measuring position were 915, 945, 985, 1095, 1300 and 1500 mm and for the reference position 2260 mm. The resonance frequency was determined every 500 msec.

For each position up to three series of measurements were taken. The results are listed in Table II as a function of the measuring position and the resonance frequency of the cavity. The uncertainty budget for systematic uncertainties of the experiment is given in Table III. The combined results from the different measuring configurations lead to a value of

$$< G > \; = \; (6.67422 \pm 0.00049 \pm 0.00085) \cdot 10^{-11} \; [\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}] \qquad (5)$$

with a statistical uncertainty of 73 ppm and a systematic uncertainty of 128 ppm. The total uncertainty of 147 ppm is the quadrature sum of the individual uncertainties. Compared with the two most accurate measurements, our result is in good agreement with that of Gundlach and Merkowitz [7] and that of S. Schlamminger et al. [8] but it does differ from that of T. Quinn et al. [9] by some 200 ppm.

Measuring position [mm]	$G_{23\,GHz}$	$G_{22\,GHz}$
915	6.67444 ± 0.00099	6.67485 ± 0.0021
	6.67461 ± 0.00340	
945	6.67299 ± 0.00150	6.67422 ± 0.0026
	6.67318 ± 0.00150	
985	6.67430 ± 0.00095	6.67490 ± 0.0019
		6.67519 ± 0.0049
1095	6.67536 ± 0.00140	
1300	6.67264 ± 0.00380	
1500	6.67332 ± 0.00720	

TABLE II: A list of the measured values for the gravitational constant with their respective statistical uncertainty in units of $1 \cdot 10^{-11}$ $[\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}]$.

The correction values for the tilt of the experiment also enable us to reanalyse the results of older measurements [10]. Due to new cognitions concerning the position accuracy of the FM in these old measurements we had to increase the systematic error to 600 ppm. Nevertheless, the value corrected for the tilt

$$G \; = \; (6.67413 \pm 0.00056 \pm 0.004) \cdot 10^{-11} \; [\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}] \qquad (6)$$

is in very good agreement with the new values listed above.

FIFTH FORCE

Much attention has been attracted by claims of deviations from Newtonian gravity. The usual convention is to parametrize the effect as an extra Yukawa potential superimposed on the Newtonian gravitational potential. Varying the ratio of gravitational constants determined at two different distances within the limits of their experimental uncertainties enables us to calculating the constrains for the strength of a composition independent fifth force via the relation given by [11]. So we were able to increase the constrains by a factor of two compared to those listed in [12].

Another possibility to check for a deviation from Newtons law is to compare the exponential distance dependence for the calculated and the measured values

$$\Delta \mathrm{f_{theo}} \propto \frac{1}{\mathrm{r^n}} \qquad (7)$$
$$\hookrightarrow \quad \log(\Delta \mathrm{f_{theo}}) \propto \mathrm{n}\log(\mathrm{r}) .$$

measuring position r_{CMS} [mm]	915	945	985	1095	1300	1500
uncertainty due to tilt [ppm]	87	88	98	182	668	1047

Source	uncertainty $\Delta G/G$ [ppm]
Attractor masses	
separation	27
geometrie	8
mass value	22
density inhomogeneity	5
Pendulum	
position	10
geometrie	6
separation	8
wire correction	14
spindle head	14
supplanted air	14
time of swing of pendulum	23
magnetic forces	10
choke-effect ($\Delta f/\Delta d$)	41 (22 GHz) / 36 (23 GHz)
mass integration	1
conversion factor	8
total	67 (22 GHz) / 64 (23 GHz)

measuring position r_{CMS} [mm]	915	945	985	1095	1300	1500
Combined uncertainty 23 GHz [ppm]	108	109	117	193	671	1049
Combined uncertainty 22 GHz [ppm]	110	111	119	194	672	1050

TABLE III: The overall uncertainy budget.

Looking at the theoretical results, the value n describes the asymptotic behaviour of our experiment. For the measured changes of the resonance frequency,

$$\begin{aligned} \Delta \mathrm{f_{data}} &\propto \frac{1}{\mathrm{r}^{(\mathrm{n}+\delta)}} \\ \hookrightarrow \quad \log(\Delta \mathrm{f_{data}}) &\propto (\mathrm{n}+\delta)\log(\mathrm{r}) \end{aligned} \tag{8}$$

any deviation δ from the expected value would indicate a possible violation of the $1/r^2$-law. We calculated for the difference of the experimental and theoretical results a value for the parameter of

$$\delta = 0.0011 \pm 0.0024\,. \tag{9}$$

This value is in good agreement with zero and a posible deviation of the exponent is smaller than $\delta \simeq 0.0047$ for the 90 % confidence level.

Whether or not the fundamental constants of nature vary with time has been a question of considerable interest since Dirac suggested that the gravitational force may be weakening with the expansion of the universe [13]. Although general relativity predicts $\dot{G}$ is identically zero, a variable G is expected in theories such as the Brans-Dicke scalar-tensor theory and its extensions [14][15], and has received renewed attention in the context of extended in ationary cosmology. By comparing two measurements with a time lag of three months, a value for the time variation of G was determined with

$$\left|\dot{G}/G\right| \leq (2.0 \pm 12.0)\cdot 10^{-4}\,\mathrm{Jahr}^{-1} \tag{10}$$

So an upper limt of $|\dot{G}/G| \leq 20.0\cdot 10^{-4}$ year^{-1} was deduced for a 90 % confidenz level.

ACKNOWLEDGMENTS

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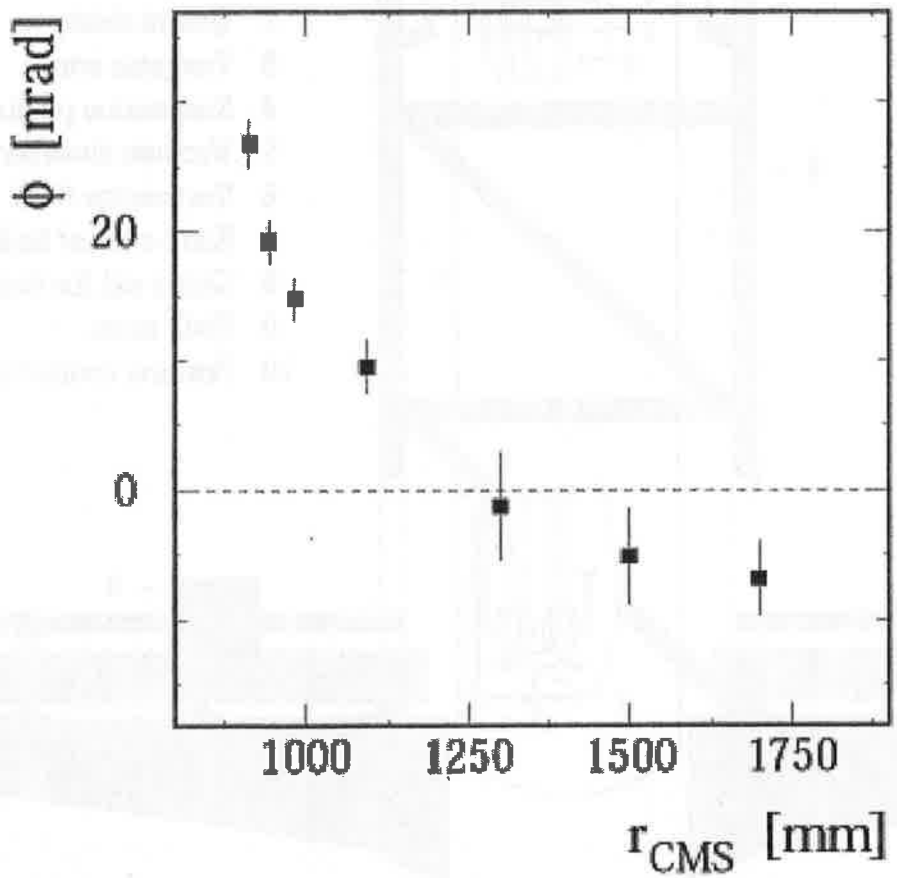
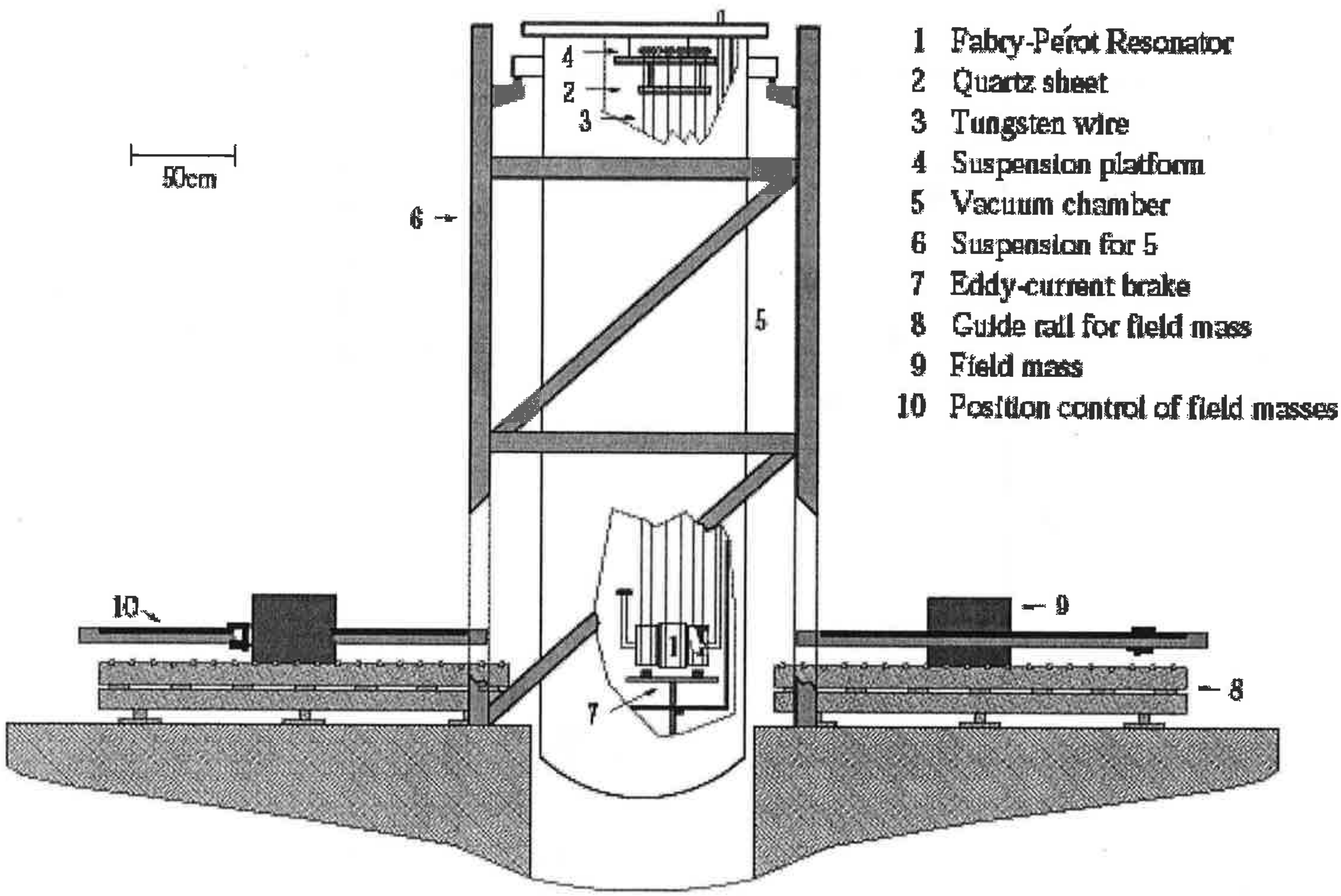
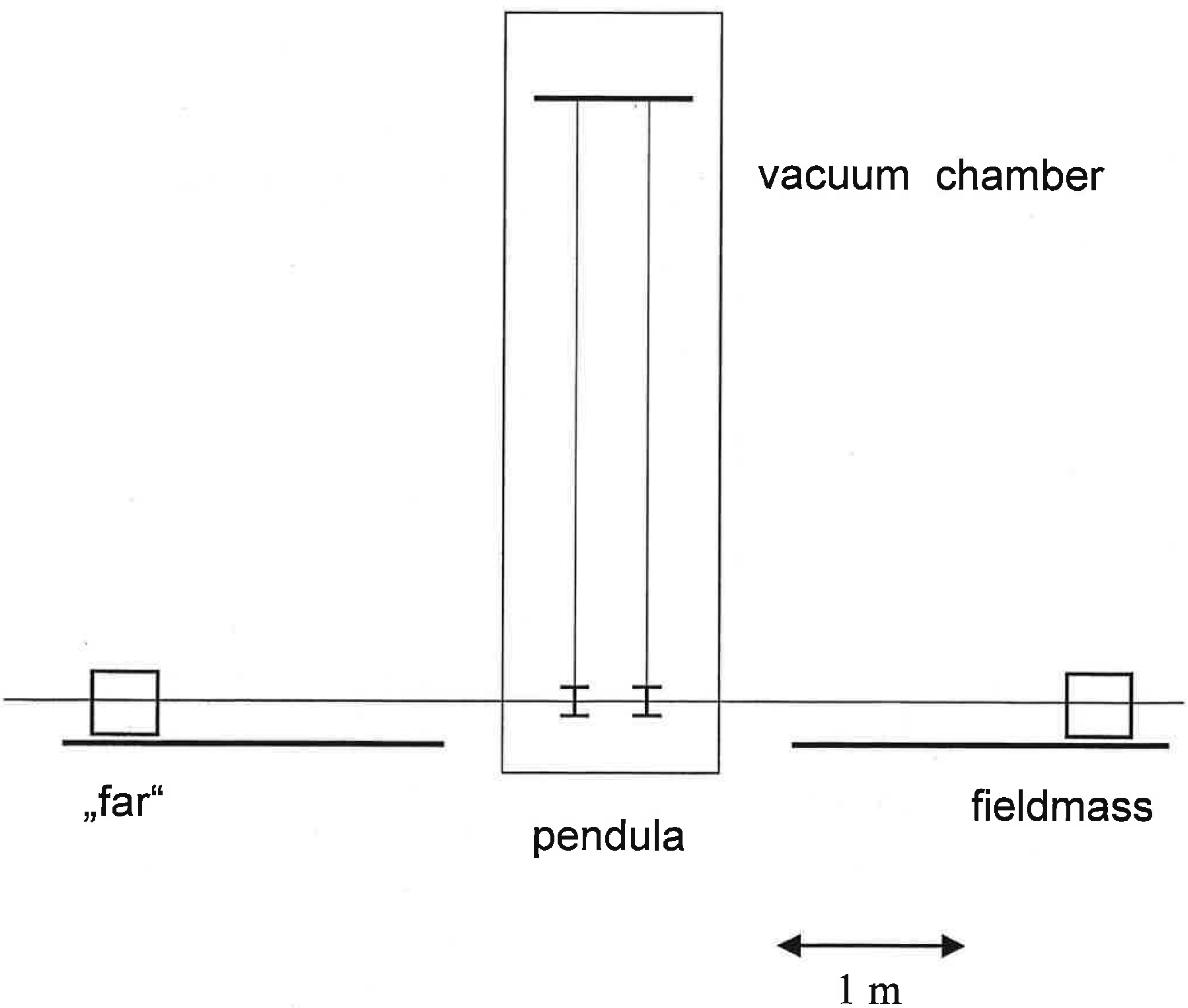



1 Fabry-Pérot Resonator
2 Quartz sheet
3 Tungsten wire
4 Suspension platform
5 Vacuum chamber
6 Suspension for 5
7 Eddy-current brake
8 Guide rail for field mass
9 Field mass
10 Position control of field masses
50cm

The wuppertaler experiment

Precept: determination of the change in the distance between two pendula due to the change in the gravitational force

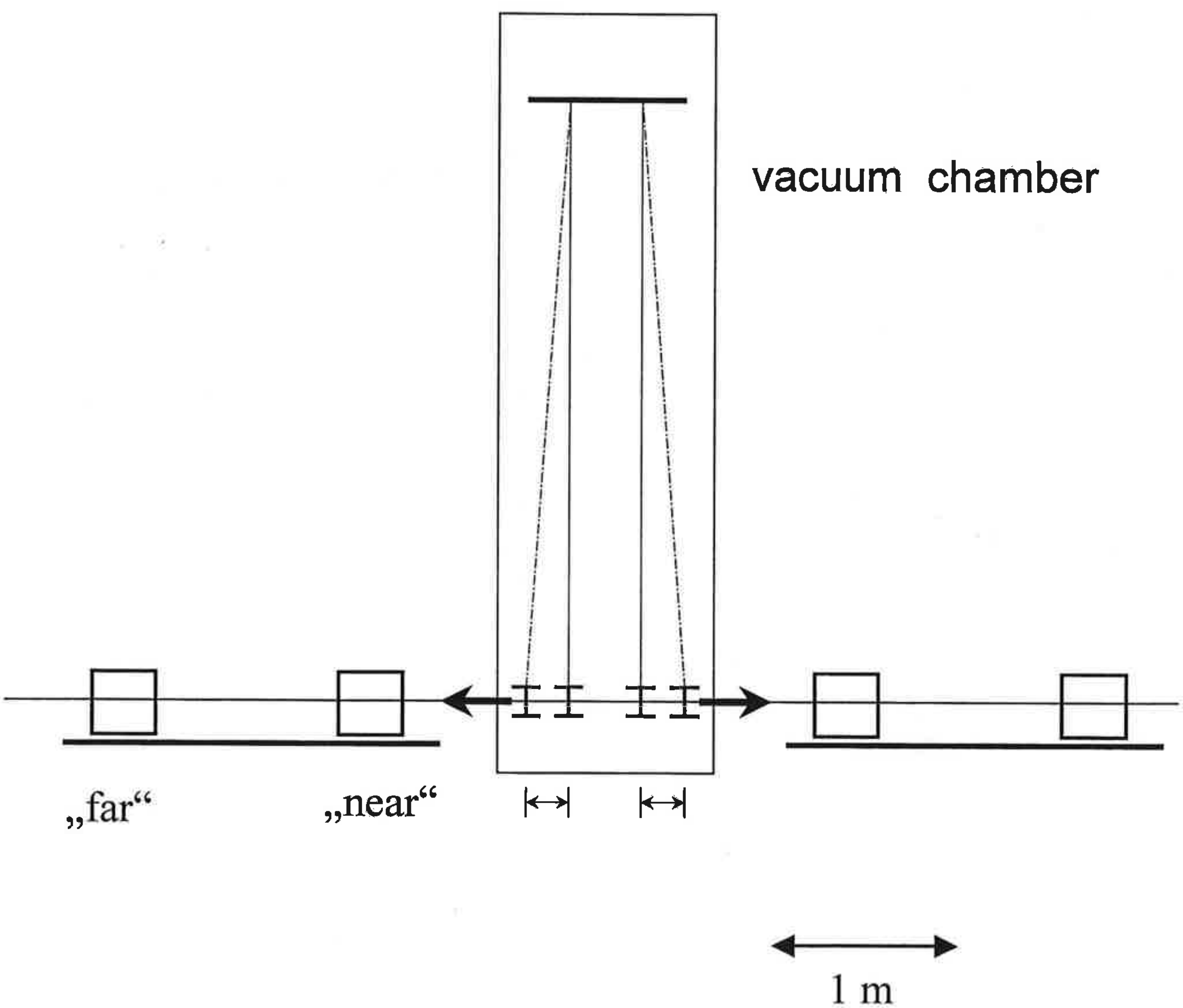


Calculable to 10^{-7}

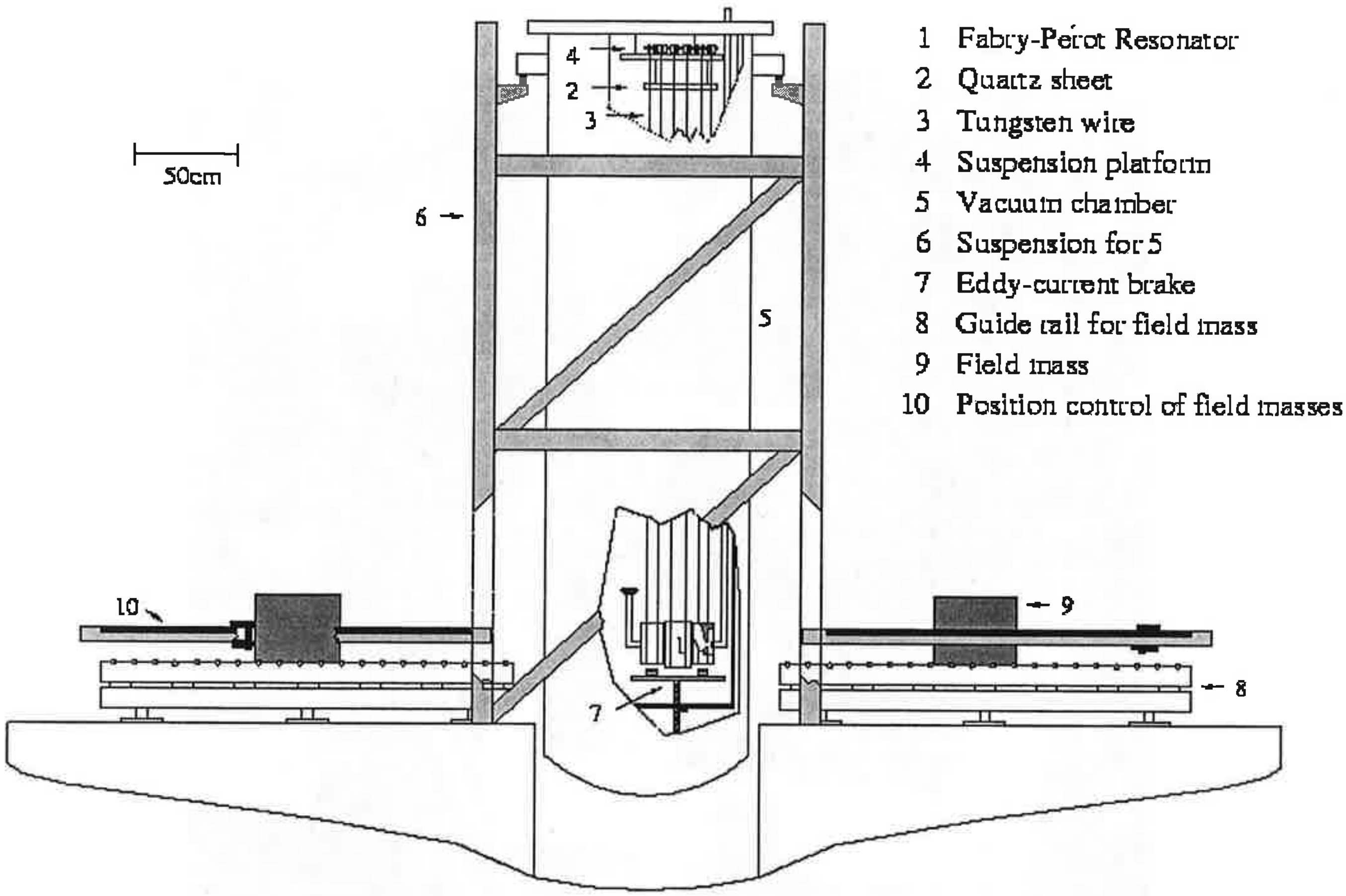
gravitational force acting on pendula = $\mathbf{G} \bullet \mathbf{Fkt}_{geom}$(FM-Position)

→ change of FM position

→ change of the distance of the pendula ($\Delta b \approx 10$ nm)



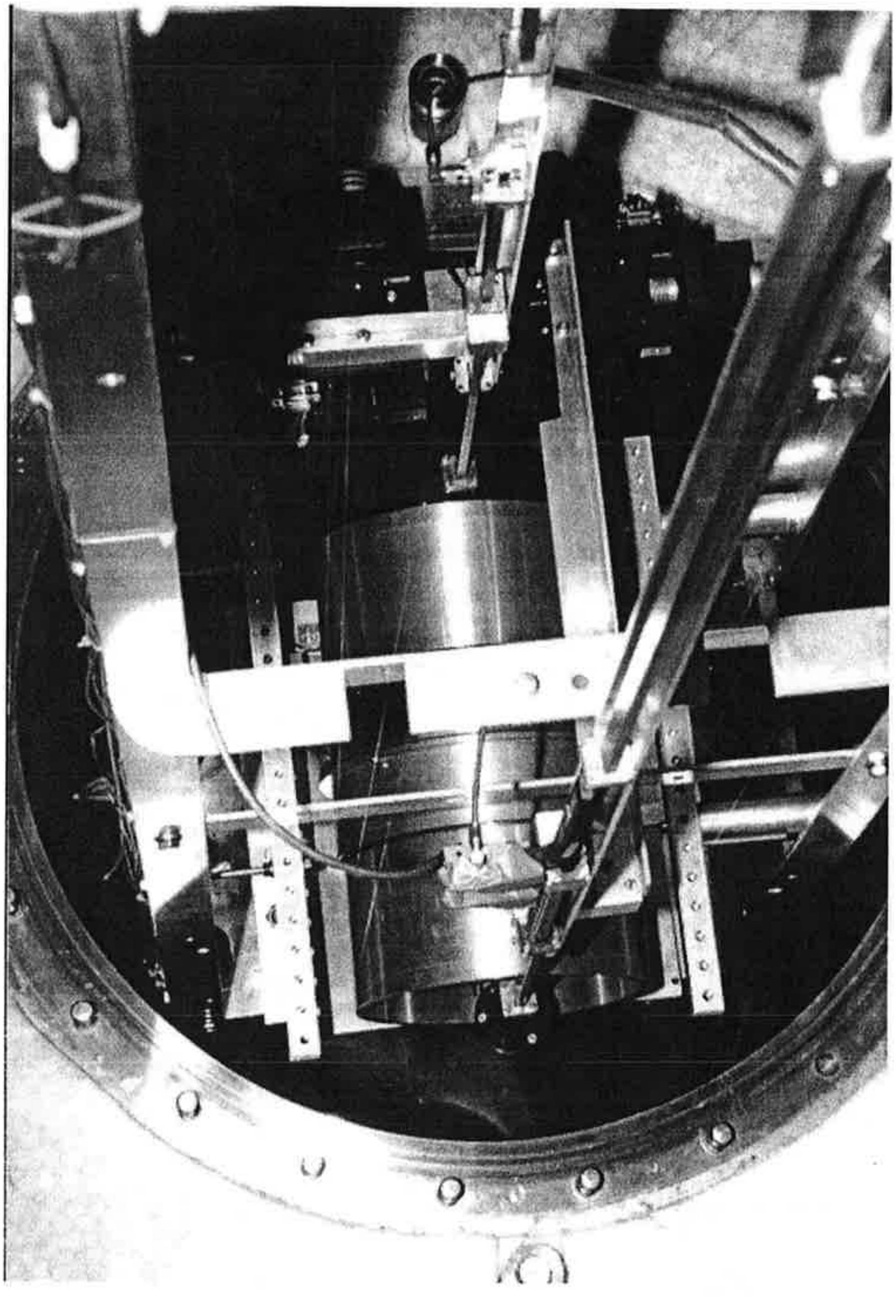
The Wuppertal Experiment

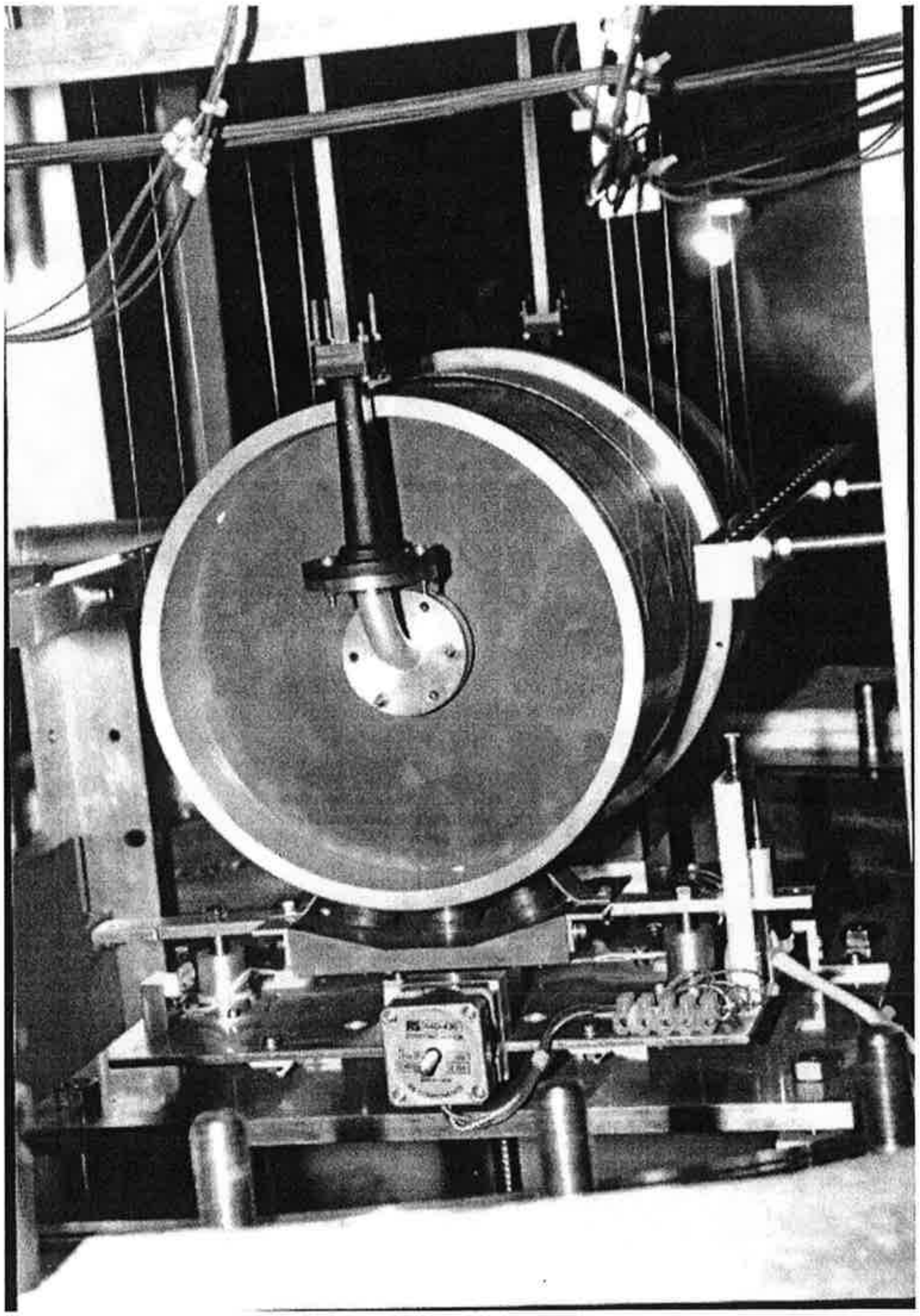


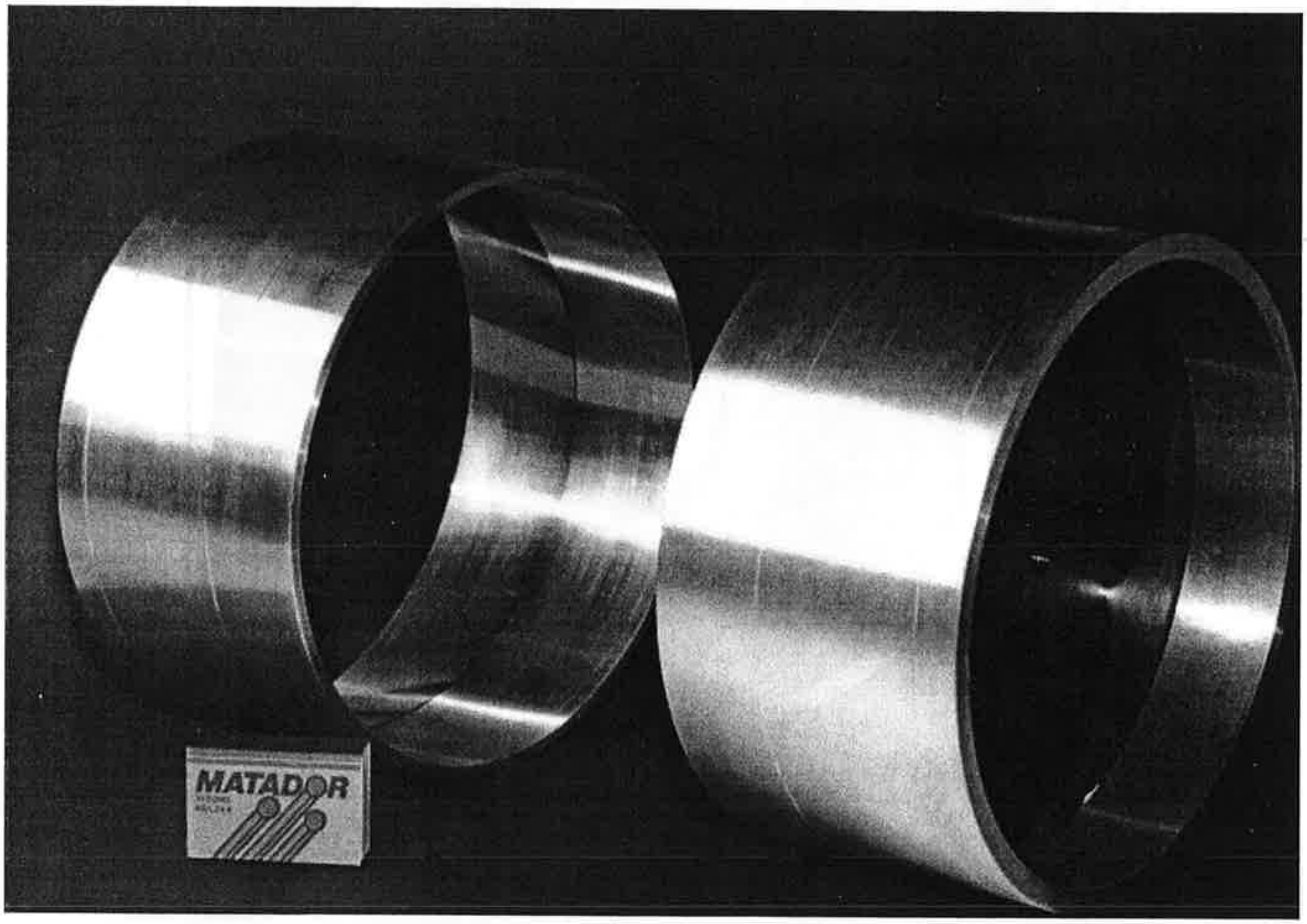
- CMS distance: **R** = 0.7 - 2.1 m
- Fieldmasses: **M** = 576 kg, brass
- Pendula: **m** = 3.2 kg, alu. & copper

↳ *grav. force* $F \approx 1 \times 10^{-7}\,N$

shift of mirror distance $\Delta b \sim 10\,nm$



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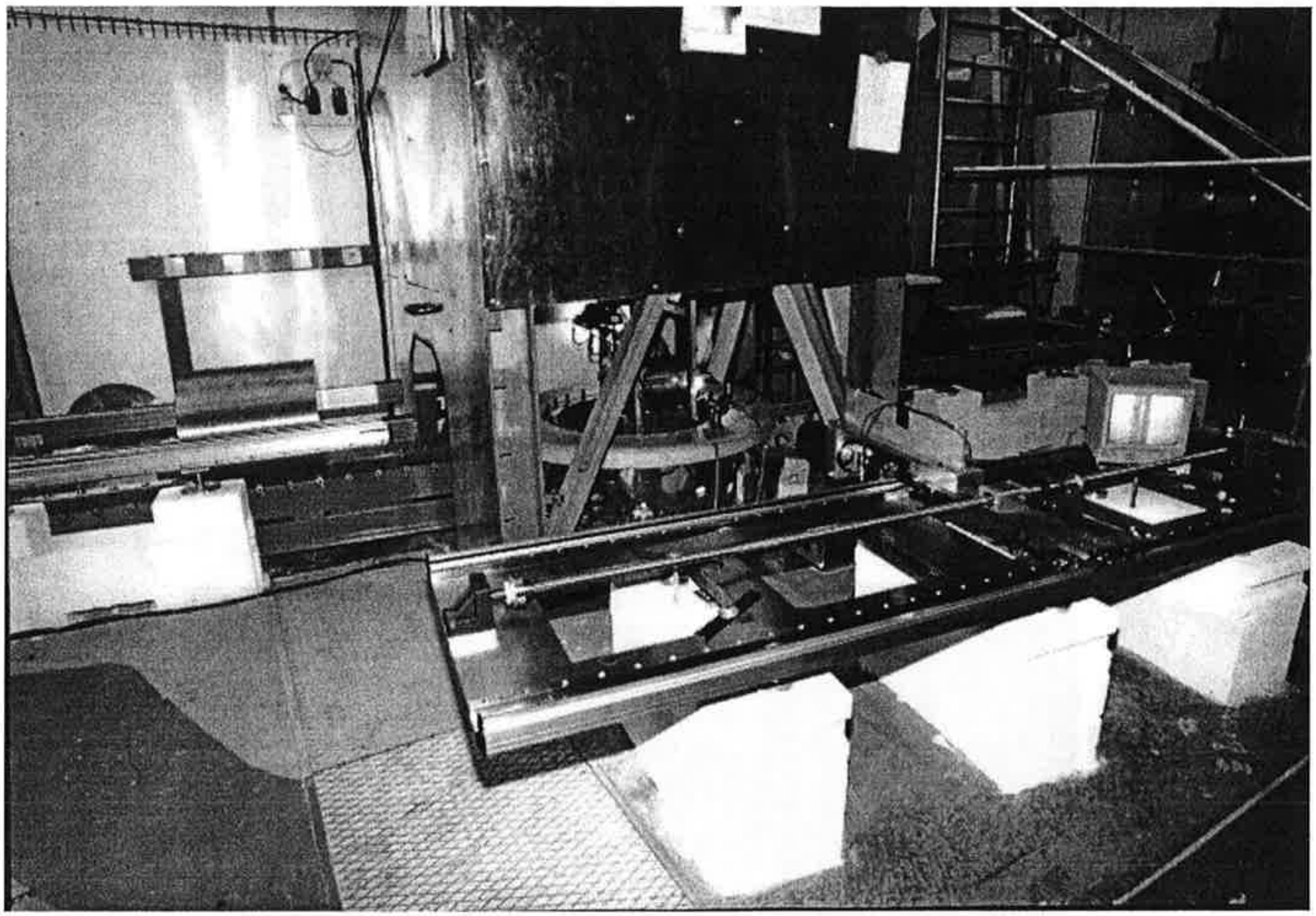
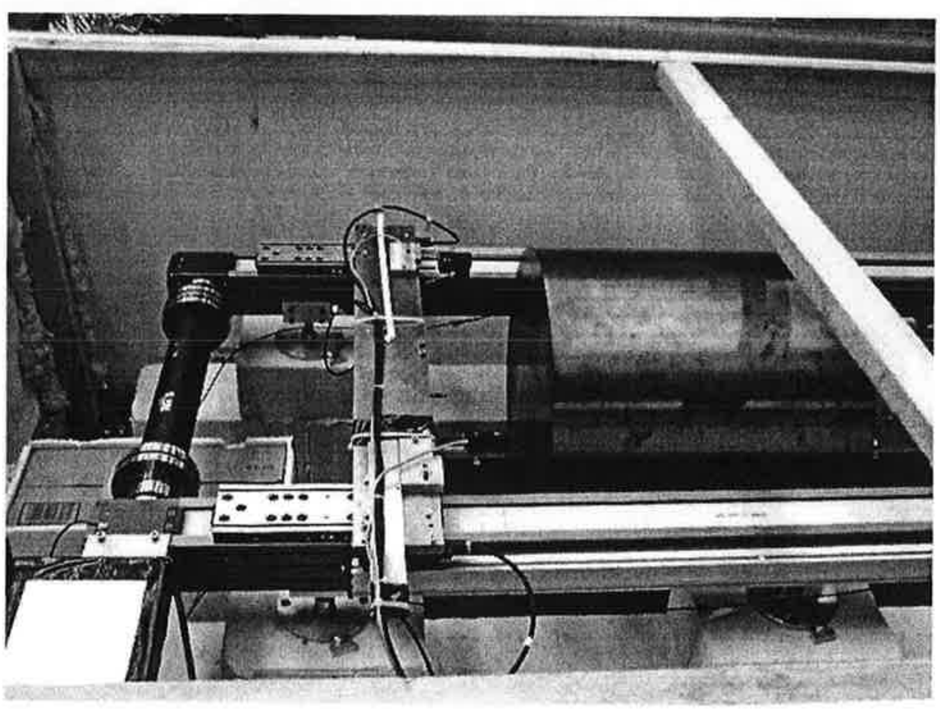


POSITIONING SYSTEM

- per FM
  - 2 sealed linear encoders
    - $\rightarrow$ acc. $\pm$ 3 μm
  - digital length gauge system
    - $\rightarrow$ acc. $\pm$ 0.5 μm

- absolute distance FM-FM
  - 2 sealed linear encoders
    - $\rightarrow$ acc. $\pm$ 2 μm

↳ total CMS accuracy $\approx$ 10 μm



The systematic errors of the measurment

Systematic error due to tilt of the experiment:

measuring position r [mm]	915	945	985	1095	1300	1500
uncertainty $\Delta G/G$ [ppm]	87	88	98	182	668	1047

Further systematic errors:

Source	uncertainty [ppm]
attractor masses	
separation	27
geometry	8
mass value	22
density	5
pendulum	
position	10
geometry	6
separation	8
wire correction	14
spindele head	14
supplant air	14
time of swing pendulum	23
mag. force	10
choke-effect ($\Delta f/\Delta d$)	41 (22 GHz) / 36 (23GHz)
mass integration	1
conversion factor	8
total	67 (22 GHz) / 64 (23 GHz)

Sum of the systematics listed as function of the measuring position

measuring position r [mm]	915	945	985	1095	1300	1500
uncertainty (23 GHz) [ppm]	108	109	117	193	671	1049
uncertainty (22 GHz) [ppm]	110	111	119	194	672	1050

Results

measuring position [mm]	**G** (23 GHz) $1 \cdot 10^{-11}\,[m^3\,kg^{-1}\,s^{-2}]$	**G** (22 GHz) $1 \cdot 10^{-11}\,[m^3\,kg^{-1}\,s^{-2}]$
915	6.67444 ± 0.00099	6.67485 ± 0.00210
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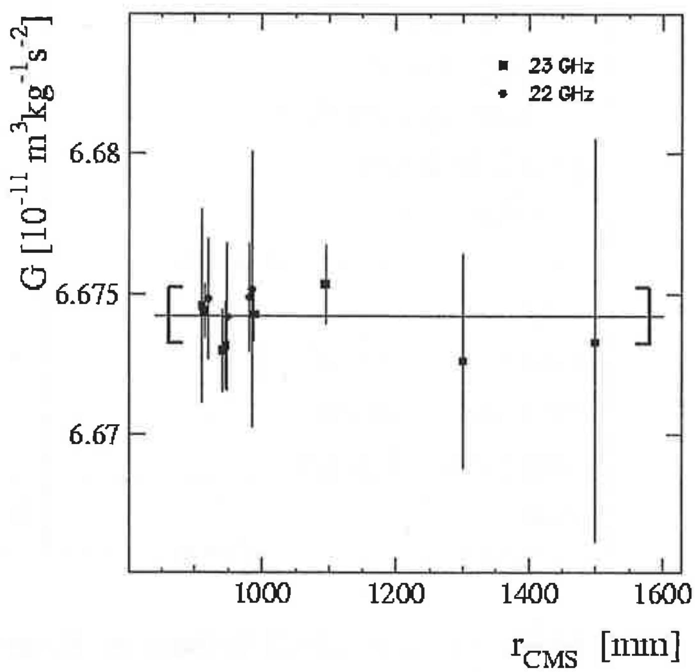

Combined result

$$G = (\,6.67422 \pm 0.00049 \pm 0.00085\,) \cdot 10^{-11} \quad [m^3\,kg^{-1}\,s^{-2}]$$

± 98