ISSN 1069-3513, Izvestiya, Physics of the Solid Earth, 2007, Vol. 43, No. 8, pp. 702–706. © Pleiades Publishing, Ltd., 2007.
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Tidal Deformations and the Electrokinetic Effect in a Two-Layer Fluid-Saturated Porous Medium

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Received June 20, 2006

Abstract—The electrokinetically induced vertical component of the electric field in a multilayer fluid-saturated porous medium caused by tidal deformations of the Earth's crust is calculated. Petrophysical properties change in a jumplike manner at the boundary between two media. The pore pressure gradient at the boundary abruptly reaches a maximum and then exponentially decays, forming a hydrodynamic skin layer. Due to the electrokinetic effect, the electric field behaves in the same way. Observations of the vertical electric field can be used, in principle, to determine the mechanical properties of the medium experiencing deformation. The magnitudes of the effects lie within a range accessible to measurement.

PACS numbers: 91.10.Tq

DOI: 10.1134/S1069351307080101

Observations of deformations provide insights into processes proceeding in the Earth's crust before and after earthquakes. Difficulties involved in direct strain measurements emphasize the significance of observations of strain-induced variations in the electromagnetic field produced by fluid flows in a deformable saturated porous medium. The electromagnetic measurements can be calibrated through the analysis of the pore pressure and its gradients caused by tidal (volumetric) deformations. The study of tidal deformations is of interest in its own right.

An equation describing the behavior of the excess pressure p of a fluid in a fluid-saturated elastic porous isotropic medium was proposed by Frenkel [1994] (see also [Surkov, 2000; Gershenzon and Bambakidis, 2001]). This equation is hyperbolic if the strain frequency ω is much lower than a certain characteristic value ω_0:

$$\omega \ll \omega_0 = \mu/(\kappa\rho_w). \tag{1}$$

Its parabolic analogue is [Garagash et al., 2005]

$$\frac{\omega_0\rho_w}{M}\frac{\partial p}{\partial t} - \nabla^2 p = -\alpha\omega_0\rho_w\frac{\partial \varepsilon}{\partial t}. \tag{2}$$

Here,

$$M = \frac{2G(\nu_u - \nu)}{\alpha^2(1-2\nu_u)(1-2\nu)}, \quad \alpha = 1 - \frac{K}{K_0},$$

$$K = \frac{2(1+\nu)}{3(1-2\nu)}G, \quad \beta = (1 - K/K_0)\frac{1}{m}, \tag{3}$$

$$\beta' = 1 + (\beta - 1)\frac{K_2}{K_0}.$$

M is the Biot modulus; K_0, K, and K_2 are the bulk moduli of the solid phase, dry porous rock, and water, respectively; m is the porosity of the medium; μ, ρ_w, and κ are the dynamic viscosity, fluid density, and permeability of the material, respectively; $\varepsilon = u_{kk}$ is the volume strain; and u_k are the components of the displacement vector. In real media, condition (1) is satisfied in a wide range of frequencies, including lunisolar tides and seismic and acoustic waves up to frequencies of the order of 10^4–10^5 Hz.

In the general case, the excess pressure of the pore fluid p depends on ε. In most rocks, variations in the excess pore pressure p do not influence the volume strain [Surkov, 2000]. Therefore, we assume that $\varepsilon = \varepsilon(t, \mathbf{r})$; i.e., ε is independent of p. As a source of periodic disturbances of the volume strain $\varepsilon(t, \mathbf{r})$, we consider lunisolar tidal waves, which are known to have a scale of the order of 10 000 km. In this case, we can neglect the influence of volume strain and gravity field gradients of tidal waves in near-surface (the upper 10 km) layers, assuming $\varepsilon = \varepsilon(t)$, and regard the near-surface medium as a half-space.

We address a two-layer fluid-saturated porous medium. At the interface, the petrophysical characteristics of the layers (density, elastic moduli, dynamic viscosity, porosity, permeability, and so on) change in a jumplike manner. Of course, electrophysical parameters of the media also experience a discontinuity at the interface, but, given that the backward effect of the electric field on the hydrodynamics of the pore fluid is small, the discontinuities of electrical parameters are usually ignored in electrokinetic phenomena. Taking this observation into account, the equations for each layer are of form (2) and are written as follows. For the first layer, we have the equation

$$\frac{\partial p_1}{\partial t} - \frac{\kappa_1}{\eta_1} M_1 \nabla^2 p_1 = -\alpha_1 M_1 \frac{\partial \varepsilon}{\partial t}. \quad (4)$$

The excess pressure is continuous at the interface. The pressure at the free surface ($z = 0$) vanishes: $p_1 = 0$. A similar equation holds for the second layer:

$$\frac{\partial p_2}{\partial t} - \frac{\kappa_2}{\eta_2} M_2 \nabla^2 p_2 = -\alpha_2 M_2 \frac{\partial \varepsilon}{\partial t}, \quad (5)$$

with the boundary condition $p_1 = p_2$ at $z = L$. In both cases, the volume strain is

$$\varepsilon(t) = \varepsilon_0 \frac{1}{2} \exp(i\omega t) + c.c. \quad (6)$$

Equations of type (4) and (5) with condition (6) are used to describe physical fields of various origins, for example, an electromagnetic wave penetrating a conductor [Landau and Lifshitz, 1982]; viscous fluid velocity fields generated by wall vibrations [Landau and Lifshitz, 1986]; or, as in our case, an excess pore pressure at the interface between two media generated by lunisolar tides. A feature common to all these cases is an exponential decay of oscillations of a physical field downward from the interface. In the case of electromagnetic waves, this phenomenon is called the skin effect. As regards the excess pore pressure in a fluid-saturated medium, the analogy with the skin effect is applicable to the pore pressure gradient; as shown below, precisely the pore pressure decays exponentially downward from the interface. The characteristic distance of this decay δ, or the depth of the skin layer (the "penetration depth" in the terminology of Landau and Lifshitz [1986]), is determined in all cases by the frequency and the coefficient at the spatial derivative. For the pore pressure gradient, we have

$$\delta = \sqrt{\frac{2\kappa M}{\eta\omega}}. \quad (7)$$

Now, we estimate the depth of the skin layer for the diurnal and semidiurnal tidal waves. Let the medium be a sandstone with the parameters $G = 4 \times 10^9$ Pa, $\kappa_1 = 10^{-12}$ m^2, and $\nu = 0.23$ [Kobranova, 1962]. Moreover, $\eta = 10^{-3}$ Pa s and $\alpha = 0.5$ [Gershenzon and Bambakidis, 2001]; the value of ν_u is taken equal to $\nu_u = 0.33$. According to formula (3), $M_1 = 1.74 \times 10^{10}$ Pa. For the diurnal tidal wave, $\omega = \frac{2\pi}{24}$ h^{-1} and we find $\delta_{1ds} = 690$ m. For the semidiurnal wave, we obtain $\delta_{0.5ds} = 490$ m. The most variable parameter of rocks is the permeability; for sandstones, clays, and limestones in oil fields, it can be lower than 0.01 mD (sandstones) and even less than 0.001 mD (10^{-18} m^2) [Kobranova, 1962]. For these rocks, the thickness of the skin layer can decrease by 10 to 100 times and reach values of the order of ten meters to one meter or smaller. In the case of limestone with the elastic constants $G = 3 \times 10^{10}$ Pa, $\nu = 0.20$, and $\nu_u = 0.30$ [8], we obtain $M_2 = 10^{11}$ Pa from formula (6) at $\alpha = 0.5$. We assume that the limestone occurs in an oil field and has the permeability $\kappa_1 = 0.1$ mD $= 10^{-16}$ m^2. In this case, we obtain the following estimates for the thickness of the skin layer: $\delta_{1dl} = 16.6$ m and $\delta_{0.5dl} = 11.7$ m.

The solution of Eq. (4) is sought in the form

$$p_1(z, t) = \frac{1}{2}[A_1(z) - \alpha_1 M_1 \varepsilon_0] \exp(i\omega t) + c.c. \quad (8)$$

The function $A_1(z)$ is described by the equation

$$A_1''(z) - i\omega \frac{\eta_1}{\kappa_1 M_1} A_1(z) = 0, \quad (9)$$

whose solution is

$$A_1(z) = A_{10} \exp(ik_1 z) + c.c. \quad (10)$$

The substitution of (10) into (9) leads to a relation determining the wave number k_1:

$$k_1^2 + i\omega \frac{\eta}{\kappa_1 M_1} = 0. \quad (11)$$

Hence, we find

$$k_1 = i(1+i)\sqrt{\frac{\omega\eta}{2\kappa_1 M_1}} = i(1+i)\frac{1}{\delta_1}. \quad (12)$$

The boundary condition $p = 0$ at $z = 0$ implies that

$$A_{10} = \alpha_1 M_1 \varepsilon_0. \quad (13)$$

As a result, the solution is reduced to the form

$$p_1(t, z) = \alpha_1 M_1 \varepsilon_0 \left\{ \exp\left[-(1+i)\frac{z}{\delta_1}\right] - 1 \right\} \times \frac{1}{2} \exp(i\omega t) + c.c. \quad (14)$$

Passing to real values, we obtain the pore pressure in the first layer

$$p_1(t, z) = \alpha_1 M_1 \varepsilon_0 \times \left[\exp\left(-\frac{z}{\delta_1}\right) \cos\left(\frac{z}{\delta_1} - \omega t\right) - \cos(\omega t) \right]. \quad (15)$$

Another important characteristic of variations in the pore pressure is its gradient. Differentiating (15), we find the pore pressure gradient

$$\frac{\partial p_1}{\partial z} = -\sqrt{2}\alpha_1 \varepsilon_0 \frac{M_1}{\delta_1} \exp\left(-\frac{z}{\delta_1}\right) \sin\left(\frac{z}{\delta_1} - \omega t + \frac{\pi}{4}\right). \quad (16)$$

As seen from this formula, the excess pore pressure gradient decays exponentially and its maximum is attained at the interface.

The solution of Eq. (5) for the second layer is sought in the form

$$p_2 = \frac{1}{2}[A_2(z) - \alpha_2 M_2 \varepsilon_0 \exp(-i\varphi)] \times \exp[i(\omega t) + \varphi] + c.c. \quad (17)$$

Here, the phase multiplier $\exp(i\varphi)$ is introduced into the solution. For the function $A_2(z)$, we obtain an equation similar to (9):

$$A_2''(z) - i\omega \frac{\eta}{\kappa_2 M_2} A_2(z) = 0. \quad (18)$$

As before, the solution is sought in the form

$$A_2(z) = A_{20}\exp(ik_2 z) + c.c. \quad (19)$$

The wave number k_2 is determined by a formula similar to (12):

$$k_2 = i(1+i)\frac{1}{\delta_2} \quad \left(\delta_2 = \sqrt{\frac{2\kappa_2 M_2}{\eta_2 \omega}}\right). \quad (20)$$

In real variables, the solution has the form

$$p_2 = A_{20}\exp\left(-\frac{z}{\delta_2}\right)\cos\left(\omega t + \varphi - \frac{z}{\delta_2}\right) - \alpha_2 M_2 \varepsilon_0 \cos\omega t. \quad (21)$$

The parameters A_{20} and φ are determined from the boundary condition $p_1 = p_2$ at $z = L$. We have

$$\alpha_1 M_1 \varepsilon_0 [\exp(-L/\delta_1)(\cos\omega t \cos(L/\delta_1) + \sin\omega t \sin(L/\delta_1) - \cos\omega t)]$$
$$= A_{20}\exp(-L/\delta_2)(\cos\omega t\cos(L/\delta_2 - \varphi) + \sin\omega t \sin(L/\delta_2 - \varphi)) - \alpha_2 M_2 \varepsilon_0 \cos\omega t.$$

Equating the coefficients at $\sin(\omega t)$ and $\cos(\omega t)$ on the right and left sides of the equation, we obtain two relations for the determination of A_{20} and φ:

$$A_{20}\exp(-L/\delta_2)\sin(L/\delta_2 - \varphi) = \alpha_1 M_1 \varepsilon_0 \exp(-L/\delta_1)\sin(L/\delta_1),$$

$$A_{20}\exp(-L/\delta_2)\cos(L/\delta_2 - \varphi) - \alpha_2 M_2 \varepsilon_0 = \alpha_1 M_1 \varepsilon_0 [\exp(-L/\delta_1)\cos(L/\delta_1) - 1].$$

Hence, we find

$$A_{20} = \alpha_1 M_1 \varepsilon_0 \frac{\exp(-L/\delta_1)\sin(L/\delta_1)}{\sin(L/\delta_2 - \varphi)}\exp(L/\delta_2), \quad (22)$$

$$\tan(L/\delta_2 - \varphi) = \frac{\alpha_1 M_1 \exp(-L/\delta_1)\sin(L/\delta_1)}{\alpha_1 M_1[\exp(-L/\delta_1)\cos(L/\delta_1) - 1] + \alpha_2 M_2}. \quad (23)$$

Solution (21) takes the form

$$p_2(z, t) = \alpha_1 M_1 \varepsilon_0 \frac{\exp(-L/\delta_1)\sin(L/\delta_1)}{\sin(L/\delta_2 - \varphi)} \times \exp(-(z-L)/\delta_2)\cos(\omega t + \varphi - z/\delta_2) - \alpha_2 M_2 \varepsilon_0 \cos\omega t. \quad (24)$$

If parameters are continuous at the interface $z = L$ (i.e., $\alpha_1 = \alpha_2$, $M_1 = M_2$, and $\delta_1 = \delta_2$), it follows from (23) that the phase is $\varphi = 0$ and solution (24) coincides with solution (14) for a homogeneous medium filling the half-space $0 \le z \le \infty$.

Differentiating (24) with respect to z, we find the gradient

$$\frac{\partial p_2}{\partial z} = \frac{\sqrt{2}\alpha_1 M_1 \varepsilon_0}{\delta_2}\frac{\exp(-L/\delta_1)\sin(L/\delta_1)}{\sin(L/\delta_2 - \varphi)} \times \exp(-(z-L)/\delta_2)\sin(\omega t - z/\delta_2 + \varphi - \pi/4). \quad (25)$$

Note that the generalization of the two-layer problem to a multilayer structure does not encounter any fundamental difficulties.

In the electrokinetic effect, the electric field intensity is determined by the formula

$$\mathbf{E} = C\nabla p. \quad (26)$$

Here C is a local coefficient of flow potential. The gradient in this case is reduced to the vertical derivative found above.

Figure 1 shows an electrical profile arising in a two-layer medium as a result of the electrokinetic effect in the field of lunisolar tidal deformations. Calculations were performed for an example in which the first and second layers consist of sandstone and limestone, respectively, in an oil field with the parameters given above; the distance between the layers is $L = 1000$ m. For the semidiurnal tidal wave, the first and second layers have skin-layer thicknesses of 490 and 11.7 m, respectively. Estimating the electric intensity E, it is necessary to specify the value of the coefficient of flow potential C. The estimates of C obtained in [Gershenzon and Bambakidis, 2001; Gershenzon and Gokhberg, 1994] lie in the range from 10^{-6} to 10^{-7} V/Pa. Surkov [2000] gives more definite values for some rocks: $C = -8 \times 10^{-7}$ V/Pa for granite, $C = -4.2 \times 10^{-6}$ V/Pa for various sandstones, and $C = -4.7 \times 10^{-6}$ V/Pa for porous rocks. In our calculations we used the value $C = -4.2 \times 10^{-6}$ V/Pa. The amplitude of tidal volume strains is equal to $\varepsilon_0 = 2 \times 10^{-8}$ [Latynina and Karmaleeva, 1978].

Figure 1 plots instantaneous values of the electric field intensity E for the semidiurnal harmonic at the time $t = 0$. In each layer, the intensity E is described by a decaying sinusoid having the spatial period $\lambda = 2\pi\delta_i$. In the first layer, $\lambda_1 = 2\pi 490 = 3079$ m and the sinusoid is unobservable on a 1000-m scale (Fig. 1a). In the second layer (Fig. 1b), the wavelength is small ($\lambda_2 = 2\pi 11.7 = 73.5$ m) and the decay is well expressed. The maximum values of the electric field intensity are $E_1 = 1.67 \times 10^{-4}$ V/m and $E_2 = 6.85 \times 10^{-2}$ V/m at the boundaries of the first and second layers, respectively.

The emf produced by the electrokinetic effect on a given base (z_0) is found by integration of the electric intensity over the spatial coordinate. The result of integration is the spatiotemporal part of the pore pressure described by formulas (14) and (24).

Estimate of the emf at the first boundary. We have

$$U_1 = \int_0^{z_0} C\frac{\partial p_1(z,t)}{\partial z}dz$$

$$= C\alpha_1 M_1 \varepsilon_0 \exp\left(-\frac{z}{\delta_1}\right)\cos\left(\omega t - \frac{z}{\delta_1}\right)\Bigg|_0^{z_0} \quad (27)$$

$$= C\alpha_1 M_1 \varepsilon_0 \left\{\exp\left(-\frac{z_0}{\delta_1}\right)\cos\left(\omega t - \frac{z_0}{\delta_1}\right) - \cos\omega t\right\}.$$

We then assume that the base of integration z_0 is the distance at which the potential difference is measured. In practice, in the case of tidal deformations, $z_0 \ll \delta_1, \delta_2$. Considering this, formula (27) can be simplified and we obtain

$$U_1 = C\frac{\sqrt{2}\alpha_1 M_1 \varepsilon_0}{\delta_1} z_0 \sin\left(\omega t - \frac{\pi}{4}\right). \quad (28)$$

Estimate of the emf at the second boundary. It is calculated in a similar way. We have

$$U_2 = C\sqrt{2}\alpha_1 M_1 \varepsilon_0 \frac{z_0}{\delta_2} \frac{\exp(-L/\delta_1)\sin(L/\delta_1)}{\sin(L/\delta_2 - \varphi)}$$
$$\times \sin\left(\omega t + \varphi - \frac{L}{\delta_2} - \frac{\pi}{4}\right). \quad (29)$$

Formula (29) contains the phase φ, determined by Eq. (23). In the two limiting cases, we can eliminate the phase and reduce formula (29) to a more illustrative form.

A thin layer with a thickness significantly smaller than the depth of the skin layer:

$$L \ll \delta_1, \delta_2. \quad (30)$$

Under this condition, formula (23) yields the relation

$$\tan(L/\delta_2 - \varphi) \approx \sin(L/\delta_2 - \varphi)$$
$$\approx \frac{\exp(-L/\delta_1)\sin(L/\delta_1)}{(\alpha_2 M_2)/(\alpha_1 M_1)} \ll 1. \quad (31)$$

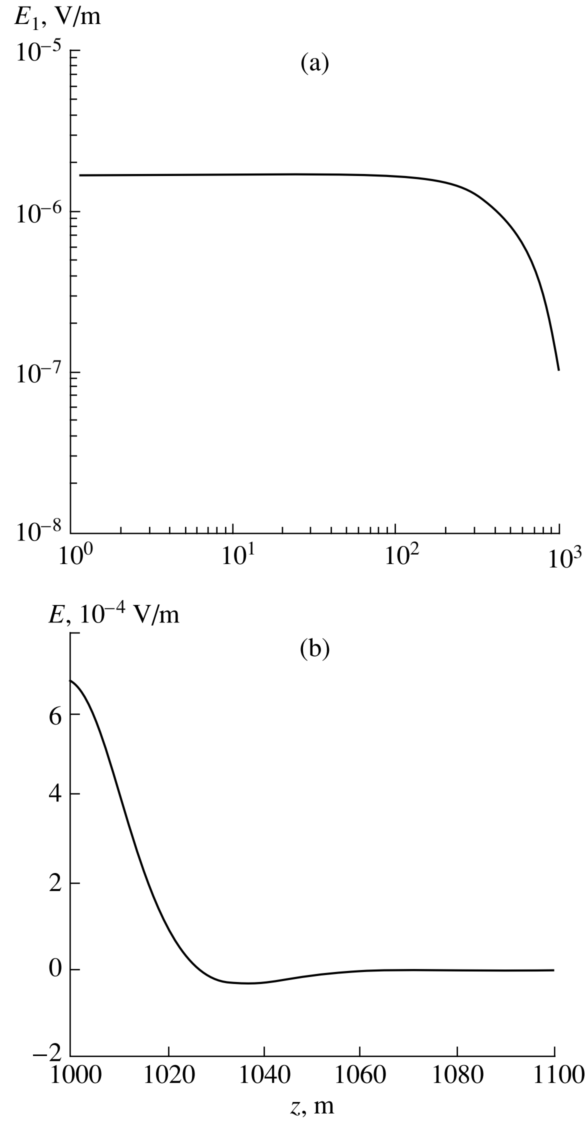

Fig. 1.

Using (30) and (31), the phase φ can be determined in an explicit form:

$$\varphi = \frac{L}{\delta_2} - \frac{\alpha_1 M_1}{\alpha_2 M_2}\frac{L}{\delta_1}. \quad (32)$$

Eliminating $\sin(L/\delta_2 - \varphi)$ and the phase φ from (29), we obtain

$$U_2 = C\frac{\sqrt{2}\alpha_2 M_2 \varepsilon_0}{\delta_2} z_0 \sin\left(\omega t - \frac{\alpha_1 M_1}{\alpha_2 M_2}\frac{L}{\delta_1} - \frac{\pi}{4}\right). \quad (33)$$

A thick layer is another limiting case, in which the inverse condition holds true:

$$L \gg \delta_1, \delta_2. \quad (34)$$

In this case, the exponent $\exp(-L/\delta_1)$ has a large negative argument and the right-hand side of formula (23) is small. This leads to the relation

$$\tan(L/\delta_2 - \varphi) \approx \sin(L/\delta_2 - \varphi)$$
$$\approx \frac{\exp(-L/\delta_1)\sin(L/\delta_1)}{(\alpha_2 M_2)/(\alpha_1 M_1) - 1} \ll 1. \quad (35)$$

In particular, (35) implies that

$$\varphi \approx L/\delta_2. \quad (36)$$

As a result, we obtain the following formula for the emf at the second boundary:

$$U_2 = C\frac{\sqrt{2}(\alpha_2 M_2 - \alpha_1 M_1)\varepsilon_0}{\delta_2} z_0 \sin\left(\omega t - \frac{\pi}{4}\right). \quad (37)$$

Using formulas (28) and (37), we can estimate the amplitude of electric field variations resulting from the electrokinetic effect due to tidal deformations in a fluid-saturated porous medium. In the example considered above for the semidiurnal wave, the potential difference on the base $z_0 = 1$ m is equal to 2.36×10^{-6} and 4.96×10^{-4} V at the first and second boundaries, respectively. The difference between these values is due to the skin layer thickness, which determines the values of the pore pressure gradient and the electrokinetic effect.

CONCLUSIONS

Our analysis has shown that tidal variations in the pore pressure significantly depend on mechanical properties of the medium, its permeability, and the viscosity of the fluid and reach a maximum near boundaries within the hydrodynamic skin layer. The maximum of the pore pressure gradient and the related maximum of the vertical component of the electric intensity are attained at the boundary. The value of the electrokinetic emf induced by tidal deformations on a 1-m base lies within the range (0.001–0.5) $\times 10^{-3}$ V, depending on mechanical properties of the medium. Measurements of the vertical component of the electric field can be used to monitor deformation processes and determine mechanical properties of the medium (permeability, elasticity, etc.).

An analysis of experimental data will be presented in our next paper.

ACKNOWLEDGMENTS

We are grateful to A.V. Guglielmi and S.M. Molodensky for discussions and constructive comments. This work was supported by the Russian Foundation for Basic Research, project no. 04-05-64667.

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