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VERTICAL DISPLACEMENTS OF THE MENDELEEVO STATION RESULTING FROM ATMOSPHERIC DEFLECTIONS OF THE EARTH'S CRUST

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A significant correlation was established between the measured vertical displacements of the Mendeleevo GPS Station and the calculated deflections of the Earth's surface caused by changes of atmospheric pressure on the Eurasian continent. A correlation was also established with the local atmospheric pressure at the Mendeleevo station.

Deflections of the Earth's surface accompanying changes in the atmospheric pressure can be estimated in terms of the Boussinesq problem concerning deformations of the surface of a semi-infinite elastic medium caused by a distributed loading [1]. Such a model is plausible from a physical point of view, but it contains undefined parameters in the form of the unknown averaged elastic characteristics of the Earth's crust, and this can cause the effects to be overestimated [2]. Another approach, developed in [3-5], is based on considering the atmospheric deflections of the spherical surface of the Earth taking account of its internal structure using a model such as, for example, the Gutenberg–Bullen model. In this case, the determination of the deflection at a given point requires a knowledge of the atmospheric pressure distribution $p(\mathbf{r})$, strictly speaking over the entire surface of the Earth. One must then take into account the specific behavior of the ocean when the change in the atmospheric pressure over the ocean is not transferred to the bottom of the ocean because of the so-called "reverse barometer" effect [6]. In practice the Green function $G(\mathbf{r} - \mathbf{r}')$ used to find the deflection $U(\mathbf{r})$ at the point of observation $\mathbf{r}$ falls off rapidly with distance, so that in the formula for calculating the effect

$$U(\mathbf{r}) = \int\int G(\mathbf{r}-\mathbf{r}')p(\mathbf{r}')dx'dy' \tag{1}$$

it is sufficient to include only the near field zone. Here $\mathbf{r}' = (\mathbf{x}', \mathbf{y}')$ is the instantaneous radius vector.

In the work presented here, formula (1) is used as a basis for analyzing the correlation of variations in the height of the GPS station in the Mendeleevo Moscow province with continuous data of the atmospheric pressure on the Eurasian continent during the period from March 18 to May 4, 1995. A Green function calculated by Farrell [5] for a spherically symmetric Earth with an internal structure corresponding to the Gutenberg–Bullen model was used in the integral of Eq. (1). In a spherical system of coordinates with its origin at the point of observation, this function depends only on the polar angle θ measured from the point of observation and represented by Farrell as

$$G(\theta) = 1 \cdot 10^{-12} \frac{f(\theta)}{R\theta},$$

where R is the radius of the Earth and $f(\theta)$ is a tabulated function [5].

In polar coordinates formula (1) assumes the form

$$U(0) = 1 \cdot 10^{-12} R \int d\lambda \int f(\theta) \frac{\sin\theta}{\theta} p(\theta, \lambda) d\theta. \tag{2}$$

Here λ has the sense of the "latitude", R is expressed in meters, and the pressure p is expressed in millibars.

The numerical integration in Eq. (2) was replaced by a summation over trapeziums with dimensions $\Delta\theta = 5°$, $\Delta\lambda = 15°$. The calculations were performed as follows. A measuring grid with a polar coordinate mesh was laid on a weather

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TABLE 1. Confidence Limits of the Coefficients r, r', and a for Different Confidence Levels

q, %	$r=0{,}686$	$r'=-0{,}633$	$a=1{,}497$
5	$+0{,}123$ -0.191	$-0{,}144$ $+0.208$	$\pm 0{,}471$
1	$+0{,}144$ -0.230	$-0{,}178$ $+0.286$	$\pm 0{,}630$
0.1	$+0{,}181$ -0.353	$-0{,}212$ $+0.383$	± 0.822

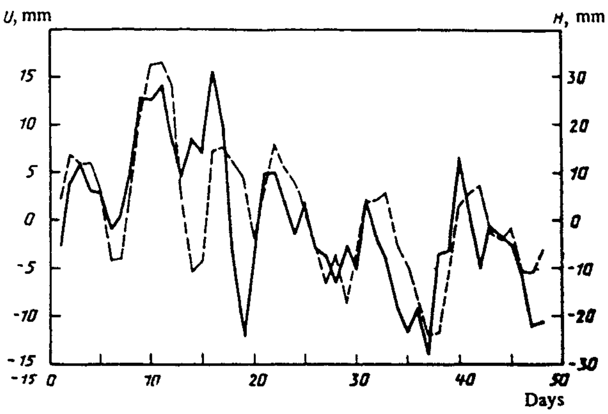

Fig. 1. Measured (H) and calculated (U) height variations of the Mendeleevo station (measurements commenced on March 18, 1995).

map showing isobars. The coordinate origin was made to coincide with the Mendeleevo observing station. The average pressure was determined in each trapezium element. The atmospheric pressure over the ocean surrounding the continent was assumed to be equal to the average atmospheric pressure (because of the "reverse barometer" effect). The summation was first performed inside a zone included between two latitudes, and then over all the zones. The difference between the average pressure in the trapeziums and the average pressure of 1013.25 mbar over the entire integrated surface was taken as the loading (the excess or deficiency).

Figure 1 gives the results of the measured height variations H and the calculated height variations U for the Mendeleevo station. In Fig. 2 the same data are presented in the form of points which are grouped around the regression line

$$H'=1{,}497U-1{,}297,$$

where H and U are expressed in millimeters; the regression coefficient $a = 1.497$ is significant with a high degree of confidence (see Table 1).

The variances of the measured and calculated heights H and of the differences $\delta H = H - H'$ were then calculated. They were found to be $D(H) = 198.8$ mm^2, $D(U) = 41.72$ mm^2, and $D(\delta H) = 105.3$ mm^2. After subtracting the calculated variations by the atmospheric pressure, the variances of the measured heights were almost halved: $D(H)/D(\delta H) = 1.89$. This change in accordance with the Fisher criterion is significant. It exceeds the 5% confidence threshold which, in the present case, is equal to 1.62. As can be seen, the change in the variance of 93.5 mm^2 considerably exceeds the variance of the correction itself, which is 41.72 mm^2. This is a characteristic feature of continental observing stations [7].

According to [8], the determined variances $D(H)$, $D(U)$, and $D(\delta H)$ can be used to estimate the variance of the so-called "atmospheric noise" in the calculated effect; the rms value of this noise is 5.1 mm.

The correlation coefficient between the measured H and calculated U values of the height variation is $r = 0.686$. For 45 degrees of freedom, this value is significant with a high confidence (see Table 1). Calculations were also made of the correlation coefficient between the measured variations in the height H and the local atmospheric pressure at the Mendeleevo

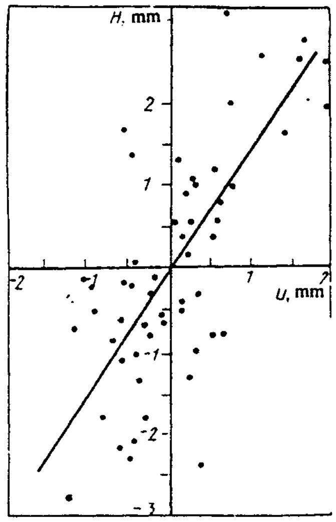

Fig. 2. Dependence $H(U)$ from the data of Fig. 1.

station. It was found to be $r' = -0.633$, 5% lower in absolute magnitude than r. The minus sign indicates anticorrelation between the measurements of the station height and the atmospheric pressure at it. The closeness in the absolute values of the correlation coefficients r and r' provides evidence of the large contribution of the pressure at the observing station and in its vicinity to the vertical deflection of the Earth's crust. It should be mentioned that the manifestation of loading deflections of the Earth's crust is of a systematic nature and exceeds, in its absolute magnitude, the random errors of contemporary instruments for radio astronomical observations, laser location of artificial satellites, and the reception of signals from navigation systems in space. This effect is of considerable importance for taking account of the deformation of the coordinate system of the Earth, fixed by observing stations when carrying out geodynamic investigations, for determining the Earth rotation parameters, and also for metrologically testing instruments for observing artificial Earth satellites.

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