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FUNDAMENTAL PROBLEMS IN METROLOGY

INTRODUCING CORRECTIONS TO THE FIELD GRADIENTS WHEN CALCULATING THE GRAVITATIONAL CONSTANT

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An analysis is made of the influence of corrections caused by gradients of the gravitational fields of closely positioned concentrated masses on the results of calculating the gravitational constant. A correction in the form of the product of the difference of the squares of the oscillations amplitudes with an experimentally determined coefficient should be added to the period for the smaller amplitude and subtracted from the period for the larger amplitude. It is shown that the systematic error is reduced by introducing this correction.

A displacement of the attracting spherical masses from the position i to the position j results in a change in the oscillation amplitude of the torsion balance which, to a first approximation, is proportional to the change in the oscillation period of the balance. By virtue of the law of the conservation of energy, the ratio $T_i/\varphi_{0i} \approx T_j/\varphi_{0j}$ can be considered to be a constant quantity. This ratio must be strictly conserved in the case when the masses are displaced instantaneously to the new position. In reality, the displacement takes place in 1–10 min and in this case the relative change in the periods, T_j/T_i, exceeds the corresponding amplitude ratio $\varphi_{0j}/\varphi_{0i}$. The difference between the amplitudes φ_{0i} and φ_{0j} when the attracting masses are fixed at the ith and jth positions makes it necessary to take account of the external gravitational fields since they influence the period of the anharmonic oscillations [1, 2]. When the attracting masses are displaced from one position to the other, the oscillation amplitude changes only very slightly, and so to a first approximation such an influence can be neglected. However, with an increase in the measurement accuracy it became clear that this source of systematic error must nevertheless be taken into account. In view of this, the problem arises of introducing a correction to the external field which is principally caused by the closeness of the working body of the balance to the concentrated masses. In a number of experiments, such gradients of the external fields were found to be significant. They were caused by the massive posts of the platform to the center of which the upper end of the auxiliary suspension was fastened while the lower end was attached to a magnetic oscillation damper in the form of an aluminum disk located in a strong magnetic field [3–5]. The small distance of the posts from the weights of the balance beam led to a significant dependence of the period T on the oscillation amplitude φ_0. The curve of $T(\varphi_0)$ was investigated experimentally in the absence of the attracting masses, and this made it possible to introduce a correction to the gradients of the external fields.

Each row of the array contains a combination of two positions of the attracting masses which have differing periods and oscillation amplitudes. The difference between the amplitudes makes it necessary to introduce a corresponding correction. If one attributes the oscillation periods to the smaller of the two amplitudes, then it is necessary to subtract from the period associated with the larger oscillation amplitude the product of some constant coefficient with the difference of the

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TABLE 1. Sums of the Two Last Significant Figures of G_{ij} for Three Coefficients c in Arrays for Four-Position Measurement Scheme in the Combinations a_1 – (1,2 + 2,1); a_2 – (2,3 + 3,2); a_3 – (3,4 + 4,1); a_4 – (1,4 + 4,1)

n_1	Array	n_2	$c = 0$				$c = 0.5$				$c = 1$			
			a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4	a_1	a_2	a_3	a_4
1	850419	911	62	61	59	66	58	58	58	58	54	55	57	50
1	850629	540	64	63	62	69	60	60	60	60	56	57	59	52
1	851211	704	66	64	63	71	60	60	60	60	54	56	59	48
1	860326	5050	61	59	59	63	58	58	58	58	55	56	58	53
2	870104	270	61	59	59	61	59	59	59	59	59	58	59	57
2	870303	1082	57	59	57	59	57	57	57	57	55	57	57	54
2	870714	150	55	56	56	57	54	54	54	54	52	53	54	50
2	870722	853	57	58	59	60	57	57	57	57	55	56	57	53
3	880802	1075	57	57	57	57	57	57	57	57	57	57	57	57
3	880805	1597	56	56	56	56	56	56	56	56	56	55	56	56
3	890309	2279	60	60	60	60	60	60	60	60	60	60	60	60
3	890606	405	57	57	57	57	57	57	57	57	57	57	57	57

squares of the amplitudes at the given positions. Of course, this correction can be added to the period for the smaller amplitude, but this variant gives no advantages since it is entirely equivalent to the former. The method proposed in [6] for measuring the gravitational constant is based on the idea that it is necessary to equalize the combinations G_{ij} with the minimum and maximum sum of the indices by changing the distance from the axis of rotation, in which case all the remaining combinations should automatically assume the same value. In particular, for a four-position scheme the condition $G_{12} + G_{21} = G_{34} + G_{43}$ must be satisfied. It was found by analyzing all the available arrays that in certain of them, when the values of G_{ij} between neighboring positions (1,2; 2,3; 3,4) were equal, those between the extreme positions (1,4) turned out to be overestimated. The disruption of this picture in a number of arrays made it necessary to make a more detailed analysis of all the possible sources of this kind of error.

It can be assumed that the introduction of a correction utilizing the difference of the squares of the oscillation amplitudes is not sufficient, and so an attempt was made to introduce a correction utilizing the difference of the amplitudes to the fourth power. All the efforts directed at weakening or completely eliminating this effect were unsuccessful. The same result was also obtained for all the other variants. These involved, for instance, selecting a constant optical indication system used when calculating G_{ij}. The systematic overestimation of the values of G_{ij} between the extreme positions led to the conclusion that there was some lack of correctness in relating the oscillation periods to the smaller amplitude for the two positions considered. The proposal arose that it would be more sensible to introduce a correction to the field gradients symmetrically in both of the considered positions, i.e., to relate the period to some average amplitude. Since it is difficult to prove that the correction must be divided strictly half-and-half, half of it being added to the period for the smaller amplitude and subtracted from the period for the larger amplitude, a supplementary coefficient c was added to the file of constants of the program for calculating G_{ij} which can be given in the range 0–1. The oscillation periods and amplitudes were determined during the time intervals between pulses from the photodetector [4, 5]. Then, to the period T_i was added

$$\Delta T_i = c\{c_1(\varphi_{0j}^2 - \varphi_{0i}^2) - c_2(\varphi_{0j}^4 - \varphi_{0i}^4)\},$$

and from the period T_j was subtracted

$$\Delta T_j = (1-c)\{c_1(\varphi_{0j}^2 - \varphi_{0i}^2) - c_2(\varphi_{0j}^4 - \varphi_{0i}^4)\},$$

TABLE 2. Sums of the Two Last Significant Figures of G_{ij} for Three Coefficients c in Arrays for Three-Position Measurement Scheme in the Combinations a_1 – (1,2 + 2,1); a_2 – (2,3 + 3,2); a_3 – (3,4 + 4,1)

n_1	Array	n_2	$c = 0$			$c = 0.5$			$c = 1$		
			a_1	a_2	a_3	a_1	a_2	a_3	a_1	a_2	a_3
2	870924	57	59	58	60	57	57	57	54	56	53
2	871111	150	62	61	62	60	60	60	57	60	56
3	890620	880	59	59	59	59	59	59	59	59	59
3	901113	4305	62	60	61	60	60	60	59	59	59
3	910516	2594	58	58	58	58	58	58	58	58	58
3	930321	4121	59	59	59	59	59	59	59	59	58
3	930622	336	57	57	57	57	57	57	57	57	57
3	931130	3642	58	58	58	58	58	58	58	58	57
3	940705	1695	56	56	56	56	56	56	55	56	54
3	941220	669	57	57	58	57	57	57	56	57	56
3	950206	1106	58	58	58	58	58	58	58	58	58
3	950525	2164	58	58	59	58	58	58	58	58	57
3	950614	142	60	60	61	60	60	60	58	60	60
3	951019	2641	57	57	57	57	57	57	57	57	57
4	960222	967	58	58	59	58	58	58	58	58	57
4	960328	717	57	57	57	57	57	57	57	57	57
5	960522	807	58	58	58	58	58	58	58	58	58
5	960605	350	60	60	60	60	60	60	60	60	60
5	960906	794	58	58	58	58	58	58	58	58	57
5	961017	388	58	58	58	58	58	58	58	58	58
5	961104	267	57	58	57	57	57	57	56	57	57
5	961125	756	58	58	58	58	58	58	58	58	57
5	970506	1609	57	56	56	56	56	56	55	56	55
5	970704	468	56	56	56	56	56	56	54	56	54
5	970815	729	57	57	57	56	56	56	55	55	55
5	970916	145	59	59	59	59	59	59	59	59	57
5	970930	2091	58	58	58	58	58	58	57	58	58
5	980112	276	59	59	59	59	59	59	59	59	59
5	980122	339	59	59	59	59	59	59	58	58	59
5	980202	980	57	57	57	57	57	57	57	57	57
5	980310	2273	60	59	59	59	59	59	59	59	59
5	980605	1587	59	59	59	59	59	59	59	59	59
5	980918	1377	58	58	58	58	58	58	58	58	57
5	990611	3673	57	57	57	57	57	57	57	57	57
5	000330	5177	58	58	58	58	58	58	58	58	58

following which, G_{ij} was calculated. An analysis of all the available arrays showed that the coefficient c should be close to the value of 0.5. Indeed, the introduction of such a more flexible correction provided equality of all combinations G_{ij} even in those arrays in which the coefficients c_1 for the difference of the squares of the amplitudes $\varphi_{0j}^2 - \varphi_{0j}^2$ had a large value. For the purposes of clarity, Tables 1 and 2 give the sums of the two figures of the values of G_{ij} for three values of the coefficient c and for two measurement schemes employed which differ by the masses being fixed in either four or three positions. The first three columns of the tables give the versions of the torsion balance n_1, the code numbers of the arrays, and the number n_2 of rows contained in them. A row of an array includes the position numbers i and j and also the ten time intervals, linked to real time, over which the oscillation periods and amplitudes and the gravitational constant G_{ij} were calculated. Since the values of G_{ij} in all possible combinations averaged over the whole array are close to the standard value, such a sum gives a full representation of the absolute values of G_{ij} in all possible combinations. If the sum of the two last figures of the G_{ij} values is equal to 58, this denotes that $G_{ij} = (6.6729 \pm 0.0005)\cdot 10^{-11}$ N·m^2·kg^{-2}. An increase of two in the sum causes an increase of unity in the fourth significant figure after the decimal point.

For the direct cycles the masses are moved away from the rotational axis, and so $i < j$. In the inverse cycles, after reversal of the electrical leads, the masses are moved closer to the beam weights and in this case $i > j$. For $c = 0$, the values of G_{ij} between the extreme positions are found to be overestimated while for $c = 1$ they are found to be underestimated. Tables 1 and 2 give the sums of the two last figures of the value of G_{ij} averaged over the whole array for the direct and inverse cycles. First shown are the sums for the adjacent positions as the sum of the indices is increased. The last sum relates to the combination of the extreme positions. For $c = 0.5$, the sums of the last two figures are equal for all combinations of G_{ij}. In a number of arrays, such a relationship persists in the whole range of variation of c from 0 to 1. In arrays containing higher values of the oscillation amplitudes of the balance or implemented for higher gradients of the external fields, the values of G_{ij} for $c = 0$ are larger than those for $c = 1$. This is especially strikingly discernable for the examples of the arrays 850419, 850629, 851211, and 860326. These arrays were obtained using a balance of variant 1 for which the field gradients correction c_1 reached 790 s/rad^2. The tendency for G_{ij} to decrease as c increases can also be noted in the arrays 870104, 870303, 870714, 870722, 870924, 871111, 901113, 940705, 941220, 970506, and 970815. The arrays 870104, 870303, 870714, 870722, 870924, and 871111 were obtained with a balance of variant 2 which possessed a relatively long oscillation period and consequently a high sensitivity. This resulted in a large change ΔT in the oscillation period when the attracting masses were displaced from one position to the other whereupon there was an increase in the difference $\varphi_{0j}^2 - \varphi_{0i}^2$ and correspondingly in ΔT_i and ΔT_j. In the array 901113 which was obtained using a balance of variant 3 the tendency for a decrease in G_{ij} is traced to be a consequence of the fact that the measurements were made with large oscillation amplitudes. A similar situation which is expressed less strikingly in the arrays 930321, 940705, 941220, 950614, 960222, 970506, 970815, and 970916 is also due to the fact that individual rows of the arrays contain large oscillation amplitudes. In the remaining arrays this tendency is either completely absent or is expressed extremely weakly. In arrays obtained for relatively large oscillation amplitudes the dependence of G_{ij} on c is seen more strikingly since the correction to the field gradients increases with an increase in the oscillation amplitude of the balance. The most vulnerable combination G_{ij} is found to be that with the extreme values of the indices, into which a substantial correction is introduced. The greater the difference in the oscillation periods in a chosen combination G_{ij} the larger is the difference in the oscillation amplitudes and consequently the more strongly expressed is the dependence on c. This is clearly seen in the example of the first four arrays in table 1 in which the combination G_{12} has the strongest dependence on c while the combination G_{34} has a markedly weaker dependence.

Conclusions. The displacement of the attracting masses from one position to the other leads to a change in the oscillation amplitude and it is necessary to introduce a correction to the gradients of the gravitational field of the closely positioned concentrated masses. The presence of such gradients, which is not related to the attracting masses, results in the period of the torsion balance being a function of the oscillation amplitude. The change in the period for a displacement from one position to the other is proportional to the product of the difference of the squares of the oscillation amplitudes with a constant coefficient. The value of the latter is determined experimentally in terms of the dependence of the period on the oscillation amplitude. This coefficient subsequently enters into the correction introduced into the oscillation periods of the balance when calculating the gravitational constant. The introduction of corrections which are equal in magnitude but opposite in sign into both periods in all the rows of the arrays (different combinations of the positions of the attracting masses) reduces the systematic error caused by the presence of external gravitational fields.

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