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Recent measurements of the gravitational constant as a function of time

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A recent publication [J. D. Anderson *et. al.*, Europhys. Lett. 110, 10002 (2015)] presented a strong correlation between the measured values of the gravitational constant G and the 5.9 year oscillation of the length of day. Here, we compile published measurements of G from the last 35 years. A least-squares regression to a sinusoid with period 5.9 years still yields a better fit than a straight line. However, our additions and corrections to the G data reported by Anderson *et al.* significantly weaken the correlation.

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I. INTRODUCTION

A recent article [1] suggested a correlation between measurements of the gravitational constant, G, and the length of day. Figure 1 in Ref. [1] shows 13 measurements of G as a function of time. Superimposed is a sinusoidal fit with an offset of $\bar{G} = 6.67390 \times 10^{-11}$ m^3 kg^{-1} s^{-2}, a period $T = 5.9$ years and amplitude $A = 0.0016 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The ratio of amplitude to offset is 2.43×10^{-4}. A second trace shows a scaled version of the change in the length of day, almost indistinguishable from the fit, suggesting a strong correlation of G measurements around the world and the observed change in the length of day.

However, several points in Ref. [1] are not plotted at the right time and one experiment [2] is missing. Here, we provide updated measured values of G with their measurement dates, as displayed in Fig. 1.

II. DATA SOURCES

It is sometimes difficult to determine the exact time of data acquisition of a published G measurement. Below we attempt to assign a best weighted average of the measurement times involved in each of the most precise G measurements in the last 35 years. In some cases, this date is the mean of the start and end dates of the data acquisition period; in others, it is an average of individual dates when data was taken. This may not always be the best measure of the effective measurement time; in fitting data we suggest assigning an uncertainty for each tabulated time equal to 20% of the time span.

NIST-82: This experiment was performed at the National Institute of Standards and Technology (then the National Bureau of Standards) in Gaithersburg, Maryland. A torsion balance used the so-called time-of-swing method in which torsional period is measured in at least two source-mass configurations. G is calculated from the difference in the squares of the periods and known mass distributions. The resulting $G = (6.6726 \pm 0.0005) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, was published in 1982 [4]. The measurement dates can be inferred from Table 1 in Ref. [5]. The first measurement was August 29, 1980 and the last was October 10, 1980. We use the average value (September 19, 1980) as the time coordinate for this measurement.

TR&D-96: This measurement, performed in Moscow by researchers at Tribotech Research and Development Company, also used a torsion balance in the time-of-swing mode, yielding $G = (6.6729 \pm 0.0005) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, published in Ref. [6]. The results of measurements spanning ten years are given in Table 3 of Ref. [6]. Unfortunately the data is given to only four decimal places. We reproduce the raw data with type A uncertainties in Table I.

The TR&D-96 data alone permits a powerful test for a dependence of G on length of day. Figure 1 shows the data (again with only type A uncertainties) as a function of time. The best fit to a sinusoid with period 5.9 years yields an offset $\bar{G} = 6.67293 \times 10^{-11}$ m^3 kg^{-1} s^{-2} and amplitude of $A = 0.000086 \times 10^{-11}$ m^3 kg^{-1} s^{-2} with uncertainty $\sigma_A = 0.000055 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. There are 23 degrees of freedom and the χ^2 is 14.3. Compared to the fit to a full G data set in Ref. [1], this fit yields an amplitude smaller by a factor of 19 and a phase differing by about 125 degrees.

In 2009, an analysis of various correlations of the TR&D measurements to solar activity and other cosmic periods was published [7]. Correlations were found, but were attributed to terrestrial effects—most probably variations in temperature and the microseismic environment. In Ref. [7] data are
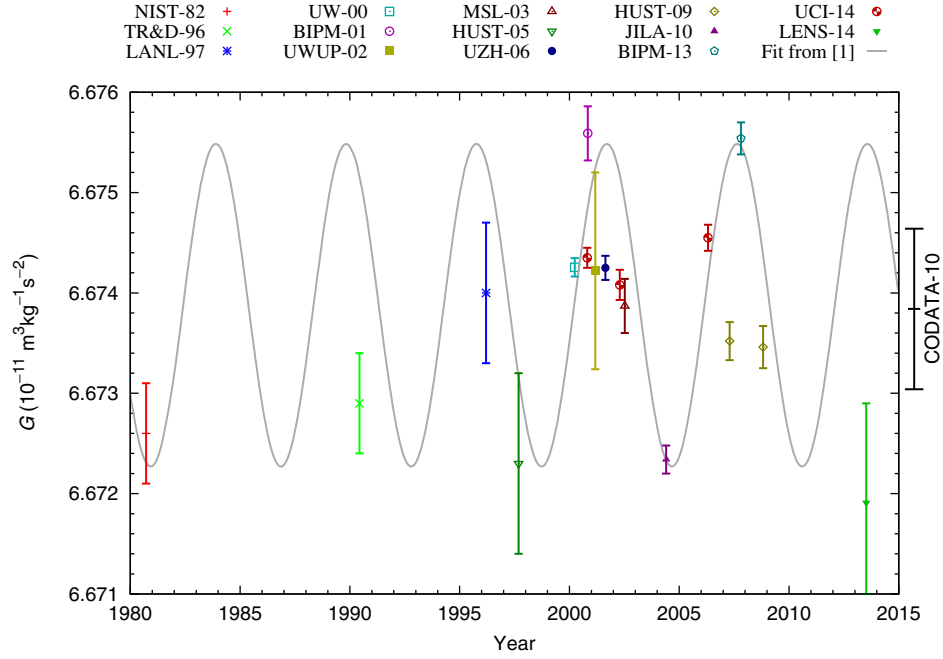
FIG. 1 (color online). Measurements of the gravitational constant, G, as a function of time. The TR&D-96 data were taken over ten years; for this plot the final TR&D-96 result is shown at the average of their measurement dates. The solid gray sinusoidal curve is the fit to the data as it appears in Ref. [1]; it is indistinguishable from the scaled length-of-day variation in the same reference. The point outside the frame gives the 2010 recommended value of G with 1-sigma uncertainties according to the Task Group on Fundamental Constants of CODATA [3].

shown ranging from 1985 to 2003. Unfortunately the data from 1995 to 2003 is not available to us.

The TR&D-96 data can be averaged to yield a single data point as displayed in Fig. 1. The average of the dates listed in table I is June 9, 1990.

LANL-97: A time-of-swing experiment was performed at the Los Alamos National Laboratory in Los Alamos, New Mexico, yielding $G = (6.6740 \pm 0.0007) \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$ [8]. The article gives no indication of when the data were taken. The thesis of Bagley [9] gives some information. Written on page 15 is "In January of 1996, I attempted a trial Heyl-type determination with this arrangement, hoping for a percent number or better". Later it is described how this measurement was much more precise, yielding the final value. On page 71 the reader learns that certain disturbances in the experiment became more frequent as the ambient temperature rose in April and May, until the data became unusable. The thesis was signed July 8, 1996. Thus we take March 15, 1996 as a time stamp for this data point.

UW-00: The measurement with the smallest uncertainty to date was performed at the University of Washington in Seattle, Washington, published in 2000 [10]. The rotation rate of a turntable supporting a torsion balance was varied such that the torsion fiber did not twist. In this angular-acceleration-feedback-mode the gravitational acceleration of a torsion pendulum towards source masses is fed back to the turntable, leaving the torsion balance motionless with respect to the turntable and adding the gravitational acceleration to the turntable motion. The gravitational constant is

TABLE I. Measurements and type A uncertainties from Table 3 of Ref. [6], with uncertainties converted to absolute values.

Date mm/dd/yyyy	$G \times 10^{11}$ $(\mathrm{m^3\,kg^{-1}\,s^{-2}})$	$\sigma_G \times 10^{11}$ $(\mathrm{m^3\,kg^{-1}\,s^{-2}})$
04/19/1985	6.673 0	0.000 60
06/29/1985	6.673 0	0.000 43
12/11/1985	6.673 0	0.000 43
03/25/1986	6.673 0	0.000 29
01/04/1987	6.673 2	0.000 93
03/03/1987	6.672 9	0.000 30
07/14/1987	6.672 9	0.000 60
07/22/1987	6.673 0	0.000 17
09/24/1987	6.672 9	0.000 51
11/11/1987	6.672 9	0.000 30
08/02/1988	6.672 7	0.000 35
08/05/1988	6.672 9	0.000 18
03/09/1989	6.673 0	0.000 15
06/06/1989	6.672 9	0.000 22
06/20/1989	6.672 7	0.000 19
11/13/1990	6.673 0	0.000 09
03/21/1993	6.673 0	0.000 13
06/22/1993	6.672 9	0.000 34
11/30/1993	6.672 9	0.000 17
07/05/1994	6.672 8	0.000 06
12/20/1994	6.672 9	0.000 09
02/06/1995	6.672 9	0.000 13
05/25/1995	6.673 0	0.000 08
06/14/1995	6.673 0	0.000 38
08/24/1995	6.673 0	0.000 17
10/19/1995	6.672 7	0.000 07

inferred from the second time derivative of the angle readout of the turntable with respect to time. The value published in 2000 must be slightly corrected due to an originally unconsidered effect of a small mass at the top of the torsion fiber which was also subject to the angular acceleration. This correction is described in Ref. [11]. After correction, the final result is $G = (6.674\,255 \pm 0.000\,092) \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The times are documented in Ref. [11]. Two sets of data were taken: one from March 10, 2000 to April 1, 2000, and the other from April 3, 2000 to April 18, 2000. We use March 29, 2000 to locate this G value.

BIPM-01: These measurements used the first torsion pendulum built at the Bureau International des Poids et Mesures (BIPM) located in Sèvres, near Paris. The experiment measured G with the same instrument operating with two methods. In the Cavendish method, the excursion of a torsion pendulum is measured for two source-mass positions. The corresponding torques are obtained using a torsion constant determined from the balance's angular moment of inertia and free angular frequency.

In the electrostatic servo method, gravitational torque on the pendulum is compensated by an electrostatic torque produced by an electric potential applied to a capacitor with one plate on the pendulum bob and the other fixed. In this phase, the applied voltage is measured. A calibration experiment measured the capacitance as a function of pendulum angle. Combining the results of both methods yielded $G = (6.675\,59 \pm 0.000\,27) \times 10^{-11}$ m^3 kg^{-1} s^{-2} [12]. The results of the Cavendish mode and servo mode are in close agreement, with $G = (6.675\,65 \pm 0.000\,45) \times 10^{-11}$ m^3 kg^{-1} s^{-2} from the Cavendish mode and $G = (6.675\,53 \pm 0.000\,40) \times 10^{-11}$ m^3 kg^{-1} s^{-2} from the servo mode. According to the authors [13], the servo data were obtained from September 29 to November 2, 2000 and the Cavendish data from November 25 to December 13, 2000. We take the effective date for the combined G to be the average of the above dates.

UWUP-02: This experiment was located at the University of Wuppertal in Germany. The separation of two simple pendulums was measured with microwave interferometry. The forces on the pendulums and, hence, their separation was modulated by external moving source masses. The final value of this measurement, $G = (6.674\,22 \pm 0.000\,98) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, was published in a PhD thesis [14]. The appendix lists the data sets used for the final value. The first data set started January 12, 2001, and the last ended June 29, 2001. Twelve data sets ranging in duration from one to six days were taken, mostly within a week of each other. A longer break occurred between March 7 and May 11 and between May 18 and June 25. Averaging the dates of the sets yields March 6.

MSL-03: This measurement, performed at the Measurement Standards Laboratory (MSL) of New Zealand, is the only recent measurement performed in the Southern Hemisphere. It employs a torsion balance in electrostatic servo mode with one difference: the calibration of the capacitance gradient is performed in an angular-acceleration experiment. The final value is $G = (6.673\,87 \pm 0.000\,27) \times 10^{-11}$ m^3 kg^{-1} s^{-2} [15]. One author [16] informed us that the data was gathered between March 21, 2002 and November 1, 2002. The average of these dates is July 11, 2002.

HUST-05: This is the first measurement of G performed at the Huazhong University of Science and Technology in Wuhan, China. A torsion balance in time-of-swing mode was used. A G value published in 1999 [17] was subsequently corrected in 2005 [18] for two small errors in mass distribution, yielding $G = (6.6723 \pm 0.0009) \times 10^{-11}$ m^{3}kg^{-1}s^{-2}. Data dates without years are given in the 1999 publication: seven sets of measurements were taken, the first starting on August 4 and the last ending on October 15. The authors report [19] that the year was 1997.

We associate September 9, 1997, equidistant in time from the start and end of the sets, with the HUST-05 G measurement.

UZH-06: The experiment, performed by researchers at the University of Zürich, was located at the Paul Scherrer Institute near Villigen, Switzerland. The gravitational force of a large mercury mass on two copper cylinders was measured with a modified commercial mass comparator, yielding $G = (6.674\,252 \pm 0.000\,12) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, published in 2006 [20]. Figure 8 in this publication shows 43 days of data beginning July 31, 2001 and ending September 9 and including a six day break. We take August 21, 2001, as the effective date of this G measurement.

HUST-09: A second torsion pendulum apparatus was constructed at HUST and used in time-of-swing mode to make two separate G measurements, whose averaged value $G = (6.673\,49 \pm 0.000\,18) \times 10^{-11}$ m^3 kg^{-1} s^{-2} was first published in 2009 [21]. A long article on the same measurements was published in 2010 [22], including the dates of the data sets used in the two experiments. The first experiment consisted of ten sets taken between March 21, 2007 and May 20, 2007. The second experiment started on October 8, 2008 and ended on November 16, 2008. The results for the first and second experiments are $G = (6.673\,52 \pm 0.000\,19) \times 10^{-11}$ m^{3}kg^{-1}s^{-2} and $G = (6.673\,46 \pm 0.000\,21) \times 10^{-11}$ m^{3}kg^{-1}s^{-2}, respectively. Averaging the start and end dates of the sets, we obtain April 20, 2007 and October 27, 2008, respectively.

JILA-10: This experiment was performed at the Joint Institute for Laboratory Astrophysics in Boulder, Colorado. Similar to UWUP-02, two simple pendulums with a separation determined by a laser interferometer were used to measure G, yielding $G = (6.672\,34 \pm 0.000\,14) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, reported in 2010 [23]. Figure 2 in this report and a table in Ref. [24] showed obtained values of G as a function of time. Thirteen G
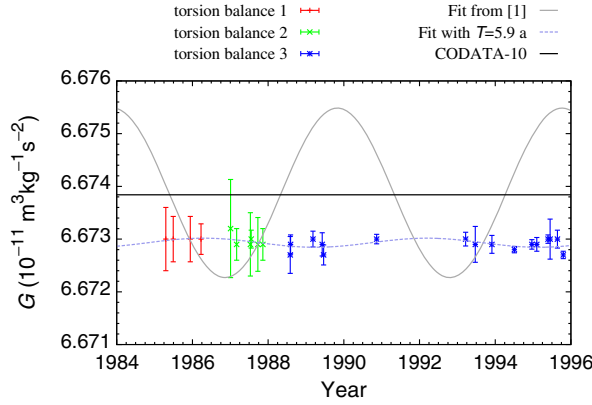
FIG. 2 (color online). Data from Ref. [6]. Karagioz and Izmailov measured over a decade using three different torsion balances. Plotted uncertainties are type A only. According to Ref. [6] the type B uncertainty associated with this experiment is $0.000\,52 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

values were obtained between May 12 to June 6, 2004. Averaging the 13 dates yields May 28, 2004.

BIPM-13: At the BIPM, a second torsion balance was constructed to measure G with two different methods.

Results were published in 2013 [25]. Combining the results of both methods yielded $G = (6.675\,54 \pm 0.000\,16) \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The Cavendish and servo methods yielded $G = (6.675\,86 \pm 0.000\,36) \times 10^{-11}$ m^3 kg^{-1} s^{-2} and $G = (6.675\,15 \pm 0.000\,41) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, respectively. These numbers include a small correction published in an erratum in 2014 [25]. Per one of the authors [13], the Cavendish data were obtained from August 31 to September 10, 2007 and the servo mode data were measured in two campaigns, with November 8, 13, 14, and 16 in 2007 for the first campaign and January 11, 12, 13, 15, and 16 in 2008 for the second campaign. Averaging these dates, we obtain October 25, 2007 as an effective time stamp for the BIPM-13 data.

UCI-14: These measurements, performed using a torsion balance at cryogenic temperatures in time-of-swing mode, were made near Hanford, Washington. Three types of fibers with differing mechanical properties (especially the amplitude dependence of the mechanical losses) were used. A result for each fiber was published in 2014 [2]: $G = (6.674\,35 \pm 0.000\,10) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, $G = (6.674\,08 \pm 0.000\,15) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, and $G = (6.674\,55 \pm 0.000\,13) \times 10^{-11}$ m^3 kg^{-1} s^{-2}. The principal investigator

TABLE II. Summary of the most precise measurements of G carried out in the last 35 years. The "Start" and "End" columns indicate our best estimate of the dates when data acquisition began and ended. The "Average" column shows our best estimate for the mean date of data acquisition. The "$e - s$" column gives the difference in days between the end and start of data acquisition, which is important in estimating the amount by which a short-period signal is attenuated. We suggest 20% of the $e - s$ duration number as a meaningful estimate of date uncertainty. We separate the two BIPM measurements into four measurements to emphasize that two different methods were used, and include data labeled BIPM-01sc and BIPM-13sc for the best G and dates combining the two methods. Particularly for the 2013 BIPM data, results with the separate methods had strongly anticorrelated uncertainties, so that a $G(t)$ fit using the combined G value can give a significantly different result from a fit treating results from the two methods separately. The BIPM data points in Fig. 1 represent the combined G data BIPM-01sc and BIPM-13sc.

Identifier	$G \times 10^{11}$ (m^3 kg^{-1} s^{-2})	$\sigma_G \times 10^{11}$ (m^3 kg^{-1} s^{-2})	Data acquisition Start	Data acquisition End	Data acquisition Average	$e - s$ (Days)	Device	Mode
NIST-82	6.672 6	0.000 5	08/29/1980	10/10/1980	09/19/1980	42	torsion balance	time-of-swing
TR&D-96	6.672 9	0.000 5	04/19/1985	10/19/1995	06/09/1990	3835	torsion balance	time-of-swing
LANL-97	6.674 0	0.000 7	01/01/1996	05/31/1996	03/15/1996	151	torsion balance	time-of-swing
UW-00	6.674 255	0.000 092	03/10/2000	04/18/2000	03/29/2000	39	torsion balance	acceleration servo
BIPM-01s	6.675 53	0.000 40	09/29/2000	11/02/2000	10/16/2000	34	torsion balance	electrostatic servo
BIPM-01c	6.675 65	0.000 45	11/25/2000	12/13/2000	12/04/2000	18	torsion balance	Cavendish
BIPM-01sc	6.675 59	0.000 27	09/29/2000	12/13/2000	11/02/2000	75	torsion balance	Cavendish & servo
UWUP-02	6.674 22	0.000 98	01/12/2001	06/29/2001	03/06/2001	168	two pendulums	
MSL-03	6.673 87	0.000 27	03/21/2002	11/01/2002	07/11/2002	225	torsion balance	electrostatic servo
HUST-05	6.672 3	0.000 9	08/04/1997	10/15/1997	09/09/1997	72	torsion balance	time-of-swing
UZH-06	6.674 25	0.000 12	07/31/2001	08/21/2001	08/21/2001	21	beam balance	
HUST-09a	6.673 52	0.000 19	03/21/2007	05/20/2007	04/20/2007	60	torsion balance	time-of-swing
HUST-09b	6.673 46	0.000 21	10/08/2008	11/16/2008	10/27/2008	39	torsion balance	time-of-swing
JILA-10	6.672 34	0.000 14	05/12/2004	06/06/2004	05/28/2004	25	two pendulums	
BIPM-13s	6.675 15	0.000 41	11/08/2007	01/16/2008	12/15/2007	69	torsion balance	electrostatic servo
BIPM-13c	6.675 86	0.000 36	08/31/2007	09/10/2007	09/05/2007	10	torsion balance	Cavendish
BIPM-13sc	6.675 54	0.000 16	08/31/2007	01/16/2008	10/25/2007	138	torsion balance	Cavendish & servo
UCI-14a	6.674 35	0.000 10	10/04/2000	11/11/2000	10/23/2000	38	torsion balance	time-of-swing
UCI-14b	6.674 08	0.000 15	03/25/2002	05/12/2002	04/18/2002	48	torsion balance	time-of-swing
UCI-14c	6.674 55	0.000 13	04/08/2006	05/14/2006	04/26/2006	36	torsion balance	time-of-swing
LENS-14	6.671 91	0.000 99	07/05/2013	07/12/2013	07/08/2013	7	atom interferometer	

TABLE III. Fits to the G data. Here the L2 norm is used exclusively. The "Maximum" column gives the date of the first maximum after 01/01/2000. The "NDF" column shows the fit's degrees of freedom.

Fit function	T (Years)	$A \times 10^{15}$ (m^3 kg^{-1} s^{-2})	$\bar{G} \times 10^{11}$ (m^3 kg^{-1} s^{-2})	Maximum	χ_f^2	NDF	$P(\chi^2 \geq \chi_f^2)$	Remarks
from Fig. 1 in Ref. [1]	5.93	16.1	6.673 88	09/13/01	381	14	10^{-72}	
sine, fixed T	5.93	10.7	6.673 59	03/14/01	132	14	10^{-21}	
sine, T free	0.77	11.2	6.673 58	02/21/00	77	13	10^{-11}	global χ^2 minimum
sine, T free	6.17	11.0	6.673 54	02/13/01	124	13	10^{-19}	local χ^2 minimum
horizontal line	n.a.	n.a.	6.674 13	n.a.	335	16	10^{-61}	

provided the following time information: data with the first fiber was first were obtained from October 4, 2000 to November 11, 2000. The average of these dates is October 23, 2000. Data with the second fiber were obtained during two disjoint intervals. About 14% of the data were obtained between December 8 and December 14, 2000, with the remainder obtained between March 25 and May 12, 2002. For simplicity we assign the average of the dates in 2002, i.e., April 18, 2002 to the result with the second fiber. The true average of all dates for this fiber would be roughly January 30, 2002. Measurements with the third fiber were collected from April 8 to May 15, 2006. The mean of this interval is April 26, 2006.

LENS-14: Following pioneering work at Stanford University [26], a precision measurement of G using a vertical atom interferometer was performed at the University of Florence, Italy. The phase shift between two paths was measured with two source-mass configurations. The G determined from the known mass distributions and the difference of the two phases is $G = (6.671\,91 \pm 0.000\,99) \times 10^{-11}$ m^3 kg^{-1} s^{-2}, published in 2014 [27]. A longer account of the experiment appears in Ref. [28], which states that data was taken between July 5 and July 12, 2013. The average of start and end dates is July 8, 2013. The experiment is currently targeting an improved measurement of G.

In Table II we summarize the precision measurements of big G in the last 35 years.

A. Discussion

The main purpose of this article is to provide an as complete as possible list of G values determined since 1980, while attempting to assign an as accurate as possible effective date for each measurement, providing data for further investigations similar to that of Anderson and collaborators.

We caution users of these data that it is very possible that much or all of the apparent G time variation simply reflects overlooked systematic error, with underestimated systematic uncertainty.

However, we have ventured to make the following fits to data presented in this article, using the combined pair of numbers for each of the two BIPM experiments.

(1) A sinusoidal function with the parameters found in Ref. [1].
(2) A sinusoidal function with free amplitude and phase but period fixed at 5.9 years.
(3) A sinusoidal function with free amplitude, phase, and period.
(4) A single time-independent parameter, $\bar{G}$.

Results of these fits are presented in Table III. These fits ignored uncertainties in the date. Including uncertainties in both coordinates did not significantly affect fit results.

Figure 3 displays goodness of fit using two different norms for sinusoidal fits as T is varied. The upper and lower graphs show fits obtained by minimizing the sum of the absolute residual (L1 norm), $\sum_i |r_i|$, and the sum of the squared residual (L2 norm), $\sum_i r_i^2$, respectively. Here, r_i is the residual of the ith data point, given by $r_i = (G_i - \bar{G} - C \cos(2\pi t_i/T) - S \sin(2\pi t_i/T))/\sigma_i$, where G_i and σ_i are the measurement and its uncertainty performed at time t_i. Fits using the L1 norm are less sensitive to outliers [29]. In this plot we note the following.

(1) There are a number of local minima.
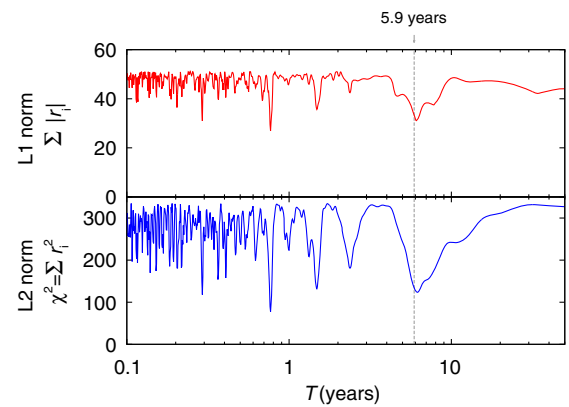(2) The lowest L1 and L2 norms are both located at $T = 0.769$ years.

FIG. 3 (color online). Goodness of fit as a function of period T. The upper graph shows the sum of the absolute residual: $\sum_i |r_i|$. The lower graph shows the sum of the squared residual: $\sum_i r_i^2$. The individual residual is given in both cases by $r_i = (G_i - \bar{G} - C \cos(2\pi t_i/T) - S \sin(2\pi t_i/T))/\sigma_i$, where G_i and σ_i are determined at time t_i.

(3) A local minimum is found at 6.1 years and 6.2 years for the L1 and L2 norms, respectively, which is near 5.9 years as found by Anderson *et al.*.

(4) There is a tantalizing local minimum in the L2 norm at 0.995 year.

We also made a least-squares regression to the data taken over a period of more than ten years by Karagioz and Izmailov [6], as discussed in the Data Sources section of this paper.

The situation is disturbing—clearly either some strange influence is affecting most *G* measurements or, probably more likely, the measurements have unrecognized large systematic errors. The need for new measurements is clear.

Scientific exchange between groups measuring *G* is necessary. The new working group on big *G* under the auspices of the International Union of Pure and Applied Physics was formed to assist experimenters who are interested in these challenging measurements and wish to discuss and understand each other's experiments.

ACKNOWLEDGMENTS

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