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Some background on the measurement of the Newtonian gravitational constant, *G*

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Abstract. This work provides some historical aspects of the determination of the Newtonian gravitational constant, G, of searches for variations in it and studies of related anomalous effects. The discussion includes a synopsis of present and planned experiments, and of the status of attempts to theoretically estimate the value of G.

Keywords: torsion balance, torsion pendulum, mechanical force detection, gravitational constant

1. Introduction

1998 marked the bicentenary of the publication of results obtained by Henry Cavendish in his attempt to use a torsion balance to determine the mean density of the Earth. The occasion provided a useful and very appropriate opportunity to reflect on the many different techniques that have been employed since that time to measure the more physically interesting quantity G, the Newtonian gravitational constant. Although such a fundamental constant was not associated with the properties of the gravitational force at the time, the work of Cavendish and his contemporaries is now universally interpreted in terms of it, and great efforts are presently underway to establish a precise value for G. Several careful histories of the measurement of G have been written since the time of Cavendish, and two in particular that date to the end of the 19th century are of special interest to those seeking details of the work done in that era (Poynting 1894, Mackenzie 1900). A detailed bibliography of the experimental literature on G published through the mid-1980s is also available (Gillies 1987), as are modern reviews of the whole field (Cook 1988, Gillies 1997).

Given the resurgent interest in G that has been produced by the ‘Fifth Force’ conjecture, the recent disparate experimental values of G, and the new work on the theoretical foundations for it, a brief résumé of some of the historical aspects of the field may be useful. To that end, what follows is a synopsis of various scientific and historical details relevant to the overall role that G plays in the structure of modern physics.

2. Newtonian gravity and the gravitational constant

It is perhaps difficult for us in modern times to estimate the impact that Isaac Newton had on his contemporaries and those who came after him, in the mid-1700s. Einstein, of course, had (and still has) a great influence on not only scientists but also the general public, yet Newton and his genius caused no less a personage than the Emperor Frederick the Great to have himself memorialized in epitaph as a ‘*Disciple of Newton*’ (figure 1). His law of universal attraction, in particular, was a fascination to natural philosophers in the 18th century and it, plus his equally fundamental laws of mechanics, formed the basis for the heroic quests to determine the mean density of the Earth via the measurement of plumb line deflections at mountains. The experiment of Cavendish marked a transformation in the nature of those studies, from one which was largely geophysical in origin to one in which instrumentation suited to benchtop use in the laboratory could be applied. The original paper by Cavendish (1798) does a masterful job of describing the torsion balance he used for this purpose, and it was immediately translated into German (Cavendish 1799) and, somewhat later, into French (Cavendish 1815). It is interesting to note that the French version, delayed and clearly influenced by the French Revolution, contains no mention of the clerical or honorific titles of Michell, Wollaston or Cavendish throughout its text. It also may be one of the first papers in which the English units used by Cavendish were deemed inappropriate and were thus followed by metric units, in parentheses!

Several repetitions of the Cavendish experiment, using more or less the same design of the torsion beam apparatus, were carried out over the next 75 years, with the focus of the experiment in each case still being the determination of the mean density of the Earth. It was not until the work of Cornu and Baille (1873) that the quantity G, then labelled f, first appeared in the literature on experimental gravitational physics and was used in the sense that we understand it today, i.e. as the scale factor in the gravitational

Figure 1. The funerary effigy of the tomb of Emperor Frederick the Great, in the Campo Santo Monumentale of the Piazza di Miracoli in Pisa, Italy. The second line from the top declares him to be a '*Disciple of Newton*'.

inverse square law. (The gravitational constant may have appeared prior to this in the literature of astronomy and celestial mechanics.) From that point on, throughout the late 19th and early 20th centuries, many workers attempted to introduce innovations into torsion balance determinations of G. Some of the more prominent of these attempts are listed in table 1. In parallel with this were the many attempts to bring altogether different measurement technologies to bear on the problem. These include a number of the approaches mentioned in table 2. Perhaps the most oft-quoted value of G obtained in that era was that of Boys (1895), which was not eclipsed until the work of Heyl and Chrzanowski (1942) at the US National Bureau of Standards almost 50 years later. The latter value formed the basis for the internationally accepted value adopted in 1973, $G = (6.6720 \pm 0.0041) \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$—a number which held until 1986. At that time, the presently accepted value of G was adopted by the CODATA Panel, on the basis of another epochal experiment also carried out at the US National Bureau of Standards (Luther and Towler 1982). This value is $G = (6.672\,59 \pm 0.000\,85) \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$. Many modern re-determinations of G have been carried out since then, and figure 2, taken from Gillies (1997), shows a comprehensive display of the results obtained over the

Table 1. Innovations in torsion balance technology for the measurement of G that appeared in the late 19th and early 20th Centuries. For references, see the reviews by Gillies (1987, 1997).

Author	Year	Innovation
Boys	1894	Introduction of the quartz fibre suspension
Boys	1895	First quantitative use of scaling arguments
Eötvös	1896	Introduction of the time-of-swing method
Braun	1897	First use of an evacuated torsion pendulum
Burgess	1902	Mercury bearing suspension for larger test masses
Cremieu	1909	Experiments with a submerged torsion pendulum
Kunz	1927	Proposed use of a resonant torsion pendulum

Table 2. Early attempts to introduce non-torsion balance-based instrumentation into the measurement of G. For references, see the review by Gillies (1987).

Author	Year	Innovation
Poynting	1879	Beam balance
von Sterneck	1882	Simple pendulum
Wilsing	1885	Vertical pendulum
Joly	1890	Resonant simple pendulum
Berget	1893	Gravimeter

past 30 years or so (references to all the publications are presented in the review by Gillies (1997)). A tabulation of the latest values of G as reported at the Cavendish Bicentenary Conference is presented elsewhere in these Proceedings. The CODATA Panel is now in the process of establishing a new value for G, a task which is made very difficult by the significantly different result obtained by workers at the Physikalisch-Technische-Bundesanstalt (PTB) in Germany (Michaelis *et al* 1995/96), see figure 2. The experiments of Michaelis *et al* have been subjected to great scrutiny, but as yet no compromising factor or effect has been found to explain the great discrepancy between the PTB result and virtually all other recent values of G. Hence, it is likely that the new CODATA value of G will have to reflect the presence of the PTB result, and this may work to increase the uncertainty in G over that accepted for the 1986 value. Fortunately, many of the re-determinations of G carried out during the last five years have been done with a variety of new instrumentation systems, including beam balances used in both laboratory and geophysical settings, simple pendulum systems used in both dc and ac configurations, and falling corner-cube gravimeters, as well as with many modern-technology torsion pendula. This opens new doors in terms of being able to compare values of G obtained by completely independent methods, and thus lessens the chance that some sort of hidden systematic effect common to all instruments of the same type (as came to light recently regarding the anelasticity of torsion fibres undergoing twist) might be masking the true value of G.

3. Searches for variations in *G*

At least as much experimental attention, if not more, has been paid to the question of the possible variability of G. Among the investigations that have been carried out are searches for a change in G with time, intermass spacing, electrical charge

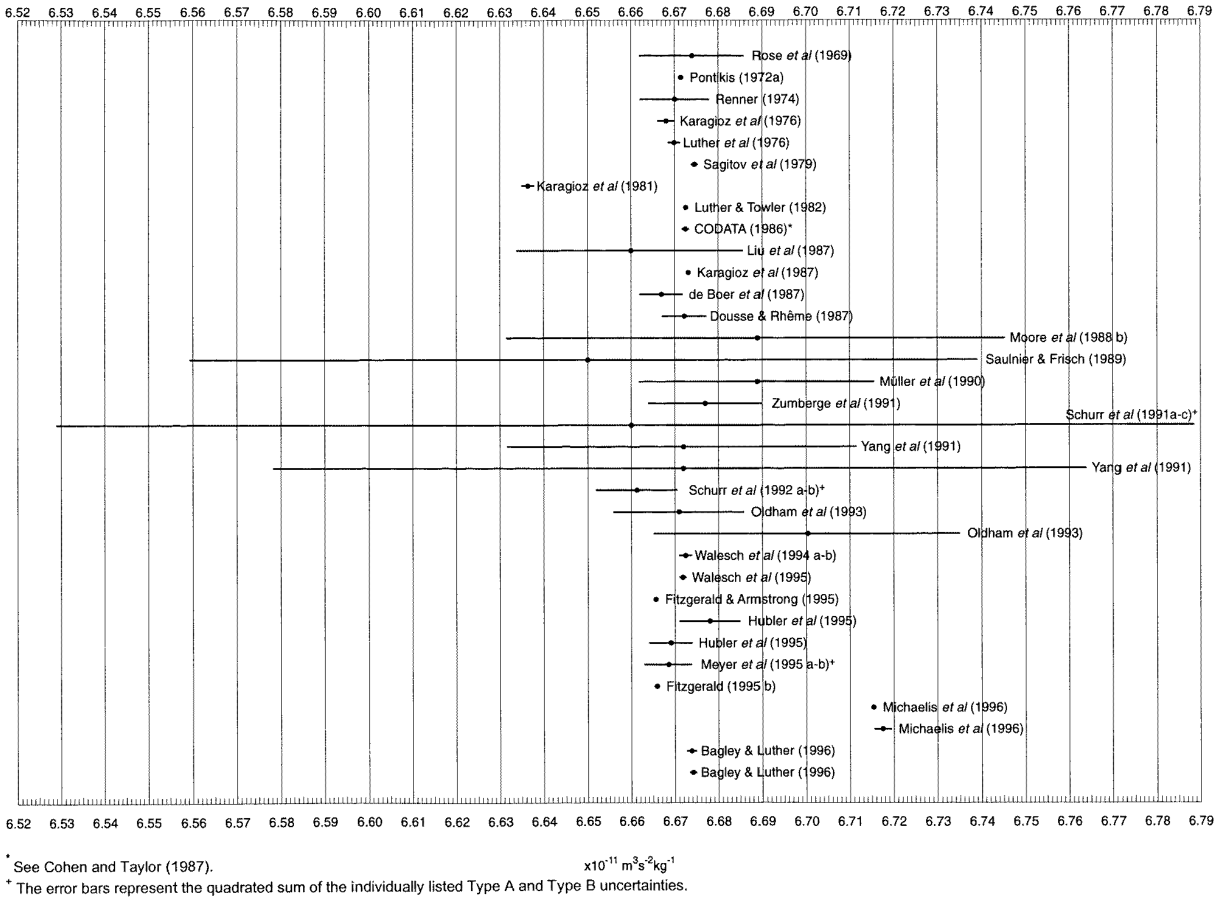

Figure 2. The various measurements of G reported since 1969. The results obtained at the PTB in Germany are those of Michaelis *et al* (1996). The results presented at the time of this conference (November 1998) are listed elsewhere in these Proceedings. (After figure 1 of Gillies (1997), p 166.)

and magnetic field of the test masses, temperature and, of course, chemical composition. With the exception of the issue of a non-zero $(\mathrm{d}G/\,\mathrm{d}t)/G$, i.e. a '$G$-dot' effect, the bulk of these studies are now interpreted as tests of the Weak Equivalence Principle of General Relativity.

For many years the physical motivation underlying such studies was simply the wish to confirm the exactness of all aspects of the inverse square law of gravity at ever higher levels of precision. Therefore, experiments searching for a scale dependence in the gravitational force were interpreted as being tests that put limits on the parameter δ in the expression $F = GmM/r^{(2+\delta)}$ or on the distance-dependent value of the gravitational constant, $G(r)$. Similarly, searches for a temperature dependence in the gravitational force were carried out by heating one of the interacting masses and then setting limits on some violation parameter, for instance an α such as that which appeared in models of the type $G(T) = G_0(1 + \alpha T)$. $G(T)$ was then meant to replace the laboratory value of G (the 'G_0') that appeared in Newton's inverse square law. Shaw (1916) was interested in this modification to Newtonian gravity and set out to test the hypothesis experimentally. Interest in temperature-dependent gravity remains to this day, with predictions for such an effect now founded on the far more orthodox basis of a relativistic increase in mass with increase in thermal energy of a test body (Assis and Clemente 1993).

Others simply sought answers to questions about the qualitative aspects of gravity, either with or without some foundation based on a theoretical prediction. Heyl (1924) weighed crystals of different materials in different orientations within the gravitational field of the Earth to see if there was some property of the crystal that was gravitationally analogous to optical birefringence. He put a limit of $\leqslant 10^{-9}$ on the size of such an effect. Zeeman and, independently, Sagnac sought evidence for a coupling between gravitation and radioactivity in matter (i.e. a 'gravito-nuclear coupling'), imposing a limit of $\leqslant 5 \times 10^{-8}$. Many others tried to determine if the gravitational force could be shielded by placing screening masses of different densities between the interacting bodies. Majorana in particular developed a theoretical modification to Newton's law that called for the presence of an 'extinction factor', λ, that he subsequently measured to be non-zero. Several others since then have contradicted that result and put ever-tighter limits on the size of λ, typically by carrying out careful gravimetry-based searches for variations in local gravitational acceleration during a total solar eclipse (i.e. the screening effect caused by the passage of the Moon between the Earth and the Sun). Complete listings of references to the literature of this field have been assembled by Gillies (1987, 1997).

In contrast to many of these studies, searches for a non-null G-dot and tests of the universality of free fall are motivated by existing theories, and a verifiable non-zero result in either type of experiment would have a profound impact not only on our understanding of the physics of the gravitational force, but also on several issues fundamental to the unification of all the forces. Because of this, the last 20 years has seen a great deal of activity on both fronts, with the former driven in part by questions about the incompleteness of General Relativity theory and the ramifications of string theory, and the latter spurred initially by the 'Fifth Force' conjecture. In fact, there have been roughly 40 different published measurements or observational inferences of a 'G-dot' effect, with the highest precision ones typically being those derived from lunar laser ranging studies, radar ranging to the inner planets, or measurements of the spin-down of binary pulsars. (See the review by Gillies (1997) for a detailed tabulation of the 40 most recent determinations of $(\mathrm{d}G/\mathrm{d}t)/G$). Recent high-precision tests of the universality of free fall are epitomized by those carried out at the University of Washington using the well known 'Eöt–Wash' apparatus, although similar studies at several other institutions have also contributed very significant findings. Adelberger *et al* (1991) published a very useful review on these topics at a time when interest in this line of work was growing rapidly. The work of Price and colleagues (see Long *et al* 1998) attempts to write a new chapter in the study of distance-dependent gravity by making measurements of the gravitational force at test body spacings below 1 cm using high-Q parallel-plate oscillators operating at cryogenic temperatures. Others have also proposed experiments to measure the size of the force at those distances, but experimental results are available only down to interaction distances of about 7 mm. Gillies (1997) gives a tabulation of the pertinent proposals and experiments.

4. Is *G* isolated from the rest of physics?

One of the most significant questions associated with the physics of gravity is that of a theoretical prediction for the value of G. In the 1980s much work was aimed at establishing the properties of a calculable 'induced' gravitational constant that would arise from theoretical considerations based on assessments of zero-point fluctuations of the vacuum, chaotic inflation scenarios, Zitterbewegung of particle motions and fluctuations in quantum gravity. In several of these studies, the gravitational constant and the Planck mass were used almost interchangeably as the quantity of interest. More recently, however, Damour (1999) and others have developed very fundamental arguments that show signs of arriving at a *bona fide* prediction for G that would finally couple this scale factor of Newtonian gravity to the rest of modern physics. If successful, this would provide experimentalists with their first 'target' for G in the history of the measurement. Interestingly, given the disagreement among measurements of G reported to date, even an estimate of G to just three significant figures would serve as a very useful backdrop for an assessment of the results. This situation stands in stark contrast to the theoretical predictions and measurements of the fine structure constant and the Planck constant, h, which was recently measured with an uncertainty of $<10^{-7}$ (Williams *et al* 1998).

5. Experiments in progress and under planning

Most of the groups presenting results at the Cavendish Bicentennial Conference are also going forward with refined versions of their experiments with the aim of decreasing the measurement uncertainties. There are also new experiments, either planned or underway, that introduce still other techniques for re-determining G, some based on the torsion pendulum, and others not. The University of Washington and University of California–Irvine experiments, for instance, take good advantage of experimental designs based on multi-pole analysis of the gravitational coupling between the interacting masses in torsion pendulums to minimize (if not eliminate altogether) several of the principle metrological concerns of such measurements. The Fabry–Pérot optical interferometer-sensed simple pendulum suspension being built by Ni and colleagues in Taiwan has many similarities to the microwave cavity-sensed instrument of the University of Wuppertal, and thus offers not only an additional independent approach to the measurement but also a means for checking on sources of systematic effects by comparing results from one such device against those of the other. The beam balance instrument being used at the University of Zürich, the bifilar suspension proposed by Luther, and the torsion-strip instrument now in use at the BIPM also implement important changes in experimental approaches to the problem. Perhaps the most different of all the present techniques is that of Faller and colleagues at JILA/University of Colorado, who are taking advantage of the very high sensitivity of their well known falling corner-cube gravimeter to measure gravitational acceleration with and without a cylindrical perturbing mass in place around the drop chamber, and thus derive a value of G from the difference. All of these and other approaches not mentioned here are either discussed elsewhere in these Proceedings or are referenced in the review of Gillies (1997). Unfortunately, there are no present plans to repeat the PTB experiment, either at the PTB itself or by replication elsewhere.

In addition to all of the terrestrial re-determinations of G that are either planned or actually underway, there have also been several space-based measurements proposed for satellites in Earth orbit. Sanders and Gillies (1996) have reviewed these. Among them, Project SEE (Sanders and Deeds 1992) is now under active study and proposes to not only make a re-determination of G, but to test the Weak Equivalence Principle, search for a non-zero G-dot, and carry out other relativistic gravity experiments as well. The rather significant expense associated with the design, construction and launch of such an experiment (even one as elegant in principle as SEE) of course dictates a substantial amount of preliminary study and the SEE Program (Headquartered at the University of Tennessee and the Oak Ridge National Laboratory) is now moving forward with an experimental agenda that begins to address this need.

If the next 20 years are as fruitful in terms of new results and the introduction of new measurement systems as the last two decades have been, much of the uncertainty in the value of G (at least at the level of 1×10^{-4}) may be resolved.

Acknowledgments

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