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FUNDAMENTAL PROBLEMS OF METROLOGY

MINI-SEE PROJECT TO MEASURE THE PARAMETERS OF GRAVITATIONAL INTERACTION USING THE ALPHA INTERNATIONAL SPACE STATION

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The behavior of two bodies interacting gravitationally onboard a drift-free Earth satellite orbiting at the altitudes of the Alpha international space station is investigated. A numerical simulation of the trajectories of the relative motion of a particle *is performed, as a result of which preliminary estimates of the precision of the experiment are obtained. An estimate of the measurement precision of the parameters of gravitational interaction is given.*

With the launching and deployment of the Alpha International Space Station, planned for 1998–2003 with the participation of Russia, the United States, the European Community, Canada, and Japan, the range of possibilities for conducting fundamental physical experiments has grown. In particular, in the course of these studies it may be possible to solve the most critical problems of modern physics and astronomy, including problems having to do with the theory of gravitation and cosmology.

The most important of these experiments at the present time are those involved in attempts to verify various elements of the relativistic theory of gravitation, including the principle of equivalence (STEP experiment [1]); second-order effects in solar-probe type experiments [2] or the Astrod experiment [3]; and effects of rotation and torsion in Gravity Probe B type experiments [4]. Other crucial experiments have to do with the measurement of the absolute value of the Newtonian gravitational constant [5], and to verify Newton's law of gravitation, deviations from which are predicted by most modern models of the unification of the fundamental interactions and, moreover, are described by the dependence of the effective gravitational constant G on time [5] and the distance between bodies [6].

One of the more promising projects designed to measure the value of G is the SEE (Satellite Energy Exchange) project [7]. The underlying idea of the SEE project involves use of the neighborhood of the turning point of the horseshoe-shaped orbits of the restricted three-body problem [10]. It is assumed that two free bodies of equal mass are traveling within a drift-free Earth satellite along similar, nearly circular orbits, with the lighter of the two bodies, henceforth referred to as "*particle*" ($m \approx 100$ g), which travels along the lower orbit, overtaking the heavier body, called "*shepherd*" ($m \approx 500$ kg). As a consequence of its interaction with *shepherd*, *particle* reaches a higher orbit and begins to lag behind. In this experiment, the parameters of gravitational interaction are determined by adjusting the theoretical and physically observed trajectories.

The trajectories of these bodies, along with other aspects of this project, have been previously studied for the case of orbits at altitudes in the range 1500-3000 km over the Earth's surface [8]. The SEE method has also been compared with other projects designed to determine the value of G [9].

In view of the proposed launching of the Alpha international space station, it would be desirable to study whether an SEE type experiment could be performed on orbits that are typical for this station, i.e., orbits at altitudes on the order of 400-500 km over the Earth, as well as with capsules shorter than the initial version of the capsule (Mini-SEE experiment). It will be necessary to use "free-floating platforms" or Progress or Shuttle-type transport spacecraft, i.e., spacecraft that can move

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TABLE 1

Perturbing factor	a, cm/sec^2	Δl, cm
Quadrupole tidal forces	$2 \cdot 10^{-7}$	1
Higher harmonies of the geopotential	$1{,}5 \cdot 10^{-9}$	$0{,}7 \cdot 10^{-2}$
Sun	$0{,}7 \cdot 10^{-10}$ $(1{,}5 \cdot 10^{-11})$	$5 \cdot 10^{-4}$ (10^{-4})
Moon	$3 \cdot 10^{-10}$ $(6 \cdot 10^{-11})$	$1{,}5 \cdot 10^{-3}$ $(3 \cdot 10^{-4})$
Jupiter	10^{-15} $(2 \cdot 10^{-16})$	10^{-8} (10^{-9})
Indeterminacy of satellite orbit ($\delta R \approx$ 1 cm)	$3 \cdot 10^{-13}$	$1{,}5 \cdot 10^{-6}$
Relativistic tidal effects	10^{-12} $(2 \cdot 10^{-13})$	$0{,}5 \cdot 10^{-5}$ (10^{-4})
Breakdown of principle of equivalence ($\zeta \approx 10^{-13}$)	10^{-10}	$0{,}5 \cdot 10^{-3}$

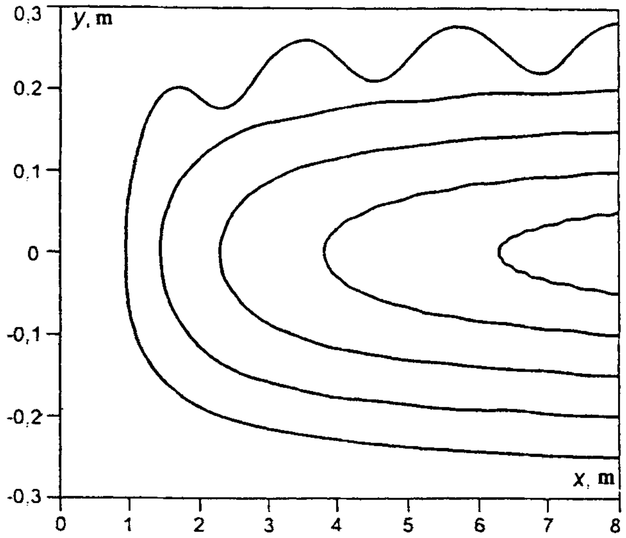

Fig. 1. Trajectories of *particle* relative to *shepherd* with H_{orb} = 500 km and x_0 = 8 m, assuming standard initial conditions for the case of *shepherd* traveling along a circular orbit.

away from the station and, thus, be protected against interference from the station while maintaining communications with the station potential.

Preliminary Estimates of the Influence of Different Factors on the Relative Movement of Bodies. It is assumed that the motion of *particle* relative to *shepherd* remains unperturbed while it is influenced solely by two factors, the factor of the spherically symmetric geopotential and the factor of the mutual Newtonian attraction of the two bodies, let us estimate the influence of other factors on this motion. Numerical estimates for H_{orb} = 500 km are presented in Table 1. Tidal effects (differential acceleration Δa) that arise due to the difference between the gravitational field of the external sources at the points (0, 0) (the position of the *shepherd*) and (x, y) (the position of *particle*) were estimated. The only exception is presented by the possible breakdown of the principle of equivalence in which differential acceleration is due to the dependence of free-fall acceleration of the bodies g on their chemical composition. Next, the least favorable case is considered, that is, the case in which acceleration Δa over the entire measurement period t is in the same direction in the plane xy, a situation that leads to

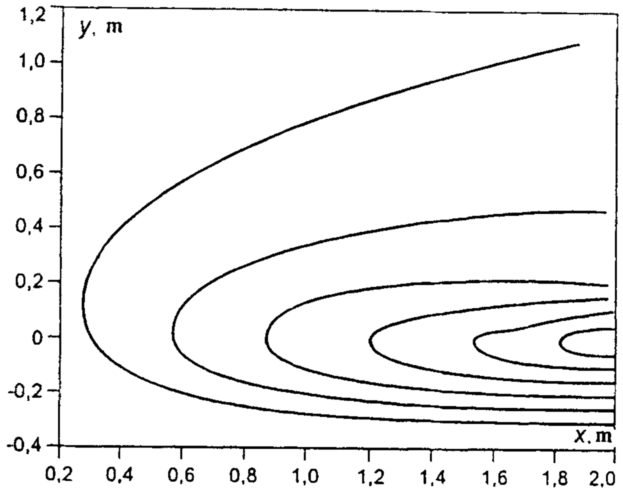

Fig. 2. Family of trajectories with $x_0 = 2$ m for *shepherd* traveling in a circular orbit under standard initial conditions.

maximal shift $\Delta l = \Delta a t^2/2$. The magnitude of this shift is estimated for time $t = 1$ h, which is approximately one-half the orbital period T.

It should be noted that the tidal influence of the external bodies (e.g., Sun, Moon, Jupiter, etc.) have period T, hence the acceleration Δa induced by these bodies lasts for approximately $T/2$ in one direction and the same length of time in the opposite direction. Thus, the estimated shifts Δl due to the influence of these bodies presented in Table 1 are valid for all $t > T/2$, whereas shifts due to other types of perturbations are proportional to t^2. In estimating the acceleration Δa due to the geopotential components, it is assumed that the "tidal" acceleration is due to a difference in altitudes of about 30 cm for all variants of the experiment. In the remaining cases, tidal acceleration is associated with the distance $s = (x^2 + y^2)^{1/2}$, which is a function of the particular variant of the SEE project; on average, $s = 10$ m and 2 m (in Table 1, the values within the parentheses correspond to $s = 2$ m).

It follows from Table 1 that, by comparison with the previously investigated case of higher orbits ($H_{\rm orb} = 1500$-3000 km) [8], the basic variations of the influence of the perturbing factors reduce to the following:

— the influence of higher harmonics of the geopotential proportional to $(R/R_{\rm E})^{n+1}$ increases rapidly; here R is the radius of the orbit, $R_{\rm E}$, the radius of the Earth, and n the multipolarity factor. Consequently, by comparison with the preceding case [8], significantly greater values of n must be taken into account;

— the absolute value of free-fall acceleration increases, and, as a consequence, the sensitivity of the trajectory to breakdown in the principle of equivalence grows;

— the sensitivity of the trajectory to a shift in the satellite's orbit as $H_{\rm orb}$ decreases grows, since the tidal effects of the geopotential depend on the radius of the orbit as $\sim R^{-3}$.

Consequently, as before, the indeterminacy δR of the radius of the satellite orbit is the basic factor limiting the length of the experiment assuming a prescribed error in the trajectory measurements δx, moreover, this limitation is even more rigid than in the case $H_{\rm orb} \geq 1500$ km.

Simulation of *Particle* Trajectories

Motion Equation and Initial Conditions. Suppose that R and φ are the polar coordinates of *shepherd* in the orbital plane; x and y, Cartesian coordinates bound with *shepherd* in the orbital plane, with the y-axis perpendicular to the tangent to the orbit of *shepherd* in the direction away from the Earth and the x-axis directed along the tangent to the orbit and pointing in the direction opposite to the direction of motion. Let us introduce the following notation: M_E, $M = 500$ kg, and $m = 100$ g are the masses of the Earth, *shepherd*, and *particle*, respectively; $r = [(R + y)^2 + x^2]^{1/2}$ is the distance of *particle* from the center of the Earth; $s = (x^2 + y^2)^{1/2}$ is the distance between *particle* and *shepherd*; η is the Eötvös coefficient, which characterizes the potential breakdown of the principle of equivalence; and α and λ are the intensity parameter and length of the hypothetical fifth interaction. The equations of relative motion of *particle* have the following form [8]:

$$\frac{d^2x}{dt^2} = 2\dot{y}\dot{\varphi} + x\left\{\dot{\varphi}^2 - \frac{GM_E}{r^3}(1+\eta)\right\} - \frac{2\dot{R}\dot{\varphi}y}{R} - \frac{G(M+m)}{s^3}x -$$
$$- \alpha x\frac{G(M+m)}{s^2}\left(\frac{1}{s}+\frac{1}{\lambda}\right)e^{-s/\lambda} - \frac{Gmxy}{s^3R} - \tag{1}$$
$$- \alpha xy\frac{Gm}{Rs^2}\left(\frac{1}{s}+\frac{1}{\lambda}\right)e^{-s/\lambda}$$

$$\frac{d^2y}{dt^2} = -2\dot{x}\dot{\varphi} + (R+y)\left\{\dot{\varphi}^2 - \frac{GM_E}{r^3}(1+\eta)\right\} +$$
$$+ \frac{2\dot{R}\dot{\varphi}x}{R} - \frac{G(M+m)}{s^3}y - \alpha y\frac{G(M+m)}{s^2}\left(\frac{1}{s}+\frac{1}{\lambda}\right)e^{-s/\lambda} + \tag{2}$$
$$+ \frac{Gmx^2}{s^3R} + \alpha x^2\frac{Gm}{Rs^2}\left(\frac{1}{s}+\frac{1}{\lambda}\right)e^{-s/\lambda}.$$

We will not be considering the motion equations of *shepherd*, since the inverse influence of *particle* on *shepherd*'s orbit is less than the computation error for the orbit.

We use the values of the velocity components of *particle* $\dot{x}(0)$ and $\dot{y}(0)$ as the standard initial conditions for Eqs. (1) and (2). These components correspond to unperturbed motion of *particle* (i.e., in the absence of $M-m$ interaction) along an orbit that differs from *shepherd*'s orbit only in terms of radius (in the case of a circular orbit) or in terms of the dimension of the major semi-axis (in the case of an elliptical orbit). Under the assumption that *particle*'s movement is initiated at the time *shepherd* crosses the orbital perigee, these conditions assume the following form:

$$x(0) = x_0;\ y(0) = y_0;\ \dot{x}(0) = \frac{\omega e' y_0}{2(1-e)^2};\ \dot{y}(0) = \frac{\omega e x_0}{e'(1-e)}, \tag{3}$$

where $\omega^2 = GM_ER_0^{-3}$, with R_0 the radius of *shepherd*'s orbit (at the perigee) and e the eccentricity of the orbit with $e' = (1 - e^2)^{1/2}$.

To better visualize these relations, they were written in a linear approximation with respect to the variables x and y. Approximations of higher orders were also used in the simulation.

A program written previously [8] for analysis of the Sanders and Deed project [7] was used to solve the system of equations (1), (2).

General Description of Trajectories. As in the case of "high" orbits with H_{orb} = 1500-3000 km and initial distance between *shepherd* and *particle* x_0 = 18 m, with H_{orb} = 500 km and x_0 = 8 m the trajectories of *particle* will be approximately *U*-shaped in the case where *shepherd* travels along circular orbits (Fig. 1), and loop-shaped in the case where it travels along elliptical orbits. There are also noticeable differences.

First, because of the reduction in the height of the orbit and the initial distance between *shepherd* and *particle* for the case of *shepherd* traveling along circular orbits under standard initial conditions, the sinusoidal component becomes marked on all trajectories, sharply increasing once $|y_0| > 20$ cm. As has been previously established [8], the nature of this effect is related to the specifics of the standard initial conditions, which correspond to "inclusion" of gravitational interaction between *shepherd* and *particle* at the initial point of the trajectory. Consequently, the form of the physical trajectory in a neighborhood of the turning point differs from an ideal horseshoe-shaped trajectory.

In the case where *shepherd* travels along elliptical orbits, the trajectories of *particle* assume the shape of loops with a *U*-shaped envelope the cross-section of which expands as the turning point is neared. Unlike the previously considered case

TABLE 2

y_0, cm	$\delta G/G$	δx_{max}, μm
−20 −20	10^{-6} 10^{-5}	1,25 12,5
−15 −15	10^{-6} 10^{-5}	2,2 21
−10 −10	10^{-6} 10^{-5}	4,5 45
−5 −5	10^{-6} 10^{-5}	4,5 45

TABLE 3

y_0, cm	α	Δx_{max}, μm			
		λ = 2 m	λ = 4 m	λ = 6 m	λ = 8 m
−20 −20	10^{-6} 10^{-5}	1,4 14	1,1 11	1,2 12	1,2 12
−15 −15	10^{-6} 10^{-5}	1,9 19	2,25 22,5	2,25 22,5	2,25 22,5
−10 −10	10^{-6} 10^{-5}	1,8 18	3,4 34	3,9 39	4,1 41
−5 −5	10^{-6} 10^{-5}	0,7 7	2,3 23	3,2 32	3,6 36

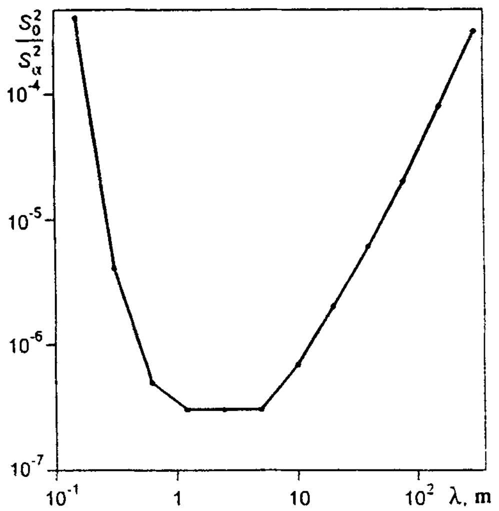

Fig. 3. Results of a calculation of the sensitivity of the SEE method based on Fisher's criterion.

[8], however, in the case H_{orb} = 1500 km and x_0 = 18 m, with $y_0 = -10$ cm and $e < 0.05$, the transverse dimension of the trajectories does not exceed the dimension of the capsule, making it possible to use orbits whose eccentricity is two or three times that of the orbits considered previously.

If the initial velocity $\dot{x}_0$ is increased beyond the standard value with *shepherd* traveling along circular orbits, the trajectories assume the form of loops with a *U*-shaped envelope that expands as the turning point is neared. In the case of elliptical orbits, the number of loops increases and the cross-sectional dimension of the trajectory decreases, making it possible to use orbits with comparatively high eccentricities.

If the initial distance between *shepherd* and *particle* is reduced to 2 m, the trajectories become noticeably asymmetric (Fig. 2). An increase in the horizontal component of the initial velocity from its "standard" values leads to the formation of a loop along the lower portion of the trajectory. With a further increase in the initial velocity, the loop at first reaches the upper portion of the trajectory and then vanishes, the trajectory meanwhile becoming symmetric. In the case where *shepherd* travels along elliptical orbits, the dependence of the trajectory on the eccentricity of the orbit is, to some degree, analogous to the dependence of the trajectory on the horizontal component of the initial velocity for the case of circular orbits. That is, as the eccentricity of the orbit along the lower portion of the trajectory increases, a loop appears, gradually reaching the upper portion and then vanishing. Unlike the case of circular orbits, here the trajectory of *particle* becomes asymmetric.

Dependence of Trajectories on the Gravitational Constant G and the Parameters of Newtonian Interaction (fifth force) α and λ. To obtain preliminary estimates of the observed effects and the sensitivity of the experiment, the dependence of the trajectories on the gravitational constant G and parameters of the fifth force α and λ were investigated. We proceeded under the assumption that the Earth's potential is known with a sufficiently high precision, hence the terms of the motion equations (1) and (2), which characterize the Earth's attraction of *shepherd* and *particle*, are assumed to be independent of variations in G, α, and λ.

The dependence of the trajectories of *particle* on the gravitational constant G and the fifth force parameters α and λ are characterized by the following shifts:

$$\Delta x = x(t; G, \alpha, \lambda) - x(t; G_0); \tag{4}$$

$$\Delta y = y(t, G, \alpha, \lambda) - y(t; G_0), \tag{5}$$

where $G = G_0 + \delta G$; $\alpha = \alpha_0 + \delta\alpha$; $\lambda = \lambda_0 + \delta\lambda$; G_0 is the currently assumed value of the gravitational constant; $\alpha_0 = 0$, $\lambda_0 = \mathrm{const} \neq 0$.

In view of the fact that the magnitude of the shifts Δy is only 1% that of the shifts Δx, our main concern will be with the dependence $x(G, \alpha, \lambda)$. It was established that the dependence of Δx on δG and $\delta\alpha$ (with fixed λ), assuming that $\delta G/G \approx 10^{-6}$-$10^{-5}$ and $\alpha \approx 10^{-6}$-10^{-5} is linear, as must also be the case for small perturbations. The maximal variations of the U-shaped trajectories due to variations in the gravitational constant G for $H_{orb} = 500$ km under standard conditions are presented in Table 2. From this table it is clear that the sensitivity of the trajectories to variations of G grow as the axis of the capsule is approached (with decreasing y_0).

It should be noted that not only the maximal shifts Δx_{max}, but the qualitative behavior of the variations $\Delta x(t; \delta G)$ as well depend strongly on the initial point of the trajectory, i.e., the coordinate y_0. Thus, with $|y_0| > 15$ cm, Δx grows along the lower part of the trajectory from 0 to Δx_{max} at the turning point, and then decreases to some terminal value Δx_e. Once $|y_0| \approx 15$ cm, the deviation Δx grows along the lower part of the trajectory until the turning point is reached and then is practically invariant along the upper portion, remaining roughly at the level Δx_{max}. Finally, when $|y_0| < 15$ cm, the deviation Δx grows along the entire trajectory, reaching a maximal value Δx_{max} at the terminal point.

Once the horizontal component of the initial velocity has increased roughly 30% from the standard value, the trajectory assumes the shape of a loop. For such trajectories, $\Delta x(\delta G)$ is monotone increasing along the entire trajectory, reaching a maximum Δx_{max} at its terminal point. Unlike the case of U-shaped trajectories, with fixed δG the value of Δx_{max} increases with increasing $|y_0|$. Thus, with $y_0 = -20$ cm, Δx_{max} varies from 5 μm with $\delta G/G = 10^{-6}$ to 50 μm with $\delta G/G = 10^{-5}$. Once $y_0 = -15$ cm, these values will be approximately one-third less (3.4 and 35 μm, respectively).

In the case where the *shepherd* is traveling along elliptical orbits, the behavior of the variations $\Delta x(\delta G)$ will be analogous to the case of loop-shaped trajectories for circular orbits. That is, the deviation $\Delta x(\delta G)$ is monotone increasing along the trajectory, reaching a maximum at its terminal point. Moreover, under the standard initial conditions, the maximal deviation Δx_{max} depends on the three parameters e, δG, and y_0. In particular, with $y_0 = -10$ cm, $e = 0.025$, and $10^{-6} \leq \delta G/G \leq 10^{-5}$, the maximal deviation of the trajectories Δx_{max} lies in the range 4-40 μm.

The maximal variations of the U-shaped trajectories of *particle* induced by variations of the fifth force parameters are presented in Table 3 for the case $H_{orb} = 500$ km, assuming the standard initial conditions.

From Table 3 and Fig. 1 it follows that if the length parameter $\lambda > x_s$, where x_s is the distance from the center of *shepherd* (coordinate origin) to the turning point, the maximal shifts in the trajectories induced by variations in the parameter α in the range $10^{-6}-10^{-5}$ are practically identical to the shifts in the trajectories induced by variations in the gravitational constant $\delta G/G \approx 10^{-6}$-$10^{-5}$. Analysis of the curves expressing the shifts $\Delta x(t; \alpha) = x(t; \alpha) - x(t)$ also shows that, depending on the value of the parameter λ, not only does the maximal shift Δx_{max} vary, but so does the qualitative behavior of the functions $\Delta x(t)$. Thus, with $\lambda = 2$ m, a monotone increase in the shifts along the trajectory is observed for all $|y_0| \leq 20$ cm. When $\lambda = 4$ m and $y_0 = -20$ cm, the maximal shift is attained at the turning point, with the magnitude of the shift remaining invariant until the end of the trajectory. In contrast, once $|y_0| < 20$ cm, the shift becomes monotone increasing along the entire trajectory. Finally, with $\lambda \geq 6$ m and $y_0 = -20$ cm, the maximal shift occurs at the turning point, after which Δx is decreasing until the terminal value is attained, though when $|y_0| < 15$ cm, it is monotone increasing along the entire trajectory.

If the initial distance between *shepherd* and *particle* is reduced down to 2 m with the variations in the gravitational constant $\delta G/G \approx 10^{-6}$-$10^{-5}$ and $|y_0| \leq 20$ cm, the maximal shift in the trajectory virtually becomes a function of y_0, the initial velocity v_{x0}, and the eccentricity of the orbit e, remaining within the range $\Delta x_{max} \approx (1\text{-}1.4)\cdot 10^{-6}$ m.

Estimated Sensitivity of the SEE Method

Experiment Set-up. To improve the estimated sensitivity of the SEE method, it is necessary to get a detailed idea of the experimental technique and to then process the results. An example of this estimate is given in the present section.

The experiment may be performed in the following way. At time $t = 0$, *particle*, which is at the point with coordinates x_0, y_0, has a velocity v_{x0}, v_{y0} imparted to it, and begins to travel in the direction towards *shepherd*. The coordinates of *particle* relative to *shepherd* are determined at specified moments of time t_i, and its trajectory $x(t_i)$, $y(t_i)$ is found. Besides this empirical trajectory, two theoretical trajectories are also considered: first, the trajectory $x^{(0)}(t_i)$, $y^{(0)}(t_i)$, which does not take into account the hypothetical fifth force, and, second, the trajectory $x^{(\alpha)}(t_i)$, $y^{(\alpha)}(t_i)$ which incorporates this force, both of which are "adjusted" to the empirical trajectory. The process of adjustment is performed by minimization of the functional

$$S_k^2 = \frac{1}{N} \sum_{i=0}^{N/2} \left[\left(x(t_i) - x^{(k)}(t_i) \right)^2 + \left(y(t_i) - y^{(k)}(t_i) \right)^2 \right] \tag{6}$$

a process that is accomplished by variation of the parameter α. Here $k = 0$ and α relates to the adjusted trajectories with $\alpha = 0$ and $\alpha \neq 0$, respectively; S_k^2 has the sense of a measure that determines the distance between the empirical and adjusted trajectories. S_k^2 may be also considered a variance characterizing the scatter of the empirical values of the coordinates relative to the adjusted trajectory. This assertion is valid whenever the theoretical model of the motion of *particle* fits the physical situation. In this case, S_α^2 has a chi-square distribution with $n_2 = N-1$ degrees of freedom. It may be assumed that S_0^2 also has a chi-square distribution with $n_1 = N$ degrees of freedom. Then the ratio $S_0^2/S_\alpha^2 = F$ will be distributed by Fisher's law with degrees of freedom n_1 and n_2. If it is found in the course of an experiment that the

$$S_0^2 / S_\alpha^2 \geq F_{n1,n2,q}, \tag{7}$$

is valid at a prescribed significance level q, we are led to conclude that a nongravitational fifth force has been discovered. The sign of the equality in (7) determines the minimal detectable force at a prescribed significance level q. We have set $q = 0.05$, which corresponds to 95% reliability.

Computer Simulation of the Experiment is Performed in the Following Way. A "model" trajectory is considered for prescribed values of the parameters α and λ. The procedure of trajectory measurements is simulated by introducing into the model trajectory noise with prescribed variance σ^2 created by a random-noise generator. Next, adjusted trajectories with $k = 0$ and $k = \alpha$ are considered, and S_0^2 and S_α^2 are determined by means of formula (6). Then Eqs. (7) are verified for distinct values of λ. As a result, the sensitivity curve, or dependence of the minimal detectable α on λ (with $q = 0.05$) is found. In the adjusted trajectory with $\alpha = 0$, the gravitational constant G is varied and the minimal value of the functional S_0^2 is found. In this case, the adjusted trajectory with $\alpha = 0$ is the best approximation of the empirical trajectory, and the distribution of the variable S_0^2 approaches a chi-square distribution. Results of a calculation of the sensitivity with respect to the Fisher criterion (7) are shown for this case in Fig. 3. It is clear that the sensitivity curve has a single extremum at $\lambda = 1.25$ m equal to $2.2\cdot 10^{-7}$. In the space experiment, the value of the sensitivity to the detection of nongravitational forces proved to be three or four orders of magnitude greater than the values that have been found by means of measurements in ground laboratories [11, 12].

Conclusion. The results that have been found give us reason to be optimistic concerning the possibility of performing the experiment proposed in [7] for measurement of the gravitational constant G and verification of Newton's law of gravitation at distances of several meters using lower orbits than those that were proposed in the latter study as well as a shorter capsule. The precision of the gravitational constant G and of the fifth force parameters α and λ thus determined depend largely on the parameters of the model, the most sensitive of which are the orbital parameters. Thus, in the case where *shepherd* travels along circular orbits, a 10-cm indeterminacy of the radius of the orbit leads to a shift in the trajectory (with $x_0 = 8$ m and $y_0 = -15$ cm) of approximately 8 μm, which is equivalent to variation of the trajectory with $\delta G/G \approx 4 \cdot 10^{-6}$. Thus, to determine the gravitational constant G with error $\delta G/G \approx 10^{-6}$, the indeterminacy of the orbit radius must not exceed 1 cm.

If the Alpha and Mir stations are brought to lower orbits, this will complicate somewhat the gravitational interaction of the Earth with *particle* and *shepherd*, which, in turn, will produce some increase in the sensitivity of the SEE method. In one possible variant, the gravitational experiments are performed in a separate space vehicle that departs from the main station (to a distance of several dozen kilometers), and then again docks with it. Along the x-axis, the scale of *particle*'s free motion is now only one-tenth as great, and along the y-axis, one-fifth. The precision of the gravitational constant thus determined is one-fifth that achieved with the initial method [8] if the instruments used to measure the trajectory of *particle* relative to *shepherd* exhibit an error of 1 μm (i.e., roughly two wavelengths of visible light). Therefore, to determine the gravitational constant with error $\Delta G/G \approx 10^{-6}$, it would be necessary to measure the trajectory of *particle* with error equal to fractions of a wavelength.

Simulation of the trajectories relative to the motion of *particle* shows that with a capsule 10 m in length, the variations $\delta x_{max}(\delta G, \alpha, \lambda)$ will depend on the initial conditions, as in the case of the SEE method. Similarly, as in the SEE method, it will be possible to distinguish between the influence of variations of the gravitational constant δG and the fifth force parameters. Thus, with $x_0 = 8$ m and under the standard initial conditions for the case of *shepherd* traveling in circular orbits, maximal sensitivity of the trajectory to the value of the gravitational constant G and the fifth force parameters is achieved with $y_0 = -10$ cm. However, with a further decrease in $|y_0|$, the sensitivity of the trajectory to the value of the gravitational constant G remains constant, whereas the sensitivity of the trajectories to the fifth force parameters decreases once $|y_0| < 10$ cm. Once x_0 has fallen down to 2 m, the sensitivity of the trajectories to the value of the gravitational constant G and to the fifth force parameters is practically independent of the type of orbit *shepherd* travels along and the initial conditions.

REFERENCES

1. Satellite Test of the Equivalence Principle. Report on the Phase A Study, ESA, SCI (96), March 5, 1996; Mini STEP. NASA-ESA Preprint (April 1996); Geo STER, CNES Preprint (April 1996).
2. V. N. Melnikov, Gravitational Measurements, Fundamental Metrology, and Constants, V. de Sabbata and V. N. Melnikov (eds.), Kluwer Academic, Dordrecht (1988), p. 283; K. A. Bronnikov and V. N. Mel'nikov, Izmerit. Tekh., No. 4, 10 (1988).
3. W.-T. Ni et al., Intern. Workshop on Gravitation and Cosmology, Taiwan (December, 1995), p. 211.
4. G. M. Keiser et al., GR-14 (abstracts), Florence, Italy (1995), p. C43.
5. V. N. Melnikov, Int. J. Theor. Phys., **33**, No. 7, 1569 (1994).
6. V. de Sabbata et al., Progr. Theor. Phys., **88**, 623 (1992).
7. A. J. Sanders and W. E. Deed, Phys. Rev. D, **46**, 480 (1992).
8. A. D. Alekseev et al., Izmerit. Tekh., No. 8, 6 (1993); No. 9, 3 (1993); No. 10, 6 (1993); No. 1, 3 (1994); Intern. J. Mod. Phys. D, **3**, No. 4, 773 (1994).
9. A. J. Sanders and G. T. Gillels, A Comparative Survey of Proposals for Space-Based Determination of the Gravitational Constant G, University of Tennessee, Knoxville, TN 37996-1200 (1994); K. A. Bronnikov, M. Yu. Konstantinov, and V. N. Mel'nikov, Izmerit. Tekh., No. 5, 3 (1996).
10. G. H. Darwin, Acta Math., **21**, 99 (1897).
11. E. Fishbach and C. Talmadge, The Sixth Marcel Grossman Meeting on General Relativity, Part B, Y. Sato and T. Nakamura (eds.), World Scientific, Singapore (1992).
12. Y. J. Paik and M. V. Moody, Class. Quantum Grav., **46**, 489 (1994).