Varying G and Other Constants

John D. Barrow
Astronomy Centre
University of Sussex
Brighton BN1 9QJ
UK

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Abstract

We review recent progress in the study of varying constants and attempts to explain the observed values of the fundamental physical constants. We describe the variation of G in Newtonian and relativistic scalar-tensor gravity theories. We highlight the behaviour of the isotropic Friedmann solutions and consider some striking features of primordial black hole formation and evaporation if G varies. We discuss attempts to explain the values of the constants and show how we can incorporate the simultaneous variations of several 'constants' exactly by using higher-dimensional unified theories. Finally, we describe some new observational limits on possible space or time variations of the fine structure constant.

1 Introduction

In this overview of some aspects of varying constants we will begin by considering the time variation of the gravitational 'constant' G in Newtonian and relativistic theories of gravity. Although the Newtonian situation is usually ignored, it provides a number of instructive parallels and contrasts with the more complex situation that prevails in scalar-tensor generalisations of general relativity. We will focus upon the behaviour of the cosmological solutions in these theories and provide prescriptions for generating the isotropic Friedmann solutions to any version of a scalar-tensor gravity theory.

Next, we shall highlight the unusual situation that seems to be created if a primordial black hole forms in the very early stages of a universe in which G is changing with time. Then we shall go on to consider some of the speculative ideas that have been put forward to explain the values of the constants of Nature. We shall discuss the problem of the simultaneous variation of several 'constants' and describe how this situation can be modelled using simple scaling invariances of physics or by exploiting the structure of unified higher-dimensional theories of the fundamental interactions. One of the most interesting quantities to appear in these discussions is α, the fine structure constant. In the final section, we describe some new observational limits on any possible space or time variations in the fine structure constant that can be deduced from spectroscopic observation of molecular and atomic hydrogen absorption lines from the gas around radio-loud quasars.

2 Some Background to Varying G

The study of gravitation theories in which Newton's gravitational constant varies in space and time has many motivations. It began in 1935 with the proposal by Milne of a theory of gravitation with two time standards (one for gravitational processes, the other for atomic processes) in which the mass within the particle horizon, $\mathrm{M}_h \propto c^3 G^{-1} t$, remains constant with respect to t-time, led to the prediction that $G \propto t$ in this time. The idea became of wider interest in 1937 with the 'Large Numbers Hypothesis' of Dirac (1937a,b 1938), that the ubiquity of certain large dimensionless numbers, with magnitudes $O(10^{39})$, which were known to arise in combinations of physical constants and cosmological quantities (Weyl 1919, Zwicky 1939, Eddington 1923) was not a coincidence but a consequence of an underlying relationship between them (Barrow and Tipler 1986, Barrow 1990a). This relationship required a linear time variation to occur in the combination $e^2 G^{-1} m_N$ (where e is the electron charge, m_N the proton mass, and G the Newtonian gravitation constant) and Dirac proposed that it was carried by $G \propto t^{-1}$, (Chandrasekhar 1937, Kothari 1938) This led to a range of new geological and palaeontological arguments being brought to bear on gravitation theories and cosmological models (Jordan 1938, 1952, Teller 1948, Dicke 1957, 1964, Gamow 1967a,b). Brans and Dicke (1961) refined the scalar-tensor theories of gravity first formulated by Jordan and, motivated by apparent discrepancies between observations and the weak-field predictions of general relativity in the solar system, proposed a generalization of general relativity that became known as Brans-Dicke theory. As the solar system and binary pulsar observations have come into close accord with the predictions of general relativity so the scope for a theory of Brans-Dicke type to make a significant difference to general relativity in other contexts, notably the cosmological, has been squeezed into the very early universe. However, more general theories with varying G exist, in which the Brans-Dicke parameter is no longer constant (Barrow 1993a). These theories possess cosmological solutions which are compatible with solar-system gravitation tests (Hellings 1984, Reasenberg 1983, Shapiro 1990, Will 1993), gravitational lensing (Krauss and White 1992), and the constraints from white-dwarf cooling (Vila 1976, García-Berro et al 1995). The crucial role that scalar fields may have played in the very early universe has been highlighted by the inflationary universe picture of its evolution. A scalar field, ϕ, which acts as the source of the gravitational coupling, $G \sim \phi^{-1}$, is a possible source for inflation and would modify the form of any inflation that occurs as a result of the universe containing weakly coupled self-interacting scalar fields of particle physics origin. There have been brief periods when experimental evidence was claimed to exist for a non-Newtonian variation in the Newtonian inverse-square law of gravitation at low energies over laboratory dimensions (Fischbach et al 1986) and speculations that non-Newtonian gravitational behaviour in the weak-field limit might explain the flatness of galaxy rotation curves (Bekenstein and Meisels 1980, Milgrom 1983, Bekenstein and Milgrom 1984, Bekenstein and Sanders 1994) usually cited as evidence for the existence of non-luminous gravitating matter in the Universe. Most recently, particle physicists have discovered that space-times with more than four dimensions have special mathematical properties which make them compelling arenas for self-consistent, finite, anomaly-free, fully-unified theories of four fundamental forces of Nature (Green and Schwarz 1984). Our observation of only three large dimensions of space means that some dimensional segregation must have occurred in the early moments of the universal expansion with the result that all but three dimensions of space became static and confined to very small dimensions $\sim 10^{-33} cm$. Any time evolution in the mean size of any extra (> 3) space dimensions will be manifested as a time evolution in the observed three-dimensional coupling constants (Freund 1982, Marciano 1984, Kolb, Perry and Walker 1986, Barrow 1987). The effect of this dimensional reduction process is to create a scalar-tensor gravity theory in which the mean size of the extra dimensional behaves like a scalar field. In particular, low-energy bosonic superstring theory bears a close relation to a particular limit of Brans-Dicke theory (see section 4).

However, despite these interconnections with modern ideas in the cosmology of the early universe, the theoretical investigation of gravity theories with time-varying G is still far from complete and, aside from the solar system and binary pulsar observations (Will 1993), there are few general observational restrictions on scalar-tensor theories which are clear-cut.

3 Newtonian Varying G

We shall begin by investigate Newtonian gravity theories with varying G, pointing out the relationships that these simple solutions have to the more complicated solutions of scalar-tensor gravity theories. In the past there has been very little discussion of the Newtonian case (see Barrow 1996). The exceptions are the rediscoveries of Meshcherskii's theorem (1893, 1949): for example, by Batyrev (1941,1949), Vinti (1974), Savedoff and Vila (1964), Duval, Gibbons and Horváthy (1991), McVittie (1978) and Lynden Bell (1982). These authors all recognised the equivalence of the Newtonian gravitational problem with time-varying G to the problem with constant G and varying masses.

Newtonian gravitation is a potential theory that is derived from the axiom that the external gravitational potential due to a sphere of mass M be equal to that of a point of mass M. This fixes the potential to be equal to

$$\Phi(r) = \frac{A}{r} + Br^2 \tag{1}$$

where A and B are constants; $A = -GM$ and $B = \frac{1}{6}\Lambda$, where Λ is the cosmological constant of Einstein. This argument shows how the cosmological constant arise naturally in Newtonian theory, as it does in general relativity. Unless otherwise stated, we shall set the cosmological constant term zero ($B = 0 = \Lambda$). In section 3.2 we shall discuss how its interpretation differs from a $p = -\rho$ fluid when G is not constant and prove some restricted cosmic no hair theorems.

Consider the Newtonian N-body problem with a time-varying gravitational 'constant' $G(t)$. If the N bodies have masses m_j and position vectors $\mathbf{r}_j$ then

$$\frac{d^2\mathbf{r}_j}{dt^2} = -\sum G(t) m_k \frac{\mathbf{r}_j - \mathbf{r}_k}{|\mathbf{r}_j - \mathbf{r}_k|^3}. \tag{2}$$

Now, if we have a solution, $\hat{\mathbf{r}}_j(\hat{t})$, of these equations with $G = G_0$ independent of time, then,

$$\mathbf{r}_j(t) = (\frac{t+b}{t_0})\hat{\mathbf{r}}_j(-\frac{t_o^2}{t+b} + c) \tag{3}$$

is an exact solution of the equations (1) with

$$G(t) = G_0 \times \left(\frac{t_0}{t-c}\right) \tag{4}$$

where b, c and t_0 are constants with $t_0 \neq 0$. Thus, given any solution of a gravitational problem (for example the output from a cosmological N-body gravitational clustering simulation) with constant G we can immediately write down an exact solution in which G varies inversely with time. For example, suppose we take the simplest Newtonian cosmological model with zero total energy, when $G = G_o$ is constant. Then, the expansion scale factor of the universe is

$$\hat{r} \propto \hat{t}^{2/3}. \tag{5}$$

By the theorem we have that

$$r(t) = (\frac{t+b}{t_0})(-\frac{t_o^2}{t+b} + c)^{2/3} \tag{6}$$

when $G(t)$ varies as

$$G(t) = G_0 \times \left(\frac{t_0}{t-c}\right) \ ; t_0 \neq 0 \tag{7}$$

and so $r(t) \propto t^{1/3}$ as $t \to \infty$.

The result (4) is also useful for modelling small variations in G over short timescales. If we expand an arbitrary analytic form for $G(t)$ to first order in t then

$$G(t) = G_0 + \dot{G}_0 t + ...O(t^2) \approx G_0(1 - t\dot{G}_0/G_0)^{-1} \tag{8}$$

and this has the form (4).

This result, a consequence of the scale invariance of the inverse-square law of force, was first found by Meshcherskii (1893). It has often been rediscovered and elaborated. Duval, Gibbons and Horváthy (1991) have explored its existence in a wider context and displayed similar invariances of the non-relativistic time-dependent Schrödinger equation with Coulomb potential (see also Barrow and Tipler 1986) which enables solutions with time-varying electron charge ($e^2 \propto t$) to be generated by transformation of known exact solutions with constant values of e. In the next section we shall prove a generalization of Meshcherskii's theorem for cases where the pressure is non-zero and the equation of state has perfect fluid form.

3.1 Newtonian Cosmologies with $\mathbf{G(t) \propto t^{-n}}$

We adopt the standard generalization of Newtonian cosmology (Milne and McCrea 1934, Heckmann and Schücking 1955, 1959) to include matter with non-zero pressure and a perfect fluid equation of state. We shall confine our attention to isotropic Newtonian solutions. This is of particular interest for the real universe in the recent past but we also know that anisotropic Newtonian cosmological models are not well posed in the sense that there are insufficient Newtonian field equations to fix the evolution of all the degrees of freedom (there are no propagation equations for the shear anisotropies (Barrow and Götz 1989a)) and this incompleteness must be repaired by supplementing the theory with extra boundary conditions or by importing shear propagation equations from a complete relativistic theory, like general relativity (Evans 1974, 1978, Shikin 1971, 1972), or by ignoring the evolution of the shear anisotropy (Narlikar 1963, Narlikar and Kembhavi 1980, Davidson and Evans 1973, 1977).

Consider a homogeneous and isotropic universe with expansion scale factor $r(t)$. The material content of the universe is a perfect fluid with pressure, p, and density ρ, obeying an equation of state (where the velocity of light has been set equal to unity)

$$p = (\gamma - 1)\rho;\ 0 \leq \gamma \leq 2, \tag{9}$$

with γ constant. If $G = G(t)$ then the equation of motion for $r(t)$ is

$$\ddot{r}(t) = -\frac{G(t)M}{r^2} = -\frac{4\pi G(t)(\rho + 3p)r}{3}. \tag{10}$$

The mass conservation equation is

$$\dot{\rho} + 3\frac{\dot{r}}{r}(\rho + p) = 0. \tag{11}$$

Hence, we have

$$\rho = \frac{\Gamma}{r^{3\gamma}}; \ \Gamma \geq 0, \ constant. \tag{12}$$

We shall initially be interested in power-law variations of $G(t)$ of the form

$$G(t) = G_0 \left(\frac{t_0}{t}\right)^n \tag{13}$$

so we have

$$\ddot{r} = -\lambda t^{-n} r^{1-3\gamma} \tag{14}$$

where λ is a constant defined by

$$\lambda = \frac{4\pi G_0 t_0^n (3\gamma - 2)\Gamma}{3} \tag{15}$$

so the sign of λ is determined by the sign of $3\gamma - 2$, as in isotropic general relativistic cosmologies. Hence, accelerating universes ($\ddot{r} > 0$) arise when $3\gamma > 2$ regardless of whether G varies or not. However, these accelerating universes need not solve the horizon and flatness problems in the way that conventional inflationary universes do; that depends upon the value of n.

A generalization of Meshcherskii's theorem can be proved for the case with $p = (\gamma - 1)\rho$. If $\hat{r}(\hat{t})$ is a solution with $G = G_0$ constant, then (Barrow 1996),

$$r(t) = (\frac{t+b}{t_0})\hat{r}(-\frac{t_o^2}{t+b} + c) \tag{16}$$

with b, c, and $t_0 \neq 0$, constants, is an exact solution of (14) with

$$G(t) = G_0 \times \left(\frac{t_0}{t-c}\right)^{4-3\gamma}; t_0 \neq 0. \tag{17}$$

These results provide a Newtonian analogue to the conformal properties of relativistic scalar-tensor theories. We can draw a number of general conclusions from them. As $t \to \infty$ we have

$$r(t) \to t, \ \text{if } c \neq 0, \ \forall\gamma \tag{18}$$

$$r(t) \to \frac{t}{t_0}\hat{r}\left(\frac{-t_0^2}{t}\right)^{2/3\gamma}, \text{ if } c=0 \text{ and } \gamma \neq 0. \tag{19}$$

In particular, if we take the solutions with constant $G = G_0$ to be the zero-curvature Friedmann solutions then, when $c \neq 0$, we have

$$\hat{r}(\hat{t}) \propto \hat{t}^{2/3\gamma}, \text{ if } \gamma \neq 0 \tag{20}$$

$$\hat{r}(\hat{t}) \propto \exp[H_0\hat{t}], \; H_0 \text{ constant, if } \gamma = 0, \tag{21}$$

and the solutions with $G(t) \propto t^{3\gamma-4}$ at large time have the form

$$\begin{aligned} r(t) &\propto t^{(3\gamma-2)/3\gamma}, \text{ if } \gamma \neq 0 \neq c \\ r(t) &\propto t, \text{ if } \gamma \neq 0, \; c = 0 \end{aligned} \tag{22}$$

$$r(t) \propto t\exp\left[H_0(c - \frac{t_0^2}{t})\right] \to t, \text{ if } \gamma = 0, \forall c. \tag{23}$$

These are particular solutions only, of course, and their properties need not be shared by the general solutions for a given value of n or γ. The $\gamma = 0$ solution, (23), does not exhibit inflation and is asymptotic to the solution of the equation $\ddot{r} = 0$. This is a result of the very rapid decay of $G(t) \propto t^{-4}$.

When $\gamma = 4/3$ there is no possible time-variation of G which preserves the scaling invariance and for other positive values of γ the expansion is slower than in universes with constant G; power-law inflation does not occur in the varying-G solutions when $0 < \gamma < 2/3$.

Equation (14) describes motion under a time-dependent force for which there need exist no time-independent energy integral. Therefore we cannot write down a Friedmann equation for $\dot{r}$ in the usual way. However, there exists a class of particular exact solution with simple power-law form:

$$r(t) \propto t^{(2-n)/3\gamma}; \; \gamma \neq 0 \tag{24}$$

$$\rho(t) = \frac{(2-n)(3\gamma-2+n)}{12\pi G_0\gamma^2 t_0^n(3\gamma-2)t^{2-n}} = \frac{(2-n)(3\gamma-2+n)}{12\pi\gamma^2(3\gamma-2)G(t)t^2} \tag{25}$$

so $\rho \geq 0$ requires that

$$\frac{(2-n)(3\gamma-2+n)}{(3\gamma-2)} \geq 0. \tag{26}$$

Clearly, when $n = 0$, these solutions reduce to the familiar zero-curvature (zero energy) solutions of general relativistic (Newtonian) cosmology with constant G. They describe expanding universes so long as $n < 2$. They are particular solutions because they do not possess the full complement of arbitrary constants of integration that specify the general solution. Solutions of this sort suggest that there may exist more general solutions that behave at early times like a solution of the form (24) with one value of $n = n_1$ for $t \leq t_1$ and with another value $n = n_2$ for $t \geq t_1$. A 'bouncing' solution would have $n_1 > 2$ and $n_2 < 2$, for example, with $t_1 << t_0$. There are many examples of scalar-tensor gravity theories with cosmological models that display this early-time behaviour (Barrow 1993b, Barrow and Parsons 1996).

We shall not go on to discuss the general solutions of the Newtonian evolution equation and the circumstances under which they approach these particular solutions. Such a analysis can be found in Barrow (1996).

3.2 Inflationary universe models with $\mathbf{p = -\rho}$

Scalar-tensor gravity theories have provided an arena in which to explore variants of the inflationary universe theory first proposed by Guth (1981) in which inflation is driven by the slow evolution of some weakly-coupled scalar field. The scalar field from which the gravitational coupling is derived can in principle be the scalar field from which the gravitational coupling is derived or it can influence the form of inflation produced by some other explicit scalar matter field. A number of studies have been made of the behaviour of inflation in scalar-tensor gravity theories (Mathiazhagen and Johri 1984, La and Steinhardt 1989, Barrow and Maeda 1990, Steinhardt and Accetta 1990, García-Bellido, Linde and Linde 1994, Barrow and Mimoso 1994, Barrow 1995). The non-linear master equation governing the evolution for $r(t)$ has interesting behaviour in the inflationary cases where $\ddot{r} > 0$.

The particular power-law solutions (24)-(26) with $2 \geq \gamma > 0$ expand when $n < 2$. Although they accelerate with time ($\ddot{r} > 0$) whenever $3\gamma - 2 < 0$, the expansion only provides a possible solution of the horizon problem when

$$2 - n > 3\gamma > 0. \tag{27}$$

So, in the case of radiation ($\gamma = 4/3$) the horizon problem can be solved if $n < -2$ in the early stages of the expansion.

In the most interesting case, when $\gamma = 0$, and ρ is constant, the perfect-fluid matter source mimics the behaviour of a slowly rolling scalar field whose evolution is dominated by its self-interaction potential. When $\gamma = 0$ the master evolution equation (14) is linear in r

$$\ddot{r} = -\lambda t^{-n} r \tag{28}$$

with $\lambda < 0$. We are interested in determining the asymptotic behaviour of this equation as $t \to \infty$ for all values of n in order to determine when there is asymptotic approach to the usual de Sitter solution that obtains when $n = 0$. The solutions fall into three classes according to the value of n. For $n < 2$, the solutions asymptote towards the WKB approximation as $t \to \infty$

$$r(t) \sim t^{n/4} \exp \left\{ \frac{2H_0 t^{\frac{2-n}{2}}}{2-n} \right\} \;;\; n < 2 \tag{29}$$

where the constant Hubble parameter, H_0, is given by

$$H_0^2 \equiv -\lambda = -\frac{4\pi G_0 t_0^n (3\gamma - 2)\Gamma}{3} \tag{30}$$

which is positive for $3\gamma - 2 < 0$.

For $n > 2$ the asymptote is

$$r(t) \sim t \;;\; n > 2. \tag{31}$$

In fact, this is a particular case of a stronger result that does not assume that $G(t)$ is a power-law. The asymptote $r \sim t$ results whenever $G(t)$ falls fast enough to satisfy (Cesari 1963)

$$\int^{\infty} t \mid G(t) \mid dt < \infty. \tag{32}$$

When $n = 2$ the asymptote is

$$\begin{aligned} r(t) &\sim t^{\alpha} \\ \alpha &= \frac{1}{2}(1 + \sqrt{1 + 4A}) \\ A &\equiv \frac{8\pi G_0 \rho}{3 t_0^n} > 0. \end{aligned} \tag{33}$$

Thus we see that if $G(t)$ falls off faster than t^{-2} the solutions of eqn. (34) approach those of $\ddot{r} = 0$ and no inflation occurs. By contrast, if $G(t)$ falls off more slowly that t^{-2}, grows ($n < 0$), or remains unchanged ($n = 0$), then inflationary solutions of the form (29) arise. We notice that in the absence of G-variation ($n = 0$) this solution reduces to the well known de Sitter expansion ($r(t) \propto \exp\{H_0 t\}$) familiar in general relativistic models of inflation with a constant vacuum energy density or positive cosmological constant. When $0 < n < 2$ it produces a form of sub-exponential inflation which is familiar from studies of scalar-tensor gravity theories with varying $G(t)$ and models of intermediate inflation studied in general relativistic cosmologies containing a wide range of scalar fields, (Barrow 1990b, Barrow and Saich 1990, Barrow and Liddle 1993). If $n < 0$ there is super-exponential inflation.

There is a Newtonian 'no hair' theorem in the general case where no particular form is assumed for the time-variation of $G(t)$ because we can make use of the general asymptotic properties of the evolution equation. We have already given this result for cases where $G(t)$ falls off faster than t^{-2} as $t \to \infty$ in eqns. (31)-(32), and as t^{-2} in eqn. (33). In the case where the fall-off is slower than t^{-2} we have a WKB approximation

$$r(t) \sim c\,[G(t)]^{-\frac{1}{4}} \exp\left\{\omega \int [G(t)]^{\frac{1}{2}}\, dt\right\} \tag{34}$$

as $t \to \infty$, where $\omega^3 = 1$ and c is a constant, so long as

$$\left|t^2 G(t)\right| \to \infty \text{ as } t \to \infty. \tag{35}$$

Clearly, eqn. (29) gives this asymptote in the special case that $G(t) \propto t^{-n}$ with $n < 2$ when we choose the $\omega = 1$ part of the linear combination of solutions.

When G is constant in general relativity and in Newtonian gravitation the presence of a perfect fluid with an equation of state $p = -\rho$ is equivalent to the addition of a cosmological constant to the field equations. However, in Newtonian gravitation with varying G and in scalar-tensor generalizations of general relativity, these two are no longer equivalent. If we had included a cosmological constant term in the Newtonian gravitational equation (10), it would become

$$\ddot{r}(t) = -\frac{4\pi G(t)(\rho + 3p)r}{3} + \frac{\Lambda r}{3} \tag{36}$$

and we can see that the choice $p = -\rho =$ constant, using (11), only makes the first term on the right-hand side of (36) equivalent to the term $\propto \Lambda r$ if $G(t)$ is constant. Clearly, the late-time may not be dominated by the cosmological constant term when $G(t) \propto t^{-n}$ and $n < 0$. We will not investigate the behaviour for general γ here, see Barrow (1996).

4 Relativistic scalar-tensor theories

Newtonian gravitation permits us to 'write in' an explicit time variation of G without the need to satisfy any further constraint. However, in general relativity the geometrical structure of space-time is determined by the sources of mass-energy it contains and so there are further constraints to be satisfied. Suppose that we take Einstein's equation in the form ($c \equiv 1$)

$$\tilde{G}^a_b \;= 8\pi G T^a_b \tag{37}$$

where $\tilde{G}^a_b$ and T^a_b are the Einstein and energy-momentum tensors, as usual, but imagine that $G = G(t)$. If we take a covariant divergence $_{;a}$ of this equation the left-hand side vanishes because of the Bianchi identities, $T^a_{b;a} = 0$ if energy-momentum conservation is assumed to hold, hence $\partial G/\partial x^a = 0$ always. In order to introduce a time-variation of G we need to derive the space and time variations from some scalar field, ψ which then contributes a stress tensor $\tilde{T}^a_b(\psi)$ to the right-hand side of the gravitational field equations.

We can express this structure by a choice of lagrangian, linear in the curvature scalar R, that generalizes the Einstein-Hilbert lagrangian of general relativity with,

$$L = -f(\psi)R + \frac{1}{2}\partial_a\psi\partial^a\psi + 16\pi L_m \tag{38}$$

where L_m is the lagrangian of the matter fields. The choice of the function $f(\psi)$ defines the theory. When ψ is constant this reduces, after rescaling of coordinates, to the Einstein-Hilbert lagrangian of general relativity. (We ignore, for simplicity the possibility of including a cosmological 'constant' term which can now be a function of the scalar field ψ). For historical reasons scalar-tensor theories have not been written with a gravitational lagrangian of this simple form (Bergmann 1968, Steinhardt and Accetta 1990, Holman et al 1991) but have followed the formulation introduced by Brans and Dicke

(1961). This can be obtained from (38) by a non-linear transformation of ψ and f. Define a new scalar field ϕ, and a new coupling function $\omega(\phi)$ by

$$\begin{aligned} \phi &= f(\psi) \\ \omega(\phi) &= \frac{f}{2f'^2} \end{aligned} \tag{39}$$

then (75) becomes

$$L = -\phi R + \frac{\omega(\phi)}{\phi}\partial_a\phi\partial^a\phi + 16\pi L_m. \tag{40}$$

This has the Brans-Dicke form. The Brans-Dicke theory arises as the special case

$$Brans - Dicke: \ \omega(\phi) = \text{ constant; } f(\psi) \propto \psi^2. \tag{41}$$

However, in general, there exists an infinite number of these theories defined by the choice of $\omega(\phi)$. One of the reasons for renewed interest in scalar-tensor gravity theories of this form is the relationship that exists between the gravitational part of lagrangian (i.e. excluding L_m) for Brans-Dicke theory and the low-energy effective action for bosonic string theory, which can be written as (Callan et al 1985)

$$L_{sst} = \exp(-2\chi)(R + 4\chi^{,a}\chi_{,a} - \frac{1}{12}H^2) \tag{42}$$

where χ is the dilaton field and $H^2 \equiv H_{abc}H^{abc}$, where H_{abc} is the totally antisymmetric 3-form field. If we identify $\phi = \exp(-2\chi)$ then L_{sst} is identical to the Brans-Dicke lagrangian, (40), when $\omega = -1$. However, differences arise in the couplings of the scalar fields to other forms of matter in the two theories.

The field equations that arise by varying the action associated with L in (40) with respect to the metric, g_{ab}, and ϕ separately, gives the field equations,

$$\begin{aligned} \tilde{G}_{ab} &= \frac{-8\pi}{\phi}T_{ab} - \frac{\omega(\phi)}{\phi^2}\left[\phi_{,a}\phi_{,b} - \frac{1}{2}g_{ab}\phi_{,i}\phi^{,i}\right] \\ &\quad -\phi^{-1}\left[\phi_{,a;b} - g_{ab}\Box\phi\right] \end{aligned} \tag{43}$$

$$[3+2\omega(\phi)]\,\Box\phi = 8\pi T_a^a - \omega'(\phi)\phi_{,i}\phi^{,i} \tag{44}$$

where the energy-momentum tensor of the matter sources obeys the conservation equation

$$T^{ab}_{;a} = 0. \tag{45}$$

These field equations reduce to those of general relativity when ϕ (and hence $\omega(\phi)$) is constant, in which case the Newtonian gravitational constant is defined by $G = \phi^{-1}$. An interesting feature of these equations is clear by inspection: when the trace of the energy-momentum tensor vanishes (this includes vacuum and radiation-dominated solutions as important particular cases) any solution of general relativity is a particular solution of the scalar-tensor theory with ϕ (and hence G) constant.

These equations are also conformally related to general relativity when the trace $T_a^a = 0$. If we conformally transform the metric

$$g_{ab} \to \Omega^{-2} g_{ab} \tag{46}$$

with $\Omega = \phi^{1/2}$ and define Ψ by

$$\Psi = \int \sqrt{\frac{2\omega(\phi)+3}{2}}\frac{d\phi}{\phi} \tag{47}$$

then the conformally transformed theory is general relativity with a matter source consisting of a scalar field Ψ with a potential $V(\Psi)$. This conformal invariance can be exploited to produce powerful solution-generating procedures for scalar-tensor theories (Barrow 1993a, Barrow and Mimoso 1994, Barrow and Parsons 1996). One can regard Meshcherskii's theorem and its generalization proved above in (16)-(17) as Newtonian analogues of these conformal invariances.

Although the coupling function $\omega(\phi)$ is unconstrained by the structure of scalar-tensor gravity theories, its choice determines the form of the cosmological models in the theory and the form of the weak-field limit. Typically, the weak-field solar-system predictions of scalar tensor gravity theories have the following relationship to those of general relativity (GR)

$$\omega(\phi) \text{ weak field result} \approx \text{ (GR result)} \times \left[1 + O\left(\frac{\omega'}{\omega^3}\right)\right]. \tag{48}$$

Specifically, the perihelion precession of Mercury, $\Delta\vartheta$, and the time-variation of G is predicted at second-order to be (Nordvedt 1970, Wagoner 1970, Will 1993)

$$\Delta\vartheta \simeq 43'' \times \left[1 - \frac{1}{12+6\omega}\left\{4 + \frac{\omega'}{(3+2\omega)^2}\right\}\right] \text{ per 100 yr} \tag{49}$$

$$\frac{\dot{G}}{G} = -\left(\frac{3+2\omega}{4+3\omega}\right)\left[\frac{G(t)}{G_0} + \frac{2\omega'(\phi)}{(3+2\omega)^2}\right]\dot{\phi} \tag{50}$$

where G_0 is the present measured value of $G(t)$. There is even a special theory for which the term in [...] brackets vanishes in (50) and $G(t) = G_0$ is constant to this weak-field order (Barker 1978). In Brans-Dicke theory we have $G \propto \phi^{-1}$. The general relativistic limit of an $\omega(\phi)$ scalar-tensor theory is obtained (if it exists) by taking the two limits

$$\omega \to \infty \text{ and } \frac{\omega'}{\omega^3} \to 0. \tag{51}$$

The observational limits on ω require $\omega > 500$ but the limit on ω' is weak, $\omega' < O(1)$, (see Will 1993). Similar limits are obtained from the binary pulsar (Damour, Gibbons & Tayler 1988), but are more model dependent.

4.1 Scalar-tensor Friedmann cosmologies

In order to examine some of the counterparts to the Newtonian solutions discussed above we give the field equations for an isotropic and homogeneous Friedmann universe with scale factor $a(t)$, curvature parameter k, and Hubble rate $H = \dot{a}/a$,

$$H^2 = \frac{8\pi\rho}{3\phi} - \frac{H\dot{\phi}}{\phi} + \frac{\omega(\phi)\dot{\phi}^2}{6\phi^2} - \frac{k}{a^2} \tag{52}$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\omega'\dot{\phi}^2}{2\omega+3} = \frac{8\pi\rho(4-3\gamma)}{2\omega+3} \tag{53}$$

$$\rho \propto a^{-3\gamma}. \tag{54}$$

We will now focus attention upon solutions of these equations with $k = 0$ in the case of Brans-Dicke theory (ω constant). Scalar-tensor theories with varying G differ from general relativity in that they admit vacuum solutions when $k = 0$ (O'Hanlon & Tupper 1970)

$$\begin{aligned} G &\propto \phi^{-1} \propto t^{-d/(1+d)} \\ a(t) &\propto t^{1/3(1+d)} \\ d &\equiv \omega^{-1}\left(1+\sqrt{1+\frac{2\omega}{3}}\right). \end{aligned} \tag{55}$$

They also possess a class of special power-law solutions for perfect-fluid universes, (Nariai 1969),

$$\begin{aligned} G &\propto \phi^{-1} \propto t^{-B} \\ a(t) &\propto t^{A} \\ p &= (\gamma-1)\rho \end{aligned} \tag{56}$$

where

$$A = \frac{2+2\omega(2-\gamma)}{4+3\omega\gamma(2-\gamma)}, \tag{57}$$

$$B = \frac{2(4-3\gamma)}{4+3\omega\gamma(2-\gamma)}. \tag{58}$$

The general solutions can be found for all γ but are rather cumbersome and opaque; exact solutions for $k = 0$ have been found by Gurevich, Finkelstein and Ruban (1973) and for all k by Barrow (1993a); a phase plane analysis has been performed by Kolitch and Eardley (1995) which includes the $k \neq 0$ models. Their general properties are as follows. As $t \to 0$ they approach the vacuum solutions (92), while as $t \to \infty$ they approach the matter-dominated solutions (93)-(95). As $\omega \to \infty$ these matter-dominated solutions approach the general relativistic results, $a(t) \propto t^{2/3\gamma}$, G constant. There is a smooth transition between these simple early and late time behaviours. Thus the power-law matter-dominated solutions (93)-(95) are unstable as $t \to 0$. The general solutions are dominated by the Brans-Dicke scalar field. A similar early-time behaviour occurs in the $\omega(\phi)$ theories although the form of the early vacuum-dominated phase depends on the detailed functional form of

$\omega(\phi)$, (see Barrow 1993a, 1993b, Barrow and Mimoso 1994, Damour and Nordvedt 1993, Serna and Alimi 1996, and Barrow and Parsons 1996 for details).

4.2 New Methods of Solution

4.2.1 Vacuum and radiation models

The general solutions to Eqs. (52) - (54) contain four arbitrary integration constants, one more than their GR counterparts, the extra degree of freedom being attached to the value of $\dot{\phi}$. When the energy-momentum tensor is trace-free there exists a conformal equivalence between the theory and GR, the right-hand side of Eq. (53) vanishes and $\dot{\phi} = 0$ is always a particular solution, corresponding to a special choice of the additional constant possessed by the model over GR. Consequently, the exact general solution of Einstein's equations when $T_{\rm ab}$ is trace-free is also a particular solution to Eqs. (52)–(54) with ϕ, and hence $\omega(\phi)$, constant.

It will seldom be the case that the particular solution obtained in this way will form the general solution for that particular choice of $\omega(\phi)$. Usually, however, it will be the late or early time attractor of the general solution. For example, in the case of Brans-Dicke theory the special GR solution is the late-time attractor for flat and open universes but not the early-time attractor. However, a method has been developed for integrating the field equations for models containing trace-free matter (Barrow 1993a). The procedure is as follows.

Eq. (54) integrates immediately to yield

$$8\pi\rho = 3\Gamma a^{-3\gamma}\,, \tag{59}$$

where $\Gamma \geq 0$ is a constant of integration; $\Gamma = 0$ describes vacuum models. Making the choice $\gamma = 4/3$, corresponding to blackbody radiation, and introducing the conformal time co-ordinate, η, defined by

$$a d\eta = dt\,, \tag{60}$$

Eq. (53) becomes

$$\phi_{\eta\eta} + \frac{2}{a}a_\eta\phi_\eta = -\frac{\omega'(\phi)}{2\omega(\phi)+3}\left(\phi_\eta\right)^2\,, \tag{61}$$

where subscript η denotes a derivative with respect to conformal time. This integrates exactly to give,

$$\phi_\eta a^2 = 3^{1/2} A(2\omega(\phi)+3)^{-1/2}\,; \quad A \text{ const.} \tag{62}$$

We now employ the variable (suggested by the conformal invariance) used by Lorenz-Petzold (1984) to study Brans-Dicke models,

$$y = \phi a^2\,, \tag{63}$$

to re-write the scalar-tensor version of the Friedmann equation, Eq. (52), as

$$(y_\eta)^2 = -4ky^2 + 4\Gamma y + \frac{1}{3}\left(\phi_\eta\right)^2 a^4 (2\omega(\phi)+3)\,. \tag{64}$$

Dividing Eq. (62) by Eq. (63), and using Eq. (62), we obtain the coupled pair of differential equations

$$\begin{aligned} \frac{\phi_\eta}{\phi} &= 3^{1/2} A y^{-1} (2\omega(\phi)+3)^{-1/2}\,, && (65)\\ (y_\eta)^2 &= -4ky^2 + 4\Gamma y + A^2\,. && (66) \end{aligned}$$

We may now obtain the general solution for a particular choice of $\omega(\phi)$, given k. Integrating Eq. (65) yields $y(\eta)$ which, in conjunction with $\omega(\phi)$, implies $\phi(\eta)$ and, without further integration, $a(\eta)$ from Eq. (63). If Eq. (60) can be integrated and inverted we may compute $\phi(t)$ and $a(t)$, so completing the solution. The vacuum models are obtained by setting $\Gamma = 0$.

4.2.2 General perfect-fluid cosmologies

When T,the trace of the energy-momentum tensor is non-vanishing, the situation is substantially more complicated. In this instance, $\dot{\phi} = 0$ is no longer a particular solution of the field equations, forcing us to resort to more elaborate methods to obtain solutions. Barrow and Mimoso(1994) have done this, for the $k = 0$ models, by generalising the method of Gurevich *et al.* (1973) for BD models to the case of non-constant $\omega(\phi)$. We outline this procedure.

Introducing the new time co-ordinate ξ, and the two new variables x and v such that

$$dt = a^{3(\gamma-1)} \sqrt{\frac{2\omega+3}{3}}\, d\xi \quad , \tag{67}$$

$$x \equiv \left[\phi a^{3(1-\gamma)} \left(a^3\right)_\xi\right] \ , \tag{68}$$

$$v \equiv \left[a^{3(2-\gamma)} \phi_\xi\right] \quad , \tag{69}$$

and confining attention to the $k = 0$ models, Eqs. (52)-(54) transform to

$$\left(\frac{2}{3}x + v\right)^2 = \left(\frac{2\omega + 3}{3}\right) \left[v^2 + 4\Gamma\, \phi\, a^{3(2-\gamma)}\right] \ , \tag{70}$$

$$v_\xi = \Gamma\ (4 - 3\gamma) \ , \tag{71}$$

and

$$x_\xi = 3\Gamma\ \left[(2-\gamma)\omega + 1\right] + \frac{3}{2}\left(\frac{2}{3(2\omega+3)}x + v\right)\omega_\xi \ , \tag{72}$$

where subscript-ξ represents a derivative with respect to ξ-time. Eqs. (71) and (72) integrate easily to yield

$$v \ = \ \Gamma(4 - 3\gamma)\,(\xi - \xi_1) \quad , \tag{73}$$

$$x \ = \ \frac{3}{2}\left[-v + \sqrt{2\omega+3}\left(C + \Gamma(2-\gamma)\int_{\xi_1}^{\xi}\sqrt{2\omega+3}\,d\bar{\xi}\right)\right] \quad , \tag{74}$$

C is an integration constant and ξ_1 fixes the origin of ξ-time. Noting the relation

$$\frac{3}{a\phi}a_\xi\phi_\xi = \frac{1}{\phi^2}\left(\phi_\xi\right)^2\ 3\frac{\phi}{a}\frac{a_\xi}{\phi_\xi} = \frac{1}{\phi^2}\ \left(\phi_\xi\right)^2\frac{x}{v} \ , \tag{75}$$

and differentiating y, with respect to ξ, yields

$$\left(\frac{\phi_\xi}{\phi}\right)_\xi + \left[\frac{3\gamma-4}{2} + \frac{1}{\xi-\xi_1}f_\xi(\xi)\right]\ \left(\frac{\phi_\xi}{\phi}\right)^2 = \frac{1}{\xi-\xi_1}\,\frac{\phi_\xi}{\phi} \quad , \tag{76}$$

where a new function $f(\xi)$, is defined by

$$f(\xi) \equiv \int_{\xi_1}^{\xi}\frac{3(2-\gamma)}{2\Gamma(4-3\gamma)}\sqrt{2\omega(\phi)+3}\ \left[C + \Gamma(2-\gamma)\int_{\xi_1}^{\xi}\sqrt{2\omega(\phi)+3}\,d\tilde{\xi}\right]\,d\bar{\xi} \ . \tag{77}$$

Solving Eq. (76), we have the solution

$$\ln\left(\frac{\phi}{\phi_0}\right) = \int_{\xi_1}^{\xi}\frac{\xi-\xi_1}{g(\xi)}d\xi \,, \tag{78}$$

with $g(\xi)$ simply related to $f(\xi)$ by

$$g(\xi) \equiv f(\xi) + \frac{3\gamma - 4}{4} (\xi - \xi_1)^2 + D\,, \tag{79}$$

where D is a constant of integration. Eq. (75) immediately reveals a simple formula for the scale-factor:

$$a^3 = a_0^3 \left(\frac{g}{\phi}\right)^{\frac{1}{2-\gamma}} \;; a_0 \text{ constant}\,. \tag{80}$$

Finally, the scalar-tensor coupling function $\omega(\phi)$ is given as a function of f by

$$2\omega\left(\phi(\xi)\right) + 3 = \frac{4 - 3\gamma}{3(2-\gamma)^2} \, \frac{(f')^2}{\left[f + \frac{4-3\gamma}{3(2-\gamma)^2}\, f_0\right]} \quad , \tag{81}$$

where f_0 is another arbitrary constant.

An important benchmark is provided by the behaviour of the BD theory, where $\omega(\phi) = \omega_0 =$ constant. In this case, the generating function, $f(\xi)$, is given by a quadratic in ξ:

$$f_{\rm BD}(\xi) = \frac{3(2-\gamma)}{2\Gamma(4-3\gamma)} \sqrt{2\omega_0 + 3} \left[C\, \xi + \frac{\Gamma(2-\gamma)}{2} \xi^2 \sqrt{2\omega_0+3}\right]. \tag{82}$$

Hence, in general ($C \neq 0 \neq \Gamma$) when $\gamma \neq 4/3\,,\ 2$, we see that $f_{\rm BD} \propto \xi^2$ as $\xi \to \infty$ and $f_{\rm BD} \propto \xi$ as $\xi \to 0$, where $dt \propto a^{3(\gamma-1)} d\xi$. If we choose $C = 0$ then $f_{\rm BD} \propto \Gamma\xi^2$ as $\xi \to 0$. The choice $C = 0$ restricts the solution to the special 'matter-dominated' solutions (termed 'Machian' by Dicke, see Weinberg (1972)) which were first found for all perfect-fluids by Nariai (1969). If $C \neq 0$ then the early-time behaviour is dominated by the dynamics of the ϕ-field; such solutions are termed 'ϕ-dominated' (or 'non-Machian' by Dicke).

Therefore if we choose a generating function $g(\xi)$ that grows slower than ξ^2 as $\xi \to \infty$ it will produce a theory that approaches BD at late times ($\phi \to$constant, $\omega(\phi) \to$ constant), whilst if $g(\xi)$ decreases slower than ξ as $\xi \to 0$ then the theory will approach the behaviour of ϕ-dominated BD theory at early-times. This means that we will find new (non-BD) late-time behaviours by studying generating functions which increase faster than $g(\xi) = \xi^2$ as $\xi \to \infty$ and new (non-BD) early-time behaviour by picking generating functions which decrease slower than $g(\xi) = \xi$ as $\xi \to 0$ or $\xi \to \xi_{\rm min}$ (if there is no zero of ξ at the minimum of $a(t)$).

4.3 Comparing Newtonian and Relativistic Cosmologies

It is now possible to consider some of the similarities and differences that exist between the Newtonian cosmologies with varying G and their curved space-time counterparts. We recall that the special Newtonian power-law solutions (24) have the form $G \propto t^{-n}$ with scale factor $r \propto t^{(2-n)/3\gamma}$. This is identical to the Brans-Dicke solution when $p = 0$ and

$$n = \frac{2}{4 + 3\omega}. \tag{83}$$

In fact, for any choice of $\omega(\phi)$ dust universes have

$$\phi a^3 \to t^2 \tag{84}$$

which corresponds to Newtonian solutions with $G \propto t^{-n}$ and scale factor $r \propto t^{(2-n)/3}$.

The vacuum solutions have a slightly different structure. If we identify

$$n = \frac{d}{d+1} \tag{85}$$

then we have

$$a(t) \propto t^{1/3(1+d)} \propto t^{(1-n)/3}. \tag{86}$$

However, there is no general correspondence for other equations of state. Most notably, Brans-Dicke radiation-dominated universes ($\gamma = 4/3$) are the same as in general relativity, $G =$ constant and $a(t) \propto t^{1/2}$, and differ from the radiation solutions. In general, the Newtonian solutions are the same as the Brans-Dicke matter-dominated solutions only if we make the choice

$$n = \frac{2(4 - 3\gamma)}{4 + 3\omega\gamma(2 - \gamma)}. \tag{87}$$

Also, as can be seen from the right-hand side of the $\phi(t)$ evolution equation, (90), the effective sign of the gravitational coupling changes sign from negative to positive when γ becomes greater than 4/3.

The Newtonian inflationary solutions with $\gamma = 0$, given in eqn. (29), do not have direct counterparts in Brans-Dicke gravity theories. However, scalar tensor theories with

$$2\omega + 3 \propto \phi^h \tag{88}$$

have solutions of similar form as $t \to \infty$ with (Barrow and Mimoso 1994, Barrow and Parsons 1996)

$$G \propto \phi^{-1} \propto t^{-2/(2h+1)} \tag{89}$$

and the scale factor evolves as

$$a(t) \propto t^{(h-1)/3(2h+1)} \exp\left\{At^{2h/(2h+1)}\right\}. \tag{90}$$

Thus, with $2h + 1 > 0$, we have $\omega \to \infty$ and $\omega'/\omega^3 \to 0$ as $t \to \infty$ and general relativity is approached in the weak-field limit. These solutions are not of identical form to the Newtonian solutions with $G \propto t^{-n}$ and $n = 2/(2h + 1)$.

5 Gravitational memory

The process of primordial black hole formation in a cosmological model with varying G creates an interesting problem. We know (Hawking 1972) that black holes in scalar-tensor gravity theories are identical to those occurring in general relativity. Suppose, for simplicity, that a Schwarzschild black hole forms in the very early universe at a time t_f when the gravitational coupling $G(t_f)$ differs from the value $G(t_0)$ that we observe at the present cosmic time, t_0. This black hole will have an horizon size $R_f = 2G(t_f)M \sim t_f$ when it forms. We now ask what happens to this black hole during the subsequent evolution of the universe as the value of G changes with time in the background universe (Barrow 1992, Barrow and Carr 1996).

There appear to be two alternatives. The value of $G(t)$ on the scale of the even horizon could change at the same rate as that in the background universe. But this would imply that the black hole was changing with time. Thus it could not be one of the black holes defined in general relativity, as required. Moreover, if $\dot{G} < 0$,the horizon area (A_{hor}) would decrease with time and so would the associated entropy which is given by (Kang 1996)

$$S_{bh} = \frac{A_{hor} \times \phi_{hor}}{4}$$

where $\phi_{hor} = G_{hor}^{-1}$ determines the value of G on the horizon. Alternatively, there might be a process of 'gravitational memory' (Barrow 1992) wherein the scalar field determining G remains constant on the scale of the black hole horizon whilst changing in the cosmological background. That is at any moment of cosmic time there would be a space variation in G. This has dramatic implications for the Hawking evaporation of primordial black holes. For the lifetime and temperature of an evaporating black hole will be determined by the value of $G(t_f)$ at the time when it formed rather than by the value $G(t_0)$ we observe in the universe today: the black hole 'remembers' the value of G at the time of its formation. Hence, its Hawking lifetime will be $\tau_{bh} \sim G_f^2 M^3$ and its temperature $T_{bh} \sim G_f^{-1} M^{-1}$.

Black holes which explode today are those whose Hawking lifetime is equal to the present age of the universe. This fixes their masses to be

$$M_{ex} \simeq 4 \times 10^{14} \times \left(\frac{G(t_0)}{G(t_f)}\right)^{\frac{2}{3}} gm$$

and their temperature when the explode is therefore given by

$$T_{ex} \simeq 24 \times \left(\frac{G(t_0)}{G(t_f)}\right)^{\frac{1}{3}} MeV.$$

Clearly, if there has been significant change in the value of G since $t_f \sim 10^{-23}s$ then the physical characteristics of exploding black holes can be very different from those predicted under the assumption that G does not change (Hawking 1974). Quite modest amounts of time evolution at unobservably early times can shift the spectral range in which we would see the evaporation products out of the gamma-ray band. This means that we should be looking for the evidence of black hole evaporation in other parts of the electromagnetic spectrum. A more detailed study of the observational evidence, seen is this light, is given by Barrow and Carr (1996).

6 Origins of the Values of Constants

In recent years there has been a good deal of speculation about mechanisms for explaining the values of the fundamental constants. At one time it was widely believed that some ultimate Theory of Everything would eventually tell us that the constants could have one, and only one, set of logically self-consistent values. Such a simple scenario now seems less and less likely (Barrow 1991). There are so many sources of randomness in the process which endow the fundamental constants with their low-energy values, and the parameters likely to be fixed by a Theory of Everything are so far removed from our three-dimensional physical constants, that many new possibilities must be taken seriously. The non-uniqueness of the ground state of any Theory of Everything would mean that fundamental constants could take on many self-consistent sets of values. We would have to use anthropic constraints in order to understand those that we observe. This creates new interpretational problems. Our underlying Theory of Everything would have quantum gravitational characteristics and its predictions about constants would have a probabilistic form. Although, formally, there would be a most probable value for the low-energy measurement of a quantity like the fine structure constant, such a value might be irrelevant for observational purposes (Barrow 1994). We would only be interested in the range of values for which the evolution of complexity, in the form that we call 'life', is possible. This may well confine us to a subset of values which, a priori, is extremely improbable. This shows that in order to make a correct comparison of the probabilistic predictions of such a theory with observation we would need to know every dependence of processes which can lead to the evolution of complexity on the values of the constants of Nature.

The fact that the observed values of many of the constants of Nature fall within a very narrow range for which life appears to be possible has elicited a variety of interpretations:

(i) **Good luck:** the constants are what they are and could be no other way. The range which allows intelligent observers to evolve and persist is narrow and we are very lucky that our universe falls within that range. No matter how improbable this sate of affairs we could observe it to be no other way.

(ii) **Life is inevitable:** we have been misled by our limited knowledge of complexity into thinking that life is restricted to universes spanned by a very narrow range of values for the constants. In fact, life may be a widespread inevitability in the phase space of all possible values of the constants. Even our own form of carbon-based life may exist in other novel forms which exploit the possibilities provided by the recently discovered fullerene chemistry. complexity of the sort that we call life may also exist in quite different forms to those we are accustomed to: for example, existing in velocity space rather than in position space.

(iii) **All possibilities exist:** whether through the actualisation of quantum-mechanical many-worlds, the realisation of all logically consistent Theories of Everything, or some elaboration of the self-reproducing universe scenarios, every possible permutation of the values of the constants exists in some universe. We live in one of the subset which allows life to exist. It is also possible that the ensemble of possibilities is played out in a single infinite Universe and we inhabit one of the life-supporting parts of it.

(iv) **Cosmic fine tuning:** some physical process brings about approach to a particular set of values for the constants over long periods of time, perhaps through many cycles of cosmic evolution. The attracting set may be predictable in certain respects.

The last of these four possibilities has attracted some interest recently. Harrison (1995) made the amusing suggestion that the fine tuning of the constants may be the end result of successive intelligent interventions by beings able to create universes in the laboratory (something discussed in the literature even by ourselves! Farhi and Guth (1987)).Aware that certain combinations of the values of fundamental constants raise the probability of life evolving, and able to engineer these values at inception, successive generations would tend to find themselves inhabiting universes in which life-supporting combinations obtained to high precision. Although Harrison refers to his as a 'natural selection' of universes, it is more akin to artificial selection, or forced breeding.

Linde (1990) has proposed generalisations of the self-reproducing eternal inflationary universe in which the values of the fundamental constants change from generation to generation. Although unobservable, this scheme has the merit of being a by-product of the standard chaotic inflationary universe scenario.

A third scenario of this sort, which has attracted a surprising amount of attention is that proposed by Smolin (1992) who suggested that a bounce, or quantum tunnelling, occurs at all final black-hole-collapse singularities which transforms them into initial singularities for new expanding universes. During this process the constants of Nature undergo small random changes. It is expected therefore that selection pressure will act so as to maximise the black holes produced in universes as time goes on (no weighting of the volume taking part in this reproduction process is introduced though, as is the case in the self-reproducing inflationary universes). Thus, if our universe is the result of the action of this selection process over many cycles of collapse and re-expansion, in which the constants have lost memory of any initial conditions they may have had, then Smolin argues that we would expect to be near a local maximum in the black-hole production. Hence, *small* changes in the constants of Nature should in general take us downhill from this local maximum and always *reduce* the amount of black hole production. By conducting such thought-experiments the general consistency of the idea can be tested.

We make three remarks about this speculative scenario. First, it is not clear that is as sharply predictive and testable as claimed. We should only expect to find ourselves residing near a local maximum in the space of constants if that maximum also provides conditions which permit living observers to exist. If those condition are unusual then we might have to exist in one of the improbable universes far from the local maxima. We can only determine if this is the case by having a complete understanding of the necessary and sufficient conditions for living complexity to exist. Second, putting this objection to one side, there may well be small changes in the values of the constants which significantly increase the production of black holes. For example a small ($70KeV$) strengthening of the strong interaction would bind the dineutron and the diproton (helium-2), so providing a direct $H + H \rightarrow He^2$ channel for nuclear burning. massive stars would run through their evolution very rapidly and end as black holes far sooner and with higher likelihood than at present (see Dyson 1971, Barrow 1987). Third, we might ask why there should be any local maxima at all for variations in certain constants. Variation would proceed to states of higher gravitational entropy by always increasing the value of $S_{bh} \propto GM^2$, and this would be effected by a random walk upwards through (over long time averaged) increasing values of G.

Finally, we should note that any scheme which relies upon random changes in the constants of Nature occurring at the endpoint of gravitational collapse must beware of the consequences of changes which prevent future collapses from occurring. A specific example is seen in the case of closed universes oscillating under the requirement that their total entropy increase from cycle to cycle. There, one finds that any positive cosmological constant (no matter how small in value) which remains constant (or falls slowly enough on average) from cycle to cycle ultimately stops the sequence of growing oscillations and leaves the Universe in a state of indefinite expansion which asymptotes towards the de Sitter state (Barrow and Dabrowski 1995). In Smolin's scenario one might consider that if the curvature of space or the value of the cosmological constant, or the magnitude of vacuum stresses associated with scalar fields which violate the strong energy condition, were to change at the collapse event in ways that prevented future collapse of some or all of the Universe, then gradually the fraction of the Universe which could gravitationally collapse and evolve the values of its constants by random reprocessing would shrink asymptotically to zero. Evolution would cease. This Universe would have 'died'.

7 Simultaneous Variations of Many Constants

The subject of varying constants is of particular current interest because of the new possibilities opened up by the structure of unified theories, like string theory and M-theory, which lead us to expect that additional compact dimensions of space may exist. Although these theories do not require traditional constants to vary, they allow a rigorous description of any variations to be provided: one which does not merely 'write in' the variation of constants into formulae derived under the assumption that they do not vary. This self-consistency is possible because of the presence of extra dimensions of space in these theories. The 'constants' seen in a three-dimensional subspace of the theory will vary at the same rate as any change occurring in the extra compact dimensions. In this way, consistent simultaneous variations of different constants can be described and searches for varying constants provide a possible observational handle on the question of whether extra dimensions exist (Marciano, 1984, Barrow 1987, Damour & Polyakov 1994).

Prior to the advent of theories of this sort, only the time variation of the gravitational constant could be consistently described using scalar-tensor gravity theories, of which the Brans-Dicke theory is the simplest example. The modelling of variations in other 'constants' was invariably carried out by assuming that the time variation of a constant quantity, like the fine structure constant, could just be written into the usual formulae that hold when it is constant. One way of avoiding this situation is to exploit the invariance properties of the non-relativistic Schrödinger equation for atomic structure, which allow it to be written in dimensionless form when atomic ('Hartree') units are chosen. It can be shown (Barrow and Tipler 1986) that any solution with an energy eigenvalue E, arising when the fine structure constant is α and the electron mass is m_e,must be related to a solution defined by a E, α', and $m_e^{'}$ by the relation

$$\frac{E}{\alpha^2 m_e c^2} = \frac{E'}{\alpha'^2 m'_e c^2} \tag{91}$$

where c is the velocity of light.

The possibility of linked variations in low-energy constants as a result of high-energy unification schemes has the added attraction of providing a more powerful means of testing those theories (Marciano 1984, Kolb, Perry, & Walker, 1986, Barrow 1987, Dixit & Sher 1988, Campbell & Olive 1995).

Higher-dimensional theories typically give rise to relationships of the following sort

$$\begin{aligned} \alpha_i(m_*) &= A_i G m_*^2 = B_i \lambda^n (\ell_{pl}/R)^k ; n, k \text{ constants} \qquad (92) \\ \alpha_i^{-1}(\mu) &= \alpha_i^{-1}(m_*) - \\ & \pi^{-1} \sum C_{ij} [\ln(m_*/m_j) + \theta(\mu - m_j)\ln(m_j/\mu)] + \Delta_i \end{aligned}$$

where $\alpha_i(..)$ are the three gauge couplings evaluated at the corresponding mass scale; μ is an arbitrary reference mass scale, m_* is a characteristic mass scale defining the theory (for example, the string scale in a heterotic string theory); λ is some dimensionless string coupling; $\ell_{pl} = G^{-1/2}$ is the Planck length, and R is a characteristic mean radius of the compact extra-dimensional manifold; C_{ij} are numbers defined by the particular theories and the constants A_i and B_i depend upon the topology of the additional (> 3) dimensions. The sum is over $j =$ leptons, quarks, gluons, $W^{\pm}$, Z and applies at energies above $\mu \sim 1GeV$ (Marciano 1984). The term Δ_i corresponds to some collection of string threshold corrections that arise in particular string theories or an over-arching M theory (Antoniadis & Quiros 1996). They contain geometrical and topological factors which are specified by the choice of theory. By differentiating these two expressions with respect to time (or space), it is possible to determine the range of self-consistent variations that are allowed. In general, for a wide range of super-symmetric unified theories, the time variation of different low-energy constants will be linked by a relationship of the form (where we consider $\dot{\beta}$ to denote the time derivative of β etc.)

$$\delta_0 \frac{\dot{\beta}}{\beta} = \delta_1 \frac{\dot{G}}{G} + \sum \delta_{2i} \frac{\dot{\alpha}_i}{\alpha_i^2} + \delta_3 \frac{\dot{m}_*}{m_*} + \sum \delta_{4j} \frac{\dot{m}_j}{m_j} + \delta_5 \frac{\dot{\lambda}}{\lambda} + ... \tag{93}$$

where $\beta \equiv m_e/m_{pr}$ (Drinkwater et al 1997). It is natural to expect that all the terms involving time derivatives of 'constants' will appear in this relation unless the constant δ prefactors vanish because of supersymmetry or some other special symmetry of the underlying theory. This relation shows that, since we might expect all terms to be of similar order (although there may be vanishing constant δ prefactors in particular theories), we might expect variations in the Newtonian gravitational 'constant', $\dot{G}/G$, to be of order $\dot{\alpha}/\alpha^2$.

8 Varying alpha – New Observational Limits

Quasar absorption systems present ideal laboratories in which to search for any temporal or spatial variation in the assumed fundamental constants of Nature. Such ideas date back to the 1930s, with the first constraints from spectroscopy of QSO absorption systems arising in the 1960s. An historical summary of the various propositions is given in Varshalovich & Potekhin (1995) and further discussion of their theoretical consequences is given in Barrow & Tipler (1986).

Recently, we have considered the bounds that can be placed on the variation of the fine structure constant and proton g factor from radio observations of atomic and molecular transitions in high redshift quasars (Drinkwater et al 1997). To do this we exploited the recent dramatic increase in quality of spectroscopic molecular absorption at radio frequencies, of gas clouds at intermediate redshift, seen against background radio-loud quasars. Elsewhere, we will consider the implications of simultaneous variations of several 'constants' and show how these observational limits can be used to constrain a class of inflationary universe theories in which small fluctuations in the fine-structure constant are predicted to occur.

The rotational transition frequencies of diatomic molecules such as CO are proportional to $\hbar/(Ma^2)$ where M is the reduced mass and $a = \hbar^2/(m_e e^2)$ is the Bohr radius. The 21 cm hyperfine transition in hydrogen has a frequency proportional to $\mu_p \mu_B/(\hbar a^3)$, where $\mu_p = g_p e\hbar/(4m_p c)$, g_p is the proton g-factor and $\mu_B = e\hbar/(2m_e c)$. Consequently (assuming m_p/M is constant) the ratio of a hyperfine frequency to a molecular rotational frequency is proportional to $g_p \alpha^2$ where $\alpha = e^2/(\hbar c)$ is the fine structure constant. Any variation in $y \equiv g_p \alpha^2$ would therefore be observed as a difference in the apparent redshifts: $\Delta z/(1+z) \approx \Delta y/y$. Redshifted molecular emission is hard to detect but absorption can be detected to quite high redshifts (see review by Combes & Wiklind, 1996). Recent measurements of molecular absorption in some radio sources corresponding to known HI 21 cm absorption systems give us the necessary combination to measure this ratio at different epochs.

Common molecular and HI 21 cm absorptions in the radio source PKS 1413+135 have previously been studied by Varshalovich & Potekhin (1996). They reported a difference in the redshifts of the CO molecular and HI 21 cm atomic absorptions which they interpreted as a mass change of $\Delta M/M = (-4 \pm 6) \times 10^{-5}$ but as we show above this comparison actually constrains $g_p\alpha^2$, not mass. Furthermore they used overestimates of both the value and error. They used the Wiklind & Combes (1994) measurement which had the CO line offset from the HI velocity by -11 kms^{-1}; a corrected CO measurement (Combes & Wiklind, 1996) shows there is no measurable offset. Furthermore, Varshalovich & Potekhin (1996) used the width of the HI line for the measurement uncertainty. Even allowing for systematic errors the true uncertainty is at least a factor of 10 smaller so these data in fact establish a limit of order 10^{-5} or better. This potential for improved limits has prompted the present investigation: previous upper limits on change in α are of order $\Delta\alpha/\alpha \approx 10^{-4}$ (Cowie & Songalia, 1995; Varshalovich, Panchuk & Ivanchik, 1996).

8.1 Comparison of HI and molecular systems

Our new more accurate redshift estimates (Drinkwater et al 1997) give a molecular redshift of 0.684680 ± 0.000006 and an a 21cm redshift of 0.684684 ± 0.000006 for the source $0218 + 357$ and a molecular redshift of 0.246710 ± 0.000005 and a 21cm redshift of 0.246710 ± 0.000004 for the source 1413+135. We can therefore combine the uncertainties in quadrature to give 1-sigma upper limits on the redshift differences. These give $|\Delta z/1 + z| < 5 \times 10^{-6}$ ($1.5\ kms^{-1}$) for both sources.

We must still consider the possibility that the molecular and atomic absorption arises in different gas clouds along the line of sight. This could explain any observed difference. However there is no measurable difference between the two velocities in our data, so we are probably detecting the same gas. The alternative would be that there was a change in the frequencies but that in both cases it was exactly balanced by the random relative velocity of the two gas clouds observed. We consider this very unlikely because of the small 1Kms^{-1} dispersion within single clouds.

We can now use the limits to $\Delta z/1 + z$ with the relationship to derive

1-sigma limits on any change in $y = g_p\alpha^2$: $|\Delta y/y| < 5\times10^{-6}$ at both $z = 0.25$ and $z = 0.68$. These are significantly lower than the previous best limit of 1×10^{-4} by Varshalovich & Potekhin (1996) (it was quoted as a limit on nucleon mass, but it actually refers to $g_p\alpha^2$).

As there are no theoretical grounds to expect that the changes in g_p and α^2 are inversely proportional, we obtain independent rate-of-change limits of $|\dot{g}_p/g_p| < 2 \times 10^{-15}\,\mathrm{y}^{-1}$ and $|\dot{\alpha}/\alpha| < 1 \times 10^{-15}\,\mathrm{y}^{-1}$ at $z = 0.25$ and $|\dot{g}_p/g_p| < 1\times10^{-15}\,\mathrm{y}^{-1}$ and $|\dot{\alpha}/\alpha| < 5\times10^{-16}\,\mathrm{y}^{-1}$ at $z = 0.68$ (for $H_0 = 75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $q_0 = 0$ assumed here). These new limits are stronger than the previous 1 sigma limit of $|\dot{\alpha}/\alpha| < 8 \times 10^{-15}\,\mathrm{y}^{-1}$ at $z \approx 3$ (Varshalovich et al. 1996).

The most stringent laboratory bound on the time variation of α comes from a comparison of hyperfine transitions in Hydrogen and Mercury atoms (Prestage et al. 1995), $|\dot{\alpha}/\alpha| < 3.7 \times 10^{-14}y^{-1}$, and is significantly weaker than our astronomical limit. The other strong terrestrial limit that we have on time variation in α comes from the analysis of the Oklo natural reactor at the present site of an open-pit Uranium mine in Gabon, West Africa. A distinctive thermal neutron capture resonance must have been in place 1.8 billion years ago when a combination of fortuitous geological conditions enriched the subterranean Uranium-235 and water concentrations to levels that enabled spontaneous nuclear chain reactions to occur (Maurette 1972). Shlyakhter (1976, 1983) used this evidence to conclude that the neutron resonance could not have shifted from its present specification by more than $5 \times 10^{-4}eV$ over the last 1.8 billion years and, assuming a simple model for the dependence of this energy level on coupling constants like α, derived a limit in the range of $|\dot{\alpha}/\alpha| < (0.5\text{–}1.0) \times 10^{-17}y^{-1}$. The chain of reasoning leading to this very strong bound is long, and involves many assumptions about the local conditions at the time when the natural reactor ran, together with modelling of the effects of any variations in electromagnetic, weak, and strong couplings. Recently, Damour & Dyson (1997) have provided a detailed reanalysis in order to place this limit on a more secure foundation. They weaken Shlyakhter's limits slightly but give a 95% confidence limit of $-6.7 \times 10^{-17}y^{-1} < \dot{\alpha}/\alpha < 5.0 \times 10^{-17}y^{-1}$. However, if there exist simultaneous variations in the electron-proton mass ratio this limit can be weakened.

These limits provide stronger limits on the time variation of α than the astronomical limits; however, the astronomical limits have the distinct advantage of resting upon a very short chain of theoretical deduction and are more closely linked to repeatable precision measurements of a simple environment. The Oklo environment is sufficiently complex for significant uncertainties to remain.

Unlike the Oklo limits, the astronomical limits also allow us to derive upper limits on any *spatial* variation in α. Spatial variation is expected from the theoretical result that the values of the constants would depend on local conditions and that they would therefore vary in both time and space (Damour & Polyakov 1994). The two sources for which we derived limits, 0218+357 and 1413+135, are separated by 131 degrees on the sky, so together with the terrestrial result, we find the same values of α to within $|\Delta\alpha/\alpha| < 3 \times 10^{-6}$ in three distinct regions of the universe separated by comoving separations up to 3000 Mpc. Limits on spatial variation of $g_p\alpha^2 m_e/m_p$ were previously discussed by Pagel (1977, 1983) and Tubbs & Wolfe (1980). We have improved on their limits by some 2 orders of magnitude but as our sources are at lower redshift, they are not causally disjoint from each other.

The high-redshift measurements are now approaching the best terrestrial measurements based on the Oklo data. These could be further improved by a factor of 2–5 with additional observations that would not be difficult to perform such as fitting the atomic and molecular data simultaneously, remeasuring the HI absorptions at higher spectral resolution.

8.2 Inflation

Inflation is something of a two-edged sword when it comes to discussing variations in constants. On the one hand there are potentials with multiple vacuum states which allow different parts of the universe to find themselves inheriting different suites of fundamental constants, with quite different values. On the other hand, if, as inflation leads us to expect, the whole of our observable universe is contained within the inflated image of a single causally connected region, then we should expect fundamental constants to reflect that single origin and to display spatial uniformity to very high precision. The key question is what precisely is that precision? A bench mark for the amplitude of possible variations is provided by the amplitude of temperature fluctuations in the microwave background, $\Delta T/T \simeq 10^{-5}$. We would expect fluctuations in the fine structure constant created at the end of inflation to have an almost constant curvature spectrum (because of the time-translation invariance of almost de Sitter inflation) with an amplitude below that of 10^{-5}. An interesting feature of the new astronomical observations described above is that, for the first time, they take the observational limits on spatial variations in α ($|\Delta\alpha/\alpha| < 3 \times 10^{-6}$) into that regime where they may be constraining the underlying theories more strongly than are the COBE observations.

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