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REVIEW ARTICLE

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Determination of the gravitational constant *G*

Abstract A precise knowledge of the Newtonian gravitational constant G has an important role in physics and is of considerable meteorological interest. Although G was the first physical constant to be introduced and measured in the history of science, it is still the least precisely determined of all the fundamental constants of nature. The 2002 CODATA recommended value for G, $G = (6.6742 \pm 0.0010) \times 10^{-11} \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$, has an uncertainty of 150 parts per million (ppm), much larger than that of all other fundamental constants. Reviewed here is the status of our knowledge of the absolute value of G, methods for determining G, and recent high precision experiments for determining G.

Keywords gravitational constant, fundamental constant

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1 Introduction

Sir Isaac Newton's gravitational law from his *Philosophiae Naturalis Principiae Mathematica* in its modern form is known as

$$|\boldsymbol{F}| = G\frac{Mm}{r^2} \tag{1}$$

It describes the attractive force between the two masses m and M separated by distance r. The strength of this force is defined by the constant of proportionality G, known as the gravitational constant. Besides the speed of light c, G has the longest history of measurements. In 1798 Henry Cavendish published results on his experiments to obtain the density of the Earth [1]. These results are nowadays known as the first precise ones on the gravitational constant.

An accurate knowledge of G is not only important from the point of view of modern physics, but is also significant for astronomy, geophysics and practical purposes, particularly when finding the density and density distributions of the interiors of the Earth, Moon, planets and stars [2]. Theorists who work on the unification of the four known forces need to know G to gauge the success of their theories. Cosmologists and astronomers are concerning astronomical structure and the early universe based on G. The factor GM of astronomical objects can be determined extremely well. A better knowledge of G leads to a better knowledge of M, which in turn leads to a better physical understanding of celestial bodies. Uncertainties of density and elastic parameters of the Earth are directly related to the uncertainties on G. The calibration of gradiometers, used for geophysical prospecting, is limited by the precision to with which G is known [3]. Furthermore, a precise knowledge of G is of considerable metrological interest, and it provides a unique as well as a valuable measurement challenge that sharpens and prepares experimental skill to better deal with a variety of precise and null experiments [4]. For those reasons, great efforts have been made over two centuries to obtain a reliable value. Since Cavendish reported the first experimental value of G, nearly 300 different measurements of G have been made over the years, including several in which the objective was to search for some type of variation in G [5].

At the start of the 20th century, the accepted value of G was 6.66×10^{-11} $\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$, which rested upon the independent work of Boys [6] and of Braun *et al*. [7]. It was displaced by the result of Heyl and Chrzanowski [8, 9], who concluded that $G = (6.673 \pm 0.003) \times10^{-11}$ $\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$. The beginning of the modern era of measurements of the absolute value of G is usually associated with the appearance of their results. During the 1960s and 1970s, many experimental works on the measurement of G were carried out. The Committee on Data for Science and Technology of the International Council for Science (CODATA) released a 1986 ac-

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cepted value $G = (6.672\ 59 \pm 0.000\ 85) \times 10^{-11} \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$ [10]. This value was obtained at NIST by Luther and Towlers in 1982 in a collaboration between NIST and the University of Virginia [11], but with its uncertainty doubled to 128 ppm. In the following 10 years, the nine measurements of G [4, 12—19] had produced values that differ wildly from each other and the 1986 CODATA value. This situation suggests a new value for G should be recommended. In 1998, the CODATA recommended value for G was $(6.673 \pm 0.010) \times 10^{-11}\ \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$ [20]. It is noted that this recommended CODATA value for G is essentially the same as the 1986 value but the relative uncertainty placed on this value has been expanded to 1500 ppm.

The 1998 recommended value was selected by the Task Group after a careful review of the status of measurements of G. After that, some new results for G have been obtained by different research groups. Based on the weighted mean of eight values obtained in the past several years, the Task Group has taken the 2002 recommended value of gravitation as [21]:

$$G = (6.6742 \pm 0.0010) \times 10^{-11} \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2} \tag{2}$$

with a relative uncertainty of 150 ppm, which exceeded the 1998 recommended value of 180 ppm by a fractional amount. Although the situation with measurements of G has improved considerably since 1998, the values in CODATA 2002 are still in poor agreement. Even for the four most precise values of G with their assigning uncertainties within 50 ppm [22, 23, 24, 25], they are only consistent with each other in the range of about 200 ppm. In spite of these many strenuous efforts, G is the least precisely determined of all the fundamental constants of nature.

Here are some reasons why measuring the gravitational constant is so difficult [26, 27]. First, gravitation is the weakest of all four known fundamental forces. For example: gravitational force is weaker than electromagnetic force between a proton and an electron by about 10^{40}. Second, gravity is a force that cannot be screened, making a precise measurement difficult to decouple from the environment. If one wanted to produce a region that was free of electric fields, one would just have to construct a conducting shell around the region. This won't work with gravitational forces. Therefore, significant efforts are made to isolate such experiments from cultural and natural sources of gravity gradients and time-varying fields. Third, there is no known quantitative theoretical relationship between the Newtonian constant of gravitation G and other fundamental constants. Hence, its value cannot be estimated in terms of other quantities and only can be determined according to the Newtonian inverse square law. By the way, Newton's law includes only point masses, no real bodies. One of the greatest difficulties in any G measurement is determining with sufficient accuracy the dimensions and density distribution of the test masses and attracting masses. Fourth, the instruments chosen for measuring G such as torsion pendula are subject to a variety of parasitic couplings and systematic effects, which ultimately limit their usefulness as transducers of the gravitational force. Beam balances, vertical and horizontal pendula, and other sensitive mechanical devices are also pressed to be limits of their performance capabilities when employed for this purpose. Finally, the absolute measurements increase the difficulties. The value of G is defined by three fundamental quantities-time, length and mass. The measurement of G requires that absolute values should be measured for the masses of attracted bodies and attracting bodies, the separation, the period of motion of the mechanical oscillator, and so on, all of which may give rise to considerable experimental difficulties.

Many reviews are of interest in pursuing the measurement of G. Earlier, Poynting [28] and Mackenzie [29] summarized the contemporary knowledge of G in 1894 and 1900, respectively. Recently, de Boer [30] made a survey of recent major experiments and Gillies [31] published a very comprehensive report giving an index of the measurement of G, containing over 1200 references. Chen and Cook [2] give an extensive discussion of laboratory techniques employed in measurements of G. Gillies [5] summarized the recent measurements of G and the search for variation in G. In this paper, we will review the methods of determining G, recent high-precision experiments of G.

2 Methods of determining the absolute value of *G*

A determination of G is conceptually easy: measure the force between two known masses arranged in a known geometry. But the measuring gravitational force with high precision in practice is a real challenge. Experiments determining G carried out hitherto can be roughly grouped as follows:

(a) Determining G with torsion pendula;
(b) Determining G with beam balances;
(c) Determining G with free-fall method;
(d) Determining G with a Fabry-Perot microwave resonator;
(e) Determining G with geophysical methods; and
(f) Determining G in space.

The geophysical methods use the Earth and mountains or parts of the Earth's crust as the attracting masses, and were commonly used in the early experiments. Using a large natural mass as source mass, geophysical methods can greatly increase the detected gravitational force. But, it is difficulty to measure a high precise value of G due to the large unknown uncertainty in source mass. Measurements in space are in progress. Sanders and Gillies [32] reviewed the various proposals for experiments to measure G in space, such as the NEWTON proposal of the University of Pisa [33], the satellite energy exchange (SEE) proposal of the University of Tennessee [34], and the G/ISL test proposed for incorporation into the STEP mission [35]. We will briefly review the other four methods used in the laboratory in the following sections.

2.1 Determining G with torsion pendula

The measurement of the weak interaction between test masses and source masses is always in competition with the gravitational forces of the Earth on the test masses being studied, and there is no shielding of this force. However, the torsion pendulum is able to place the Earth's gravitation force in an orthogonal relationship to the horizontal plane in which the signal of interest occurs. The torsion pendulum was designed by Rev. John Mitchell in around 1750 and independently by Charles Augustin de Coulomb in 1785. It was used by Cavendish to determine the mean density of the Earth in 1798 [1]. Since then, the torsion pendulum is used as the sensor to test gravitational force in most cases, and the principle of the apparatus remained practically the same, whereas big efforts were taken on the improvement of experimental details. There are only a few methods, which do not rely on a torsion pendulum [5].

In general the torsion pendulum method uses two test masses at each end of a torsion beam that is suspended with a torsion fiber in its middle as in Fig. 1. Optical instruments, like an optical lever, are often used to monitor the movement of the pendulum. Torsion pendulum experiments measure the torque produced by source masses interacting with a dipole or higher order moment of test masses. Torsion pendulum can be used in a static or a dynamic mode to measure G as shown in Fig. 2. In the static mode method, the gravitation constant G is determined by measuring deflection of the pendulum, while the dynamic mode method determines G by measuring the change in period.

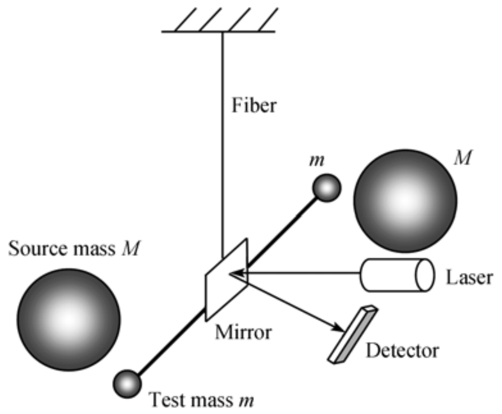

Fig. 1 Torsion pendulum used in measuring the gravitational interaction between source masses and test masses. The two small spheres are suspended with a torsion fiber as the test masses. An optical lever is used to monitor the movement of the pendulum.

There are two ways in the static mode, the direct deflection method and compensated torque method, to measure the gravitational torque. The direct deflection method detects the pendulum equilibrium position. The first determination of G using the direct deflection method was made by Cavendish in 1798. After Cavendish, the most careful work was done by Boys in 1895 [6]. The direct deflection method is sensitive to gravitational torque, but it also suffers from many shortcomings, such as absolute measurement of the deflection angular, the inelastic behaviors of the fiber and the local gravitational gradient.

The compensated method uses a known compensated torque acting on the pendulum to balance the detected gravitational torque to keep it at the same equilibrium position. A compensated torsion pendulum method was employed by Fitzgerald and colleagues to measure G at the Measurement Standards Laboratory, Industrial Research Ltd, in New Zealand. They used a static electronic torque produced by the electrometer to balance the gravitational torque acting on the test masses to maintain it at a constant angular position. The static electronic torque was calibrated by using an acceleration method. They measured G with a combined standard uncertainty of 95 ppm, 90 pmm, and 41 ppm, in 1995, 1999, and 2003 [13, 25, 36], respectively. The main features of a compensated method are that the torsion fiber does not twist during the measurement and the suspended mass remains stationary–these can reduce the effects of the inelastic behaviors of the fiber and the local gravitational gradient on the measurement.

The test mass can be supported in other ways than by a torsion fiber provided a very small restoring force comparable to the gravitational attraction is produced by the source masses. At the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig, de Boer, Haars and Michaelis [37] carried out a similar experiment for which planning began in 1976. They used a compensated torsion pendulum with a mercury bearing, rather than a fiber, to support the test mass. This type of bearing is nearly free of static friction. The experiment was carried out by means of an apparatus consisting of a fibreless torsion pendulum carried by the buoyancy of a floater in mercury. The motion of the pendulum is sensed by a differential laser interferometer. Movable attracting masses can be used to apply a calculable alternating torque to the pendulum via their gravitational interaction with the attracted masses fixed to it. Control signals applied to the vanes of a quadrant electrometer provide an electrostatic restoring torque used to counteract the gravitational torque and the null motion of the pendulum. The difference in the electrostatic torques measured when the attracting masses are moved from one of their test positions to another, will be equal to the associated change in the gravitational torque acting on the pendulum. They finally obtained the value of G in 1994 with a combined standard uncertainty of 83 ppm [12], but this value is larger than all other group values. It was in substantial disagreement with the 1986 recommended value. The Task Group concluded that the PTB result should not be taken into account in the determination of the 2002 recommended value [21].

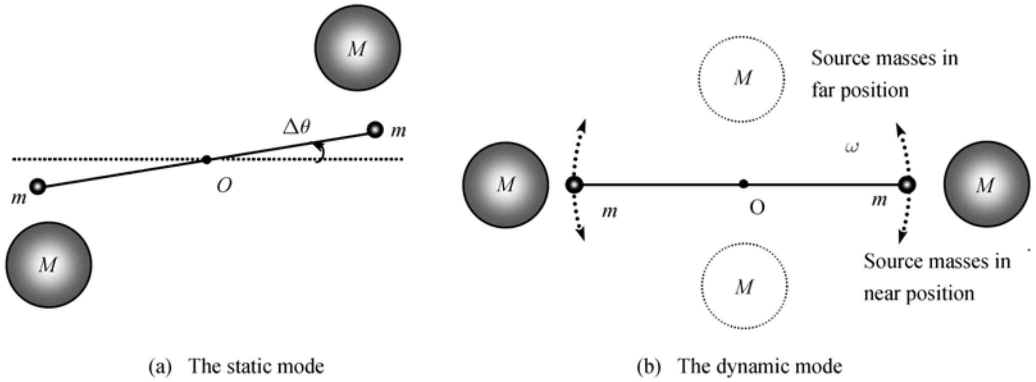

Fig. 2 Torsion pendulum used in measuring the gravitational interaction between source masses and test masses. In static mode method, the gravitation constant *G* is determined by measuring the deflection of the pendulum, while the dynamic mode method is determining *G* by measuring the change in period.

The high sensitivity torsion pendulum means that there is a substantial change of the free period when the test mass is attracted by source masses. In the dynamic mode, the change in the natural frequency of oscillation of the torsion pendulum due to the source masses is determined with the masses in different configurations, referred to as "near" position (the pendulum is in line with the source masses, or represented as $\theta = 0$) and "far" position ($\theta = \pi/2$) in Fig. 2. This method also called time-of-swing method or period method. At both configurations, the torsion pendulum oscillates in the gravitational field produced by the source masses and hence a "gravitational torsion constant" $K_g(\theta)$ is added to the torsion coefficient of the fiber K_f. As a result, the oscillation frequency is changed. In the near position, the gravitational attraction of source masses reduces the period of the oscillation and the period is extended with source masses in the far position. The value of gravitational constant *G* can be calculated by determining the change in the oscillation period due to the source masses's gravitational attraction.

The time-of-swing method was first used by Reich in 1852 [38], then developed by Heyl and co-workers in 1930 and 1942 [8, 9], and is popularly used by most recent high precision measurements [5]. As compared with the direct deflection method, the time-of-swing method is not so sensitive to the drift of the equilibrium position of the pendulum. But it will suffer the nonlinear effect and anelasticity effect at the same time.

In the deflection method, the gravitation torque is determined by measuring displacements, while the period method determines *G* by measuring velocities. Another dynamic method is the angular acceleration method. This method was first proposed by Beam *et al*. in 1965 [39], and Rose *et al*. had performed an experiment and determined *G* with an uncertainty of 1800 ppm in 1969 [40]. Gundlach and Merkowitz used the angular acceleration feedback method to measure *G* with an uncertainty of 14 ppm at the University of Washington in 2000 [22].

The most precise measurements of *G* are relied on the torsion pendulum. The beam balance, the gravimeter and the Fabry-Perot microwave resonator are also adopted in experiments of determining the value of *G*.

2.2 Determining *G* with beam balances

The beam balance is a device commonly used for mass comparisons. The principle of the beam balance method is using a known force to balance the gravitational interaction between the test masses and the source masses by means of a high sensitive beam balance. With a properly designed beam balance, a resolution of $10^{-10}\mathrm{m \cdot s^{-2}}$ can be achieved [41]. However, it is very hard to reach a torsion pendulum's resolution of $10^{-12}\mathrm{m \cdot s^{-2}}$. Because the resolution of a beam balance is limited, a heavy source mass should be used to produce an adequate gravitational attraction. But moving a heavy mass will cause serious local tilting of the ground. Reducing the effects of the tilt and vibration of the ground is very important in the beam balance method. High accuracy calibration of the balance force is another difficultly problem in this method. Determining *G* with beam balances has been used by Poynting in 1891 [28], Richaz and Krigar-Menzel in 1898 [42], Speake and Gillies in 1987 [41], and Kunding *et al*. in 1998 and 2002 [18, 24].

2.3 Determining *G* with free-fall method

The free-fall method depends on our ability to measure the amount that an external source mass changes the acceleration of a freely falling object. First a source mass is placed above the region in which the test mass falls. Here the gravitational pull of the source mass acts in the opposite direction to the attraction of the Earth, decreasing the downward acceleration of the test mass. Second the source mass is placed below the drop region, where it increases the

Earth's attraction and the acceleration of the falling mass. If the change in acceleration can be measured accurately, and if the geometry of the source and test masses are well known, then one can determine G. The free fall method is unique in that it uses an unsupported test mass to sense the gravitational force. This method, therefore, has a very different set of systematic errors than other methods.

A research group at the Joint Institute for Laboratory Astrophysics in Boulder, Colorado, reported a free-fall determination of G [4]. Their method involves using a laser interferometer system to track the motion of a test mass that is repeatedly dropped in the presence of a locally induced perturbing gravity field. This gravitational field is produced by a 500 kg tungsten ring-shaped source mass located alternately above and below the dropping region. The experiment was concluded in a differential mode in which the acceleration of the test mass was alternately increased and then decreased by the source masses. The acceleration of the test mass was measured with a FG-5 absolute gravimeter, and the local background gravity concurrent with the G experiment was measured by a superconducting relative gravimeter. Their final result is $G = (6.6873 \pm 0.0094)\times 10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$ with a relative uncertainty of 1400 ppm.

Based on the techniques of the atomic interferometry, a new free-fall experiment with cold atoms is used in determining G. A further experiment for the determination of G to a precision of 10^{-3} with the existing gravity gradiometer of M. Kasevich is in progress at Stanford University [43]. The MAGIA (Misura Accurata di G mediante Interferometria Atomica) is being proposed by Tino *et al*. at University Firenze [44]. They are setting up an experiment in which free-falling atoms are used to probe the gravitational acceleration originating from nearby source masses. The aim of the MAGIA experiment is the measurement of G with a precision of 10^{-4}. At present these techniques are not competitive with mechanical detectors, such as the torsion pendulum.

2.4 Determining G with a Fabry-Perot pendulum

The principle of the Fabry-Perot pendulum method is as follows. Two mirrors forming a Fabry-Perot microwave resonator are suspended as the test masses. By placing a source mass system on the axis defined by the line joining the two test masses, the distance between the two suspended test masses was changed. The induced change of resonance frequency can be measured with a high resolution and is used to calculate the gravitational constant G. The Fabry-Perot Pendulum gravimeter was used to measure G at the University of Wuppertal (UW), Germany, by Meyer *et al.* [14, 45, 46]. The schematic view of the UW experiment is shown in Fig. 3. Two source masses provide a gravitational field to influence the relative positions of two pendula that constitute the defining walls of an open microwave cavity. Both pendula are suspended by two loops of tungsten wire, which are mounted from a suspension platform. Different separations between the test mass and the resonator in the range 0.6—2.1 m have been chosen to measure the gravitational force. No significant deviations from the inverse square law are observed on a level of a few parts in 10^4 in the near range, and finally the result for G is $G = (6.6719 \pm 0.0008) \times 10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$ with a combined standard uncertainty of 110 ppm. In Ref. [46], they reported a new preliminary value for G from 1998 measurements by the improved experiment: $G = (6.6735 \pm 0.0011 \pm 0.0026) \times 10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, where the first uncertainty is the statistical one and the second value is the systematic one.

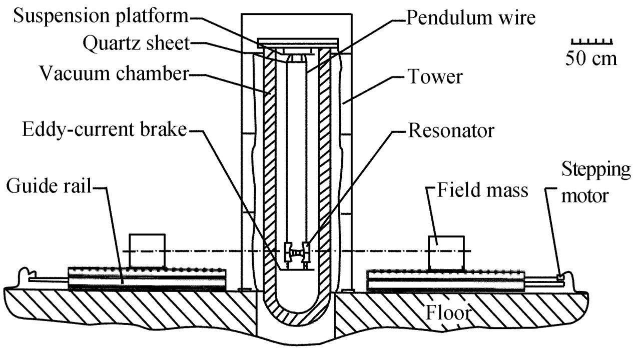

Fig. 3 The Fabry-Perot Pendulum gravimeter was used to measure G at the University of Wuppertal. Two source masses provide a gravitational field to influence the relative positions of two pendula that constitute the defining walls of an open microwave cavity. The induced change of resonance frequency can be measured with a high resolution and is used to calculate the gravitational constant G. (Schematic diagrams from Ref. [46], copyright of IOP Publishing Ltd)

3 Recent high-precision experiments

The absolute value of the gravitational constant G is the least precisely known among the fundamental physical constants. In 1998 the CODATA Task Group on Fundamental Constants decided to increase the uncertainty in the recommended value of G to 1500 parts per million (ppm). Since then, four new values of G have been reported with relative uncertainties below 50 ppm. These, with their assigned uncertainties, are the experiments carried out by Gundlach and Merkowitz with 14 ppm at the University of Washington in 2000 (UWash-00) [22], Quinn *et al*. with 41 ppm at the Bureau International des Poids et Measures in 2001 (BIPM-01) [23], Schlamminger *et al*. with 33 ppm at University of Zurich in 2002 (UZur-02) [24], and Armstrong and Fitzgerald with 40 ppm at the Measurement Standards Laboratory in 2003 (MSL-03) [25]. We are performing new experiments aimed at determining G within 50 ppm at Huazhong University of Science and Technology from 1998.

3.1 UWash-00 experiment

Gundlach and Merkowitz used the angular acceleration feedback method to measure G at the University of Washington in 2000. At the heart of the apparatus is a torsion pendulum placed on a turntable located between a set of source masses. The turntable is first rotated at a constant rate so that the pendulum experiences a sinusoidal torque due to the gravitational interaction with the source masses. A feedback is then turned on which changes the rotation rate so as to minimize the torsion fiber twist. The resulting angular acceleration of the turntable, which is now equal to the gravitational angular acceleration of the pendulum, is determined from the second time derivative of the turntable angle readout.

The schematic view of the Uwash-00 experiment is shown in Fig. 4. The torsion pendulum turntable consists of an air bearing, a precision angle encoder, and an eddy- current motor. The torsion pendulum is located in an aluminum vacuum chamber and is surrounded by a μ-metal shield. The pendulum is hung from a 41.5 cm long, 17 μm diameter tungsten fiber, which is attached to a swing damper. The pendulum is a 1.506 mm thick, 76 mm wide, and 41.6 mm high glass plate with a thin gold coating. The small pendulum deflection angle is sensed with an autocollimator using four reflections off the pendulum plate. The vacuum chamber is fastened to a well controlled turntable that, when activated, rotates the torsion pendulum between four stainless steel source masses with 124.89 mm in diameter and of mass $\approx$ 8.140 kg. Two of the source masses rest on a horizontal plate around and just above the pendulum, and two on a horizontal plate around and just below the pendulum.

They performed two sets of experimental data with different source masses. Three different values of G resulted from each set, each value being from a combination of a pair of source masses configurations that together eliminate the effect of accelerations due to the attractor plates and turntable themselves. The three values in a set were obtained using different, optimally chosen orientations of the spherical source masses in order to reduce the influence of imperfections in the shape of the spheres and possible nonuniform densities. After combining the two G values obtained with different source masses, their experimental value for the gravitational constant is

$$G = (6.674\ 215 \pm 0.000\ 092) \times 10^{-11}\,\mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2} \tag{3}$$

with a standard uncertainty of 14 ppm [22].

The authors think that this method has the following advantages [22, 47, 48]: Possible systematic effects arising from mass distribution could be minimized by choosing a plate as a test mass, which can be assumed as two-dimensional so that mass distribution becomes insignificant. Since the torsion fiber does not experience any appreciable deflection, this technique is independent of many torsion fiber properties including anelasticity.

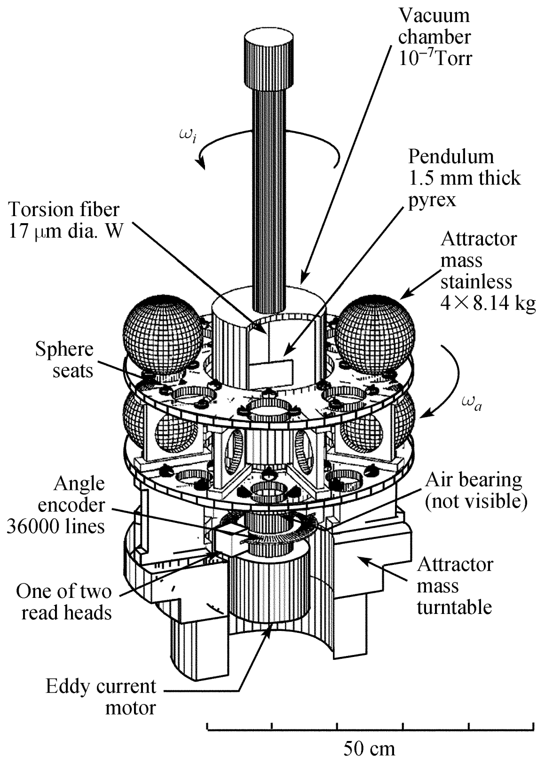

Fig. 4 The schematic view of the UWash-00 experiment. A plate as a test mass is accelerated by rotating four spherical source masses and its swing is balanced by a servo on the suspension platform of the test masses. (Schematic diagrams from Ref. [22], copyright of the American Physical Society)

3.2 BIPM-01 experiment

The BIPM-01 experiment for the measurement of G used a torsion-strip pendulum worked in two substantially independent ways, *e.g.*, the electrostatic compensation method and the direct deflection method.

The schematic view of the BIPM-01 experiment is shown in Fig. 5. The test masses are mounted on a radius of about 120 mm around the periphery of an aluminum-alloy disk suspended from the torsion strip inside a vacuum chamber. Outside the vacuum chamber, the four source masses are mounted symmetrically on a radius of about 214 mm on an aluminum-alloy carousel belt-driven by a stepping motor. When aligned with the test masses, the source masses produce no torque on the pendulum. When rotated in either direction by 18.7° the gravitational torque is at its maximum. In the electrostatic compensation method, the gravitational torque of the source masses is balanced by an electrostatic torque acting directly on the test masses. In the direct deflection method the torsion balance is allowed to move in response to the movements of the source masses. At equilibrium, the applied gravitational torque is balanced by the suspension stiffness. The angular deflection is related to the applied gravitational torque.

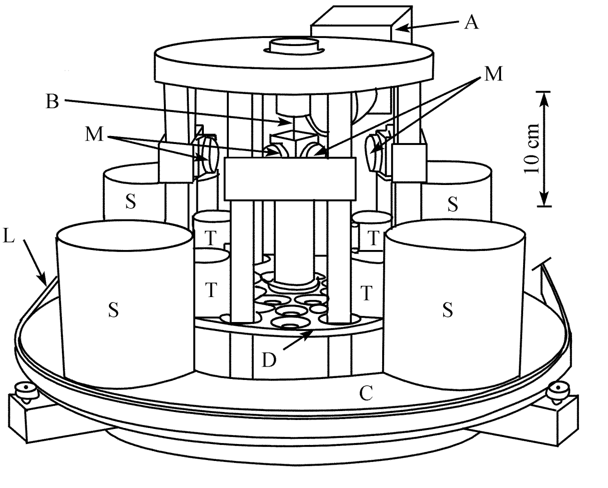

Fig. 5 The schematic view of the BIPM-01 experiment. The test masses (T) are mounted on an aluminum-alloy disk (D) suspended from the torsion strip (B) inside a vacuum chamber. Outside the vacuum chamber, the four source masses (S) are mounted symmetrically on an aluminum-alloy carousel (C) belt-driven (L) by a stepping motor. The torsion-strip pendulum worked in the electrostatic compensation method and the direct deflection method. (Schematic diagrams from Ref. [23], copyright of the American Physical Society)

Their result for the electrostatic compensation method is $G = 6.675\ 53\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$ with a standard uncertainty of 60 ppm and for the direct deflection method is $G = 6.675\ 65\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$ with a standard uncertainty of 67 ppm. The two measurements have a correlation coefficient of −0.18. This coefficient, defined as the covariance of the two measurements divided by the geometric mean of their variances, is a measure of the independence of the two methods. They combined these two values, and the obtained weighted mean of G is

$$G = (6.675\ 59 \pm 0.000\ 27)\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2} \tag{4}$$

with a standard uncertainty of 41 ppm [23].

The BIPM-01 experiment has the following principal features [23]: (1) A four-mass configuration to give a much reduced sensitivity to external gravitational fields; (2) A torsion strip to give much improved stability with practically no dependence on the material properties of the strip; (3) A gravitational signal torque larger by about 4 orders of magnitude than in most previous comparable experiments; (4) The result is based on two methods of operation, the electrostatic compensation method and the direct deflection method; (5) Dimensional metrology that is quick and accurate by having the whole apparatus mounted on the base of a coordinate measuring machine.

3.3 UZur-02 experiment

In the UZur-02 experiment of measuring G, a modified commercial single-pan balance is used to measure the change in the difference in weight of two cylindrical test masses when the position of two source masses is changed.

The schematic view of the UZur-02 experiment is shown in Fig. 6. The mass setup consists of two movable tanks, labeled field masses (FM) and two smaller masses, called test masses (TM). The FM's are hollow cylinders that can be moved vertically between two positions such that the TM's pass through the central hole. A device, called a mass exchanger, allows either one of the two TM's to be connected to the beam balance. In each FM position the weight difference of the two TM's is determined. The weight difference is the signal of interest. From the known mass distribution, the amplitude of the signal, and the value of the local acceleration, a value for the gravitational constant G can be determined.

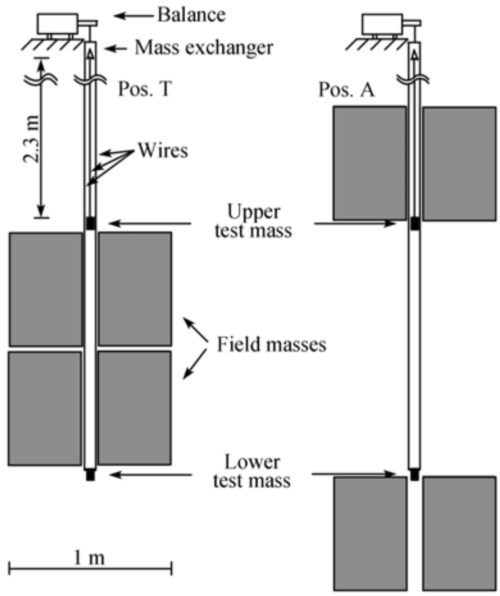

Fig. 6 The schematic view of the UZur-02 experiment. A modified commercial single-pan balance is used to measure the change in the difference in weight of two cylindrical test masses when the position of two source masses is changed in two positions together and apart. (Schematic diagrams from Ref. [24], copyright of the American Physical Society)

Their preliminary result was published in 1999 with a relative uncertainty of 220 ppm [18]. In 2002, their new result is the weighted mean, with correlations appropriately taken into account, of three values obtained from three series of measurements denoted Cu, Ta I, and Ta II, respectively. The designation Cu means that the test masses were gold plated copper, and the designation Ta means that they were tantalum. The position of the field masses was the same for the Cu and Ta II series of measurements and different for the Ta I series. The three values obtained are $G = 6.674\ 03 \times 10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, $G = 6.674\ 09\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, and $G = 6.674\,10\times 10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, respectively, all in good agreement. They averaged these values weighted with their uncertainties. As a result the final value of G was found to be

$$G = (6.674\ 07 \pm 0.000\ 22)\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2} \tag{5}$$

with a relative standard uncertainty of 33 ppm [24].

The UZur-02 experiment differs from the torsion pendulum experiments in the following points [17, 24]: (1) The measurement was performed parallel to the local acceleration, (2) The gravitational attraction was measured at an effective distance of about 1 m, and (3) The gravitational force was several orders of magnitude larger due to large masses. The authors think that the systematic uncertainties were considerably different than those of the torsion pendulum experiments.

3.4 MSL-03 experiment

The MSL-03 experiment for determining G is based on a torsion pendulum compensated such that the torque produced by the gravitational attraction between the torsion pendulum masses is balanced by an electrostatically induced torque.

The schematic view of the MSL-03 experiment is shown in Fig. 7. It uses two large cylindrical source masses (~27 kg each) to produce a gravitational attraction on a ~500 g cylindrical small test mass made of copper. The test mass is suspended from a ~1 m long tungsten fiber with a rectangular cross section of 300 μm×17 μm so that it is free to rotate in response to the gravitational attraction of the source masses. This rotation is detected by an autocollimator viewing a mirror attached to the test mass. The signal from the autocollimator goes to a feedback control system to generate a voltage applied to an electrometer. This produces an electrostatic force on the test mass that compensates the gravitational attraction so that the fiber is not required to twist. At each large source mass position, positive and negative voltages are used to permit the calculation of the contact potential V_c between the stainless steel of the electrometer plates and the copper small test mass. The electrostatic torque constant is determined in a separate experiment by measuring the angular acceleration of the test mass when the source masses are removed and a voltage U_A is applied to the electrometer.

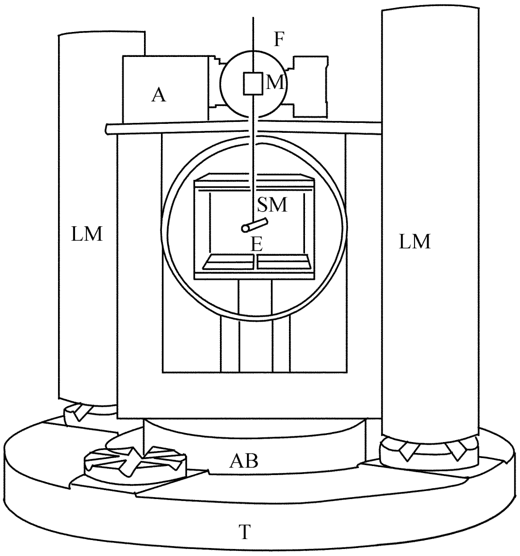

Fig. 7 The schematic view of the MSL-03 experiment. The two large cylindrical source masses (LM) are used to produce a gravitational attraction on a cylindrical small test mass (SM) which is suspended by a tungsten torsion-strip fiber (F). A voltage is applied to an electrometer (E) to produce an electrostatic force on the test mass that compensates the gravitational attraction. (Schematic diagrams from Ref. [25], copyright of the American Physical Society).

The MSL group have been making precise measurements of G using the Measurement MSL torsion pendulum for over ten years [13, 25, 36]. Over that time, they have improved the apparatus, the measurement method, and the analysis of the results. In 2003, they reported their recent four measurement results as: $G=6.673\ 59\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, $G=6.673\ 98\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, $G=6.673\ 99\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, and $G=6.673\ 92\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$, where the first three were obtained using the copper source masses and the last using the stainless steel source masses. They had combined these four separate values, and obtained their final result of:

$$G=(6.673\ 87\pm0.000\ 27)\times10^{-11}\mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2} \tag{6}$$

with a relative standard uncertainty of 41 ppm [25].

The MSL-03 experiment uses a compensated method. Hence, the torsion fiber does not twist during the measurement and the suspended mass remains stationary. This was done to reduce the effects of anelastic behaviour of the fiber and the local gravitational gradient on the measurement.

3.5 HUST experiment

Based on time-of-swing method, a preliminary result of HUST-99 was obtained with a relative uncertainty of 105 ppm [19, 49]. Following the publication of the HUST-99 result, it was decided to look further for systematic effects in the measurement. We have studied the physical properties of torsion pendula, such as the nonlinearity and thermoelasticity of the torsion fiber [50, 51].

Our new HUST experiment also adopts the time-of-swing method but with a flat plate shape test mass. The change in the oscillation period of the torsion is measured to calculate the value of G with the source masses alternately in the near and far positions by rotation of a turntable. Our HUST experiment has the following features: (1) A flat plate torsion pendulum has less vibration modes and improves the stability of the period as well as minimizes the uncertainty of the inertial momentum of the pendulum; (2) The spherical source masses easily determine and minimize the uncertainties of the eccentricity of the mass center from the geometrical one; (3) Both the test and source masses are all set in a vacuum vessel to improve the accuracy of measuring the relative positions between them; (4) The configuration of the pendulum system is remotely operated to avoid the disturbances due to the experimenter approach. Furthermore, the temperature is continuously monitored with an accuracy of

0.01 °C during the experiment to minimize the thermal effect. The experimental data, which were taken alternately with the source masses in "near" and "far" positions, shows that the statistical uncertainty of G is less than 50 ppm. A more detailed systematic error is still being investigated.

4 Conclusions

The measurement of the Newtonian gravitational constant G is a problem that has been under study for many years. Many experimental values of G have been reported recently by using a variety of methods, but these results fail to converge within their error. The large spread in results compared to small error estimates indicates that there are unknown systematic errors in various results, which could be: disturbances of the ground; influence of the ambient temperature; inhomogeneity of the masses; gradients in the gravitational field; changes of the surrounding field; magnetic and electrical influences.

We hope that the experimental values for G will agree with each other due to the effort of the experimenters.

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