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Project SEE (Satellite Energy Exchange): an international effort to develop a space-based mission for precise measurements of gravitation

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Abstract. Project SEE (Satellite Energy Exchange) is an international effort to develop a space-based mission for precise measurements of gravitation. Gravity is the missing link in unification theory. Because of the unique paucity of knowledge about this, the weakest of all known forces, and because gravity must have a key role in any unification theory, many aspects of gravity need to be understood in greater depth. A SEE mission would extend our knowledge of a number of gravitational parameters and effects, which are needed to test unification theories and various modern theories of gravity.

SEE is a comprehensive gravitation experiment. A SEE mission would test for violations of the equivalence principle (EP), both by inverse-square-law (ISL) violations and by composition dependence (CD), both at ranges of the order of metres and at ranges on the order of R_E. A SEE mission would also determine the gravitational constant G, test for time variation of G, and possibly test for post-Einsteinian orbital resonances. The potential finding of a non-zero time variation of G is perhaps the most important aspect of SEE. A SEE mission will also involve a search for new particles with very low masses, since any evidence of violations of the EP would be analysed in terms of a putative new Yukawa-like particle.

Thus, SEE does not merely test for violations of general relativity (GR); SEE is a next-generation gravity mission.

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1. Introduction

Gravity is the missing link in unification theory. The proposals and experiments reported here in this special issue of *Classical and Quantum Gravity* are aimed at providing the experimental

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data which are needed to better characterize this elusive force so that it can be incorporated into a scheme which embraces all the known forces.

Project SEE (Satellite Energy Exchange) is a comprehensive gravitation experiment. The main ideas for a SEE mission are laid out in the original SEE proposal published in 1992 (Sanders and Deeds 1992a, b, 1993, Sanders *et al* 1993). A series of early analyses by the Moscow group of the Russian Gravitational Society quickly followed (Alexeev *et al* 1993a, b, 1994, Bronnikov *et al* 1993a, b, Melnikov *et al* 1993). The most comprehensive published paper to date on the subsequent evolution of the SEE concept is the report to the Cavendish commemorative conference in November 1998 (Sanders *et al* 1999). In the present paper we recap on these earlier papers very briefly and then concentrate on $\dot{G}$.

The discovery that a fundamental 'constant' of nature is not in fact constant would have profound significance scientifically; few things could do more could do more to invigorate interest in the new theories, most of which do, in fact, predict time variation of G and other fundamental 'constants'. A finding of non-zero $\dot{G}$ would of course require extensions of general relativity, since it assumes a constant value of G. More broadly, this would clearly mark the boundaries where general relativity is valid, and signify the onset of new physics.

Of almost equal importance is that the SEE satellite is a powerful instrument in the search for new, very-low-mass particles. The relatively small separation between the two SEE test bodies makes it possible to observe a new Yukawa particle with masses of up to $\sim 10^{-6}$ eV. SEE will also search for evidence of a new Yukawa-type particle with a range of R_E or more, which requires much smaller masses. The interaction between the Earth and an orbiting test body can detect a new Yukawa particle only if its mass is very tiny indeed (about 10^{-11} eV), i.e. small enough to satisfy the Heisenberg uncertainty principle if the particle is travelling at close to the speed of light. Thus, the short-range EP tests by SEE are crucial complements to the long-range ($\sim R_E$) tests.

There is already some experimental evidence that existing theories may not adequately describe gravity, or what appears to be gravity (see, in particular, Thieberger 1987, Michaelis 1995, Michaelis *et al* 1995, Achilli *et al* 1997). Unless all of these experiments are simply wrong, they must be taken as clues for the existence of a new particle or for fundamental flaws in the theory of gravity.

2. A SEE mission

SEE is a comprehensive gravity experiment. A SEE mission would:

(a) Test for violations of the equivalence principle (EP) by inverse-square-law (ISL) violations, both at ranges on the order of metres and at ranges on the order of R_E.

(b) Test for violations of the equivalence principle by composition dependence (CD), both at ranges on the order of metres and at ranges on the order of R_E, using a multiplicity of combinations of materials.

(c) Determine the gravitational constant G.

(d) Test for time variation of G.

(e) Possibly test for post-Einsteinian orbital resonances.

The expected accuracies of the various tests of a SEE mission are given in table 1.

Four measurements and tests of a SEE mission—G, $\dot{G}$, and the ISL tests at both ranges (metres and R_E)—are by far the best of any experiment, either extant or under consideration.

Table 1. Expected accuracy of SEE tests and measurements.

Test/measurement	Expected accuracy
EP/ISL at $\sim$ few metres	2×10^{-7}
EP/CD at $\sim$ few metres	$<10^{-7}$ ($\therefore \alpha < 10^{-4}$)
EP/ISL at $\sim R_E$	$<10^{-10}$
EP/CD at $\sim R_E$	$<10^{-16}$ ($\therefore \alpha < 10^{-13}$)
G	0.3 ppm (300 ppb)
$(\dot{G})/G$	A few parts in 10^{14} yr^{-1}
Post-Einsteinian resonances	Undetermined

Here we note that, in table 1, α does not mean the fine-structure constant, but rather the interaction strength of a putative non-Newtonian contribution to a potential of the form

$$U = \left(\frac{GMm}{r}\right)\left[1 + \alpha \exp\left(\frac{-r}{\lambda}\right)\right]. \tag{1}$$

3. The drag-free-satellite concept

The initial concept of the drag-free satellite was developed independently by Lange of Stanford University (1964) and by the Space Physics Group of the Applied Physics Laboratory (APL) of Johns Hopkins University (1974). Their basic concept was to sense acceleration of the entire satellite by the relative movement between it and a small 'proof mass' in a capacitive cage and then to activate small thrusters in such a manner as to keep the proof mass nearly centred in its cage.

Decoupling. At the beginning of this decade, a group from the University of Pisa working on project 'Newton' (Nobili *et al* 1989, 1990, 1993) made a major advance in the drag-free concept by proposing that the test body would be free-floating within a large chamber, thus obviating the necessity to continually apply forces to the satellite *per se*. In short, the Newton team recognized the immense advantage of *decoupling* the test body from the satellite; no longer was it necessary to continuously sense accelerations and continually make corrections.

Satellite mass distribution. Project SEE made two more major improvements in 1992 (Sanders and Deeds 1992, Sanders *et al* 1999). First, the mass of the SEE satellite is to be distributed so that it exerts no gravitational force on the test bodies within, which is possible in principle if the forces are Newtonian. That is, the satellite becomes 'gravitationally invisible' to the test bodies floating within it. This removes the restriction that a test body must be near the centre of mass of the satellite (which is almost universally prescribed by all previous proposals for space-based gravitational measurements (Sanders and Gillies 1996)). Needless to say, this approach is efficacious only to the extent that other extraneous forces can also be limited, such as those due to radiation pressure and electromagnetic interaction, as discussed in the appendix and Sanders *et al* (1999). When extraneous forces are thus controlled, very sensitive gravitational measurements can be accurately made.

Automatic force cancellation. The second major improvement by the SEE team is that the relative positions and orientations of the Shepherd and the satellite body are to be varied systematically in such a manner as to effect automatic cancellation of forces at a level *far below the measurement threshold*. In principle this can be done exactly in a time-averaged sense, and in practice it can be done at a level to cancel forces several orders of magnitude below the size of forces which can be measured and on a time scale of a few weeks to a few months. We emphasize the value of being able to achieve several more orders of magnitude of force reduction using this cancellation scheme.

AZTAD satellites. We give the term 'almost-zero-time-averaged-drag' (AZTAD) to the sense in which an orbiting body is drag-free using the above-described principles; it is vastly more nearly drag-free than can be achieved using the basic Lange/APL concept. The SEE Shepherd, as an AZTAD satellite, is uniquely suited to making a measurement of $\dot{G}$ (Sanders and Deeds 1992a, b, Sanders *et al* 1999).

4. Aspects of a $\dot{G}$ measurement on a SEE mission

The main idea is to use the SEE Shepherd's orbital motion as a clock to detect any time variation in the strength of its gravitational interaction with the Earth. Centimetre-level tracking (post-fit residuals $\sim$1 cm) means that the Shepherd's position is known well enough to correspond to 1.4 ns resolution in time, which is the time required for a satellite in low-Earth orbit to travel 1 cm. Thus, the time of the crossing of the equator of an AZTAD satellite will be known approximately hourly with a precision of less than 2 ns.

$\dot{G}$ is important for two reasons. First, few things could be more important than finding a time variation in a fundamental 'constant'. Second, the putative variation of G is one of the few windows on the various unification theories, such as extra-dimension theories, which generally have specific predictions for the value of $\dot{G}$ (Marciano 1984). Determination of a secular $\dot{G}$ to within a few parts in 10^{12} yr^{-1} is sufficient to distinguish among various unification theories, which typically predict that $(\dot{G})/G$ is a small multiple of 10^{-11} yr^{-1} or somewhat less (Marciano 1984, Bronnikov *et al* 1988, Melnikov 1994, Drinkwater *et al* 1999, Ivashchuk and Melnikov 2000).

A great many investigators have turned their attention recently to observational estimates of $\dot{G}$ resulting in the publication of at least 40 papers during the past decade (Gillies 1997). Most investigators claim limits on $|\dot{G}/G|$ of a part in 10^{11} or somewhat less. We note that lunar laser ranging (LLR) will soon be able to make a determination of $\dot{G}$ to within one part in 10^{12} yr^{-1}.

The primary goal of our $\dot{G}$ tests is to look for a *secular* change in G. Such a change would mean that the position of the satellite in orbit would not be a linear function of time, but rather it would include a quadratic term.

We will also look for possible anisotropy of space as a test of general relativity. This would be manifested as an annual fluctuation of the apparent value of G, as inferred from the Shepherd's orbital period, as the nodes of the plane of the Shepherd's orbit precess in inertial space, since the average strength of the gravitational force between the Earth and the Shepherd over one orbital revolution is indicative of the force along the various directions of the Earth–Shepherd line for that particular orientation of the orbital plane.

Table 2 is an error budget for the determination of $\dot{G}$ in terms of the putative linear secular variation. It indicates that a test of $(\dot{G})/G$ to a few parts in 10^{14} is possible. We believe that this is by far the best of any proposed experiment.

Figure 1 summarizes table 2 according to three major classes of errors: tracking, timekeeping, and unmodelled forces. Figure 1 shows that the tracking error and unmodelled forces make roughly equal contributions to the $\mathrm{d}(\dot{G})/G$ error if the observation time τ is about a

Table 2. Error budget for $\dot{G}$. 1 and 4 year observation periods.

Error source	Average force ($\times 10^{-17}$ N)	$\delta\,(\dot{G}/G)$ ($\times 10^{-15}$) 1 yr	4 yr	Brief comments (details below)
Tracking error	NA	15.6	2.0	GPS/SLR accuracy = 1 cm
Timekeeping error	NA	1	1	Future atomic clocks
Black-body radiation	10.0	8.6	4.3	$\Delta\Theta < 0.1$ mK
Electrostatic forces	<15	<10	<5	Surface potential < 6.4 mV
Lorentz forces	Small	0	0	Perpendicular to velocity
Earth's field	<1.4	<0.9	<0.5	With GRACE or equivalent
Capsule mass defects	22.2	15	7.4	Many defects $\sim$ 10 mg
Gravity of particle	<0.22	<0.15	<0.08	Newton's third law
Shepherd's moments	Small	Small?	Small?	Not evaluated yet
Outgassing jets	Small	Small	Small	Obviate by baking
Total	NA	25	10	

year. The tracking error falls as $\tau^{-3/2}$, and unmodelled-force errors fall as $\tau^{-1/2}$. Timekeeping errors are not expected to be a significant part of the error budget for the SEE $\dot{G}$ measurement.

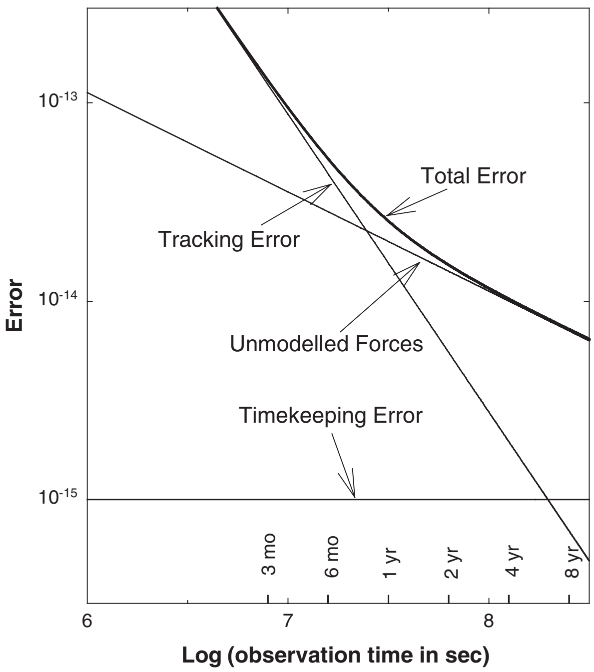

Figure 1. Error budget for $\dot{G}/G$ by class of source.

As shown in the appendix, we expect to be able to detect an annual fluctuation of G, presumably due to anisotropy of space, at the level of about 2 parts in 10^{14}.

We note that, at present, timekeeping errors would contribute significantly to the total error in any very accurate $\dot{G}$ experiment, since even the best atomic clocks have problems with long-term (greater than one year) stability. For example, as shown in the appendix, HP model

5071 caesium-beam clocks are good to about 2 parts in 10^{14} for observation periods of a few years (Matsakis and Josties 1997, Matsakis *et al* 1998). However, among the new generation of atomic clocks that are now being tested, there are fortunately several promising candidates which will be good to about 1 part in 10^{15} for long observation periods (at least several years) (Prestage 2000).

The long-term stability of international time standards may at some point be further augmented by the incorporation of millisecond-pulsar data. This prospect has attracted considerable interest over the past 20 years because of the extraordinary regularity in their rotation rates and spin-down rates (Backer *et al* 1982, Bronnikov and Melnikov 1989, Taylor 1991, Guinot and Petit 1991, Petit 1996, Foster and Matsakis 1996, Matsakis and Josties 1997). Moreover, pulsars depend essentially on the conservation of angular momentum and, hence, are suitable for independent measurements of changes in fundamental constants, such as G and the fine-structure constant α. Although there are a number of potential problems associated with using millisecond pulsars as time standards (D W Allan, discussion in Foster and Matsakis 1996), many of these problems can be circumvented by the use of ensembles of a large number (20 or more) of pulsars (see, for example, Foster and Matsakis 1996).

At a very fundamental level, the long-term timekeeping issues raised by very accurate $\dot{G}$ tests will highlight the need and the opportunity to compare clocks which rely on three entirely different physical processes:

- Caesium beam clocks, which depend on atomic processes and, hence, on the fine-structure constant α.
- Orbital-rotation clocks such as the SEE Shepherd, which can be argued to be a 'gravitational clock' since its dynamics are governed by the gravitational interaction (with the Earth).
- Millisecond-pulsars, which are 'gravitational-inertial clocks' since their behaviour is governed by strong gravitational and inertial forces.

Thus, the orbital motion of a decoupled almost-zero-time-averaged-drag (AZTAD) satellite, of the Newton-SEE type, such as the SEE Shepherd, has the potential to provide a useful comparison of clocks which operate on different physical principles and hence to probe very fundamental questions about the behaviour of clocks in gravitational fields (and thus provide an independent test of various quantities in parameterized relativistic theories).

5. Conclusion

Gravitation is the missing link in unification theories. The proposals and experiments reported in this special issue aim to remedy this situation. Those of us at this conference hope and believe that our work will soon enable gravitation to become the keystone in developing a satisfactory unified theory of physics.

A Satellite Energy Exchange mission would be a significant step in this direction by providing very precise data on a number of different gravitational phenomena.

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Appendix. Error budget for a $\dot{G}$ measurement in the SEE satellite

Summary

The basic idea for measuring $\dot{G}$ is to ensure that the Shepherd is in a nearly force-free environment and to observe whether and how its orbital period changes. A secular increase in the period would indicate a secular decrease in the product $M_E G$. An annual fluctuation in the apparent G would indicate possible anisotropy of space. Any of these results, that $M_E G$ is zero within tight limits, that it is changing secularly or that the apparent G is fluctuating annually, would be useful.

A cosmologically significant measurement of $(\dot{M_E G})$ will require three things:

(a) sufficient tracking accuracy;
(b) sufficient timekeeping accuracy;
(c) sufficiently small extraneous (unmodelled) forces.

We find that if G has a linear secular variation (and therefore the distance travelled has a quadratic term in time, which is proportional to $\dot{G}$), then the error in $(\dot{G})/G$ will be about 25×10^{-15} in a 1 yr observation period of the SEE Shepherd. For a 4 yr period, the result will be 10×10^{-15}. If G is fluctuating annually due to anisotropy of space, then the amplitude can be determined to within an uncertainty of about 14×10^{-15}.

Tracking error

The availability of 1 cm level tracking provides very fine resolution in the angular positions of satellites as well as their angular velocities and angular accelerations. This situation, plus the fact that large statistics are typically available, means that the tracking error makes only a modest contribution to the total $\dot{G}$ error for observation periods over 1 year.

The position of the Shepherd will be nearly linear as a function of time, and a constant secular change in G would introduce a quadratic term (in time) to the angular position of the Shepherd. Therefore, the position in radians as a function of time would be (in the absence of Newtonian perturbations)

$$\phi(t) = \phi(0) + at + bt^2. \tag{A1}$$

Here the coefficient b is proportional to $\dot{G}$. Thus, a non-zero value of $\dot{G}$ is indicated by curvature of ϕ versus t. We note that the difference between (1) the value of ϕ at the midpoint in time and (2) the average of the values of ϕ at the two end points is $\frac{1}{4}bt^2$. This difference is simply b if we normalize time so that the period of observation runs from $t = -1$ to 1.

The contribution to the uncertainty in $\dot{G}$ is clearly proportional to the uncertainty in the curvature,

$$[\delta(\dot{G})/G]_{\text{tracking}} = \tfrac{3}{2}\sqrt{5}\sigma(\tau\Phi)^{-1}N^{-1/2} \tag{A2}$$

where σ is the tracking error in radians, τ is the observation period in years, Φ is the mean motion of the Shepherd in radians/yr and N is the number of independent data points. Here we take $R = 7.878 \times 10^6$ m as the Shepherd's orbital radius (therefore altitude = 1500 km), so $n = 12.418\ \mathrm{d}^{-1}$ is the mean motion of the Shepherd (in orbital revolutions per day) at this altitude. Taking $\sigma = (1\ \text{cm})/R = 1.26 \times 10^{-9}$, $\Phi = 2\pi n \times 365.25 = 28\,499$ and $N = 365.25\tau$ (dimensionless), equation (A4) yields the values shown in line 1 of table 2.

The number of independent data points N is currently limited to the number of days of observation of the satellite because of the daily oscillations in the errors in the orbital elements. If better geopotential models become available, then the number of data points N can be taken as equal to the total number of orbital revolutions.

Since data are taken at a nearly constant rate, the number of data points will be proportional to the observation time τ, and therefore the position-resolution contribution to the uncertainty in $\dot{G}$ is proportional to the $-3/2$ power of the observation time:

$$[\delta(\dot{G})/G]_{\text{tracking}} \propto \tau^{-3/2}. \tag{A3}$$

We note that heuristic considerations also make it clear that position resolution is better than one part in $10^{13}\ \text{yr}^{-1}$. To see this, we need merely to note that the total path travelled by the Shepherd in 1 year is

$$L = 2\pi Rn \times (365.25\ \text{d}) = 2.245 \times 10^{11}\ \text{m}. \tag{A4}$$

Thus, if 1 cm level tracking is available, then, using *only two* data points, the angular-velocity resolution in 1 year would be

$$\epsilon = \sqrt{\tfrac{1}{2}} \times [(0.01\ \text{m})/R]/\Phi_0 = 3.15 \times 10^{-14}. \tag{A5}$$

This value of ϵ is approximately equal to the position contribution to $\delta(\dot{G})/G$ shown in table 2. This is to be expected, since equation (A5) is essentially a first difference, from which the second differences can be constructed, albeit with some loss of accuracy, and this loss is offset by the large number of statistics available, which tend to improve the accuracy.

Anisotropy of space. The foregoing discussion deals with the case where any variation of G is linear and secular. If space is anisotropic, that is if the parametrized post-Newtonian (PPN) parameter α_1 is non-zero, then the effective G experienced by a satellite would fluctuate according to the direction between it and the Earth. The precession of the nodes of the orbit would lead to a periodic fluctuation of the satellite's orbital period. For a sun-synchronous orbit, the fluctuation would have a period of 1 yr. We may estimate the error in determining the amplitude of fluctuation as follows. During a 6 month period of observation, from a maximum to a minimum of effective G, the mathematics of employing curvature in satellite position to infer $\dot{G}$ should yield results of the same order of magnitude as if the variation in G were linear for six months. Therefore, from equation (A3), the error would be larger than the 1 yr value by a factor of $2^{3/2} = 2.8$. However, the observation period would, in fact, be over a number of years, and we would expect the error to decline as the one-half power of time, specifically, as the one-half power of the *number of 6 month intervals*. Thus, after 4 years, the relative size of this error to the 6 month error would be roughly $8^{-1/2} = 1/2.8$. Thus, after 4 years of observation, the tracking contribution to the amplitude of a fluctuation in $\dot{G}$ would be about the same as the 1 year error in the case of a secular linear variation of G, namely about two parts in 10^{14}.

Timekeeping error

Timekeeping, in terms of accuracy and stability, is probably sufficient since the new generation of atomic clocks will be good to about 1 part in 10^{15}. However, with the present generation of atomic clocks timekeeping would be problematic. Table A1 shows the results of recent tests of Hewlett-Packard Model 5071 Cs standards and tuned Sigma-Tau hydrogen maser standards at the US Naval Observatory. The results are summarized in figure A1 (based on figures 3(*a*) and (*b*) of Matsakis and Josties (1997)). The timing error is expressed as the square root of the Hadamard variance, which is similar to the Allan variance, except that the Hadamard variance is based on third differences rather than second differences. Here the Hadamard deviation is normalized so that, for white phase noise, the data shown in table A1 and figure A1 correspond to the rms deviation of the original phase data.

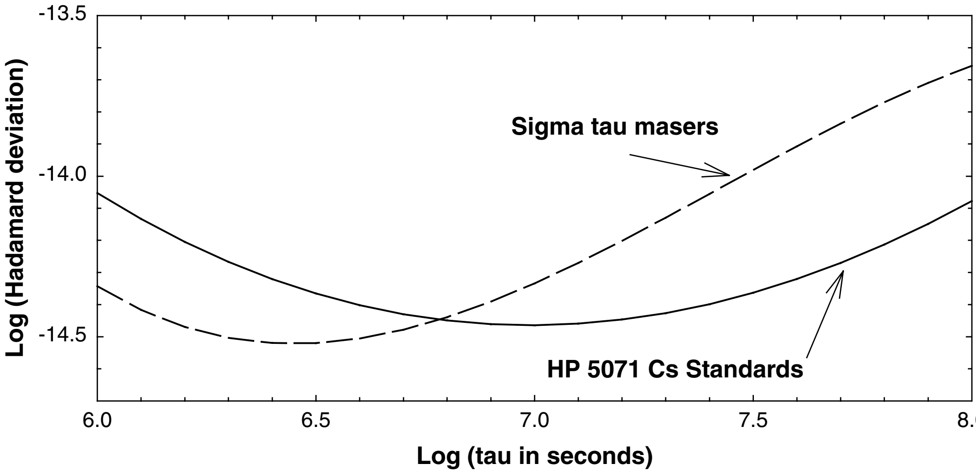

Figure A1. Approximate errors (schematic) of current time standards.

Table A1 shows that the timekeeping error with present atomic clocks is an increasing function of the duration of the observation period τ for periods longer than a few months. Although the variation is complex, we note that the increase goes very roughly as the 0.6 power of the observation period τ,

$$[\delta\dot{G}/G]_{\mathrm{timekeeping}} \propto \tau^{0.6} \tag{A6}$$

for 1 yr $< \tau <$ 6 yr. Thus, table A1 and figure A1 suggest that, until the next generation of atomic clocks is in service, timekeeping problems might prove to be a fundamental limitation on any $\dot{G}$ experiment based on long observation periods.

Table A1. Estimates of timing error for long observation times.

	Approximate Hadamard deviation ($\times 10^{-15}$)	
Observation time	5071 Cs standards	Sigma-Tau masers
3 months	3.5	3.7
6 months	3.6	5.7
1 yr	4.3	9.5
2 yr	6.1	15
3 yr	8.0	19
4 yr	10.0	22
5 yr	12	24
6 yr	14	25

Unmodelled forces

The extraneous forces on the Shepherd arise from several sources.

(a) Anisotropy of black-body radiation from the walls of the chamber.
(b) Electrostatic forces, especially due to the image charge of the Shepherd.
(c) Lorentz forces.
(d) Uncertainties in the Earth's field due chiefly to seasonal hydrological shifts.
(e) Mass defects in the capsule.
(f) The gravitational forces of the particles on the Shepherd.
(g) Interaction of the Shepherd's quadrupole moment with Earth's field.
(h) Outgassing jets.

Estimates of all unmodelled forces are provided in table 2 for 1 and 4 year observation periods. Here we have assumed that every such errors follows a random walk. Thus, its contribution to the position error of the Shepherd grows as the square root of the observation time τ, while the *relative* error, which is what contributes to $\delta(\dot{G})/G$, is inversely proportional to the square root of the time.

$$[\delta(\dot{G})/G]_{\text{unmodelled}} \propto \tau^{-1/2}. \tag{A7}$$

Discussion of unmodelled forces error budget

First, we find the size of a force which would mimic a change in G at the level of one part in 10^{14} yr^{-1}. Thus the requirement on unmodelled forces must be that all such forces should be below this amount. Then we evaluate the forces due to the sources listed in table 2.

By Kepler's third law, the orbital period is related to the semi-major axis thus

$$T \sim R^{3/2}. \tag{A8}$$

Thus, the change in the semi-major axis R during 1 year due to unmodelled forces must be small in the sense that

$$|\Delta R/R| < \tfrac{2}{3} \times 1 \times 10^{-14} = 6.67 \times 10^{-15}. \tag{A9}$$

Since the total orbital energy E of the Shepherd is proportional to the semi-major axis R, it follows that the same limit applies to energy

$$|\Delta E/E| < 6.67 \times 10^{-15}. \tag{A10}$$

Since ΔE is also the work done in 1 year by the unmodelled forces, we may now obtain the upper limit on the average unmodelled along-track force

$$\Delta E = W_{\text{year}} = \int F\,\mathrm{d}l = \langle F\rangle_{\max} L. \tag{A11}$$

Here $\langle F\rangle_{\max}$ is the size of the average unmodelled force which would cause the period to change by 6.7×10^{-15} in 1 year, thus mimicking a change in G of one part in 10^{14} yr^{-1}. To obtain a number for $\langle F\rangle_{\max}$, we note that the orbital energy, for $M = 200$ kg, is

$$E = -\tfrac{1}{2} M M_{\mathrm{E}} G/R = -5.06 \times 10^{9}\ \mathrm{J} \tag{A12}$$

and that the total orbital path length L is already calculated above as 2.25×10^{11} m, so

$$\begin{aligned} \langle F\rangle_{\max} &= \Delta E/L = (\Delta E/E)E/L = (\Delta E/E)(\tfrac{1}{2} M M_{\mathrm{E}} G/R)/L \\ &= 6.67 \times 10^{-15} \times 5.06 \times 10^{9}\ \mathrm{J}/2.25 \times 10^{11}\ \mathrm{m} \end{aligned} \tag{A13}$$

i.e.

$$\langle F \rangle_{\max} = 1.50 \times 10^{-16}\ \mathrm{N}. \tag{A14}$$

This is a threshold value in the sense that the average unmodelled force should be kept below this amount. We now examine each source of unmodelled forces.

Radiation pressure

For radiation pressure, the net axial force due to a temperature difference of $\Delta\Theta = 10^{-4}$ K between the two ends of the capsule would be

$$f = 4\Theta^3 \Delta\Theta \epsilon\sigma/c \times \pi R_c^2 = 1.41 \times 10^{-15}\ \mathrm{N} \tag{A15}$$

if there were no internal reflections. Here ϵ is the emissivity, σ is the Stefan–Boltzmann constant, Θ is the temperature, c is the speed of light and R_s is the radius of the chamber (50 cm). We take $\epsilon = 0.05$ and $\Theta = 78$ K.

In fact there are multiple internal reflections, with the result that the radiation which originates at one point in the capsule is reflected multiple times and strikes the test bodies almost isotropically. Thus, the net force is approximately 7% of what it would be without the internal reflections. Specifically, the net force is

$$f = 1.00 \times 10^{-16}\ \mathrm{N} \tag{A16}$$

which is slightly below the threshold of concern, $\langle F \rangle_{\max}$.

The remainder of this section is an explanation of this result. The radius of the Shepherd is about $R_s = 18$ cm (consistent with assuming that Shepherd has the density of stainless steel and a mass of 200 kg). Thus, the cross sectional area of the Shepherd is about 13% of the cross sectional area of the experimental chamber perpendicular to the chamber axis. Therefore, approximately 87% of the photons originally radiated from one end of the capsule will bypass the Shepherd and roughly 95% $(1 - \epsilon)$ of these will be reflected from the other end towards the 'back side' of the Shepherd. However, approximately 87% of these photons will also bypass the Shepherd, etc. This process may be expressed as a power series: the fraction of the photons originating from one end which eventually strike the Shepherd on the 'front side' is

$$p = (1 - b) \sum_{n=0}^{\infty} [b(1 - \epsilon)]^{2n} \approx 0.4102 \tag{A17}$$

where $b = 0.87$. The fraction of the original photons that strikes the 'back side' of the Shepherd is $p[b(1 - \epsilon)] \approx 0.3390$.

So the *excess* proportion of photons striking the 'front' side is

$$p_{\mathrm{net}} = p\{1 - [b(1 - \epsilon)]\} \approx 0.0712. \tag{A18}$$

Thus, the net radiation-pressure force on the Shepherd due to a temperature difference between the two ends of the capsule is about 7% of what it would be without reflections. This result is very insensitive to the values of b and ϵ; e.g., for $\epsilon = 0.01$ or 0.10, the values of p_{net} are 0.0698 and 0.0729, respectively. □

Electrostatic forces

The electrostatic force on the Shepherd is chiefly due to its own image charge in the capsule wall. If we treat the end wall as an infinite plane, the size of the image charge is the same as that on the Shepherd. Thus, the electrostatic force is

$$f = 1/(4\pi\epsilon_0)Q^2/r^2. \tag{A19}$$

To make f smaller than $\langle F\rangle_{\max}$, we must have

$$Q < 1.29 \times 10^{-13}\ \mathrm{C} = 0.129\ \mathrm{pC} \tag{A20}$$

if $r = 1$ m (Shepherd 50 cm from the end wall in the worst case). The corresponding potential at the surface of the Shepherd is

$$V < 1/(4\pi\epsilon_0)Q/R_{\mathrm{s}} = 6.44 \times 10^{-3}\ \mathrm{V} = 6.44\ \mathrm{mV}. \tag{A21}$$

This is a large voltage; it should be easy to detect. We note that the above treatment neglects the induced dipole moment.

Lorentz forces

Lorentz forces $q\boldsymbol{v}\times\boldsymbol{B}$ are transverse to velocity. Therefore, they can do no work, so they cannot change the orbital energy, the semi-major axis or the period. We have done no calculations on other effects but expect them to be small.

Earth's gravitational field

The seasonal changes in the Earth's gravitational field, due chiefly to hydrological shifts, would be a very serious problem if unknown. We assume that precise time-varying determinations of the Earth's field will be available when SEE is flown. A candidate mission is GRACE, which would give changes accurate to the equivalent of a change of a few microns in sea level over an area 1000 km wide on a monthly basis. We note that such data are needed with reference to both an Earth-fixed coordinate system and an inertial coordinate system.

The fractional increase in the mass of the Earth due to a *uniform* 5 μm addition to sea level would be 4.28×10^{-16}. The impact on the period T is reduced by a factor of two by Kepler's third law. Moreover, the impact of a given uncertainty in the amplitude of the nth harmonic on T is lower by roughly a factor of n than the impact of a uniform addition to sea level of the same amplitude. Thus, the uncertainty in the period is

$$\delta|\Delta T/T| < 4.28 \times 10^{-16}/(2n) < 1.1 \times 10^{-16} \quad \text{for } n = 2 \tag{A22}$$

if the Earth's potential is known equivalent to the output of the GRACE mission. This is well below the threshold of concern.

Mass defects in the capsule

In principle the internal gravitational field of the capsule is zero. However, it will, in fact, be non-zero because of mass defects. We have shown that all defects smaller than 1 g may be treated as rings, in the sense that the periodic components of their force due to capsule rotation will cause test-body motion on the order of 1 nm (Corcovilos and Gadfort 1998).

We have simulated the unknown ('unmodelled') force by randomly distributing 100 to 1000 rings of arbitrary positive mass and an equal number of rings of the same negative mass along the capsules and analysing the resulting spatial distribution of the force spectrum. The results are described in Sanders *et al* (1999).

Here we wish to emphasize the considerable benefits of employing four distinct flight configurations. The time-average gravitational force due to mass defects in the capsule would in principle be exactly zero if the capsule and the Shepherd were flown in four distinct configurations. The Shepherd would have its time divided into four equal parts, so that it were placed exactly (a) at one point in the capsule for $\frac{1}{4}$ of its time in the orbit, (b) at the symmetric (mirror-image) location for $\frac{1}{4}$ of the time, and then the capsule were flipped and the Shepherd could again be placed (c) and (d) for $\frac{1}{4}$ of the time at each of the same two points. This is done because the two symmetric locations obviously causes cancellation of *even* potential terms, and the two orientations of the capsule causes cancellation of the *odd* terms.

In practice the location of the Shepherd cannot be made exact. There will be some error in the attempt to replicate the positions of the Shepherd, resulting in incomplete force cancellation. The error would be proportional to the (unknown) derivative of the unmodelled force at the chosen point. It turns out that rather than try to position the Shepherd at a particular *point*, a more accurate average-force cancellation may be effected by moving the Shepherd uniformly back and forth in a 'roaming interval' whose length is chosen as roughly equal to one quasi-wavelength of the force fluctuation. Statistical simulations yield the following result.

> The largest error in average force that could result from a 1 mm error in locating the roaming interval of the Shepherd would be 0.002 75 times the peak force due to a single ring. This is 0.11% of the rms deviation of the force.

That is, this procedure of changing the position of the Shepherd within the capsule and flipping the capsule reduces the average unmodelled force by two orders of magnitude, assuming the Shepherd's roaming interval can be replicated to within 1 mm. We also need to simulate irregularities in the 'dwell time' at various locations within the roaming interval.

The force due to a ring of 10 mg mass may be taken as the approximate limit of detection (Sanders and Gillies 1996). This is conservative—we could justify 1 mg. The peak on-axis force due to such a mass ring is 2.05×10^{-13} N. We may essentially equate this with the rms amplitude of the unknown force. Then the roaming/flipping procedure described above leads to an average force not larger than 0.108% of this, or 2.22×10^{-16}. This is slightly above the threshold force, $\langle F \rangle_{\max} = 1.50 \times 10^{-16}$ N.

Finally, we note that in principle the effects of the unmodelled forces can be described by two parameters, namely an 'even' and an 'odd' deviation, corresponding to the even and odd parts of the unmodelled forces. By representing this effect in the overall fit with two dummy variables, it may be possible after several years in orbit to evaluate these two parameters. This would make free-standing monthly determinations of $\dot{G}$ possible retrospectively, since the deviation for each month could be described as the sum +/− 'even' +/− 'odd'.

Gravitational forces due to particles

The $\dot{G}$ determination could be done while the Shepherd is alone in the experimental chamber, rather than simultaneously with SEE encounters (the particles would be stowed at the ends of the capsule in such a manner as to produce essentially zero force on the Shepherd). However, since Newton's third law will accurately give us the forces of the particles upon the Shepherd, we should be able to perform $\dot{G}$ and other measurements simultaneously. Thus, $\dot{G}$ can be carried out over a period of years while all other measurements are also in progress.

The gravitational forces on the Shepherd due to the particle(s) may be calculated *a priori* almost accurately enough to account sufficiently for these forces, assuming G is actually known to one part in 10^4. This uncertainty is equivalent to a 10 mg mass defect at the separation of the test bodies. Moreover, we will actually measure these forces on the particles to much better than 1 part in 10^6 (equivalent to a mass defect of less than 100 μg at this separation). Using Newton's third law, we will know the forces on the Shepherd to the same level of accuracy. Therefore, the unmodelled force from this source will be more than two orders of magnitude below that due to the mass defects in the capsule.

Interaction of the Shepherd's quadrupole moment with the Earth's field

These effects are believed to be negligible but have not yet been evaluated. There is some possibility that the whole-orbit average of the Hamiltonian of the Shepherd in the Earth's gravity will vary slightly with the orientation of the Shepherd's rotation axis. This could result in a small seasonal fluctuation of the orbital energy, and, hence, a fluctuation in the orbital period.

Outgassing jets

We have done no calculations on outgassing. We expect that this problem can be eliminated by baking out the test bodies and the experimental chamber at 200 °C or higher. We note that outgassing at other locations on the satellite (outside the experimental chamber) is irrelevant, largely because SEE employs the time-average drag-free concept.

Conclusion to appendix

SEE should be able to detect non-zero $\dot{G}$ at the level of 25 parts in 10^{15} after 1 year of observation and 10 parts in 10^{15} after 4 years. We expect to be able to see annual fluctuations at the level of $\sim$two parts in 10^{14} after 4 years of observation. We are limited by tracking errors for observation periods for less than 1 year and by unmodelled forces for observation periods longer than a few years.

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