A new precise determination of Newton's gravitational constant

1995 Metrologia 32 267

(http://iopscience.iop.org/0026-1394/32/4/4)

A new precise determination of Newton's gravitational constant

W. Michaelis, H. Haars and R. Augustin

Abstract. A new determination of the numerical value of the gravitational constant G was carried out by means of an apparatus consisting of a fibreless torsion balance which was carried by the buoyancy of a floater in a liquid; the angular deflection of the balance was compensated with the aid of a control loop and an electrostatic torque transmitter; the reacting bodies were cylinders. The result obtained for G differs from the CODATA value by about $+6 \cdot 10^{-3}$ in relative value. No dependence on the materials or on the distance between the cylinders was found. The apparatus is described, the methods of evaluation and the results are reported, the possible sources of errors are discussed.

1. Introduction

Newton's gravitational constant G is that fundamental constant of classical physics charged with the greatest relative uncertainty (about 10^{-4}), although it has been determined in many important experiments in the past. Reviews are given in [1] and [2].

A new precise determination of Newton's gravitational constant was carried out at the Physikalisch-Technische Bundesanstalt (PTB). The aim of the experiment was also to examine the $1/r^2$ dependence of the gravitational force at close range and to detect a possible dependence of the gravitational constant on the material used. In the course of this work, begun in 1986, the fundamental principle of the experiment has already been described [3]. The first measurements, however, are not consistent with those described here. In recent years we have done much to improve the apparatus and to detect systematic errors. We revealed errors of the autocollimation system which was used for angle measurements in connection with the electrometer calibration (see the determination of $\mathrm{d}C/\mathrm{d}\alpha$ in Section 2). We found an unexpected attenuation factor in the registration system introduced by a low-pass filter. Finally we detected an error in the calibration of the digital voltmeter. Today we cannot apply further corrections to our 1987 results as parts of the original instrumentation no longer exist.

As essential parts of the equipment have been fundamentally changed, the equipment is described again in the present paper.

W. Michaelis, H. Haars and R. Augustin: Physikalisch-Technische Bundesanstalt, Postfach 33 45, D-38023 Braunschweig, Germany.

2. Equipment and measuring method

The basic element of the setup used for the experiment is a static torsion balance in which the torque generated by the gravitational forces is compensated by a counter-torque. In this experiment, the torsion wire generally used both to support the torsion balance and to measure the torque is replaced by a liquid bearing running without static friction [4] and an electrostatic arrangement similar to Maxwell's quadrant electrometer [5] (for circuit, see Figure 2a) used in conjunction with a control circuit (Figure. 1). We use two electrometers with a single shaft so that their torques add, only one electrometer being required, however, for the actual measurement of G. The second electrometer serves to compensate possible voltages on the moving electrode (Figure 2c), and is used in secondary experiments to determine unavoidable contact voltages.

The torque M_e generated by the electrometers cannot be calculated with the required accuracy from the geometric data so it is determined from values of the quantity $\mathrm{d}C/\mathrm{d}\alpha$ obtained by measurement, on the basis of the elementary relation

$$M_e = \mathrm{d}E/\mathrm{d}\alpha,$$

$E = 1/2\, CU^2$ being the energy of the electric field between the electrodes, U the voltage applied and α the angular position of the moving electrode or of the torsion balance. A separate calibration, i.e. a measurement of capacitances at different angular positions, is therefore required. In contrast with the calibration of a torsion wire, this method can be applied and repeated at any time without intervention in the equipment [5]. The capacitance is measured with the

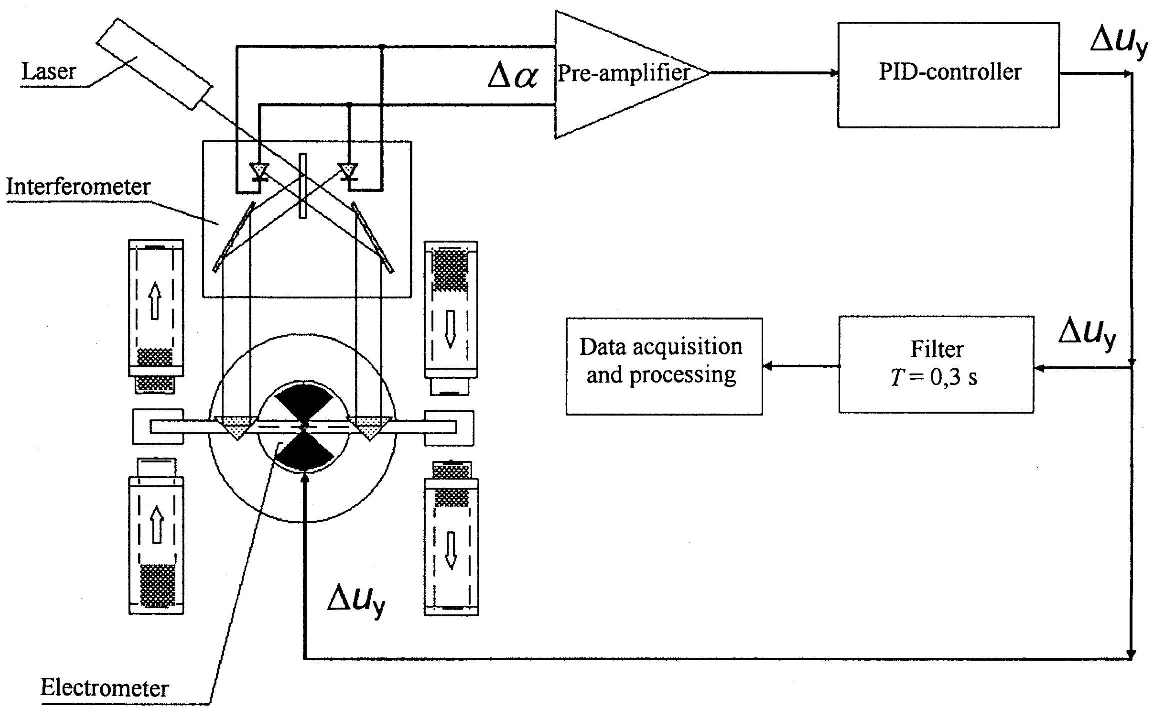

Figure 1. Schematic representation of the control circuit: compensation of gravitational forces.

aid of a precision capacitance bridge; for the angle measurement, a laser angular interferometer is used.

Figure 3 gives a simplified general view of the equipment. The gravitational force to be measured acts between cylindrically ground Zerodur bodies (masses of about 120 g) which are fixed to the torsion balance serving as test masses, and tungsten cylinders (40 mm in diameter, $m = 900$ g) as source masses arranged outside the torsion balance casing. The partner bodies are so arranged that their parallel end-faces are opposite one another.

The source masses are contained in displacement devices which are basically similar to thick-walled brass tubes. Within 10 s, and with the aid of compressed air, the cylinders can be moved to and fro between two limit stops. For the close-up positions, these stops are ruby balls, 2,000 mm in diameter. The source masses can be moved over a length of 105 mm. Two of these displacement devices are mounted together on a solid block, which may be adjusted in three coordinates. With the aid of the adjusting devices, the tungsten cylinders may be so aligned that their axes coincide with the axes of the test masses fixed to the balance.

For the measurement, four source masses were moved to and fro in the way shown in Figure 4a; in relation to the test masses, two source masses were in the close-up position (distance between the centres of gravity 37 mm), the other two in the distant position (distance between the centres of gravity 142 mm).

As the positions selected were closely symmetrical, a precise measurement of the distance between the source masses and the test masses (which, as parts of the balance, are accessible only with difficulty) could be dispensed with. It was sufficient to perform exact measurements of the short distances d and the displacements l, and to adjust the idle position of the test masses in such a way that they were in the centre with an uncertainty of $\pm 0,1$ mm. Compliance with this condition was easily checked by gently moving the source masses against each other in such a way that all of them were in the distant position or in the close-up position at the same time (Figure 4b). In the symmetric case, this mode of operation does not induce torque that can be detected by the electrometer. Otherwise, a correction of the alignment is feasible with the aid of the movable adjusting devices.

The torsion balance, the mercury bearing and the electrometer are enclosed by a vacuum-tight housing; this housing is filled with helium under reduced pressure (0,1 bar). A second housing, which surrounds the whole equipment, serves to provide thermal isolation, magnetic shielding and protection from rapid variations of the air pressure. The whole arrangement rests on a pillar anchored in natural ground to a depth of 3 m. Attempts to diminish the influence of vibrations of the ground by insulators failed to reduce the noise in the signal so we carried out the measurements without them.

3. Execution of the measurements

Between November 1992 and May 1993, we used the final version of the equipment to carry out measurement series of different scopes. For each measurement series,

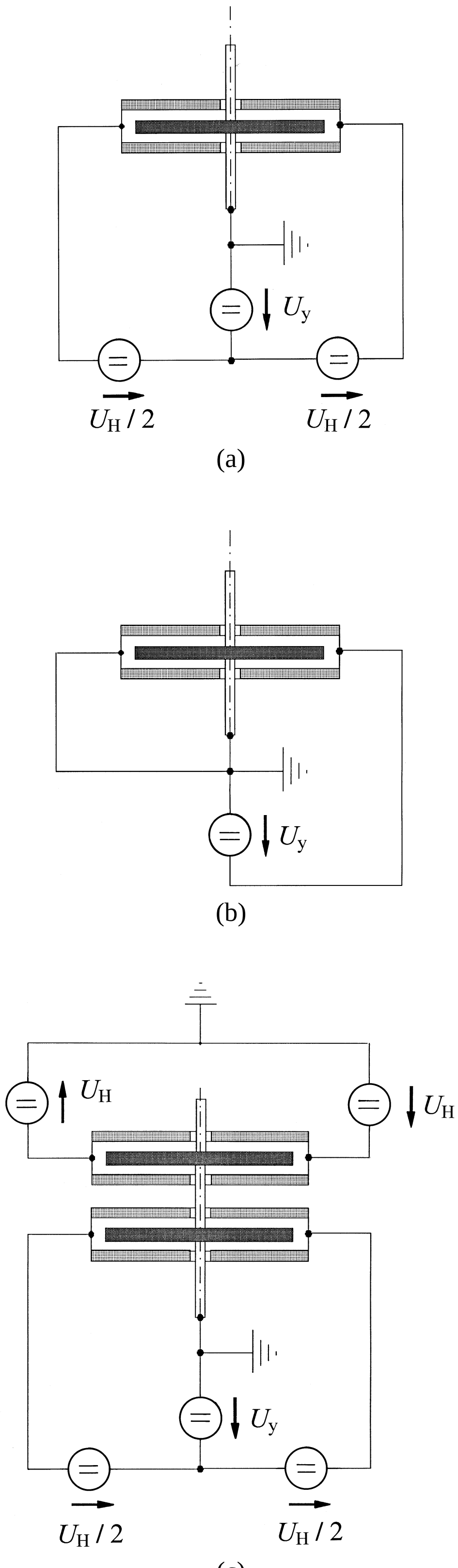

Figure 2. Electrometer circuits. (a) Symmetrical heterostatic circuit for the measurements. U_H, auxiliary voltage; U_y, control voltage. (b) Isostatic circuit for auxiliary experiments. (c) Compensation of voltages on the needle with the second electrometer.

the equipment was dismounted and then adjusted and set up again according to a well-contrived scheme; at the same time, various quantities required for the evaluation (distances between the masses, auxiliary voltage at the electrometer, calibration of the angular interferometer) were measured. In six measurement series, we used tungsten cylinders as source masses; in another five series, these were replaced by Zerodur cylinders.

Each measurement series essentially comprised continuous measurements of the control voltage over time intervals of 10 s, carried out with an integrating digital voltmeter; every 50 min the source masses were moved to the other position, this process being computer-controlled. We carried out measurements only at night or at weekends; in the afternoons before a measurement was carried out and in the mornings following a measurement, the electrometer was calibrated, i.e. the $C(\alpha)$ dependence was measured.

The control voltages measured were in a range between −2,5 V and 2,5 V; they were superimposed on a noise voltage of about 10 mV. We could not find all the sources of this noise, but it seemed to be stationary, so it could be reduced by analogue and digital filtering. Despite this filtering, there were times when – due to the weather, or as a result of increased microseism or earthquakes – the noise voltage temporarily increased to such a level that the measurement had to be excluded from the evaluation.

It was an advantage of our experiment that all quantities important for the evaluation could be traced back to the base units in specialized PTB laboratories. Before the cylinders were installed, their geometry was measured in the Dimensional Standards laboratory and their masses were determined by the Unit of Mass laboratory. At regular intervals, the digital voltmeters and the capacitance bridge were calibrated by the Unit of Voltage and Unit of Capacitance laboratories. The angle measurement required for the calibration of the electrometer was carried out with an angular interferometer. For each new mounting of the equipment, the angular interferometer was calibrated using an autocollimator which in turn had been calibrated at regular intervals in the Angle and Gearing laboratory. The close-up distances $d_{1,2}$ and $d_{3,4}$ were determined with the aid of a calibrated set of gauge blocks each time before the equipment was mounted again.

4. Evaluation of the measurements

In the experiment, a torque M_G generated by gravitation is compensated by an equal electrostatic torque M_e. The torque generated by gravitation is the sum of the torque M_N generated by the gravitational forces between adjacent reaction masses (test and source masses), and the torque M_k generated by other gravitational forces. These include the attraction between the source masses and the test masses on the opposite side of the

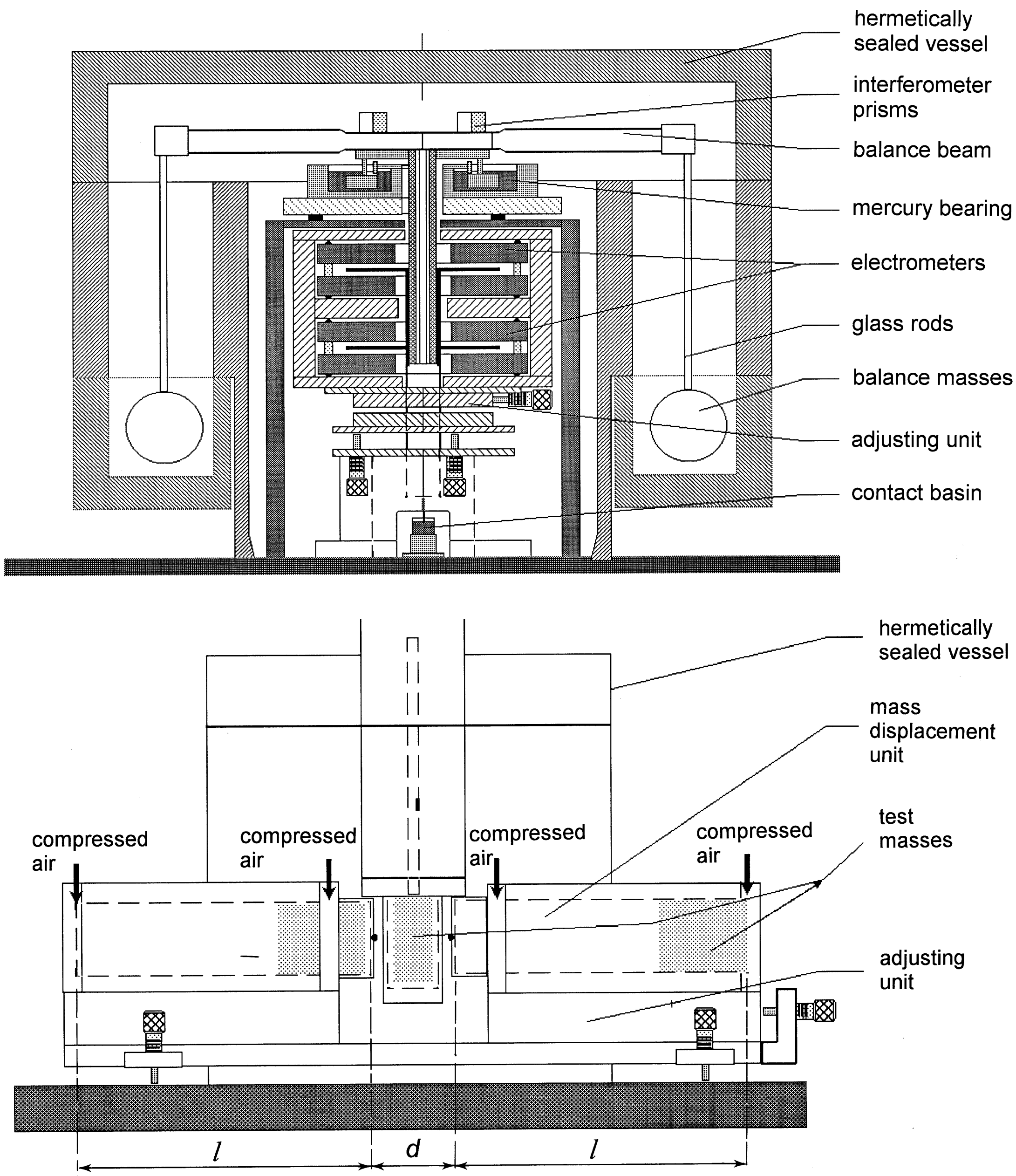

Figure 3. Simplified representation of the equipment, two side views.

balance, and the attraction between the source masses and those parts of the balance bearing the test masses. As only the difference between the torques for the two end positions of the movable masses enters into the evaluation, torques generated by gravitational fields of masses, which are invariable during the experiment, need not be taken into account. Consequently, the difference of the torques is

$$\Delta M_{\mathrm{G}} = \Delta M_{\mathrm{N}} + \Delta M_{\mathrm{k}} = \Delta M_{\mathrm{e}}. \tag{1}$$

On the basis of the law of gravitation, the following results:

$$G\,\Delta\mu = G\,(\Delta\mu_{\mathrm{N}} + \Delta\mu_{\mathrm{k}}) = \Delta M_{\mathrm{e}}. \tag{2}$$

The calculation of the difference $\Delta\mu$ which, but for the factor G, represents the gravitation torque, makes it necessary to calculate a six-fold integral over the volumes of the sample masses. Four of the six integrations were made in a closed form which resulted in a complicated formalism. The other two integrations were carried out numerically.

We used tungsten masses to determine the gravitational constant. In another test series we gradually increased the close-up distances $d_{1,2}$ and $d_{3,4}$ (Figure 4) to 197 mm to check the $1/r^2$ law. To examine the material independence of the gravitational constant, it had been planned to use different materials for the masses. However, for reasons of time, the only comparison made was between tungsten and Zerodur.

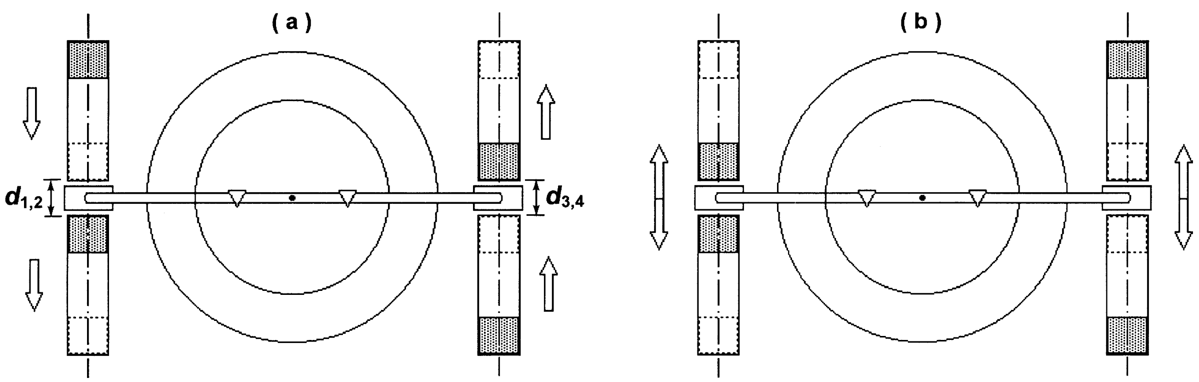

Figure 4. Position of the masses displaced (a) during measurement and (b) during mounting of the equipment.

Table 1. Summary of the results for G.

Results of the measurements with tungsten masses

Series	G/N·m^2·kg^{-2}	u_G	u_e	u
A	$6{,}716\,425 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$43 \cdot 10^{-6}$	$68 \cdot 10^{-6}$
B	$6{,}713\,350 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$31 \cdot 10^{-6}$	$62 \cdot 10^{-6}$
E	$6{,}715\,567 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$52 \cdot 10^{-6}$	$74 \cdot 10^{-6}$
F	$6{,}714\,594 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$41 \cdot 10^{-6}$	$68 \cdot 10^{-6}$
G	$6{,}716\,114 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$59 \cdot 10^{-6}$	$80 \cdot 10^{-6}$
H	$6{,}716\,638 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$30 \cdot 10^{-6}$	$61 \cdot 10^{-6}$

Results of the measurements with Zerodur masses

Series	G/N·m^2·kg^{-2}	u_G	u_e	u
A	$6{,}718\,156 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$178 \cdot 10^{-6}$	$186 \cdot 10^{-6}$
B	$6{,}717\,350 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$648 \cdot 10^{-6}$	$650 \cdot 10^{-6}$
E	$6{,}715\,856 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$124 \cdot 10^{-6}$	$135 \cdot 10^{-6}$
F	$6{,}725\,854 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$234 \cdot 10^{-6}$	$240 \cdot 10^{-6}$
H	$6{,}712\,487 \cdot 10^{-11}$	$53 \cdot 10^{-6}$	$230 \cdot 10^{-6}$	$237 \cdot 10^{-6}$

Notes:
The apparatus was completely remounted for each series. G is the mean value of the results of the series. u represents the relative uncertainty, calculated from the error budgets given in Tables 2 and 3: u_G is the relative uncertainty of the gravitational torque M_G, u_e is the relative uncertainty of the electrostatic torque M_e. The relative combined incertainty is u, the geometric sum of the relative uncertainties u_G and u_e.

Final results

Tungsten: $G = 6{,}715\,40 \cdot (1 \pm 83 \cdot 10^{-6}) \cdot 10^{-11}$ N · m^2 · kg^{-2}

Zerodur: $G = 6{,}717\,40 \cdot (1 \pm 30 \cdot 10^{-5}) \cdot 10^{-11}$ N · m^2 · kg^{-2}

These results were calculated as weighted means of the mean values of G weighted as the inverse squares of their uncertainties u:

$$G = \frac{\sum p_i\, G_i}{\sum p_i}, \qquad u^2 = \frac{\sum p_i\,(G_i - G)^2}{(N-1)\cdot \sum p_i},$$

with

$$p_i = \frac{1}{u_2}, \qquad i = 1, \ldots, N.$$

4.1 Measurements carried out with tungsten masses

The results of the measurement series are shown in Table 1. The final result was calculated as the weighted mean from the results obtained in six measurement series, with the standard deviations of the individual results used as weighting factors:

$$G = (6{,}715\,40 \pm 0{,}000\,56) \cdot 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}},$$

or, expressed differently:

$$G = 6{,}715\,40 \cdot (1 \pm 83 \cdot 10^{-6}) \cdot 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}.$$

The uncertainty stated corresponds to the single standard deviation and includes all systematic errors known to us.

The expression of the uncertainty of measurement is explained in Section 5.

4.2 Measurements carried out with Zerodur masses

Five G-values and their mean value were calculated from five measurement series carried out with Zerodur masses (Table 1):

$$G = (6{,}7174 \pm 0{,}0020) \cdot 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}.$$

The uncertainty stated is the standard deviation. As the measurement signal is much smaller due to the much lower gravitational force ($\rho_{\text{tungsten}} = 19{,}3 \cdot 10^3$ kg · m^{-3}, $\rho_{\text{Zerodur}} = 2{,}5 \cdot 10^3$ kg · m^{-3}), the standard deviation of the measurement result is much higher. The two gravitational constants measured are not significantly different.

4.3 Measurement of the distance dependence

When the distances between the interacting masses are greater, the gravitational forces can be measured with a torsion balance only in a limited range. In additional experiments, we installed our displacement device at four different distances from the torsion balance. The results are shown in Figure 5 in the form of a diagram. They show no significant dependence of G on distance.

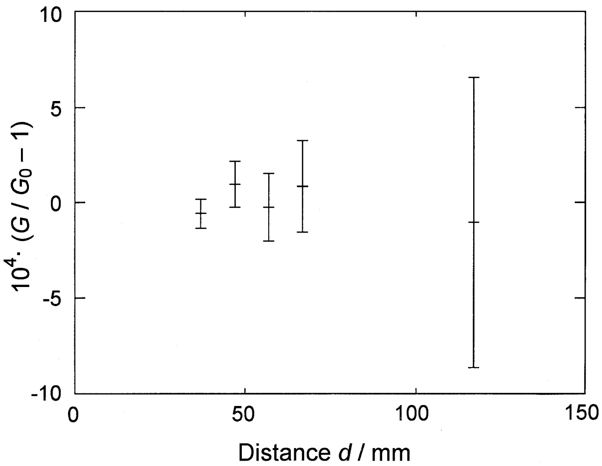

Figure 5. Results of measurements at greater distances, related to $G_0 = 6{,}7154 \cdot 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}$.

5. Uncertainty of measurement

The principal contributions to the uncertainty are listed in Tables 2 and 3. Columns headed s_x show the uncertainties of individual quantities. These values are based on the information provided by the laboratories concerned or on our own estimates if the quantities were determined by us. The influence which these individual uncertainties exert on the total result differs, however. The uncertainty of the displacement, for example – although it is considerably greater – influences the total uncertainty much less than the uncertainty of the short distance. In general, these influences cannot be calculated by a closed formula. We have therefore chosen to determine the uncertainty by calculation in the following way.

All quantities listed in the table are contained in a numerical program for the calculation of the values μ and M_e. In individual calculations, the quantities x were increased or decreased by the standard deviation s_x, and the resulting changes $\mathrm{d}\mu$ and $\mathrm{d}M_e$ were determined. These changes in relation to the mean value for $\Delta\mu$ or ΔM_e are given in the columns headed $(\mathrm{d}\mu/\mathrm{d}x) \cdot \Delta\mu$ and $(\mathrm{d}M_e/\mathrm{d}x) \cdot \Delta M_e$, respectively.

Some uncertainties are not listed, as they cannot be determined numerically. By completely dismounting the equipment and setting it up again before a new measurement series was started we tried to randomize these uncertainties and thus determine them globally.

The following uncertainties in particular are involved:

(a) Inhomogeneities of the density and shape deviations of the masses. The resulting effects were randomized by changing the orientation of the cylinders in each new setup by turning them about their axes and changing the position of their end-faces.

Table 2. Uncertainty budget for the gravitational torques obtained using tungsten masses. The gravitational torque is $M_G = G \cdot \mu$: we measure differences $\Delta\mu$. The coefficients $(\mathrm{d}\mu/\mathrm{d}x)/\Delta\mu$ describe the influence of the input quantities x on the results of the gravitational torques $\Delta\mu$; s_x is the standard deviation of the input quantity, N_x is the number of quantities of the same species; u_i is the resulting relative uncertainty.

No.	Name of quantity	N_x	$\frac{1}{\Delta\mu} \cdot \frac{\mathrm{d}\mu}{\mathrm{d}x}$	s_x	$u_i = \frac{\sqrt{N_x}}{\Delta\mu} \cdot \frac{\mathrm{d}\mu}{\mathrm{d}x} \cdot s_x$
Displaced masses					
1	Mass	4	0,28 kg^{-1}	6 mg	$3{,}4 \cdot 10^{-6}$
2	Diameter	4	1,78 m^{-1}	1,2 µm	$4{,}3 \cdot 10^{-6}$
3	Length	4	4,10 m^{-1}	0,6 µm	$4{,}9 \cdot 10^{-6}$
Displacement devices					
4	Distance moved	4	0,34 m^{-1}	6 µm	$4{,}1 \cdot 10^{-6}$
5	Close-up distance	2	11,26 m^{-1}	2 µm	$31{,}8 \cdot 10^{-6}$
Balance masses					
6	Mass	2	4,17 kg^{-1}	6 mg	$35{,}4 \cdot 10^{-6}$
7	Diameter	2	5,61 m^{-1}	1,2 µm	$9{,}5 \cdot 10^{-6}$
8	Length	2	2,59 m^{-1}	0,6 µm	$2{,}2 \cdot 10^{-6}$
Balance beam					
9	Length (lever arm)	1	2,38 m^{-1}	6 µm	$14{,}3 \cdot 10^{-6}$
10	Altitude	1	0,02 m^{-1}	30 µm	$0{,}6 \cdot 10^{-6}$
11	Mass of the tube	1	0,01 kg^{-1}	30 mg	$0{,}3 \cdot 10^{-6}$
12	Mass of mounting parts	2	0,03 kg^{-1}	30 mg	$1{,}3 \cdot 10^{-6}$
Rods for mounting the balance masses					
13	Diameter	2	1,78 m^{-1}	6 µm	$15{,}1 \cdot 10^{-6}$
14	Mass	2	0,35 kg^{-1}	6 mg	$3{,}0 \cdot 10^{-6}$
Electrometer					
15	Diameter of moving electrodes	2	0,01 m^{-1}	30 µm	$0{,}4 \cdot 10^{-6}$

Uncertainty of the result $\Delta\mu$
(error propagation, as there is no indication of a correlation between the quantities):

$$\Sigma\, u_i^2 = 2,86 \cdot 10^{-9} = (s_\mu/\Delta\mu)^2$$

$$s_\mu = 53,3 \cdot 10^{-6}\, \Delta\mu \qquad \text{with} \qquad \Delta\mu = 47,7\ \mathrm{kg^2/m}$$

Notes:
The standard deviations are estimated with the assumption of a rectangular distribution of errors within maximum bounds.
For Nos. 1, 4, 5, 6, and 9, the laboratories executing the measurements declared maximum bounds.
For Nos. 2 and 7, the roundness of the cylinders shows maximum deviations of ± 2 µm. The uncertainty in the measurements of diameter is much smaller.
For Nos. 3 and 8, the parallelism of the faces of the cylinders shows maximum deviations of ± 1 µm. The uncertainty of the measurements of height is much smaller. These values also influence quantities in the calculations of the distances of reacting masses.

(b) Deficiencies in the setup: parallel displacement or tilting of the axes of the interacting cylinders. These, too, differ at each new setup.

Table 3. Uncertainty budget for the electrostatic compensation torques. The coefficients $(\mathrm{d}M_\mathrm{e}/\mathrm{d}x)/\Delta M_\mathrm{e}$ describe the influences of the input quantities x on the results of the electrostatic torques ΔM_e; s_x is the standard deviation of the input quantity; u_i is the relative uncertainty of ΔM_e.

No. Name of quantity x	$\frac{1}{\Delta M_\mathrm{e}} \cdot \frac{\mathrm{d}M_\mathrm{e}}{\mathrm{d}x}$	s_x	$u_i = \frac{1}{\Delta M_\mathrm{e}} \cdot \frac{\mathrm{d}M_\mathrm{e}}{\mathrm{d}x} \cdot s_x$
Coefficients of capacity			
1 Coefficient k_1	$2{,}91 \cdot 10^{-2}$ pF^{-1} rad	$6{,}6 \cdot 10^{-4}$ pF/rad	$19{,}2 \cdot 10^{-6}$
2 Coefficient k_2	$2{,}22 \cdot 10^{-2}$ pF^{-1} rad	$6{,}6 \cdot 10^{-4}$ pF/rad	$0{,}01 \cdot 10^{-6}$
Voltages			
3 Auxiliary voltage	$5{,}00 \cdot 10^{-2}$ V^{-1}	0,3 mV	$15{,}0 \cdot 10^{-6}$
4 Mean of the control voltage	$1{,}46 \cdot 10^{-4}$ V^{-1}	3,0 mV	$0{,}4 \cdot 10^{-6}$
5 Contact voltage (needle)	$1{,}46 \cdot 10^{-4}$ V^{-1}	…	…
6 Contact voltage (quadrants)	$5{,}00 \cdot 10^{-2}$ V^{-1}	…	…
7 Differences of control voltage	$2{,}14 \cdot 10^{-1}$ V^{-1}	…	…

The standard deviations of quantities Nos. 5, 6 and 7 vary:

Name of series	A	B	E	F	G	H
5 Contact voltage (needle)/mV	0	5	3	2	10	2
6 Contact voltage (quadrants)/mV	600	300	500	300	350	300
7 Contact voltage/(differences)/μV	82	52	176	139	238	33

Relative standard deviation of the resulting quantity ΔM_e:

	A	B	E	F	G	H
$\frac{1}{\Delta M_\mathrm{e}} \cdot \frac{\mathrm{d}M_\mathrm{e}}{\mathrm{d}x} \cdot 10^{-6}$	43	31	52	41	59	30

Notes:
For Nos. 1 and 2, the coefficients are calculated from measurements of the capacitances C_A and C_B with varying angles α. They are defined as

$k_1 = (\mathrm{d}C_\mathrm{A}/\mathrm{d}\alpha - \mathrm{d}C_\mathrm{B}/\mathrm{d}\alpha)$ and $k_2 = (\mathrm{d}C_\mathrm{A}/\mathrm{d}\alpha + \mathrm{d}C_\mathrm{B}/\mathrm{d}\alpha)$.

The uncertainty was estimated from the specifications of the capacitance bridge and the autocollimator, and from the random fluctuations of the observed data.
For No. 3, the uncertainty of the control voltage was estimated from the specifications of the digital voltmeter.
For Nos. 4 to 7, the standard deviations are calculated as random fluctuations from multiple measurements of these quantities.

We determined our results for G as the quotient $G = \Delta M_\mathrm{e}/\Delta\mu$. There is no reason to assume a dependence between the uncertainties of ΔM_e and $\Delta\mu$, so the standard deviations determined from s_μ and $s_{M\mathrm{e}}$ were combined following the usual law of error propagation to give the total result for G.

6. Search for systematic errors

As the value of the gravitational constant determined in our measurements substantially deviates from the current CODATA value [7], we searched intensively for potential systematic errors in our equipment.

6.1 Checking of the torque measurements

6.1.1 Error sources in the capacitance measurement

The capacitances of the electrometer were measured with a calibrated precision bridge; a significant error caused by the capacitance measurement is therefore not to be expected. However, as the capacitance measurement was carried out at a frequency of 1 kHz while a steady voltage was used for torque compensation, we suspected a difference between the ac and dc capacitances. Although it has already been pointed out [8] that a relative deviation of less than 10^{-7} should be expected between the dc and ac capacitance, we nevertheless took into consideration that nonconducting layers might exist on the surfaces of the electrodes and that charge carriers could accumulate on these. In the case of dc voltage, a capacitance differing from that measured with the capacitance bridge would then be effective. Using a measurement frequency below 1 kHz would mean tolerating a loss in sensitivity and, thus, a loss in accuracy. At the lowest acceptable measurement frequency (100 Hz), no significant change of the capacitance was measured.

To check the case of dc voltage, we first measured the capacitance with an ac voltage of 1 kHz and then with a dc voltage of 9 V, superimposed on the measurement voltage. The two measurements showed very good agreement so that it may be assumed that there were no nonconducting layers on the electrode plates.

Although the leads to the electrodes inside the electrometer are carefully shielded, charging of the isolating layers, and thus uncontrollable additional torques, could not be excluded. By periodic reversal of the polarity of the control voltage and of the auxiliary voltage, this charging effect would, however, have become apparent on the control voltage. These tests also showed no irregularities.

To further support the capacitance measurement, we used the second electrometer system as a torque transmitter, as shown in Figure 2. In this experiment, the transmitting electrometer was operated with a sinusoidal ac voltage of variable frequency. Due to the square-law characteristic, this led to a torque which was independent of the frequency. In this experiment, a change of the capacitances by charging or charge reversal of isolating layers would have become directly apparent. However, frequency variations in the range 50 Hz to 1000 Hz did not cause any perceptible changes.

6.1.2 Nonlinearities in the electrometer

To investigate a potential nonlinear relation between the voltage and the torque, for example as a result of marginal effects of the electrostatic field, we changed the mean torque in several experiments. In one experiment, we fed a constant voltage into the second electrometer system so that an additional constant torque acted on the torsion balance. In a second experiment, we placed large masses (as sources of constant gravitational forces) in the vicinity of the torsion balance. Both procedures allowed sufficiently large additional torques to be generated that the required control voltage was outside the range originally covered. Nevertheless, the results obtained for G showed no significant deviations.

6.1.3 Verification of the electrometer calculation

Our derivation of the relationship between control voltage and generated torque is based on the calculation of the changes in the electric field energies in the capacitances, which take place when the torsion balance is rotated. To check this relationship in a practical experiment, it would have been desirable to test our measuring equipment using an accurately known torque. As such a torque was not available, we used the second electrometer system as a torque transmitter. To avoid the same error being produced by both electrometers, the transmitter electrometer was operated in a different circuit, the isostatic circuit of Figure 2b.

Between the voltage fed and the torque generated there is theoretically a quadratic relationship. This was checked with the second electrometer. This experiment showed excellent agreement between calculated and measured torque.

6.1.4 Dependence of the measured value on the auxiliary voltage

Although a dependence of the measured value on the auxiliary voltage is not to be expected from the theoretical point of view, we also investigated this aspect by halving the auxiliary voltage at the electrometer with the aid of a precision voltage divider. The result was an approximate duplication of the control voltage; the complete evaluation formula did not reveal a deviation in the results obtained for G.

After having carried out the experiments described, we are convinced that our torque transmitter is without error source.

6.2 Calculation of the torques of the gravitational forces

As we use cylindrical reaction masses and as the mass of the bearing parts of our torsion balance is relatively great, the gravitational forces exert torques on the torsion balance which are difficult to calculate. A significant number of tests were therefore required to eliminate errors in the evaluation of the measurement.

6.2.1 Calculation of the torque

Extensive calculations are required to determine the useful force, i.e. the attraction of adjacent cylindrical masses. To check the algorithm, the calculation was repeated numerically in an elementary way by imagining the masses to be divided into a great number of small parts and applying the point-mass formula to each pair of small masses. The time required for the calculation increased greatly as the division was refined, but convergence on the value originally obtained became obvious.

6.2.2 Calculation of correction torques

We had to calculate the additional torques M_{k} generated by forces between the source masses and different parts of the balance: the test masses on opposite sides of the balance give rise to a torque of about $2 \cdot 10^{-3} \cdot M_{\mathrm{N}}$, the rods carrying the test masses induce $7 \cdot 10^{-3} \cdot M_{\mathrm{N}}$, the balance beam induces $2 \cdot 10^{-3} \cdot M_{\mathrm{N}}$ and the needles of the electrometers induce $0{,}7 \cdot 10^{-3} \cdot M_{\mathrm{N}}$, where M_{N} is the main torque originating from the source masses and the adjacent test masses as explained in Section 4. The relevant torques were calculated by applying approximations, in which the parts were treated as point masses or line masses.

Correctness was checked by an experiment, in which the cylindrical reaction masses were removed from the torsion balance; their mass was replaced by means of a cylinder situated as low as possible at the bottom of the balance shaft with its axis congruent to the axis of rotation. This experiment allowed the correction torque to be determined directly. We observed agreement between the calculated and the measured values within the statistical uncertainty of the torque measurements of 10^{-12} Nm.

The experiments carried out to check the dependence on the distance also serve to check the correctness of our approximation formulas for M_{k}; the effect of an error in these calculations would have been quite different for different distances.

6.3 Influence of thermal interference

On the one hand, our equipment is susceptible to thermal interference, as the torsion balance operates in a gas atmosphere and as it is subject to the dynamic effects exerted by the gas atoms. On the other hand, thermal effects can falsify the torque differences only if they coincide with the movements of the masses shifted.

At an earlier stage of our experiment, we used tube-like devices of transparent synthetic material (polyacryl) to move the source masses. At that time, we observed warm-up effects: after displacement of the masses, which took about 10 s, the control voltage increased almost exponentially to its final value, with a time constant of about 40 s; during this time, it generally varied between 2 mV and 5 mV (about 10^{-3} of our useful signal). This effect was no longer seen when thick-walled brass tubes were used as displacement devices. Moreover, with the old device, we noticed that our results varied with the gas pressure in the compartment of the torsion balance; the result obtained for G was increased by about $4 \cdot 10^{-4}$ when the helium pressure was increased from 10^4 Pa to 10^5 Pa. This effect was no longer observed either.

Despite our efforts to obtain good thermal isolation, the temperature in the interior of our casing was obviously not sufficiently uniform, and the masses returned from the distant position at a temperature differing slightly from that in the close-up position. On the displacement device made of synthetic material, we measured a temperature difference of about 0,03 K between the close-up and the distant position; at the surface of the pendulum casing, the temperature changed by about 0,5 mK when the masses were moved to and fro. When using the brass displacement devices, we were unable to measure the small temperature differences.

Our explanation is supported by the fact that similar, though much greater, effects were observed when the displacement device was intentionally heated or cooled. Thermal contact between the front wall of the displacement device and the wall of the torsion balance casing also considerably increased the effects. On the other hand, an experiment in which each displacement was repeated five times in succession did not show a noticeable difference from the normal case: the generation of heat during motion cannot play an important role.

6.4 Influence of magnetic interferences

A significant falsification of our measurement results by magnetic effects is conceivable, as the tungsten and Zerodur masses have slightly different permeabilities. These inevitably change the existing magnetic fields, so the effects of the magnetic force vary when the positions of the masses are changed.

To check the existence of such influences, we carried out a series of auxiliary experiments with the aid of a large Helmholtz coil, which allowed the direction and strength of the magnetic field in the vicinity of our equipment to be changed drastically. Variations in the mean level of the control voltages, but no changes in the voltage difference, were observed when the masses were moved to and fro. There is obviously a dynamic effect of the Earth's field on the torsion balance, but these forces do not change significantly when the masses are moved.

6.5 Influence of electric fields

The torsion balance was earthed via a platinum wire and a cup filled with sulphuric acid. Even so, contact potentials could not be prevented altogether. Differences typically of 0,15 V occurred.

This potential has two effects. First, it causes a deviation of the electrometer torque from the value calculated for a grounded needle. We applied the second electrometer with an inverse auxiliary voltage, as shown in Figure 2c, so the second system compensates the deviation. Second, it causes electrostatic fields between the torsion balance and the casing. These exert no influence, as the torsion balance does not move and the additional torque is constant. We nevertheless made the resulting torque negligibly small by carrying out preliminary tests and selecting a working position for the torsion balance which is symmetric with respect to the casing walls. Intentional charges of up to 10 V and more did not show measurable effects.

The calibration of the electrometer is carried out by altering the angular position of the balance, as noted above. This additional procedure requires a determination of the angular variation of the capacitances, $\mathrm{d}C/\mathrm{d}\alpha$, a quantity which can be measured, whether or not other forces are acting.

7. Discussion of the results

Our result for the gravitational constant G,

$$G = 6{,}715\,40 \cdot (1 \pm\ 83 \cdot 10^{-6}) \cdot 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}},$$

deviates significantly from the CODATA value [7],

$$G = 6{,}672\,59 \cdot (1 \pm 128 \cdot 10^{-6}) \cdot 10^{-11}\ \mathrm{N \cdot m^2 \cdot kg^{-2}}.$$

The difference amounts to 0,6 % or 50 standard deviations. However, although we have spent much time and effort on these measurements, and despite an intensive search, we have found no error in our concept, our measurements or our evaluation.*

The 1986 CODATA value is based on a single measurement, i.e. the measurement carried out by Luther and Towler [6], as no other experiment with a sufficient and reliable estimate of the uncertainty was available. Nevertheless, the majority of older measurements [1, 2], are consistent and in good agreement with this value. Many determinations of G report uncertainties better than 1 %, but are not included in the CODATA adjustment.

We therefore looked for differences between our measurements and those made by Luther and Towler which, in conjunction with unknown deviations from the classical law of gravitation, could explain the discrepancy. As we used cylindrical masses of comparable dimensions, while Luther used relatively large tungsten balls and tiny torsion balance masses, the distances between the sources of the gravitational fields for our experiments were smaller in the close-up position: the distances between our centres of gravity were about 37 mm in the close-up position

*The authors have prepared an internal report, giving details of the experimental setup, the evaluation of corrections and additional investigations. This report will be made available to interested readers.

and about 63 mm in Luther's experiment. The results of our additional experiment at greater distances are inconsistent, however, with the explanation that the discrepancy is due to a deviation from the law of gravitation.

Another important difference from Luther's experiment is that we did not measure the gravitational force at a defined distance. Instead, we determined the difference between the forces for two distances which, measured between the mass centres, always differed by 105 mm (our displacement length). It is possible to construct a gravitational potential which increases at short range so that the same difference would always be measured for different distance ranges. Such a potential would show greater relative changes at long range, where the force differences are smaller, than at short range. This characteristic is, however, hardly conceivable.

The conclusion to be drawn, therefore, is that the uncertainty in the value for the gravitational constant is currently much greater than had been assumed and that further precise determinations of its numerical value are required.

References

1. Gillies G. T., The Newtonian Gravitational Constant. An Index of Measurements, *Metrologia*, 1987, **24 (Suppl.)**, 1-56.
2. de Boer H., Experiments Related to the Newtonian Gravitational Constant, In *Precision Measurements and Fundamental Constants II* (Edited by B.N. Taylor and W. D. Phillips), Natl. Bur. Stand. (US) Special. Pub., 1984, **617**, 561-572.
3. de Boer H., Haars H., Michaelis W., A New Experiment for the Determination of the Newtonian Gravitational Constant, *Metrologia*, 1987, **24**, 171-174.
4. Augustin R., de Boer H., Haars H., Michaelis W., Ein selbstzentrierendes Quecksilberlager, *Feinwerktechnik & Messtechnik*, 1982, **90**, 411-413.
5. de Boer H., Haars H., Michaelis W., Schlimme E., Quadrantenelektrometer als Drehmomentmesser für kleine Drehmomente, *Feinwerktechnik & Messtechnik*, 1980, **88**, 237-241.
6. Luther G. G., Towler W. R., Redetermination of the Newtonian Gravitational Constant G, *Phys. Rev. Lett.*, 1982, **48**, 121-123; Luther G. G., Towler W. R., Redetermination of the Newtonian Gravitational Constant G, In *Precision Measurement and Fundamental Constants II* (Edited by B.N. Taylor and W.D. Phillips), Natl. Bur. Stand. (US) Spec. Publ., 1984, **617**, 573-576.
7. Cohen E. R., Taylor B. N., The 1986 Adjustment of the Fundamental Physical Constants, *Rev. Mod. Phys.*, 1987, **59**, 1121-1148.
8. Funck T., Eine elektrostatische Messeinrichtung zur Darstellung der Einheit der elektrischen Spannung, dissertation TU Braunschweig, 1992.
9. *Guide to the Expression of Uncertainty in Measurement*, Geneva, ISO/TAG 4/WG 3, 1992.

Received on 11 August 1994 and in revised form on 22 February 1995.