Determination of G by Means of a Beam Balance

Frithjof Nolting, Jürgen Schurr, and Walter Kündig

***Abstract*— We are carrying out a high-precision measurement of the gravitational constant G by means of a beam balance. Test measurements with water as field masses have been made and G has been determined with an uncertainty of 240×10^{-6} [1]. In the next step 13.5 tons of mercury will be used as field masses. The preparations for these measurements are briefly described in this paper.**

***Index Terms*— Beam balance, fundamental constant, gravitational constant, mercury.**

I. Introduction

The presently accepted value for the gravitational constant G has a claimed uncertainty of 128×10^{-6} [2]. Recent attempts to improve the precision have lead to large and unexplained discrepancies indicating that there are systematic effects which are not understood.

The most often used device for measuring G is the torsion balance in one of its various forms. We have designed an experiment using a beam balance to measure G [3]. The basic idea is to measure the weight difference of two test masses. The weight difference is changed by the gravitational force of two moveable field masses. G can be calculated from the measured change of the weight difference. The general principles of the experiment will be explained in detail in the following section. The idea arose from our experiment at the Gigerwald storage lake [4], [5]. The water of the Gigerwald storage lake was used to measure G at an effective distance of about 100 m. This experiment was carried out in order to set new constraints on the strength and range of a conjectured fifth force. G was determined with an uncertainty of 750×10^{-6}, the most precise value at this effective distance.

There have been only few attempts to measure G by means of a beam balance [6], none of which reached a precision comparable to the torsion balance measurements. Due to the recent developments in the balance technology, an optimized mass arrangement, and a favorable measuring procedure, a precision of 10×10^{-6} seems to be within reach. A detailed description of the measurements made with water can be found elsewhere [1], [7].

II. General Principles

In this experiment the "lake" of the previous measurement is taken into the laboratory (see Fig. 1). Two test masses are suspended on separate long wires from the beam balance so that they hang on the same axis but at different levels. The weight difference of these two test masses is measured with the beam balance. Effects that are equal for both test masses such as tidal forces or zero point drifts of the balance cancel. The weight difference is modulated by the gravitational force of two movable field masses, which are moved periodically between two positions. In the first position they rest between the test masses. In the second, one is above the upper and one is below the lower test mass. The resulting change of the weight difference is the gravitational signal. All effects that are not correlated with the position of the field masses cancel in the average.

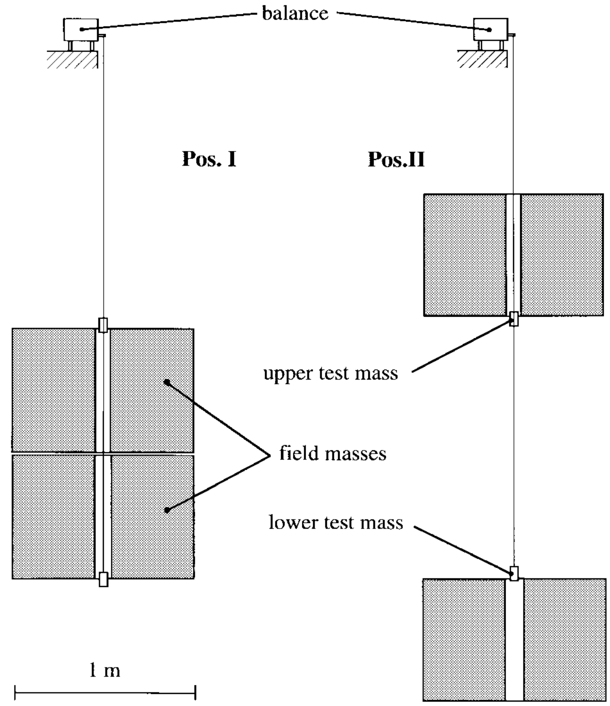

Fig. 1. Schematic view of the mass arrangement. The two field masses are alternately positioned in two states.

The field masses are cylinders with a central hole. The gravitational force of each field mass has extrema close to both ends of the hole. Each test mass is placed at an extremum. Therefore, the gravitational force acting on a test mass is to first order independent of the test mass position. This reduces the required accuracy of the distances between the field and test masses. Furthermore, only a moderate homogeneity is needed for the density of the test masses.

The field masses are tanks, which can be filled with a liquid. A liquid is used, because the density of liquids is more homogeneous than that of solids. Mercury is used because of

Manuscript received July 2, 1998. This work was supported by the Swiss National Science Foundation, the Dr. Tomalla Foundation, and the Scientific Research Foundation of the University of Zurich.

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Publisher Item Identifier S 0018-9456(99)02878-8.

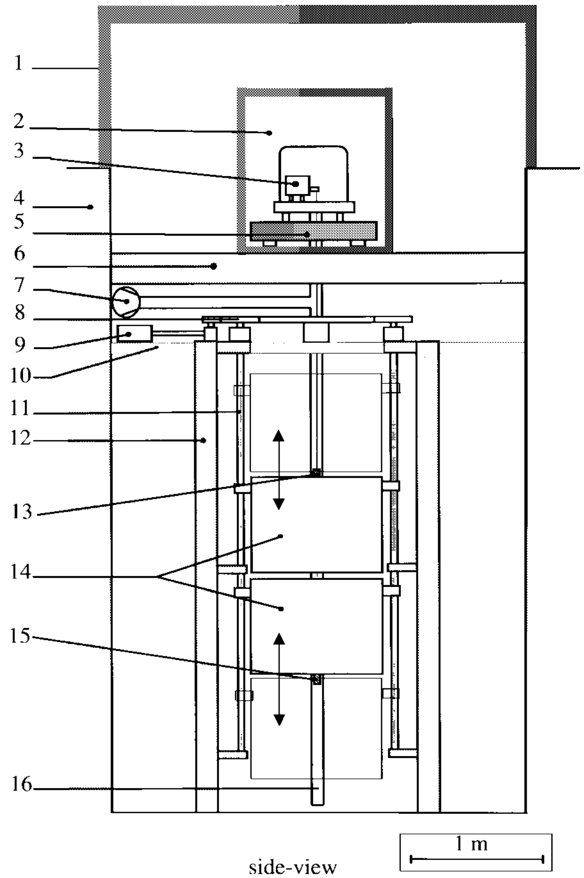

Fig. 2. Side view of the experimental setup. Legend: 1 = hut, 2 = thermally insulated chamber, 3 = balance inside the vacuum chamber, 4 = concrete walls of the pit, 5 = granite plate, 6 = steel girders supporting the balance, 7 = vacuum pumps, 8 = gear drive, 9 = motor, 10 = working platform, 11 = spindle, 12 = steel pillar, 13 = upper test mass, 14 = field masses, 15 = lower test mass, and 16 = vacuum tube.

its high and well-known density. Measurements made with water allow an important consistency check. The effect of the tanks can be measured with empty tanks and subtracted if necessary.

There are two further important features of this experiment resulting from the use of a beam balance. First, the beam balance can be directly calibrated in terms of mass by putting known standard masses on the balance. Second, the beam balance has a high load capacity compared with a torsion balance [8], [9]. As a result, the gravitational signal can be made much larger and thereby less sensitive to disturbing forces.

III. Experimental Setup

The experiment is located in a 4.5 m deep pit inside a machine hall at the Paul–Scherrer Institut. This pit has thick concrete walls and good mechanical and thermal stability. A schematic view is shown in Fig. 2. The experiment extends over two rooms. The upper contains the balance, the electronics and the auxiliary equipment. The lower houses the field masses. The rooms are thermally and mechanically insulated from each other.

The tanks are made of stainless steel, each having an outer diameter of 1046 mm, a volume of 500 liters and a weight of 800 kg. Their shapes and weights were measured with high precision. They are suspended by means of ball bearings on three spindles with left- and right-hand threads, so that they can be moved vertically and in opposite directions. The spindles are coupled by a gear-drive and driven by a synchronous motor.

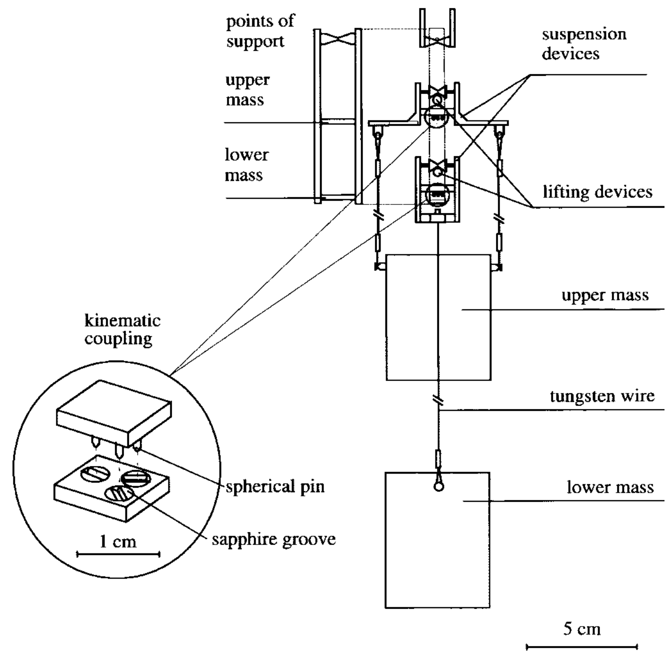

Fig. 3. Schematic side view of test masses and the suspension devices.

The balance is placed on a granite plate, which is supported by two steel girders fixed to the wall of the pit. In order to avoid errors due to convection, buoyancy and other gas pressure forces, the balance is operated in vacuum. The vacuum system consists of a 0.5 m diameter chamber housing the balance and a long tube enclosing the test masses. The pressure is about 1×10^{-4} Pa.

The balance is a modified version of the AT1006 Comparator of Mettler Toledo with an increased resolution of 100 ng. This balance was constructed for high precision comparison of 1 kg standard masses. It is a single-pan, flexure-strip balance with a servo-controlled beam. It works on the substitution principle, which means that the masses to be compared are placed one after the other on the same pan. The exchange of the test masses is a critical part because the reproducibility of the weighing depends strongly on the performance of the exchange mechanism. Our design is shown in Fig. 3. For the exchange of the test masses, two suspension devices are alternately connected to the balance via a kind of a ladder, labeled as points of support in Fig. 3. Each test mass is suspended from the device by two tungsten wires. The wires are 0.1 mm in diameter and have a length of 2.3 m for the upper and 3.7 m for the lower test mass, respectively. By means of a lifting device, the suspensions of the test masses can be lowered onto or lifted from the point of support. The lifting devices are moved hydraulically by stepping motor pumps, which are placed outside the vacuum chamber. Each test mass has its own, independent lifting device. The test masses are moved only vertically and by at most 0.8 mm. The stepping motors are controlled by a computer and during the exchange the reading of the balance is used to maintain a load which is constant to within 1 g. This is important in order to reduce effects resulting from anelasticity in the flexure-strips of the balance [8]. One exchange takes typically 130 s. The horizontal position of the suspension device with its kinematic coupling (see Fig. 3) [10] is reproducible to better than 10 μm. The test masses are cylinders, 45 mm in diameter and 77 mm high. Their shape has been measured by a coordinate measuring machine. They are made of copper and are equipped with two small hooks of Cu-Be for the suspending wires. The test masses are gold plated. Their masses are 1 097.607 0(3) g and 1 095.073 4(3) g for the upper and lower test mass, respectively, and the long term stability is checked every few months.

The zero point drift of the balance is mainly caused by temperature changes. If the drift is small and slow in comparison with the time between successive readings, it cancels in the weight difference as mentioned above. Therefore, it is desirable to make the short time variations of the temperature as small as possible, whereas slow drifts of the temperature are tolerable. For this purpose the balance is placed inside a heavy copper box (45 kg). The copper box is inside the vacuum chamber and the vacuum serves for thermal insulation. Further, the vacuum chamber is inside a thermally insulated box and only the temperature of the surrounding is actively stabilized. With the present system, the temperature of the balance is observed to drift by typically 10 mK per day.

Also the temperature of the vacuum tube in the vicinity of a test mass was found to be important [1], [7]. Therefore, a ventilating system is installed in the lower part of the experiment and the setup is covered by radiation shields.

Due to these precautions a reproducibility of 300 ng is reached. This is the standard deviation of ten weight comparisons. No effects were found that are correlated with day-night, week-weekend, or vacuum pumps on–off differences.

For the calibration a 1 g standard mass is placed on the balance in addition to a test mass. The statistical uncertainty of the calibration is 0.5×10^{-6}. The calibration has been tested with two 0.5 g standard masses and no discrepancy was found. To convert the balance reading into force, the local gravity at the balance, $g = 9.807\,233\,5(6)$ ms^{-2} [11] is used.

IV. MEASUREMENTS

Measurements with empty and with water filled tanks have been carried out during 1997. The whole measuring procedure is fully automated and computer controlled. First, the weight difference of the two test masses is determined while the field masses are in position I. The test masses are alternately connected to the balance and the readings for each test mass are averaged over 120 s. The weight difference is calculated by linear interpolation between two successive measurements. One complete cycle takes 11 min. After approximately 4 h the field masses are moved to the second position, which takes about 5 min. Then the weight difference is determined again. After another 4 h, the tanks are moved back to the first position. This procedure is repeated many times. The balance is calibrated in each field mass position. No variation of the calibration factor was found. Some data obtained with water filled tanks are shown in Fig. 4. The raw readings for the upper and lower test mass are plotted in (a). The determined weight difference is shown in (b). As can been seen the balance reading is influenced by drift effects, but these cancel in the weight difference. Further effects cancel in the gravitational signal, which is the change of the weight difference being 111 μg for water filled tanks.

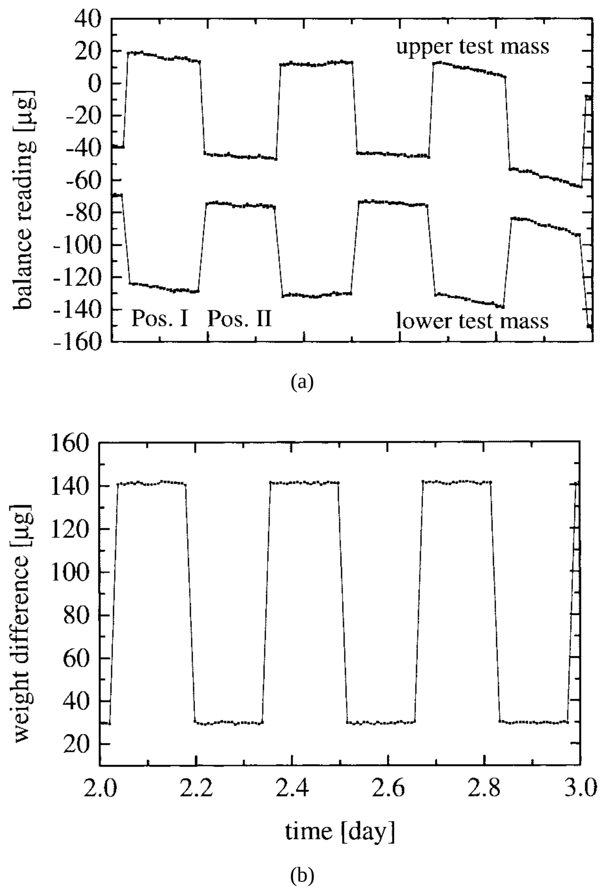

Fig. 4. (a) Reading of the balance for the upper and lower test mass while the water filled tanks are alternately positioned in Pos. I and II, and (b) the determined weight difference.

With a measuring time of 20 days the signal has been measured with a statistical uncertainty of 9 ng. To determine G, the gravitational force from the field masses on the test masses were computed by numerical integration. The results from measurements made with full and empty tanks were found to be consistent within the statistical uncertainty. The weighted average is

$$G = 6.675\,4(16) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}.$$

The combined standard uncertainty of 240×10^{-6} is mainly due to our present knowledge of the balance and the integration of mass distribution [1]. This result, which differs from the presently accepted value by about two of our standard deviations, represents the current state of our experiment.

During the winter of 1997–1998 the tanks were filled with mercury. The mercury was purchased in 400 flasks, each with a volume of 2.5 l. The filling procedure was similar to the procedure used for filling the tanks with water [1], [7]. To

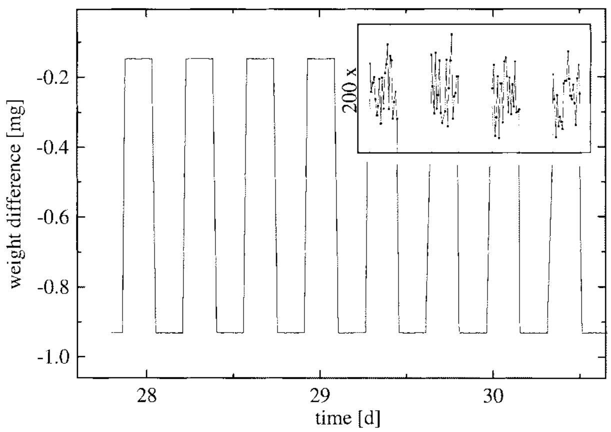

Fig. 5. Modulation of the weight difference due to the mercury filled tanks. In the inset the weight difference is shown enlarged 200-fold while the field masses are in Pos. I.

prevent air bubbles from being locked in the tanks, the tanks have been evacuated before the filling. To determine the mass of the mercury, each flask was weighed before and after the filling. The amount of remaining mercury in the filling system was found in a similar manner. In this way the total mass of mercury in the tanks was determined to be 13 520.635(27) kg. The position of the extrema of the gravitational force differs by about 40 mm for water and mercury filled tanks. The positions of the test masses have been changed accordingly.

A first measurement with mercury has been started. The resulting modulation of the weight difference is shown in Fig. 5. The change of the weight difference is about 785 μg and the resulting gravitational constant is in agreement with the measurement made with water filled tanks. Currently we are investigating possible systematic errors, mainly to improve our knowledge of the balance. Several measurements and improvements are in preparation and we expected to be able to give an improved value of G in the near future.

ACKNOWLEDGMENT

The authors would like to thank the Paul–Scherrer Institut for helpful support. They also wish to thank Mettler–Toledo for providing the balance and for mass determinations. They gratefully acknowledge the excellent work of the machine shop of our institute. They also thank E. Holzschuh for helpful discussions and all the metrological institutes that have helped them.

REFERENCES

[1] J. Schurr, F. Nolting, and W. Kündig, "Gravitational constant measured by means of a beam balance," *Phys. Rev. Lett.*, vol. 80, pp. 1142–1145, 1998.

[2] E. R. Cohen and B. N. Taylor, "The 1986 adjustment of the fundamental physical constants," *Rev. Mod. Phys.*, vol. 59, pp. 1121–1148, 1987.

[3] F. Nolting, J. Schurr, and W. Kündig, "A new experiment to measure G by means of a beam balance," presented at *1996 Conf. Precision Electromagnetic Measurements*, Braunschweig, Germany, 1996.

[4] B. Hubler, A. Cornaz, and W. Kündig, "Determination of the gravitational constant with a lake experiment: New constrains for non-Newtonian gravity," *Phys. Rev. D*, vol. 51, pp. 4005–4016, 1995.

[5] A. Cornaz, B. Hubler, and W. Kündig, "Determination of the gravitational constant at an effective interaction distance of 112 m," *Phys. Rev. Lett.*, vol. 72, pp. 1152–1155, 1994.

[6] G. T. Gillies, "The Newtonian gravitational constant: Recent measurements and related studies," *Rep. Prog. Phys.*, vol. 60, p. 151, 1997.

[7] J. Schurr, F. Nolting, and W. Kündig, "Measurement of the gravitational constant G by means of a beam balance," *Phys. Lett. A*, in press, 1998.

[8] T. J. Quinn, "The beam balance as an instrument for very precise weighing," *Meas. Sci. Technol.*, vol. 3, pp. 141–159, 1992.

[9] C. C. Speake and G. T. Gillies, "The beam balance as a detector in experimental gravitation," *Proc. R. Soc. Lond. A*, vol. 414, pp. 315–332, 1987.

[10] J. E. Furse, "Kinematic design of fine mechanisms in instruments," *J. Phys. E: Sci. Instrum.*, vol. 14, pp. 264–272, 1981.

[11] Measurement made by E. E. Klingelé, ETH Zürich, Switzerland, 1996.

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