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LETTER TO THE EDITOR

Supplementary investigations to PTB's evaluation of *G*

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Abstract
In addition to earlier experiments on the determination of G that were carried out in the PTB, supplementary investigations of the electrostatic torque transmitter unit have been performed. We have identified a previously overlooked effect that is capable of reducing the deviation from other determinations by an order of magnitude. Additionally, the uncertainty associated with the former value has to be enlarged.

1. Introduction

1.1. Motivation

The measurements of the Newtonian gravitational constant G performed in PTB [1] proved to be in disagreement with other determinations [2–5] and the CODATA adjustment of the fundamental constants [6]. The value of G obtained by PTB was about 0.6% larger than the CODATA value. Previously, it has been pointed out [7, 8] that the influence of the frequency dependence in the electrostatic torque transmitter may have been overlooked. This was the initiation for supplementary investigations, using the parts of the apparatus still existing from the previous measurements.

1.2. Description of previous work

The former determination of G [1] used a torsion balance, which was carried by the buoyancy of a floater in mercury. The angular deflection of the balance by the gravitational force was compensated by using a control loop and an electrostatic torque transmitter. During the experiment, attracting masses were moved between two fixed positions, and the resulting variation of the control voltage was used to calculate G.

Here, the torque transmitter is the object of further investigations. Its critical physical quantities are the derivatives of capacitances (between two pairs of electrodes and a movable electrode) with respect to the angle.

2. New investigations

A particular reason for the deviation could be the difference between the calibration method of the electrostatic torque transmitter, which operates by means of ac, on one side and the operating mode of the control circuit with nearly dc voltages on the other side. As it is impossible to compare both methods directly, since there exists no way to measure capacitances by means of dc voltages with the precision required for this purpose, supplementary investigations of related effects have been performed.

Additional errors may result from incomplete linearity of the dependence between capacitance and angle, possible dielectric layers carrying charges on the electrodes and the influence of the contact basin.

Another possibility results from contributions of the system of partial capacitances.

We performed three sets of additional measurements with the original electrometer used in [1].

2.1. Comparison of two methods for capacitance measurement

First we mounted our electrostatic torque transmitter (a kind of a quadrant electrometer) on top of the PTB's high precision device for angle measurements, an air-bearing turntable combined with an angle decoder with 2^{17} steps. The uncertainty for the determination of angles using this device is 1.2×10^{-8} rad.

For a series of angle values in the range of about $\pm 0.4°$, as used in the former G experiment, we determined the capacitances C_A and C_B with the help of two instruments: on the one hand with a high precision capacitance bridge of the type AH 2500A, operating in a manner similar to the one originally used (General Radio bridge, but with an improved circuit and with higher accuracy) and determining successively each of the two capacitances, while the other one is grounded; on the other hand we used a special device consisting of an inductive divider and an fine-tunable capacitor which enabled us to determine the ratio C_A/C_B simultaneously in a symmetric circuit.

The values of the capacitances and their slopes could not be reproduced exactly, compared with the former G measurements, as the vertical coordinate of the movable electrode has no fixed value. We observed a drift that appeared between forward and backward motion during the recent measurements. Up to now the origin for this drift is not identified.

In the measurements with the AH 2500 bridge, linear fits for the function $C_A(\alpha)$ and $C_B(\alpha)$ result in slopes of $dC_A/d\alpha = 34.4805\,\mathrm{pF\,rad^{-1}}$ and $dC_B/d\alpha = -34.5532\,\mathrm{pF\,rad^{-1}}$, respectively; hence, the coefficient c of the torque transmitter is calculated as $c = 690.337 \times 10^{-12}\,\mathrm{N\,m\,V^{-1}}$, $u(c) = 0.063 \times 10^{-12}\,\mathrm{N\,m\,V^{-1}}$.

In the measurements with the divider bridge the linear fit leads to an expression for the ratio $C_A/(C_A + C_B)$. We calculated the resulting torque transmitter coefficient c and obtained $c = 690.233 \times 10^{-12}\,\mathrm{N\,m\,V^{-1}}$, $u(c) = 0.119 \times 10^{-12}\,\mathrm{N\,m\,V^{-1}}$.

In both cases there is no indication of a lack of linearity, and the resulting linear fits coincide within their uncertainties. The coefficients obtained with both methods are in agreement within the limits of their uncertainties and no significant difference was observed.

2.2. *Variation with frequency in the range from 60 Hz to 10 kHz*

In a second experiment we mounted the electrostatic torque transmitter inside the housing of the G apparatus, with a fixed position of the movable part (because the mercury bearing was no longer available). This movable part was connected to the electric ground by means of a basin with an electrolytic liquid (sulfuric acid), and the atmosphere was helium with reduced pressure, as in the original experiments. So the original state was reproduced as closely as possible. We performed measurements of the two capacitances with another Andeen–Hagerling bridge of the type AH 2700. This time we varied the frequency in the range from 60 Hz to 10 kHz. The sensitivity and the accuracy of the bridge measurement decrease if the frequency deviates from the optimal value of 1000 Hz.

In this experiment we obtained values of the capacitance that increased to a maximum of 7×10^{-5} relatively to the 1000 Hz value with low and with high frequencies.

The uncertainty related to these measurements considerably increases towards lower frequency, comprising the increase of the capacitance values. More importantly, the dissipation factor shows no rise towards lower frequencies. Therefore, the rise of the capacitance value to lower frequency is not significant for the experiment but is regarded as an artefact of the bridge.

Simple model calculations assuming a dielectric resonance at a higher frequency are not capable of describing the observed frequency dependence of the capacitance or the loss tangent. The rise with increasing frequency proves to be in agreement with a serial resistance of about 140 Ω, which we attribute to the resistance of the contact basin. This serial resistance leads to a dissipation factor of about $D = 3 \times 10^{-5}$ at $f = 1000\,\mathrm{Hz}$, which causes a relative difference of the capacity values for the serial and the parallel RC models of 10^{-9}. At lower frequencies, the difference is smaller so that an error in the calibration of the torque transmitter cannot arise from the serial resistance.

These measurements show that there is no relevant effect of any variation of the capacitance or dissipation to be seen between 1000 Hz and 60 Hz. The range down to 0 Hz cannot be examined without considerable effort.

2.3. *Contributions of previously overlooked partial capacitances*

In our previous interpretation of the apparatus we concluded that the torque M would result as the derivative of the electrical field energy with respect to the angle of rotation, i.e.

$$M = \frac{1}{2} \sum_{i,j=1}^{N} \frac{\partial C_{ij}}{\partial \alpha} U_{ij}^2,$$

where i and j denote all the electrodes including the needle and the surrounding parts of the housing, C the capacitance, U the voltage and α the angle of rotation.

However, only capacitances C_{AN} and C_{BN} between the electrode plates A, B and the moving electrode N (‘Needle’) were taken into consideration. It was overseen that additional capacitances involved in the field of the torque transmitter may vary with rotation, too.

To investigate the full system of partial capacitances we used the assembly of our electrometer in the old apparatus, without a balance beam but with an angle encoder instead of the liquid bearing. The electrometer housing was electrically insulated from the support and the shielding case, so that we were able to lead an additional cable to the capacitance bridge in order to get a facility to measure capacitances between the electrodes and the housing with the aid of an AH 2550A precision capacitance bridge. As our device consists of two electrometer systems, we now had to use six plugs to perform the capacitance measurements: two electrodes in the top system (A, B) and two electrodes in the bottom system (C, D), the common movable electrode (N, in this experiment connected by means of a wire) and the housing of the electrometer (O) (figure 1). We measured all partial capacitances (while all unused electrodes were grounded) against the angular position of the movable electrode.

We measured the following values C_i of the partial capacitance:

$$\begin{aligned} &C_{AN},\ C_{BN},\ C_{CN},\ C_{DN} && \approx 30\,\mathrm{pF}, \\ &C_{AO},\ C_{BO},\ C_{CO},\ C_{DO} && \approx 780\,\mathrm{pF}, \\ &C_{AB},\ C_{CD} && \approx 18\,\mathrm{pF}, \\ &C_{AC},\ C_{AD},\ C_{BC},\ C_{BD} && \approx 0\,\mathrm{pF}. \end{aligned}$$

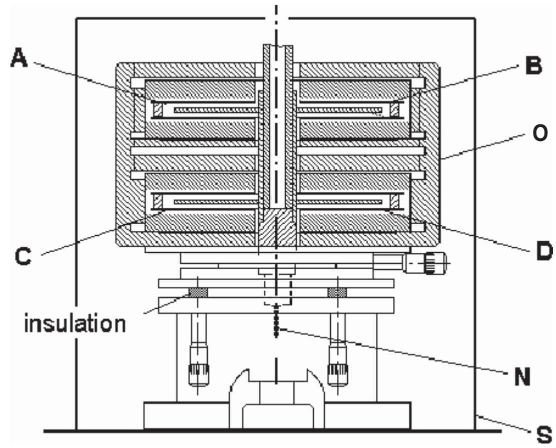

Figure 1. A drawing of the double electrometer system used in the G experiment with the various electrodes.

For the slopes $\partial C_i/\partial\alpha$ we obtained

$$\frac{\partial C_{\mathrm{BN}}}{\partial\alpha}, \frac{\partial C_{\mathrm{DN}}}{\partial\alpha} \approx +34.5\,\mathrm{pF\,rad^{-1}},$$
$$\frac{\partial C_{\mathrm{AN}}}{\partial\alpha}, \frac{\partial C_{\mathrm{CN}}}{\partial\alpha} \approx -34.5\,\mathrm{pF\,rad^{-1}},$$
$$\frac{\partial C_{\mathrm{BO}}}{\partial\alpha}, \frac{\partial C_{\mathrm{DO}}}{\partial\alpha} \approx -0.25\,\mathrm{pF\,rad^{-1}},$$
$$\frac{\partial C_{\mathrm{AO}}}{\partial\alpha}, \frac{\partial C_{\mathrm{CO}}}{\partial\alpha} \approx +0.25\,\mathrm{pF\,rad^{-1}},$$
$$\frac{\partial C_{\mathrm{AB}}}{\partial\alpha}, \frac{\partial C_{\mathrm{CD}}}{\partial\alpha}, \frac{\partial C_{\mathrm{AC}}}{\partial\alpha}, \frac{\partial C_{\mathrm{AD}}}{\partial\alpha}, \frac{\partial C_{\mathrm{BC}}}{\partial\alpha}, \frac{\partial C_{\mathrm{BD}}}{\partial\alpha} \quad \text{non-measurable.}$$

These values are approximate ones, because they vary with each remounting cycle of the apparatus. In our apparatus it was not possible to find the central position in the radial and axial directions absolutely. Thus, in the G experiment, we performed a calibration of the electrometer system before each measurement.

Contrary to our expectation and the model used before, the capacitance between the electrode plates and the housing varied: we explain this behaviour by the fact that the field components between the plates and the grounded border are influenced by the position of the moving electrode as illustrated in figure 2.

The electric field between the electrode plates and the housing is not rotationally symmetric because the plates are divided into four segments forming two pairs of electrodes A and B. Therefore the moving electrode N shields varying parts of these edge fields. As a consequence there is an additional contribution to the torque that was not taken into account in our former experiment and which amounts to about -0.71%. Hence this effect is capable of reducing the difference between the previous result [1] and the CODATA value [6] by an order of magnitude.

Unfortunately, this additional contribution cannot be determined with the accuracy required for a meaningful correction of our old result, because the conditions of the original apparatus cannot be reconstructed in detail. Furthermore, the influence of deviations on these contributions cannot be estimated, as the values of the slopes varied with test displacements in radial or axial directions in a complex manner. Their origin in edge field effects seems to be responsible for this complication.

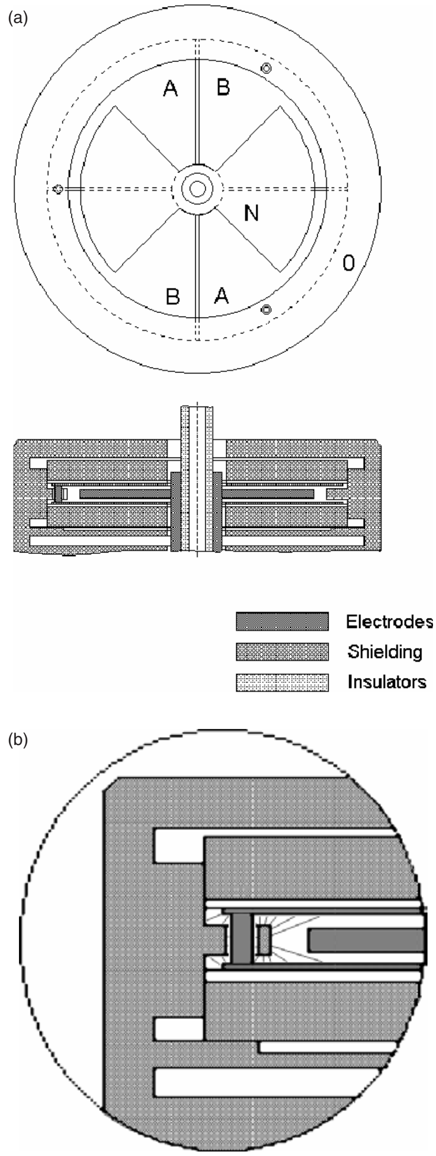

Figure 2. (a) Top view of the torque transmitter and a cross-section of one system. (b) A detail of the cross-section with some electric flux lines.

3. Requirement of a re-estimation of the uncertainty

Apart from the missing part in the torque transmitter equation, the contribution of the capacitance measurement to the uncertainty has been re-investigated. The measurement values were determined from the capacitance difference $\Delta C \approx 0.2$ pF derived from two capacitance measurements of about 30 pF. In the former evaluation [1] it has been assumed that the uncertainty of the measurement of the capacitance difference was $u(\Delta C/C) = 2 \times 10^{-5}$. This uncertainty was based on the assumption that the uncertainty of the capacitance bridge was determined by a set of high-precision standard capacitors in the bridge. In this scheme, the covariance between two measurements would reduce the uncertainty of the capacitance difference to a high degree.

A re-investigation of the circuits in the bridge used, however, shows that the internal couplings of the bridge do not allow us to apply this model. As a result of the coupling the divider ratios are not kept constant when switching from one particular capacitance measurement range to the next one. This nonlinearity of the bridge in the worst case would lead to the difference of two totally uncorrelated capacitance measurements. In this case the relative uncertainty would be as large as $u(\Delta C/C) = 2.7 \times 10^{-3}$, leading to the same fractional uncertainty of the measurement of the gravitational constant G. As this uncertainty has a magnitude close to the difference between our results and the CODATA value [6], this contribution to the uncertainty could largely explain the deviations. However, there are hints from experiments that lead us to assume that this effect is much smaller. The six measurements obtained for tungsten masses and the five measurements of the Zerodur masses were obtained over a period of six months, where the apparatus was repeatedly re-assembled, thereby leading to different capacitance values. Despite the different capacitances measured, the scatter of the resulting torque transmitter coefficient of relatively 10^{-4} (standard deviation) was much smaller. From these findings it seems that the two capacitance measurements were correlated to some extent. As the particular capacitance bridge used in the experiments of G is no longer available, the degree of correlation of both measurements cannot be determined. An alternative, more optimistic model of the capacitance bridge would lead to a relative uncertainty which is an order of magnitude lower than the worst case, but still an order of magnitude higher than the previous estimation of the uncertainty given in [1].

4. Summary and conclusions

A re-analysis of PTB's previous measurement of G [1] has been performed. Experimentally it has been found that the influence of partial capacitances in the electrostatic torque transmitter, not taken into account in the former measurement, is great enough to explain the difference between the former result of PTB's measurement and the CODATA value. As the measurements presented here have required a serious modification of the previously used apparatus, this effect cannot be calculated with the accuracy required for a meaningful correction of the previous value.

In addition the estimate of the contribution to the uncertainty associated with the measurement of the capacitance differences was too small. Its final contribution would depend on the covariance of the measurement of the two particular capacitances, which is not known.

In conclusion, both effects require that the value of G and its uncertainty presented in [1] can no longer be considered to be correct.

References

[1] Michaelis W, Haars H and Augustin R 1995/96 *Metrologia* **32** 267–76

[2] Schlamminger St, Holzschuh E and Kündig W 2002 *Phys. Rev. Lett.* **89** 161102

[3] Gundlach J H and Merkowitz S M 2000 *Phys. Rev. Lett.* **85** 2869–72

[4] Quinn T J, Speake C C, Richman S J, Davis R S and Picard A 2001 *Phys. Rev. Lett.* **87** 111101

[5] Armstrong T R and Fitzgerald M P 2003 *Phys. Rev. Lett.* **91** 201101

[6] Mohr P J and Taylor B N 2000 *Rev. Mod. Phys.* **72** 351–495

[7] Speake C C, Davis R S, Quinn T J and Richman S J 1999 *Phys. Lett.* A **263** 219–25

[8] Moore J 2001 *PhysicsWeb News* 3 August