Thermoelastic property of the torsion fiber in the gravitational experiments

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The thermoelastic and the nonlinear properties of a torsion fiber were studied. A symmetric disk torsion pendulum was designed to measure the temperature coefficient of the torsion spring constant of a tungsten fiber at room temperature, and the result shows that the ambient temperature fluctuation with $\pm 1\,°\mathrm{C}$ would introduce a considerable uncertainty about $\mp 165\,\mathrm{ppm}$ in the torsion spring constant of the fiber. It is suggested that the thermoelasticity of the torsion fiber should be measured in a precision torsion pendulum experiment. © *2000 American Institute of Physics.* [S0034-6748(00)03203-2]

I. INTRODUCTION

The Newtonian gravitational constant G has an important place in physics.[1] Since the first measurement by Cavendish,[2] there have been several hundred laboratory measurements of G by talented experimenters working in prestigious institutions.[3] The 1986 CODATA value for G was $6.67259\times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ with uncertainty of 128 parts per million (ppm), much larger than that of all other fundamental constants.[4] Furthermore, recent determinations of G wildly differ from each other and the 1986 CODATA value,[5] which are listed in Table I. This situation–disagreement far in excess of error estimate–suggests a new value for G should be recommended. In 1998, the CODATA recommended value for G is $6.673\times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$ with uncertainty of 1500 ppm.[6] It seems clear that investigations on all kinds of the systematic errors of different methods are desirable.

In the determination of G, the torsion pendulum is one of the most precise instruments, and works in two common modes: static mode and dynamic mode.[2,8,10,11,14–18] For the static mode, such as the direct deflection method, the gravitational constant G is determined by

$$G = K\Delta\theta/C, \tag{1}$$

where K is the torsion spring constant, C is a constant determined by the masses and the geometries of the attracting masses and the torsion pendulum, $\Delta\theta$ is the deflective angle of the pendulum due to the attracting masses. For the dynamic method, such as the time-of-swing method, G is determined by

$$G = \Delta K/(C_2 - C_1) = I\Delta(\omega_0^2)/(C_2 - C_1), \tag{2}$$

where ΔK and $\Delta(\omega_0^2)$ are the differences of the torsion spring constant and the frequency squares measured at two configurations of attracting masses, C_1 and C_2 are two constants determined by the masses and the geometries of the attracting masses and the torsion pendulum at the cases, respectively, and I is the inertial moment of the pendulum.

The accuracy of those experiments depends on the constancy of the torsion spring constant of the torsion fiber. While the research works about the physical properties of the torsion fibers indicate that the torsion spring constant is dependent on the oscillation frequency and amplitude due to the anelasticity and the nonlinearity, respectively.[11,17,19,20] In this article, we will show a new kind of inconstancy, the torsion spring constant of the torsion fiber is temperature dependent, and this will directly lead a systematic error to the uncertainty of a gravitational experiment with the torsion pendulum. Miscellaneous thermal effects except for the thermoelasticity in the gravitational experiments have been discussed by many authors. Chen and Cook discussed the thermal noise of the physical oscillator, thermal expansion of the system, heat treatment to remove the residual stress, and the temperature dependence of the contact potential in the gravitation experiments.[18] Adelberger *et al.* measured the fiber drift of the torsion balance with the temperature fluctuation and temperature gradients.[21,22] Gillies and Ritter reviewed the history of the fiber-suspended apparatus and assessed the fundamental limits of performance of the dumbbell pendulum, such as the thermal noise, the seismic noise, and the gravitational gradient effect.[23] Most of the authors studied the thermal expansion effect in the torsion balance experiments, but few of them considered the thermoelastic effect. In the following parts, we will show that the thermoelastic

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TABLE I. Recent results of the experimental measurements of G.

Source	G value $\times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$	Error estimate, ppm	Deviation from 1986 CODATA value, ppm
1986 CODATA value (Ref. 4)	6.67259	128	0
1998 CODATA value (Ref. 6)	6.673	1500	+61
Michaelis *et al.* (Ref. 7)	6.7154	83	+6420
Fitzgerald *et al.* (Ref. 8)	6.6656	95	−1050
Walesch *et al.* (Ref. 9)	6.6719	83	−105
Karagioz *et al.* (Ref. 10)	6.6729	78	+45
Bagley *et al.* (Ref. 11)	6.6740	102	+210
Schurr *et al.* (Ref. 12)	6.6754	240	+420
Schwarz *et al.* (Ref. 13)	6.6873	1400	+2210
J. Luo *et al.* (Ref. 14)	6.6699	105	−405

effect is much larger than the thermal expansion effect and it should be measured in the torsion balance experiments.

II. THERMOELASTIC PROPERTY OF THE TORSION FIBER

The torsion spring constant is an intrinsic property of a torsion fiber. For an ideal uniform circular fiber of length l and radius r, the torsion spring constant can be expressed as[18,24]

$$K=\frac{\pi r^4 S}{2l}, \tag{3}$$

where $S=E/2(1+\mu)$ is the shear modulus of the fiber, E and μ are Young's modulus and Poisson's ratio of the fiber, respectively. The l and r will depart from their initial values l_0 and r_0 due to the temperature change Δt as follows:

$$l=l_0(1+\alpha_l\Delta t), \tag{4}$$

$$r=r_0(1+\alpha_l\Delta t), \tag{5}$$

where α_l is the temperature coefficients of linear expansion. Since the shear and Young's moduli are associated with interatomic force which depends on the distance between atoms in the crystal lattice, the moduli S and E depend on the ambient temperature.[25] Therefore we can define the temperature coefficient of shear modulus of the fiber α_S for a small temperature change Δt near temperature t_0, and the shear modulus S at temperature $t_0+\Delta t$ can be expressed as

$$S=S_0(1+\alpha_S\Delta t), \tag{6}$$

where S_0 is the shear modulus at temperature t_0. From above Eqs. (4)–(6), the value of K changing with the temperature can be expressed as

$$K=K_0(1+\alpha_K\Delta t), \tag{7}$$

where

$$\alpha_K=3\alpha_l+\alpha_S \tag{8}$$

is the temperature coefficient of the K, and K_0 is the torsion spring constant at temperature t_0.

From Eqs. (1), (2), (7), and (8), we can show that the uncertainty of measuring G due to the thermoelastic property of the fiber is

$$\delta G/G=\delta K/K=\alpha_K\Delta t. \tag{9}$$

We can define the temperature coefficients of the Young's modulus and Poisson's ratio α_E and α_μ as same as α_S in Eq. (6). Using the equation $S=E/2(1+\mu)$, we knew that α_S, α_E, and α_μ satisfy with the following relation approximately

$$\alpha_S=\alpha_E-\alpha_\mu\frac{\mu_0}{1+\mu_0}, \tag{10}$$

where μ_0 is the Poisson's ratio at temperature t_0. The experiential value of α_E is about $-25\alpha_l$ and the values of α_μ and μ usually are positive.[25] So the value of α_K is at least one order larger than that of α_l, and should be determined for precise measuring G by a torsion pendulum. The α_K can be determined through measuring the temperature coefficient of the pendulum period α_T, which is defined as

$$\alpha_T=\frac{\Delta T}{T\Delta t}, \tag{11}$$

where ΔT is the change in the period due to the change in the temperature Δt. Because the period of the pendulum is $T=2\pi\sqrt{I/K}$, so α_T and α_K satisfy with the following relation:

$$\alpha_K=\alpha_I-2\alpha_T, \tag{12}$$

where α_I is the temperature coefficient of the inertial moment of the pendulum, which can be calculated according to the α_l of the pendulum. Therefore, we can measure the period varying with the variation of the room temperature and then determine α_T and α_K of the torsion pendulum, correspondingly.

A schematic of the apparatus used in our measurement is shown in Fig. 1. A symmetric disk aluminum torsion pendulum of 57.415 mm in diameter and 9.988 mm in thickness was designed to measure temperature coefficient of the period α_T. The background gravitational effect can be eliminated by the symmetric design of the pendulum. The pendulum was suspended by a tungsten fiber and put in a stainless vacuum vessel maintained at vacuum of 1.5×10^{-5} Pa by an ion pump. The thickness and length of the fiber are 25 μm and 513 mm, respectively, which has been suspended for more than three years and has been used in our experiment of determining G.[14] The present drift of the fiber was less than 2×10^{-6} rad/day. The height and the initial amplitude of the torsion pendulum can be adjusted by a vacuum feedthrough.

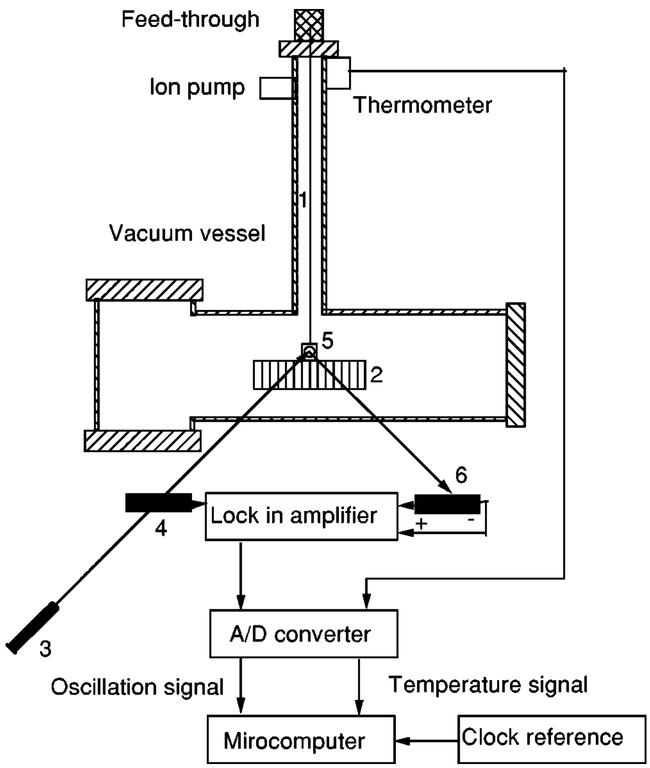

FIG. 1. A schematic of the apparatus used in measurement of the thermoelasticity of a tungsten fiber, where 1 is the tungsten fiber, 2 is the torsion pendulum, 3 is the laser source, 4 is the optical chopper, 5 is the mirror, and 6 is the position sensor used to measure the pendulum oscillation.

The apparatus was located in an electromagnetic shield room of 5 m$\times$3 m$\times$3.5 m in dimension, which stands on a heavy vibration isolation platform in our cave laboratory.[14]

The rotation of the torsion pendulum was detected by an optical lever. The beam from a frequency-stabilizer He-Ne laser 3 ($\Delta\nu/\nu \leqslant 1\times10^{-8}$/day, $P=0.6$ mW) passed through an optical chopper 4 (SR540, made by Stanford Research Systems Company, USA), and then was reflected by the mirror 5 attached on the center of the pendulum with a reflection angle about 5°. The beam finally fell on a position sensor 6 (SD-1161-21-11-391, made by Silicon Detector Corporation), which was placed about 450 mm far from the mirror. The difference signal voltage from the position sensor went through a lock-in amplifier (SR830) and then was converted into a series of time-angle data through a 16-bit A/D converter, and finally recorded by a microcomputer. Four bulbs with power 40 W were concealed in four metal boxes suspended in the outside of the shield room as the heat sources to adjust the shield room temperature. A quartz thermometer with resolution of 0.001 °C (SW-2, Changsha Instrument Works, China), whose probe was attached on the vacuum vessel near the suspension point, was used to monitor the temperature of the pendulum approximately. A square wave of 2 Hz from a time interval counter (SR620, $\Delta\nu/\nu \leqslant 5\times10^{-10}$/day) was used as the trigger signal. Data were taken for each set in the following procedure. We first let the pendulum oscillate at usual temperature t_1 about 19.8 °C for about 10 hs, then heated the room to a high temperature t_2 about 20.9 °C by means of the bulbs, and this duration lasted about 60 hs. Finally, the bulbs were turned off and the temperature in the room decreased gradually. The recording system sampled the oscillation signal of the pendulum and the output of the thermometer continuously each half-second for five days. A modulation of the ambient temperature was shown in Fig. 2. The quality factor of the pendulum is about 1520 and the amplitude attenuated from 0.016 to 0.001 rad in this duration. After that we repeated the above procedure again for the next set of the experimental data. We obtained three sets of the experimental data from May. 28 to Jun. 15 in 1999.

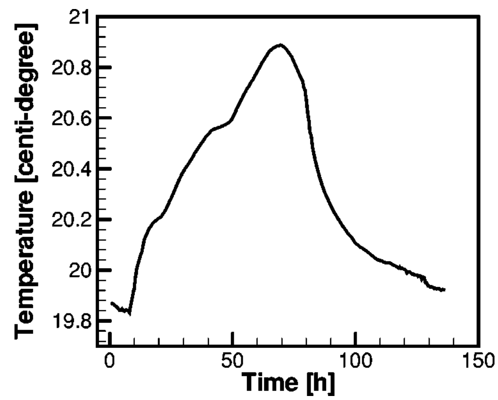

FIG. 2. The modulation the ambient temperature during Jun. 6 to Jun. 11 in 1999. We first let the pendulum oscillate at usual temperature t_1 about 19.8 °C for about 10 h, then heated the room to a high temperature t_2 about 20.9 °C by means of the bulbs, and this duration lasted about 60 h. Finally, the bulbs were turned off and the temperature in the room decreased gradually.

The period of the pendulum oscillation was fit by means of the period fitting method[26] for each half an hour data. Figure 3 shows a typical oscillation period modulated by the ambient temperature. Figure 4 shows a typical period of the pendulum versus the ambient temperature. The circle points represent the experimental values of the period T at different temperature t, and the solid line represents the linear fitting result of T varying with t. The slope of the fitted line is 0.0391 s/°C which indicates the rate of the change in the period with the temperature, and the intercept is 359.803 s which represents the period of the pendulum at the temperature 0 °C. The statistical root-mean-square of the fitted line is 0.001 s. Therefore, the value of α_T of the pendulum is 108×10^{-6}/ °C when the temperature of the vessel is 20 °C,

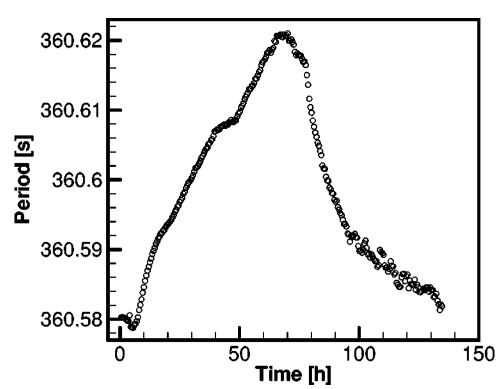

FIG. 3. A typical oscillation period modulated by changing the ambient temperature. Each circle point represents the period fitted by means of the period fitting method for each half an hour data.

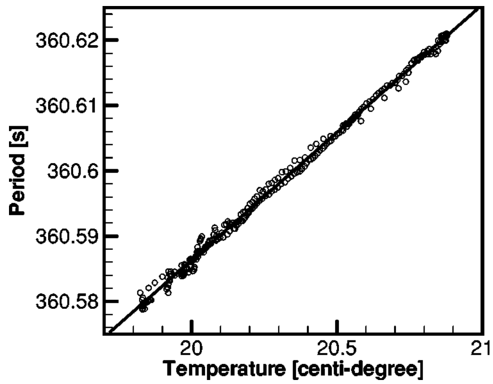

FIG. 4. The period of the pendulum vs the ambient temperature near 20 °C. The circle points represent the experimental values of the period T at different temperature t, and the solid line represent the linear fitting result of T varying with t.

and the uncertainty of α_T is $3\times10^{-6}/\,°\mathrm{C}$ according to Eq. (11). The fitting results for three sets experimental data indicate that the values of α_T are $103\times10^{-6}/\,°\mathrm{C}$, $108\times10^{-6}/\,°\mathrm{C}$, and $107\times10^{-6}/\,°\mathrm{C}$, respectively, and the mean value is $\alpha_T=(106\pm3)\times10^{-6}/\,°\mathrm{C}$.

In our experiment, $I=mR^2/2$, where m and R are the mass and the radius of the aluminum disk, respectively, so $\alpha_I=2\alpha_l(\mathrm{Al})=47\times10^{-6}/\,°\mathrm{C}$. According Eq. (12), the temperature coefficient of the torsion spring constant of the tungsten fiber is $\alpha_K(\mathrm{W})=-(165\pm6)\times10^{-6}/\,°\mathrm{C}$. It means that the 1 °C variation of the ambient temperature would introduce 165 ppm relative uncertainty in the torsion pendulum experiments both in the static mode and the dynamic mode. Some experiments also indicate the existence of this phenomenon, such as in Ref. 11, the authors observed a diurnal variation of torsion pendulum frequency about ∓70 ppm as shown in Fig. 2 in Ref. 11, it may be due to the ambient temperature variation of about ±0.5 °C in a day. Of course, the systematic error due to the variation of the torsion spring constant with the temperature will be less than 70 ppm because the time span of measuring $\Delta(\omega_0^2)$ was limited in an hour in Ref. 11. In Ref. 8, there is a downward bias about 33 ppm in measuring G because the temperature of the electrometer calibrations was 0.2 °C higher than that of measurement of V_G^2. In Refs. 10 and 14, the temperature fluctuations are ±0.1 and ±0.005 °C, and the corresponding systematic errors due to these are about 17 ppm and less than 1 ppm, respectively.

To determine G with uncertainty of 100 ppm or less, we should control the ambient temperature within ±0.1 °C and monitor the temperature variation for correcting the experimental result. To reduce the temperature coefficient of the torsion spring constant, we can let the torsion pendulum operate at cryogenic temperature such as in Ref. 17, or select the better constant elastic alloy material as the torsion fiber. However, the thermoelastic property of the torsion fibers should be tested in a high precision gravitational experiment with the torsion pendulum both at room temperature and at the cryogenic one.

III. CHECK THE NONLINEAR PROPERTIES OF THE TORSION FIBER

As stated above, the amplitude of the pendulum oscillation was attenuated from 0.016 to 0.001 rad in the experiments. While the nonlinearity of the oscillation indicates that the period of the pendulum oscillation varies with its amplitude, so we should check the nonlinear properties of the fiber. The typical equation of the oscillation can be written as

$$I\ddot{\theta}+\gamma\dot{\theta}+K_1\theta+K_3\theta^3=0, \tag{13}$$

where I, θ, and γ are the inertial moment, the angle displacement, and the damping factor of the pendulum, respectively. The coefficient K_1 is the total equivalent torsion constant of the pendulum system, which usually includes three parts: the torsion constant of the fiber K_{1f}, the gravitational torsion constant of the attracting masses K_{1a}, and the torsion constant of the background gravitational field K_{1b}.[14] The coefficient K_3 represents the nonlinearity of the pendulum system, which can also be classified into three parts K_{3f}, K_{3a}, and K_{3b}, correspondingly. Using harmonic balance method,[27] we can obtain the approximate solution of Eq. (14) at small amplitude oscillation as follows:

$$\theta(t)\approx\theta_0 e^{-\beta t}\cos\omega t+\frac{K_3}{32K_1}\theta_0^3 e^{-3\beta t}\cos 3\omega t, \tag{14}$$

where $\beta=\gamma/2I$, θ_0 is the initial amplitude. The frequency ω^2 is shifted from its unperturbed value $\omega_0^2=K_1/I$ to

$$\omega^2(A)=\omega_0^2-\beta^2+\frac{3K_3}{4I}A^2, \tag{15}$$

where $A=\theta_0 e^{-\beta t}$ is the amplitude of the oscillation at time t. Correspondingly, the period of the oscillation can be written as

$$T(A)=T_0\left[1-\frac{I\beta^2}{K_1}+\frac{3K_3}{4K_1}A^2\right]^{-1/2}, \tag{16}$$

where $T_0=2\pi/\omega_0$ is the unperturbed period of the pendulum.

In Ref. 20, we measured the nonlinear properties of the fiber, and obtained $K_3/K_1=(-0.026\pm0.008)\,\mathrm{rad}^{-2}$. This result included a systematic error due to the variation of the ambient temperature. Therefore, we measured the ambient temperature synchronously and rechecked the nonlinear properties of the fiber during Aug. 10 to Sept. 5 in 1999. Three sets of data were obtained and a typical record of the period versus the oscillation amplitude was shown in Fig. 5. The solid squares represent the experimental values of the period without correction of the ambient temperature variation, and the circles represent the corrected values of the period. And the dashdot line and the solid line represent the fitting results with Eq. (16) in the two cases, respectively. The corresponding values of K_3/K_1 are shifted from -0.026, -0.028 and -0.023 to -0.0010, -0.0011 and -0.0011, respectively, after considering the variation of the ambient temperature, and the mean value of K_3/K_1 is about $-0.0011\,\mathrm{rad}^{-2}$. The uncertainty of K_3/K_1 due to the period uncertainty of 0.15 ms is $0.0018\,\mathrm{rad}^{-2}$ at amplitude 0.015 rad according to Eq. (16). It means that the nonlinearity of the

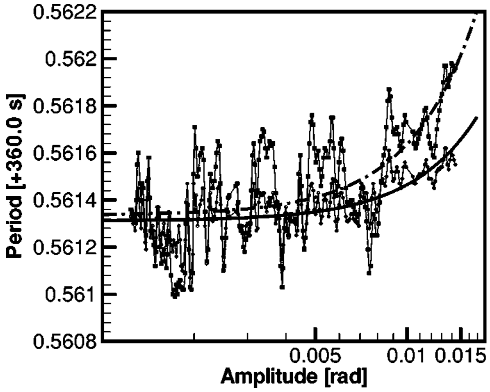

FIG. 5. The period of the pendulum versus the oscillation amplitude. The solid squares represent the experimental values of the period without correction of the ambient temperature variation, and the circles represent the corrected values of the period. And the dashdot and the solid lines represent the fitting results with Eq. (16) in the two cases, respectively.

fiber at most contributes a systematic error of $0.2 \times 10^{-6}/\,^{\circ}\mathrm{C}$ to the measurement of α_T. It is about 15-fold lesser than the experimental statistical uncertainty of α_T ($3 \times 10^{-6}/\,^{\circ}\mathrm{C}$).

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