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Absolute measurement of the Newtonian force and a determination of *G*

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Abstract. We present the status and preliminary results of the Wuppertal gravitational experiment which is based on a microwave resonator. The gravitational force of two test masses acting on the resonator is measured as a function of distance. From new data taken recently we determine the gravitational constant G and test Newton's inverse square law in a distance range between 0.7 m and 2.2 m. From our measurements we obtain a preliminary value for $G = 6.6735 \times 10^{-11}$ m^3 kg^{-1} s^{-2} with an uncertainty of 432 ppm dominated by systematic considerations.

Keywords: gravitational constant G, microwave resonator (GHz)

1. Introduction

The Fabry–Pérot Pendulum gravimeter at the University of Wuppertal has been in development since 1988 for absolute experiments on Newtonian gravity. Accounts of those developments can be found in several earlier papers [1, 2], contributions to conferences [3] and in the theses of N Klein [4], J Schurr [5] and H Walesch [6]. The value of G, the gravitational constant, was determined with nominal precision of a few times 10^{-4}, however we found the absolute value to differ by 10^{-3} between different sets of measurements and in addition to depend on the distance of the fieldmasses from the Fabry–Pérot gravimeter [7]. The following investigations revealed a rather large systematic shift in the determination of the absolute distance of the field masses that could qualitatively explain the distance dependence of the G measurements [8]. In 1997 a new method for measuring the absolute position of the field masses was installed that allowed for a sufficiently precise determination of the absolute distances in our set-up [9]. Here we report on a first set of measurements using the improved experimental arrangement and give a new preliminary value for Newton's constant G.

2. The Fabry–Pérot gravimeter

In the Fabry–Pérot gravimeter two fieldmasses provide a gravitational field to influence the relative positions of two pendula that constitute the defining walls of an open microwave cavity. Both pendula are suspended by two loops of tungsten wire, which are mounted from a suspension platform. The pendula are placed inside a vacuum tank with

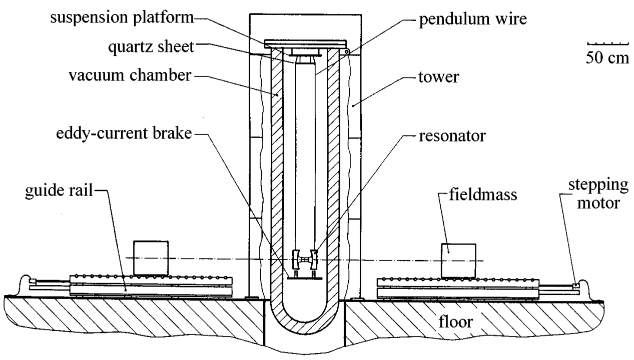

Figure 1. Schematic view of the experimental set-up with the Fabry–Pérot resonator and the two fieldmasses.

a pressure of $p \leqslant 5 \times 10^{-5}$ mbar. The position change of the two cavity mirrors is determined by measuring the resonance frequency of the cavity to high precision. The separation of the fieldmasses is changed periodically between a reference distance (r_{ref}) and a measuring position (r_i). The basic signal is then a change in the resonance frequency of the cavity

$$\Delta f = f_{r_i}^{resonance} - f_{r_{ref}}^{resonance} \tag{1}$$

which is related with high precision to the change in the separation of the two pendula using cavity theory [10]. Figure 1 shows the set-up and in table 1 lists values of some parameters.

The position of the pendula (and therefore the resonance frequency of the cavity) is rather sensitive to changes in the ambient temperature which is, however, a very smooth and slow effect and is quantitatively described by a polynomial of fourth order. Noise affecting the pendula positions is

Table 1. Some parameters for the components of the Fabry–Pérot gravimeter.

Item	Material	Dimensions	Mass
Fieldmasses	Brass	∅ 440 mm, l 430 mm	576 kg
Cavity	Copper and aluminium	∅ 200 mm, l 300 mm	6.7 kg
Wire	Tungsten	∅ 200 μm	3.25 g

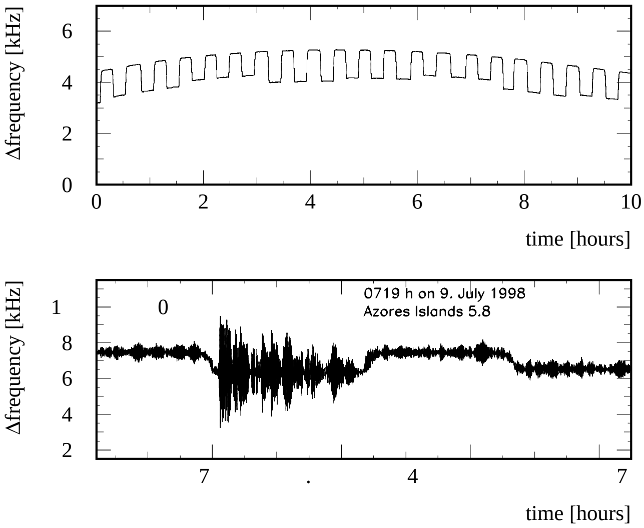

Figure 2. Measured resonance frequency change Δf plotted against time. The upper trace shows a 10 h portion of the measurements and lower trace shows a rather large earthquake.

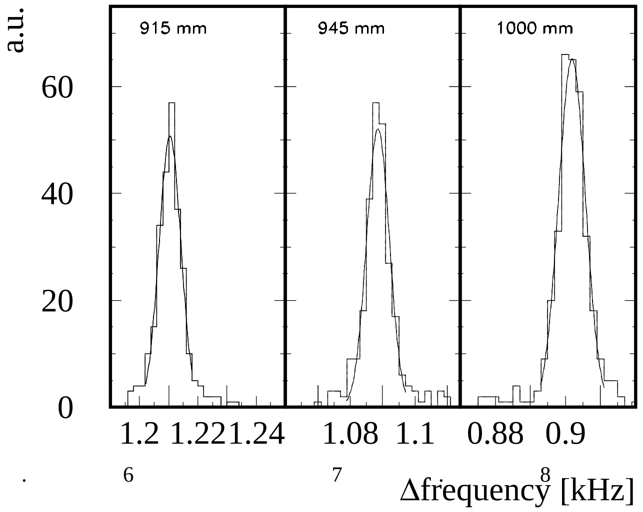

Figure 3. The measured frequency changes Δf for three different CM distances (915, 945 and 1000 mm) of the fieldmasses and the cavity.

mainly from seismic events with an amplitude range from microseismic to large earthquake. A set-up of eddy current brakes that can be adjusted by remote control effectively damps this noise. Furthermore, the regular movements of the pendula average the noise out to a very high degree of accuracy. So far no influence of the noise level on the final results from our measurements has been detected. The measurements consist then of a large number of frequency determinations while changing every twelve minutes the positions of the field masses between the measuring position and the reference position. A typical data set is shown in figure 2 where the measured resonance frequency change Δf is plotted against time for three choices of the position r_i.

The gravitational constant G is determined from

$$\Delta f = \left(\frac{\mathrm{d}f}{\mathrm{d}b}\right) G \frac{M}{\omega_2^0}\left[\left(\frac{1}{r_i^2} - \frac{1}{(r_i+b)^2}\right)K_r - \left(\frac{1}{r_{ref}^2} - \frac{1}{(r_{ref}+b)^2}\right)K_{ref}\right] \qquad (2)$$

with M the field masses (measured at PTB in Braunschweig), ω_0 the fundamental (undamped) frequency of the pendula, b the cavity length and K_r and K_{ref} two correction factors to be calculated from the mass distributions in the experiment, which essentially describe the deviation from a point mass geometry. To determine the length of the cavity b and the radius of the cavity mirrors R, a number of modes of the cavity are excited and their frequencies measured and compared with cavity theory. Enough modes of the cavity are easily obtained to provide a determination of b and R with sufficient precision.

Table 2. Results.

G ($\times 10^{-11}$ m^3 kg^{-1} s^{-2})	CM distance (mm)
6.6751 ± 0.0015	915
6.6721 ± 0.0016	945
6.6733 ± 0.0016	1000

The resonance frequency is typically centred in the range of 20 GHz $< \omega <$ 26 GHz. A shift of $\Delta b = 10$ nm (typical for the distances of the field masses) in cavity length changes the resonance frequency by about 1 kHz. This frequency shift can be resolved to about 1/10 Hz by averaging over a larger set of measurements (a few hundreds).

3. New measurements

As compared to our earlier measurements, the data of 1998 should benefit from

- the much improved position control of the field masses
- new cavity mirrors of higher quality
- better coupling of the RF to the cavity
- the new adjustable eddy current brake
- new program for mass integration [11].

We consider the above items to have successfully improved the experiment. Only the attempt to reduce the temperature dependence of the cavity was essentially unsuccessful. We replaced the quartz spacer in the suspension platform with a Zerodur plate to limit thermal expansion related drift in the mirror separation, but the temperature dependence remained at the same level as before. This effect is not yet understood.

The new measurements were performed from 24 June –30 July 1998. The field masses were moved simultaneously and symmetrically at 12 min intervals between a measuring position and a reference position. The distances between the centre of momenta (CM) of the cavity to the CM of the field masses at the measuring positions were either 915, 945 or 1000 mm; the reference position was at 2260 mm. The resonance frequency was determined every 500 ms. At 915, 945 and 1000 mm, 264, 262 and 347 measurements of Δf were taken. Figure 3 shows the distributions of the individual frequency shifts. At this stage of the analysis most of the values follow a clear Gaussian profile with only a very few showing larger deviations. The mean values of Δf at each position are determined and, using the values for the static parameters in relation (2), G is determined. The three values obtained are comparable (see table 2) and can therefore be averaged to give a preliminary value for G from the 1998 measurements:

$$G = 6.6735 \pm 0.0011 \pm 0.0026 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}.$$

The first uncertainty is purely statistical (from the Gaussian distribution of the individual measurements) and the second accounts for our present estimate of possible systematic effects on G.

More measurements are planned for the near future and we hope to finally achieve $\Delta G/G < 10^{-4}$ using the present version of the gravimeter.

Acknowledgments

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