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Preliminary determination of *G* using the BIPM torsion strip balance

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Abstract. The BIPM is undertaking a determination of the Newtonian gravitational constant, G using a torsion balance with a thin, heavily loaded strip as the suspension element. This apparatus uses four 1.2 kg test masses and four 15.5 kg source masses to produce a gravitational torque of 2×10^{-8} N m. A preliminary determination of G has been made with a relative standard uncertainty of about 1.7×10^{-3}, set primarily by the uncertainty in the measurement of the balance deflection by an autocollimator. The system will soon be used in a servo-controlled configuration, in which the gravitational torque will be balanced by an electrostatic torque applied between fixed thin cylindrical electrodes and the test masses. Operating in the servo mode should yield a significantly lower uncertainty by extending the autocollimator measurements over a larger range. A parallel effort will aim to lower the uncertainty of the unservoed measurement by using multiple reflections of the optical signal from the autocollimator.

Keywords: metrology, gravitational constant, torsion strip, autocollimator, torsion balance, capacitive transducer, fundamental constants

1. Introduction

A number of experimenters have determined G, the Newtonian gravitational constant, with stated relative standard uncertainties of the order of 10^{-4} (Fitzgerald and Armstrong 1995, Michaelis *et al* 1995/96, Schurr *et al* 1998; for a recent review, see Gillies 1997). However, the differences in the value of G obtained in these measurements are as great as 0.7%. There is thus renewed interest in performing various experiments that could shed light on these discrepancies and reduce the overall uncertainty in determinations of G taken as a group. We have undertaken a programme to determine G with a relative standard uncertainty of 10^{-4} or better using a torsion balance with a novel suspension. A prototype of the balance has been described elsewhere (Quinn *et al* 1997b); in this paper we report preliminary measurements with the Mark II version.

The designs of many torsion balances of this century have been guided by a scaling law attributed to Boys (1895). He noted that while the mass that could be supported by a torsion wire (and hence the gravitational torque) increases as the square of the wire diameter, the stiffness increases as the fourth power. The displacement signal is then inversely proportional to the square of the wire diameter, and up to some limit the sensitivity of a torsion balance increases rapidly as its size is reduced. While this relaxes the problem of sensing the balance motion, it makes the balance increasingly susceptible to spurious torques, gravitational and otherwise. Our design philosophy has been to create a system that can produce a very large gravitational torque for a table-top experiment, recognizing that a good modern angle sensor can make up for the reduced balance sensitivity. Noise torques thus become a much smaller problem.

2. Apparatus

At the heart of the balance is a thin, heavily loaded torsion strip. It is made from a copper–2% beryllium (Cu–Be) alloy and is 160 mm long, 2.5 mm wide and 30 μm thick. The stiffness of a strip length L, width b, and thickness t, where $L \gg b \gg t$ is given by

$$c = \frac{bt^3F}{3L} + \frac{Mgb^2}{12L} \quad (1)$$

where Mg is the load on the strip and F its shear modulus. The existence of the second, load-dependent term has been known for most of this century (Buckley 1914) but has been repeatedly overlooked or forgotten. It is this term that gives a torsion strip two advantages over a traditional circular cross section fibre. The ratio of the stiffness of the strip to that of a wire of the same properties (length, cross-section, shear modulus and load) is given by

$$\frac{c}{c_w} = \frac{2\pi t}{3b} + \frac{\pi b}{6t}\frac{\sigma}{F} \quad (2)$$

where σ is the stress in each. For this apparatus, we have $\sigma \simeq 800$ MPa and $F = 1100$ MPa, giving a ratio of c/c_w of

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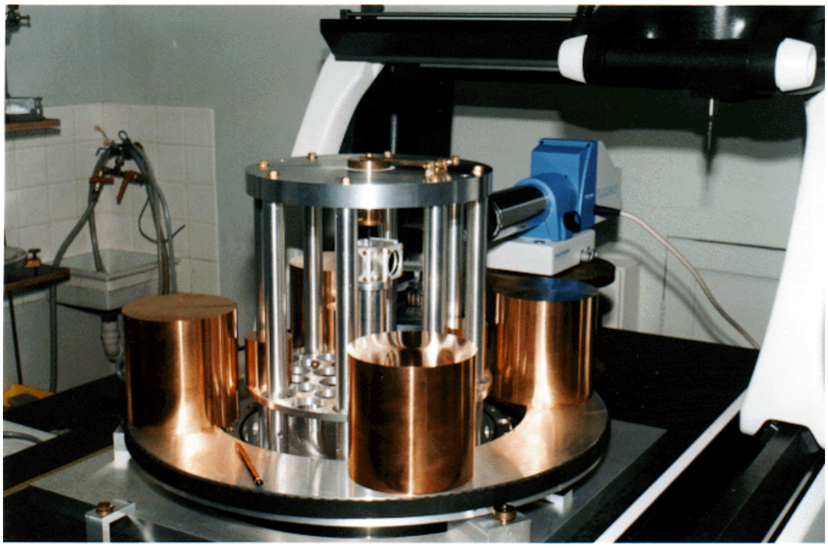

Figure 1. Photograph of Mark II G apparatus. The torsion strip is mostly concealed by the post carrying the mirrors for optical readout. The autocollimator is visible at the rear of the balance, and the tip of the coordinate measuring machine is at the upper right of the photo.

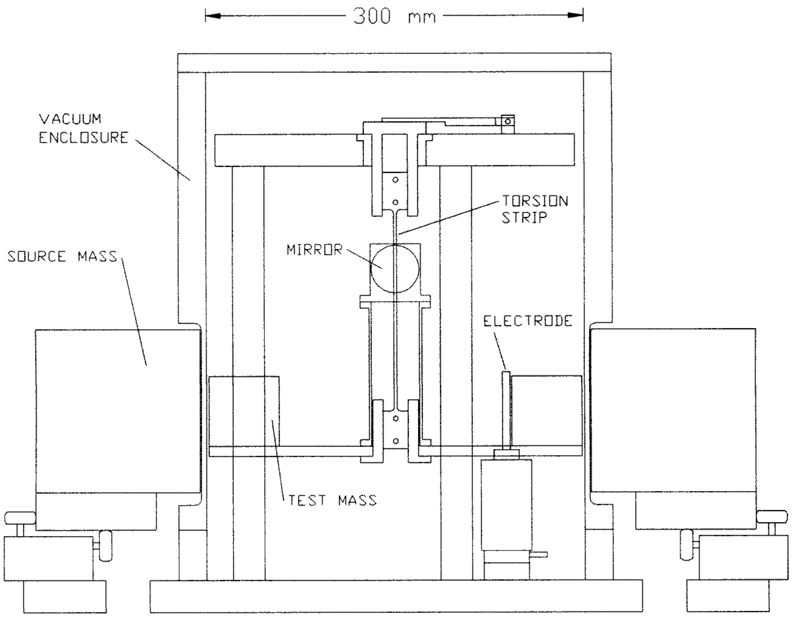

Figure 2. Schematic elevation view of Mark II G apparatus. Some of the important pieces are labelled.

about 0.3. This strip thus has about three times the sensitivity of the corresponding wire. Moreover, for this suspension, the gravitational stiffness comprises about 90% of the total, and the elastic stiffness about 10%. It is thus largely free from problems associated with anelasticity. These may be manifested as errors in the balance position caused by the anelastic after-effect (Quinn *et al* 1992), biases arising from a frequency-dependent torsion constant (Kuroda 1995) and drift in the balance equilibrium position.

Figure 1 is a photograph of the apparatus used for the determination of G. Figure 2 shows an elevation view with some of the important components labelled. Eight 25 mm diameter Dural columns bolted into thick Dural plates at either end form the support structure. The strip is clamped within a sleeve mounted to the top plate. The strip's ends flare out to 30 mm by 20 mm flanges by which the strip is clamped. The large clamping surface helps to reduce losses associated with stick–slip motion between the strip and the clamping blocks (Quinn *et al* 1995, 1997a). At the lower end, the strip is clamped within a sleeve that is in turn bolted to a wide Dural disc. The 8 mm thick, 296 mm diameter disc serves to support the test masses at a wide separation. Several series of holes are cut through the disc to make it lighter and to allow passage of the support columns and electrodes. These holes are arranged with as high a degree of azimuthal symmetry as possible to reduce the gravitational torque between the source masses and the disc.

Each of the four test masses is a 1.2 kg cylinder of a copper–0.5% tellurium (Cu–Te) alloy, of diameter and length both 55.6 mm. They are arranged symmetrically on the disc, each pressed against two locating pins. The centre-to-centre separation between opposite masses is 240 mm. This hexadecapole configuration of test masses renders the system quite insensitive to the motions of distant masses, its response falling off as the fifth power of the distance to such masses. The four source masses are cylinders machined from the same Cu–Te alloy. They each have a mass of about 15.5 kg and length and diameter of 130 mm. The radial spacing between the edge of each source mass and its corresponding test mass is about 7 mm. The source masses are placed on a Dural carousel that is supported and located by a set of six rollers. The carousel is turned by a belt driven by a stepping motor. When aligned radially with the test masses, the source masses produce no torque on the balance. When rotated in either direction by 19°, the source masses exert a maximum torque of about 2×10^{-8} N m.

The angular position of the balance is measured by a commercial autocollimator, model Elcomat-2000 manufactured by Möller-Wedel. This device has been calibrated by the Physikalisch-Technische Bundesanstalt (PTB) and can measure angles over a range of 10 mrad with an uncertainty of about 0.25 μrad. Fixed to the centre of the suspended disc is a post upon which are mounted four thick circular mirrors, approximately 140 mm above the top surface of the disc. These 38 mm diameter mirrors have a flatness of $\lambda/10$. Only one mirror is used for the optical readout in the measurements reported here; the others symmetrize the mass distribution. We plan to use all four mirrors in an angle-multiplication scheme for future measurements.

The entire apparatus is mounted on the marble bed of a coordinate measuring machine (CMM). This allows for the easy measurement of mass dimensions and positions with an accuracy of a few micrometres.

3. Free measurement principle

In the free measurement, the torsion balance is allowed to deflect under the influence of the source masses. At equilibrium, the applied gravitational torque is balanced by the (mostly gravitational) suspension stiffness. The angular deflection of the balance about its symmetry axis, θ, is related to the applied torque τ by Hooke's law,

$$\tau = c\theta \tag{3}$$

where c is the suspension stiffness. The applied torque can also be expressed as

$$\tau = G\Gamma(\rho) \tag{4}$$

where $\Gamma(\rho)$ is a complicated function of the mass distribution ρ in the apparatus, to be calculated by a computer. The stiffness cannot be accurately calculated from measurements of the strip dimensions and the load, but it may be derived via the simple harmonic oscillator relation

$$c = I\left(\frac{2\pi}{T}\right)^2 \tag{5}$$

where I is the balance's moment of inertia about its symmetry axis and T its period of oscillation. Combining equations (3)–(5) gives an expression for the gravitational constant

$$G = I\left(\frac{2\pi}{T}\right)^2 \frac{\theta}{\Gamma}. \tag{6}$$

Γ and I are calculated from dimensional and mass measurements of the components of the apparatus, while θ and T are determined from observations of the balance motion. In practice, θ is measured at a frequency about an order of magnitude smaller than $1/T$. The previous derivation is thus valid only if c is a negligible function of frequency. This is in fact the case for the nearly lossless suspension described above, at least at the level of interest of the measurement reported here.

4. Free measurement procedure

The disc carrying the test masses is clamped as close as possible to its suspended position, and measurements of the positions of the test masses are made with the CMM. The autocollimator is zeroed to provide an angular reference. The disc is then released and the vacuum chamber placed over the balance. The system is evacuated with a turbomolecular pump attached via a flexible tube held in a moulded lead collar to reduce transmitted vibrations. While air inside the chamber does not significantly increase the short-term position noise of the balance, experience with the prototype balance indicated that operation at atmospheric pressure suffered from large (of order 1%) drifts in the amplitude of the gravitational signal. It is thought that this arises from a very small systematic change in the thermal environment due to the motion of the source masses. Running at high vacuum alleviates this problem.

Next, the source masses are placed in position on the carousel. Their positions are measured in the two angular orientations of the carousel corresponding to maximum clockwise and counter-clockwise torque. Multiple measurements with the CMM have indicated the carousel angular orientation is repeatable to better than 100 μrad.

To measure the period of the balance, its angular position is read by the autocollimator and recorded by a computer once every 4 s for several cycles of motion. For this nearly lossless mechanical oscillator (measurements of the prototype yielded Q values in excess of 40 000), it suffices to fit an undamped sine function to extract the period. This measurement is generally carried out using an amplitude of the rotation of about 100 μrad, although the result is the same within the measurement uncertainty for amplitudes several times as great. One may move the source masses in such a fashion as to damp or excite gravitationally the motion of the balance to reach the desired amplitude. This method is used to reduce the motion to about 30 μrad or less for the deflection measurement.

The deflection measurement is accomplished by recording the balance position with the source masses alternately at the two positions of maximum torque. This measurement is usually done at night when noise from local activities is at a minimum. The results from a typical data

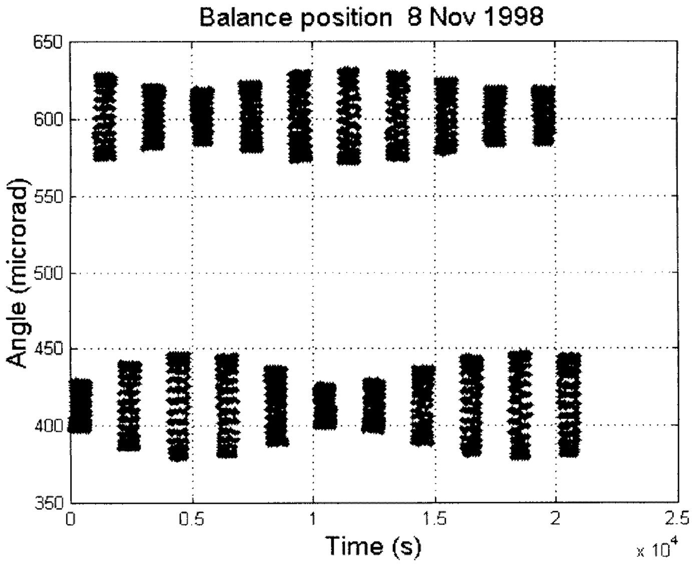

Figure 3. Angular position of the balance over several hours as the source masses are moved. Although the amplitude of oscillation varies considerably between positions, the drift in the zero point is less than 0.2 μrad over the whole data set.

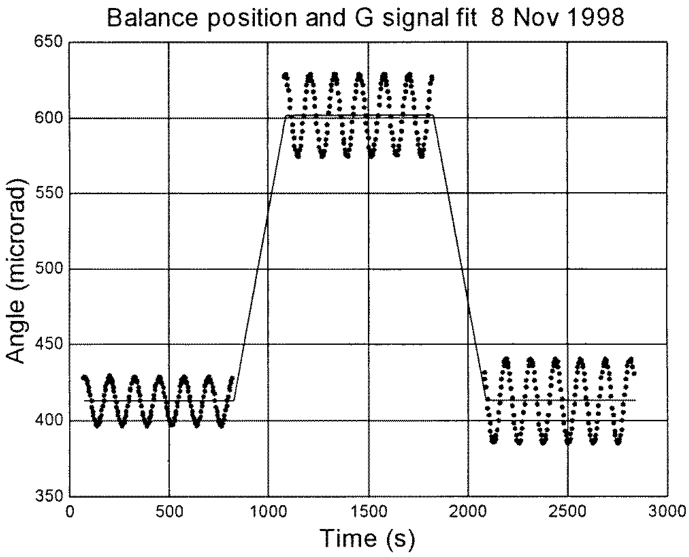

Figure 4. Angular position of the balance for the first three source mass positions of figure 3, and the G signal fit of a square wave plus drift. Six full cycles of balance oscillation are used at each position.

run are displayed in figure 3, which is a plot of the balance angular position over several hours as the source masses are moved. In this set, 200 readings at 4 s intervals were taken at each source mass position, and the time to move between positions was about 170 s. The amplitude of the balance oscillation is modulated depending on the relative phase between its motion and the source mass movement. This plot demonstrates one of the significant advantages of the heavily loaded torsion strip: very little zero drift over long times. The rate is typically less than 2 μrad per day.

The balance deflection θ is determined by finding the difference in balance position for the two source mass positions. A data set like that in figure 3 is divided into subsets of three positions as shown in figure 4. A few points at the beginning of each position are discarded to leave an integer number of balance oscillations. These data are then fitted with a square wave including a linear drift. The amplitude of this square wave is the basic θ signal. This calculation is then repeated for the rest of the subsets of a given data run, and these results averaged. To get an estimate of the statistical uncertainty of the measurement, one may fit a sine wave with a DC offset to the data from a single source mass position. The offset calculated for one position using six oscillation cycles (as shown in figure 4) is consistent with that calculated with the square wave method, and its uncertainty is about 100 ppm. As this is already significantly less than the systematic uncertainty in the calibration of the autocollimator, we did not make a more detailed analysis.

One may calculate I in a straightforward manner from the masses and dimensional measurements of the suspended pieces. The test masses and disc have shapes with simple analytic expressions for the moments. The few pieces near

Table 1. Primary quantities involved in determination of G.

Quantity	Value	Contribution to relative standard uncertainty in G
θ	188.97 μrad	1.6×10^{-3}
Γ	565.78 kg m^{-1} rad^{-1}	5.3×10^{-4}
I	0.078 453 kg m^2	2.7×10^{-4}
T	124.411 s	1.5×10^{-4}

the centre with more irregular shapes (like clamping screws) contribute less than 0.1% of the total.

The calculation of Γ is a more complicated affair. We start with an intermediate expression found in the derivation of the radial acceleration field of a right cylinder (Chen and Cook 1993). This is integrated numerically and summed over the volume of a test cylinder to find the force between a source and test mass. This is calculated for all 16 source and test mass combinations, and the torque determined using the appropriate geometric coefficients. The accuracy of this method has been checked by comparing the result with direct point-to-point integrations using different grid forms and spacings. All methods seem to converge to the same answer.

Various minor corrections to the total torque are also calculated. These include the influence of the source masses on the disc, the test mass locating pins and the small balls used to level the disc. In the free measurement, there is also the possibility of a systematic torque on the balance caused by static local field gradients coupling to the small (189 μrad) difference in angular position for the two source mass positions. This was calculated for the support columns (rather small masses at close range) and estimated for the hill near the laboratory (rather large mass at distant range). In both cases the torque contribution was less than 0.01% of the total.

The influence of uncertainties in various dimensional measurements on the calculated torque was determined. The largest effect comes from uncertainty in the relative positions of the test masses. For the measurements reported here, this effect was exacerbated in that geometrical constraints prevented us from realizing the full accuracy of the CMM during *in situ* measurements of the test mass positions. We expect to address this problem in the future by using a different measuring tip on the CMM and perhaps incorporating kinematic mounts for repeatable positioning of the masses.

5. Results

The four quantities in equation (6) were determined and they are shown in table 1 along with their contributions to the relative standard uncertainty in G.

The resulting value for the gravitational constant is

$$G = 6.683 \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$$

with a combined standard uncertainty $u_c = 1.1 \times 10^{-13}$ m^3 s^{-2} kg^{-1}. This corresponds to a relative combined standard uncertainty of 1.7×10^{-3}. This should be considered a preliminary value for the experiment. Our analysis for this measurement included all major known uncertainties. Other sources of uncertainty have been estimated to be too small to affect this result. Note that the uncertainty in the deflection angle θ is by far the largest contribution to the uncertainty in G. This limit should be reduced considerably by operating the balance in a servo-controlled mode, as described in the following section, and in a parallel project to increase the optical signal by multiple bounces.

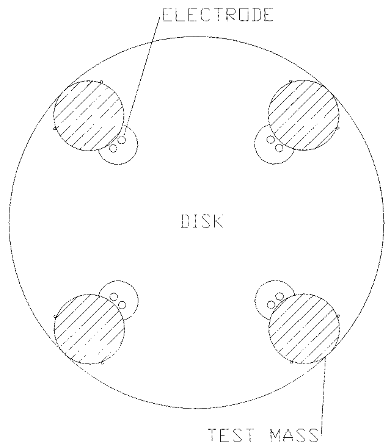

Figure 5. Schematic plan view of the electrode geometry for the servoed measurement. The test masses are hatched. For clarity, the only disc holes shown are those for the passage of electrodes.

6. Servo measurement

The torsion strip balance has also been designed to operate under servo control, whereby the gravitational torque is balanced by an electrostatic torque. This is accomplished via voltages applied to stationary electrodes in the vicinity of the test masses. For a single electrode pair, the torque generated may be expressed as

$$\tau = \frac{1}{2}\frac{\mathrm{d}C}{\mathrm{d}\theta}V^2 \tag{7}$$

where V is the potential difference between the electrode and the test mass and $\mathrm{d}C/\mathrm{d}\theta$ is the derivative of the capacitance between the two with respect to the rotation angle of the balance. One must then accurately measure the voltages used and the capacitance as a function of angle. We expect this to be possible using a commercial voltmeter and precision capacitance bridge. One advantage of the servo measurement is that the balance always remains in the same position as the gravitational torque is varied. This greatly reduces effects due to gravity gradients and any possible problems associated with anelastic effects resulting from twisting of the strip. Additionally, the servo addresses the major limitation in the free measurement—the uncertainty in the angular deflection of the balance. Whereas the free measurement (without optical multiplication) could only use a small fraction of the range of the autocollimator (roughly 200 μrad out of 10 mrad), the servo calibration can use the entire range.

We chose to use the test masses themselves as the electrodes on the balance. The connection is made through the torsion strip. This preserves the austere geometry that helps in calculating the moment of inertia and gravitational torque. It also has the desirable effect that the electrostatic force is applied at very nearly the same place as the gravitational force. One electrode is shown in the elevation view of figure 2 and a good idea of the total geometry can be gained from the schematic plan view in figure 5. The electrodes are 6 mm diameter cylinders made from the same Cu–Te alloy as the source and test masses. They are held in pairs on single-axis translation stages mounted to the vacuum baseplate, and protrude through holes in the disc to run up along the lengths of the test masses. The test mass–electrode gap is about 1.5 mm, adjustable via the translation stage. A voltage of either sign applied to an electrode will attract the nearby test mass. Thus two electrodes 180 degrees apart are always driven together, so that there is a cancellation of the radial force.

This geometry satisfies a number of important criteria. Consider the curves of the capacitance and its derivatives as a function of balance angle shown in figure 6. The $\mathrm{d}C/\mathrm{d}\theta$ curve has a maximum at the operating position of the balance, rendering the calibration constant insensitive to the balance angular position to first order. This also means that $\mathrm{d}^2C/\mathrm{d}\theta^2$ is zero at the operating point and thus the electrodes do not contribute any stiffness to the suspension. Note that C is very nearly linear throughout the 10 mrad range of the autocollimator, so that measurements of the capacitance and angle are only needed at a few points in this range to determine $\mathrm{d}C/\mathrm{d}\theta$ at the operating point. The magnitude of $\mathrm{d}C/\mathrm{d}\theta$, about 17 pF/rad for each electrode, requires potential differences of about 30 V to balance the gravitational torque. This is a desirable voltage to use—small enough that one may conveniently use stable, low-noise, inexpensive amplifiers, and still very large compared to anticipated variations in contact potential differences (Fitzgerald *et al* 1994).

To linearize the system, the servo will be operated with biases applied to each of the electrodes. These biases may be varied in both magnitude and sign to evaluate contact potentials. In the measurement mode, the error signal for the servo will be provided by a high-resolution autocollimator on loan to the BIPM from the PTB, capable of an angular resolution of 1 nrad in a 1 s integration time (Windisch and Ebeling 1990). In the calibration mode, the balance will be allowed to oscillate freely, and the capacitance and angle will be recorded simultaneously near the turning points of the motion. The capacitance will be measured with a precision bridge, and the angle with the Elcomat-2000 autocollimator referred to previously.

7. Conclusion

The Mark II version of the BIPM torsion strip balance has produced a very stable, low-noise gravitational signal in the presence of large source masses. Drifts of the order of 1% of the signal amplitude that had been consistently observed in the prototype balance operating at atmospheric pressure have been eliminated by running at high vacuum. Measuring the deflection of the balance with an autocollimator calibrated against an absolute angle standard, we have made a preliminary determination of the gravitational constant G with a relative standard uncertainty of 1.7×10^{-3}.

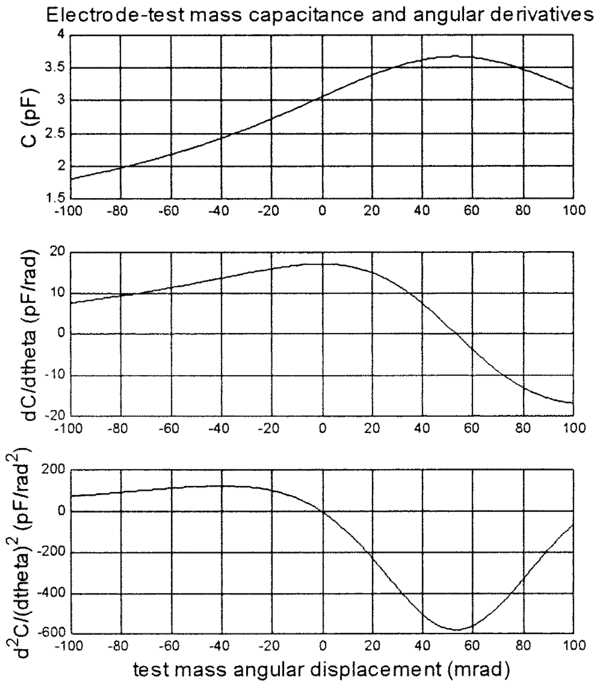

Figure 6. Capacitance between a single electrode–test mass pair as a function of balance angular position, and derivatives. Note maximum in $\mathrm{d}C/\mathrm{d}\theta$ at the operating position, $\theta = 0$. The torque transducer will be calibrated at a few points within the range of the autocollimator, ± 5 mrad. At 53 mrad, the electrode and test mass are aligned along a radial line of the disc and the capacitance is a maximum.

We expect to reduce this uncertainty in the very near future in two different ways. One is the use of an optical multiplication scheme whereby the angle measured by the autocollimator will be increased via multiple reflections from the balance. The other is the incorporation of an electrostatic servo, which will not only allow for calibration over a greater angular range but will also reduce uncertainties associated with small deflections of the strip. Our goal is to determine G with a relative uncertainty of 10^{-4} or better. Achieving a consistent result with the two different techniques will increase confidence in the accuracy of both.

Acknowledgments

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