

FYS-STK3155 Project 1

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By comparing predictions of a two-dimensional function by three different methods – Ordinary Least Squares, Ridge Regression and Lasso – we will confirm their degree of accuracy up against theory. Thereafter we go on to do a bias-variance trade-off and cross validation. Here we find that ...

I. INTRODUCTION

Many have seen the amazing Norwegian nature. The fjords are remnants of the iceage shaping the rock, and the mountains are a memory of the forces of tectonic activity that shaped the land as we know it.

In this report we want to explore the Norwegian topography using basic machine learning methods. Essentially, we are interested in predicting the topography using real data. For the specific methods we are going to use Ordinary Least Squares (OLS), Ridge and Lasso regression. We will exploring these regression methods and compare them. For this, we will use metrics such as bias-variance tradeoff, cross-validation and in addition we are going to explore the resulting Mean Squared Errors (MSE). The point of this will be to see what methods produce the best fit to the actual topography.

Before actually using the topography data we will test the code on the Franke's function (See *Franke's function* 1).

II. METHODS

We begin by defining the function we are going to do regressions on. This is called a Franke function, and is a two dimensional weighted function over four exponentials:

$$\begin{aligned} f(x, y) = & \frac{3}{4} \exp \left(-\frac{(9x-2)^2}{4} - \frac{(9y-2)^2}{4} \right) \\ & + \frac{3}{4} \exp \left(-\frac{(9x+1)^2}{49} - \frac{(9y+1)^2}{10} \right) \\ & + \frac{1}{2} \exp \left(-\frac{(9x-7)^2}{4} - \frac{(9y-3)^2}{4} \right) \\ & - \frac{1}{5} \exp \left(-(9x-4)^2 - (9y-7)^2 \right) \end{aligned} \quad (1)$$

Firstly we are going to use our own code doing the Ordinary Least Squares (OLS) for a linear regression of the Franke function up to it's fifth order (link to code is in the appendix section A). Linear regression using OLS is given from

$$\tilde{\mathbf{z}} = X\beta \quad (2)$$

and

$$\beta_{OLS} = (X^T X)^{-1} X^T \mathbf{z} \quad (3)$$

where β_{OLS} is the optimal predictor for our function with function-output \mathbf{z} , and X is the design matrix for our two-dimensional function.

After defining this, we are able to calculate the Mean Square Error (MSE) defined by

$$MSE(\mathbf{z}, \tilde{\mathbf{z}}_i) = \frac{1}{n} \sum_{i=0}^{n-1} (z_i - \tilde{z}_i)^2, \quad (4)$$

where the mean value of \mathbf{z}

$$\bar{z} = \frac{1}{n} \sum_{i=0}^{n-1} z_i \quad (5)$$

Can also calculate the R^2 -score, given by

$$R^2(\mathbf{z}, \tilde{\mathbf{z}}_i) = 1 - \frac{\sum_{i=0}^{n-1} (z_i - \tilde{z}_i)^2}{\sum_{i=0}^{n-1} (z_i - \bar{z})^2} \quad (6)$$

To understand the concept of the linear regression, both β_{OLS} , MSE and the R^2 -score should be plotted with respect to the polynomial complexity.

III. RESULTS

test [1]

IV. DISCUSSION

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V. CONCLUSION

Appendix A: Github repository

<https://github.com/LassePladsen/FYS-STK3155-projects/tree/main/project1>

[1] Hjorth-Jensen, M. (2023). Project 1 on machine learning. <https://compphysics.github.io/MachineLearning/>

[doc/LectureNotes/_build/html/project1.html](#). [Online; accessed 26-September-2023].