On numerical approximations to solutions of Laplace's equation for different boundary conditions

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Contents

Contents

Introduction

Purpose of the program is to solve electrostatic systems for which analytical solution does not exist.
Accuracy is compared against a system for which analytical solution does exist (System A).
Computational performance compared against various parameters...

Contents

Iterations

Computing system A and comparing the results with different number of iterations to the analytical solution:

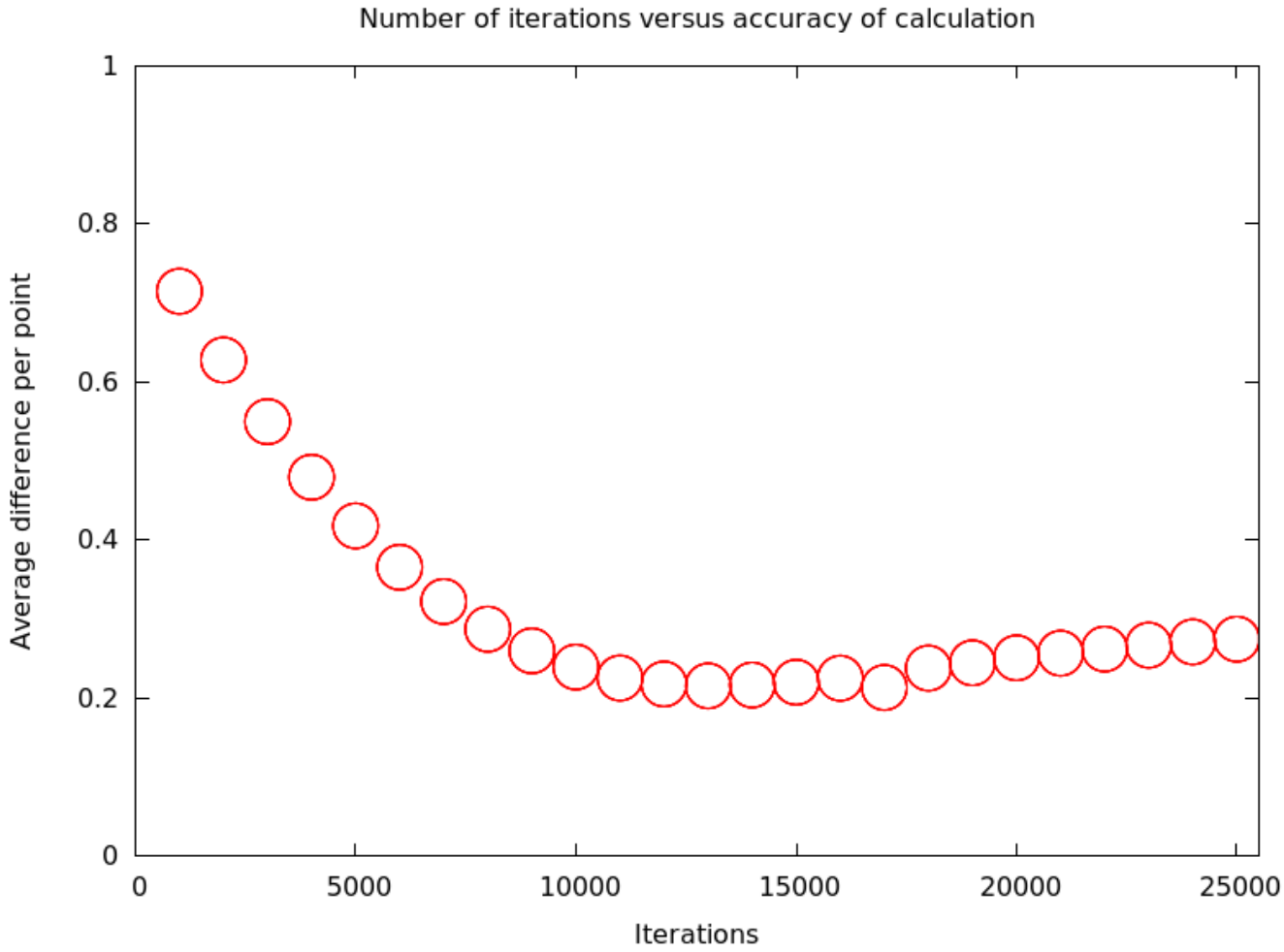


Figure : Relaxation parameter = 1, desired convergence = 1e-20.

Iterations continued

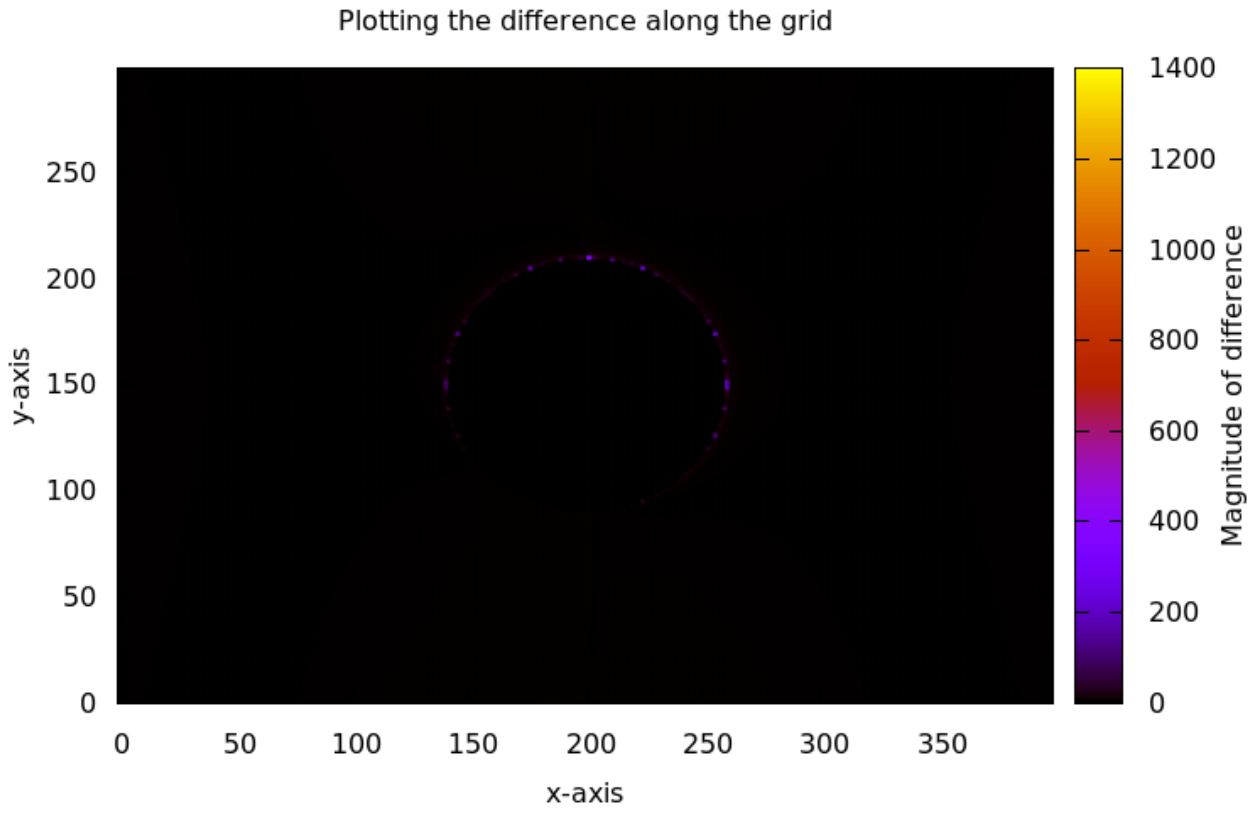


Figure : Computation done with 12000 iterations

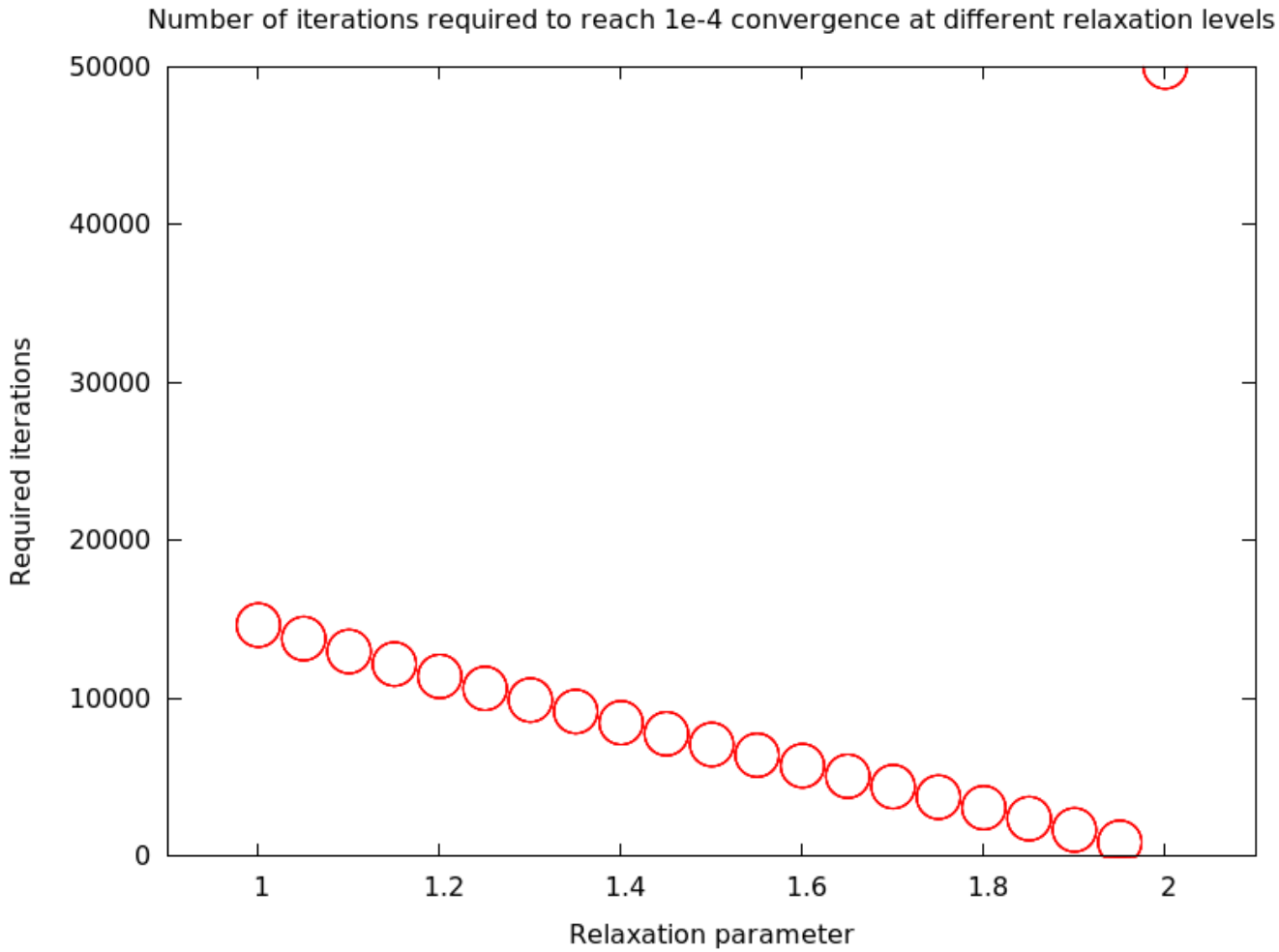
Contents

Relaxation parameter

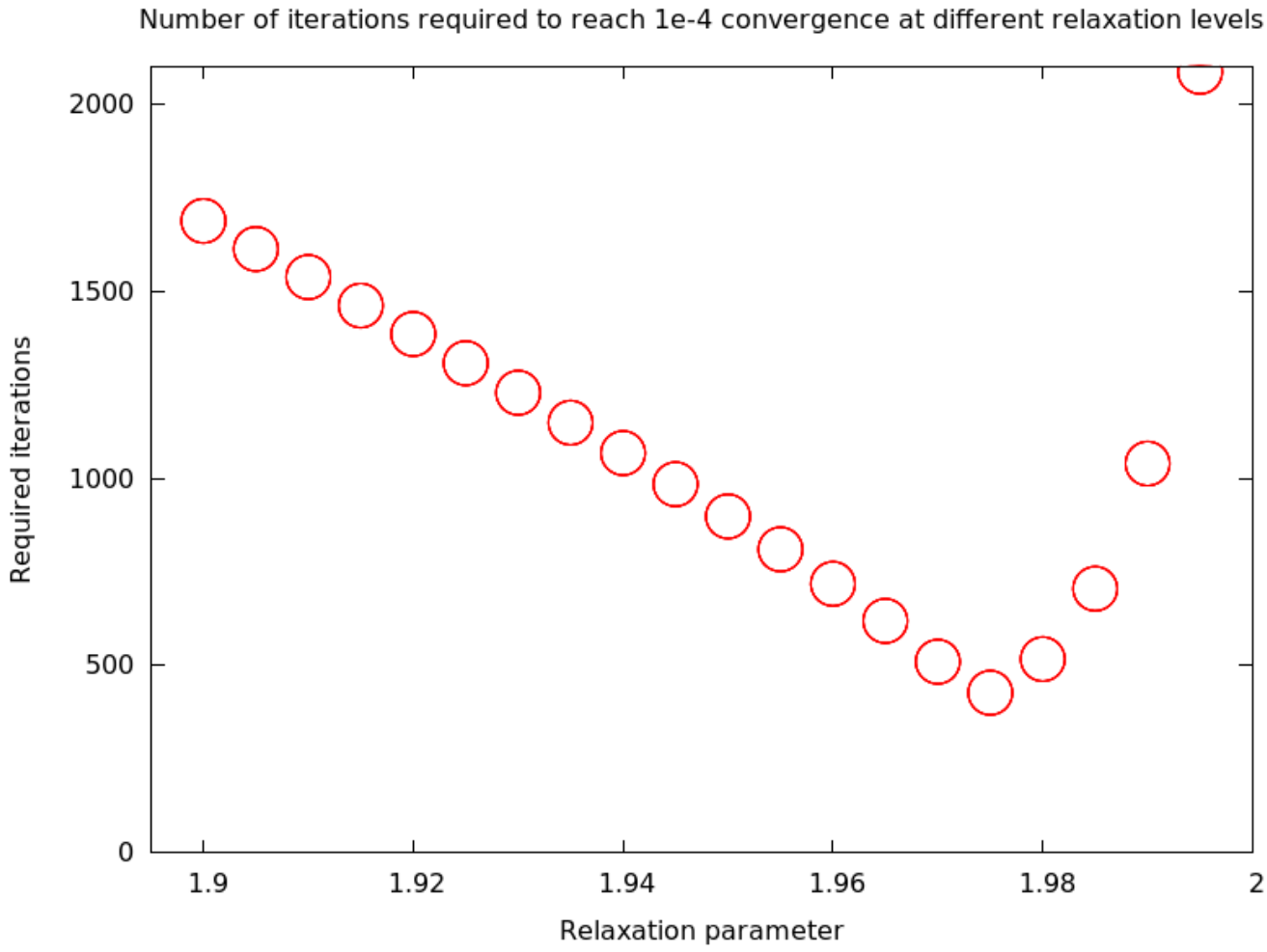
Running the *eStatics* package with a varying relaxation parameter to a desired convergence shows that there is an optimal value, which is dependent on the electrostatic system.

Relaxation parameter continued

Computing system A to $1 * 10^{-4}$ convergence with different relaxation parameters to determine optimal value:



Relaxation parameter continued



Optimal value of about 1.975 for System A.

Relaxation parameter continued

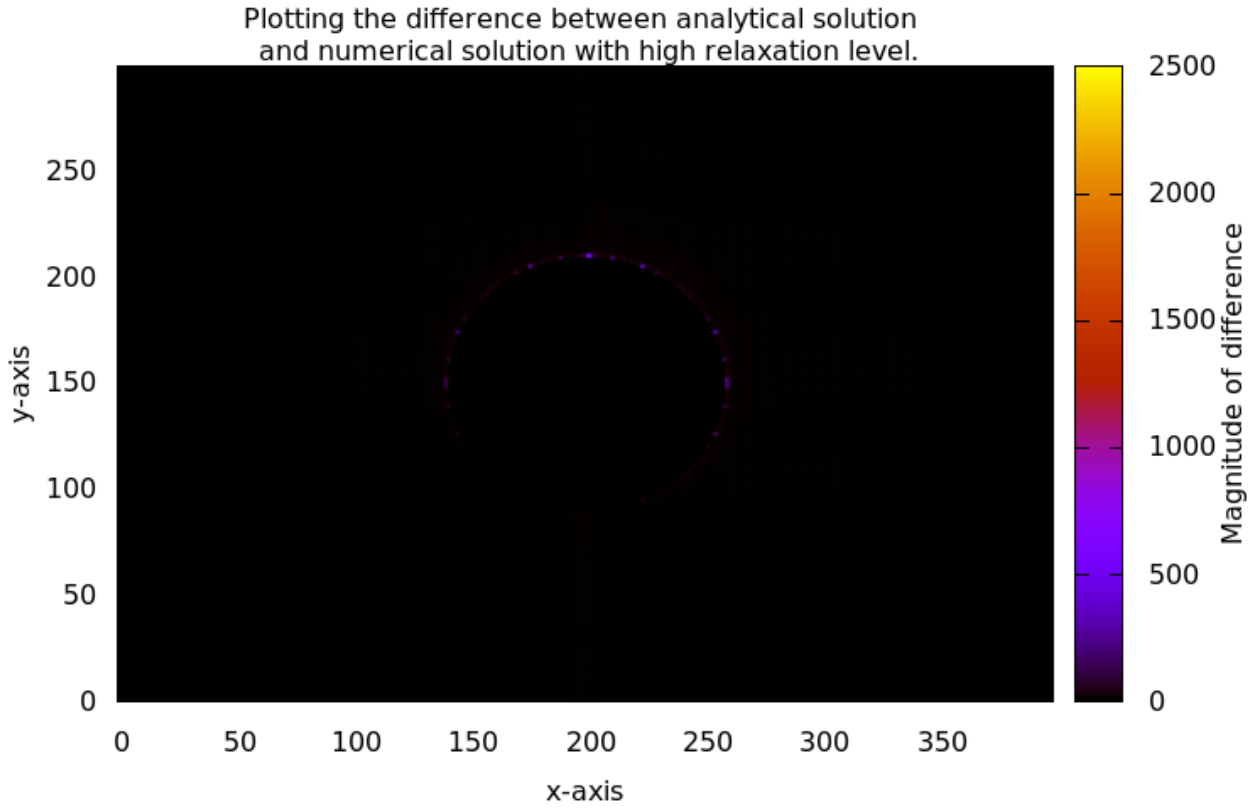


Figure : Figure showing difference between analytical solution and numerical solution with relaxation parameter of 1.975 and 428 iterations

Contents

Convergence

Running the *eStatics* package with a varying number of iterations and a varying level of desired convergence shows that the required iterations vs desired convergence is rougly inversely exponential.

Convergence continued

Computing system A to a desired convergence and comparing with time taken:

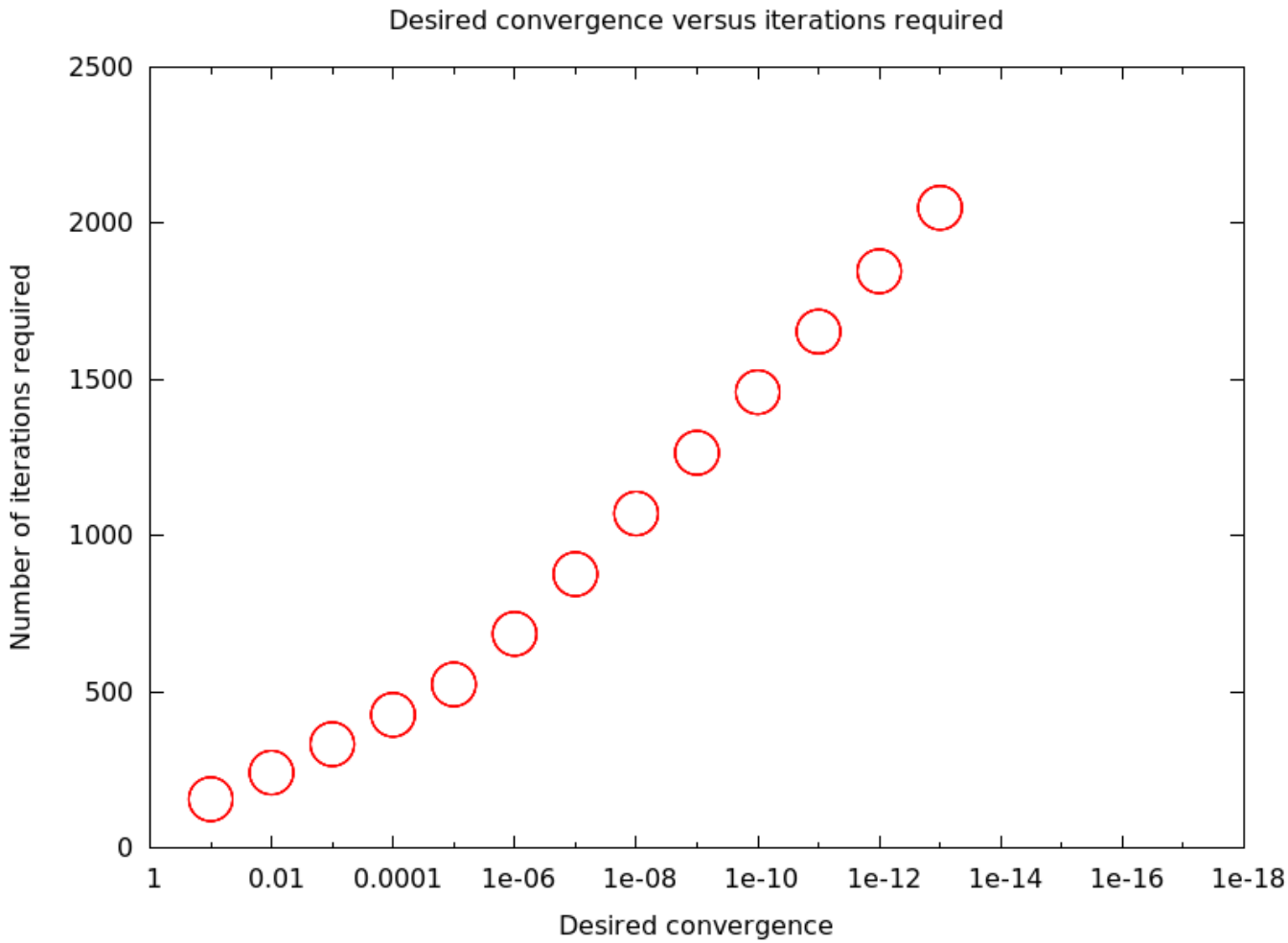
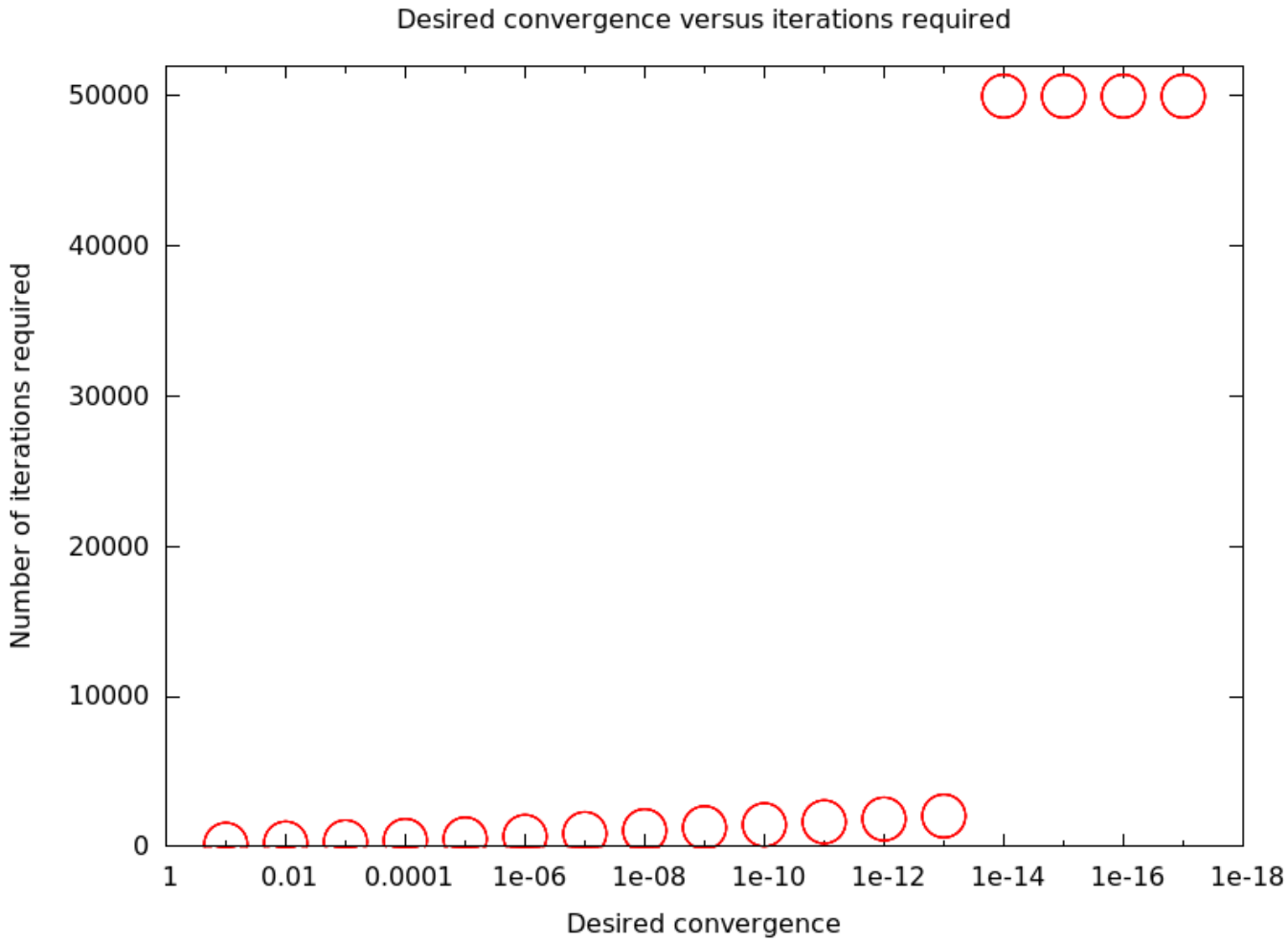


Figure : Relaxation parameter = 1

Convergence continued

The rest of the data points are off the scale:



Contents

Multithreading

Multithreading has an effect on the computational speed, but not on the number of iterations.
Allows a higher number of iterations to be run in the same time, possibly allowing larger input images to be used while keeping the length of the computation reasonable.

Multithreading continued

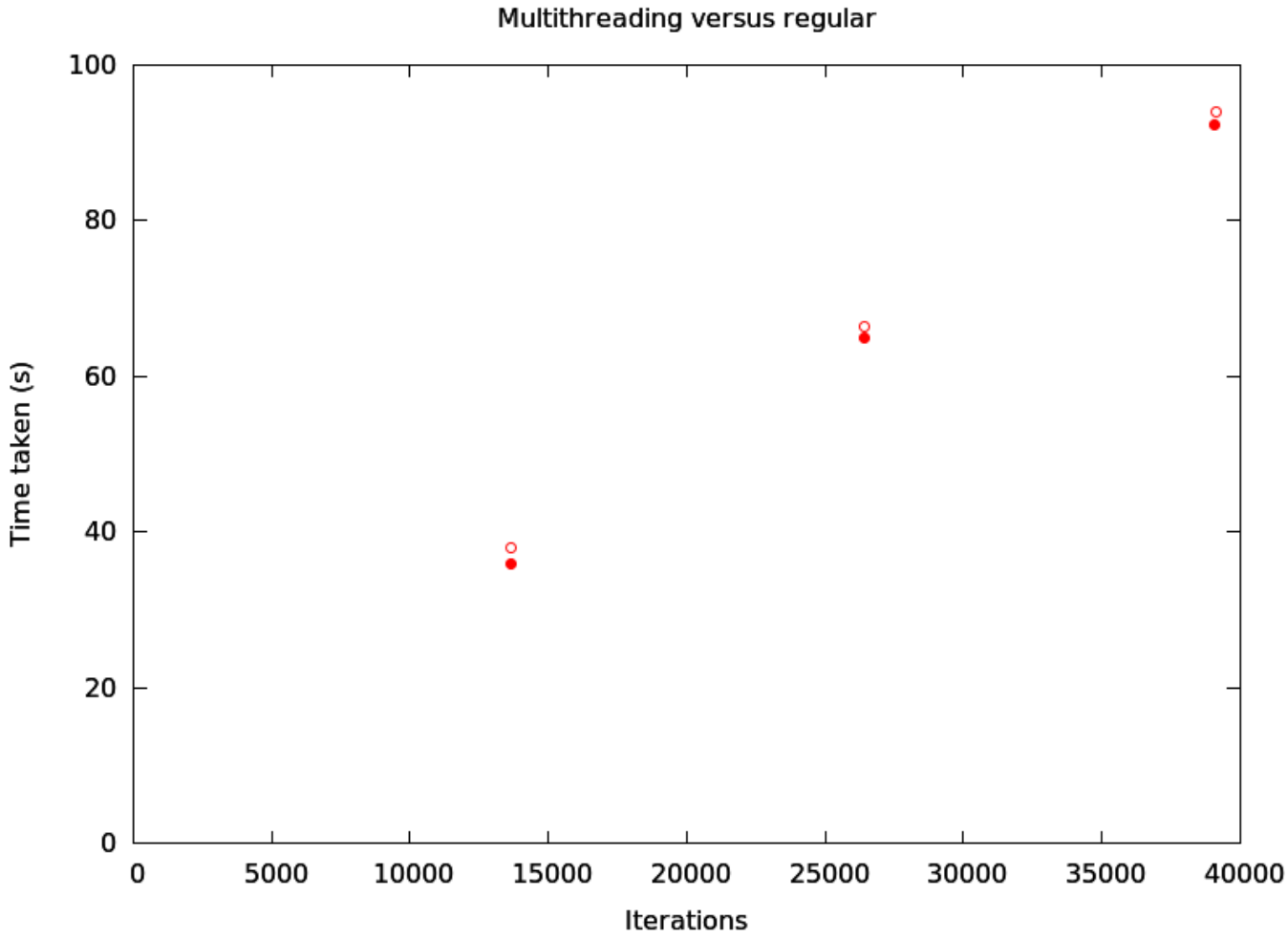


Figure : Filled circles = multithreaded, open circles = single thread. Computation done on a $400 * 300$ grid.

Multithreading continued

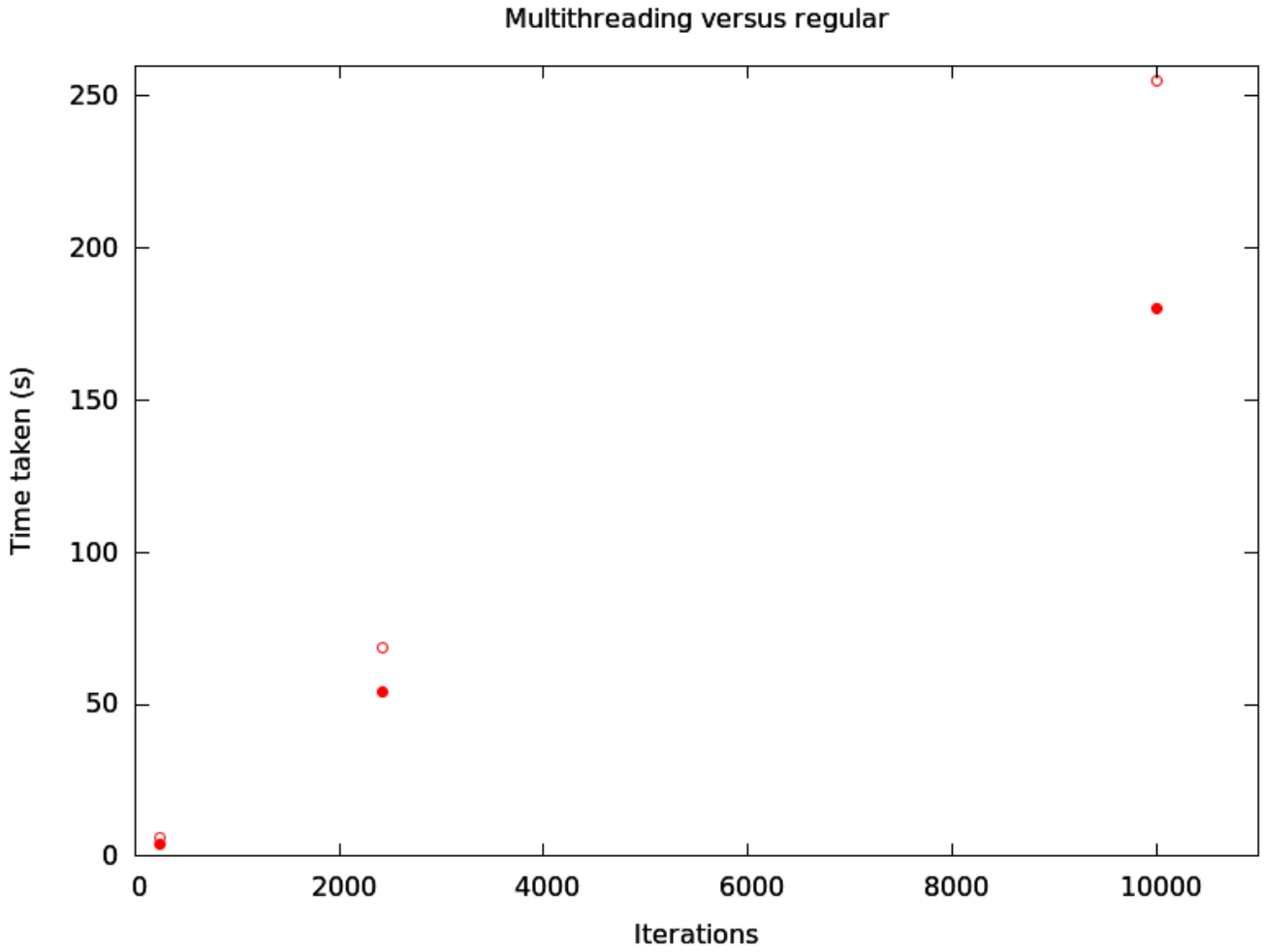


Figure : Filled circles = multithreaded, open circles = single thread. Computation done on a $1280 * 720$ grid.

Contents

Further work

The *eStatics* package could be generalized to apply to a wider range of applications:

- Temperature distributions
- Gravitational fields
- Anisotropic blurring (image processing)

Basically anything that the Laplace equation applies to. Could also be increased to three dimensions to solve more complicated electrostatic problems.

Questions?

Thank you for your attention.