

Given the definition of Discrete Fourier Transform:

$$\alpha[k] = \sum_{m=0}^7 x(m) W_8^{mk}$$

Rewrite the $x(m)$ into the addition of real and imaginative part.

$$\begin{aligned} \alpha[k] &= \sum_{m=0}^7 [x_{RE}(m) + jx_{IM}(m)] \times e^{-j\frac{2\pi}{8}mk} \\ &= \sum_{m=0}^7 [x_{RE}(m) + jx_{IM}(m)] \times [\cos\left(\frac{mk\pi}{4}\right) - j\sin\left(\frac{mk\pi}{4}\right)] \\ &= \sum_{m=0}^7 \{x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right) \\ &\quad + jx_{IM}(m) \cos\left(\frac{mk\pi}{4}\right) - jx_{RE}(m) \sin\left(\frac{mk\pi}{4}\right)\} \\ &= \sum_{m=0}^7 \{[x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right)] \\ &\quad - j[x_{RE}(m) \sin\left(\frac{mk\pi}{4}\right) - x_{IM}(m) \cos\left(\frac{mk\pi}{4}\right)]\} \end{aligned}$$

Now calculate its real and imaginative part respectively. Take the real part as an example:

$$\begin{aligned} \alpha(k)_{RE} &= \sum_{m=0}^7 [x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right)] \\ &= \sum_{m=0}^3 [x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right)] \\ &\quad + \sum_{m=4}^7 [x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right)] \end{aligned}$$

As an FFT algorithm, do decimation-in-time (DIT) folding,

$$\begin{aligned} &= \sum_{m=0}^3 [x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right)] \\ &\quad + \sum_{m=0}^3 [x_{RE}(m+4) \cos\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right) + x_{IM}(m+4) \sin\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right)] \end{aligned}$$

Denote the second term as S,

$$\begin{aligned} S &= \sum_{m=0}^3 [x_{RE}(m+4) \cos\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right) + x_{IM}(m+4) \sin\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right)] \\ &= \sum_{m=0}^3 \{x_{RE}(m+4) [\cos\left(\frac{mk\pi}{4}\right) \cos(k\pi) - \sin\left(\frac{mk\pi}{4}\right) \sin(k\pi)] \\ &\quad + x_{IM}(m+4) [\sin\left(\frac{mk\pi}{4}\right) \cos(k\pi) + \cos\left(\frac{mk\pi}{4}\right) \sin(k\pi)]\} \\ &= \sum_{m=0}^3 [x_{RE}(m+4) (-1)^k \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m+4) (-1)^k \sin\left(\frac{mk\pi}{4}\right)] \end{aligned}$$

Thus,

$$\begin{aligned} \alpha(k)_{RE} &= \sum_{m=0}^3 [x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right)] \\ &\quad + \sum_{m=0}^3 [x_{RE}(m+4) (-1)^k \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m+4) (-1)^k \sin\left(\frac{mk\pi}{4}\right)] \end{aligned}$$

$$= \sum_{m=0}^3 \left\{ [x_{RE}(m) + (-1)^k x_{RE}(m+4)] \cos\left(\frac{mk\pi}{4}\right) \right. \\ \left. + [x_{IM}(m) + (-1)^k x_{IM}(m+4)] \sin\left(\frac{mk\pi}{4}\right) \right\}$$

By similar derivation, we have $\alpha(k)_{IM}$,

$$\alpha(k)_{IM} = (-j) * \sum_{m=0}^3 \left\{ [x_{RE}(m) + (-1)^k x_{RE}(m+4)] \sin\left(\frac{mk\pi}{4}\right) \right. \\ \left. - [x_{IM}(m) + (-1)^k x_{IM}(m+4)] \cos\left(\frac{mk\pi}{4}\right) \right\}$$

And finally,

$$\alpha(k) = \alpha(k)_{RE} + \alpha(k)_{IM}$$

As the equation shows, $\alpha(k)_{RE}$ is sum of two 4-point inner dot. Otherwise, by rearranging the coefficient vectors, it could be derived by an 8-point inner dot. These two algorithms requires different look-up-tables(LUT).