

By the definition of Discrete Fourier Transform,

$$\alpha[k] = \sum_{m=0}^7 x(m) W_8^{mk}$$

Do some math,

$$\begin{aligned} &= \sum_{m=0}^7 [x_{RE}(m) + jx_{IM}(m)] \times e^{-j\frac{2\pi}{8}mk} \\ &= \sum_{m=0}^7 [x_{RE}(m) + jx_{IM}(m)] \times [\cos\left(\frac{mk\pi}{4}\right) - j\sin\left(\frac{mk\pi}{4}\right)] \\ &= \sum_{m=0}^7 \{x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right) \\ &\quad + jx_{IM}(m) \cos\left(\frac{mk\pi}{4}\right) - jx_{RE}(m) \sin\left(\frac{mk\pi}{4}\right)\} \\ &= \sum_{m=0}^7 \{[x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right)] \\ &\quad - j[x_{RE}(m) \sin\left(\frac{mk\pi}{4}\right) - x_{IM}(m) \cos\left(\frac{mk\pi}{4}\right)]\} \end{aligned}$$

Take real part as an example,

$$\begin{aligned} \alpha(k)_{RE} &= \sum_{m=0}^7 \left[x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right) \right] \\ &= \sum_{m=0}^3 \left[x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right) \right] \\ &\quad + \sum_{m=4}^7 \left[x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right) \right] \\ &= \sum_{m=0}^3 \left[x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right) \right] \\ &\quad + \sum_{m=0}^3 \left[x_{RE}(m+4) \cos\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right) + x_{IM}(m+4) \sin\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right) \right] \end{aligned}$$

Denote the second term as S,

$$\begin{aligned} S &= \sum_{m=0}^3 \left[x_{RE}(m+4) \cos\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right) + x_{IM}(m+4) \sin\left(\frac{mk\pi}{4} + \frac{4k\pi}{4}\right) \right] \\ &= \sum_{m=0}^3 \{ x_{RE}(m+4) \left[\cos\left(\frac{mk\pi}{4}\right) \cos(k\pi) - \sin\left(\frac{mk\pi}{4}\right) \sin(k\pi) \right] \} \\ &\quad + \sum_{m=0}^3 \{ x_{IM}(m+4) \left[\sin\left(\frac{mk\pi}{4}\right) \cos(k\pi) + \cos\left(\frac{mk\pi}{4}\right) \sin(k\pi) \right] \} \\ &= \sum_{m=0}^3 \left[x_{RE}(m+4) (-1)^k \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m+4) (-1)^k \sin\left(\frac{mk\pi}{4}\right) \right] \end{aligned}$$

Thus,

$$\begin{aligned} \alpha(k)_{RE} &= \sum_{m=0}^3 \left[x_{RE}(m) \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m) \sin\left(\frac{mk\pi}{4}\right) \right] \\ &\quad + \sum_{m=0}^3 \left[x_{RE}(m+4) (-1)^k \cos\left(\frac{mk\pi}{4}\right) + x_{IM}(m+4) (-1)^k \sin\left(\frac{mk\pi}{4}\right) \right] \end{aligned}$$

$$\begin{aligned}
&= \sum_{m=0}^3 \left\{ [x_{RE}(m) + (-1)^k x_{RE}(m+4)] \cos\left(\frac{mk\pi}{4}\right) \right. \\
&\quad \left. + [x_{IM}(m) + (-1)^k x_{IM}(m+4)] \sin\left(\frac{mk\pi}{4}\right) \right\}
\end{aligned}$$

Similarly,

$$\begin{aligned}
\alpha(k)_{IM} &= (-j) * \sum_{m=0}^3 \left\{ [x_{RE}(m) + (-1)^k x_{RE}(m+4)] \sin\left(\frac{mk\pi}{4}\right) \right. \\
&\quad \left. - [x_{IM}(m) + (-1)^k x_{IM}(m+4)] \cos\left(\frac{mk\pi}{4}\right) \right\}
\end{aligned}$$

We can see that $\alpha(k)_{RE}$ is the sum of two 4-point dot products. However, by merging the coefficient vectors, it could be derived by an 8-point dot product, also. These two methods corresponds two functions, and they require different look-up-tables(LUT).