ESTIMATING TRAFFIC STREAM SPACE-MEAN SPEED AND RELIABILITY FROM DUAL AND SINGLE LOOP DETECTORS

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ABSTRACT

The relationship between time-mean and space-mean speed that was derived by Wardrop (*1*) and presented in several textbooks (e.g. Hobbs and Richardson (*2*); May (*3*); Garber and Hoel (*4*)) is suitable for estimating time-mean speeds from space-mean speeds. However, in most cases it is desired to estimate the space-mean speed from time-mean speed measurements. Consequently, the paper develops a new formulation, which utilizes the variance about the time-mean speed as opposed to the variance about the space-mean speed 与……截然相反, for the estimation of space-mean speeds. The paper demonstrates that the space-mean speeds are estimated within a margin of error from 0 to 1 percent. Furthermore, the paper develops a relationship between the space and time-mean speed variances and between the space-mean speed and the spatial travel time variance.

In addition, the paper demonstrates that both the Hall and Persaud (*5*) and the Dailey (*6*) formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation computes the space-mean speed (harmonic mean) while the Dailey formulation computes the time-mean speed (arithmetic mean).

1. INTRODUCTION

1.1 Background

Lighthill and Witham (*7*) derived the classical steady-state traffic flow relationship between the traffic stream flow rate (q), the traffic stream density (k), and the traffic stream space-mean-speed ($\bar{u}_S$) as

$$q = k \cdot \bar{u}_S . \qquad [1.]$$

Traffic stream speeds are typically measured in the field using a variety of spot speed measurement technologies. The most common of these spot speed measurement technologies is a presence-type loop detector, which identifies the presence and passage of vehicles over a short segment of roadway (typically 5 to 20 meters long). When a vehicle enters the detection zone, the sensor is activated and remains activated until the vehicle leaves the detection zone. These surveillance detectors measure the traffic stream flow rate (number of actuations per unit time), traffic stream speed (in the case of dual loop detectors), and percentage of time that the detector is occupied (detector occupancy). The traditional practice for estimating speeds from single loop detectors is based on the assumption of a constant average

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effective vehicle length. Studies, however, have shown that this assumption provides speed estimates that are sufficiently inaccurate as to severely limit the usefulness of these speed estimates for real-time traffic management and traveler information systems (Hellinga (*8*)). In addressing these issues researchers have investigated the use of filtering techniques. For example, Dailey (*6*) developed a Kalman filter on vehicle length estimates while Hellinga (*8*) used exponentially smoothed adjacent dual loop detector vehicle length measurements to enhance the speed estimates of single loop detectors. Hellinga demonstrated that the exponential smoothing of 20-s average vehicle length measurements from adjacent dual loop detectors enhanced the accuracy of the speed estimates by approximately 20 percent. Wang and Nihan (*9*) using screening procedures to remove intervals with long vehicles and space-mean speed estimates were derived from the intervals with passenger cars only. Alternatively, researchers have investigated the use of median as opposed to mean statistics in order to enhance the robustness of the statistics by ensuring that the measures are not influenced by outlier observations. For example, Lin *et al*. (*10*) used the median vehicle passage time as opposed to the mean passage time to estimate speeds from single loop detectors. Similarly, Coifman *et al*. (*11*) computed the median speed from the median occupancy in order to reduce speed estimate errors when a wide range of vehicle lengths are present in the traffic stream.

Dailey (*6*) and Wang and Nihan (*12*) claimed that the traditional speed estimation method proposed by Hall and Persaud (*5*) is biased. However, Coifman (*13*) refuted this claim and demonstrated that the speed estimates are not biased. This paper demonstrates that the conclusions of Dailey (*6*) and Hall and Persaud (*5*) are both valid and that the differences in the conclusions result from the use of time-mean versus space-mean speeds, as will be discussed in detail in the paper.

The average traffic stream speed can be computed in two different ways: a time-mean speed and a space-mean speed. The difference in speed computations is attributed to the fact that the space-mean speed reflects the average speed over a spatial section of roadway, while the time-mean speed reflects the average speed of the traffic stream passing a specific stationary point. Specifically, Daganzo (*14*) demonstrates that the space-mean speed is a density weighted average speed, while the time-mean speed is a flow weighted average speed. Given that a stationary observer will observe faster vehicles more often than slower vehicles while an aerial photograph would show more slow moving vehicles than faster vehicles over a fixed roadway length, it should come as no surprise that the time-mean speed is greater than or equal to the space-mean speed.

1.2 Paper Objectives and Layout

The objectives of this paper are two-fold. First, the paper modifies the Wardrop (*1*) formulation to estimate the space-mean speed as a function of the time-mean speed. Second, the paper derives the relationship between the time-speed and space-speed variances, as well as the relationship between space-speed and travel time variance. Consequently, the paper provides a means for estimating the reliability of travel times for use within the context of traveler information systems. Subsequently, the paper presents two formulations that are documented in the literature for estimating traffic stream speed from single loop detectors that may appear to be inconsistent at first glance. The paper demonstrates that both formulations are correct and that differences in the formulations arise from the fact that the Hall and Persaud (*5*) formulation estimates the traffic space-mean speed while the Dailey (*6*) formulation estimates the time-mean speed.

The significance of this research effort is three-fold. First, because the reality is that Traffic Management Center (TMC) controllers are designed to estimate time-mean speeds the proposed formulation provides an efficient approach for estimating space-mean speeds. Space-mean speed, as opposed to time-mean speed, is used within state-of-the-practice traffic stream models and thus is critical to the accurate modeling of traffic stream behavior. Second, the paper provides a means for quantifying the reliability of space-mean speed and travel time estimates. Third, the paper demonstrates the consistency and differences between the Hall and Persaud (*5*) and Dailey (*6*) formulations for estimating traffic stream speed from single loop detectors.

Initially, the problems with the Wardrop formulation are discussed and the relationship between space-mean speed and time-mean speed is derived using the statistics of the estimates. Subsequently, two formulations for estimating traffic stream speed from single loop detector volume to occupancy measurements are presented. Subsequently, we demonstrate that both formulations are consistent and that differences arise because of differences in estimating time versus space-mean speeds. Finally the conclusions of the paper are presented.

2. RELATIONSHIP BETWEEN TIME-MEAN AND SPACE-MEAN SPEEDS

As was mentioned earlier, time-mean speed is the arithmetic mean of the speeds of vehicles passing a point on a highway during an interval of time. Alternatively, the space-mean speed is the harmonic mean of the speeds of vehicles passing a point on a highway during an interval of time. The space-mean speed is a traffic density speed estimate and reflects the spatial dimension of speed and thus is utilized in the standard speed-flow-density relationships.

2.1 State-of-Practice Relationships

Wardrop (*1*) derived the relationship between the time-mean speed ($\bar{u}_T$) and the space-mean speed ($\bar{u}_S$) as

$$\bar{u}_T = \bar{u}_S + \frac{\sigma_s^2}{\bar{u}_S}, \qquad [2.]$$

where σ^2_S is the variance in vehicle speeds about the space-mean speed. Consequently, Equation 2 is applied to estimate the time-mean speed from the space-mean speed. However, in most cases the time-mean speed, as opposed to the space-mean speed, is available and it is desirable to estimate the space-mean speed from the time-mean speed. Because TMCs do not measure/estimate space-mean speeds (harmonic mean), but instead measure/estimate time-mean speeds, there is a need to compute space-mean speeds from time-mean speeds. The importance of space-mean speed lies in the fact that Equation 1 requires the use of this variable as opposed to time-mean speed. Consequently, a new formulation is required to address this need.

Studies have shown that the difference between time-mean speed and space-mean speed estimates are on the order of 1 to 5 percent with greater differences occurring when the coefficient of variation (CV) is large and the mean speed is small (May (*3*)). Figure 1 and Figure 2 illustrate the difference between time-mean speed and space-mean speed measurements for data gathered from a dual loop detector located on the I-880 freeway in Los Angeles (Coifman *et al*., (*15*)). The data are unique in that they include individual detector activations at a resolution of 1/60th of a second as opposed to aggregated 20- or 30-s estimates. The data were gathered over an entire day on the median lane (lane 1) and the lane adjacent to the shoulder lane (lane 4). The data demonstrates that while the differences between space- and time-mean speeds are typically in the range of 1 to 5 percent, larger differences can be observed when the traffic stream speed is lower (during congestion). Specifically, differences in the range of 10 to 30 percent are not uncommon.

Garber and Hoel (*4*) describe a more direct relationship between time-mean and space-mean speed as

$$\bar{u}_T = 0.966\,\bar{u}_S + 3.541\,. \qquad [3.]$$

However, the model parameters are specific to the local roadway and traffic stream characteristics. For example, data from the I-880 freeway resulted in different model parameters when a regression line was fit to the data. Specifically, the optimum model constant was 2.389, as opposed to 3.541, and the model slope was 0.986, as opposed to 0.966. Consequently, the model proposed by Garber and Hoel would require calibration to local roadway and traffic conditions and could not be generalized.

In this paper, we shall demonstrate that the relationship between time-mean and space-mean speed that was derived by Wardrop (*1*) and presented in several textbooks (e.g. Hobbs and Richardson (*2*); May (*3*); Garber and Hoel (*4*)) produces an error in the range of 0 to 1 percent in time-mean speed estimates. We also propose a new formulation for estimating space-mean speeds from time-mean speeds with a similar margin of error (within 0 to 1 percent). Specifically, we use the statistics of the estimates to derive

$$\bar{u}_S \approx \bar{u}_T - \frac{\sigma_T^2}{\bar{u}_T}. \qquad [4.]$$

It should be noted that after developing this relationship it was recognized that Khisty and Lall (*16*) presented a similar relationship and demonstrated it was valid using 3 observations; however, Khisty and Lall provided no description of how the relationship was derived analytically. Consequently, the paper expands the state of knowledge by deriving this relationship from the statistics of measurements.

To test the Wardrop and proposed formulation, the aforementioned I-880 data samples are utilized. The I-880 section that is analyzed is a five-lane section with lanes numbered in ascending order from median to shoulder lane. Using these data, the speed and length of each individual vehicle was computed, as illustrated in Figure 3 and Figure 4. The solid black line represents a moving average of 50 observations while the light grey lines represent the actual field measurements. The median lane data clearly demonstrates a high degree of variability in speed and vehicle length measurements; however the mean vehicle length appears to remain fairly constant throughout the entire day. The median lane only had 1.19 percent of the vehicles with lengths that exceeded 8 meters. Considering a threshold of 8 meters for the classification of trucks, the median lane (lane 1) only had a 1 percent truck volume. Alternatively, second rightmost lane (lane 4) was composed of approximately a 12 percent truck volume considering a vehicle length threshold of 8 meters. The data of the median lane and lane 4 also demonstrates the onset of congestion during the PM peak period with a significant decrease in vehicle speeds. Using these raw field data, 5-minute time-mean (arithmetic mean) and space-mean (harmonic mean) speeds were computed. In addition, the speed variance about the time-mean and space-mean speeds was computed. Furthermore, using these aggregated data, an estimate of the time-mean and space-mean speeds was made using Equations 2 and 4, respectively. The results of Figure 5 and Figure 6 demonstrate a high degree of correlation between the measured and computed time-mean speeds (R^2 in excess of 99%) on the median lane and lane 4, respectively. A further analysis of the estimate errors revealed that the error increased as a function of the speed coefficient of variation, as illustrated in Figure 7. However, the speed estimate errors did not exceed 4 km/h for the entire range of speed coefficients of variation. These CVs ranged from 0 to 50 percent as illustrated in Figure 7 with a higher speed CV in the median lane compared to the inner lanes (lane 4).

The results that were presented demonstrate that the proposed formulation maintains the accuracy of parameter estimates while estimating the space-mean speed from the time-mean speed. Consequently, if current loop detector technologies were to store not only the mean speed within a polling interval but also the speed variance, it would be possible to estimate the space-mean speed from the time-mean speed to a high degree of accuracy, as demonstrated in Equation 4.

2.2 Proposed Model for Estimating Space-Mean Speed

In deriving the proposed relationship between the time-mean speed and the space-mean speed, we will consider the statistics of estimates similar to an earlier publication by Dailey (*6*) in which he attempted to estimate the traffic stream speed from single loop detectors as is presented later in this paper.

The speed of the j^{th} vehicle in a polling interval can be computed as

$$u_j = \frac{D}{t_j}. \qquad [5.]$$

Equation 5 assumes that the distance of travel between the two reference points (D) is sufficiently long enough that differences in vehicle lengths can be ignored in computing the vehicle speed (u_j). Specifically, the speed of vehicle j within a polling interval is computed as the travel distance (D) divided by the time it takes the vehicle to travel between the two reference points (t_j).

The time-mean speed is computed as the expected speed over all observations within the polling interval as

$$\bar{u}_T = E\{u_j\} = 3.6 \cdot E\left\{\frac{D}{t_j}\right\} = [3.6 \cdot D] \cdot E\left\{\frac{1}{t_j}\right\} = d \cdot E\left\{\frac{1}{t_j}\right\}, \tag{6.}$$

where E{*} is the expectation operator. The operator E is the expectation over all realizations within the polling interval. It should be noted that the constant 3.6 is used to convert the speed from units of m/s to km/h. In summary, Equation 6 demonstrates that the time-mean speed is equal to the product of the distance between the two observation points and the geometric mean of the travel times between these two reference points within the polling interval i.

We can express the travel time measurements as the expected value (mean) and some deviation (Δt_j) that occurs for this observation j,

$$t_j = \bar{t} + \Delta t_j \tag{7.}$$

where the statistics of the deviation term are selected such that the $E\{\Delta t_j\} = 0$.

Substituting Equation 7 in Equation 6, we get

$$\bar{u}_T = d \cdot E\left\{\frac{1}{\bar{t}_i + \Delta t_j}\right\} = \frac{d}{\bar{t}} \cdot E\left\{\frac{1}{1 + \frac{\Delta t_j}{\bar{t}}}\right\}. \tag{8.}$$

Expanding the right-hand-side (RHS) using the power series we get

$$\bar{u}_T = \frac{d}{\bar{t}} \cdot E\left\{1 - \frac{\Delta t_j}{\bar{t}} + \frac{\Delta t_j^2}{\bar{t}^2} - \frac{\Delta t_j^3}{\bar{t}^3} + \cdots\right\}. \tag{9.}$$

Alternatively, the space-mean speed is computed as the distance of travel divided by the expected travel time, as follows:

$$\bar{u}_S = \frac{d}{E\{t_j\}} = \frac{d}{E\{\bar{t} + \Delta t_j\}} = \frac{d}{\bar{t}}. \tag{10.}$$

Inserting Equation 10 in Equation 9 and approximating the series for the first three terms, we get

$$\bar{u}_T = \bar{u}_S \cdot E\left\{1 - \frac{\Delta t_j}{\bar{t}} + \frac{\Delta t_j^2}{\bar{t}^2} - \frac{\Delta t_j^3}{\bar{t}^3} + \cdots\right\} \approx \bar{u}_S \cdot E\left\{1 + \frac{\Delta t_j^2}{\bar{t}^2}\right\}. \tag{11.}$$

Dailey (*6*) demonstrated that the vehicle length and speed observations could be considered independent (coefficient of correlation of 0.018) using sample field data. If we consider no differences in vehicle lengths (i.e. all vehicles are of equal lengths), the travel time and speed measurements are highly negatively correlated (Pearson product moment correlation coefficient of -0.975 in the case of the sample data that were described earlier). However, if we consider potential differences in vehicle lengths that are independent of the variability in vehicle speeds, the travel time and speed measurements while continuing to be negatively correlated are less correlated. For example, considering a detection length of 5 meters, an average vehicle length of 8 meters, and a vehicle length coefficient of variation of 0.25, results are a Pearson product moment correlation coefficient of -0.70. Consequently, we may assume that the speed and travel time observations are negatively but not highly correlated and thus relate the travel times to the time-mean speed as

$$t_j = \frac{d}{u_j} \qquad \Rightarrow \qquad \bar{t} + \Delta t_j = \frac{d}{\bar{u}_T - \Delta u_j}. \qquad [12.]$$

Equation 12 considers Δu_j as the deviation is vehicle specific speeds about the time-mean speed and is selected such that $E\{\Delta u_j\} = 0$. It should be noted that in the case that the deviations are zero ($\Delta u_j = 0$), Equation 12 reverts to Equation 10 given that the time-mean and space-mean speeds would be equal in magnitude. In addition, if we assume vehicle travel times and speeds to be highly negatively correlated, we may use the space-mean speed instead of the time-mean speed and thus derive the Wardrop formulation that was presented in Equation 2.

Re-arranging Equation 10, the mean travel time for polling interval *i* can be computed as

$$\bar{t} = \frac{d}{\bar{u}_S}. \qquad [13.]$$

Inserting Equation 13 in Equation 12 the deviation in vehicle speed can be approximated for

$$\Delta t_j = \frac{d}{\bar{u}_T - \Delta u_j} - \bar{t} = \frac{d}{\bar{u}_T - \Delta u_j} - \frac{d}{\bar{u}_S} = d \cdot \left[\frac{(\bar{u}_S - \bar{u}_T)}{(\bar{u}_T - \Delta u_j)} + \frac{\Delta u_j}{\bar{u}_S (\bar{u}_T - \Delta u_j)} \right]. \qquad [14.]$$

Recognizing that the difference between the space-mean and time-mean speeds is minor (1 to 5%) and that the deviation in speed is typically small relative to the mean speed, we can approximate Equation 14 for

$$\Delta t_j \approx d \cdot \left[\frac{\Delta u_j}{\bar{u}_S \cdot \bar{u}_T} \right] = \frac{d}{\bar{u}_S} \cdot \frac{\Delta u_j}{\bar{u}_T}. \qquad [15.]$$

Incorporating Equation 13 in Equation 15, we get

$$\frac{\Delta t_j}{\bar{t}} \approx \frac{\Delta u_j}{\bar{u}_T}. \qquad [16.]$$

Inserting Equation 16 in Equation 11 and solving for the expectation, we get

$$\bar{u}_T \approx \bar{u}_S \cdot E\left\{ 1 + \frac{\Delta u_j^2}{\bar{u}_T \cdot \bar{u}_T} \right\} = \bar{u}_S + \frac{\bar{u}_S}{\bar{u}_T} \cdot \frac{E\{\Delta u_j^2\}}{\bar{u}_T} \approx \bar{u}_S + \frac{\sigma_T^2}{\bar{u}_T}, \qquad [17.]$$

where $\frac{\bar{u}_S}{\bar{u}_T} \approx 1.0$.

It should be noted that the variance (σ_T^2) in Equation 17 is the variance with respect to the time-mean speed for all realizations within the polling interval. Alternatively, Equation 17 can be written as Equation 4 by substituting the space-mean speed in the denominator for the time-mean speed and solving for the space-mean speed. The advantage of Equation 4 is that all terms on the RHS are computed using the time-mean speed for the computation of the space-mean speed.

2.3 Relationship between Time-Mean and Space-Mean Speed Variance

In this paper, we also attempt to derive the relationship between the time-mean speed and space-mean speed variance. Specifically, considering the formulation of the space-mean speed variance as

$$\sigma_S^2 = E\left(u_j - \bar{u}_S\right)^2, \tag{18.}$$

the space-mean speed variance can be formulated as

$$\sigma_S^2 \approx E\left(u_j - \bar{u}_T + \frac{\sigma_T^2}{\bar{u}_T}\right)^2 = E\left(u_j - \bar{u}_T\right)^2 + E\left(\frac{\sigma_T^2}{\bar{u}_T}\right)^2 + 2E\left[\left(u_j - \bar{u}_T\right)\frac{\sigma_T^2}{\bar{u}_T}\right]. \tag{19.}$$

Equation 19 substitutes the space-mean speed for the relationship that was proposed earlier in Equation 4. Recognizing that $E\left(u_j - \bar{u}_T\right) = 0$, Equation 19 can be reduced to

$$\sigma_S^2 = \sigma_T^2 + \left(\frac{\sigma_T^2}{\bar{u}_T}\right)^2. \tag{20.}$$

Using the I-880 field data space-speed, variances were computed and compared against the estimates of Equation 20, as illustrated in Figure 8. The figure clearly demonstrates a high degree of correlation between the field-computed and estimated variances (slope of 0.99 and R^2 in excess of 99 percent). Equation 20 demonstrates that the space-speed variance is typically greater than the time-speed variance.

3. ESTIMATING TRAFFIC STREAM SPEED FROM SINGLE LOOP DETECTORS

Unfortunately, a significant number of loop detectors in Freeway Traffic Management Systems (FTMSs) are single loop detectors. Consequently, these detectors do not measure vehicle speeds; instead, the detectors measure the traffic volume that passes the detection station and the occupancy (percentage of time the detector is occupied) of detectors. This section describes a number of procedures, documented in the literature, for estimating traffic stream speed from single loop detector volume and occupancy measurements.

3.1 Hall and Persaud Procedures

Hall and Persaud (*5*) developed a procedure for estimating traffic stream speed from single loop detector volume and occupancy measurements. These procedures are discussed in detail in this section.

Specifically, Hall and Persaud computed the speed of the j^{th} vehicle within polling interval as

$$u_j = 3.6\left(\frac{l_j + l_D}{t_j}\right). \tag{21.}$$

Equation 21 assumes that the vehicle length (l_j) and detection zone length (l_D) are in meters while the time that the loop detector is activated (t_j) is in units of seconds. The speed of the vehicle (u_j) is then computed in km/h by multiplying by the constant 3.6.

Summing up the time that loop detector is occupied within the polling interval

$$\sum_{j=1}^{N_i} t_j = 3.6\left(\bar{l} + l_D\right)\sum_{j=1}^{N} \frac{1}{u_j}, \tag{22.}$$

the occupancy for a polling interval can be computed as

$$O = \frac{\sum_{j=1}^{N} t_j}{T} = 3.6\left(\frac{\bar{l} + l_D}{T}\right)\sum_{j=1}^{N} \frac{1}{u_j}. \tag{23.}$$

Substituting the vehicle speeds for the distance divided by the travel time and assuming the vehicle lengths are constant, we get

$$O = 3.6\left(\frac{\bar{l} + l_D}{T}\right)\sum_{j=1}^{N}\frac{1}{u_j} = 3.6\left(\frac{\bar{l} + l_D}{T}\right)\frac{\sum_{j=1}^{N} t_j}{d}. \quad [24.]$$

Multiplying and dividing Equation 24 by the number of observations within the polling interval *i*, we get

$$O = 3.6\left(\frac{\bar{l} + l_D}{T}\right)\frac{\sum_{j=1}^{N_i} t_j}{d}\cdot\frac{N}{N} = 3.6\left(\frac{\bar{l} + l_D}{T}\right)\frac{\sum_{j=1}^{N} t_j}{dN}\cdot N. \quad [25.]$$

where: $E\{t_j\} = \bar{t} = \frac{\sum_{j=1}^{N} t_j}{N}$.

Replacing the distance divided by the mean travel time for the space-mean speed, we get

$$N_i = 3.6\left(\frac{\bar{l}_i + l_D}{T}\right)\cdot\frac{\bar{t}}{d}N = 3.6\left(\frac{\bar{l} + l_D}{T}\right)\cdot\frac{N}{\bar{u}_S}. \quad [26.]$$

Solving for the space-mean speed, we get

$$\bar{u}_S = 3.6\left(\frac{\bar{l} + l_D}{T}\right)\cdot\frac{N}{O}. \quad [27.]$$

It should be noted that Equation 27 estimates the space-mean speed from a single loop detector as a constant multiplied by the volume to occupancy ratio within a given polling interval. This constant may vary depending on the average vehicle length within a polling interval.

3.2 Dailey Procedures

Dailey (*6*) explicitly considered the statistics of estimates from loop detector measurements, including volume (*N)* and occupancy (*O*). Specifically, using the relationship between occupancy and the j^{th} vehicle speed (u_j) and length (l_j) the occupancy for a polling interval can be computed as follows, as was described earlier:

$$O = \frac{3.6}{T}\cdot\sum_{j=1}^{N}\frac{l_j + l_D}{u_j} \quad [28.]$$

Dailey expressed the speed and length observations as the expected value (mean) for the polling interval plus some deviation (Δl_j, Δu_j) around that mean. By substituting the sum of the vehicle length and the constant detection length ($l_j + l_D$) for L_j, the following can be derived:

$$L_j = l_j + l_D = \left[\bar{l} + l_D\right] + \Delta l_j = \bar{L} + \Delta l_j \quad [29.]$$

$$u_j = \bar{u} + \Delta u_j \quad [30.]$$

It should be noted that in Equation 30, the mean speed is the time-mean speed for a polling interval given that it is the expectation of the speed over all realizations within the polling interval.

The expected occupancy within a polling interval can be computed using the expected value operator (*E*{*}), as follows:

$$E\{O\} = E\left\{\frac{3.6}{T} \cdot \sum_{j=1}^{N_i} \frac{l_j + l_D}{u_j}\right\} = E_i\left\{\frac{3.6}{T} \cdot \sum_{j=1}^{N_i} \frac{L_j}{u_j}\right\} = 3.6 \cdot \frac{N}{T} \cdot E\left\{\frac{L_j}{u_j}\right\} \quad [31.]$$

It should be noted that each measurement produces a pair of volume (*N*) and occupancy values (*O*). Dailey denoted *E* as the expectation over all realizations that have the volume *N* to compute the expected occupancy for a polling interval, as follows:

$$E\{O\} = 3.6\frac{N}{T} \cdot E\left\{\frac{L_j}{u_j}\right\}. \quad [32.]$$

Inserting Equations 29 and 30 into Equation 32, we get

$$E\{O\} = 3.6\frac{N}{T} \cdot E_i\left\{\frac{\overline{L} + \Delta l_j}{\overline{u} + \Delta u_j}\right\} = 3.6\frac{N}{T} \cdot E_i\left\{\frac{\overline{L}}{\overline{u} + \Delta u_j} + \frac{\Delta l_j}{\overline{u} + \Delta u_j}\right\}, \quad [33.]$$

where the statistics of the deviation terms are selected such that $E\{\Delta l_j\} = E\{\Delta u_j\} = 0$. Consequently, Equation 33 can be simplified assuming that Δl_j and Δu_j are independent

$$E\{O\} = 3.6\frac{N}{T} \cdot E\left\{\frac{\overline{L}}{\overline{u} + \Delta u_j}\right\} = 3.6\frac{N}{T} \cdot \frac{\overline{L}}{\overline{u}} E\left\{\frac{1}{1 + \frac{\Delta u_j}{\overline{u}}}\right\}. \quad [34.]$$

Dailey demonstrated that the correlation coefficient between Δl_j and Δu_j was very low using some sample field data (r = 0.018) and thus, concluded that such an assumption of independency was reasonable. Dailey then expanded the RHS of Equation 34 using a power series to derive

$$E\{O\} = 3.6\frac{N}{T} \cdot \frac{\overline{L}}{\overline{u}} \cdot E\left\{1 - \frac{\Delta u_j}{\overline{u}} + \frac{\Delta u_j^2}{\overline{u}^2} - \frac{\Delta u_j^3}{\overline{u}^2} + \ldots\right\}. \quad [35.]$$

Noting that $E\{\Delta u_j\} = 0$ and approximating the series for three terms, Dailey derived the following:

$$E\{O\} = 3.6\frac{N}{T} \cdot \frac{\overline{L}}{\overline{u}} \cdot \left[1 + \frac{E\{\Delta u_j^2\}}{\overline{u}^2}\right]. \quad [36.]$$

Substituting $E\{\Delta u_j^2\}$ for the speed variance within the polling interval *i* (σ_i^2), Dailey derived

$$E\{O\} = 3.6\frac{N}{T} \cdot \frac{\overline{L}}{\overline{u}} \cdot \left[1 + \frac{\sigma^2}{\overline{u}^2}\right]. \quad [37.]$$

Rearranging the terms and solving for *N* Dailey derived

$$N = \frac{\overline{u}T}{3.6\overline{L}} \cdot E\{O\} \cdot \left[\frac{\overline{u}^2}{\sigma^2 + \overline{u}^2}\right]. \quad [38.]$$

If the measurements over the different polling intervals are considered to have a constant mean, then the expected occupancy for a polling interval can be expressed as

$$E\{O\} = E\{\overline{O} + \Delta O\} = \overline{O} \quad [39.]$$

Considering the polling interval occupancy to be an estimate of the expected polling interval occupancy, the traffic stream speed over the polling interval can be computed as

$$\bar{u} = 3.6\left(\frac{\bar{L}}{T}\right) \cdot \frac{N_i}{O_i} \cdot \left[\frac{\sigma^2 + \bar{u}^2}{\bar{u}^2}\right] = 3.6\left(\frac{\bar{l} + l_D}{T}\right) \cdot \frac{N}{O} \cdot \left[1 + \frac{\sigma^2}{\bar{u}^2}\right]. \qquad [40.]$$

Dailey mentioned that "*previous authors have asserted that a ratio of measured volumes and occupancies converted to density by a constant can be used to estimate speed*" (Hall and Persaud (*5*); Persaud and Hurdle (*17*); Hall and Gunter (*18*); Ross (*19*)). Dailey then concluded that "*an estimate which does not consider the variability of the speed contained in the variance (σ^2) has a bias.*" While this conclusion is correct, the speed that is estimated in this formulation is the time-mean speed and not the space-mean speed, as is typically utilized in traffic stream analysis.

3.3 Comparison of Procedures

As was mentioned earlier, the speed that is computed in Equation 40 is the time-mean speed and not the space-mean speed. Consequently inserting Equation 27 (Hall and Persaud's derivation of space-mean speed) in Equation 40, we obtain

$$\bar{u}_T = \bar{u}_S \cdot \left[1 + \frac{\sigma_T^2}{\bar{u}_T \cdot \bar{u}_T}\right] \approx \bar{u}_S + \frac{\sigma_T^2}{\bar{u}_T}. \qquad [41.]$$

In conclusion, we demonstrate that both the Hall and Persaud (*5*) and the Dailey (*6*) formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation compute the space-mean speed while the Dailey formulation computes the time-mean speed.

4. RELATIONSHIP BETWEEN SPACE-MEAN SPEED AND TRAVEL TIME RELIABILITY

Typically, space-mean speed is measured at specific locations along a highway in order to estimate roadway travel times. Alternatively, travel times can be measured using license plate recognition cameras or Automatic Vehicle Identification (AVI) technologies and the desire is to not only estimate average travel speeds but also the reliability of these speeds. Consequently, this section attempts to relate space-mean speed variability to travel time variability in order to estimate either travel time or travel speed confidence limits.

The variance of travel times for all observations *j* within a polling interval can be computed as

$$\sigma_t^2 = \sum_j \frac{\left(t_j - \bar{t}\right)^2}{n}, \qquad [42.]$$

where $t_j = \frac{D}{u_j}$ and $\bar{t} = \frac{D}{\bar{u}}$. It should be noted that the mean speed ($\bar{u}$) is the space-mean speed. Expanding Equation 42 we derive

$$\sigma_t^2 = \sum_j \frac{\left(t_j - \bar{t}\right)^2}{n} = \sum_j D^2 \frac{\left(\frac{1}{u_j} - \frac{1}{\bar{u}}\right)^2}{n} = \frac{D^2}{n}\sum_j \frac{\left(\bar{u} - u_j\right)^2}{\bar{u}^2 u_j^2}. \qquad [43.]$$

Recognizing that the speed coefficient of variation (standard deviation divided by mean) is typically small (less than 10 percent), we can ignore differences in speeds with minimum effect on the formulation to derive

$$\sigma_t^2 \approx \frac{D^2}{\bar{u}^4}\sum_j \frac{(u_j - \bar{u})^2}{n}. \qquad [44.]$$

Re-arranging the terms of Equation 44, the travel time variance can be related to the variance in speeds about the space-mean speed as

$$\sigma_t^2 \approx \frac{\bar{t}^2}{\bar{u}^2}\sum_j \frac{(u_j - \bar{u})^2}{n} = \frac{\bar{t}^2}{\bar{u}^2}\sigma_s^2. \qquad [45.]$$

Solving Equation 45 we derive

$$\sigma_t \approx \frac{\bar{t}}{\bar{u}}\sigma_s. \qquad [46.]$$

Consequently, the final relationship relates the travel time and space-mean speed coefficients of variations as

$$CV_t \approx CV_s. \qquad [47.]$$

Equation 47 demonstrates that the coefficient of variation of space-mean speeds is approximately equal to the coefficient of variation of travel times. In other words, a standard deviation in vehicle speeds of 10 percent the space-mean speed results in a standard deviation of roadway travel times that is 10 percent the mean travel time.

5. CONCLUSIONS

The paper demonstrates that the relationship between time-mean and space-mean speed that was derived by Wardrop (*1*) and presented in several textbooks (e.g. May (*3*)) produces an error in the range of 1 percent in time-mean speed estimates. However, the formulation estimates the time-mean speed from the space-mean speed, which is typically the reverse of what is required. Specifically, the objective is to estimate the space-mean speed from the time-mean speed. Consequently, using the statistics of the estimates, the paper derives a modified relationship between space-mean speed and time-mean speed which computes space-mean speed as a function of time-mean speed. The paper demonstrates that the proposed formulation, which utilizes the variance about the time-mean speed as opposed to the variance about the space-mean speed, produces an estimate error to within 0 to 1 percent, as is the case for the Wardrop formulation.

In addition, the paper demonstrates that both the Hall and Persaud (*5*) and the Dailey (*6*) formulations for estimating traffic stream speed from single loop detectors are valid. However, the differences in the derivations are attributed to the fact that the Hall and Persaud formulation computes the space-mean speed (harmonic mean) while the Dailey formulation computes the time-mean speed (arithmetic mean).

Finally, the paper demonstrates that the space-mean speed coefficient of variation (standard deviation divided by mean) is approximately equal to the coefficient of variation of roadway travel times. Using this relationship it would be possible to estimate travel speed confidence limits based on field measurements of travel times.

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LIST OF FIGURES

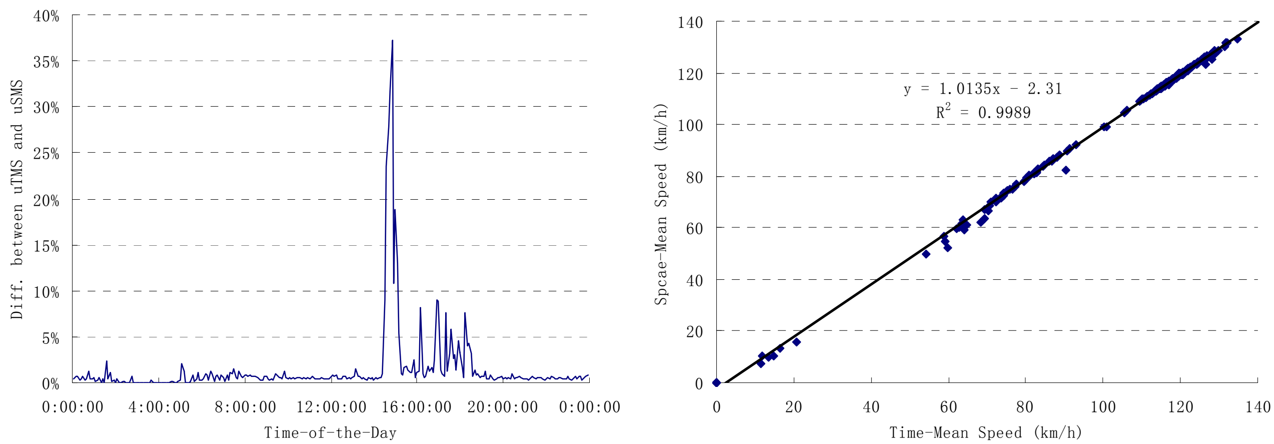


Figure 1 Comparison between Time-Mean and Space-Mean Speed Estimates (Median Lane).

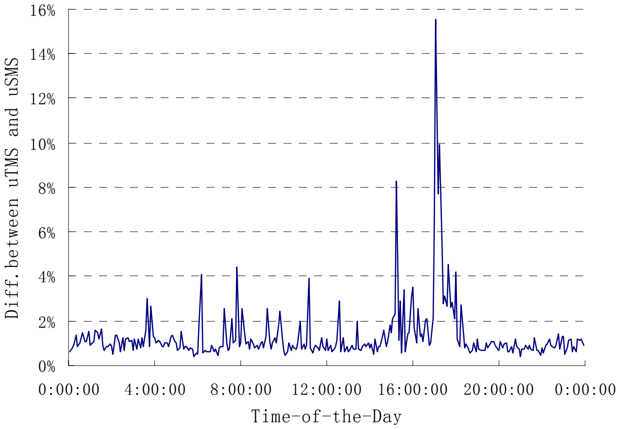
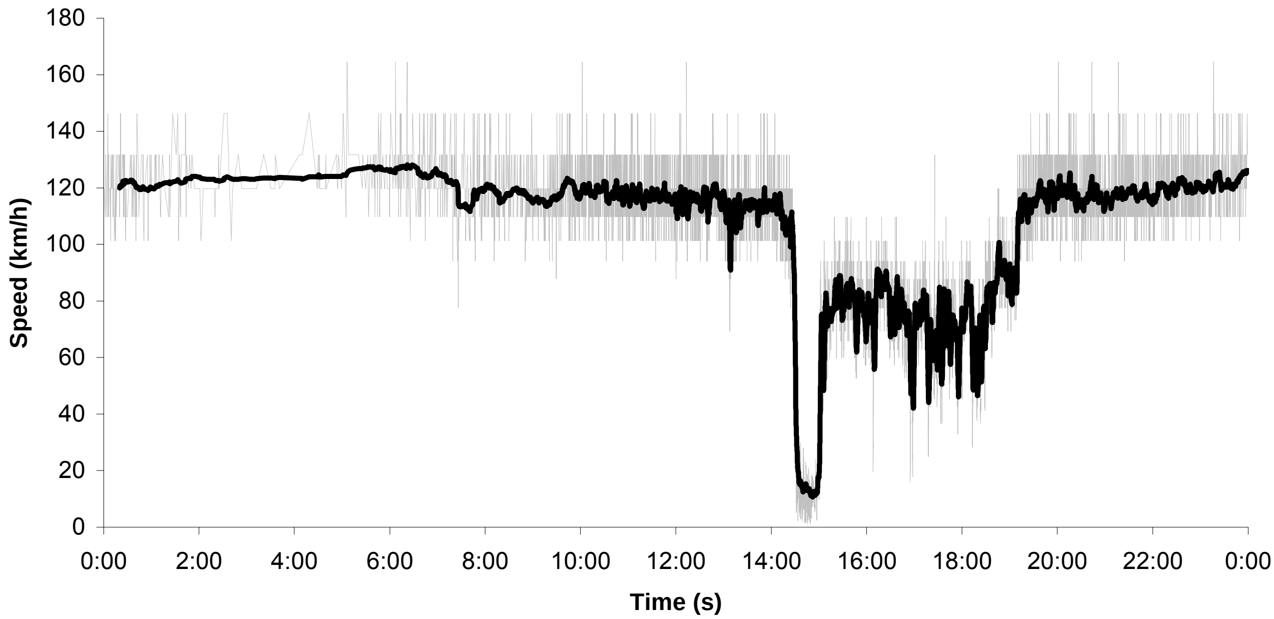


Figure 2 Comparison between Time-Mean and Space-Mean Speed Estimates (Lane 4).



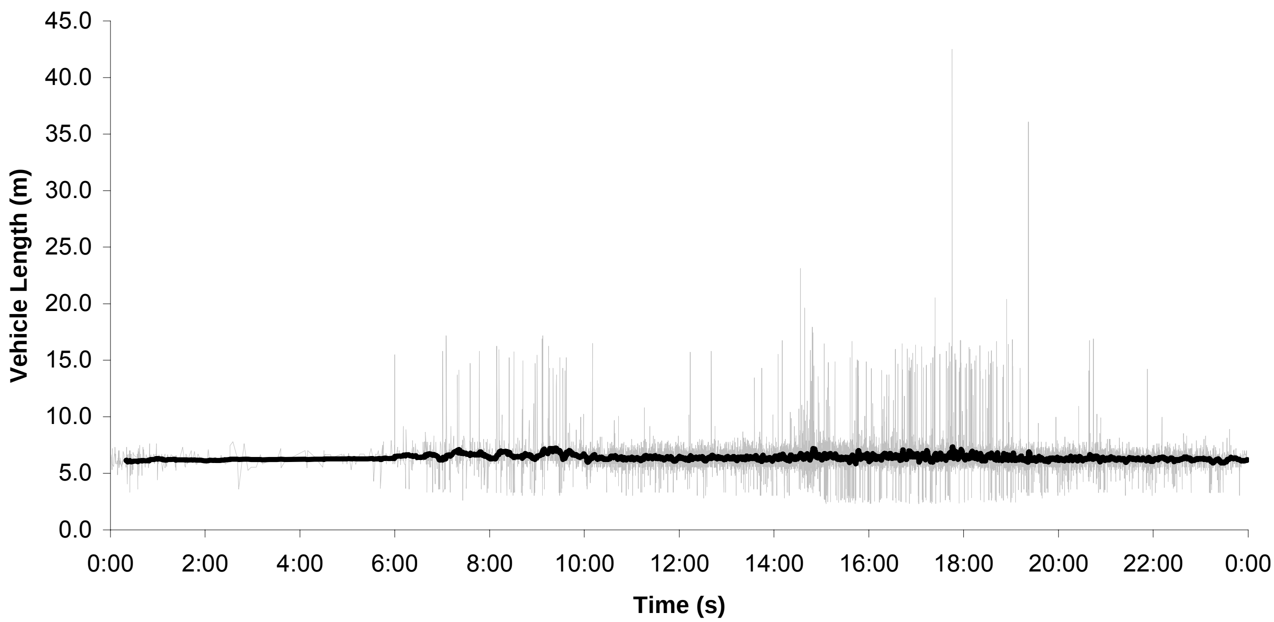


Figure 3 Temporal Variation in Vehicle Speed and Length Measurements on the Median Lane.

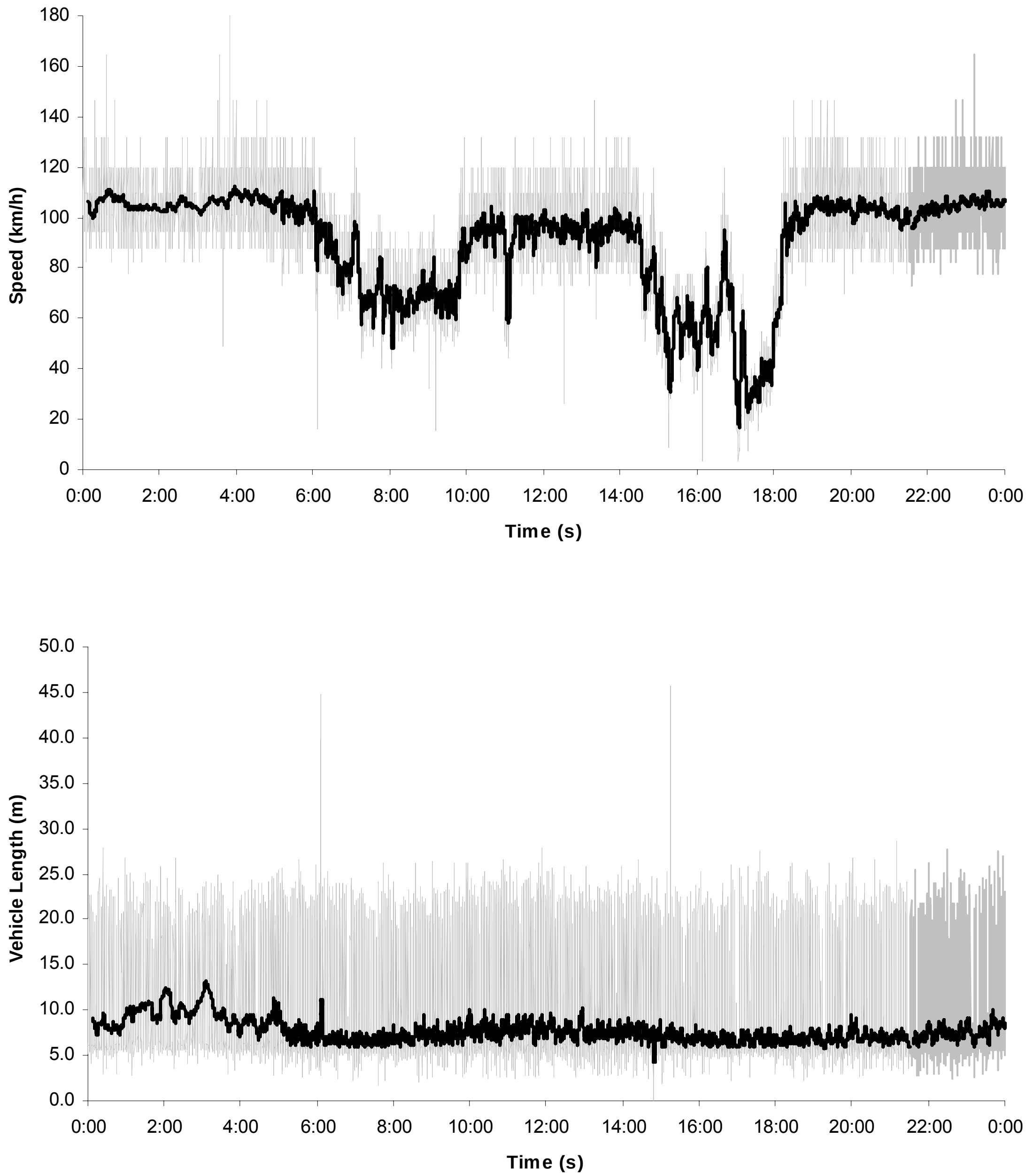


Figure 4 Temporal Variation in Vehicle Speed and Length Measurements on Lane 4.

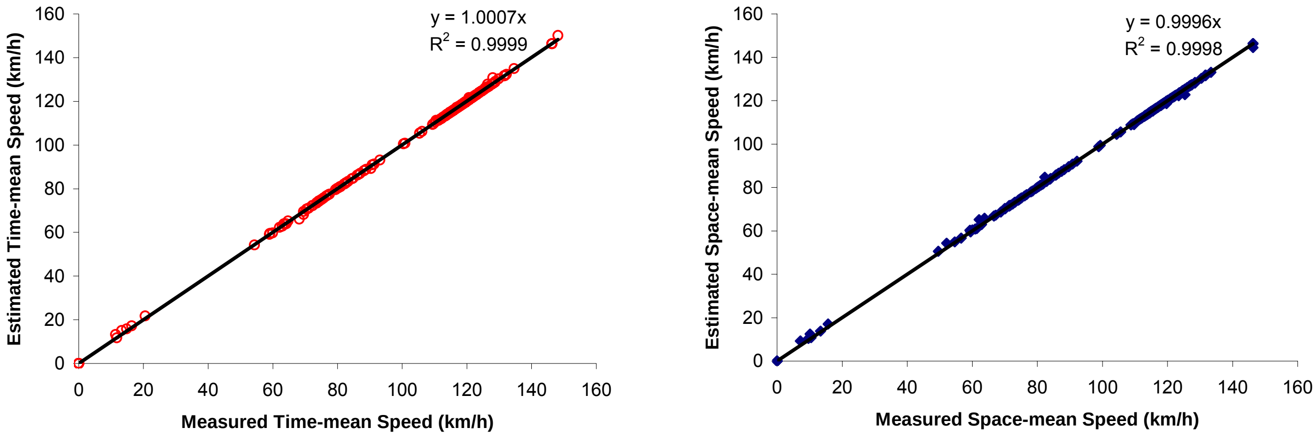


Figure 5 Correlation between Estimated and Measured Mean Speeds on the Median Lane.

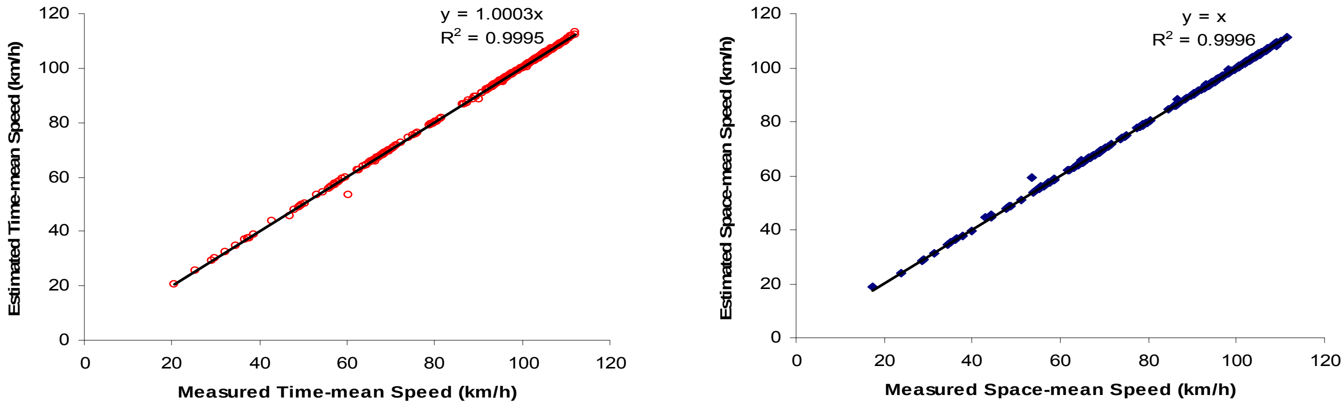
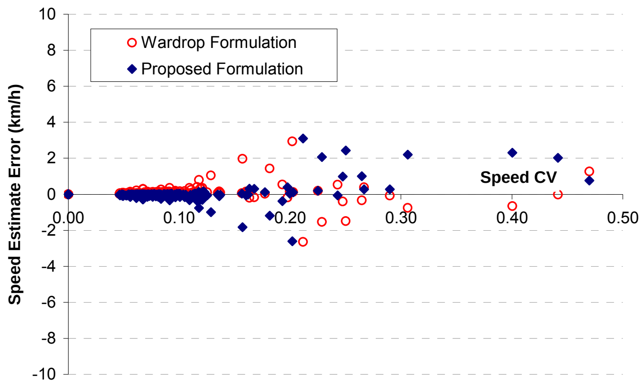


Figure 6 Correlation between Estimated and Measured Mean Speeds on Lane 4.



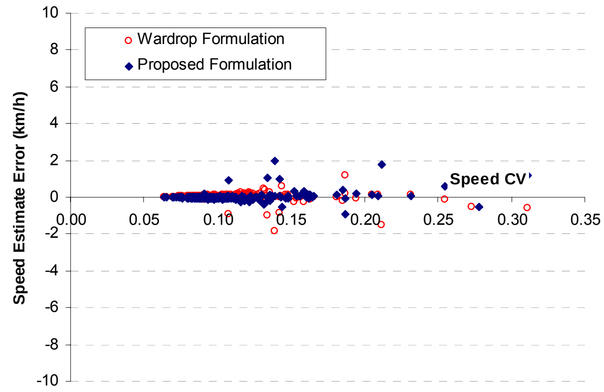


Figure 7 Variation in Estimated Error as a Function of Speed Variance and CV (Median Lane (left) and Lane 4 (right)).

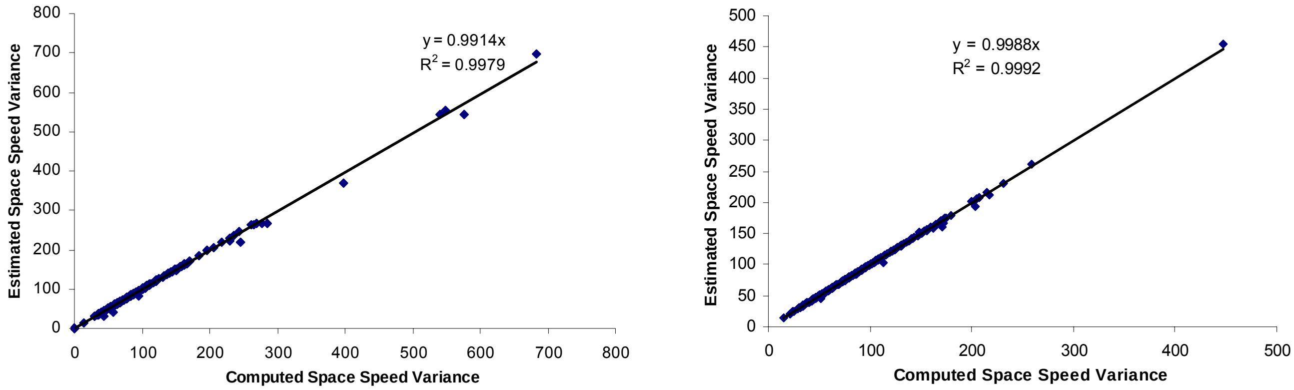


Figure 8 Correlation between Estimated and Measured Space Speed Variance (Median Lane (left) and Lane 4 (right)).