FACULTY OF INFORMATION TECHNOLOGY CTU IN PRAGUE

Supervisor's statement of a final thesis

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Student:	Petr Laštovička
Thesis title:	NAC-colorings search: complexity and algorithms
Branch / specialization:	Computer Science 2021
Created on:	–

Evaluation criteria

1. Fulfillment of the assignment

- ▸ [1] **assignment fulfilled**
  [2] assignment fulfilled with minor objections
  [3] assignment fulfilled with major objections
  [4] assignment not fulfilled

The assignment has been (more than) fulfilled. The student recalls necessary concepts from rigidity theory and how NAC-colorings are related to the existence of flexible realizations. He proves that the existence of a NAC-coloring is NP-complete on graphs with maximum degree five. Algorithms to list all NAC-colorings of a given graph have been proposed and implemented.
Beyond the assignment, it is shown in two different ways that the existence of a NAC-coloring is fixed parameter tractable by treewidth. Moreover, the student has implemented a known algorithm giving a NAC-coloring for a flexible graph.

2. Main written part 98 / 100 (A)

The thesis is well structured, the level of detail is appropriate. Keeping Chapter 3 aside as it is beyond the scope of the assignment, see more below, the text is very well written and nice to read, there only a few minor language issues. All formal requirements are satisfied, used references are properly cited. The used terminology is correctly defined and well formulated statements are rigorously proven. The performance of the implemented algorithms and heuristics is carefully compared and analysed.
In Chapter 3 about FPT of NAC-coloring existence, which goes far beyond the scope of the assignment, some justifications are not so easy to follow and there are some minor issues. Still, I am convinced that the main results are correct and having in mind the level of difficulty and technicality, the student has done good job.

3. Non-written part, attachments 100/100 (A)

The provided implementation in Python covers all approaches to find NAC-colorings described in the thesis and also many other that did not perform so well. The code is well structured into modules and sufficiently tested. A simple Jupyter notebook illustrates how to get a single or all NAC-colorings of a graph. Another notebook collects even more detailed benchmarks than in the written part. The implementation significantly outperforms the only previous one available in SageMath package FlexRiLoG.
Moreover, a separate module implements a known algorithm to find a stable cut of a flexible graph.
The core algorithms suggested by the student based on the benchmarks have been already merged to PyRigi, which is a general-purpose Python package for research in rigidity theory.

4. Evaluation of results, publication outputs and awards 100/100 (A)

The proof of NP-completeness on graphs with maximum degree five extends the previously known result that the existence of a NAC-coloring is NP-complete. This narrows the gap (in terms of the number of edges w.r.t. vertices) for which it is unknown whether the existence of a NAC-coloring can be determined in polynomial time. FPT by treewidth is the first FPT result about NAC-colorings. Also the provided implementation is by orders of magnitude faster than the previously available one. A preprint with these results is already available on arXiv and after extending it by the detailed FPT analysis shall be submitted to a journal.

5. Activity of the student

- ▸ [1] **excellent activity**
  [2] very good activity
  [3] average activity
  [4] weaker, but still sufficient activity
  [5] insufficient activity

The student has started to work on the topic within VýLeT in the summer 2024 and has spent incredible effort in this research. He has been working beyond the scope of the assignment not only regarding the FPT, but also helping to improve PyRigi and merge the code into it.

6. Self-reliance of the student

- ▸ [1] **excellent self-reliance**
  [2] very good self-reliance
  [3] average self-reliance
  [4] weaker, but still sufficient self-reliance
  [5] insufficient self-reliance

The student has shown to be able to both create efficient code and prove theoretical results.

The overall evaluation

100 /100 (A)

This is an excellent thesis that could very well be a Master thesis.

Instructions

Fulfillment of the assignment

Assess whether the submitted FT defines the objectives sufficiently and in line with the assignment; whether the objectives are formulated correctly and fulfilled sufficiently. In the comment, specify the points of the assignment that have not been met, assess the severity, impact, and, if appropriate, also the cause of the deficiencies. If the assignment differs substantially from the standards for the FT or if the student has developed the FT beyond the assignment, describe the way it got reflected on the quality of the assignment's fulfilment and the way it affected your final evaluation.

Main written part

Evaluate whether the extent of the FT is adequate to its content and scope: are all the parts of the FT contentful and necessary? Next, consider whether the submitted FT is actually correct – are there factual errors or inaccuracies?

Evaluate the logical structure of the FT, the thematic flow between chapters and whether the text is comprehensible to the reader. Assess whether the formal notations in the FT are used correctly. Assess the typographic and language aspects of the FT, follow the Dean's Directive No. 52/2021, Art. 3.

Evaluate whether the relevant sources are properly used, quoted and cited. Verify that all quotes are properly distinguished from the results achieved in the FT, thus, that the citation ethics has not been violated and that the citations are complete and in accordance with citation practices and standards. Finally, evaluate whether the software and other copyrighted works have been used in accordance with their license terms.

Non-written part, attachments

Depending on the nature of the FT, comment on the non-written part of the thesis. For example: SW work – the overall quality of the program. Is the technology used (from the development to deployment) suitable and adequate? HW – functional sample. Evaluate the technology and tools used. Research and experimental work – repeatability of the experiment.

Evaluation of results, publication outputs and awards

Depending on the nature of the thesis, estimate whether the thesis results could be deployed in practice; alternatively, evaluate whether the results of the FT extend the already published/known results or whether they bring in completely new findings.

Activity of the student

From your experience with the course of the work on the thesis and its outcome, review the student's activity while working on the thesis, his/her punctuality when meeting the deadlines and whether he/she consulted you as he/she went along and also, whether he/she was well prepared for these consultations.

Self-reliance of the student

From your experience with the course of the work on the thesis and its outcome, assess the student's ability to develop independent creative work.

The overall evaluation

Summarize which of the aspects of the FT affected your grading process the most. The overall grade does not need to be an arithmetic mean (or other value) calculated from the evaluation in the previous criteria. Generally, a well-fulfilled assignment is assessed by grade A.