

# Review report of a final thesis

**Reviewer:** Mgr. Michal Opler, Ph.D.

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Thesis title: NAC-colorings search: complexity and algorithms

Branch / specialization: Computer Science 2021

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## **Evaluation** criteria

## 1. Fulfillment of the assignment

- ▶ [1] assignment fulfilled
  - [2] assignment fulfilled with minor objections
  - [3] assignment fulfilled with major objections
  - [4] assignment not fulfilled

The assignment was extensively surpassed by the student. All the outlined tasks have been without doubt satisfied and on top of that, a non-trivial FPT algorithm is designed that counts NAC colorings on graphs of small tree-width.

#### 2. Main written part

95/100 (A)

The structure of the thesis is very natural and the text reads well going from chapter to chapter. The overall level of writing in English is very good with only a few typos, albeit there are few awkward sentences here and there with Czech-like word order. I have not found any factual mistakes and I believe all the statements (theorems) to be correct. That being said, the proofs in Chapter 3 could be improved as some of them are quite hard to follow and validate in their current form. Part of that stems from not distinguishing properly between the semantics of a cache function and its actual computation. It is worth noting that dynamic programming algorithms of this complexity on treewidth tend to be really cumbersome to formally write and prove their correctness. So the shortcomings of this chapter are very understandable given this is a bachelor thesis.

In the second part of the thesis, a modular algorithm for computing the existence (or the number of) NAC-colorings is given with various parts allowing the choice from multiple heuristics. These are then implemented and properly compared on different natural classes of graphs. I would like to highlight the extensive benchmarking that compares these heuristics among themselves as well with the previously existing algorithms that is supported by large number of graphs. This allows for a substantiated argument about the most useful choice for practical usage in the PyRigi library.

## 3. Non-written part, attachments

The algorithm for counting NAC-colorings is implemented in Python, including the many heuristics described in the thesis. The code is very well-organized and properly tested. Moreover, several Jupyter notebooks are provided that really help understanding the available methods and their usage. I was quite surprised when I found out that the notebook "NAC\_presentation.ipynb" contains even more thorough comparison and benchmarking of the different heuristics than what is included in the thesis. Although I am not actively involved in research on rigidity theory, I can imagine that the code and provided benchmarks will be highly valuable for researchers working in that area.

## 4. Evaluation of results, publication outputs and awards 100/100 (A)

From the practical standpoint, part of the implementation is already merged into the PyRigi library and thus, can be used by the whole community of rigidity research. So this part is, in fact, already deployed in practice. From a theoretical perspective, some of the novel results were already uploaded as a preprint to arXiv and are intended for future journal submission. This concerns, in particular, the proof of NP-hardness for the existence of NAC-colorings in graphs with maximum degree 5. Therefore, this thesis impressively delivers on both practical and theoretical fronts.

## The overall evaluation

100<sub>/100</sub> (A)

This is an excellent thesis with both theoretical and implementation contributions on very high level that far exceeds the usual expectations of an A-level bachelor thesis - consistently demonstrating a steadfast commitment to the topic, never faltering, letting go, or straying from academic rigor.

## **Ouestions for the defense**

Is it typically the case that the color classes in a NAC-coloring are balanced with respect to the two colors (especially when the graph has only few NAC-colorings)? Or does it often happen (in practice) that there exists some NAC-coloring that is largely biased towards one of the two colors?

#### Instructions

#### Fulfillment of the assignment

Assess whether the submitted FT defines the objectives sufficiently and in line with the assignment; whether the objectives are formulated correctly and fulfilled sufficiently. In the comment, specify the points of the assignment that have not been met, assess the severity, impact, and, if appropriate, also the cause of the deficiencies. If the assignment differs substantially from the standards for the FT or if the student has developed the FT beyond the assignment, describe the way it got reflected on the quality of the assignment's fulfilment and the way it affected your final evaluation.

#### Main written part

Evaluate whether the extent of the FT is adequate to its content and scope: are all the parts of the FT contentful and necessary? Next, consider whether the submitted FT is actually correct – are there factual errors or inaccuracies?

Evaluate the logical structure of the FT, the thematic flow between chapters and whether the text is comprehensible to the reader. Assess whether the formal notations in the FT are used correctly. Assess the typographic and language aspects of the FT, follow the Dean's Directive No. 52/2021, Art. 3.

Evaluate whether the relevant sources are properly used, quoted and cited. Verify that all quotes are properly distinguished from the results achieved in the FT, thus, that the citation ethics has not been violated and that the citations are complete and in accordance with citation practices and standards. Finally, evaluate whether the software and other copyrighted works have been used in accordance with their license terms.

#### Non-written part, attachments

Depending on the nature of the FT, comment on the non-written part of the thesis. For example: SW work – the overall quality of the program. Is the technology used (from the development to deployment) suitable and adequate? HW – functional sample. Evaluate the technology and tools used. Research and experimental work – repeatability of the experiment.

## Evaluation of results, publication outputs and awards

Depending on the nature of the thesis, estimate whether the thesis results could be deployed in practice; alternatively, evaluate whether the results of the FT extend the already published/known results or whether they bring in completely new findings.

#### The overall evaluation

Summarize which of the aspects of the FT affected your grading process the most. The overall grade does not need to be an arithmetic mean (or other value) calculated from the evaluation in the previous criteria. Generally, a well-fulfilled assignment is assessed by grade A.