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Assignment - 6

$$f(x) = \frac{1}{2\sqrt{\pi}6^2} e^{-\frac{(x-\mu)^2}{26^2}}$$

$x_1, x_2 \ldots x_n)$ $f \Rightarrow$ samplesize of n

$$L(x_1, x_2, x_3, x_n) = f(x_1) \cdot f(x_2) \ldots f(x_n)$$

$$\Rightarrow \left(\frac{1}{\sqrt{2\pi 6^2}} e^{-\left(\frac{x-\mu}{26}\right)^2}\right) \cdot \left(\frac{1}{\sqrt{2\pi 6}} \cdot e^{-\frac{(x_2-\mu)^2}{26}}\right)$$

taking ln on bothside

$$\ln(L) = -\frac{n}{2} \ln(2\pi 6^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{26^2}\right) \text{—(1)}$$

take portal derivative w.r.t μ of the above equation

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i-\mu)}{26^2}\right) 0$$

$$= \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

hence $\theta = \bar{x}$ is therfore sample ~~...~~ mean

taking derivative w.r.t. 6^2 of eq (1)

$$\partial \ln(L) = \frac{-n}{26} + \sum_{i=1}^{n} \frac{-(x_i-\mu)}{2(6)^2} = 0$$

$$= -n + \sum_{i=1}^{n} \frac{-(x_i-\mu)^2}{26^2} = 0$$

$$n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{6}$$ hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu^2)$

#2

Binomial distribution → ${}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log({}^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right)$$

$$\log L = \sum_{i=1}^{n} \log({}^{n}C_{x_i}) + \log \theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n-x_i)$$

differentiate wrt θ

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n-x_i) = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta} \Rightarrow \boxed{\theta = \frac{\sum x_i}{n^2}}$$

Ans