PROBLEM #1

Re-write the chain rule in Lagrange notation,

Leibniz notation — $\dfrac{dg}{dx} = \dfrac{dg}{dh}\dfrac{dh}{dx}$

Lagrange,

$$f'(x) = f'(h(x))\,h'(x)$$

PROBLEM #2.

Use the chain rule to differentiate $\exp(x^2-3)$.

Let $u = e^{v(x)}$

Let $v(x) = x^2 - 3$

$\dfrac{d}{dx} u(v(x)) = \dfrac{du}{dv}\dfrac{dv}{dx}$

$\qquad = e^{v(x)}\, 2x$

$\qquad = e^{(x^2-3)}\, 2x$

PROBLEM #3.

Use chain rule to derive $\sin^3(x)$.

Let $f(x) = \sin^3(x)$

$\qquad = (\sin x)^3$

Let $u(h) = h^3$, $h(x) = \sin x$.

Derivative of u relative to h,

$\qquad \dfrac{d}{dh} h^3 = 3h^2$

Derivative of h relative to x,

$\qquad \dfrac{d}{dx} \sin x = \cos x$.

Derivative of u relative to x,

$$\frac{du}{dx} = \frac{d}{dx} f(x)$$

$$= \frac{du}{dh} \frac{dh}{dx}$$

$$= 3h^2 \cos x$$

$$= 3 \sin^2 x \cos x.$$

PROBLEM # 4.

Find the derivative of $\tan x$.

Let $f(x) = \tan x$,

$$f(x) = \frac{r \sin \theta}{r \cos \theta} \text{, where } \underline{r} \text{ is the hypothetical hyppotenuse.}$$

$$= \sin \theta \cos^{-1} \theta$$

Derivative of $f(x)$ relative to x,

Let $u = \sin \theta$, $g(\theta) = \cos^{-1} \theta$

$$\frac{d}{d\theta} f(\theta) = u(\theta) \frac{d}{d\theta} \cos^{-1} \theta + \cos^{-1} \theta \frac{du}{d\theta}$$

$$= \sin \theta \frac{-1}{\cos^2 \theta} (-\sin \theta) + \cos^{-1} \theta \cos \theta$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos \theta}{\cos \theta}$$

$$= 1 + \tan^2 \theta.$$

PROBLEM #5.

Derive the function $f(x) = e^{\sin(x^2)}$

Let $u(h) = e^h$, $h(v) = \sin v$, $v(x) = x^2$

$$\frac{d}{dx} u(h(v(x))) = \frac{du}{dg}\frac{dg}{dv} \text{ , where } u(g) = u(h(g))$$

$$= \frac{du}{dg}\frac{dg}{dg} \cdot \frac{dg}{dv} \text{ , where } \frac{du}{dg} = \frac{du}{dh}\frac{dh}{dg}$$

Therefore $\frac{du}{dx} = \frac{du}{dh}\frac{dh}{dg}\frac{dg}{dx}$, when $u(x) = u(h(g(x)))$.

□

The derivative of the original $f(x)$,

$$\frac{d}{dx} f(x) = \frac{du}{dh}\frac{dh}{dv}\frac{dv}{dx}$$

$$= e^h \cos v \, 2x$$

$$= e^{\sin v} \cos(x^2) 2x$$

$$= e^{\sin(x^2)} \cos(x^2) 2x.$$