PROBLEM #1

Use the Maclaurin series to determine if $f(x) = \tan^{-1}(x)$ is even, odd or neither.

Suppose $\tan^{-1}(x)$ can be modelled with the Maclaurin series,

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$f^{(1)}(x) = \frac{d}{dx}(\tan^{-1}x)$$

Proof of $\frac{d}{dx}\arctan(x)$,

$$y = \arctan x$$

$$\tan y = x$$

$$\frac{d}{dx}\tan y = \frac{d}{dx}x$$

$$\sec^2 y \frac{d}{dx}y = 1 \Leftrightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

Since $\sec^2 y = 1 + \tan^2 y$, since $y = \arctan x$, $\tan^2 y = x^2$,

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$= \frac{1}{1 + \tan^2 y}$$

$$= \frac{1}{1+x^2}.$$

$$g_1(x) = \frac{f'(0)}{1!}x^1 + g_0(x)$$

$$= \frac{x}{(1)} + \arctan(0)$$

$$g_2(x) = -2x^3 + x + 0$$

Based on $f^{(2)}(0)x^3$ the series is odd.

$$g(x) = \sum_{n=0}^{\infty} \frac{(n \cdot 2)!(-1)^n x^{(n \cdot 2+1)}}{(n \cdot 2+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{(n \cdot 2+1)}}{n \cdot 2+1}$$, powers always odd!

PROBLEM #2.

Given $f(x) = 2/(x^2 - x)$, which ranges can a power series be formed for?

There are asymptotes at the roots of $x^2 - x$.

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(0)}}{2(1)}$$

$$= \frac{1 \pm 1}{2}$$, roots at 0, 1.

Taylor series can be found between $[-\infty, 0), (0, 1), (1, \infty]$.

PROBLEM #3

For $f(x) = 2/(x^2 - x)$ which statements are true about it's power series representation.

- ☐ approx. properly captures asymptotes
- ☐ this is a well behaved function
- ☑ approx ignores segments of function
- ☐ approx converges quickly → sign flip in approximation
  ↳ not quick conv.
- ☑ approx ignores asymptotes.

PROBLEM #4

Given $f(x) = \frac{1}{(1+x^2)}$, performing taylor expansions for the first 3 terms around point $x=2$ are,

$$f(x) = \frac{1}{5} - \frac{4(x-2)}{25} + \frac{22(x-2)^2}{125} + \dots$$

Why does the expansion perform poorly @ $x=0$.

- ☑ asymptotes are in the complex plane.
- ☐ function does not differentiate well.
- ☐ the function has no ~~real~~ real roots
- ☐ none of these options.
- ☑ it is a discontinuous function in the complex plane.

quadratic form. $= \frac{-(0) \pm \sqrt{(0)^2 - 4(1)(1)}}{2(1)}$

$= \frac{0 \pm \sqrt{-4}}{2}$, roots @ $\sqrt{-4}/2 = \frac{\pm 4i}{2}$

Runge phenomenon. - any attempt to approx. with a polynomial fails b.c. poles in complex plane.

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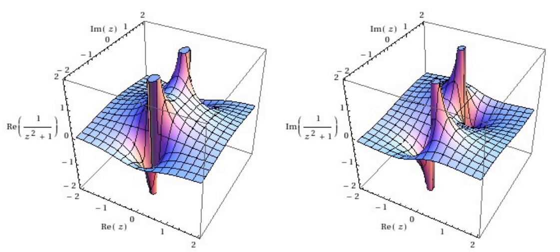
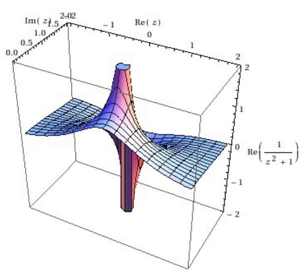


These are plots of the real and imaginary parts of the function $\frac{1}{z^2+1}$, where $z = x + iy$. The poles of the function at $z = \pm i$ are easily seen in the plots. Since we're limited to seeing (a two-dimensional projection of) three dimensions, we are forced here to illustrate the poles by plotting the real and imaginary parts of the function separately.

Now, look at a "slice" of the real part:



PROBLEM #5.

Given $f(x) = 1/x$, calculate the taylor expansions at $x = 4$ in the order of 2 degrees of accuracy.

$$f'(x) = -1/x^2$$

$$f''(x) = 2/x^3$$

$$g(x) = \sum_{n=0}^{\infty} \frac{\cancel{n!}\,(-1)^n x^{-(n+1)}}{\cancel{n!}} (x-4)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n (4)^{-(n+1)} (x-4)^n$$

$$g_1(x) = \frac{1}{4} - \frac{(x-4)}{4^2(1)!} + O(\Delta x^2)$$

$$= \frac{1}{4} - \frac{x-4}{16} + O(\Delta x^2)$$