
PROBLEM #1.

Compute length of vector x .

$$x = (1, -1, 3)^T$$

The length of x is the norm of x .

$$\begin{aligned}\|x\| &= \sqrt{x \cdot x} \\ &= \sqrt{x^T x} \\ &= \sqrt{\sum_i^n x_i^2}\end{aligned}$$

$$\begin{aligned}\|x\| &= \sqrt{(1)^2 + (-1)^2 + (3)^2} \\ &= \sqrt{11}\end{aligned}$$

PROBLEM #2

Compute the angle in radians between x and y vectors using the dot product.

The angle between x and y can be found using the cosine rule.

$$\begin{aligned}c^2 &= a^2 + b^2 - 2ab \cos\theta \Leftrightarrow r \cdot s = \|r\| \|s\| \cos\theta \\ \Leftrightarrow \cos\theta &= \frac{r \cdot s}{\|r\| \|s\|}\end{aligned}$$

$$x = (3, 4)^T \quad y = (-1, -1)^T$$

$$\begin{aligned}\cos\theta &= \frac{(3)(-1) + (4)(-1)}{\sqrt{3^2 + 4^2} \sqrt{(-1)^2 + (-1)^2}} \\ &= \frac{-7}{5\sqrt{2}}\end{aligned}$$

$$\theta = \cos^{-1} \left(-\frac{7}{5\sqrt{2}} \right)$$

$$= 2.9996956 \text{ rad}$$

PROBLEM #3

Compute the distance between x and y .

$$x = (3, 4)^T \quad y = (1, -1)^T$$

The distance between x and y is the norm of their difference

$$\text{diff between } x \text{ & } y = x - y$$

$$\begin{aligned} \|x - y\| &= \sqrt{(x-y)^T(x-y)} \\ &= \sqrt{(x-y) \cdot (x-y)} \\ &= \sqrt{\begin{bmatrix} 3-1 & 4-(-1) \end{bmatrix} \begin{bmatrix} 3-1 \\ 4-(-1) \end{bmatrix}} \end{aligned}$$

$$= \sqrt{(2)^2 + (5)^2}$$

$$= \sqrt{29}$$

$$= 5.38516480713$$

PROBLEM #4

Write code to compute the length of a vector.

```
def length(x):
    return np.linalg.norm(x)
```

PROBLEM #5

Calculate the angle between x and $(x-y)$ - in rad.

$$x = (1, 2, 3)^T \quad y = (-1, 0, 8)^T$$

Can use the cosine rule to calculate the angle between two intersecting lines.

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

$$\begin{aligned} r \cdot s &= \|r\| \|s\| \cos\theta \iff \cos\theta = \frac{r \cdot s}{\|r\| \|s\|} \\ &\iff \theta = \cos^{-1}\left(\frac{r \cdot s}{\|r\| \|s\|}\right) \quad (1) \end{aligned}$$

Can use x or y in the first line of the intersection, the second line by requirement of the question is $(x-y)$.

Let line #1 = x .

Let line #2 = $(x-y)$.

$$x-y = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - (-1) \\ 2 - 0 \\ 3 - 8 \end{bmatrix}$$

$$= (2, 2, -5)^T$$

Thus the θ between x and $(x-y)$ is

$$\theta = \cos^{-1} \left(\frac{(1, 2, 3)^T \cdot (2, 2, -5)^T}{\|(1, 2, 3)^T\| \|(2, 2, -5)^T\|} \right)$$

$$= \cos^{-1} \left(\frac{1(2) + 2(2) + 3(-5)}{\sqrt{1^2 + 2^2 + 3^2} \sqrt{1+4+25}} \right)$$

$$= \cos^{-1} \left(\frac{-10}{\sqrt{14} \sqrt{33}} \right)$$

$$= 2.05470443$$