
PROBLEM #1

Calc. Jacobian of $f(x,y) = x^2y + \frac{3}{4}xy + 10$,

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (x^2y + \frac{3}{4}xy + 10) \\&= y2x + x\frac{\partial}{\partial x}(\frac{3}{4}y) + \frac{\partial}{\partial x}(10) + \frac{3}{4}y\frac{\partial}{\partial x}x \\&= y2x + x(0) + (0) + \frac{3}{4}y(1) \\&= y2x + \frac{3y}{4}\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (x^2y + \frac{3}{4}xy + 10) \\&= y\frac{\partial}{\partial y}x^2 + x^2\frac{\partial}{\partial y}y + y\frac{\partial}{\partial y}(\frac{3}{4}x) + \frac{3}{4}x\frac{\partial}{\partial y}(y) \\&= y(0) + x^2(1) + y(0) + \frac{3}{4}x(1) \\&= x^2 + \frac{3x}{4}\end{aligned}$$

$$\begin{aligned}J &= \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \\&= \left[y2x + \frac{3y}{4}, x^2 + \frac{3x}{4} \right]\end{aligned}$$

PROBLEM #2.

Calc Jacobian row vector for $f(x,y) = e^x \cos(y) + xe^{3y} - 2$

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (e^x \cos(y) + xe^{3y} - 2) \\&= e^x \frac{\partial}{\partial x} \cos(y) + \cos(y) \frac{\partial}{\partial x} e^x + x \frac{\partial}{\partial x} e^{3y} + e^{3y} \frac{\partial}{\partial x} x + 0 \\&= e^x(0) + \cos(y)e^x(1) + x(0) + e^{3y}(1) \\&= \cos(y)e^x + e^{3y}\end{aligned}$$

$$\begin{aligned}
 \frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (e^x \cos(y) + x e^{3y} - 2) \\
 &= \cos(y) \frac{\partial}{\partial y} e^x + e^x \frac{\partial}{\partial y} \cos(y) + e^{3y} \frac{\partial}{\partial y} x + x \frac{\partial}{\partial y} e^{3y} + 0 \\
 &= \cos(y)(0) + e^x(-\sin(y)) + e^{3y}(0) + x e^{3y} \cdot 3 \\
 &= -\sin(y)e^x + 3x e^{3y}
 \end{aligned}$$

$$J = \begin{bmatrix} \cos(y)e^x + e^{3y}, & -\sin(y)e^x + 3x e^{3y} \end{bmatrix}$$

PROBLEM #3.

Calculate Jacobian row vector

$$f(x, y, z) = e^x \cos(y) + x^2 y^2 z^2$$

$$\begin{aligned}
 \frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (e^x \cos(y) + x^2 y^2 z^2) \\
 &= e^x \frac{\partial}{\partial x} \cos(y) + \cos(y) \frac{\partial}{\partial x} e^x + x^2 \frac{\partial}{\partial x} y^2 z^2 + y^2 z^2 \frac{\partial}{\partial x} x^2 \\
 &= \cos(y)e^x + y^2 z^2 \cdot 2x
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (e^x \cos(y) + x^2 y^2 z^2) \\
 &= \cos(y) \frac{\partial}{\partial y} e^x + e^x \frac{\partial}{\partial y} \cos(y) + y^2 \frac{\partial}{\partial y} x^2 z^2 + x^2 z^2 \frac{\partial}{\partial y} y^2 \\
 &= e^x(-\sin(y)) + x^2 z^2 \cdot 2y
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} (e^x \cos(y) + x^2 y^2 z^2) \\
 &= \frac{\partial}{\partial z} (z^2) \cdot x^2 y^2 \\
 &= x^2 y^2 \cdot 2z
 \end{aligned}$$

$$J = [\cos(y)e^x + y^2 z^2 2x, e^x(-\sin y) + x^2 z^2 2y, x^2 y^2 2z]$$

PROBLEM # 4.

Calculate Jacob. row vec and eval at $\vec{0}$.

$$f(x, y, z) = x^2 + 3e^y e^z + \cos x \sin z$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} x^2 + \frac{\partial}{\partial x} (3e^y e^z) + \frac{\partial}{\partial x} (\cos x) \cdot \sin z \\ &= 2x - \sin x \cdot \sin z \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (x^2 + 3e^y e^z + \cos x \sin z) \\ &= e^z \frac{\partial}{\partial y} e^y + 3e^y \frac{\partial}{\partial y} e^z + \frac{\partial}{\partial y} (\cos x \sin z) \\ &= 3e^z e^y \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} (x^2 + 3e^y e^z + \cos x \sin z) \\ &= \frac{\partial}{\partial z} (x^2) + 3e^y \frac{\partial}{\partial z} e^z + \cos x \frac{\partial}{\partial z} \sin z \\ &= 3e^y e^z + \cos x \cos z \end{aligned}$$

$$J = [2x - \sin x \cdot \sin z, 3e^z e^y, 3e^y e^z + \cos x \cos z]$$

Eval @ $u = \vec{0}$:

$$Ju = [0, 3, 1]$$

PROBLEM #5

(calc Jacobian row vector and eval at $u = 0$.

$$f(x, y, z) = xe^y \cos z + 5x^2 \sin(y) e^z$$

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (xe^y \cos z + 5x^2 \sin(y) e^z) \\&= e^y \cos z \frac{\partial}{\partial x} x + 5 \sin(y) e^z \frac{\partial}{\partial x} x^2 \\&= e^y \cos z (1) + 5 \sin(y) e^z 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (xe^y \cos z + 5x^2 \sin(y) e^z) \\&= x \cos z \frac{\partial}{\partial y} e^y + 5x^2 e^z \frac{\partial}{\partial y} \sin y \\&= x \cos(z) e^y + 5x^2 e^z \cos y\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} (xe^y \cos z + 5x^2 \sin(y) e^z) \\&= xe^y \frac{\partial}{\partial z} \cos z + 5x^2 \sin(y) \frac{\partial}{\partial z} e^z \\&= xe^y (-\sin z) + 5x^2 \sin y e^z\end{aligned}$$

$$\begin{aligned}J = \left[\begin{array}{c} e^y \cos z (1) + 5 \sin(y) e^z 2x, \\ x \cos(z) e^y + 5x^2 e^z \cos y, \\ xe^y (-\sin z) + 5x^2 \sin y e^z \end{array} \right]\end{aligned}$$

Eval J u @ u = $\vec{0}$,

$$Ju = [1, 0, 0]$$