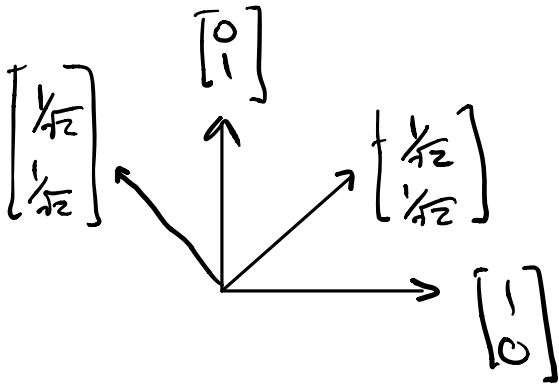Doing transformations f-basis using e basis.

To rotate a vector u_f in the f-basis,

$$= B^{-1} R B u_f,$$ where B is the matrix of f's basis vectors in the frame of e.

R is a rotation in e.

B^{-1} is the matrix of e's basis vectors in the frame of f.

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \quad \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Example:

$$\underbrace{\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}}_{B^{-1}} \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{u_f} = u_f \text{ rotated.}$$

$$B^{-1}RB = \frac{1}{2\sqrt{2}}\begin{bmatrix} -2 & -2 \\ 10 & 6 \end{bmatrix}$$