

PROBLEM #1

Calc. Jacobian of $f(x,y) = x^2y + \frac{3}{4}xy + 10$.

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (x^2y + \frac{3}{4}xy + 10) \\ &= y2x + x \frac{\partial}{\partial x} (\frac{3}{4}y) + \frac{\partial}{\partial x} (10) + \frac{3}{4}y \frac{\partial}{\partial x} x \\ &= y2x + x(0) + (0) + \frac{3}{4}y(1) \\ &= y2x + \frac{3y}{4}\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (x^2y + \frac{3}{4}xy + 10) \\ &= y \frac{\partial}{\partial y} x^2 + x^2 \frac{\partial}{\partial y} y + y \frac{\partial}{\partial y} (\frac{3}{4}x) + \frac{3}{4}x \frac{\partial}{\partial y} (y) \\ &= y(0) + x^2(1) + y(0) + \frac{3}{4}x(1) \\ &= x^2 + \frac{3x}{4}\end{aligned}$$

$$\begin{aligned}J &= \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \\ &= \left[y2x + \frac{3y}{4}, x^2 + \frac{3x}{4} \right]\end{aligned}$$

PROBLEM #2.

Calc Jacobian row vector for $f(x,y) = e^x \cos(y) + xe^{3y} - 2$

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (e^x \cos(y) + xe^{3y} - 2) \\ &= e^x \frac{\partial}{\partial x} \cos(y) + \cos(y) \frac{\partial}{\partial x} e^x + x \frac{\partial}{\partial x} e^{3y} + e^{3y} \frac{\partial}{\partial x} x + 0 \\ &= e^x(0) + \cos(y)e^x(1) + x(0) + e^{3y}(1) \\ &= \cos(y)e^x + e^{3y}\end{aligned}$$

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (e^x \cos(y) + x e^{3y} - 2) \\
&= \cos y \frac{\partial}{\partial y} e^x + e^x \frac{\partial}{\partial y} \cos y + e^{3y} \frac{\partial}{\partial y} x + x \frac{\partial}{\partial y} e^{3y} + 0 \\
&= \cos y(0) + e^x(-\sin y) + e^{3y}(0) + x e^{3y} 3 \\
&= -\sin y e^x + 3x e^{3y}
\end{aligned}$$

$$J = [\cos y e^x + e^{3y}, -\sin y e^x + 3x e^{3y}]$$

PROBLEM #13.

Calculate Jacobian row vector

$$f(x, y, z) = e^x \cos y + x^2 y^2 z^2$$

$$\begin{aligned}
\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (e^x \cos y + x^2 y^2 z^2) \\
&= e^x \frac{\partial}{\partial x} \cos y + \cos y \frac{\partial}{\partial x} e^x + x^2 \frac{\partial}{\partial x} y^2 z^2 + y^2 z^2 \frac{\partial}{\partial x} x^2 \\
&= \cos y e^x + y^2 z^2 2x
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (e^x \cos y + x^2 y^2 z^2) \\
&= \cos y \frac{\partial}{\partial y} e^x + e^x \frac{\partial}{\partial y} \cos y + y^2 \frac{\partial}{\partial y} x^2 z^2 + x^2 z^2 \frac{\partial}{\partial y} y^2 \\
&= e^x(-\sin y) + x^2 z^2 2y
\end{aligned}$$

$$\begin{aligned}
\frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} (e^x \cos y + x^2 y^2 z^2) \\
&= \frac{\partial}{\partial z} (z^2) \cdot x^2 y^2 \\
&= x^2 y^2 2z.
\end{aligned}$$

$$J = [\cos(y)e^x + y^2z^22x, e^x(-\sin y) + x^2z^22y, x^2y^22z]$$

PROBLEM # 4.

Calculate Jacob. row vec and eval at $\vec{0}$.

$$f(x, y, z) = x^2 + 3e^y e^z + \cos x \sin z$$

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} x^2 + \frac{\partial}{\partial x} (3e^y e^z) + \frac{\partial}{\partial x} (\cos x) \cdot \sin z \\ &= 2x - \sin x \cdot \sin z\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (x^2 + 3e^y e^z + \cos x \sin z) \\ &= e^z \frac{\partial}{\partial y} e^y + 3e^y \frac{\partial}{\partial y} e^z + \frac{\partial}{\partial y} (\cos x \sin z) \\ &= 3e^z e^y\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} (x^2 + 3e^y e^z + \cos x \sin z) \\ &= \frac{\partial}{\partial z} (x^2) + 3e^y \frac{\partial}{\partial z} e^z + \cos x \frac{\partial}{\partial z} \sin z \\ &= 3e^y e^z + \cos x \cos z\end{aligned}$$

$$J = [2x - \sin x \cdot \sin z, 3e^z e^y, 3e^y e^z + \cos x \cos z]$$

Eval @ $u = \vec{0}$:

$$J_u = [0, 3, 4]$$

PROBLEM #5

(calc Jacobian row vector and eval at $u = 0$.

$$f(x, y, z) = x e^y \cos z + 5x^2 \sin y e^z$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (x e^y \cos z + 5x^2 \sin(y) e^z)$$

$$= e^y \cos z \frac{\partial}{\partial x} x + 5 \sin(y) e^z \frac{\partial}{\partial x} x^2$$

$$= e^y \cos z (1) + 5 \sin(y) e^z 2x$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} (x e^y \cos z + 5x^2 \sin(y) e^z)$$

$$= x \cos z \frac{\partial}{\partial y} e^y + 5x^2 e^z \frac{\partial}{\partial y} \sin y$$

$$= x \cos(z) e^y + 5x^2 e^z \cos y$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} (x e^y \cos z + 5x^2 \sin(y) e^z)$$

$$= x e^y \frac{\partial}{\partial z} \cos z + 5x^2 \sin(y) \frac{\partial}{\partial z} e^z$$

$$= x e^y (-\sin z) + 5x^2 \sin y e^z$$

$$J = \begin{bmatrix} e^y \cos z (1) + 5 \sin(y) e^z 2x, \\ x \cos(z) e^y + 5x^2 e^z \cos y, \\ x e^y (-\sin z) + 5x^2 \sin y e^z \end{bmatrix}$$

Eval J_u @ $u = \vec{0}$,

$$J_u = [1, 0, 0]$$