PROBLEM #1

Given the transformation matrix T and the eigenbasis matrix C – which are eigenvectors of T, calculate $D=C^{-1}TC$.

$$T=\begin{bmatrix}6 & -1\\ 2 & 3\end{bmatrix} \quad C=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$$

$$D=C^{-1}TC$$

where C^{-1} is

$$C^{-1}=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}, \text{ where } C=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$=\begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}6 & -1\\ 2 & 3\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}5 & 0\\ 0 & 4\end{bmatrix}$$

PROBLEM #2.

Given the transformation matrix T and the eigenbasis matrix C – which are eigenvectors of T, calculate $D=C^{-1}TC$.

$$T=\begin{bmatrix}2 & 7\\ 0 & -1\end{bmatrix} \quad C=\begin{bmatrix}7 & 1\\ -3 & 0\end{bmatrix}$$

$$D=C^{-1}TC$$

where C^{-1} is

$$C^{-1}=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}, \text{ where } C=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1\\ 3 & 7\end{bmatrix} \frac{1}{(0)(7)-(-1)(3)}$$

$$= \begin{bmatrix} 0 & -1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 \\ 0 & 6 \end{bmatrix} \times \frac{1}{3}$$

PROBLEM #3.

Given the transformation matrix T and the eigenbasis matrix C — which are eigenvectors of T, calculate $D = C^{-1}TC$.

$$T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$D = C^{-1}TC$

where C^{-1} is

$$C^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ where } C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \frac{1}{(1)(1) - (0)(-1)}$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

PROBLEM #4.

Given a diagonal matrix $D = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$, and a change of basis matrix $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ with inverse, $C^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$, calculate $T = CDC^{-1}$!

$$T=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}a&0\\0&a\end{bmatrix}\begin{bmatrix}1&-2\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}a&2a\\0&a\end{bmatrix}\begin{bmatrix}1&-2\\0&1\end{bmatrix}$$

$$=\begin{bmatrix}a&0\\0&a\end{bmatrix}.$$

PROBLEM #5.

Given $T=\begin{bmatrix}6&-1\\2&3\end{bmatrix}=\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}5&0\\0&4\end{bmatrix}\begin{bmatrix}2&-1\\-1&1\end{bmatrix}$, calculate T^3.

$$T^3=\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}5&0\\0&4\end{bmatrix}^3\begin{bmatrix}2&-1\\-1&1\end{bmatrix}$$

$$=\begin{bmatrix}1&1\\1&2\end{bmatrix}\begin{bmatrix}125&0\\0&64\end{bmatrix}\begin{bmatrix}2&-1\\-1&1\end{bmatrix}$$

$$=\begin{bmatrix}125&64\\125&128\end{bmatrix}\begin{bmatrix}2&-1\\-1&1\end{bmatrix}$$

$$=\begin{bmatrix}186&-61\\122&3\end{bmatrix}.$$

PROBLEM #6.

Given that $T=\begin{bmatrix}2&7\\0&-1\end{bmatrix}=\begin{bmatrix}7&1\\-3&0\end{bmatrix}\begin{bmatrix}-1&0\\0&2\end{bmatrix}\begin{bmatrix}0&-1/3\\1&7/3\end{bmatrix}$, calculate T^3.

$$T^3=\begin{bmatrix}7&1\\-3&0\end{bmatrix}\begin{bmatrix}-1&0\\0&2\end{bmatrix}^3\begin{bmatrix}0&-1/3\\1&7/3\end{bmatrix}$$

$$=\begin{bmatrix}7&1\\-3&0\end{bmatrix}\begin{bmatrix}-1&0\\0&8\end{bmatrix}\begin{bmatrix}0&-1/3\\1&7/3\end{bmatrix}$$

$$=\begin{bmatrix}-7&8\\3&0\end{bmatrix}\begin{bmatrix}0&-1/3\\1&7/3\end{bmatrix}$$

$$=\begin{bmatrix}8&63/3\\0&-1\end{bmatrix}=\begin{bmatrix}8&21\\0&-1\end{bmatrix}$$

PROBLEM # 7.

Given $T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, calculate T^5.

$$T^5 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^5\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$