
PROBLEM #1.

Construct a series of simultaneous equations to represent these variables and their historical values.

Imagine,

On Monday, you bought for €15.

1 apple
1 banana
1 carrot

On Tuesday, you bought for €28.

3 apples
2 bananas
1 carrot

On Wednesday, you bought for €23.

2 apples
1 banana
2 carrots.

Solution: create columns for each lecture and tabulate their quantities-express the combination of those lectures as having a total price.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

PROBLEM #2.

Manipulate the expression $Mx=b$ into RREF.

$$M = \begin{bmatrix} 4 & 6 & 2 \\ 3 & 4 & 1 \\ 2 & 8 & 15 \end{bmatrix} \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ 7 \\ 2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 2 & 1 & | & 2 \\ 3 & 2 & 2 & | & 20 \\ 0 & 4 & 11 & | & -16 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 2 & 1 & | & 2 \\ 0 & 1 & 1 & | & 10 \\ 0 & 0 & 7 & | & -56 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 2 & 1 & | & 2 \\ 0 & 1 & 0 & | & 10 \\ 0 & 0 & 1 & | & -8 \end{bmatrix}$$

PROBLEM #3 . b #4.

Same answer as problem #2.

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 8 & 13 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ -\frac{1}{2} \\ 2 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 5 & 12 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ -\frac{1}{2} \\ -\frac{5}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ -\frac{1}{2} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ -\frac{1}{2} \\ 0 \end{bmatrix}$$

PROBLEM #5

Convert this simultaneous equation to echelon form.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & -1 & 0 \end{bmatrix} x = \begin{bmatrix} 15 \\ -17 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 15 \\ 17 \\ 5 \end{bmatrix}$$

PROBLEM #6.

Solve these simultaneous equations w- back substitution.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 15 \\ 17 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 3 \\ 7 \\ 5 \end{bmatrix}$$

PROBLEM #7.

Find the inverse of this matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{array}{c|ccc} 1 & 1 & 1 & 100 \\ 0 & 1 & 2 & -310 \\ 2 & 1 & 2 & 001 \end{array}$$

$$= \begin{array}{c|ccc} 1 & 1 & 1 & 100 \\ 1 & 0 & 0 & -3/2 y_2 y_2 \\ 0 & 1 & 2 & 3 - 10 \end{array}$$

$$= \begin{array}{c|ccc} 1 & 0 & 0 & -3/2 y_2 y_2 \\ 0 & 1 & 1 & 5/2 y_2 - y_2 \\ 0 & 1 & 2 & 3 - 10 \end{array}$$

$$= \begin{array}{c|ccc} 1 & 0 & 0 & -3/2 y_2 y_2 \\ 0 & 1 & 1 & 5/2 y_2 - y_2 \\ 0 & 0 & 1 & y_2 - y_2 y_2 \end{array}$$

$$= \begin{array}{c|ccc} 1 & 0 & 0 & -3/2 y_2 y_2 \\ 0 & 1 & 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & y_2 & -y_2 & y_2 \end{array}$$

