The power of Shapley Values are the properties which it guarantees. When you are looking to disseminate the proceeds of a collective effort to the members of the collective - and you are constrained in the way you split up the profit. if your constraints are an improper subset of the shapley properties - then Shapley Values is the only possible method of profit assignment which can meet those constraints across all games.

Why would I want to use Shapley Values
(PROPERTIES GUARNTEED BY SHAPLEY VALUES)

- ☐ Efficiency: the sum of the attributed profits is the value of having all members of the coalition work together.

  $\sum_{i \in N} \varphi_i(v) = v(N)$, where N is the grand coalition.

- ☐ Symmetry: members which contribute the same value will recieve the same payout.

  $\forall_{S \subseteq N;\, S \not\ni i} \; v(S \cup \{i\}) = v(S \cup \{j\}) \Rightarrow \varphi_i(v) = \varphi_j(v)$.

☐ Linearity: the distribution of profits of one game (by Shapley Values) have the intrinsic property of being normalized with the distributions of other games. Games are comparable to one-another in an apples-to-apples kind of way. This allows you to combine or normalize games.

Additivity Property.

$$\varphi_i(v + w) = \varphi_i(v) + \varphi_i(w),$$ v and w are characteristic functions.

Homogeneity Property.

$$\varphi_i(\alpha v) = \varphi_i(v) \cdot \alpha$$

☐ Null Player: members which do not contribute any value recieve zero payout.

$$\forall (S \subseteq N \land S \not\ni i) : v(S \cup \{i\}) = v(S)$$

☐ Anonymity: the payout of players of a game is solely derived from their role and value in the game; it is not influenced by the happenstance in the way the game is described.

If $\underline{v}$ and $\underline{w}$ are characteristic functions that have exchanged the roles of $\underline{i}$ and $\underline{j}$ the $\varphi_i(v) = \varphi_j(w)$.

☐ Marginalism: the payout distribution can be calculated from only the marginal contributions of a player to coalition.

☐ Stand-Alone Test: the distribution of profits will increase as working together becomes more useful — and diminishes as cooperation loses value.

Additive Set Function

$$v(S \cup T) = v(S) + v(T)$$

Superadditive Set Function

$$v(S \cup T) \geq v(S) + v(T)$$

Subadditive Set Function

$$v(S \cup T) \leq v(S) + v(T).$$

WHAT ARE SHAPLEY VALUES

TODO:

- describe algebraic structure of the characteristic function
    - additivity, homogeneity of 1.
    - axioms
- homomorphism of Shapley ϕ_i
    - Linear Map, · proof of single solution to a Linear Map homomorph.