

Matrix multiplication is one way of forming compositions or combinations of matrix transformations.

These ops are associative but do not possess commutativity.

$$(A_1 A_2) A_3 x = y$$

$$A_1 (A_2 A_3) x = y$$

$$A_1 A_2 A_3 x = y$$

$A_3 A_1 A_2 x \neq y$, not commutative

Matrix multiplications are sequential applications of algebra which is robust in additivity and scalar multiplication.

Example. $A_2 A_1 x = y$.

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\xrightarrow{\text{A}_1}$ Transformation to \hat{e}_1 ,

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$\xrightarrow{\text{A}_2}$ Transformation to $[x \ y]^T$,

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

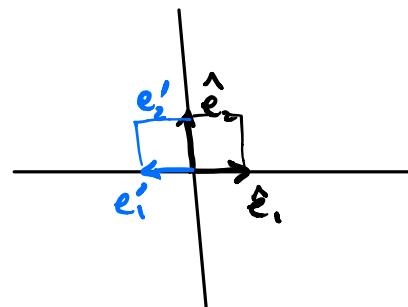
$\xrightarrow{\text{A}_1}$ Transformation to \hat{e}_2 ,

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$\xrightarrow{\text{A}_2}$ is a mirror over y-axis.



$\xrightarrow{\text{A}_1}$ is a clockwise rotation



Commutativity check,

$A_1 A_2 x \rightarrow$ mirror over y, then rot-clock

$A_2 A_1 x \rightarrow$ rot-clock, mirror over y.

COMMUTIVITY FAILS

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A_1 A_2 x \rightarrow \cancel{x}$$

$$A_2 A_1 x \rightarrow \cancel{x}$$