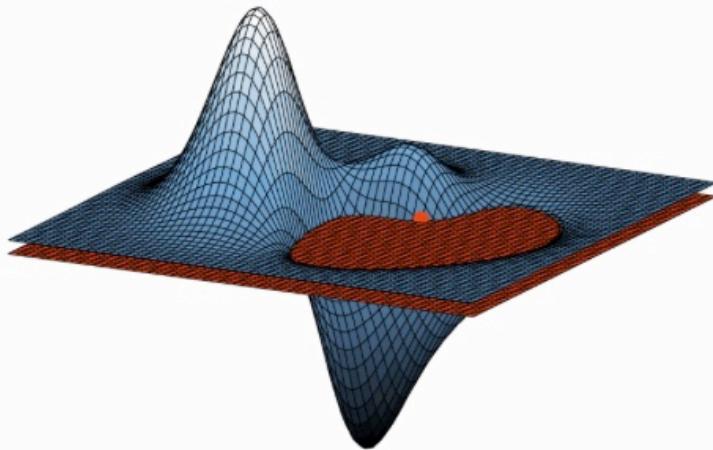

PROBLEM #1.

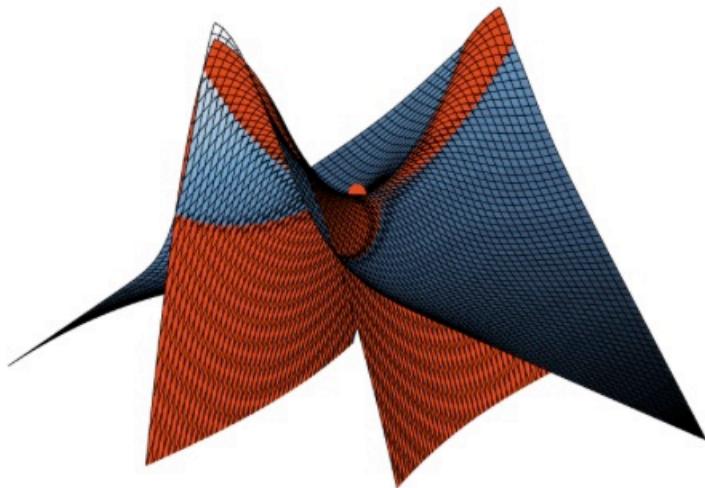
What order is this Taylor series approx. (in red)?



This is a zeroth order approximation where z is a constant value.

PROBLEM #2

What order Taylor series expansion is this?



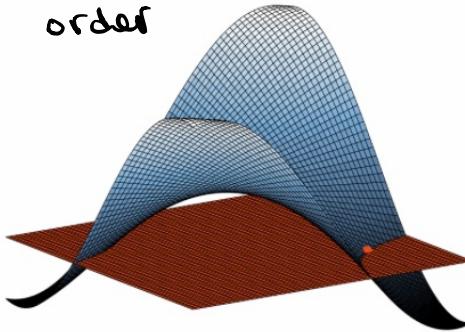
2nd Order - parabolic shape.

PROBLEM #3

which of the following are first order Taylor Series approx
blue is original func.

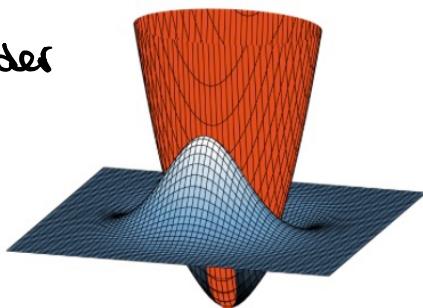
$$f(x, y) = \sin(xy/5)$$

zeroth
order



$$f(x, y) = xe^{-x^2-y^2}$$

2nd
order



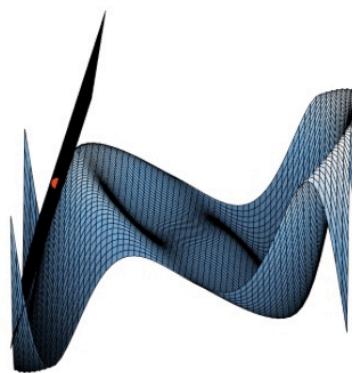
$$f(x, y) = (x^2 + 2x)e^{-x^2-y^2/5}$$

(not an approx.)



$$f(x, y) = x \sin(x^2/2 + y^2/4)$$

(first order)



PROBLEM #4.

What is the first order Taylor Expansion of

$$f(x, y) = xy^2 e^{-x^4-y^2/2} \text{ around the point } (-1, 2).$$

The second order expansion can be written.

$$f(x + \Delta x) = f(x) + \nabla f \Delta x + \frac{1}{2} \Delta x^T H f \Delta x + \dots$$

$$\begin{aligned} \nabla f &= \left[y^2 e^{-x^4-y^2/2} + xy^2 e^{-x^4-y^2/2} (-4x^3), \right. \\ &\quad \left. 2yx e^{-x^4-y^2/2} + xy^2 e^{-x^4-y^2/2} (-y) \right] \end{aligned}$$

$$= \left[4e^{-1-2} + (-1)(4)e^{-1-2}(4), 2(2)(-1)e^{-1-2} + (-1)(4)e^{-1-2}(-2) \right]$$

$$= [4e^{-3} - 4e^{-3}(4), -4e^{-3} + 8e^{-3}]$$

$$= [-12e^{-3}, 4e^{-3}]$$

$$\begin{aligned}
 g_1(x) &= f(x) + J_f \Delta x \\
 &= (-1)(2)^2 e^{-3} + J_f \Delta x \\
 &= -4e^{-3} + J_f \Delta x \\
 &= -4e^{-3} - 12e^{-3} \Delta x + 4e^{-3} \Delta y
 \end{aligned}$$

PROBLEM #5

What is the second order Hessian of the Taylor expansion of $f(x, y) = \sin(\pi x - x^2 y)$ around point $(1, \pi)$.

$$J_f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$$

$$J_f = \left[\cos(\pi x - x^2 y)(\pi - 2xy), \cos(\pi x - x^2 y)(-x^2) \right]$$

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} \partial_{xx} f & \partial_{xx} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f \end{bmatrix}$$

$$\partial_{xx} f = [-\sin(\pi x - x^2 y)(\pi - 2xy)^2 + \cos(\pi x - x^2 y)(-2y)]$$

Should be same Hessian usually symmetric.

$$\begin{cases} \partial_{x_1 x_2} f = [-\sin(\pi x - x^2 y)(-x^2)(\pi - 2xy) + \cos(\pi x - x^2 y)(-2x)] \\ \partial_{x_2 x_1} f = [-\sin(\pi x - x^2 y)(\pi - 2xy)(-x^2) + (-2x)\cos(\pi x - x^2 y)] \end{cases}$$

$$\partial_{x_2 x_2} f = [-\sin(\pi x - x^2 y)(-x^2)^2]$$

$$H_f = \begin{bmatrix} -\sin(\pi x - x^2 y)(\pi - 2xy)^2 + \cos(\pi x - x^2 y)(-2y), \\ -\sin(\pi x - x^2 y)(-x^2)(\pi - 2xy) + \cos(\pi x - x^2 y)(-2x) \\ -\sin(\pi x - x^2 y)(\pi - 2xy)(-x^2) + (-2x)\cos(\pi x - x^2 y), \\ -\sin(\pi x - x^2 y)(-x^2)^2 \end{bmatrix}$$

@(x,y) = (1,π):

$$H_f = \begin{bmatrix} -\sin(\pi - \pi)(\pi - 2\pi)^2 + \cos(\pi - \pi)(-2\pi), -\sin(\pi - \pi)(-1)(\pi - 2\pi) + \cos(0)(-2) \\ -\sin(0)(\pi - 2\pi)(-1) + (-2)\cos(\pi - \pi), -\sin(\pi - \pi)(1) \end{bmatrix}$$

$$= \begin{bmatrix} -2\pi & -2 \\ -2 & 0 \end{bmatrix}$$