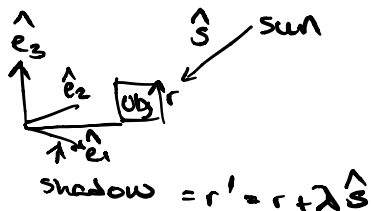


PROBLEM #1.



Get r' in terms of r .

$$\text{eq 1. } r' = r + \lambda \hat{S}$$

$$\text{eq 2. } r'_3 = 0, \text{ since } r' \cdot \hat{e}_3 = 0$$

$$\text{eq 3. } 0 = r \cdot \hat{e}_3 + \lambda (\hat{S} \cdot \hat{e}_3)$$

① Get lambda in terms of

$r \rightarrow$ that is

$$r \cdot \hat{e}_3 = (-1) \text{proj } r \hat{e}_3$$

$$0 = r \cdot \hat{e}_3 + \lambda (\hat{S} \cdot \hat{e}_3)$$

$$\lambda = \frac{-r \cdot \hat{e}_3}{\hat{S} \cdot \hat{e}_3}, \text{ only makes sense}$$

because \hat{e}_3 is unit orthogonal vector.

\rightarrow basically how many vertical comps of \hat{e}_3 needed to match vert of r .

$$\text{② } r' = r - \hat{S} \left(\frac{r \cdot \hat{e}_3}{\hat{S} \cdot \hat{e}_3} \right)$$

□.

PROBLEM #2.

Write $r' = r - \hat{S} \left(\frac{r \cdot \hat{e}_3}{\hat{S} \cdot \hat{e}_3} \right)$ as a linear transformation of r .
(e.g. $Ar = r'$)

Also write einstein summation convention of $Ar = r'$.

As vectors:

$$r' = r - \hat{S} \frac{r_3}{S_3}$$

Einstein:

$$r'_i = r_i - \frac{S_i r_3}{S_3} \quad \rightarrow \quad r'_i = (I_{ij} - S_i S_j / S_3^2) r_j$$

As matrix:

$$\begin{bmatrix} r_1 - \frac{S_1 r_3}{S_3} & r_2 - \frac{S_2 r_3}{S_3} & r_3 - \frac{S_3 r_3}{S_3} \end{bmatrix}^T = \begin{bmatrix} r_1 - \frac{S_1 r_3}{S_3} \\ r_2 - \frac{S_2 r_3}{S_3} \\ 0 \end{bmatrix}$$

PROBLEM #3.

Given the einstein sum. conv. for the matrix

$$r'_i = (I_{ij} - s_i I_{sj} / s_3) r_j$$

Write the component form of A.

$$\begin{bmatrix} 1 & 0 & -s_i/s_3 \\ 0 & 1 & -s_i/s_3 \end{bmatrix}$$

PROBLEM #4.

Write the third row of A from Prob #2 if the matrix went from $\mathbb{R}^3 \rightarrow \mathbb{R}^3$:

$$\begin{aligned} A_3 &= [0 \quad 0 \quad (1 - s_i/s_3)] \\ &= [0 \quad 0 \quad (1 - 1)] \\ &= [0 \quad 0 \quad 0] \end{aligned}$$

PROBLEM #5

Calculate b from $Ax = b$.

$$A = \begin{bmatrix} 1 & 0 & -s_i/s_3 \\ 0 & 1 & -s_i/s_3 \end{bmatrix} \quad x = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} \quad s = \begin{bmatrix} 4/13 \\ -3/13 \\ -12/13 \end{bmatrix}$$

$$\begin{aligned} b &= \begin{bmatrix} (1)(6) + (0)(2) + (3)(-1)(\frac{4}{13}) \\ (0)(6) + (1)(2) + (3)(-1)(\frac{-3}{13}) \end{bmatrix} \\ &= \begin{bmatrix} 6 + 1 \\ 2 - 3/4 \end{bmatrix} = \begin{bmatrix} 7 \\ 5/4 \end{bmatrix} \end{aligned}$$

PROBLEM #6

A transformation $r' = Ar$ can be generalized to a matrix equation,

$$R' = AR$$

where R' and R are matrices where each col. corresponds to r' and r vectors.

$$\underbrace{\begin{bmatrix} r'_1 & s'_1 & t'_1 & u'_1 \\ r'_2 & s'_2 & t'_2 & u'_2 \dots \end{bmatrix}}_{R'} = A \begin{bmatrix} r_1 & s_1 & t_1 & u_1 \\ r_2 & s_2 & t_2 & u_2 \dots \\ r_3 & s_3 & t_3 & u_3 \end{bmatrix}$$

In Einstein's notation

$$r'_i = A_{ij}r_j \quad \text{becomes} \quad R'_{ia} = A_{ij}R_{ja}. \quad \text{Cool!}$$

For the same s as previous question, apply A to the matrix,

$$R = \begin{bmatrix} 5 & -1 & -3 & 7 \\ 4 & -4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{bmatrix} \quad s = \begin{bmatrix} 4/13 \\ -3/13 \\ -12/13 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & -s_1/s_3 \\ 0 & 1 & -s_2/s_3 \end{bmatrix}$$

$$R' = \begin{bmatrix} 1 & 0 & -s_1/s_3 \\ 0 & 1 & -s_2/s_3 \end{bmatrix} \begin{bmatrix} 5 & -1 & -3 & 7 \\ 4 & -4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{bmatrix}, \quad \text{using } s_i = \vec{s}_i$$

$$\text{Let } c_1 = -s_1/s_3 = (4/13)(-1)(13/-12) = 1/3, \quad c_2 = -s_2/s_3 = (-3/13)(-1)(13/12) = -1/4$$

$$\begin{aligned} R' &= \begin{bmatrix} (1)(5) + (9)c_1^3 & (-1) + 3c_1^1 & -3 + 0c_1^0 & 7 + 12c_1^4 \\ 4 + 9c_2^{-7/4} & -4 + 3c_2^{-3/4} & 1 + 0c_2^0 & -2 + 12c_1^{-3} \end{bmatrix} \\ &= \begin{bmatrix} 8 & 0 & -3 & 11 \\ 7/4 & -19/4 & 1 & -5 \end{bmatrix} \end{aligned}$$