

Functions of arbitrary complexity can be fitted - can find the parameters which minimize some fitness function.

### Notation

A function is parameterized.

$$y(x_i; \alpha_k) = (x - \alpha_1)^2 + \alpha_2, \text{ where } k=1 \dots m$$

for  $m$  possible parameters

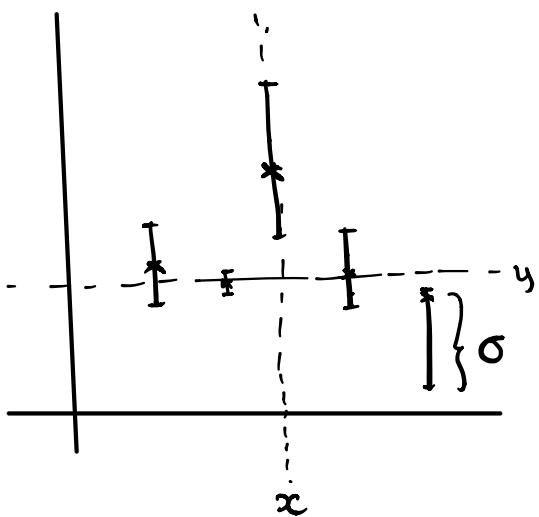
A datapoint can be written.

$(y_i, x_i, \sigma_i)$  - where  $\sigma_i$  quantifies the uncertainty

### Generalized Chi-Squared Test

$$\chi^2 = \sum_{i=1}^n \frac{[y_i - y(x_i; \alpha_k)]^2}{\sigma_i^2}, \text{ the magnitude of the residual is proportionate to the certainty } \sigma_i^2.$$

↳ placing less emphasis on less certain deviations.



Can drop the  $\sigma$  term for (1) if the certainty is equitable.

The extrema of the  $\chi^2$ -Chi-squared function can be found using the gradient where zero.

$$\nabla \chi^2 = 0$$

There are two ways to solve this gradient formula, algebraically or via gradient (steepest) descent.

Sometimes you can't solve algebraically.

To solve by steepest descent, can iteratively update the parameters to minimize the chi-squared test,

$\alpha_{\text{next}} = \alpha_{\text{cur}} - C \cdot \nabla \chi^2$ , where  $C$  is some constant controlling the magnitude of the step.

To get the gradient must differentiate  $\chi^2$ .

$$\begin{aligned} \frac{d\chi^2}{d\alpha_k} &= \sum_{i=1}^n -2 \left[ \frac{y_i - y(x_i; \alpha_k)}{\sigma^2} \right] \frac{dy}{dx}, \text{ the } (-2) \text{ can} \\ &= \nabla \chi^2 \text{ be moved into} \\ & \quad \text{the } C \text{ constant} \end{aligned}$$

Example: calculating the gradient for  $y(x_i; \alpha_k) = (x - \alpha_1)^2 + \alpha_2$ .

$$\frac{dy}{d\alpha_1} = -2(x - \alpha_1)$$

$$\frac{dy}{d\alpha_2} = 1.$$

These derivatives can then be used to form the gradient of the weighted chi-square test.

This is called "nonlinear least squares fitting", for fitting of a nonlinear function to nonlinear parameters.