PROBLEM #1

Is vector a, and b independent?

$$Xa = b$$

$$X \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$2 [1 \ 1]^T = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Therefor vector a is lenearly dependent on b.

PROBLEM #2

Are a and b independent?

$$a = [1 \ 1]^T, \quad b = [2 \ 1]^T$$

$a \cdot b = |a||b| \cos\theta$, if a and b are orthogonal their dot product will be zero.

$$a \cdot b = \frac{(1)(2) + (1)(1)}{\sqrt{2} \sqrt{5}}$$

Therefor a and be aren't orthogonal. They aren't dependent either.

$$\forall_c \text{ in } \mathbb{R}: c \cdot a \neq b.$$

PROBLEM #3

$$a = [2 \ 2]^T, \quad b = [1 \ -2], \quad c = [-1 \ 0]$$

What is q_1, q_2 in $a = q_1 b + q_2 c$

$$q_1 = -1$$
$$q_2 = -3$$

PROBLEM #4.

Are a, b, c linearly independent?

$a = [1\ 0\ 0]^T$, $b = [1\ 1\ 0]^T$, $c = [1\ 0\ 1]^T$

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$, since a, b, c form a matrix M in RREF. their vectors are independent.

PROBLEM #5

Are $[1 0 1]^T$, $[2\ -1\ 1]^T$ and $[-3\ 1\ -2]^T$ independent?

$$M = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix} = \begin{matrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{matrix}$$, adding $-1 * \text{Row}1$.

$$= \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$, Row #2 and #3 are dependent.

PROBLEM #6.

The vectors a, b, c can be used as a basis for $\mathbb{R}^3$? Why?

- ☑ vectors are linearly independent
- ☐ vectors are not linearly independent
- ☐ vectors do not span $\mathbb{R}^3$.
- ☐ there are too many vectors for the basis of $\mathbb{R}^3$.

If $a = [1\ 2\ 0]^T$, $b = [-2\ 1\ 3]^T$ and $c = [4\ 3\ -3]^T$, vectors would be linearly dependent and could not span $\mathbb{R}^3$.