

Matrices can manipulate the basis objects in several different ways,

- rotations
- shears
- inversions
- mirrors
- stretches

There is a special matrix which does not manipulate an applied vector,

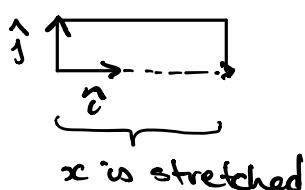
The identity matrix, I .

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}, \text{ this generalizes to any dimension.}$$



STRETCH

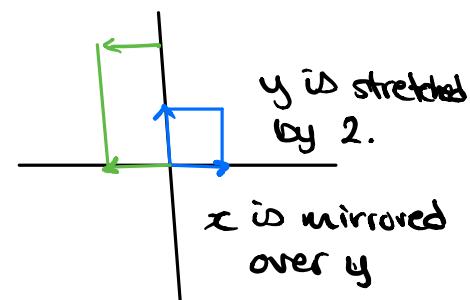
$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3x \\ y \end{bmatrix}$$



x is stretched

MIRRORING

$$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ 2y \end{bmatrix}$$



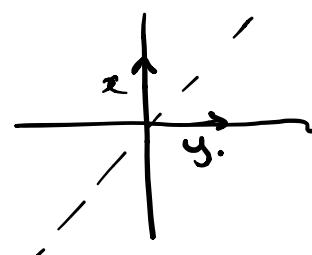
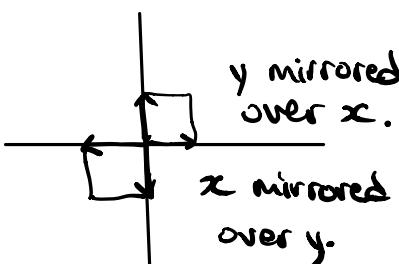
y is stretched by 2.

x is mirrored over y

INVERSION

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}: \text{flips over } 45^\circ$$



ROTATION

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix}$$



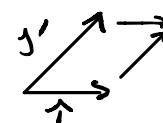
Can generalize.

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\sin\theta = -1$, when $\theta=90^\circ$.

SHEAR

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ y \end{bmatrix}$$



$$\rightarrow \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} : \text{rotate around } z \text{ in } \mathbb{R}^3$$

Some sense of matrix geometry and the ways of manipulating space is useful for data science. It will allow for the representation of data for easier or more tower characterizations.