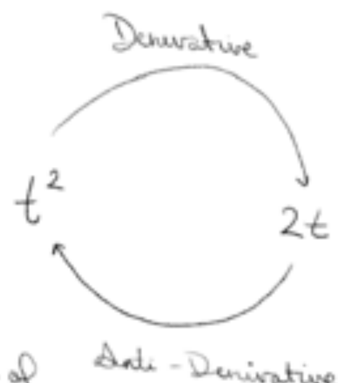
"One should never to prove anything
that is not almost obvious"

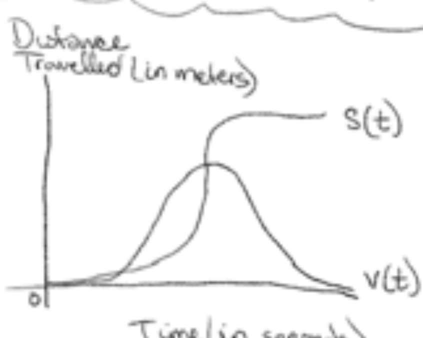
— Alexander Grothendieck

Integrals are the inverse of a derivative.

$$\frac{d}{dT}\int_0^T v(t)\,dt = v(t)$$

From a distance (change) function
to a velocity (rate of change) function.





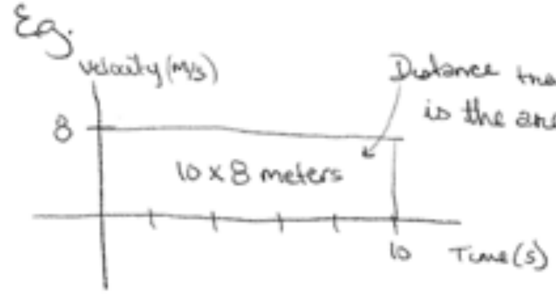
The derivative of
a distance versus
time function,
gives a velocity
versus time function.

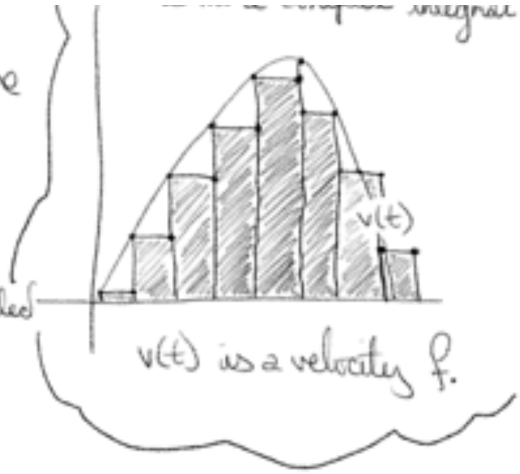
The methods for doing
are directly
related to determining
the area bounded by the derivative function. (Integrals)

→ You can also use the velocity function
to derive the distance function.

Integrals can be complex: (1) A more complex integral can be complex, however they are simpler when the velocity function is constant:

Eg.

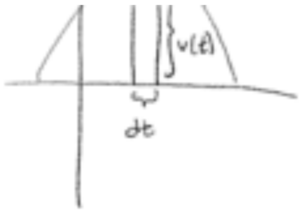




The general strategy for solving this generic constant velocity function is to take the product of the velocity and time where during that time velocity is constant; in this case there is only one velocity but in the general case these products would all be summed.

When a velocity function is continually changing; this change can be reframed as a sequence of arbitrarily small and finite changes — to then employ the same strategy of $\sum^{n} \frac{d}{dt} v(t)\, dt$.

Ideology is to treat continously changing velocity as a discontinuos sequence of progressively changing velocities. (The integration of many values).



$\int$ is the sum.

— rather what it approaches —

* signed integral *

An integral is expressed $\boxed{\int_{-\infty}^{\infty} \frac{d}{dt} v(t) dt}$

dt implies two responsabilities:

$\boxed{1}$ a factor in the quantities being summed.

$\boxed{2}$ the granularity of sampling;

which have a proportional relationship to one another.

$\int$ is the approximation of a sum; not the actual sum — which is why the sigma notation is not used.

Σ is not $\int$.

To find the distance travelled you can go from velocity and covert the method to a distance function via the anti-derivative; you can also calculate the integral of the velocity function → the area underneath a graph's curve bounded by certain axises.

Calculating area is a common problem inherent in a multitude of applications, which is why it is useful to know.

$$s(T) = \int_0^T v(t)\,dt, \quad \text{a distance function.}$$

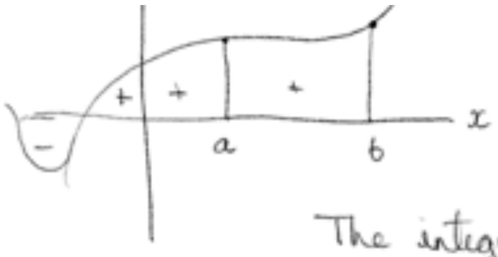
$$\begin{tikzcd} 4t^2 - \frac{1}{3}t^3 + C \arrow[r, bend left, "\text{Derivative}"] & 8t - t^2 \arrow[l, bend left, "\text{Antiderivative.}"] \end{tikzcd}$$

There are infinite number of integrals, when unbounded.

To determine which of the many integrals correspond to the current velocity function — subtract the lower bound.

$$\int_0^T t(8-t)\,dt = \left(4T^2 - \frac{1}{3}T^3\right) - \left(4(0)^2 - \frac{1}{3}(0)^3\right)$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$



$$\frac{dF}{dx}(x) = f(x)$$

The integration of numerous discontinuos arbitrarily granular approximations can be calculated by computing only the upper and lower bound integrals.

The antiderivative encapsulates everything required to integrate ever smaller divisions of [illegible]