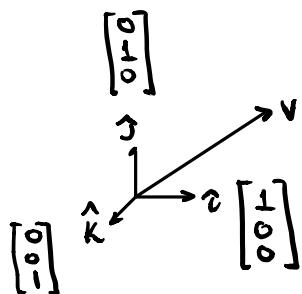


All vectors are located in a coordinate space.  
 When performing projection, the vector upon which you are projecting is a "new" coordinate space.

## Standard Basis

where  $e_1, e_2, \dots, e_n$  are (can also use " $\hat{e}_i$ ")  
 unit vectors on their axes  
 → corollarily orthonormal



The hat in vectors  
 terminology represents  
 a basis vector.

A vector  $v$  can be represented as  
 a typical vector,

$$v = [\alpha e_1, \beta e_2, \dots, \gamma e_n]^T$$

Or as a sum of basis vector,

$$v = \alpha e_1 + \beta e_2 + \gamma e_3 + \dots + \zeta e_n$$

By expressing a vector as a combination of other vectors — this expression can be re-expressed in the frame of a different set of vectors.

$$\text{changing } E = \{\vec{e}_i\} \rightarrow F = \{\vec{f}_i\}$$

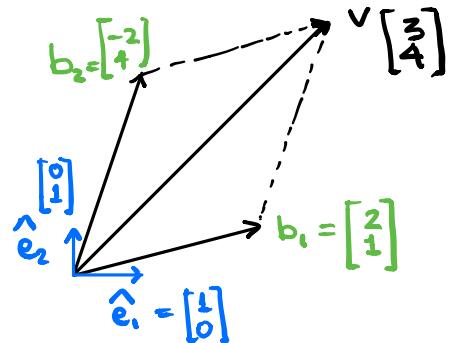
These sets of vectors are known as basis vectors.

→ The vector  $v$ , will take on different values depending on the coord-sys;  
 the vector is independent of the the coord. sys..

⇒ This is extremely easy when the new coord. sys. are orthogonal to each other.

Use of the Projectional Dot-Product  
 can use the projectional dot-product to find the vector  $v$  in the new coord. sys.  
 — provided you know how the new basis vectors relate to the old.

Translation of Vector  $v$   
into second coord. sys.:



① Test basis of new coord sys  
are orthogonal (i.e.  $\cos\theta = 1$  for  
all basis vectors).

$$b_1 \cdot b_2 = \|b_1\| \|b_2\| \cos\theta$$

$$\cos\theta = \frac{b_1 \cdot b_2}{\|b_1\| \|b_2\|}$$

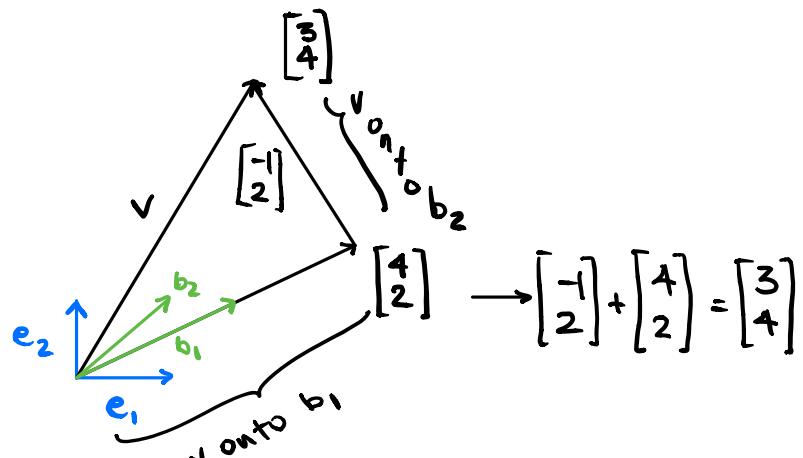
(\*) using the example,  $b_1 = [2 1]^T$   
 $b_2 = [-2 4]^T$

$$\cos(\theta) = \frac{2(-2) + 1(4)}{\sqrt{5} \sqrt{20}}$$

$$= 0.$$

$$\begin{aligned} \text{Vector proj of } b_1 &= \underbrace{\frac{b_1}{\|b_1\|}}_{\text{Scalar Proj.}} \times \underbrace{\frac{v \cdot b_1}{\|b_1\|}}_{\text{normalized projection}} \\ &= \left[ \begin{array}{c} 2 \\ 1 \end{array} \right] \frac{10}{(\sqrt{2^2+1^2})^2} \text{ in the direction of } b_1. \\ &= \left[ \begin{array}{c} 4 \\ 2 \end{array} \right] \end{aligned}$$

$$\begin{aligned} \text{vector proj of } b_2 &= \frac{b_2}{\|b_2\|} \times \frac{v \cdot b_2}{\|b_2\|} \\ &= \left[ \begin{array}{c} -2 \\ 4 \end{array} \right] \times \frac{10}{20} \\ &= \left[ \begin{array}{c} -1 \\ 2 \end{array} \right] \end{aligned}$$



Summary,

All vectors aren't tied to their axes, they can be reexpressed by other bases. Those bases describe the space of those vectors; the selection of those bases is very important.

This translation can be done very easily using the dot product when the bases are orthogonal to one another.

