
PROBLEM #1

$$A'(x) = f'(x)g(x) + f(x)g'(x).$$

Write the product rule using Leibniz notation.

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x).$$

PROBLEM #2.

Deconstruct the $q(x)$ function into two functions f and g , and use the product rule to calculate their derivatives.

$$q(x) = (x+2)(3x-3)$$

$$q(x) = f(x)g(x), \text{ where}$$

$$f(x) = (x+2)$$

$$g(x) = (3x-3)$$

$$\frac{d}{dx}q(x) = \frac{d}{dx}(f(x)g(x))$$

$$= f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x), \text{ by the product rule.}$$

$$= (x+2)(3) + (3x-3)(1)$$

$$= 3(x+2) + (x-1)$$

$$= 3(2x+1)$$

PROBLEM #3

Differentiate $f(x) = x^3 \sin x$

$$\begin{aligned}\frac{d}{dx}f(x) &= x^3 \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x^3 \\ &= x^3 \cos x + \sin x \cdot 3x^2\end{aligned}$$

PROBLEM #4.

Differentiate $f(x) = \frac{e^x}{x}$.

$$\begin{aligned}\frac{d}{dx} f(x) &= e^x \frac{d}{dx} x^{-1} + \frac{1}{x} \frac{d}{dx} e^x \\ &= e^x \left(-\frac{1}{x^2}\right) + \frac{e^x}{x} \\ &= e^x \left(\frac{1}{x} - \frac{1}{x^2}\right)\end{aligned}$$

PROBLEM #5.

Find the derivative of the generalized three term product $f(x)$ - using substitution.

$$f(x) = f_1(x)f_2(x)f_3(x)$$

$$\begin{aligned}\frac{d}{dx} f(x) &= \frac{d}{dx} f_1 f_2 f_3 \\ &= u \frac{d}{dx} f_3(x) + f_3(x) \frac{d}{dx} u, \text{ where } u = f_1(x)f_2(x) \\ &= u \frac{d}{dx} f_3(x) + f_3(x) \left[f_1(x) \frac{d}{dx} f_2(x) + f_2(x) \frac{d}{dx} f_1(x) \right] \\ &= f_1(x)f_2(x) \frac{d}{dx} f_3(x) + f_3(x)f_1(x) \frac{d}{dx} f_2(x) + f_3(x)f_2(x) \frac{d}{dx} f_1(x)\end{aligned}$$

PROBLEM #6.

Differentiate $f(x) = x e^x \cos x$.

$$\frac{d}{dx} f(x) = \frac{d}{dx} (x e^x \cos x)$$

$$= xe^x \frac{d}{dx} \cos x + x \cos x \frac{d}{dx} e^x + e^x \cos x \frac{d}{dx} x$$

$$= xe^x (-\sin x) + x \cos x e^x + e^x \cos x$$

$$= e^x [-x \sin x + x \cos x + \cos x]$$

$$= e^x [-x \sin x + \cos x (x+1)]$$