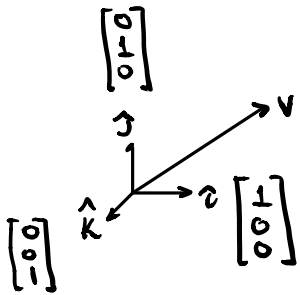


All vectors are located in a coordinate space.
When performing projection, the vector upon which you are projecting is a "new" coordinate space.

Standard Basis

where $\hat{i}, \hat{j}, \hat{k} \dots \hat{z}$ are (can also use \hat{e}_i)
unit vectors on their axes
→ collaterally orthonormal



The hat in vectors terminology represents a basis vector.

A vector v can be represented as a typical vector,

$$v = [\alpha e_1 \ \beta e_2 \ \dots \ \gamma e_n]^T$$

or as a sum of basis vector,

$$v = \alpha e_1 + \beta e_2 + \gamma e_3 \dots + \gamma e_n$$

By expressing a vector as a combination of other vectors — this expression can be re-expressed in the frame of a different set of vectors.

changing $E = \{e_i\} \rightarrow F = \{f_i\}$.

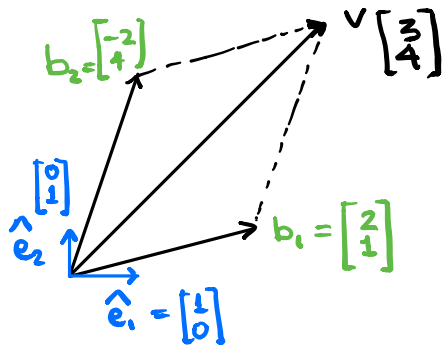
These sets of vectors are known as basis vectors.

Use of the Projectional Dot-Product
can use the projectional dot-product to find the vector v in the new coord. sys.
— provided you know how the new basis vectors relate to the old.

⇒ This is extremely easy when the new coord. sys. _{vectors} are orthogonal to each other.

↳ The vector v , will take on different values depending on the coord-sys; the vector is independent of the the coord. sys..

Translation of Vector v
into second coord. sys.:



① Test basis of new coord sys
are orthogonal (i.e. $\cos\theta = 1$ for
all basis vectors).

$$b_1 \cdot b_2 = |b_1| |b_2| \cos\theta$$

$$\cos\theta = \frac{b_1 \cdot b_2}{|b_1| |b_2|}$$

(*) using the example, $b_1 = [2 \ 1]^T$
 $b_2 = [-2 \ 4]^T$

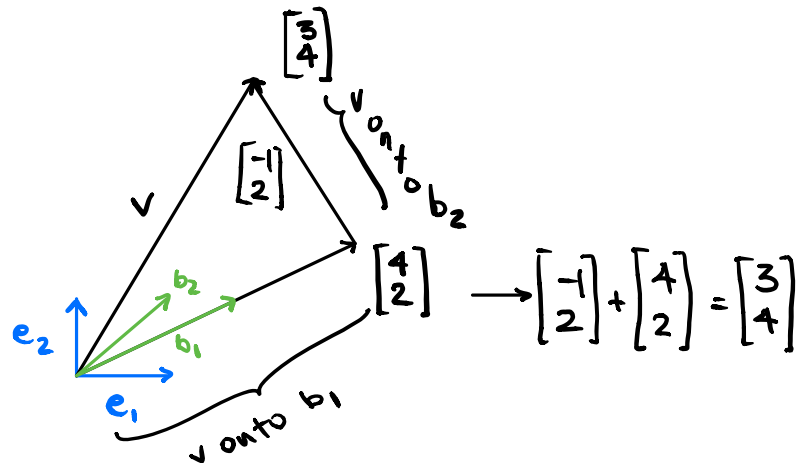
$$\cos(\theta) = \frac{2(-2) + 1(4)}{\sqrt{5} \sqrt{20}}$$

$$= 0.$$

② Perform vector projection
of the vector v onto the
basis vectors.

$$\begin{aligned} \text{vector proj} &= \frac{b_1}{|b_1|} \times \frac{v \cdot b_1}{|b_1|} \\ \text{of } b_1 & \\ &= \underbrace{\frac{b_1}{|b_1|}}_{\text{Scalar Proj.}} \times \underbrace{\frac{v \cdot b_1}{|b_1|}}_{\text{normalized projection in the direction of } b_1} \\ &= \begin{bmatrix} 2 \\ 1 \end{bmatrix} \frac{10}{(\sqrt{2^2+1^2})^2} \\ &= \begin{bmatrix} 4 \\ 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{vector proj} &= \frac{b_2}{|b_2|} \times \frac{v \cdot b_2}{|b_2|} \\ \text{of } b_2 & \\ &= \begin{bmatrix} -2 \\ 4 \end{bmatrix} \times \frac{10}{20} \\ &= \begin{bmatrix} -1 \\ 2 \end{bmatrix} \end{aligned}$$



Summary,

All vectors aren't tied to their axes, they can be
reexpressed by other bases. These bases describe the space
of those vectors; the selection of those bases is very important.
This translation can be done very easily using the dot product when
the bases are orthogonal to one another.

