

### PROBLEM #1.

Construct a series of simultaneous equations to represent these variables and their historical values.

Imagine,

On Monday, you bought for €15.

1 apple  
1 banana  
1 carrot

On Tuesday, you bought for €28.

3 apples  
2 bananas  
1 carrot

On Wednesday, you bought for €23.

2 apples  
1 banana  
2 carrots.

Solution: create columns for each feature and tabulate their quantities—express the combination of those features as having a total price.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

### PROBLEM #2.

Manipulate the expression  $Mx=b$  into RREF.

$$M = \begin{bmatrix} 4 & 6 & 2 \\ 3 & 4 & 1 \\ 2 & 8 & 13 \end{bmatrix} \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ 7 \\ 2 \end{bmatrix} \Leftrightarrow \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & 2 & -20 \\ 0 & 4 & 11 & -16 \end{array} \right]$$
$$\Leftrightarrow \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & -10 \\ 0 & 0 & 7 & -56 \end{array} \right] \Leftrightarrow \left[ \begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & -10 \\ 0 & 0 & 1 & -8 \end{array} \right]$$

### PROBLEM #3. b #4.

Same answer as problem #2.

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 2 & 8 & 13 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 0 & 5 & 12 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ -5/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ -1/2 \\ 0 \end{bmatrix}$$

### PROBLEM #5

Convert this simultaneous equation to echelon form.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & -1 & 0 \end{bmatrix} x = \begin{bmatrix} 15 \\ -17 \\ -17 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 15 \\ 17 \\ 5 \end{bmatrix}$$

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PROBLEM #6.

Solve these simultaneous equations w- back substitution.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 15 \\ 17 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 3 \\ 7 \\ 5 \end{bmatrix}$$

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PROBLEM #7.

Find the inverse of this matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \left\| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \left\| \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right. = \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -2 & -3 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}$$

$$= \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & -3/2 & 1/2 & 1/2 \\ 0 & 1 & 2 & 3 & -1 & 0 \end{array}$$

$$= \begin{array}{ccc|ccc} 1 & 0 & 0 & -3/2 & 1/2 & 1/2 \\ 0 & 1 & 1 & 5/2 & -1/2 & -1/2 \\ 0 & 1 & 2 & 3 & -1 & 0 \end{array}$$

$$= \begin{array}{ccc|ccc} 1 & 0 & 0 & -3/2 & 1/2 & 1/2 \\ 0 & 1 & 1 & 5/2 & -1/2 & -1/2 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}$$

$$= \begin{array}{ccc|ccc} 1 & 0 & 0 & -3/2 & 1/2 & 1/2 \\ 0 & 1 & 0 & 2 & 0 & -1 \\ 0 & 0 & 1 & 1/2 & -1/2 & 1/2 \end{array}$$

