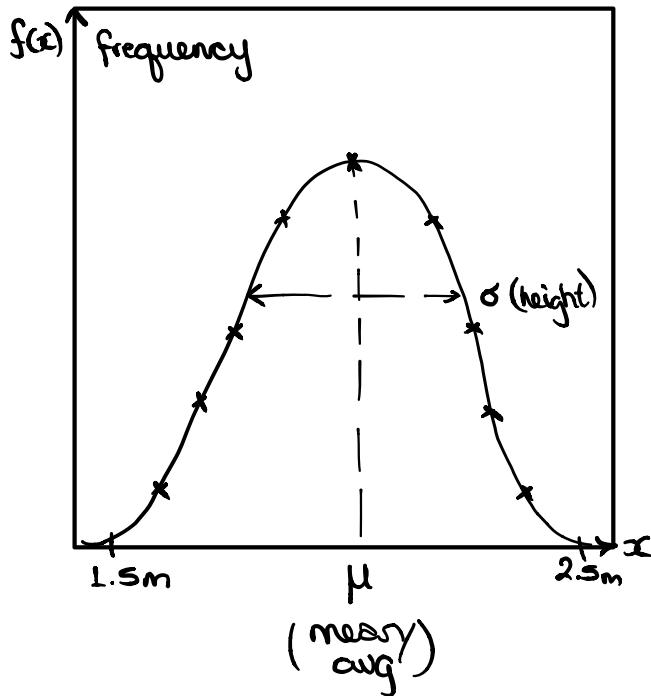


The Newton-Raphson method can be used to solve equations.

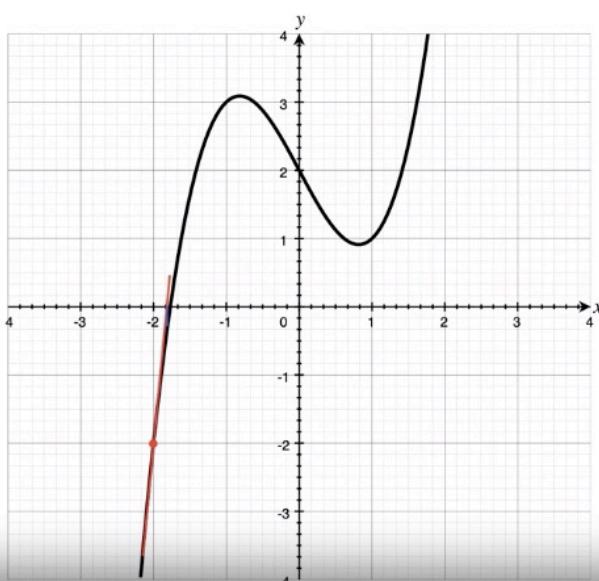


$$f(x) = x^3 - 2x + 2.$$

#### NEWTON-RAPHSON ITERATION

TABLE.

i	$x_i$	$y(x_i)$	$\frac{dy(x_i)}{dx}$
0	-2	-2	10
1	-1.8	-0.23	7.7
2	-1.77	-0.005	7.4
3	-1.769	-2.3E-6	



$$f(x) = x^3 - 2x + 2.$$

By solving the equation - a model for the distribution can be produced; often working with a model is more desirable than working with the raw data points.

Newton-Raphson produces a degree of error; and the parameter values are subject to a goodness of fit.

Example: solving for  $y=0$  in

$$f(x) = x^3 - 2x + 2.$$

①  $\frac{df}{dx} = 3x^2 - 2$ , linearise the function

②  $g_1(-2) = 10$ , compute the gradient

③  $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ , estimate the parameters which will solve the equation.

$$-1.8 = -2 - \frac{(-2)}{10}$$

④  $-0.23 = f(-1.8)$ , evaluate the new function.

(\*) The key requirements of the Newton-Raphson method for solving an equation are: (1) the ability to evaluate the function, and (2) the ability to derive the func.

Complex functions with massive datasets are often too complex to be solved analytically - or be graphed out.

An algorithmic corner case in Newton-Raphson,

$$y = x^3 - 2x + 2$$

Newton-Raphson

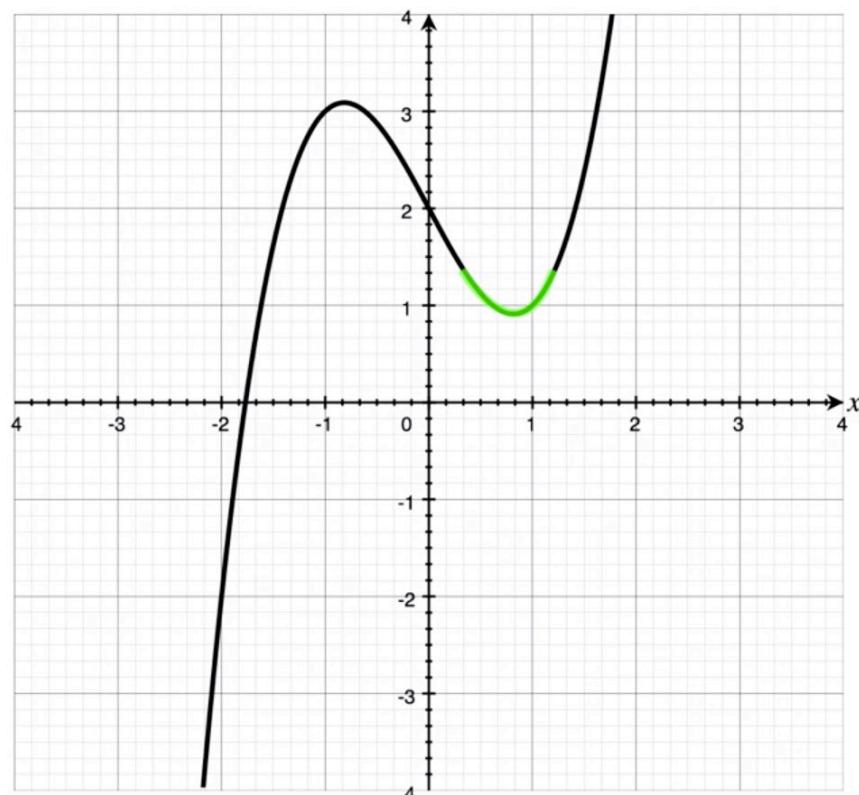
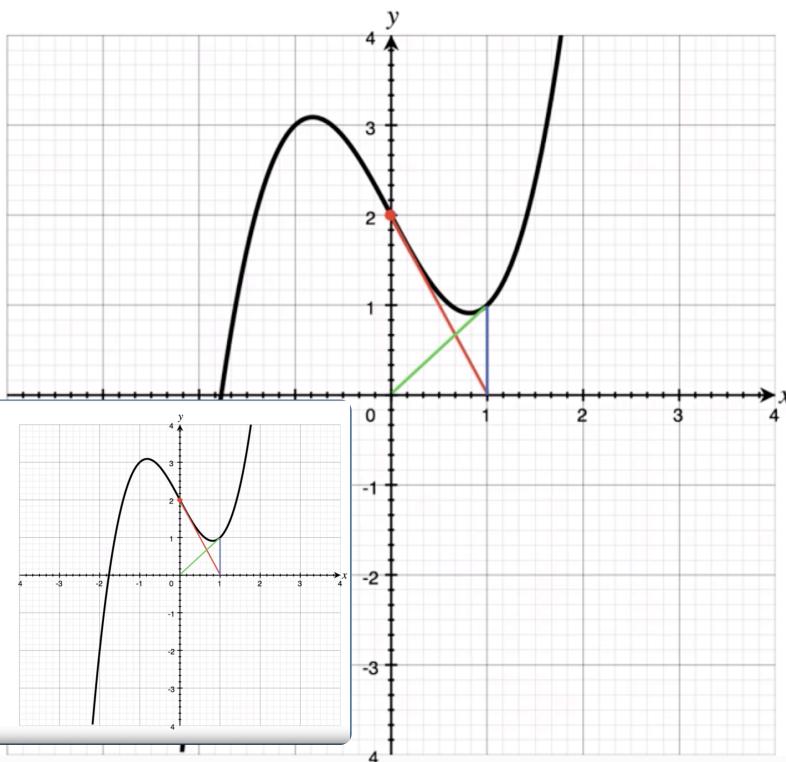
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\frac{\partial y}{\partial x} = 3x^2 - 2$$

$i$	$x_i$	$y(x_i)$	$\frac{\partial y(x_i)}{\partial x}$
0	0	2	
1	1	1	
2	0	2	

$y = x^3 - 2x + 2$   
 Newton-Raphson  
 $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$   
 $\frac{\partial y}{\partial x} = 3x^2 - 2$

Some points can "loop"



Secondly, when estimating/iterating with Newton-Raphson near a minima/maxima the division by the gradient will produce an inflated jump to the next estimate. Zero also is problematic.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

In a multivariate context newton-raphson requires a contour plot.

Newton-Raphson works by linearisation.

Assuming the root is nearby a certain point — like  $(x_0 + \delta x)$ .

$$\begin{aligned}f(x_0 + \delta x) &= f(x_0) + f'(x_0) \delta x \\&= 0, \text{ the jump slightly over will be the root.}\end{aligned}$$

By rearranging this equation; and assuming  $f(x_0 + \delta x)$  is 0.

$$f(x_0 + \delta x) = f(x_0) + f'(x_0) \delta x$$

$$0 = f(x_0) + f'(x_0) \delta x$$

$$f'(x_0) \delta x = -f(x_0)$$

$$\delta x = -\frac{f(x_0)}{f'(x_0)}$$

• the delta to shift for the  
next Newton-Raphson  
estimation.

Hence,  $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ .