The process of manipulating a simultaneous equation into reduced row echelon form, then performing backsubstitution works on expressions involving a variety of mathematical objects.

Formerly it was seen RREF & BS works for,

$$Ax = y \longrightarrow A^{-1}Ax = A^{-1}y$$
$$x = A^{-1}y.$$

This same process can also be applied to,

$AB = I \iff AA^{-1} = I$, note A and it's inverse are commutative — and as normal associative.

RREF & Backsub:

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ \underbrace{b_{31}} & \underbrace{b_{32}} & \underbrace{b_{23}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

the basis vectors / linear transformations applied to the input vectors.

$$I = A^{-1}A$$
$$A = A(A^{-1}A)$$
$$A^{-1} = A^{-1}(AA^{-1})$$
$$= (A^{-1}A)A^{-1}$$
$$= A^{-1}(AA^{-1})$$

, I is representative of either sequence.

① $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \Bigg| \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

② $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \Bigg| \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, RREF

③ $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Bigg| \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$

④ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Bigg| \begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$, Backsubstitute.

$\begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ is B: the inverse of A.