Matrix multiplication is one way of forming compositions or combinations of matrix transformations.

These ops are associative but do not possess commutativity.

$$(A_1 A_2) A_3 x = y \qquad A_1 A_2 A_3 x = y$$

$$A_1 (A_2 A_3) x = y \qquad A_3 A_1 A_2 x \neq y, \text{ not commutative}$$

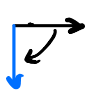
Matrix multiplications are sequential applications of algebra which is robust in additivity and scalar multiplication.

Example. $A_2 A_1 r = y_1$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

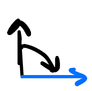
A_1 Transformation to $\hat{e}_1$,

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$



A_1 Transformation to $\hat{e}_2$,

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

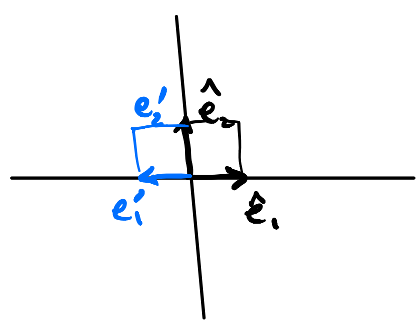


A_1 is a clockwise rotation

A_2 transformation to $[x\ y]^T$,

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix}$$

A_2 is a mirror over y-axis.



Commutativity check,

$A_1 A_2 x \longrightarrow$ mirror over y, then rot-clock

$A_2 A_1 x \longrightarrow$ rot-clock, mirror over y.

COMMUTIVITY FAILS

$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A_1 A_2 x \rightarrow$ 

$A_2 A_1 x \rightarrow$