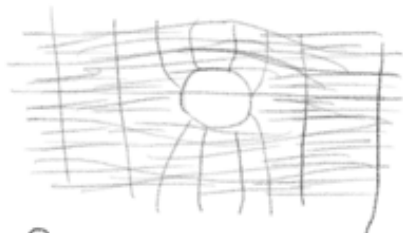


Complex Plane



$$f(z) = z + \frac{1}{z}$$

- Fast where vertical lines are close
- Slow where vertical lines far

Two things at Play

Divergence



$$\nabla \cdot \vec{v}$$

Curl



$$\nabla \times \vec{v}$$

Vector Field — associate each point in space with a vector (magnitude & direction)

Vector Field Function

$$\text{div } \vec{F}(x, y, z) = \dots$$

(x, y) } measures how much (x, y) generates fluid out (+) or into (-).
 Some Point

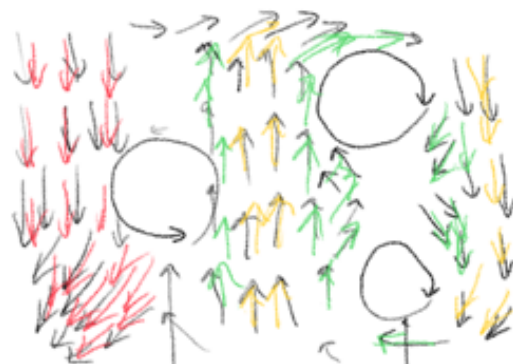
$$F(x, y) = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

\Rightarrow Analogous to a derivative.

* Side note if the divergence is 0 everywhere the fluid is incompressible.

$$\text{div} F = 0.$$

CURL



$\text{curl} F > 0$

* counter clockwise

$\text{curl} F < 0$

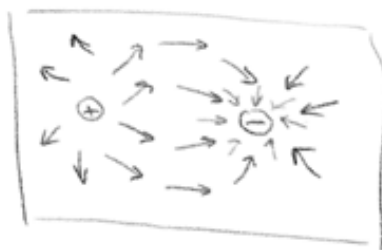
* clockwise



Maxwell's Equations

Electric Field: E Magnetic Field: B

$$\text{div } E = \frac{\rho}{\epsilon_0} \quad \left\{ \begin{array}{l} \text{charge Density} \\ \text{Gauss' Law} \end{array} \right.$$



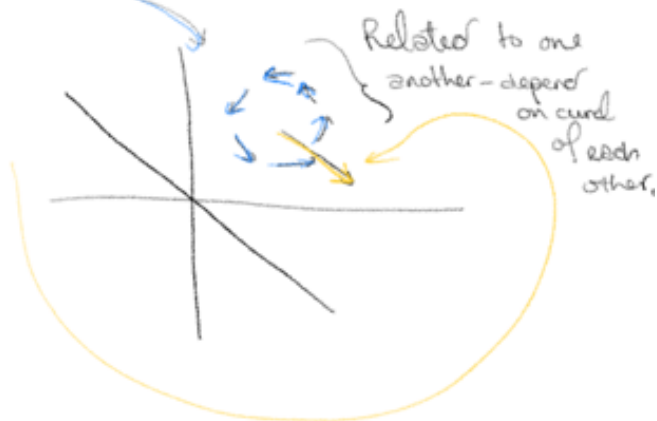
$$\text{div } B = 0$$



$$\text{curl } E = -\frac{\partial B}{\partial t}$$

The field is incompressible
no sources or sinks.

$$\text{curl } B = \mu_0 \left(J + \epsilon_0 \frac{\partial E}{\partial t} \right)$$



* Divergence and curl
are independent of flow

Divergence, Curl, Dot Product and Cross Product.

Divergence = $\nabla \cdot \vec{v}$, and the dot product calculates
how similar two vectors are



$$\vec{u} \cdot \vec{v} = \text{+ diff}$$

+ is outward flow
- is sink.



dot is +

$$\vec{u} \cdot \vec{v} = \text{+ diff}$$

Cross Product Measures how
perpendicular two
vectors are.

$\vec{a} \cdot \vec{b}$ dot $\vec{a} \cdot \vec{b}$

cross prod is +, when clockwise



C.P is -.

Divergence, curl and incompressible irrotational
flow

$$\text{div } F = 0$$

$$\text{curl } F = 0.$$

• Like no charges or current in a vacuum.