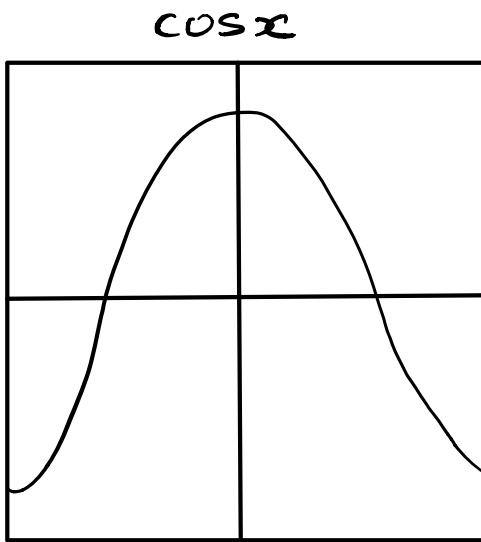


Building Taylor Series approximations, including the Maclaurin specific approximation, have interesting properties in themselves and sometimes belie interesting properties about the functions they seek to approximate!

Cosine is a well behaved function - it is continuous everywhere and infinitely differentiable.



$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Differentiating  $\cos x$ .

$f^{(0)}(x) = \cos x$	$@x=0: 1$
$f^{(1)}(x) = -\sin x$	$0$
$f^{(2)}(x) = -\cos x$	$-1$
$f^{(3)}(x) = \sin x$	$0$
$f^{(4)}(x) = \cos x$	$1$

Since every other term, more specifically - the odd terms of the power series has a zero coefficient, only the even terms are coefficients of the even powers.

(\*) where the cos terms are  $\pm 1$  and sin terms are zero.

This means each term of the cos approximation is a symmetrical function oriented to the vertical axis!

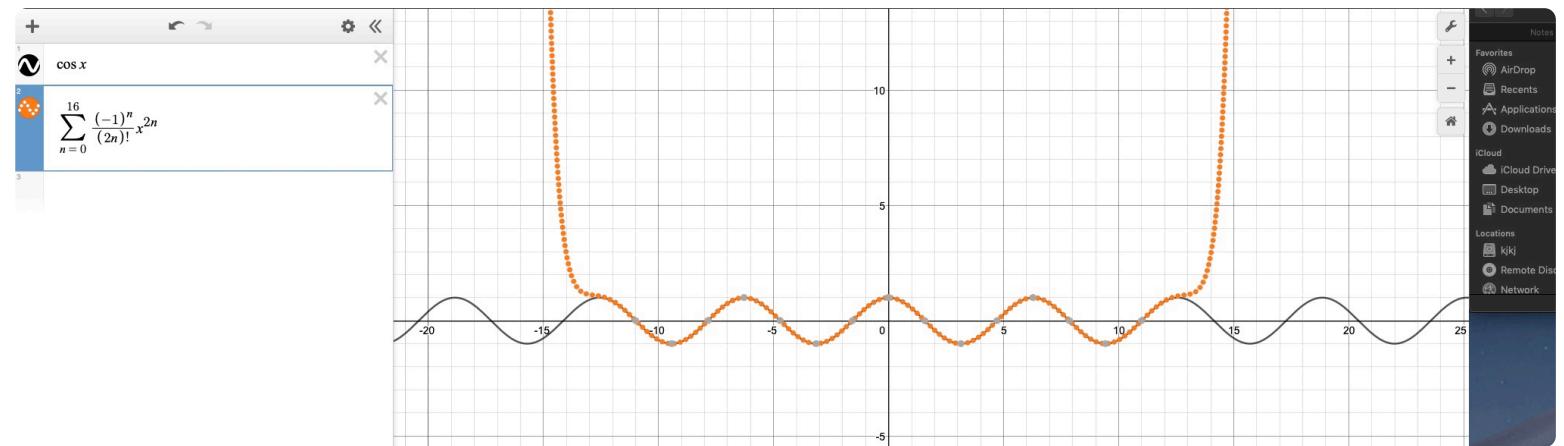
The Maclaurin series approximation for  $\cos x$  is,

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$$

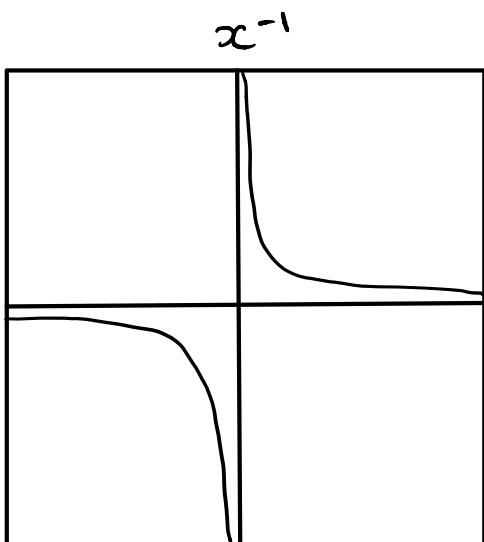
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

, where  
n is the  
n-th nonzero  
term of the series!

the  $(-1)^n$  implements the oscillating behaviour!



The second example features  $f(x) = x^{-1}$ .



Instead form the Taylor Series at a well defined point like  $x=1$ ,

$$\begin{aligned} g(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(\rho)}{n!} (x-\rho)^n \\ &= 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots \\ &= \sum_{n=0}^{\infty} (-1)^n (x-1)^n \end{aligned}$$

This reflects the difficulty with which a power series approximates a poorly behaved function.

$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Differentiating  $x^{-1}$ ,

$$f^{(0)}(x) = x^{-1} \quad @x=0: \text{undefined}$$

$$f^{(1)}(x) = -1/x^2 \quad @x=1: 1$$

$$f^{(2)}(x) = 2/x^3 \quad -1$$

$$f^{(3)}(x) = -6/x^4 \quad 2$$

$$f^{(4)}(x) = 24/x^5 \quad -6$$

$$f^{(5)}(x) = -120/x^6 \quad 24$$

It is interesting that power series approximations ignore asymptotes entirely; perhaps the power series can not be defined.

In fact the function is not approximated at zero or for any values of  $x$  less than zero (when  $\rho=1$ ).

Also, interestingly the tail ends of the approximation oscillate with asymptotically dominating inaccuracy.

