Definition <u>Generating Set | Span</u>:

Given a vector space V.

A set of vectors $A = \{x_1, \dots, x_k\} \subseteq V$

If every vector $v \in V$ is a linear combination of $x_1, \dots, x_k$ then A is called a generating set or span.

This can be written $V = [A]$ or $V = [x_1, \dots, x_k]$

Definition <u>Basis</u>:

- Given a vector space V, a generating set A of V is minimal if there is no smaller minimal set $\tilde{A} \subseteq A \subseteq V$ that spans V.

- Every linearly independent generating set of V is minimal and called the basis V.

Given the definition above, and a the subset B agreeing with that definition.

Let $B \subseteq V$, $B \neq 0$. The following statements are equivalent.

I. B is a basis of V.

II. B is a minimal generating set

III. B is a maximally linear independent subset of vectors in V.

IV. Every vector $x \in V$ is a linear combination of vectors from B, and every linear combination is unique.

$$x = \sum_{i=1}^{k} \lambda_i b_i$$

$$= \sum_{i=1}^{k} \psi_i b_i \,, \quad \text{where } \lambda_i, \psi_i \in \mathbb{R}$$

$$b_i \in B$$

If this is true then $\lambda_i = \psi_i$, $i = 1, \dots, k$.

Example of the Canonical/Standard Basis in $\mathbb{R}^3$

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

A Counter Example of A Non-Basis

$$A = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ -4 \end{bmatrix} \right\}.$$

Linear independent but not a generating set.

$v = [1\ 0\ 0\ 0]^T$ can not be created from vectors in A.

Basis Vector Dimensions

Every vector space V has a basis B.
All bases have the same number of elements.

In finite-dimensional space V,
$\dim(U) \leq \dim(V)$, when $U \subseteq V$

$\dim(U) = \dim(V)$, when $U = V$

Can be thought of as the number of basis vectors, or the number of independent directions in the vector space.