

PMF visualize relationships and patterns well, when the number of values are few.

When the dispersion and the amount of data grows the plot becomes more generalized (smaller probabilities) and the proportion of noise to data grows.

Furthermore the crossection of one dataset on to others often becomes convoluted.

It becomes hard to distinguish between data sources and determine qualities of any distribution even by itself.

→ This can be mitigated by grouping data into non-overlapping buckets - a technique called "binning".

however this can generalize real patterns + relationships, but when done appropriately remove noise.

- it's also just painstaking.

Cumulative Distribution Functions is a method which solves this problem without the friction of Binning.

### Percentiles & Percentile Rank.

Percentile rank - the fraction of people who scored lower than or equal to a given score.

$$pr = \frac{\text{size}(S_L)}{\text{size}(S)}, \text{ where } S \text{ is the set of all data points.}$$

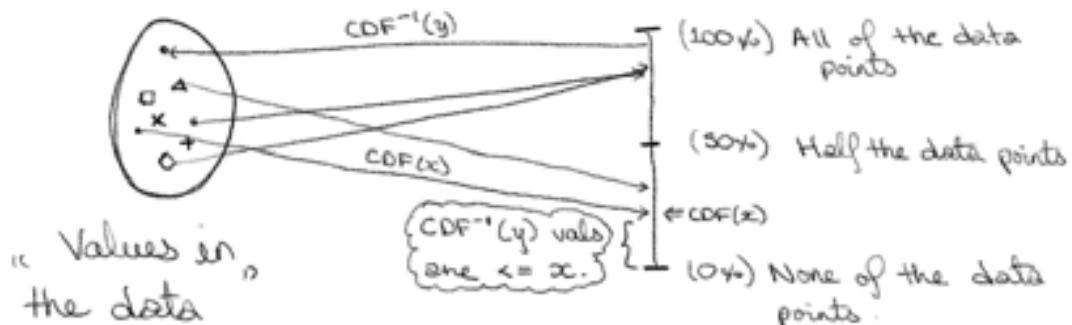
Percentile - the maximum percentile rank's where  $S_L$  is the corresponding score which does not exceed the definition defined by percentile rank.

\* To get a percentile must sort data an access index based on percentile rank's ceiling index.

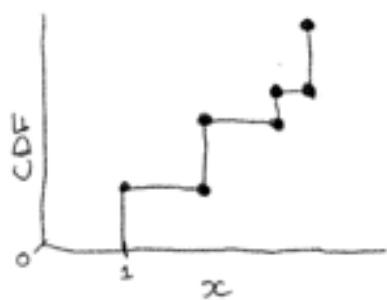
### Cumulative Distribution Function

Distrib. Vals

Percentile Rk.



Graph of CDF



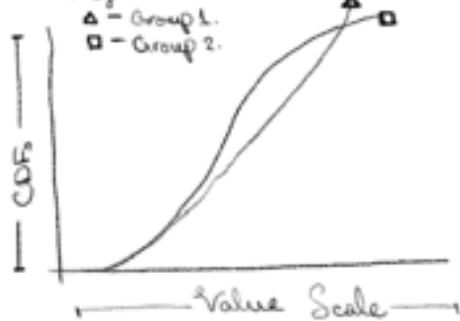
Graphed using  
a step function.

If  $x_2$  is greater  
than the longest  $x$ ,  
 $CDF(x_2) = 1$ .

The fraction of values  
in the distribution  
less than or equal  
to some value.

Comparing CDFs

Legend:  
▲ - Group 1.  
■ - Group 2.



Interpreting a CDF, can use as a lookup; where the percentile rank indicates the values and fraction of the distribution which falls below those values maximum.

Vertical sections indicate density and common values in the distribution.

L ⇒ If there are many common values in a distribution this means significant fractions of the distrib. will be less than or equal to that value. Due to this significant ranges of the CDF (the vertical part) will be occupied by the preceding value.

CDFs are superior regularizations for competing Distrib.

They present the change of the distrib. in a continuous and undisruptive way. In addition they have the faculty to observe singular elements in

relation to the distribution.  
Due to this distibl  
can be graphed alongside,  
then evaluated at  
an aggregate level,  
and then dissected at  
a granular level if need  
be.

### Percentile-based Summary Statistics

with CDFs in hand summary stats  
can be computed through use of the CDF  
to form percentile based descriptions.

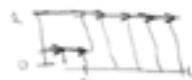
50<sup>th</sup> percentile - val. which divs.  
distrib. in half.  
called - median -.

inner quantile - the measure of  
range (var) a distrib. spread  
to dispersion.  
It's the diff. between  
the 25<sup>th</sup> and 75<sup>th</sup> percentile.

quartiles - often segmentation of a distrib into continuous  
ranges of percentiles, can be used to describe  
a distrib.

→ quintillion is percentile ranges of 20:  
20<sup>th</sup>, 40<sup>th</sup>, 60<sup>th</sup>, ...

### Random Numbers



Population

SAMPLE

Percentile  
Ranks

\* w-replacement - without  
removing the value from  
the population.

Meaning

10% of sample  $\leq$  10<sup>th</sup> percentile

20% ...  $\leq$  20<sup>th</sup> percentile

C  
D  
F

The random  
selection of samples

100% of samp  $\leq$  100<sup>th</sup> percentile.

Percentile Rk.

will uniformly span  
the original CDF,  
its percentiles.

Generating Random Numbers  
w- a Cumulative Distrib.  
Function.

1. Choose a percentile rank  
uniformly from  $[0, 100]$ .
2.  $CDF^{-1}(y)$  the percentile  
rank to find the percentile  
score.

Comparing Percentile  
Ranks.

Can compare across  
groups. via ranking.

