PROBLEM # 1

Compute the length of x using the inner product $\langle a, b \rangle$, where,

$$x = (1, -1, 3) \quad \text{and} \quad \langle a, b \rangle = a^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} b$$

$$\|x\| = \sqrt{\langle x, x \rangle}$$

$$= \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \\ 7 \end{bmatrix}$$

$$= \sqrt{26}$$

PROBLEM # 2.

Compute the squared distance between x and y using the inner product $\langle a, b \rangle$

$$x = (1/2, -1, -1/2) \qquad \langle a, b \rangle = a^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} b$$

$$y = (0, 1, 0)$$

$$\|x - y\|^2 = \langle x - y, x - y \rangle$$

what do you fear
— A cage — to stay behind bars until use or old age accept them; and all chance

$$= \begin{bmatrix} 1/2 & -2 & -1/2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1/2 \\ -2 \\ -1/2 \end{bmatrix}$$

of valor has gone beyond recall or desire.

$$= \begin{bmatrix} 1/2 & -2 & -1/2 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}$$

$$= 5$$

PROBLEM # 3

Compute the length of $\underline{x}$ using the inner product defined by $\langle a, b \rangle$, where

$$x = (-1, 1) \quad \langle x, x \rangle = a^T \frac{1}{2} \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} b$$

The length of x is it's norm. Computed by

$$\|x\| = \sqrt{\langle x, x \rangle}$$

$$= \sqrt{\begin{bmatrix} -1 & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}}$$

$$= \sqrt{\begin{bmatrix} -1 & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} -6 \\ 6 \end{bmatrix}}$$

$$= \sqrt{6}$$

PROBLEM # 4.

Compute the distance between $\underline{x}$ and $\underline{y}$ (not squared) using the inner product $\langle a, b \rangle$, where

$$x = (4, 2, 1)$$
$$y = (0, 1, 1) \qquad \langle a, b \rangle = a^T \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} b$$

The distance between x and y, is the norm of the distance vector.

$$\|x - y\| = \sqrt{\langle x-y, x-y \rangle}$$

$$= \left(\begin{bmatrix} 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} \right)^{1/2}$$

$$= \left(\begin{bmatrix} 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ -1 \end{bmatrix} \right)^{1/2}$$

$$= \sqrt{42}$$

$$= 6.48074069841$$

PROBLEM #5

Compute the length of $\underline{x}$ with the inner product $\langle a, b \rangle$ where,

$$x = (-1, -1, -1) \qquad \langle a, b \rangle = a^T I b,$$ where I is the identity matrix

$$\|x\| = \sqrt{\langle x, x \rangle}$$

$$= \sqrt{3}$$