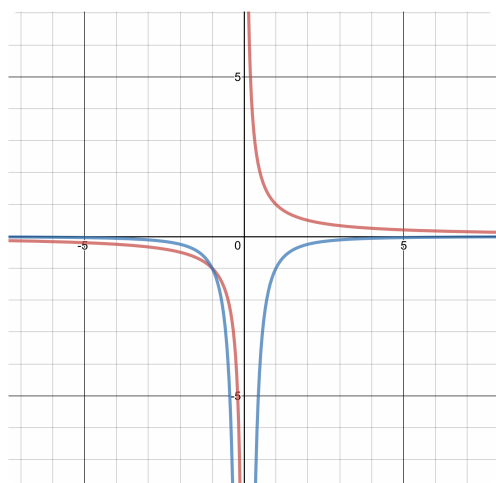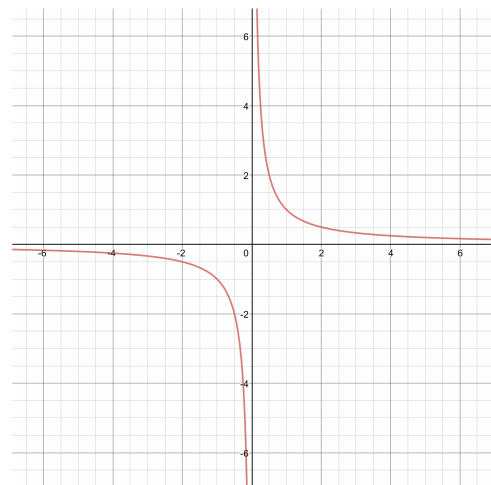The first special case is the function,
$$f(x) = \frac{1}{x}.$$

(*) The gradient of this function is negative everywhere, except at zero.

(**) Discontinuity — gradient isn't defined at zero; and the function itself is not defined at zero.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x + \Delta x} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x - (x + \Delta x)}{x(x + \Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x \cdot x(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x^2}$$

$$= \frac{1}{x^2}.$$

The second function equals it's own derivative.

Properties:

I. The function must always be positive/negative.
   If the function even tried to cross the x-axis, then both the function and the gradient will be zero which would mean the derivative has to equal zero.

II. By always increasing or decreasing, the same value can never be achieved.

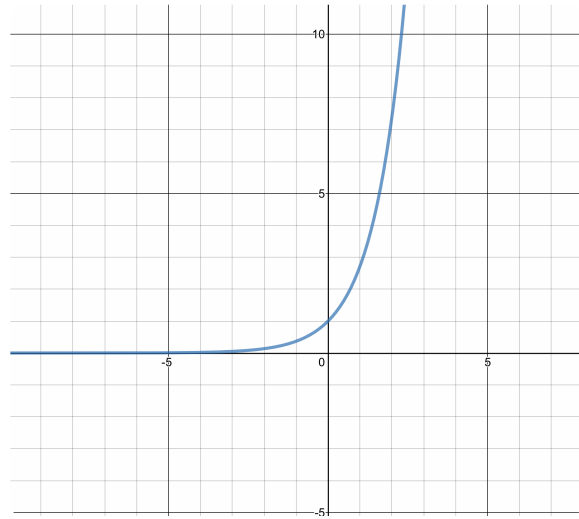Function is $f(x) = e^x$, where e is Euler's number $= 2.71828...$
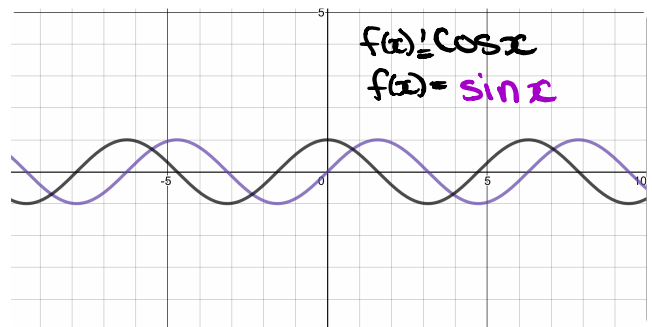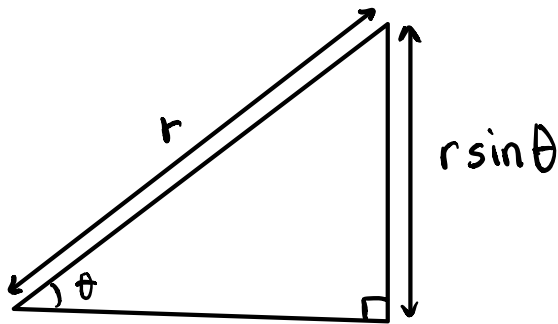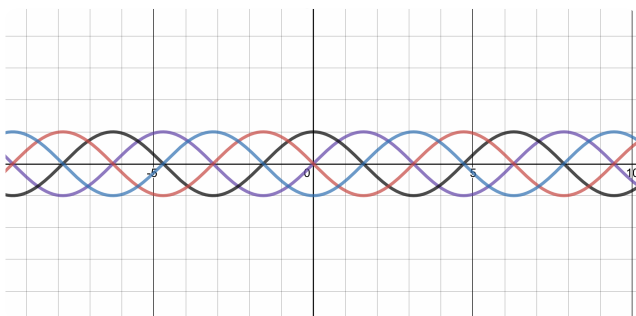
$f(x) = e^x$

$f^1(x) = e^x$

$f^2(x) = e^x$

...

$f^n(x) = e^x$

The third function is rather a class of functions the trigonometric functions: sin and cosine.

$f(x) = \cos x$
$f(x) = \sin x$

$f'(\sin(x)) = \cos x$

$f(x) = \sin x$
$f'(x) = \cos x$
$f''(x) = -\sin x$
$f^3(x) = -\cos x$
$f^4(x) = \sin x$.

Trig functions are Euler exponentials.

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$