

PROBLEM #1

Given the transformation matrix T and the eigenvector basis matrix C — which are eigenvectors of T , calculate $D = C^{-1}TC$.

$$T = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$D = C^{-1}TC$$

where C^{-1} is

$$C^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ where } C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$= \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$$

PROBLEM #2.

Given the transformation matrix T and the eigenvector basis matrix C — which are eigenvectors of T , calculate $D = C^{-1}TC$.

$$T = \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}$$

$$D = C^{-1}TC$$

where C^{-1} is

$$C^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ where } C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$= \begin{bmatrix} 0 & -1 \\ 3 & 7 \end{bmatrix} \frac{1}{(0)(7) - (-1)(3)}$$

$$\begin{aligned}
 &= \begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 1 \\ 6 & 14 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} -3 & 0 \\ 0 & 6 \end{bmatrix} \times \frac{1}{3}
 \end{aligned}$$

PROBLEM #3.

Given the transformation matrix T and the eigenvector matrix C — which are eigenvectors of T , calculate $D = C^{-1}TC$.

$$T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$D = C^{-1}TC$$

where C^{-1} is

$$\begin{aligned}
 C^{-1} &= \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \text{ where } C = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \frac{1}{(1)(1) - (0)(-1)}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}
 \end{aligned}$$

PROBLEM #4.

Given a diagonal matrix $D = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$, and a change of basis matrix $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ with inverse, $C^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$, calculate $T = CDC^{-1}$.

$$\begin{aligned}
 T &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} a & 2a \\ 0 & a \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}.
 \end{aligned}$$

PROBLEM #5.

Given $T = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$, calculate T^3 .

$$\begin{aligned}
 T^3 &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}^3 \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 125 & 0 \\ 0 & 64 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 125 & 64 \\ 125 & 128 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 186 & -61 \\ 122 & 3 \end{bmatrix}.
 \end{aligned}$$

PROBLEM #6.

Given that $T = \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & -1/3 \\ 1 & 7/3 \end{bmatrix}$, calculate T^3 .

$$\begin{aligned}
 T^3 &= \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}^3 \begin{bmatrix} 0 & -1/3 \\ 1 & 7/3 \end{bmatrix} \\
 &= \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} 0 & -1/3 \\ 1 & 7/3 \end{bmatrix} \\
 &= \begin{bmatrix} -7 & 8 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1/3 \\ 1 & 7/3 \end{bmatrix} \\
 &= \begin{bmatrix} 8 & 6\frac{2}{3} \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 21 \\ 0 & -1 \end{bmatrix}
 \end{aligned}$$

PROBLEM #7.

Given $T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, calculate T^5 .

$$T^5 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}^5 \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$$