

**- PROBLEM -**

Prove by induction that for all  $n \geq \emptyset$ ,

$$\binom{n}{\emptyset} + \binom{n}{1} + \dots + \binom{n}{n} = 2^n.$$

In the inductive step use Pascal's Identity,

$$\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$$

**- INDUCTIVE PROOF -****Basic Step,**

$$\binom{\emptyset}{\emptyset} = \frac{\emptyset!}{\emptyset! \emptyset!} = 2^{\emptyset}$$

**Inductive Step,**

Let  $k \in \mathbb{Z}_+$ , where  $k \neq \emptyset$   
Let  $\ell \in [\emptyset, k]$ ,

$$H_\ell \rightarrow \sum_{i=0}^{\ell} \binom{\ell}{i} = 2^\ell$$

**- INDUCTIVE PROOF CONT. -**

Using Pascal's Identity, two equivalences can easily be calculated,

$$\begin{aligned} ① \quad \binom{n+1}{n+1} &= \binom{n}{n} + \binom{n}{n+1} \\ &= \binom{n}{n} + \emptyset \end{aligned}$$

$$\begin{aligned} ② \quad \binom{k+1}{\emptyset} &= \binom{k}{-1} + \binom{k}{\emptyset} \\ &= \emptyset + \binom{k}{\emptyset} \end{aligned}$$

① Frame the summation in similar terms of Pascal's Identity.

$$\binom{k+1}{0} + \binom{k+1}{1} + \binom{k+1}{2} + \dots + \binom{k+1}{k} + \binom{k+1}{k+1}$$

② Replace intermediate terms with Pascal equivalents,

$$\begin{array}{c} \text{Begin} \\ \boxed{\binom{k+1}{0}} \quad \binom{k}{0} + \binom{k}{1} + \dots + \underbrace{\binom{k}{k-1} + \binom{k}{k}}_{\text{TERM #2}} + \boxed{\binom{k+1}{k+1}} \\ \text{End} \end{array}$$

③ Replace terminal terms,

$$\begin{array}{c} \text{Begin} \quad \text{same} \quad \text{same} \\ \boxed{\binom{k}{\emptyset}} + \boxed{\binom{k}{\emptyset}} + \underbrace{\binom{k+1}{1} + \binom{k+1}{1}}_{T \# 1} + \underbrace{\binom{k+1}{2} + \binom{k+1}{2}}_{T \# 2} \\ \text{End} \end{array}$$

⑦ Simplify the (geometric?) series

$$\sum_{i=\emptyset}^k \binom{k}{i} = 2 \cdot \underbrace{2 \cdot \dots \cdot 2}_{k-1 \text{ times}}$$

⑧ State the equivalence,

$$\sum_{i=\emptyset}^k \binom{k}{i} = 2^k \quad \text{QED}$$

④ Remove grouped terms via factoring.

$$2 \cdot (\binom{k}{0} + \binom{k}{1} + \binom{k}{2} + \dots + \binom{k}{k})$$

⑤ Re-express the summation as contiguous

$$2 \cdot \sum_{i=\emptyset}^k \binom{k}{i}$$

⑥ State the recurrence,

$$\sum_{i=0}^k \binom{k}{i} = 2 \cdot \sum_{i=\emptyset}^{k-1} \binom{k-1}{i}$$