

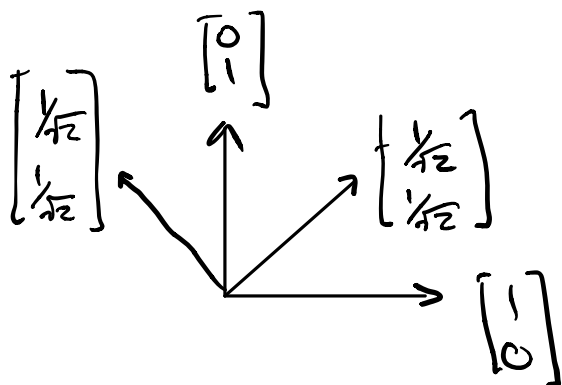
Doing transformations  $f$ -basis using  $e$  basis.

To rotate a vector  $u_f$  in the  $f$ -basis,

$= B^{-1} R B u_f$ , where  $B$  is the matrix of  $f$ 's basis vectors in the frame of  $e$ .

$R$  is a rotation in  $e$ .

$B^{-1}$  is the matrix of  $e$ 's basis vectors in the frame of  $f$ .



Example:

$$\underbrace{\frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}}_{B^{-1}} \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}}_R \underbrace{\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}}_B \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{u_f} = u_f \text{ rotated}.$$

$$B^{-1} R B = \frac{1}{2\sqrt{2}} \begin{bmatrix} -2 & -2 \\ 10 & 6 \end{bmatrix}$$