What is the mathematical concept for describing a relationship of subsets to a secondary set?

How do you relate the powerset to a set like $\mathbb{R}$?

$$f: P(S) \to \mathbb{R}$$

What is this morphism.

The interpretation of a Partially Ordered Set (Poset).

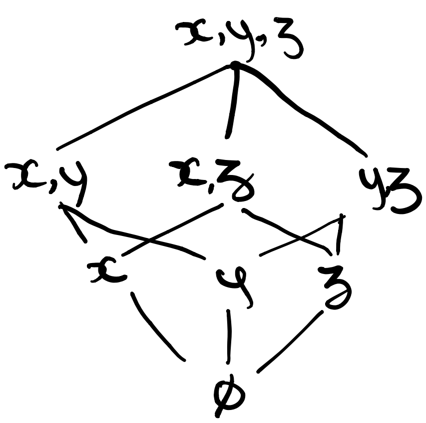


Diagram 1: Hasse diagram of a Partially Ordered Set.

- Posets are a construction of elements and a binary relation: <u>xRy.</u>

  - Those elements: $\emptyset, \{x\}, \{x,y\}$.. have their dimensionality eroded. As vectors

$$\dim(\emptyset) = 0$$
$$\dim([x]) = 1$$
$$\dim([x,y]^T) = 2$$

Interpretation as Set-Function

This is just a relation defined on many sets to a set.

Let F be a family of sets.
Let D be an arbitrary set

There is a morphism f where

$$\forall_{S \in F}: f: S \to D \text{ is defined.}$$

This is interesting but treats the sets $\underline{S}$ in the family uniformly and independent of one another. It doesn't capture the differing dimensionalities of the elements within those sets.

Function Spaces

Also interesting - seems more encompassing, but lacking any sort of framework, ecosystem or backing theory.

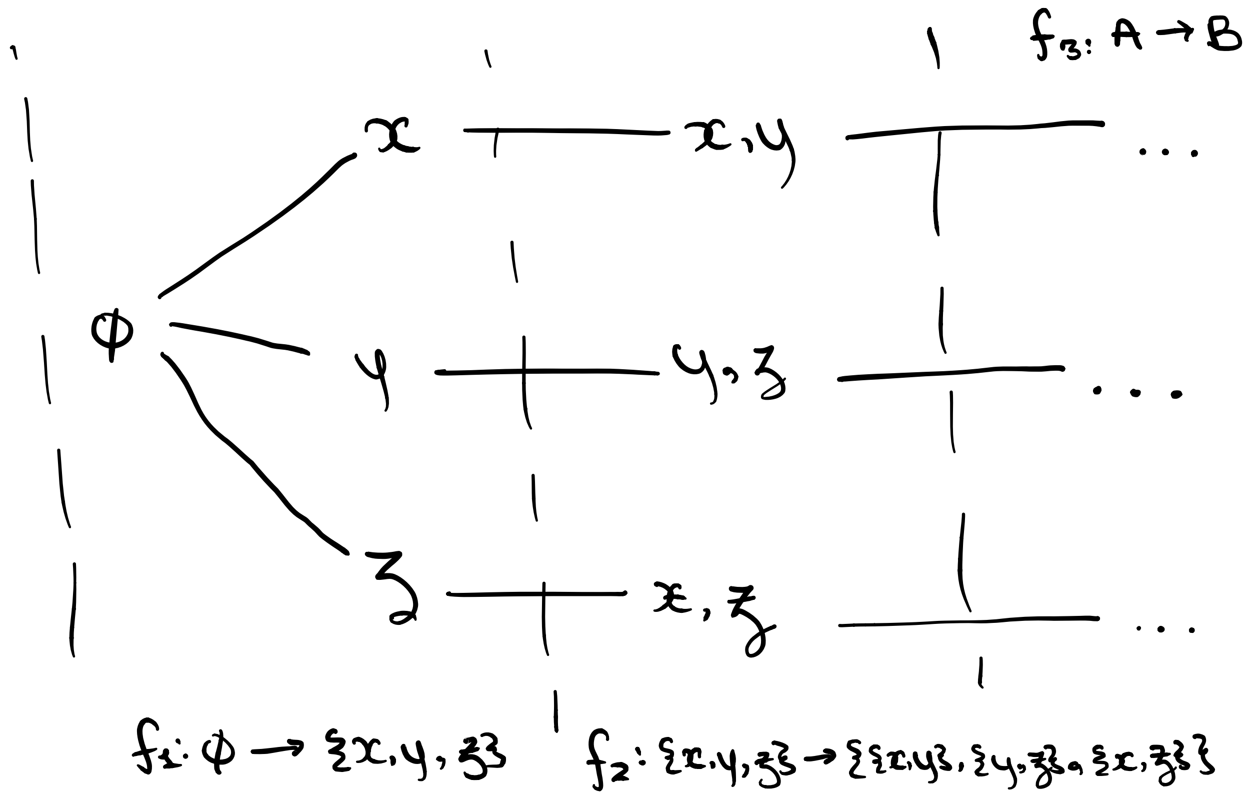
The idea is to construct a collection of functions between two sets.

Let G be a collection of morphisms
Let X and Y be two arbitrary sets.
Then,

$\forall_{g \in G}: g: X \to Y$ is a k-ary operation that is defined.

A Collection of Partial Binary Relations

Flimsy and too involved. Too much work to construct.



The collection of $f = \{f_1, f_2, ...\}$ can be used to form the relationships between a Powerset and an arbitrary set.

Conclusion

Need to find a better Theory for relationships between subsets and an arbitrary set.

Posets, Set-Functions, Function spaces, Collections of binary relationships are poor representations.

- Function Spaces are probaby the most precise representation of those options.