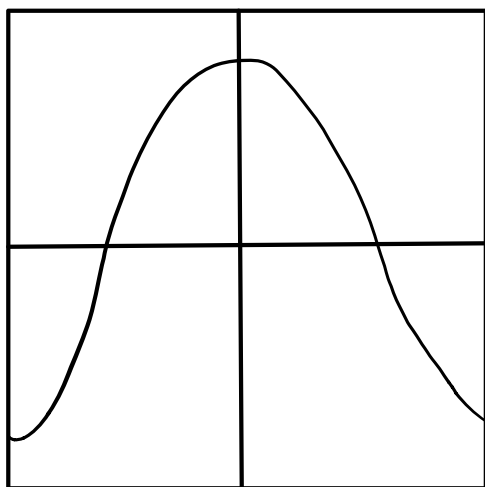


Building Taylor Series approximations, including the Maclaurin specific approximation, have interesting properties in themselves and sometimes belie interesting properties about the functions they seek to approximate!

Cosine is a well behaved function - it is continuous everywhere and infinitely differentiable.

$\cos x$



$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Differentiating $\cos x$.

$$f^{(0)}(x) = \cos x \quad @x=0: 1$$

$$f^{(1)}(x) = -\sin x \quad 0$$

$$f^{(2)}(x) = -\cos x \quad -1$$

$$f^{(3)}(x) = \sin x \quad 0$$

$$f^{(4)}(x) = \cos x \quad 1$$

Since every other term, more specifically - the odd terms of the power series has a zero coefficient, only the even terms are coefficients of the even powers.

This means each term of the cos approximation is a symmetrical function oriented to the vertical axis!

(*) where the cos terms are ± 1 and sin terms are zero.

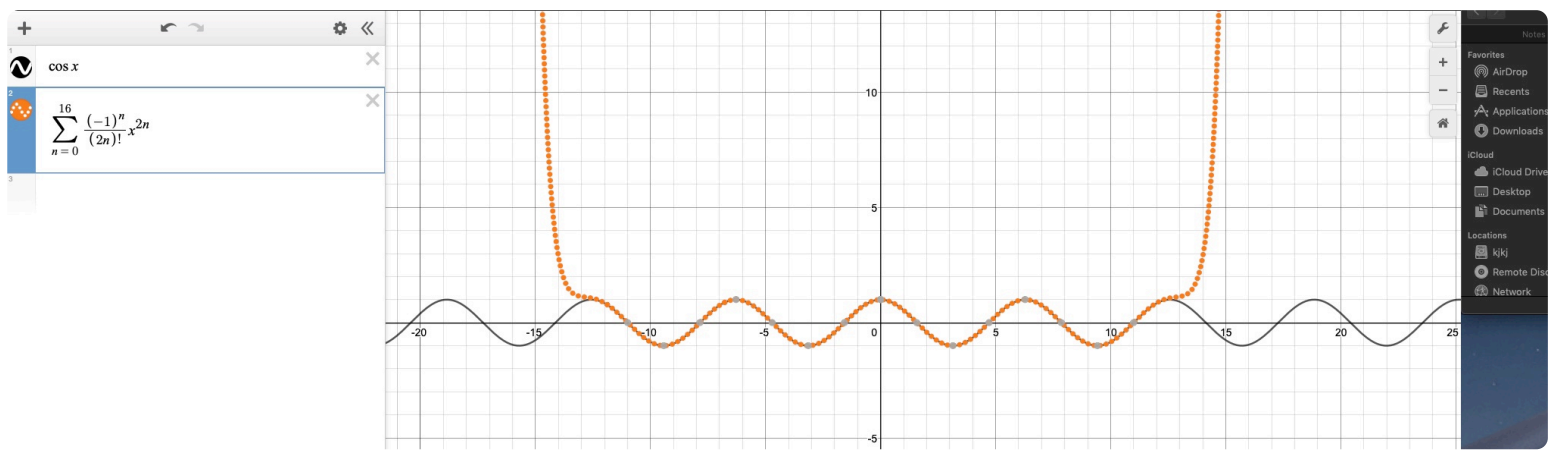
The Maclaurin series approximation for $\cos x$ is,

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$$

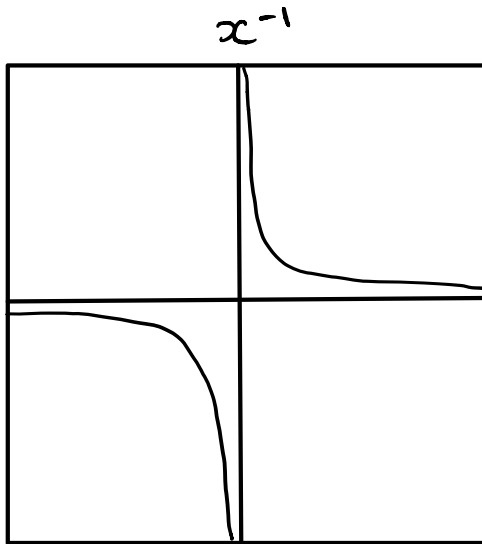
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

the $(-1)^n$ implements the oscillating behaviour!

where n is the n -th non-zero term of the series!



The second example features $f(x) = x^{-1}$.



$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Differentiating x^{-1} ,

$$f^{(0)}(x) = x^{-1}$$

@x=0: undefined

$$f^{(1)}(x) = x^{-1}$$

@x=1: 1

$$f^{(2)}(x) = -1/x^2$$

-1

$$f^{(3)}(x) = 2/x^3$$

2

$$f^{(4)}(x) = -6/x^4$$

-6

$$f^{(5)}(x) = 24/x^5$$

24

Instead form the Taylor Series at a well defined point like $x=1$,

$$\begin{aligned} g(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(p)}{n!} (x-p)^n \\ &= 1 - (x-1) + (x-1)^2 - (x-1)^3 + \dots \\ &= \sum_{n=0}^{\infty} (-1)^n (x-1)^n \end{aligned}$$

It is interesting that power series approximations ignore asymptotes entirely; perhaps the power series can not be defined.

In fact the function is not approximated at zero or for any values of x less than zero (when $p=1$).

This reflects the difficulty with which a power series approximates a poorly behaved function. Also, interestingly the tail ends of the approximation oscillate with asymptotically dominating inaccuracy.

1

$\frac{1}{x}$

2

$\sum_{n=0}^{16} (-1)^n (x-1)^n$

3

