## PROBLEM #1.

Calc H.

$f(x,y) = x^3y + x + 2y \qquad J = [3x^2y + 1, \; x^3 + 2]$

$$H = \begin{bmatrix} \partial x_1x_1 f & \partial x_1x_2 f \\ \partial x_2x_1 f & \partial x_2x_2 f \end{bmatrix} \qquad H = \begin{bmatrix} 6xy & 3x^2 \\ 3x^2 & 0 \end{bmatrix}$$

## PROBLEM #2.

Calc. H.

$f(x,y) = e^x \cos(y) \qquad J = [e^x\cos(y), \; -\sin(y)e^x]$

$$H = \begin{bmatrix} \partial x_1x_1 f & \partial x_1x_2 f \\ \partial x_2x_1 f & \partial x_2x_2 f \end{bmatrix} \qquad H = \begin{bmatrix} e^x\cos(y) & -\sin y\, e^x \\ -e^x\sin y & -\cos y\, e^x \end{bmatrix}$$

## PROBLEM #3.

Calc. H. calc $\frac{1}{2}[x,y]\,H\begin{bmatrix} x \\ y \end{bmatrix}$.

$f(x,y) = \frac{x^2}{2} + xy + \frac{y^2}{2} \qquad J = [x + y, \; x + y]$

$$H = \begin{bmatrix} \partial x_1x_1 f & \partial x_1x_2 f \\ \partial x_2x_1 f & \partial x_2x_2 f \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\frac{1}{2}[x, \; y]\begin{bmatrix} x+y \\ x+y \end{bmatrix} = \frac{x(x+y)}{2} + \frac{y(x+y)}{2}$$

$$= \frac{x^2 + xy + y^2 + xy}{2}$$

$$= \frac{x^2}{2} + xy + \frac{y^2}{2}.$$

(*) 2nd order derivative equals $f(x)$. similar to $e^x$...

## PROBLEM #4.

Calc H.

$$f(x,y,z) = x^2 e^{-y}\cos z$$

$$J = \left[2xe^{-y}\cos z,\ -e^{-y}x^2\cos z,\ -\sin z\, x^2 e^{-y}\right]$$

$$H = \begin{bmatrix} \partial x_1x_1 f & \partial x_1x_2 f & \partial x_1x_3 f \\ \partial x_2x_1 f & \partial x_2x_2 f & \partial x_2x_3 f \\ \partial x_3x_1 f & \partial x_3x_2 f & \partial x_3x_3 f \end{bmatrix}$$

$$H = \begin{bmatrix} 2e^{-y}\cos z & -e^{-y}2x\cos z & -\sin z\, 2xe^{-y} \\ -2xe^{-y}\cos z & e^{-y}x^2\cos z & \sin z\, x^2 e^{-y} \\ -2x\sin z\, e^{-y} & e^{-y}x^2\sin z & -\cos z\, x^2 e^{-y} \end{bmatrix}$$

## PROBLEM #5.

Calc. H.

$$f(x,y,z) = xe^y + y^2\cos(z)$$

$$J = \left[e^y,\ e^y x + 2y\cos z,\ -\sin z\, y^2\right]$$

$$H = \begin{bmatrix} \partial x_1x_1 f & \partial x_1x_2 f & \partial x_1x_3 f \\ \partial x_2x_1 f & \partial x_2x_2 f & \partial x_2x_3 f \\ \partial x_3x_1 f & \partial x_3x_2 f & \partial x_3x_3 f \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & e^y & 0 \\ e^y & e^y x + 2\cos z & 2y(-\sin z) \\ 0 & -2y\sin z & -\cos z\, y^2 \end{bmatrix}$$