22: 16 22 11 34 17 52 26 13 40 20 10 5 16 8 4 2 1.

Without expansion,

$a_0 = x$

$a_1 = 3(x)+1$

$a_2 = 3(3x+1)+1$

$a_3 = 3(3(3x+1)+1)+1$

With expansion,

$a_0 = x$

$a_1 = 3x+1$

$a_2 = 3^2x + 3 + 1$

$a_3 = 3^3x + 3(3) + (1)3 + 1$

$a_4 = 3^4x + 3(3)(3) + (1)3(3) + (1)3 + 1$

$a_5 = 3^5x + 3^4 + 3^3 + 3^2 + 3^1 + 1$

$$S_n = 3^n x + \sum_{i=0}^{n-1} 3^i$$

Find closed form of the Series Sum,

$$S_n = 3^n x + \sum_{i=0}^{n-1} 3^i$$

$$3S_n = 3^{(n+1)} x + \sum_{i=1}^{n} 3^i$$

The sum should be equivalent to $(3S_n - S_n)/2$.

$$3S_n - S_n = 3^{(n+1)} x + \sum_{i=1}^{n} 3^i - \left[3^n x + \sum_{i=0}^{n-1} 3^i\right]$$

$$= 3^n x(3-1) + 3^n - 3$$

$$= 2 \cdot 3^n x + 3^n - 3$$

$$2S_n = 3(2 \cdot 3^{n-1} x + 3^{n-1} - 1)$$

Find a power of 2.

$$2^y = 3^n(2x+1)$$

$$y = \lg[3^n \cdot (2x+1)]$$

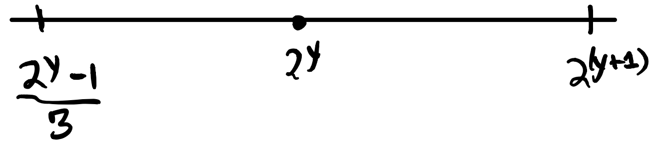
$$= \lg(3^n) + \lg(2x+1)$$

$$\lg(3^n) = y - \lg(2x+1)$$

$$3^n = 2^{(y-\lg(2x+1))}$$

$$n = \frac{\lg(2^{(y-\lg(2x+1))})}{\lg(3)}$$

$$n = \frac{y-\lg(2x+1)}{\lg(3)}$$



$$2^y = \frac{3(2\cdot 3^{n-1}x + 3^{n-1} - 1)}{2}$$

$$\frac{2^{y+1}}{3} = 2\cdot 3^{n-1}x + 3^{n-1} - 1$$

$$\frac{2^{y+1}+3}{3} = 3^{n-1}(2x+1)$$

$$2^{y+1}+3 = 3^n(2x+1)$$

$$= 3(3^{n-1}(2x+1) - 1)$$

Given $\underline{n}$, the number of iterations in the series and $\underline{y}$, the power of 2 you are trying to achieve, the number of cycles required equals $\underline{y+n}$.

Tests

Powers of 2,

$2 \rightarrow 1$ Let $y=2$