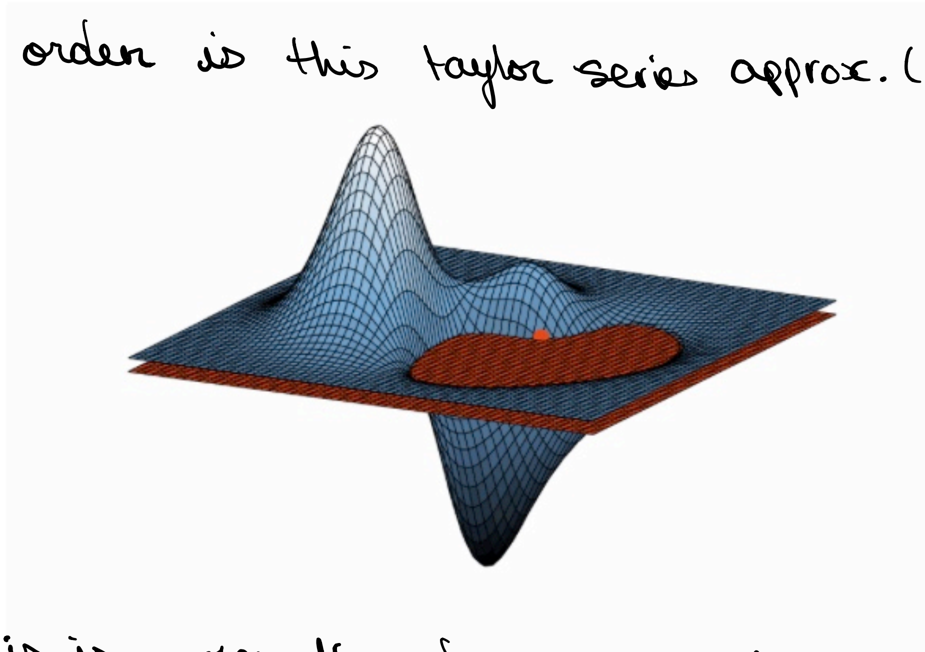
PROBLEM #1.

What order is this taylor series approx. (in red)?



This is a zeroeth order approximation where z is a constant value.

PROBLEM #2

What order taylor series expansion is this?

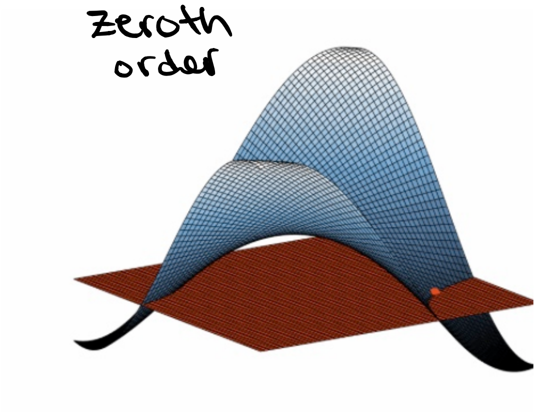
2nd Order - parabollic shape.

PROBLEM #3

which of the following are first order Taylor Series approx
blue is original func.

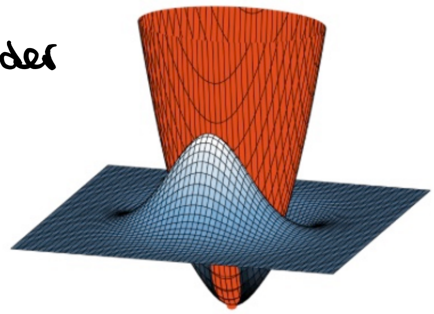
$f(x,y) = \sin(xy/5)$

Zeroth order



$f(x,y) = xe^{-x^2-y^2}$

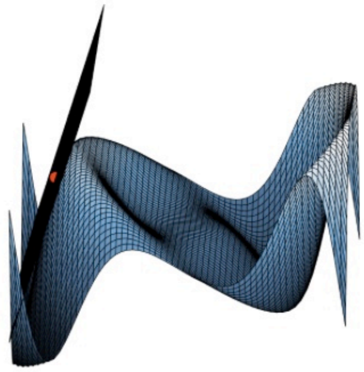
2nd order



$f(x,y) = (x^2+2x)e^{-x^2-y^2/5}$

(not an approx.)

$f(x,y) = x\sin(x^2/2 + y^2/4)$ (first order)



PROBLEM #4.

What is the first order Taylor Expansion of

$f(x,y) = xy^2e^{-x^4-y^2/2}$ around the point $(-1,2)$.

The second order expansion can be written.

$$f(x+\Delta x) = f(x) + J_f\Delta x + \tfrac{1}{2}\Delta x^T H_f \Delta x + \dots$$

$$J_f = \left[y^2e^{-x^4-y^2/2} + xy^2e^{-x^4-y^2/2}(-4x^3),\ 2yxe^{-x^4-y^2/2} + xy^2e^{-x^4-y^2/2}(-y)\right]$$

$$= \left[4e^{-1-2} + (-1)(4)e^{-1-2}(4),\ 2(2)(-1)e^{-1-2} + (-1)(4)e^{-1-2}(-2)\right]$$

$$= \left[4e^{-3} - 4e^{-3}(4),\ -4e^{-3} + 8e^{-3}\right]$$

$$= \left[-12e^{-3},\ 4e^{-3}\right]$$

$$g_1(x) = f(x) + J_f \Delta x$$
$$= (-1)(2)^2 e^{-3} + J_f \Delta x$$
$$= -4e^{-3} + J_f \Delta x$$
$$= -4e^{-3} - 12e^{-3}\Delta x + 4e^{-3}\Delta y$$

PROBLEM #5

What is the second order Hessian of the Taylor Expansion of $f(x,y) = \sin(\pi x - x^2 y)$ around point $(1, \pi)$.

$$J_f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

$$J_f = \left[\cos(\pi x - x^2 y)(\pi - 2xy), \cos(\pi x - x^2 y)(-x^2)\right]$$

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 x_2} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$= \begin{bmatrix} \partial_{x_1 x_1} f & \partial_{x_1 x_2} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f \end{bmatrix}$$

$$\partial_{x_1 x_1} f = \left[-\sin(\pi x - x^2 y)(\pi - 2xy)^2 + \cos(\pi x - x^2 y)(-2y)\right]$$

should be same Hessian usually symmetric.

$$\left\{\begin{array}{l} \partial_{x_1 x_2} f = \left[-\sin(\pi x - x^2 y)(-x^2)(\pi - 2xy) + \cos(\pi x - x^2 y)(-2x)\right] \\ \partial_{x_2 x_1} f = \left[-\sin(\pi x - x^2 y)(\pi - 2xy)(-x^2) + (-2x)\cos(\pi x - x^2 y)\right] \end{array}\right.$$

$$\partial_{x_2 x_2} f = \left[-\sin(\pi x - x^2 y)(-x^2)^2\right]$$

$$H_f = \begin{bmatrix} -\sin(\pi x - x^2 y)(\pi - 2xy)^2 + \cos(\pi x - x^2 y)(-2y), & -\sin(\pi x - x^2 y)(-x^2)(\pi - 2xy) + \cos(\pi x - x^2 y)(-2x) \\ -\sin(\pi x - x^2 y)(\pi - 2xy)(-x^2) + (-2x)\cos(\pi x - x^2 y), & -\sin(\pi x - x^2 y)(-x^2)^2 \end{bmatrix}$$

$@(x,y) = (1,\pi)$:

$$H_f = \begin{bmatrix} -\sin(\pi - \pi)(\pi - 2\pi)^2 + \cos(\pi - \pi)(-2\pi), & -\sin(\pi - \pi)(-1)(\pi - 2\pi) + \cos(0)(-2) \\ -\sin(0)(\pi - 2\pi)(-1) + (-2)\cos(\pi - \pi), & -\sin(\pi - \pi)(1) \end{bmatrix}$$

$$= \begin{bmatrix} -2\pi & -2 \\ -2 & 0 \end{bmatrix}$$