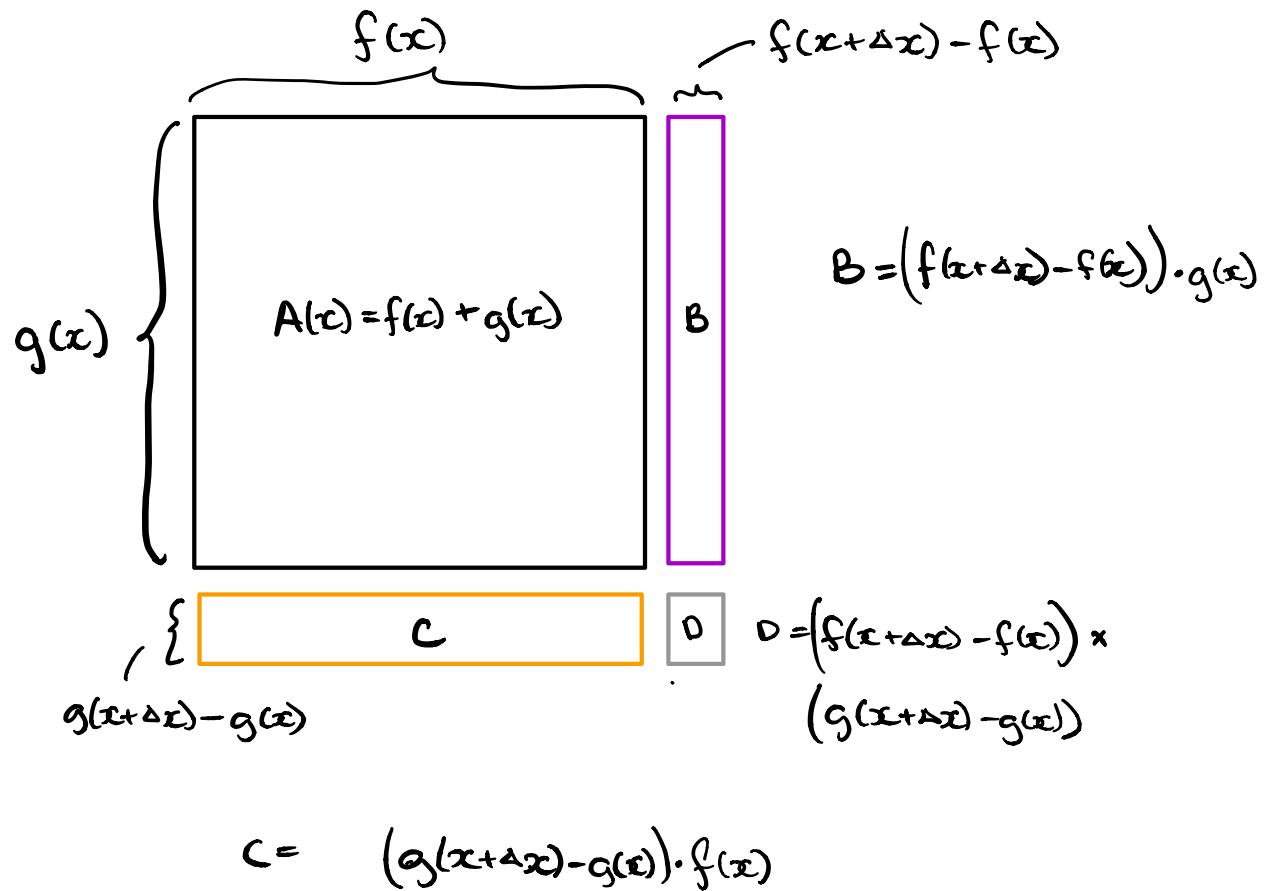


If you imagine two functions which relative to  $x$  produce nearly the same outputs you could graph those functions like:



$$\lim_{\Delta x \rightarrow 0} (\Delta A(x)) = \lim_{\Delta x \rightarrow 0} \left[ f(x)(g(x+\Delta x) - g(x)) + g(x)(f(x+\Delta x) - f(x)) + (f(x+\Delta x) - f(x)) \cdot (g(x+\Delta x) + g(x)) \right]$$

$$= \lim_{\Delta x \rightarrow 0} \left[ f(x)(g(x+\Delta x) - g(x)) + g(x)(f(x+\Delta x) - f(x)) \right].$$

could drop D because  
 $\Delta x \rightarrow 0, f(\Delta x) \rightarrow 0.$

Hawking figured out what  $\lim_{\Delta x \rightarrow 0} (\Delta A(x))$ , can now

$$\text{determine } \frac{d}{dx}(f(x)g(x)) = \lim_{\Delta x \rightarrow 0} \left[ \frac{\Delta A(x)}{\Delta x} \right]$$

$$= \lim_{\Delta x \rightarrow 0} \frac{f(x)(g(x+\Delta x) - g(x)) + g(x)(f(x+\Delta x) - f(x))}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \left( f(x) \frac{g(x+\Delta x) - g(x)}{\Delta x} + g(x) \frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

$$= f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$$