PROBLEM #1.

Calc Jacol. Matrix

$$u(x,y) = x^2 - y^2$$
$$v(x,y) = 2xy$$

$$J = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\[2ex] \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{bmatrix}$$

$$= \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2)$$
$$= 2x$$

$$\frac{\partial u}{\partial y} = -2y$$

$$\frac{\partial v}{\partial x} = 2y$$

$$\frac{\partial v}{\partial y} = 2x$$

PROBLEM #2

Calculate Jacobian matrix

$$u(x,y,z) = 2x + 3y$$
$$v(x,y,z) = \cos x \sin z$$
$$w(x,y,z) = e^{x+y+z}$$

$$J = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} & \dfrac{\partial u}{\partial z} \\[2ex] \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} & \dfrac{\partial v}{\partial z} \\[2ex] \dfrac{\partial w}{\partial x} & \dfrac{\partial w}{\partial y} & \dfrac{\partial w}{\partial z} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 0 \\ -\sin x \sin z & 0 & \cos z \cos x \\ e^{x+y+z} & e^{x+y+z} & e^{x+y+z} \end{bmatrix}$$

$$\frac{\partial u}{\partial x} = 2 \qquad \frac{\partial v}{\partial x} = -\sin x \sin z$$

$$\frac{\partial u}{\partial y} = 3 \qquad \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial z} = 0 \qquad \frac{\partial v}{\partial z} = \cos z \cos x$$

$$\frac{\partial w}{\partial x} = e^{x+y+z}$$

$$\frac{\partial w}{\partial y} = e^{x+y+z}$$

$$\frac{\partial w}{\partial z} = e^{x+y+z}$$

PROBLEM #3.

Calc Jacobian matrix.

$$u(x,y) = ax + by$$
$$v(x,y) = cx + dy \quad \text{, where } a, b, c, d \text{ are const.}$$

$$J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \text{just equals coefficients of the concerned var of the partial derivative.}$$

PROBLEM #4.

Calc Jacobian matrix & eval at $\vec{0}$.

$$u(x,y,z) = 9x^2y^2 + ze^x \qquad\qquad \frac{\partial u}{\partial x} = 18xy^2 + e^x z$$
$$v(x,y,z) = xy + x^2y^3 + 2z \qquad\qquad \frac{\partial u}{\partial y} = 2y9x^2$$
$$w(x,y,z) = \cos x \sin z \, e^y \qquad\qquad \frac{\partial u}{\partial z} = e^x$$

$$J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial w} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

$$\frac{\partial v}{\partial x} = y + 2xy^3$$
$$\frac{\partial v}{\partial y} = x + 3y^2x^2$$
$$\frac{\partial v}{\partial z} = 2$$

$$\frac{\partial w}{\partial x} = -\sin x \sin z \, e^y$$
$$\frac{\partial w}{\partial y} = e^y \cos x \sin z \qquad \frac{\partial w}{\partial z} = \cos z \cos x \, e^y$$

$$J = \begin{bmatrix} 18xy^2 + e^x z & 2y^9 z^2 & e^x \\ y + 2xy^3 & x + 3y^2x^2 & 2 \\ -\sin x \sin z e^y & e^y \cos x \sin z & \cos z \cos x e^y \end{bmatrix}$$

$$J\vec{O} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

PROBLEM #5.

Calc. Jacobian matrix..

$x(r,\theta,\phi) = r\cos\theta \sin\phi$

$y(r,\theta,\phi) = r\sin\theta \sin\phi$

$z(r,\theta,\phi) = r\cos\phi$

$$J = \begin{bmatrix} \dfrac{\partial x}{\partial r} & \dfrac{\partial x}{\partial \theta} & \dfrac{\partial x}{\partial \phi} \\[2mm] \dfrac{\partial y}{\partial r} & \dfrac{\partial y}{\partial \theta} & \dfrac{\partial y}{\partial \phi} \\[2mm] \dfrac{\partial z}{\partial r} & \dfrac{\partial z}{\partial \theta} & \dfrac{\partial z}{\partial \phi} \end{bmatrix}$$

$=$

$\dfrac{\partial x}{\partial r} = \cos\theta \sin\phi$

$\dfrac{\partial x}{\partial \theta} = -\sin\theta\, r\sin\phi$

$\dfrac{\partial x}{\partial \phi} = r\cos\theta \cos\phi$

$\dfrac{\partial y}{\partial r} = \sin\theta \sin\phi$

$\dfrac{\partial y}{\partial \theta} = r\cos\theta \sin\phi$

$\dfrac{\partial y}{\partial \phi} = r\sin\theta \cos\phi$

$\dfrac{\partial z}{\partial r} = \cos\phi$

$\dfrac{\partial z}{\partial \theta} = 0$

$\dfrac{\partial z}{\partial \phi} = -r\sin\phi.$

$$J = \begin{bmatrix} \cos\theta\sin\phi & -\sin\theta\, r\sin\phi & r\cos\theta\cos\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\phi & 0 & -r\sin\phi \end{bmatrix}$$