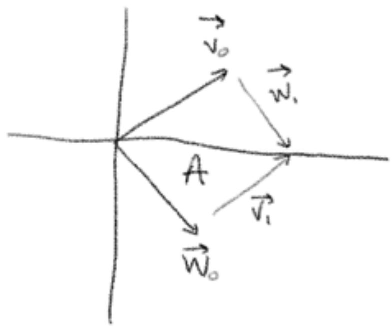
The cross product calculates the area of $\vec{v}$ and $\vec{w}$



Where A is the area.

Cross product has orientation

if $\vec{v}$ is right of $\vec{w}$ → +

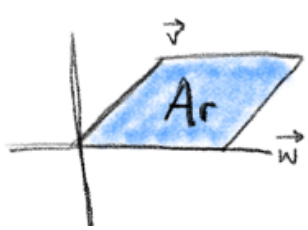
$\vec{v}$ is left of $\vec{w}$ → −

The cross product of two vectors can be calculated using the determinant.

The central idea is to conceive a process to calculate the area of ~~two~~ several vectors — which are basis vectors, then transform those vectors and apply the same area (parallelepiped) calculation.

This is just the determinant,

$$\det(A) = \text{Area}, \text{ where } A = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \end{bmatrix}$$



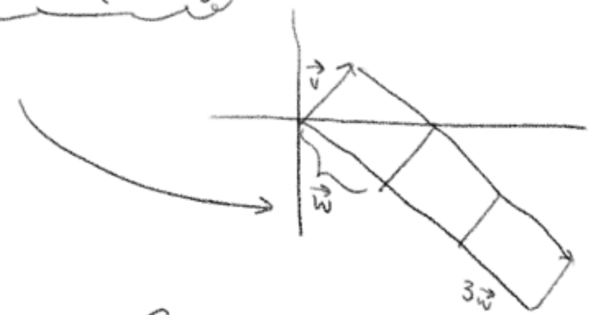
NOTES:

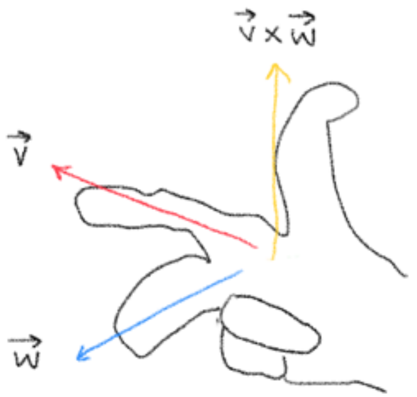
- Perpendicular

vectors have bigger cross products.

Equation Equality,

$$(3\vec{v}) \times \vec{w} = 3(\vec{v} \times \vec{w})$$





Cross Product Short Form

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \rightarrow \det \begin{bmatrix} \hat{\imath} & v_1 & w_1 \\ \hat{\jmath} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix}$$

$$\hat{\imath}(v_2 w_3 - v_3 w_2) + \hat{\jmath}(v_3 w_1 - v_1 w_3) + \hat{k}(v_1 w_2 - v_2 w_1)$$

Length = area of parallelogram $\Rightarrow$

Direction perpendicular to $\vec{v}$ and $\vec{w}$.

$$\det\left(\begin{bmatrix} \hat{\imath} & v_1 & w_1 \\ \hat{\jmath} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix}\right) = f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)$$

$\leftarrow$ This is a linear function and is a way of collapsing $\mathbb{R}^3$ into the number line $\mathbb{R}^1$.

$$= \begin{bmatrix} ? & ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

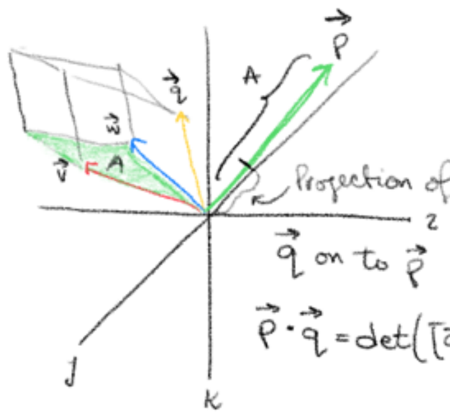
$\leftarrow$ Therefor a linear transformation must exist.

$$= \underbrace{\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}}_{\vec{p}} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

← Degeneration to $\mathbb{R}^1$ is the same as projection. Which is the same as the dot product.

This expands to,

$$p_1 x + p_2 \cdot y + p_3 z = x(v_2 w_3 - v_3 w_2) + y(v_3 w_1 - v_1 w_3) + z(v_1 w_2 - v_2 w_1)$$



$$\vec{p} \cdot \vec{q} = \det([\vec{q}\,\vec{v}\,\vec{w}])$$

Therefor,

$$p_1 = v_2 w_3 - v_3 w_2$$

$$p_2 = v_3 w_1 - v_1 w_3$$

$$p_3 = v_1 w_2 - v_2 \cdot w_1$$

This means that the volume of a parallelepiped is equal to $\underbrace{\vec{v} \times \vec{w}}_{\text{in } \mathbb{R}^2}$ as the length of an orthogonal vector to $\vec{v}$ and $\vec{w}$.