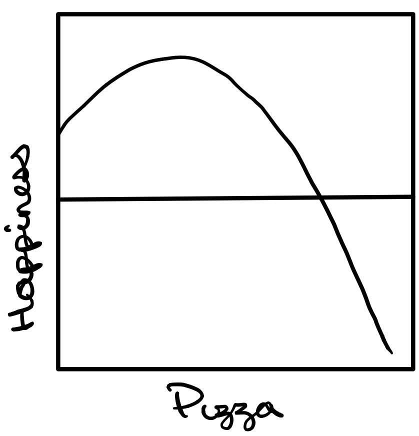
Sometimes when performing derivatives, all you have are the components of the final derivative. When the total derivative is formed by composition Its derivative can be found using the component derivative and the chain rule.

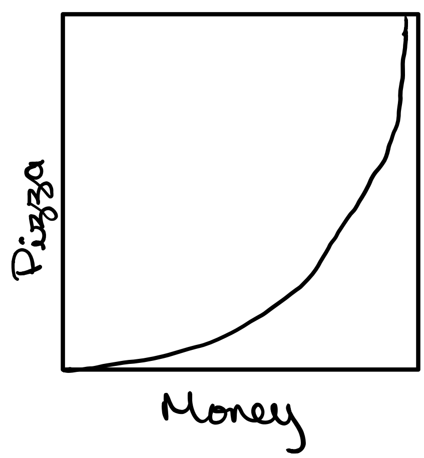
Example of Chain Rule:

Imagine two functions

$h(p)$: happiness over number of consumed pizzas.

$p(m)$: the number of consumed pizzas over the amount of available money.





$h(p) = -\frac{1}{3}p^2 + p + \frac{1}{5}$

$p(m) = e^m - 1$

Sometimes you can simply sub the embedded function in and differentiate,

$$h(p(m)) = -\frac{1}{3}(e^m - 1)^2 + (e^m - 1) + \frac{1}{5}$$

$$\frac{dh}{dm} = \frac{1}{3}e^m(5 - 2e^m)$$

However - sometimes complexity or circumstance inhibit this.

Instead you can differentiate each function relative to its input rather than relative to the absolute compositions input. With these relative derivatives you can chain their behaviours into continuity.

$$\frac{dh}{dm} = \frac{dh}{\cancel{dp}} \times \frac{\cancel{dp}}{dm}$$

$$= \frac{dh}{dm}$$

$$h(p) = -\frac{1}{3}p^2 + p + \frac{1}{5}$$

$$\frac{d}{dp}h(p) = 1 - \frac{2}{3}p$$

$$p(m) = e^m - 1$$

$$\frac{d}{dp}p(m) = e^m$$

To calculate $\frac{dh}{dm}$ with the with the chain rule.

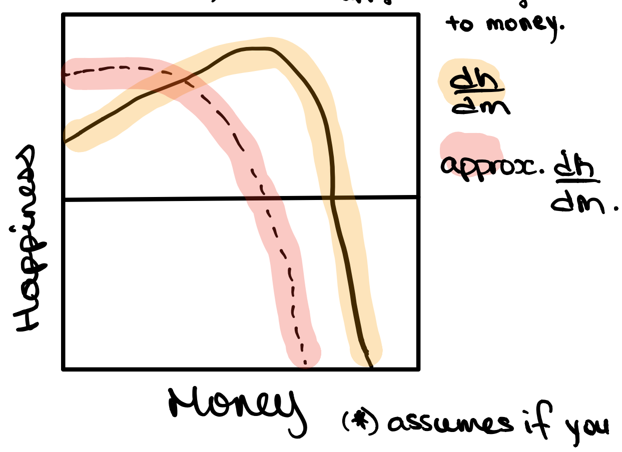
$$\frac{dh}{dm} = \frac{dh}{dp} \times \frac{dp}{dm}$$

$$= \left[1 - \frac{2}{3}p\right]e^m,$$ derivative is relative to p.

$$= \left[1 - \frac{2}{3}(e^m - 1)\right]e^m,$$ substituting $(e^m - 1)$ for p.

$$= \frac{1}{3}e^m(5 - 2e^m)$$

Graph of $\frac{dh}{dm}$: how the rate of happyness changes according to money.



(*) assumes if you have money, you will buy all the pizza you can and eat that pizza.

In this way the derivatives can be manipulated to calculate the chain rule, without having to know the actual functions which were derived.