
PROBLEM #1.

What are the partial derivatives with respect to x and y ?

$$f(x, y) = \pi x^3 + xy^2 + my^4$$

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (\pi x^3 + xy^2 + my^4) \\ &= \pi 3x^2 + y^2\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (\pi x^3 + xy^2 + my^4) \\ &= (0) + 2xy + 4my^3\end{aligned}$$

PROBLEM #2.

Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$ for $f(x, y, z) = x^2y + y^2z + z^2x$.

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (x^2y + y^2z + z^2x) \\ &= 2xy + (0) + z^2\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (x^2y + y^2z + z^2x) \\ &= x^2 + 2yz + (0)\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} (x^2y + y^2z + z^2x) \\ &= (0) + y^2 + 2zx\end{aligned}$$

PROBLEM #3.

Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial f}{\partial z}$ for $f(x, y, z) = e^{2x} \sin(y)z^2 + \cos(z)e^x e^y$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (e^{2x} \sin(y)z^2 + \cos(z)e^x e^y)$$

$$\begin{aligned}
&= e^{2x} \frac{\partial}{\partial x} (\sin(y)z^2) + \sin(y)z^2 \frac{\partial}{\partial x} e^{2x} + e^x \frac{\partial}{\partial x} \cos(z) e^y \\
&\quad + \cos(z) e^y \frac{\partial}{\partial x} e^x
\end{aligned}$$

$$\begin{aligned}
&= e^{2x}(0) + \sin(y)z^2 e^{2x} \cdot 2 + e^x(0) + \cos(z) e^y e^x(1) \\
&= \sin(y)z^2 e^{2x}(2) + \cos(z) e^y e^x
\end{aligned}$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (e^{2x} \sin(y)z^2 + \cos(z) e^x e^y), \text{ with factoring}$$

$$= \frac{\partial f}{\partial x} e^x (e^x \sin y z^2 + \cos z e^y)$$

$$= e^x \frac{\partial f}{\partial x} (e^x \sin y z^2 + \cos z e^y) + (e^x \sin y z^2 + \cos z e^y) \cdot \frac{\partial f}{\partial x} e^x$$

$$= e^x \left[\frac{\partial f}{\partial x} (e^x \sin y z^2) + \frac{\partial f}{\partial x} (\cos z e^y) \right] + (e^x \sin y z^2 + \cos z e^y) e^x$$

$$= e^{2x} \sin y z^2 + e^x (e^x \sin y z^2 + \cos z e^y)$$

$$= e^x \left[(e^x \sin y z^2 + \cos z e^y) + e^x \sin y z^2 \right]$$

$$(*) \underline{\text{way easier}}. \quad = e^x (e^x \sin y z^2(2) + \cos z e^y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} [e^x (e^x \sin y z^2 + \cos z e^y)]$$

$$\begin{aligned}
&= e^x \frac{\partial}{\partial y} (e^x \sin y z^2 + \cos z e^y) \\
&\quad + (e^x \sin y z^2 + \cos z e^y) \frac{\partial}{\partial y} (e^x)
\end{aligned}$$

$$= e^x \left[\frac{\partial}{\partial y} (e^x \sin y z^2) + \frac{\partial}{\partial y} (\cos z e^y) \right] + (e^x \sin y z^2 + \cos z e^y)(0)$$

$$= e^x [e^x z^2 \cos y + e^y \cos z(1)]$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} \left[e^x (e^x \sin y z^2 + \cos z e^y) \right]$$

$$= e^x \left[\frac{\partial}{\partial z} (e^x \sin y z^2) + \frac{\partial}{\partial z} (\cos z e^y) \right] + (e^x \sin y z^2 + \cos z e^y) \frac{\partial}{\partial z} e^x$$

$$= e^x \left[2z e^x \sin y + (-\sin z e^y) \right]$$

PROBLEM # 4.

Calculate the total derivative of $f(x, y) = \frac{\sqrt{x}}{y}$. $x(t) = t$
 $y(t) = \sin(t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

The f -partial derivative to y ,

$$\begin{aligned} \frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} \left(\frac{\sqrt{x}}{y} \right) \\ &= \frac{\sqrt{x}}{y} \frac{\partial}{\partial y} (y^{-1}) + y^{-1} \frac{\partial}{\partial y} (\sqrt{x}) \\ &= \frac{\sqrt{x}}{y} (-y^{-2}) + y^{-1} (0) \\ &= -\frac{\sqrt{x}}{y^2} \end{aligned}$$

The f -partial derivative to x ,

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} \left(\frac{\sqrt{x}}{y} \right) \\ &= \frac{\sqrt{x}}{y} \frac{\partial}{\partial x} (y^{-1}) + y^{-1} \frac{\partial}{\partial x} (\sqrt{x}) \\ &= \frac{\sqrt{x}}{y} (0) + y^{-1} \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x}y} \end{aligned}$$

The derivative of y to $-t$,

$$\begin{aligned} \frac{dy}{dt} &= \frac{d}{dt} \sin t \\ &= \cos t \end{aligned}$$

The derivative of x -to- t ,

$$\frac{dx}{dt} = \frac{d}{dt} t \\ = 1$$

The total derivative is,

$$\begin{aligned}\frac{df}{dt} &= \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} \\ &= \frac{1}{2\sqrt{xy}} (1) + \left(-\frac{\sqrt{x}}{y^2} \cos t \right) \\ &= \frac{1}{2\sqrt{t} \sin t} - \frac{t \cos t}{\sin^2 t} \text{, substituting t definitions for } x \text{ and } y.\end{aligned}$$

PROBLEM #5.

Calculate the total derivative of

$$f(x, y, z) = \cos(x) \sin(y) e^{2z}$$

$$x(t) = t + 1$$

$$y(t) = t - 1$$

$$z(t) = t^2$$

The total derivative is,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

The f -partial derivative to x is,

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} \cos(x) \sin(y) e^{2z} \\ &= \cos x \frac{\partial}{\partial x} \sin(y) e^{2z} + \sin(y) e^{2z} \frac{\partial}{\partial x} \cos x \\ &= \cos x (0) + \sin(y) e^{2z} (-\sin x) \\ &= -\sin x \sin y e^{2z}\end{aligned}$$

The f -partial derivative to y is,

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \cos(x) \sin(y) e^{2z}$$

$$\begin{aligned}
 &= \sin y \frac{\partial}{\partial y} \cos x e^{2z} + \cos x e^{2z} \frac{\partial}{\partial y} \sin y \\
 &= \sin y(0) + \cos x e^{2z} \cos y \\
 &= \cos x \cos y e^{2z}
 \end{aligned}$$

The f -partial derivative to z is,

$$\begin{aligned}
 \frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} \cos(x) \sin(y) e^{2z} \\
 &= e^{2z} \frac{\partial}{\partial z} \cos x \sin y + \cos x \sin y \frac{\partial}{\partial z} e^{2z} \\
 &= e^{2z}(0) + \cos x \sin y e^{2z} \cdot 2 \\
 &= 2 \cos x \sin y e^{2z}
 \end{aligned}$$

The derivative of x -to- t ,

$$\begin{aligned}
 \frac{dx}{dt} &= \frac{d}{dt}(t+1) \\
 &= 1
 \end{aligned}$$

The derivative of y -to- t ,

$$\begin{aligned}
 \frac{dy}{dt} &= \frac{d}{dt}(t-1) \\
 &= 1
 \end{aligned}$$

The derivative of z -to- t ,

$$\begin{aligned}
 \frac{dz}{dt} &= \frac{d}{dt}(t^2) \\
 &= 2t
 \end{aligned}$$

The total derivative is,

$$\begin{aligned}
 \frac{df}{dt} &= \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt} \\
 &= -\sin x \sin y e^{2z}(1) + \cos x \cos y e^{2z}(1) + 2 \cos x \sin y e^{2z} \cdot 2t \\
 &= -\sin(t+1) \sin(t-1) e^{2t^2} + \cos(t+1) \cos(t-1) e^{2t^2} \\
 &\quad + 2 \cos(t+1) \sin(t-1) e^{2t^2} \cdot 2t \\
 &= e^{2t^2} [-\sin(t+1) \sin(t-1) + \cos(t+1) \cos(t-1) + 4t \cos(t+1) \sin(t-1)]
 \end{aligned}$$