

Simultaneous equations can be solved at once using a matrix to represent those simultaneous equations, a solution, and the result of that solution — expressing those components in a singular expression and then manipulating that expression with the matrix inverse which in turn will produce an equation equating the numerical values of the solution.

$A^{-1}A = I$, where I is the identity matrix

Suppose A is an invertible matrix — then the numerical solution to A can be found,

$$\begin{aligned} Ax &= y \\ A^{-1}Ax &= A^{-1}y \\ Ix &= A^{-1}y \\ x &= A^{-1}y \end{aligned}$$

This approach can be algebraically by changing the matrix into a reduced-row-echelon-form and using backpropagation to form a diagonal matrix.

Matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

Reduced Row Echelon Form

$$A \Leftrightarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \text{ subtract row\#1 from row\#2}$$

$$\Leftrightarrow \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \text{ subtract row\#1 from row\#3.}$$

- Apply the same ops to y .

Backpropagation.

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 6 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 15 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 2 \end{bmatrix}.$$

□.