⇒ Goal to change from standard basis to orthogonal basis.

PROBLEM # 1.

What is v in the basis of b_1 and b_2, given

$$v = [5\ -1]^T,\ b_1 = [1\ 1],\ b_2 = [1\ -1]^T.$$

1. Check if bases vectors are orthogonal,

$$b_1 \cdot b_2 = |b_1||b_2| \cos\theta$$

$$\frac{b_1 \cdot b_2}{|b_1||b_2|} = \cos\theta$$

If the basis vectors are orthogonal they will be 90° apart from one another.

$$0 = \cos 90$$
$$= \frac{(1)(1) + (1)(-1)}{\sqrt{2}\sqrt{2}}$$
$$= 0/2.$$

Thus b_1 and b_2 are orthogonal.

2. Translate vector's bases,

Project v onto b_1,

$$\frac{b_1}{|b_1|} \times \frac{v \cdot b_1}{|b_1|} = v_{b_1}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \times \left(\frac{1}{\sqrt{2}}\right)^2 \times ((5)(1) + (-1)(1))$$

$$= 2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The scalar projection of v onto b_1 is 2.

Project v onto b_2,

$$v_{b_2} = \frac{b_2}{|b_2|} \times \frac{v \cdot b_2}{|b_2|}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \times \left(\frac{1}{\sqrt{2}}\right)^2 \times ((5)(1) + (1)(1))$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \frac{6}{2}$$

The scalar projection of v onto b_2 is 3.

$$v_b = \begin{bmatrix} v_{b_1} \\ v_{b_2} \end{bmatrix},\ [2\ 3]^T.$$

PROBLEM #2

What is v defined in the basis b_1 and b_2.

Given $v = \begin{bmatrix} 10 \\ -5 \end{bmatrix}, b_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, b_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$.

Project v onto b_1,

$$v_{b_1} = \frac{b_1}{|b_1|} \times \frac{b_1 \cdot v}{|b_1|}$$

$$= \begin{bmatrix} 3 \\ 4 \end{bmatrix} \times \frac{(10)(3) + (-5)(4)}{(\sqrt{3^2 + 4^2})^2}$$

$$= \frac{10}{25} [3 \ 4]^T.$$

Project v onto b_2.

$$v_{b_2} = \frac{b_2}{|b_2|} \times \frac{b_2 \cdot v}{|b_2|}$$

$$= \begin{bmatrix} 4 \\ -3 \end{bmatrix} \times \frac{((4)(10) + (-5)(-3))}{(\sqrt{(4)^2 + (-3)^2})^2}$$

$$= \frac{55}{25} \begin{bmatrix} 4 \\ -3 \end{bmatrix}.$$

$$v = \begin{bmatrix} v_{b_1} \\ v_{b_2} \end{bmatrix}$$

$$= \begin{bmatrix} 2/5 \\ 11/5 \end{bmatrix}.$$

PROBLEM #3.

Where $v = [2 \ 2]$, $b_1 = [-3 \ 1]$
$b_2 = [1 \ 3]$.

$$v_b = \begin{bmatrix} \frac{b_1}{|b_1|} \times \frac{b_1 \cdot v}{|b_1|} \\ \frac{b_2}{|b_2|} \times \frac{b_2 \cdot v}{|b_2|} \end{bmatrix}$$

$$= \begin{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix} \times \frac{((2)(-3) + (2)(1))}{(\sqrt{(-3)^2 + (1)^2})^2} \\ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \times \frac{((1)(2) + (2)(3))}{(\sqrt{(1)^2 + (3)^2})^2} \end{bmatrix}$$

$$= \left[-\tfrac{4}{10}[-3 \ 1]^T \ \ \tfrac{8}{10}[1 \ 3]^T \right] \longrightarrow \left[-\tfrac{2}{5} \ \ \tfrac{4}{5} \right]^T$$

PROBLEM #4.

$$V_{b_1} = \frac{((1)(2) + (1)(1) + (1)(0))}{(\sqrt{2^2 + 1^2 + 0^2})^2} = \frac{3}{5}$$

$$V_{b_2} = \frac{(1)(1) + (1)(-2) + (1)(-1)}{(\sqrt{(1)^2 + (-2)^2 + (-1)^2})^2}$$

$$= \frac{-2}{6}$$

$$V_{b_3} = \frac{((1)(-1) + (1)(2) + (1)(-5))}{(\sqrt{(-1)^2 + (2)^2 + (-5)^2})^2}$$

$$= \frac{-4}{30}$$

$$= \frac{-2}{15}$$

PROBLEM #5.

$$V_b = [1 \quad 0 \quad 1 \quad 1]^T$$