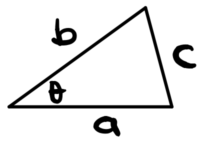
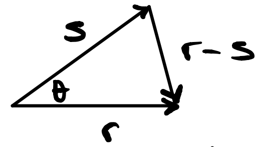
Cosine Rule:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

This can be drawn like,



This can be expressed by vectors,



The cosine rule transforms to,

$$|r-s|^2 = |r|^2 + |s|^2 - 2|r||s|\cos\theta$$

(*) $|r-s||r-s| = (r-s)\cdot(r-s)$, because we know $|v|^2 = v\cdot v$

$$(r-s)\cdot(r-s) = r\cdot r + 2(-s\cdot r) + (-s\cdot -s)$$

$$= |r|^2 - 2(s\cdot r) + |s|^2$$

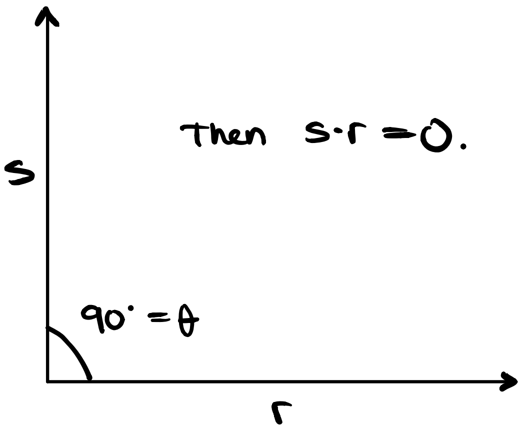
$$|r|^2 + |s|^2 - 2|r||s|\cos\theta \Longleftrightarrow |r|^2 - 2(s\cdot r) + |s|^2$$

$$-2|r||s|\cos\theta \Leftrightarrow -2(s\cdot r)$$

$$|r||s|\cos\theta \Leftrightarrow s\cdot r$$

WOW!

- if $\cos\theta = 0$ then $s\cdot r = 0$, by symmetry. (when $\theta = \frac{\pi}{2}$).
- if $\cos\theta = 1$ then $s\cdot r = |r||s|$ e.g ($\theta = 0$)
- if $\cos\theta = -1$ then $s\cdot r = -|r||s|$ e.g ($\theta = 180$)



Then $s\cdot r = 0$.

INTERPRETING THE DOT PRODUCT.

The important realization is,

1. when vectors are going same direction $s\cdot r$ is positive
2. when vectors are going different direction $s\cdot r$ is negative
3. when vectors are orthogonal, $s\cdot r$ is 0.