

### PROBLEM # 1.

Select graphs which can be modelled by nonlinear least squares.



### PROBLEM #2.

Given the Chi-Squared formula which are true?

$$\chi^2 = \sum_{i=1}^n \frac{[y_i - y(x_i, a)]^2}{\sigma_i^2}$$

- ☐ param  $\chi^2$  is the uncertainty value of the vars
- ☒ param  $\chi$  is squared so the effect of bad uncertainties are minimized
- ☒ take gradient of  $\chi^2$  set this to zero to determine fitting params
- ☐ when  $\chi^2$  is large the params used are a good fit for the data.

### PROBLEM #3.

The gradient of Chi-Squared with uncertainty is, <sup>with respect to fitting params</sup>

$$\frac{\partial \chi^2}{\partial a_j} = -2 \sum_{i=1}^n \frac{y_i - f(x_i, a)}{\sigma_i^2} \frac{\partial f(x_i, a)}{\partial a_j} \text{ for } j=1 \dots n$$

Can define  $z_j = \frac{\partial f(x_i, a)}{\partial a_j}$

Assuming  $f(x_i, a) = a_1 x^3 - a_2 x^2 + e^{-a_3 x}$

Differentiate  $f(x_i, a)$  — partially w- respect to the fitting params.

$$\frac{\partial f}{\partial a_1} = x^3$$

$$\frac{\partial f}{\partial a_2} = -x^2$$

$$\frac{\partial f}{\partial a_3} = e^{-a_3 x} \cdot (-x)$$

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#### PROBLEM #4.

Calculate the Jacobian of the Chi-Squared test of the function  $y(x_i; a) = a_1(1 - e^{-a_2 x_i^2})$ , assume  $\sigma^2 = 1$ .

$$\chi^2 = \sum_{i=1}^n \frac{[y_i - y(x_i; a)]^2}{(\sigma)^2}, \quad k = 1 \dots n$$

$\sigma = 1.$

$$\frac{\partial(\chi^2)}{\partial(a_1)} = -2 \sum_{i=1}^n \frac{(y_i - y(x_i; a))}{(1)^2} (1 - e^{-a_2 x_i^2})$$

$$\frac{\partial(\chi^2)}{\partial(a_2)} = -2 \sum_{i=1}^n \frac{[y_i - a_1(1 - e^{-a_2 x_i^2})]^2}{(1)^2} (x_i^2 e^{-a_2 x_i^2}) a_1$$

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#### PROBLEM #5.

Find  $\frac{\partial y}{\partial x_p}$  for  $y(x; \sigma, x_p, I, b) = b + \frac{1}{\sigma \sqrt{2\pi}} \exp\left\{\frac{-(x-x_p)^2}{2\sigma^2}\right\}$

$$\frac{\partial y}{\partial x_p} = \frac{1}{\sigma \sqrt{2\pi}} \exp\left\{\frac{-(x-x_p)^2}{2\sigma^2}\right\} \cdot \frac{(2)(-1)(x-x_p)}{2\sigma^2} \cdot (-1)$$

$$= \frac{1}{\sigma \sqrt{2\pi}} \exp\left\{\frac{-(x-x_p)^2}{2\sigma^2}\right\} \cdot \frac{x-x_p}{\sigma^2}$$