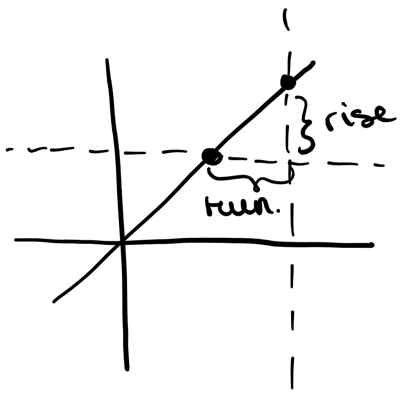
Gradients are derivatives.

- horizontal lines having a gradient of zero
- downward/upward sloping lines have +/- gradients.

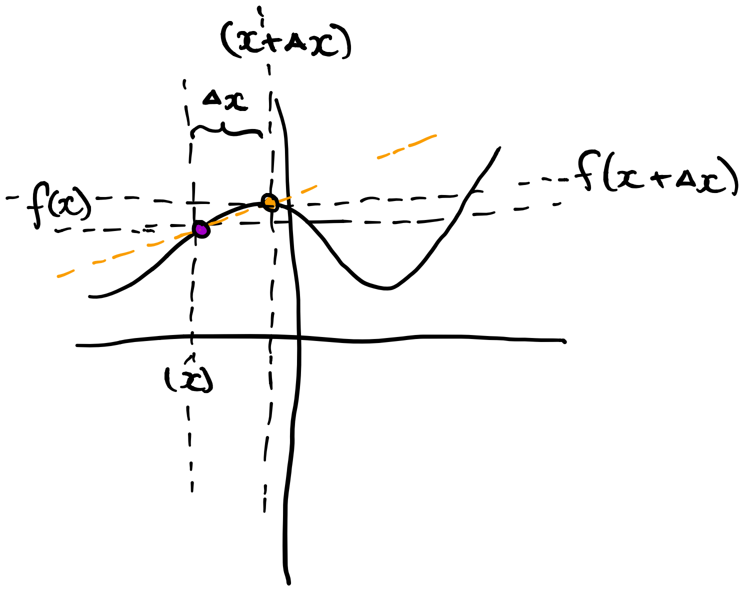
The gradient of a function,



Is the amount a function changes divided by the length of the interval (average).

gradient = "rise-over-run"

The gradient can be applied on very small intervals to approximate the true rate of change.



By decreasing delta x, the approximation becomes more accurate.

$$\text{Gradient at } x = \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

The gradient can also be written as,

$$\text{Gradient at } x = f'(x) = \frac{df}{dx}.$$

The $\lim_{\Delta x \to 0}$ means as x becomes extremely close to zero.

<u>Not</u> x is zero.

Example: Computing the derivative of $f(x) = 3x + 2$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[3(x+\Delta x) + 2] - [3x + 2]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x + 3\Delta x + 2 - 3x - 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3x} + 3\cancel{\Delta x} + \cancel{2} - \cancel{3x} - \cancel{2}}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 3$$

$$= 3.$$

(*) This derivative makes sense because the gradient of a line (e.g $3x+2 \Leftrightarrow mx+b$), is of a constant rate of change.

(*₂) The function <u>$3x$</u> + <u>2</u> has two components and the derivative/gradient can be computed for composite expressions.

($*_3$) This is known as the sum rule.

The Sum Rule - $\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$

Example: Computing the derivative of $5x^2$.

$$f'(x) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x) - f(x)}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0}\left(\frac{5(x+\Delta x)^2 - 5x^2}{\Delta x}\right)$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{5}(\cancel{x^2} + 2\Delta x + \Delta x^2) - \cancel{5x^2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{10\cancel{\Delta x} + 5\Delta x^{\cancel{2}}}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 10 + 5\Delta x$$

$= 10$, as x approaches zero $5\Delta x \to 0$.

($*$) This can be generalized for all functions of the form

$f(x) = ax^b \Leftrightarrow f'(x) = bax^{(b-1)}$.