PROBLEM #1.

Construct a series of simultaneous equations to represent these variables and their historical values.

Imagine,

On monday, you bought for €15.
1 apple
1 banana
1 carrot

On tuesday, you bought for €28.
3 apples
2 bananas
1 carrot

On wednesday, you bought for €23.
2 apples
1 banana
2 carrots.

Solution: create columns for each feature and tabulate their quantities—express the combination of those features as having a total price.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

PROBLEM #2.

Manipulate the expression $Mx = b$ into RREF.

$$M = \begin{bmatrix} 4 & 6 & 2 \\ 3 & 4 & 1 \\ 2 & 8 & 13 \end{bmatrix} \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ 7 \\ 2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & -2 \\ 0 & 4 & 11 \end{bmatrix} \begin{bmatrix} 2 \\ -20 \\ -16 \end{bmatrix}$$

$$\left(\Leftrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 10 \\ -56 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 10 \\ -8 \end{bmatrix} \right.$$

PROBLEM #3.b#4.

Same answer as problem #2.

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 2 & 8 & 13 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 0 & 5 & 12 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ -5/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ -1/2 \\ 0 \end{bmatrix}$$

PROBLEM #5

Convert this simultaneous equation to echelon form.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & -1 & 0 \end{bmatrix} x = \begin{bmatrix} 15 \\ -17 \\ -7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 15 \\ 17 \\ 5 \end{bmatrix}$$

PROBLEM #6.

Solve these simultaneous equations w- back substitution.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 15 \\ 17 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 3 \\ 7 \\ 5 \end{bmatrix}$$

PROBLEM #7.

Find the inverse of this matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} x = \begin{bmatrix} 15 \\ 28 \\ 23 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \left. \begin{matrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 2 & 1 & 2 \end{matrix} \right| \begin{matrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$= \left. \begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \end{matrix} \right| \begin{matrix} 1 & 0 & 0 \\ -3/2 & 1/2 & 1/2 \\ 3 & -1 & 0 \end{matrix}$$

$$= \left. \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{matrix} \right| \begin{matrix} -3/2 & 1/2 & 1/2 \\ 5/2 & -1/2 & -1/2 \\ 3 & -1 & 0 \end{matrix}$$

$$= \left. \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{matrix} \right| \begin{matrix} -3/2 & 1/2 & 1/2 \\ 5/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \end{matrix}$$

$$= \left. \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \right| \begin{matrix} -3/2 & 1/2 & 1/2 \\ 2 & 0 & -1 \\ 1/2 & -1/2 & 1/2 \end{matrix}$$