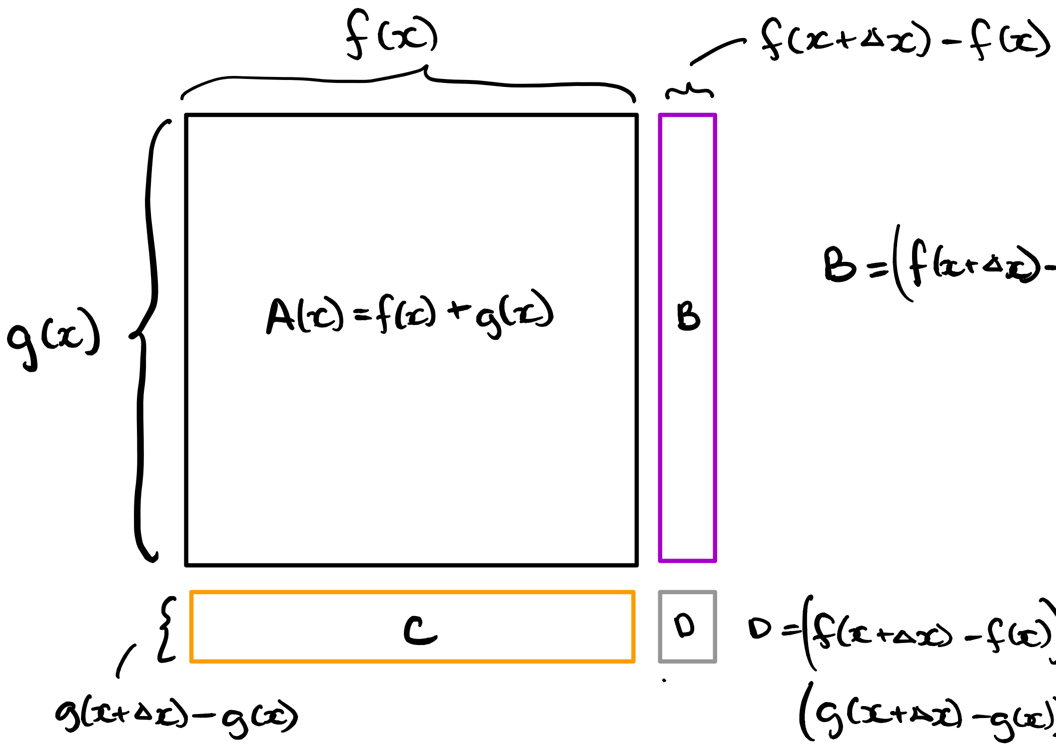
If you imagine two functions which relative to x produce nearly the same outputs you could graph those functions like:



$$B = \big(f(x+\Delta x) - f(x)\big) \cdot g(x)$$

$$D = \big(f(x+\Delta x) - f(x)\big) \times \big(g(x+\Delta x) - g(x)\big)$$

$$C = \big(g(x+\Delta x) - g(x)\big) \cdot f(x)$$

$$\lim_{\Delta x \to 0} (\Delta A(x)) = \lim_{\Delta x \to 0} \Big[f(x)\big(g(x+\Delta x) - g(x)\big) + g(x)\big(f(x+\Delta x) - f(x)\big) + \big(f(x+\Delta x) - f(x)\big) \cdot \big(g(x+\Delta x) + g(x)\big) \Big]$$

$$= \lim_{\Delta x \to 0} \Big[f(x)\big(g(x+\Delta x) - g(x)\big) + g(x)\big(f(x+\Delta x) - f(x)\big) \Big].$$

could drop $\underline{D}$ because $\Delta x \to 0$, $f(\Delta x) \to 0$.

Having figured out what $\lim_{\Delta x \to 0} (\Delta A(x))$, can now determine $\frac{d}{dx}(f(x)g(x)) = \lim_{\Delta x \to 0} \left[\frac{\Delta A(x)}{\Delta x} \right]$

$$= \lim_{\Delta x \to 0} \frac{f(x)(g(x+\Delta x) - g(x)) + g(x)(f(x+\Delta x) - f(x))}{\Delta x}$$

$$= \lim_{\Delta x \to} \left(f(x) \cdot \frac{g(x+\Delta x) - g(x)}{\Delta x} + g(x) \frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

$$= f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$$