
PROBLEM #1

Re-write the chain rule in Lagrange notation,

Leibniz notation - $\frac{dg}{dx} = \frac{dg}{dh} \frac{dh}{dx}$

Lagrange,

$$f'(x) = f'(h(x)) h'(x)$$

PROBLEM #2.

Use the chain rule to differentiate $\exp(x^2 - 3)$.

$$\text{Let } u = e^{v(x)}$$

$$\text{Let } v(x) = x^2 - 3$$

$$\frac{d}{dx} u(v(x)) = \frac{du}{dv} \frac{dv}{dx}$$

$$= e^{v(x)} 2x$$

$$= e^{(x^2-3)} 2x$$

PROBLEM #3.

Use chain rule to derive $\sin^3(x)$.

$$\text{Let } f(x) = \sin^3(x)$$

$$= (\sin x)^3$$

$$\text{Let } u(h) = h^3, h(x) = \sin x.$$

Derivative of u relative to h ,

$$\frac{d}{dh} h^3 = 3h^2$$

Derivative of h relative to x ,

$$\frac{d}{dx} \sin x = \cos x.$$

Derivative of u relative to x ,

$$\begin{aligned}\frac{du}{dx} &= \frac{d}{dx} f(x) \\ &= \frac{du}{dh} \frac{dh}{dx} \\ &= 3h^2 \cos x \\ &= 3 \sin^2 x \cos x.\end{aligned}$$

PROBLEM # 4.

Find the derivative of $\tan x$.

Let $f(x) = \tan x$.

$$\begin{aligned}f(x) &= \frac{r \sin \theta}{r \cos \theta}, \text{ where } r \text{ is the hypothetical hypotenuse.} \\ &= \sin \theta \cos^{-1} \theta\end{aligned}$$

Derivative of $f(x)$ relative to x ,

Let $u = \sin \theta$, $g(\theta) = \cos^{-1} \theta$

$$\begin{aligned}\frac{d}{dt} f(t) &= u(t) \frac{d}{dt} \cos^{-1} \theta + \cos^{-1} \theta \frac{du}{dt} \\ &= \sin \theta \frac{-1}{\cos^2 \theta} (-\sin \theta) + \cos^{-1} \theta \cos \theta \\ &= \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos \theta}{\cos \theta} \\ &= 1 + \tan^2 \theta.\end{aligned}$$

PROBLEM # 5.

Derive the function $f(x) = e^{\sin(x^2)}$

Let $u(h) = e^h$, $h(v) = \sin v$, $v(x) = x^2$

$$\frac{d}{dx} u(h(v(x))) = \frac{du}{dg} \frac{dg}{dv} , \text{ where } u(g) = u(h(g))$$

$$= \frac{du}{dg} \frac{dg}{dv} \cdot \frac{dg}{dv} , \text{ where } \frac{du}{dg} = \frac{du}{dh} \frac{dh}{dg}$$

Therefore $\frac{du}{dx} = \frac{du}{dh} \frac{dh}{dg} \frac{dg}{dx}$, when $u(x) = u(h(g(x)))$.

□

The derivative of the original $f(x)$,

$$\begin{aligned}\frac{d}{dx} f(x) &= \frac{du}{dh} \frac{dh}{dv} \frac{dv}{dx} \\ &= e^h \cos v 2x \\ &= e^{\sin v} \cos(x^2) 2x \\ &= e^{\sin(x^2)} \cos(x^2) 2x.\end{aligned}$$