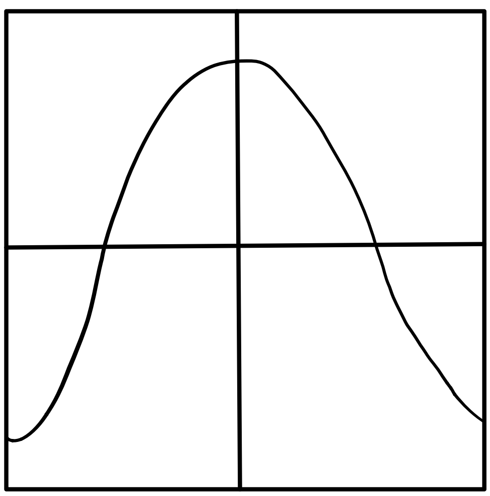
Building Taylor Series approximations, including the Maclaurin specific approximation, have interesting properties in themselves and sometimes belie interesting properties about the functions they seek to approximate!

Cosine is a well behaved function – it is continuous everywhere and infinitely differentiable.

cos x



$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Differentiating cos x.

$f^{(0)}(x) = \cos x$	@ $x=0$:	1
$f^{(1)}(x) = -\sin x$		0
$f^{(2)}(x) = -\cos x$		-1
$f^{(3)}(x) = \sin x$		0
$f^{(4)}(x) = \cos x$		1

(*) where the cos terms are 1/-1 and sin terms are zero.

Since every other term, more specifically – the odd terms of the power series has a zero coefficient, only the even terms are are coefficients of the even powers. This means each term of the cos approximation is a symmetrical function oriented to the vertical axis!

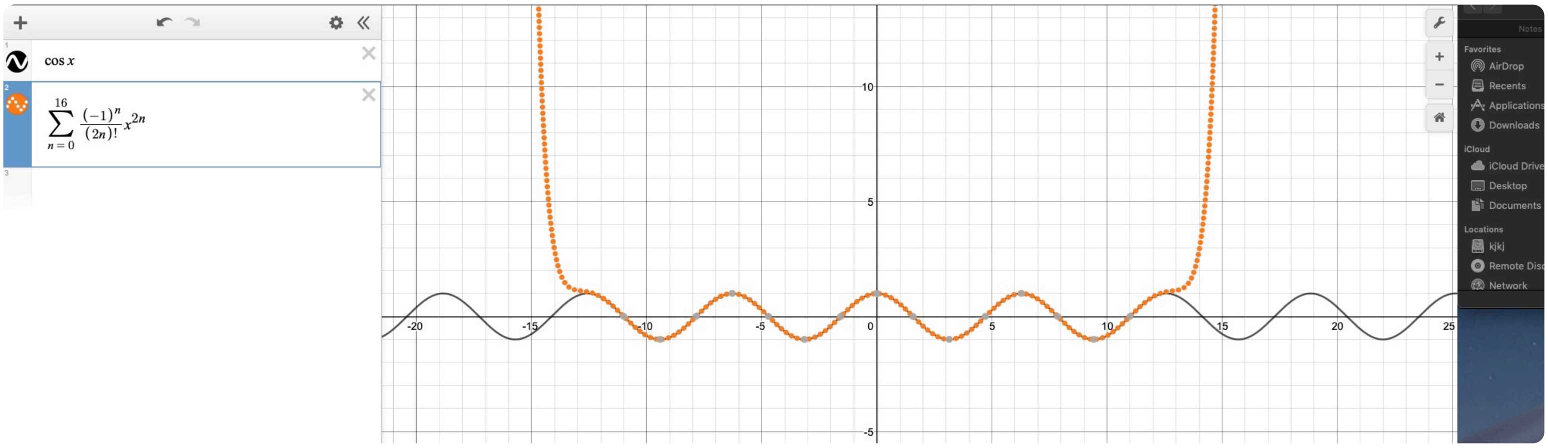
The Maclaurin series approximation for cos x is,

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$$

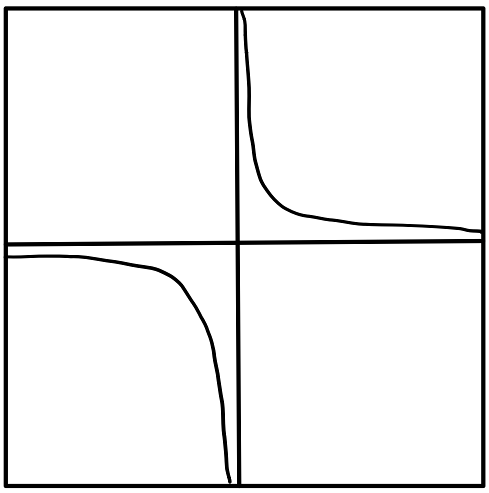
, where n is the n-th nonzero term of the series!

the $(-1)^n$ implements the oscillating behaviour!



The second example features $f(x) = x^{-1}$.

x^{-1}



$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Differentiating x^{-1},

$f^{(0)}(x) = x^{-1}$ @ $x=0$: undefined

$f^{(0)}(x) = x^{-1}$	@ $x=1$: 1
$f^{(1)}(x) = -1/x^2$	-1
$f^{(2)}(x) = 2/x^3$	2
$f^{(3)}(x) = -6/x^4$	-6
$f^{(4)}(x) = 24/x^5$	24

Instead form the Taylor Series at a well defined point like $x=1$.

$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(p)}{n!} (x-p)^n$$

$$= 1-(x-1)+(x-1)^2-(x-1)^3+\ldots$$

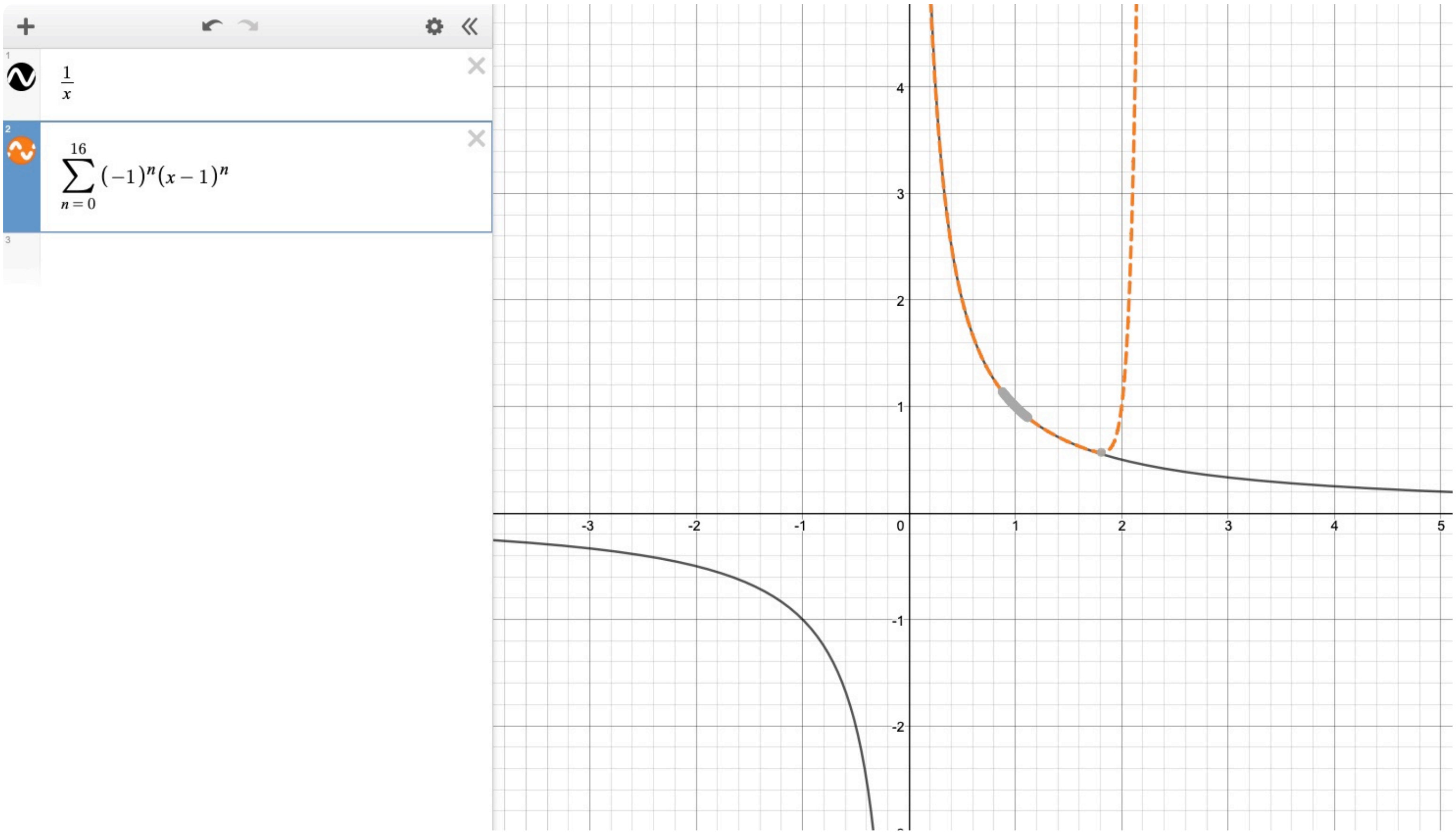
$$= \sum_{n=0}^{\infty} (-1)^n (x-1)^n$$

It is interesting that power series approximations ignore asymptotes entirely; perhaps the power series can not be defined.

In fact the function is not approximated at zero or for any values of x less than zero (when $p=1$).

This reflects the difficulty with which a power series approximates a poorly behaved function.

Also, interestingly the tail ends of the approximation oscillate with asymptotically dominating inaccuracy.



$\frac{1}{x}$
$\sum_{n=0}^{16} (-1)^n (x-1)^n$
-3
-2
-1
0
1
2
3
4
5