Cramer's rule is not always the best way (efficient).
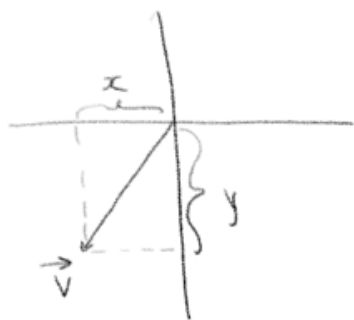
→ Gaussian is better

→ Row echelon & RREF is an alternative too.

Orthonormal — if $T(\vec{v}) \cdot T(\vec{w}) = \vec{v} \cdot \vec{w}$, for all $\vec{v}$ and $\vec{w}$

↳ Known as "rotation"

Obtaining A Solution to Linear Equations which use the Basis Vector — via Dot Product.

Both x and y can be calculated by dotting the basis vectors.

HOWEVER —

Whilst, non-applic. to stretched matrices, dotting can work for Orthonormal transformat; this is because the

$$\begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = x \qquad \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = y$$

!! This can not be extended to transformed vectors and basis vectors.

factors making up the

linear equations represent the current form of the old
vector space's basis vectors. As such, by orthonormal
definition those transformed basis vectors are relative
to the [output] vector in the same proportions to the
basis vectors to the original input vector.

___

Given the orthonormal example below
the input vector could be derived,

$$\begin{bmatrix} \cos(30°) & -\sin(30°) \\ \sin(30°) & \cos(30°) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x = \begin{bmatrix} \cos(30°) \\ \sin(30°) \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y = \begin{bmatrix} -\sin(30°) \\ \cos(30°) \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

___

A Similar Idea Can be Employed with
   Determinants.

This is because the properties of determinants remain relative to the original bases vectors even upon transformation.

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

$\underbrace{\qquad}_{A}$

Transf. y comp.

$\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Area $= \det(A) \, y$

$\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

Sub in
$x = 1$
transformed ver

$$\Rightarrow y = \frac{\text{Area}}{\det(A)} = \frac{\det\left(\begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\right)}$$

CRAMER'S RULE

$$\Rightarrow x = \frac{\text{Area}_{(y=1)}}{\det(A)} = \frac{\det\left(\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}\right)}$$

This rule can be generalized to all dimensions via constructing a parallelepiped (area) with the transformed input vector and a transformed basis vector.