

Partial differentiation is an extension of single variable.  
There is also a total derivative.

$$f(x, y, z) = \sin x e^{yz^2}$$

$$\frac{\partial f}{\partial x} = \cos x e^{yz^2}$$

$$\frac{\partial f}{\partial y} = \sin x e^{yz^2} z^2$$

$$\frac{\partial f}{\partial z} = \sin x e^{yz^2} 2yz$$

Can form a total derivative by normalizing the independent variables in relation to one another.

$$f(x) = \sin x e^{yz^2}, \text{ where } x = t - 1 \\ y = t^2 \\ z = \frac{1}{t}$$

If the function's definition is available or if it is analytically possible, can perform direct substitution into the function or take the derivative to get the total derivative.

$$\begin{aligned} f(x) &= \sin x e^{yz^2} \\ &= \sin(t-1) e^{t^2 \cdot \left(\frac{1}{t}\right)^2} \\ &= \sin(t-1) e^1 \end{aligned}$$

$$\frac{df}{dt} = \cos(t-1) e^1$$

When analytically that is not possible can instead

$$\frac{df(x,y,z)}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

(\*) note the use of single variate  $\frac{d}{dt} x$ . Partial notation had to be used for function  $f$  relative to several variables but the variables  $x, y$  and  $z$  are each relative to a single variable. In this case all of them are related  $t$ .

$$\frac{\partial f}{\partial x} = \cos x e^{yz^2} \quad \frac{\partial f}{\partial y} = \sin x e^{yz^2} z^2 \quad \frac{\partial f}{\partial z} = \sin x e^{yz^2} 2yz$$

$$\frac{dx}{dt} = 1 \quad \frac{dy}{dt} = 2t \quad \frac{dz}{dt} = -t^{-2}$$

On chaining the derivatives,

$$\frac{df(x,y,z)}{dt} = \cos x e^{yz^2} 1 + \sin x e^{yz^2} z^2 2t + \sin x e^{yz^2} 2yz(-t^{-2})$$

Then re-expressing in terms of  $t$  (substitution),

$$\begin{aligned} \frac{df(x,y,z)}{dt} &= \cos(t-1)e + 2^{t-1} \sin(t-1)e - 2^{t-1} \sin(t-1)e \\ &= \cos(t-1)e \end{aligned}$$