PROBLEM # 1

Calculate the partial derivatives of the function and its constraint and form the Lagrangian.

$$f(x) = \exp\left(-\frac{2x^2 + y^2 - xy}{2}\right).$$ does not have any minima

$$g(x) = x^2 + 3(y+1)^2 - 1$$
$$= 0$$

Since the contours of the function and the constraint are parallel — so too will their gradients be parallel.

$$\nabla f = \lambda \nabla g$$

This can be written more explicitly,

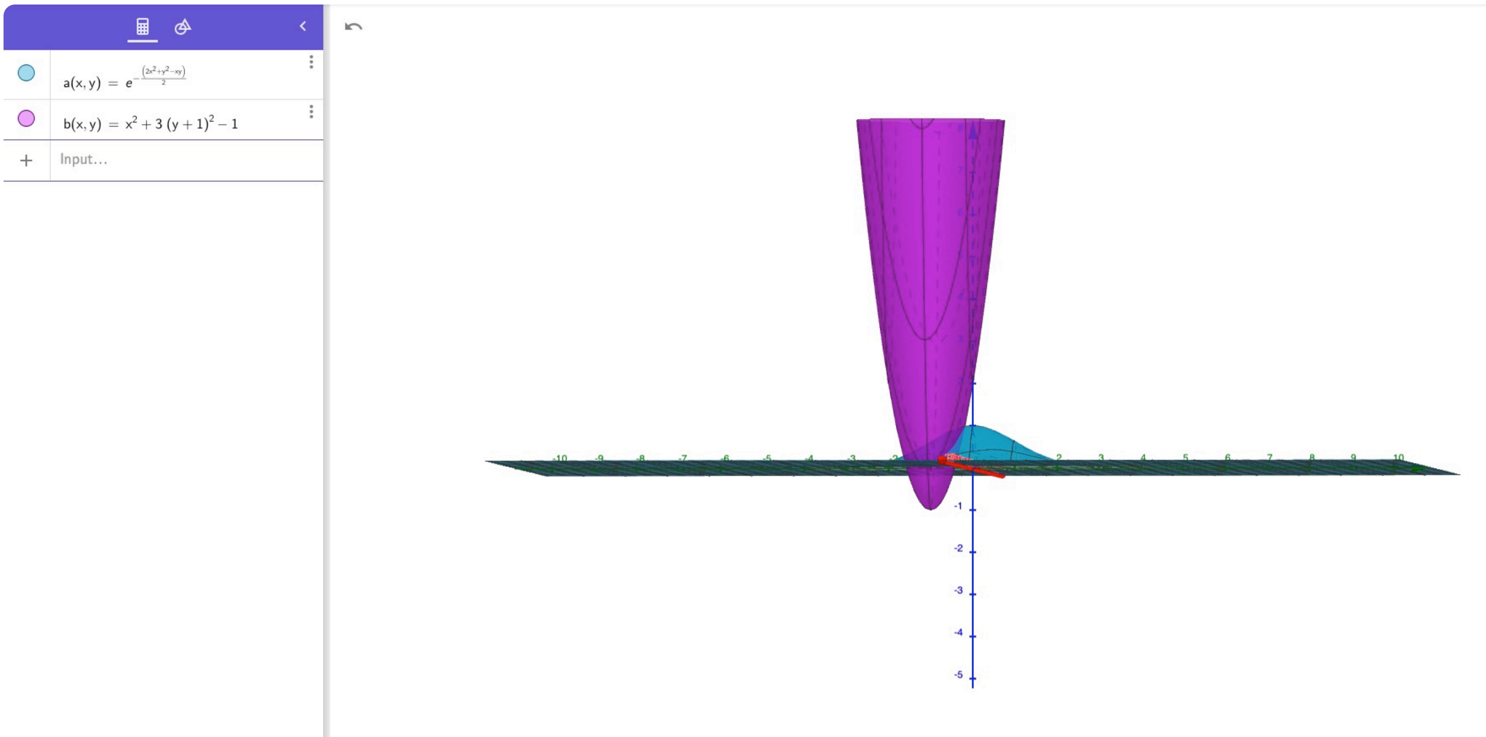
$$\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \lambda \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \end{bmatrix}$$

This can be combined into a Lagrangian,

$$\nabla \mathcal{L}(x, y, \lambda) = \begin{bmatrix} \frac{\partial f}{\partial x} - \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} - \lambda \frac{\partial g}{\partial y} \\ -g(x) \end{bmatrix}$$ given $g(x) = 0$.

- maybe $-\lambda g(x) = 0$
  $= g(x)$
- this combined a 2D function and a 1D constraint into a 3D problem.
- in collating the relationships of the func & constraint

Potentially via Newton-Raphson $\left\{ \begin{array}{l} \text{the problem has become} \\ \text{solvable as a linear eq.s} \end{array} \right. \leftarrow$



Partial derivatives.

$$g(x) = x^2 + 3(y+1)^2 - 1$$
$$= 0$$

$$\frac{\partial g}{\partial x} = 2x$$

$$\frac{\partial g}{\partial y} = 6(y+1)$$

PROBLEM #2

Find a zero-root of the Lagrangian.

Next let's define the vector, $\nabla\mathcal{L}$, that we are to find the zeros of; we'll call this "DL" in the code. Then we can use a pre-written root finding methodin scipy to solve.

```
from scipy import optimize

def DL (xyλ) :
    [x, y, λ] = xyλ
    return np.array([
            dfdx(x, y) - λ * dgdx(x, y),
            dfdy(x, y) - λ * dgdy(x, y),
            - g(x, y)
        ])

(x0, y0, λ0) = (-1, -1, 0)
x, y, λ = optimize.root(DL, [x0, y0, λ0]).x
print("x = %g" % x)
print("y = %g" % y)
print("λ = %g" % λ)
print("f(x, y) = %g" % f(x, y))
```

Run

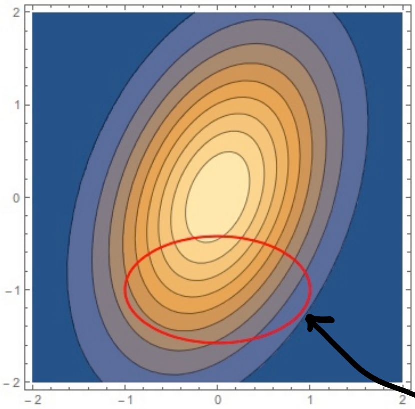
Reset

$$y = -1.21$$
$$x = 0.930942$$

PROBLEM #3

Find global minimum x-coord.

$x = 0.93$, using newton-raphson.



min is near outer contour

PROBLEM #4.

The gradient of $\mathcal{L}(x,y,\lambda)$ over x, y, λ is $\nabla\mathcal{L}$.

What function would be equal to $\mathcal{L}(x,y,\lambda)$ given it's $\nabla\mathcal{L}$ — the gradient.

$$\mathcal{L}(x,y,z) = f(x) - \lambda g(x).$$

PROBLEM #5.

Calculate the minimum of $f(x,y)$ given the constraint $g(x,y)$.

$$f(x,y) = \exp(x - y^2 + xy)$$
$$g(x,y) = \cosh(y) + x - 2 = 0$$

Write out the Langrangian,

$$\mathcal{L}(x, y, z) = f(x, y) - \lambda g(x, y)$$

$$\nabla \mathcal{L} = \begin{bmatrix} \frac{\partial f}{\partial x} - \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} - \lambda \frac{\partial g}{\partial y} \\ -g(x) \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = e^{(x - y^2 + xy)}(1 + y)$$

$$\frac{\partial f}{\partial y} = e^{(x - y^2 + xy)}(-2y + x)$$

$$\frac{\partial g}{\partial x} = 1$$

$$\frac{\partial g}{\partial y} = \sinh y$$