PROBLEM #1

The function $\beta(x, y) = x^T \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} y$ is.

- [x] positive definite
- [ ] not bilinear
- [ ] not symmetric
- [ ] not an inner product
- [x] symmetric
- [x] bilinear
- [ ] an inner product
- [ ] not positive definite

The function is similar to a inner product.

Inner products are,
- positive definite
- symmetrical
- bilinear

$$\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$
$$= 3 \pm \sqrt{5}/2$$

The inner product is only valid if it's matrix is
- positive definite
- symmetrical

$$|A - \lambda I| = 0$$
$$= \left| \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$$
$$= (2-\lambda)(1-\lambda) - (-1)(-1)$$
$$= \lambda^2 - 3\lambda + 1$$

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

<u>Symmetry</u>

A is symmetric on the left-to-right diag.

<u>Double Checking Symmetry</u>

$$z = x^T A y$$
$$= [x_1 \ x_2] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$
$$= [2x_1 - x_2 \quad x_2 - x_1] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$
$$= y_1(2x_1 - x_2) + y_2(x_2 - x_1)$$

<u>Positive Definite</u> (Definite Quadratic Form)

A matrix is positive definite if $z^T M z$ is greater than zero for all $z \in \mathbb{R}$.

$$y = z^T M z$$
$$= [z_1 \ z_2] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$
$$= [2z_1 - z_2 \quad -z_1 + z_2] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$
$$= 2z_1^2 - z_1 z_2 - z_1 z_2 + z_2^2$$
$$= (z_1 - z_2)^2 + z_1^2$$. this function is always squared

and greater than equal to zero.

Thus y is positive semidefinite

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u}\frac{du}{dt}$$

$$= \underbrace{\begin{bmatrix} \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2} \end{bmatrix}}_{J_f} \underbrace{\begin{bmatrix} \frac{\partial x_1}{\partial u_1} & \frac{\partial x_1}{\partial u_2} \\ \frac{\partial x_2}{\partial u_1} & \frac{\partial x_2}{\partial u_2} \end{bmatrix}}_{J_x} \underbrace{\begin{bmatrix} \frac{du_1}{t} \\ \frac{du_2}{t} \end{bmatrix}}$$

derivative vector u

PROBLEM #2

The function $\beta(x, y) = x^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} y$ is.

- [ ] positive definite
- [ ] not bilinear
- [ ] not symmetric
- [x] not an inner product
- [x] symmetric
- [x] bilinear
- [ ] an inner product
- [x] not positive definite

Double Checking

Symmetry

$$z = x^T A y$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 - x_2 & x_2 - x_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= y_1(x_1 - x_2) + y_2(x_2 - x_1)$$

PROBLEM #3

The function $\beta(x,y) = x^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} y$ is.

- [ ] positive definite
- [ ] not bilinear
- [x] not symmetric
- [x] not an inner product
- [ ] symmetric
- [x] bilinear
- [ ] an inner product
- [x] not positive definite

Double Checking

<u>Symmetry</u>

$$z = x^T A y$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2x_1 + x_2 & x_2 - x_1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= y_1(2x_1 + x_2) + y_2(x_2 - x_1)$$

PROBLEM #4

The function $\beta(x,y) = x^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} y$ is.

- [x] positive definite
- [ ] not bilinear
- [ ] not symmetric
- [ ] not an inner product
- [x] symmetric
- [x] bilinear
- [x] an inner product
- [ ] not positive definite

Double Checking

<u>Symmetry</u>

$$z = x^T A y$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= y_1(x_1) + y_2(x_2)$$

PROBLEM #5

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1 point

5. For any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ write a short piece of code that defines a valid inner product.

```python
import numpy as np

def dot(a, b):
  """Compute dot product between a and b.
  Args:
    a, b: (2,) ndarray as R^2 vectors

  Returns:
    a number which is the dot product between a, b
  """

  dot_product = abs(a - b)

  return dot_product

# Test your code before you submit.
a = np.array([1,0])
b = np.array([0,1])
print(dot(a,b))
```

Run

Reset

```
[1 1]
```