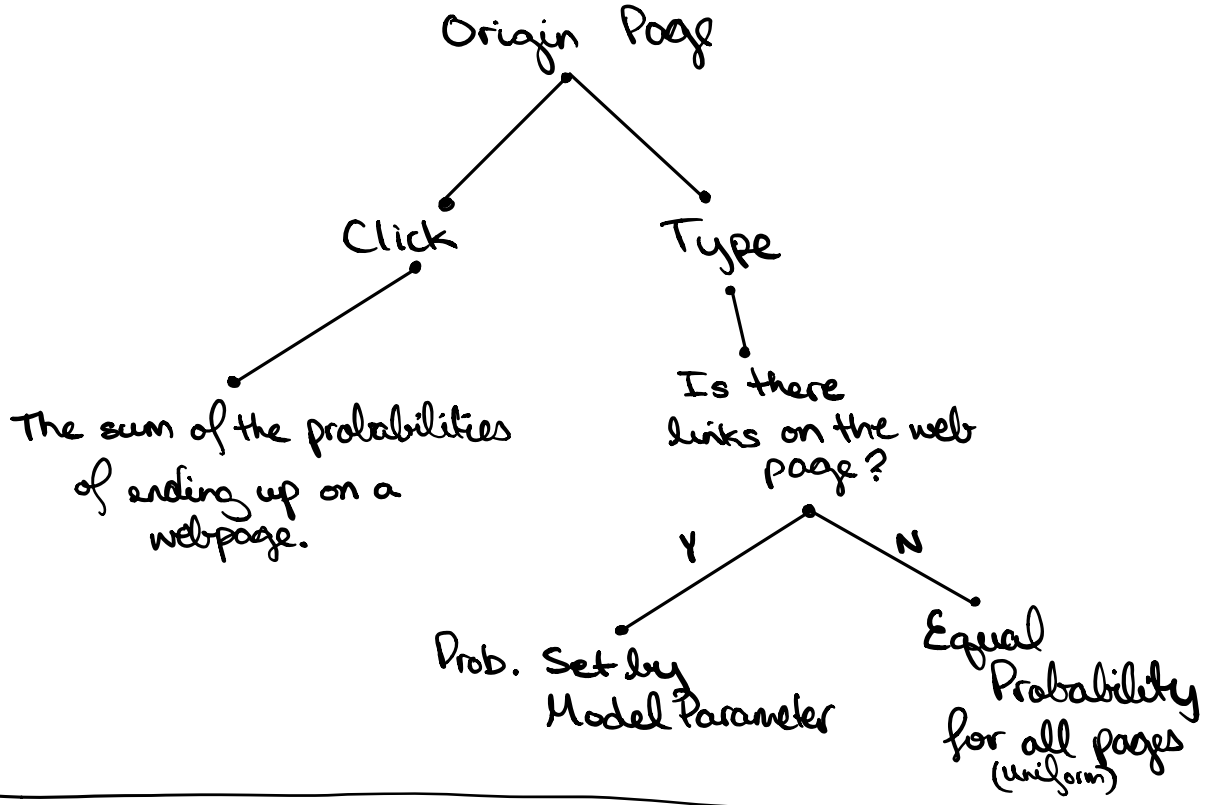
Google produced search results based on the connectivity of webpages.

A web-surfer navigates the web through these actions.

① Clicking on webpage links

② Navigating to a new page by typing.

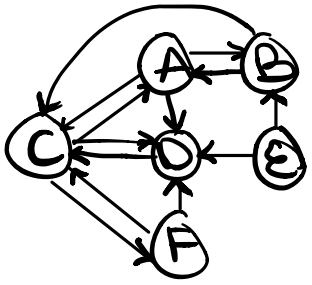
A workflow would look like.



Inherent in the design of PageRank algorithm is.

- the underlying connectivity is reflective a page's search rank.
- second is that important websites link to other important websites

A Graph of Page Connectivity



Each node represents a page and each directed edge is a link that connects the source webpage to the target.

Page surfers are modelled as making random walks choosing edges travel with a uniform probability dictated by the number of outbound edges of the current node.

Page surfers can also teleport (by typing).

Given the random walks, the graph, and a random initial state of users in the network - the traffic through those cycles will converge. Users can then be found predictably on certain pages with certain ratios/densities.

The number of page surfers on each page once the network activity converges is expressed by the vector $\underline{r}$.

$$r = \begin{bmatrix} r_A \\ r_B \\ r_C \\ r_D \\ r_E \\ r_F \end{bmatrix}$$

The number of users on each website can be generalized with the expression

$$r^{i+1} = Lr^i$$

This models the number of users per page at minute $\underline{i}$. It holds true before and after convergence.

After convergence $r^{(i+1)} = r^i$, can instead write

$$Lr = r$$

This is an eigenequation for matrix L with eigenvalue 1, determined by the probabilistic structure of the matrix L. (i.e network has no loss & normalized pr. & const. traffic)

The matrix L.

$$\begin{bmatrix} L_{A\to A} & L_{B\to A} & L_{C\to A} & L_{D\to A} & L_{E\to A} & L_{F\to A} \\ L_{A\to B} & L_{B\to B} & L_{C\to B} & L_{D\to B} & L_{E\to B} & L_{F\to B} \\ L_{A\to C} & L_{B\to C} & L_{C\to C} & L_{D\to C} & L_{E\to C} & L_{F\to C} \\ L_{A\to D} & L_{B\to D} & L_{C\to D} & L_{D\to D} & L_{E\to D} & L_{F\to D} \\ L_{A\to E} & L_{B\to E} & L_{C\to E} & L_{D\to E} & L_{E\to E} & L_{F\to E} \\ L_{A\to F} & L_{B\to F} & L_{C\to F} & L_{D\to F} & L_{E\to F} & L_{F\to F} \end{bmatrix}$$

Each column represent the (sum=1) probability leaving a page for another page. Each row represents how likely you are to enter a website from another (doesn't need to sum = 1).

Can try to calculate each of the eigenvectors of the network graph; however this isn't scalable for high dimension systems. Instead power iteration can be used — the power series.

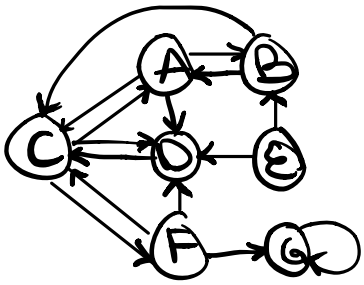
To add the opportunity for walks to jump/teleport to another vertice.
The model can be adjusted to include additional variables

$$M = dL + \frac{1-d}{n} J$$

- d is the prob of clicking a link
- $1-d$ is the prob of typing
- $J_{n \times n}$ is a matrix of 1 elements computing to equal probabilities of leaving any page for any other page.

Typically $1-d$ is small.

THE DAMPING PARAM



The power iteration applied to the network graph will yield (G) as the highest rank page. This is due to an oversimplified model which only allows for walks — once a path enters the reflexive (G) node the walk only has the opportunity to revisit the (G) node.