PROBLEM #1.

What is the characteristic polynomial and it's solutions?

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$Ax = \lambda x$$
$$0 = Ax - \lambda x$$
$$= (A - \lambda I)x$$
$$= \begin{bmatrix} (1-\lambda) & 0 \\ 0 & (2-\lambda) \end{bmatrix} x$$

Assume that matrix A has eigenvalues, use determinant as a structure with which to determine those eigenvalues,

$$|A - \lambda I| = 0$$
$$= \left| \begin{bmatrix} (1-\lambda) & 0 \\ 0 & 2-\lambda \end{bmatrix} \right|$$
$$= (1-\lambda)(2-\lambda) - (0)(0)$$
$$= \lambda^2 - 3\lambda + 2$$

The eigenvalues are $\lambda = 1, 2$.

PROBLEM #2.

Find the eigenvectors of A, $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

From question #1, eigenvalues are

$\lambda = 1, 2$

Substitute λs in to the eigen-matrix expression,

$$@\lambda=1: \begin{bmatrix} 1-1 & 0 \\ 0 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$$

$$@\lambda=2: \begin{bmatrix} 1-2 & 0 \\ 0 & 2-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Therefor $x_2 = 0$.

Therefor $x_1 = 0$.

With the constraints found from the backsubstitution get the eigenvalues,

$@\lambda=1: x = [x_1\ 0]^T$ $@\lambda=2: x = [0\ x_2]^T$

PROBLEM #3.

What is the characteristic polynomial it's solution?

$$A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$$

Refer to Problem #1 for full solution steps.

$$|A - \lambda I| = 0$$
$$= (3-\lambda)(5-\lambda) - (4)(0)$$
$$= \lambda^2 - (3+5)\lambda + 15 \text{, characteristic polynomial}$$

Solutions

$\lambda = 3, 5$

PROBLEM #4.

Find eigenvectors for $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$

$$0 = (A - \lambda I)x$$
$$= \begin{bmatrix} (3-3) & 4 \\ 0 & (5-3) \end{bmatrix} x \text{, } @\lambda = 3.$$
$$= \begin{bmatrix} 0 & 4 \\ 0 & 2 \end{bmatrix} x$$
$$= 2\begin{bmatrix} 2x_2 \\ x_2 \end{bmatrix}$$

Therefor x_2 must be zero.

$$0 = (A - \lambda I)x$$
$$= \begin{bmatrix} (3-5) & 4 \\ 0 & (5-5) \end{bmatrix} x \text{, } @\lambda = 5.$$
$$= \begin{bmatrix} -2 & 4 \\ 0 & 0 \end{bmatrix} x$$
$$= \begin{bmatrix} -2x_1 + 4x_2 \\ 0 \end{bmatrix}$$

Therefor $2x_1$ must be equal to x_2.