Previously looked @

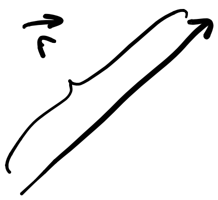
1. vector addition
2. scaling vector by number } mathematical properties a vector has (i.e a linear sys.).

Can now define,

1. length of vector (aka size)
2. dot product. (aka inner-scale or projection product)

Coordinate Systems

A vector is just a direction; it depends on a coord. sys. to manifest.



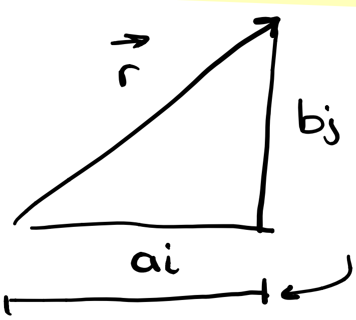
If the coord. sys. was made from $\hat{i}, \hat{j}$ where,

$i = [1 \;\; 0]^T$

$j = [0 \;\; 1]^T$

$\longrightarrow \quad r = ai + bj$

Calculating Length of Vector



height $= |b|$

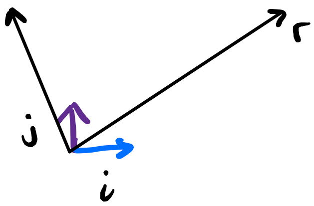
width $= |a|$

by the Pythagoreum Theorem $a^2 + b^2 = c^2$

Therefor <u>length</u> of r is

$$|r| = \sqrt{a^2 + b^2}$$

$\begin{bmatrix} s_i \\ s_j \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} s$



$r \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} r_i \\ r_j \end{bmatrix}$

Dot Product:

$$r \cdot s = r_i s_i + r_j + s_j$$
$$= (3)(-1) + (2)(2)$$
$$= 1$$

Properties of Dot Product

1. Commutative, → $s \cdot r = r \cdot s$
2. Distributive by Addition, → $r \cdot (s+t) = (r \cdot s) + (r \cdot t)$
3. Associative over Scalar → $r \cdot (as) = a(r \cdot s)$

Reflexive Dot Product.

$$r \cdot r = r_1 r_1 + r_2 r_2 + \dots r_n r_n$$
$$= r_1^2 + r_2^2 \dots + r_n^2$$
$$= |r|^2$$

Proof:

PROOF OF DOT PROD DISTRIB BY ADD.

$r = [r_1 \ r_2 \dots r_n]^T$

$s = [s_1 \ s_2 \dots s_n]^T$

$t = [t_1 \ t_2 \dots t_n]^T$

$$r \cdot (s+t) = r_1(s_1+t_1) + r_2(s_2+t_2) + \dots + r_n(s_n+t_n)$$
$$= r_1 s_1 + r_1 t_1 + r_2 s_2 + r_2 t_2 + \dots + r_n s_n + r_n t_n$$
$$= r_1 s_1 + r_1 s_2 \dots r_n s_n + r_1 t_1 + r_1 t_2 \dots r_n t_n$$
$$= r \cdot s + r \cdot t$$

PROOF DOT PROD ASSOC OVER SCALAR

$r = [r_1 \ r_2 \dots r_n]^T$

$s = [s_1 \ s_2 \dots s_n]^T$

$a =$ some const/scalar

$$\vec{r} \cdot (a\vec{s}) = r_1(as_1) + r_2(as_2) + \dots + r_n(as_n)$$
$$= ar_1 s_1 + ar_2 s_2 + \dots ar_n s_n$$
$$= a(r \cdot s)$$