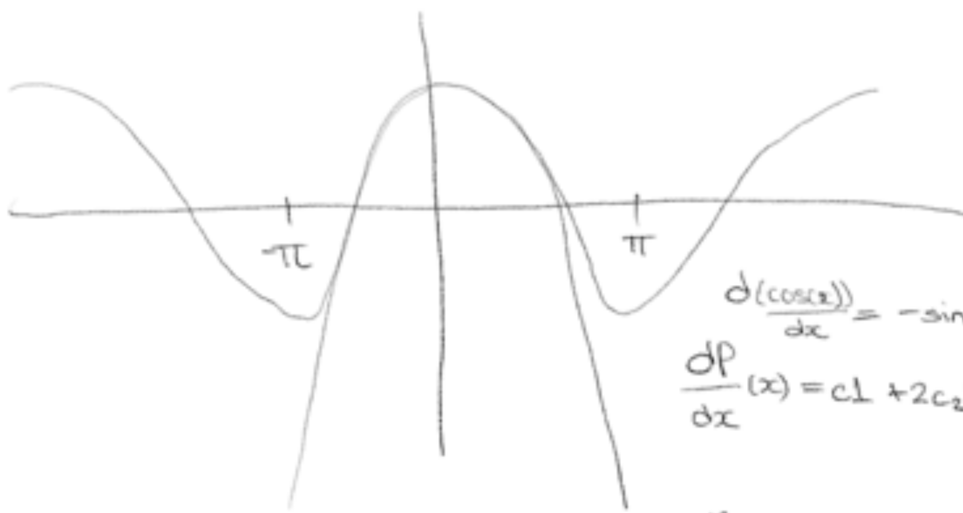
Taylor Series is good for approximating functions.

→ Finding polynomials to fit some non-polynomial function.

$\cos(x) \xrightarrow{x=0} 1$

$P(x) = c_0 + c_1 x + c_2 x^2$

$P(0) = 1 + \ldots$



$\frac{d(\cos(x))}{dx} = -\sin(x)$

$\frac{dP}{dx}(x) = c1 + 2c_2 x$

$\frac{d^2\cos(x)}{dx^2} = -\cos(x)$

The Taylor Polynomial

$P(x) =$

$1 + 0x - \frac{1}{2}(x^2)$ $\Longleftarrow$

$\frac{d^2P}{dx}(x) = 2c_2$

$\frac{d^2\cos}{dx^2}(0) = -1$

$\underbrace{\frac{d^2P}{dx}(0) = 2c_2}_{\frac{d^2P}{dx}(0) = \frac{d^2\cos}{dx^2}(0) \Leftrightarrow c_2 = \frac{-1}{2}.}$

This same process can be applied to higher order terms (x^n).

By analyzing the rate of changing change and then constructing a polynomial with a similar trend in velocity, an approximating function can be formed.

Observations about the process.

1. Factorial terms are produced naturally from subsequent derivations.

$$\frac{d^2}{dx^2}(c_8x^8) = 7\cdot 8x^6$$

$$\frac{d^3}{dx^3}(c_8x^8) = 6\cdot 7\cdot 8x^5$$

...

$$\frac{d^n}{dx^n}(c_8x^8) = n!\,x^0$$ → Then the approximation function's polynomial is set to.

Set $c_8 = \dfrac{\text{desired derivative value}}{K!}$

$K=n$ Divided by some factorial

2. Adding more terms to the approximating function does not affect / is independent of previously added terms.

$$P(x) = 1 + \left(\frac{-1}{2}\right)x^2 + c_4x^4$$ Doesn't affect previous terms.

* when plugging in $x=0$.
since x will render the term as
zero. (e.g $\underbrace{3 \cdot 4\, c_4(0)^2}_{\text{Product is 0.}}$).

If you were doing this for non-zero, would
have to orient function to that delta

Generalized Taylor Polynomial $\left\{ \quad P_\pi(x) = c_0 + c_1(x-\pi)^1 + c_2(x-\pi)^2 + c_3(x-\pi)^3 \ldots \right.$

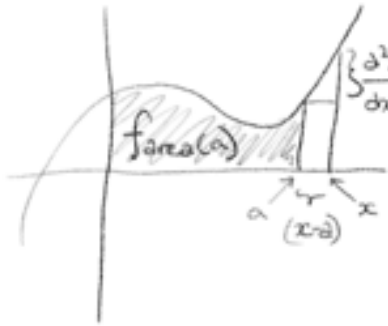
3. Using derivative information to figure out data about the function we are trying to approximate.

$\Longrightarrow$ Similar to finding $s(t)$ (distance) given some velocity graph.

Higher-Order Derivative Information at a point $\longrightarrow$ Output information near that point.

Can apply Taylor Polynomials to the approximation of integrals.

$f(x) =$



$$\frac{d^2 f_{area}}{dx^2}(a)(x-a) \approx f(a) + \frac{df}{dx}(a)(x-a) + \frac{1}{2}\frac{d^2f}{dx^2}(a)(x-a)^2$$

Infinite sum $\Leftrightarrow$ series.

Adding all infinitely many terms of the Taylor Polynomial gives the Taylor Series.

This increase the convergence of the series.

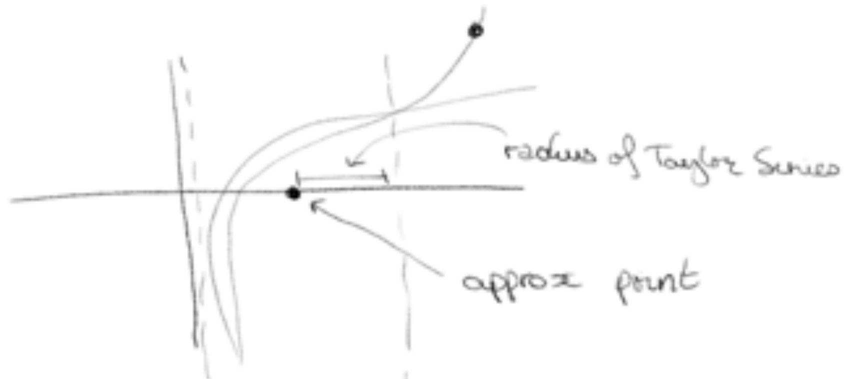
$$e^x \longrightarrow 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

and euler is special in that the convergence is e^x,

$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Taylor Series can be bounded because some functions are asymptotically to different.

$$\ln(x) \rightarrow (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \cdots$$



Whilst the series can diverges after some bounds,
can form Taylor Series around some point.