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### PROBLEM #1.

What are the partial derivatives with respect to  $x$  and  $y$ ?

$$f(x,y) = \pi x^3 + xy^2 + my^4$$

$$\begin{aligned}\frac{df}{dx} &= \frac{d}{dx} (\pi x^3 + xy^2 + my^4) \\ &= \pi 3x^2 + y^2\end{aligned}$$

$$\begin{aligned}\frac{df}{dy} &= \frac{d}{dy} (\pi x^3 + xy^2 + my^4) \\ &= (0) + 2xy + 4my^3\end{aligned}$$

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### PROBLEM #2.

Find  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial f}{\partial z}$  for  $f(x,y,z) = x^2y + y^2z + z^2x$ .

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (x^2y + y^2z + z^2x) \\ &= 2xy + (0) + z^2\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (x^2y + y^2z + z^2x) \\ &= x^2 + 2yz + (0)\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} (x^2y + y^2z + z^2x) \\ &= (0) + y^2 + 2zx\end{aligned}$$

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### PROBLEM #3.

Find  $\frac{\partial f}{\partial x}$ ,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial f}{\partial z}$  for  $f(x,y,z) = e^{2x} \sin(y) z^2 + \cos(z) e^x e^y$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (e^{2x} \sin(y) z^2 + \cos(z) e^x e^y)$$

$$= e^{2x} \frac{\partial}{\partial x} (\sin(y) z^2) + \sin(y) z^2 \frac{\partial}{\partial x} e^{2x} + e^x \frac{\partial}{\partial x} \cos(z) e^y + \cos(z) e^y \frac{\partial}{\partial x} e^x$$

$$= e^{2x}(0) + \sin(y) z^2 e^{2x} \cdot 2 + e^x(0) + \cos(z) e^y e^x(1)$$

$$= \sin(y) z^2 e^{2x}(2) + \cos(z) e^y e^x$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} (e^{2x} \sin(y) z^2 + \cos(z) e^x e^y), \text{ with factoring}$$

$$= \frac{\partial f}{\partial x} e^x (e^x \sin y z^2 + \cos z e^y)$$

$$= e^x \frac{\partial f}{\partial x} (e^x \sin y z^2 + \cos z e^y) + (e^x \sin y z^2 + \cos z e^y) \times \frac{\partial f}{\partial x} e^x$$

$$= e^x \left[ \frac{\partial f}{\partial x} (e^x \sin y z^2) + \frac{\partial f}{\partial x} (\cos z e^y) \right] + (e^x \sin y z^2 + \cos z e^y) e^x$$

$$= e^{2x} \sin y z^2 + e^x (e^x \sin y z^2 + \cos z e^y)$$

$$= e^x [e^x \sin y z^2 + \cos z e^y] + e^x \sin y z^2$$

$$= e^x (e^x \sin y z^2(2) + \cos z e^y)$$

(\*) way easier.

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} [e^x (e^x \sin y z^2 + \cos z e^y)]$$

$$= e^x \frac{\partial}{\partial y} (e^x \sin y z^2 + \cos z e^y)$$

$$+ (e^x \sin y z^2 + \cos z e^y) \frac{\partial}{\partial y} (e^x)$$

$$= e^x \left[ \frac{\partial}{\partial y} (e^x \sin y z^2) + \frac{\partial}{\partial y} (\cos z e^y) \right] + (e^x \sin y z^2 + \cos z e^y)(0)$$

$$= e^x [e^x z^2 \cos y + e^y \cos z(1)]$$

$$\begin{aligned}
 \frac{\partial f}{\partial z} &= \frac{\partial}{\partial z} [e^x (e^x \sin y z^2 + \cos z e^y)] \\
 &= e^x \left[ \frac{\partial}{\partial z} (e^x \sin y z^2) + \frac{\partial}{\partial z} (\cos z e^y) \right] + (e^x \sin y z^2 + \cos z e^y) \frac{\partial}{\partial z} e^x \\
 &= e^x [2z e^x \sin y + (-\sin z e^y)]
 \end{aligned}$$

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PROBLEM # 4.

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Calculate the total derivative of  $f(x, y) = \frac{\sqrt{x}}{y}$ .  $x(t) = t$   
 $y(t) = \sin(t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

The f-partial derivative to y,

$$\begin{aligned}
 \frac{\partial f}{\partial y} &= \frac{\partial}{\partial y} (\sqrt{x} y^{-1}) \\
 &= \sqrt{x} \frac{\partial}{\partial y} (y^{-1}) + y^{-1} \frac{\partial}{\partial y} (\sqrt{x}) \\
 &= \sqrt{x} (-y^{-2}) + y^{-1} (0) \\
 &= -\frac{\sqrt{x}}{y^2}
 \end{aligned}$$

The f-partial derivative to x,

$$\begin{aligned}
 \frac{\partial f}{\partial x} &= \frac{\partial}{\partial x} (\sqrt{x} y^{-1}) \\
 &= \sqrt{x} \frac{\partial}{\partial x} (y^{-1}) + y^{-1} \frac{\partial}{\partial x} (\sqrt{x}) \\
 &= \sqrt{x} (0) + y^{-1} \frac{1}{2\sqrt{x}} \\
 &= \frac{1}{2\sqrt{x}y}
 \end{aligned}$$

The derivative of y-to-t,

$$\begin{aligned}
 \frac{dy}{dt} &= \frac{d}{dt} \sin t \\
 &= \cos t
 \end{aligned}$$

The derivative of  $x = t - t^2$ ,

$$\frac{dx}{dt} = \frac{d}{dt} t - t^2$$
$$= 1 - 2t$$

The total derivative is,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$
$$= \frac{1}{2\sqrt{x}y} (1) + \left( -\frac{\sqrt{x}}{y^2} \cos t \right)$$
$$= \frac{1}{2\sqrt{t} \sin t} - \frac{\sqrt{t} \cos t}{\sin^2 t}, \text{ substituting } t \text{ definitions for } x \text{ and } y.$$

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### PROBLEM #5.

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Calculate the total derivative of

$$f(x, y, z) = \cos(x) \sin(y) e^{2z}$$

$$x(t) = t + 1$$

$$y(t) = t - 1$$

$$z(t) = t^2$$

The total derivative is,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

The  $f$ -partial derivative to  $x$  is,

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \cos(x) \sin(y) e^{2z}$$
$$= \cos x \frac{\partial}{\partial x} \sin(y) e^{2z} + \sin(y) e^{2z} \frac{\partial}{\partial x} \cos x$$
$$= \cos x (0) + \sin(y) e^{2z} (-\sin x)$$
$$= -\sin x \sin y e^{2z}$$

The  $f$ -partial derivative to  $y$  is,

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \cos(x) \sin(y) e^{2z}$$

$$= \sin y \frac{\partial}{\partial y} \cos x e^{2z} + \cos x e^{2z} \frac{\partial}{\partial y} \sin y$$

$$= \sin y(0) + \cos x e^{2z} \cos y$$

$$= \cos x \cos y e^{2z}$$

The  $f$ -partial derivative to  $z$  is,

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} \cos(x) \sin(y) e^{2z}$$

$$= e^{2z} \frac{\partial}{\partial z} \cos x \sin y + \cos x \sin y \frac{\partial}{\partial z} e^{2z}$$

$$= e^{2z}(0) + \cos x \sin y e^{2z} \cdot 2$$

$$= 2 \cos x \sin y e^{2z}$$

The derivative of  $x$ -to- $t$ ,

$$\frac{dx}{dt} = \frac{d}{dt} (t+1)$$

$$= 1$$

The derivative of  $y$ -to- $t$ ,

$$\frac{dy}{dt} = \frac{d}{dt} (t-1)$$

$$= 1$$

The derivative of  $z$ -to- $t$ ,

$$\frac{dz}{dt} = \frac{d}{dt} (t^2)$$

$$= 2t$$

The total derivative is,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

$$= -\sin x \sin y e^{2z} (1) + \cos x \cos y e^{2z} (1) + 2 \cos x \sin y e^{2z} 2t$$

$$= -\sin(t+1) \sin(t-1) e^{2t^2} + \cos(t+1) \cos(t-1) e^{2t^2} + 2 \cos(t+1) \sin(t-1) e^{2t^2} 2t$$

$$= e^{2t^2} [-\sin(t+1) \sin(t-1) + \cos(t+1) \cos(t-1) + 4t \cos(t+1) \sin(t-1)]$$