

Index notation is better for differentiation with respect to spatial coordinates; it also simplifies the presentation and manipulation of differential geometry.

Symbolic notation is good when the matrix calculus is simple, while the matrix algebra and matrix arithmetic is messy.

□ Nomenclature

Definition. Real Matrix of dimension $M \times N$, is when $a_{ij} \in \mathbb{R}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, and the ordered rectangular matrix is:

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & & \ddots & \\ \vdots & & \ddots & \ddots \\ a_{M1} & \dots & & a_{MN} \end{bmatrix}$$

A short formed version of A can be written

$$A = [a_{ij}] . \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

where the typical element is a_{ij} .

□ Matrix Multiplication

Definition. Matrix Product. The matrix product AB can be represented by the matrix C , where C :

$$\begin{bmatrix} C_{11} & C_{12} & \dots & C_{1p} \\ C_{21} & & \dots & \\ \vdots & \vdots & & \\ C_{m1} & \dots & \dots & C_{mp} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & \dots & \\ \vdots & \vdots & & \\ a_{m1} & \dots & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1p} \\ b_{21} & & \dots & \\ \vdots & \vdots & & \\ b_{m1} & \dots & \dots & b_{mp} \end{bmatrix}$$

↑
i-th row
↑
j-th column

this holds for all $i = 1, 2, \dots, m$, $j = 1, 2, \dots, p$. It can be written algebraically:

$$C_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$