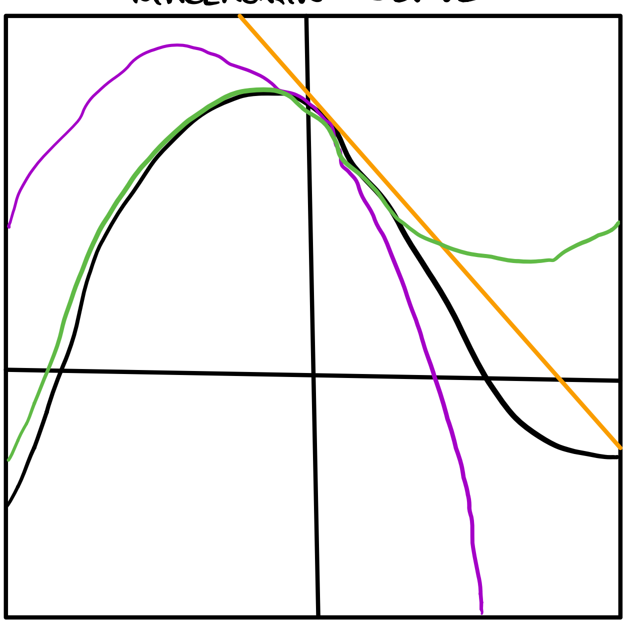
A cool thing happens when you differentiate $e^x$ – also equivalent to maclaurin series

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

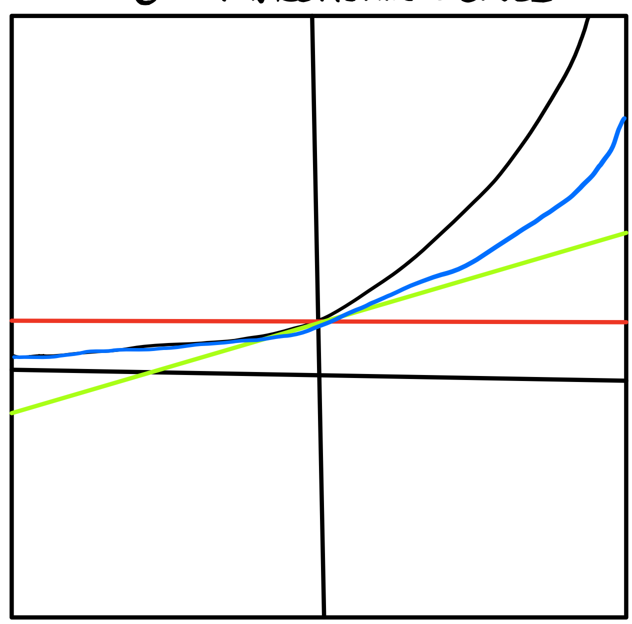
$$\frac{d}{dx} e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

The derivative of the infinitly long series of $e^x$ is the very same series!

MACLAURIN SERIES



$e^x$ – MACLAURIN SERIES



$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

"If you know everything about a function at $x=0$, you can reconstruct everything about the function everywhere."

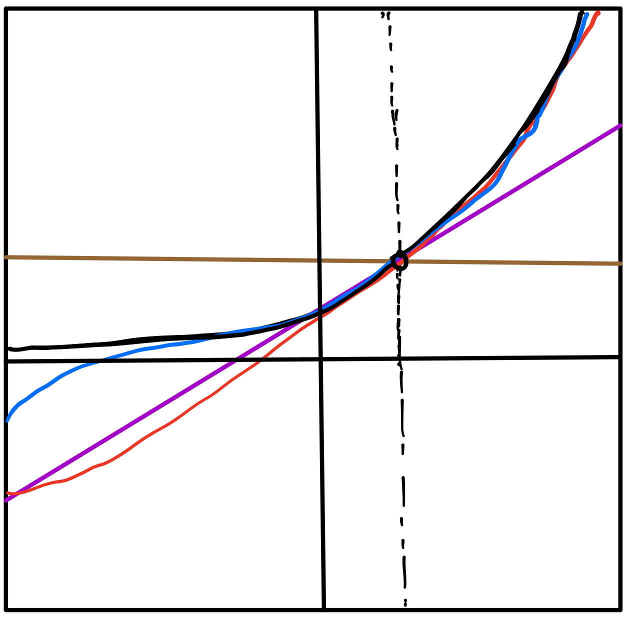
$$e^x = 1 + x + \frac{x^2}{2} \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Similarily expand around $x=0$.

"Instead of expanding around $x=0$, to expand around $x=p$!"

$e^x$ Taylor Series




The first order derivation at point $p$ of a Taylor Series yields the process by which you can produce the other derivation orders.

Approximate the first order - linear component - of $e^x$,

$$y = mx + c$$
$$= f'(x)x + c, \text{ where } f'(x) \text{ is the first order deriv.}$$

Calculation of c:

$$y = f'(p)p + c$$
$$= f'(p)p + c, \text{ where } p \text{ is the point we are deriving around}$$
$$\Leftrightarrow f(p) = f'(p)p + c$$
$$\Leftrightarrow c = f(p) - f'(p)p$$

Substituting c into the linear model,

$$= f'(p)x + f(p) - f'(p)p$$
$$= f'(p)(x-p) + f(p)$$

In approximating around point p, the first order derivative shows that the gradient is relative to the point $(x-p)$; that is p's distance from $\underline{x}$.

Approximation functions around $p$,

$$g_0(x) = f(p)$$
$$g_1(x) = f(p) + f'(p)(x-p)$$
$$g_2(x) = f(p) + f'(p)(x-p) + \frac{f''(p)(x-p)^2}{2}$$, where $g_2(x)$ in the Maclaurin Series is:

$$g_2(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2}$$

From which generalizes the Taylor Series,

$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(p)}{n!}(x-p)^n$$

Brook Taylor (1685-1731)

Note if you sub in $p=0$, Taylor becomes Maclaurin.