Homomorphisms are just <u>maps</u> between algebraic structures or sets; these maps must maintain the operations that are in the alegbraic structures — their operations applied to the domain of the algebraic structure must map to the same image as the operation applied to the mapping images of the domain elements.

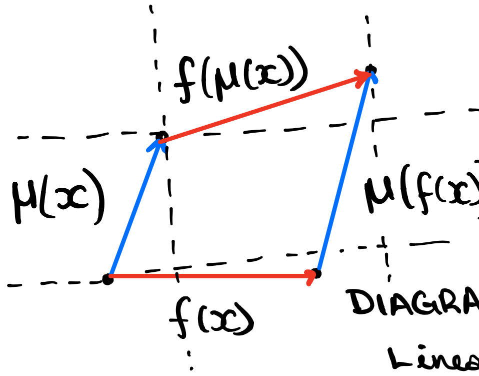

DIAGRAM 1.

Linear Map is a homomorphism.

FORMAL DEFINITION

Let $f: A \to B$ be a map.

Let μ be an operation of k-arity, defined on A and B.

If A, B have an identity elemenent then the domain identity must map to the image identity

If the map satisfies:

$$\forall a_1,\dots,a_k \in A: f(\mu_A(a_1,\dots,a_k)) = \mu_B(f(a_1),\dots,f(a_k)).$$

where $\{k \in \mathbb{N} \cup 0\}$.

ALGEBRAIC STRUCTURES

- happens on a set A, called the <u>Carrier</u>
- has collection of finite arity operations on A.
- has finite identities (axioms) which the operations satisfy

Homomorphism Examples

Let $A, B \in \mathbb{R}$.

Let A, B be defined under the operation $f: A \to B$

Let $f(z) = |z|$.

$f(z_1 z_2) = |z_1 z_2| = |z_1||z_2|$ $\square$