

PROBLEM #1

Use the MacLaurin series to determine if  $f(x) = \tan^{-1}(x)$  is even, odd or neither.

Suppose  $\tan^{-1}(x)$  can be modelled with the MacLaurin series,

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$f'(x) = \frac{d}{dx} (\tan^{-1} x)$$

Proof of  $\frac{d}{dx} \arctan(x)$ ,

$$y = \arctan x$$

$$\tan y = x$$

$$\frac{d}{dx} \tan y = \frac{d}{dx} x$$

$$\sec^2 y \frac{d}{dx} y = 1 \Leftrightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

Since  $\sec^2 y = 1 + \tan^2 y$ , since  $y = \arctan x$ ,  
 $\tan^2 y = x^2$ .

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\ &= \frac{1}{1 + \tan^2 y} \\ &= \frac{1}{1 + x^2}. \end{aligned}$$

$$g_1(x) = \frac{f'(0)}{1!} x^1 + g_0(x)$$

Based on  $f^{(n)}(0)x^n$  the series is odd.

$$= \frac{x}{1} + \arctan(0)$$

$$g_2(x) = -2x^3 + x + 0$$

$$g(x) = \sum_{n=0}^{\infty} \frac{(n \cdot 2)! (-1)^n}{(n \cdot 2 + 1)!} x^{(n \cdot 2 + 1)}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{(n+2)+1}}{(n+2)+1}, \text{ powers always odd!}$$

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### PROBLEM #2.

Given  $f(x) = 2/(x^2 - x)$ , which ranges can a power series be formed for?

There are asymptotes at the roots of  $x^2 - x$ .

$$= \frac{-(-1)^{\pm} \sqrt{(-1)^2 - 4(1)x_0}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1}}{2}, \text{ roots at } 0, 1.$$

Taylor series can be 'found' between  $[-\infty, 0), (0, 1), (1, \infty]$ .

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### PROBLEM #3

For  $f(x) = 2/(x^2 - x)$  which statements are true about its power series representation.

- approx. properly captures asymptotes
- this is a well behaved function
- approx ignores segments of function
- approx converges quickly  $\rightarrow$  sign flip in approximation
- approx ignores asymptotes.  $\rightarrow$  not quick conv.

## PROBLEM #4

Given  $f(x) = \frac{1}{(1+x^2)}$ , performing Taylor expansions

for the first 3 terms around point  $x=2$  are,

$$f(x) = \frac{1}{5} - \frac{4(x-2)}{25} + \frac{22(x-2)^2}{125} + \dots$$

Why does the expansion perform poorly @  $x=0$ .

asymptotes are in the complex plane.

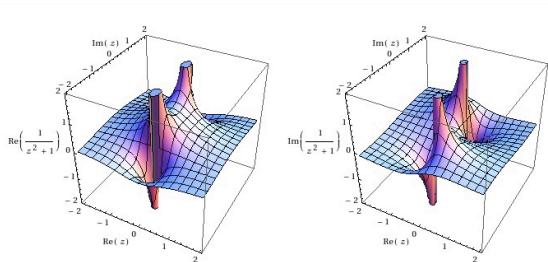
function does not differentiate well.

the function has no real real roots

none of these options.

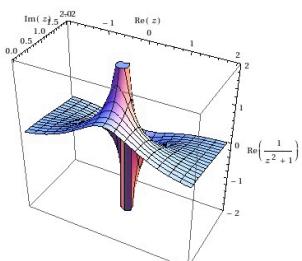
it is a discontinuous function in the complex plane.

quadratic form.  $= \frac{-10 \pm \sqrt{100 - 4(0)}}{2(1)}$



These are plots of the real and imaginary parts of the function  $\frac{1}{z^2 + 1}$ , where  $z = x + iy$ . The poles of the function at  $z = \pm i$  are easily seen in the plots. Since we're limited to seeing a two-dimensional projection of three dimensions, we are forced here to illustrate the poles by plotting the real and imaginary parts of the function separately.

Now, look at a "slice" of the real part:



Runge phenomenon. - any attempt to approx. with a polynomial fails bc. poles in complex plane.

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PROBLEM #5.

Given  $f(x) = \ln x$ , calculate the Taylor expansions at  $x=4$  in the order of 2 degrees of accuracy.

$$f'(x) = -\frac{1}{x^2}$$

$$f''(x) = \frac{2}{x^3}$$

$$g_1(x) = \frac{1}{4} - \frac{(x-4)}{4^2(1)!} + O(\Delta x^2)$$

$$= \frac{1}{4} - \frac{x-4}{16} + O(\Delta x^2)$$

$$g(x) = \sum_{n=0}^{\infty} \frac{A^n (-1)^n}{n!} x^{-(n+1)} (x-4)^n$$

$$= \sum_{n=0}^{\infty} (-1)^n (4)^{-(n+1)} (x-4)^n$$