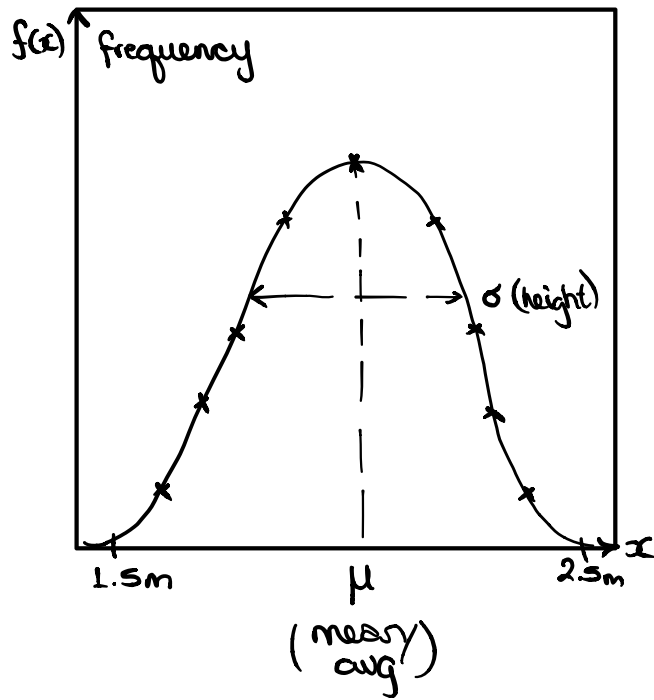


The Newton-Raphson method can be used to solve equations.



By solving the equation - a model for the distribution can be produced; often working with a model is more desirable than working with the raw data points.

Newton-Raphson produces a degree of error; and the parameter values are subject to a goodness of fit.

$f(x) = x^3 - 2x + 2$ .  
NEWTON-RAPHSON ITERATION  
TABLE.

$i$	$x_i$	$y(x_i)$	$\frac{\partial y(x_i)}{\partial x}$
0	-2	-2	10
1	-1.8	-0.23	7.7
2	-1.77	-0.005	7.4
3	-1.769	-2.3E-6	

Example: solving for  $y=0$  in  
 $f(x) = x^3 - 2x + 2$ .

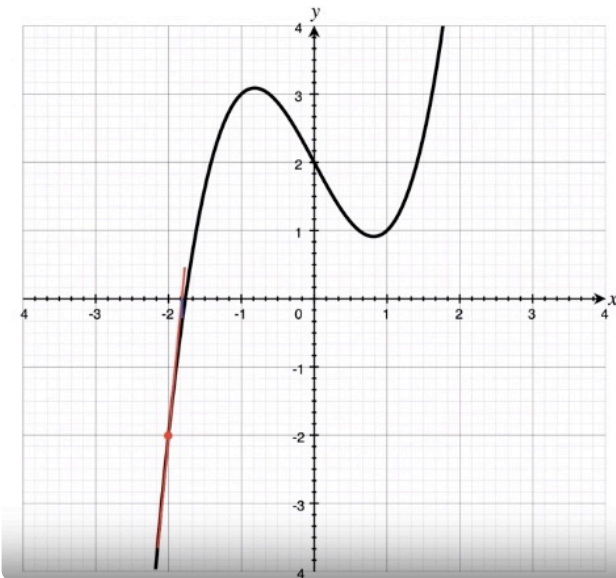
①  $\frac{\partial f}{\partial x} = 3x^2 - 2$ , linearise the function

②  $g_1(-2) = 10$ , compute the gradient

③  $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ , estimate the parameters which will solve the equation.

$$-1.8 = -2 - \frac{(-2)}{10}$$

④  $-0.23 = f(-1.8)$ , evaluate the new function.



$$f(x) = x^3 - 2x + 2.$$

(\*) The key requirements of the Newton-Raphson method for solving an equation are: (1) the ability to evaluate the function, and (2) the ability to derivate the func.

Complex functions with massive datasets are often too complex to be solved analytically - or be graphed out.

An Algorithmic corner case in Newton-Raphson.

$$y = x^3 - 2x + 2$$

Newton-Raphson

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\frac{\partial y}{\partial x} = 3x^2 - 2$$

$i$	$x_i$	$y(x_i)$
0	0	2
1	1	1
2	0	2

Some points can "loop"

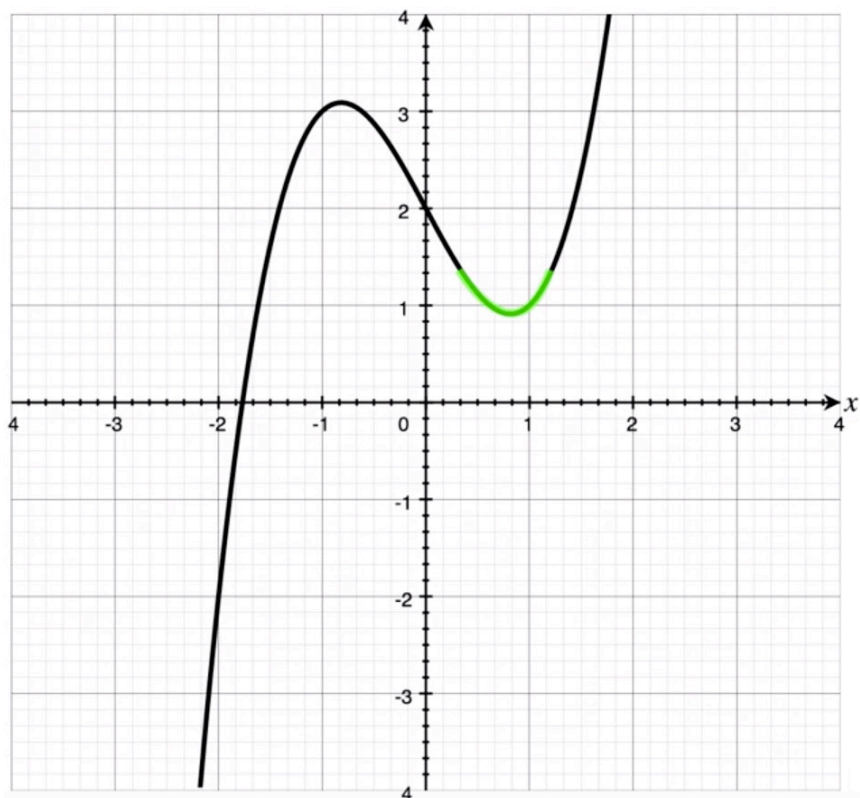
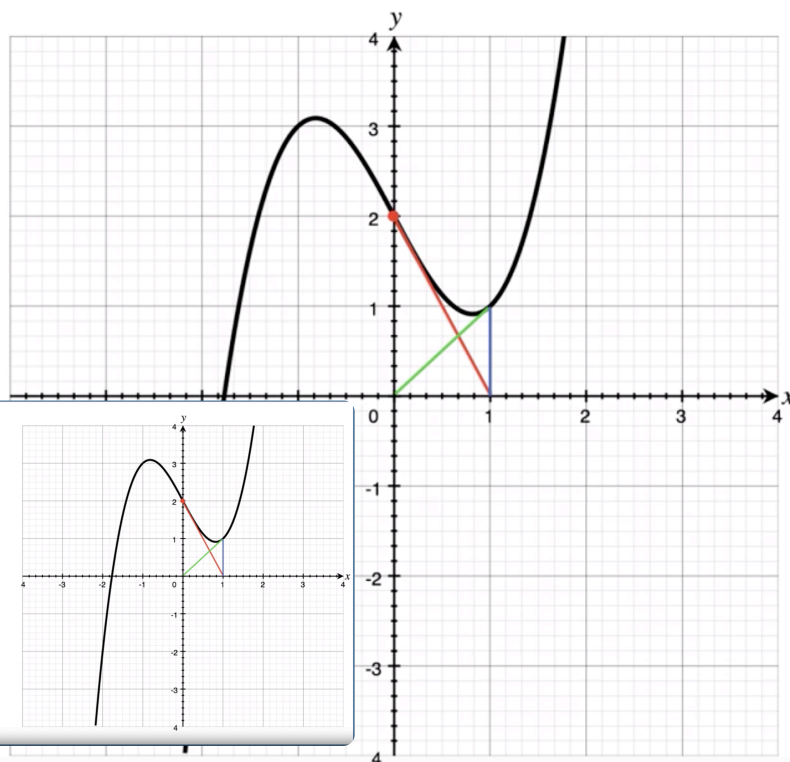
$y = x^3 - 2x + 2$

Newton-Raphson

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$\frac{\partial y}{\partial x} = 3x^2 - 2$$

$i$	$x_i$	$y(x_i)$	$\frac{\partial y(x_i)}{\partial x}$
0	0	2	-2
1	1	1	1
2	0	2	-2



Secondly, when estimating/iterating with Newton-Raphson near a minima/maxima the division by the gradient will produce an inflated jump to the next estimate. Zero also is problematic.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

In a multivariate context newton-raphson requires a contour plot.

Newton-Raphson works by linearisation.

Assuming the root is nearby a certain point — like  $(x_0 + \delta x)$ .

$$f(x_0 + \delta x) = f(x_0) + f'(x_0)\delta x$$

$= 0$ , the jump slightly over will be the root.

By rearranging this equation; and assuming  $f(x_0 + \delta x)$  is 0.

$$f(x_0 + \delta x) = f(x_0) + f'(x_0)\delta x$$

$$0 = f(x_0) + f'(x_0)\delta x$$

$$f'(x_0)\delta x = -f(x_0)$$

$$\delta x = -\frac{f(x_0)}{f'(x_0)} \quad \text{• the delta to shift for the next Newton-Raphson estimation.}$$

$$\text{Hence, } x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$