Why is important to know how to calculate derivatives explicity especially derivatives of abstract functions.

$f(x) = 2x^2 - x^3$

$f(x) = \sin(x)$

$f(x) = e^x$

Because many abstract concepts are actually used to describe worldly rates of change problems.

The process does not just involve memorizing rules,

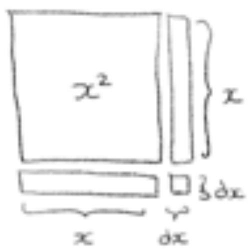
$$\frac{d}{dx} x^n = n x^{n-1} \qquad \frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x) \qquad \frac{d}{dx} a^x = \ln(a) a^x$$

...

Tiny "nudges", the small changes in input are the heart of derivatives.

Calculating the derivative of x^2.



where $df = \square \times 2 + \square$

$$= dx(x)2 + dx^2$$

Thus df is proportional to,

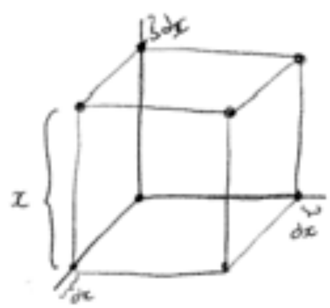
$$df \approx dx(x)2, \quad dx^2 \approx 0 \text{ while } x \to 0$$

Calculating x^3.

This can proportionality can be expressed as an approximation,

$$df = dx(x)2$$

$$\Leftrightarrow \frac{df}{dx} = 2x$$



$$x^2 dx(3) + dx^2 x(3) + dx^3 = df$$

since $dx^2 \approx 0$ as $dx \to 0$, then that equation can be simplified to,

$$df = x^2 dx(3)$$

$$\frac{df}{dx} = x^2(3)$$

Generalized x^n.

$(x+dx)^n = (x+dx)(x+dx)$

$$\underbrace{\cdots x(x+dx)\ldots}_{n \text{ times}}$$

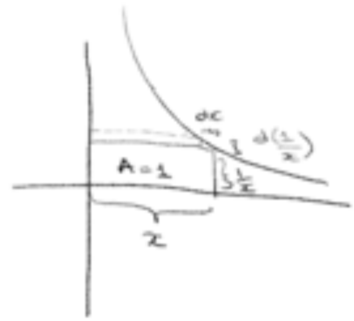
$$= x^n + nx^{n-1}dx + (\text{Mult of } dx^2)$$

$$\overbrace{\begin{aligned} &+ dx \cdot x \cdot x \ldots x \\ &+ x \cdot dx \cdot x \ldots x \\ &+ x \cdot x \cdot dx \ldots x \\ &\ \vdots \\ &+ x \cdot x \cdot x \ldots dx \end{aligned}}$$

$$= x^n + nx^{n-1}dx, \text{ since } dx^2 \to 0.$$

$$\boxed{\frac{d}{dx}x^n = nx^{n-1}}$$

$\hookrightarrow$ Called "Power Rule".



when $f(x) = \frac{1}{x} = x^{-1}$

$$\frac{d}{dx}x^{-1} = \frac{-1}{x^2}$$

$$df = -\left(\frac{1}{x} - \frac{1}{x+dx}\right)$$

$$= -\frac{1}{x} + \frac{1}{x+dx}$$

$$= \frac{-1(x+dx)}{x(x+dx)} + \frac{x}{x(x+dx)}$$

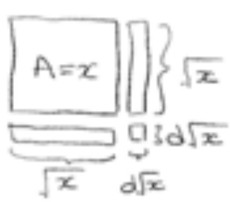
$$= \frac{-dx - x + x}{x^2 + xdx}$$

$$= \frac{-dx}{x^2+xdx}$$

$$\frac{df}{dx} = \frac{-1(dx)}{(x^2+xdx)(dx)}$$

$$= \frac{-1}{x^2}, \quad xdx \to 0.$$

Derivative of $\sqrt{x}$.



$$\frac{1}{2\sqrt{x}} = \frac{d}{dx}\sqrt{x}$$

$$dx = 2\sqrt{x}\,d\sqrt{x} + (d\sqrt{x})^2$$

$$df = \sqrt{x+dx} - \sqrt{x}$$

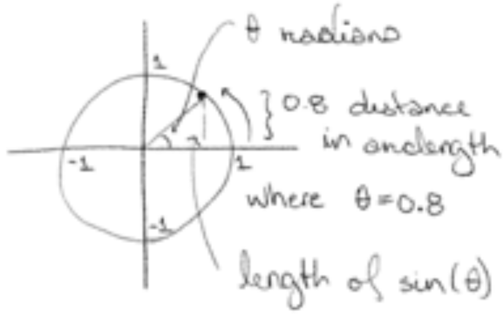
$$\frac{df}{dx} = \frac{\sqrt{x+dx} - \sqrt{x}}{2\sqrt{x}\,d\sqrt{x} + (d\sqrt{x})^2}$$

$$= \frac{d\sqrt{x}}{2\sqrt{x}\,d\sqrt{x}}$$

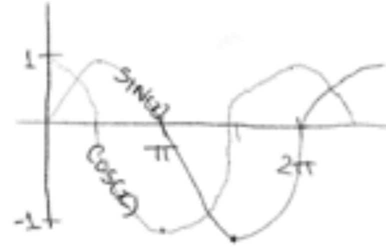
$d\sqrt{x} = \sqrt{x+dx} - \sqrt{x}$. $(d\sqrt{x})^2 \to 0$ as $dx \to 0$.

$$= \frac{1}{2\sqrt{x}}$$

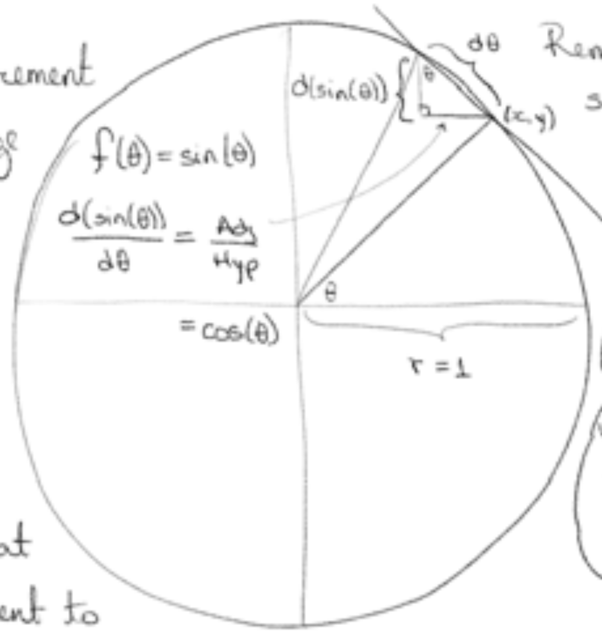
$\square$



$$\frac{d}{dx}\sin(x) = \cos(x)$$

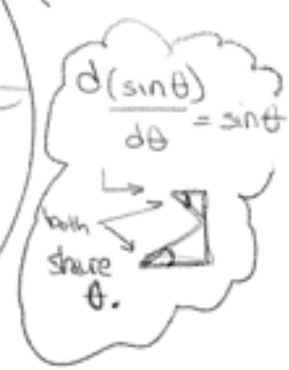


Since the increment produces a change approximated by a triangle, that triangle's height can be computed as the change in output − f(x); that height is equivalent to



Remember $\sin\left(\frac{\pi}{2}\right) = 1$

$\therefore \sin(\theta) = y$



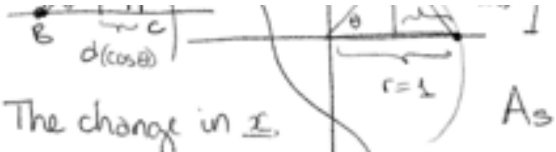
the adjacent side divided by the hypotenuse. This calculation is the trigonometric function cosine.

$\Longrightarrow$ Consequently, the derivative of sin is <u>cos.</u>

Derivative of $\cos(\theta) = -\sin(\theta)$.

cosine = adj / hypotonuse.

$$\frac{d}{d\theta}\cos(\theta) = \frac{d(\cos(\theta))}{\ldots}$$



$d\theta$

The change in $\underline{x}$, $d\cos\theta$ is the same as $\frac{\sin\theta}{hyp}$, but $hyp = \text{radius} = 1$.

PROOF

$$\frac{d}{d\theta}\cos\theta = \frac{dy}{d\theta}$$

$$\lim_{\Delta\theta \to 0} \frac{\Delta y}{\Delta\theta} \approx \frac{AE}{AD} \approx \frac{AC}{AB}$$

Thus $d(\cos\theta) = -\sin(d\theta)$.

Equivalently, $-\frac{opp}{hyp}$.

where,

$opp = \sin\theta$

$hyp = 1 \Rightarrow \frac{\sin\theta}{hyp} \cdot -1$

As the cosine of an angle grows smaller, the sin grows greater.

$-\cos(\theta) : \sin(\theta)$

Therefor the derivative is

$\sin\theta = \frac{opp}{hyp}$, $r = 1$

$\sin\theta = opp$

$\frac{opp}{d\theta}$