PROBLEM #1

What is the mean of $D = \{1, 2, 3\}$.

$$E[X] = \frac{1}{N}\sum_i x_i$$

$$E[D] = \frac{1}{3}(1+2+3)$$

$$= 2$$

PROBLEM #2.

What is the mean of $D = \left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$.

$$E[X] = \frac{1}{N}\sum_i x_i.$$

Since vectors are objects which follow the axioms of a linear system,

$$E[D] = \frac{1}{3}\begin{bmatrix} 1+2+3 \\ 4+5+6 \\ 7+8+9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

PROBLEM #3

What is the mean of the dataset after multiplying each sample by $\underline{2}$.

$$D = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \right\}$$

$$E[X] = \frac{1}{N} \sum_i x_i$$

Account for multiplier per sample,

$$E[X] = \frac{1}{N} \sum_i 2x_i$$

$$= \frac{2}{N} \sum_i x_i$$

$$= \frac{2}{3} \begin{bmatrix} 1+3+5 \\ 2+4+3 \\ 3+5+1 \end{bmatrix}$$

$$= 6 [1 \ 1 \ 1]^T$$

PROBLEM #4.

What is the mean of the dataset D after adding u to each sample?

$$D = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \right\} \quad u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$E[X] = \frac{1}{N} \sum_i x_i$$

Account for addition,

$$E[X] = \frac{1}{N} \sum_i (u + x_i)$$

$$= \frac{1}{N}\sum_i u + \frac{1}{N}\sum_i x_i$$

$$= \frac{1}{N} N u + \frac{1}{N}\sum_i x_i$$, the sum of u N-times is $N * u$.

$$= u + \frac{1}{N}\sum_i x_i$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 1+3+5 \\ 2+4+3 \\ 3+5+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3[1\ 1\ 1]^T$$

$$= [4\ 5\ 6]^T$$

PROBLEM #5

Select the correct formula for $\overline{x}_n$, given

$D_n = D_{n-1} \cup \{x_*\}$, D_{n-1} has $n-1$ datapoints

x_* is one datapoint.

- ☒ $\overline{x}_n = \overline{x}_{n-1} + \frac{1}{n}(x_* - \overline{x}_{n-1})$
- ☐ $\overline{x}_n = \overline{x}_{n-1} + \frac{1}{n+1}(x_* - \overline{x}_{n-1})$
- ☐ $\overline{x}_n = \overline{x}_{n-1} + \frac{1}{n-1}(x_* - \overline{x}_{n-1})$
- ☐ $\overline{x}_n = \overline{x}_{n-1} + \frac{1}{n+1}(\overline{x}_{n-1} - x_*)$

$E[X] = \frac{1}{N}\sum_i x_i$

Account for second datapoint,

$$E[X] = \frac{1}{N}\left(\sum_j x_j + x_*\right)$$

$$= \frac{1}{N}\cdot\frac{(N-1)}{(N-1)}\left(\sum_j x + x_*\right)$$

$$= \frac{(N-1)}{N} \times \frac{\sum_i x_j}{(N-1)} + \frac{(N-1)}{N(N-1)} x_*$$

$$\frac{1}{|i|} > \frac{1}{|i+1|}$$

$$= \frac{(N-1)}{N} \overline{x}_{n-1} + \frac{1}{N} x_*$$

$$= \frac{N\overline{x}_{n-1}}{N} - \frac{x_{n-1}}{N} + \frac{1}{N} x_*$$

$$= \overline{x}_{n-1} + \frac{1}{N}(x_* - \overline{x}_{n-1})$$