What is the derevative of exponential functions.

$$\frac{dM}{dt}(t) = \frac{2^{\overbrace{(t+dt)}^{\text{Additive}}} - 2^t}{dt}$$

Can relate additive and multiplicative ideas.

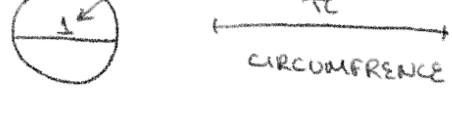
$$= \frac{\overbrace{2^t \cdot 2^{dt}}^{\text{Multiplicative}} - 2^t}{dt}$$

$$= 2^t \cdot \underbrace{\left(\frac{2^{dt} - 2^t}{dt}\right)}_{\text{Proportionality Constant.}}, \quad dt \to 0.$$

Meaning the derivative of an exponential is itself by some constan
↳ its proportional to itself.

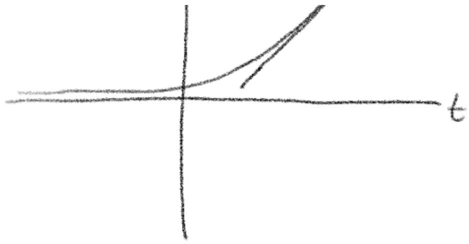
There is a base with a proportionality constant. That base is e (2.71828...).

↳ This is the definition of e; much in the same way π is the ratio of a circles circumfrence to it's diameter.



$y = e^t(1) + b$

$$\boxed{\frac{d}{dt}(e^t) = e^t}$$



Euler's number can be used to express exponentials is a different way; that is using the chain rule.

$$\frac{d(e^{ct})}{dt} = ce^{ct}$$

$$2^t = e^{\ln(2)t}, \text{ knowing } 2 = e^{\ln(2)}$$

$$\frac{d}{dt}(2^t) = \frac{d}{dt}\left(e^{\ln(2)t}\right)$$

$$= \ln(2) \cdot e^{\ln(2)t}$$

$$= \ln(2) \cdot 2^t$$

It is important to note that the size of a changing variable (e^{z}) and the rate of change ($c \cdot e^{z}$) is proportional to the constant expressed in e-based exponentials ($f(t) = e^{vt}$).