The ability to evaluate the value in playing a game seems to be a property which follows from the axiom which purports all players as able to assess the value of all their available actions. Whether this holds true or not – the ability to evaluate the value of playing a game is necessary in order to comment on larger compositional game of games.

Evaluating Von-Neumann and Morgenstern's "essential" games are intractable; Shapley values remove this obstacle.

Will make these assumptions:

1. utility is objective and transferable
2. games are cooperative affairs
3. games which comply with #1 and #2 are suitably represented by their characteristic function.

4. Do not assume von Neumann's & Morgenstern's qualities of rational behaviour

THE CONCEPTION OF A GAME

<u>Game</u> – a set of rules w- players in the playing position

<u>Abstract Game</u> – defined by rules; playable by roles like "dealer", "home team".

Game Theory usually refers to Abstract Games. Value of game based on Abstract

DEFINITIONS

$\underline{U}$ is the universe of players.

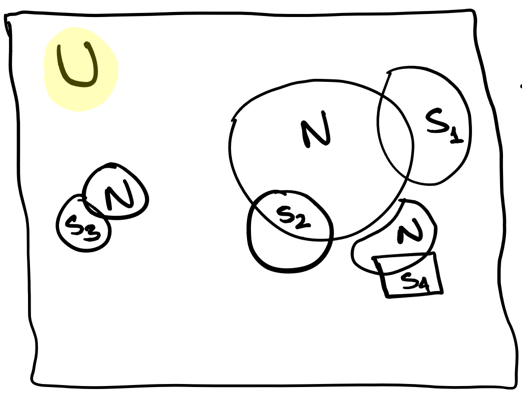
A game is a superadditive set function $\underline{v}$ from U to $\mathbb{R}$.

(1) $v(\emptyset) = 0$.

(2) $v(S) \geq v(S \cap T) + v(S - T), \quad \forall S,T \in U$

A carrier of v is any set $N \subseteq U$ with

$$v(S) = v(N \cap S), \quad \{S \subseteq U\}.$$



The carrier $\underline{N}$ satisfy the function $\underline{v}$; and contains members which are defined with $\mathbb{R}^+$ values for the domain $\underline{U}$.

The superset of a Carrier on $\underline{v}$ is also a Carrier of $\underline{v}$. The carrier terminology implies minimum satisficing requirements – and operate as a lower bounding criterion.

Sum/superposition of a game, when two games are combined with independent rules – allowing for the same players to play; when the games have disjunct carriers the game's sum is named a composition.

Abstract Games Corresponding to $\underline{v}$.

Let $\Pi(U)$ be the permutations of U — these are a collection of bijective <u>mappings</u>.

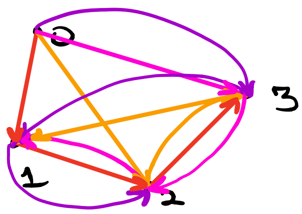


FIGURE #1

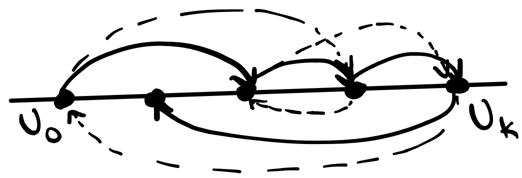


FIGURE #2

Permutations of U.

For example $\pi \in \Pi(U)$ is a mapping of $f: U \to U$.

The image of the mapping $\underline{\pi}$ can be denoted $\underline{\pi S}$.

Given the mapping can define a abstract game:

$$\pi v(\pi S) = v(S). \quad \text{for all } S \subseteq U.$$

The abstract game corresponding to the game $\underline{v}$, is defined by the class of games:

$$\pi v, \quad \pi \in \Pi(U).$$

The additive operation can not be applied to abstract games.