



Velocity is  $\vec{v}$  (i.e. the vector projection of ship onto current)

$$s = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} s \text{ onto } c &= \frac{c}{\|c\|} \times \frac{s \cdot c}{\|c\|} \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{(1 \cdot 1) + (2 \cdot 1)}{(\sqrt{2})^2} \\ &= \frac{3}{2} [1 \ 1]^T \end{aligned}$$

### PROBLEM #2.

ball has velocity  $[21]^T$ , wind has direction  $[3 -4]^T$ .  
what is the magnitude of velocity of ball in direction of the wind? (i.e scalar projection)

$$b = [2 \ 1]^T, w = [3 \ -4]^T$$

$$\begin{aligned} \text{scalar proj } b \text{ onto } w &= \frac{b \cdot w}{\|w\|} \\ &= \frac{(2 \cdot 3) + (1 \cdot -4)}{\sqrt{(3^2) + (-4)^2}} \\ &= \frac{2}{5} \end{aligned}$$

### PROBLEM #3.

$v = [-4 \ -3 \ 8]$ ,  $a = [1 \ 2 \ 3]$ ,  $b = [-2 \ 1 \ 0]$ ,  $b_3 = [-3 \ -6 \ 5]$ .  
 $b$  is pairwise orthogonal.

$$\begin{aligned}
 v \text{ onto } a &= \frac{a}{|a|} \times \frac{v \cdot a}{|a|} \\
 &= a \times \frac{v \cdot a}{a \cdot a} \\
 &= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \frac{((-4)(1) + (-3)(2) + (8)(3))}{(1)^2 + (2)^2 + (3)^2} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 v \text{ onto } b &= b \times \frac{b \cdot v}{b \cdot b} \\
 &= \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \frac{((-4)(-2) + (-3)(1) + (8)(0))}{(-2)^2 + 1^2 + 0^2} \\
 &= \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \textcircled{1} = \frac{b \cdot v}{|b|} = \frac{1}{14}
 \end{aligned}$$

$$\begin{aligned}
 v \text{ onto } c &= \frac{c \cdot v}{|c|} = \frac{(-4)(-3) + (-3)(-6) + (8)(5)}{(-3)^2 + (-6)^2 + 5^2} \\
 &= 1
 \end{aligned}$$

#### PROBLEM # 4

Is  $[1 \ 2 \ -1]^T, [3 \ -4 \ 5]^T, [1 \ -8 \ 7]^T$  independent?

$$M = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -4 & -8 \\ -1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -4 & -8 \\ 0 & 8 & 8 \end{bmatrix}, \text{ adding row } \#1$$

$$= \begin{bmatrix} 1 & 3 & 1 \\ 0 & 10 & 10 \\ 0 & 8 & 8 \end{bmatrix}, \text{ adding } -2 \times \text{row } \#1 \\
 \rightarrow \text{row } \#2 \text{ and } \#3 \text{ are dependent.}$$

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PROBLEM #5.

Obj. moving w- velocity  $\begin{bmatrix} 1 & 2 & -3 \end{bmatrix}^T$  km/h, starting at  $\begin{bmatrix} 3 & 2 & 4 \end{bmatrix}^T$ . Where after 2 hours?

$$d = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} 2$$

$$= \begin{bmatrix} 1 \\ 6 \\ -2 \end{bmatrix}, \text{ where the ship ended up after 2 hours.}$$