Inverse Matrices, Column Space & Null Space

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- Gaussian Elimination
- Row Echelon Form

Linear Algebra is useful for solving certain systems of linear equations.

$\underbrace{x \quad y \quad z}$

List of variables

$6x - 3y + 2z = 7$
$x + 2y + 5z = 0$
$2x - 8y - z = -2$

List of equations.

$$\to \overset{A}{\begin{bmatrix} 2 & 5 & 3 \\ 4 & 0 & 8 \\ 1 & 3 & 0 \end{bmatrix}} \overset{\vec{x}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$$

Applying a transformation in reverse, is called the inverse. (A^{-1})

$$A^{-1}A = I,$$

$$\vec{x} = A^{-1}\vec{v}$$

As long as transformations do not squish the vector space into a lower dimension, Transformation matrix will always have an inverse.

When the dimensions have been collapsed, an inverse can still exist, but the vector of origin must have already existed on that subspace.

The dimensionality of a transformation can be indicated by its determinant, it can better be described by rank.

1^d = rank 1
2^d = rank 2
3^d = rank 3

"Number of dimensions in the output"

The set of all possible outputs, ($A\vec{v}$)

- line
- plane
- 3^d space

$\longleftrightarrow$ Column Space of M - a matrix

REMEMBER – the columns of a matrix declare where the transformed basis vectors land and the span of the transformed basis vectors – gives you all possible outputs. (i.e the column space).

$$\text{Span}(M) = \text{Column Space}(M)$$

When rank is equivalent to the number of columns making up the transformation vector that is Full Rank.

$\vec{0}$ will always be in the column space.

When transformations cause dimensionality to collapse, linear combinations form swaths of points which reduce down to singularities

All vectors which become the $\vec{0}$ after transformation is known as the null space (or kernel).

$$A\underbrace{\vec{x}}_{\uparrow \text{ Null space.}} = 0$$