In univariate calculus gradients taken at each point in a function can be used to geometrically intuit the derivative. Those derivatives can be calculated more easily by use of rules like the sum, product, chain and quotient rule.

These same ideas can be applied in multivariate systems; however care should be taken to understand what are the Variables, Constants and their Contexts.

Variables

Calculus begins with differentiation of dependent variables according to some independent variables - like time.

Depending on the context - these variables and others might actually be constants.

For example, consider the equation for force,

$F = ma + dv^2$, where $\underline{m}$ is mass,
$\underline{a}$ is acceleration,
$\underline{d}$ is drag,
$\underline{v}$ is velocity,
$\underline{F}$ is force.

Both mass and drag are constants, and acceleration and velocity are outcomes of the amount of force being applied.

F is independent
m, d are constant
a, v are dependent variables

However, if you were designing a car to meet a certain velocity and acceleration then,

v, a are constant
m, d are dependent
F is independent

(*) In engineering disciplines this is more thought of as exploring a class of similar functions rather than treating those class attributes as functions.

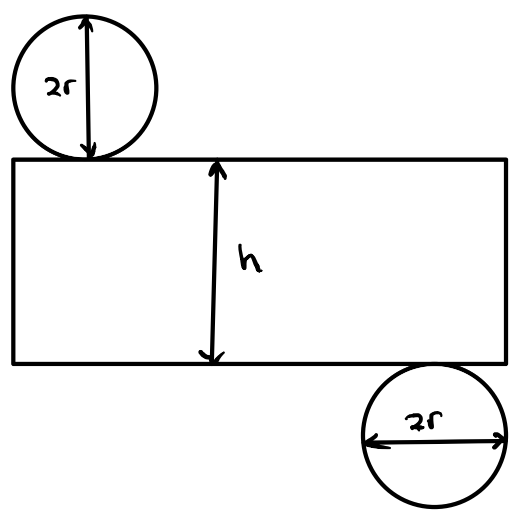
⇒ The central idea is that any term can be differentiated relative to another even sometimes when conceptually with out the right context it might not make sense.

Example: Cylinder mass

$$m = 2\pi r^2 tp + 2\pi rhtp,$$

πr^2 is the area of the circle
$2\pi r$ is the circumference of the circle.
t is the thickness of the material
p is the density of the material

Note must use the curly partial symbol to denote the differentiation of a function of more than one variable.



$$\underbrace{\begin{aligned} \frac{\partial m}{\partial h} &= 2\pi rtp \\ \frac{\partial m}{\partial r} &= 4\pi rtp + 2\pi htp \\ \frac{\partial m}{\partial t} &= 2\pi r^2 p + 2\pi rhp \\ \frac{\partial m}{\partial p} &= 2\pi r^2 t + 2\pi rht \end{aligned}}_{\text{Partial Derivative.}}$$