

### PROBLEM #1

What is the mean of  $D = \{1, 2, 3\}$ .

$$E[X] = \frac{1}{N} \sum_i x_i$$

$$\begin{aligned} E[D] &= \frac{1}{3} (1 + 2 + 3) \\ &= 2 \end{aligned}$$

### PROBLEM #2.

What is the mean of  $D = \left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$ .

$$E[X] = \frac{1}{N} \sum_i x_i.$$

Since vectors are objects which follow the axioms of a linear system,

$$\begin{aligned} E[D] &= \frac{1}{3} \begin{bmatrix} 1+2+3 \\ 4+5+6 \\ 7+8+9 \end{bmatrix} \\ &= \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} \end{aligned}$$

### PROBLEM #3

What is the mean of the dataset after multiplying each sample by 2.

$$D = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \right\}$$

$$E[x] = \frac{1}{N} \sum_i x_i$$

Account for multiplier per sample,

$$E[x] = \frac{1}{N} \sum_i 2x_i$$

$$= \frac{2}{N} \sum_i x_i$$

$$= \frac{2}{3} \begin{bmatrix} 1+3+5 \\ 2+4+3 \\ 3+5+1 \end{bmatrix}$$

$$= 6[1 \ 1 \ 1]^T$$

---

#### PROBLEM #4.

What is the mean of the dataset  $D$  after adding  $u$  to each sample?

$$D = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \right\} \quad u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$E[x] = \frac{1}{N} \sum_i x_i$$

Account for addition,

$$E[x] = \frac{1}{N} \sum_i (u + x_i)$$

$$= \frac{1}{N} \sum_i u + \frac{1}{N} \sum_i x_i$$

$$= \frac{1}{N} Nu + \frac{1}{N} \sum_i x_i \rightarrow \text{the sum of } u \text{ } N\text{-times is } N \times u.$$

$$= u + \frac{1}{N} \sum_i x_i$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1+3+5 \\ 2+4+3 \\ 3+5+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3[1 \ 1 \ 1]^T$$

$$= [4 \ 5 \ 6]^T$$

### PROBLEM #5

Select the correct formula for  $\bar{x}_n$ , given

$D_n = D_{n-1} \cup \{x_*\}$ ,  $D_{n-1}$  has  $n-1$  datapoints

$x_*$  is one datapoint.

☒  $\bar{x}_n = \bar{x}_{n-1} + \frac{1}{n}(x_* - \bar{x}_{n-1})$

$$E[X] = \frac{1}{N} \sum_i x_i$$

☐  $\bar{x}_n = \bar{x}_{n-1} + \frac{1}{n+1}(x_* - \bar{x}_{n-1})$

Account for second datapoint.

☐  $\bar{x}_n = \bar{x}_{n-1} + \frac{1}{n-1}(x_* - \bar{x}_{n-1})$

$$E[X] = \frac{1}{N} \left( \sum_j x_j + x_* \right)$$

☐  $\bar{x}_n = \bar{x}_{n-1} + \frac{1}{n+1}(\bar{x}_{n-1} - x_*)$

$$= \frac{1}{N} \cdot \frac{(N+1)}{(N+1)} \left( \sum_j x_j + x_* \right)$$

$$= \frac{(N-1)}{N} \times \frac{\sum_j x_j}{(N-1)} + \frac{(N-1)}{N(N-1)} x_*$$

$$\frac{1}{|i|} > \frac{1}{|i+1|}$$

$$= \frac{(N-1)}{N} \bar{x}_{n-1} + \frac{1}{N} x_*$$

$$= \frac{N\bar{x}_{n-1} - x_{n-1}}{N} + \frac{1}{N} x_*$$

$$= \bar{x}_{n-1} + \frac{1}{N} (x_* - \bar{x}_{n-1})$$