PROBLEM #1.

What is the characteristic polynomial and it's solutions?

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$Ax = \lambda x$$
$$0 = Ax - \lambda x$$
$$= (A - \lambda I)x$$
$$= \begin{bmatrix} (1-\lambda) & 0 \\ 0 & (2-\lambda) \end{bmatrix} x$$

Assume that matrix A has eigenvalues, use determinant as a structure with which to determine those eigenvalues,

$$|A - \lambda I| = 0$$
$$= \left| \begin{bmatrix} (1-\lambda) & 0 \\ 0 & 2-\lambda \end{bmatrix} \right|$$
$$= (1-\lambda)(2-\lambda) - (0)(0)$$
$$= \lambda^2 - 3\lambda + 2$$

The eigenvalues are $\lambda = 1, 2$.

PROBLEM #2.

Find the eigenvectors of A, $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

From question #1, eigenvalues are

$\lambda = 1, 2$

Substitute λs in to the eigen-matrix expression,

$$@\lambda=1: \begin{bmatrix} 1-1 & 0 \\ 0 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 = \begin{bmatrix} 0 \\ x_2 \end{bmatrix}$$

$$@\lambda=2: \begin{bmatrix} 1-2 & 0 \\ 0 & 2-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Therefor $x_2 = 0$.

Therefor $x_1 = 0$.

With the constraints found from the backsubstitution get the eigenvalues,

$@\lambda=1: x = [x_1\ 0]^T$ $@\lambda=2: x = [0\ x_2]^T$

PROBLEM #3.

What is the characteristic polynomial it's solution?

$$A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$$

Refer to Problem #1 for full solution steps.

$$|A - \lambda I| = 0$$
$$= (3-\lambda)(5-\lambda) - (4)(0)$$
$$= \lambda^2 - (3+5)\lambda + 15 \text{, characteristic polynomial}$$

Solutions

$\lambda = 3, 5$

PROBLEM #4.

Find eigenvectors for $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$

$$0 = (A - \lambda I)x$$
$$= \begin{bmatrix} (3-3) & 4 \\ 0 & (5-3) \end{bmatrix} x \text{, } @\lambda = 3.$$
$$= \begin{bmatrix} 0 & 4 \\ 0 & 2 \end{bmatrix} x$$
$$= 2\begin{bmatrix} 2x_2 \\ x_2 \end{bmatrix}$$

Therefor x_2 must be zero.

$$0 = (A - \lambda I)x$$
$$= \begin{bmatrix} (3-5) & 4 \\ 0 & (5-5) \end{bmatrix} x \text{, } @\lambda = 5.$$
$$= \begin{bmatrix} -2 & 4 \\ 0 & 0 \end{bmatrix} x$$
$$= \begin{bmatrix} -2x_1 + 4x_2 \\ 0 \end{bmatrix}$$

Therefor $2x_1$ must be equal to x_2.

PROBLEM #5.

What is the characteristic polynomial it's solution?

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

$$C.P = \left| \begin{bmatrix} (1-\lambda) & 0 \\ -1 & (4-\lambda) \end{bmatrix} \right|$$

$= (1-\lambda)(4-\lambda) - (-1)(0)$. solutions are $\lambda = 1, 4$.

$= \lambda^2 - 5\lambda + 4$

$= 0$

PROBLEM #6.

Solve for eigenvectors of A

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

Given the matrix lambda expression, back-substitute the lambda values which will negate the transformational rotations, shearing or mirroring.

$Ax = \lambda x$.

$0 = (A - \lambda I)x$

$$@\lambda = 1: \begin{bmatrix} (1-1) & 0 \\ -1 & (4-1) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 0 \\ -x_1 + 3x_2 \end{bmatrix}$$

Therefor x_1 must be equal to $3x_2$.

$$@\lambda = 4: \begin{bmatrix} 1-4 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$= \begin{bmatrix} -3x_1 \\ -x_1 \end{bmatrix}$$

Therefor if x_1 is zero x_2 can be anything.

PROBLEM #7.

What is the characteristic polynomial it's solution?

$$A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$$

$$0 = \left| \begin{bmatrix} (-3-\lambda) & 8 \\ 2 & (3-\lambda) \end{bmatrix} \right|$$

$$= (-3-\lambda)(3-\lambda) - (8)(2)$$

$$= \lambda^2 - 25$$

$$= (\lambda-5)(\lambda+5)$$

Therefor the lambda solutions are $\lambda = 5, -5$.

PROBLEM #8

Solve for the eigenvectors of A.

$$A = \begin{bmatrix} -3 & 8 \\ 2 & 3 \end{bmatrix}$$

$$0 = (A - \lambda I)x$$

@$\lambda = 5$: $\begin{bmatrix} (-3-5) & 8 \\ 2 & (3-5) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$$\begin{bmatrix} -8x_1 + 8x_2 \\ 2x_1 - 2x_2 \end{bmatrix} = 0$$

Therefor x_1 must equal x_2.

$$u_1 = \begin{bmatrix} t \\ t \end{bmatrix}$$

@$\lambda = -5$: $= \begin{bmatrix} (-3+5) & 8 \\ 2 & (3+5) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$= \begin{bmatrix} 2x_1 + 8x_2 \\ 2x_1 + 8x_2 \end{bmatrix}.$$

Therefore x_1 must be equal to $-4x_2$

$$u_2 = \begin{bmatrix} -4t \\ t \end{bmatrix}$$

PROBLEM #9.

What is the characteristic polynomial it's solution?

$$A = \begin{bmatrix} 5 & 4 \\ -4 & -3 \end{bmatrix}$$

$$0 = (A - \lambda I)x$$

$$\Leftrightarrow \det(A - \lambda I) = 0$$

$$= \det\left(\begin{bmatrix} (5-\lambda) & 4 \\ -4 & (-3-\lambda) \end{bmatrix}\right)$$

$$= (5-\lambda)(-3-\lambda) - (-4)(4)$$

$$= \lambda^2 - 2\lambda + 1$$

Solving for λ,

$$\lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)}$$, find roots by quadratic eq.

$$= \frac{2 \pm \sqrt{0}}{2}, \quad \lambda = 1, 1$$

PROBLEM #10.

What is the characteristic polynomial it's solution?

$$A = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}$$

The matrix equation which posits eigenvectors is,

$$Ax = \lambda x \Leftrightarrow 0 = (A - \lambda I)x$$

$$= \begin{bmatrix} (-2-\lambda) & -3 \\ 1 & (1-\lambda) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Use the determinant to create a polynomial which roots represent the lambda values which when coupled with their eigenvectors will make $(A-\lambda I)x = 0$.

$$\det(A - \lambda I) = 0$$

$$= (-2-\lambda)(1-\lambda) - (-3)(1)$$

$$= \lambda^2 + \lambda + 1$$, the characteristic poly.

Can do shortcut,
using intuition a quadratic $f(x)$,
is Ax: $\min f \leq f(x)$. $\because$ No roots that intersect x-axis.

Use the quadratic equation to solve for λ,

$$\text{roots of } |A-\lambda I| = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{(-3)}}{2}$$

The roots of the characteristic polynomial equation are complex; $\lambda = \frac{-1+\sqrt{-3}}{2}, \frac{-1-\sqrt{-3}}{2}$.