A series of increasing powers of x can be used to represent functions. This can be achieved through the derivation of the Power Series.

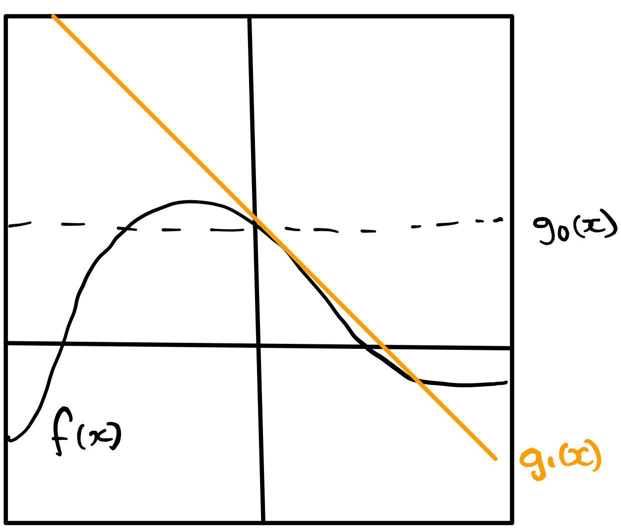
For Example,

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \dots$$

Euler exponential can be approximated by a power series.

If the derivatives of every order of a function can be found at a point and the value of the function at that point is known, that information can be used to reconstruct the function at every other point.

This is only true for well behaved functions,
- funcs that are continuous
- and are differentiable an arbitrary number of times.



$$g_0(x) = f(0)$$
$$g_1(x) = f(0) + f'(0)x$$
$$g_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$
$$g_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f^{(3)}(0)}{6}x^3$$
$$g_4(x) = \frac{f^{(0)}(0)}{0!} + \frac{f^{(1)}(0)x}{1!} + \frac{f^{(2)}(0)x^2}{2!} + \frac{f^{(3)}(0)x^3}{3!} + \frac{f^{(4)}(0)x^4}{4!}$$

$y = ax^2 + bx + c$

$y' = 2ax + b$

$y'' = 2a \iff a = \frac{y''}{2}$

$\iff a = \frac{f''(0)}{2}$, since the double derivative at zero of the function $f(x)$ should be approximately equal to the 2nd order derivative of a quadratic.

FINDING THE COEFF OF 1ST ORDER TERM.

Assume the function is best modelled by a quadratic.

$y = ax^2 + bx + c$

Find the first derivative,

$y' = 2ax + b$

Evaluate the function at $f(0)$ to determine the gradient,

$$
\begin{aligned}
y' &= 2ax + b \\
&= 2a(0) + b \\
&= b
\end{aligned}
$$

Assume that the quadratic is a good approx. of the function $f(x)$,

$y' = b \iff y' \approx f'(0)$

$\iff f'(0) = b$

FINDING THE COEFF OF 0TH ORDER TERM.

$y = ax^2 + bx + c \iff y \approx f(x)$, assume a quadratic is a good representation of the function

$\iff f(0) = a(0)^2 + b(0) + c$

$\iff f(0) = c$

Having determined the coefficients as functions of the derivatives of the model function.

$a = \frac{1}{2}$, $b = 1$, $c = 1$

Can express the second order Power Series Approximation,

$$g_2(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2$$

$\boxed{\text{3rd Order Approximation}}$

$$y = ax^3 + bx^2 + cx + d$$

Can take derivatives,

$$y' = 3ax^2 + 2bx + c$$
$$y'' = 6ax + 2b$$
$$y^{(3)} = 6a$$

} solving for the coefficients of the previous low-order power series will remain the same

→ this is because the data, the point and its derivatives, that is the function - hasn't changed

Then can solve for

$$y^{(3)} = 6a \Leftrightarrow y^{(3)} \approx f^{(3)}(0)$$
$$\Leftrightarrow f^{(3)}(0) = 6a$$
$$a = \frac{f^{(3)}}{6}$$

The implication is that the higher third order is too generalized to better represent/be a substitute for the lower order derivatives.

Conversely this also means unadorned functions - that only consist of a coefficient and higher order function (e.g $f(x) = x^6$) can not be fully approximated by lower order terms of x.

There is information loss.

The iterative process of derivation, due to the calculus power rule, forms a coefficient proportional to the reciprocal of the factorial equal to the order of the derivative.

One Taylor Series is the Maclaurin Series (Colin Maclaurin, 1698 - 1746),

$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

, when specifically calculating derivatives at $f(0)$.