
PROBLEM #1

Re-write the chain rule in Lagrange notation,

Leibniz notation - $\frac{dg}{dx} = \frac{dg}{dh} \frac{dh}{dx}$

Lagrange,

$$f'(x) = f'(h(x))h'(x)$$

PROBLEM #2.

Use the chain rule to differentiate $\exp(x^2-3)$.

$$\text{Let } u = e^{v(x)}$$

$$\text{Let } v(x) = x^2 - 3$$

$$\begin{aligned} \frac{d}{dx} u(v(x)) &= \frac{du}{dv} \frac{dv}{dx} \\ &= e^{v(x)} 2x \\ &= e^{(x^2-3)} 2x \end{aligned}$$

PROBLEM #3.

Use chain rule to derive $\sin^3(x)$.

$$\begin{aligned} \text{Let } f(x) &= \sin^3(x) \\ &= (\sin x)^3 \end{aligned}$$

$$\text{Let } u(h) = h^3, \quad h(x) = \sin x.$$

Derivative of u relative to h ,

$$\frac{d}{dh} h^3 = 3h^2$$

Derivative of h relative to x ,

$$\frac{d}{dx} \sin x = \cos x.$$

Derivative of u relative to x ,

$$\frac{du}{dx} = \frac{d}{dx} f(x)$$

$$= \frac{du}{dh} \frac{dh}{dx}$$

$$= 3h^2 \cos x$$

$$= 3 \sin^2 x \cos x.$$

PROBLEM # 4.

Find the derivative of $\tan x$.

Let $f(x) = \tan x$,

$$f(x) = \frac{r \sin \theta}{r \cos \theta}, \text{ where } \underline{r} \text{ is the hypothetical hypotenuse.}$$
$$= \sin \theta \cos^{-1} \theta$$

Derivative of $f(x)$ relative to x ,

Let $u = \sin \theta$, $g(\theta) = \cos^{-1} \theta$

$$\frac{d}{d\theta} f(\theta) = u(\theta) \frac{d}{d\theta} \cos^{-1} \theta + \cos^{-1} \theta \frac{du}{d\theta}$$

$$= \sin \theta \frac{-1}{\cos^2 \theta} (-\sin \theta) + \cos^{-1} \theta \cos \theta$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos \theta}{\cos \theta}$$

$$= 1 + \tan^2 \theta.$$

PROBLEM # 5.

Derive the function $f(x) = e^{\sin(x^2)}$

Let $u(h) = e^h$, $h(v) = \sin v$, $v(x) = x^2$

$$\frac{d}{dx} u(h(v(x))) = \frac{du}{dg} \frac{dg}{dv}, \text{ where } u(g) = u(h(g))$$

$$= \frac{du}{dg} \frac{dg}{dh} \cdot \frac{dh}{dv}, \text{ where } \frac{du}{dg} = \frac{du}{dh} \frac{dh}{dg}$$

$$\text{Therefore } \frac{du}{dx} = \frac{du}{dh} \frac{dh}{dg} \frac{dg}{dx}, \text{ when } u(x) = u(h(g(x))).$$

□

The derivative of the original $f(x)$,

$$\frac{d}{dx} f(x) = \frac{du}{dh} \frac{dh}{dv} \frac{dv}{dx}$$

$$= e^h \cos v \cdot 2x$$

$$= e^{\sin v} \cos(x^2) \cdot 2x$$

$$= e^{\sin(x^2)} \cos(x^2) \cdot 2x.$$