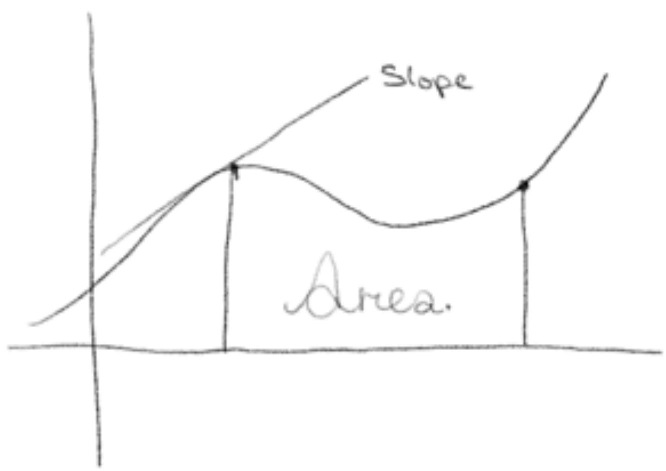
Most of preliminary Calculus is based on graphing.

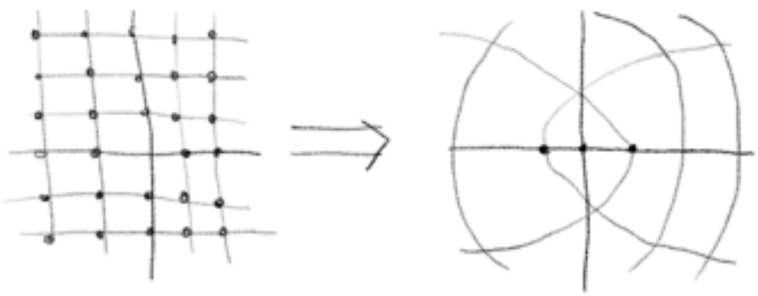
For example,

- Derivatives - are visualized as the slope of a graph.
- Integrals - are visualized as the area underneath the graph.



However generalized Calculus is not always graphable.

Complex $\longrightarrow$ Complex.

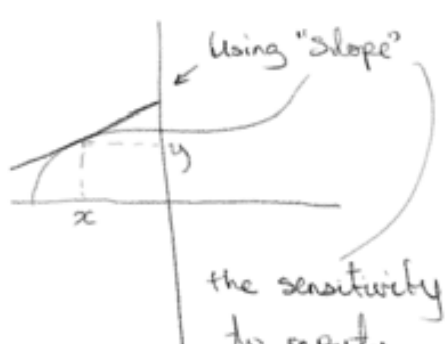


Don't conceptualize Calculus in terms of graphing → because Caculus extends beyond that mental model (e.g multivariable calculus, complex analysis, differential geometry).

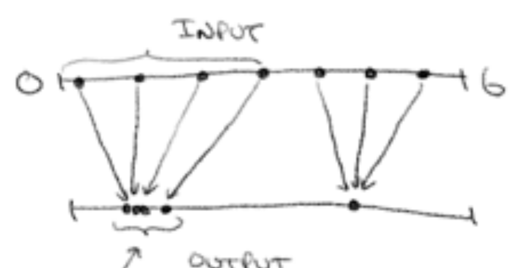
$$(x, y, t) \longrightarrow (x', y')$$

Derivatives can be conceptualized transformationally.

Typical Geometric Vizualization (Graph)

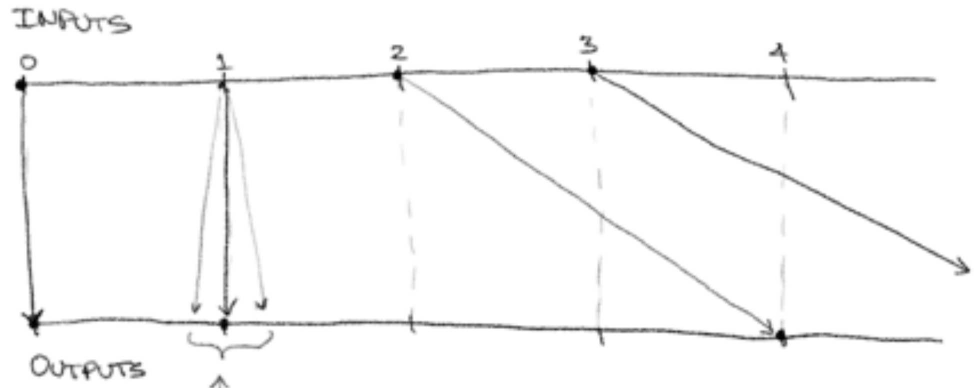


Number Line Vizualization



$\frac{df}{dx}(x)$ measures stretching/squishing.

THE NUMBER LINE OF x^2:



If you were to look at a evenly distributed range of inputs near $(x=1)$, you would find the function (x^2) spreads/stretches those values by a factor of two.

This is what it means for the derivative of x^2 to be 2.

$$f(x) = x^2$$

$$\Longleftrightarrow \quad \frac{df}{dx}(1) = 2(1)$$

and what it looks like in terms of transformations.

- Except with 0. The points aren't stretched or squished. As you get closer to $f^{-1}(y) = 0$, the values of x collapse into zero.

This is what it means for the derivative to be like 0. While the points don't always = 0, the limiting behaviour is 0.
(due to the derivative).

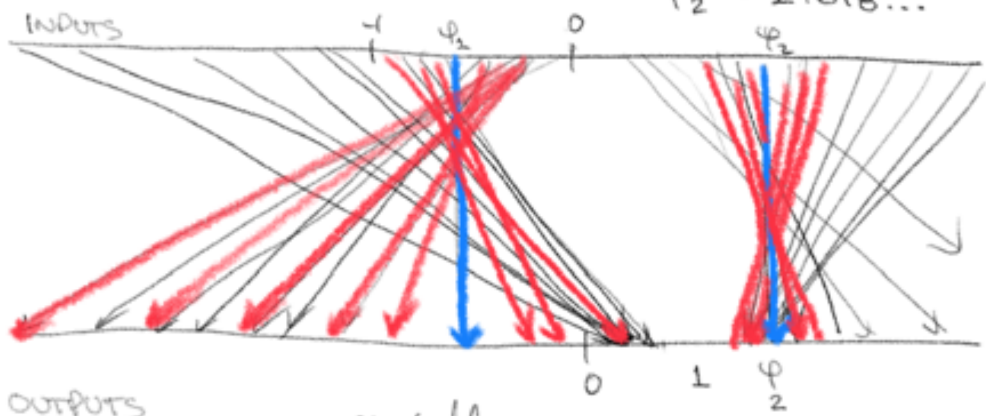
$$\frac{df}{dx}(0) = 2(0).$$

THE NUMBER LINE OF $1+\frac{1}{x}$.

Imagine $f(x) = 1 + \frac{1}{x}$

$\varphi_1 = -0.618\ldots$

$\varphi_2 = 1.618\ldots$



Unstable Fixed Point

Stable Fixed Point

$$\left|\frac{df}{dx}(-0.618\ldots)\right| > 1$$

$$\left|\frac{df}{dx}(1.618\ldots)\right| < 1$$

Meaning the points get stretched.

Meaning the points are shrunk — collapsi[ng] towards φ_2