Differentiation Rules

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

Sum Rule

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{g'(x)h(x) - h'(x)g(x)}{[h(x)]^2}$$

Chain Rule

$$\frac{d}{dx}f(g(x)) = \frac{d}{dg}f(g)\cdot\frac{dg}{dx}g(x)$$

$$= \frac{df}{dg}\cdot\frac{dg}{df}$$

Example: Differentiate function $f(x)$; this will require the derivative sum, product, chain and optionally the quotient rule.

Let $f(x) = \dfrac{\sin(2x^5+3x)}{e^{7x}}$.

Let $g(h) = \sin(h(x))$

Let $h(x) = 2x^5+3x$

The derivative of $g(h)$,

$$\frac{d}{dh}g(h) = \frac{d}{dh}\sin h$$

$$= \cos h$$

The derivative of $h(x)$,

$$\frac{d}{dx}h(x) = \frac{d}{dx}(2x^3+3x)$$

$$= \frac{d}{dx}2x^5 + \frac{d}{dx}3x$$

$$= (5)2x^4 + 3.$$

The derivative of $g(x)$,

$$\frac{d}{dx}g(x) = \frac{d}{dh}g(h) \cdot \frac{d}{dx}h(x)$$

$$= \cos h\left[5(2x^4)+3\right]$$

$$= \cos(2x^5+3x)\left[(5)2x^4+3\right]$$

Let $u(j) = e^j$

Let $j(x) = 7x$

The derivative of e^j,

$$\frac{d}{dj}u(j) = \frac{d}{dj}e^j$$

$$= e^j$$

The derivative of $7x$,

$$\frac{d}{dx}j(x) = \frac{d}{dx}7x.$$

$$= 7(1)$$

The derivative of e^{7x},

$$\frac{d}{dx}e^{-7x} = \frac{d}{dj}u(j) \cdot \frac{d}{dx}j(x)$$

$$=_- e^j \cdot 7$$

$$=_- e^{7x} \cdot 7$$

The derivative of $f(x)$,

$$f(x) = \frac{\sin(2x^5+3x)}{e^{7x}}.$$

$$\frac{d}{dx} f(x) = g(x) \frac{d}{dx} u(x) + u(x) \frac{d}{dx} g(x)$$

$$= \sin(2x^5 + 3x)(-7e^{-7x})$$
$$+ \frac{1}{e^{7x}} \cos(2x^5 + 3x)\left[(5) 2x^4 + 3\right]$$

(*) In real life don't pre-optimize by factorizing.
It might be more helpful to leave the expression in its expanded form.