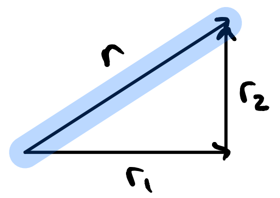
1. The size of a vector is the root of the squared sum of all it's components



$$|r| = \sqrt{|r|_1^2 + |r|_2^2}$$
$$= \sqrt{\sum_i^d |r|_i^2}$$

, more generally this distance (euclidean) holds a cross any number of dimensions $\underline{d}$.

What is the size of $v = [1\ 3\ 4\ 2]^T$

$$|v| = \sqrt{(1)^2 + (3)^2 + (4)^2 + (2)^2}$$
$$= \sqrt{30}$$

PROBLEM #2

What is $r = [-5\ 3\ 2\ 8]^T$ dotted with $s = [1\ 2\ -1\ 0]^T$

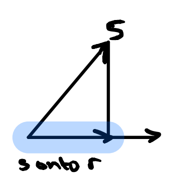
$$r \cdot s = -5(1) + 3(2) + 2(-1) + 8(0)$$
$$= -1$$

PROBLEM #3

Scaler projection can happen in any dimension

What is the scaler proj. of $r = [3\ -4\ 0]^T$ onto $s = [10\ 5\ -6]^T$

scaler proj: $\dfrac{r \cdot s}{|r|} = |s| \cos\theta$

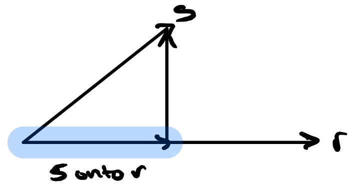


"Want to find the size of the blue vector" - scalar proj.

$$r \cdot s = 3(10) + (-4)(5) + 0(-6)$$
$$|r| = \sqrt{3^2 + (-4)^2 + (0)^2}$$

$$\frac{r \cdot s}{|r|} = \frac{10}{5}$$
$$= 2$$

PROBLEM #4.



The vector proj. is the blue vector.

what is the vector projection of $r = [3 \ -4 \ 0]^T$ onto $s = [10 \ 5 \ -6]^T$?

$$\text{vector proj.} = \frac{r}{|r|} \times \text{scalar proj.}$$

$$= \frac{r}{|r|} \times \frac{r \cdot s}{|r|}, \quad \text{or} \quad \frac{r}{|r|} \times |s| \cos\theta$$

$$= r \, \frac{r \cdot s}{r \cdot r}$$

$$r \cdot s = 3(10) + (-4)(5) + (0)(-6)$$
$$= 10$$

$$r \cdot r = (3)^2 + (-4)^2 + (0)^2$$
$$= 25$$

$$r \, \frac{r \cdot s}{r \cdot r} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix} \frac{10}{25}$$

$$= \begin{bmatrix} 6/5 \\ -8/5 \\ 0 \end{bmatrix}$$

PROBLEM #5.

Which is larger given $a = [3 \ 0 \ 4]^T$ and $b = [0 \ 5 \ 12]^T$ is $|a+b|$ or $|a| + |b|$?

Options:

(in this case).

I. $|a+b| < |a| + |b|$

II. $|a+b| = |a| + |b|$

III. $|a+b| > |a| + |b|$

Given the two vectors conjoined have an angle between them from $[0° - 180°]$, their addition will either be synergistic or otherwise.

Therefor $|a+b| \leq |a| + |b|$.

$$|a| = \sqrt{(3)^2 + (0)^2 + (4)^2} = 5$$

$$|b| = \sqrt{(0)^2 + (5)^2 + (12)^2} = 13$$

$$a+b = [(3+0) \ \ (0+5) \ \ (4+12)]^T$$

$$|a+b| = \sqrt{3^2 + (5)^2 + 16^2} = \sqrt{290}$$

PROBLEM #6.

Which of the following statements about the dot-product are correct?

☐ The dot product is not commutative (i.e $s \cdot r \neq r \cdot s$) $\Rightarrow$ dot prod $= \forall_{i \in d} |s_i| \times |r_i|$, multiplication is alway commutative

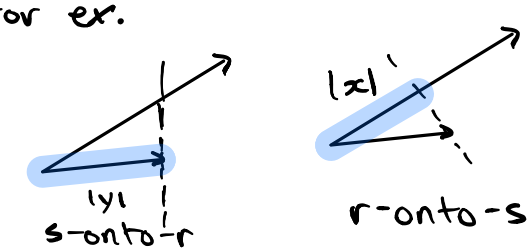
☐ The scalar projection of s-onto-r is always the same as r-onto-s.

scalar proj of r-onto-s $= \dfrac{s \cdot r}{|s|} = |r|\cos\theta$

scalar proj of s-onto-r $= \dfrac{r \cdot s}{|r|} = |s|\cos\theta$

Both equations share $r \cdot s$, which is commutative. But $|s|$ is not always equal to $|r|$. Therefor the scalar projections are not always equal.

For ex.



$|x| \neq |y|$

☑ The vector proj of s-onto-r is equal to scalar proj of s-onto-r multiplied by vector of unit length with direction of r.

vector proj $= \dfrac{r}{|r|} \cdot$ scalar proj., given this formula the statement is true.

$= \dfrac{r}{|r|} \times \dfrac{r \cdot s}{|r|}$

$= r \, \dfrac{r \cdot s}{r \cdot r}$

☑ We can find the angle between two vectors using the dot product.

$$\text{dot prod} = \forall i \in d \quad d_i = s_i \times r_i$$

$$= |r||s|\cos\theta$$, by equating the dot product and cosine rule (using vectors) and simplifying.

to get the angle between two vectors can

dot prod = x, some val

$$r \cdot s = x \iff x = |r||s|\cos\theta$$

$$\iff \cos\theta = x \times \frac{1}{|r||s|}$$

$$\theta = \cos^{-1}\left(\frac{x}{|r||s|}\right)$$

Thus, you could find the angle using the dot prod and moduli of participating vectors.

☑ The size of the vector is equal to the square root of the dot product of the vector with itself.

Assume the converse,

$$|r| \neq \sqrt{r \cdot r}$$

$$|r| = \sqrt{\sum_{i}^{d} |r|_i^2}$$

$$\sqrt{r \cdot r} = \sqrt{|r||r|\cos\theta}$$

$$= \sqrt{|r||r|\cos(0)}$$

$$= \sqrt{|r||r|(1)}$$

$$= \sqrt{|r|^2}$$

$= |r|$, Therefor $|r| \neq \sqrt{r \cdot r}$ is false and the hypothesis must be true!