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PROBLEM #1

What is the mean of  $D = \{1, 2, 3\}$ .

$$E[X] = \frac{1}{N} \sum_i x_i$$

$$\begin{aligned} E[D] &= \frac{1}{3} (1 + 2 + 3) \\ &= 2 \end{aligned}$$

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PROBLEM #2.

What is the mean of  $D = \left\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$ .

$$E[X] = \frac{1}{N} \sum_i x_i.$$

Since vectors are objects which follow the axioms of a linear system,

$$E[D] = \frac{1}{3} \begin{bmatrix} 1+2+3 \\ 4+5+6 \\ 7+8+9 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

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PROBLEM #3

What is the mean of the dataset after multiplying each sample by 2.

$$D = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \right\}$$

$$E[x] = \frac{1}{N} \sum_i x_i$$

Account for multiplier per sample,

$$E[x] = \frac{1}{N} \sum_i 2x_i$$

$$= \frac{2}{N} \sum_i x_i$$

$$= \frac{2}{3} \begin{bmatrix} 1+3+5 \\ 2+4+3 \\ 3+5+1 \end{bmatrix}$$

$$= 6 [1 \ 1]^T$$

#### PROBLEM #4.

What is the mean of the dataset D after adding u to each sample?

$$D = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \right\} \quad u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$E[x] = \frac{1}{N} \sum_i x_i$$

Account for addition,

$$E[x] = \frac{1}{N} \sum_i (u + x_i)$$

$$= \frac{1}{N} \sum_i u + \frac{1}{N} \sum_i x_i$$

$= \frac{1}{N} N u + \frac{1}{N} \sum_i x_i \rightarrow$  the sum of  $u$   $N$ -times is  $N \cdot u$ .

$$= u + \frac{1}{N} \sum_i x_i$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1+3+5 \\ 2+4+3 \\ 3+5+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 3[1+1]^T$$

$$=[4 \ 5 \ 6]^T$$

### PROBLEM #5

Select the correct formula for  $\bar{x}_n$ , given

$D_n = D_{n-1} \cup \{x_*\}$ ,  $D_{n-1}$  has  $n-1$  datapoints

$x_*$  is one datapoint.

$\bar{x}_n = \bar{x}_{n-1} + \frac{1}{n}(x_* - \bar{x}_{n-1})$        $E[X] = \frac{1}{N} \sum_i x_i$

$\bar{x}_n = \bar{x}_{n-1} + \frac{1}{n+1}(x_* - \bar{x}_{n-1})$       Account for second datapoint,

$\bar{x}_n = \bar{x}_{n-1} + \frac{1}{n-1}(x_* - \bar{x}_{n-1})$        $E[X] = \frac{1}{N} (\sum_j x_j + x_*)$

$$\frac{1}{n+1} (\bar{x}_{n-1} - x_*)$$

$$= \frac{1}{N} \cdot \frac{(N-1)}{(N+1)} (\sum_j x_j + x_*)$$

$$= \frac{(N-1)}{N} \times \frac{\sum_{j \neq i} x_j}{(N-1)} + \frac{(N-1)}{N(N-1)} x_*$$
$$\frac{1}{|i|} > \frac{1}{|i+1|}$$

$$= \frac{(N-1)}{N} \bar{x}_{n-1} + \frac{1}{N} x_*$$

$$= \frac{N\bar{x}_{n-1} - \bar{x}_{n-1}}{N} + \frac{1}{N} x_*$$

$$= \bar{x}_{n-1} + \frac{1}{N} (x_* - \bar{x}_{n-1})$$