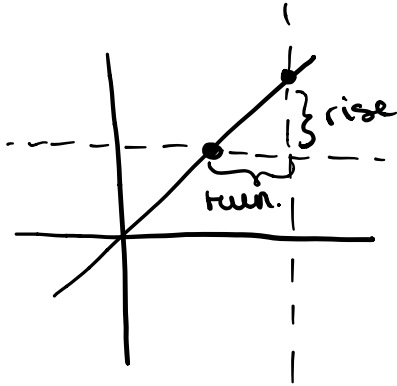Gradients are derivatives.

- horizontal lines having a gradient of zero
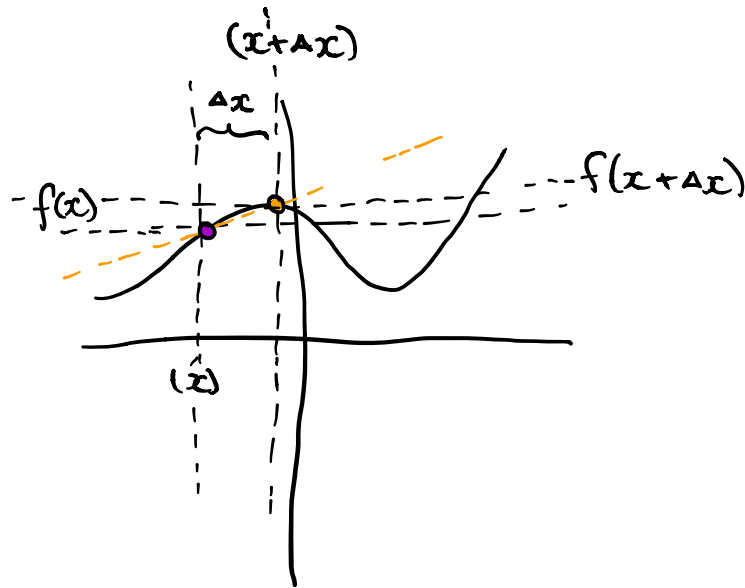- downward/upward sloping lines have +/- gradients.

The gradient of a function,

Is the amount a function changes divided by the length of the interval (average).

gradient = "rise-over-run"

The gradient can be applied on very small intervals to approximate the true rate of change.

By decreasing delta x, the approximation becomes more accurate.

$$\text{Gradient at } x = \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

The gradient can also be written as,

$$\text{Gradient at } x = f'(x) = \frac{df}{dx}.$$

The $\lim_{\Delta x \to 0}$ means as x becomes extremely close to zero.

__Not__ x is zero.

Example: Computing the derivative of $f(x) = 3x + 2$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[3(x + \Delta x) + 2] - [3x + 2]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x + 3\Delta x + 2 - 3x - 2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{3x} + 3\Delta x + \cancel{2} - \cancel{3x} - \cancel{2}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 3$$

$$= 3.$$

(*) This derivative makes __sense__ because the gradient of a line (e.g $3x + 2 \Leftrightarrow mx + b$), is of a constant rate of change.

(*₂) The function $\underline{3x} + \underline{2}$ has two components and the derivative/gradient can be computed for composite expressions.

(*₃) This is known as the sum rule.

The Sum Rule $- \dfrac{d}{dx}\left(f(x) + g(x) \right) = \dfrac{df(x)}{dx} + \dfrac{dg(x)}{dx}$

Example: Computing the derivative of $5x^2$.

$f'(x) = \lim\limits_{\Delta x \to 0} \left(\dfrac{f(x+\Delta x) - f(x)}{\Delta x} \right)$

$= \lim\limits_{\Delta x \to 0} \left(\dfrac{5(x+\Delta x)^2 - 5x^2}{\Delta x} \right)$

$= \lim\limits_{\Delta x \to 0} \dfrac{5(x^2 + 2\Delta x + \Delta x^2) - 5x^2}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{10\Delta x + 5\Delta x^2}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} 10 + 5\Delta x$

$= 10$, as x approaches zero $5\Delta x \to 0$.

(*) This can be generalized for all functions of the form
$f(x) = ax^b \iff f'(x) = bax^{(b-1)}$.