PROBLEM #1.

What are the partial derivatives with respect to x and y?

$$f(x,y) = \pi x^3 + xy^2 + my^4$$

$$\frac{df}{dx} = \frac{d}{dx}(\pi x^3 + xy^2 + my^4)$$

$$= \pi 3x^2 + y^2$$

$$\frac{df}{dy} = \frac{d}{dy}(\pi x^3 + xy^2 + my^4)$$

$$= (0) + 2xy + 4my^3$$

PROBLEM #2.

Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ for $f(x,y,z) = x^2y + y^2z + z^2x$.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^2y + y^2z + z^2x)$$

$$= 2xy + (0) + z^2$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2y + y^2z + z^2x)$$

$$= x^2 + 2yz + (0)$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}(x^2y + y^2z + z^2x)$$

$$= (0) + y^2 + 2zx$$

PROBLEM #3.

Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$ for $f(x,y,z) = e^{2x}\sin(y)z^2 + \cos(z)e^xe^y$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(e^{2x}\sin(y)z^2 + \cos(z)e^xe^y)$$

$$= e^{2x}\frac{\partial}{\partial x}(\sin(y)z^2) + \sin(y)z^2\frac{\partial}{\partial x}e^{2x} + e^x\frac{\partial}{\partial x}\cos(z)e^y$$

$$+ \cos z\, e^y\frac{\partial}{\partial x}e^x$$

$$= e^{2x}(0) + \sin(y)z^2 e^{2x}\cdot 2 + e^x(0) + \cos(z)e^y e^x(1)$$

$$= \sin(y)z^2e^{2x}(2) + \cos(z)e^ye^x$$

$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(e^{2x}\sin(y)z^2 + \cos(z)e^xe^y)$, with factoring

$$= \frac{\partial f}{\partial x}\ e^x(e^x\sin y\ z^2 + \cos z\ e^y)$$

$$= e^x\frac{\partial f}{\partial x}(e^x\sin y\ z^2 + \cos z\ e^y) + (e^x\sin y\ z^2 + \cos z\ e^y)$$

$$\times\frac{\partial f}{\partial x}e^x$$

$$= e^x\left[\frac{\partial f}{\partial x}(e^x\sin y\ z^2) + \frac{\partial f}{\partial x}(\cos z\ e^y)\right] + (e^x\sin y\ z^2 + \cos z\ e^y)e^x$$

$$= e^{2x}\sin y\, z^2 + e^x(e^x\sin y\, z^2 + \cos z\, e^y)$$

$$= e^x\left[(e^x\sin y\, z^2 + \cos z\, e^y) + e^x\sin y\ z^2\right]$$

$$= e^x(e^x\sin y\, z^2(2) + \cos z\, e^y)$$

(*) <u>way easier.</u>

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[e^x(e^x\sin y\ z^2 + \cos z\ e^y)\right]$$

$$= e^x\frac{\partial}{\partial y}(e^x\sin y\ z^2 + \cos z\ e^y)$$

$$+ (e^x\sin y\ z^2 + \cos z\ e^y)\frac{\partial}{\partial y}(e^x)$$

$$= e^x\left[\frac{\partial}{\partial y}(e^x\sin y\ z^2) + \frac{\partial}{\partial y}(\cos z\ e^y)\right] + (e^x\sin y\ z^2 + \cos z\ e^y)(0)$$

$$= e^x\left[e^x z^2\cos y + e^y\cos z(1)\right]$$

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left[e^{x}(e^{x}\sin y\, z^2 + \cos z\, e^{y})\right]$$

$$= e^{x}\left[\frac{\partial}{\partial z}(e^{x}\sin y\, z^2) + \frac{\partial}{\partial z}(\cos z\, e^{y})\right] + (e^{x}\sin y\, z^2 + \cos z\, e^{y})\frac{\partial}{\partial z}e^{x}$$

$$= e^{x}\left[2z e^{x}\sin y + (-\sin z\, e^{y})\right]$$

PROBLEM # 4.

Calculate the total derivative of $f(x,y) = \frac{\sqrt{x}}{y}$, $x(t)=t$, $y(t)=\sin(t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

The f-partial derivative to y,

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\sqrt{x}\, y^{-1})$$

$$= \sqrt{x}\frac{\partial}{\partial y}(y^{-1}) + y^{-1}\frac{\partial}{\partial y}(\sqrt{x})$$

$$= \sqrt{x}(-y^{-2}) + y^{-1}(0)$$

$$= -\frac{\sqrt{x}}{y^2}$$

The f-partial derivative to x,

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(\sqrt{x}\, y^{-1})$$

$$= \sqrt{x}\frac{\partial}{\partial x}(y^{-1}) + y^{-1}\frac{\partial}{\partial x}(\sqrt{x})$$

$$= \sqrt{x}(0) + y^{-1}\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}\, y}$$

The derivative of y-to-t,

$$\frac{dy}{dt} = \frac{d}{dt}\sin t$$

$$= \cos t$$

The derivative of x-to-t,

$$\frac{dx}{dt} = \frac{d}{dt} t$$

$$= 1$$

The total derivative is,

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$= \frac{1}{2\sqrt{x}y}(1) + \left(-\frac{\sqrt{x}}{y^2}\cos t\right)$$

$$= \frac{1}{2\sqrt{t}\sin t} - \frac{\sqrt{t}\cos t}{\sin^2 t}$$, substituting t definitions for x and y.

PROBLEM #5.

Calculate the total derivative of

$$f(x, y, z) = \cos(x)\sin(y)e^{2z}$$

$$x(t) = t+1$$
$$y(t) = t-1$$
$$z(t) = t^2$$

The total derivative is,

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

The f-partial derivative to x is,

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\cos(x)\sin(y)e^{2z}$$

$$= \cos x \frac{\partial}{\partial x}\sin(y)e^{2z} + \sin(y)e^{2z}\frac{\partial}{\partial x}\cos x$$

$$= \cos x(0) + \sin(y)e^{2z}(-\sin x)$$

$$= -\sin x \sin y e^{2z}$$

The f-partial derivative to y is,

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\cos(x)\sin(y)e^{2z}$$

$$= \sin y \frac{\partial}{\partial y} \cos x e^{2z} + \cos x e^{2z} \frac{\partial}{\partial y} \sin y$$

$$= \sin y(0) + \cos x e^{2z} \cos y$$

$$= \cos x \cos y e^{2z}$$

The f-partial derivative to z is,

$$\frac{\partial f}{\partial z} = \frac{\partial}{\partial z} \cos(x) \sin(y) e^{2z}$$

$$= e^{2z} \frac{\partial}{\partial z} \cos x \sin y + \cos x \sin y \frac{\partial}{\partial z} e^{2z}$$

$$= e^{2z}(0) + \cos x \sin y e^{2z} \cdot 2$$

$$= 2 \cos x \sin y e^{2z}$$

The derivative of x-to-t,

$$\frac{dx}{dt} = \frac{d}{dt}(t+1)$$

$$= 1$$

The derivative of y-to-t,

$$\frac{dy}{dt} = \frac{d}{dt}(t-1)$$

$$= 1$$

The derivative of z-to-t,

$$\frac{dz}{dt} = \frac{d}{dt}(t^2)$$

$$= 2t$$

The total derivative is,

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

$$= -\sin x \sin y e^{2z}(1) + \cos x \cos y e^{2z}(1) + 2 \cos x \sin y e^{2z} 2t$$

$$= -\sin(t+1)\sin(t-1) e^{2t^2} + \cos(t+1)\cos(t-1) e^{2t^2} + 2\cos(t+1)\sin(t-1) e^{2t^2} 2t$$

$$= e^{2t^2}\left[-\sin(t+1)\sin(t-1) + \cos(t+1)\cos(t-1) + 4t\cos(t+1)\sin(t-1)\right]$$