PROBLEM #1.

Use the Maclaurin series to formulate approximations of the first three non-zero terms for the function,

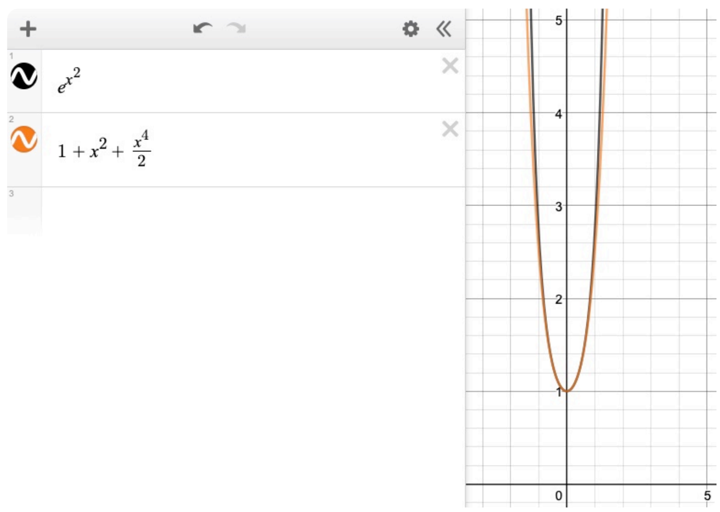
$$f(x) = e^{x^2}$$

The Maclaurin series is

$$g(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$g_0(0) = \frac{f^{(0)}(0)}{0!} x^{(0)}$$

$$= \frac{1}{1}(1)$$



$$g_1(x) = 1 + \frac{f^{(1)}(0)}{1!} x^1$$

$$g_1(x) = 1 + e^{x^2} 2x \cdot x^1$$

$$= 1 + e^0 2(0) \cdot x$$

$$= 1 + 0$$

$$g_2(x) = 1 + 0 + \frac{f^{(2)}(0)}{2!} x^2$$

$$= 1 + \left[e^{x^2} 4x^2 + 2e^{x^2}\right] x^2$$

$$= 1 + 2x^2/2$$

$$g_3(x) = 1 + x^2 + \frac{f^{(3)}(0)}{3!} x^3$$

$$= 1 + x^2 + \left[e^{x^2} 8x^3 + 8x e^{x^2} + 2e^{x^2} 2x\right] x^3$$

$$= 1 + x^2$$

$$g_4(x) = 1 + x^2 + \left[e^{x^2} 16x^4 + 24x^2 e^{x^2} + 8e^{x^2} + e^{x^2} 16x^2 + 4e^{x^2} + 8x^2 e^{x^2}\right] x^4$$

$$= 1 + x^2 + 12(1) x^4 / 4!$$

PROBLEM #2.

Use Taylor Series to approximate the first three terms of $f(x) = 1/x$ at $x = p = 4$.

Taylor Series: $\sum_{n=0}^{\infty} \frac{f^{(n)}(p)}{n!}(x-p)^n$

$$y^{0}(1/x) = 1/x$$

$$\frac{d}{dx}(1/x) = -x^{-2}$$

$$y''(1/x) = 2x^{-3}$$

$$y^{(3)}(1/x) = -6x^{-4}$$

$$g_0(x) = \frac{1/x}{0!}(x-4)^0$$

$$= \frac{\frac{1}{4}}{1}(1)$$

$$= 1/4$$

$$g_1(x) = \frac{-(p)^{-2}}{1!}(x-4)^1 + g_0(x)$$

$$= -\frac{(x-4)}{4^2} + g_0(x)$$

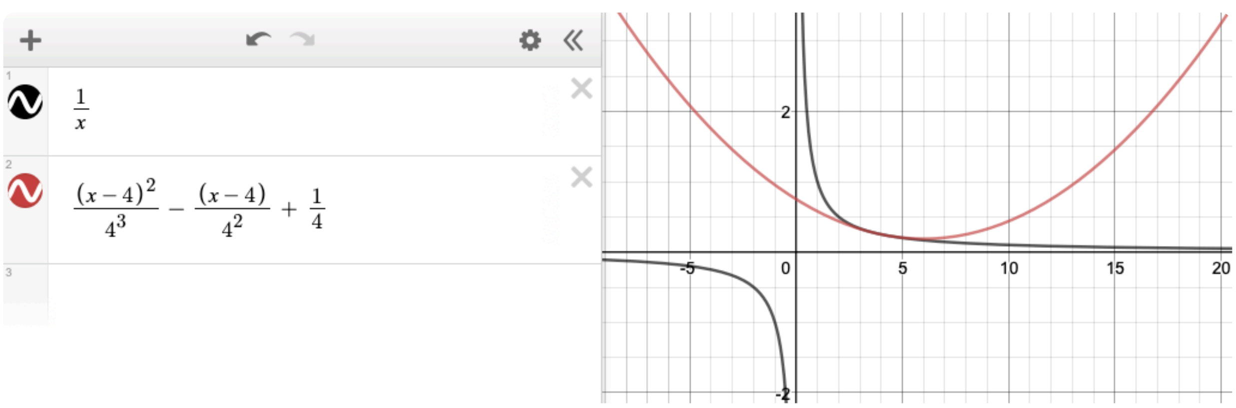
$$= -\frac{(x-4)}{16} + 1/4$$

$$g_2(x) = \frac{2(p)^{-3}}{2!}(x-p)^2 + g_1(x)$$

$$= 4^{-3}(x-4)^2 + g_1(x)$$

$$= \frac{(x-4)^2}{4^3} + g_1(x)$$

$$= \frac{(x-4)^2}{4^3} - \frac{(x-4)}{4^2} + \frac{1}{4}$$



PROBLEM #3

Determine the magnitude of difference, between the second order and first order expansion of the Taylor Series when approximating to find the value of $f(2)$, use $f(x) = \ln x$ at the point $x = 10$.

$$f^{(0)}(x) = \ln x$$

$$f^{(1)}(x) = \frac{d}{dx} \ln x$$

The derivative of $\ln x$,

$$e^y = x \Leftrightarrow \frac{d}{dx} e^y = \frac{d}{dx} x$$

$$\Leftrightarrow e^y \frac{d}{dx}(y) = 1,$$ provided has an object relative to x.

$$\Leftrightarrow \frac{dy}{dx} = \frac{1}{e^y}$$

$$\Leftrightarrow \frac{dy}{dx} = \frac{1}{x},$$ given $e^y = x$.

Therefor the derivative of $\ln x$ and the first order derivative of $f(x) = \ln x$ is,

$$= x^{-1}.$$

$$f^{(2)}(x) = \frac{d}{dx} x^{-1}$$

$$= -x^{-2}$$

Taylor Series Approximations. $\sum_{n=0}^{\infty} \frac{f^{(n)}(p)}{n!}(x-p)^n$,

$$g_0(x) = \frac{\ln(p)(x-10)^0}{1!}$$

$$= \ln 10$$

$$g_1(x) = \frac{1}{p \cdot 1!}(x-p)^1 + g_0(x)$$

$$= \frac{(x-10)}{10} + \ln 10$$

$$g_2(x) = \frac{f^{(2)}(p)}{n!}(x-p)^n + g_1(x)$$

$$= -\frac{(x-10)^2}{(10)^2 2!} + \frac{x-10}{10} + \ln 10.$$

The difference between the first order and second order approximation is $g_2(x) - g_1(x)$ at $x = 2$.

$$\Delta f(2) = g_2(x) - g_1(x), \text{ where } x = 2.$$

$$= -\frac{(x-10)^2}{2 \cdot 10^2} + \frac{x-10}{10} + \ln 10 - \left[\frac{x-10}{10} + \ln 10\right]$$

$$= -\frac{(x-10)^2}{10^2 \cdot 2!}$$

$$= -\frac{(2-10)^2}{10^2 \cdot 2}$$

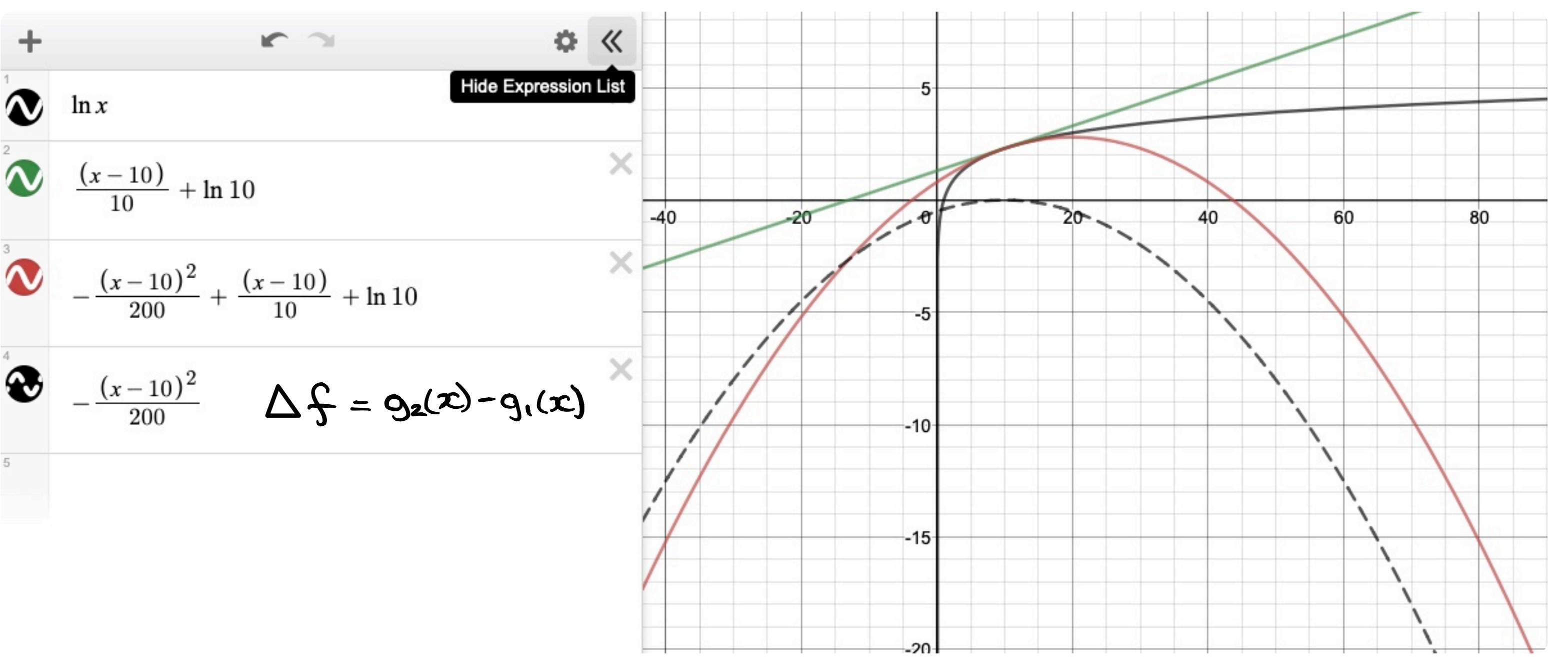
$$= -\frac{(8)^2}{10^2 \cdot 2}$$

$$= -\frac{\left(\frac{8}{10}\right)^2}{2}$$

$$= -\frac{1}{2} \times \left(\frac{4}{5}\right)^2$$

$$= -\frac{(16/2)}{25}$$

$$= -8/25$$



PROBLEM #4.

Find the general taylor series approximation for $f(x) = \frac{1}{(1-x)^2}$ where $x=0$.

$f^{(0)}(x) = \frac{1}{(1-x)^2}$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(p)}{n!}(x-p)^n$, where $p=0$.

$f^{(1)}(x) = \frac{-2(-1)}{(1-x)^3}$

$g_0(x) = \frac{\frac{1}{(1-0)^2}}{0!}(x-0)^0$

$= 1$

$f^{(2)}(x) = \frac{3(2)}{(1-x)^4}$

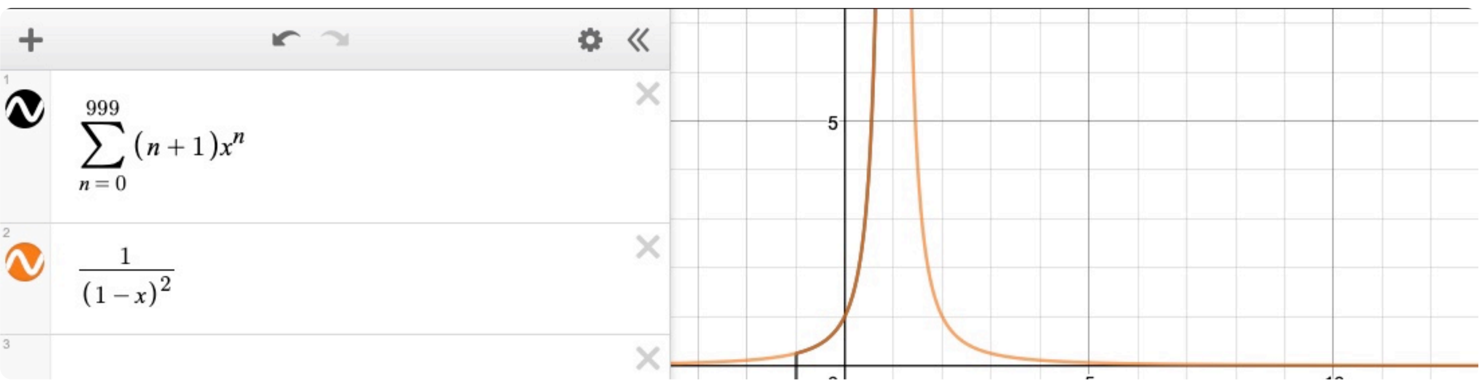
$g_1(x) = \frac{\frac{2}{(1-0)^3}}{1!}(x-0)^1 + g_0(x)$

$f^{(3)}(x) = \frac{4(3)(2)}{(1-x)^5}$

This can be generalized to,

$f(x) \approx \sum_{n=0}^{\infty} \frac{(n+1)!}{n!} x^n$

$\approx \sum_{n=0}^{\infty} (n+1)x^n$



PROBLEM # 5.

Use the Maclaurin series to approximate $f(x) = \sqrt{4-x}$. as a quadratic equation.

$f^{(0)}(x) = \sqrt{4-x}$

$M = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$f^{(1)}(x) = \frac{1}{2}(4-x)^{-1/2}(-1)$

$g_0(x) = \frac{\sqrt{4-x}}{0!} x^0$

$g_2(x) = \frac{-\frac{1}{4}(4-x)^{\frac{-3}{2}}}{2!} x^2 - \frac{1}{2}(4-x)^{-1/2} x + \sqrt{4-x}$

$f^{(2)}(x) = +\frac{1}{4}(4-x)^{-3/2}(-1)$

$g_1(x) = \frac{\frac{1}{2}(4-x)^{-1/2}}{1!} x + \sqrt{4-x}$

$$g_2(x) = \frac{-x^2}{64} - \frac{x}{4} + 2$$