PROBLEM #1.

Calc Jacob. Matrix

$u(x,y) = x^2 - y^2$
$v(x,y) = 2xy$

$$J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

$$= \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$$

$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2 - y^2)$

$= 2x$

$\frac{\partial u}{\partial y} = -2y$

$\frac{\partial v}{\partial x} = 2y$

$\frac{\partial v}{\partial y} = 2x$

PROBLEM #2

Calculate Jacobian matrix

$u(x,y,z) = 2x + 3y$
$v(x,y,z) = \cos x \sin z$
$w(x,y,z) = e^{x+y+z}$

$$J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 0 \\ -\sin x \sin z & 0 & \cos z \cos x \\ e^{x+y+z} & e^{x+y+z} & e^{x+y+z} \end{bmatrix}$$

$\frac{\partial u}{\partial x} = 2$ $\qquad \frac{\partial v}{\partial x} = -\sin x \sin z$

$\frac{\partial u}{\partial y} = 3$ $\qquad \frac{\partial v}{\partial y} = 0$

$\frac{\partial u}{\partial z} = 0$ $\qquad \frac{\partial v}{\partial z} = \cos z \cos x$

$\frac{\partial w}{\partial x} = e^{x+y+z}$

$\frac{\partial w}{\partial y} = e^{x+y+z}$

$\frac{\partial w}{\partial z} = e^{x+y+z}$

PROBLEM #3.

Calc Jacobian matrix.

$$u(x,y) = ax + by$$
$$v(x,y) = cx + dy$$, where a, b, c, d are const.

$$J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

$$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ → just equals coefficients of the concerned var of the partial derivative.

PROBLEM #4.

Calc Jacobian matrix & eval at $\vec{0}$.

$$u(x,y,z) = 9x^2y^2 + ze^x$$
$$v(x,y,z) = xy + x^2y^3 + 2z$$
$$w(x,y,z) = \cos x \sin z \, e^y$$

$$J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial w} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

$$\frac{\partial u}{\partial x} = 18xy^2 + e^x z$$
$$\frac{\partial u}{\partial y} = 2y9x^2$$
$$\frac{\partial u}{\partial z} = e^x$$

$$\frac{\partial v}{\partial x} = y + 2xy^3$$
$$\frac{\partial v}{\partial y} = x + 3y^2x^2$$
$$\frac{\partial v}{\partial z} = 2$$

$$\frac{\partial w}{\partial x} = -\sin x \sin z e^y$$
$$\frac{\partial w}{\partial y} = e^y \cos x \sin z \quad \frac{\partial w}{\partial z} = \cos z \cos x e^y$$

$$J = \begin{bmatrix} 18xy^2 + e^x z & 2y9x^2 & e^x \\ y + 2xy^3 & x + 3y^2x^2 & 2 \\ -\sin x \sin z e^y & e^y \cos x \sin z & \cos z \cos x e^y \end{bmatrix}$$

$$J\vec{0} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

PROBLEM #5.

Calc. Jacobian matrix..

$$x(r,\theta,\phi) = r\cos\theta \sin\phi$$
$$y(r,\theta,\phi) = r\sin\theta \sin\phi$$
$$z(r,\theta,\phi) = r\cos\phi$$

$$J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{bmatrix}$$

$=$

$$\frac{\partial x}{\partial r} = \cos\theta \sin\phi$$
$$\frac{\partial x}{\partial \theta} = -\sin\theta \, r\sin\phi$$
$$\frac{\partial x}{\partial \phi} = r\cos\theta \cos\phi$$
$$\frac{\partial y}{\partial r} = \sin\theta \sin\phi$$
$$\frac{\partial y}{\partial \theta} = r\cos\theta \sin\phi$$
$$\frac{\partial y}{\partial \phi} = r\sin\theta \cos\phi$$
$$\frac{\partial z}{\partial r} = \cos\phi$$
$$\frac{\partial z}{\partial \theta} = 0$$
$$\frac{\partial z}{\partial \phi} = -r\sin\phi.$$

$$J = \begin{bmatrix} \cos\theta \sin\phi & -\sin\theta\, r\sin\phi & r\cos\theta \cos\phi \\ \sin\theta \sin\phi & r\cos\theta \sin\phi & r\sin\theta \cos\phi \\ \cos\phi & 0 & -r\sin\phi \end{bmatrix}$$