

Partial differentiation is an extension of single variable. There is also a total derivative.

$$f(x, y, z) = \sin x e^{yz^2}$$

$$\frac{\partial f}{\partial x} = \cos x e^{yz^2}$$

$$\frac{\partial f}{\partial y} = \sin x e^{yz^2} z^2$$

$$\frac{\partial f}{\partial z} = \sin x e^{yz^2} 2yz$$

Can form a total derivative by normalizing the independent variables in relation to one another.

$$f(x) = \sin x e^{yz^2}, \text{ where } x = t-1$$
$$y = t^2$$
$$z = \frac{1}{t}$$

If the functions definition is available or if it is analytically possible, can perform direct substitution into the function or take the derivative to get the total derivative.

$$f(x) = \sin x e^{yz^2}$$
$$= \sin(t-1) e^{t^2 \cdot (\frac{1}{t})^2}$$
$$= \sin(t-1) e^1$$

$$\frac{df}{dt} = \cos(t-1) e$$

When analytically that is not possible can instead

$$\frac{df(x,y,z)}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}$$

(*) note the use of single variate $\frac{d}{dt}x$. Partial notation had to be use for function f relative to several variables but the variables x, y and z are each relative to a single variable. In this case all of them are related t .

$$\frac{\partial f}{\partial x} = \cos x e^{yz^2} \quad \frac{\partial f}{\partial y} = \sin x e^{yz^2} z^2 \quad \frac{\partial f}{\partial z} = \sin x e^{yz^2} 2yz$$

$$\frac{dx}{dt} = 1$$

$$\frac{dy}{dt} = 2t$$

$$\frac{dz}{dt} = -t^{-2}$$

On chaining the derivatives,

$$\frac{df(x,y,z)}{dt} = \cos x e^{yz^2} 1 + \sin x e^{yz^2} z^2 2t + \sin x e^{yz^2} 2yz(-t^{-2})$$

Then re-expressing in terms of t (substitution),

$$\begin{aligned} \frac{df(x,y,z)}{dt} &= \cos(t-1)e + \cancel{2^{t+1} \sin(t-1)e} - \cancel{2^{t+1} \sin(t-1)e} \\ &= \cos(t-1)e \end{aligned}$$