

In a thought experiment, imagine (3) strategies for finding the deepest area of a graded sandpit.

### I. Simple

Arbitrary

### II. Steepest Descent

Use the Jacobian with an aggression parameter — for how big each step is,

$$\delta x = -\gamma J$$

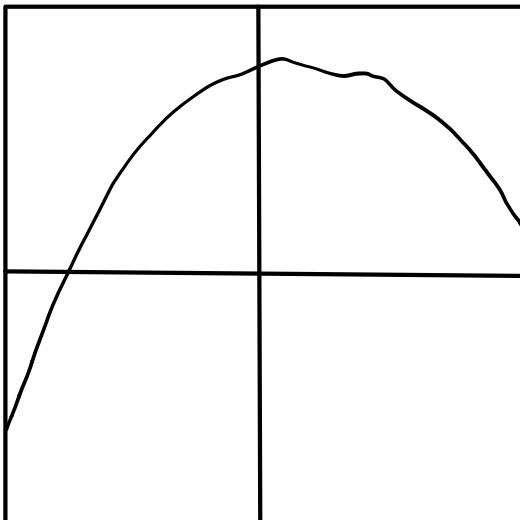
### III. Hessian - Approach (second derivative matrix).

It is difficult to know how large to set  $\gamma$  (gamma). Setting the parameter inappropriately will either prevent the gradient descent from converging (when  $\gamma$  is too small) or entirely skipping over the gradient (if too large).

The Hessian can be used to set the  $\gamma$ -value applied to the Jacobian.

$$\delta x = -H^{-1} J. \text{ where } \delta x \text{ is a small step on the } x\text{-axis.}$$

(\*) liable to find maxima or minima! Changes direction and size.



$$f(x) = -x^2 + 10$$

$$f'(x) = -2x, \text{ Jacobian } 1 \times 1$$

$$f''(x) = -2, \text{ Hessian } 1 \times 1$$

The hessian expresses the rate of change which the rates change relative to a variable(s).

By inverting the rate of change which the rates change relative to a variable, and applying it to the Jacobian the jacobian's gradient is transformed to an inverse - what Jacobian (i.e the gradient) would have been before being influenced by the Hessian (i.e the underlying 2nd-order characteristic of the polynomial).

Does this just normalize/linearize the steps.

Is the change in direction due to too big of a step function?

#### IV. Hybrid Approach

use hessian and gradient descent.

if hessian step is too big/wrong direction use gradient descent.

- too big = when  $-H^{-1}J(-J) \leq 0$  or  $\| -H^{-1}J \| > 2$ .