

Differentiation Rules

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{g(x)}{h(x)}\right) = \frac{g'(x)h(x) - h'(x)g(x)}{[h(x)]^2}$$

Sum Rule

$$\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

Chain Rule

$$\begin{aligned}\frac{d}{dx}f(g(x)) &= \frac{d}{dg}f(g) \cdot \frac{dg}{dx}g(x) \\ &= \frac{df}{dg} \cdot \frac{dg}{dx}\end{aligned}$$

Example: Differentiate function $f(x)$; this will require the derivative sum, product, chain and optionally the quotient rule.

Let $f(x) = \frac{\sin(2x^5 + 3x)}{e^{7x}}$.

Let $g(h) = \sin(h(x))$

Let $h(x) = 2x^5 + 3x$

The derivative of $g(h)$,

$$\begin{aligned}\frac{d}{dh}g(h) &= \frac{d}{dh}\sin h \\ &= \cosh\end{aligned}$$

The derivative of $h(x)$,

$$\begin{aligned}\frac{d}{dx}h(x) &= \frac{d}{dx}(2x^5 + 3x) \\ &= \frac{d}{dx}2x^5 + \frac{d}{dx}3x \\ &= (5)2x^4 + 3.\end{aligned}$$

The derivative of $g(x)$,

$$\begin{aligned}\frac{d}{dx} g(x) &= \frac{d}{dh} g(h) \cdot \frac{d}{dx} h(x) \\ &= \cosh [5(2x^4) + 3] \\ &= \cos(2x^5 + 3x)[(5)2x^4 + 3]\end{aligned}$$

Let $u(j) = e^j$

Let $j(x) = 7x$

The derivative of e^j ,

$$\begin{aligned}\frac{d}{dj} u(j) &= \frac{d}{dj} e^j \\ &= e^j\end{aligned}$$

The derivative of $7x$,

$$\begin{aligned}\frac{d}{dx} j(x) &= \frac{d}{dx} 7x \\ &= 7(1)\end{aligned}$$

The derivative of e^{7x} ,

$$\begin{aligned}\frac{d}{dx} e^{7x} &= \frac{d}{dj} u(j) \cdot \frac{d}{dx} j(x) \\ &= e^j \cdot 7 \\ &= e^{7x} \cdot 7\end{aligned}$$

The derivative of $f(x)$,

$$f(x) = \frac{\sin(2x^5 + 3x)}{e^{7x}}.$$

$$\begin{aligned}
 \frac{d}{dx} f(x) &= g(x) \frac{d}{dx} u(x) + u(x) \frac{d}{dx} g(x) \\
 &= \sin(2x^5 + 3x)(-7e^{-7x}) \\
 &\quad + \frac{1}{e^{-7x}} \cos(2x^5 + 3x) [(5) 2x^4 + 3]
 \end{aligned}$$

(*) In real life don't pre-optimize by factorizing.

It might be more helpful to leave the expression in its expanded form.