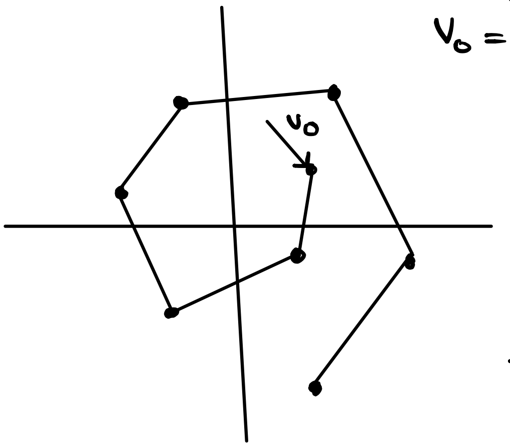
Can use eigenvectors and eigenvalues to form a basis set.

↳ called eigenbasis

An eigenbasis can be used to perform efficient matrix operations called diagonilization.



$$v_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \qquad v_1 = Tv_0 \qquad T = \begin{bmatrix} -0.9 & 0.8 \\ -1 & 0.35 \end{bmatrix}$$

$$v_2 = T^2 v_0$$

$$v_n = T^n v_0$$

If the matrix is diagonizable can apply the matrix exponent to the elements on the diagonal.

When the matrix isn't diagonalized, you can change to a basis where the transformation is diagonilizable (the eigenbasis), apply a number of transformations by an exponential then transforming back to the basis which you originally came from.

$$A^n = \begin{bmatrix} 1^n & 0 \\ 0 & 2^n \end{bmatrix}$$

In the transformation matrix the colum vectors will be the new location of the transformed unit vectors.

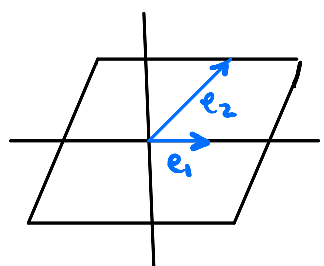
To build the eigenbasis conversion matrix, just use eigenvectors,

$$C = \begin{bmatrix} e_1 \,|\, e_2 \,|\, \cdots \,|\, e_n \end{bmatrix}$$

$$D \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

To achieve the same effect as the original transformation,

$$T = CDC^{-1}$$



- The C^{-1} matrix converts the input vector into the necessary quantities of each eigenbasis vector.
- The lambda values act as the exponential multiplyers.
- Then C converts back to the original coordinate system.

To apply the transformation multiple times can use the diagonalizable matrix multiple times,

$$T = CDC^{-1}$$
$$T^2 = CD\cancel{C^{-1}}\cancel{C}DC^{-1}$$
$$\dots$$
$$T^n = CD^nC^{-1}$$

Seems like many 2x2 transformations, can be expressed using <u>cos</u> and <u>sin</u>.

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} \cos -27^\circ & \cos -135^\circ \\ \sin -27^\circ & \sin -135^\circ \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} \cos -27^\circ & \cos -135^\circ \\ \sin -27^\circ & \sin -135^\circ \end{bmatrix}^{-1}$$

Example:

Given the matrix T, apply the matrix twice to the vector u using both T and a second time via an eigenbasis.

<u>2-Power w- T.</u>

$$T = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad u = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$$

$$Tu = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$TTu = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

<u>2-Power w- Eigenbasis.</u>

$$C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad @\lambda=1: \ x = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$
$$\qquad @\lambda=2: \ x = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$$

$$C^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$T^2 = CD^2C^{-1}$$
$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}^2 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix}$$

$$T^2u = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 4 \end{bmatrix}^T$$

Coursera Math for ML course does not cover all areas of eigen-theory.

Like:
- un-diagonalizable matrices
- complex eigenvalues.