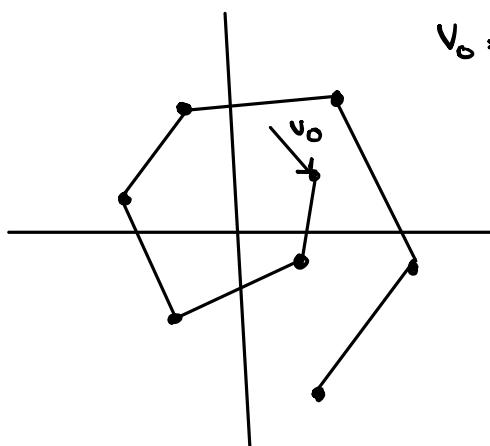


Can use eigenvectors and eigenvalues to form a basis set.

↳ called eigenbasis

An eigenbasis can be used to perform efficient matrix operations called diagonalization.



$$v_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \quad v_1 = Tv_0 \quad T = \begin{bmatrix} -0.9 & 0.8 \\ -1 & 0.35 \end{bmatrix}$$
$$v_2 = T^2v_0$$
$$v_n = T^n v_0$$

If the matrix is diagonalizable can apply the matrix exponent to the elements on the diagonal.

When the matrix isn't diagonalized, you can change to a basis where the transformation is diagonalizable (the eigenbasis), apply a number of transformations by an exponential then transforming back to the basis which you originally came from.

To build the eigenbasis conversion matrix, just use eigenvectors,

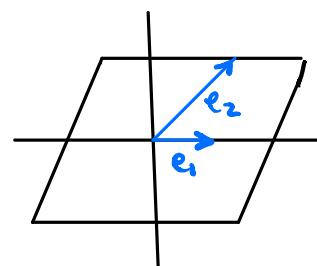
$$C = [e_1 | e_2 | \dots | e_n]$$

$$D = [\lambda_1, \lambda_2, \dots, \lambda_n]$$

To achieve the same effect as the original transformation,

$$T = C D C^{-1}$$

- The C^{-1} matrix converts the input vector into the necessary quantities of each eigenbasis vector.
- The lambda values act as the exponential multipliers.
- Then C converts back to the original coordinate system.



To apply the transformation multiple times can use the diagonalizable matrix multiple times.

$$\left. \begin{array}{l} T = CDC^{-1} \\ T^2 = CD\lambda^{-1}\lambda DC^{-1} \\ \dots \\ T^n = CD^nC^{-1} \end{array} \right\}$$

Example:

Given the matrix T , apply the matrix twice to the vector u using both T and a second time via an eigenbasis.

2-Power w- T .

$$T = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \cdot u = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$$

$$\begin{aligned} Tu &= \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 2 \end{bmatrix} \end{aligned}$$

$$TTu = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

2-Power w- Eigenbasis.

$$C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad @\lambda=1: x = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$$

$$C^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \quad @\lambda=2: x = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$T^2 = CD^2C^{-1}$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}^2 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} T^2u &= \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ &= [2 \ 4]^T \end{aligned}$$

Coursera Math for ML course does not cover all areas of eigen-theory.

Like:

- un-diagonalizable matrices
- complex eigenvalues.