What happens to summary statistics like the mean when the dataset is transformed – by scale or by shifting; how does our description of the dataset change when the dataset is linearly transformed?

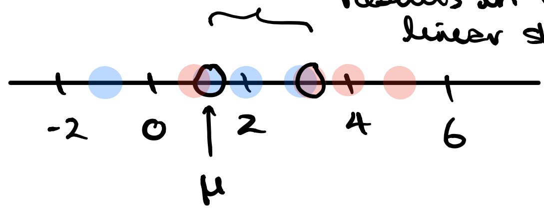
Example:

$D = \{-1, 2, 3\}$

$E[D] = \dfrac{-1, 2, 3}{3}$

$= 4/3$

A linear shift of every element results in the same linear shift to the mean.



- Blue is the original dataset
- Red is the shifted dataset

$D' = \{-1+2, 2+2, 3+2\}$

$= \{1, 4, 5\}$

$E[D'] = \dfrac{1+4+5}{3}$

$= 10/3$

$= \dfrac{6}{3} + 4/3$

$= 2 + 4/3.$

Generalized Expression of Mean with Shift.

$E[D+a] = E[D] + E[a]$

$= E[D] + a$, where a is a shift and D is a dataset.

Generalized Expression of Mean with Scaling

$D'' = \{-1(2), 2(2), 3(2)\}$

$= \{-2, 4, 6\}$

$E[D''] = \dfrac{-2+4+6}{3}$

$= 8/3$

$= (4/3)2.$

$E[\alpha D] = \alpha E[D].$

$\square$

Generalized Expression of Mean with Linear Transformations

$E[\alpha D + a] = E[\alpha D] + E[a]$

$= \alpha E[D] + a$

Where α is the scale and a is the shift.