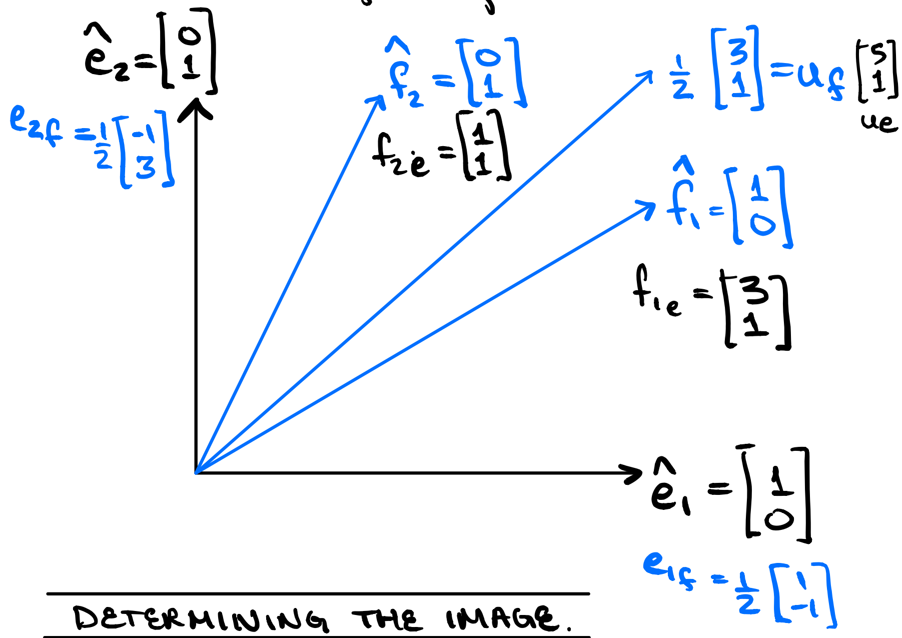
How to transform from one basis vectors to another.



DETERMINING THE PRE-IMAGE

From the $\underline{e}$ frame f's basis vectors are $\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

If the vector u_f in the basis of $\underline{f}$, needs to be translated into the basis of $\underline{e}$ do,

$$u_e = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 1 \end{bmatrix} \text{ in basis of } e.$$

DETERMINING THE IMAGE.

To convert from a basis e to another basis f – given the translation matrix $T(x): x_f \to x_e$, the inverse of that translation will be the applicative matrix required to go from $e \to f$.

$$T^{-1}(x): x_e \to x_f$$

In this example,

$$T(x) = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

$$T^{-1}(x) = \frac{1}{\det(T(x))}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

This means that e's basis will be,

$$T(\hat{e}_1): \hat{e}_1 \to \tfrac{1}{2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$T(\hat{e}_2): \hat{e}_2 \to \tfrac{1}{2}\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

TESTING CONVERT u TO f-BASIS

$$u \text{ in } e = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$Y(x): x_e \to x_f$$

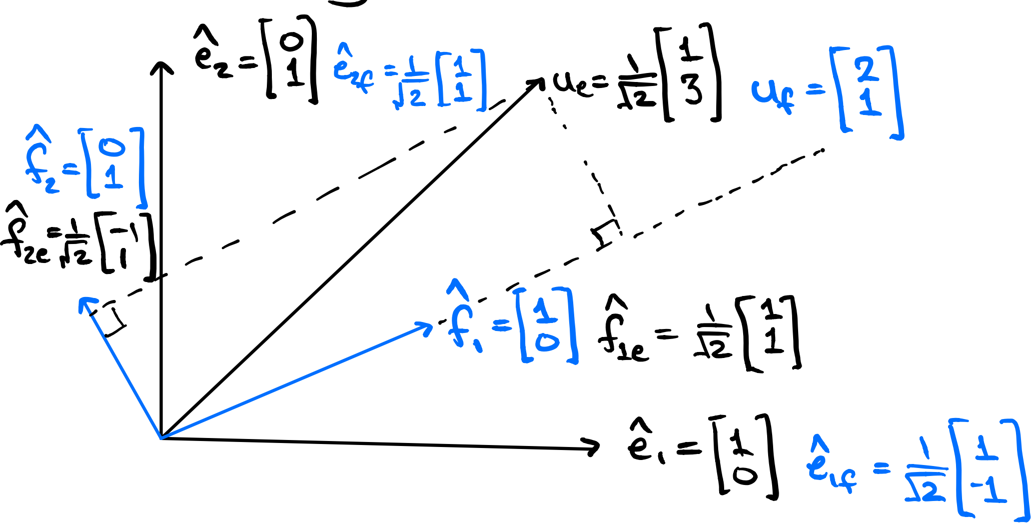
$$: \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} x_e$$

$$u_f = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}_f . \quad \square$$

Translating to An Orthonormal Basis Vector Set.



TRANSFORMING FROM f-to-e USING f basis in e

$T(x): x_f \to x_e$

$$: \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}_f\right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Big\} \; Au_f = u_e$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

e-to-f using inverse of f-basis in e.

$$A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Big\} \; A^{-1} u_e = u_f$$

Since the vectors are orthogonal; can do changes of basis really easily with projections (that simplify to the dot-product).

① Get the basis vectors of f in terms of e.

$$\hat{f}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ in } e$$

$$\hat{f}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}_f = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ in } e$$

$$u_f = \mathrm{proj}_{u_e} \hat{f}_1 + \mathrm{proj}_{u_e} \hat{f}_2$$

Basically a vector is the sum of its components, and to figure out its components in the basis f, project the vector onto equivalent e basis representations then project the scaler proj in the direction of the basis of the f component basis vectors.

② Project the vector to translate u, onto the e-translated basis vectors of f.

$$\mathrm{proj}_{u_e} \hat{f}_1 = \frac{\hat{f}_1}{\|\hat{f}_1\|} \frac{\hat{f}_{1e} \cdot u_e}{\|\hat{f}_1\|} \xrightarrow{\text{simplifies to}} \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}{1} \times \frac{\hat{f}_{1e} \cdot u_e}{1} \to \begin{bmatrix} \hat{f}_{1e} \cdot u_e \\ 0 \end{bmatrix}$$