

Previously looked @

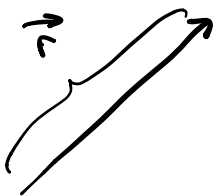
1. vector addition
 2. scaling vector by number
- } mathematical properties a vector has (i.e a linear sys.).

Can now define,

1. length of vector (aka size)
2. dot product. (aka
inner-scale
or
projection product)

Coordinate Systems

A vector is just a direction; it depends on a coord. sys. to manifest.



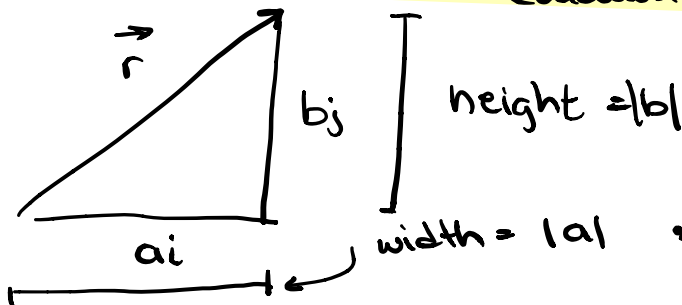
If the coord. sys. was made from \hat{i}, \hat{j} where,

$$\hat{i} = [1 \ 0]^T$$

$$\hat{j} = [0 \ 1]^T$$

$$\rightarrow r = a\hat{i} + b\hat{j}$$

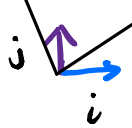
Calculating Length of Vector



by the Pythagorean Theorem
 $a^2 + b^2 = c^2$

Therefore length of r is

$$|r| = \sqrt{a^2 + b^2}$$

$$\begin{bmatrix} s_i \\ s_j \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} s$$


$$r = \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} r_i \\ r_j \end{bmatrix}$$

Dot Product:

$$\begin{aligned} r \cdot s &= r_i s_i + r_j s_j \\ &= (3)(-1) + (2)(2) \\ &= 1 \end{aligned}$$

Properties of Dot Product

1. Commutative, $\rightarrow s \cdot r = r \cdot s$
2. Distributive by Addition, $\rightarrow r \cdot (s+t) = (r \cdot s) + (r \cdot t)$
3. Associative over Scalar $\rightarrow r \cdot (as) = a(r \cdot s)$

Reflexive Dot Product.

$$\begin{aligned} r \cdot r &= r_1 r_1 + r_2 r_2 + \dots + r_n r_n \\ &= r_1^2 + r_2^2 + \dots + r_n^2 \\ &= |r|^2 \end{aligned}$$

Proof: PROOF OF DOT PROD DISTRIB BY ADD.

$$\begin{aligned} r &= [r_1 \ r_2 \ \dots \ r_n]^T \\ s &= [s_1 \ s_2 \ \dots \ s_n]^T \\ t &= [t_1 \ t_2 \ \dots \ t_n]^T \end{aligned}$$

$$\begin{aligned} r \cdot (s+t) &= r_1(s_1+t_1) + r_2(s_2+t_2) + \dots + r_n(s_n+t_n) \\ &= r_1 s_1 + r_1 t_1 + r_2 s_2 + r_2 t_2 \\ &\quad + \dots + r_n s_n + r_n t_n \\ &= r_1 s_1 + r_1 s_2 + \dots + r_n s_n \\ &\quad + r_1 t_1 + r_1 t_2 + \dots + r_n t_n \\ &= r \cdot s + r \cdot t \end{aligned}$$

PROOF DOT PROD ASSOC OVER SCALAR

$$\begin{aligned} r &= [r_1 \ r_2 \ \dots \ r_n]^T \\ s &= [s_1 \ s_2 \ \dots \ s_n]^T \\ a &= \text{some const/scalar} \\ r \cdot (as) &= r_1(as_1) + r_2(as_2) + \dots + r_n(as_n) \\ &= ar_1 s_1 + ar_2 s_2 + \dots + ar_n s_n \\ &= a(r \cdot s) \end{aligned}$$

