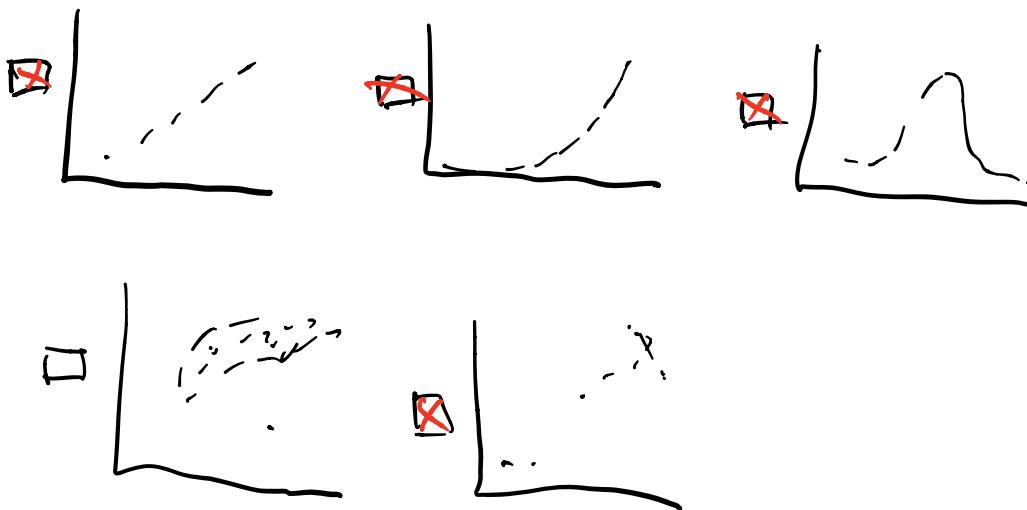


### PROBLEM # 1.

Select graphs which can be modelled by nonlinear least squares.



### PROBLEM #2.

Given the Chi-Squared formula which are true?

$$\chi^2 = \sum_{i=1}^n \frac{[y_i - y(x_i; \alpha)]^2}{\sigma_i^2}$$

- param  $\chi^2$  is the uncertainty value of the vars
- param  $\chi^2$  is squared so the effect of bad uncertainties are minimized
- take gradient of  $\chi^2$  set this to zero to determine fitting params
- when  $\chi^2$  is large the params used are a good fit for the data.

### PROBLEM # 3.

The gradient of Chi-Squared with uncertainty is, with respect to fitting params

$$\frac{\partial \chi^2}{\partial \alpha_j} = -2 \sum_{i=1}^n \frac{y_i - f(x_i, \alpha)}{\sigma_i^2} \frac{\partial f(x_i, \alpha)}{\partial \alpha_j} \text{ for } j = 1 \dots n$$

Can define  $Z_j = \frac{\partial f(x_i, a)}{\partial a_j}$

$$\text{Assuming } f(x_i, a) = a_1 x^3 - a_2 x^2 + e^{-a_3 x}$$

Differentiate  $f(x_i, a)$  — partially w-respect to the fitting params.

$$\frac{\partial f}{\partial a_1} = x^3$$

$$\frac{\partial f}{\partial a_2} = -x^2$$

$$\frac{\partial f}{\partial a_3} = e^{-a_3 x} \cdot (-x)$$

#### PROBLEM #4.

Calculate the Jacobian of the Chi-Squared test of the function  $y(x_i; a) = a_1(1 - e^{-a_2 x_i^2})$ , assume  $\sigma^2 = 1$ .

$$X^2 = \sum_{i=1}^n \frac{[y_i - y(x_i; a)]^2}{(\sigma)^2}, \quad k = 1 \dots n$$

$\sigma = 1$ .

$$\frac{\partial(X^2)}{\partial(a_1)} = -2 \sum_{i=1}^n \frac{(y_i - y(x_i; a))(1 - e^{-a_2 x_i^2})}{(\sigma)^2}$$

$$\frac{\partial(X^2)}{\partial(a_2)} = -2 \sum_{i=1}^n \frac{[y_i - a_1(1 - e^{-a_2 x_i^2})]^2}{(\sigma)^2} (x_i^2 e^{-a_2 x_i^2}) a_1$$

#### PROBLEM #5.

$$\text{Find } \frac{\partial y}{\partial x_p} \text{ for } y(x; \sigma, x_p, I, b) = b + \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(x-x_p)^2}{2\sigma^2} \right\}$$

$$\frac{\partial y}{\partial x_p} = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(x-x_p)^2}{2\sigma^2} \right\} \frac{(2)(-(x-x_p))}{2\sigma^2} \cdot (-1)$$

$$= \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(x-x_p)^2}{2\sigma^2} \right\} \cdot \frac{x-x_p}{\sigma^2}$$