These lessons will cover:

1. using Einstein summation Convention to do matrix multiplication and other operations.
2. how matrices transform a vector from one basis to another.
   - ↳ in turn how to apply reflection to an image
   - ↳ how to construct a convenient basis to do this

- Identify matrices as operators.
- Relate the transformation matrix to a set of new basis vectors.
- Mapping based on matrix transformations
- Finding an orthonormal basis set.

Einstein Summation Equation

$$\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{n1} & \cdots & & a_{nn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & & & \vdots \\ \vdots & & & \vdots \\ b_{n1} & \cdots & & b_{nn} \end{bmatrix} = AB$$

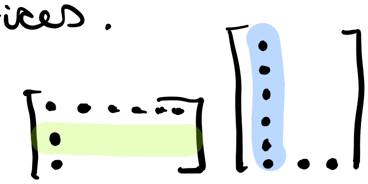
row col

$$(ab)_{23} = a_{21}b_{13} + a_{22}b_{23} + \dots + a_{2n}b_{n3}$$

$$= \sum_j a_{ij}b_{jk}$$

$ab_{ik} = a_{ij}b_{jk}$, the summation can be shortened in the convention

This convention plainly expresses how to multiply non-square matrices.



Dot Product w- Einstein Summation Convention

$$u \cdot v = \begin{bmatrix} u_i \\ \\ \\ \end{bmatrix} \cdot \begin{bmatrix} v_i \\ \\ \\ \end{bmatrix}$$

$$= u_i v_i$$

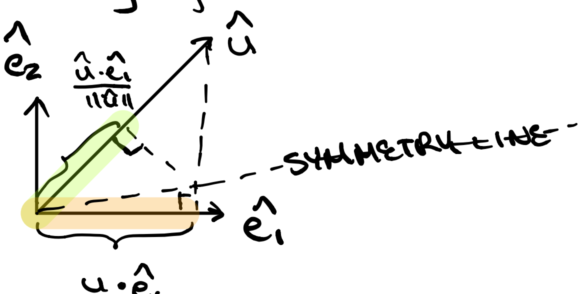
Matrix Dot-Product w transpose.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}$$

$$= \sum a_{ij} b_{jk}$$

$$(uv)_{11} = a_{ij} b_{jk}$$

Symmetry of Dot Product



$\frac{u \cdot \hat{e}_1}{\|\hat{e}_1\|}$

(*) $\hat{u}$ onto $\hat{e}_1$ = $\hat{e}_1$ onto $\hat{u}$

Why when multiplying a vector onto a matrix it is a projection of the vector onto the vectors composing that matrix.