

Previously looked @

1. vector addition
2. scaling vector by number

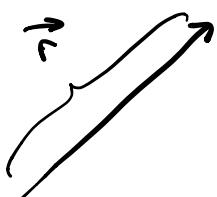
} mathematical properties a vector has (i.e a linear sys.).

Can now define,

1. length of vector (aka size)
2. dot product. (aka
inner-scale
or
projection product)

Coordinate Systems

A vector is just a direction; it depends on a coord. sys. to manifest.

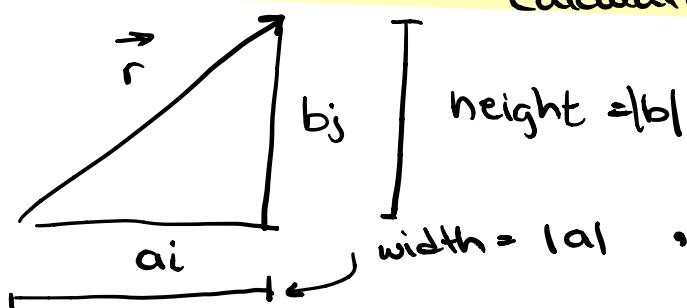


If the coord. sys.
was made from \hat{i}, \hat{j}
where,

$$\hat{i} = [1 \ 0]^T$$

$$\hat{j} = [0 \ 1]^T$$

$$\rightarrow r = ai + bj$$



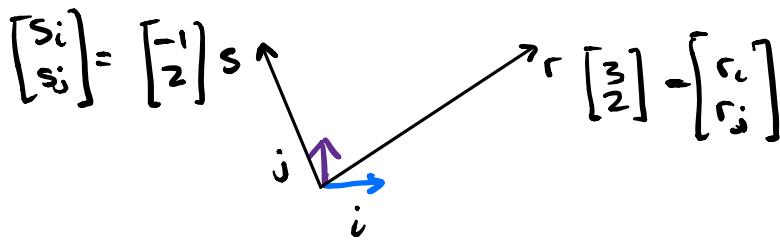
Calculating Length of Vector

by the Pythagorean Theorem
 $a^2 + b^2 = c^2$

Therefore length of r is

$$|r| = \sqrt{a^2 + b^2}$$

$$\begin{bmatrix} s_i \\ s_j \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} s$$



Dot Product:

$$\begin{aligned} r \cdot s &= r_i s_i + r_j s_j \\ &= (3)(-1) + (2)(2) \\ &= 1 \end{aligned}$$

Properties of Dot Product

1. Commutative, $\rightarrow s \cdot r = r \cdot s$

2. Distributive

$$\text{by Addition, } \rightarrow r \cdot (s+t) = (r \cdot s) + (r \cdot t)$$

3. Associative over Scalar $\rightarrow r \cdot (as) = a(r \cdot s)$

Reflexive Dot Product.

$$\begin{aligned} r \cdot r &= r_1 r_1 + r_2 r_2 + \dots + r_n r_n \\ &= r_1^2 + r_2^2 + \dots + r_n^2 \\ &= |r|^2 \end{aligned}$$

Proof: PROOF OF DOT PROD. DISTRIB BY ADD.

$$r = [r_1, r_2, \dots, r_n]^T$$

$$s = [s_1, s_2, \dots, s_n]^T$$

$$t = [t_1, t_2, \dots, t_n]^T$$

$$\begin{aligned} r \cdot (s+t) &= r_1(s_1+t_1) + r_2(s_2+t_2) + \dots + \\ &\quad r_n(s_n+t_n) \end{aligned}$$

$$\begin{aligned} &= r_1 s_1 + r_1 t_1 + r_2 s_2 + r_2 t_2 \\ &\quad + \dots + r_n s_n + r_n t_n \end{aligned}$$

$$\begin{aligned} &= r_1 s_1 + r_1 s_2 + \dots + r_n s_n \\ &\quad + r_1 t_1 + r_1 t_2 + \dots + r_n t_n \end{aligned}$$

$$= r \cdot s + r \cdot t$$

PROOF DOT PROD ASSOC OVER SCALAR

$$r = [r_1, r_2, \dots, r_n]^T$$

$$s = [s_1, s_2, \dots, s_n]^T$$

a = some const/scalar

$$\begin{aligned} \vec{r} \cdot (\vec{as}) &= r_1(as_1) + \\ &\quad r_2(as_2) + \\ &\quad \dots + r_n(as_n) \end{aligned}$$

$$\begin{aligned} &= ar_1 s_1 + ar_2 s_2 \\ &\quad + \dots + ar_n s_n \end{aligned}$$

$$= a(r \cdot s)$$

