

A matrix is positive definite if it is symmetric (i.e. square and equals its transpose) and if the scalar value of $x^T M x$ is positive for every non-zero column vector.

Positive definiteness means the output always has a positive inner product with the input.

Real Definitions:

M positive definite $\iff x^T M x > 0$ for all $x \in \mathbb{R}^n \setminus 0$.

M positive semi-definite $\iff x^T M x \geq 0$ for all $x \in \mathbb{R}^n \setminus 0$.

M negative definite $\iff x^T M x < 0$ for all $x \in \mathbb{R}^n \setminus 0$.

M negative semi-definite $\iff x^T M x \leq 0$ for all $x \in \mathbb{R}^n \setminus 0$.

(*) A $n \times n$ symmetric real matrix that is neither positive semidefinite or negative semidefinite is called indefinite.