
PROBLEM #1.

Calc H .

$$f(x, y) = x^3y + x + 2y$$

$$J = [3x^2y + 1, x^3 + 2]$$

$$H = \begin{bmatrix} \partial_{x_1} x_1 f & \partial_{x_1} x_2 f \\ \partial_{x_2} x_1 f & \partial_{x_2} x_2 f \end{bmatrix}$$

$$H = \begin{bmatrix} 6xy & 3x^2 \\ 3x^2 & 0 \end{bmatrix}$$

PROBLEM #2.

Calc. H .

$$f(x, y) = e^x \cos(y)$$

$$J = [e^x \cos(y), -\sin(y)e^x]$$

$$H = \begin{bmatrix} \partial_{x_1} x_1 f & \partial_{x_1} x_2 f \\ \partial_{x_2} x_1 f & \partial_{x_2} x_2 f \end{bmatrix}$$

$$H = \begin{bmatrix} e^x \cos(y) & -\sin y e^x \\ -e^x \sin y & -\cos y e^x \end{bmatrix}$$

PROBLEM #3.

Calc. H . Calc $\frac{1}{2}[x, y] H \begin{bmatrix} x \\ y \end{bmatrix}$.

$$f(x, y) = \frac{x^2}{2} + xy + \frac{y^2}{2}$$

$$J = [x + y, x + y]$$

$$H = \begin{bmatrix} \partial_{x_1} x_1 f & \partial_{x_1} x_2 f \\ \partial_{x_2} x_1 f & \partial_{x_2} x_2 f \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{aligned} \frac{1}{2} [x, y] \begin{bmatrix} x+y \\ x+y \end{bmatrix} &= \frac{x(x+y)}{2} + \frac{y(x+y)}{2} \\ &= \frac{x^2 + xy + y^2 + xy}{2} \\ &= \frac{x^2}{2} + xy + \frac{y^2}{2} \end{aligned}$$

(*) 2nd order derivative equals $f(x)$. similar to $e^x \dots$

PROBLEM #4.

Calc. H.

$$f(x, y, z) = x^2 e^{-y} \cos z$$

$$J = [2x e^{-y} \cos z, \\ -e^{-y} x^2 \cos z, \\ -\sin z x^2 e^{-y}]$$

$$H = \begin{bmatrix} \partial x_1 x_1 f & \partial x_1 x_2 f & \partial x_1 x_3 f \\ \partial x_2 x_1 f & \partial x_2 x_2 f & \partial x_2 x_3 f \\ \partial x_3 x_1 f & \partial x_3 x_2 f & \partial x_3 x_3 f \end{bmatrix}$$

$$H = \begin{bmatrix} 2e^{-y} \cos z & -e^{-y} 2x \cos z & -\sin z 2x e^{-y} \\ -2x e^{-y} \cos z & e^{-y} x^2 \cos z & \sin z x^2 e^{-y} \\ -2x \sin z e^{-y} & e^{-y} x^2 \sin z & -\cos z x^2 e^{-y} \end{bmatrix}$$

PROBLEM #5.

Calc. H.

$$f(x, y, z) = x e^y + y^2 \cos(z) \quad J = [e^y, e^y x + 2y \cos z, -\sin z y^2]$$

$$H = \begin{bmatrix} \partial x_1 x_1 f & \partial x_1 x_2 f & \partial x_1 x_3 f \\ \partial x_2 x_1 f & \partial x_2 x_2 f & \partial x_2 x_3 f \\ \partial x_3 x_1 f & \partial x_3 x_2 f & \partial x_3 x_3 f \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & e^y & 0 \\ e^y & e^y x + 2y \cos z & 2y(-\sin z) \\ 0 & -2y \sin z & -\cos z y^2 \end{bmatrix}$$