

A matrix is positive definite if it is symmetric (i.e square and equals its transpose) and if the scalar value of  $x^T M x$  is positive for every non-zero column vector.

Positive definiteness means the output always has a positive inner product with the input.

Real Definitions:

$M$  positive definite  $\Leftrightarrow x^T M x > 0$  for all  $x \in \mathbb{R}^n \setminus \{0\}$ .

$M$  positive semi  
definite  $\Leftrightarrow x^T M x \geq 0$  for all  $x \in \mathbb{R}^n \setminus \{0\}$ .

$M$  negative definite  $\Leftrightarrow x^T M x < 0$  for all  $x \in \mathbb{R}^n \setminus \{0\}$ .

$M$  negative semi  
definite  $\Leftrightarrow x^T M x \leq 0$  for all  $x \in \mathbb{R}^n \setminus \{0\}$

(\*) A  $n \times n$  symmetric real matrix that is neither positive semidefinite or negative semidefinite is called indefinite.