PROBLEM #1.

Given vector set $u = \{u_1, u_2, u_3\}$ and the Transformation T, which vectors in u are eigenvectors?

$$u = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad T = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

Proof:

All vectors in u are eigenvectors.

$$\forall u_i \in u: Tu_i \in \text{Span}(u_i)$$

Therefor all vectors in u are eigenvectors.

□.

PROBLEM #2.

Given vector set $u = \{u_1, u_2, u_3\}$ and the Transformation T, which vectors in u are eigenvectors?

$$u = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad T = \begin{bmatrix}3 & 0\\0 & 2\end{bmatrix}$$

both u_1 and u_3,

$$Tu_i \in \text{Span}(u_i).$$

u_2 is not an eigenvector,

$$Tu_2 \notin \text{Span}(u_2).$$

PROBLEM #3.

Given vector set $u = \{u_1, u_2, u_3\}$ and the Transformation T, which vectors in u are eigenvectors?

$$u = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad T = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}$$

u_1 is the only eigen vector

$\forall u_i \in \{u_2, u_3\}$: $Tu_i \notin \text{Span}(u_i)$.

PROBLEM #4.

Given vector set $u = \{u_1, u_2, u_3\}$ and the Transformation T, which vectors in u are eigenvectors?

$$u = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad T = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

Answer is none.

PROBLEM #5.

Given vector set $u = \{u_1, u_2, u_3\}$ and the Transformation T, which vectors in u are eigenvectors?

$$u = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad T = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

Answer is all.

PROBLEM #6.

Given vector set $u = \{u_1, u_2, u_3\}$ and the Transformation T, which vectors in u are eigenvectors?

$$u = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\} \quad T = \begin{bmatrix}2 & 1\\ 0 & 2\end{bmatrix}$$

Answer is only u_1.