$$\frac{df}{dx}(2), \quad dx \rightarrow 0.$$

The derivative is formed by "nudging" dx and subsequently df.

Can be expressed,

$$\frac{f(2+dx) - f(2)}{dx}, \quad dx \rightarrow 0$$

There is a difference between a infintesmals and the behaviour as differences approach finilely small values.
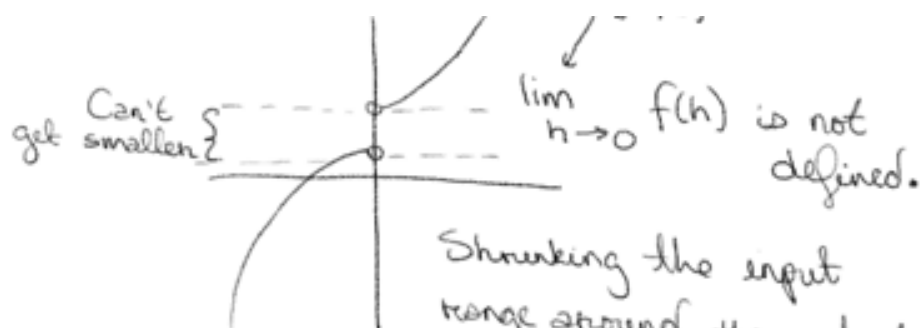
More formally,   Common to use Δx or h for dx.

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

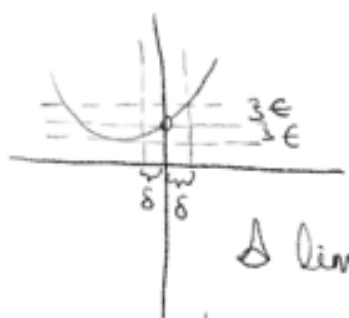Derivatives concern the finite small differences approaching zero.

← undefined, $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = 12$

(ϵ, δ)

get Can't smaller } $\lim_{h \to 0} f(h)$ is not defined.
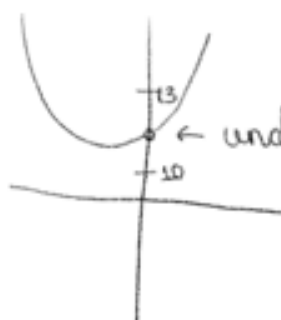
Shrinking the input range around the output range and evaluating convergence is similar to the Epsilon-delta definition of limits.

$\underbrace{\qquad\qquad}$

Real Analysis.

(Epsilon-Delta)

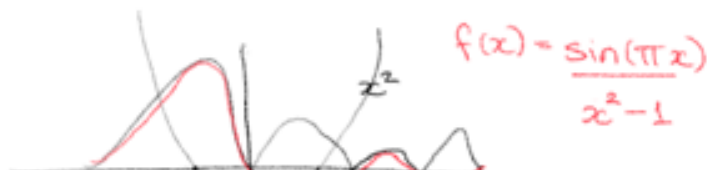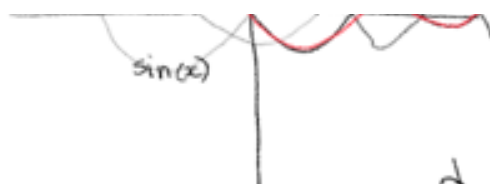$\frac{3\epsilon}{3\epsilon}$

$\frac{\delta}{\delta}$

A limit which exists, has a range of inputs around the limiting point of some distance (δ), so that any such input corresponds to an output within the range of epsilon. Critically — for any epsilon (ϵ).

Given the function $f(x) = \dfrac{\sin(\pi x)}{x^2 - 1}$.

· This function is undefined at 1 and -1.

x^2

$f(x) = \dfrac{\sin(\pi x)}{x^2 - 1}$

sin(x)

The rate of change
(i.e the derivative) is

$$\frac{d}{dx}(\sin(\pi x)) = \cos(\pi x) \cdot \pi$$

$$\frac{d}{dx}(x^2 - 1) = 2x$$

$$\frac{\cos(\pi x) \cdot \pi}{2x}.$$

At the undefined value of 1 or -1, the derivative —
"that is the rate of change", is

The idea is
$$\frac{\cos(\pi(1)) \cdot \pi}{2(1)} \Rightarrow \frac{(-1) \cdot \pi}{2}$$

that if the limit is
$$\Rightarrow \frac{-\pi}{2}.$$

defined then so too is the derivative.

If the derivative is found then the limit can be computed
for undefined values by computing a prediction for that
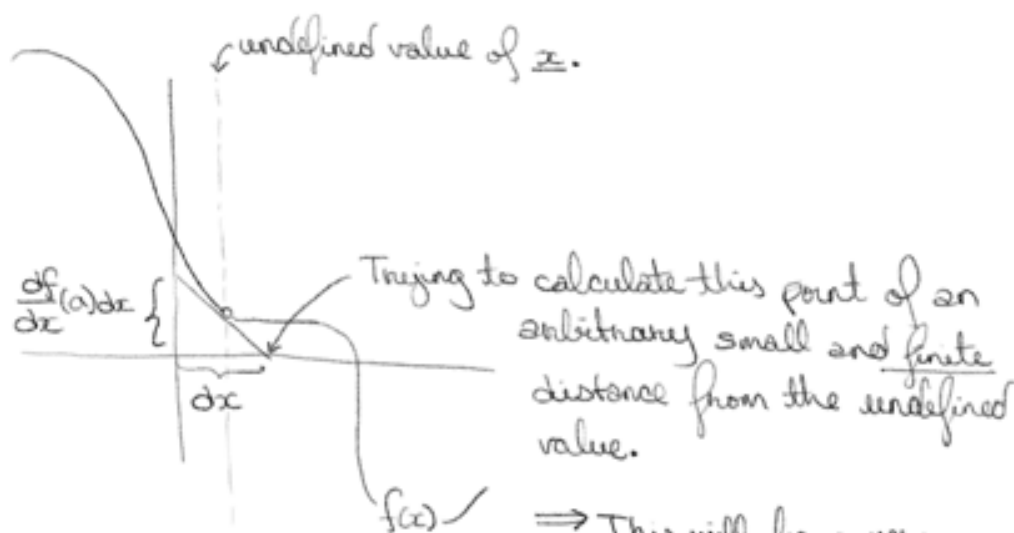limit using arbitrarily near locations to compute rates of
change.

Generalization — L'Hopital's Rule

Even though $\frac{f(x)}{g(x)}$ is sometimes undefined ($\frac{0}{0}$).

their derivatives are well defined — and so to are the limits.

That is, $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)}$ is well defined.

One can use that derivative in conjuction with arbitrarily close approaches (dx) to the target "a", to approximate the undefined value.

undefined value of $\underline{x}$.

$\dfrac{df}{dx}(a)dx \bigg\{$

dx

Trying to calculate this point of an arbitrary small and $\underline{finite}$ distance from the undefined value.

$f(x)$

$\Rightarrow$ This will be a very accurate approximation of the limit which can be substituded for the undifined value.

Through the application of the derivative to a dx (from zero) that derivative in conjunction with the other derivatives of the other involved functions,

$$\left\{ 3\frac{dg}{dx}(a)\,dx \right.$$

$$\left. 3\frac{df}{dx}(a)\,dx \right\}$$

dx

$\checkmark$
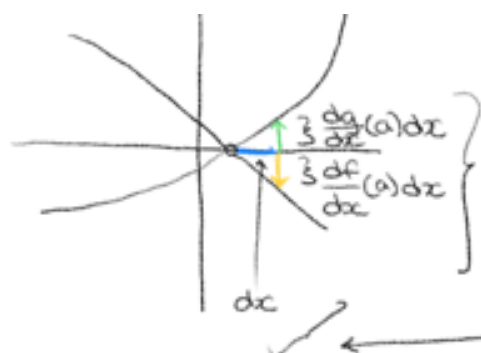
└┤ l'opital's
Rule
(*BUT ACTUALLY BERNOULLI)

This and this become
arbitrarily small.