Implicit Differentiation.

Procedure to find $\frac{dy}{dx}$, is:

$$\frac{dy}{dx}(5^2) = \frac{dy}{dx}(x^2+y^2)$$

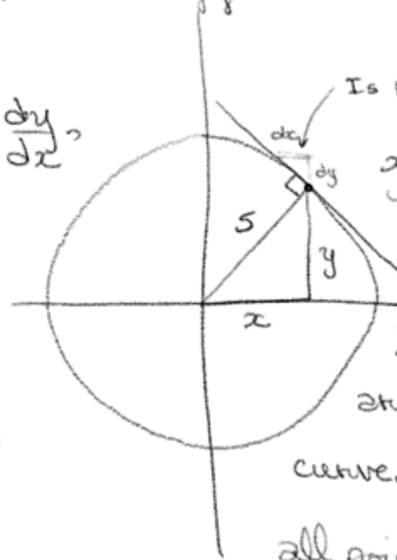
$$0 = 2x\,dx + 2y\,dy$$

$$-2x\,dx = 2y\,dy$$

$$-2x = \frac{2y\,dy}{dx}$$

$$\frac{dy}{dx} = 2y/2x.$$

$$= -y \cdot x^{-1}.$$



$\underbrace{x^2+y^2}=5^2$

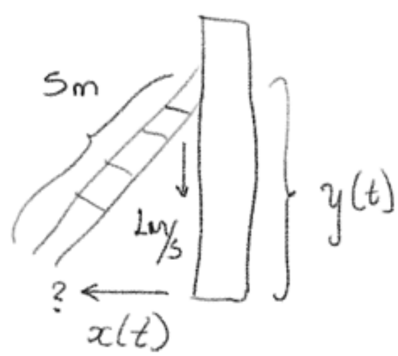
x and y are not inputs - they are interdependent, and are known as an implicit curve. That is the set of all points which satisfy some property.

$$x(t)^2 + y(t)^2 = 5^2$$

$$x(t) = \sqrt{(5^2 - y(t)^2)}$$

Could then find $x(t)$, s

$$\frac{dx}{dt} = \frac{d}{dt}\left(\sqrt{5^2 - y(t)^2}\right)$$



However, there is a second way,

$$\underbrace{x(t)^2 + y(t)^2} = 5^2$$

Is a function producing a constant value.
It is also a function of time.

Due to the function being a constant, you can take the derivative which in totality would be zero; although its components might be non-zero.

$$x(t)^2 + y(t)^2 = 5^2$$

$$\frac{d}{dt} 5^2 = \frac{2x(t)dx + 2y(t)dy}{dt}$$

$$\frac{d}{dt} 0 = \ldots$$

By plugging in the initial width and height,

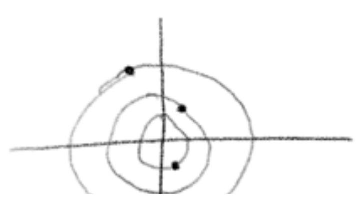
$$2(3)\frac{dx}{dt} + 2(4)(-1) = 0$$

$$\frac{dx}{dt} = \frac{4}{3}.$$

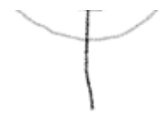
Generalized $S(x,y) = x^2+y^2$.

This formula associates points with a number

Often many points share the same number.



$$dS = 2x\,dx + 2y\,dy$$



$\sin(x)y^2 = x$

The derivative is $\sin(x)2y\,dy + y^2\cos(x)\,dx = dx$.

Rate of change relative to y and x.

Not relative to x.

Can use implicit derivative formulas to figure out new derivatives.

$$y = \ln(x) \iff e^y = x$$

$$e^y dy = dx, \text{ since } \frac{d}{dx} e^x = (1)\, e^x$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$= \frac{1}{x}, \text{ since } e^y = x \text{ from line \#1.}$$

$$\therefore \frac{d}{dx}(\ln(x)) = \frac{1}{x}.$$