

To explain dot products require linear transformation

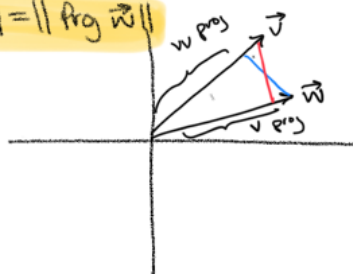
$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 2 \cdot 1 + 4 \cdot 0 = 2$$

$$= (\text{length of projected } \vec{w}) \cdot (\text{length of } \vec{v})$$

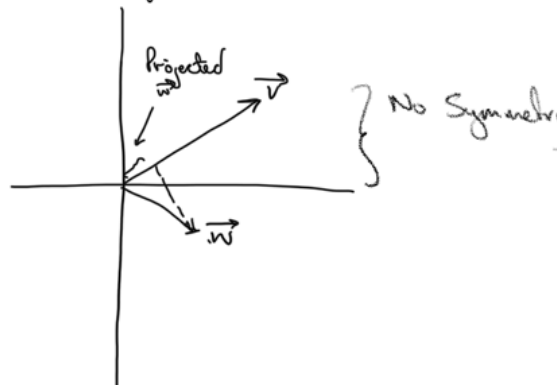
A Symmetrical Approach.

$$\|\vec{v}\| = \|\vec{w}\|$$

$$\|\text{Proj}_{\vec{v}} \vec{w}\| = \|\text{Proj}_{\vec{w}} \vec{v}\|$$



A Projection,



Asymmetric Dot Products work because there is independence between the projection and the other asymmetric vector.

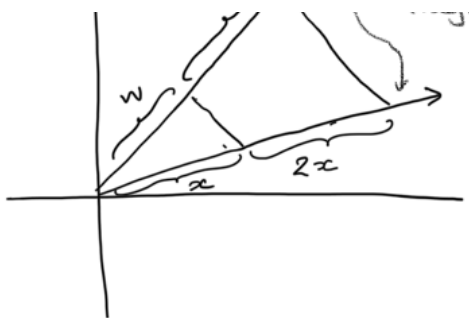
Therefore,

$$(2\vec{v}) \cdot \vec{w} = 2(\vec{v} \cdot \vec{w})$$

On this case the \vec{w} projection doesn't change length; although

Asymmetry doesn't matter. You are either just scaling the projection or the vector which you are projecting on too which are equal.

Only when aligned, $\vec{v} = \vec{w}$
 $2w \swarrow \searrow$ $2 \times \text{hypo} \rightarrow 2 \times \text{adj}$



Can do proof using
perpendicular vectors
because projection is 0.

the \vec{v} vector which it is projecting
on to does. The effect is doubled.

Similarly, projecting a doubled
vector \vec{v} onto the original vector
is equal to doubling the
original product. \rightarrow this is because
the hypotenuse grows proportionate
to the adjacent line segment.

Reconciling Euclidean Distance Commutative Vector Projection

$$\begin{bmatrix} u_x & u_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = u_x \cdot x + u_y \cdot y$$

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = u_x \cdot x + u_y \cdot y$$

THE DOT PRODUCT IS THE SAME AS APPLYING
THE VECTOR AS A LINEAR TRANSFORMATION.

\hookrightarrow The columns making up the vector are
transformations of the original vectors basis vectors.

Power of Dot.

A geometric tool for

- understanding projections
- testing orientation

With more meaning, dotting is the collapse of a vector onto the real number line (of the original vector).