

Exercises 12.5

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1. Use Roster notation for

$$A = \{x \mid x^2 - 1 = 0\}$$

$$\text{r.n.} \rightarrow \{-1, 1\}$$

$$B = \{x \mid (x-1)^2 = 0\}$$

$$\text{r.n.} \rightarrow \{1\}$$

$$C = \{x \mid x+8=9\}$$

$$\text{r.n.} \rightarrow \{1\}$$

$$D = \{x \mid x^3 - 2x^2 + x = 2\}$$

$$\text{r.n.} \rightarrow \{2\}$$

$$(x-2)$$

2. $B \subseteq A$, $B \subseteq C$, $C \subseteq A$, $C \subseteq A$

3. Prove $A \subseteq B$, $\frac{A = \{1\}}{B = \{1, 2\}}$

$$\textcircled{1} \forall x \in A \rightarrow x \in B$$

$$\textcircled{2} A \neq B$$

Therefore $A \subseteq B$ and $A \subseteq B$. \square

Prove $A \subseteq B$, where $A = \{1\}$ and $B = \{1, 2\}$.

$\textcircled{1} \forall x \in A \rightarrow x \in B$ * whenever every element of A also belongs to B .

Disprove $A \subseteq B$,

$$\textcircled{1} \forall x \in B \rightarrow x \in A$$

Prove $1 \in A$.

$$\textcircled{1} 1 \in B.$$

$$B \subseteq A$$

Therefore $1 \in A$, or B could not be a subset of A .

Disprove $1 \subseteq A$.

A is a set, 1 is a number.

Subsets are not possible with numbers.

7. Prove the following,

a) $\{a, a\} = \{a\}$ Sets are equal when every element from every set can be found in every other set.
 Let $A = \{a, a\}$
 Let $B = \{a\}$

$$|A| = |B| = |A \cup B|.$$

$$\textcircled{b) } \{a, b\} = \{b, a\}$$

$$\forall x \in B \rightarrow x \in A$$

$$\forall x \in A \rightarrow x \in B$$

c) $\{a\} = \{b, c\}$ $\forall x \in A \rightarrow \exists y \in B, x = y$
 $A = \{a\}$
 $B = \{b, c\}$ And vice-versa.

8. Commutative laws: $A \cup B = B \cup A$, $A \cap B = B \cap A$

→ $A \cup B$, is the result of combining elements in A , B or in both.

$$C = A \cup B \rightarrow \forall x \in C, x \in A \text{ or } x \in B$$

$$\Leftrightarrow D = B \cup A \rightarrow \forall x \in D, x \in B \text{ or } x \in A$$

$$\Leftrightarrow \forall x \in D \text{ s.t. } x \in C$$

• Associative Laws, $A \cup (B \cup C) = (A \cup B) \cup C$

$$\forall x \in A \rightarrow x \in A \cup (B \cup C), x \in (A \cup B) \cup C$$

$$\forall x \in B \rightarrow x \in A \cup (B \cup C), x \in (A \cup B) \cup C$$

$$\forall x \in C \rightarrow x \in A \cup (B \cup C), x \in (A \cup B) \cup C$$

$$\forall x \in A \cup B \rightarrow x \in A \cup (B \cup C), x \in (A \cup B) \cup C$$

$$\forall x \in B \cup C \rightarrow x \in A \cup (B \cup C), x \in (A \cup B) \cup C$$

$$\forall x \in C \cup A \rightarrow x \in A \cup (B \cup C), x \in (A \cup B) \cup C$$

$$\forall x \in A \cup (B \cup C) \rightarrow x \in A \cup B \text{ or } x \in C \text{ or } x \in A$$

$$\forall x \in (A \cup B) \cup C \rightarrow x \in A \cup B \text{ or } x \in C \text{ or } x \in A$$

$$\forall x \in (A \cup B) \cup C \rightarrow x \in A \cup (B \cup C), x \in (A \cup B) \cup C$$