PROBLEM # 1.

Select graphs which can be modelled by nonlinear least squares.

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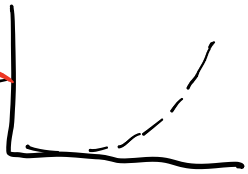
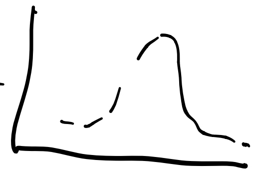
☒ 

☒ 

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PROBLEM #2.

Given the Chi-Squared formula which are true?

$$\chi^2 = \sum_{i=1}^{n} \frac{[y_i - y(x_i; a_k)]^2}{\sigma^2}$$

- ☐ param χ^2 is the uncertainty value of the vars
- ☒ param χ is squared so the effect of bad uncertainties are minimized
- ☒ take gradient of χ^2 set this to zero to determine fitting params
- ☐ when χ^2 is large the params used are a good fit for the data.

PROBLEM # 3.

The gradient of Chi-Squared with uncertainty is, with respect to fitting params

$$\frac{\partial \chi^2}{\partial a_j} = -2 \sum_{i=1}^{n} \frac{y_i - f(x_i, a)}{\sigma_i^2} \frac{\partial f(x_i, a)}{\partial a_j} \quad \text{for } j = 1 \ldots n$$

Can define $Z_j = \frac{\partial f(x_i, a)}{\partial a_j}$

Assuming $f(x_i, a) = a_1 x^3 - a_2 x^2 + e^{-a_3 x}$

Differentiate $f(x_i, a)$ — partially w- respect to the fitting params.

$$\frac{\partial f}{\partial a_1} = x^3$$

$$\frac{\partial f}{\partial a_2} = -x^2$$

$$\frac{\partial f}{\partial a_3} = e^{-a_3 x} \cdot (-x)$$

PROBLEM # 4.

Calculate the Jacobian of the Chi-Squared test of the function $y(x_i; a) = a_1(1 - e^{-a_2 x_i^2})$, assume $\sigma^2 = 1$.

$$X^2 = \sum_{i=1}^{n} \frac{[y_i - y(x_i; a_k)]^2}{(\sigma)^2}, \quad k = 1 \ldots n, \quad \sigma = 1.$$

$$\frac{\partial(x^2)}{\partial(a_1)} = -2 \sum_{i=1}^{n} \frac{(y_i - y(x_i; a_k))}{(1)^2}(1 - e^{-a_2 x_i^2})$$

$$\frac{\partial(x^2)}{\partial(a_2)} = -2 \sum_{i=1}^{n} \frac{[y_i - a_1(1 - e^{-a_2 x_i^2})]^2}{(1)^2} (x_i^2 e^{-a_2 x_i^2}) a_1$$

PROBLEM # 5.

Find $\frac{\partial y}{\partial x_p}$ for $y(x; \sigma, x_p, I, b) = b + \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(x - x_p)^2}{2\sigma^2}\right\}$

$$\frac{\partial y}{\partial x_p} = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-(x - x_p)^2}{2\sigma^2}\right\} \frac{(2)(-1)(x - x_p)}{2\sigma^2} \cdot (-1)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x - x_p)^2}{2\sigma^2}\right\} \cdot \frac{x - x_p}{\sigma^2}$$