PROBLEM #1.

Calc H.

$$f(x,y) = x^3 y + x + 2y \qquad J = [3x^2 y + 1, \ x^3 + 2]$$

$$H = \begin{bmatrix} \partial_{x_1 x_1} f & \partial_{x_1 x_2} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f \end{bmatrix} \qquad H = \begin{bmatrix} 6xy & 3x^2 \\ 3x^2 & 0 \end{bmatrix}$$

PROBLEM #2.

Calc. H.

$$f(x,y) = e^x \cos(y) \qquad J = [e^x \cos(y), \ -\sin(y) e^x]$$

$$H = \begin{bmatrix} \partial_{x_1 x_1} f & \partial_{x_1 x_2} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f \end{bmatrix} \qquad H = \begin{bmatrix} e^x \cos(y) & -\sin y\, e^x \\ -e^x \sin y & -\cos y\, e^x \end{bmatrix}$$

PROBLEM #3.

Calc. H. calc $\frac{1}{2}[x, y] H \begin{bmatrix} x \\ y \end{bmatrix}$.

$$f(x,y) = \frac{x^2}{2} + xy + \frac{y^2}{2}$$

$$J = [x + y, \ x + y]$$

$$H = \begin{bmatrix} \partial_{x_1 x_1} f & \partial_{x_1 x_2} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\frac{1}{2}[x, y]\begin{bmatrix} x + y \\ x + y \end{bmatrix} = \frac{x(x+y)}{2} + \frac{y(x+y)}{2}$$

$$= \frac{x^2 + xy + y^2 + xy}{2}$$

$$= \frac{x^2}{2} + xy + \frac{y^2}{2}.$$

(*) 2nd order derivative equals $f(x)$. similar to e^x ...

PROBLEM #4.

Calc. H.

$$f(x,y,z) = x^2 e^{-y} \cos z$$

$$J = \left[2x e^{-y} \cos z, \quad -e^{-y} x^2 \cos z, \quad -\sin z\, x^2 e^{-y} \right]$$

$$H = \begin{bmatrix} \partial_{x_1 x_1} f & \partial_{x_1 x_2} f & \partial_{x_1 x_3} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f & \partial_{x_2 x_3} f \\ \partial_{x_3 x_1} f & \partial_{x_3 x_2} f & \partial_{x_3 x_3} f \end{bmatrix}$$

$$H = \begin{bmatrix} 2 e^{-y} \cos z & -e^{-y} 2x \cos z & -\sin z\, 2x e^{-y} \\ -2x e^{-y} \cos z & e^{-y} x^2 \cos z & \sin z\, x^2 e^{-y} \\ -2x \sin z\, e^{-y} & e^{-y} x^2 \sin z & -\cos z\, x^2 e^{-y} \end{bmatrix}$$

PROBLEM #5.

Calc. H.

$$f(x,y,z) = x e^{y} + y^2 \cos(z) \qquad J = \left[e^{y}, \quad e^{y} x + 2y \cos z, \quad -\sin z\, y^2 \right]$$

$$H = \begin{bmatrix} \partial_{x_1 x_1} f & \partial_{x_1 x_2} f & \partial_{x_1 x_3} f \\ \partial_{x_2 x_1} f & \partial_{x_2 x_2} f & \partial_{x_2 x_3} f \\ \partial_{x_3 x_1} f & \partial_{x_3 x_2} f & \partial_{x_3 x_3} f \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & e^{y} & 0 \\ e^{y} & e^{y} x + 2 \cos z & 2y(-\sin z) \\ 0 & -2y \sin z & -\cos z\, y^2 \end{bmatrix}$$