

The process of manipulating a simultaneous equation into reduced row echelon form, then performing backsubstitution works on expressions involving a variety of mathematical objects.

Formerly it was seen RREF & BS works for,

$$Ax = y \rightarrow A^{-1}Ax = A^{-1}y \\ x = A^{-1}y.$$

This same process can also be applied to,

$$AB = I \Leftrightarrow AA^{-1} = I, \text{ note } A \text{ and its inverse are commutative — and as normal associative.}$$

RREF & Backsub:

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

↳ the basis vectors/linear transformations applied to the input vectors.

$$I = A^{-1}A$$

$$A = A(A^{-1}A)$$

$$A^{-1} = A^{-1}(AA^{-1}) \\ = (A^{-1}A)A^{-1}$$

$$= A^{-1}(AA^{-1}), I \text{ is representative of either sequence.}$$

$$\textcircled{1} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \left| \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right.$$

$$\textcircled{2} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \right., \text{ RREF}$$

$$\textcircled{3} \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \right.$$

$$\textcircled{4} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left| \begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \right., \text{ Backsubstitute.}$$

$$\begin{bmatrix} 0 & -1 & 2 \\ -2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ is } B: \text{ the inverse of } A.$$