**The lattice spin**

The difference between the imposed velocity gradient (), and the one produced by the slips () is actually a spin tensor, called the lattice spin :

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where

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Eq. (1) is the basic equation to calculate the orientation change of crystals during plastic deformation.

The fact that  is a spin tensor, i.e. antisymmetric, can be proved as follows.

The strain rate tensor corresponding to the imposed velocity gradient is:

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This strain is produced by crystallographic slip, so the same strain rate tensor can be written by the velocity gradients corresponding to slips:

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From the equality of Eqs. (3) and (4), we obtain:

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Replacing this relation into Eq. (1), we get:

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Now compare this equation with the following, obtained from Eq. (1):

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It is clear then from Eqs. (6) and (7) that:

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which is the basic property of an antisymmetric tensor.

In order to obtain ,  has to be computed, which is done by solving the ‘strain’ equation for the slip rates :

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The solution of Eq. (9) is possible if the viscoplastic constitutive law is employed:

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which relates the slip rates to the resolved shear stresses . *m* is the strain rate sensitivity index for slip,  and  are reference values. As the resolved shear stress is obtained from the stress tensor:

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eq. (9) becomes:

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In this equation the unknowns are the components of the stress tensor. When some strain components are relaxed, the corresponding stress components are zero, so the number of unknows is reduced. Eq. (12) is usually solved by the Newton-Raphson technique. Once the stress tensor is known, then the slip rates  are obtained from Eqs. (10-11). Knowing the slip rates, the velocity gradient of slip  can be fully calculated (Eq. 2).

The lattice spin can be readily obtained from Eq. (1) when  is fully known. However, for mixed boundary conditions  is not fully defined, so one has to apply a specific procedure to obtain . Indeed, to fully define , only three non-diagonal components of  need to be known from the maximum possible six, because  is antisymmetrique. For many testings, there are usually at least three shear components known, from which  can be calculated. When both  and  are computed, Eq. (1) provides all the unknown components of the velocity gradient , so  becomes fully known:

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We would like to calculate the finite strain values of the unknown strain components. For this, we have to obtain the strain gradient tensor . As the velocity gradient is known, we can obtain it from the equation:

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This relation provides  :

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Therefore,  can be incremented as follows:

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For the incrementation, one needs the initial value of , before deformation, which is simply the identity tensor.

From , one can obtain the dimension changes of a grain for any initial direction, as  is relating the dimensions before () and after the deformation ():

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