



GRUNDLAGEN DER ELEKTROTECHNIK II

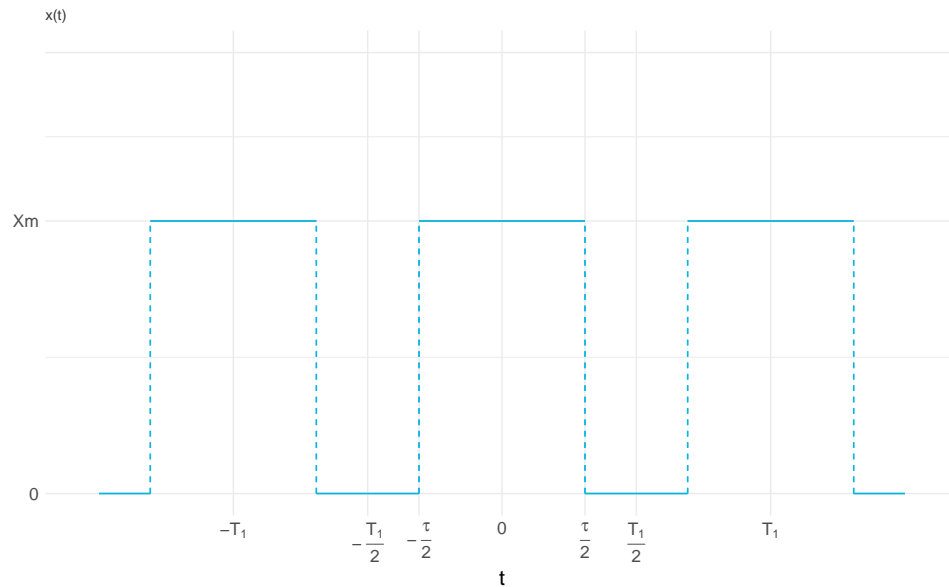
Frequenzanalyse periodischer Signale

Studien- und Versuchsaufgaben

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1 Vorbereitungsaufgaben

1.1



$$\begin{aligned}
 \underline{X}_\nu &= \frac{1}{T_1} \cdot \int_{T_1} x(t) \cdot e^{-j\nu \cdot \omega_1 t} dt \\
 &= \frac{1}{T_1} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} X_m \cdot e^{-j\nu \cdot \omega_1 t} dt \\
 &= -\frac{X_m}{T_1 \cdot j\nu \omega_1} \cdot \left[e^{-j\nu \cdot \omega_1 t} \right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \\
 &= -\frac{X_m}{T_1 \cdot j\nu \omega_1} \cdot \left(e^{-j\nu \cdot \omega_1 \frac{\tau}{2}} - e^{j\nu \cdot \omega_1 \frac{\tau}{2}} \right)
 \end{aligned}$$

$\omega_1 = \frac{2\pi}{T_1}$ und Erweiterung mit $\frac{-1}{-1}$:

$$\begin{aligned}
 \underline{X}_\nu &= \frac{X_m}{2j\pi\nu} \cdot \left(e^{j\nu \cdot \pi \frac{\tau}{T_1}} - e^{-j\nu \cdot \pi \frac{\tau}{T_1}} \right) \\
 &= \frac{X_m}{\pi\nu} \cdot \frac{\left(e^{j\nu \cdot \pi \frac{\tau}{T_1}} - e^{-j\nu \cdot \pi \frac{\tau}{T_1}} \right)}{2j}
 \end{aligned}$$

mit $\frac{(e^{jx} - e^{-jx})}{2j} = \sin(x)$ und $\frac{\tau}{T_1} = D$:

$$\underline{X}_\nu = \frac{X_m}{\pi\nu} \cdot \sin(\pi\nu D)$$

Erweitert man wieder mit $\frac{D}{D}$ erhält man das Bild einer Spaltfunktion $\text{si}(x) = \frac{\sin x}{x}$:

$$\underline{X}_\nu = D \cdot X_m \cdot \frac{\sin(\pi\nu D)}{\pi\nu D} = D \cdot X_m \cdot \text{si}(\pi\nu D)$$

Als reelle Reihe:

$$x(t) = X_0 + \sum_{\nu=1}^{\infty} \hat{X}_\nu \cos(\nu \cdot \omega_1 t + \phi_\nu)$$

$$X_0 = \frac{1}{T_1} \cdot \int_{T_1} x(t) dt = \frac{X_m}{2}$$

Aus der komplexen Reihendarstellung folgt

$$b_\nu = -2 \cdot \text{Im}(\underline{X}_\nu) = 0$$

$$\hat{X}_\nu = \sqrt{a_\nu^2 + b_\nu^2} = 2 \cdot |\underline{X}_\nu| \implies a_\nu = 2 \cdot |D \cdot X_m \cdot \text{si}(\nu\pi D)|$$

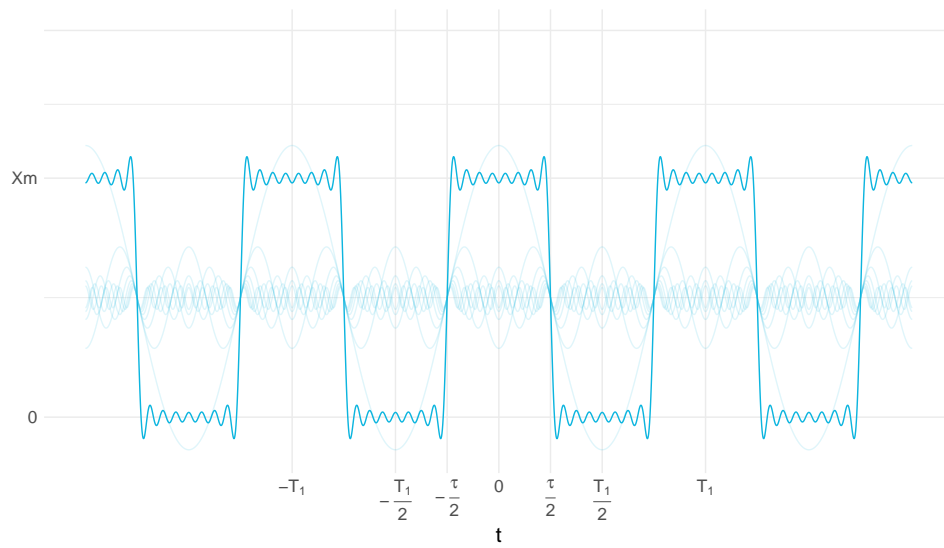
ϕ_ν hängt nur vom Wert von $\text{si}(\nu\pi D)$ ab, da \underline{X}_ν rein reell ist:

$$\phi_\nu = \begin{cases} 0 & ; \nu = \frac{4k+1}{2D} \\ \pi & ; \nu = \frac{4k-1}{2D} \\ \text{n.d.} & ; \text{sonst} \end{cases}$$

Somit ist

$$x(t) = \frac{X_m}{2} + \sum_{\nu=1}^{\infty} 2DX_m \cdot |\operatorname{si}(\pi\nu D)| \cdot \cos\left(\nu \cdot \frac{2\pi}{T_1} \cdot t + \phi_\nu\right)$$

Reihenentwicklung von $x(t)$ bis zur 16. Oberwelle
 $D=0.5$



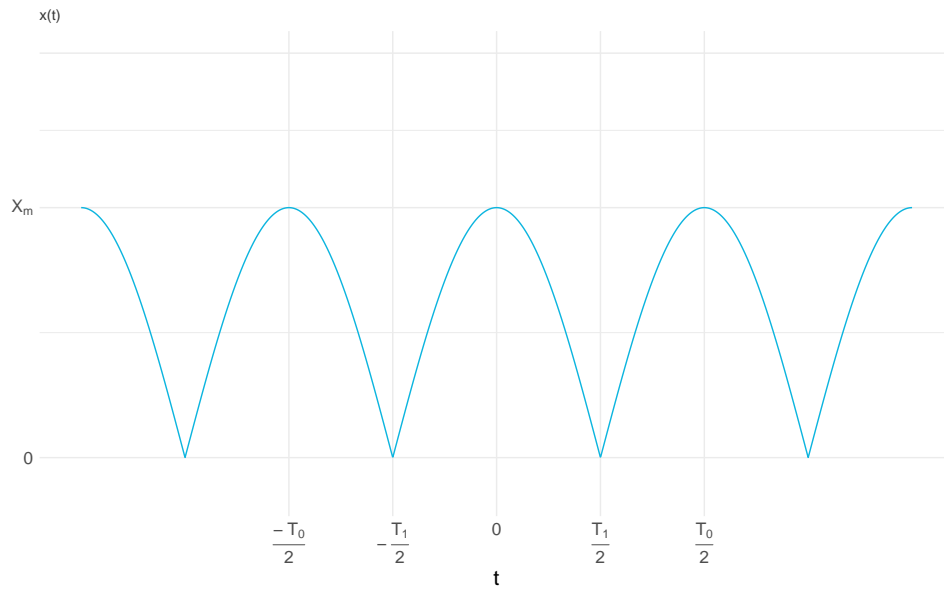
Effektivwert:

$$X_{\text{eff}} = \sqrt{\frac{1}{T_1} \cdot \int_{T_1} x^2(t) dt} = \sqrt{\frac{X_m^2}{T_1} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1 dt}$$

$$X_{\text{eff}} = X_m \cdot \sqrt{\frac{\tau}{T_1}} = X_m \cdot \sqrt{D}$$

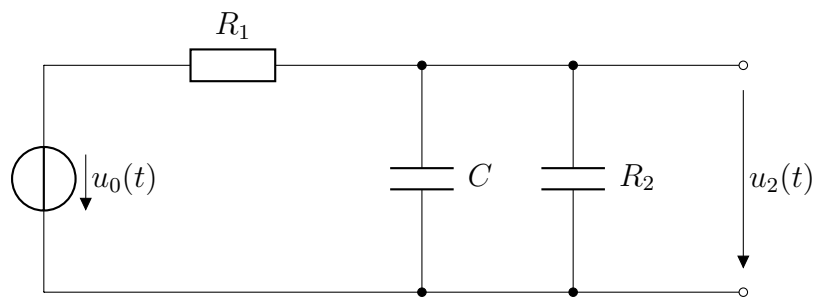
$D = \frac{1}{2}, X_{\text{eff}} = \frac{X_m}{\sqrt{2}}$				$D = \frac{1}{4}, X_{\text{eff}} = \frac{X_m}{2}$			$D = \frac{1}{8}, X_{\text{eff}} = \frac{X_m}{\sqrt{8}}$		
ν	\underline{X}_ν	\hat{X}_ν	ϕ_ν	ν	\hat{X}_ν	ϕ_ν	ν	\hat{X}_ν	ϕ_ν
1	$\frac{1}{\pi}X_m$	$\frac{2}{\pi}X_m$	0	1	$\frac{\sqrt{2}}{\pi}X_m$	0	1	$\frac{\sqrt{2-\sqrt{2}}}{\pi}X_m$	0
2	—	—	—	2	$\frac{1}{\pi}X_m$	0	2	$\frac{\sqrt{2}}{2\pi}X_m$	0
3	$-\frac{1}{3\pi}X_m$	$\frac{2}{3\pi}X_m$	π	3	$\frac{\sqrt{2}}{3\pi}X_m$	0	3	$\frac{\sqrt{2+\sqrt{2}}}{3\pi}X_m$	0
4	—	—	—	4	—	—	4	$\frac{1}{2}X_m$	0
5	$\frac{1}{5\pi}X_m$	$\frac{2}{5\pi}X_m$	0	5	$\frac{\sqrt{2}}{5\pi}X_m$	π	5	$\frac{\sqrt{2+\sqrt{2}}}{5\pi}X_m$	0
6	—	—	—	6	$\frac{1}{3\pi}X_m$	π	6	$\frac{\sqrt{2}}{16\pi}X_m$	0
7	$-\frac{1}{7\pi}X_m$	$\frac{2}{7\pi}X_m$	π	7	$\frac{\sqrt{2}}{7\pi}X_m$	π	7	$\frac{\sqrt{2-\sqrt{2}}}{7\pi}X_m$	0
8	—	—	—	8	—	—	8	—	—
9	$\frac{1}{9\pi}X_m$	$\frac{2}{9\pi}X_m$	0	9	$\frac{\sqrt{2}}{9\pi}X_m$	0	9	$\frac{\sqrt{2-\sqrt{2}}}{9\pi}X_m$	π
10	—	—	—	10	$\frac{1}{5\pi}X_m$	0	10	$\frac{\sqrt{2}}{5\pi}X_m$	π
11	$-\frac{1}{11\pi}X_m$	$\frac{2}{11\pi}X_m$	π	11	$\frac{\sqrt{2}}{11\pi}X_m$	0	11	$\frac{\sqrt{2+\sqrt{2}}}{11\pi}X_m$	π
12	—	—	—	12	—	—	12	$\frac{1}{6\pi}X_m$	π
13	$\frac{1}{13\pi}X_m$	$\frac{2}{13\pi}X_m$	0	13	$\frac{\sqrt{2}}{13\pi}X_m$	π	13	$\frac{\sqrt{2+\sqrt{2}}}{13\pi}X_m$	π
14	—	—	—	14	$\frac{1}{7\pi}X_m$	π	14	$\frac{\sqrt{2}}{7\pi}X_m$	π
15	$-\frac{1}{15\pi}X_m$	$\frac{2}{15\pi}X_m$	π	15	$\frac{\sqrt{2}}{15\pi}X_m$	π	15	$\frac{\sqrt{2-\sqrt{2}}}{15\pi}X_m$	π
16	—	—	—	16	—	—	16	—	—

1.2



$$\underline{X}_\nu = \frac{1}{T_1} \cdot \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} X_m \cos(\omega_0 t) \cdot e^{-j\nu\omega_0 t} dt = \frac{X_m}{T_1} \cdot \left[\frac{e^{-j\nu\omega_0 t}}{1} \right]_{-\frac{T_1}{2}}^{\frac{T_1}{2}}$$

1.3



$$\underline{U}_2 = \underline{U}_0 \cdot \frac{\frac{1}{j\omega C + \frac{1}{R_2}}}{R_1 + \frac{1}{j\omega C + \frac{1}{R_2}}} = \frac{\underline{U}_0 \cdot R_2}{R_1 + R_2 + j\omega C R_1 R_2}$$

Betrag:

$$\hat{U}_2 = \frac{\hat{U}_0 \cdot R_2}{\sqrt{(R_1 + R_2)^2 + (\omega C R_1 R_2)^2}}$$

Phase:

$$\phi_{\underline{U}_2} = \phi_{\underline{U}_0} - \arctan \frac{\omega C R_1 R_2}{R_1 + R_2}$$

2 Versuchsaufgaben

2.1

Messwerte der Aufgabe 3.1 für $T = 10 \text{ } \mu\text{s}$:

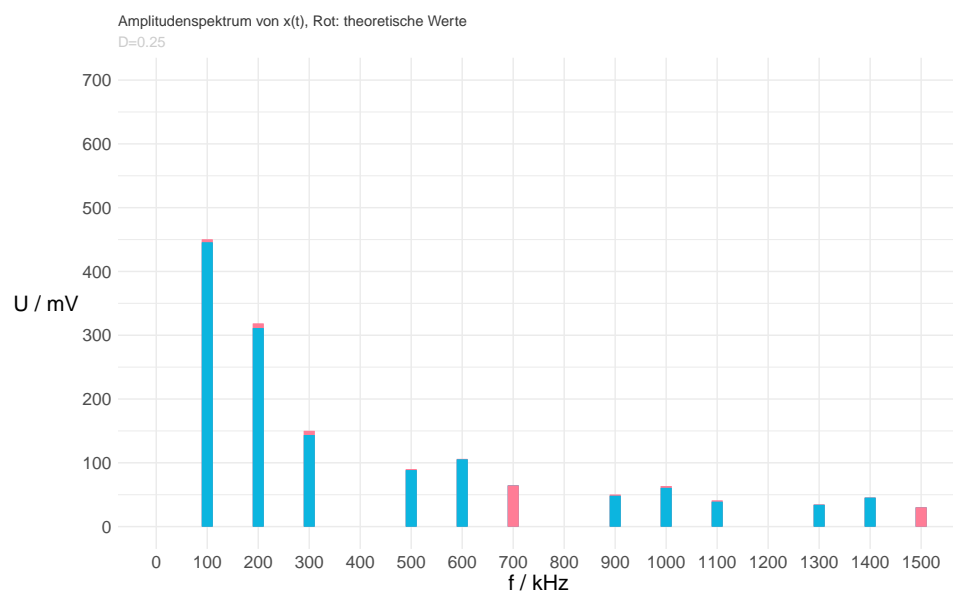
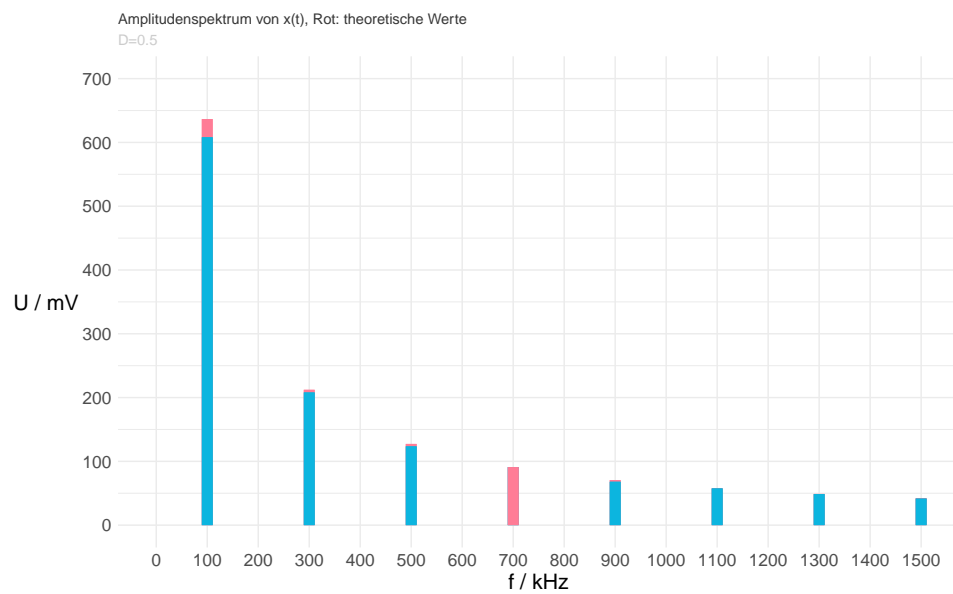
$D = \frac{1}{2}, X_{\text{eff}} = ?$			
ν	\hat{X}_ν/mV	$X_{\nu_{\text{eff}}}/\text{mV}$	f_ν/kHz
1	608.112	430	100
2	-	-	-
3	208.045	147.11	300
4	-	-	-
5	123.546	87.36	500
6	-	-	-
7	91.040	64.375	700
8	-	-	-
9	68.059	48.125	900
10	-	-	-
11	57.544	40.69	1100
12	-	-	-
13	48.691	34.43	1300
14	-	-	-
15	41.606	29.42	1500

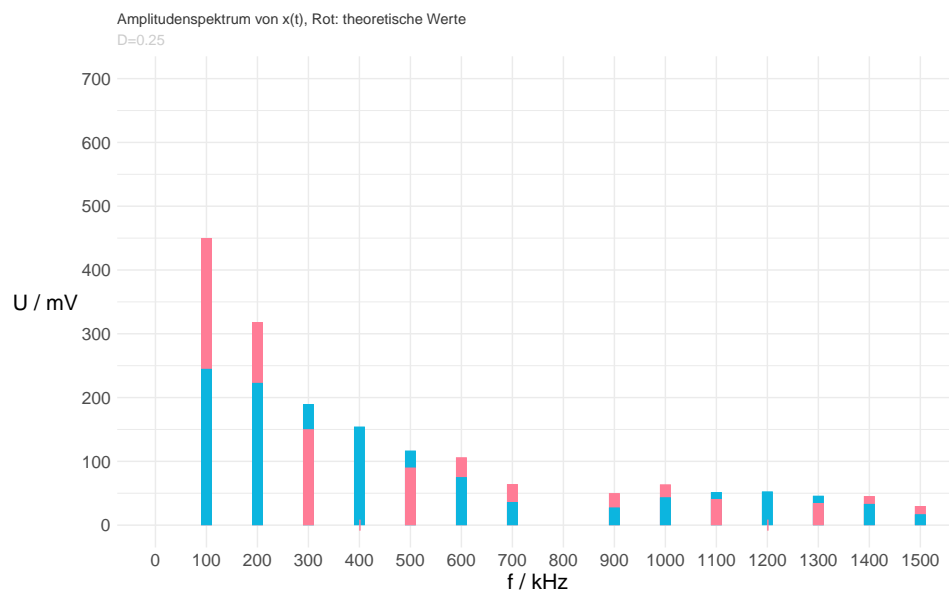
$$D = \frac{1}{4}, X_{\text{eff}} = 500 \text{ mV}$$

ν	\hat{X}_ν/mV	$X_{\nu_{\text{eff}}}/\text{mV}$	f_ν/kHz
1	445.477	315	100
2	311.070	219.96	200
3	143.401	101.4	300
4	-	-	-
5	88.388	62.5	500
6	105.217	74.4	600
7	64.488	45.6	700
8	-	-	-
9	48.225	34.1	900
10	60.670	42.9	1000
11	38.891	27.5	1100
12	-	-	-
13	34.083	24.1	1300
14	45.255	32	1400
15	30.123	21.3	1500

$$D = \frac{1}{8}, X_{\text{eff}} = 343 \text{ mV}$$

ν	\hat{X}_ν/mV	$X_{\nu_{\text{eff}}}/\text{mV}$	f_ν/kHz
1	244.942	173.2	100
2	222.965	157.66	200
3	189.787	134.2	300
4	154.291	109.1	400
5	116.673	82.5	500
6	75.095	53.1	600
7	36.204	25.6	700
8	-	-	-
9	27.436	19.4	900
10	43.416	30.7	1000
11	51.760	36.6	1100
12	52.609	37.2	1200
13	45.962	32.5	1300
14	33.234	23.5	1400
15	16.829	11.9	1500





2.2

2.3