Grundlagen der Elektrotechnik II

Schwingkreise

Studien- und Versuchsaufgaben

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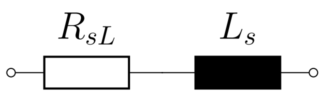
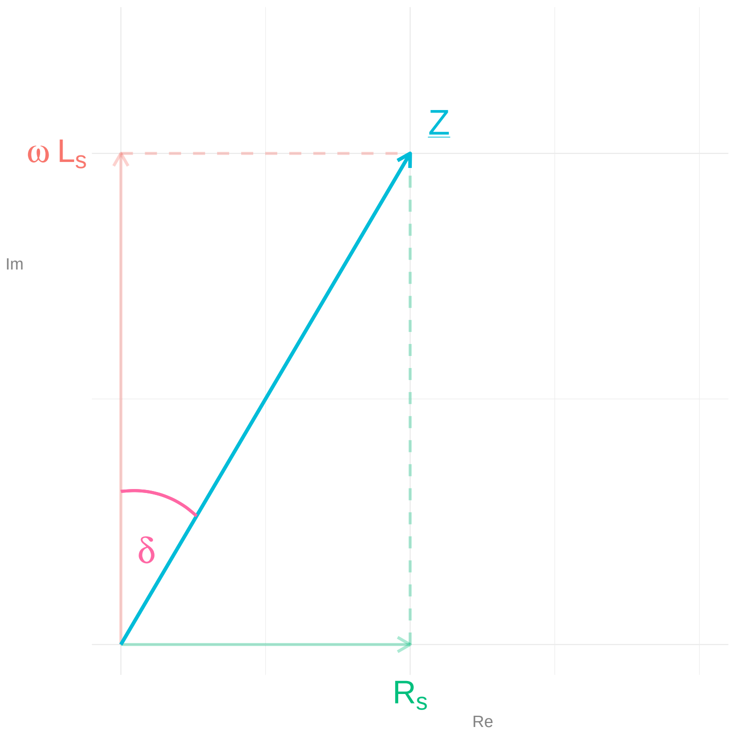
1 Vorbereitungsaufgaben

1.1

Spule

Reihenmodell





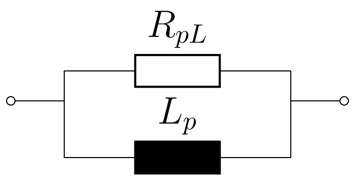
Der Verlustwinkel δ ist der Winkel der Spulenimpedanz $\underline{Z}$ mit der imaginären Achse der gaußschen Zahlenebene. $\tan\delta$ wird auch Verlustfaktor d genannt.

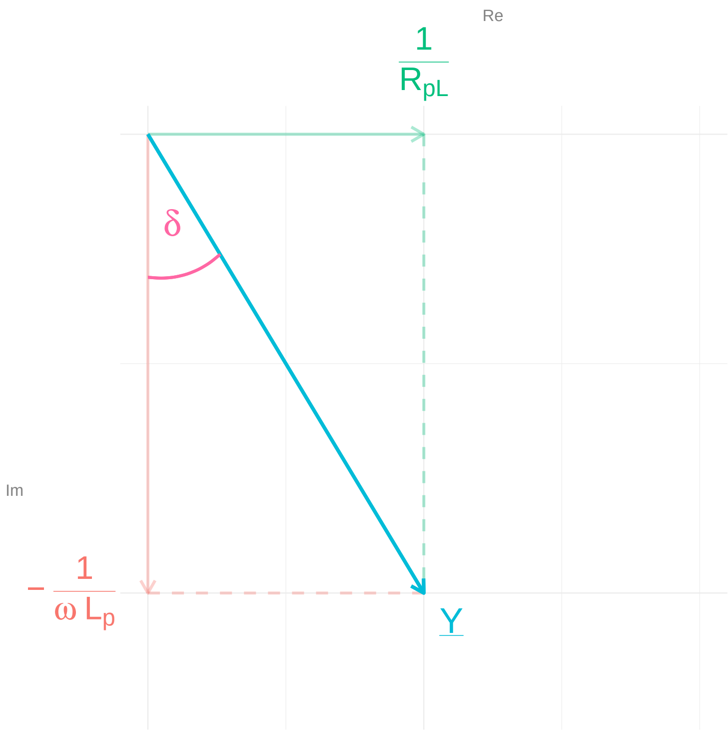
$$\tan\delta = \frac{\omega L_s}{R_{sL}}$$

Die Güte Q der realen Induktivität ist demnach als Kehrwert des Verlustfaktors definiert:

$$Q_{Ls} = \frac{1}{\tan\delta} = \frac{R_{sL}}{\omega L_s}$$

Parallelmodell





Der Verlustwinkel δ ist der Winkel der Spulenadmittanz $\underline{Y}$ mit der imaginären Achse der gaußschen Zahlenebene (Betrag).

$$\mid \tan\delta \mid = \frac{\frac{1}{R_{pL}}}{\frac{1}{\omega L_p}} = \frac{\omega L_p}{R_{pL}}$$

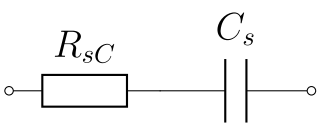
Kehrwert des Verlustfaktors:

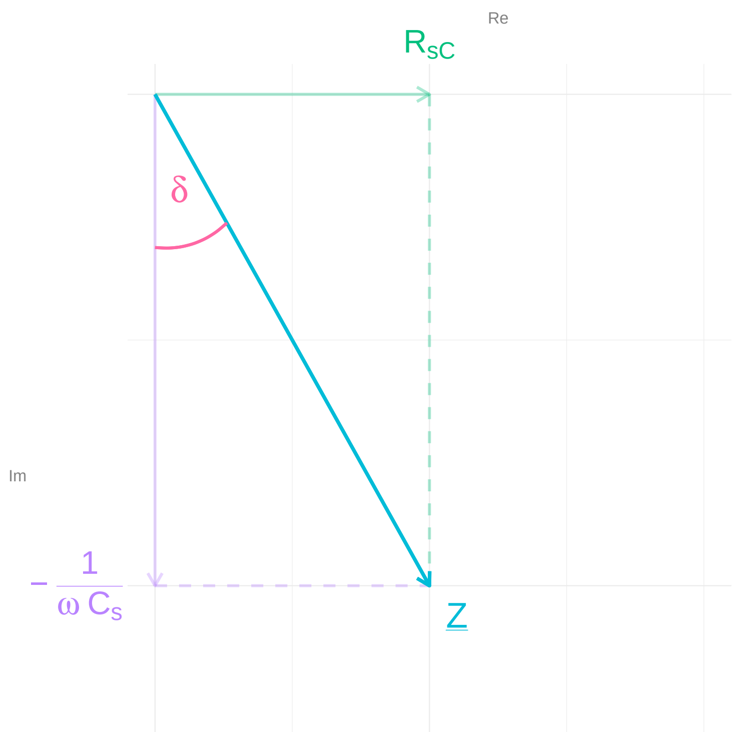
$$Q_{Lp} = \frac{1}{\tan\delta} = \frac{R_{pL}}{\omega L_p}$$

$$Q_{Lp} = Q_{Ls} = \frac{R_{pL}}{\omega L_p} = \frac{\omega L_s}{R_{sL}}$$

Kondensator

Reihenmodell





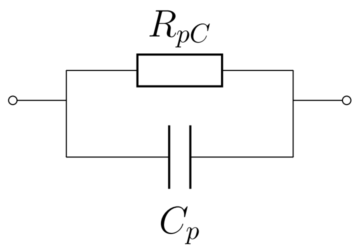
Der Verlustwinkel δ ist der Winkel der Kondensatorimpedanz $\underline{Z}$ mit der imaginären Achse der gaußschen Zahlenebene (Betrag).

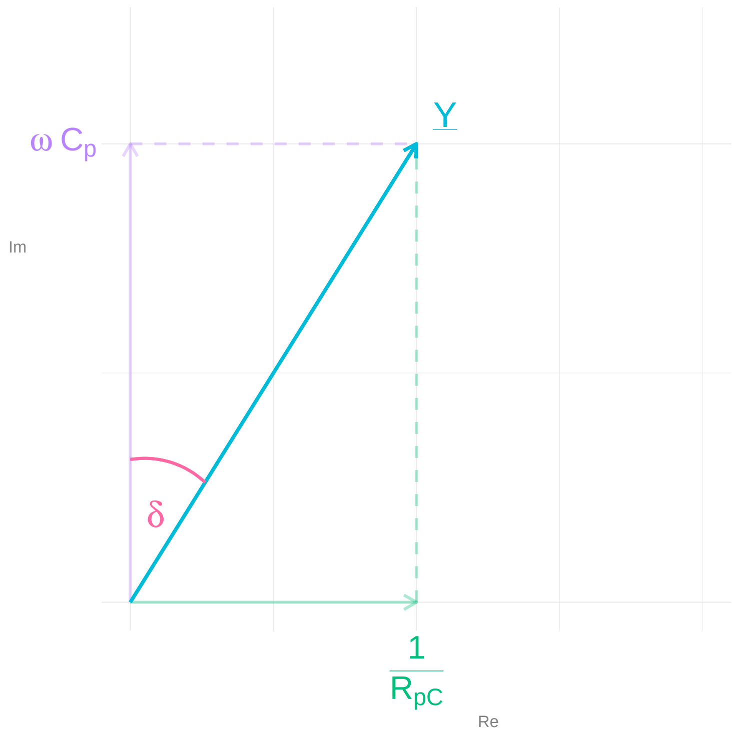
$$|\tan\delta| = \frac{R_{sC}}{\frac{1}{\omega C_s}} = R_{sC} \cdot \omega C_s$$

Somit ist die Kondensatorgüte des Reihenmodells:

$$Q_{Cs} = \frac{1}{\tan\delta} = \frac{1}{R_{sC} \cdot \omega C_s}$$

Parallelmodell





Der Verlustwinkel δ ist der Winkel der Kondensatoradmittanz $\underline{Y}$ mit der imaginären Achse der gaußschen Zahlenebene.

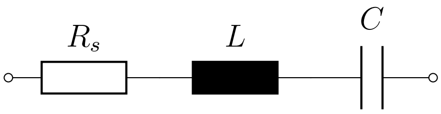
$$\tan\delta = \frac{\frac{1}{R_{pC}}}{\omega C_p} = \frac{1}{R_{pC}\cdot\omega C_p}$$

Kehrwert des Verlustfaktors:

$$Q_{Cp} = \frac{1}{\tan\delta} = R_{pC} \cdot \omega C_p$$

$$Q_{Cs} = Q_{Cp} = \frac{1}{R_{sC} \cdot \omega C_s} = R_{pC} \cdot \omega C_p$$

1.2



Gleichung (4):

$$\begin{aligned} \underline{Z} &= R_s + j(\omega L - \frac{1}{\omega C}) \qquad (4') \\ &= R_s + jX \\ &= \mid \underline{Z} \mid \cdot e^{j\phi_{\underline{Z}}} \end{aligned}$$

Gleichung (5):

$$\begin{aligned} \phi_{\underline{Z}} = \arg(\underline{Z}) &= \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R_s}\right) \qquad (5') \\ &= \arctan\left(\frac{X}{R_s}\right) \end{aligned}$$

Gleichung (6):

$$\mid \underline{Z} \mid = \sqrt{R_s^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad (6')$$

Gleichung (7):

$$\omega_0 = \frac{1}{\sqrt{LC}} \implies \omega_0 = \frac{1}{\sqrt{LC}} \tag{7´}$$

Gleichung (8):

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \tag{8´}$$

Gleichung (9):

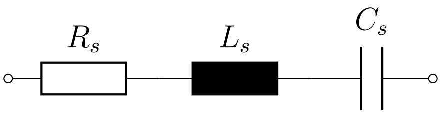
$$\begin{aligned} \mid \underline{I} \mid &= \frac{\mid \underline{U} \mid}{\mid \underline{Z} \mid} \qquad (9´) \\ &= \frac{\mid \underline{U} \mid}{\sqrt{R_s^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \end{aligned}$$

Gleichung (10):

$$\underline{I} = \underline{U} \cdot G_s = \frac{\underline{U}}{R_s} \tag{10´}$$

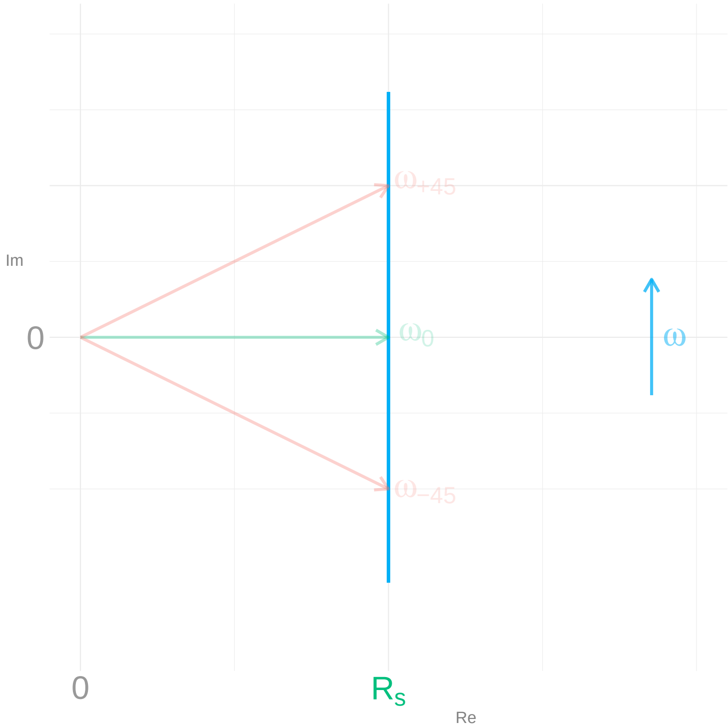
1.3

Reihenschwingkreis



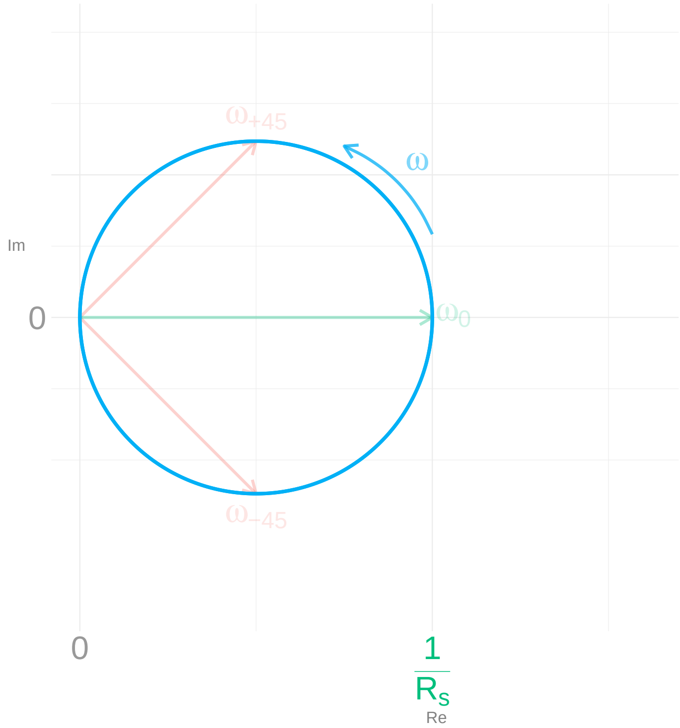
Impedanzortskurve

$$\underline{Z} = R_s + j(\omega L_s - \frac{1}{\omega C_s})$$

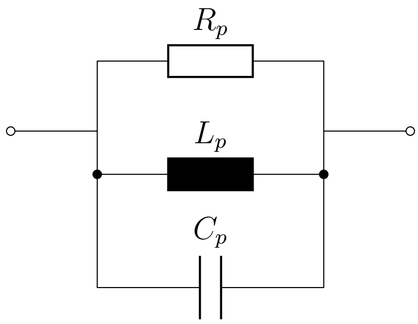


Admittanzortskurve

$$\underline{Y} = \frac{1}{R_s + j(\omega L_s - \frac{1}{\omega C_s})}$$

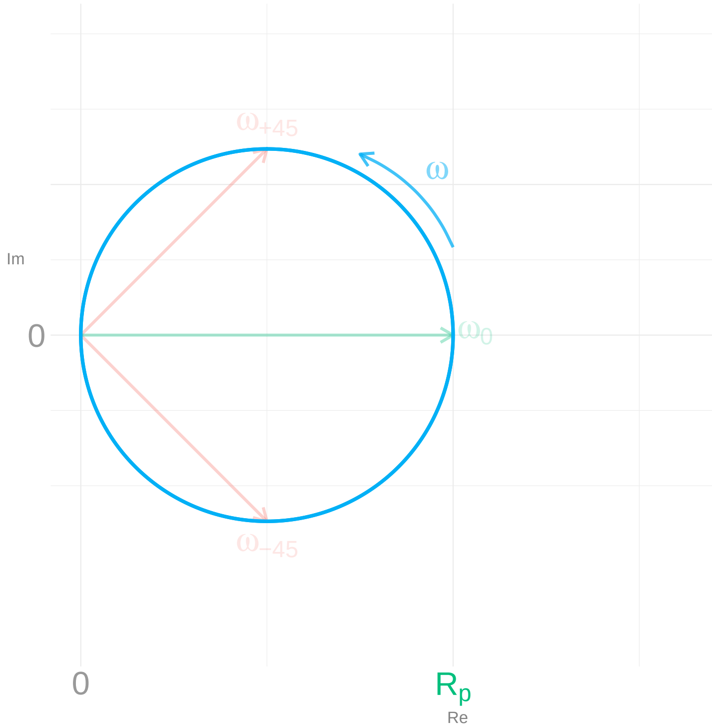


Parallelschwingkreis



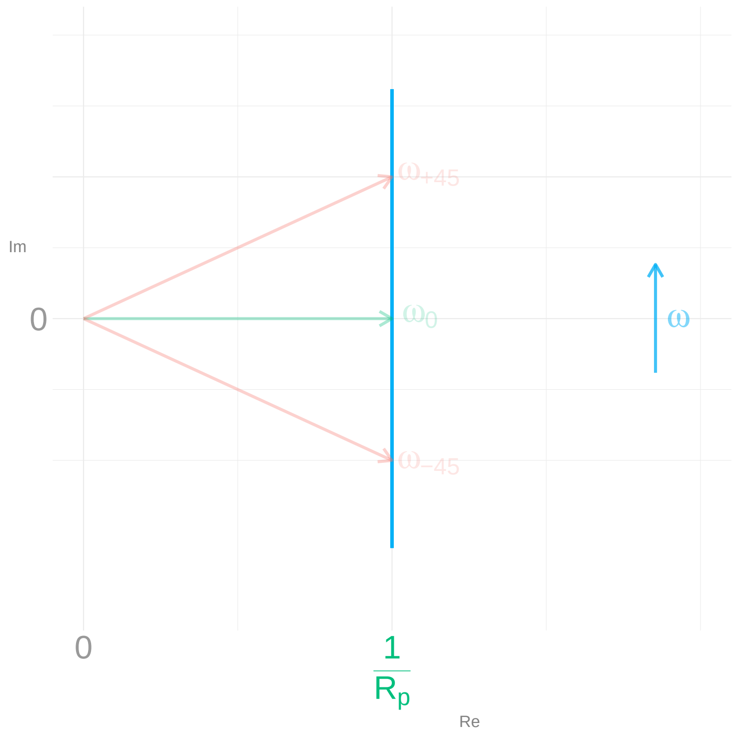
Impedanzortskurve

$$\underline{Z} = \frac{1}{\frac{1}{R_p} + j(\omega C_p - \frac{1}{\omega L_p})}$$

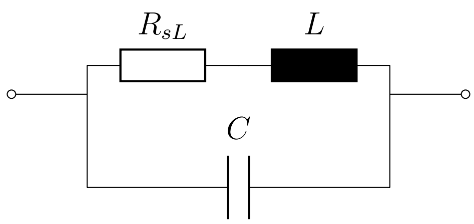


Admittanzortskurve

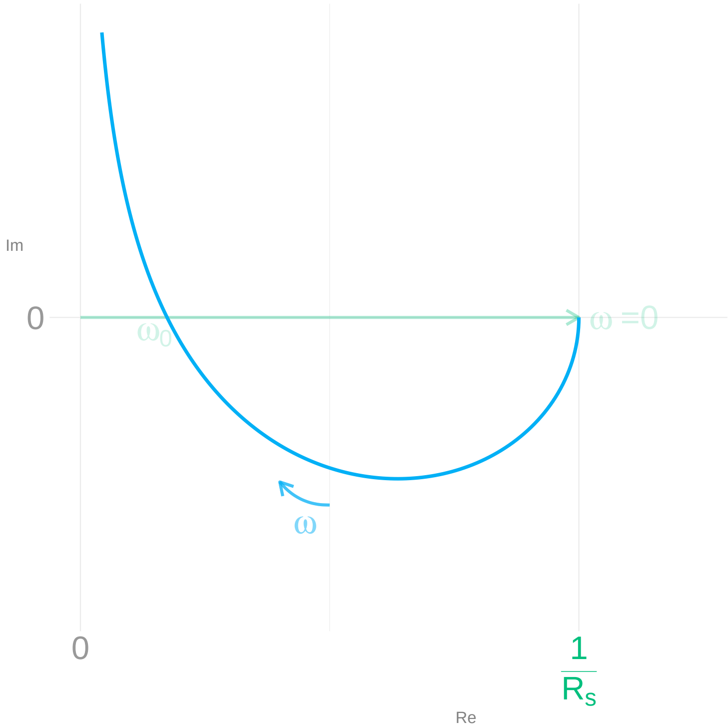
$$\underline{Y} = \frac{1}{R_p} + j(\omega C_p - \frac{1}{\omega L_p})$$



1.4



$$
\begin{aligned}
\underline{Y} &= \frac{1}{R_{sL} + j\omega L} + j\omega C \\
&= \frac{R_{sL}}{R_{sL}^2 + \omega^2 L^2} + j\omega \left(C - \frac{L}{R_{sL}^2 + \omega^2 L^2} \right)
\end{aligned}
$$



Resonanzfrequenz:

$$\mathrm{Im}(\underline{Y}) = 0$$

$$\omega_0' C - \frac{\omega_0' L}{R_{sL}^2 + \omega_0'^2 + L_s^2} = 0$$

$$R_{sL}^2 \cdot C + \omega_0'^2 + L^2 \cdot C^2 = L$$

$$\omega_0' = \sqrt{\underbrace{\frac{1}{LC}}_{\omega_0^2} - \frac{R_{sL}^2}{L^2}}$$

$$\boxed{\omega_0' = \omega_0 \sqrt{1 - \frac{R_{sL}}{\omega_0^2 \cdot L^2}} = \omega_0 \sqrt{1 - \frac{1}{Q_L^2}}}$$

1.5

1.6

1.7

1.8

1.9

2 Versuchsaufgaben