Grundlagen der Elektrotechnik II

Schwingkreise

Studien- und Versuchsaufgaben

Autor: Richard Grünert

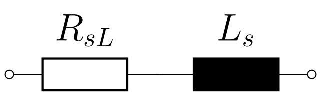
11.6.2019

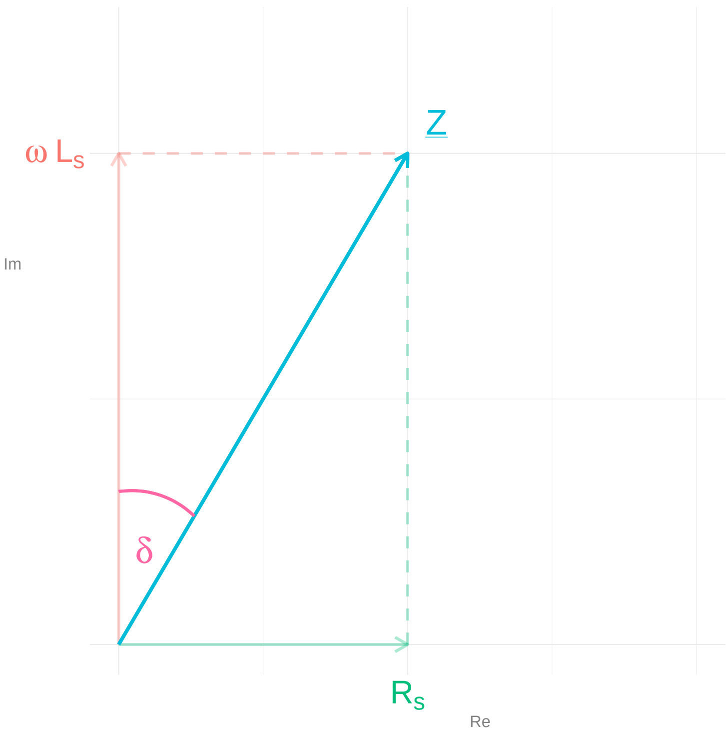
1 | Vorbereitungsaufgaben

1.1

Spule

Reihenmodell





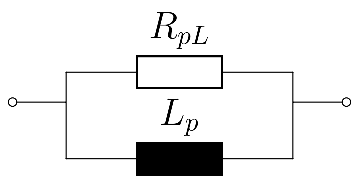
Der Verlustwinkel δ ist der Winkel der Spulenimpedanz $\underline{Z}$ mit der imaginären Achse der gaußschen Zahlenebene. $\tan\delta$ wird auch Verlustfaktor d genannt.

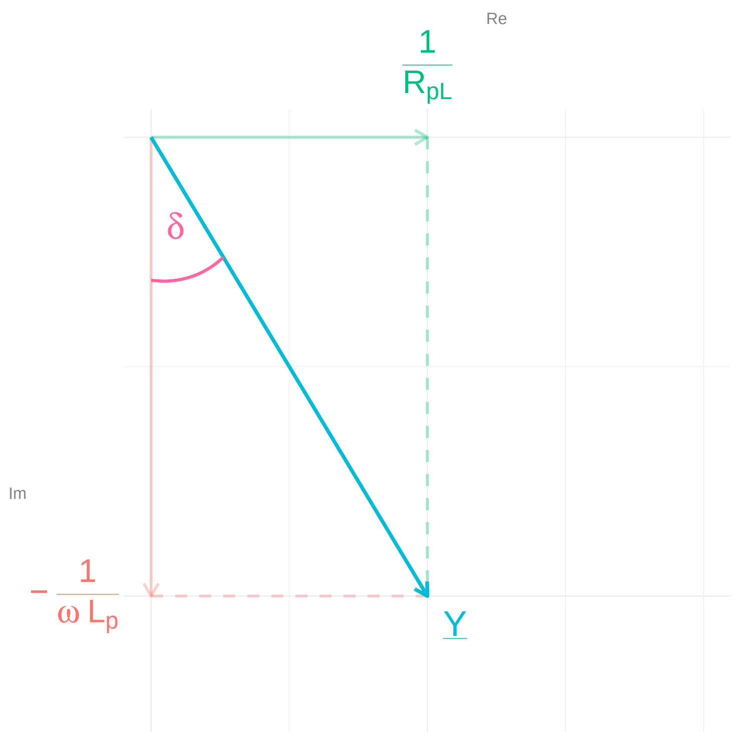
$$\tan\delta = \frac{\omega L_s}{R_{sL}}$$

Die Güte Q der realen Induktivität ist demnach als Kehrwert des Verlustfaktors definiert:

$$Q_{Ls} = \frac{1}{\tan\delta} = \frac{R_{sL}}{\omega L_s}$$

Parallelmodell





Der Verlustwinkel δ ist der Winkel der Spulenadmittanz $\underline{Y}$ mit der imaginären Achse der gaußschen Zahlenebene (Betrag).

$$\mid \tan\delta \mid = \frac{\frac{1}{R_{pL}}}{\frac{1}{\omega L_p}} = \frac{\omega L_p}{R_{pL}}$$

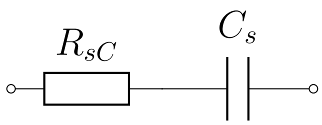
Kehrwert des Verlustfaktors:

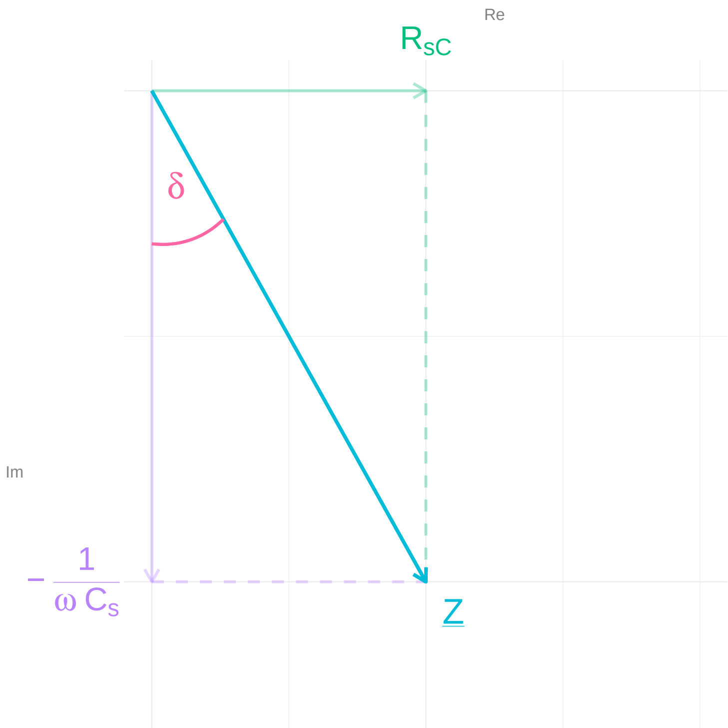
$$Q_{Lp} = \frac{1}{\tan\delta} = \frac{R_{pL}}{\omega L_p}$$

$$Q_{Lp} = Q_{Ls} = \frac{R_{pL}}{\omega L_p} = \frac{\omega L_s}{R_{sL}}$$

Kondensator

Reihenmodell





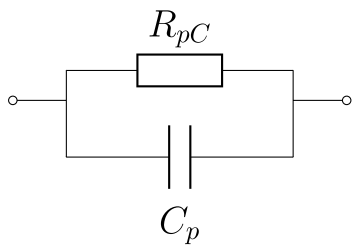
Der Verlustwinkel δ ist der Winkel der Kondensatorimpedanz $\underline{Z}$ mit der imaginären Achse der gaußschen Zahlenebene (Betrag).

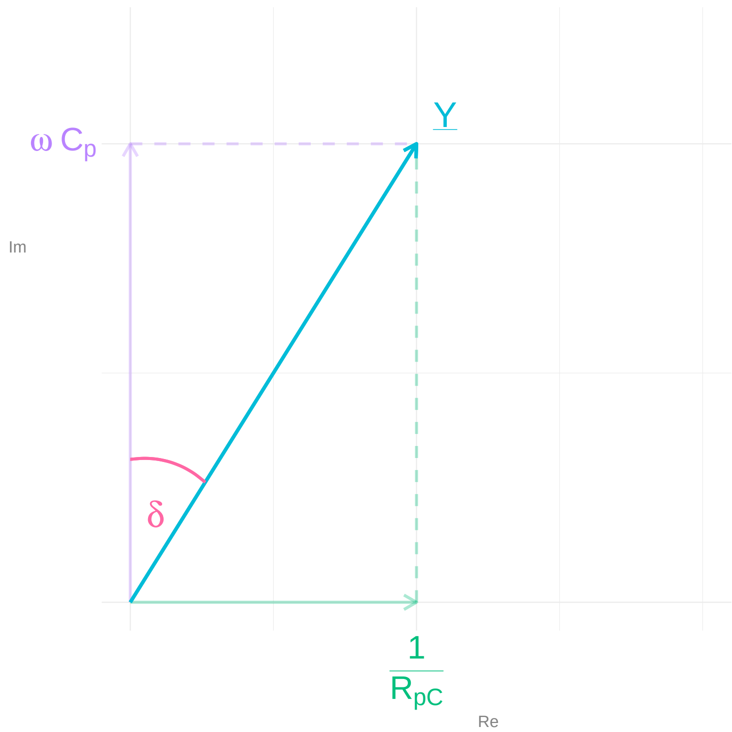
$$\mid \tan\delta \mid = \frac{R_{sC}}{\frac{1}{\omega C_s}} = R_{sC} \cdot \omega C_s$$

Somit ist die Kondensatorgüte des Reihenmodells:

$$Q_{Cs} = \frac{1}{\tan\delta} = \frac{1}{R_{sC} \cdot \omega C_s}$$

Parallelmodell





Der Verlustwinkel δ ist der Winkel der Kondensatoradmittanz $\underline{Y}$ mit der imaginären Achse der gaußschen Zahlenebene.

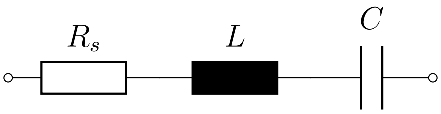
$$\tan\delta = \frac{\frac{1}{R_{pC}}}{\omega C_p} = \frac{1}{R_{pC} \cdot \omega C_p}$$

Kehrwert des Verlustfaktors:

$$Q_{Cp} = \frac{1}{\tan\delta} = R_{pC} \cdot \omega C_p$$

$$Q_{Cs} = Q_{Cp} = \frac{1}{R_{sC} \cdot \omega C_s} = R_{pC} \cdot \omega C_p$$

1.2



Gleichung (4):

$$\begin{aligned} \underline{Z} &= R_s + j(\omega L - \frac{1}{\omega C}) \\ &= R_s + jX \\ &= \mid \underline{Z} \mid \cdot e^{j\phi_{\underline{Z}}} \end{aligned} \tag{4´}$$

Gleichung (5):

$$\begin{aligned} \phi_{\underline{Z}} = \arg(\underline{Z}) &= \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R_s}\right) \\ &= \arctan\left(\frac{X}{R_s}\right) \end{aligned} \tag{5´}$$

Gleichung (6):

$$\mid \underline{Z} \mid = \sqrt{R_s^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{6´}$$

Gleichung (7):

$$\omega_0 = \frac{1}{\sqrt{LC}} \implies \omega_0 = \frac{1}{\sqrt{LC}} \tag{7'}$$

Gleichung (8):

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \tag{8'}$$

Gleichung (9):

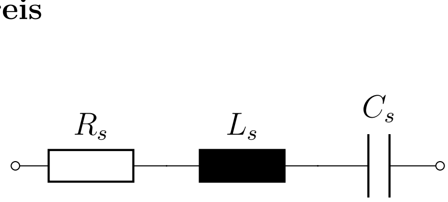
$$\begin{aligned} \mid \underline{I} \mid &= \frac{\mid \underline{U} \mid}{\mid \underline{Z} \mid} \\ &= \frac{\mid \underline{U} \mid}{\sqrt{R_s^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \end{aligned} \tag{9'}$$

Gleichung (10):

$$\underline{I} = \underline{U} \cdot G_s = \frac{\underline{U}}{R_s} \tag{10'}$$

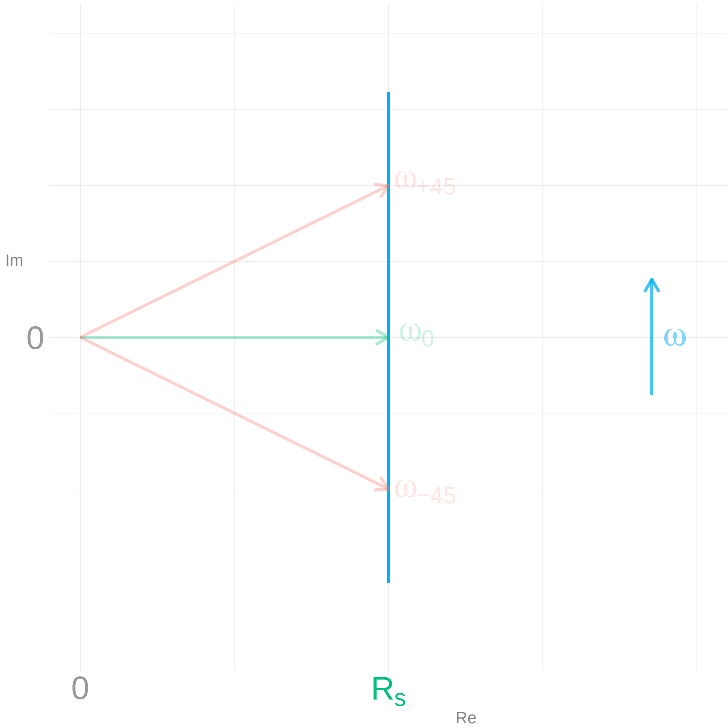
1.3

Reihenschwingkreis



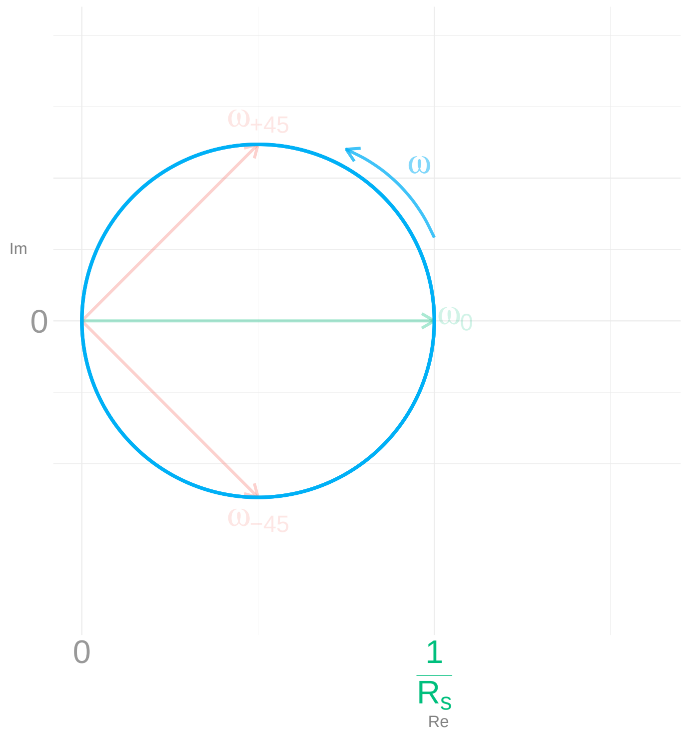
Impedanzortskurve

$$\underline{Z} = R_s + j(\omega L_s - \frac{1}{\omega C_s})$$

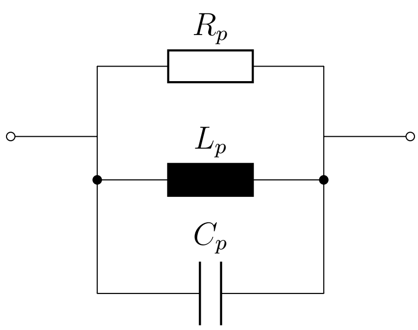


Admittanzortskurve

$$\underline{Y} = \frac{1}{R_s + j(\omega L_s - \frac{1}{\omega C_s})}$$

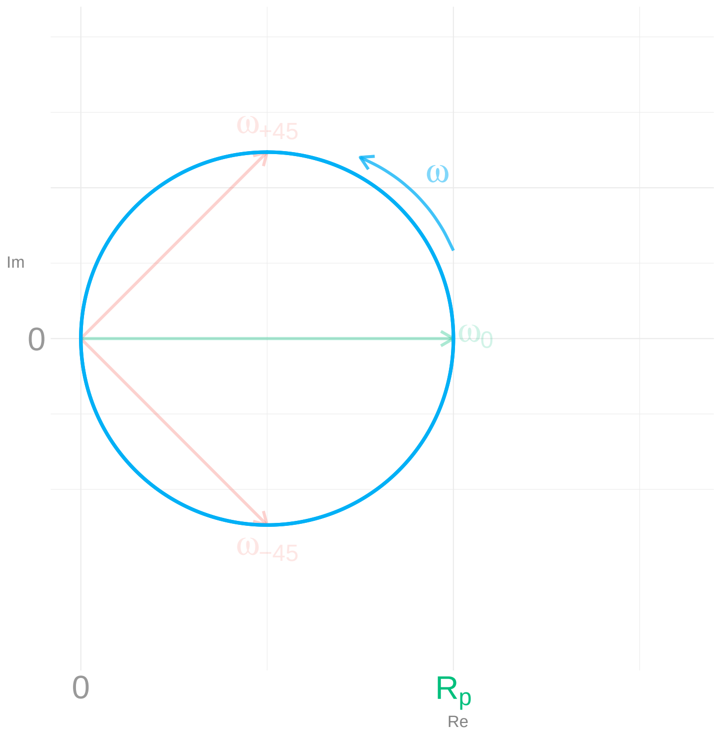


Parallelschwingkreis



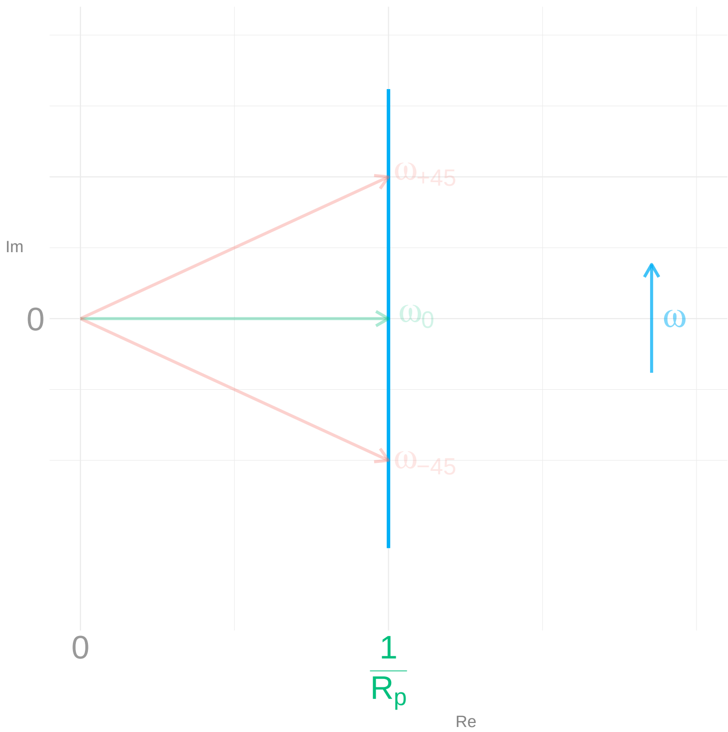
Impedanzortskurve

$$\underline{Z} = \frac{1}{\frac{1}{R_p} + j(\omega C_p - \frac{1}{\omega L_p})}$$

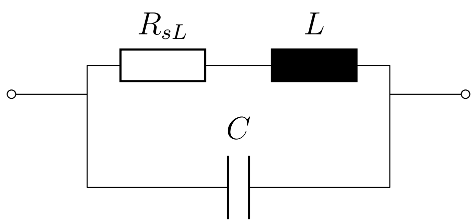


Admittanzortskurve

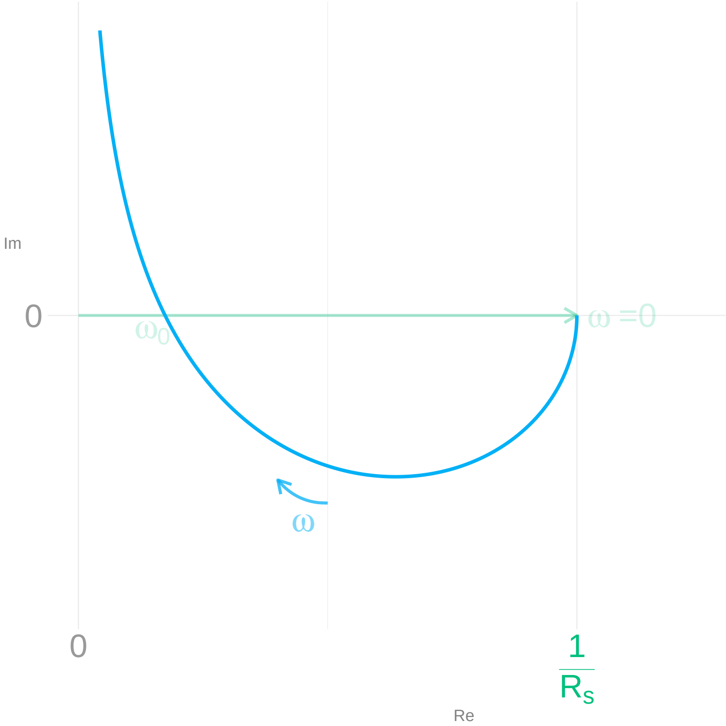
$$\underline{Y} = \frac{1}{R_p} + j(\omega C_p - \frac{1}{\omega L_p})$$



1.4



$$
\begin{aligned}
\underline{Y} &= \frac{1}{R_{sL} + j\omega L} + j\omega C \\
&= \frac{R_{sL}}{R_{sL}^2 + \omega^2 L^2} + j\omega \left(C - \frac{L}{R_{sL}^2 + \omega^2 L^2} \right)
\end{aligned}
$$



Resonanzfrequenz:

$$\mathrm{Im}(\underline{Y}) = 0$$

$$\omega_0^{'} C - \frac{\omega_0^{'} L}{R_{sL}^2 + \omega_0^{'2} + L_s^2} = 0$$

$$R_{sL}^2 \cdot C + \omega_0^{'2} + L^2 \cdot C^2 = L$$

$$\omega_0^{'} = \sqrt{\underbrace{\frac{1}{LC}}_{\omega_0^2} - \frac{R_{sL}^2}{L^2}}$$

$$\boxed{\omega_0^{'} = \omega_0 \sqrt{1 - \frac{R_{sL}}{\omega_0^2 \cdot L^2}} = \omega_0 \sqrt{1 - \frac{1}{Q_L^2}}}$$

1.5

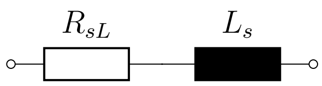
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.3\text{H} \cdot 22.5\text{nF}}} = 5834.1 \;\; \text{s}^{-1}$$

$$\omega_0^{'} = \omega_0 \cdot \sqrt{1 - \frac{R_{sL^2}}{\omega_0^2 \cdot L^2}} = 5834.1\text{s}^{-1} \cdot \sqrt{1 - \frac{(100\Omega)^2}{(5834.1\text{s}^{-1})^2 \cdot (1.3\text{H})^2}}$$

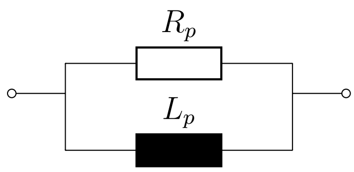
$$\omega_0^{'} = 5833.6 \;\; \text{s}^{-1}$$

$$\Longrightarrow \text{ geringe Differenz } \Longrightarrow \text{ hohe Spulengüte}$$

Zur Berechnung von Güte und Bandbreite wird der Schwingkreis in einen idealen Parallelschwingkreis umgewandelt:



$$\Longrightarrow$$



$$\underline{Y}_s = \underline{Y}_p$$

$$\frac{1}{R_s + j\omega_0 L_s} = \frac{1}{R_p} - j\frac{1}{\omega_0 L_p}$$

$$\frac{R_s}{R_s^2 + \omega_0^2 L_s^2} - j\frac{\omega_0 L_s}{R_s^2 L_s + \omega_0^2 L_s} = \frac{1}{R_p} - j\frac{1}{\omega_0 L_p}$$

Realteilvergleich:

$$R_p = 1 + \frac{\omega_0^2 L_s^2}{R_s^2} = 1 + Q_L^2$$

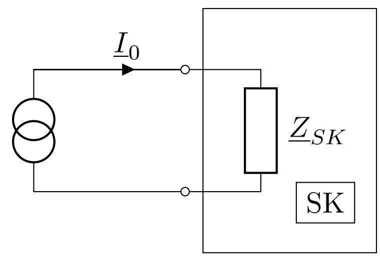
Imaginärteilvergleich:

$$L_p = L_s \left(1 + \frac{R_s^2}{\omega_0^2 L_s^2}\right) = 1 + \frac{1}{Q_L^2}$$

$$R_p = 575.3 \ \text{k}\Omega, \ L_p = 1.30023 \ \text{H} \approx L_s$$

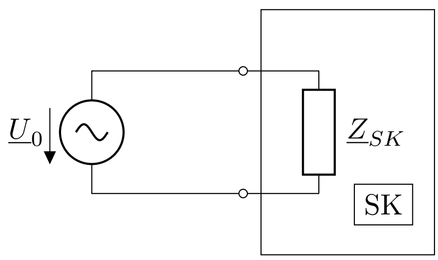
1.6

Stromspeisung



$$\underline{U} = \underline{Z}_{SK} \cdot \underline{I}_0, \quad \underline{I}_0 = \text{konst.} \implies \underline{U} \sim \underline{Z}_{SK}$$

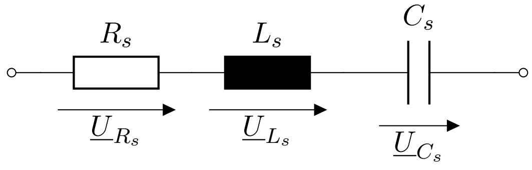
Spannungsspeisung



$$\underline{I} = \underline{U} \cdot \underline{Y}_{SK}, \quad \underline{U}_0 = \text{konst.} \implies \underline{I} \sim \underline{Y}_{SK}$$

1.7

Spannungsüberhöhung
(Nur im Reihenschwingkreis)



R_s:

$$\underline{U}_{R_s} = \frac{\underline{U} \cdot R_s}{R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)}$$

L_s:

$$\underline{U}_{L_s} = \frac{\underline{U} \cdot j\omega L_s}{R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)}$$

C_s:

$$\underline{U}_{C_s} = \frac{\underline{U}}{j\omega C_s\left(R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)\right)}$$

bei $\omega = \omega_0$:

$$\underline{U}_{R_s} = \frac{\underline{U} \cdot R_s}{R_s} = \underline{U}$$

$$\underline{U}_{L_s} = j\underline{U} \cdot \frac{\omega_0 L_s}{R_s} = j\underline{U} \cdot Q_L$$

$$\underline{U}_{C_s} = -j\underline{U} \cdot \frac{1}{\omega_0 C_s R_s}$$

Beträge bei ω_0:

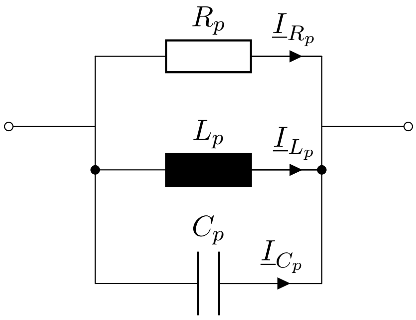
$$\hat{U}_{R_s} = \hat{U}$$

$$\hat{U}_{L_s} = Q_L \cdot \underline{U}$$

$$\hat{U}_{C_s} = Q_C \cdot \underline{U}$$

Stromüberhöhung

(Nur im Parallelschwingkreis)



R_p:

$$\underline{I}_{R_p} = \frac{\underline{I} \cdot \frac{1}{R_p}}{\frac{1}{R_p} + j\left(\omega C_p - \frac{1}{\omega L_p}\right)}$$

L_p:

$$\underline{I}_{L_p} = \frac{\underline{I}}{j\omega L_p \left(\frac{1}{R_p} + j\left(\omega C_p - \frac{1}{\omega L_p}\right)\right)}$$

C_p:

$$\underline{I}_{C_p} = \frac{\underline{I} \cdot j\omega C_p}{\frac{1}{R_p} + j\left(\omega C_p - \frac{1}{\omega L_p}\right)}$$

bei $\omega = \omega_0$:

$$\underline{I}_{R_p} = \underline{I}$$

$$\underline{I}_{L_p} = j\underline{I} \cdot \frac{R_p}{\omega_0 L_p} = j\underline{I} \cdot Q_L$$

$$\underline{I}_{C_p} = -j\underline{I} \cdot \omega_0 R_p C_p = j\underline{I} \cdot Q_C$$

Beträge bei ω_0:

$$\hat{I}_{R_p} = \hat{I}$$

$$\hat{I}_{L_p} = Q_L \cdot \underline{I}$$

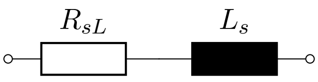
$$\hat{I}_{C_p} = Q_C \cdot \underline{I}$$

Die Strom- bzw. Spannungsüberhöhungen betragen demnach bei Resonanz das $Q_{L/C}$-fache der Quellgrößen.

1.8

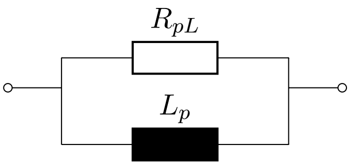
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1\mathrm{H} \cdot 100\mathrm{nF}}} = 10000 \ \mathrm{s}^{-1} \implies f_0 = 1.592 \ \mathrm{kHz}$$

Spule



$$Q_L = \frac{\omega_0 L_s}{R_{sL}}$$

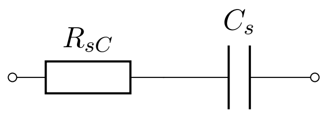
$$R_{sL} = \frac{\omega_0 L_s}{Q_L} = 20 \ \ \Omega$$



$$Q_L = \frac{R_{pL}}{\omega_0 L_p}$$

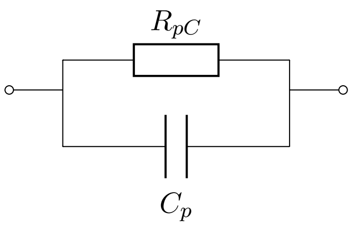
$$R_{pC} = Q_L \cdot \omega_0 L_p = 50 \ \ \mathrm{k}\Omega$$

Kondensator



$$Q_C = \frac{1}{\omega_0 R_{sC} C_s}$$

$$R_{sC} = \frac{1}{\omega_0 Q_C C_s} = 3.3 \ \ \Omega$$



$$Q_C = \omega_0 R_{pC} C_p$$

$$R_{pC} = \frac{Q_C}{\omega_0 C_p} = 300 \ \ \mathrm{k}\Omega$$

$$Q_{SK} = \frac{1}{\frac{1}{Q_c} + \frac{1}{Q_L}} = \frac{Q_C \cdot Q_L}{Q_C + Q_L}$$

$$Q_{SK} = \frac{300}{7} \approx 42.86$$

$$B_\omega = \frac{\omega_0}{Q_{SK}} = \frac{10000\text{s}^{-1}}{42.86} = 232.3 \ \text{s}^{-1}$$

$$B_f = \frac{223.2\text{s}^{-1}}{2\pi} = 37.14 \ \text{Hz}$$

1.9

2 Versuchsaufgaben