Grundlagen der Elektrotechnik II

Frequenzanalyse periodischer Signale

Studien- und Versuchsaufgaben

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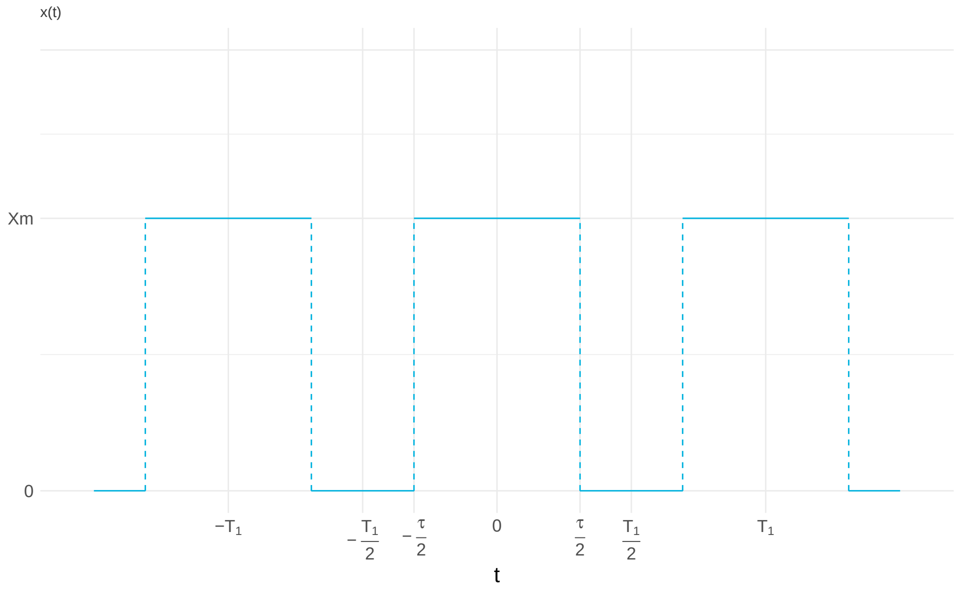
1 Vorbereitungsaufgaben

1.1



$$
\begin{aligned}
\underline{X}_\nu &= \frac{1}{T_1} \cdot \int_{T_1} x(t) \cdot e^{-(j\nu \cdot \omega_1 t)} \, \mathrm{d}t \\
&= \frac{1}{T_1} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} X_m \cdot e^{-(j\nu \cdot \omega_1 t)} \, \mathrm{d}t \\
&= -\frac{X_m}{T_1 \cdot j\nu\omega_1} \cdot \left[e^{-(j\nu \cdot \omega_1 t)} \right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \\
&= -\frac{X_m}{T_1 \cdot j\nu\omega_1} \cdot \left(e^{-j\nu \cdot \omega_1 \frac{\tau}{2}} - e^{j\nu \cdot \omega_1 \frac{\tau}{2}} \right)
\end{aligned}
$$

$\omega_1 = \frac{2\pi}{T_1}$ und Erweiterung mit $\frac{-1}{-1}$:

$$
\begin{aligned}
\underline{X}_\nu &= \frac{X_m}{2j\pi\nu} \cdot \left(e^{j\nu \cdot \pi \frac{\tau}{T_1}} - e^{-j\nu \cdot \pi \frac{\tau}{T_1}} \right) \\
&= \frac{X_m}{\pi\nu} \cdot \frac{\left(e^{j\nu \cdot \pi \frac{\tau}{T_1}} - e^{-j\nu \cdot \pi \frac{\tau}{T_1}} \right)}{2j}
\end{aligned}
$$

mit $\dfrac{\left(e^{jx} - e^{-jx}\right)}{2j} = \sin(x)$ und $\dfrac{\tau}{T_1} = D$:

$$\underline{X}_\nu = \frac{X_m}{\pi\nu} \cdot \sin(\pi\nu D)$$

Erweitert man wieder mit $\dfrac{D}{D}$ erhält man das Bild einer Spaltfunktion $\mathrm{si}(x) = \dfrac{\sin x}{x}$:

$$\underline{X}_\nu = D \cdot X_m \cdot \frac{\sin(\pi\nu D)}{\pi\nu D} = D \cdot X_m \cdot \mathrm{si}(\pi\nu D)$$

Als reele Reihe:

$$x(t) = X_0 + \sum_{\nu=1}^{\infty} \hat{X}_\nu \cos(\nu \cdot \omega_1 t + \phi_\nu)$$

$$X_0 = \frac{1}{T_1} \cdot \int_{T_1} x(t)\,\mathrm{d}t = \frac{X_m}{2}$$

Aus der komplexen Reihendarstellung folgt

$$b_\nu = -2 \cdot \mathrm{Im}(\underline{X}_\nu) = 0$$

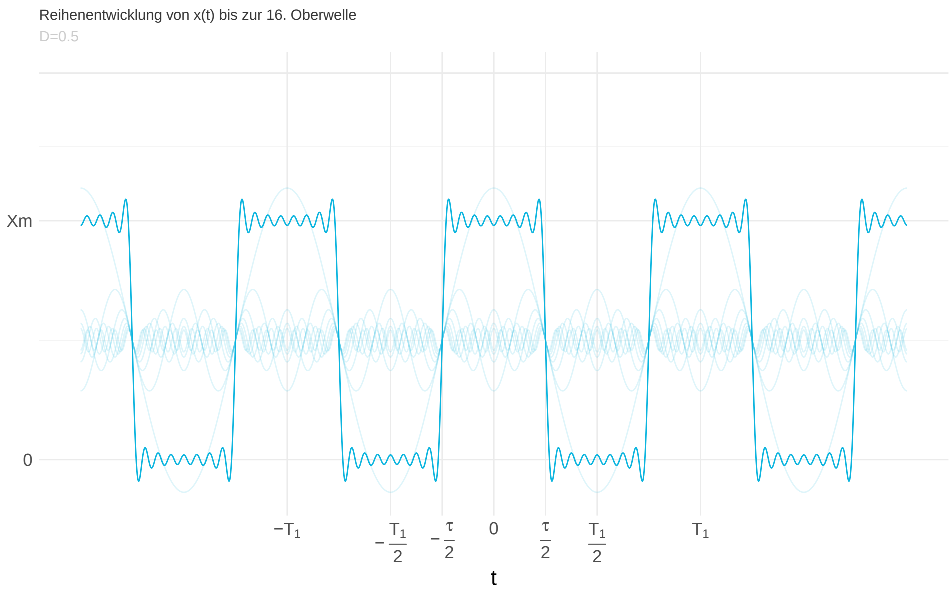
$$\hat{X}_\nu = \sqrt{a_\nu^2 + b_\nu^2} = 2 \cdot \mid \underline{X}_\nu \mid \Longrightarrow\ a_\nu = 2 \cdot \mid D \cdot X_m \cdot \mathrm{si}(\nu\pi D) \mid$$

ϕ_ν hängt nur vom Wert von $\mathrm{si}(\nu\pi D)$ ab, da $\underline{X}_\nu$ rein reell ist:

$$\phi_\nu = \begin{cases} 0 & ; \nu = \dfrac{4k+1}{2D} \\ \pi & ; \nu = \dfrac{4k-1}{2D} \\ \text{n.d.} & ; \text{sonst} \end{cases}$$

Somit ist

$$x(t) = \frac{X_m}{2} + \sum_{\nu=1}^{\infty} 2DX_m \cdot |\operatorname{si}(\pi\nu D)| \cdot \cos(\nu \cdot \frac{2\pi}{T_1} \cdot t + \phi_\nu)$$



Effektivwert:

$$X_{\text{eff}} = \sqrt{\frac{1}{T_1} \cdot \int_{T_1} x^2(t)\,\mathrm{d}t} = \sqrt{\frac{X_m^2}{T_1} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1\,\mathrm{d}t}$$

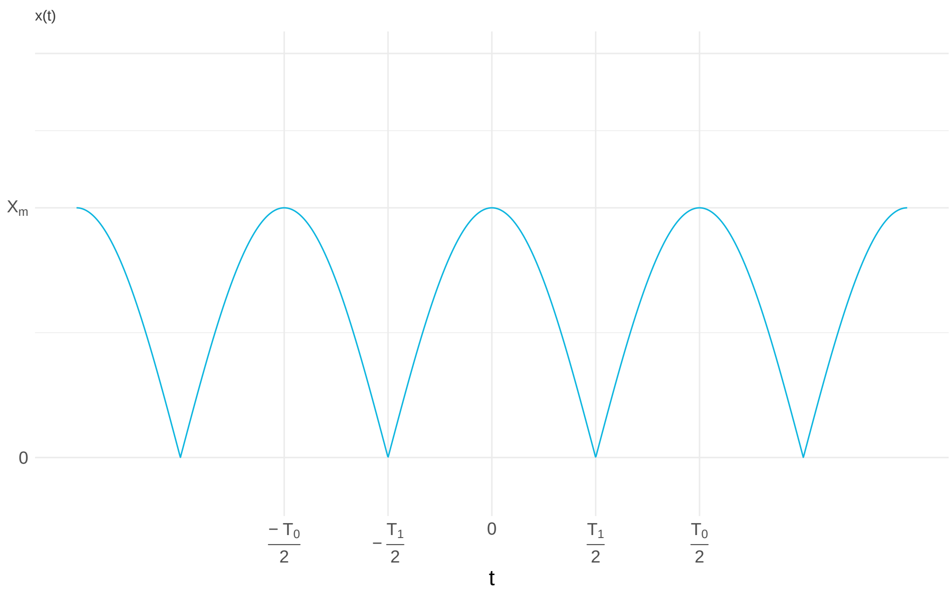
$$X_{\text{eff}} = X_m \cdot \sqrt{\frac{\tau}{T_1}} = X_m \cdot \sqrt{D}$$

$D=\frac{1}{2}, X_{\text{eff}}=\frac{X_m}{\sqrt{2}}$			
ν	$\underline{X}_\nu$	$\hat{X}_\nu$	ϕ_ν
1	$\frac{1}{\pi}X_m$	$\frac{2}{\pi}X_m$	0
2	–	–	–
3	$-\frac{1}{3\pi}X_m$	$\frac{2}{3\pi}X_m$	π
4	–	–	–
5	$\frac{1}{5\pi}X_m$	$\frac{2}{5\pi}X_m$	0
6	–	–	–
7	$-\frac{1}{7\pi}X_m$	$\frac{2}{7\pi}X_m$	π
8	–	–	–
9	$\frac{1}{9\pi}X_m$	$\frac{2}{9\pi}X_m$	0
10	–	–	–
11	$-\frac{1}{11\pi}X_m$	$\frac{2}{11\pi}X_m$	π
12	–	–	–
13	$\frac{1}{13\pi}X_m$	$\frac{2}{13\pi}X_m$	0
14	–	–	–
15	$-\frac{1}{15\pi}X_m$	$\frac{2}{15\pi}X_m$	π
16	–	–	–

$D=\frac{1}{4}, X_{\text{eff}}=\frac{X_m}{2}$		
ν	$\hat{X}_\nu$	ϕ_ν
1	$\frac{\sqrt{2}}{\pi}X_m$	0
2	$\frac{1}{\pi}X_m$	0
3	$\frac{\sqrt{2}}{3\pi}X_m$	0
4	–	–
5	$\frac{\sqrt{2}}{5\pi}X_m$	π
6	$\frac{1}{3\pi}X_m$	π
7	$\frac{\sqrt{2}}{7\pi}X_m$	π
8	–	–
9	$\frac{\sqrt{2}}{9\pi}X_m$	0
10	$\frac{1}{5\pi}X_m$	0
11	$\frac{\sqrt{2}}{11\pi}X_m$	0
12	–	–
13	$\frac{\sqrt{2}}{13\pi}X_m$	π
14	$\frac{1}{7\pi}X_m$	π
15	$\frac{\sqrt{2}}{15\pi}X_m$	π
16	–	–

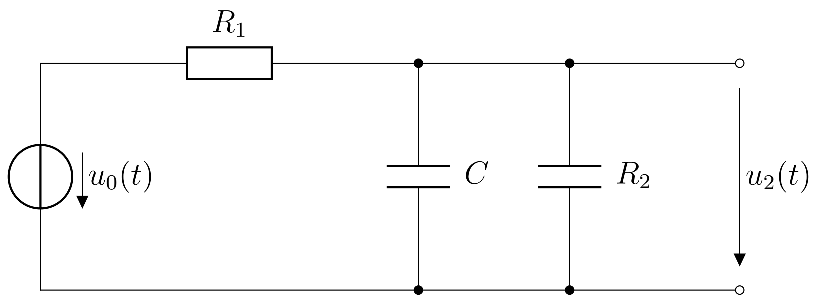
$D=\frac{1}{8}, X_{\text{eff}}=\frac{X_m}{\sqrt{8}}$		
ν	$\hat{X}_\nu$	ϕ_ν
1	$\frac{\sqrt{2-\sqrt{2}}}{\pi}X_m$	0
2	$\frac{\sqrt{2}}{2\pi}X_m$	0
3	$\frac{\sqrt{2+\sqrt{2}}}{3\pi}X_m$	0
4	$\frac{1}{2}X_m$	0
5	$\frac{\sqrt{2+\sqrt{2}}}{5\pi}X_m$	0
6	$\frac{\sqrt{2}}{16\pi}X_m$	0
7	$\frac{\sqrt{2-\sqrt{2}}}{7\pi}X_m$	0
8	–	–
9	$\frac{\sqrt{2-\sqrt{2}}}{9\pi}X_m$	π
10	$\frac{\sqrt{2}}{5\pi}X_m$	π
11	$\frac{\sqrt{2+\sqrt{2}}}{11\pi}X_m$	π
12	$\frac{1}{6\pi}X_m$	π
13	$\frac{\sqrt{2+\sqrt{2}}}{13\pi}X_m$	π
14	$\frac{\sqrt{2}}{7\pi}X_m$	π
15	$\frac{\sqrt{2-\sqrt{2}}}{15\pi}X_m$	π
16	–	–

1.2



$$\underline{X}_\nu = \frac{1}{T_1} \cdot \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} X_m \cos(\omega_0 t) \cdot e^{-(j\nu\omega_0 t)} \, \mathrm{d}t \qquad = \frac{X_m}{T_1} \cdot \left[\frac{e^{-(j\nu\omega_0 t)}}{1} \right]_{-\frac{T_1}{2}}^{\frac{T_1}{2}}$$

1.3



$$\underline{U}_2 = \underline{U_0} \cdot \frac{\frac{1}{j\omega C + \frac{1}{R_2}}}{R_1 + \frac{1}{j\omega C + \frac{1}{R_2}}} = \frac{\underline{U}_0 \cdot R_2}{R_1 + R_2 + j\omega C R_1 R_2}$$

Betrag:

$$\hat{U}_2 = \frac{\hat{U}_0 \cdot R_2}{\sqrt{(R_1 + R_2)^2 + (\omega C R_1 R_2)^2}}$$

Phase:

$$\phi_{\underline{U}_2} = \phi_{\underline{U}_0} - \arctan \frac{\omega C R_1 R_2}{R_1 + R_2}$$

2 Versuchsaufgaben

2.1

2.2

2.3