Communication Systems

Equalisation of Frequency Selective Channels

Homework 1 CS Summer Term 2023

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13.4.2023

1 Procedure / Method

Using MATLAB SIMULINK™, the provided model file `M5_T_EQL_FIR_pre.slx` as well as the `initial_.m` file have been extended to include an additional equalizer at the receiver side. Figure 1 shows a screenshot of the model in Simulink.

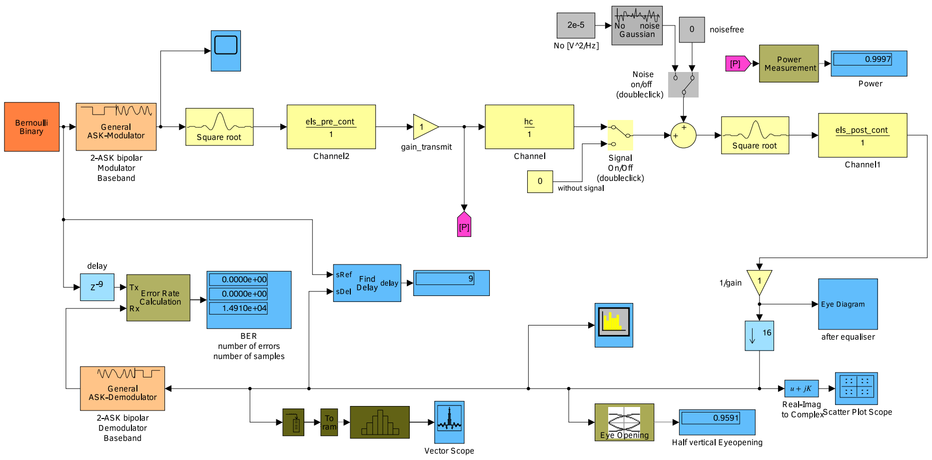

Figure 1: Simulation System

Either of the equalizers can be put into “thru”-mode by setting the coefficients of the respective FIR-Block to `[ 1 0 0 ... 0 0 0 ]`. The equalization was done using 5 coefficients.

2 Model Check

The AWGN conditions with a non-frequency-selective channel have been checked and without any noise addition, the two symbols were clearly distinguishable, as expected.

With the simulated power measurement, the signal power shows $P_s = 1\,\mathrm{V}^2$. Multiplying by the symbol time $T_{sym} = \frac{1}{5000}\,\mathrm{s} = 2 \cdot 10^{-4}\,\mathrm{s}$, gives a symbol energy of $E_s = 2 \cdot 10^{-4}\,\mathrm{V}^2\,\mathrm{s}$. With the set noise power density of $N_0 = 2 \cdot 10^{-5}\,\mathrm{V}^2\,\mathrm{Hz}^{-1}$, the channel SNR becomes

$$\frac{E_s}{N_0} = \frac{2 \cdot 10^{-4}\,\mathrm{V}^2\,\mathrm{s}}{2 \cdot 10^{-5}\,\mathrm{V}^2\,\mathrm{Hz}^{-1}} = 10 \rightarrow 10\,\mathrm{dB}$$

as required.

2.1 Procedure for each Channel Type

With the provided spreadsheet, each channel type could be characterised. The best nyquist vector for each situation was noted.

The simulation time was 30 s in each case. The simulation results can be found in Table 1 as well as in the accompanying spreadsheet. The modified simulation model and Matlab script file are also attached.

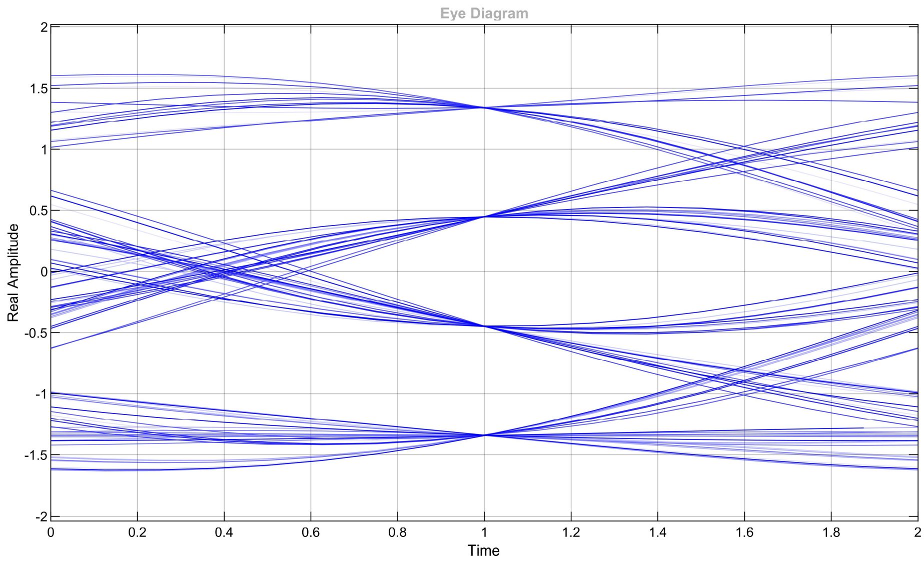

Figure 2: Eye diagram of system with two-tap minimum phase channel without noise.

3 Channel I: 2-Tap Minimum Phase Channel

3.1 Difference between calculated and simulated BER

In the case of not using any equalizer, one finds that the BER resulting from the simulation is approximately half of that which was calculated.

This is due to the fact that the ISI introduced by the channel causes four possible signal values to appear at the sampling point (without any noise influence). This can be easily seen in the eye diagram in Figure 2.

In this case, a sent symbol (1 or 0 in this case) could be moved by the ISI to have a magnitude of 0.45 or 1.35. 50 % of the sent symbols will be received as either 0.45 or −0.45 and the other 50 % will be either 1.35 or −1.35.

Since the symbols with the greater magnitude (1.35) will have a greater U_A their error probability will be lower under the infuence of noise, given the same noise power. We get two different error probabilities:

$$P_{e1} = \frac{1}{2} \cdot \mathrm{erfc}\left(\frac{0.45\,\mathrm{V}}{\sqrt{2} \cdot \sqrt{0.1\,\mathrm{V}^2}}\right) = 7.7 \cdot 10^{-2}$$

$$P_{e2} = \frac{1}{2} \cdot \mathrm{erfc}\left(\frac{1.35\,\mathrm{V}}{\sqrt{2} \cdot \sqrt{0.1\,\mathrm{V}^2}}\right) = 1.1 \cdot 10^{-5}$$

i.e. the symbols with magnitude 1.35 will be less likely to cause errors. Given that the error probability is much lower for the higher-magnitude symbols, their errors can be neglected. This means that 50 % of the symbols will not cause any errors which explains that the calculated BER is approximately two times as large as the simulated BER since the equation does not model this ISI-behaviour.

The total error rate is:

$$P_e = 0.5 \cdot P_{e1} + 0.5 \cdot P_{e2} = 0.5 \cdot 7.7011 \cdot 10^{-2} \approx 3.9 \cdot 10^{-2}$$

which matches the simulated value $4.0 \cdot 10^{-2}$ well.

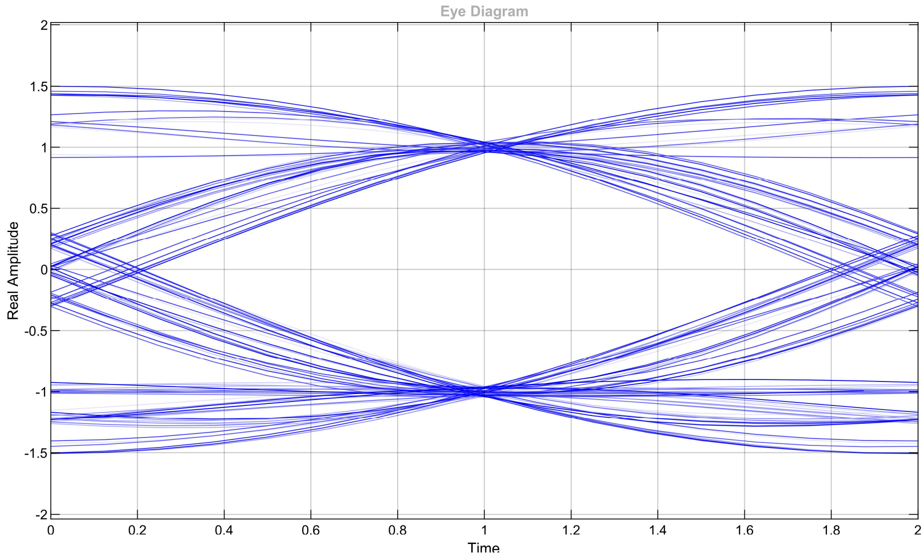

Figure 3: Eye of equalized two-tap channel system (post-coding).

3.2 Equalization

Using the equalizer yields a lowered BER, as expected. The equalization is not perfect, however, as only a finite number of coefficients can be used. Figure 3 shows the eye diagram and the remaining spread of values at the sample point which causes a lower U_A than under AWGN conditions.

The simulated BER is also smaller than the calculated one. This can be explained by the fact that the calculation uses the minimal half vertical eye opening. However, as seen in Figure 3 there exist also symbol amplitudes greater than this value causing the actual BER to be smaller. The highest of these is approximately $U_{A_{\max}} = 1.05\,\mathrm{V}$ resulting in a BER of

$$P_{e_{\min}} = 5.4 \cdot 10^{-3}$$

while the highest using the minimal eye opening is

$$P_{e_{\max}} = 9.9 \cdot 10^{-3}$$

so that the actual BER has to be in the interval $[P_{e_{\min}}, P_{e_{\max}}]$, which is the case ($P_{e_{\mathrm{sim}}} = 7.3 \cdot 10^{-3}$).

3.2.1 Post- vs Pre-Coding

In the post-coding case, the noise path will receive an increase in power by the greater-than-unity gain of the equalizer. The power gain can be calculated by taking the sum of the squared coefficients and is $G_{\mathrm{EQ}} = 1.654$ so that the noise power after the equalizer will be $P_{\mathrm{n_{calc}}} = 0.1654\,\mathrm{V}^2$. This matches the measurement from the simulation ($P_{\mathrm{n_{sim}}} = 0.17\,\mathrm{V}^2$).

By moving the equalizer to the sender side, it cannot increase the noise power as it is not part of the noise path. This is reflected in the BER simulation / measurement. This way, the BER could be improved by a factor of approximately 50 compared to the case without any equalizer and is only slightly higher than the AWGN BER.

4 Channel II: 2-Tap Maximum Phase Channel

For the maximum phase channel, the second channel coefficient (previous symbol) is larger than the first (current symbol). This results in an eye diagram as illustrated in figure 4. Starting from the two possible

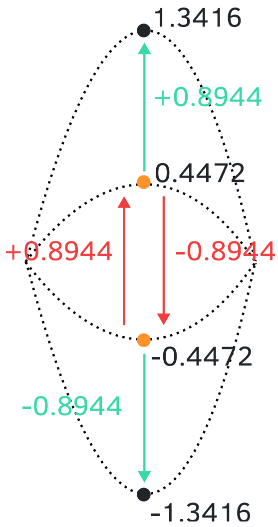

Figure 4: Illustration of how the max-phase eye comes about. The orange points are the starting positions. The red arrows show the ISI-influences that cause bit errors. $\frac{1}{\sqrt{5}} \approx 0.4472$, $\frac{2}{\sqrt{5}} \approx 0.8944$.

reduced positions ($1\,\mathrm{V} \cdot h_0 = 0.4472\,\mathrm{V}$), the influence of the previous symbol can be $1\,\mathrm{V} \cdot h_1 = +0.8944\,\mathrm{V}$ or $-1\,\mathrm{V} \cdot h_1 = -0.8944\,\mathrm{V}$. Two problematic situations arise:

1. After a symbol with an amplitude of $-1\,\mathrm{V}$, a symbol of amplitude $1\,\mathrm{V}$ is sent
2. After a symbol with an amplitude of $1\,\mathrm{V}$, a symbol of amplitude $-1\,\mathrm{V}$ is sent

The first situation results in a received amplitude of

$$\underbrace{1\,\mathrm{V} \cdot 0.4472}_{\text{current symbol}} + \underbrace{-1\,\mathrm{V} \cdot 0.8944}_{\text{previous symbol}} = -0.4472\,\mathrm{V}$$

even though a symbol with an amplitude of $1\,\mathrm{V}$ has been sent. Similarly, the second situation results in

$$\underbrace{-1\,\mathrm{V} \cdot 0.4472}_{\text{current symbol}} + \underbrace{1\,\mathrm{V} \cdot 0.8944}_{\text{previous symbol}} = 0.4472\,\mathrm{V}$$

even though a symbol with an amplitude of $-1\,\mathrm{V}$ has been sent. So for these two situations, errors arise because the symbol amplitudes moved over the detector threshold. Assuming a uniformly distributed set of sent symbols, these error inducing transitions occur for $50\,\%$ of all sent symbols. So a BER of 0.5 is to be expected. The BER measurement from the simulation confirms this.

Using equalization does not help in this case. The BER stays at 0.5. Whether post- or pre-coding is used does not matter, as 0.5 is already the worst case. One explanation why this channel type is not equalizable can be seen by looking at the eye diagram. The eye diagram of this channel actually looks identical to the one from the minimum phase channel (see Figure 2). This means that from the receiver's point of view, it is impossible to distinguish between these two situations to develop an equalizer using the inverse-method. One way to solve this problem is to introduce additional delay at the receiver side.

5 Channel II: 3-Tap Mixed Phase Channel

The channel mechanism is similar to the previous one. The additional influence of the symbol previous to the previous one leads the eye to have additional discrete values, see Figure 5. In this case, the eye

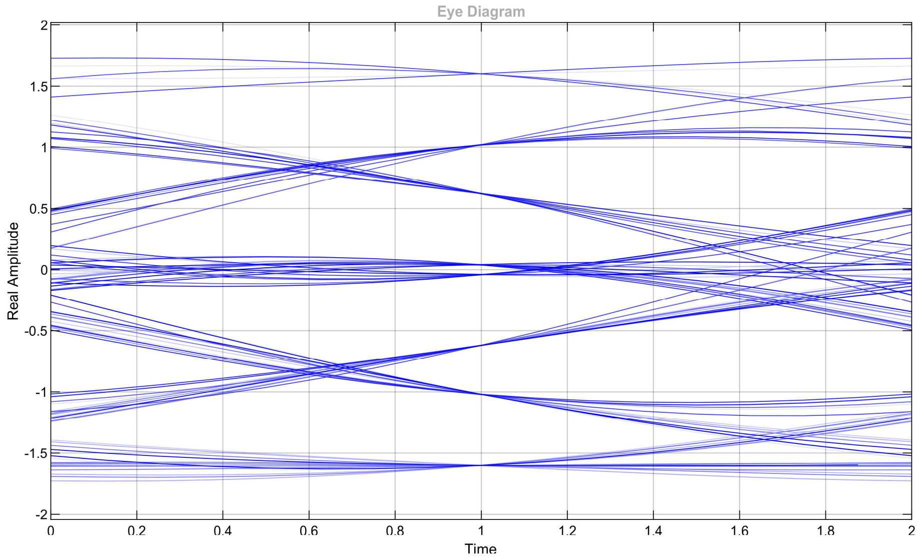

Figure 5: Eye of two-tap mixed phase channel system (no noise).

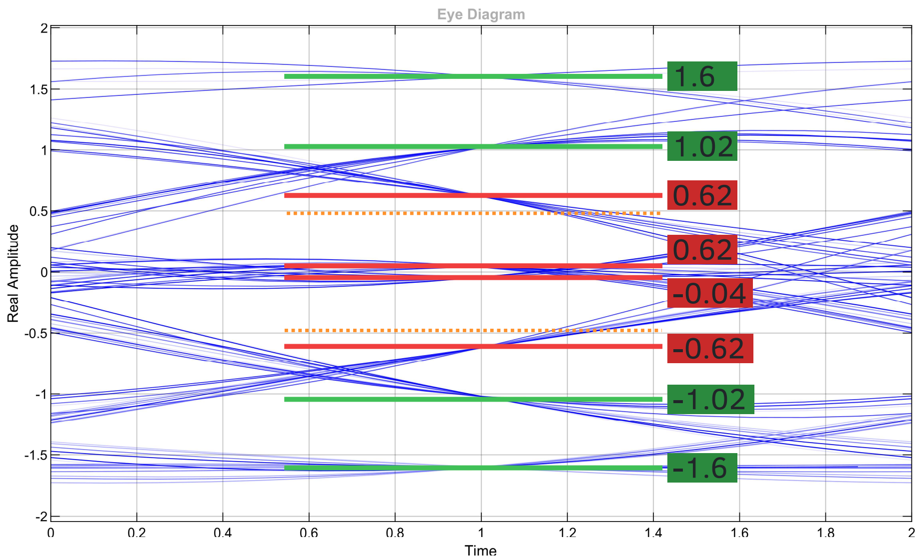

Figure 6: Annotated eye of mixed phase channel. Values that are erroneous are colored in red.

opening is really small. Figure 6 shows the possible symbol amplitude levels. Again, 50 % of them are erroneous, leading to a BER of 0.5.

Again, the inverse / zero-forcing strategy of equalization is insufficient in this case and another solution would have to be found.

The best vector of the nyquist conditions that minimizes the sum of the squared differences of the filter coefficients to itself is

$$z_1 = [0, 1, 0, 0, 0, 0, 0]$$

while the best vector that minimizes the simulated BER is

$$z_2 = [1, 0, 0, 0, 0, 0, 0]$$

with equalizer coefficients

$$f_2 = [1.2948, -1.5365, 1.2833, -0.8282, 0.3486]$$

which results in a BER of 0.25.

Channel	Equalisation	U_a/V	U_a^2/V^2	P_N/V	ρ	BERcalc	BERSim	Nyquist Vector	EQ Coefficients
AWGN	-	1	1	0.1	10	7.8E-04	8.1E-04	-	-
[2/sqrt(5), 1/sqrt(5)]	none	0.45	0.2025	0.1	2.025	7.7E-02	4.0E-02	-	-
	post	0.96	0.9216	0.17	5.421	9.9E-03	7.3E-03	[1 0 0 0 0 0 0]	[1.1172, -0.5570, 0.2752, -0.1311, 0.0524]
	pre	0.96	0.9216	0.1	9.216	1.2E-03	8.5E-04	[1 0 0 0 0 0 0]	[1.1172, -0.5570, 0.2752, -0.1311, 0.0524]
[1/sqrt(5), 2/sqrt(5)]	none	0.44	0.1936	0.1	1.936	8.2E-02	4.6E-01	-	-
	post	0.96	0.9216	0.17	5.421	9.9E-03	5.0E-01	[0 0 0 0 0 1 0]	[0.0524 -0.1311 0.2752 -0.5570 1.1172]
	pre	0.96	0.9216	0.1	9.216	1.2E-03	5.0E-01	[0 0 0 0 0 1 0]	[0.0524 -0.1311 0.2752 -0.5570 1.1172]
[0.49 0.82 0.29]	none	0.044	0.001936	0.1	0.019	4.4E-01	3.8E-01	-	-
	post	0.44	0.1936	0.6	0.323	2.9E-01	5.4E-01	[0 0 0 0 1 0 0]	[-0.2779 0.7073 -1.2342 1.8458 -0.5170]

Table 1: Simulation results.