Logarithms

if

$$b^x = N$$

then

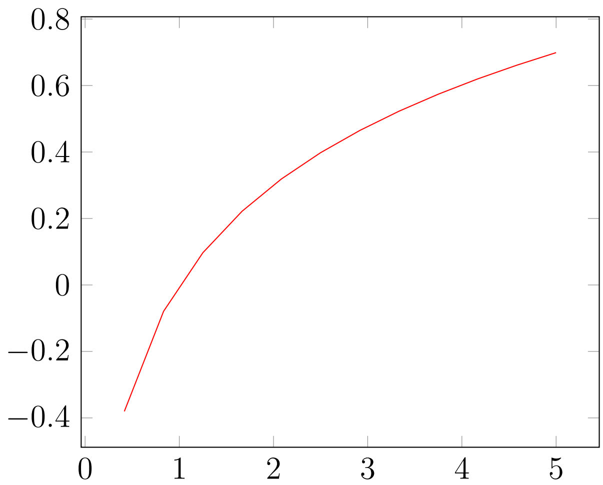
$$x = \log_b N$$

Logarithms return exponents.

$$\log_{10} 100 = 2$$

because $10^2 = 100$

$$\begin{aligned}
\log 0 &= \text{undefined}(\rightarrow -\infty) \\
\log a &= \text{undefined}; a < 1 \\
\log 1 &= 0 \\
\log (a \cdot b &= \log(a) + \log(b) \\
\log \frac{a}{b} &= \log(a) - \log(b) \\
\log a^b &= b \cdot \log(a)
\end{aligned}$$



Logarithms allow for a graphical representation of both very small and very large values in a plot. Using a linearily scaled x- (or y-)axis results in fixed Δx (Δy) which also results in a fixed physical distance (i.e. on a piece of paper or a screen) between each axis ticks. So as the range of values increases, the space needed for the plot increases linearily.

With logarithmic scales, each tick is one power of 10 greater than its predecessor, i.e. $10^1 = 10, 10^2 = 100, 10^3 = 1000$.

When choosing to plot raw data, say i.e. the datum 500, with a log-scale of i.e. base 10, one has to take the log of each datum to find the distance of the point from 0.

$$\log 500 \approx 2.7$$

which means that the datum 500 is 2.7 units from the 0 point. This is also where the name 'logarithmic' scale comes from. You get the exponent of 10 on the axis you want to plot the value on.

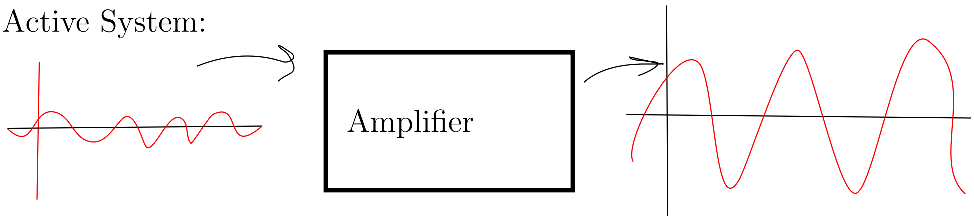
Gain and dB

Systems are divided into active and passive.

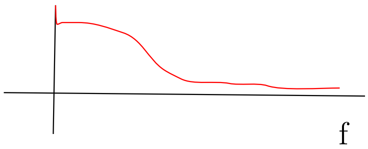
Active systems produce an output that is *greater* than their input. They require external supply for amplification

Passive systems produce an output that is *less* than their input

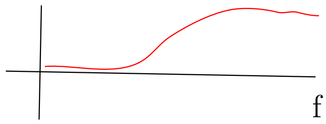
Active System:



Passive System:



low pass



high pass

The Gain of active and passive Systems make use of logarithmic units.

The standard (logarithmic) unit for gain is the Bel.

$$\text{GAIN/Bel} = \log \frac{P_{out}}{P_{in}}$$

$$10\text{dB} = 1Bel$$

so

$$\text{GAIN/deciBel} = 10 \cdot \log \frac{P_{out}}{P_{in}}$$

It sometimes makes more sense to use voltage or current in order to calculate gain, so express power accordingly:

$$\text{GAIN/dB} = 10 \cdot \log \frac{\frac{V_1^2}{R}}{\frac{V_2^2}{R}} = 20 \cdot \log \frac{V_1}{V_2}$$

that is if we assume Resistance to be the same!
Examples:

$$V_{in} = 5V, V_{out} = 12V, V_{out}/V_{in} = 2.4$$

$$\rightarrow 20 \log \frac{12V}{5V} = 7.6dB$$

$$V_{in} = 0.005V, V_{out} = 12V, V_{out}/V_{in} = 2400$$
$$\rightarrow 20 \log \frac{12V}{0.005V} = 67.6dB$$

Relation to Active/Passive Systems:

Passive Systems: Output < Input → - GAIN
Active Systems: Output > Input → + GAIN

For filters, the critical frequency describes the situation where output Power is half of input Power

$$V_{out} = V_{in}/\sqrt{2} \rightarrow 1/2 \cdot P_{max}, GAIN : -3dB$$

$$V_{out} = V_{in}/10 \rightarrow 1/100 \cdot P_{max}, GAIN : -20dB$$