Grundlagen der Elektrotechnik II

Schwingkreise

Studien- und Versuchsaufgaben

Autor: Richard Grünert

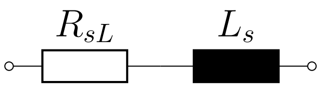
17.6.2019

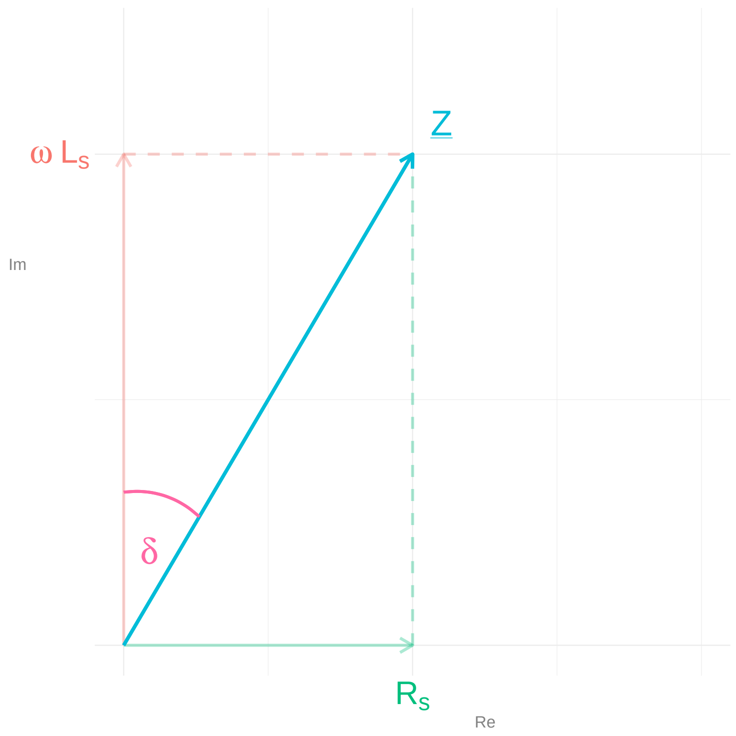
1 | Vorbereitungsaufgaben

1.1

Spule

Reihenmodell





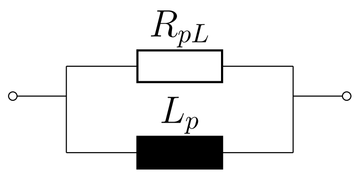
Der Verlustwinkel δ ist der Winkel der Spulenimpedanz $\underline{Z}$ mit der imaginären Achse der gaußschen Zahlenebene. $\tan \delta$ wird auch Verlustfaktor d genannt.

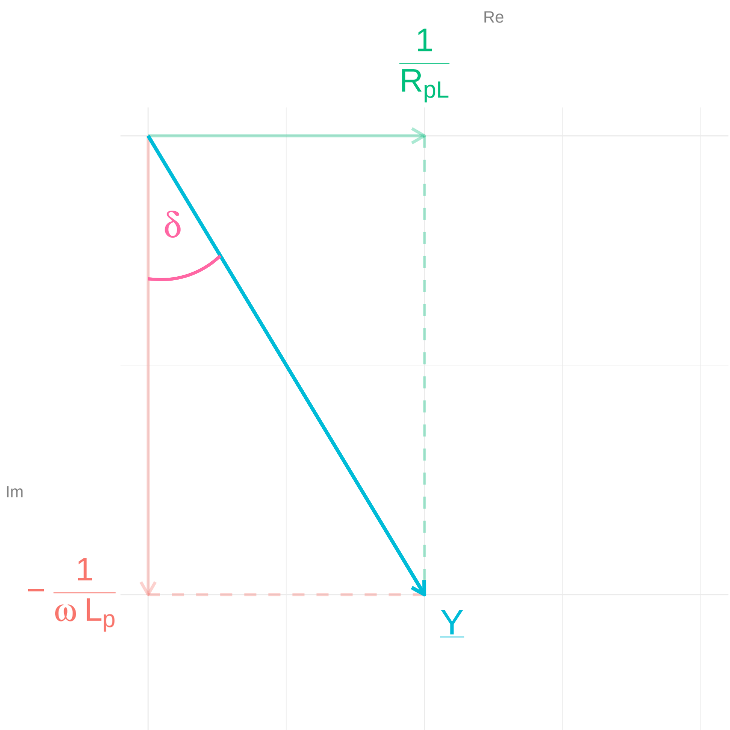
$$\tan \delta = \frac{\omega L_s}{R_{sL}}$$

Die Güte Q der realen Induktivität ist demnach als Kehrwert des Verlustfaktors definiert:

$$Q_{Ls} = \frac{1}{\tan\delta} = \frac{R_{sL}}{\omega L_s}$$

Parallelmodell





Der Verlustwinkel δ ist der Winkel der Spulenadmittanz $\underline{Y}$ mit der imaginären Achse der gaußschen Zahlenebene (Betrag).

$$| \tan\delta |= \frac{\frac{1}{R_{pL}}}{\frac{1}{\omega L_p}} = \frac{\omega L_p}{R_{pL}}$$

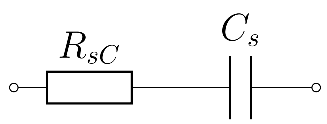
Kehrwert des Verlustfaktors:

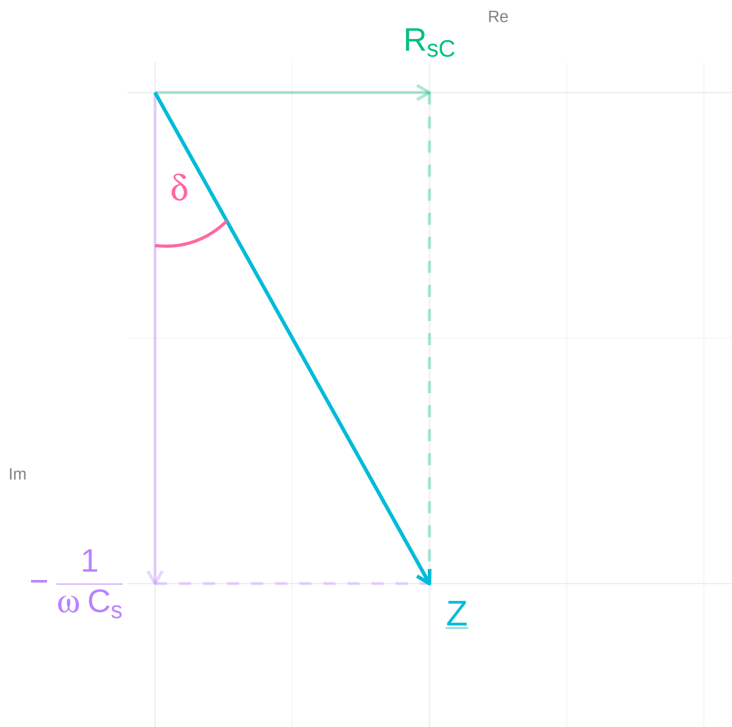
$$Q_{Lp} = \frac{1}{\tan\delta} = \frac{R_{pL}}{\omega L_p}$$

$$Q_{Lp} = Q_{Ls} = \frac{R_{pL}}{\omega L_p} = \frac{\omega L_s}{R_{sL}}$$

Kondensator

Reihenmodell





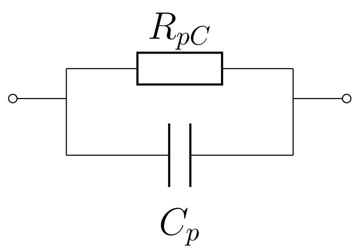
Der Verlustwinkel δ ist der Winkel der Kondensatorimpedanz $\underline{Z}$ mit der imaginären Achse der gaußschen Zahlenebene (Betrag).

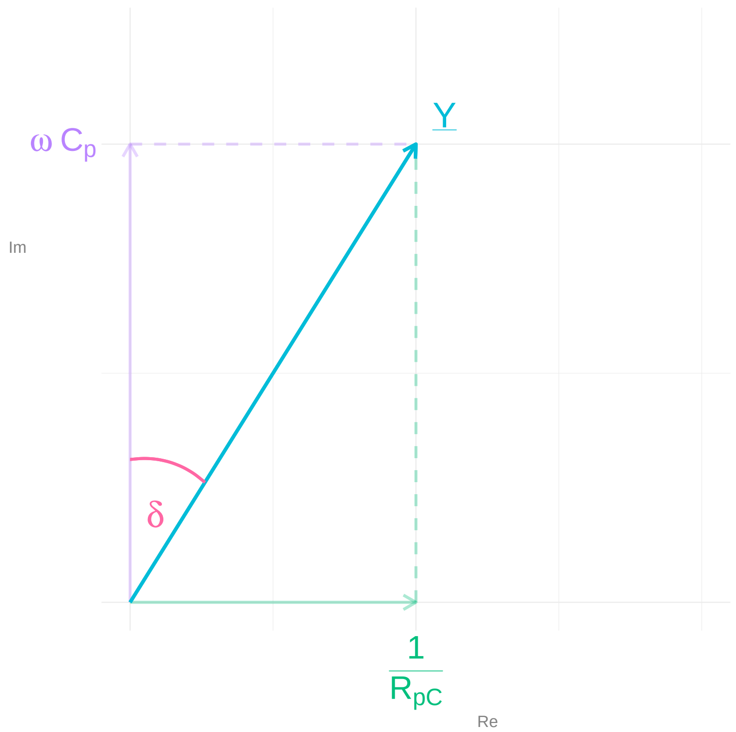
$$|\tan\delta| = \frac{R_{sC}}{\frac{1}{\omega C_s}} = R_{sC} \cdot \omega C_s$$

Somit ist die Kondensatorgüte des Reihenmodells:

$$Q_{Cs} = \frac{1}{\tan\delta} = \frac{1}{R_{sC} \cdot \omega C_s}$$

Parallelmodell





Der Verlustwinkel δ ist der Winkel der Kondensatoradmittanz $\underline{Y}$ mit der imaginären Achse der gaußschen Zahlenebene.

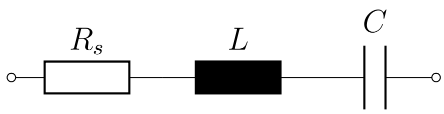
$$\tan\delta = \frac{\frac{1}{R_{pC}}}{\omega C_p} = \frac{1}{R_{pC}\cdot\omega C_p}$$

Kehrwert des Verlustfaktors:

$$Q_{Cp} = \frac{1}{\tan\delta} = R_{pC}\cdot\omega C_p$$

$$Q_{Cs} = Q_{Cp} = \frac{1}{R_{sC}\cdot\omega C_s} = R_{pC}\cdot\omega C_p$$

1.2



Gleichung (4):

$$\begin{aligned}\underline{Z} &= R_s + j(\omega L - \frac{1}{\omega C}) \qquad (4') \\ &= R_s + jX \\ &= \mid \underline{Z} \mid \cdot e^{j\phi_{\underline{Z}}}\end{aligned}$$

Gleichung (5):

$$\begin{aligned}\phi_{\underline{Z}} = \arg(\underline{Z}) &= \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R_s}\right) \qquad (5') \\ &= \arctan\left(\frac{X}{R_s}\right)\end{aligned}$$

Gleichung (6):

$$\mid \underline{Z} \mid = \sqrt{R_s^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{6´}$$

Gleichung (7):

$$\omega_0 = \frac{1}{\sqrt{LC}} \implies \omega_0 = \frac{1}{\sqrt{LC}} \tag{7´}$$

Gleichung (8):

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \tag{8´}$$

Gleichung (9):

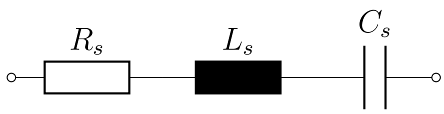
$$\begin{aligned} \mid \underline{I} \mid &= \frac{\mid \underline{U} \mid}{\mid \underline{Z} \mid} \qquad (9´) \\ &= \frac{\mid \underline{U} \mid}{\sqrt{R_s^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \end{aligned}$$

Gleichung (10):

$$\underline{I} = \underline{U} \cdot G_s = \frac{\underline{U}}{R_s} \tag{10´}$$

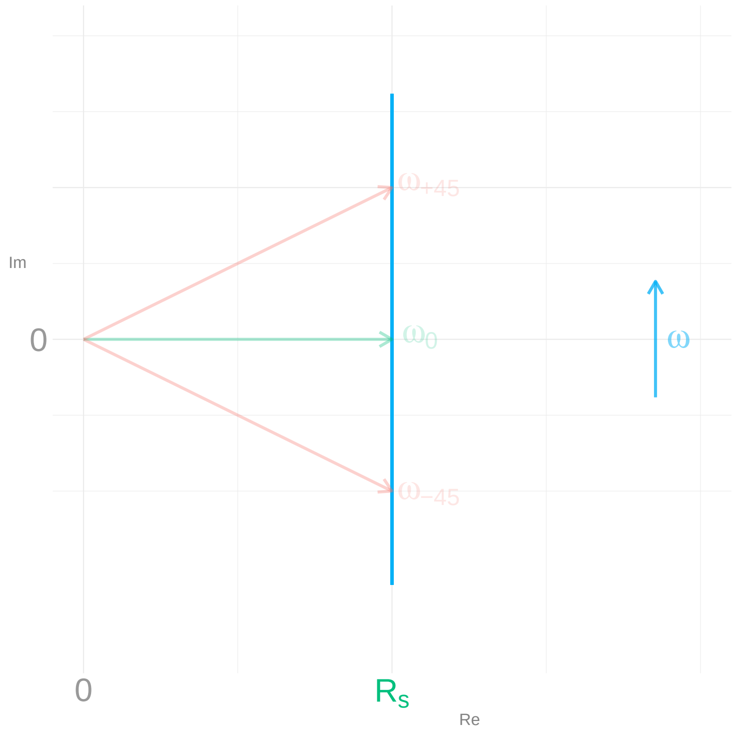
1.3

Reihenschwingkreis



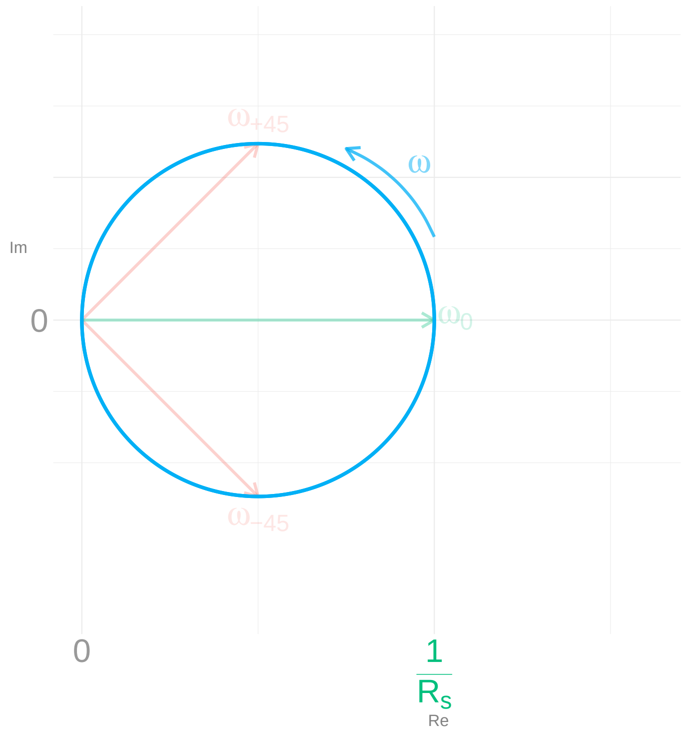
Impedanzortskurve

$$\underline{Z} = R_s + j(\omega L_s - \frac{1}{\omega C_s})$$

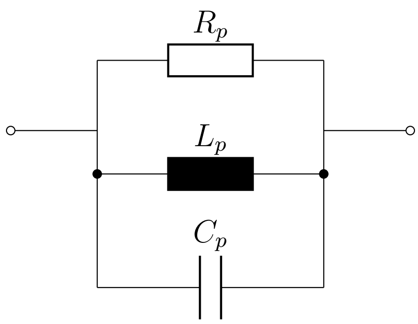


Admittanzortskurve

$$\underline{Y} = \frac{1}{R_s + j(\omega L_s - \frac{1}{\omega C_s})}$$

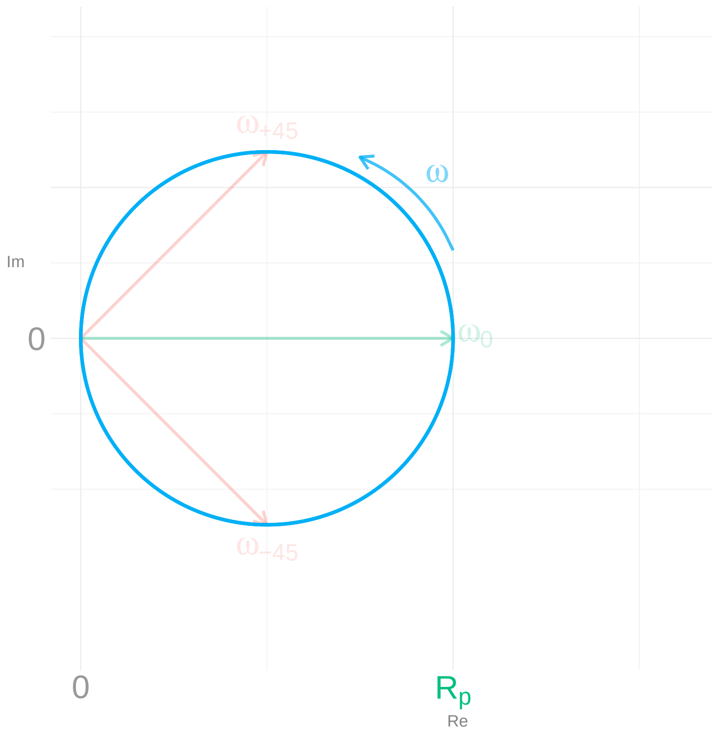


Parallelschwingkreis



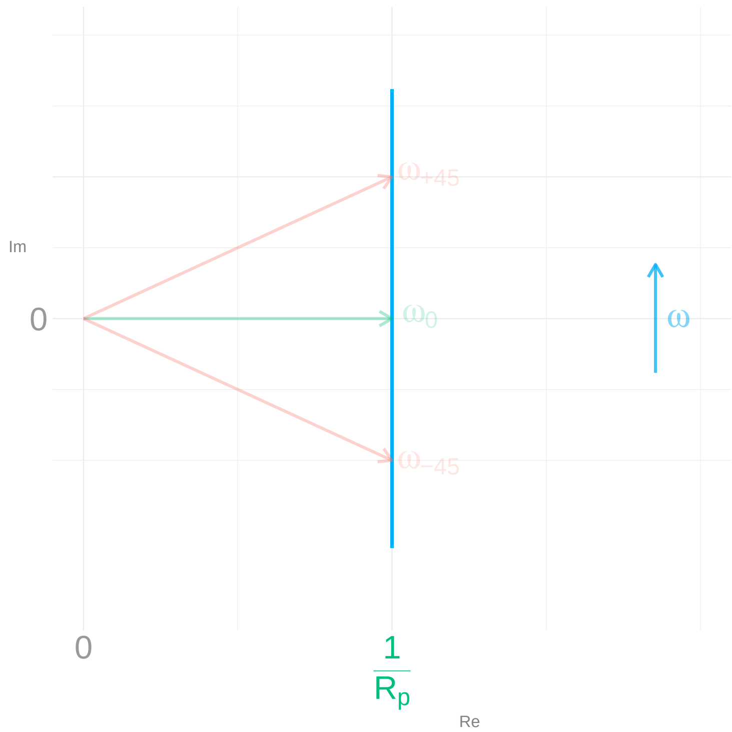
Impedanzortskurve

$$\underline{Z} = \frac{1}{\frac{1}{R_p} + j(\omega C_p - \frac{1}{\omega L_p})}$$

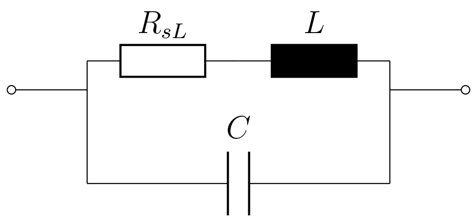


Admittanzortskurve

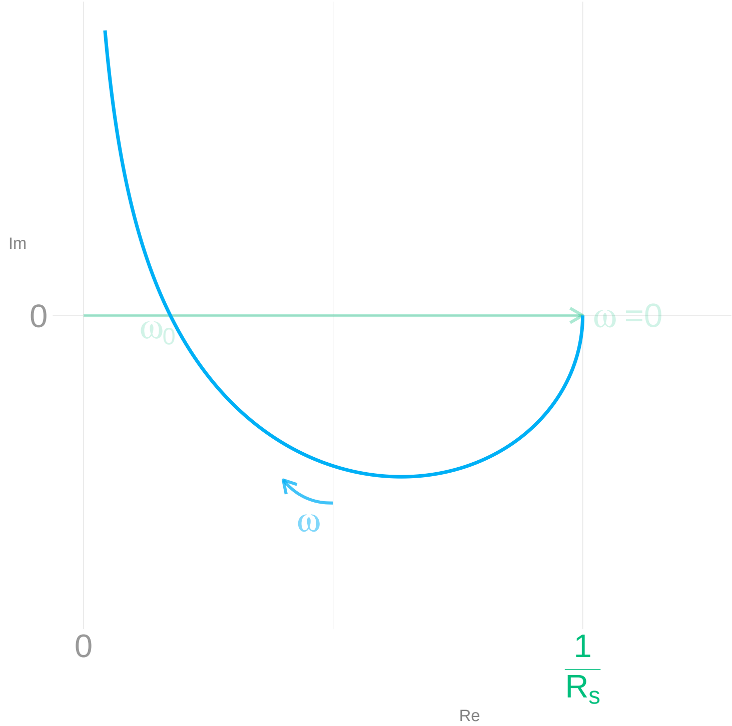
$$\underline{Y} = \frac{1}{R_p} + j(\omega C_p - \frac{1}{\omega L_p})$$



1.4



$$
\begin{aligned}
\underline{Y} &= \frac{1}{R_{sL} + j\omega L} + j\omega C \\
&= \frac{R_{sL}}{R_{sL}^2 + \omega^2 L^2} + j\omega \left(C - \frac{L}{R_{sL}^2 + \omega^2 L^2} \right)
\end{aligned}
$$



Resonanzfrequenz:

$$\mathrm{Im}(\underline{Y}) = 0$$

$$\omega_0^{'} C - \frac{\omega_0^{'} L}{R_{sL}^2 + \omega_0^{'2} + L_s^2} = 0$$

$$R_{sL}^2 \cdot C + \omega_0^{'2} + L^2 \cdot C^2 = L$$

$$\omega_0^{'} = \sqrt{\underbrace{\frac{1}{LC}}_{\omega_0^2} - \frac{R_{sL}^2}{L^2}}$$

$$\boxed{\omega_0^{'} = \omega_0 \sqrt{1 - \frac{R_{sL}}{\omega_0^2 \cdot L^2}} = \omega_0 \sqrt{1 - \frac{1}{Q_L^2}}}$$

1.5

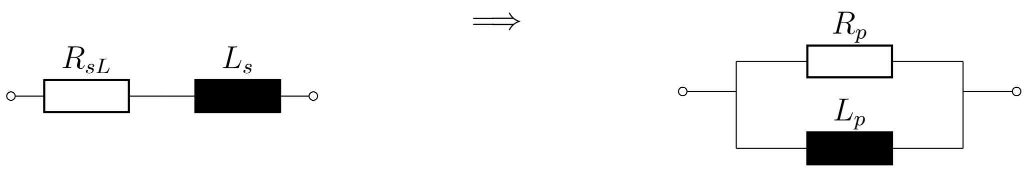
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1.3\text{H} \cdot 22.5\text{nF}}} = 5834.1 \ \text{s}^{-1}$$

$$\omega_0^{'} = \omega_0 \cdot \sqrt{1 - \frac{R_{sL^2}}{\omega_0^2 \cdot L^2}} = 5834.1\text{s}^{-1} \cdot \sqrt{1 - \frac{(100\Omega)^2}{(5834.1\text{s}^{-1})^2 \cdot (1.3\text{H})^2}}$$

$$\omega_0^{'} = 5833.6 \ \text{s}^{-1}$$

$$\implies \text{ geringe Differenz } \implies \text{ hohe Spulengüte}$$

Zur Berechnung von Güte und Bandbreite wird der Schwingkreis in einen idealen Parallelschwingkreis umgewandelt:



$$\underline{Y}_s = \underline{Y}_p$$

$$\frac{1}{R_s + j\omega_0 L_s} = \frac{1}{R_p} - j\frac{1}{\omega_0 L_p}$$

$$\frac{R_s}{R_s^2 + \omega_0^2 L_s^2} - j\frac{\omega_0 L_s}{R_s^2 L_s + \omega_0^2 L_s} = \frac{1}{R_p} - j\frac{1}{\omega_0 L_p}$$

Realteilvergleich:

$$R_p = 1 + \frac{\omega_0^2 L_s^2}{R_s^2} = 1 + Q_L^2$$

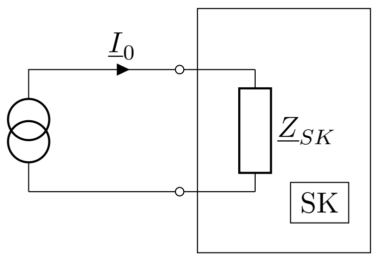
Imaginärteilvergleich:

$$L_p = L_s \left(1 + \frac{R_s^2}{\omega_0^2 L_s^2}\right) = 1 + \frac{1}{Q_L^2}$$

$$R_p = 575.3\ \text{k}\Omega,\ L_p = 1.30023\ \text{H} \approx L_s$$

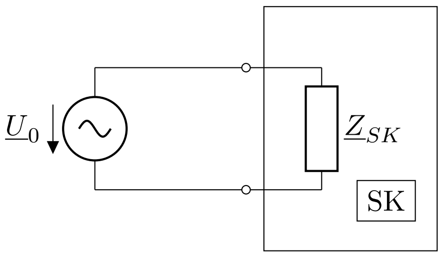
1.6

Stromspeisung



$$\underline{U} = \underline{Z}_{SK} \cdot \underline{I}_0, \quad \underline{I}_0 = \text{konst.} \implies \underline{U} \sim \underline{Z}_{SK}$$

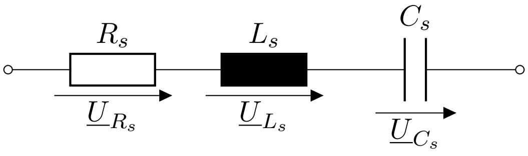
Spannungsspeisung



$$\underline{I} = \underline{U} \cdot \underline{Y}_{SK}, \quad \underline{U}_0 = \text{konst.} \implies \underline{I} \sim \underline{Y}_{SK}$$

1.7

Spannungsüberhöhung
(Nur im Reihenschwingkreis)



R_s:

$$\underline{U}_{R_s} = \frac{\underline{U} \cdot R_s}{R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)}$$

L_s:

$$\underline{U}_{L_s} = \frac{\underline{U} \cdot j\omega L_s}{R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)}$$

C_s:

$$\underline{U}_{C_s} = \frac{\underline{U}}{j\omega C_s\left(R_s + j\left(\omega L_s - \frac{1}{\omega C_s}\right)\right)}$$

bei $\omega = \omega_0$:

$$\underline{U}_{R_s} = \frac{\underline{U} \cdot R_s}{R_s} = \underline{U}$$

$$\underline{U}_{L_s} = j\underline{U} \cdot \frac{\omega_0 L_s}{R_s} = j\underline{U} \cdot Q_L$$

$$\underline{U}_{C_s} = -j\underline{U} \cdot \frac{1}{\omega_0 C_s R_s}$$

Beträge bei ω_0:

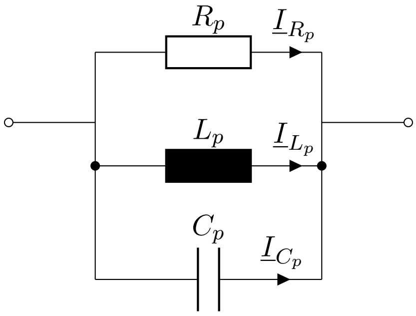
$$\hat{U}_{R_s} = \hat{U}$$

$$\hat{U}_{L_s} = Q_L \cdot \underline{U}$$

$$\hat{U}_{C_s} = Q_C \cdot \underline{U}$$

Stromüberhöhung

(Nur im Parallelschwingkreis)



R_p:

$$\underline{I}_{R_p} = \frac{\underline{I} \cdot \frac{1}{R_p}}{\frac{1}{R_p} + j\left(\omega C_p - \frac{1}{\omega L_p}\right)}$$

L_p:

$$\underline{I}_{L_p} = \frac{\underline{I}}{j\omega L_p \left(\frac{1}{R_p} + j\left(\omega C_p - \frac{1}{\omega L_p}\right)\right)}$$

C_p:

$$\underline{I}_{C_p} = \frac{\underline{I} \cdot j\omega C_p}{\frac{1}{R_p} + j\left(\omega C_p - \frac{1}{\omega L_p}\right)}$$

bei $\omega = \omega_0$:

$$\underline{I}_{R_p} = \underline{I}$$

$$\underline{I}_{L_p} = j\underline{I} \cdot \frac{R_p}{\omega_0 L_p} = j\underline{I} \cdot Q_L$$

$$\underline{I}_{C_p} = -j\underline{I} \cdot \omega_0 R_p C_p = j\underline{I} \cdot Q_C$$

Beträge bei ω_0:

$$\hat{I}_{R_p} = \hat{I}$$

$$\hat{I}_{L_p} = Q_L \cdot \underline{I}$$

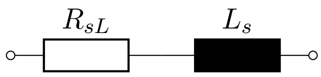
$$\hat{I}_{C_p} = Q_C \cdot \underline{I}$$

Die Strom- bzw. Spannungsüberhöhungen betragen demnach bei Resonanz das $Q_{L/C}$-fache der Quellgrößen.

1.8

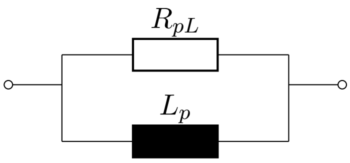
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1\text{H} \cdot 100\text{nF}}} = 10000 \ \text{s}^{-1} \implies f_0 = 1.592 \ \text{kHz}$$

Spule



$$Q_L = \frac{\omega_0 L_s}{R_{sL}}$$

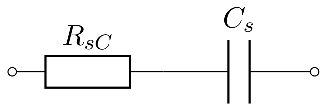
$$R_{sL} = \frac{\omega_0 L_s}{Q_L} = 20 \ \ \Omega$$



$$Q_L = \frac{R_{pL}}{\omega_0 L_p}$$

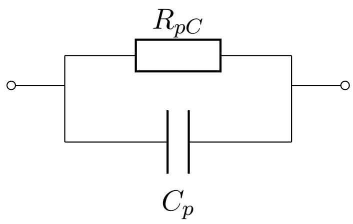
$$R_{pC} = Q_L \cdot \omega_0 L_p = 50 \ \ \mathrm{k}\Omega$$

Kondensator



$$Q_C = \frac{1}{\omega_0 R_{sC} C_s}$$

$$R_{sC} = \frac{1}{\omega_0 Q_C C_s} = 3.3 \ \ \Omega$$



$$Q_C = \omega_0 R_{pC} C_p$$

$$R_{pC} = \frac{Q_C}{\omega_0 C_p} = 300 \ \ \mathrm{k}\Omega$$

$$Q_{SK} = \frac{1}{\frac{1}{Q_c} + \frac{1}{Q_L}} = \frac{Q_C \cdot Q_L}{Q_C + Q_L}$$

$$Q_{SK} = \frac{300}{7} \approx 42.86$$

$$B_\omega = \frac{\omega_0}{Q_{SK}} = \frac{10000\text{s}^{-1}}{42.86} = 232.3 \;\; \text{s}^{-1}$$

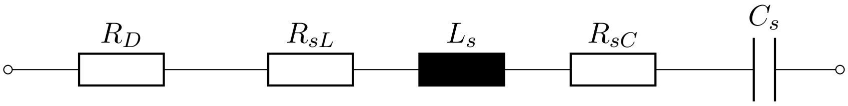
$$B_f = \frac{223.2\text{s}^{-1}}{2\pi} = 37.14 \;\; \text{Hz}$$

Messfrequenz:

$$f_0 = 1.59 \;\; \text{kHz} \implies \text{geringere Güte} \implies B_f = 116.6 \;\; \text{Hz}$$

$$Q_{\text{mess}} = \frac{f_0}{B_f} = 13.64$$

Modell:



$$B_\omega = \frac{R_{\text{res.}}}{L_s} \implies R_{\text{res.}} = 2\pi \cdot B_f \cdot L_s = 73.26 \;\; \Omega$$

$$R_D = R_{\text{res.}} - R_{sC} - R_{sL} = 49.9 \;\; \Omega \approx 50 \;\; \Omega$$

1.9

Anwendungen von Schwingkreisen:

- Filter
- Blindstromkompensation

2 Versuchsaufgaben

2.1

C	R_{pC}	R_{sC}	Q_C	ϕ_C
995.97 nF	4.7278 kΩ	213.42 mΩ	153.374233	89.624°

Werte des Kondensators

L	R_{sL}	R_{pL}	Q_L	ϕ_L
1.0263 mH	1.0466 Ω	994.19 Ω	30.838	88.14°

Werte der Spule

Reihenschwingkreis:

$$\omega_0 = 31278.1 \ \mathrm{s}^{-1}$$

$$Q_{RSK} = \frac{153.37 \cdot 30.838}{153.37 + 30.838} = 25.74$$

$$B_\omega = \frac{31278.1\mathrm{s}^{-1}}{25.74} = 1215.15 \ \mathrm{s}^{-1}$$

Parallelschwingkreis:

$$\omega_0 = 31278.1 \ \mathrm{s}^{-1}$$

$$Q_{PSK} = \frac{153.37 \cdot 30.838}{153.37 + 30.838} = 25.74$$

$$B_\omega = \frac{31278.1\mathrm{s}^{-1}}{25.74} = 1215.15 \ \mathrm{s}^{-1}$$

2.2

(a)

Für die Resonanzfrequenz wurde $f_0 = 5000$ Hz ermittelt. Die Grenzfrequenzen wurden zu $f_{+45} = 5200$ Hz und $f_{-45} = 4830$ Hz messtechnisch ermittelt. Zur Konstanthaltung des Stromes wurde die Spannung über $R_V = 1\ \Omega$ auf 1 V eingestellt und gemessen.

f in Hz	U_V in V	U in V	ϕ in °
2500	1.0076	0.0215	83.4
3000	1.0078	0.02965	82
3500	1.0075	0.004296	79.9
4000	1.0212	0.06958	75.3
4500	1.0572	0.13935	63.3
4830	1.2505	0.3148	33.8
5000	1.447	0.4452	0
5200	1.2432	0.3148	-36.5
5500	1.0711	0.16457	-61.79
6000	1.0205	0.0878	-75.2
6500	1.0132	0.06084	-80.1
7000	1.0062	0.04674	-82.2
7500	1.0061	0.0384	-84.3

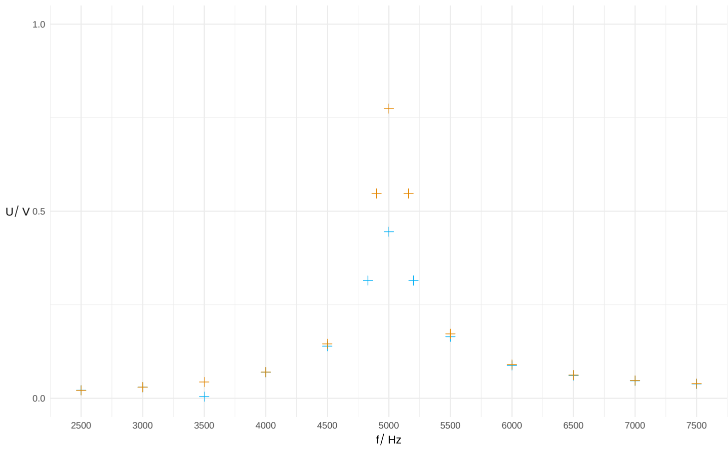
Messwerte aus (a)

(b)

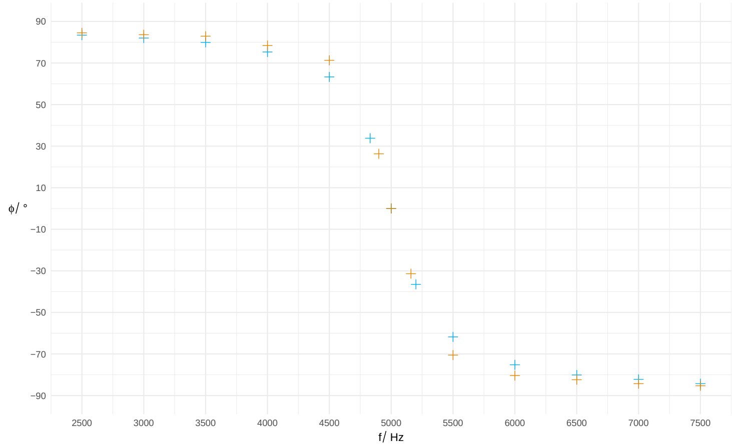
f in Hz	U_V in V	U in V	ϕ in °
2500	1.0074	0.02124	84.5
3000	1.0073	0.02974	83.6
3500	1.0073	0.04356	82.9
4000	1.0142	0.07003	78.4
4500	1.043	0.14542	71.3
4900	1.4921	0.5474	26.3
5000	1.768	0.7742	0
5160	1.421	0.5474	-31.4
5500	1.0494	0.1722	-70.54
6000	1.0133	0.0903	-80.4
6500	1.0065	0.0621	-82.4
7000	1.0058	0.0475	-84.3
7500	1.0054	0.0391	-85.3

Messwerte aus (b)

(c)

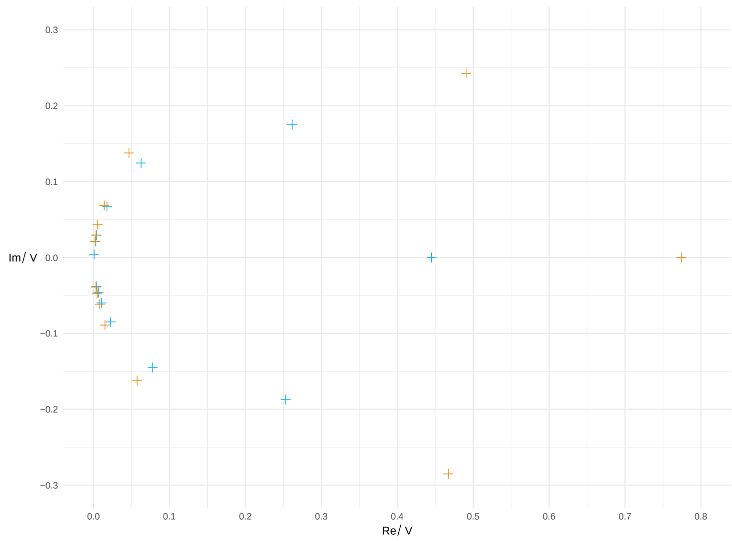


Spannungsverlauf. Blau: (a), orange: (b)



Phasenverlauf. Blau: (a), orange: (b)

(d)



Ortskurve der Spannung. Blau: (a), orange: (b)

2.3

Aus 3.1:

$$f_0 = 4978.04 \text{ Hz}$$
$$B_f = 193.397 \text{ Hz}$$
$$Q = 25.74$$

Aus 3.2:

$$f_0 = 5000 \text{ Hz}$$
$$B_f = (5200 - 4830)\text{Hz} = 370 \text{ Hz}$$
$$Q = 13.51$$

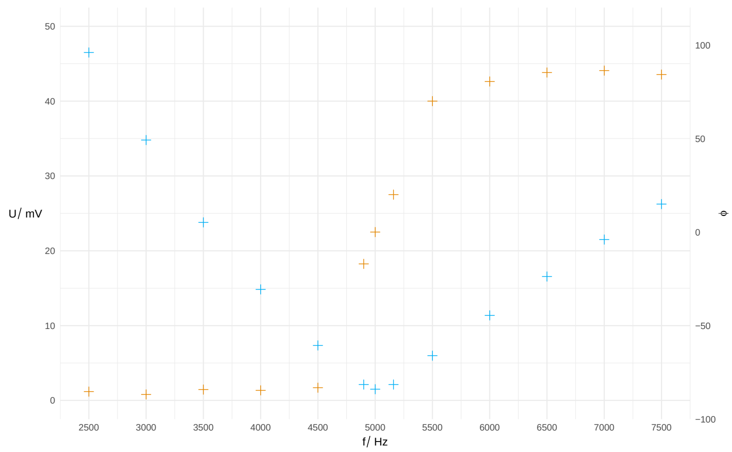
Die hohe Abweichung kam hier aufgrund der ungenauen Bestimmung der Resonanzfrequenz zustande.

2.4

Zur Konstanthaltung des Stroms wurde die Spannung über einem Widerstand von $R_V = 1 \text{ k}\Omega$ auf 1 V eingestellt und gemessen. Der Widerstand R_s des Serienschwingkreises wurde zudem kurzgeschlossen.

f in Hz	U_1 in V	U in mV	phi in °
2500	0.96	46.5	85.3
3000	1.008	34.8	86.8
3500	1.006	23.8	84.2
4000	1.006	14.84	84.6
4500	1.0064	7.35	83.2
4900	1.0064	2.121	17
5000	1.0063	1.5	0
5160	1.0064	2.121	-20
5500	1.0072	5.98	-70
6000	1.007	11.37	-80.5
6500	1.0072	16.56	-85.3
7000	1.0064	21.5	-86.3
7500	1.0064	26.24	-84.2

Messwerte der Aufgabe 3.4



Spannungsverlauf: blau, Phasenverlauf: orange

2.5

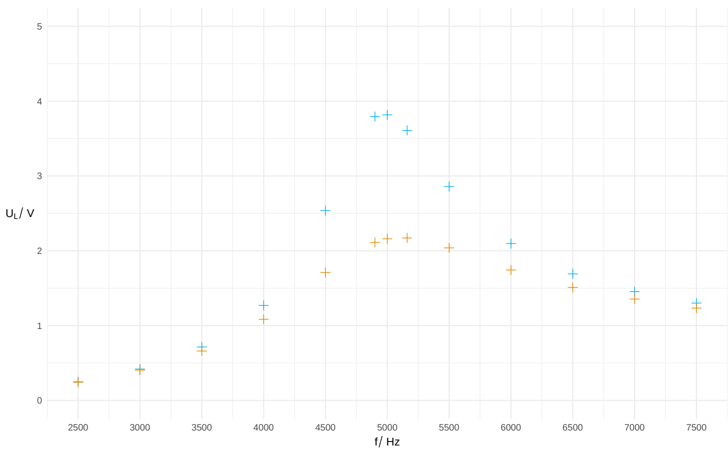
Die Spannung wurde hier konstant auf 750 mV gehalten. Die Werte der Reihenwiderstände R_s sind 5Ω und 10Ω.

f in Hz	U_L in V	U_C in V
2500	0.2485	0.9954
3000	0.4194	1.1611
3500	0.715	1.457
4000	1.27	1.968
4500	2.537	3.068
4900	3.794	3.851
5000	3.817	3.722
5160	3.609	3.309
5500	2.859	2.321
6000	2.097	1.444
6500	1.692	0.9916
7000	1.454	0.7376
7500	1.301	0.5774

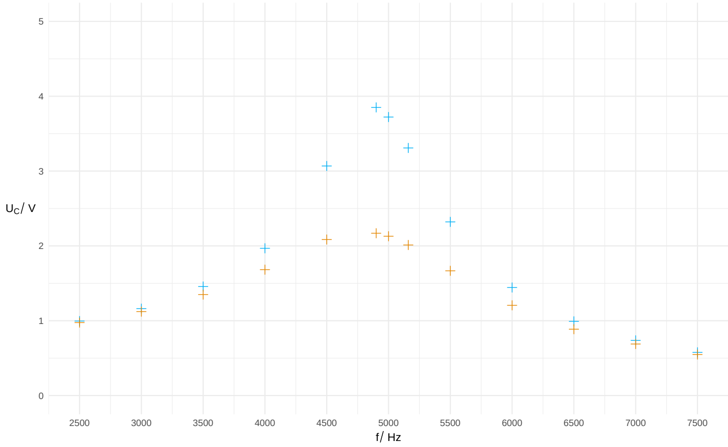
Messwerte für $R_s = 5\ \Omega$

f in Hz	U_L in V	U_C in V
2500	0.2432	0.9744
3000	0.4048	1.1211
3500	0.6607	1.349
4000	1.0844	1.682
4500	1.71	2.085
4900	2.113	2.169
5000	2.16	2.129
5160	2.172	2.012
5500	2.039	1.667
6000	1.743	1.206
6500	1.51	0.8865
7000	1.355	0.6886
7500	1.234	0.5481

Messwerte für $R_s = 10\ \Omega$



U_L. Blau: $R_s = 5\ \Omega$, orange: $R_s = 10\ \Omega$



U_C. Blau: $R_s = 5\ \Omega$, orange: $R_s = 10\ \Omega$