Hochschule Wismar
Fakultät für Ingenieurwissenschaften
Bereich Elektrotechnik und Informatik

Laborprotokoll

Studiengang: Informations- und Elektrotechnik, Ba...

Laborpraktikum: Grundlagen der Elektrotechnik II...

Versuchsbezeichnung: Frequenzanalyse periodischer Signale...

Datum: 28.05.2019..

Raum: Haus 16...

Versuchsbetreuer: Prof. Dr.-Ing. Ansgar Wego...

Teilnehmer:

Name, Vorname (Blockschrift)	Matrikelnummer	Versuchsgruppe
Grünert, Richard	289427	Gruppe 1

Protokollführer (Unterschrift): R. Grünert...

(Wird vom Betreuer ausgefüllt)
Abgabedatum: ....

Bemerkungen:
....
....
....

Bestätigung: ....

Grundlagen der Elektrotechnik II

Frequenzanalyse periodischer Signale

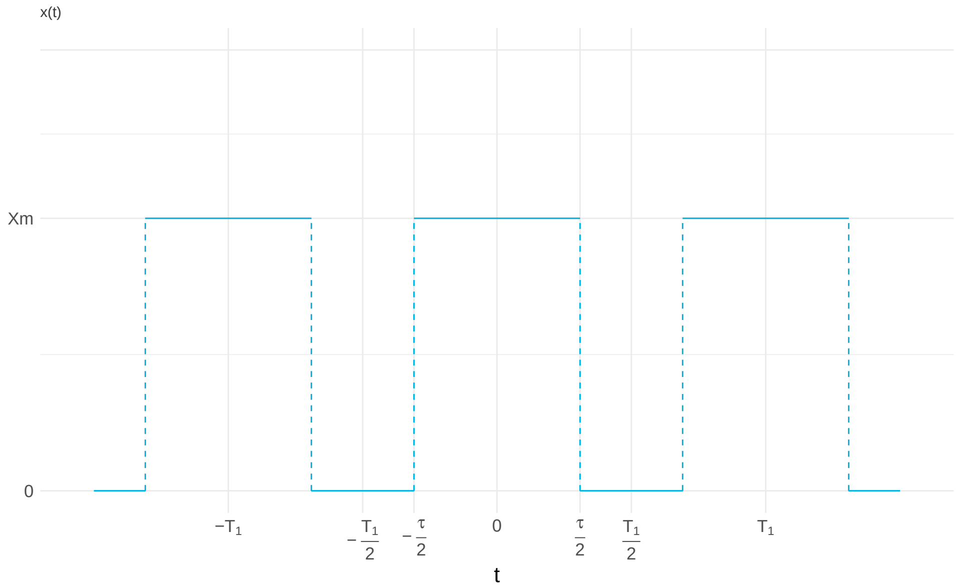
Studien- und Versuchsaufgaben

Autor: Richard Grünert

3.6.2019

1 Vorbereitungsaufgaben

1.1



$$
\begin{aligned}
\underline{X}_\nu &= \frac{1}{T_1} \cdot \int_{T_1} x(t) \cdot e^{-(j\nu \cdot \omega_1 t)}\,\mathrm{d}t \\
&= \frac{1}{T_1} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} X_m \cdot e^{-(j\nu \cdot \omega_1 t)}\,\mathrm{d}t \\
&= -\frac{X_m}{T_1 \cdot j\nu\omega_1} \cdot \left[e^{-(j\nu \cdot \omega_1 t)}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \\
&= -\frac{X_m}{T_1 \cdot j\nu\omega_1} \cdot \left(e^{-j\nu \cdot \omega_1 \frac{\tau}{2}} - e^{j\nu \cdot \omega_1 \frac{\tau}{2}}\right)
\end{aligned}
$$

$\omega_1 = \frac{2\pi}{T_1}$ und Erweiterung mit $\frac{-1}{-1}$:

$$
\begin{aligned}
\underline{X}_\nu &= \frac{X_m}{2j\pi\nu} \cdot \left(e^{j\nu \cdot \pi \frac{\tau}{T_1}} - e^{-j\nu \cdot \pi \frac{\tau}{T_1}}\right) \\
&= \frac{X_m}{\pi\nu} \cdot \frac{\left(e^{j\nu \cdot \pi \frac{\tau}{T_1}} - e^{-j\nu \cdot \pi \frac{\tau}{T_1}}\right)}{2j}
\end{aligned}
$$

$$\text{mit } \frac{\left(e^{jx} - e^{-jx}\right)}{2j} = \sin(x) \text{ und } \frac{\tau}{T_1} = D\text{:}$$

$$\underline{X}_\nu = \frac{X_m}{\pi\nu} \cdot \sin(\pi\nu D)$$

Erweitert man wieder mit $\frac{D}{D}$ erhält man das Bild einer Spaltfunktion $\mathrm{si}(x) = \frac{\sin x}{x}$:

$$\underline{X}_\nu = D \cdot X_m \cdot \frac{\sin(\pi\nu D)}{\pi\nu D} = D \cdot X_m \cdot \mathrm{si}(\pi\nu D)$$

Als reele Reihe:

$$x(t) = X_0 + \sum_{\nu=1}^{\infty} \hat{X}_\nu \cos(\nu \cdot \omega_1 t + \phi_\nu)$$

$$X_0 = \frac{1}{T_1} \cdot \int_{T_1} x(t)\,\mathrm{d}t = \frac{X_m}{2}$$

Aus der komplexen Reihendarstellung folgt

$$b_\nu = -2 \cdot \mathrm{Im}(\underline{X}_\nu) = 0$$

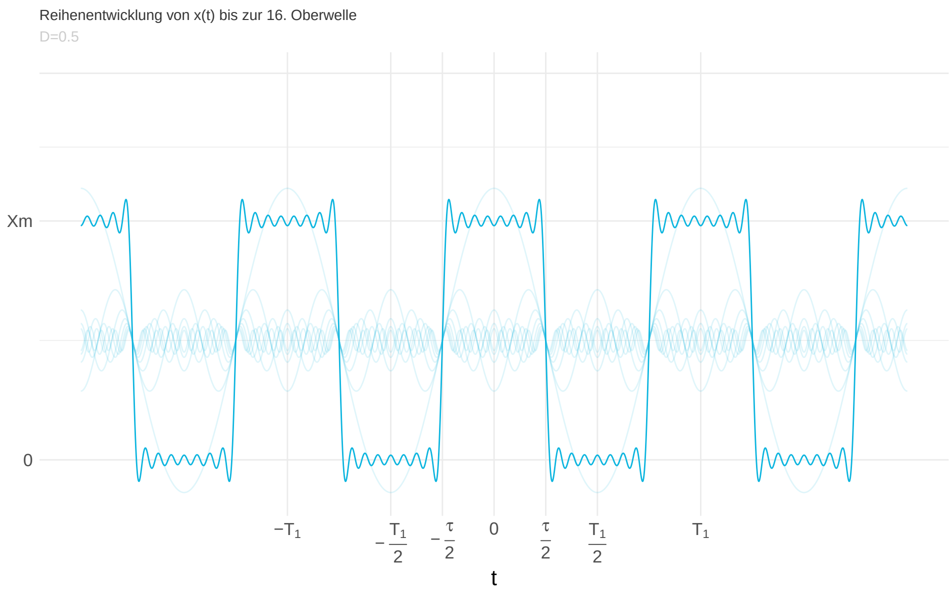
$$\hat{X}_\nu = \sqrt{a_\nu^2 + b_\nu^2} = 2 \cdot \mid \underline{X}_\nu \mid \Longrightarrow a_\nu = 2 \cdot \mid D \cdot X_m \cdot \mathrm{si}(\nu\pi D) \mid$$

ϕ_ν hängt nur vom Wert von $\mathrm{si}(\nu\pi D)$ ab, da $\underline{X}_\nu$ rein reell ist:

$$\phi_\nu = \begin{cases} 0 & ; \nu = \frac{4k+1}{2D} \\ \pi & ; \nu = \frac{4k-1}{2D} \\ \text{n.d.} & ; \text{sonst} \end{cases}$$

Somit ist

$$x(t) = \frac{X_m}{2} + \sum_{\nu=1}^{\infty} 2DX_m \cdot \mid \mathrm{si}(\pi\nu D) \mid \cdot \cos(\nu \cdot \frac{2\pi}{T_1} \cdot t + \phi_\nu)$$



Effektivwert:

$$X_{\text{eff}} = \sqrt{\frac{1}{T_1} \cdot \int_{T_1} x^2(t)\,\mathrm{d}t} = \sqrt{\frac{X_m^2}{T_1} \cdot \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} 1\,\mathrm{d}t}$$

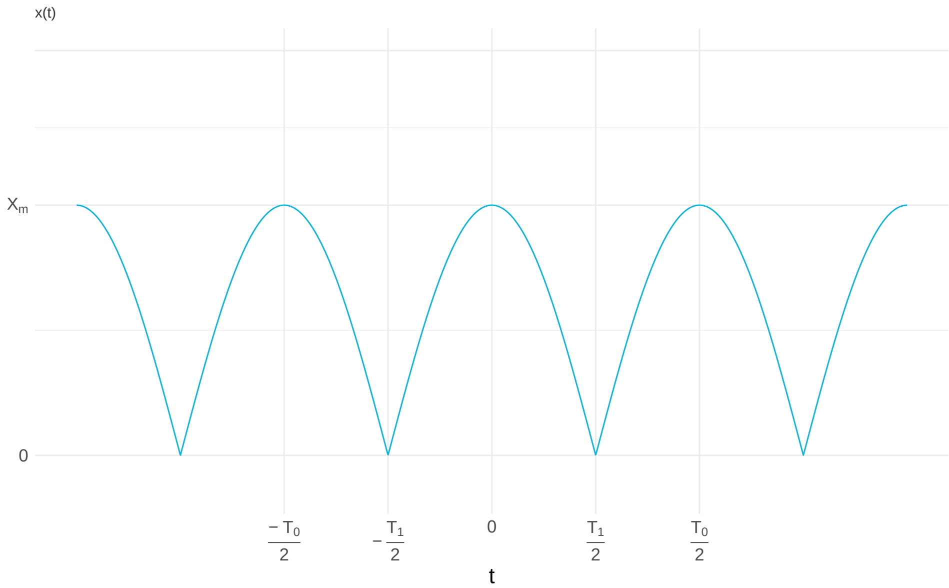
$$X_{\text{eff}} = X_m \cdot \sqrt{\frac{\tau}{T_1}} = X_m \cdot \sqrt{D}$$

$D = \frac{1}{2}, X_{\text{eff}} = \frac{X_m}{\sqrt{2}}$			
ν	$\underline{X}_\nu$	$\hat{X}_\nu$	ϕ_ν
1	$\frac{1}{\pi}X_m$	$\frac{2}{\pi}X_m$	0
2	–	–	–
3	-$\frac{1}{3\pi}X_m$	$\frac{2}{3\pi}X_m$	π
4	–	–	–
5	$\frac{1}{5\pi}X_m$	$\frac{2}{5\pi}X_m$	0
6	–	–	–
7	-$\frac{1}{7\pi}X_m$	$\frac{2}{7\pi}X_m$	π
8	–	–	–
9	$\frac{1}{9\pi}X_m$	$\frac{2}{9\pi}X_m$	0
10	–	–	–
11	-$\frac{1}{11\pi}X_m$	$\frac{2}{11\pi}X_m$	π
12	–	–	–
13	$\frac{1}{13\pi}X_m$	$\frac{2}{13\pi}X_m$	0
14	–	–	–
15	-$\frac{1}{15\pi}X_m$	$\frac{2}{15\pi}X_m$	π
16	–	–	–

$D = \frac{1}{4}, X_{\text{eff}} = \frac{X_m}{2}$		
ν	$\hat{X}_\nu$	ϕ_ν
1	$\frac{\sqrt{2}}{\pi}X_m$	0
2	$\frac{1}{\pi}X_m$	0
3	$\frac{\sqrt{2}}{3\pi}X_m$	0
4	–	–
5	$\frac{\sqrt{2}}{5\pi}X_m$	π
6	$\frac{1}{3\pi}X_m$	π
7	$\frac{\sqrt{2}}{7\pi}X_m$	π
8	–	–
9	$\frac{\sqrt{2}}{9\pi}X_m$	0
10	$\frac{1}{5\pi}X_m$	0
11	$\frac{\sqrt{2}}{11\pi}X_m$	0
12	–	–
13	$\frac{\sqrt{2}}{13\pi}X_m$	π
14	$\frac{1}{7\pi}X_m$	π
15	$\frac{\sqrt{2}}{15\pi}X_m$	π
16	–	–

$D = \frac{1}{8}, X_{\text{eff}} = \frac{X_m}{\sqrt{8}}$		
ν	$\hat{X}_\nu$	ϕ_ν
1	$\frac{\sqrt{2-\sqrt{2}}}{\pi}X_m$	0
2	$\frac{\sqrt{2}}{2\pi}X_m$	0
3	$\frac{\sqrt{2+\sqrt{2}}}{3\pi}X_m$	0
4	$\frac{1}{2}X_m$	0
5	$\frac{\sqrt{2+\sqrt{2}}}{5\pi}X_m$	0
6	$\frac{\sqrt{2}}{16\pi}X_m$	0
7	$\frac{\sqrt{2-\sqrt{2}}}{7\pi}X_m$	0
8	–	–
9	$\frac{\sqrt{2-\sqrt{2}}}{9\pi}X_m$	π
10	$\frac{\sqrt{2}}{5\pi}X_m$	π
11	$\frac{\sqrt{2+\sqrt{2}}}{11\pi}X_m$	π
12	$\frac{1}{6\pi}X_m$	π
13	$\frac{\sqrt{2+\sqrt{2}}}{13\pi}X_m$	π
14	$\frac{\sqrt{2}}{7\pi}X_m$	π
15	$\frac{\sqrt{2-\sqrt{2}}}{15\pi}X_m$	π
16	–	–

1.2



$$\underline{X}_\nu = \frac{1}{T_1} \cdot \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} X_m \cos\left(\frac{\pi}{T_1}t\right) \cdot e^{-(j\nu\frac{\pi}{T_1}t)} \, \mathrm{d}t$$

$$= \frac{X_m}{T_1} \cdot \left[\frac{e^{-(j\nu\frac{\pi}{T_1}t)}}{(-j\nu\frac{\pi}{T_1})^2 + (\frac{\pi}{T_1})^2} \cdot \left((-j\nu\frac{\pi}{T_1}) \cdot \cos\left(\frac{\pi}{T_1}t\right) + \frac{\pi}{T_1} \cdot \sin\left(\frac{\pi}{T_1}t\right) \right) \right]_{-\frac{T_1}{2}}^{\frac{T_1}{2}}$$

$$= \frac{X_m}{\pi(1-4\nu^2)} \cdot \left[e^{-j\nu\pi} \cdot (\underbrace{-j2\nu \cdot \cos\left(\frac{\pi}{2}\right)}_{0} + 1) - e^{j\nu\pi} \cdot (\underbrace{-j2\nu \cdot \cos\left(-\frac{\pi}{2}\right)}_{0} - 1) \right]$$

$$= \frac{X_m}{\pi(1-4\nu^2)} \cdot (e^{-j\nu\pi} + e^{j\nu\pi}) = 2\frac{X_m}{\pi(1-4\nu^2)} \cdot \left(\frac{e^{j\nu\pi} + e^{-j\nu\pi}}{2} \right)$$

$$\underline{X}_\nu = \frac{2X_m}{\pi(1-4\nu^2)} \cdot \cos(\nu\pi)$$

$$\hat{X}_\nu = 2 \cdot |\underline{X}_\nu| = \frac{4 \cdot X_m}{\pi(1-4\nu^2)} \cdot \cos(\nu\pi)$$

Mittelwert:

$$X_0 = \frac{1}{T_1} \cdot \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} X_m \cdot \cos{(\frac{\pi}{T_1} t)} \, \mathrm{d}t = \frac{X_m}{\pi} \cdot \left[\sin{(\frac{\pi}{T_1} t)} \right]_{-\frac{T_1}{2}}^{\frac{T_1}{2}}$$

$$X_0 = \frac{2X_m}{\pi}$$

Effektivwert:

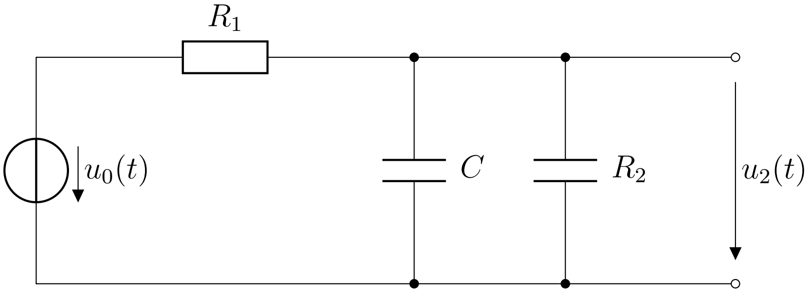
$$X_{\text{eff}} = \sqrt{\frac{X_m^2}{T_1} \cdot \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} (\cos{(\frac{\pi}{T_1} t)})^2 \, \mathrm{d}t}$$

$$X_{\text{eff}} = \sqrt{\frac{X_m^2}{T_1} \cdot \left[\frac{t}{2} + \frac{\sin{(\frac{2\pi}{T_1})t}}{4 \cdot \frac{\pi}{T_1}} \right]_{-\frac{T_1}{2}}^{\frac{T_1}{2}}}$$

$$X_{\text{eff}} = \frac{X_m}{\sqrt{2}}$$

$D = \frac{1}{4}, X_{\text{eff}} = \frac{X_m}{2}$		
ν	$\hat{X}_\nu$	ϕ_ν
1	$\frac{4}{3\pi}X_m$	0
2	$\frac{4}{15\pi}X_m$	π
3	$\frac{4}{35\pi}X_m$	0
4	$\frac{4}{63\pi}X_m$	π
5	$\frac{4}{99\pi}X_m$	0
6	$\frac{4}{143\pi}X_m$	π
7	$\frac{4}{195\pi}X_m$	0
8	$\frac{4}{255\pi}X_m$	π
9	$\frac{4}{323\pi}X_m$	0
10	$\frac{4}{399\pi}X_m$	π
11	$\frac{4}{483\pi}X_m$	0
12	$\frac{4}{575\pi}X_m$	π
13	$\frac{4}{675\pi}X_m$	0
14	$\frac{4}{783\pi}X_m$	π
15	$\frac{4}{899\pi}X_m$	0

1.3



$$\underline{U}_2 = \underline{U_0} \cdot \frac{\frac{1}{j\omega C + \frac{1}{R_2}}}{R_1 + \frac{1}{j\omega C + \frac{1}{R_2}}} = \frac{\underline{U}_0 \cdot R_2}{R_1 + R_2 + j\omega C R_1 R_2}$$

Betrag:

$$\hat{U}_2 = \frac{\hat{U}_0 \cdot R_2}{\sqrt{(R_1 + R_2)^2 + (\omega C R_1 R_2)^2}}$$

Phase:

$$\phi_{\underline{U}_2} = \phi_{\underline{U}_0} - \arctan \frac{\omega C R_1 R_2}{R_1 + R_2}$$

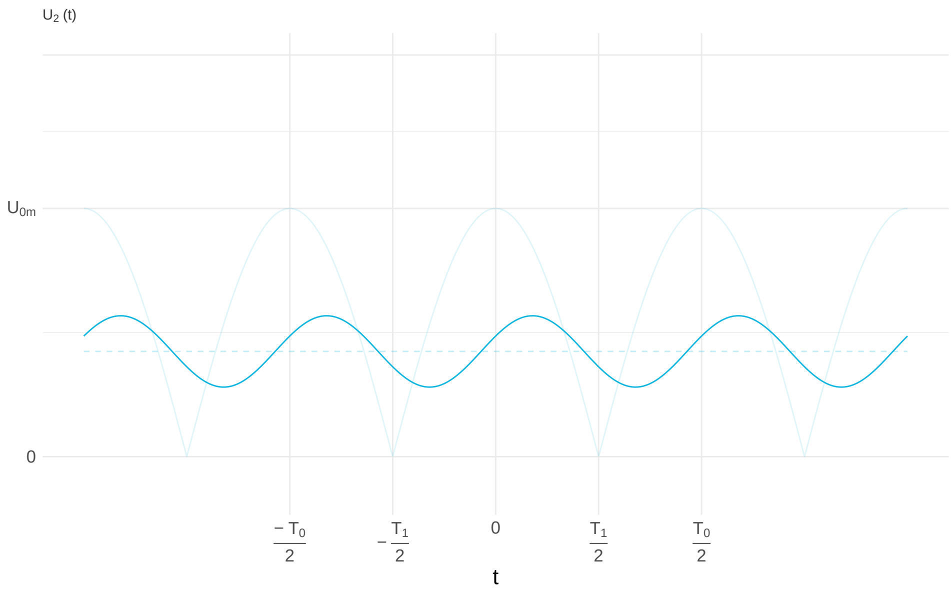
Mittelwert:

$$\underline{U}_{0_\nu}(\nu = 0) = \hat{U}_{0_\nu}(\nu = 0) \cdot \frac{R_2}{R_1 + R_2} = \frac{2 \cdot 1\text{V} \cdot 1\text{k}\Omega}{\pi \cdot (1\text{k}\Omega + 400\text{k}\Omega)} \approx 0.424 \text{ V}$$

ν	$\underline{U}_{0_\nu}/\mathrm{V}$	$\hat{U}_{0_\nu}/\mathrm{V}$	ϕ	$\hat{U}_{2_\nu}/\mathrm{V}$	$\phi_{U_{2_\nu}}/°$	$U_{2_{\text{eff}}}/\mathrm{V}$
1	0.2122	-0.4244	0	0.1219	-60.88	0.0862
2	-0.4244	0.08488	π	0.0131	105.56	0.0093
3	0.0182	0.03638	0	0.0038	-79.48	0.0027
4	-0.0101	0.02021	π	0.0016	97.93	0.0011
5	0.0064	0.01286	0	0.0008	-83.64	0.0006
6	-0.0045	0.00890	π	0.0005	95.3	0.0004
7	0.0033	0.00652	0	0.0003	-85.45	0.0002
8	-0.0025	0.00499	π	0.0002	93.19	0.0001
9	0.0020	0.00394	0	-	-86.45	-
10	-0.0016	0.00319	π	-	93.19	-
11	0.0013	0.00264	0	-	-87.1	-
12	-0.0011	0.00221	π	-	92.28	-
13	-0.0009	0.00189	0	-	-87.55	-
14	-0.0008	0.00163	π	-	92.28	-
15	0.0007	0.00142	0	-	-87.87	-
16	-0.0006	0.0012	π	-	91.99	-

Effektivwert aus Summe der einzelnen Effektivwerte:

$$U_{2_{\text{eff}}} = \sqrt{U_{2_\nu}^2 + \sum_{\nu=1}^{\infty} U_{2_{\text{eff}_\nu}}^2} \approx 0.433 \ \mathrm{V}$$



U2 (t)
U0m
0
− T0
2
− T1
2
0
T1
2
T0
2
t

2 Versuchsaufgaben

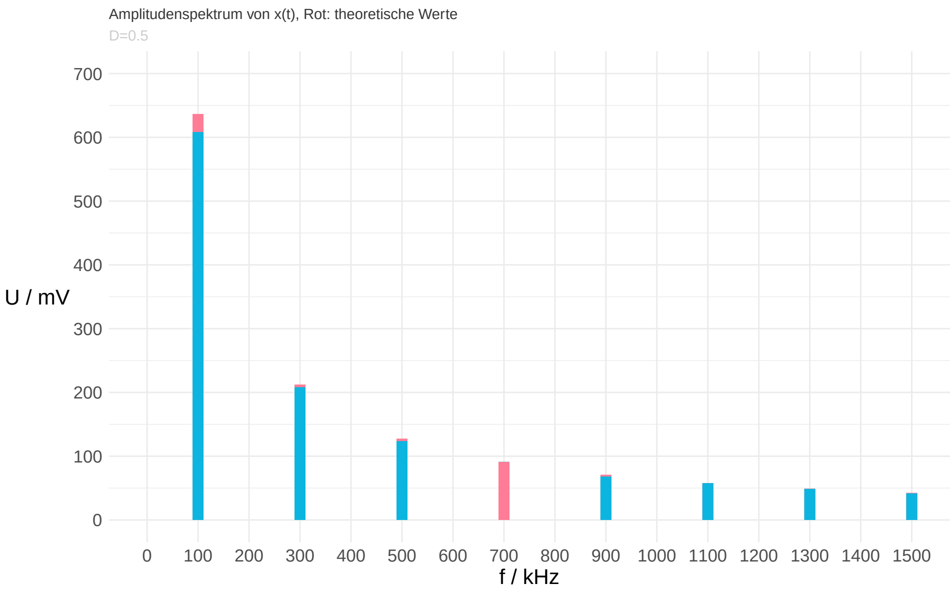
2.1

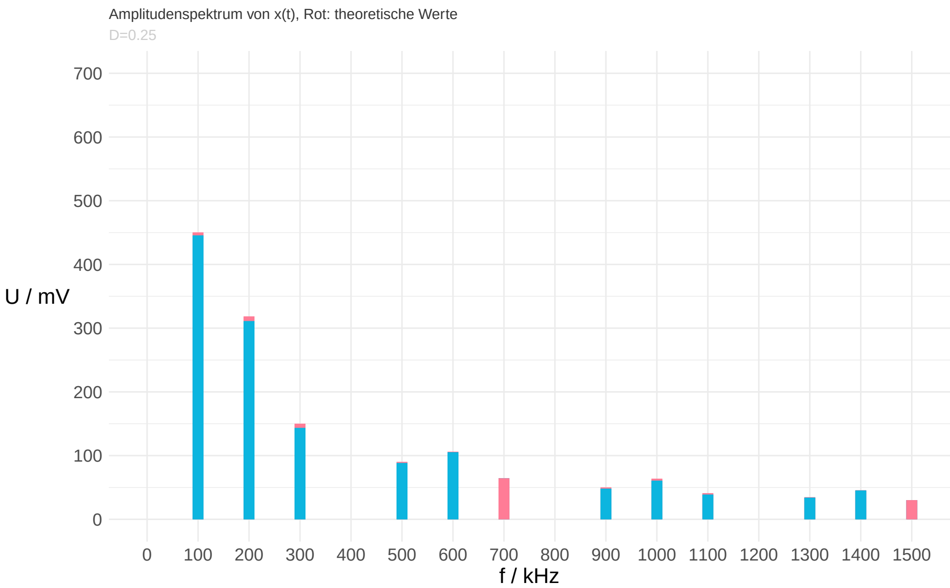
Messwerte der Aufgabe 3.1 für $T = 10$ µs:

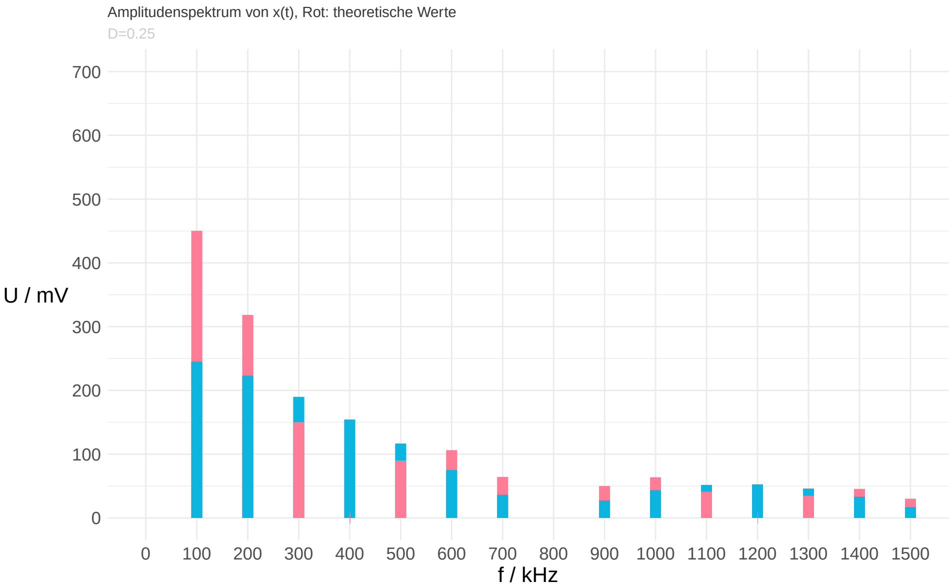
$D = \frac{1}{2}, X_{\text{eff}} =?$			
ν	$\hat{X}_\nu$/mV	$X_{\nu_{\text{eff}}}$/mV	f_ν/kHz
1	608.112	430	100
2	-	-	-
3	208.045	147.11	300
4	-	-	-
5	123.546	87.36	500
6	-	-	-
7	91.040	64.375	700
8	-	-	-
9	68.059	48.125	900
10	-	-	-
11	57.544	40.69	1100
12	-	-	-
13	48.691	34.43	1300
14	-	-	-
15	41.606	29.42	1500

$D = \frac{1}{4}, X_{\text{eff}} = 500\ \text{mV}$			
ν	$\hat{X}_\nu/\text{mV}$	$X_{\nu_{\text{eff}}}/\text{mV}$	f_ν/kHz
1	445.477	315	100
2	311.070	219.96	200
3	143.401	101.4	300
4	-	-	-
5	88.388	62.5	500
6	105.217	74.4	600
7	64.488	45.6	700
8	-	-	-
9	48.225	34.1	900
10	60.670	42.9	1000
11	38.891	27.5	1100
12	-	-	-
13	34.083	24.1	1300
14	45.255	32	1400
15	30.123	21.3	1500

$D = \frac{1}{8}, X_{\text{eff}} = 343$ mV			
ν	$\hat{X}_\nu$/mV	$X_{\nu_{\text{eff}}}$/mV	f_ν/kHz
1	244.942	173.2	100
2	222.965	157.66	200
3	189.787	134.2	300
4	154.291	109.1	400
5	116.673	82.5	500
6	75.095	53.1	600
7	36.204	25.6	700
8	-	-	-
9	27.436	19.4	900
10	43.416	30.7	1000
11	51.760	36.6	1100
12	52.609	37.2	1200
13	45.962	32.5	1300
14	33.234	23.5	1400
15	16.829	11.9	1500


Amplitudenspektrum von x(t), Rot: theoretische Werte
D=0.5
U / mV
700
600
500
400
300
200
100
0
0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500
f / kHz

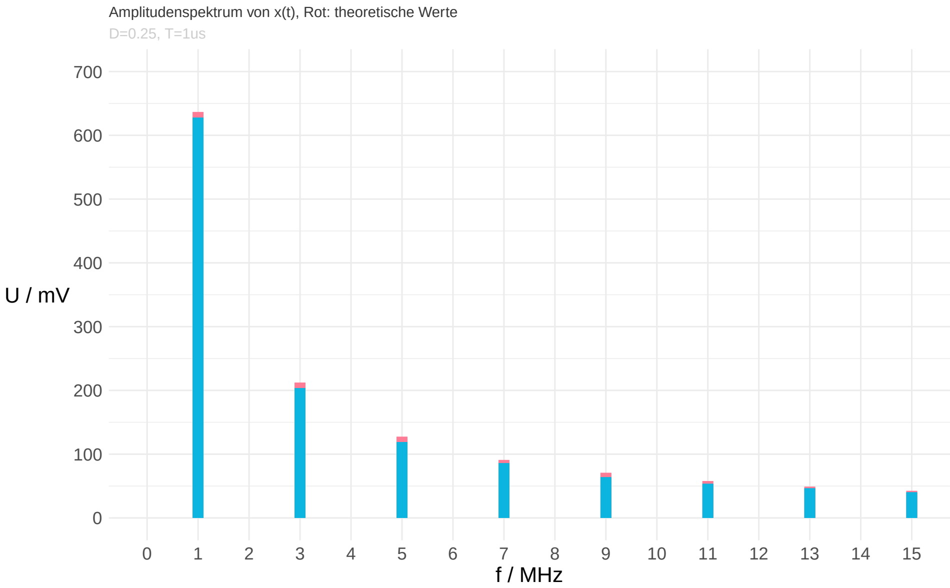

Amplitudenspektrum von x(t), Rot: theoretische Werte
D=0.25
U / mV
700
600
500
400
300
200
100
0
0 100 200 300 400 500 600 700 800 900 1000 1100 1200 1300 1400 1500
f / kHz



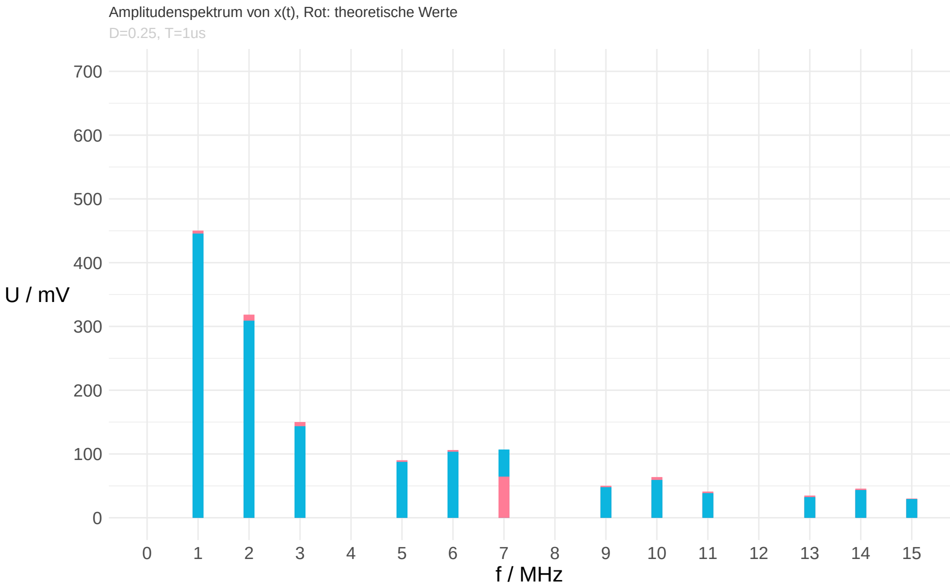
Messwerte der Aufgabe 3.1 für $T = 1$ µs:

$D = \frac{1}{2}, X_{\text{eff}} = 692$ mV				$D = \frac{1}{4}, X_{\text{eff}} = 489$ mV			
ν	$\hat{X}_\nu$/mV	$X_{\nu_{\text{eff}}}$/mV	f_ν/MHz	ν	$\hat{X}_\nu$/mV	$X_{\nu_{\text{eff}}}$/mV	f_ν/MHz
1	627.911	444	1	1	445.477	315	1
2	-	-	-	2	308.864	218.4	2
3	203.647	144	3	3	143.401	101.4	3
4	-	-	-	4	-	-	-
5	119.077	84.2	5	5	87.540	61.9	5
6	-	-	-	6	103.379	73.1	6
7	85.984	60.8	7	7	106.915	75.6	7
8	-	-	-	8	-	-	-
9	63.922	45.2	9	9	47.800	33.8	9
10	-	-	-	10	59.256	41.9	10
11	53.599	37.9	11	11	38.467	27.2	11
12	-	-	-	12	-	-	-
13	46.528	32.9	13	13	32.244	22.8	13
14	-	-	-	14	43.416	30.7	14
15	40.305	28.5	15	15	29.274	20.7	15

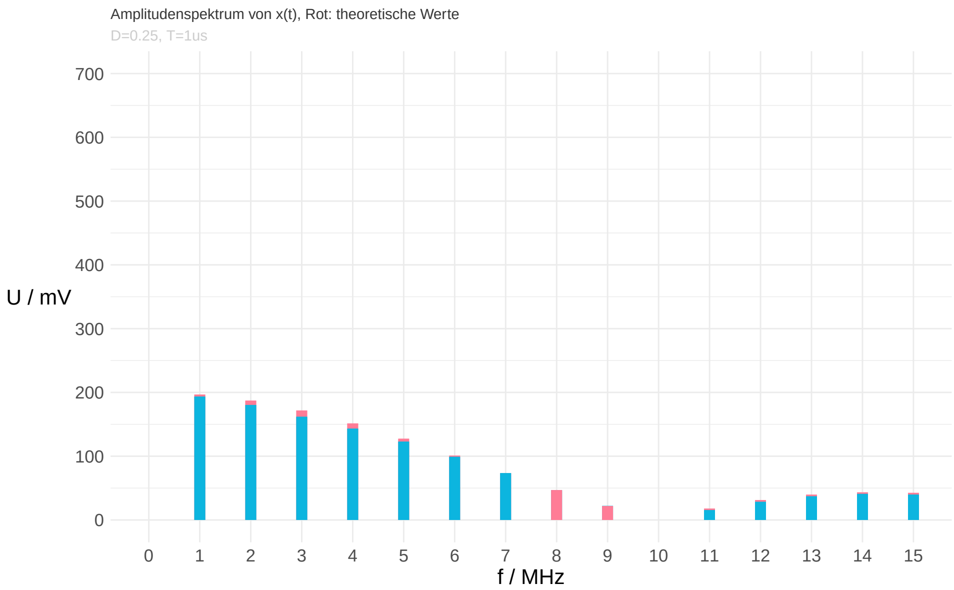
$D = \frac{1}{10}, X_{\text{eff}} = 305\ \text{mV}$			
ν	$\hat{X}_\nu$/mV	$X_{\nu_{\text{eff}}}$/mV	f_ν/MHz
1	193.606	136.9	1
2	180.312	127.5	2
3	161.786	114.4	3
4	143.260	101.3	4
5	122.895	86.9	5
6	98.995	70	6
7	73.398	51.9	7
8	46.810	33.1	8
9	22.062	15.6	9
10	-	-	-
11	15.415	10.9	11
12	28.426	20.1	12
13	37.052	26.2	13
14	40.871	28.9	14
15	39.881	28.2	15



Amplitudenspektrum von x(t), Rot: theoretische Werte
D=0.25, T=1us
U / mV
700
600
500
400
300
200
100
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
f / MHz



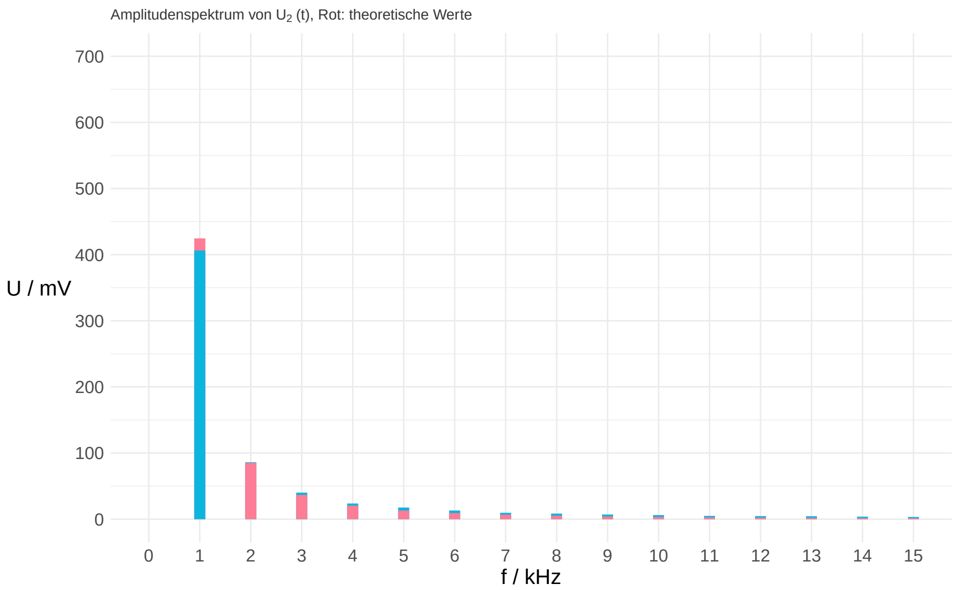
Amplitudenspektrum von x(t), Rot: theoretische Werte
D=0.25, T=1us
U / mV
700
600
500
400
300
200
100
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
f / MHz


Amplitudenspektrum von x(t), Rot: theoretische Werte
D=0.25, T=1us
U / mV
700
600
500
400
300
200
100
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
f / MHz

2.2

$X_{\text{eff}} = 698$ mV, $X_0 = 627$ mV			
ν	$\hat{X}_\nu$/mV	$X_{\nu_{\text{eff}}}$/mV	f_ν/kHz
1	406.021	287.1	1
2	85.843	60.7	2
3	39.881	28.2	3
4	23.617	16.7	4
5	17.395	12.3	5
6	13.011	9.2	6
7	9.475	6.7	7
8	8.202	5.8	8
9	6.930	4.9	9
10	5.940	4.2	10
11	4.808	3.4	11
12	4.384	3.1	12
13	4.243	3	13
14	3.536	2.5	14
15	3.253	2.3	15
16	2.828	2	16

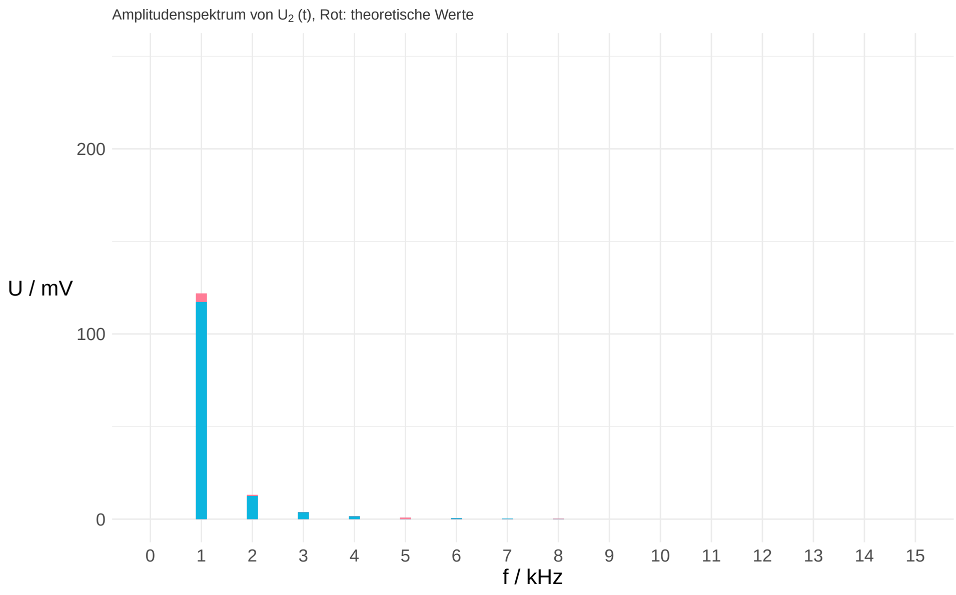
Messwerte der Aufgabe 3.2

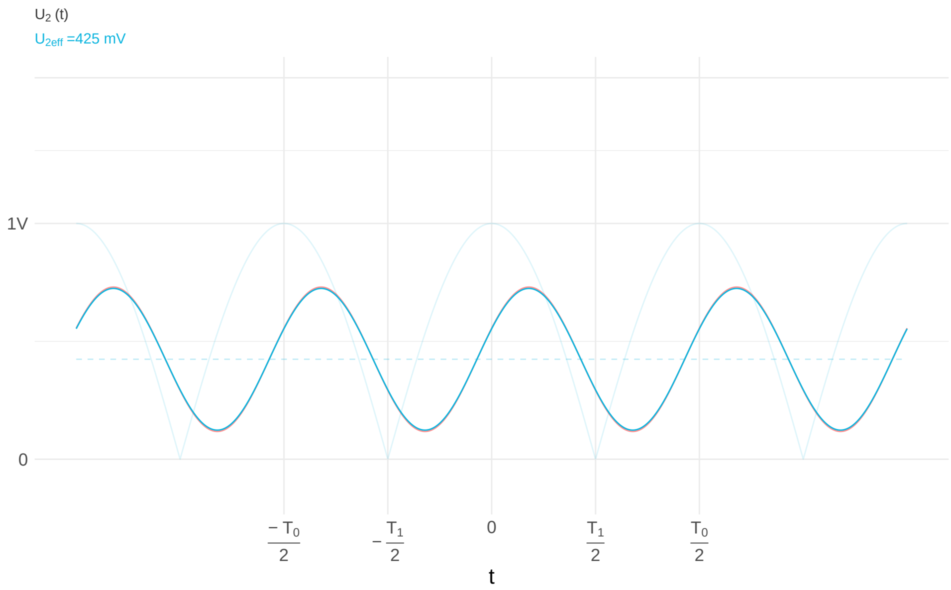


2.3

$U_{2_{\mathrm{eff}}} = 425$ mV, $U_0 = 416$ mV			
ν	$\hat{X}_\nu/\mathrm{mV}$	$X_{\nu_{\mathrm{eff}}}/\mathrm{mV}$	f_ν/kHz
1	117.238	82.9	1
2	12.445	8.8	2
3	3.677	2.6	3
4	1.485	1.05	4
5	0.813	0.575	5
6	0.443	0.313	6
7	0.252	0.178	7
8	0.208	0.147	8

Messwerte der Aufgabe 3.3


Amplitudenspektrum von U2 (t), Rot: theoretische Werte
U / mV
200
100
0
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
f / kHz


U2 (t)
U2eff =425 mV
1V
0
− T0/2
− T1/2
0
T1/2
T0/2
t