Fourth Semester B.E. Degree Examination, July/August 2022

Complex Analysis, Probability and Statistical Methods

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive Cauchy-Riemann equation in Polar form. **(06 Marks)**

b. Find the analytic function f(z) whose real part is
$x \sin x \cosh y - y \cos x \sinh y$ **(07 Marks)**

c. If f(z) is analytic show that

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right] |f(z)|^2 = 4|f'(z)|^2$$ **(07 Marks)**

OR

2 a. Find the analytic function f(z) given that the sum of its real and imaginary part is
$x^3 - y^3 + 3xy(x-y)$ **(06 Marks)**

b. Find the analytic function $f(z) = u + iv$ if
$v = r^2 \cos 2\theta - r\cos\theta + 2$ **(07 Marks)**

c. If f(z) is analytic function then show that

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$$ **(07 Marks)**

Module-2

3 a. State and prove Cauchy's Integral formula. **(06 Marks)**

b. Evaluate $\int_0^{2+i} \bar{z}^2 dz$ along (i) the line $y = \frac{x}{2}$ (ii) The real axis to 2 and then vertically to 2 + i. **(07 Marks)**

c. Find the bilinear transformation which maps the points 1, i, −1 onto the points i, 0, −i respectively. **(07 Marks)**

OR

4 a. Discuss the transformation $w = e^z$, with respect to straight lines parallel to x and y axis. **(06 Marks)**

b. Using Cauchy's integral formula evaluate

$$\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz \text{ , where } c : |z| = 3$$ **(07 Marks)**

c. Find the bilinear transformation which maps the points 0, 1, ∞ on to the points −5, −1, 3 respectively. **(07 Marks)**

Module-3

5 a. A random variable X has the following probability function for various values of X.

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find i) k ii) $P(X < 6)$ iii) $P(3 < X \le 6)$ **(06 Marks)**

b. Out of 800 families with 5 children each, how many families would you expect to have (i) 3 boys (ii) 5 girls (iii) either 2 or 3 boys (iv) atmost 2 girls, assuming equal probabilities for boys and girls. **(07 Marks)**

c. The length in time (minutes) that a certain lady speaks on a telephone is a random variable with probability density function

$$f(x)=\begin{cases} Ae^{-x/5} & \text{for } x>0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of the constant A. What is the probability that she will speak over the phone for (i) More than 10 minutes (ii) Less than 5 minutes (iii) Between 5 and 10 minutes. **(07 Marks)**

OR

6 a. Find the constant C such that the function

$$f(x)=\begin{cases} Cx^2, & 0<x<3 \\ 0 & \text{otherwise} \end{cases}$$

is a probability density function. Also compute $P(1<x<2)$, $P(x\le 1)$ and $P(x>1)$ **(06 Marks)**

b. 2% fuses manufactured by a firm are found to be defective. Find the probability that the box containing 200 fuses contains
(i) No defective fuses (ii) 3 or more defective fuses (iii) At least one defective fuse. **(07 Marks)**

c. If x is a normal variate with mean 30 and standard deviation 5 find the probabilities that
(i) $26\le x\le 40$ (ii) $x\ge 45$ (iii) $|x-30|>5$
Given that $\phi(1)=0.3413$, $\phi(0.8)=0.2881$, $\phi(2)=0.4772$, $\phi(3)=0.4987$ **(07 Marks)**

Module-4

7 a. The following table gives the ages (in years) of 10 married couples. Calculate Karl Pearson's coefficient of correlation between their ages:

Age of husband (x)	23	27	28	29	30	31	33	35	36	39
Age of wife (y)	18	22	23	24	25	26	28	29	30	32

(06 Marks)

b. In a partially destroyed laboratory record of correlation data only the following results are available:
Variance of x is 9 and regression lines are $8x-10y+66=0$, $40x-18y=214$. Find
(i) Mean value of x and y
(ii) Standard deviation of y
(iii) Coefficient of correlation between x and y. **(07 Marks)**

c. Fit a parabola of the form $y=ax^2+bx+c$ for the data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

(07 Marks)

OR

8 a. Obtain the lines of regression and hence find the coefficient of correlation of the data:

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

(06 Marks)

b. Show that if θ is the angle between the lines of regression

$$\tan\theta=\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}\left(\frac{1-r^2}{r}\right)$$

(07 Marks)

c. Fit a straight line y = a + bx to the data

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(07 Marks)

Module-5

9 a. The joint probability distribution of the random variables X and Y is given below.

X \ Y	-4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Find (i) E[X] and E[Y] (ii) E[XY] (iii) cov(X, Y) iv) $\rho(X, Y)$.
Also, show that X and Y are not independent. (06 Marks)

b. A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory confirmed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5% ($z_{0.05} = 1.96$, $z_{0.01} = 2.58$). (07 Marks)

c. A certain stimulus administered to each of the 12 patients resulted in the following change in blood pressure 5, 2, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 d.f. is 2.201) (07 Marks)

OR

10 a. Define the terms :
(i) Null hypothesis (ii) Type-I and Type – II errors (iii) Significance level (06 Marks)

b. In an experiment of pea breeding the following frequencies of seeds were obtained:

Round Yellow	Wrinkled Yellow	Round Green	Wrinkled Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportions 9:3:3:1
Is the experiment in agreement with theory ($\chi^2_{0.5}$ for 3 d.f is 7.815) (07 Marks)

c. The joint probability distribution of two discrete random variable X and Y is given by f(x, y) = k(2x + y) where x and y are integers such that $0 \le x \le 2$, $0 \le y \le 3$. Find k and the marginal probability distribution of X and Y. Show that the random variables X and Y are dependent. Also, find $P(X \ge 1, Y \le 2)$. (07 Marks)

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