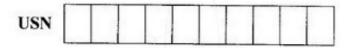
CBCS SCHEME



18MATDIP31

Third Semester B.E. Degree Examination, Aug./Sept.2020 Additional Mathematics - I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove that
$$(1+i)^n + (1-i)^n = 2^{n/2+1} \cos \frac{n\pi}{4}$$

(08 Marks)

b. Expression the complex number
$$(2+3i)+\frac{1}{1-i}$$
 in the form $a+ib$.

(06 Marks)

Find the modulus and amplitude of the complex number 1 - cosα + i sinα

(06 Marks)

a. If
$$\vec{A} = i + 2j - 3k$$
, $\vec{B} = 3i - j + 2k$ show that $\vec{A} + \vec{B}$ is perpendicular to $\vec{A} - \vec{B}$. Also find the angle between $2\vec{A} + 3\vec{B}$ and $\vec{A} + 2\vec{B}$.

b. Show that the vectors
$$i-2j+3k$$
, $2i+j+k$, $3i+4j-k$ are coplanar.

(06 Marks)

c. Find the sine of the angle between
$$\vec{A} = 4i - j + 3k$$
 and $\vec{B} = -2i + j - 2k$. (06 Marks)

b. If
$$u = \sin^{-1} \left[\frac{x^2 + y^2}{x - y} \right]$$
 prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. (06 Marks)

c. If
$$u = f(x - y, y - z, z - x)$$
 prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)

4 a. Prove that
$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$
 by using Maclaurin's series. (08 Marks)

b. If
$$x = r \cos \theta$$
, $y = r \sin \theta$ find $\frac{\partial(x, y)}{\partial(r, \theta)}$. (06 Marks)

c. If
$$z = e^{ax + by}$$
 f(ax - by) then show that $b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$. (06 Marks)

5 a. Find the angle between the surfaces
$$x^2 + y^2 + z^2 = 9$$
 and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ (08 Marks)

b. Find the unit vector normal to the surface
$$x^2y + 2xz = 4$$
 at $(2, -2, 3)$. (06 Marks)
c. Show that the vector $(-x^2 + yz)i + (4y - z^2x)j + (2xz - 4z)k$ is solenoidal. (06 Marks)

Important Note: I. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.

2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

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- a. A particle moves along the curve $x = t^3 + 1$, $y = t^2$, z = 2t + 3 where t is the time. Find the components of its velocity and acceleration at t = 1 in the direction i + j + 3k.
 - b. Find the values of a, b, c such that $\vec{F} = (x + y + az)i + (bx + 2y z)j + (x + cy + 2z)k$ (06 Marks) is irrotational.
 - c. Find div \vec{F} and curl \vec{F} where $\vec{F} = \nabla(x^3 + y^3 + z^3 3xyz)$.

(06 Marks)

a. Obtain the reduction formula for $\int_{-\pi}^{\pi/2} \cos^n x \, dx$, n > 0.

(08 Marks)

b. Evaluate $\int \frac{x^9}{\sqrt{1-x^2}} dx$

- (06 Marks)
- c. Evaluate $\iint xy(x+y)dxdy$ over the area between $y=x^2$ and y=x
- (06 Marks)

OR

8 a. Obtain the reduction formula for

$$\int_{0}^{\pi/2} \sin^{n} x \, dx \, , \quad n > 0.$$

(08 Marks)

b. Evaluate $\int \frac{x^2}{(1-x^2)^{7/2}} dx$

(06 Marks)

c. Evaluate $\int_{0}^{4} \int_{0}^{\infty} \int_{0}^{\infty} e^{x+y+z} dz dy dx$

(06 Marks)

9 a. Solve $y(\log y)dx + (x \log y)dy = 0$

(08 Marks)

b. Solve $x \cdot \frac{dy}{dx} + y = x$

(06 Marks)

c. Solve $(xy^2 - e^{(x^2)})dx - x^2y dy = 0$

(06 Marks)

10 a. Solve $(5x^4 + 3x^2y^2 - 2xy^3) dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$

(08 Marks)

b. Solve $\frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$ c. Solve $(xy^3 + y)dx + 2(x^2y^2 + x + y^4) dy = 0$

(06 Marks)

(06 Marks)