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а

$$f(x) = e^{|x|}, \quad [-\pi, \pi]$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} e^{|x|}\,dx = \frac{2}{\pi}\int\limits_{0}^{\pi} e^{x}\,dx = \frac{2}{\pi}\left(e^x\right)\Big|_0^{\pi} = \frac{2(e^{\pi}-1)}{\pi}$$

$$a_n = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} e^{|x|}\cos nx\,dx = \frac{2}{\pi}\int\limits_{0}^{\pi} e^{x}\cos nx\,dx$$

отдельно посчитаем первообразную

$$\int e^x \cos nx\,dx = [u = \cos nx, du = -n\sin nx, dv = e^x\,dx, v = e^x] = e^x\cos nx + n\int e^x \sin nx\,dx =$$

$$= [u = \sin nx, du = n\cos nx\,dx, dv = e^x\,dx, v = e^x] = e^x\cos nx + n\left(e^x \sin nx - n\int e^x\cos nx\,dx\right) =$$

$$= e^x\cos nx + ne^x\sin nx - n^2\int e^x\cos nx\,dx$$

$$\int e^x\cos nx\,dx = e^x\cos nx + ne^x\sin nx - n^2\int e^x\cos nx\,dx$$

$$\int e^x\cos nx\,dx = \frac{1}{1+n^2}e^x\cos nx + \frac{n}{1+n^2}e^x\sin nx$$

$$\frac{2}{\pi}\int\limits_0^{\pi} e^x\cos nx\,dx = \frac{2}{\pi}\left(\frac{1}{1+n^2}e^x\cos nx + \frac{n}{1+n^2}e^x\sin nx\right)\Bigg|_0^{\pi} =$$

$$= \frac{2}{\pi}\left(\frac{e^{\pi}\cos\pi n}{1+n^2} - \frac{1}{1+n^2}\right) = \frac{2((-1)^n e^{\pi}-1)}{\pi(1+n^2)}$$

$$b_n = \frac{1}{\pi}\int\limits_{-\pi}^{\pi} e^{|x|}\sin nx\,dx = 0$$

потому что под интегралом нечётная функция и симметричный промежуток

$$f(x) \sim \frac{e^{\pi}-1}{\pi} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n e^{\pi}-1}{(1+n^2)}\cos nx$$

б

$$f(x) \sim \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int\limits_0^{\pi} e^{|x|} \sin nx \, dx = \frac{2}{\pi} \int\limits_0^{\pi} e^{x} \sin nx \, dx$$

отдельно посчитаем первообразную

$$\int e^x \sin nx \, dx = [u = \sin nx, du = n \cos nx, dv = e^x \, dx, v = e^x] = e^x \sin nx - n \int e^x \cos nx \, dx =$$

$$= [u = \cos nx, du = -n \sin nx \, dx, dv = e^x \, dx, v = e^x] = e^x \sin nx - n \left(e^x \cos nx + n \int e^x \sin nx \, dx \right) =$$

$$= e^x \sin nx - ne^x \cos nx - n^2 \int e^x \sin nx \, dx$$

$$\int e^x \sin nx \, dx = e^x \sin nx - ne^x \cos nx - n^2 \int e^x \sin nx \, dx$$

$$\int e^x \cos nx \, dx = \frac{1}{1+n^2} e^x \sin nx - \frac{n}{1+n^2} e^x \cos nx$$

$$b_n = \frac{2}{\pi} \int\limits_0^{\pi} e^x \sin nx \, dx = \frac{2}{\pi} \left(\frac{1}{1+n^2} e^x \sin nx - \frac{n}{1+n^2} e^x \cos nx \right) \Bigg|_0^{\pi} =$$

$$= \frac{2}{\pi} \left(- \frac{n}{1+n^2} e^{\pi} \cos \pi n + \frac{n}{1+n^2} \right) = \frac{2n(1 - e^{\pi}(-1)^n)}{\pi(1+n^2)}$$

$$f(x) \sim \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{n(1 - e^{\pi}(-1)^n)}{1+n^2} \sin nx$$

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