

а

$$f(x) = e^{|x|}, \quad [-\pi, \pi]$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} e^{|x|} dx = \frac{2}{\pi} \int_0^{\pi} e^x dx = \frac{2}{\pi} (e^x) \Big|_0^{\pi} = \frac{2(e^{\pi} - 1)}{\pi}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} e^{|x|} \cos nx dx = \frac{2}{\pi} \int_0^{\pi} e^x \cos nx dx$$

отдельно посчитаем первообразную

$$\begin{aligned} \int e^x \cos nx dx &= [u = \cos nx, du = -n \sin nx, dv = e^x dx, v = e^x] = e^x \cos nx + n \int e^x \sin nx dx = \\ &= [u = \sin nx, du = n \cos nx dx, dv = e^x dx, v = e^x] = e^x \cos nx + n \left(e^x \sin nx - n \int e^x \cos nx dx \right) = \end{aligned}$$

$$= e^x \cos nx + ne^x \sin nx - n^2 \int e^x \cos nx dx$$

$$\int e^x \cos nx dx = e^x \cos nx + ne^x \sin nx - n^2 \int e^x \cos nx dx$$

$$\int e^x \cos nx dx = \frac{1}{1+n^2} e^x \cos nx + \frac{n}{1+n^2} e^x \sin nx$$

$$\frac{2}{\pi} \int_0^{\pi} e^x \cos nx dx = \frac{2}{\pi} \left(\frac{1}{1+n^2} e^x \cos nx + \frac{n}{1+n^2} e^x \sin nx \right) \Big|_0^{\pi} =$$

$$= \frac{2}{\pi} \left(\frac{e^{\pi} \cos \pi n}{1+n^2} - \frac{1}{1+n^2} \right) = \frac{2((-1)^n e^{\pi} - 1)}{\pi(1+n^2)}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} e^{|x|} \sin nx dx = 0,$$

потому что под интегралом нечётная функция и симметричный промежуток

$$f(x) \sim \frac{e^{\pi} - 1}{\pi} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n e^{\pi} - 1}{(1+n^2)} \cos nx$$

6

$$f(x) \sim \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} e^{|x|} \sin nx \, dx = \frac{2}{\pi} \int_0^{\pi} e^x \sin nx \, dx$$

отдельно посчитаем первообразную

$$\begin{aligned} \int e^x \sin nx \, dx &= [u = \sin nx, du = n \cos nx, dv = e^x dx, v = e^x] = e^x \sin nx - n \int e^x \cos nx \, dx = \\ &= [u = \cos nx, du = -n \sin nx \, dx, dv = e^x dx, v = e^x] = e^x \sin nx - n \left(e^x \cos nx + n \int e^x \sin nx \, dx \right) = \\ &= e^x \sin nx - ne^x \cos nx - n^2 \int e^x \sin nx \, dx \\ \int e^x \sin nx \, dx &= e^x \sin nx - ne^x \cos nx - n^2 \int e^x \sin nx \, dx \\ \int e^x \cos nx \, dx &= \frac{1}{1+n^2} e^x \sin nx - \frac{n}{1+n^2} e^x \cos nx \\ b_n &= \frac{2}{\pi} \int_0^{\pi} e^x \sin nx \, dx = \frac{2}{\pi} \left(\frac{1}{1+n^2} e^x \sin nx - \frac{n}{1+n^2} e^x \cos nx \right) \Big|_0^{\pi} = \\ &= \frac{2}{\pi} \left(-\frac{n}{1+n^2} e^{\pi} \cos \pi n + \frac{n}{1+n^2} \right) = \frac{2n(1 - e^{\pi}(-1)^n)}{\pi(1+n^2)} \\ f(x) &\sim \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{n(1 - e^{\pi}(-1)^n)}{1+n^2} \sin nx \end{aligned}$$

69

$$\iint_D \frac{\ln(x^2 + y^2)}{x^2 + y^2} \, dx dy, \quad D : e \leq x^2 + y^2 \leq e^4, x > 0$$

Перейдём к полярным координатам

$$\begin{aligned} x &= r \cos \varphi, y = r \sin \varphi, |J| = r, x^2 + y^2 = (r \cos \varphi)^2 + (r \sin \varphi)^2 = r^2 \\ e &\leq x^2 + y^2 \leq e^4 \implies e \leq r^2 \leq e^4 \\ x > 0 &\implies r \cos \varphi > 0 \implies \cos \varphi > 0 \implies -\frac{\pi}{2} < \varphi < \frac{\pi}{2} \\ \iint_D \frac{\ln(x^2 + y^2)}{x^2 + y^2} \, dx dy &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_e^{e^4} \frac{\ln r^2}{r^2} \cdot r \, dr = 2\pi \int_e^{e^4} \frac{\ln r}{r} \, dr = 2\pi \left(\frac{\ln^2 r}{2} \right) \Big|_e^{e^4} = \\ &= 2\pi \cdot \left(\frac{4^2}{2} - \frac{1^2}{2} \right) = 2\pi \cdot \frac{15}{2} = 15\pi \end{aligned}$$

$$z = \sqrt{xy}, \quad y = 2x^2, y = x^3, z = 0$$

$$\begin{aligned} V &= \iint_D z \, dx dy = \iint_D \sqrt{xy} \, dx dy = \int_0^2 dx \int_{x^3}^{2x^2} \sqrt{xy} \, dy = \int_0^2 \sqrt{x} \, dx \cdot \left(\frac{2}{3} y \sqrt{y} \right) \Big|_{x^3}^{2x^2} = \\ &= \int_0^2 \sqrt{x} \cdot \left(\frac{2}{3} \cdot 2x^2 \cdot \sqrt{2x^2} - \frac{2}{3} x^3 \sqrt{x^3} \right) dx = \int_0^2 \left(\frac{4\sqrt{2}}{3} x^{\frac{7}{2}} - \frac{2}{3} x^5 \right) dx = \\ &= \left(\frac{4\sqrt{2}}{3} \cdot \frac{2}{9} x^{\frac{9}{2}} - \frac{2}{3} \cdot \frac{1}{6} x^6 \right) \Big|_0^2 = \left(\frac{8\sqrt{2}}{27} x^{\frac{9}{2}} - \frac{1}{9} x^6 \right) \Big|_0^2 = \frac{8\sqrt{2} \cdot 2^{\frac{9}{2}}}{27} - \frac{2^6}{9} = \frac{8\sqrt{2} \cdot 2^4 \sqrt{2}}{27} - \frac{64}{9} = \\ &= \frac{256}{27} - \frac{64}{9} = \frac{64}{27} \end{aligned}$$

259

a

$$f(x) = \begin{cases} \frac{\pi}{4}, & 0 \leq x < \pi \\ -\frac{\pi}{4}, & -\pi < x < 0 \end{cases}$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \, dx = \frac{1}{\pi} \int_{-\pi}^0 \left(-\frac{\pi}{4} \right) dx + \frac{1}{\pi} \int_0^{\pi} \frac{\pi}{4} dx = \frac{1}{\pi} \cdot \left(-\frac{\pi}{4} \cdot \pi \right) + \frac{1}{\pi} \cdot \left(\frac{\pi}{4} \cdot \pi \right) = 0$$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = \frac{1}{\pi} \int_{-\pi}^0 \left(-\frac{\pi}{4} \right) \cos nx \, dx + \frac{1}{\pi} \int_0^{\pi} \frac{\pi}{4} \cos nx \, dx = \\ &= -\frac{1}{4} \int_{-\pi}^0 \cos nx \, dx + \frac{1}{4} \int_0^{\pi} \cos nx \, dx = -\frac{1}{4} \left(\frac{\sin nx}{n} \right) \Big|_{-\pi}^0 + \frac{1}{4} \left(\frac{\sin nx}{n} \right) \Big|_0^{\pi} = 0 + 0 = 0 \end{aligned}$$

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx = \frac{1}{\pi} \int_{-\pi}^0 \left(-\frac{\pi}{4} \right) \sin nx \, dx + \frac{1}{\pi} \int_0^{\pi} \frac{\pi}{4} \sin nx \, dx = \\ &= -\frac{1}{4} \int_{-\pi}^0 \sin nx \, dx + \frac{1}{4} \int_0^{\pi} \sin nx \, dx = -\frac{1}{4} \left(-\frac{\cos nx}{n} \right) \Big|_{-\pi}^0 + \frac{1}{4} \left(-\frac{\cos nx}{n} \right) \Big|_0^{\pi} = \end{aligned}$$

$$= -\frac{1}{4} \left(-\frac{1}{n} + \frac{\cos \pi n}{n} \right) + \frac{1}{4} \left(-\frac{\cos \pi n}{n} + \frac{1}{n} \right) = \frac{1}{4n} - \frac{(-1)^n}{4n} - \frac{(-1)^n}{4n} + \frac{1}{4n} = \frac{1 - (-1)^n}{2n}$$

$$f(x) \sim \frac{1}{2} \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n} \sin nx$$

6

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} \frac{\pi}{4} dx = \frac{2}{\pi} \cdot \frac{\pi}{4} \cdot \pi = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_0^{\pi} \frac{\pi}{4} \cos nx dx = \frac{1}{2} \int_0^{\pi} \cos nx dx = \frac{1}{2} \left(\frac{\sin nx}{n} \right) \Big|_0^{\pi} =$$

$$= \frac{1}{2} \left(\frac{\sin \pi n}{n} - 0 \right) = 0$$

$$f(x) \sim \frac{\pi}{4}$$

86

$$z = \sqrt{xy}, \quad z = 0, y = x^3, y = x^4$$

$$V = \iint_D z dx dy = \iint_D \sqrt{xy} dx dy = \int_0^1 dx \int_{x^4}^{x^3} \sqrt{xy} dy = \int_0^1 \sqrt{x} dx \cdot \left(\frac{2}{3} y \sqrt{y} \right)_{x^4}^{x^3} =$$

$$= \int_0^1 \sqrt{x} \left(\frac{2}{3} x^3 \sqrt{x^3} - \frac{2}{3} x^4 \sqrt{x^4} \right) dx = \int_0^1 \left(\frac{2}{3} x^5 - \frac{2}{3} x^{\frac{13}{2}} \right) dx =$$

$$= \left(\frac{2}{3} \cdot \frac{1}{6} x^6 - \frac{2}{3} \cdot \frac{2}{15} x^{\frac{15}{2}} \right) \Big|_0^1 = \left(\frac{1}{9} x^6 - \frac{4}{45} x^{\frac{15}{2}} \right) \Big|_0^1 = \frac{1}{9} - \frac{4}{45} = \frac{1}{45}$$