Online Round Exam

- Correct answers count as $+1$.
- Wrong answers count as $-\frac{1}{2}$.
- The exam duration is only 1 hour & 10 minutes.
- Insert the answers in the form carefully, following the instructions.
- Place the question's answers in order in a string, Place Numerical answers between two brackets, and if you don't want to answer a question type 0.

1. Let A be the set of positive integer divisors of 2025. Let B be a randomly selected subset of A. The probability that B is a nonempty set with the property that the least common multiple of its elements is 2025 is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

   (Please provide a numerical answer between two () brackets)

2. The number of diagonals that can be drawn in a polygon of 100 sides is:

   (A) 4850

   (B) 4950

   (C) 9900

   (D) 98

   (E) 8800

3. If $\log(x - y) - \log 5 - \frac{1}{2}\log x - \frac{1}{2}\log y = 0$, then $\frac{x}{y} + \frac{y}{x} =$

   (a) 25

   (b) 26

   (c) 27

(d) 28

4. Let m and n be 2 integers such that $m > n$. Suppose $m + n = 20$ and $m^2 + n^2 = 328$. Find $m^2 - n^2$.

   (A) 280

   (B) 292

   (C) 300

   (D) 320

   (E) 340

5. If $x = \frac{2\sin\theta}{1+\cos\theta+\sin\theta}$, then $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta}$ is:

   (a) $1 + x$

   (b) $1 - x$

   (c) x

   (d) $\frac{1}{x}$

6. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y-axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \ldots, T_n$ returns the point $(1, 0)$ back to itself?

   (A) 359

   (B) 360

   (C) 719

   (D) 720

   (E) 721

7. Let x, y, and z be positive real numbers satisfying the system of equations:

$$\sqrt{2x - xy} + \sqrt{2y - xy} = 1$$
$$\sqrt{2y - yz} + \sqrt{2z - yz} = \sqrt{2}$$
$$\sqrt{2z - zx} + \sqrt{2x - zx} = \sqrt{3}$$

Then $[(1-x)(1-y)(1-z)]^2$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

(Please provide a numerical answer between two () brackets)

8. The ratio of w to x is $4:3$, the ratio of y to x is $3:2$, and the ratio of z to x is $1:6$. What is the ratio of w to y?

   (A) $4:3$

   (B) $3:2$

   (C) $8:3$

   (D) $4:1$

   (E) $16:3$

9. A game show offers a contestant three prizes A, B and C, each of which is worth a whole number of dollars from \$1 to \$9999 inclusive. The contestant wins the prizes by correctly guessing the price of each prize in the order A, B, C. As a hint, the digits of the three prices are given. On a particular day, the digits given were 1, 1, 1, 1, 3, 3, 3. Find the total number of possible guesses for all three prizes consistent with the hint.

   (Please provide a numerical answer between two () brackets)

10. Reduced to lowest terms, $\frac{a^2-b^2}{ab} - \frac{ab-b^2}{ab-a^2}$ is equal to:

    (A) $\frac{a}{b}$

    (B) $\frac{a^2-2b^2}{ab}$

    (C) a^2

    (D) $a-2b$

    (E) None of these

11. What is the maximum value of $\frac{(2^t-3t)t}{4^t}$ for real values of t?

    (A) $\frac{1}{16}$

    (B) $\frac{1}{15}$

    (C) $\frac{1}{12}$

    (D) $\frac{1}{10}$

(E) $\frac{1}{9}$

12. If $(\sqrt{8}+i)^{50} = 3^{49}(a+ib)$ then a^2+b^2 is:

    (a) 3

    (b) 8

    (c) 9

    (d) $\sqrt{8}$

13. Ellina has twelve blocks, two each of red (R), blue (B), yellow (Y), green (G), orange (O), and purple (P). Call an arrangement of blocks *even* if there is an even number of blocks between each pair of blocks of the same color. For example, the arrangement R B B Y G G Y R O P P O is even. Ellina arranges her blocks in a row in random order. The probability that her arrangement is even is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

    (Please provide a numerical answer between two () brackets)

14. Find the smallest positive integer k such that $2^{91}+k$ is divisible by 127.

    (A) 122

    (B) 123

    (C) 124

    (D) 125

    (E) 126

15. A straight river that is 264 meters wide flows from west to east at a rate of 14 meters per minute. Amr and Mohamed sit on the south bank of the river with Amr a distance of D meters downstream from Mohamed. Relative to the water, Amr swims at 80 meters per minute, and Mohamed swims at 60 meters per minute. At the same time, Amr and Mohamed begin swimming in straight lines to a point on the north bank of the river that is equidistant from their starting positions. The two men arrive at this point simultaneously. Find D.

    (Please provide a numerical answer between two () brackets)

16. If $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \ldots$ to n terms is S, then S is equal to:

(a) $\frac{n(n+3)}{4}$

(b) $\frac{n(n+2)}{4}$

(c) $\frac{n(n+1)(n+2)}{6}$

(d) n^2

17. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then which of the following is not correct?

    (a) $\sin 2\theta - \cos 2\alpha = 0$

    (b) $\sin\alpha + \cos\alpha = \pm\sqrt{2}\cos\theta$

    (c) $\cos 2\theta = \sin 2\alpha$

    (d) $\sin\theta - \cos\theta = \pm\sqrt{2}\sin\theta$

18. The lengths of the sides of a triangle with positive area are $\log_{10} 12$, $\log_{10} 75$, and $\log_{10} n$, where n is a positive integer. Find the number of possible values for n.

    (A) 2

    (B) 3

    (C) 4

    (D) 5

    (E) 6

19. When $x^{13} + 1$ is divided by $x - 1$, the remainder is:

    (A) 1

    (B) -1

    (C) 0

    (D) 2

    (E) None of these answers

20. The roots of the polynomial $10x^3 - 39x^2 + 29x - 6 = 0$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

    (A) $\frac{24}{5}$

(B) $\frac{42}{5}$

(C) $\frac{81}{5}$

(D) 30

(E) 48

21. If $\tan x = \frac{b}{a}$ then $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$ where $a > b > 0$, is equal to:

(a) $2 \sin x / \sqrt{\sin 2x}$

(b) $2 \cos x / \sqrt{\cos 2x}$

(c) $2 \cos x / \sqrt{\sin 2x}$

(d) $2 \sin x / \sqrt{\cos 2x}$

22. An aquarium has a rectangular base that measures 100 cm by 40 cm and has a height of 50 cm. It is filled with water to a height of 40 cm. A brick with a rectangular base that measures 40 cm by 20 cm and a height of 10 cm is placed in the aquarium. By how many centimeters does the water rise?

(A) 0.5

(B) 1

(C) 1.5

(D) 2

(E) 2.5

23. Positive real numbers a and b have the property that

$$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$$

and all four terms on the left are positive integers, where log denotes the base 10 logarithm. What is ab?

(A) 10^{52}

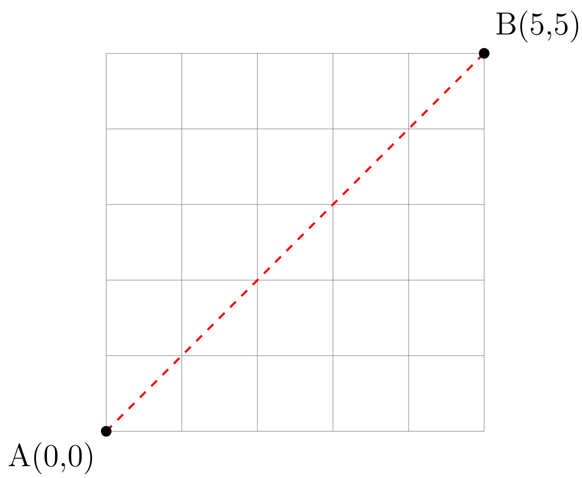
(B) 10^{100}

(C) 10^{144}

(D) 10^{164}

(E) 10^{200}

24. How many paths are there from point A(0,0) to point B(5,5) on the grid below, using only steps to the right or up, such that the path never goes above the main diagonal line $y = x$?



(A) 35

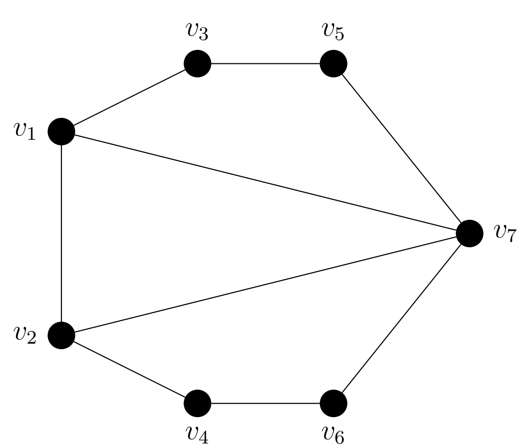
(B) 42

(C) 126

(D) 252

(E) 56

25. Consider the graph G shown below. How many spanning trees does G have?



(This is an open-ended question. Please provide a numerical answer.)