Final Round Exam

- Each answer is worth 10 points.
- The answer is evaluated based on the solution, and the mathematical writing.
- You should upload the answers as one scanned PDF.
- You have only 10 minutes right before you submit your answers, any switching tabs violation before the 10 minutes will be counted.

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1. For all positive real numbers x, y, z, prove that

$$x^4 + y^4 + z^2 \geq \sqrt{8}xyz.$$

2. Prove that $\log_2(3)$ is irrational.

3. Find all triples (a, b, c) of positive integers with $a \leq b \leq c$ such that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1.$$

4. Suppose that P is a polyhedron, all of whose faces are triangles. Suppose that P has F faces and E edges. Prove that $2E = 3F$.

5. Let $ABCD$ be a convex quadrilateral with $\angle CBD = 2\angle ADB$, $\angle ABD = 2\angle CDB$ and $AB = CB$. Prove that $AD = CD$.

6. Consider an equilateral triangle of sidelength n, broken down into equilateral triangles of sidelength 1, as shown in the figure below when $n = 5$. How many paths are there from the top triangle to the middle triangle (or the left of middle in the case that n is even) in the bottom row, such that adjacent triangles in the path share a common edge and never go up or revisit a triangle? One such path is shown in the Figure below.