Clasificación de E.D.

a) Verificar que cada función dada es una solución de la E.D.

13. $y'' + y = \sec t, \quad 0 < t < \pi/2; \quad y = (\cos t)\ln \cos t + t \sin t$

$y' = -(\sin t)\ln \cos t + (\cos t)\frac{1}{\cos t}(\sin t) + \sin t + t\cos t = -(\sin t)\ln \cos t + t\cos t$

$y'' = -\cos(t)\ln(\cos(t)) - \sin(t)\frac{1}{\cos t}\cdot(-\sin t) + \cos t - t\sin t$

$y'' + y = \cos(t)\ln(\cos t) + t\sin(t) - \cos(t)\ln(\cos(t)) + \tan(t)\sin(t) + \cos(t) - t\sin t$

$y'' + y = \tan(t)\sin(t) + \cos(t) = \frac{\sin^2(t)}{\cos(t)} + \cos(t) = \frac{\sin^2(t) + \cos^2(t)}{\cos(t)} = \frac{1}{\cos(t)} = \sec t$

14. $y' - 2ty = 1 \qquad y = e^{t^2}\underbrace{\int_0^t e^{-s^2}ds}_{z} + e^{t^2}$

$y' = \frac{d}{dt}(e^{t^2}z + e^{t^2})$

$= 2te^{t^2}\int_0^t e^{-s^2}ds + e^{t^2}e^{-t^2} + 2te^{t^2} = 2te^{t^2}\int_0^t e^{-s^2}ds + 2te^{t^2} + 1$

$y' - 2ty = 2te^{t^2}\int_0^t e^{-s^2}ds + 2te^{t^2} + 1 - 2te^{t^2}\int_0^t e^{-s^2}ds - 2te^{t^2} = 1$

b) Determine los valores de r para que la E.D tenga solución $y = e^{rt}$

$y' = e^{rt}\cdot r$

$y' = e^{rt}\cdot r^2$

17. $y'' + y' - 6y = 0 = r^2e^{rt} + re^{rt} - 6e^{rt}$

Note que $e^{rt} \neq 0$

$0 = r^2 + r - 6 = (r+3)(r-2)$

$r + 3 = 0 \quad o \quad r - 2 = 0$

$r = -3 \quad o \quad r = 2$

$r = -3 \quad y = e^{-3t} \quad y' = -3e^{-3t} \quad y'' = 9e^{-3t}$

$9e^{-3t} - 3e^{-3t} - 6e^{-3t} = e^{-3t}(9-3-6) = 0$

$r = 2 \quad y = e^{2t} \quad y' = e^{2t}\cdot 2 \quad y'' = 4e^{2t}$

$4e^{2t} + 2e^{2t} - 6e^{2t} = e^{2t}(4+2-6) = 0$

Tanto $r=3$ y $r=2$ son soluciones. $y(t) = C_1e^{-3t} + C_2e^{2t}$

$$y'' - 5y' + 6y = 0$$

$$r^2 e^{rt} - 5 r e^{rt} + 6 e^{rt}$$

$$r^2 - 5r + 6$$

$$(r-3)(r-2) \quad r=3 \quad r=1$$

$$r=3 \quad 9e^{3t} - 15e^{3t} + 6e^{3t} = 0$$

$$r=2 \quad 4e^{2t} - 10e^{2t} + 6e^{2t} = 0$$

$$\frac{dy}{dx} = 2y+3 \quad \int \frac{1}{2y+3}\,dy = \int dx \qquad \begin{matrix} u = 2y+3 \\ du = 2dy \end{matrix}$$

$$\frac{1}{2}\ln(2y+3) = x + C$$

$$\ln(2y+3) = 2(x+c) \quad 2y+3 = e^{2(x+c)} \quad y = \frac{e^{2x+2c} - 3}{2}$$

$$y = e^{2x} \cdot \frac{e^{2c_1}}{2} - \frac{3}{2} \qquad K = \frac{e^{2c_1}}{2} \quad K > 0$$

$$y = Ke^{2x} - \frac{3}{2}$$

c. Determine r para la solución $y=t^r \quad t>0$

$y'=rt^{r-1}$

$y''=r(r-1)t^{r-2}$

19. $t^2y''+4ty'+2y=0$

$t^2r(r-1)t^{r-2}+4trt^{r-1}+2t^r$

$r(r-1)t^r+4rt^r+2t^r=0 \qquad t^r\neq 0 \quad t>0$

$r(r-1)+4r+2)=0$

$r^2-r+4r+2=0$

$r^2+3r+2=0$

$(r+2)(r+1)=0$

$r+2=0 \quad o \quad r+1=0$

$r=-2 \quad o \quad r=-1$

Si $r=-2 \quad t^{-2}=y$

$y'=-2t^{-3} \quad y''=6t^{-4}$

$t^26t^{-4}-4tt^{-3}+2t^{-2}$

$=6t^{-2}-4t^{-2}+2t^{-2}=4t^{-2}\neq 0$

Si $r=-1 \quad y=t^{-1}$

$y'=-t^{-2} \quad y''=2t^{-3}$

$2t^2t^{-3}-4t\,t^{-2}+2t^{-1}$

$=2t^{-1}-4t^{-1}+2t^{-1}=0$

$\boxed{r=-1}$

Factores de Integración

a) Solucione el problema de valor inicial

18. $ty'+2y=\sin t \qquad y(\pi/2)=1 \quad t>0$

$$\frac{dy}{dt}+\frac{2}{t}y=\frac{\sin t}{t} \qquad P=\frac{2}{t} \quad Q=\frac{\sin t}{t}$$

$$\int P(t)dt=\int\frac{2}{t}dt=2\ln(t) \qquad \mu(t)=e^{\int P(t)dt}=e^{2\ln(t)}=e^{\ln(t^2)}=t^2$$

$$\int Q\mu dt=\int\frac{\sin t}{t}\cdot t^2dt=\int t\cdot\sin t=-t\cos t+\sin t \qquad \begin{matrix} u=x & du=dx \\ dv=\sin t & v=\int\sin t=-\cos t\end{matrix}$$

$$y=\frac{\int Q(t)\mu(t)dt}{\mu(t)}+\frac{c}{\mu(t)}=\frac{-t\cos t+\sin t}{t^2}+\frac{C}{t^2}=-t^{-1}\cos t+t^{-2}\sin t+t^{-2}c=y$$

$$y(\pi/2)=1=\left(\tfrac{\pi}{2}\right)^{-2}\cdot 1+\left(\tfrac{\pi}{2}\right)^{-2}C$$

$$1-\frac{4}{\pi^2}=\frac{4}{\pi^2}C \qquad \frac{\pi^2}{4}-1=C$$

20. $ty'+(t+1)y=t \qquad \frac{dy}{dt}+\frac{t+1}{t}y=1 \qquad P=\frac{t+1}{t} \qquad Q=1$

$y(\ln 2)=1 \qquad t>0$

$$\int P(t)dt=\int\frac{t+1}{t}dt=\int 1+\frac{1}{t}dt=t+\ln t \qquad \mu(t)=e^{\int Pdt}=e^{t+\ln t}=e^t\cdot t$$

$$\int Q\cdot\mu\, dt = \int t e^{t} dt = \begin{matrix} u=t & du=dt \\ dv=e^{t} & \\ v=\int e^{t}dt=e^{t} & \end{matrix} = t\cdot e^{t} - \int e^{t} dt = t\cdot e^{t} - e^{t}$$

$$y = \frac{\int Q\mu dt}{\mu} + \frac{C}{\mu} = \frac{t\cdot e^{t} - e^{t}}{e^{t}\cdot t} + \frac{C}{e^{t}\cdot t} = 1 - t^{-1} + Ce^{-t}t^{-1}$$

$$y(\ln 2) = 1 = 1 - (\ln 2)^{-1} + C e^{-\ln 2}\cdot(\ln 2)^{-1}$$
$$= 1 - \frac{1}{\ln 2} + C\frac{1}{2\ln 2}$$

$$\frac{1}{\ln 2} = C\frac{1}{2\ln 2} \to C = 2$$

b. Halle el valor de y_0 para que la solución permanezca finita mientras $t \to \infty$.

30. $y' - y = 1 + 3\sin t$, $y(0) = y_0$ $\quad P = -1 \quad Q = 1 + 3\sin t$

$\int P dt = -\int dt = -t \qquad \mu(t) = e^{-t}$

Inv Ln Alg Trig Exp
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$$\int Q\mu dt = \int (1+3\sin t)e^{-t} dt = \int e^{-t} + 3e^{-t}\sin t\, dt \overset{\substack{u=-t \\ du=-dt}}{=} -e^{-t} + 3\int e^{-t}\sin t \quad \begin{matrix} u=e^{-t} & du=-e^{-t}dt \\ dv=\sin t & v=\int \sin t = -\cos t \end{matrix}$$

$$= -e^{-t}\cos t - \int e^{-t}\cos t\, dt \quad \begin{matrix} u=e^{-t} & du=-e^{-t}dt \\ dv=\cos t & v=\int\cos t dt = \sin t \end{matrix} = -e^{-t}\cos t - (e^{-t}\sin t - \int -\sin t e^{-t} dt$$

$$\int e^{-t}\sin t = -e^{-t}\cos t - e^{-t}\sin t - \int \sin t e^{-t} dt \Rightarrow \int e^{-t}\sin t = -\tfrac{1}{2}e^{-t}\cos t - \tfrac{1}{2}e^{-t}\sin t$$

$$\int (1+3\sin t)e^{-t}dt = -e^{-t} - \tfrac{3}{2}e^{-t}\cos t - \tfrac{3}{2}e^{-t}\sin t$$

$$y = \frac{-e^{-t} - \frac{3}{2}e^{-t}\cos t - \frac{3}{2}e^{-t}\sin t}{e^{-t}} + \frac{C}{e^{-t}} = -1 - \tfrac{3}{2}\cos t - \tfrac{3}{2}\sin t + Ce^{t}$$

$$y(0) = y_0 = -1 - \tfrac{3}{2} + C \qquad y_0 = -\tfrac{5}{2} + C$$

31. $y' - \frac{3}{2}y = 3t + 2e^{t}$ $\quad y(0) = y_0 \quad P = -\frac{3}{2} \quad Q = 3t + 2e^{t}$

$\int P dt = \int -\frac{3}{2} dt = -\frac{3}{2}t \qquad \mu(t) = e^{-\frac{3}{2}t}$

$$\int Q\mu dt = \int (3t + 2e^{t})e^{-\frac{3}{2}t} dt = \int 3te^{-\frac{3}{2}t} + 2e^{-\frac{t}{2}} dt = -2e^{-\frac{3}{2}t}t - \tfrac{4}{3}e^{-\frac{3}{2}t} - 4e^{-\frac{t}{2}}$$

$$y = \frac{-2te^{-\frac{3}{2}t} - \frac{4}{3}e^{-\frac{3}{2}t} - 4e^{-\frac{t}{2}}}{e^{-3/2t}} + \frac{C}{e^{-3/2t}} = -2t - \tfrac{4}{3} - 4e^{t} + Ce^{3/2t}$$

$$y(0) = y_0 = -\tfrac{4}{3} - 4 + C = C - \tfrac{16}{3} = y_0 \qquad C = y_0 + \tfrac{16}{3}$$

$$y_0 = -2t - \tfrac{4}{3} - 4e^{t} + (y_0 + \tfrac{16}{3})e^{3/2t}$$

Otro Libro

1. $\frac{dy}{dt}-5y=0 \quad P(t)=-5 \quad Q(t)=0$

$\int P(t)dt=\int -5\,dt=-5t \quad \mu(t)=e^{-5t} \quad \int Q(t)\cdot\mu(t)dt=\int 0\,dt=0+C \quad y=e^{5t}+C$

2. $\frac{dy}{dt}+2y=0 \quad P(t)=2 \quad Q(t)=0$

$\int P(t)dt=\int 2dt=2t \quad \mu(t)=e^{2t} \quad \int Q(t)\cdot\mu(t)dt=0+C \quad y=e^{-2t}\cdot C$

3. $\frac{dy}{dt}+y=e^{3t} \quad P(t)=1 \quad Q(t)=e^{3t}$

$\int P(t)dt=\int dt=t \quad \mu(t)=e^{t} \quad \int Q(t)\mu(t)dt=\int e^{3t}\cdot e^{t}dt=\int e^{4t}dt \;\; \begin{smallmatrix}u=4t\\ du=4dt\end{smallmatrix} =\frac{1}{4}e^{4t}+C \quad y=\frac{\frac{1}{4}e^{4t}}{e^{t}}+\frac{C}{e^{t}}=\frac{1}{4}e^{3t}+Ce^{-t}$

4. $3y'+12y=4 \quad y'+4y=\frac{4}{3} \quad P(t)=4 \quad Q(t)=\frac{4}{3}$

$\int P(t)dt=\int 4dt=4t \quad \mu(t)=e^{4t} \quad \int Q(t)\mu(t)dt=\int\frac{4}{3}e^{4t}dt=\frac{1}{3}e^{4t}+C \quad y=\frac{\frac{1}{3}e^{4t}+C}{e^{4t}}=\frac{1}{3}+Ce^{-4t}$

5. $y'+3t^2y=t^2 \quad P(t)=3t^2 \quad Q(t)=t^2$

$\int P(t)dt=\int 3t^2dt=t^3 \quad \mu(t)=e^{t^3} \quad \int Q(t)\mu(t)dt=\int t^2e^{t^3}dt \;\; \begin{smallmatrix}u=t^3\\ du=3t^2dt\end{smallmatrix} =\frac{1}{3}e^{t^3}+C \quad y=\frac{\frac{1}{3}e^{t^3}+C}{e^{t^3}}=\frac{1}{3}+Ce^{-t^3}$

6. $y'+2ty=t^3 \quad P(t)=2t \quad Q(t)=t^3$

$\int P(t)dt=\int 2tdt=t^2 \quad \mu(t)=e^{t^2} \quad \int t^3e^{t^2}dt \;\; \begin{smallmatrix}u=x^2 & dv=2xdx\\ dv=xe^{x^2} & \int xe^{x^2}dx\end{smallmatrix} \;\; \begin{smallmatrix}w=x^2\\ dw=2xdx\end{smallmatrix} \;\; v=\frac{1}{2}e^{x^2} =\frac{1}{2}e^{x^2}x^2-\int\frac{1}{2}e^{x^2}2xdx=\frac{1}{2}x^2e^{x^2}-\frac{1}{2}e^{x^2}+C=\frac{1}{2}(x^2e^{x^2}-e^{x^2})+C$

$y=\frac{\frac{1}{2}x^2e^{x^2}-\frac{1}{2}e^{x^2}+C}{e^{x^2}}=\frac{1}{2}x^2-\frac{1}{2}+Ce^{-x^2}$

7. $y'+\frac{y}{t}=\frac{1}{t^2} \quad P(t)=\frac{1}{t} \quad Q(t)=\frac{1}{t^2}$

$\int P(t)dt=\int\frac{1}{t}dt=Ln(t) \quad \mu(t)=e^{Ln(t)}=t \quad \int Q(t)\mu(t)dt=\int t\cdot\frac{1}{t^2}dt=\int\frac{1}{t}dt=Ln(t)+C \quad y=\frac{Ln(t)+C}{t}=t^{-1}Ln(t)+Ct^{-1}$

8. $y'-2y=t^2+5 \quad P(t)=-2 \quad Q(t)=t^2+5 \quad \begin{smallmatrix}t^2 & e^{-2t}\\ 2t & -\frac{1}{2}e^{-2t}\\ 0 & \frac{1}{4}e^{-2t}\end{smallmatrix}$

$\int P(t)dt=\int -2dt=-2t \quad \mu(t)=e^{-2t} \quad \int Q(t)\mu(t)dt=\int e^{-2t}(t^2+5)dt=\int t^2e^{-2t}+5e^{-2t}dt=-\frac{1}{2}t^2e^{-2t}-\frac{1}{2}te^{-2t}-\frac{5}{2}e^{-2t}+C$

$y=\frac{-\frac{1}{2}t^2e^{-2t}-\frac{1}{2}te^{-2t}-\frac{1}{2}e^{-2t}+C}{e^{-2t}} \quad y=-\frac{1}{2}t^2-\frac{1}{2}t-\frac{1}{2}+Ce^{2t}$

9. $y'-\frac{y}{t}=t\,\text{sen}\,t \quad P(t)=-\frac{1}{t} \quad Q(t)=t\,\text{sen}(t)$

$\int P\,dt=\int -\frac{1}{t}dt=-Ln(t) \quad \mu=e^{-Ln(t)}=e^{Ln(t^{-1})}=t^{-1} \quad \int Q\mu dt=\int \text{sen}\,t\,dt=-\cos t+C \quad y=\frac{C-\cos t}{t^{-1}}=Ct-t\cos t$

10. $y'+\frac{2}{t}y=\frac{3}{t} \quad P(t)=\frac{2}{t} \quad Q(t)=\frac{3}{t}$

$\int P\,dt=\int\frac{2}{t}dt=2Ln\,t \quad \mu=e^{2Ln\,t}=e^{Ln\,t^2}=t^2 \quad \int Q\cdot\mu dt=\int\frac{3}{t}\cdot t^2dt=\int 3tdt=\frac{3}{2}t^2+C \quad y=\frac{\frac{3}{2}t^2+C}{t^2}=\frac{3}{2}+Ct^{-2}$

11. $y'+\frac{4}{t}y=t^2-1 \quad P=\frac{4}{t} \quad Q=t^2-1$

$\int P\,dt=\int\frac{4}{t}dt=4Ln(t) \quad \mu=e^{4Ln\,t}=e^{Ln\,t^4}=t^4 \quad \int Q\mu dt=\int t^4(t^2-1)dt=\int t^6-t^4dt=\frac{t^7}{7}-\frac{t^5}{5}+C \quad y=\frac{\frac{t^7}{7}-\frac{t^5}{5}+C}{t^4}=\frac{t^3}{7}-\frac{t}{5}+Ct^{-4}$

12. $y'-\frac{t}{1+t}y=t \quad P=\frac{t}{1+t} \quad Q=t$

$\int P\,dt=\int\frac{t}{1+t}dt \;\; \begin{smallmatrix}u=1+t\\ du=dt\end{smallmatrix} =\int\frac{u-1}{u}du=\int 1-\frac{1}{u}du=\int u-Ln(u)=1+t-Ln(1+t) \quad \mu=e^{1+t-Ln(1+t)}=\frac{e^{1+t}}{1+t} \quad \int Q\mu dt=\int\frac{e^{1+t}}{1+t}t\,dt \;\; \begin{smallmatrix}u=1+t\\ du=dt\end{smallmatrix} \int\frac{e^u}{u}(u-1)du$

$\int e^u-\frac{e^u}{u}du=e^u-\int\frac{e^u}{u}du+C \quad y=\frac{e^u-\int\frac{e^u}{u}du+C}{\frac{e^{1+t}}{1+t}}=\frac{1+t}{e^{1+t}}\left(e^u-\int\frac{e^u}{u}du\right)+C$

Variable Separable

$u=3y \quad du=3dy \qquad v=2x \quad dv=2dx$

19. $\sin 2x\,dx + \cos 3y\,dy = 0 \qquad \int \cos 3y\,dy = \int \sin 2x\,dx \qquad \frac{1}{3}\cdot\sin(3y) = \frac{1}{2}\cos 2x + C$

$\sin(3y) = \frac{3}{2}\cos(2x) + 3C \qquad 3y = \sin^{-1}\left(\frac{3}{2}\cos(2x) + 3C\right) \qquad y = \frac{1}{3}\cdot\sin^{-1}\left(\frac{3}{2}\cos(2x)+3C\right)$

$y(\pi/2) = \pi/3 = \frac{1}{3}\cdot\sin^{-1}\left(\frac{3}{2}\cos\left(\frac{\pi}{2}\right) + 3C\right)$

$\pi/3 = \frac{1}{3}\cdot\sin^{-1}\left(-\frac{3}{2} + 3C\right)$

$\pi = \sin^{-1}\left(-\frac{3}{2} + 3C\right)$

$0 = -\frac{3}{2} + 3C \qquad C = \frac{1}{2} \qquad y = \frac{1}{3}\cdot\sin^{-1}\left(\frac{3}{2}\cos(2x) + \frac{3}{2}\right)$

25. $y' = \frac{2\cos 2x}{3+2y} \qquad y(0) = -1 \qquad \frac{dy}{dx} = \frac{2\cos(2x)}{3+2y} \qquad 3+2y\,dy = 2\cos 2x\,dx$

$u=2x \quad du=2dx$

$\int 3+2y\,dy = \int 2\cos 2x\,dx \qquad 3y + y^2 = \sin(2x) + C \qquad y^2 + 3y + \frac{9}{4} = \sin(2x) + C + \frac{9}{4} \qquad \left(y+\frac{3}{2}\right)^2 = \sin(2x) + C + \frac{9}{4}$

$y + \frac{3}{2} = \pm\sqrt{\frac{9}{4} + \sin(2x) + C} \qquad y = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \sin(2x) + C}$

$y(0) = -1 = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \sin(0) + C} \qquad \left(\frac{1}{2}\right)^2 = \frac{9}{4} + C \qquad -2 = C$

26. $y' = 2(1+x)(1+y^2),\ y(0) = 0 \qquad \frac{dy}{dx} = 2(1+x)(1+y^2) \qquad dy = 2(1+x)(1+y^2)\,dx$

$\int \frac{1}{1+y^2}\,dy = \int 2(1+x)\,dx \qquad \tan^{-1}(y) = 2x + x^2 + C \qquad y = \tan(2x + x^2 + C)$

$y(0) = 0 = \tan(C) \qquad C = 0 \qquad y = \tan(2x + x^2)$

Ecuaciones Exactas

1. $(2x+3)+(2y-2)\frac{dy}{dx}=0$ $(2y-2)\frac{dy}{dx}=-2x-3$ $(2y-2)dy+(2x+3)dx=0$ $N=2y-2$ $M=2x+3$

$\frac{\partial M}{\partial y}=0 = \frac{\partial N}{\partial x}=0$

$\int 2x+3dx=x^2+3x+C(y)$ $C'(y)=2y-2$ $C(y)=y^2-2y$

$f(x,y)=x^2+3x+y^2-2y$ $\frac{\partial f}{\partial x}=2x+3=M$ $\frac{\partial f}{\partial y}=2y-2=N$

3. $(3x^2-2xy+2)dx+(6y^2-x^2+3)dy=0$ $M=3x^2-2xy+2$ $N=6y^2-x^2+3$

$\frac{\partial M}{\partial y}=-2x$ $\frac{\partial N}{\partial x}=-2x$

$\int 6y^2-x^2+3dy=2y^3-x^2y+3y+C(x)=f(x,y)$ $\frac{\partial f}{\partial x}=-2xy+C'(x)=3x^2-2xy+2$ $C'(x)=3x^2+2$ $C(x)=x^3+2x$

$f(x,y)=2y^3-x^2y+3y+x^3+2x$ $\frac{\partial f}{\partial x}=-2xy+3x^2+2=M$ $\frac{\partial f}{\partial y}=6y^2-x^2+3$

7. $(e^x\sin y-2y\sin x)dx+(e^x\cos y+2\cos x)dy=0$ $M=e^x\sin y-2y\sin x$ $N=e^x\cos y+2\cos x$

$\frac{\partial M}{\partial y}=e^x\cos y-2\sin x = \frac{\partial M}{\partial x}=e^x\cos y-2\sin x$

$\int e^x\sin y-2y\sin x\,dx=e^x\sin y+2y\cos x+C(y)=f(x,y)$ $\frac{\partial f}{\partial y}=e^x\cos y+2\cos x+C'(y)=e^x\cos y+2\cos x$ $C'(y)=0$ $C(y)=0$

$f(x,y)=e^x\sin y+2y\cos x$ $\frac{\partial f}{\partial x}=e^x\sin y-2y\sin x$ $\frac{\partial f}{\partial y}=e^x\cos y+2\cos x$

15. $(xy^2+bx^2y)dx+(x+y)x^2dy=0$ $M=xy^2+bx^2y$ $N=x^3+yx^2$

$\frac{\partial M}{\partial y}=2xy+bx^2$ $\frac{\partial N}{\partial x}=3x^2+2xy$ $\therefore$ $b=3$ y ser ecuación exacta

$\int xy^2+3x^2y\,dx=\frac{x^2y^2}{2}+x^3y+C(y)=f(x,y)$ $\frac{\partial f}{\partial y}=x^2y+x^3+C'(y)=x^3+x^2y$ $C'(y)=0$ $C(y)=C_2$ cte

$f(x,y)=\frac{x^2y^2}{2}+x^3y+C_2$ $\frac{\partial f}{\partial x}=xy^2+3x^2y$ $\frac{\partial f}{\partial y}=x^2y+x^3$

20. $\left(\frac{\sin y}{y}-2e^{-x}\sin x\right)dx+\left(\frac{\cos y+2e^{-x}\cos x}{y}\right)dy=0$ $\mu(x,y)=ye^x$ $M=\frac{\sin y}{y}-2e^{-x}\sin x$

$\mu Mdx+\mu Ndy=0 \rightarrow (e^x\sin y-2y\sin x)dx+(e^x\cos y+2\cos x)\ dy=0$ $M=e^x\sin x-2y\sin x$

$\frac{\partial M}{\partial y}=e^x\cos y-2\sin x$ $\frac{\partial N}{\partial x}=e^x\cos y-2\sin x$

$\int e^x\cos y+2\cos x\,dy=-e^x\operatorname{sen} y+2y\cos x+C(x)=f(x,y)$ $\frac{\partial f}{\partial x}=-e^x\operatorname{sen} y-2y\operatorname{sen} x+C'(x)=e^x\sin y-2y\sin x$ $C'(x)=2e^x\sin y$ $C(x)=2e^x\sin y$

$f(x,y)=e^x\operatorname{sen} y+2y\cos x$ $\frac{\partial f}{\partial x}=e^x\sin y-2y\sin x$ $\frac{\partial f}{\partial y}=e^x\cos y+2\cos x$

27. $dx+\left(\frac{x}{y}-\sin y\right)dy=0$ $M=1$ $N=\frac{x}{y}-\sin y$

$\frac{\partial M}{\partial y}=0$ $\frac{\partial N}{\partial x}=\frac{1}{y}$

$\frac{M_y-N_x}{N}=\frac{0-\frac{1}{y}}{\frac{x}{y}-\sin y}=\frac{-1}{x-y\sin y}$ ← No $\frac{N_x-M_y}{M}=\frac{\frac{1}{y}}{1}=\frac{1}{y}$ ✓ $\int\frac{1}{y}dy=\ln(y)$ $\mu=e^{\ln(y)}=y$

$\mu Mdx+\mu Ndy=0$ $ydx+(x-y\sin y)dy=0$ $\frac{\partial M}{\partial y}=1 = \frac{\partial N}{\partial x}=1$

$\int ydx=xy+C(y)=f(x,y)$ $\frac{\partial f}{\partial y}=x+C'(y)=x-y\sin y$ $C'(y)=-y\sin y$ $C(y)=\int -y\sin y\,dy=y\cos y-\sin y$

$f(x,y)=xy+y\cos(y)-\sin(y)$ $\frac{\partial f}{\partial x}=y$ ✓ $\frac{\partial f}{\partial y}=x+\cos(y)-y\sin y-\cos(y)=x-y\sin y$ ✓

29. $e^xdx+(e^x\cot y+2y\csc y)dy=0$ $M=e^x$ $N=e^x\cot y+2y\csc y$ $\frac{\partial M}{\partial y}=0$ $\frac{\partial N}{\partial x}=e^x\cot y$

$\frac{M_y-N_x}{N}=\frac{0-e^x\cot y}{e^x\cot y+2y\csc y}=?$ $\frac{N_x-M_y}{M}=\frac{e^x\cot y-0}{e^x}=\cot y$ $\int\cot y\,dy=\ln\sin y$ $\mu=e^{\ln\sin y}=\sin y$ $2y\tan y$ $2\ln\sec y$

$\mu Mdx+\mu Ndy=0$ $e^x\operatorname{sen} y\,dx+(\cos y\, e^x+2y\quad)dy$ $\frac{\partial M}{\partial y}=e^x\cos y$ $\frac{\partial N}{\partial x}=e^x\cos y$

$\int e^x\operatorname{sen} y\,dx=e^x\operatorname{sen} y+C(y)=f(x,y)$ $\frac{\partial f}{\partial y}=e^x\cos y+C'(y)=e^x\cos y+2$ $C'(y)=2y$ $C(y)=y^2$

$f(x,y)=e^x\operatorname{sen} y+y^2$

30. $\left[4\left(\frac{x^3}{y^2}\right)+\left(\frac{3}{y}\right)\right]dx+\left[3\left(\frac{x}{y^2}\right)+4y\right]dy=0$ $\quad \frac{\partial M}{\partial y}=-8x^3y^{-3}-3y^{-2}$ $\quad \frac{\partial N}{\partial x}=3y^{-2}$

$\frac{N_x-M_y}{M}=\frac{3y^{-2}+3y^{-2}+8x^3y^{-3}}{4x^3y^{-2}+3y^{-1}}=\frac{6y^{-2}+8x^3y^{-3}}{4x^3y^{-2}+3y^{-1}}=\frac{\frac{6}{y^2}+\frac{8x^3}{y^3}}{\frac{4x^3}{y^2}+\frac{3}{y}}=\frac{\frac{6y+8x^3}{y^3}}{\frac{4x^3+3y}{y^2}}=\frac{y^2(6y+8x^3)}{y^3(4x^3+3y)}=\frac{(3y+4x^3)2}{(4x^3+3x)y}=\frac{2}{y}$ $\quad \int\frac{2}{y}dy=2\ln(y)$ $\quad N=e^{2\ln(y)}=y^2$

$(4x^3+3y)dx+(3x+4y^3)dy=0$ $\quad M=4x^3+3y$ $\quad N=3x+4y^3$ $\quad \frac{\partial M}{\partial y}=3$ $\quad \frac{\partial N}{\partial x}=3$

$\int 4x^3+3y\,dx=x^4+3yx+C'(y)$ $\quad \frac{\partial f}{\partial y}=3x+C'(y)=3x+4y^3$ $\quad C'(y)=4y^3$ $\quad C(y)=y^4$

$f(x,y)=x^4+3xy+y^4$ $\quad \frac{\partial f}{\partial x}=4x^3+3x$ $\quad \frac{\partial f}{\partial y}=3x+4y^3$

Otro Libro

1. $(2x-1)dx+(3y+7)dy=0$ $\quad M=2x-1$ $\quad N=3y+7$ $\quad \frac{\partial M}{\partial y}=0$ $\quad \frac{\partial N}{\partial x}=0$

$\int 2x-1\,dx=x^2-x+C(y)=f(x,y)$ $\quad \frac{\partial f}{\partial y}=C'(y)=3y+7$ $\quad C(y)=\frac{3}{2}y^2+7y$

$f(x,y)=x^2-x+\frac{3}{2}y^2+7y$ $\quad \frac{\partial f}{\partial x}=2x-1$ $\quad \frac{\partial f}{\partial y}=3y+7$

3. $(5x+4y)dx+(4x-8y^3)dy=0$ $\quad M=5x+4y$ $\quad N=4x-8y^3$ $\quad \frac{\partial M}{\partial y}=4$ $\quad \frac{\partial N}{\partial x}=4$

$\int 5x+4y\,dx=\frac{5}{2}x^2+4yx+C(y)=f(x,y)$ $\quad \frac{\partial f}{\partial y}=4x+C'(y)=4x-8y^3$

4. $(\text{sen}\,y-y\,\text{sen}\,x)dx+(\cos x+x\cos y-y$

31. $\left(3x+\frac{6}{y}\right)dx+\left(\frac{x^2}{y}+3\frac{y}{x}\right)dy$ $\quad \frac{\partial M}{\partial y}=-6y^{-2}$ $\quad \frac{\partial N}{\partial x}=2xy^{-1}-3y^{2}$

$$\frac{M_y-N_x}{N}=\frac{-\frac{6}{y^2}-\frac{2x}{y}-\frac{3}{y^2}}{x^2/y+3y/x}=\frac{\frac{-9-2xy}{y}}{\frac{x^3+3y^2}{xy}}=\frac{xy(-9-2xy)}{y(x^3+3y^2)}$$

$$\frac{N_x-M_y}{M}=\frac{\frac{2x}{y}-\frac{3}{y^2}+\frac{6}{y^2}}{3x+\frac{6}{y}}=\left(\frac{\frac{2xy+3}{y^2}}{\frac{3xy+6}{y}}\right)=\frac{y(2xy+3)}{y^2(3xy+6)}=y(2xy+3)$$