Transformada de Laplace

↳ Definición

① → Bilateral

→ ROC

→ estabilidad

→ Unilateral

• Nuevos tipos de señales

· left-sided

· Right-sided

· bilateral (Finite time)

· Positive Time

· Negative Time

→ Transformada de Fourier

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

$$\rightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega$$

Transformada inversa.

Transformada de Laplace

$$\boxed{X(s) := \mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt}$$

$$s = \sigma + j\omega \Rightarrow s = j\omega, \quad \sigma = 0$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} \, dt$$

Convergencia ⟷ Existencia de la transformada

$$X(j\omega) = \mathcal{F}\{x(t)\}$$

$$\rightsquigarrow \quad x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$$

$$\underline{s = j\omega}$$

$$\boxed{s = \sigma + j\omega \,, \quad \sigma = 0}$$

$$X(s) = \mathcal{F}\{x(t)\}$$

$$x(t) \xleftrightarrow{\mathcal{F}} X(s)$$

$$X(s) = \mathcal{L}\{x(t)\}$$

$$X(s) \xleftrightarrow{\mathcal{L}} x(t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\,dt < \infty$$

$t \longleftrightarrow \omega$

- Transf
- Rap

$\boxed{x(t)} \longleftrightarrow$

ω_0 — Series Fourier - Espectro

ω — Transformada de Fourier

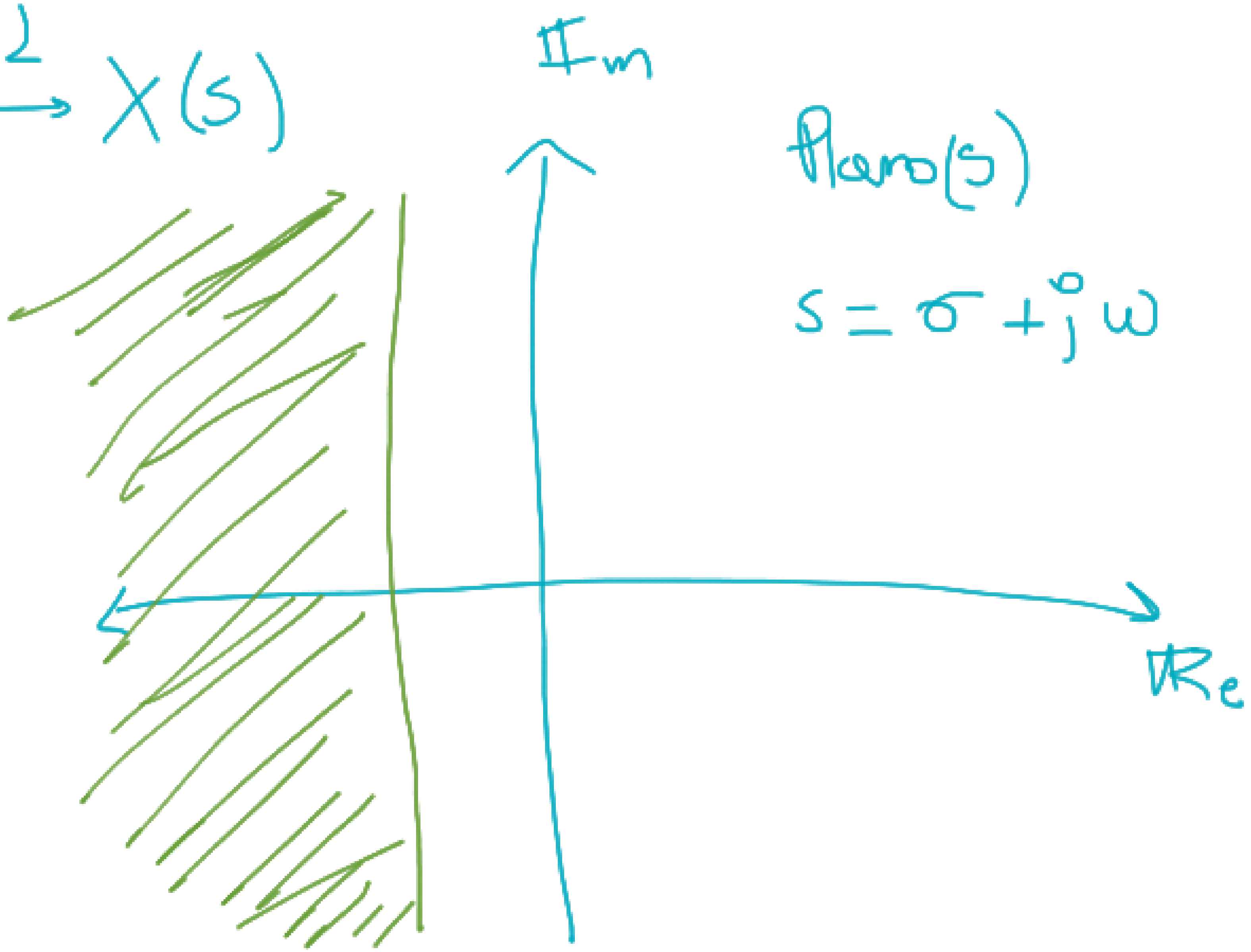
Convergencia

Existencia.

$x(t) \xleftrightarrow{L} X(s)$

Plano(s)

$s = \sigma + j\omega$



ROC

$\rightarrow s \rightarrow$ Intervalo de valores para los cuales la integral de la transformada de laplace converge convergencia

$$X(j\omega) = \frac{1}{j\omega + a}, \quad a > 0$$

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

$$X(j\omega) = \mathcal{F}\{x(t)\} =$$

$$x(t) = e^{-at} u(t)$$

$a \to >0, <0, =0$

$$\int_{-\infty}^{\infty} |e^{-at} u(t)|\, dt \quad \xrightarrow{a>0} \quad \int_{0}^{\infty} e^{-at} dt = -\frac{1}{a} e^{-at} \Big|_{0}^{\infty} = -\frac{1}{a}(0-1) = \frac{1}{a} < \infty$$

$$\int_{0}^{\infty} e^{-at} dt \quad \rightarrow \quad \boxed{a < 0} \quad \int_{0}^{\infty} e^{at} dt = -\frac{1}{a} e^{at} \Big|_{0}^{\infty} = \infty \not< \infty$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} e^{-at} u(t)\, e^{-st}\, dt = \int_{0}^{\infty} e^{-at} \cdot e^{-st}\, dt$$

$$\rightsquigarrow \boxed{x(t) = e^{-at} u(t)}$$

$$= \int_0^{\infty} e^{-(s+a)t}\, dt = -\frac{1}{s+a}\left(e^{-(s+a)t}\right)\Big|_0^{\infty} = -\frac{1}{s+a}\left(e^{-\infty} - 1\right)$$

$$X(s) = -\frac{1}{\sigma+j\omega+a} \cdot e^{-(\sigma+j\omega+a)t}\Big|_0^{\infty} = -\frac{1}{\sigma+j\omega+a} \cdot e^{-(\sigma+a)t} \cdot e^{-j\omega t}$$

$$= -\frac{1}{\sigma+j\omega+a}\Big($$

$$= -\frac{1}{\sigma+j\omega+a}\, e^{-(\sigma+a)t} \cdot e^{-j\omega t}\Big|_0^{\infty} = \boxed{\frac{1}{s+a}}, \quad s+a > 0$$

$$= -\frac{1}{\sigma+j\omega+a} \cdot \boxed{e^{-(\sigma+a)t}} \cdot e^{-j\omega t}$$

$$\downarrow$$

$$\cos(\omega t) - j\sin(\omega t)$$

$$\sigma + a > 0$$

Convergencia ← $a \to ?$ $s \to ?$

$$= \frac{1}{\sigma+j\omega+a} = \frac{1}{j\omega+(\sigma+a)}$$

$$\sigma + a > 0$$

Fourier → $\hat{x}(t) = e^{-\sigma t}\, x(t)$

laplace → $x(t) = e^{-at}\, u(t)$

$$\boxed{\int_{-\infty}^{\infty} |\hat{x}(t)|\, dt < \infty}$$

$$\int_{-\infty}^{\infty} e^{-\sigma t} \cdot e^{-at} \cdot u(t)\, dt$$

$$\int_{0}^{\infty} e^{-\sigma t} \cdot e^{-at}\, dt = \int_{0}^{\infty} e^{-(\sigma+a)t}\, dt = -\frac{1}{\sigma+a} \cdot e^{-(\sigma+a)t}\Big|_{0}^{\infty} = \frac{1}{\sigma+a}, \quad \sigma+a>0$$

- $\sigma + a < 0$ → no converge.
- $\sigma + a > 0$

la región de convergencia de la transformada de Laplace para $x(t)$, son aquellos valores de s para los cuales. la transformada de Fourier de la señal $x(t)e^{-\sigma t}$ converge.

$x(t)$

Laplace

Fourier

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\int_{-\infty}^{\infty} |x(t)\, e^{-\sigma t}|\, dt < \infty$$

$$\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$$

$$\int_{-\infty}^{\infty} |\hat{x}(t)|\, dt < \infty$$