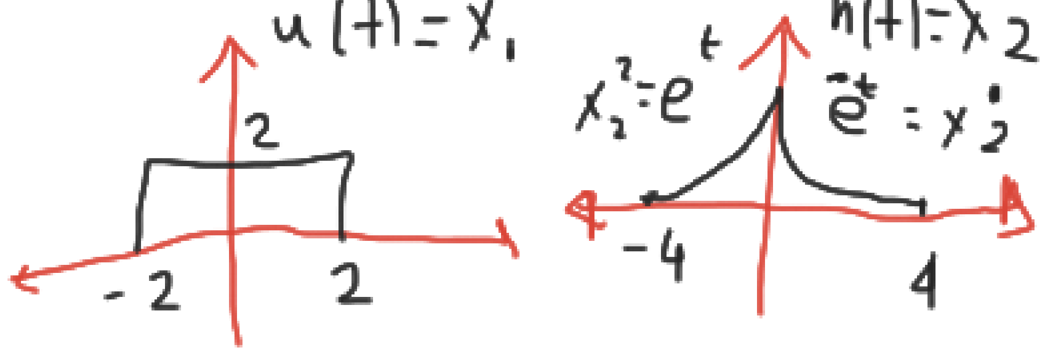
$$x_1 = 2[\hat{u}(t+2) - \hat{u}(t-2)]$$

$$x_2 = \begin{cases} 0 & t < -4 \\ e^{-|t|}, & -4 \le t \le 4 \\ 0, & 4 < t \end{cases}$$

$x_1 * x_2$?

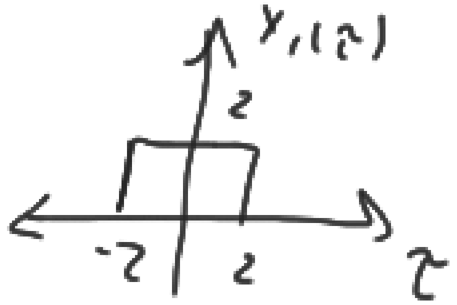
$\Rightarrow u(t) * h(t) = y(t)$

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$$

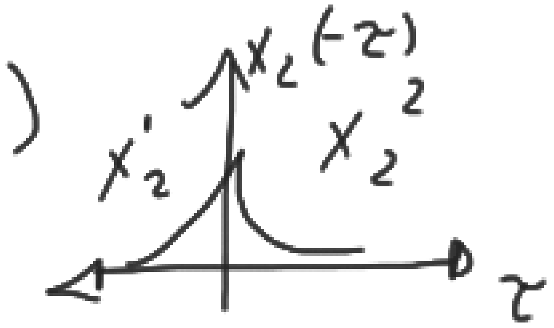


$\Rightarrow$ Encontrar intervalos de tiempo

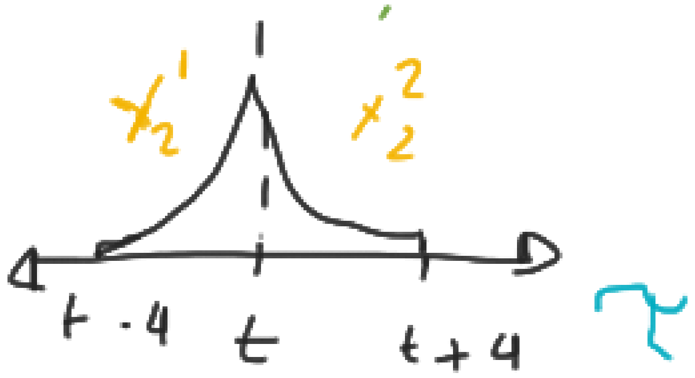
$\rightarrow x_1(\tau)$



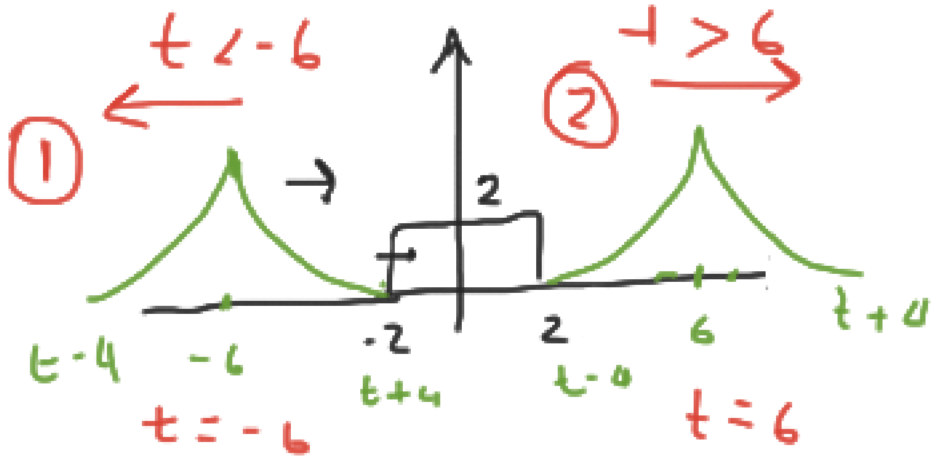
$\rightarrow x_2(\tau)$



$\rightarrow x_2(t-\tau)$



$\rightarrow$ Defini los intervalos



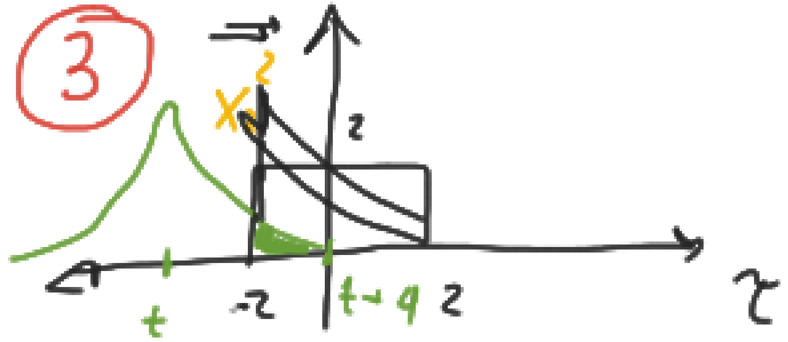
① $t < -6$

$y(t) = 0$

② $t > 6$

$y(t) = 0$

3



$$\int_{-2}^{t+4} 2e^{(t-\tau)}\, d\tau = 2e^{t} \int^{t+4} e^{-\tau}\, d\tau$$

$$= -2e^{t}\left(e^{-\tau}\right)\Big|_{-2}^{t+4}$$

$$= -2e^{t}\left[e^{-(t+4)} - e^{2}\right]$$

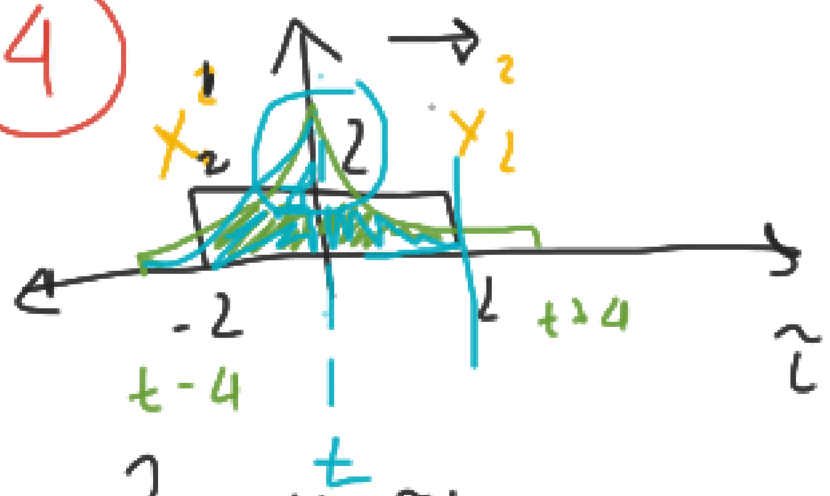
$$= -2e^{(t-t-4)} + 2e^{t}e^{2}$$

$$= -2e^{-4} + 2e^{t+2}$$

$\Rightarrow$ 3 $\quad -6 \le t < 2$

$$y(t) = -2e^{-4} + 2e^{t+2}$$

4



$$\int_{-2}^{2} 2e^{-|t-\tau|}\,d\tau$$

$$= 2\int_{-2}^{t} e^{-(t-\tau)}\,d\tau + 2\int_{t}^{2} e^{t-\tau}\,d\tau$$

$$= 2e^{-t}\int_{-2}^{t} e^{\tau}\,d\tau + 2e^{t}\int_{t}^{2} e^{-\tau}\,d\tau$$

$$= 2e^{-t}\left[e^{\tau}\right]_{-2}^{t} - 2e^{t}\left[e^{-\tau}\right]_{t}^{2}$$

$$= 2e^{-t}\left[e^{t} - e^{-2}\right] - 2e^{t}\left[e^{-2} - e^{-t}\right]$$

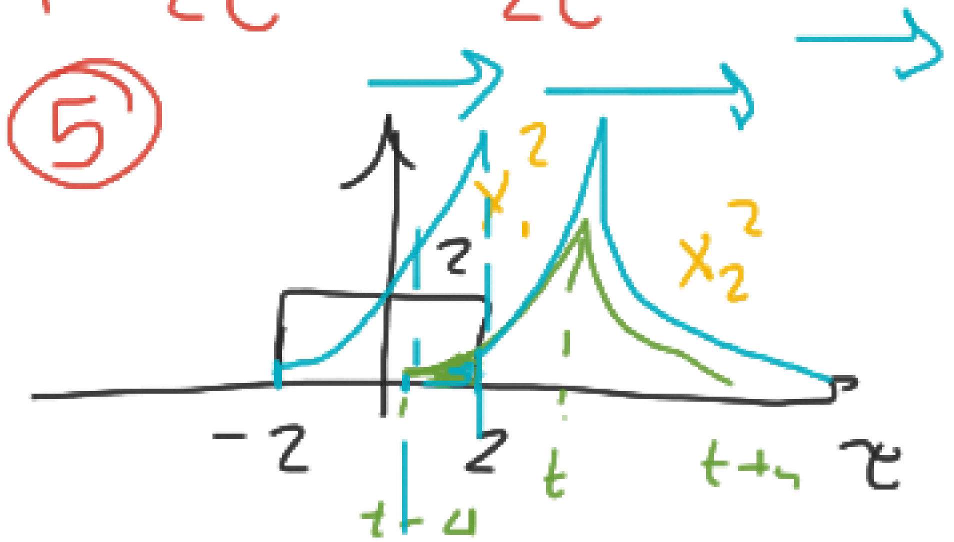
$$= 2 - 2e^{-(t+2)} - 2e^{t-2} + 2$$

$$= 4 - 2e^{-(t+2)} - 2e^{t-2}$$

4 $-2 < t \leq 2$

$$y(t) = 4 - 2e^{-(t+2)} - 2e^{t-2}$$

5



$$\int_{t-4}^{2} 2e^{-(t-\tau)}\,d\tau$$

$$= 2e^{-t}\int_{t-4}^{2} e^{\tau}\,d\tau$$

$$= 2e^{-t}\left[e^{\tau}\right]_{t-4}^{2}$$

$$= 2e^{-t}\left[e^{2} - e^{t-4}\right]$$

$$= 2e^{-(t-2)} - 2e^{-4}$$

5 $2 < t \leq 6$

$$y(t) = 2e^{-(t-2)} - 2e^{-4}$$

$\Rightarrow$ Solución

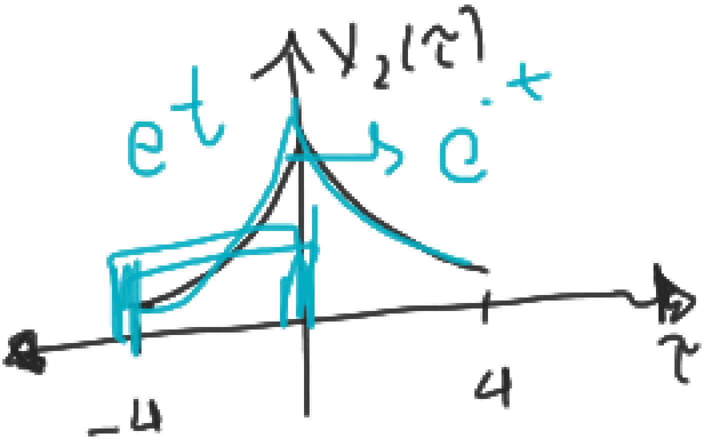
$$y(t) = \begin{cases} (1)\ \ 0, & \forall t \in (-\infty, -6) \\ (3)\ \ -2e^{-4} + 2e^{t+2}, & \forall t \in [-6, -2] \\ (4)\ \ 4 - 2e^{-(t+2)} - 2e^{t-2}, & \forall t \in [-2, 2] \\ (5)\ \ 2e^{-(t-2)} - 2e^{-4}, & \forall t \in [2, 6) \\ (2)\ \ 0, & \forall t \in [6, \infty) \end{cases}$$

La solución debe ser igual si

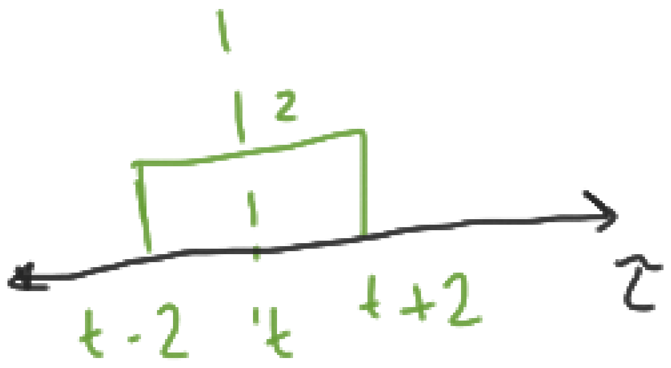
$x_2 * x_1 \rightarrow$

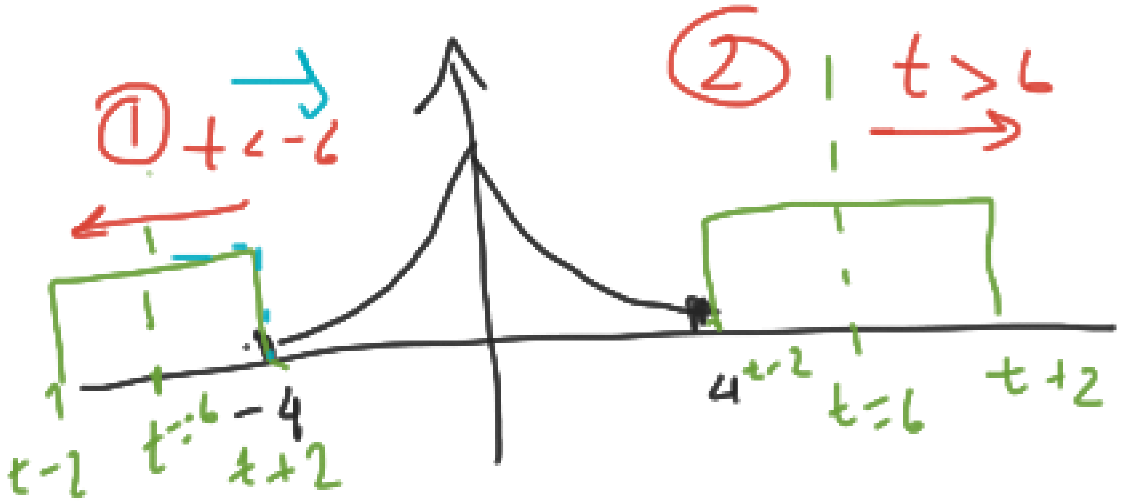
$x_2 * x_1$

$$y(t) = \int_{-\infty}^{\infty} x_2(\tau)\, x_1(t-\tau)\, d\tau$$



$x_1(t-\tau)$





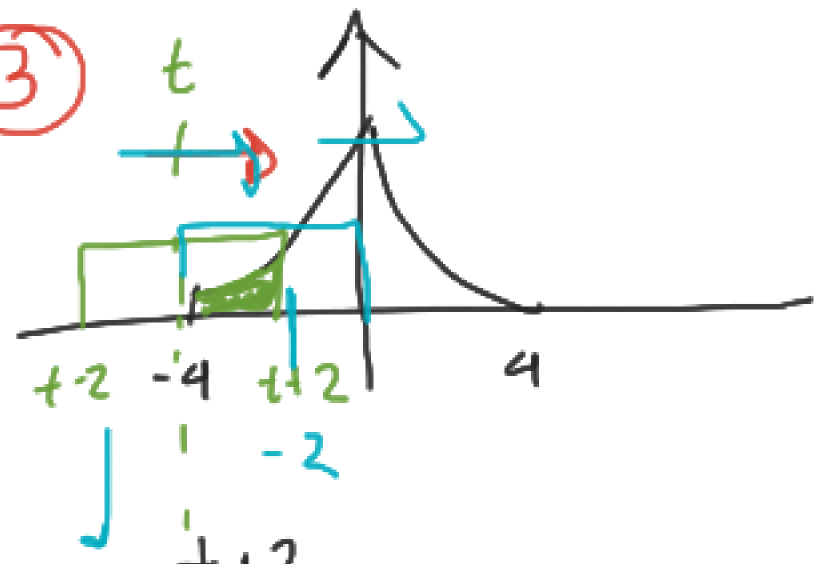
① $t < -6$

$y(t) = 0$

② $t > 6$

$y(t) = 0$

③



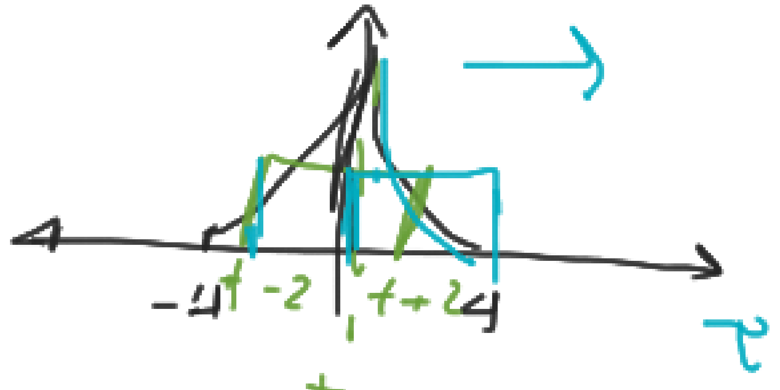
$$\int_{-4}^{t+2} 2\cdot e^{\tau}\, d\tau = 2\left[e^{t+2} - e^{-4}\right]$$

$$= 2e^{t+2} - 2e^{-4}$$

③ $-6 \le t < -2$

$y(t) = 2e^{t+2} - 2e^{-4}$

④



$$\int_{t-2}^{t+2} 2e^{-|\tau|}\, d\tau$$

$$= \int_{t-2}^{0} 2e^{\tau}\, d\tau + \int_{0}^{t+2} 2e^{-\tau}\, d\tau$$

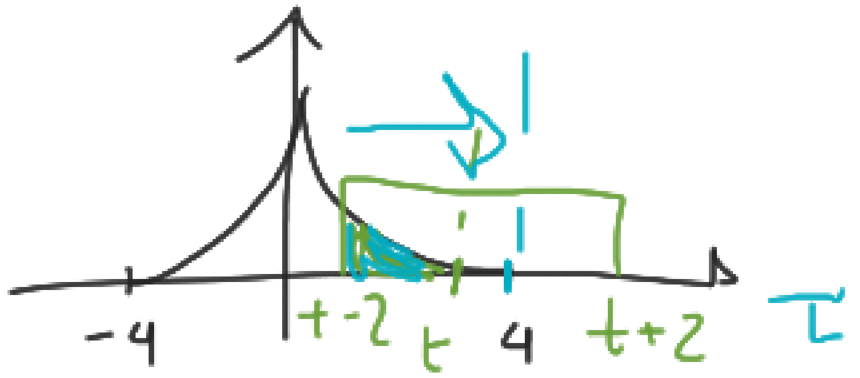
$$= 2 - 2e^{t-2} - 2e^{-(t+2)} + 2$$

$$= 4 - 2e^{t-2} - 2e^{-(t+2)}$$

④ $-2 \le t < 2$

$y(t) = 4 - 2e^{t-2} - 2e^{-(t+2)}$

5



$$\int_{t-2}^{4} 2e^{-\tau}d\tau = -2\left[e^{-\tau}\right]_{t-2}^{4}$$

$$= -2\left[e^{-4} - e^{-(t-2)}\right]$$

$$= -2e^{-4} + 2e^{-(t-2)}$$

5 $2 \le t < 6$

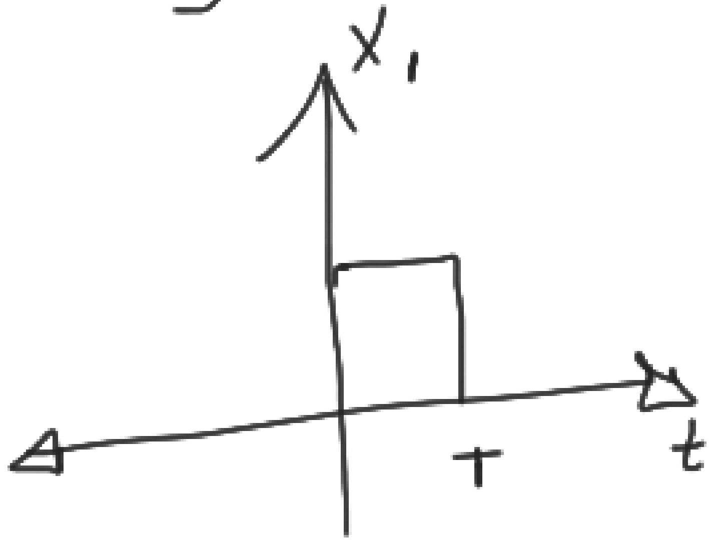
$$y(t) = -2e^{-4} + 2e^{-(t-2)}$$

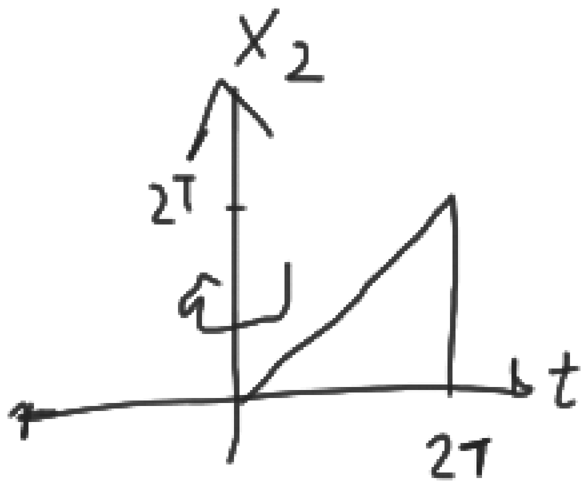
$$y(t) = \begin{cases} 0 & \forall\, t \in (-\infty, -6) \\ 2e^{t+2} - 2e^{-4} & \forall\, t \in [-6, -2) \\ 4 - 2e^{(-t+2)} - 2e^{t-2} & \forall\, t \in [-2, 2) \\ -2e^{-4} + 2e^{-(t-2)} & \forall\, t \in [2, 6) \\ 0 & \forall\, t \in [6, \infty) \end{cases}$$

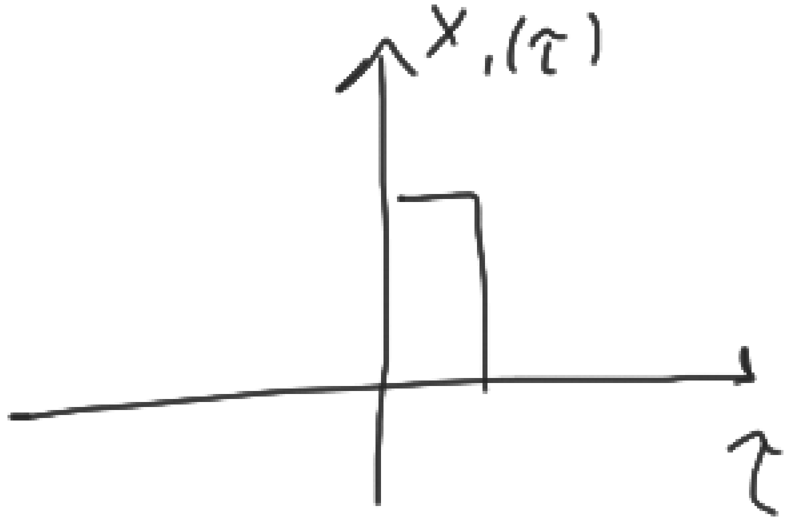
Entonces $x_1 * x_2 = x_2 * x_1$.

Ver video case 2.

Caso 3





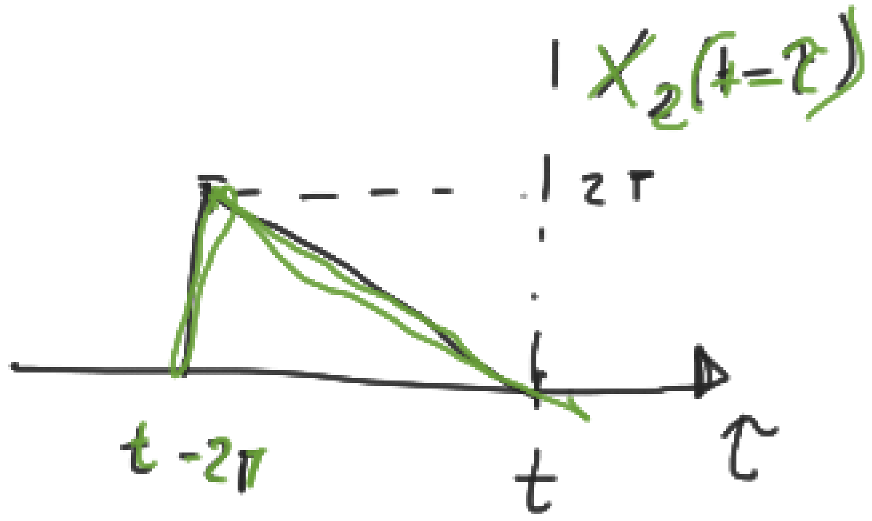


① $t<0$

$y(t)=0$

② $t>3T$

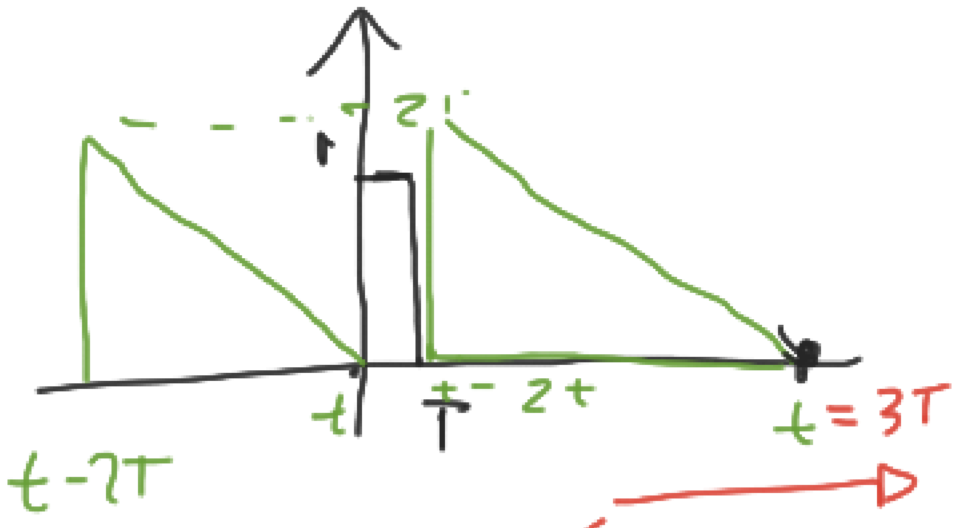
$y(t)=0$



$$\Rightarrow x_1(t)=\begin{cases}1 & \forall\, t\in[0,T]\\ 0 & \forall\, t\notin[0,T]\end{cases}$$

$$x_2(t)=\begin{cases}t & \forall\, t\in[0,2T]\\ 0 & \forall\, t\notin[0,2T]\end{cases}$$

$x_1 * x_2$?



① $t<0$

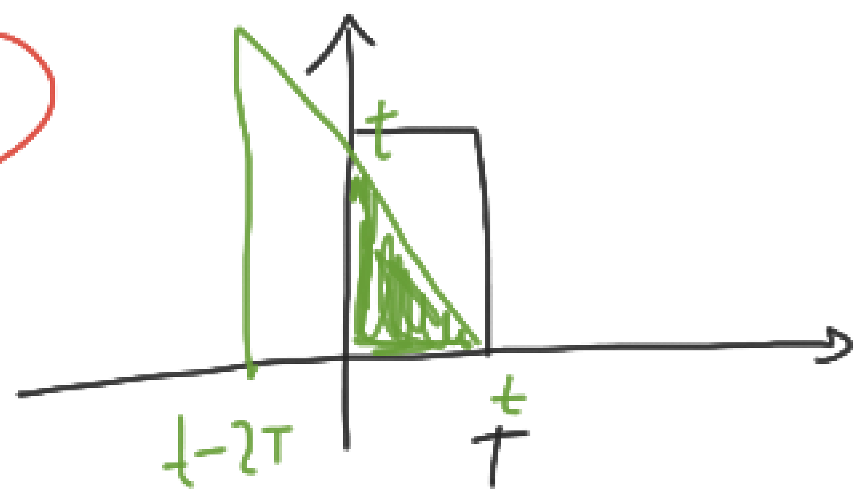
② $t>3T$

$t-2T>T$

$\rightarrow y(t)=0$

$t>3T$

3

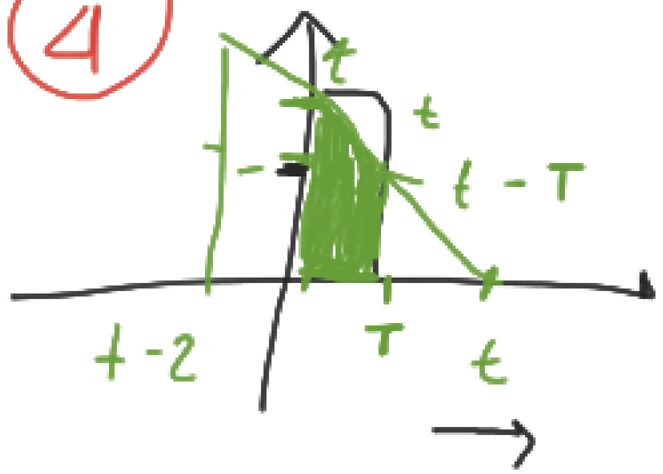


$\rightarrow$ $y(t) = t \cdot t \cdot \frac{1}{2}$

$= \frac{t^2}{2}$

$\forall\ 0 < t < T$

4



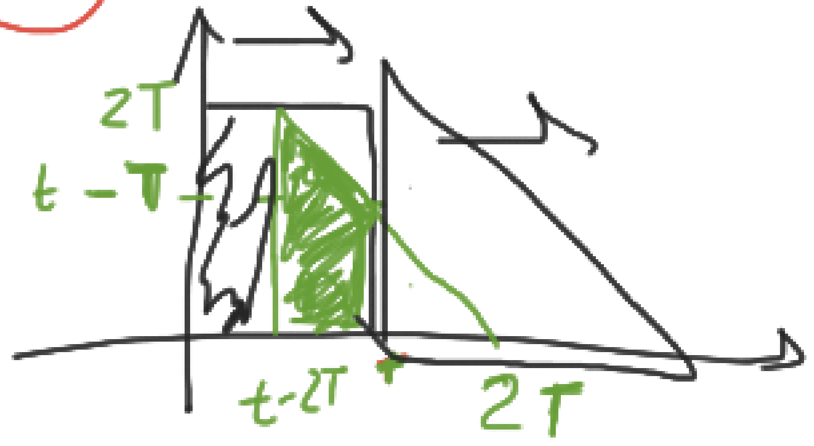
$y(t) = (t - T) \cdot T + (t - (t - T)) \cdot T \cdot \frac{1}{2}$

$= tT - T^2 + \frac{T^2}{2}$

$= tT - \frac{T^2}{2}$

$\forall\ T < t < 2T$

5



$y(t) = \frac{(2T - (t - T))(T - (t - 2T))}{2}$

$+ (T - (t - 2T))(t - T)$

$y(t) = (3T - t)(3T - t) \cdot \frac{1}{2}$

$+ (3T - t)(t - T)$

$y(t) = (9T^2 - 6Tt + t^2)\frac{1}{2}$

$+ 3Tt - 3T^2 - t^2 + T^2$

$y(t) = \frac{9T^2}{2} - \frac{6Tt}{2} + \frac{t^2}{2} + 4Tt - 3T^2 - t^2$

$= \frac{9T^2}{2} - 3Tt - \frac{t^2}{2} + 4Tt - 3T^2$

$= \frac{9T^2}{2} + Tt - 3T^2 - \frac{t^2}{2}$

$y(t) = -\frac{t^2}{2} + Tt + \frac{3T^3}{2}$

$\forall\ 2T < t < 3T$

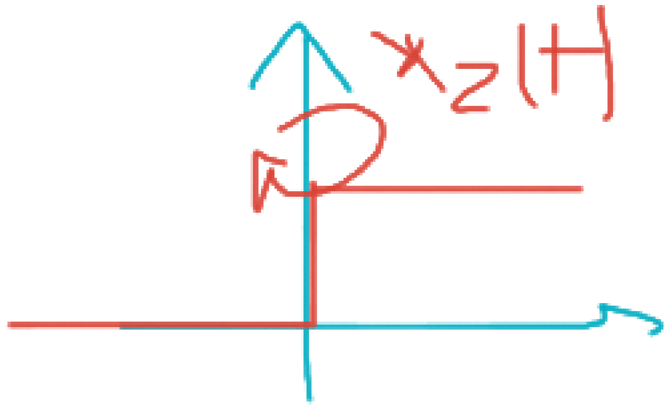
$$y(t)=\begin{cases} 0, & \forall\ t<0 \\ \dfrac{t^2}{2}, & \forall\ 0\le t<T \\ tT-\dfrac{T^2}{2}, & \forall\ T\le t<2T \\ -\dfrac{t^2}{2}+Tt+\dfrac{3T^2}{2}, & \forall\ 2T\le t\le 3T \\ 0, & \forall\ t>3T \end{cases}$$

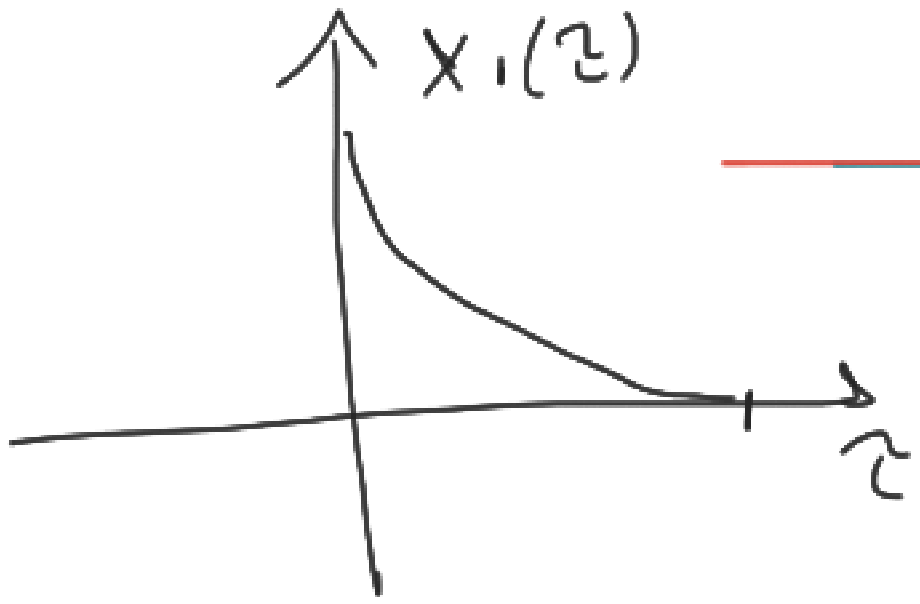
Case 1

$x_1 = e^{-t}\hat{u}(t)$

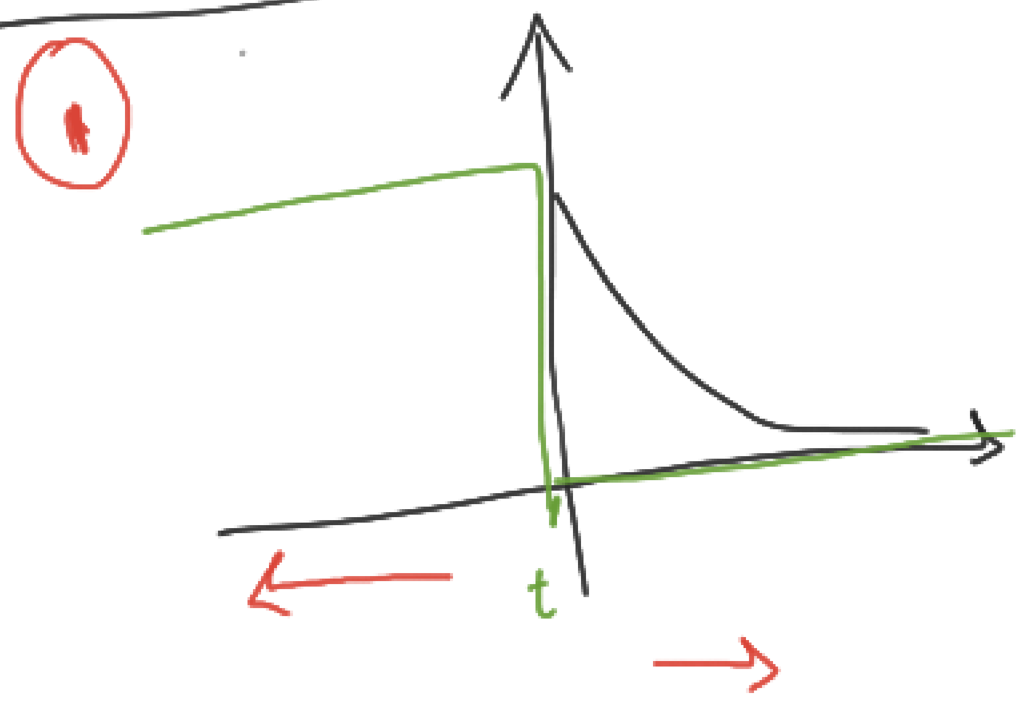
$x_2 = \hat{u}(t)$

$x_1 * x_2$



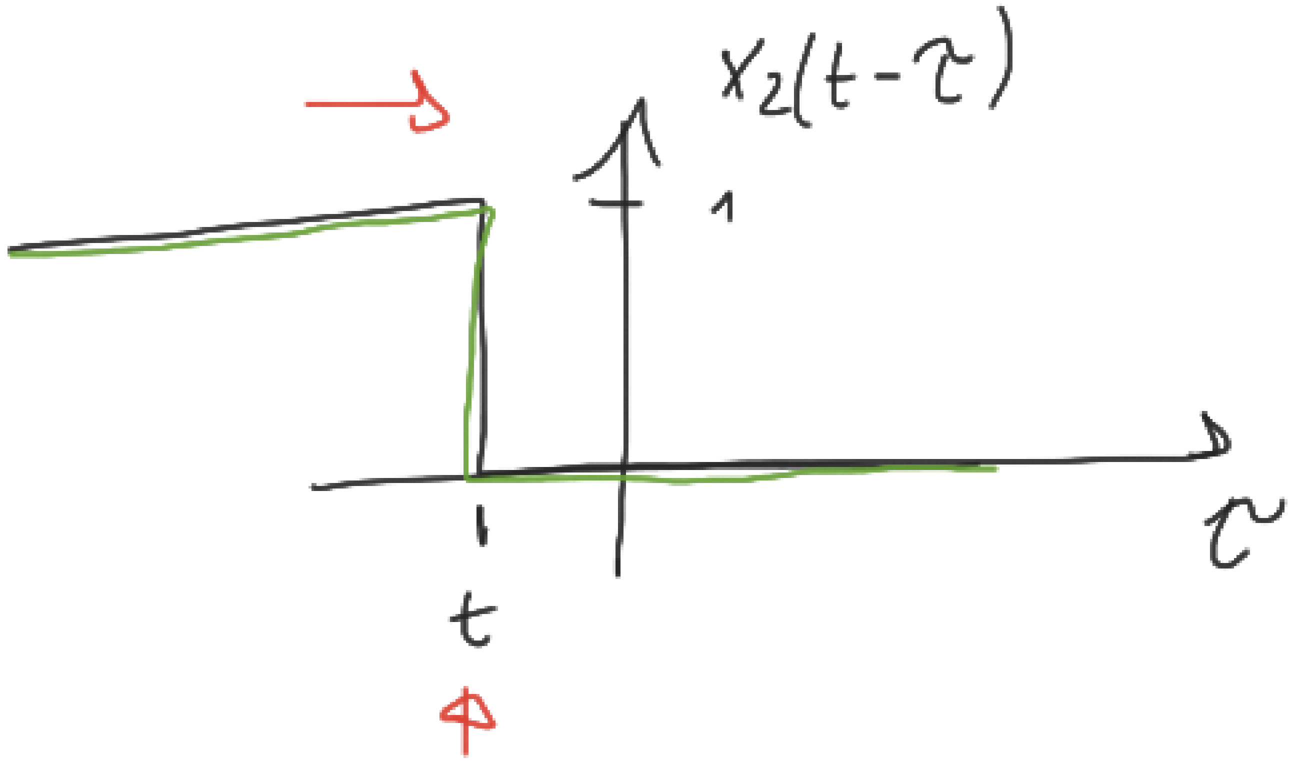


Ver video Case 3

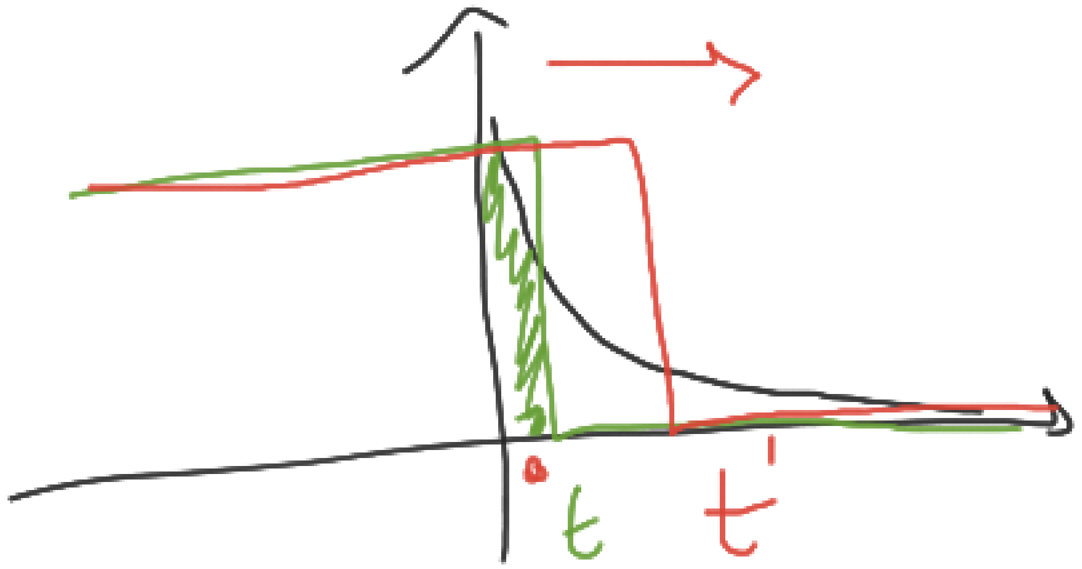


$\forall\ t<0$

$y(t)=0$



2



$$y(t) = \begin{cases} 0 & \forall t < 0 \\ 1 - e^{-t} & \forall t \geq 0 \end{cases}$$

$$\int_0^t e^{-\tau} d\tau = -e^{-t} + 1$$

$$= 1 - e^{-t}$$

$\forall t > 0$

Ver video case 1

Determine la estabilidad de

$$y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau$$

① Encuentre la respuesta impulso del sistema

$$h(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau = \hat{u}(t)$$

Evalúe la condición

$$\int_{-\infty}^{\infty} |h(\tau)|\,d\tau < \infty$$

$$\Rightarrow \int_{0}^{\infty} \hat{u}(\tau)\,d\tau = \tau \Big|_{0}^{\infty} = \infty$$

El sistema es inestable