ED Homogenea $a_0(x)y + a_1(x)\frac{dy}{dx} + \ldots + a_n(x)\frac{d^ny}{dx^n} = 0$

Principio de superposición: Si $y_1, \ldots, y_n$ son soluciones $\to y = C_1y_1 + \ldots + C_ny_n$

Teorema Sea $y_1, \ldots, y_n$ soluciones, son linealmente independientes sii

$W(y_1, y_2, \ldots, y_n) \neq 0 \quad | \quad W(y_1, y_2, \ldots, y_n) = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ \vdots & \vdots & & \vdots \\ y_1^{n-1} & y_2^{n-1} & \cdots & y_n^{n-1} \end{vmatrix}$

Coeficientes Constantes $a_0y + a_1y' + \ldots + a_ny^{(n)} = 0$

$ay'' + by' + cy = 0 \to am^2 + bm + c = 0 \qquad m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

1. $b^2 - 4ac > 0 \to m_1 \neq m_2 \quad y = C_1e^{m_1 \cdot x} + C_2e^{m_2 \cdot x}$
2. $b^2 - 4ac = 0 \quad m_1 = m_2 \quad y = C_1e^{m_1x} + C_2xe^{m_1x} + \ldots + C_nx^{n-1}e^{m_1 \cdot x}$
3. $b^2 - 4ac < 0 \quad \alpha = Re(m),\ \beta = Img(m) \quad y = C_1e^{\alpha x}\cos(\beta x) + e^{\alpha x}sen(\beta x)$ Salen 2.

Reducción de Orden

Llevar a la forma $y'' + P(x)y' + q(x)y = 0$ y y_1 una solución

$y_2 = y_1 \cdot \int \frac{e^{-\int P(x)dx}}{y_1(x)^2}dx$

ED No Homogenea $ay'' + by' + cy' = g(x)$

1. $ay'' + by' + cy = 0$ (Resolver) $= y_c$ (Complementaria)
2. Dos tecnicas para y_p solución particular

3) $y = y_c + y_p$

Coeficientes indeterminados

Polinomios, senos y euler $\to + \cdot -$

$10 \to A \quad 5 - x^2 \to Ax^2 + Bx + C \quad 7x^2e^{3x} \to (Ax^2 + Bx + C)e^{3x}$

$\cos(6x) = A\cos(6x) + B\,sen(6x) \quad 4x^2sen(x) \to (Ax^2 + Bx + C)sen(x) + (Dx^2 + Cx + E)\cos(x)$.

Se sacan las derivadas, se reemplazan en la ED y se igualan $g(x)$

Se sacan coeficientes de ecuaciones "x" y se despeja A, B. Reemplazar.

No funciona en tan, sec, $\sqrt{\ }$, log, $\frac{g(x)}{f(x)}$,

Variación de Parámetros. $a_0y + a_1y' + a_2y'' = g(x)$

1. Halle y_c. $y_1, y_2, y_n \ldots$ acompañan a las C_n.
2. $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \quad W_1 = \begin{vmatrix} 0 & y_2 \\ g(x) & y_2' \end{vmatrix} \quad u_1 = \int \frac{W_1}{W} \quad W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & g(x) \end{vmatrix} \quad u_2 = \int \frac{W_2}{W}$
3. $y_p = u_1y_1 + u_2y_2$

$y'' + 2y' + 5y = 4e^{-x}\cos(2x) \quad y_c: m^2 + 2m + 5 = 0 \quad m = -1 \pm 2i \quad \alpha = -1,\ \beta = 2 \quad y_c: C_1e^{-x}sen(2x) + C_2e^{-x}\cos(2x)$

$y_p = Axe^{-x}\cos(2x) + Bxe^{-x}\sin(2x)$

$y_p' = A[e^{-x}\cos(2x) - xe^{-x}\cos(2x) - 2xe^{-x}\sin(2x)] + B[e^{-x}\sin(2x) - xe^{-x}\sin(2x) + 2xe^{-x}\cos(2x)]$

$y_p'' = A[-2e^{-x}\cos(2x) - 4e^{-x}\sin(2x) + 4xe^{-x}\sin(2x) - 3xe^{-x}\cos(2x)] + B[-2e^{-x}\sin(2x) + 4e^{-x}\cos(2x) - 4xe^{-x}\cos(2x) - 3xe^{-x}\sin(2x)]$

$-4Ae^{-x}\sin(2x) + 4Be^{-x}\cos(2x) = 4e^{-x}\cos(2x) \quad e^{-x}\sin(2x): -4A = 0,\ A = 0$

$e^{-x}\cos(2x): 4B = 4,\ B = 1. \quad y = C_1e^{-x}sen(2x) + C_2e^{-x}\cos(2x) + xe^{-x}\sin(2x)$

$y' = C_1[-e^{-x}sen(2x) + 2e^{-x}\cos(2x)] + C_2[-e^{-x}\cos(2x) - 2e^{-x}\sin(2x)] + e^{-x}\sin(2x) - xe^{-x}\sin(2x) + 2xe^{-x}\cos(2x)$

$y(0) = 1 = C_2 \to C_2 = 1 \quad y'(0) = 0 = 2C_1 - C_2 \to C_1 = \frac{1}{2}$

$\therefore y = \frac{1}{2}e^{-x}sen(2x) + e^{-x}\cos(2x) + xe^{-x}\sin(2x)$

$xy'' - (1+x)y' + y = x^2e^{2x}$

- Si $y_1 = 1 + x \quad y_1' = 1 \quad y'' = 0$. Si $xy'' - (1+x)y' + y = 0 - (1+x) + (1+x) = 0$ ✓
- Sea $y'' + (-\frac{1+x}{x})y' + (\frac{1}{x})y = xe^{2x}$ y $y_1(x) = 1 + x$

$y_2 = y_1\int \frac{e^{-\int P(x)dx}}{y_1^2(x)}dx = (1+x)\int \frac{e^{-\int -\frac{1+x}{x}dx}}{(1+x)^2}dx = (1+x)\int \frac{e^{\ln(x)+x}}{(1+x)^2}dx = (1+x)\int \frac{e^xx}{(1+x)^2}dx \quad u = xe^x,\ du = (e^x + xe^x)dx \quad dv = (1+x)^{-2}dx,\ v = -\frac{1}{1+x}$

$= (1+x)\left[xe^x\left(\frac{-1}{1+x}\right) - \int -\frac{1}{1+x}(e^x + xe^x)dx\right] = (1+x)\left[\frac{-xe^x}{1+x} + \int e^xdx\right] = (1+x)\left[e^x - \frac{xe^x}{1+x}\right] = e^x + xe^x - xe^x = e^x$

$y_c = C_1(1+x) + C_2(e^x) \qquad W = \begin{vmatrix} 1+x & e^x \\ 1 & e^x \end{vmatrix} = e^x(1+x) - e^x = e^x + xe^x - e^x = xe^x$

$W_1 = \begin{vmatrix} 0 & e^x \\ xe^{2x} & e^x \end{vmatrix} = -e^x(xe^{2x}) \qquad u_1 = \int \frac{-e^x(xe^{2x})}{xe^x}dx = \int -e^{2x}dx = -\frac{1}{2}e^{2x}$

$W_2 = \begin{vmatrix} 1+x & 0 \\ 1 & xe^{2x} \end{vmatrix} = (1+x)xe^{2x} \qquad u_2 = \int \frac{(1+x)xe^{2x}}{xe^x}dx = \int e^x + xe^xdx = e^x + e^xx - e^x = xe^x$

$y_p = u_1y_1 + u_2y_2 = (1+x)(-\frac{1}{2}e^{2x}) + e^x(xe^x) = -\frac{1}{2}e^{2x} - \frac{1}{2}xe^{2x} + xe^{2x} = -\frac{1}{2}e^{2x} + \frac{1}{2}xe^{2x} = \frac{-e^{2x} + xe^{2x}}{2}$

$2x^{(5)} - 7x^{(4)} + 12x^{(3)} + 8x^{(2)} \to 2m^5 - 7m^4 + 12m^3 + 8m^2 = 0 \quad m^2(2m^3 - 7m^2 + 12m + 8) = 0$

$2m^3 - 7m^2 + 12m + 8 \mid 2m + 1 \quad m^2 - 4m + 8$

$m^2(2m+1)(m^2 - 4m + 8) \quad m_{1,2} = 0,\ m_3 = -\frac{1}{2},\ m_{4,5}: \alpha = 2,\ \beta = 2 \quad y = C_1 + C_2x + C_3e^{-\frac{1}{2}x} + C_4e^{2x}sen(2x) + C_5e^{2x}\cos(2x)$.

$y^{(3)} + 12y^{(2)} + 36y^{(1)} = 0,\ y(0) = 0,\ y'(0) = 1,\ y''(0) = -7 \to m^3 + 12m^2 + 36m = 0 \quad m(m^2 + 12m + 36) = 0 \quad m(m+6)^2$

$m_1 = 0,\ m_{2,3} = -6 \quad y = C_1 + C_2e^{-6x} + C_3xe^{-6x} \quad y(0) = 0 = C_1 + C_2 \quad y' = -6C_2e^{-6x} + C_3e^{-6x} - 6C_3xe^{-6x} \quad y'(0) = 1 = -6C_2 + C_3$

$y'' = 36C_2e^{-6x} - 12C_3e^{-6x} + 36xC_3e^{-6x} \quad y''(0) = -7 = 36C_2 - 12C_3 \therefore y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}e^{-6x}x$

Método Lineal

$\frac{dy}{dt} + P(t)y = Q(t)$

$\mu(t) = e^{\int P(t)dt}$ $\quad y = \frac{\int Q(t)\mu(t)dt}{\mu(t)} + \frac{C}{\mu(t)}$

Variables Separables

$y' = g(t) \cdot h(y)$

$\int \frac{1}{h(y)} dy = \int g(t)dt$. Despejar y.

Ecuaciones Exactas

$M dx + N dy = 0$

$* \frac{\partial f}{\partial x} = M \quad \frac{\partial f}{\partial y} = N *$

1) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

2) $\int M dx = f(x) + C(y) \quad \frac{\partial f}{\partial y} = N$ ó $\int N dy = f(y) + C(x) \quad \frac{\partial f}{\partial x} = M$

3) $f(x,y) = f(x) + C(y)$ ó $f(y) + C(x)$. $C(x)$ se encuentra y *

Factor Integrante

$\frac{M_y - N_x}{N} = h(x)$ ó $\frac{N_x - M_y}{M} = g(y)$ $\quad \int h(x)dx = \int g(y)dy \rightarrow \mu^{\int d}$

$\mu(?) M dx + \mu(?) N dy = 0$ (Resolver)

Modelos Matemáticos

Dinamica Poblacional

$\frac{dP}{dt} = KP$ $\quad P(t)$ población en t.

$P(t) = P_0 e^{Kt}$. Hallar P_0 y K despejando.

Decaimiento Radioactivo

$\frac{dA}{dt} = KA$ $\quad A(t)$ sustancia que queda.

$P(t) = P_0 e^{Kt}$ $\quad$ Vida Media $A(t) = \frac{1}{2} A_0$ $\quad \frac{1}{2} = e^{Kt}$ $\quad t = \frac{\ln(0.5)}{K}$

Ley de Enfriamiento

$\frac{dT}{dt} = K(T - T_m)$

$T(t)$ = temperatura en t. dT/dt rapidez cambio,
T_m es la temperatura ambiente

$T(t) = T_m + Ce^{Kt}$ $\quad$ C y K

Mezclas

$\frac{dA}{dt} = \text{Razon Entra}\left[\frac{m_a}{t}\right] - \text{Razón Sale}\left[\frac{m_a}{t}\right]$

Cantidad sal $[m_a]$

Si $\text{vel}_{\text{entra}} = \text{vel}_{\text{salida}}$ $\quad \frac{dA}{dt} = \text{Vel}_{\text{Entra}}\left[\frac{V_0}{t}\right] \cdot \text{Concen}_{\text{Entra}}\left[\frac{m_a}{V}\right] - \text{Vel}_{\text{Sali}}\left[\frac{V_0}{t}\right] \cdot \frac{A}{\text{Cap. Tanque}[V_0]} \rightarrow \frac{\partial A}{\partial t} + P(t)A = Q(t)$

Si $\text{vel}_{\text{entra}} \neq \text{vel}_{\text{sale}}$ $\quad \frac{dA}{dt} = \text{Vel}_{\text{Entra}}\left[\frac{V_0}{t}\right] \cdot \text{Concen}_{\text{Entra}}\left[\frac{m_a}{V}\right] - \text{Vel}_{\text{Sali}}\left[\frac{V_0}{t}\right] \cdot \frac{A}{\text{Cap. Tanque}[V_0] + (\text{Vel}_{\text{entra}} - \text{Vel}_{\text{Sale}})t}$

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