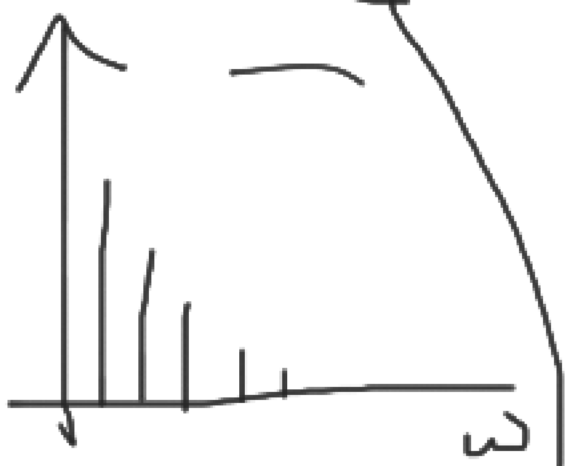
$$\underbrace{x(t)}_{\sim} \approx \underbrace{\sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t}}$$



* Convergencia.

$k = \underset{k}{\operatorname{argmin}}\ e$

(Periódica.)

$$e = \Big| \cancel{x}(t) - \underbrace{\sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t}} \Big| < \delta$$

x(t)
1 - integrabilidad absoluta $\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$ -
2 - # finito max - min en t finito -
3 - # " discontinuidades en t finito. -

Serie de Fourier.

$$X(\omega) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\,dt$$

* Convergencia —

· 1.
· 2.
· 3.

} x(t)

Existe la transf. Fourier para la señal x(t) si y solo si se cumplen cond. 1-3

$a \in (-\infty, \infty)$

$$\int_{-\infty}^{\infty} |e^{-at} \cdot u(t)|\,dt < \infty$$

$\int_0^{\infty} |e^{-at}|\,dt$

$-\frac{1}{a}(e^{-at})\Big|_0^{\infty}$

∞ × No

∞

$< \infty$ si

Transf. Fourier.

Convergencia

1.

Integrabilidad absoluta $\int_{-\infty}^{\infty} |e^{-at}\hat{u}(t)| dt = \int_{0}^{\infty} |e^{-at}| dt = -\frac{1}{a} e^{-at}\Big|_{0}^{\infty}$

si $a \in (-\infty, \infty)$

para $a = ?$

$\int_{-\infty}^{\infty} |x(t)| < \infty$

Fourier

$\rightarrow$ Para $\boxed{a>0}$ $\rightarrow$ converge

$$= -\frac{1}{a}(e^{-\infty} - e^{0}) = 1/a < \infty$$

$\rightarrow$ para $a<0$ $\rightarrow$ no converge

$$= -\frac{1}{a}(e^{\infty} - e^{0}) = \infty$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} (e^{-at}\hat{u}(t)) e^{-j\omega t} dt = \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} dt = \int_{0}^{\infty} e^{-(a+j\omega)t} dt = -\frac{e^{-(a+j\omega)t}}{a+j\omega t}\Bigg|_{0}^{\infty}$$

$$= -\frac{1}{a+j\omega t}(e^{-\infty} - e^{0}) = \frac{1}{a+j\omega t}, \quad \boxed{a>0}$$

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt}$$

Transformada de Laplace

- Convergencia de LT para $x(t)$ → Exista solución la integral

⤳ Condiciones que debe cumplir la señal.

$x(t)e^{-\sigma t}$

tales que la transformada de Fourier converja

$\leadsto s = \sigma + j\omega$

$$\rightarrow X(\sigma + j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-(\sigma + j\omega)t}\, dt$$

$$\int_{-\infty}^{\infty} \boxed{x(t)e^{-\sigma t}}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \hat{x}(t)\, e^{-j\omega t}\, dt$$

$\rightarrow \sigma \in (-\infty, \infty)$

1. $\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$ $\quad s = j\omega$, σ → Integrabilidad absoluta conv. Fourier

→ 1*. $\int_{-\infty}^{\infty} \| \underbrace{x(t) e^{-\sigma t}}_{\hat{x}} \|\, dt < \infty$ → Integrabilidad absoluta conv. Laplace

→ Integrabilidad absoluta conv. Fourier para $\hat{x}(t) = x(t)e^{-\sigma t}$

Transformada de Laplace (Bilateral)

$$x(t) = e^{-at} u(t)$$

ROC - TL

→ todos los valores de $\mathrm{Re}\{s\}$ para los cuales

$$\boxed{\int_{-\infty}^{\infty} |\underbrace{x(t)\, e^{-\sigma t}}_{\hat{x}}|\, dt < \infty}$$

$x_2(t) \Rightarrow X(s) = \frac{1}{a+s}$, $\mathrm{Re}\{s\} < -a$

σ

$x(t)$

$$\boxed{X(s) = \frac{1}{a+s},\ \boxed{\mathrm{Re}\{s\} > -a}}$$

→ $x(t) = e^{-at} u(t)$

∘ ROC

$$\int_{-\infty}^{\infty} |e^{-at}\hat{u}(t) \cdot e^{-\sigma t}|\, dt = \int_{0}^{\infty} |e^{-at} \cdot e^{-\sigma t}|\, dt = \int_{0}^{\infty} e^{-(a+\sigma)t}\, dt < \infty$$

∘ Transf.

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-at}\hat{u}(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-st}\, dt = \int_{0}^{\infty} \underbrace{e^{-at}\, e^{-\sigma t}}\, e^{-j\omega t}\, dt$$

$$= -\frac{1}{a+s}$$

$$\frac{\left(e^{-(a+s)t}\right)\Big|_{0}^{\infty}}{a+\sigma}$$

$$\sigma = \boxed{\mathrm{Re}\{s\} > -a}$$

$a + \sigma > 0$

$\sigma > -a$

Pasa bajas

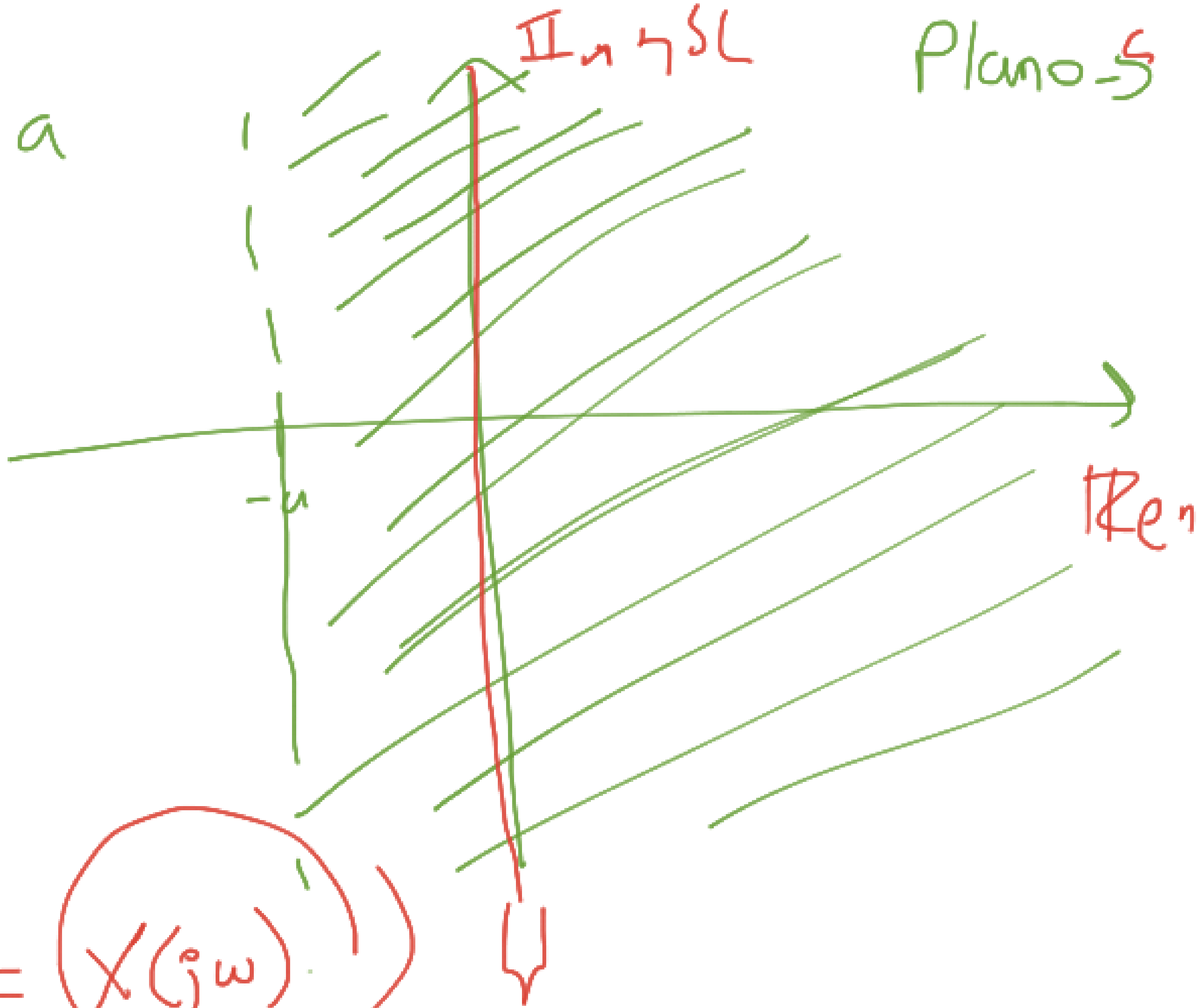
$$H(j\omega) = \begin{cases} 1 & \forall\ \omega < \omega_c \\ 0 & \forall\ \omega > \omega_c \end{cases}$$

Pasa altas

$$H(j\omega) = \begin{cases} 0 & \forall\ \omega < \omega_c \\ 1 & \forall\ \omega > \omega_c \end{cases}$$

$$\mathcal{L}\{x(t)\} = \quad X(s) = \underbrace{\frac{1}{s+a}} \quad \mathrm{Re}\{s\} > -a$$

$a > 0$



$$X(s) \triangleq \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$\sigma = 0$

$$X(s)\Big|_{s=j\omega} = X(j\omega)$$

$\sigma = 0$

$s = 0 + j\omega$

$s = j\omega$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

BIBO

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$$

SLIT

↳ T.L.R

ROC

$\sigma = 0$

$\sigma < 0$

$\sigma > 0$

$$\int_{-\infty}^{\infty} |h(t)|\, e^{-\sigma t}\, dt < \infty$$

todo

σ,

integral $< \infty$

ROC incluya $\boxed{\sigma = 0}$

$s = j\omega$

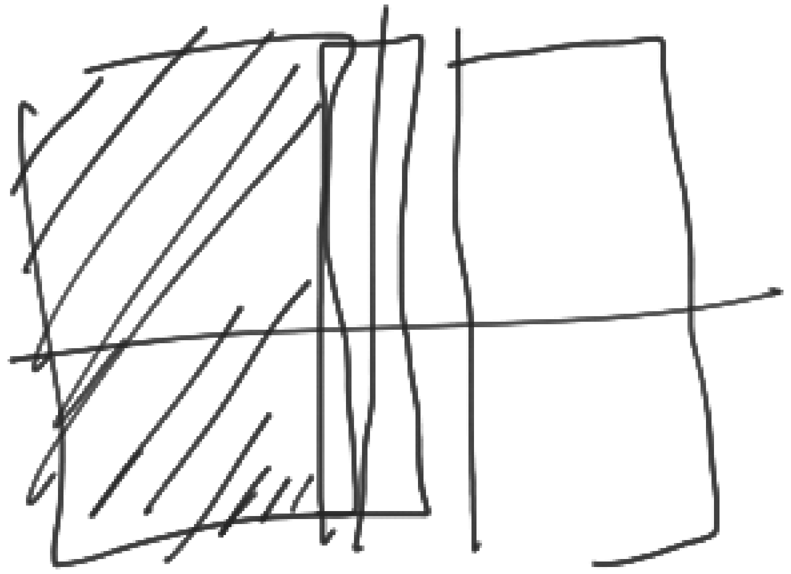
$H(s) = \int$

Estabilidad es convergencia de la transformada de ~~Fourier~~ Laplace para $\sigma = 0$

$H(s) = \mathcal{L}\{h(t)\}$

$\mathrm{Re}\{s\}$ incluye $\sigma = 0$

$$\mathcal{F}\{h(t)\} = H(s)\Big|_{s=j\omega} = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt$$



- ROC , $s = \sigma + j\omega$

$\sigma = 0$