Transformada de Laplace

Se usa para resolver ED. Si se tiene $ay'' + by' + cy = g(t) \rightarrow y = y_c + y_p$

Definición

Sea $f(t)$ una función integrable en $[0,\infty)$. Se define $\mathcal{L}(f(t))(s) = \int_0^\infty e^{-st} f(t)\,dt$

Ejercicios

1. $f(t) = e^{3t} \rightarrow \mathcal{L}(f(t))(s) = \int_0^\infty e^{-st} \cdot e^{3t}\,dt = \int_0^\infty e^{-st+3t}\,dt = \int_0^\infty e^{(3-s)t}\,dt = \frac{1}{3-s} e^{(3-s)t} \Big|_0^\infty = \frac{-1}{3-s} = \frac{1}{s-3} \ \forall s>3$

2. $f(t) = t^2 \quad \mathcal{L}(f(t))(s) = \int_0^\infty e^{-st} \cdot t^2\,dt = \left(-t^2 s^{-1} e^{-st} - 2t s^{-2} e^{-st} - 2 s^{-3} e^{-st}\right) \Big|_0^\infty = \frac{2}{s^3}$

3. $f(t) = 1. \quad \mathcal{L}(f(t))(s) = \int_0^\infty e^{-st}\,dt = -\frac{1}{s} e^{-st} \Big|_0^\infty = \frac{1}{s}$

4. $\mathcal{L}(\text{sen}(Kt)) = \frac{K}{s^2+K^2} \rightarrow \int_0^\infty e^{-st} \cdot \text{sen}(Kt) = -\frac{1}{K} e^{st} \cos(Kx) - \int -\frac{1}{K}\cos(Kx) \cdot -s e^{-st} = -\frac{1}{K} e^{-st}\cos(Kx) - \frac{s}{K}\int e^{-st}\cos(Kt)\,dt =$

$= -\frac{1}{K} e^{st}\cos(Kx) - \frac{s}{K}\left(\frac{1}{K} e^{-st}\sin(Kt) - \int \frac{1}{K}\sin(Kx) \cdot -s e^{st}\,dt \rightarrow \int e^{-st} \cdot \text{sen}(Kt)\,dt = \frac{-1}{K} e^{-st}\cos(Kt) - \frac{s}{K^2} e^{-st}\sin(Kt) - \frac{s}{K^2}\int e^{-st}\sin(Kt)\,dt\right.$

$\rightarrow \left(1 + \frac{s^2}{K^2}\right)\int e^{-st}\text{sen}(Kt)\,dt = \frac{-1}{K} e^{-st}\cos(Kt) - \frac{s}{K^2} e^{-st}\sin(Kt) \rightarrow \int e^{-st}\text{sen}(Kt)\,dt = \frac{-K}{K^2+s^2} e^{-st}\cos(Kt) - \frac{s}{K^2+s^2} e^{-st}\sin(Kt)$

$\int_0^\infty e^{-st}\text{sen}(Kt)\,dt = \left(\frac{-K}{K^2+s^2} e^{-st}\cos(Kt) - \frac{s}{K^2+s^2} e^{-st}\sin(Kt)\right)\Big|_0^\infty = \frac{s}{K^2+s^2}$

$+\frac{s}{K}\int e^{-st}\text{sen}(Kt)\,dt \quad u = e^{-st} \quad dv = \text{sen}(Kt) \quad du = -se^{-st}dt \quad v = -\frac{1}{K}\cos(Kt)$

5. $\mathcal{L}(\cos(Kt)) = \frac{s}{s^2+K^2} \rightarrow \int_0^\infty e^{-st}\cos(Kt)\,dt \quad u = e^{-st} \quad dv = \cos(Kt) \quad du = -se^{-st}dt \quad v = \frac{1}{K}\text{sen}(Kt) \rightarrow = \frac{1}{K} e^{-st}\text{sen}(Kt) - \int \frac{1}{K}\text{sen}(Kt) \cdot -se^{-st}\,dt = \frac{1}{K} e^{-st}\text{sen}(Kt) + \frac{s}{K^2} e^{-st}\cos(Kt) - \frac{s^2}{K^2}\int e^{-st}\cos(Kt)\,dt$

$\rightarrow \left(1 + \frac{s^2}{K^2}\right)\int e^{-st}\cos(Kt)\,dt = \frac{1}{K} e^{-st}\text{sen}(Kt) + \frac{s}{K^2} e^{-st}\cos(Kt) \rightarrow \int e^{-st}\cos(Kt)\,dt = \frac{K}{K^2+s^2} e^{-st}\text{sen}(Kt) - \frac{s}{K^2+s^2} e^{-st}\cos(Kt)$

$\int_0^\infty e^{-st}\cos(Kt)\,dt = \left(\frac{K}{K^2+s} e^{-st}\text{sen}(Kt) - \frac{s}{K^2+s^2} e^{-st}\cos(Kt)\right)\Big|_0^\infty = -\left(\frac{K}{K^2+s^2} e^{-st}\text{sen}\,Kt - \frac{s}{K^2+s^2} e^{-st}\cos(Kt)\right)\Big|_0 = \frac{s}{K^2+s^2}$

Tabla de TL fundamentales

$f(t)$	$\mathcal{L}(f(t))$	$f(t)$	$\mathcal{L}(f(t))$	$f(t)$	$\mathcal{L}(f(t))$
1	$\frac{1}{s}$	$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$	e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$	$\sin(Kt)$	$\frac{K}{s^2+K^2}$	$a\cos(Kt) + b/K\,\text{sen}(Kt)$	$\frac{as+b}{s^2+K^2}$
$\frac{t^{n-1}}{(n-1)!}$	$\frac{1}{s^n}$	$\cos(Kt)$	$\frac{s}{s^2+K^2}$	$e^{-at}\text{sen}(Kt)$	$\frac{K}{(s+a)^2+K^2}$
e^{-at}	$\frac{1}{s+a}$	$\sinh(Kt)$	$\frac{K}{s^2-K^2}$	$e^{-at}\cos(Kt)$	$\frac{s+a}{(s+a)^2+K^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$\cosh(Kt)$	$\frac{s}{s^2-K^2}$		

Propiedades de TL.

- $\mathcal{L}(c \cdot f(t)) = c\,\mathcal{L}(f(t))$.
- $\mathcal{L}(f(t)+g(t)) = \mathcal{L}(f(t))$

Ejercicios

1. $\mathcal{L}(5t^4-6t^3+4t+8) = \frac{120}{s^5} - \frac{36}{s^4} + \frac{4}{s^2} + \frac{8}{s}$

2. $\mathcal{L}(\text{sen}(6t)) = \frac{6}{s^2+36}$ $\quad \mathcal{L}(\cos(7t)) = \frac{s}{s^2+49}$ $\quad \mathcal{L}(\text{sen}(\sqrt{5}\cdot t)) = \frac{\sqrt{5}}{s^2+5}$

3. $\mathcal{L}(\text{senh}(t)) = \mathcal{L}\left(\frac{e^t-e^{-t}}{2}\right) = \frac{1}{2}\left[\mathcal{L}(e^t)-\mathcal{L}(e^{-t})\right] = \frac{1}{2}\left[\frac{1}{s-1} - \frac{1}{s+1}\right] = \frac{1}{2}\left[\frac{s+1-s+1}{(s-1)(s+1)}\right] = \frac{1}{s^2-1}$

4. $\mathcal{L}(2t^4) = \frac{48}{s^5}$ $\quad \mathcal{L}\left[(e^t-e^{-t})^2\right] = \mathcal{L}(e^{2t}-2+e^{-2t}) = \frac{1}{s-2} - \frac{2}{s} + \frac{1}{s+2}$

5. $\mathcal{L}(\text{sen}\,2t\cos 2t) = \mathcal{L}(\frac{1}{2}\text{sen}(4t)) = \frac{1}{2}\cdot\frac{4}{s^2+16} = \frac{2}{s^2+16}$

6. $\mathcal{L}\left(\begin{cases} 0 & \text{si } 0\le t\le 1 \\ 2t-2 & \text{si } 1<t \end{cases}\right)$ $\begin{matrix} m=2 \\ 0=2+b \\ b=-2 \end{matrix}$ $\rightarrow \int_0^\infty e^{-st}dt = \int_0^1 e^{-2t}\cdot 0\,dt + \int_1^\infty e^{-2t}(2t-2)dt =$

Transformada Inversa de Laplace

Sea $F(s)$ una función. La TL inversa de F es una función $f(t)$ tal que: $\mathcal{L}(f(t)) = F(s)$. Denotando la inversa como $\mathcal{L}^{-1}(F(s))$.

Propiedades

- $\mathcal{L}^{-1}(c\cdot F(s)) = c\cdot\mathcal{L}^{-1}(F(s))$
- $\mathcal{L}^{-1}(F(s)+G(s)) = \mathcal{L}^{-1}(F(s)) + \mathcal{L}^{-1}(G(s))$

Ejemplos

$\mathcal{L}^{-1}\left(\frac{1}{s^2}\right) = t$ $\quad \mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$ $\quad \mathcal{L}^{-1}\left(\frac{1}{s^6}\right) = \frac{t^5}{5!}$ $\quad \mathcal{L}^{-1}\left(\frac{8}{s^3}\right) = 8\frac{t^2}{2!} = 4t^2$ $\quad \mathcal{L}^{-1}\left(\frac{5}{s}+\frac{3}{s^2}\right) = 5+3t$

$\mathcal{L}^{-1}\left(\frac{5s+3}{(s+4)(s-5)}\right) = \frac{A}{s+4} + \frac{B}{s-5} \rightarrow$ $5s+3 = A(s-5)+B(s+4)$ Si $s=5$ $28=9B$ $B=\frac{28}{9}$ Si $s=-4$ $-20=-9A$ $A=\frac{20}{9}$ $\rightarrow \mathcal{L}^{-1}\left(\frac{\frac{20}{9}}{s+4} + \frac{\frac{28}{9}}{s-5}\right) = \frac{20}{9}e^{-4t} + \frac{28}{9}e^{5t}$

Taller de Transformada de Laplace

1. Calcular las siguientes transformadas de Laplace

a) $\mathcal{L}\{(t^2+3)^2\} = \mathcal{L}\{t^4+6t^2+9\} = \frac{24}{s^5} + 6\cdot\frac{2}{s^3} + \frac{9}{s} = \frac{24}{s^5} + \frac{12}{s^3} + \frac{9}{s}$

b) $\mathcal{L}\{e^{3t}\cos(5t)\} = \int_0^\infty e^{-st}e^{3t}\cos(5t)\,dt = \int_0^\infty e^{-t(s-3)}\cos(5t)\,dt$ $\quad u = e^{-t(s-3)}$, $du = -(s-3)e^{-t(s-3)}$, $dv = \cos(5t)$, $v = \frac{1}{5}\sin(5t)$ $= \frac{1}{5}\sin(5t)e^{-t(3-s)} + \frac{s-3}{5}\int \sin(5t)e^{-t(s-3)}dt$

$u = e^{-t(s-3)}$, $du = -(s-3)e^{-t(s-3)}$, $dv = \sin(5t)$, $v = -\frac{1}{5}\cos(5t)$ $= \frac{1}{5}\sin(5t)e^{-t(s-3)} - \frac{s-3}{25}\cos(5t)e^{-t(s-3)} - \frac{(s+3)^2}{25}\int \cos(5t)e^{-t(s-3)}dt \Rightarrow \left(1+\frac{(s-3)^2}{25}\right)\int \cos(5t)e^{-t(s-3)}dt = \frac{1}{5}\sin(5t)e^{-t(s-3)} - \frac{s-3}{25}\cos(5t)e^{-t(s-3)}$

$\int_0^\infty e^{-t(s-3)}\cos(5t)\,dt = \left(\frac{5}{25+(s-3)^2}e^{-t(s-3)}\sin(5t) - \frac{s-3}{25+(s-3)^2}\cos(5t)e^{-t(s-3)}\right)\Big|_0^\infty = \frac{s-3}{25+(s-3)^2}$

$\mathcal{L}(e^{-at}\cos Kt) = \frac{s+a}{(s+a)^2+K^2} \rightarrow \mathcal{L}(e^{3t}\cos 5t) = \frac{s-3}{(s-3)^2+25}$

c) $\mathcal{L}\{t^4e^{6t} + (t-8)^4u(t-8)\} = \frac{24}{(s-6)^5} + \frac{24e^{-8s}}{s^5}$

$\mathcal{L}\{t^4e^{6t}\} = \frac{24}{(s-6)^5}$ $\quad \mathcal{L}((t-8)^4u(t-8)) = e^{-8s}\mathcal{L}(t^4) = e^{-8s}\frac{4!}{s^5} = \frac{24}{s^5}e^{-8s}$

d) $\mathcal{L}\{\cos(t-4)u(t-4)\} = e^{-4s}\mathcal{L}(\cos(t)) = e^{-4s}\cdot\frac{s}{s^2+1}$

2. Calcular las siguientes transformadas inversas

a) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+7s+10}\right\} \rightarrow \frac{s+1}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5} \rightarrow s+1 = A(s+5)+B(s+2)$ $\quad$ Si $s=-5 \Rightarrow -4=-3B$ $\quad B=\frac{4}{3}$ $\quad$ Si $s=-2 \Rightarrow -1=3A$ $\quad A=\frac{-1}{3}$ $\rightarrow \mathcal{L}^{-1}\left\{\frac{-\frac{1}{3}}{s+2} + \frac{\frac{4}{3}}{s+5}\right\}$

$= -\frac{1}{3}e^{-2t} + \frac{4}{3}e^{-5t}$

$As^2+9A+Bs^2+Cs \Rightarrow s^2(A+B)+sC+9A$

b) $\mathcal{L}^{-1}\left\{\frac{s^2+4}{s^3+9s}\right\} \rightarrow \frac{s^2+4}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9} \rightarrow s^2+4 = A(s^2+9)+Bs^2+Cs$ $\quad s^2+4 = s^2(A+B)+sC+(9A)$ $\quad A=\frac{4}{9}$ $\quad C=0$ $\quad 1=A+B$ $\quad B=\frac{-5}{4}$ $\rightarrow \frac{\frac{4}{9}}{s} + \frac{\frac{5}{9}s}{s^2+9}$

$\mathcal{L}^{-1}\left\{\frac{4}{9}\cdot\frac{1}{s} + \frac{5}{9}\frac{s}{s^2+9}\right\} = \frac{4}{9} + \frac{5}{9}\cos(3t)$ $\quad s^2+4 \approx s^2(A+B)$

c) $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+6s+13}\right\} = \frac{(s+3)-2}{(s+3)^2+2^2} \rightarrow e^{-3t}\cos(2t) - e^{-3t}\operatorname{sen}(2t)$

d) $\mathcal{L}^{-1}\left\{\frac{e^{2s}}{s^2-4}\right\} = \mathcal{L}^{-1}\left\{\frac{e^{2s}\cdot 2}{(s^2-2^2)2}\right\} = u(t+2)\operatorname{senh}(2(t+2))\cdot\frac{1}{2}$

e) $\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2(s-2)}\right\} \rightarrow \frac{2s-1}{s^2(s-2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-2} \rightarrow 2s-1 = As(s-2)+B(s-2)+Cs^2$ $\quad$ Si $s=2$, $3=4C$ $\quad C=\frac{3}{4}$ $\quad 2s-1 = As(s-2)+\frac{1}{2}(s-2)+\frac{3}{4}s^2$ $\quad$ Si $s=0$ $\quad -1=-2B$ $\quad B=\frac{1}{2}$ $\quad 2s-1 = s^2(A+\frac{3}{4})+s(\frac{1}{2}-2A)-1 \Rightarrow A+\frac{3}{4}=0$ $\quad A=-\frac{3}{4}$

$\mathcal{L}^{-1}\left\{\frac{-\frac{3}{4}}{s} + \frac{\frac{1}{2}}{s^2} + \frac{\frac{3}{4}}{s-2}\right\} = -\frac{3}{4} + \frac{1}{2}t + \frac{3}{4}e^{2t}$

f) $\mathcal{L}^{-1}\left\{\frac{s}{(3-s)^3}\right\} = \mathcal{L}^{-1}\left\{-\frac{1}{(s-3)^3}\right\} = -\frac{1}{2}t^2e^{3t}$

3. Use la transformada de Laplace para encontrar la solución del problema de valor inicial.

a) $y'' - 4y = e^t$, $\quad y(0) = y'(0) = 0$

$\mathcal{L}(y''-4y) = \mathcal{L}(e^t) \rightarrow s^2\mathcal{L}(y) - s\,y(0) - y'(0) = \frac{1}{s-1} \rightarrow s^2\mathcal{L}(y) - 4\mathcal{L}(y) = \frac{1}{s-1} \rightarrow s^2Y - 4Y = \frac{1}{s-1} \rightarrow Y(s^2-4) = \frac{1}{s-1}$

$Y = \frac{1}{(s-1)(s+2)(s-2)} \rightarrow 1 = A(s+2)(s-2)+B(s-1)(s-2)+C(s-1)(s+2)$ $\quad s=2 \Rightarrow 1=4C$ $\quad C=\frac{1}{4}$, $s=1 \Rightarrow 1=-3A$ $\quad A=-\frac{1}{3}$, $\quad s=-2$ $\quad 1=12B$ $\quad B=1/12$ $\rightarrow Y = \frac{-\frac{1}{3}}{s-1} + \frac{\frac{1}{12}}{s+2} + \frac{\frac{1}{4}}{s-2}$ $\quad y = \mathcal{L}^{-1}(Y) = -\frac{1}{3}e^t + \frac{1}{12}e^{-2t} + \frac{1}{4}e^{2t}$

b) $y''+9y=1 \quad y(0)=0,\ y'(0)=1$

$\mathcal{L}(y''+9y)=\mathcal{L}(1)\to s^2\mathcal{L}(y)-s(y(0))-y'(0)+9\mathcal{L}(y)=\frac{1}{s}\to s^2Y-1+9Y=\frac{1}{s}\to Y(s^2+9)=\frac{1+s}{s}\to Y=\frac{1+s}{s(s^2+9)}$

$\frac{1+s}{s(s^2+9)}=\frac{A}{s}+\frac{Bs+C}{s^2+9}\to 1+s=A(s^2+9)+s(Bs+C)\to 1+s=\frac{1}{9}(s^2+9)+Bs^2+Cs \to C=1$
$s=0\to 1=9A\to A=\frac{1}{9} \quad \to 1+s=s^2(\frac{1}{9}+B)+sC+1 \to \frac{1}{9}+B=0 \ B=-\frac{1}{9} \quad Y=\frac{\frac{1}{9}}{s}+\frac{-\frac{1}{9}s+1}{s^2+9} \quad K=3,\ a=-\frac{1}{9},\ b=1.$

$y=\mathcal{L}^{-1}(Y)=\frac{1}{9}-\frac{1}{9}\cos(3t)+\frac{1}{3}\operatorname{sen}(3t)$

c) $y''-5y'+6y=u(t-1),\ y(0)=0,\ y'(0)=1.$

$\mathcal{L}(y''-5y'+6y)=\mathcal{L}(u(t-1))\to s^2\mathcal{L}(y)-sy(0)-y'(0)-5[s\mathcal{L}(y)-y(0)]+6\mathcal{L}(y)=\frac{e^{-s}}{s}\to s^2Y-1-5sY+6Y=\frac{e^{-s}}{s}+1$

$Y(s^2-5s+6)=\frac{e^{-s}}{s}+1 \quad Y=\frac{e^{-s}}{s(s-3)(s-2)}+\frac{1}{(s-3)(s-2)}=e^{-s}\left[\frac{\frac{1}{6}}{s}-\frac{\frac{1}{2}}{s-2}+\frac{\frac{1}{3}}{s-3}\right]+\frac{1}{s-3}-\frac{1}{s-2}$

$y=\mathcal{L}^{-1}(Y)=e^{3t}-e^{2t}+u(t-1)\left[\frac{1}{6}(t-1)-\frac{1}{2}e^{2(t-1)}+\frac{1}{3}e^{3(t-1)}\right]$

d) $y''-y'=e^t\cos t,\ y(0)=y'(0)=0.$

$\mathcal{L}(y''-y')=\mathcal{L}(e^t\cos t)\to s^2\mathcal{L}(y)-s\,y(0)-y'(0)-[s\mathcal{L}(y)-y(0)]=\frac{s-1}{(s-1)^2+1}\to s^2Y-sY=\frac{s-1}{(s-1)^2+1}\to Y(s^2-s)=\frac{s-1}{(s-1)^2+1}$

$Y=\frac{s-1}{s(s-1)((s-1)^2+1)}=\frac{1}{s[(s-1)^2+1]}=\frac{1}{s(s^2-2s+2)}=\frac{A}{s}+\frac{Bs+C}{s^2-2s+2}\to \begin{matrix}1=A(s^2-2s+2)+Bs^2+Cs \to 1=\frac{1}{2}s^2-s+1+Bs^2+Cs \quad C=1\\ s=0\to 1=2A \quad A=\frac{1}{2} \qquad 1=s^2(\frac{1}{2}+B)+s(-1+C) \quad B=-\frac{1}{2}\end{matrix}$

$Y=\frac{\frac{1}{2}}{s}+\frac{-\frac{1}{2}s+1}{(s-1)^2+1}=\frac{\frac{1}{2}}{s}+\frac{(s-1)(-\frac{1}{2})+\frac{1}{2}}{(s-1)^2-1}=\frac{\frac{1}{2}}{s}-\frac{1}{2}\frac{s-1}{(s-1)^2-1}+\frac{1}{2}\frac{1}{(s-1)^2-1} \quad y=\mathcal{L}^{-1}(Y)=\frac{1}{2}-\frac{1}{2}e^t\cos(t)+\frac{1}{2}e^t\operatorname{sen}(t)$

e) $y''+2y'+y=2(t-3)u(t-3),\ y(0)=2,\ y'(0)=1.$

$\mathcal{L}(y''+2y'+y)=s^2\mathcal{L}(y)-sy(0)-y'(0)+2[s\mathcal{L}(y)-y(0)]+\mathcal{L}(y)=s^2Y-2s-1+2sY-2+Y=Y(s^2+2s+1)-3$

$\mathcal{L}(2(t-3)u(t-3))=e^{-3s}\frac{2}{s}\to Y(s^2+2s+1)=\frac{2e^{-3s}}{s}+3 \quad Y=\frac{2e^{-3s}}{s(s+1)^2}+\frac{3}{(s+1)^2}=e^{-3s}\left[\frac{2}{s}-\frac{2}{s+1}-\frac{2}{(s+1)^2}\right]+\frac{3}{(s+1)^2}$

$\frac{2}{s(s+1)^2}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{(s+1)^2}\to 2=A(s+1)^2+B(s+1)s+Cs\to 2=As^2+2sA+A+Bs^2+Bs-2s$
$s=-1\to C=-2 \quad A=2 \qquad 2=s^2(A+B)+s(2A+B-2)+A \quad B=-2 \to \frac{2}{s(s+1)^2}=\frac{2}{s}-\frac{2}{s+1}-\frac{2}{(s+1)^2}$

$y=\mathcal{L}^{-1}(Y)=3\cdot t\cdot e^{-t}+u(t-3)\left[2(t-3)-2e^{-(t-3)}-6(t-3)e^{-(t-3)}\right].$

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X → 1

g) $y''-4y'+4y=t^2e^t,\ y(0)=0,\ y'(0)=1.$

$\mathcal{L}(y''-4y'+4y)=s^2Y-1-4sY+4Y=Y(s^2-4s+4)-1. \quad \mathcal{L}(t^2e^t)=\frac{2}{(s-1)^3}\to Y(s-2)^2=\frac{2}{(s-1)^3}+1$

$Y=\frac{2}{(s-2)^2(s-1)^3}+\frac{1}{(s-2)^2} \quad \frac{2}{(s-2)^2(s-1)^3}=\frac{A}{s-2}+\frac{B}{(s-2)^2}+\frac{C}{s-1}+\frac{D}{(s-1)^2}+\frac{E}{(s-1)^3}\to 2=A(s-2)(s-1)^3+B(s-1)^3+C(s-2)^2(s-1)^2+D(s-2)^2(s-1)+E(s-2)^2$
$s=2\to 2=B,\ s=1\to 2=E\ \ldots\ C=6,\ D=4,\ A=-6$

$Y=\frac{-6}{s-2}+\frac{3}{(s-2)^2}+\frac{6}{s-1}+\frac{4}{(s-1)^2}+\frac{2}{(s-1)^3}\to y=-6e^{2t}+3te^{2t}+6e^t+4te^t+t^2\cdot e^t$

h) $x'+3y=t,\ y'+3x=-1,\ x(0)=0,\ y(0)=1.$

$sX+3Y=\frac{1}{s^2},\ sY-1+3X=\frac{-1}{s}\to 3X+sY=\frac{-1}{s}+1 \quad A=\begin{bmatrix}s&3\\3&s\end{bmatrix} \quad A_1=\begin{bmatrix}\frac{1}{s^2}&3\\\frac{s-1}{s}&s\end{bmatrix} \quad A_2=\begin{bmatrix}s&\frac{1}{s^2}\\3&\frac{s-1}{s}\end{bmatrix}$

$|A|=s^2-9=(s+3)(s-3) \quad |A_1|=\frac{1}{s}-\frac{3+3s}{s}=\frac{-2-3s}{s} \quad |A_2|=\frac{3+3s}{s}-\frac{1}{s}=\frac{2+3s}{s}$

$X=\frac{|A_1|}{|A|}=\frac{-2-3s}{s(s+3)(s-3)}=\frac{A}{s}+\frac{B}{s+3}+\frac{C}{s-3}\to -2-3s=A(s+3)(s-3)+Bs(s-3)+Cs(s+3)$
$s=0\to A=\frac{2}{9},\ s=3\to C=-\frac{11}{18},\ s=-3\to B=\frac{7}{18} \quad X=\frac{\frac{2}{9}}{s}+\frac{\frac{7}{18}}{s+3}+\frac{-\frac{11}{18}}{s-3}\to x=\frac{2}{9}+\frac{7}{18}e^{-3t}-\frac{11}{18}e^{3t}$

$Y=\frac{|A_2|}{|A|}=\frac{2+3s}{s(s+3)(s-3)}=\frac{A}{s}+\frac{B}{s+3}+\frac{C}{s-3}\to 2+3s=AC$