

Table A-1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab} \left[1 + \frac{1}{a-b}(be^{-bt} - ae^{-at}) \right]$	$\frac{1}{s(s+a)(s+b)}$

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Table A-2 Properties of Laplace Transforms

1	$\mathcal{L}[Af(t)] = AF(s)$
2	$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathcal{L}_s \left[\frac{d}{dt} f(t) \right] = sF(s) - f(0^\pm)$
4	$\mathcal{L}_s \left[\frac{d^2}{dt^2} f(t) \right] = s^2 F(s) - sf(0^\pm) - \dot{f}(0^\pm)$
5	$\mathcal{L}_s \left[\frac{d^n}{dt^n} f(t) \right] = s^n F(s) - \sum_{k=1}^n s^{n-k} f^{(k-1)}(0^\pm)$ where $f^{(k)}(t) = \frac{d^k}{dt^k} f(t)$
6	$\mathcal{L}_s \left[\int_0^t f(t) dt \right] = \frac{F(s)}{s} + \frac{1}{s} \left[\int f(t) dt \right]_{t=0^+}$
7	$\mathcal{L}_s \left[\int \dots \int f(t) (dt)^n \right] = \frac{F(s)}{s^n} + \sum_{k=1}^n \frac{1}{s^{n-k+1}} \left[\int \dots \int f(t) (dt)^k \right]_{t=0^+}$
8	$\mathcal{L} \left[\int_0^t f(t) dt \right] = \frac{F(s)}{s}$
9	$\int_0^\infty f(t) dt = \lim_{s \rightarrow 0} F(s) \quad \text{if } \int_0^\infty f(t) dt \text{ exists}$
10	$\mathcal{L}[e^{-at} f(t)] = F(s+a)$
11	$\mathcal{L}[f(t-\alpha)1(t-\alpha)] = e^{-s\alpha} F(s) \quad \alpha \geq 0$
12	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
13	$\mathcal{L}[t^2 f(t)] = \frac{d^2}{ds^2} F(s)$
14	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s) \quad (n = 1, 2, 3, \dots)$
15	$\mathcal{L} \left[\frac{1}{t} f(t) \right] = \int_s^\infty F(s) ds \quad \text{if } \lim_{t \rightarrow 0} \frac{1}{t} f(t) \text{ exists}$
16	$\mathcal{L} \left[f \left(\frac{1}{a} \right) \right] = aF(as)$
17	$\mathcal{L} \left[\int_0^t f_1(t-\tau) f_2(\tau) d\tau \right] = F_1(s) F_2(s)$
18	$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s-p) dp$

Table A-1 (continued)

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s^2(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
22	$\frac{\omega_\zeta}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_\zeta t} \sin \omega_\zeta \sqrt{1-\zeta^2} t \quad (0 < \zeta < 1)$	$\frac{\omega_\zeta^2}{s^2 + 2\zeta\omega_\zeta s + \omega_\zeta^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_\zeta t} \sin(\omega_\zeta \sqrt{1-\zeta^2} t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{s}{s^2 + 2\zeta\omega_\zeta s + \omega_\zeta^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_\zeta t} \sin(\omega_\zeta \sqrt{1-\zeta^2} t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{\omega_\zeta^2}{s(s^2 + 2\zeta\omega_\zeta s + \omega_\zeta^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
26	$\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)}$
27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$
28	$\frac{1}{2\omega} t \sin \omega t$	$\frac{s}{(s^2 + \omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) \quad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$
31	$\frac{1}{2\omega} (\sin \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2 + \omega^2)^2}$

Finally, we present two frequently used theorems, together with Laplace transforms of the pulse function and impulse function.

Initial value theorem	$f(0^+) = \lim_{t \rightarrow 0^+} f(t) = \lim_{s \rightarrow \infty} sF(s)$
Final value theorem	$f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$
Pulse function $f(t) = \frac{A}{t_0} 1(t) - \frac{A}{t_0} 1(t - t_0)$	$\mathcal{L}[f(t)] = \frac{A}{t_0 s} - \frac{A}{t_0 s} e^{-st_0}$
Impulse function $g(t) = \lim_{\epsilon \rightarrow 0} \frac{A}{\epsilon} 1_\epsilon(t)$ for $0 < t < t_0$ $= 0$, for $t < 0, t_0 < t$	$\mathcal{L}[g(t)] = \lim_{\epsilon \rightarrow 0} \left[\frac{A}{t_0 s} (1 - e^{-st_0}) \right]$ $= \lim_{\epsilon \rightarrow 0} \frac{\frac{d}{dt_0} [A(1 - e^{-st_0})]}{\frac{d}{dt_0} (t_0 s)}$ $= \frac{As}{s} = A$