TALLER CORTE 2

LAURA GONZÁLEZ y DAFNE CASTELLANOS

1. Encuentre la solución del problema de valor inicial.

a) $9y'' - 12y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = -1$

→ Ecuación auxiliar: $9m^2 - 12m + 4 = 0$

$$m = \frac{12 \pm \sqrt{144-144}}{18} = \frac{12}{18} = \frac{2}{3}$$

$$y = C_1 e^{2/3x} + C_2 x e^{2/3x}$$
$$y' = \frac{2}{3} C_1 e^{2/3x} + \frac{2}{3} C_2 x e^{2/3x} + C_2 e^{2/3x}$$

Si $y(0) = 2 \Rightarrow 2 = C_1$

Si $y'(0) = -1 \Rightarrow -1 = \frac{4}{3} + C_2 \Rightarrow C_2 = -\frac{7}{3}$

→ Solución del P.V.I. $y = 2e^{2/3x} - \frac{7}{3} x e^{2/3x}$

b) $y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2$

→ Ecuación auxiliar $m^2 - 2m + 5 = 0$

$$m = \frac{2 \pm \sqrt{4-20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \begin{cases} m_1 = 1+2i \\ m_2 = 1-2i \end{cases} \qquad \begin{matrix} \alpha = 1 \\ \beta = 2 \end{matrix}$$

$$y = C_1 e^x \operatorname{sen}(2x) + C_2 e^x \cos(2x)$$
$$y' = C_1 2\cos(2x) e^x + C_1 e^x \operatorname{sen}(2x) - C_2 2e^x \operatorname{sen}(2x) + C_2 e^x \cos(2x)$$

Si $y(\pi/2) = 0 \Rightarrow 0 = C_1 e^{\pi/2} \operatorname{sen}(\pi) + C_2 e^{\pi/2} \cos(\pi)$
$\Rightarrow 0 = -C_2 e^{\pi/2} \Rightarrow C_2 = 0$

Si $y'(\pi/2) = 2 \Rightarrow 2 = C_1 2\cos(\pi) e^{\pi/2} + C_1 e^{\pi/2} \operatorname{sen}(\pi)$
$-e^{-\pi/2} = C_1$

→ Solución del P.V.I. $y = -e^{-\pi/2} e^{x} \operatorname{sen}(2x)$

2. Muestre que $W(e^{\lambda t}\cos(\mu t), e^{\lambda t}\operatorname{sen}(\mu t)) = \mu e^{2\lambda t}$

$$W = \begin{vmatrix} e^{\lambda t}\cos(\mu t) & e^{\lambda t}\operatorname{sen}(\mu t) \\ -\mu e^{\lambda t}\operatorname{sen}(\mu t) + \lambda e^{\lambda t}\cos(\mu t) & \mu e^{\lambda t}\cos(\mu t) + \lambda e^{\lambda t}\operatorname{sen}(\mu t) \end{vmatrix}$$

$$W = e^{\lambda t}\cos(\mu t)\left(\mu e^{\lambda t}\cos(\mu t) + \lambda e^{\lambda t}\operatorname{sen}(\mu t)\right) - e^{\lambda t}\operatorname{sen}(\mu t)\left(-\mu e^{\lambda t}\operatorname{sen}(\mu t) + \lambda e^{\lambda t}\cos(\mu t)\right)$$

$$W = \mu e^{2\lambda t}\cos^2(\mu t) + \cancel{\lambda e^{2\lambda t}\cos(\mu t)\operatorname{sen}(\mu t)} + \mu e^{2\lambda t}\operatorname{sen}^2(\mu t) - \cancel{\lambda e^{2\lambda t}\cos(\mu t)\operatorname{sen}(\mu t)}$$

$$W = \mu e^{2\lambda t}\left(\operatorname{sen}^2(\mu t) + \cos^2(\mu t)\right)$$

$$W = \mu e^{2\lambda t}$$

3. Si el wronskiano de f y g es $3e^{4x}$ y si $f(x) = e^{2x}$, encuentre $g(x)$.

$$W = \begin{vmatrix} e^{2x} & g(x) \\ 2e^{2x} & g'(x) \end{vmatrix} = e^{2x}g' - 2e^{2x}g = 3e^{4x}$$

Resolviendo la ED lineal $g' - 2g = 3e^{2x}$

→ $P(x) = -2 \qquad Q(x) = 3e^{2x}$

→ $\int P(x)\,dx = -2x \qquad \mu(x) = e^{-2x}$

→ $\int Q(x)\,\mu(x)\,dx = 3\int dx = 3x$

Por lo cual, $g(x) = \dfrac{3x + C}{e^{-2x}}$

4. Encuentre la solución del problema del valor inicial dado utilizando coeficientes indeterminados.

a) $y'' + 4y = x^2 + 3e^x, \quad y(0) = 0, \quad y'(0) = 2$

$$y_p = Ax^2 + Bx + C + De^x$$
$$y_p' = 2Ax + B + De^x$$
$$y_p'' = 2A + De^x$$

$$\cancel{2A} + \cancel{De^x} + 4(\cancel{Ax^2} + \cancel{Bx} + \cancel{C} + \cancel{De^x}) = x^2 + 3e^x$$

"coef. de x^2" $\rightarrow$ $4A = 1 \Rightarrow A = 1/4$

"coef. de x" $\rightarrow$ $4B = 0 \Rightarrow B = 0$

"coef. de e^x" $\rightarrow$ $4D + D = 3 \Rightarrow D = 3/5$

"sin coef." $\rightarrow$ $2A + 4C = 0 \Rightarrow C = -1/8$

$$\underbrace{y_p = \frac{x^2}{4} - \frac{1}{8} + \frac{3e^x}{5}}$$

→ Hallando y_c

$m^2 + 4 = 0 \quad m = \sqrt{-4} = 2i \quad \underline{y_c = C_1 e^x \operatorname{sen}(2x) + C_2 e^x \cos(2x)}$

→ Solución general: $y = C_1 \operatorname{sen}(2x) + C_2 \cos(2x) + \frac{x^2}{4} - \frac{1}{8} + \frac{3e^x}{5}$

→ Si $y(0) = 0$

$$0 = C_2 - \frac{1}{8} + \frac{3}{5} \Rightarrow C_2 = \frac{1}{8} - \frac{3}{5} = \frac{5-24}{40} \Rightarrow C_2 = \frac{-19}{40}$$

→ Si $y'(0) = 2$

$$y' = 2C_1 \cos(2x) - 2C_2 \operatorname{sen}(2x) + \frac{x}{2} + \frac{3}{5}e^x$$

$$2 = 2C_1 + \frac{3}{5} \Rightarrow C_1 = \frac{7}{10}$$

→ Solución del P.V.I. $y = \frac{7}{10}\operatorname{sen}(2x) - \frac{19}{40}\cos(2x) + \frac{x^2}{4} - \frac{1}{8} + \frac{3e^x}{5}$

b) $y'' - 2y' + y = xe^x + 4, \quad y(0)=0, \quad y'(0)=1$

$y_p = (Ax+B)e^x + C$
$y_p' = (Ax+B)e^x + Ae^x$
$y_p'' = (Ax+B)e^x + Ae^x + Ae^x$

$2Ae^x + (Ax+B)e^x - 2((Ax+B)e^x + Ae^x) + (Ax+B)e^x = 0$

→ Multiplicando por x $\quad y_p = (Ax+B)xe^x = (Ax^2+Bx)e^x + C$

$y_p' = (Ax^2+Bx)e^x + (2Ax+B)e^x = e^x(Ax^2 + Bx + 2Ax + B)$
$y_p'' = e^x(4Ax + 2B + 2A + Ax^2 + Bx)$

$e^x(\cancel{4Ax} + \cancel{2B} + 2A + \cancel{Ax^2} + \cancel{Bx} - \cancel{2Ax^2} - \cancel{2Bx} - \cancel{4Ax} - \cancel{2B} + \cancel{Ax^2} + \cancel{Bx}) + C$
$2Ae^x + C = xe^x + 4$

→ Multiplicando por x^2 $\quad y_p = (Ax+B)x^2e^x = (Ax^3+Bx^2)e^x + C$

$y_p' = (Ax^3+Bx^2)e^x + (3Ax^2+2Bx)e^x$
$y_p'' = (Ax^3 + Bx^2 + 6Ax^2 + 4Bx + 6Ax + 2B)e^x$

$e^x(\cancel{Ax^3} + \cancel{Bx^2} + \cancel{6Ax^2} + \cancel{4Bx} + 6Ax + 2B - \cancel{2Ax^3} - \cancel{2Bx^2} - \cancel{6Ax^2} - \cancel{4Bx} + \cancel{Ax^3} + \cancel{Bx^2}) + C = xe^x + 4$

$e^x(\cancel{6A}x + \cancel{2B}) + \cancel{C} = xe^x + 4$

"Coef. de xe^x" → $6A = 1 \Rightarrow A = 1/6$
"Coef. de e^x" → $2B = 0 \Rightarrow B = 0$
"Sin coef." → $C = 4$

$$y_p = \frac{1}{6}x^3e^x + 4$$

→ Solucionando la homogenea

$m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0 \Rightarrow m = 1 \qquad y_c = C_1e^x + C_2xe^x$

→ Solución general: $y = C_1e^x + C_2xe^x + \frac{x^3}{6}e^x + 4$

→ Si $y(0)=0 \Rightarrow 0 = C_1 + 4 \Rightarrow C_1 = -4$

→ Si $y'(0)=1 \Rightarrow y' = C_1e^x + C_2e^x(1+x) + e^x\left(\frac{x^2}{2} + \frac{x^3}{6}\right)$

$$1 = -4 + C_2 \Rightarrow C_2 = 5$$

→ Solución del P.V.I. $y = -4e^x + 5xe^x + \frac{x^3}{6}e^x + 4$

c) $y'' + 4y = 3\,sen(2x), \quad y(0) = 2, \quad y'(0) = -1$

$y_p = A\,sen(2x) + B\cos(2x)$
$y_p' = 2A\cos(2x) - 2B\,sen(2x)$
$y_p'' = -4A\,sen(2x) - 4B\cos(2x)$

$-4(A\,sen(2x) + B\cos(2x)) + 4(A\,sen(2x) + B\cos(2x)) = 0$

→ Multiplicando por x $\quad y_p = Ax\,sen(2x) + Bx\cos(2x)$

$y_p' = 2Ax\cos(2x) + A\,sen(2x) - 2Bx\,sen(2x) + B\cos(2x)$
$y_p'' = -4Ax\,sen(2x) + 4A\cos(2x) - 4Bx\cos(2x) - 4B\,sen(2x)$
$= 4A(\cos(2x) - x\,sen(2x)) - 4B(x\cos(2x) + sen(2x))$

$\cancel{4A}\cos(2x) - \cancel{4B}\,sen(2x) = 3\,sen(2x)$

"Coef. de $\cos(2x)$" → $4A = 0 \Rightarrow A = 0$
"coef. de $sen(2x)$" → $-4B = 3 \Rightarrow B = -3/4$

$y_p = \frac{-3}{4}x\cos(2x)$

→ Solucionando la homogenea
$m^2 + 4 = 0 \quad m = \sqrt{-4} \quad m = 2i \quad y_c = C_1\,sen(2x) + C_2\cos(2x)$

→ Solución General: $y = C_1\,sen(2x) + C_2\cos(2x) - \frac{3}{4}x\cos(2x)$

→ Si $y(0) = 2 \Rightarrow C_2 = 2$
→ Si $y'(0) = -1 \Rightarrow -1 = 2C_1\cos(2x) - \frac{3}{4}\cos(2x) \Rightarrow C_1 = -\frac{1}{8}$

→ Solución del P.V.I. $y = -\frac{1}{8}sen(2x) + 2\cos(2x) - \frac{3}{4}x\cos(2x)$

d) $y'' + 2y' + 5y = 4e^{-x}\cos(2x), \quad y(0)=1, \quad y'(0)=0$

$$y_p = (A\cos(2x) + B\,\text{sen}(2x))\,e^{-x}$$
$$y_p' = (-2A\,\text{sen}(2x) + 2B\cos(2x) - A\cos(2x) - B\,\text{sen}(2x))\,e^{-x}$$
$$y_p'' = (2A\,\text{sen}(2x) - 2B\cos(2x) + A\cos(2x) + B\,\text{sen}(2x) - 4A\cos(2x) - 4B\,\text{sen}(2x)$$
$$+ 2A\,\text{sen}(2x) - 2B\cos(2x))\,e^{-x}$$
$$(4(A\,\text{sen}(2x) - B\cos(2x)) - 3(B\,\text{sen}(2x) + A\cos(2x))$$
$$-4A\,\text{sen}(2x) + 4B\cos(2x) - 2A\cos(2x) - 2B\,\text{sen}(2x)$$
$$+5A\cos(2x) + 5B\,\text{sen}(2x))\,e^{-x} = 4e^{-x}\cos(2x)$$
$$0 = 4e^{-x}\cos(2x)$$

→ Multiplicando por x $\quad y_p = (Ax\cos(2x) + Bx\,\text{sen}(2x))e^{-x}$

$$y_p' = (-Ax\cos(2x) - Bx\,\text{sen}(2x) - 2Ax\,\text{sen}(2x) + A\cos(2x) + 2Bx\cos(2x)$$
$$+B\,\text{sen}(2x))\,e^{-x}$$
$$y_p'' = (Ax\cos(2x) + Bx\,\text{sen}(2x) + 2Ax\,\text{sen}(2x) - A\cos(2x) - 2Bx\cos(2x)$$
$$- B\,\text{sen}(2x) + 2Ax\,\text{sen}(2x) - A\cos(2x) - 2Bx\cos(2x) - B\,\text{sen}(2x)$$
$$-4Ax\cos(2x) - 2A\,\text{sen}(2x) - 2A\,\text{sen}(2x) - 4Bx\,\text{sen}(2x)$$
$$+2B\cos(2x) + 2B\cos(2x))\,e^{-x}$$
$$y_p'' = -3Ax\cos(2x) - 3Bx\,\text{sen}(2x) + 4Ax\,\text{sen}(2x) - 2A\cos(2x)$$
$$-4Bx\cos(2x) - 2B\,\text{sen}(2x) - 4A\,\text{sen}(2x) + 4B\cos(2x))\,e^{-x}$$

$$[\,A(-3x\cos(2x) + 4x\,\text{sen}(2x) - 2\cos(2x) - 4\,\text{sen}(2x)) +$$
$$B(-3x\,\text{sen}(2x) - 4x\cos(2x) - 2\,\text{sen}(2x) + 4\cos(2x)) +$$
$$2A(-x\cos(2x) - 2x\,\text{sen}(2x) + \cos(2x)) +$$
$$2B(-x\,\text{sen}(2x) + 2x\cos(2x) + \text{sen}(2x)) +$$
$$5(Ax\cos(2x) + Bx\,\text{sen}(2x))\,]\,e^{-x} = 4e^{-x}\cos(2x)$$

"Coef. $e^{-x}x\cos(2x)$" → $-3A - 4B - 2A + 4B + 5A = 0$

"coef. $e^{-x}x\,\text{sen}(2x)$" → $4A - 3B - 4A + 5B - 2B = 0$

"coef. $e^{-x}\cos(2x)$" → $-2A + 4B + 2A = 4 \Rightarrow 4B = 4 \Rightarrow B = 1$

"coef. $e^{-x}\,\text{sen}(2x)$" → $-4A - 2B + 2B = 0 \Rightarrow -4A = 0 \Rightarrow A = 0$

$$y_p = e^{-x}x\,\text{sen}(2x)$$

→ Solucionando la homogenea:

$m^2 + 2m + 5 = 0 \qquad m = \dfrac{-2 \pm \sqrt{4-20}}{2} \longrightarrow m_1 = -1 + 2i \qquad \alpha = -1$

$m_2 = -1 - 2i \qquad \beta = 2$

$$y_c = C_1 e^{-x} \operatorname{sen}(2x) + C_2 e^{-x} \cos(2x)$$

→ Solución General: $y = C_1 e^{-x} \operatorname{sen}(2x) + C_2 e^{-x} \cos(2x) + e^{-x} x \operatorname{sen}(2x)$

→ Si $y(0) = 1 \Rightarrow \quad 1 = C_2$

→ Si $y'(0) = 0 \Rightarrow$

$$y' = 2C_1 e^{-x} \cos(2x) - \cancel{C_1 e^{-x} \operatorname{sen}(2x)} - C_2 e^{-x} (\cancel{2\operatorname{sen}(2x)} + \cos(2x))$$
$$+ \cancel{e^{-x} \operatorname{sen}(2x)} - \cancel{e^{-x} x \operatorname{sen}(2x)} - \cancel{2e^{-x} x \cos(2x)}$$

$$0 = 2C_1 - C_2 \Rightarrow 0 = 2C_1 - 1 \Rightarrow C_1 = 1/2$$

→ Solución del P.V.I. $\quad y = \left[\frac{1}{2} \operatorname{sen}(2x) + \cos(2x) + x \operatorname{sen}(2x)\right] e^{-x}$

5. En cada uno de los ejercicios, verifique que las funciones dadas y_1 y y_2 satisfacen la correspondiente ecuación diferencial homogénea. Luego, encuentre una solución particular por el método de variación de parámetros.

a) $x^2 y'' - 2y = 3x^2 - 1, \quad x > 0, \quad y_1(x) = x^2, \quad y_2(x) = x^{-1}$.

→ Para $y_1(x) = x^2 \Rightarrow y_1' = 2x, \quad y_1'' = 2 \Rightarrow 2x^2 - 2x^2 = 0$ ✓
→ Para $y_2(x) = x^{-1} \Rightarrow y_2' = -x^{-2}, \quad y_2'' = 2x^{-3} \Rightarrow 2x^{-1} - 2x^{-1} = 0$ ✓

→ Dividiendo por x^2 $\quad y'' - \frac{2}{x^2} y = 3 - \frac{1}{x^2}$

→ Encontrando una solución por variación de parámetros

$$W = \begin{vmatrix} x^2 & x^{-1} \\ 2x & -x^{-2} \end{vmatrix} = -1 - 2 = -3 \qquad W_1 = \begin{vmatrix} 0 & x^{-1} \\ 3 - x^{-2} & -x^{-2} \end{vmatrix} = -\frac{3}{x} - \frac{1}{x^3}$$

$$W_2 = \begin{vmatrix} x^2 & 0 \\ 2x & 3 - x^{-2} \end{vmatrix} = 3x^2 - 1$$

$$u_1 = \int x^{-1} + \frac{x^{-3}}{3}\,dx = \int \frac{1}{x}\,dx + \frac{1}{3}\int x^{-3}dx = \ln x + \frac{1}{6x^2}$$

$$u_2 = -\int x^2 + \int \frac{dx}{3} = -\frac{x^3}{3} + \frac{x}{3}$$

$$y_p = y_1 u_1 + y_2 u_2 = x^2 \ln x + \frac{1}{2} - \frac{x^2}{3}$$

b) $x^2 y'' - 3xy' + 4y = x^2 \ln x;\quad x>0,\quad y_1(x) = x^2;\quad y_2(x) = x^2 \ln x$

→ Para $y_1(x) = x^2 \Rightarrow y_1' = 2x,\ y_1'' = 2 \Rightarrow 2x^2 - 6x^2 + 4x^2 = 0$ ✓

→ Para $y_2(x) = x^2 \ln x \Rightarrow y_2' = 2x\ln x + x,\ y_2'' = 2\ln x + 3$

$\Rightarrow (2\ln x + 3)x^2 - 6x^2\ln x - 3x^2 + 4x^2 \ln x = 0$ ✓

→ Dividiendo x^2 $\quad y'' - \frac{3}{x}y' + \frac{4}{x^2}y = \ln x$

→ Solucionando por variación de parámetros

$$W = \begin{vmatrix} x^2 & x^2\ln x \\ 2x & 2x\ln x + x \end{vmatrix} = 2x^3\ln x + x^3 - 2x^3 \ln x = x^3$$

$$W_1 = \begin{vmatrix} 0 & x^2\ln x \\ \ln x & 2x\ln x + x \end{vmatrix} = -x^2\ln^2 x$$

$$W_2 = \begin{vmatrix} x^2 & 0 \\ 2x & \ln x \end{vmatrix} = x^2 \ln x$$

$$u_1 = \int \frac{-x^2\ln^2 x}{x^3}\,dx = -\int \frac{\ln^2 x}{x}\,dx \qquad u = \ln x,\quad du = \frac{1}{x}dx$$

$$u_1 = -\int u^2 du = -\frac{u^3}{3} = \frac{-\ln^3 x}{3}$$

$$u_2 = \int \frac{x^2 \ln x}{x^3}\,dx = \int \frac{\ln x}{x}\,dx \qquad u = \ln x,\quad du = \frac{1}{x}dx$$

$$u_2 = \int u\,du = \frac{u^2}{2} = \frac{\ln^2 x}{2}$$

$$y_p = \frac{-x^2\ln^3 x}{3} + \frac{x^2\ln^3 x}{2} = \frac{-2x^2\ln^3 x + 3x^2\ln^3 x}{6}$$

$$y_p = \frac{x^2\ln^3 x}{6}$$

6. En cada uno de los ejercicios, haga lo siguiente:

1) Verifique que la función dada, y_1 satisface la ED homogenea asociada.

2) Halle una segunda solución y_2 linealmente independiente con y_1 con el método de reducción de orden.

3) Encuentre una solución particular por el método de variación de parámetros.

a) $x^2y'' - 2xy' + 2y = 4x^2 - 1$, $x > 0$, $y_1(x) = x$

1) Para $y_1(x) = x \Rightarrow y_1' = 1$, $y_1'' = 0 \Rightarrow -2x + 2x = 0$ ✓

2) Reducción de orden

→ Forma estandar: $y'' + P(x)y' + q(x)y = 0$

$$y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0$$

→ Segunda ecuación dada por: $y_2 = y_1(x)\int \frac{e^{-\int P(x)dx}}{(y_1(x))^2}dx$

→ $-\int P(x)dx = 2\int x^{-1}dx = 2\ln x$

→ $\int \frac{e^{2\ln x}}{x^2}dx = \int \frac{x^2}{x^2}dx = x$

→ $y_2 = x^2$

→ Variación de parámetros

→ Dividiendo por x^2 $\quad y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 4 - \frac{1}{x^2}$

$$W = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 = x^2$$

$$W_1 = \begin{vmatrix} 0 & x^2 \\ 4-x^{-2} & 2x \end{vmatrix} = -4x^2 + 1$$

$$W_2 = \begin{vmatrix} x & 0 \\ 1 & 4-x^{-2} \end{vmatrix} = 4x - x^{-1}$$

$$u_1 = \int \frac{-4x^2+1}{x^2}\,dx = -4\int dx + \int x^{-2}\,dx = -4x - x^{-1}$$

$$u_2 = \int \frac{4x - x^{-1}}{x^2}\,dx = 4\int \frac{1}{x}\,dx - \int x^{-3}\,dx = 4\ln x + \frac{x^{-2}}{2}$$

$$y_p = -4x^2 - \frac{1}{2} + 4x^2 \ln x$$

b) $xy'' - (1+x)y' + y = x^2 e^{2x}, \quad x > 0, \quad y_1(x) = 1 + x$

1) Para $y_1(x) = 1+x \Rightarrow y_1' = 1,\ y_1'' = 0 \Rightarrow -(1+x) + 1 + x = 0$ ✓

2) Reducción de orden

→ Forma estandar: $\quad y'' - \left(\frac{1}{x} + 1\right)y' + \frac{1}{x}y = xe^{2x}$

→ Segunda ecuación dada por: $\quad y_2 = y_1(x)\displaystyle\int \frac{e^{-\int P(x)dx}}{(y_1(x))^2}\,dx$

$$\rightarrow -\int P(x)dx = \int \frac{dx}{x} + \int dx = \ln x + x$$

$$\rightarrow \int \frac{xe^x}{(1+x)^2}\,dx = -xe^x(1+x)^{-1} + \int (1+x)^{-1}(e^x + xe^x)\,dx$$

$u = xe^x \qquad dv = \frac{1}{(1+x)^2}$

$du = (e^x + xe^x)dx \qquad V = -(1+x)^{-1}$

$$\int \frac{e^x + xe^x}{1+x}\,dx = \int \frac{e^x(1+x)}{1+x}\,dx = \int e^x dx = e^x$$

$$\int \frac{xe^x}{(1+x)^2}\,dx = -xe^x(1+x)^{-1} + e^x = e^x\left(\frac{-x}{1+x} + 1\right)$$

$$\rightarrow y_2 = e^x(1+x)\left(\frac{-x}{1+x} + 1\right) = e^x(-x+1+x) = e^x \Rightarrow y_2 = e^x$$

→ Variación de parámetros

$$W = \begin{vmatrix} 1+x & e^x \\ 1 & e^x \end{vmatrix} = e^x + xe^x - e^x = xe^x$$

$$W_1 = \begin{vmatrix} 0 & e^x \\ xe^{2x} & e^x \end{vmatrix} = -xe^{3x}$$

$$W_2 = \begin{vmatrix} 1+x & 0 \\ 1 & xe^{2x} \end{vmatrix} = xe^{2x} + x^2e^{2x} = e^{2x}(x+x^2)$$

$$u_1 = \int \frac{-xe^{3x}}{xe^x}\,dx = -\int e^{2x} = -\frac{1}{2}e^{2x}$$

$$u_2 = \int \frac{e^{2x}(x+x^2)}{xe^x}\,dx = \int e^x dx + \int xe^x dx = e^x + xe^x - e^x = xe^x$$

$$y_p = -\frac{e^{2x}}{2} - \frac{xe^{2x}}{2} + xe^{2x}$$

$$y_p = \frac{e^{2x}}{2}(-1+x)$$