Taller Quiz

1. Use coeficientes indeterminados.

a) $y'' + 4y' + 3y = 4x^2 - 5x + 6$

$y_p = Ax^2 + Bx + C \quad y_p' = 2Ax + B \quad y_p'' = 2A$. Reemplazando $2A + 4(2Ax+B) + 3(Ax^2+Bx+C)$

$2A + 8Ax + 4B + 3Ax^2 + 3Bx + 3C = x^2(3A) + x(8A+3B) + (2A+4B+3C) = 4x^2 - 5x + 6. \quad y_p = A(\overset{Ec1.}{x^2}) + B(\overset{Ec2}{x}) + C$

Coeficientes de x^2 $\quad 3A = 4 \quad A = \frac{4}{3}$

Coeficientes x: $8A + 3B = -5 \quad \frac{32}{3} + 3B = -5 \quad 3B = -\frac{47}{3} \quad B = -\frac{47}{9}$

Coeficientes "1" $\quad 2A + 4B + 3C = 6 \quad \frac{8}{3} - \frac{188}{9} + 3C = 6 \quad 3C = \frac{218}{9} \quad C = \frac{218}{27}$

y_c: $m^2 + 4m + 3 = 0 \quad (m+3)(m+1) = 0 \quad \begin{matrix} m_1 = -3 \\ m_2 = -1 \end{matrix} \quad y_c = C_1 e^{-3x} + C_2 e^{-x}$

$\therefore \quad y = C_1 e^{-3x} + C_2 e^{-x} + \frac{4}{3}x^2 - \frac{47}{9}x + \frac{218}{27}$

b) $y'' + y' = (3x^2 + x)e^{5x}$

y_c: $m^2 + m = 0 \quad m(m+1) = 0 \quad \begin{matrix} m = 0 \\ m_1 = -1 \end{matrix} \quad y_c = C_1 + C_2 e^{-x}$

$y_p = (Ax^2 + Bx + C)e^{5x} = A(\overset{Ec1}{x^2 e^{5x}}) + B(\overset{Ec2}{x e^{5x}}) + C(\overset{Ec3}{e^{5x}})$

$y_p' = A(2xe^{5x} + 5x^2e^{5x}) + B(e^{5x} + 5xe^{5x}) + C(5e^{5x})$

$y_p'' = A(2e^{5x} + 20xe^{5x} + 25x^2e^{5x}) + B(10e^{5x} + 25xe^{5x}) + C(25e^{5x})$ Reemplazando

$2Ae^{5x} + 22Axe^{5x} + 30Ax^2e^{5x} + 11Be^{5x} + 30Bxe^{5x} + 30Ce^{5x}$

$x^2e^{5x}(30A) + xe^{5x}(22A + 30B) + e^{5x}(2A + 11B + 30C) = 3(x^2e^{5x}) + (xe^{5x}) + 0(e^{5x})$

Ecuación "x^2e^{5x}": $30A = 3 \quad A = \frac{1}{10}$

Ecuación "xe^{5x}": $22A + 30B = 1 \quad \frac{11}{5} + 30B = 1 \quad B = -\frac{1}{25}$

Ecuación "e^{5x}": $2A + 11B + 30C = 0 \quad \frac{1}{5} - \frac{11}{25} + 30C = 0 \quad C = \frac{1}{125}$

$\hookrightarrow \therefore \quad y = C_1 + C_2 e^{-x} + \left(\frac{1}{10} \cdot x^2 - \frac{1}{25} \cdot x + \frac{1}{125}\right)e^{5x}$

c) $y'' - 4y = \cos(2x)$

y_c: $m^2 - 4 = 0 \quad m = \pm 2 \quad y_c = C_1 e^{2x} + C_2 e^{-2x}$

$y_p = A\overset{Ec1}{\cos(2x)} + B\overset{Ec2.}{\text{sen}(2x)} \quad y_p' = -2A\sin(2x) + 2B\cos(2x) \quad y_p'' = -4A\cos(2x) - 4B\sin(2x)$. Reemplazando:

$-4A\cos(2x) - 4B\sin(2x) - 4A\cos(2x) - 4B\,\text{sen}(2x) = \cos(2x)[-8A] + \text{sen}(2x)[-8B] = \cos(2x)$

Ecuación "$\cos(2x)$": $-8A = 1 \quad A = -\frac{1}{8}$ $\qquad$ Ecuación "$\text{sen}(2x)$": $-8B = 0 \quad B = 0$

$y = C_1 e^{2x} + C_2 e^{-2x} - \frac{1}{8}\cos(2x)$

d) $y'' + 5y' + 6y = 4x\,\text{sen}(2x)$

y_c: $m^2 + 5m + 6m = 0 \quad (m+3)(m+2) = 0. \quad \begin{matrix} m = -3 \\ m = -2 \end{matrix} \quad y_c = C_1 e^{-3x} + C_2 e^{-2x}$

$y_p = (Ax+B)\,\text{sen}(2x) + (Cx+D)\cos(2x) = A(x\,\text{sen}(2x)) + B(\text{sen}\,2x) + C(x\cos(2x)) + D(\cos(2x)).$ $\sim 6Ax\sin(2x) + 6B\sin(2x) + 6Cx\cos(2x) + 6D\cos(2x)$

$y_p' = A\,\text{sen}(2x) + 2(Ax+B)\cos(2x) + C\cos(2x) - 2(Cx+D)\sin(2x)$ $\sim 5A\sin(2x) + 10Ax\cos(2x) + 10B\cos(2x) + 10C\cos(2x) - 10Cx\sin(2x) - 10D\sin(2x)$

$y_p'' = 4A\cos(2x) - 4(Ax+B)\sin(2x) - 4C\sin(2x) - 4(Cx+D)\cos(2x)$ $\sim 4A\cos(2x) - 4Ax\sin(2x) - 4B\sin(2x) - 4C\sin(2x) - 4Cx\cos(2x) - 4D\cos(2x)$

$4A\cos(2x) + 2Ax\sin(2x) + 2B\sin(2x) - 4C\sin(2x) + 2Cx\cos(2x) + 2D\cos(2x) + 5A\sin(2x) + 10Ax\cos(2x) + 10B\cos(2x) + 5C\cos(2x) - 10Cx\sin(2x) - 10D\sin(2x)$

$= x\,\text{sen}(2x)[2A - 10C] + \text{sen}(2x)[2B - 4C + 5A - 10D] + x\cos(2x)[2C + 10A] + \cos(2x)[4A + 2D + 10B + 5C] = 4x\,\text{sen}(2x)$

$2A-10C=4 \quad 2A-10(-5A)=4 \quad 2A+50A=4 \quad A=\frac{1}{13}$

$2B-4C-10D+5A=0 \quad 2B-10D=\frac{-25}{13} \quad 2B-10(\frac{21}{26}-5B)=\frac{-25}{13} \quad B=\frac{20}{169}$

$2C+10A=0 \quad C=-5A \quad C=\frac{-5}{13}$

$4A+2D+10B+5C=0 \quad 2D+10B=\frac{21}{13} \quad D=\frac{21}{26}-5B \quad D=\frac{73}{338}$

$D=\frac{73}{338}$

$B=\frac{20}{169}$

e) $y''-4y'+4y=3e^{2x}$

y_c: $m^2-4m+4=0 \quad (m-2)^2=0 \quad m=2 \quad y=\overbrace{C_1e^{2x}+C_2xe^{2x}}^{y_c}+\overbrace{C_3x^2e^{2x}}^{y_p}$

$y_p=C_3x^2e^{2x} \quad y_p'=C_3(2xe^{2x}+2x^2e^{2x}) \quad y_p''=C_3(2e^{2x}+8xe^{2x}+4x^2e^{2x})$. Reemplazando

$2C_3e^{2x}=3e^{2x} \quad 2C_3=3 \quad C_3=3/2$

$\hookrightarrow \therefore y=C_1e^{2x}+C_2xe^{2x}+\frac{3}{2}\cdot x^2e^{2x}$

2. Use variación de parámetros.

a) $y''+y=\tan(x)$

y_c: $m^2+1=0 \quad m=\sqrt{-1} \quad \begin{matrix}\alpha=0\\ \beta=1\end{matrix} \quad y_c=C_1\operatorname{sen}(x)+C_2\cos(x) \quad y_1=\operatorname{sen}(x) \quad y_2=\cos(x)$

$$W=\begin{vmatrix}\operatorname{sen}(x) & \cos(x)\\ \cos(x) & -\operatorname{sen}(x)\end{vmatrix}=-\operatorname{sen}^2(x)-\cos^2(x)=-(\operatorname{sen}^2(x)+\cos^2(x))=-1$$

$$W_1=\begin{vmatrix}0 & \cos(x)\\ \tan(x) & -\operatorname{sen}(x)\end{vmatrix}=-\cos(x)\cdot\frac{\operatorname{sen}(x)}{\cos(x)}=-\operatorname{sen}(x)$$

$$W_2=\begin{vmatrix}\operatorname{sen}(x) & 0\\ \cos(x) & \tan(x)\end{vmatrix}=\operatorname{sen}(x)\cdot\frac{\operatorname{sen}(x)}{\cos(x)}=\frac{\operatorname{sen}^2(x)}{\cos(x)}$$

$$u_1=\int\frac{W_1}{W}=\int\frac{-\operatorname{sen}(x)}{-1}=-\cos(x)$$

$$u_2=\int\frac{W_2}{W}=-\int\frac{\operatorname{sen}^2(x)}{\cos(x)}=-\int\frac{1-\cos^2(x)}{\cos(x)}dx=-\int\sec(x)-\cos(x)dx=-\ln|\sec x+\tan x|-\sin(x)$$

$$y_p=-\cos(x)\operatorname{sen}(x)-\cos(x)(\ln|\sec x+\operatorname{tg}x|+\cos x\operatorname{sen}x=-\cos x\ln|\sec x+\operatorname{tg}x|$$

$$y=y_c+y_p=C_1\operatorname{sen}x+C_2\cos x-\cos x\ln|\sec x+\operatorname{tg}x|$$

b) $y''+4y'+4y=x^{-2}e^{-2x}$

y_c: $m^2+4m+4=0 \quad (m+2)^2=0 \quad m=-2 \quad y=C_1e^{-2x}+C_2xe^{-2x} \quad y_1=e^{-2x} \quad y_2=xe^{-2x}$

$$W=\begin{vmatrix}e^{-2x} & xe^{-2x}\\ -2e^{-2x} & e^{-2x}-2xe^{-2x}\end{vmatrix}=e^{-2x}(e^{-2x}-2xe^{-2x})+2e^{-2x}(xe^{-2x})=e^{-4x}-2xe^{-4x}+2xe^{-4x}=e^{-4x}$$

$$w_1 = \begin{vmatrix} 0 & xe^{-2x} \\ x^{-2}e^{-2x} & e^{-2x}-2xe^{-2x} \end{vmatrix} = -x^{-2}e^{-2x} \cdot xe^{-2x} = -x^{-1}e^{-4x}$$

$$w_2 = \begin{vmatrix} e^{-2x} & 0 \\ -2e^{-2x} & x^{-2}e^{-2x} \end{vmatrix} = x^{-2}e^{-2x} \cdot e^{-2x} = x^{-2}e^{-4x}$$

$$u_1 = \int \frac{w_1}{w} = \int \frac{-x^{-1}e^{-4x}}{e^{-4x}} dx = \int -x^{-1} dx = -\ln|x|$$

$$u_2 = \int \frac{w_2}{w} dx = \int \frac{x^{-2}e^{-4x}}{e^{-4x}} dx = \int x^{-2} dx = -\frac{1}{x}$$

$$y_p = u_1 y_1 + u_2 y_2 = -\ln|x| e^{-2x} - \frac{1}{x} \cdot xe^{-2x} = e^{-2x}(-\ln(x) - 1)$$

$$y = y_p + y_c = C_1 e^{-2x} + C_2 xe^{-2x} + e^{-2x}(-\ln(x) - 1)$$

c) $y'' + 6y' + 9y = e^{3x}$

y_c: $m^2 + 6m + 9 = 0$ $(m+3)^2 = 0$ $m = -3$ $y_c = C_1 e^{-3x} + C_2 xe^{-3x}$ $y_1 = e^{-3x}$ $y_2 = xe^{-3x}$

$$W = \begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & e^{-3x}-3xe^{-3x} \end{vmatrix} = e^{-6x} - 3xe^{-6x} + 3xe^{-6x} = e^{-6x}$$

$x = \frac{u}{6}$
$u = 6x \quad du = 6dx$

$$W_1 = \begin{vmatrix} 0 & xe^{-3x} \\ e^{3x} & e^{-3x}-3xe^{-3x} \end{vmatrix} = -x \qquad u_1 = \int \frac{W_1}{W} dx = \int \frac{-x}{e^{-6x}} dx = \int -xe^{6x} dx = -\int \frac{e^u u}{36} du = -\frac{1}{6}xe^{6x} + \frac{e^{6x}}{36}$$

$$W_2 = \begin{vmatrix} e^{-3x} & 0 \\ -3e^{-3x} & e^{3x} \end{vmatrix} = 1 \quad u_2 = \int \frac{W_2}{W} dx = \int \frac{1}{e^{-6x}} dx = \int e^{6x} dx = \frac{1}{6} e^{6x}$$

$$y_p = u_1 y_1 + u_2 y_2 = -\frac{1}{6} xe^{3x} + \frac{1}{36} e^{3x} + \frac{1}{6} xe^{3x} = \frac{1}{36} e^{3x}$$

$$y = y_c + y_p = C_1 e^{-3x} + C_2 xe^{-3x} + \frac{1}{36} \cdot e^{3x}$$

d) $y'' + 3y' + 2y = \frac{1}{1+e^x}$

y_c: $m^2 + 3m + 2 = 0$ $(m+2)(m+1) = 0$ $y_c = C_1 e^{-2x} + C_2 e^{-x}$ $y_1 = e^{-2x}$ $y_2 = e^{-x}$

$$W = \begin{vmatrix} e^{-2x} & e^{-x} \\ -2e^{-2x} & -e^{-x} \end{vmatrix} = -e^{-3x} + 2e^{-3x} = e^{-3x}$$

$\frac{e^{2x}}{u} \frac{du}{e^x}$ $\frac{e^x}{u} du$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ \frac{1}{1+e^x} & -e^{-x} \end{vmatrix} = -\frac{e^{-x}}{1+e^x} \quad u_1 = \int \frac{W_1}{W} dx = \int \frac{-\frac{e^{-x}}{1+e^x}}{\frac{e^{-3x}}{1}} dx = \int \frac{-e^{-x}}{e^{-3x}(1+e^x)} dx = \int \frac{-e^{2x}}{1+e^x} dx = -\int \frac{u-1}{u} du = \int 1 - \frac{1}{u} du = (u - \ln|u|) - 1$$

$$= -1 - e^x + \ln|1+e^x|$$

$u = 1+e^x$
$du = e^x dx$
$u - 1 = e^x$

$$W_2=\begin{vmatrix} e^{-2x} & 0 \\ -2e^{-2x} & \frac{1}{1+e^x}\end{vmatrix}=\frac{e^{-2x}}{1+e^x} \quad u_2=\int\frac{W_2}{W}dx=\int\frac{\frac{e^{-2x}}{1+e^x}}{\frac{e^{-3x}}{1}}dx=\int\frac{e^{-2x}}{e^{-3x}(1+e^x)}=\int\frac{e^x}{1+e^x}dx=\int\frac{1}{u}du=Ln|u|=Ln|1+e^x|$$

$u=1+e^x$
$du=e^x dx$

$$y_p=u_1y_1+u_2y_2=(ln|1+e^x|-1-e^x)e^{-2x}+e^{-x}Ln|1+e^x|=$$

$$y=C_1e^{-2x}+C_2e^{-x}+e^{-x}Ln|1+e^x|+e^{-2x}(ln|1+e^x|-1-e^x)$$

e) $y''+2y'+y=e^{-x}Ln\,x$

y_c: $m^2+2m+1=0 \quad (m+1)^2=0 \quad m=-1 \quad y_c=C_1e^{-x}+C_2xe^{-x} \quad y_1=e^{-x} \quad y_2=xe^{-x}$

$$W=\begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & e^{-x}-xe^{-x}\end{vmatrix}=e^{-x}(e^{-x}-xe^{-x})+e^{-x}(xe^{-x})=e^{-2x}-xe^{-2x}+xe^{-2x}=e^{-2x}$$

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$u=\ln x \quad du=\frac{1}{x}dx \quad dv=x \quad v=\int x dx=\frac{x^2}{2}$

$$W_1=\begin{vmatrix} 0 & xe^{-x} \\ e^{-x}\ln x & e^{-x}-xe^{-x}\end{vmatrix}=-e^{-x}\ln x\cdot xe^{-x}=-xe^{-2x}\ln x \quad u_1=\int\frac{W_1}{W}dx=\int\frac{-xe^{-2x}\ln x}{e^{-2x}}dx=-\int x\ln x=-\left(\frac{x^2}{2}\cdot\ln x-\int\frac{x^2}{2}\cdot\frac{1}{x}dx\right)=\left(\frac{x^2}{2}\cdot\ln x-\frac{x^2}{4}\right)\cdot -1=\frac{x^2}{4}-\frac{x^2}{2}\ln x$$

$$W_2=\begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{-x}\ln x\end{vmatrix}=e^{-2x}\ln x \quad u_2=\int\frac{W_2}{W}dx=\int\frac{e^{-2x}\ln x}{e^{-2x}}dx=\int Ln\,x=xLn\,x-x$$

$$y_p=u_1y_1+u_2y_2=e^{-x}\left(\frac{x^2}{4}-\frac{x^2}{2}Ln\,x\right)+xe^{-x}(xLn\,x-x)=\frac{x^2e^{-x}-2x^2e^{-x}Ln\,x+4x^2e^{-x}Ln\,x-4x^2e^{-x}}{4}=\frac{-3x^2e^{-x}+2x^2e^{-x}Ln\,x}{4}$$

$$y=y_c+y_p=C_1e^{-x}+C_2xe^{-x}+\frac{2x^2e^{-x}\ln x-3e^{-x}x^2}{4}$$

f) $y''-9y=\frac{9x}{e^{3x}}$.

y_c: $m^2-9=0 \quad m^2=9 \quad m=\pm3 \quad y_c=C_1e^{3x}+C_2e^{-3x} \quad y_1=e^{3x} \quad y_2=e^{-3x}$

$$W=\begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x}\end{vmatrix}=-3e^{-3x}e^{3x}-3e^{3x}\cdot e^{-3x}=-3-3=-6$$

$\frac{3}{2}x \quad e^{-6x}$
$\frac{3}{2} \quad \frac{1}{-6}e^{-6x}$
$0 \quad \frac{1}{36}e^{-6x}$

$$W_1=\begin{vmatrix} 0 & e^{-3x} \\ \frac{9x}{e^{3x}} & -3e^{-3x}\end{vmatrix}=-e^{-3x}\cdot\frac{9x}{e^{3x}}=-e^{-6x}9x \quad u_1=\int\frac{W_1}{W}=\int\frac{-9xe^{-6x}}{-6}dx=\int\frac{3}{2}xe^{-6x}dx=-\frac{1}{4}xe^{-6x}-\frac{1}{24}e^{-6x}$$

$$W_2=\begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & \frac{9x}{e^{3x}}\end{vmatrix}=e^{3x}\cdot\frac{9x}{e^{3x}}=9x \quad u_2=\int\frac{W_2}{W}dx=\int\frac{9x}{-6}dx=-\frac{3}{4}x^2$$

$$y_p=u_1y_1+u_2y_2=e^{3x}\left(-\frac{1}{4}xe^{-6x}-\frac{1}{24}e^{-6x}\right)-\frac{3}{4}x^2e^{-3x}=-\frac{1}{4}xe^{-3x}-\frac{1}{24}e^{-3x}-\frac{3}{4}x^2e^{-3x}$$

$$y=y_c+y_p=C_1e^{3x}+C_2e^{-3x}-\frac{3}{4}x^2e^{-3x}-\frac{1}{4}xe^{-3x}-\frac{1}{24}e^{-3x}$$

g) $y''-3y'-10y=7xe^{-2x}$

$y_c: m^2-3m-10=0 \quad (m-5)(m+2)=0 \quad \begin{matrix} m=5 \\ m=-2 \end{matrix} \quad y=C_1e^{5x}+C_2e^{-2x} \quad y_1=e^{5x} \quad y_2=e^{-2x}$

$W=\begin{vmatrix} e^{5x} & e^{-2x} \\ 5e^{5x} & -2e^{-2x} \end{vmatrix} = e^{5x}(-2e^{-2x})-(e^{-2x}\cdot 5e^{5x}) = -2e^{3x}-5e^{3x}=-7e^{3x}$

$\begin{matrix} x \searrow + e^{-7x} \\ 1 \searrow -\frac{1}{7}e^{-7x} \\ 0 \searrow \frac{1}{49}e^{-7x} \end{matrix}$

$W_1=\begin{vmatrix} 0 & e^{-2x} \\ 7xe^{-2x} & -2e^{-2x} \end{vmatrix} = -e^{-2x}\cdot 7xe^{-2x}=-7xe^{-4x} \quad u_1=\int \frac{W_1}{W}dx=\int \frac{-7xe^{-4x}}{-7e^{3x}}dx=\int xe^{-7x}dx=-\frac{1}{7}e^{-7x}x-\frac{1}{49}e^{-7x}$

$W_2=\begin{vmatrix} e^{5x} & 0 \\ 5e^{5x} & 7xe^{-2x} \end{vmatrix} = e^{5x}\cdot 7xe^{-2x}=7xe^{3x} \quad u_2=\int \frac{W_2}{W}dx=\int \frac{7xe^{3x}}{-7e^{3x}}dx=\int -x\,dx=\frac{-x^2}{2}$

$y_p=u_1y_1+u_2y_2=e^{5x}(-\frac{1}{7}e^{-7x}x-\frac{1}{49}e^{-7x})+e^{-2x}(\frac{-x^2}{2})=\frac{-1}{7}xe^{-2x}-\frac{1}{49}e^{-2x}-\frac{x^2}{2}e^{-2x}$

$y=y_p+y_c=C_1e^{5x}+Ce^{-2x}-\frac{1}{7}xe^{-2x}-\frac{x^2}{2}e^{-2x} \quad C=C_1-\frac{1}{49}$

h) $y''+9y=2\sec(3x)$

$y_c: m^2+9=0 \quad m=\sqrt{-9} \quad \begin{matrix}\alpha=0\\ \beta=3\end{matrix} \quad y=C_1\cos(3x)+C_2\,\mathrm{sen}(3x) \quad y_1=\cos(3x) \quad y_2=\mathrm{sen}(3x)$

$W=\begin{vmatrix} \cos(3x) & \mathrm{sen}(3x) \\ -3\sin(3x) & 3\cos(3x) \end{vmatrix} = 3\cos^2(3x)+3\,\mathrm{sen}^2(3x)=3$

$W_1=\begin{vmatrix} 0 & \mathrm{sen}(3x) \\ 2\sec(3x) & 3\cos(3x) \end{vmatrix} = -2\sec(3x)\cdot\mathrm{sen}(3x)=-2\,\mathrm{tg}(3x) \quad u_1=\int \frac{-2\,\mathrm{tg}(3x)}{3}dx=\frac{2}{9}\ln|\cos(3x)|$

$W_2=\begin{vmatrix} \cos(3x) & 0 \\ -3\sin(3x) & 2\sec(3x) \end{vmatrix} = 2 \quad u_2=\int \frac{2}{3}dx=\frac{2}{3}x$

$y_p=u_1y_1+u_2y_2=\frac{2}{9}\cos(3x)\ln|\cos(3x)|+\frac{2}{3}x\,\mathrm{sen}(3x)$

$y=C_1\cos(3x)+C_2\,\mathrm{sen}(3x)+\frac{2}{9}\cos(3x)\ln|\cos(3x)|+\frac{2}{3}x\,\mathrm{sen}(3x)$