**Método lineal** $y'+P(t)y=Q(t)$ | $\mu(t)=e^{\int P(t)dt}$ | $y=\frac{\int Q(t)\mu(t)dt}{\mu(t)}+\frac{C}{\mu(t)}$

**Variables Separables** $y'=g(t)\cdot h(y)$ | $\int \frac{1}{h(y)}dy=\int g(t)dt$ Despejar y.

**Ecuaciones Exactas** $Mdx+Ndy=0$ | $\frac{\partial F}{\partial x}=M$ $\frac{\partial f}{\partial y}=N$ | $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ | $\int Mdx=f(x)+C(y)$ $\frac{\partial f}{\partial y}=N$

ó $\int Ndy=f(y)+C(y)$ $\to \frac{\partial f}{\partial x}=M$ | $f(x,y)=f(x)+C(y)$ ó $f(x)+C(x)$

→**Factor Integrante** $\frac{M_y-N_x}{N}=h(x)$ ó $\frac{N_x-M_y}{M}=g(y)$ | $\mu$ $^{\mu\ \text{segundo}}_{\text{primero o}}$ | $\mu(?)Mdx+\mu(?)Ndy=0$

# Modelos Matemáticos

**Dinámica Poblacional** $\frac{dP}{dt}=KP_{(t)}$ | $P(t)=P_0e^{Kt}$ | $P(t)$ población en t | Hallar $P_0$ y K despejando

$P(3)=400$ $400=P_0e^{3K}\to P_0=\frac{400}{e^{3K}}$ ; $P(10)=2000$ $2000=P_0e^{10K}\to P_0=\frac{2000}{e^{10K}}$ } $\frac{400}{e^{3K}}=\frac{2000}{e^{10K}}\to\frac{e^{10K}}{e^{3K}}=\frac{2000}{400}\Rightarrow e^{7K}=5$ $K=\frac{\ln 5}{7}$

$P_0=\frac{400}{e^{3(\ln 5/7)}}=200{,}678=80\cdot 5^{4/7}$ Bacterias

**Decaimiento Radioactivo** $\frac{dA}{dt}=KA_{(t)}$ | $P(t)=P_0e^{Kt}$ | Vida Media $A(t)=\frac{1}{2}A_0\to\frac{1}{2}=e^{Kt}$ $t=\frac{\ln(0{,}5)}{K}$ | $A(t)$ sustancia que queda

$P(0)=100$ $P(6)=3\%=97\to 97=100e^{6K}$ $K=\frac{\ln(0{,}97)}{6}=-0{,}0051$ $P(t)=100e^{-0{,}0051t}$

$P(24)=88{,}47\,mg$ $t=\frac{\ln(0{,}5)}{-0{,}0051}=135{,}91\ h.$

**Ley de Enfriamento** $\frac{dT}{dt}=K(T-T_m)$ | $T(t)=T_m+Ce^{Kt}$ | $T(t)$ = temperatura en t. $dT/dt$ rapidez cambio, $T_m$ es la temperatura ambiente

$T(0)=100°C$ $T_m=30°C$ $T(3)=70°C$ $T(t)=31°C\to T(t)=30+Ce^{Kt}$ $T(0)=100=30+C\to C=70\to T=30+70e^{Kt}$

$T(3)=70=30+70e^{3K}\to\frac{4}{7}=e^{3K}$ $K=\frac{\ln(4/7)}{3}=-0{,}1865\to T(t)=30+70e^{-0{,}1865t}=31\to t=22{,}78$

**Mezclas** $\frac{dA}{dt}=$ Razón Entra − Razón Sale | $A(t)$ = Cantidad de sal en t.

• Si vel entra = vel salida $\frac{dA}{dt}=\text{Vel Entra}\left[\frac{V_0}{t}\right]\cdot\text{Concen Entra}\left[\frac{M_a}{V}\right]-\text{Vel Sali}\left[\frac{V_0}{t}\right]\cdot\frac{A}{\text{Cap. Tanque}[V_0]}\to\frac{\partial A}{\partial t}+P(t)A=Q(t)$

"$A(t)=C_eV-C_1e^{-\left(\frac{V_s}{V}\cdot t\right)}$"

$V=200L$ $A(0)=30$ $C_e=1\%$ $V_e=4\ l/min$ $V_s=4\ l/min$ $\frac{dA}{dt}=4-\frac{A}{50}$ $\int Pdt=\frac{t}{50}$ $\int Q(t)\cdot\mu(t)=200e^{\frac{t}{50}}$ $A(t)=200+\frac{C}{e^{t/50}}$ $A(0)=30$ $C=170$

• Si vel entra ≠ vel sale $\frac{dA}{dt}=\text{Vel Entra}\left[\frac{V_0}{t}\right]\cdot\text{Concen Entra}\left[\frac{M_a}{V}\right]-\text{Vel Sali}\left[\frac{V_0}{t}\right]\cdot\frac{A}{\text{Cap. Tanque}[V_0]+(\text{Vel entra}-\text{Vel Sale})t}$

$X:1000$ $V:500$ $A(0)=0$ $C_e=2$ $V_e=5$ $V_s=10$ | $\frac{dA}{dt}=10-\frac{2A}{100-t}$ $\int Pdt=-2\ln(100-t)$ $\int Q\mu dt=10(100-t)^{-1}$ $\to A(t)=10(100-t)+C(100-t)^2$ $A(0)=0$ $C=-\frac{1}{10}$

"$A(t)=C_e(Vol+(V_e-V_s)\cdot t)+C_1(V+(V_e-V_s)t)^{\frac{-V_s}{V_e-V_s}}$" *

$X:1000$ $V:500$ $A(0)=0$ $C_e=2$ $V_e=5$ $V_s=10$ | $A(t)=2(500-5t)+\frac{C_1}{(500-5t)^{-2}}\to 10(100-t)+\frac{\cancel{C_1}\to C}{5(100-t)^{-2}}$ $C=\frac{C_1}{5}$

# ED Homogenea $a_0(x)y+a_1(x)y'+\dots+a_nx\,y^{(n)}=0$

**Principio de Superposición**: Si $y_1,\dots,y_n$ son soluciones $\to y=C_1y_1+\dots+C_ny$

**Teorema** Sean $y_1,y_2,\dots,y_n$ soluciones, son linealmente independientes Sii $W(y_1,\dots,y_n)\neq 0$ | $W(y_1,y_2,\dots,y_n)=\begin{vmatrix} y_1 & y_2 & \cdots & y_n\\ y_1' & y_2' & \cdots & y_n'\\ \vdots & \vdots & & \vdots\\ y_1^{n-1} & y_2^{n-1} & & y_n^{n-1}\end{vmatrix}$

**Coeficientes Constantes** $ay''+by+cy=0\to am^2+bm+C=0$ $m=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

1) $b^2-4ac>0\to m_1\neq m_2$: $y=C_1e^{m_1x}+C_2e^{m_2x}$ | 2) $b^2-4ac=0\to m_1=m_2$: $y=C_1e^{mx}+C_2xe^{mx}+C_3x^2e^{mx}$ | 3) $b^2-4ac<0\to\alpha=Re(m)$ $B=Img(m)$: $y=C_1e^{\alpha x}\cos(Bx)+C_2e^{\alpha x}\operatorname{sen}(Bx)$

**Reducción de Orden** llevar a la forma $y''+P(x)y'+Q(x)y=0$ | $y_1$ una solución | $y_2=y_1\int\frac{e^{-\int P(x)dx}}{y_1(x)^2}dx$

# ED NO Homogenea $ay''+by'+cy=g(x)$

1) Resolver $ay''+by'+cy=0\to y_c$ complementaria | 2) Coeficientes / Parámetros $=y_p$ | 3) $y=y_c+y_p$

**Coeficientes Indeterminados**

| $g(x)$ | $10\to$ | $5-x^2\to$ | $7x^2e^{3x}$ | $\cos(6x)$ | $e^{3x}\to$ | $4x^2\operatorname{sen}x$ |
|---|---|---|---|---|---|---|
| Será: | $A$ | $Ax^2+Bx+C$ | $(Ax^2+Bx+C)e^{3x}$ | $A\cos(6x)+B\operatorname{sen}(6x)$ | $Ae^{3x}$ | $(Ax^2+Bx+C)\operatorname{sen}x+(Dx^2+Ex+F)\cos x$ |

Se sacan las derivadas, se reemplazan en la ED y se iguala a $g(x)$. Se despejan terminos $(3A-4B)x=3x$ y se despejan las letras, se reemplazan. No funciona en $\tan(x)$, $\sec(x)$, $\sqrt{x}$, $\log(x)$, $\frac{g(x)}{f(x)}$

$y''+2y'+5y=4e^{-x}\cos(2x)$ $y_c$: $m^2+2m+5=0$ $m=-1\pm 2i$ $\alpha=-1$ $B=2$ $y_c$: $C_1e^{-x}\operatorname{sen}(2x)+C_2e^{-x}\cos(2x)$

$y_p=Axe^{-x}\cos(2x)+Bxe^{-x}\sin(2x)$ $y_p'=A[e^{-x}\cos(2x)-xe^{-x}\cos(2x)-2xe^{-x}\sin(2x)]+B[e^{-x}\sin(2x)-xe^{-x}\sin(2x)+2xe^{-x}\cos(2x)]$

$y_p''=A[2e^{-x}\cos(2x)-4e^{-x}\sin(2x)+4xe^{-x}\sin(2x)-3xe^{-x}\cos(2x)]+B[-2e^{-x}\sin(2x)+4e^{-x}\cos(2x)-4xe^{-x}\cos(2x)-3xe^{-x}\sin(2x)]$

$-4Ae^{-x}\sin(2x)+4Be^{-x}\cos(2x)=4e^{-x}\cos(2x)$ $e^{-x}\sin(2x)$: $-4A=0$, $A=0$

$e^{-x}\cos(2x)$: $4B=4$, $B=1$. $y=C_1e^{-x}\operatorname{sen}(2x)+C_2e^{-x}\cos(2x)+xe^{-x}\sin(2x)$

$y'=C_1[-e^{-x}\operatorname{sen}(2x)+2e^{-x}\cos(2x)]+C_2[-e^{-x}\cos(2x)-2e^{-x}\sin(2x)]+e^{-x}\sin(2x)-xe^{-x}\sin(2x)+2xe^{-x}\cos(2x)$

$y(0)=1=C_2\to C_2=1$ $y'(0)=0=2C_1-C_2\to C_1=\frac{1}{2}$

$\therefore y=\frac{1}{2}e^{-x}\operatorname{sen}(2x)+e^{-x}\cos(2x)+xe^{-x}\sin(2x)$

**Variación de Parámetros** 1) Hallar $y_c$. $y_1,y_2,\dots,y_n$ acompañan $C_n$. | 2) $W=\begin{vmatrix}y_1 & y_2\\ y_1' & y_2'\end{vmatrix}$ $W_1=\begin{vmatrix}0 & y_2\\ g(x) & y_2'\end{vmatrix}$ $u_1=\int\frac{W_1}{W}$, $W_2=\begin{vmatrix}y_1 & 0\\ y_1' & g(x)\end{vmatrix}$ $u_2=\int\frac{W_2}{W}$ | 3) $y_p=u_1y_1+u_2y_2$

$xy''-(1+x)y'+y=x^2e^{2x}$ • Si $y_1=1+x$ $y_1'=1$ $y''=0$ Si $xy''-(1+x)y'+y=0-(1+x)+(1+x)=0$ ✓

• Sea $y''+\left(-\frac{1+x}{x}\right)y'+\left(\frac{1}{x}\right)y=xe^{2x}$ y $y_1(x)=1+x$

$y_2=y_1\int\frac{e^{-\int P(x)dx}}{y_1^2(x)}dx=(1+x)\int\frac{e^{-\int-\frac{1+x}{x}dx}}{(1+x)^2}dx=(1+x)\int\frac{e^{\ln(x)+x}}{(1+x)^2}dx=(1+x)\int\frac{e^xx}{(1+x)^2}dx$ $u=xe^x$ $du=(e^x+xe^x)dx$ $dv=(1+x)^{-2}dx$ $v=-\frac{1}{1+x}$

$=(1+x)\left[xe^x\left(\frac{-1}{1+x}\right)-\int-\frac{1}{1+x}(e^x+xe^x)dx\right]=(1+x)\left[\frac{-xe^x}{1+x}+\int e^xdx\right]=(1+x)\left[e^x-\frac{xe^x}{1+x}\right]=e^x+xe^x-xe^x=e^x$

$y_c=C_1(1+x)+C_2(e^x)$ $W=\begin{vmatrix}1+x & e^x\\ 1 & e^x\end{vmatrix}=e^x(1+x)-e^x=e^x+xe^x-e^x=xe^x$

$W_1=\begin{vmatrix}0 & e^x\\ xe^{2x} & e^x\end{vmatrix}=-e^x(xe^{2x})$ $u_1=\int\frac{-e^x(xe^{2x})}{xe^x}dx=\int-e^{2x}dx=-\frac{1}{2}e^{2x}$

$W_2=\begin{vmatrix}1+x & 0\\ 1 & xe^{2x}\end{vmatrix}=(1+x)xe^{2x}$ $u_2=\int\frac{(1+x)xe^{2x}}{xe^x}dx=\int e^x+xe^xdx=e^x+e^xx-e^x=xe^x$

$y_p=u_1y_1+u_2y_2=(1+x)(-\frac{1}{2}e^{2x})+e^x(xe^x)=-\frac{1}{2}e^{2x}-\frac{1}{2}xe^{2x}+xe^{2x}=-\frac{1}{2}e^{2x}+\frac{1}{2}xe^{2x}=\frac{-e^{2x}+xe^{2x}}{2}$

# Sistemas de ED 2x2 homogéneos y no homogeneos

Valores Propios $|A-\lambda I|$ Vectores Propios $Ax=\lambda x$

## Variación de Parámetros $X_p = u_1 V_1 e^{\lambda_1 t} + u_2 V_2 e^{\lambda_2 t}$

$x' = 3x - y + 4e^{2t}$, $y' = -x + 3y + 4e^{2t}$ | $X' = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix} x + \begin{pmatrix} 4e^{2t} \\ 4e^{2t} \end{pmatrix}$ | $\lambda = 4$, $\lambda = 2$ $V_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ | $X_c = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + C_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{4t}$

$X_p = u_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + u_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{4t} \to \begin{pmatrix} e^{2t} \\ e^{2t} \end{pmatrix} u_1' + \begin{pmatrix} e^{4t} \\ -e^{4t} \end{pmatrix} u_2' = \begin{pmatrix} 4e^{2t} \\ 4e^{2t} \end{pmatrix}$ Regla de Cramer

$u_1' = \dfrac{\begin{vmatrix} 4e^{2t} & e^{4t} \\ 4e^{2t} & -e^{4t} \end{vmatrix}}{\begin{vmatrix} e^{2t} & e^{4t} \\ e^{2t} & -e^{4t} \end{vmatrix}} = 4$ $u_1 = \int 4\,dt = 4t$ | $u_2' = \dfrac{\begin{vmatrix} e^{2t} & 4e^{2t} \\ e^{2t} & 4e^{2t} \end{vmatrix}}{\begin{vmatrix} e^{2t} & e^{4t} \\ e^{2t} & -e^{4t} \end{vmatrix}} = 0$ $u_2 = \int 0\,dt = 0 \Rightarrow X_p = \begin{pmatrix} 4t \\ 4t \end{pmatrix} e^{2t}$ $X = X_c + X_p$

## Coeficientes Indeterminados

$X' = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} x + \begin{pmatrix} -2t^2 \\ t+5 \end{pmatrix} \to \begin{vmatrix} 1-\lambda & 3 \\ 3 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 9$, $\lambda_1 = -2$, $\lambda_2 = 4$ $\lambda = 4$ $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \vec{0}$ $3a + 3b = 0$ $V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\lambda = -2$ $\begin{bmatrix} -3 & 3 \\ 3 & -3 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \vec{0}$ $-3 - 3b = 0$ $V_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $X_c = C_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t} + C_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{4t}$ $X_p = \begin{pmatrix} A \\ B \end{pmatrix}$

$\begin{pmatrix} -2t^2 \\ t+5 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} t^2 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} t + \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ $X_p = \begin{pmatrix} a \\ b \end{pmatrix} t^2 + \begin{pmatrix} b \\ c \end{pmatrix} t + \begin{pmatrix} e \\ f \end{pmatrix}$ $X_p' = 2 \begin{pmatrix} a \\ b \end{pmatrix} t + \begin{pmatrix} c \\ d \end{pmatrix}$

$2 \begin{pmatrix} a \\ b \end{pmatrix} t + \begin{pmatrix} c \\ a \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \left[ \begin{pmatrix} a \\ b \end{pmatrix} t^2 + \begin{pmatrix} b \\ c \end{pmatrix} t + \begin{pmatrix} e \\ f \end{pmatrix} \right] + \begin{pmatrix} -2t^2 \\ t+5 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} at^2 + ct + e \\ bt^2 + dt + f \end{pmatrix} + \begin{pmatrix} -2t^2 \\ t+5 \end{pmatrix} =$ *

$\begin{pmatrix} at^2 + ct + e + 3bt^2 + 3dt + 3f - 2t^2 \\ 3at^2 + 3ct + 3e + bt^2 + dt + f + t + 5 \end{pmatrix} = \begin{pmatrix} (a+3b-2)t^2 + (c+3d)t + (e+3f) \\ (3a+b)t^2 + (3c+d+1)t\,(3e+f+5) \end{pmatrix} = \begin{pmatrix} 2at + c \\ 2bt + a \end{pmatrix}$

"$t^2$" $0 = a + 3b - 2$, $0 = 3a + b$ | "$t$" $2a = c + 3d$, $2b = 3c + d + 1$ | "$t^0$" $c = e + 3f$, $d = 3e + f + 5$ | $b = 3/4$ $a = -1/4$ $c = 1/4$ $d = -1/4$ $f = 3/4$ $e = -2$ | $X = X_c + X_p$

## Transformada de Laplace

$\mathcal{L}\ \dfrac{t^{n-1} e^{at}}{(n-1)!} = \dfrac{1}{(s-a)^n}\ \mathcal{L}^{-1}$

| $f(t)$ | $\mathcal{L}(f)$ | $f(t)$ | $\mathcal{L}(f)$ | $f(t)$ | $\mathcal{L}(f)$ |
|---|---|---|---|---|---|
| $1$ | $\frac{1}{s}$ | $t^n e^{at}$ | $\frac{n!}{(s-a)^{n+1}}$ | $a\cos(Kt) + \frac{b}{K}\operatorname{sen}(Kt)$ | $\frac{as+b}{s^2+K^2}$ |
| $t^n$ | $\frac{n!}{s^{n+1}}$ | $\sin(Kt)$ | $\frac{K}{s^2+K^2}$ | $e^{at}\operatorname{sen}(Kt)$ | $\frac{K}{(s-a)^2+K^2}$ |
| $\frac{t^{n-1}}{(n-1)!}$ | $\frac{1}{s^n}$ | $\cos(Kt)$ | $\frac{s}{s^2+K^2}$ | $e^{at}\cos(Kt)$ | $\frac{(s-a)}{(s-a)^2+K^2}$ |
| $e^{at}$ | $\frac{1}{s-a}$ | $\sinh(Kt)$ | $\frac{K}{s^2-K^2}$ | $t\operatorname{sen}(Kt)$ | $\frac{2Ks}{(s^2+K^2)}$ |
| $e^{-at}$ | $\frac{1}{s+a}$ | $\cosh(Kt)$ | $\frac{s}{s^2-K^2}$ | $t\cos(Kt)$ | $\frac{s^2-K^2}{(s^2+K^2)^2}$ |

☆ $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$ ☆ $\mathcal{L}(af(t) + bg(t)) = a\mathcal{L}(f) + b\mathcal{L}(g)$

**Traslación 1:** • $\mathcal{L}\{e^{at} f(t)\} = \mathcal{L}\{f(t)\}\big|_{s \to s-a}$ • $\mathcal{L}^{-1}\{F(s-a)\} = e^{at}\mathcal{L}^{-1}(F(s))$.

**Transformada Derivada** $\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$ $\mathcal{L}(f') = sF - f(0)$ $\mathcal{L}(f'') = s^2 F - s f(0) - f'(0)$

**Escalon Unitario** $u(t-a) = \begin{cases} 0, & 0 \le t < a \\ 1, & t \ge a \end{cases}$ $\mathcal{L}(u(t-a)) = \dfrac{e^{-as}}{s}$

**Trozos** $f(g) = \begin{cases} g(t), & 0 \le t < a \\ h(t), & t \ge a \end{cases} \to f(t) = g(t) - g(t)u(t-a) + h(t)u(t-a)$

$f(t) = \begin{cases} 0, & 0 \le t < a \\ g(t), & a \le t < b \\ 0, & b \le t \end{cases} \to f(t) = g(t)u(t-a) - g(t)u(t-b)$

**Traslación 2:** $\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$ ó $\mathcal{L}\{g(t)u(t-a)\} = e^{-as}\mathcal{L}\{g(t+a)\}$

**Convolución** $f * g = \int_0^t f(\tau)\, g(t-\tau)\,d\tau$

**Teorema Convolución** • $\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}(f(t)) \cdot g(t) = F(s)G(s)$

• $\mathcal{L}^{-1}\{F(s) \cdot G(s)\} = \mathcal{L}^{-1}\{F(s)\} * \mathcal{L}^{-1}\{G(s)\} = f * g$

**Integrales** $\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \dfrac{F(s)}{s}$ $\mathcal{L}^{-1}\left\{\dfrac{F(s)}{s}\right\} = \int_0^t f(\tau)\,d\tau$

**Integral de Volterra** $f(t) = g(t) + \int_0^t f(\tau)\,h(t-\tau)\,d\tau$

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$\mathcal{L}\{\operatorname{sen} t \cdot u(t-\frac{\pi}{2})\} = e^{-\frac{\pi}{2}s} \cdot \mathcal{L}\{\operatorname{sen}(t+\frac{\pi}{2})\} = e^{-\frac{\pi}{2}s}\mathcal{L}(\sin(t)\cancel{\cos(\frac{\pi}{2})} + \cos(t)\cdot \sin(\frac{\pi}{2})) = \dfrac{e^{-\frac{\pi}{2}s} \cdot s}{s^2+1}$

$\mathcal{L}\{\cos t\, u(t-\pi)\} = e^{-\pi s} \cdot \mathcal{L}\{\cos(t+\pi)\} = e^{-\pi s} \cdot -\dfrac{s}{s^2+1}$

$\mathcal{L}\{\cos(t-4) \cdot u(t-4)\} = e^{-4s} \cdot \dfrac{s}{s^2+1}$ $\mathcal{L}^{-1}\left\{\dfrac{e^{-s}}{s}\right\} = u(t-1)$

$\mathcal{L}^{-1}\left\{\dfrac{e^{2s}}{s^2-4}\right\}$ $e^{2s} = u(t+2)$ y $\mathcal{L}^{-1}\left(\dfrac{1}{s^2-4}\right) = \dfrac{1}{2}\operatorname{senh}(2t) \to \mathcal{L}^{-1}\left\{\dfrac{e^{2s}}{s^2-4}\right\} = \dfrac{1}{2}u(t+2)\operatorname{senh}(2(t+2))$

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$y' + y = f(x)$, $y(0) = 5$, $f(x) = \begin{cases} 0, & 0 \le t < \pi \\ 3\cos, & t \ge \pi \end{cases} \to f(t) = 3\cos t \cdot u(t-\pi)$

$sY - 5 + Y = -3e^{-\pi s} \cdot \dfrac{s}{s^2+1} \to (s+1)Y = 5 - 3e^{-\pi s} \cdot \dfrac{s}{s^2+1} \to Y = \dfrac{5}{s+1} - \dfrac{3e^{-\pi s} \cdot s}{(s^2+1)(s+1)}$

$A = \frac{1}{2}$ $B = \frac{1}{2}$ $C = -\frac{1}{2}$ $y = \frac{3}{2}u(t-\pi)(\cos(t-\pi) + \operatorname{sen}(t-\pi)) + 5e^{-t}$

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$x' + 3y = t$, $y' + 3x = -1$, $x(0) = 0$, $y(0) = 1$. $\to sX + 3Y = \dfrac{1}{s^2}$, $3X + sY = \dfrac{s-1}{s}$

$A = \begin{bmatrix} s & 3 \\ 3 & s \end{bmatrix}$ $A_1 = \begin{bmatrix} \frac{1}{s^2} & 3 \\ \frac{s-1}{s} & s \end{bmatrix}$ $A_2 = \begin{bmatrix} s & \frac{1}{s^2} \\ 3 & \frac{s-1}{s} \end{bmatrix}$ $|A_1| = \dfrac{4-3s}{s}$ $|A_2| = s - 1 - \dfrac{3}{s^2}$ $|A| = s^2 - 9$

• $X = \dfrac{\frac{4-3s}{s}}{s^2-9} = \dfrac{4}{s(s^2+9)} - \dfrac{3}{s^2-9}$ $x = \dfrac{4}{9}[-1 + \cosh(3t)] - \operatorname{senh}(3t)$

$x = -\dfrac{4}{9} + \dfrac{4}{9}\left[\dfrac{e^{3t} + e^{-3t}}{2}\right] - \dfrac{e^{3t} - e^{-3t}}{2} = -\dfrac{4}{9} + \dfrac{5}{18}e^{3t} + \dfrac{13}{18}e^{-3t} = x(t)$

• $y = \dfrac{s - 1 - \frac{3}{s^2}}{s^2-9} = \dfrac{s}{s^2-9} - \dfrac{1}{s^2-9} - \dfrac{3}{s^2(s^2-9)} = \dfrac{s}{s^2-9} - \dfrac{1}{s^2-9} - 3\left(-\dfrac{1}{9s^2} + \dfrac{1}{9(s^2-9)}\right)$

$y(t) = \cosh(3t) - \dfrac{1}{3}\operatorname{senh}(3t) - \dfrac{1}{9}[-3t + \operatorname{senh}(3t)]$

$y(t) = \dfrac{5}{18}e^{3t} + \dfrac{13}{18}e^{-3t} + \dfrac{t}{3}$