

```
In[37]:= A = {{3, 2, 2, 2, 1}, {2, 3, 2, 2, 1}, {2, 2, 3, 1, 1}, {2, 2, 1, 3, 0}, {1, 1, 1, 0, 1}}
```

```
Out[37]= {{3, 2, 2, 2, 1}, {2, 3, 2, 2, 1}, {2, 2, 3, 1, 1}, {2, 2, 1, 3, 0}, {1, 1, 1, 0, 1}}
```

```
In[19]:= n = 5;
```

```
In[35]:= A // MatrixForm
```

```
Det[A]
```

```
Out[35]//MatrixForm=
```

$$\begin{pmatrix} 3 & 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 2 & 2 & 3 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

```
Out[36]= -5
```

```
In[38]:= L = ConstantArray[0, {5, 5}]
```

```
U = L;
```

```
Out[38]= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

```
In[40]:= For[k = 1, k ≤ n, k++,
```

```
    L[[k, k]] = A[[k, k]] - Sum[L[[k, m]] * U[[m, k]], {m, k - 1}];
```

```
    For[j = k, j ≤ n, j++,
```

```
        U[[k, j]] = 1 / L[[k, k]] * (A[[k, j]] - Sum[L[[k, m]] * U[[m, j]], {m, k - 1}]);
```

```
    ];
```

```
    For[i = k + 1, i ≤ n, i++,
```

```
        L[[i, k]] = 1 / U[[k, k]] * (A[[i, k]] - Sum[L[[i, m]] * U[[m, k]], {m, k - 1}])
```

```
    ];
```

```
]
```

```
MatrixForm[U]
```

```
Out[41]//MatrixForm=
```

$$\begin{pmatrix} 1 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 1 & -\frac{3}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & -\frac{5}{8} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[42]:= MatrixForm[L]
```

```
Out[42]//MatrixForm=
```

$$\begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 2 & \frac{5}{3} & 0 & 0 & 0 \\ 2 & \frac{2}{3} & \frac{7}{5} & 0 & 0 \\ 2 & \frac{2}{3} & -\frac{3}{5} & \frac{8}{7} & 0 \\ 1 & \frac{1}{3} & \frac{1}{5} & -\frac{5}{7} & \frac{1}{8} \end{pmatrix}$$

```
In[45]:= A = L.U
Print[A // MatrixForm, "=", L // MatrixForm, "*", U // MatrixForm, "=", L.U // MatrixForm]
```

```
Out[45]= {{3, 2, 2, 2, 1}, {2, 3, 2, 2, 1}, {2, 2, 3, 1, 1}, {2, 2, 1, 3, 0}, {1, 1, 1, 0, 1}}
```

$$\begin{pmatrix} 3 & 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 2 & 2 & 1 & 3 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 & 0 & 0 \\ 2 & \frac{5}{3} & 0 & 0 & 0 \\ 2 & \frac{2}{3} & \frac{7}{5} & 0 & 0 \\ 2 & \frac{2}{3} & -\frac{3}{5} & \frac{8}{7} & 0 \\ 1 & \frac{1}{3} & \frac{1}{5} & -\frac{5}{7} & \frac{1}{8} \end{pmatrix} * \begin{pmatrix} 1 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \\ 0 & 0 & 1 & -\frac{3}{7} & \frac{1}{7} \\ 0 & 0 & 0 & 1 & -\frac{5}{8} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 2 & 2 & 1 & 3 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

```
In[47]:=
```

```
L = ConstantArray[0, {5, 5}]
U = L;
```

```
Out[47]= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

```
In[170]:= For[k = 1, k ≤ n, k++,
  L[[k, k]] = 1;
  For[j = k, j ≤ n, j++,
    U[[k, j]] = A[[k, j]] - Sum[L[[k, m]] * U[[m, j]], {m, k - 1}]
  ];
  For[i = k + 1, i ≤ n, i++,
    L[[i, k]] = 1 / U[[k, k]] * (A[[i, k]] - Sum[L[[i, m]] * U[[m, k]], {m, k - 1}))
  ];
];
```

MatrixForm[U]

```
Out[171]//MatrixForm=
```

$$\begin{pmatrix} 3 & 2 & 2 & 2 & 1 \\ 0 & \frac{5}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & \frac{7}{5} & -\frac{3}{5} & \frac{1}{5} \\ 0 & 0 & 0 & \frac{8}{7} & -\frac{5}{7} \\ 0 & 0 & 0 & 0 & \frac{1}{8} \end{pmatrix}$$

```
In[51]:= MatrixForm[L]
```

```
Out[51]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & 1 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{5} & 1 & 0 & 0 \\ \frac{2}{3} & \frac{2}{5} & -\frac{3}{7} & 1 & 0 \\ \frac{1}{3} & \frac{1}{5} & \frac{1}{7} & -\frac{5}{8} & 1 \end{pmatrix}$$

```
In[168]:= A = L.U;
Print[A // MatrixForm, "=", L // MatrixForm, "*", U // MatrixForm, "=", L.U // MatrixForm]
```

$$\begin{pmatrix} 3 & 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 2 & 2 & 1 & 3 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3} & 0 & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}} & \sqrt{\frac{5}{3}} & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{15}} & \sqrt{\frac{7}{5}} & 0 & 0 \\ \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{15}} & -\frac{3}{\sqrt{35}} & 2\sqrt{\frac{2}{7}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{35}} & -\frac{5}{2\sqrt{14}} & \frac{1}{2\sqrt{2}} \end{pmatrix} * \begin{pmatrix} \sqrt{3} & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{5}{3}} & \frac{2}{\sqrt{15}} & \frac{2}{\sqrt{15}} & \frac{1}{\sqrt{15}} \\ 0 & 0 & \sqrt{\frac{7}{5}} & -\frac{3}{\sqrt{35}} & \frac{1}{\sqrt{35}} \\ 0 & 0 & 0 & 2\sqrt{\frac{2}{7}} & -\frac{5}{2\sqrt{14}} \\ 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 2 & 2 & 1 & 3 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

```
In[156]:= L = ConstantArray[0, {5, 5}]
L // MatrixForm
A = {{3, 2, 2, 2, 1}, {2, 3, 2, 2, 1}, {2, 2, 3, 1, 1}, {2, 2, 1, 3, 0}, {1, 1, 1, 0, 1}}
For[k = 1, k ≤ n, k++,
L[[k, k]] = Sqrt[A[[k, k]] - Sum[L[[k, m]] ^ 2, {m, k - 1}]];
For[i = k + 1, i ≤ n, i++,
L[[i, k]] = 1 / L[[k, k]] * (A[[i, k]] - Sum[L[[i, m]] * L[[k, m]], {m, k - 1}])
];
];
U = Transpose[L];
L // MatrixForm
U // MatrixForm
A = L.U;
A // MatrixForm
A == L * U
```

```
Out[156]= {{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}
```

```
Out[157]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
Out[158]= {{3, 2, 2, 2, 1}, {2, 3, 2, 2, 1}, {2, 2, 3, 1, 1}, {2, 2, 1, 3, 0}, {1, 1, 1, 0, 1}}
```

Out[161]//MatrixForm=

$$\begin{pmatrix} \sqrt{3} & 0 & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}} & \sqrt{\frac{5}{3}} & 0 & 0 & 0 \\ \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{15}} & \sqrt{\frac{7}{5}} & 0 & 0 \\ \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{15}} & -\frac{3}{\sqrt{35}} & 2\sqrt{\frac{2}{7}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{35}} & -\frac{5}{2\sqrt{14}} & \frac{1}{2\sqrt{2}} \end{pmatrix}$$

Out[162]//MatrixForm=

$$\begin{pmatrix} \sqrt{3} & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} & \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \sqrt{\frac{5}{3}} & \frac{2}{\sqrt{15}} & \frac{2}{\sqrt{15}} & \frac{1}{\sqrt{15}} \\ 0 & 0 & \sqrt{\frac{7}{5}} & -\frac{3}{\sqrt{35}} & \frac{1}{\sqrt{35}} \\ 0 & 0 & 0 & 2\sqrt{\frac{2}{7}} & -\frac{5}{2\sqrt{14}} \\ 0 & 0 & 0 & 0 & \frac{1}{2\sqrt{2}} \end{pmatrix}$$

Out[164]//MatrixForm=

$$\begin{pmatrix} 3 & 2 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 & 1 \\ 2 & 2 & 3 & 1 & 1 \\ 2 & 2 & 1 & 3 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

Out[165]=

False

```

In[208]:= B = RandomInteger[{1, 5}, {6, 6}]
L = ConstantArray[0, {6, 6}];
U = L;
B = B . Transpose[B];
B // MatrixForm
n = 6;
A = B;
For[k = 1, k ≤ n, k++,
  L[[k, k]] = A[[k, k]] - Sum[L[[k, m]] * U[[m, k]], {m, k - 1}];
  For[j = k, j ≤ n, j++,
    U[[k, j]] = 1 / L[[k, k]] * (A[[k, j]] - Sum[L[[k, m]] * U[[m, j]], {m, k - 1}));
  ];
  For[i = k + 1, i ≤ n, i++,
    L[[i, k]] = 1 / U[[k, k]] * (A[[i, k]] - Sum[L[[i, m]] * U[[m, k]], {m, k - 1}));
  ];
]
L // MatrixForm
U // MatrixForm
A == L.U

```

Out[208]=

```

{{4, 4, 2, 2, 1, 2}, {3, 5, 1, 1, 1, 2}, {2, 3, 1, 4, 4, 2},
 {3, 4, 3, 4, 2, 2}, {4, 1, 5, 1, 1, 5}, {3, 3, 2, 1, 4, 5}}

```

Out[212]//MatrixForm=

$$\begin{pmatrix} 45 & 41 & 38 & 48 & 43 & 44 \\ 41 & 41 & 34 & 42 & 34 & 41 \\ 38 & 34 & 50 & 49 & 34 & 47 \\ 48 & 42 & 49 & 58 & 47 & 49 \\ 43 & 34 & 34 & 47 & 69 & 55 \\ 44 & 41 & 47 & 49 & 55 & 64 \end{pmatrix}$$

Out[216]//MatrixForm=

$$\begin{pmatrix} 45 & 0 & 0 & 0 & 0 & 0 \\ 41 & \frac{164}{45} & 0 & 0 & 0 & 0 \\ 38 & -\frac{28}{45} & \frac{730}{41} & 0 & 0 & 0 \\ 48 & -\frac{26}{15} & \frac{335}{41} & \frac{325}{146} & 0 & 0 \\ 43 & -\frac{233}{45} & -\frac{131}{41} & \frac{10}{73} & \frac{5193}{260} & 0 \\ 44 & \frac{41}{45} & 10 & -\frac{305}{146} & \frac{841}{52} & \frac{400}{5193} \end{pmatrix}$$

Out[217]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{41}{45} & \frac{38}{45} & \frac{16}{15} & \frac{43}{45} & \frac{44}{45} \\ 0 & 1 & -\frac{7}{41} & -\frac{39}{82} & -\frac{233}{164} & \frac{1}{4} \\ 0 & 0 & 1 & \frac{67}{146} & -\frac{131}{730} & \frac{41}{73} \\ 0 & 0 & 0 & 1 & \frac{4}{65} & -\frac{61}{65} \\ 0 & 0 & 0 & 0 & 1 & \frac{4205}{5193} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Out[218]=

True

In[257]:= **A = {{3, 2, 2, 2, 1}, {2, 3, 2, 2, 1}, {2, 2, 3, 1, 1}, {2, 2, 1, 3, 0}, {1, 1, 1, 0, 1}};**
{lu, p, c} = LUDecomposition[A]

Out[258]=

$$\left\{ \left\{ \{1, 1, 1, 0, 1\}, \{2, 1, 0, 2, -1\}, \{2, 0, 1, 1, -1\}, \{2, 0, -1, 4, -3\}, \left\{3, -1, -1, \frac{5}{4}, -\frac{1}{4}\right\} \right\}, \right. \\ \left. \{5, 2, 3, 4, 1\}, 0 \right\}$$

In[354]:=

Clear[n]
Clear[a]
RSolve[{a[n] == 2 * a[n - 1] + 5 * a[n - 2] - 6 * a[n - 3] + 1}, a[n], n]
Clear[a, n]
f[n_] =
RSolve[{a[n] == 2 * a[n - 1] + 5 * a[n - 2] - 6 * a[n - 3] + 1, a[0] == 1, a[1] == 4, a[2] == 7}, a[n], n]
Clear[a, n]
f[10]

Out[356]=

$$\left\{ \left\{ a[n] \rightarrow \frac{1}{180} (-9 + 4 (-1)^{2n} - 30 n) + (-2)^n c_1 + c_2 + 3^n c_3 \right\} \right\}$$

Out[358]=

$$\left\{ \left\{ a[n] \rightarrow \frac{1}{180} (81 + 4 (-1)^{2n} - 19 (-1)^n 2^{2+n} + 19 \times 3^{2+n} - 30 n) \right\} \right\}$$

Out[360]=

{{a[10] → 55 663}}

In[58]:=

```
g[x_, y_] := (x + y) / (1 + x * y);
```

```
result = g[1/2, 1/3];
For[i = 4, i ≤ 1000, i++,
result = g[result, 1/i];
];
```

```
Print[result]
```

```
500 499
-----
500 501
```

In[93]:=

```
Clear[a, n, k]
n = 2;
k = 1000;
FullSimplify[RSolve[{a[n, k] == g[a[n, k - 1], 1 / (n + k)], a[n, 0] == 1 / n}, a[n, k], n, k]]
RSolve: 2 cannot be used as a variable.
```

```
Out[96]= RSolve[{{ $\frac{1004003}{1002 + a[2, 999]} + a[2, 1000] == 1002$ ,  $2 a[2, 0] == 1$ }}, a[2, 1000], 2, 1000]
```