GINS

member function

prediction

$$\hat{\boldsymbol{x}}_{k/k-1} = \boldsymbol{\Phi}_{k/k-1}\hat{\boldsymbol{x}}_{k-1}$$

$$\mathbf{P}_{k/k-1} = \boldsymbol{\Phi}_{k/k-1}\mathbf{P}_{k-1}\boldsymbol{\Phi}_{k/k-1}^T + \boldsymbol{\Gamma}_{k-1}\mathbf{Q}_{k-1}\boldsymbol{\Gamma}_{k-1}^T$$

$$\delta\boldsymbol{x}_k = \boldsymbol{\Phi}_{k/k-1}\delta\boldsymbol{x}_{k-1} + \boldsymbol{w}_{k-1}$$

$$\boldsymbol{\Phi}_{k/k-1} = \exp\{\mathbf{F}(t_{k-1})\Delta t\} \approx \mathbf{I} + \mathbf{F}(t_k)\Delta t$$

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{rr} & \mathbf{I}_{3\times3} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{F}_{vr} & \mathbf{F}_{vv} & [(\mathbf{C}_b^n\boldsymbol{f}^b)\times] & \mathbf{0} & \mathbf{C}_b^n & \mathbf{0} & \mathbf{C}_b^n\mathrm{diag}(\boldsymbol{f}^b) \\ \mathbf{F}_{\phi r} & \mathbf{F}_{\phi v} & -(\boldsymbol{\omega}_{in}^n\times) & -\mathbf{C}_b^n & \mathbf{0} & -\mathbf{C}_b^n\mathrm{diag}(\boldsymbol{\omega}_{ib}^b) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{-1}{T_{gb}}\mathbf{I}_{3\times3} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{-1}{T_{ab}}\mathbf{I}_{3\times3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{-1}{T_{gs}}\mathbf{I}_{3\times3} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \frac{-1}{T_{as}}\mathbf{I}_{3\times3} \end{bmatrix}$$

$$\mathbf{F}_{rr} = \begin{bmatrix} -\frac{v_D}{R_M+h} & 0 & \frac{v_N}{R_M+h} \\ \frac{v_E\tan\varphi}{R_N+h} & -\frac{v_D+v_N\tan\varphi}{R_N+h} & \frac{v_E}{R_N+h} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{F}_{vr} = \begin{bmatrix} \frac{-2v_E\omega_e\cos\varphi}{R_M+h} - \frac{v_E^2\sec^2\varphi}{(R_M+h)(R_N+h)} & 0 & \frac{v_Nv_D}{(R_M+h)^2} - \frac{v_E^2\tan\varphi}{(R_N+h)^2} \\ \frac{2\omega_e(v_N\cos\varphi - v_D\sin\varphi)}{R_M+h} + \frac{v_Nv_E\sec^2\varphi}{(R_M+h)(R_N+h)} & 0 & \frac{v_Ev_D+v_Nv_E\tan\varphi}{(R_N+h)^2} \\ \frac{2\omega_ev_E\sin\varphi}{R_M+h} & 0 & -\frac{v_E^2}{(R_N+h)^2} - \frac{v_N^2}{(R_M+h)^2} + \frac{2g_p}{\sqrt{R_MR_N}+h} \end{bmatrix}$$

$$\mathbf{F}_{vv} = \begin{bmatrix} \frac{v_D}{R_M+h} & -2\left(\omega_e\sin\varphi + \frac{v_E\tan\varphi}{R_N+h}\right) & \frac{v_N}{R_M+h} \\ 2\omega_e\sin\varphi + \frac{v_E\tan\varphi}{R_N+h} & \frac{v_D+v_N\tan\varphi}{R_N+h} & 2\omega_e\cos\varphi + \frac{v_E}{R_N+h} \\ -\frac{2v_N}{R_M+h} & -2\left(\omega_e\cos\varphi + \frac{v_E}{R_N+h}\right) & 0 \end{bmatrix}$$

$$\mathbf{F}_{\phi r} = \begin{bmatrix} -\frac{\omega_e\sin\varphi}{R_M+h} & 0 & \frac{v_E}{(R_N+h)^2} \\ 0 & 0 & -\frac{v_N}{(R_M+h)^2} \\ -\frac{\omega_e\cos\varphi}{R_M+h} - \frac{v_E\sec^2\varphi}{(R_M+h)(R_N+h)} & 0 & -\frac{v_E\tan\varphi}{(R_N+h)^2} \end{bmatrix}, \quad \mathbf{F}_{\phi v} = \begin{bmatrix} 0 & \frac{1}{R_N+h} & 0 \\ -\frac{1}{R_M+h} & 0 & 0 \\ 0 & -\frac{\tan\varphi}{R_N+h} & 0 \end{bmatrix}$$

$$\underset{21\times18}{\mathbf{G}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{C}_b^n & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{C}_b^n & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I}_{3\times3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{I}_{3\times3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I}_{3\times3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{I}_{3\times3} \end{bmatrix}$$

$$\boldsymbol{w}_{k-1} = \int_{t_{k-1}}^{t_k} \boldsymbol{\Phi}_{k/t}\mathbf{G}(t)\boldsymbol{w}(t)dt$$

$$\underset{18\times18}{\mathbf{q}}(t) = \begin{bmatrix} \mathrm{VRW}^2\,\mathbf{I}_{3\times3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathrm{ARW}^2\,\mathbf{I}_{3\times3} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2\sigma_{gb}^2}{T_{gb}}\mathbf{I}_{3\times3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\sigma_{ab}^2}{T_{ab}}\mathbf{I}_{3\times3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2\sigma_{gs}^2}{T_{gs}}\mathbf{I}_{3\times3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2\sigma_{as}^2}{T_{as}}\mathbf{I}_{3\times3} \end{bmatrix}$$

$$\mathbf{Q}_k \approx \frac{1}{2}\left[\boldsymbol{\Phi}_{k/k-1}\mathbf{G}(t_{k-1})\mathbf{q}(t_{k-1})\mathbf{G}^{\mathrm{T}}(t_{k-1})\boldsymbol{\Phi}_{k/k-1}^{\mathrm{T}} + \mathbf{G}(t_k)\mathbf{q}(t_k)\mathbf{G}^{\mathrm{T}}(t_k)\right]\Delta t$$

GPS update

$$\mathbf{K}_k = \mathbf{P}_{k/k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k/k-1}\mathbf{H}_k^T + \mathbf{R}_k)^{-1}$$

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_{k/k-1} + \mathbf{K}_k(z_k - \mathbf{H}_k\hat{\boldsymbol{x}}_{k/k-1})$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{P}_{k/k-1}\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^T + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^T$$

$$\mathbf{H}_r = \begin{bmatrix}\mathbf{I}_3 & \mathbf{0}_3 & (\mathbf{C}_b^n\boldsymbol{l}^b\times) & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3\end{bmatrix}$$

$$\mathbf{H}_v = \begin{bmatrix}\mathbf{0}_3 & \mathbf{I}_3 & \mathbf{H}_{v3} & -(\mathbf{C}_b^n\boldsymbol{l}^b\times) & \mathbf{0}_3 & \mathbf{H}_{v6} & \mathbf{0}_3\end{bmatrix}$$

$$\mathbf{H}_{v3} = -(\boldsymbol{\omega}_{in}^n\times)(\mathbf{C}_b^n\boldsymbol{l}^b\times) - [\mathbf{C}_b^n(\boldsymbol{l}^b \times \boldsymbol{\omega}_{ib}^b)\times]$$

$$\mathbf{H}_{v6} = -\mathbf{C}_b^n(\boldsymbol{l}^b\times)\mathrm{diag}(\boldsymbol{\omega}_{ib}^b)$$

feedback

attitude

$$C_b^t = \mathrm{C}_p^t C_b^p = \left(C_t^p\right)^{-1} C_b^n$$

$$\mathrm{C}_t^p = I - (\Phi_{rp}\times)$$

$$C_b^p = \begin{pmatrix} c\theta c\psi & -c\phi s\psi + s\phi s\theta c\psi & s\phi s\psi + c\phi s\theta c\psi \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & -s\phi c\psi + c\phi s\theta s\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{pmatrix}$$

position

$$\mathrm{r}_{IMU,BLH}^n = \hat{\mathrm{r}}_{IMU,BLH}^n - \delta\mathrm{r}_{IMU,BLH}^n{}^{-}$$

$$\delta\mathrm{r}_{IMU,BLH}^n = \widehat{D}_R^{-1}\delta\mathrm{r}_{IMU,NED}^n{}^{-}$$

$$\widehat{D}_R^{-1} = \begin{bmatrix} \frac{1}{R_M+h} & 0 & 0 \\ 0 & \frac{1}{(R_N+h)cos\varphi} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

velocity

$$v_{IMU}^n = \hat{v}_{IMU}^n - \delta v_{IMU}^n{}^{+}$$

$$\hat{v}_{IMU}^n = \mathrm{C}_p^t\hat{v}_{IMU}^p$$

bias and scalar

$$b_g(\mathrm{t}_k) = b_g(\mathrm{t}_{k-1}) + \left(1 + \mathrm{s}_g(\mathrm{t}_{k-1})\right)\hat{b}_g(t_k)$$

$$\mathrm{s}_g(\mathrm{t}_k) = \left(1 + \mathrm{s}_g(\mathrm{t}_{k-1})\right)\left(1 + \hat{\mathrm{s}}_g(\mathrm{t}_k)\right) - 1$$

$$b_a(\mathrm{t}_k) = b_a(\mathrm{t}_{k-1}) + \left(1 + \mathrm{s}_a(\mathrm{t}_{k-1})\right)\hat{b}_a(t_k)$$

$$\mathrm{s}_a(\mathrm{t}_k) = \left(1 + \mathrm{s}_a(\mathrm{t}_{k-1})\right)\left(1 + \hat{\mathrm{s}}_a(\mathrm{t}_k)\right) - 1$$

variable

- typedef Matrix<double, 21, 1> ErrorType
- typedef Matrix<double, 18, 1> NoiseType
- typedef Matrix<double, 18, 18> qType
- typedef Matrix<double, 21, 18> GType
- typedef Matrix<double, 21, 21> SevenDimType
- ErrorType delta_x_, last_delta_x_ $\delta x(t) = \left[(\delta r^n)^T \quad (\delta v^n)^T \quad \phi^T \quad b_g^T \quad b_a^T \quad s_g^T \quad s_a^T\right]^T$
- NoiseType w_, last_w_ $\boldsymbol{w} = \left[\boldsymbol{w}_v^T \quad \boldsymbol{w}_\phi^T \quad \boldsymbol{w}_{gb}^T \quad \boldsymbol{w}_{ab}^T \quad \boldsymbol{w}_{gs}^T \quad \boldsymbol{w}_{as}^T\right]^T$
- SevenDimType P_, last_P_, G_, last_G_