UTM
UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 3

SECI1013 – Discrete Structure

Semester 2024/2025-1

Instructions:

This is a group assignment. Each group should consist of no more than 3 members.

Please write all your answers by hand using a pen. Ensure that your answers are well-structured and that your handwriting is neat and easy to read. Submissions in printed form will not be accepted.

Please submit your assignment by **17/1/2025, 5:00 PM**, in room **N28-346-05**.

Question 1 **[10 marks]**

A company is organizing a raffle with 30 tickets. The tickets are divided into three categories:

- **Category A**: 10 tickets, numbered from 1 to 10.
- **Category B**: 12 tickets, numbered from 11 to 22.
- **Category C**: 8 tickets, numbered from 23 to 30.

A participant purchases 3 tickets randomly **without replacement**. After the tickets are drawn, the company will offer the participant a prize based on the following conditions:

- If the participant selects **at least two tickets from Category B**, they will win a special prize.
- If the participant selects **exactly one ticket from Category A** and the other two tickets from Category C**, they will win a consolation prize.
- If the participant selects tickets from **all three categories**, they will win a grand prize.

a. What is the probability that the participant wins a special prize? Show all your working.

b. What is the probability that the participant wins a consolation prize? Show all your working.

c. What is the probability that the participant wins the grand prize? Show all your working.

d. What is the probability that the participant does not win any prize? Show all your working.

e. Given that the participant wins a special prize, what is the probability that they selected exactly two tickets from Category B and one from Category C? Show all your working.

Question 2 **[10 marks]**

A delivery company operates three warehouses (X, Y, and Z) that store various types of goods. Items are selected for delivery based on the following rules:

- Warehouse X holds 40% of the total items, Warehouse Y holds 35%, and Warehouse Z holds 25%.
- Items from Warehouse X have a 30% chance of being defective, items from Warehouse Y have a 20% chance, and items from Warehouse Z have a 10% chance.
- If an item is delivered to a customer, there is a 15% chance the customer will be dissatisfied if the item is defective and a 5% chance if the item is not defective.

A customer has just lodged a complaint about being dissatisfied.

a. What is the probability that the item delivered to the customer came from Warehouse X? Show all your working.
b. Given that the item came from Warehouse X, what is the probability that the customer lodged a complaint? Show all your working.
c. Overall, what is the probability that a customer lodges a complaint? Show all your working.

Question 3 **[10 marks]**

In a large computer network, there are 100 servers. Each server has an independent probability of 0.03 of being infected with malware during a given time period. The infection is transmitted from one infected server to another if the two servers are directly connected in the network. If a server is infected, it has a 0.1 probability of infecting each of its neighbors (servers directly connected to it) in the next time period.

The network is structured such that each server has 4 neighbors (directly connected). A system administrator randomly selects 10 servers to monitor, and you're interested in the following:

a. What is the probability that exactly 2 of the monitored servers are infected by malware after 1 time period? Show all your working.
b. What is the probability that at least 3 servers in the monitored group are infected after 1 time period? Show all your working.

c. Given that exactly 2 servers in the monitored group are infected, what is the expected number of neighboring servers (out of the remaining 10) that will be infected in the next time period? Show all your working.

d. What is the probability that **none** of the monitored servers get infected after 1 time period? Show all your working.

e. If exactly 5 servers are infected in the monitored group after 1 time period, what is the expected number of **new infections** (in neighboring servers) during the next time period? Show all your working.

Question 4 **[10 marks]**

a. Consider the following graph G with the set of vertices V={A, B, C, D, E, F, G, H} and the set of edges E:

E={(A,B), (A,C), (A,D), (B,D), (B,E), (C,F), (D,F), (E,G), (F,G), (G,H), (H,A)}

Determine whether the graph has an Euler Path, an Euler Circuit, or neither. If it has one, provide a specific example of the path or circuit. Justify your answer thoroughly.

b. Consider the graph G with the set of vertices V={A, B, C, D, E, F} and the set of edges E:

E={(A,B), (A,C), (B,D), (C,D), (C,E), (D,F), (E,F)}

Determine whether the graph has an Euler Path, an Euler Circuit, or neither. If it has one, provide an example of the path or circuit. Justify your answer thoroughly.

Question 5 **[10 marks]**

Given an incomplete graph with 10 vertices (A, B, C, D, E, F, G, H, I, J) and edges as follows:

A↔B, A↔C, B↔C, B↔G, B↔D, C↔D, C↔E, C↔H, D↔E, D↔F, D↔J, E↔F, E↔H, E↔I, F↔I, F↔J

a. Draw the graph.

b. Determine if the graph has a **Hamiltonian Path**.

c. Does it have a **Hamiltonian Circuit**? Explain your reasoning.

Question 6 **[10 marks]**

Consider the following graph GGG, where each vertex represents a city, and the edges represent the distances between them (in kilometers). The graph is undirected, and the distances between the cities are given by the weights on the edges.

- **Vertices (Cities):** A, B, C, D, E
- **Edges (Distances):**
  - A-B: 10
  - A-C: 15
  - A-D: 20
  - A-E: 25
  - B-C: 35
  - B-D: 30
  - B-E: 40
  - C-D: 50
  - C-E: 45
  - D-E: 55

a. Draw the graph.

b. Find if there exists a Hamiltonian path in the graph that visits every city exactly once. If such a path exists, provide the sequence of cities and the total distance. Show all your working.

c. Solve the Traveling Salesman Problem (TSP) for the given graph by finding the shortest possible route that visits each city exactly once and returns to the starting city. What is the total distance of this shortest cycle? Show all your working.