Modeling Population Dynamics of Monarch Butterflies with an Emphasis on Climate Change

Laura Hu, Sebastian Griego, Meg Bucich

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Abstract

The monarch butterfly population has been declining due to factors such as climate change, and changes in migration and breeding patterns. Their main source of food, milkweed, is invasive and is being eradicated. The analysis in this paper focuses on investigating the effects of temperature changes and its relationship to the population of monarchs and milkweed. The model consists of 2 stages; the first stage tracks monarch breeding, and the second stage focuses on the interaction of milkweed and monarch caterpillars. Using an existing model, we contributed by creating a temperature dependent milkweed growth rate and caterpillar mortality rate. We analyze the effects of temperature on milkweed growth, the monarch's reproductive cycle, and population dynamics using equilibrium and stability analysis. Other analysis done includes stability analysis along with solution curves on phase plans. 3-dimensional phase spaces show relationships between variables and allow for a conclusion that temperature greatly impacts both monarchs and milkweed populations. Results show that increased temperatures have had a negative impact on milkweed growth and caterpillar survival rate. This study focuses on the importance of temperature regulation efforts and highlights how increasing temperatures are increasing the monarch population decline.

1 Introduction

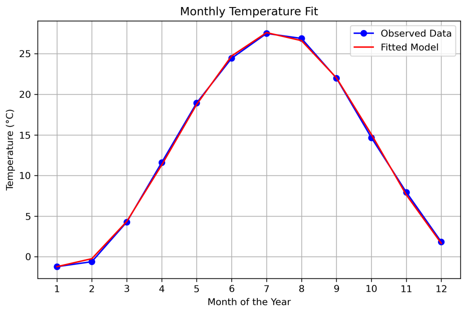
There are existing models that attempt to show the dynamics of the monarch butterfly population using a variety of components. The goal of this paper is to take an existing model and add components based on fitted temperature data to see how their populations change through the effects of climate change. We fit temperature data from Toronto, Canada to create a sine function to predict accurate temperature values. This is used to create a mortality rate for caterpillars and milkweed growth rates. After adding original contributions to an existing model, graphical solutions were created to analyze the dynamics over time. Other analysis done includes stability analysis along with solution curves on phase plans. 3-dimensional phase spaces show relationships between variables and allow for a conclusion that temperature greatly impacts both monarchs and milkweed populations. Modeling this can help push for more conservation efforts.

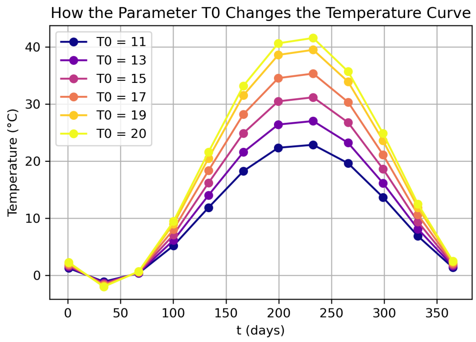
2 Model-system Description

2.1 Preliminaries

In the Stage 2 of the model, we introduce 3 different functions: The temperature function of Toronto, Canada; The growth rate of the milkweed as the function of temperature; The death rate of the larvae as the function of temperature.

To obtain the temperature function, we fitted the monthly average temperature data of Toronto, Canada from 2011- 2022 to the equation $T = T_0(1 + a\sin(\frac{2\pi t}{12} + b))$. The parameters we obtained are $T_0 = 11.98, a = -1.102, b = 0.917$.





The milkweed growth rate function is obtained from the paper by *Yan et al.* The growth rate of any plant can be modeled by the function.

$$R(t) = R_{max}(\frac{T_{max} - t}{T_{max} - T_{opt}})(\frac{t}{T_{opt}})^{\frac{T_{opt}}{T_{max}-T_{opt}}}$$

For milkweed, the parameter values are $R_{max} = 0.07/day, T_{max} = 35, T_{opt} = 27$.

The survival rate function of the larvae as a function of temperature is obtained from the paper by []. The survival rate of the larvae as a function of temperature is given in the form:

$$Y(t, S) = 0.003t^2 + 0.1411t - 0.0911S - 0.67$$

S is the stage of the larvae, and we define S = 3 in this paper. In order to keep the models consistent, we define the death rate the larvae to be

$$\delta(t) = 1 - Y(t)$$

2.2 Stage 1

This model describes the beginning of the Monarchs' migration to North America in Spring. Between March to June, Monarchs make their way to the United States region of the North America and continuously reproduce and travel around the United States. This model does not account for the effect of temperature nor the milkweed population.

$$
\begin{aligned}
\frac{dM_0}{dt} &= -\mu_0 M_0 \\
\frac{dI_1}{dt} &= \alpha M_0 - \mu_1 I_1 - \gamma I_1 \\
\frac{dM_1}{dt} &= \gamma I_1 - \mu_2 M_1 \\
\frac{dI_2}{dt} &= \alpha M_1 - \mu_1 I_2 - \gamma I_2 \\
\frac{dM_2}{dt} &= \gamma I_2 - \mu_2 M_2
\end{aligned}
$$

Parameter	Biological Meaning	Value
μ_0	Death rate of overwintering monarchs	0.1198/day
α	Reproductive rate	2.232/day
γ	Maturation rate	0.03571/day
μ_1	Death rate of larvae	0.0902/day
μ_2	Death rate of adult monarchs	0.07143/day

Table 1: Parameters for monarch butterfly population model in stage 1

2.3 Stage 2

This model describes the end of the spring migration where the butterflies stop at their final destination in North America and go through the last reproductive phase and slowly go into diapause. In this stage, we changed the larvae death rate as a function of temperature and incorporates the milkweed population. The initial values of I_2 and M_2 are obtained from the end values of stage 1.

$$
\begin{aligned}
\frac{dI_2}{dt} &= -(\gamma + \delta(T))I_2 \\
\frac{dM_2}{dt} &= -\mu_2 M_2 + \gamma I_2 \\
\frac{dI_s}{dt} &= \alpha A M_2 - (\gamma + \delta(T))I_s \\
\frac{dM_s}{dt} &= \gamma L_s - \mu_3 M_s \\
\frac{dA}{dt} &= R(T)(1 - \frac{A}{K}) - \beta A I_s
\end{aligned}
$$

Parameter	Biological Meaning	Value
$\delta(T)$	Larvae death rate as a function of temperature	N/A
$R(t)$	Milkweed growth rate as a function of temperature	N/A
β	consumption rate of milkweed by larvae	5×10^-9/day
μ_3	Death rate of diapause butterflies	0.005/day
K	Carrying capacity	1.79188

Table 2: Additional parameters for monarch butterfly population model in stage 2

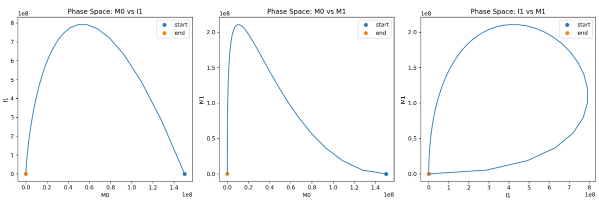
3 Model Analysis

3.1 Stability Analysis of Stage 1

To analyze the stability of the breeding stage in model 1, we take the Jacobian of the 5-D system and get the following Jacobian.

$$J = \begin{bmatrix} -\mu_1 & 0 & 0 & 0 & 0 \\ \alpha & -(\mu_1+\gamma) & 0 & 0 & 0 \\ 0 & \gamma & -\mu_2 & 0 & 0 \\ 0 & 0 & \alpha & -(\mu_1+\gamma) & 0 \\ 0 & 0 & 0 & \gamma & -\mu_2 \end{bmatrix}$$

Since this is an upper triangular matrix, we get the eigenvalues from the diagonal. Since all parameters are positive, we get all negative eigenvalues, which makes the Jacobian a stability matrix. Notice that all variables disappear, and only parameters remain when the Jacobian is taken. This indicates that we have stability at all equilibria. However, the only equilibrium in this system is (0,0,0,0,0). Below are the phase portraits:



3.2 Stability Analysis of Stage 2

To analyze the stability of model 2, we take the Jacobian of the 5-D system and get the following result.

$$J = \begin{bmatrix} -(\gamma+Y(T)) & 0 & 0 & 0 & 0 \\ \gamma & -\mu_2 & 0 & 0 & 0 \\ 0 & \alpha A & -(\gamma+Y(T)) & 0 & \alpha M_{in} \\ 0 & 0 & \gamma & -\mu_3 & 0 \\ 0 & 0 & -\beta A & 0 & R(T)(1-\frac{2A}{K})-\beta I_s \end{bmatrix}$$

$$\begin{aligned} \lambda_1 &= -(\gamma+Y(T)) \\ \lambda_2 &= -\mu_2 \\ \lambda_3 &= -\mu_3 \\ \lambda_{4,5} &= \frac{(R(T)(1-\frac{2A}{K})-\beta I_s)+(\gamma+Y(T))}{2} \\ &\pm \frac{\sqrt{[(R(T)(1-\frac{2A}{K})-\beta I_s)+(\gamma+Y(T))]^2-4[(R(T)(1-\frac{2A}{K})-\beta I_s)(\gamma+Y(T))+\beta A\alpha M_{in}]}}{2} \end{aligned}$$

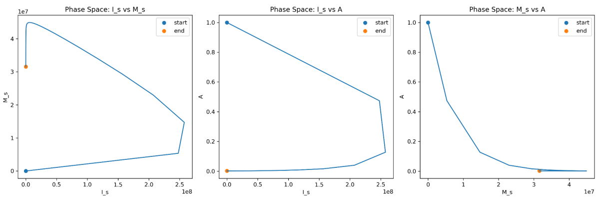
Trivial equilibrium: $(I_{in}, M_{in}, I_s, M_s, A) = (0,0,0,0,0)$
Works out to be:

$$\lambda_4 = R(T)$$
$$\lambda_5 = \gamma + Y(T)$$

There are positive eigenvalues, so this is asymptotically unstable.
Carrying capacity equilibrium: $(I_{in}, M_{in}, I_s, M_s, A) = (0, 0, 0, 0, K)$
Then:

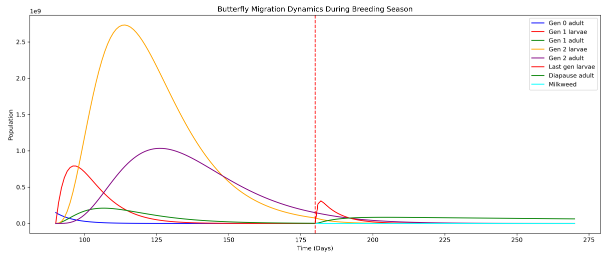
$$\lambda_4 = \frac{-R(T) + (\gamma + Y(T))}{2} + \frac{R(T) + (\gamma + Y(T))}{2} = (\gamma + Y(T))$$
$$\lambda_5 = \frac{-R(T) + (\gamma + Y(T))}{2} - \frac{R(T) + (\gamma + Y(T))}{2} = -R(T)$$

$\lambda_4 > 0$, so the equilibrium of carrying capacity is always asymptotically unstable.



4 Numerical Results

The solution curves to our models are as follows:



The red line represents where stage 1 ends and stage 2 begins. The initial conditions of stage 2 are the ending conditions from stage 1. In stage 1 we see how the mature and immature populations fluctuate throughout the breeding season. We see a spike in the immature population during peak breeding season but then decrease. All populations die out because monarch's live relatively small life spans but the cycle repeats itself every year.

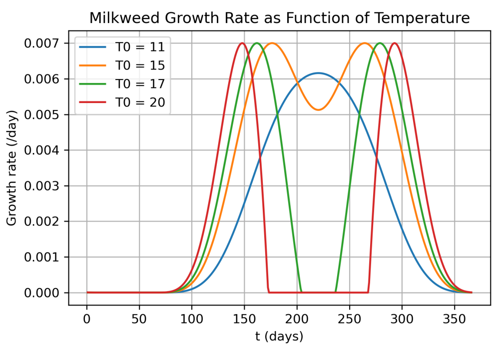


Figure 1: This graph shows how milkweed growth rates fluctuates with different values of t0.

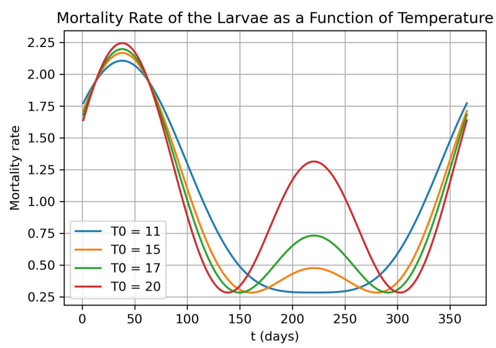


Figure 2: This graph shows how larvae mortality rates fluctuates with different values of t0.

In these graphs, we see how temperature affects both milkweed growth and caterpillar mortality. For milkweed (left), the rate is impacted by extreme temperatures. Each curve represents a different T0, which affects the amplitude of the curve. Extreme values such as the green and red curve show populations thriving but quickly dying off in the summer where the blue and orange curve show populations thriving more consistently throughout the year. For caterpillar mortality (right), we see that larger values of T0, i.e. higher average temperatures, results in a higher mortality rate for the caterpillars, where lower temperatures result in lower mortality rates.

5 Discussion

Our analysis reveals the important relationship between temperature changes and the monarch butterfly population. As seen in 3, the increasing temperatures due to climate change will first impact the butterfly population and will eventually cause a decrease in the milkweed population as well. Other analysis done includes stability analysis along with solution curves on phase plans. 3-dimensional phase spaces show relationships between variables and allow for a conclusion that temperature greatly impacts both monarchs and milkweed populations.

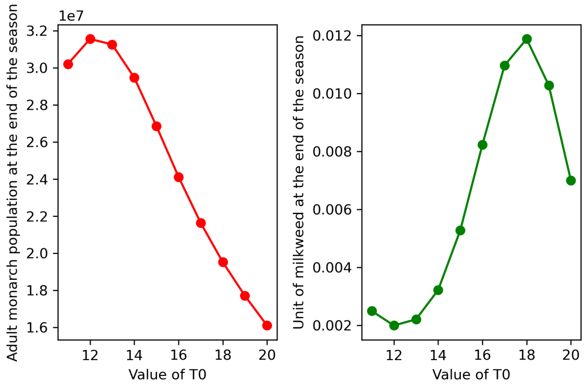


Figure 3: These graphs show a decline in both Monarch and Milkweed population during extreme weather.

The stability analysis of the first stage, which includes just two generations of larvae and butterflies, makes sense in an ecological context. All species in each generation will eventually die out. Before they die out, the butterflies and larvae follow the behavior of a predator-prey dynamic in the cyclical nature of the populations. This same pattern follows in the second model for the larvae and monarchs, but there is a non-trivial equilibrium for the milkweed. Both the trivial and carrying capacity equilibria for the milkweed were unstable saddle points, so this population is sensitive to perturbations and environmental changes.

The temperature sensitivity of both the butterflies and the milkeed stress the need for conservation efforts. Too much change in local temperature patterns will have a strong negative impact on the populations of both species.

6 Conclusion

This paper has analyzed the relationship between larvae, butterflies, and milkweed. Primarily, we have modeled how temperature impacts the populations of the endangered monarch butterflies and milkweed. The model predicts that even moderate increases in average temperature can lead to substantial increases in larval mortality and decreased milkweed growth rate. The findings of this study have important implications for both theoretical ecology and practical conservation efforts. Modeling this can help push for more conservation efforts, as results show that increased temperatures have had a negative impact on milkweed growth and caterpillar survival rate.

References

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