Bilag fra Maple

```
> restart;
  with(plots):
  with(LinearAlgebra):
```

Indledning

```
> l1:=a->x+2*a*y+a*z=a^2:
  l2:=a->x+a*y+a*z=a:
  l3:=a->x+a^2*z=a^3:
  l4:=a->a*x+a*y+a^2*z=a:
```

koefficientmatrix

```
> A:=a-><1,2*a,a;1,a,a;1,0,a^2;a,a,a^2>:
  A=A(a)
```

$$A=\begin{bmatrix} 1 & 2a & a \\ 1 & a & a \\ 1 & 0 & a^2 \\ a & a & a^2 \end{bmatrix} \tag{1.1}$$

højresiden

```
> b:=a-><a^2,a,a^3,a>:
  b=b(a)
```

$$b=\begin{bmatrix} a^2 \\ a \\ a^3 \\ a \end{bmatrix} \tag{1.2}$$

Totalmatrix

```
> T:=a-><A(a)|b(a)>:
  T=T(a)
```

$$T=\begin{bmatrix} 1 & 2a & a & a^2 \\ 1 & a & a & a \\ 1 & 0 & a^2 & a^3 \\ a & a & a^2 & a \end{bmatrix} \tag{1.3}$$

Finder determinanten af totalmatricen T ved opløsning.

Vi opløser efter række 3, da den indeholder et 0 - Så behøver vi ikke at bestemme determinanten til snitmatricen T32.

Vi finder determinanten af snimatricen T31 ved opløsning til 2x2 matricer

```
> T31:=DeleteRow(DeleteColumn(T(a),1),3);
  T3111:=DeleteRow(DeleteColumn(T31,1),1);
  T3112:=DeleteRow(DeleteColumn(T31,2),1);
```

```
T3113:=DeleteRow(DeleteColumn(T31,3),1);
```

$$T31 := \begin{bmatrix} 2\,a & a & a^2 \\ a & a & a \\ a & a^2 & a \end{bmatrix}$$

$$T3111 := \begin{bmatrix} a & a \\ a^2 & a \end{bmatrix}$$

$$T3112 := \begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

$$T3113 := \begin{bmatrix} a & a \\ a & a^2 \end{bmatrix} \tag{1.1.1}$$

```
> detT31:=simplify(2*a*(a^2-a^3)-a*(a^2-a^2)+a^2*(a^3-a^2))
```

$$detT31 := a^5 - 3\,a^4 + 2\,a^3 \tag{1.1.2}$$

Vi finder determinanten af snimatricen T33 ved opløsning til 2x2 matricer

```
> T33:=DeleteRow(DeleteColumn(T(a),3),3);
  T3311:=DeleteRow(DeleteColumn(T33,1),1);
  T3312:=DeleteRow(DeleteColumn(T33,2),1);
  T3313:=DeleteRow(DeleteColumn(T33,3),1);
```

$$T33 := \begin{bmatrix} 1 & 2\,a & a^2 \\ 1 & a & a \\ a & a & a \end{bmatrix}$$

$$T3311 := \begin{bmatrix} a & a \\ a & a \end{bmatrix}$$

$$T3312 := \begin{bmatrix} 1 & a \\ a & a \end{bmatrix}$$

$$T3313 := \begin{bmatrix} 1 & a \\ a & a \end{bmatrix} \tag{1.1.3}$$

```
> detT33:=simplify(1*(a^2-a^2)-2*a*(a-a^2)+a^2*(a-a^2))
```

$$detT33 := -a^4 + 3\,a^3 - 2\,a^2 \tag{1.1.4}$$

Vi finder determinanten af snimatricen T34 ved opløsning til 2x2 matricer.

```
> T34:=DeleteRow(DeleteColumn(T(a),4),3);
  T3411:=DeleteRow(DeleteColumn(T34,1),1);
  T3412:=DeleteRow(DeleteColumn(T34,2),1);
  T3413:=DeleteRow(DeleteColumn(T34,3),1);
```

$$T34 := \begin{bmatrix} 1 & 2\,a & a \\ 1 & a & a \\ a & a & a^2 \end{bmatrix}$$

$$T3411 := \begin{bmatrix} a & a \\ a & a^2 \end{bmatrix}$$

$$T3412 := \begin{bmatrix} 1 & a \\ a & a^2 \end{bmatrix}$$

$$T3413 := \begin{bmatrix} 1 & a \\ a & a \end{bmatrix} \tag{1.1.5}$$

```
> detT34:=simplify((a^3-a^2)-2*a*(a^2-a^2)+a*(a-a^2))
```

$$detT34 := 0 \tag{1.1.6}$$

Vi kan nu beregne determinanten, idet vi benytter determinanten af de snitmatricer vi har fundet.

```
> detT:=expand(1*detT31-0*detT32+a^2*detT33-a^3*detT34)
```

$$detT := -a^6 + 4\,a^5 - 5\,a^4 + 2\,a^3 \tag{1.1.7}$$

Vi finder rødder i determinant

```
> solve(detT)
```

$$0, 0, 0, 2, 1, 1 \tag{1.1.8}$$

GaussJordan-elimination på T0, T1 og T2

Substituerer hhv 0,1, og 2 ind på a's plads i T og får de tre matricer T0, T1 og T1:

```
> T0=T(0);
  T1=T(1);
  T2=T(2);
```

$$T0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T1 = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$T2 = \begin{bmatrix} 1 & 4 & 2 & 4 \\ 1 & 2 & 2 & 2 \\ 1 & 0 & 4 & 8 \\ 2 & 2 & 4 & 2 \end{bmatrix} \tag{1.2.1}$$

GaussJordan-elimination på T0

```
> T(0)
```

(1.2.2)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1.2.2)$$

```
> T01:=RowOperation(T(0),[2,1],-1)
```

$$T01 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1.2.3)$$

```
> T02:=RowOperation(T01,[3,1],-1)
```

$$T02 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1.2.4)$$

GaussJordan-elimination på T1

```
> T(1)
```

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (1.2.5)$$

```
> T11:=RowOperation(T(1),[4,2],-1)
```

$$T11 := \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1.2.6)$$

```
> T12:=RowOperation(T11,[2,3],-1)
```

$$T12 := \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1.2.7)$$

```
> T13:=RowOperation(T12,[1,2],-2)
```

$$T13 := \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1.2.8)$$

```
> T14:=RowOperation(T13,[3,1],-1)
```

$$T14 := \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1.2.9}$$

GaussJordan-elimination på T2

```
> T(2)
```

$$\begin{bmatrix} 1 & 4 & 2 & 4 \\ 1 & 2 & 2 & 2 \\ 1 & 0 & 4 & 8 \\ 2 & 2 & 4 & 2 \end{bmatrix} \tag{1.2.10}$$

```
> T21:=RowOperation(T(2),[1,4],1)
```

$$T21 := \begin{bmatrix} 3 & 6 & 6 & 6 \\ 1 & 2 & 2 & 2 \\ 1 & 0 & 4 & 8 \\ 2 & 2 & 4 & 2 \end{bmatrix} \tag{1.2.11}$$

```
> T22:=RowOperation(T21,[1,2],-3)
```

$$T22 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 \\ 1 & 0 & 4 & 8 \\ 2 & 2 & 4 & 2 \end{bmatrix} \tag{1.2.12}$$

```
> T23:=RowOperation(T22,[4,2],-2)
```

$$T23 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 \\ 1 & 0 & 4 & 8 \\ 0 & -2 & 0 & -2 \end{bmatrix} \tag{1.2.13}$$

```
> T24:=RowOperation(T23,[2,4],1)
```

$$T24 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 4 & 8 \\ 0 & -2 & 0 & -2 \end{bmatrix} \tag{1.2.14}$$

```
> T25:=RowOperation(T24,[3,2],-1)
```

(1.2.15)

$$T25 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 2 & 8 \\ 0 & -2 & 0 & -2 \end{bmatrix} \tag{1.2.15}$$

```
> T26:=RowOperation(T25,[2,3],-1)
```

$$T26 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -8 \\ 0 & 0 & 2 & 8 \\ 0 & -2 & 0 & -2 \end{bmatrix} \tag{1.2.16}$$

```
> T26:=RowOperation(T26,4,-1/2)
```

$$T26 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -8 \\ 0 & 0 & 2 & 8 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{1.2.17}$$

```
> T27:=RowOperation(T26,3,1/2)
```

$$T27 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{1.2.18}$$

De tre matricer på trapperform:

```
> trapT0:=T02;
  trapT1:=T14;
  trapT2:=T27
```

$$trapT0 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$trapT1 := \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(1.2.19)

$$trapT2 := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix} \tag{1.2.19}$$

Viser, at plan 4 ikke er defineret for a=0, mens de tre resterende får ens ligninger

```
> l1(0);l2(0);l3(0);l4(0)
```

$$\begin{gathered} x=0 \\ x=0 \\ x=0 \\ 0=0 \end{gathered} \tag{1.4}$$

Viser, at alle fire planer er defineret for a=1 og a=2

a=1

```
> l1(1);l2(1);l3(1);l4(1)
```

$$\begin{gathered} x+2y+z=1 \\ x+y+z=1 \\ x+z=1 \\ x+y+z=1 \end{gathered} \tag{1.5}$$

a=2

```
> l1(2);l2(2);l3(2);l4(2)
```

$$\begin{gathered} x+4y+2z=4 \\ x+2y+2z=2 \\ x+4z=8 \\ 2x+2y+4z=2 \end{gathered} \tag{1.6}$$

Aflæser løsningerne.

```
> X:=<x,y,z>;
  L:=T->DeleteColumn(T,4).X=DeleteColumn(T,[1..3])
```

$$X := \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$L := T \mapsto LinearAlgebra:-DeleteColumn(T,4) \cdot X = LinearAlgebra:-DeleteColumn(T,[1..3]) \tag{1.7}$$

```
> L(trapT0)
```

$$\begin{bmatrix} x \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1.8}$$

```
> L(trapT1)
```

$$\begin{bmatrix} x+z \\ y \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1.9)$$

```
> L(trapT2)
```

$$\begin{bmatrix} 0 \\ x \\ z \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -8 \\ 4 \\ 1 \end{bmatrix} \quad (1.10)$$

d)

Viser at alle fire planer er defineret og ikke sammenfaldende idet deres ortogonalvektorer peger i hver sin retning, hvis a=-1

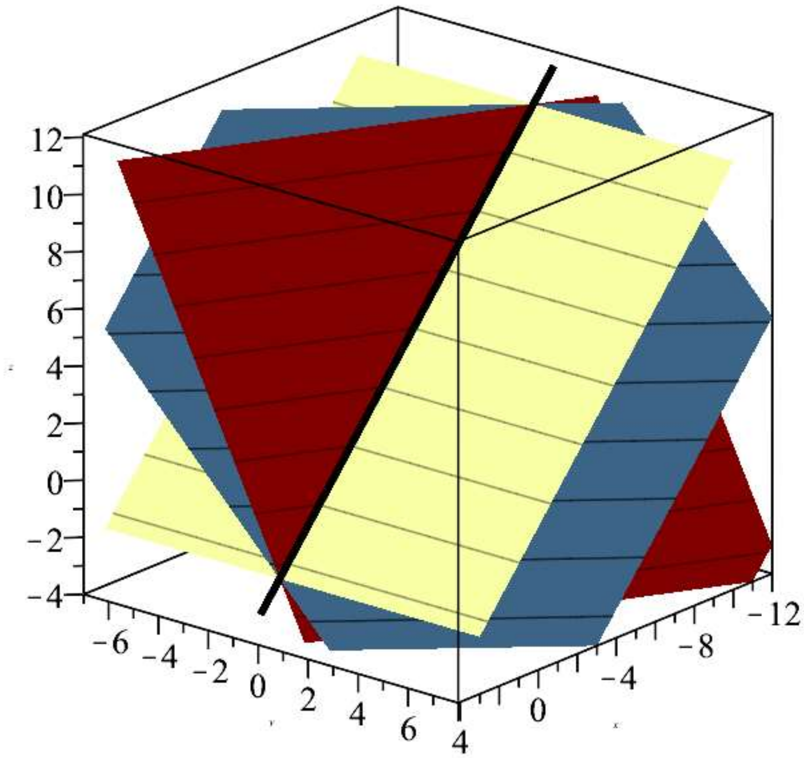
```
> l1(-1);l2(-1);l3(-1);l4(-1)
```

$$x-2y-z=1$$
$$x-y-z=-1$$
$$x+z=-1$$
$$-x-y+z=-1 \quad (2.1)$$

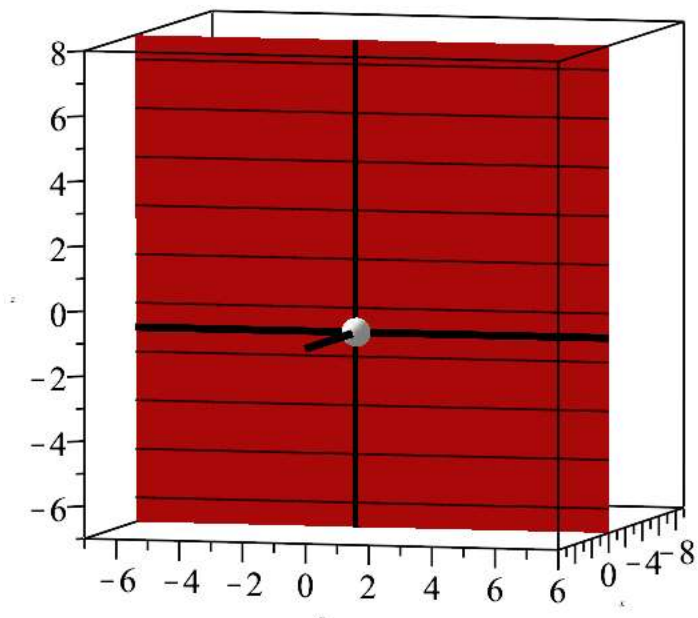
e)

Illustration af opgave c. B
Bemærk, at vi i linjens parameterfremstilling har lagt konstanten 0.1 til i alle koordinaternes udtryk. Dette er for at gøre linjen tydeligere. Havde vi ikke gjort det, ville linjen have været dækket af det røde og det gule plan, og så ville den være sværere at se i plottet. Når vi lægger 0.1 til, er det nok til at linjen er tydelig, men ikke så meget, at vi kan se, at linjen ligger et andet sted, end den burde.

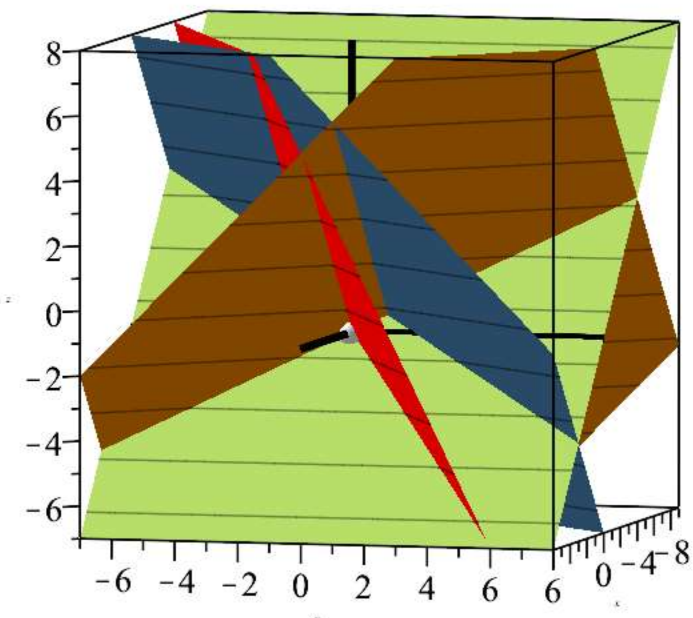
```
> rangexs:=s->s..s+15:
  rangeys:=s->s..s+15:
  rangezs:=s->s..s+15:
  rx:=40:
  ry:=70:
  rangex:=rangexs(-12):
  rangey:=rangeys(-7):
  rangez:=rangezs(-4):
  plot1:=k->implicitplot3d([l1(k),l2(k),l3(k),l4(k)],x=rangex,y=
  rangey,z=rangez, color=["Red","SteelBlue","OliveDrab",
  "DarkOrange"],  style=surfacecontour,orientation=[rx,ry,0]):
  plot2:=plot3d([1-t+0.1,0.1,t+0.1],t=-3..12,thickness=5):
  plottemp:=plot3d(<1,0,0>+<-1,0,1>*t,t=-3..12):
  display(plot2,plot1(1));
```




```
> plot3:=pointplot3d(<0,0,-1>, symbol=solidsphere, symbolsize = 30,
  color = white):
  rangex:=rangexs(-9):
  rangey:=rangeys(-7):
  rangez:=rangezs(-7):
  plotx:=plot3d([t,0,-1],t=rangex,thickness=5):
  ploty:=plot3d([0,t,-1],t=rangey,thickness=5):
  plotz:=plot3d([0,0,t],t=rangez,thickness=5):
  rx:=15:
  ry:=85:
  display(plot1(0),plot3,plotx,ploty,plotz);
  display(plot1(-1),plot3,plotx,ploty,plotz)
```



8
6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6
0 −4 −8


8
6
4
2
0
−2
−4
−6
−6 −4 −2 0 2 4 6 6
0 −4 −8