02417: Time Series Analysis
Rune Grønborg Junker
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DTU Compute
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Assignment 1: Atmospheric CO2

In this assignment you will estimate different linear models for the development of atmospheric CO_2 over the past 60 years. Monthly observations from Mauna Loa, Hawaii, are provided by NOAA and the goal is to predict the concentration a couple of years ahead.
The data is provided in `A1_co2.txt` and includes four columns:

year: Year measurement was made

month: Month measurement was made

time: Decimal year and month

co2: Atmospheric CO2 [ppm]

The data is provided by NOAA/ESRL: `www.esrl.noaa.gov/gmd/ccgg/trends/`
You should not use the observations for years 2018 and 2019 (Last 20 observations) for estimations/training - only for comparisons/testing.
In this assignment you are expected to code your own implementations of the relevant algorithms. Do include the code representing your algorithms in your main report.

Question 1.1: Plotting Read the data and plot the CO_2 as a function of time. Do indicate which data is used for training and testing. Comment on the evolution of the values over time. It is OK if the plot is combined with results from the following questions.

Question 1.2: OLS and WLS First OLS and WLS models are to be tested. The models should contain an intercept, a slope and an annual harmonic part. The anticipated model is of the form:

$$Y_t = \alpha + \beta_t t + \beta_s \sin\left(\frac{2\pi}{p} t\right) + \beta_c \cos\left(\frac{2\pi}{p} t\right) + \varepsilon_t$$

1. Estimate the parameters in the above linear regression model (OLS).
2. Present the estimated parameters including a measure of uncertainty.
3. Plot the fitted values together with the data.
4. Now, we assume that the correlation structure of the residiuals is an exponential decaying function of the time distance between two observations, i.e.

$$Cor(\varepsilon_{t_i}, \varepsilon_{t_j}) = \rho^{|t_i - t_j|} \tag{1}$$

for some $0 < \rho < 1$, which results in the following correlation matrix:

$$\Sigma = \begin{pmatrix} 1 & \rho & \rho^2 & \cdots \\ \rho & 1 & \rho & \cdots \\ \rho^2 & \rho & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{2}$$

5. Estimate ρ using five iterations of the relaxation algorithm.
6. Present the estimated parameters based on this WLS model. Provide a measure of uncertainty for each of the estimates.
7. Again plot the fitted values.
8. Compare the two estimates and comment on the results.

Question 1.3: Local linear trend Use a local linear trend model with $\lambda = 0.9$.

1. Provide L and $f(0)$ for the trend model corresponding to the linear model in the previous question.
2. Filter the data with the chosen model.
3. Plot the one step prediction errors and the resulting estimates of σ for each observation (Do skip the first 10 observations as transient).
4. Plot the data and the corresponding one step predictions for all observations in the training data. Include a 95% prediction interval.
5. Zoom in on the time since 2010 and include predictions for the test data. Again including a 95% prediction interval.
6. Make a table presenting the predictions 1, 2, 6, 12 and 20 months ahead.
7. Compare with the test data and comment on the results
8. Plot the data and the estimated mean for each time step (Typically the first element in θ_t for all t).

Question 1.4: Optimal λ In the previous question you used a given λ. In this question you should find the optimal λ (minimizing squared one step prediction errors) with a burnin period of 100 months.

1. Make a plot with the optimal λ showing the data and predictions from 2010 and onwards (As in the previous question).
2. Make a table predicting the test data as in the previous question.
3. Comment on the results.

Question 1.5: Over all Which model do you prefer and why?

Suggest extensions to the model and how you find they can improve the model.