081203M04001H - Algorithm Design and Analysis

## Assignment 1

September 17, 2021

Notice:

1. Please submit your answer in hard copy AND submit a digital version to UCAS website

[http://sep.ucas.ac.cn](http://sep.ucas.ac.cn/).

Hard copy should be submitted before 9 am. October 8 and digital version should be submitted before 11 pm. October 8.

1. You can choose **three** from problems 1-6.
2. For problems 1-6, you should do at least the following things:
   1. Describe your algorithm in natural language **AND** pseudo-code;
   2. Draw a “subproblem reduction graph”, where nodes represent subproblems, and edges describe the “reduction relationship” between them for every problem you choose in problems 1-6;
   3. Prove the correctness of your algorithm;
   4. Analyse the complexity of your algorithm.

(a)用自然语言和伪代码描述你的算法;

(b)绘制“子问题约简图”，其中节点表示子问题，边描述子问题与问题1-6中选择的每个问题的“约简关系”;

(c)证明算法的正确性;

(d)分析算法的复杂性。

# Divide and Conquer

Given an integer array nums and an integer k, please return the k-th largest element in the array.

给定一个整数数组nums和一个整数k，请返回数组中第k大的元素。

Your algorithm’s runtime complexity must be in the order of O(n), prove the correctnes- sand analyze the complexity.(k is much smaller than n, n is the length of the array.)

你的算法的运行复杂度必须在O(n)的顺序，证明正确-分析复杂性。(k比n小得多，n是数组的长度)

# Divide and Conquer

Consider an *n*-node complete binary tree *T* , where *n* = 2*d −* 1 for some *d*. Each node *v* of *T* is labeled with a real number *xv*. You may assume that the real numbers labeling the nodes are all distinct. A node *v* of *T* is a local minimum if the label *xv* is less than the label *xw* for all nodes *w* that are joined to *v* by an edge.

You are given such a complete binary tree *T* , but the labeling is only specified in the following: implicit way: for each node *v*, you can determine the value *xv* by probing the node *v*.

Show how to find a local minimum of *T* using only *O*(*logn*) probes to the nodes of *T* .

# Divide and Conquer

Given an integer array, one or more consecutive integers in the array form a sub-array. Find the maximum value of the sum of all subarrays.

给定一个整数数组，数组中的一个或多个连续整数构成子数组。找出所有子数组的和的最大值。

Please give an algorithm with O(nlogn) complexity

# Divide and Conquer

Given an array of integers nums sorted in ascending order, find the starting and ending position of a given target value. If the target is not found in the array, return [-1, -1]. For example, if the array is [5*,* 7*,* 7*,* 8*,* 8*,* 10] and the target is 8, then the output should be [3*,* 4].Your algorithm’s runtime complexity must be in the order of *O*(log *n*), prove the correctness and analyze the complexity.

# Divide and Conquer

Given a convex polygon with *n* vertices, we can divide it into several separated pieces, such that every piece is a triangle. When *n* = 4, there are two different ways to divide the polygon; When *n* = 5, there are five different ways.

Give an algorithm that decides how many ways we can divide a convex polygon with *n*

vertices into triangles.

给定一个有n个顶点的凸多边形，我们可以把它分成几个独立的部分，这样每个部分都是一个三角形。当n = 4时，有两种不同的方法来划分多边形;当n = 5时，有五种不同的方法。

给出一个算法来决定我们可以用多少种方法来划分一个带有n的凸多边形

顶点成三角形。

# Divide and Conquer

Given an array of k linked-lists lists, each linked-list is sorted in ascending order. Given an O(knlogk) algorithm to merge all the linked-lists into one sorted linked-list. (Note that the length of a linked-lists is n)

给定一个由k个链表组成的数组，每个链表按升序排序。给出了一个O(knlogk)算法，将所有链表合并为一个已排序链表。(注意链表的长度是n)