$X_{dim} = (3,2) \mid w_{dim} = (\quad)$

1.1

House

SQFT : 2,000 x_1
Bedrooms : 3 x_2

weights

[150, 10000] (\$/sqft, \$/bedroom)

bias

b = 50,000

A. Dot Product

$w \bullet x = 150 \cdot 2000 + 3 \cdot 10000$
$= 30000 + 30000$

$\Rightarrow \hat{y} = w \bullet x + b = 60000 + 50000$
$\Rightarrow \hat{y} = 110000$

A 2,000 sqft house with three bedrooms would cost \$110,000.

B. Multiple Predictions

n = 3 houses

$$X = \begin{array}{c} \text{sqft} \quad \text{beds} \\ \begin{bmatrix} 2000 & 3 \\ 1500 & 2 \\ 2500 & 4 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Keep w & b the same.

$$X \cdot w^T = \begin{bmatrix} 2000 & 3 \\ 1500 & 2 \\ 2500 & 4 \end{bmatrix} \begin{bmatrix} 150 \\ 10000 \end{bmatrix}$$

$$= \begin{bmatrix} 2000(150) + 3(10000) \\ 1500(150) + 2(10000) \\ 2500(150) + 4(10000) \end{bmatrix} = \begin{bmatrix} 30000 + 30000 \\ 22500 + 20000 \\ 37500 + 40000 \end{bmatrix} = \begin{bmatrix} 60000 \\ 42500 \\ 77500 \end{bmatrix}$$

$$\Rightarrow \hat{y} = Xw^T + b = \begin{bmatrix} 60000 \\ 42500 \\ 77500 \end{bmatrix} + \begin{bmatrix} 50000 \\ 50000 \\ 50000 \end{bmatrix} = \begin{bmatrix} 110000 \\ 92500 \\ 127500 \end{bmatrix}$$

1.2 For $A = (m \times n)$, $B = (p \times q)$,

A.

AB iff $n = p \xrightarrow{\text{row} \times \text{col}}$ col = col

- $(100 \times 5)(5 \times 1) \to$ Valid, $(100, 1)$
- $(100 \times 5)(3 \times 1) \to$ Invalid
- $(1 \times 5)(5 \times 100) \to$ Valid, (1×100)
- $(100 \times 1)(1 \times 5) \to$ Valid, (100×5)

B.

$$L = \frac{1}{n}\sum_i (y_i - \hat{y}_i)^2 = \frac{1}{n}\sum_i (e)^2$$

I = 5 (observations)
K = 2 (features)
out = 1 (MPG)

$X : (5 \times 2)$
$y : (5 \times 1)$
$e : (5 \times 1)$
$w : (1 \times 2)$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial e} \cdot \frac{\partial e}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w}$$

$\Rightarrow \frac{\partial}{\partial e} L = \frac{1}{n} \frac{\partial}{\partial e}(e)^2 = \frac{1}{n} \cdot 2e = \frac{2}{n} e$

$\Rightarrow \frac{\partial}{\partial \hat{y}} e = \frac{\partial}{\partial \hat{y}}(y - \hat{y}) = -1$

$\Rightarrow \frac{\partial \hat{y}}{\partial w} = \frac{\partial}{\partial w}(wx + b) = x$

Since we're working with vectors, $e \to e^T$

$\frac{2}{n} e^T \cdot -1 \cdot X$

$= -\frac{2}{n} e^T X$

Q.E.D.

$\Rightarrow e^T_{dim} = (1, 5)$
$\Rightarrow (e^T X)_{dim} = (1, 2)$
$\Rightarrow (e^T X)_{dim} = (w)_{dim}$

This operation accomplishes gradient descent because each operation is based on the same values as the loop approach. When we multiply the matrix X by e^T, it is the same as $x_1 e_1 + x_2 e_2 + \dots + x_i e_i$ since dot products have the same shape.
The (1×2) result is the relative effect of each weight on the loss function in vector form.
By adding them together, we see the collective effect that "votes" for the direction to change

Data 2

$$X = \begin{bmatrix} 3.0 & 1.1 \\ 2.5 & 0.9 \\ 4.0 & 1.5 \\ 3.5 & 1.2 \\ 2.8 & 1.0 \end{bmatrix} \quad y = \begin{bmatrix} 25 \\ 30 \\ 18 \\ 22 \\ 28 \end{bmatrix}$$

weight, hp
x_1, x_2

$$w^T = \begin{bmatrix} 1.2 \\ -0.8 \end{bmatrix} \quad \begin{array}{l} b = 17 \\ \alpha = 0.01 \\ n = 5 \end{array}$$

$\hat{y} = w^T X + b$

$$\Rightarrow w^T X = \begin{bmatrix} 3.0 & 1.1 \\ 2.5 & 0.9 \\ 4.0 & 1.5 \\ 3.5 & 1.2 \\ 2.8 & 1.0 \end{bmatrix} \begin{bmatrix} 1.0 \\ -0.8 \end{bmatrix} = \begin{bmatrix} (3.0 \cdot 1.0) + (1.1 \cdot -0.8) \\ (2.5 \cdot 1.0) + (0.9 \cdot -0.8) \\ (4.0 \cdot 1.0) + (1.5 \cdot -0.8) \\ (3.5 \cdot 1.0) + (1.2 \cdot -0.8) \\ (2.8 \cdot 1.0) + (1.0 \cdot -0.8) \end{bmatrix} = \begin{bmatrix} 2.12 \\ 1.78 \\ 2.8 \\ 2.54 \\ 2 \end{bmatrix}$$

$$\Rightarrow \hat{y} = \begin{bmatrix} 2.12 \\ 1.78 \\ 2.80 \\ 2.54 \\ 2.00 \end{bmatrix} + [17] = \begin{bmatrix} 19.12 \\ 18.78 \\ 19.80 \\ 19.54 \\ 19.00 \end{bmatrix}$$

Predicted MPG

$$\Rightarrow \frac{\partial L}{\partial w} = -\frac{2}{n} e^T X$$

$e = y - \hat{y}$

$$\Rightarrow e^T = \begin{bmatrix} 25 - 19.12 \\ 30 - 18.78 \\ 18 - 19.80 \\ 22 - 19.54 \\ 28 - 19.00 \end{bmatrix} = \begin{bmatrix} 5.88 \\ 11.22 \\ -1.8 \\ 2.46 \\ 9 \end{bmatrix}$$

$(5, 1)$

$$\Rightarrow e^T X = \begin{bmatrix} 3.0 \cdot 5.88 & 1.1 \cdot 5.88 \\ 2.5 \cdot 11.22 & 0.9 \cdot 11.22 \\ 4.0 \cdot -1.8 & 1.5 \cdot -1.8 \\ 3.5 \cdot 2.46 & 1.2 \cdot 2.46 \\ 2.8 \cdot 9 & 1.0 \cdot 9 \end{bmatrix}$$

$\overline{72.3} \quad \overline{25.816}$

$x_{dim} = (2, 5)$

$e^T X_{dim} = (2, 1)$

Gradient Descent

$$W_{n+1} = W_n - \alpha \frac{\partial L}{\partial w}$$

$$\Rightarrow W_{n+1} = \begin{bmatrix} 1.2 \\ -0.8 \end{bmatrix} - \left(0.01 \cdot \begin{bmatrix} 72.3 \\ 25.818 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 1.2 \\ -0.8 \end{bmatrix} - \left(\begin{bmatrix} 0.723 \\ 0.25818 \end{bmatrix}\right)$$

$$\Rightarrow W_{n+1} = \begin{bmatrix} 0.477 \\ -1.05818 \end{bmatrix}$$

Reduce both w_1 & w_2.

This spreadsheet is to be used to perform a single step of multiple linear regression.

DATA

x1_i (weight)	x2_i(hp)
3	1.1
2.5	0.9
4	1.5
3.5	1.2
2.8	1

Hyper-parameters

α
0.01

Current parameters

w1_0 (weight)	w2_0 (hp)
1.2	-0.8

Model Inference

x1_i (weight)	x2_i(hp)	*
3	1.1	
2.5	0.9	
4	1.5	
3.5	1.2	
2.8	1	

Gradient Descent Epoch 1	Gradient Computation	Gradient of loss with respect to m	$\partial L/\partial \mathbf{w}$
			72.3
		Gradient of loss with respect to b	$\partial L/\partial b$
			-10.704
	Update parameters	**w_1**	=
		1.48	-0.69

MPG
25
30
18
22
28

b_0

17

$\mathbf{w}^T$	=	$\mathbf{xw}^T$	+	b_0
1.2		2.12		17
-0.8		1.78		
		2.8		
		2.54		
		2		

	=	∂e_i/∂y_hat_i	*	$(\partial L\_i/\partial e\_i)^T$
25.818		-1		2.35
	=	∂e_i/∂y_hat_i	*	$(\partial L\_i/\partial e\_i)^T$
		-1		-2.352

w_0-α*∂L/∂**w**	b_1	=	b_0-α*∂L/∂b
	17.1		

=	y_hat_i	e_i = y_i - y_hat_i
	19.12	5.88
	18.78	11.22
	19.8	-1.8
	19.54	2.46
	19	9

*

4.49	-0.72	0.984	3.6

*

-4.49	0.72	-0.984	-3.6

L_i = 1/n e^2	∂L_i/∂e_i
6.91	2.35
25.18	4.49
0.65	−0.72
1.21	0.98
16.20	3.60

_hat_i/ **w**

∂y ∂

3	1.1
2.5	0.9
4	1.5
3.5	1.2
2.8	1

∂y_hat_i/∂b

1
1
1
1
1

In [67]:
```
import numpy as np

class VectorizedMLR:
    """
    Multiple Linear Regression: ŷ = X@wᵀ + b

    Works for ANY number of features:
        - p = 1: Simple Linear Regression
        - p > 1: Multiple Linear Regression

    REQUIRED Dimensions:
        X: (n, p) - n observations, p features
        w: (1, p) - 1 row, p features (ROW VECTOR)
        b: scalar
        ŷ: (n,1) - n predictions
    """

    def __init__(self, learning_rate=0.01, num_iterations=1000):
        self.learning_rate = learning_rate
        self.num_iterations = num_iterations
        self.w = None  # Will be (1, p)
        self.b = 0.0
        self.loss_history = []

    def predict(self, X):
        """
        Matrix-based prediction: ŷ = X@wᵀ + b

        Input:
            X: numpy array (n, p)
        Returns:
            y_pred: numpy array (n,1) - predictions
        """
        y_pred = X @ self.w.T + self.b
        return y_pred

    def compute_loss(self, X, y):
        """Mean Squared Error: (1/n) * sum((y_pred - y)^2)"""
        n = X.shape[0]
        y_pred = self.predict(X)
        loss = (1 / n) * np.sum((y_pred - y) ** 2)
        return loss

    def compute_gradients(self, X, y):
        """
        Computes gradients for w and b.
        grad_w = dJ/dw
        grad_b = dJ/db
        """
        n = X.shape[0]
        y_pred = self.predict(X)
        error = y_pred - y # Shape (n, 1)

        # Gradient w:
        # Shape must be (1, p) to match self.w
        # Formula: (2/n) * error.T @ X
        # (1, n) @ (n, p) -> (1, p)
        grad_w = (2 / n) * (error.T @ X)

        # Gradient b:
        # Scalar mean of errors * 2
        grad_b = (2 / n) * np.sum(error)

        return grad_w, grad_b
```

```
    def fit(self, X, y):
        """
        Apply gradient descent to optimize the parameters of the model.

        Input:
            X: numpy array (n, p)
            y: numpy array (n, 1)
        Returns:
            self for chained calls.
        """
        n_samples, n_features = X.shape
        self.w = np.zeros((1, n_features))
        self.b = 0.0
        self.loss_history = []

        # Gradient Descent Loop
        for i in range(self.num_iterations):
            # Calculate gradients
            grad_w, grad_b = self.compute_gradients(X, y)

            # Update parameters
            self.w = self.w - (self.learning_rate * grad_w)
            self.b = self.b - (self.learning_rate * grad_b)

            # Track loss
            current_loss = self.compute_loss(X, y)
            self.loss_history.append(current_loss)

        return self
```

In [68]:
```
# Load Auto MPG dataset
from urllib.request import urlretrieve
import pandas as pd

url = 'https://archive.ics.uci.edu/ml/machine-learning-databases/auto-mpg/auto-mpg.da
urlretrieve(url, 'auto-mpg.data')

column_names = ['mpg', 'cylinders', 'displacement', 'horsepower', 'weight',
                'acceleration', 'model_year', 'origin', 'car_name']
data = pd.read_csv('auto-mpg.data', names=column_names,
                   delim_whitespace=True, na_values='?')
data = data.dropna()

# Extract target
y = data['mpg'].values.reshape(-1,1)

print(f"Dataset: {len(y)} cars")
print(f"y shape: {y.shape}")
```

```
Dataset: 392 cars
y shape: (392, 1)
```

```
/tmp/ipython-input-322/2949022177.py:10: FutureWarning: The 'delim_whitespace' keyword
in pd.read_csv is deprecated and will be removed in a future version. Use ``sep='\s+'`
` instead
  data = pd.read_csv('auto-mpg.data', names=column_names,
```

In [69]:
```
# ============================================================
# Simple Linear Regression: One feature (weight → mpg)
# ============================================================
from sklearn.preprocessing import MinMaxScaler

# Extract weight and reshape to (n, 1) - MUST be 2D!
X_slr = data['weight'].values / 1000  # Scale to thousands
X_slr = X_slr.reshape(-1, 1)  # Shape: (n, 1) - single feature
```

```
# normalize
scaler = MinMaxScaler()
scaler.fit(X_slr)
X_slr = scaler.transform(X_slr)

print(f"\n" + "="*60)
print("SIMPLE LINEAR REGRESSION (1 feature)")
print("="*60)
print(f"X_slr shape: {X_slr.shape}")  # Should be (n, 1)
print(f"y shape: {y.shape}")          # Should be (n,1)

# Train model
slr_model = VectorizedMLR(learning_rate=0.1, num_iterations=1000)
slr_model.fit(X_slr, y)

print(f"\nResults:")
print(f"weights shape: {slr_model.w.shape}")  # Should be (1, 1)
print(f"Weight (w): {slr_model.w[0, 0]:.4f} (MPG per 1000 lbs)")
print(f"Bias (b): {slr_model.b:.4f}")
print(f"Final MSE: {slr_model.loss_history[-1]:.4f}")
```

```
============================================================
SIMPLE LINEAR REGRESSION (1 feature)
============================================================
X_slr shape: (392, 1)
y shape: (392, 1)

Results:
weights shape: (1, 1)
Weight (w): -26.9707 (MPG per 1000 lbs)
Bias (b): 33.8807
Final MSE: 18.6766
```

In [70]:

```
import numpy as np

class minMaxScaler:
    def __init__(self):
        self.min = None # this should be a row vector sized (1,features)
        self.max = None # this should be a row vector sized (1,features)

    def fit(self, X):
        """
        get min and max from training data.
        X: (n, p) numpy array
        """
        # be sure to find the min and max along the correct axis
        # Want min and max in each column (features)
        # numpy refers to this as finding the min or max ALONG the row (axis=0)
        self.min = X.min(axis=0).reshape(1,-1)
        self.max = X.max(axis=0).reshape(1,-1)
        # the reshape says we want a 2D matrix with
        # 1 row that has "don't care" how many columns

    def transform(self, X):
        """
        Apply the learned scaling to data.
        X: (n, p) numpy array
        Returns: scaled X with same shape
        """
        X_scaled = (X - self.min) / (self.max - self.min)
        return X_scaled
```

In [71]:

```
# ============================================================
# Multiple Linear Regression: three features → mpg
# ============================================================
```

```python
# Extract multiple features - already 2D!
X_mlr = data[['weight', 'horsepower']].values

scaler = minMaxScaler()
scaler.fit(X_mlr)
X_mlr = scaler.transform(X_mlr)

print(f"\n" + "="*60)
print("MULTIPLE LINEAR REGRESSION (2 features)")
print("="*60)
print(f"X_mlr shape: {X_mlr.shape}")  # Should be (n, 3)
print(f"y shape: {y.shape}")          # Should be (n,1)

# Train model - EXACT SAME CODE!
mlr_model = VectorizedMLR(learning_rate=0.1, num_iterations=1000)
mlr_model.fit(X_mlr, y)

print(f"\nResults:")
print(f"Theta shape: {mlr_model.w.shape}")  # Should be (1, 3)
print(f"Weights (w):")
print(f"  Weight:       {mlr_model.w[0, 0]:7.4f} (per 1000 lbs)")
print(f"  Horsepower:   {mlr_model.w [0, 1]:7.4f} (per 100 hp)")
print(f"Bias (b): {mlr_model.b:.4f}")
print(f"Final MSE: {mlr_model.loss_history[-1]:.4f}")
```

```
============================================================
MULTIPLE LINEAR REGRESSION (2 features)
============================================================
X_mlr shape: (392, 2)
y shape: (392, 1)

Results:
Theta shape: (1, 2)
Weights (w):
  Weight:       -19.2731 (per 1000 lbs)
  Horsepower:   -10.0712 (per 100 hp)
Bias (b): 34.1028
Final MSE: 17.8631
```

In [72]:

```python
# ============================================================
# Multiple Linear Regression: all features → mpg
# ============================================================

# Extract multiple features - already 2D!
# correct this to select the appropriate columns from the pandas dataframe.
# Refer to the selection code used above and
# the feature list shown in the code for downloading the dataset.
# do not include the MPG column or the car_name column.
X_mlr_all = data[['cylinders', 'displacement', 'horsepower', 'weight',
                  'acceleration', 'model_year']].values

# normalize
scaler = minMaxScaler()
scaler.fit(X_mlr_all)
X_mlr_all = scaler.transform(X_mlr_all)

print(f"\n" + "="*60)
print("MULTIPLE LINEAR REGRESSION (all features)")
print("="*60)
print(f"X_mlr_all shape: {X_mlr_all.shape}")  # Should be (n, 7)
print(f"y shape: {y.shape}")          # Should be (n,1)

# Train model - EXACT SAME CODE!
mlr_all_model = VectorizedMLR(learning_rate=0.1, num_iterations=1000)
mlr_all_model.fit(X_mlr_all, y)
```

```python
print(f"\nResults:")
print(f"Theta shape: {mlr_all_model.w.shape}")  # Should be (1, 7)
print(f"Final MSE: {mlr_all_model.loss_history[-1]:.4f}")
```

```
============================================================
MULTIPLE LINEAR REGRESSION (all features)
============================================================
X_mlr_all shape: (392, 6)
y shape: (392, 1)

Results:
Theta shape: (1, 6)
Final MSE: 11.8372
```

In [73]:
```python
def compute_mse(y_true, y_pred):
    """
    Mean Squared Error

    Inputs:
        y_true: numpy array (n,1)
        y_pred: numpy array (n,1)
    Returns:
        mse: scalar
    """
    # Calculate the squared differences
    squared_errors = (y_true - y_pred) ** 2

    # Calculate the mean
    mse = np.mean(squared_errors)

    return mse


def compute_r_squared(y_true, y_pred):
    """
    Coefficient of Determination (R²)

    R² = 1 - (MSE_model / MSE_baseline)

    Where:
        MSE_model = mean((y_true - y_pred)²)
        MSE_baseline = mean((y_true - ȳ)²) : ȳ is mean of y values

    Interpretation:
        R² = 1.0  → perfect predictions
        R² = 0.0  → model is only as good as always predicting the mean
        R² < 0.0  → model is worse than predicting the mean (bad!)

    R² measures the proportion of variance in y explained by the model.

    Inputs:
        y_true: numpy array (n,) or (n, 1)
        y_pred: numpy array (n,) or (n, 1)
    Returns:
        r_squared: scalar
    """

    # Calculate MSE_model (Variance of the residuals)
    mse_model = np.mean((y_true - y_pred) ** 2)

    # Calculate MSE_baseline (Variance of the data)
    # The baseline model always predicts the mean of the target (y_true)
    y_mean = np.mean(y_true)
    mse_baseline = np.mean((y_true - y_mean) ** 2)
```

```python
    # Calculate R

    r_squared = 1 - (mse_model / mse_baseline)

    return r_squared
```

In [74]:

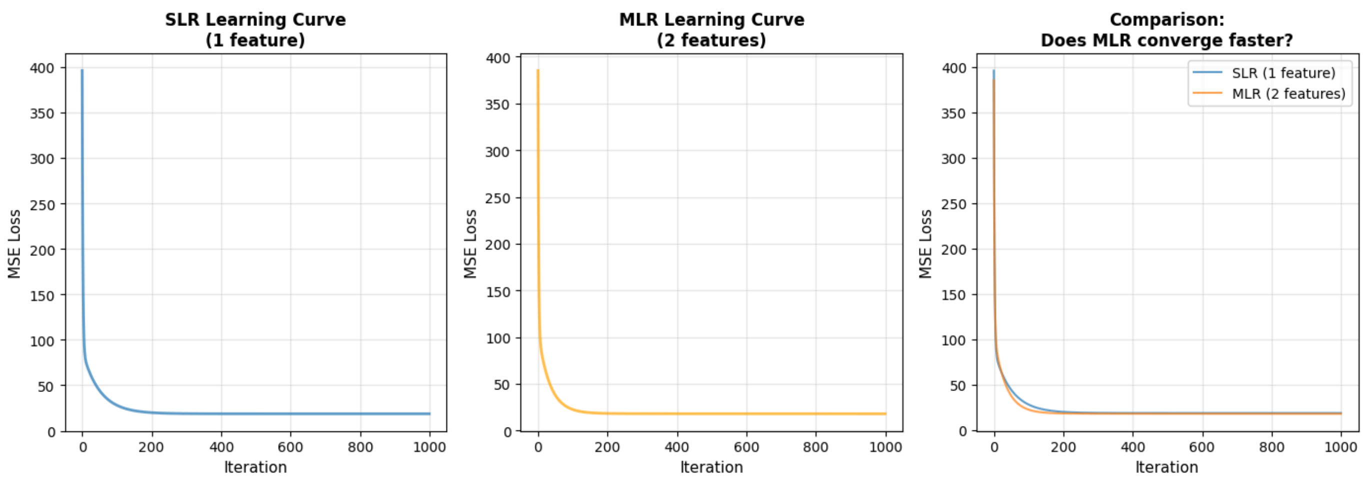
```python
# Get predictions
y_pred_slr = slr_model.predict(X_slr)
y_pred_mlr = mlr_model.predict(X_mlr)
y_pred_mlr_all = mlr_all_model.predict(X_mlr_all)

# Compute metrics
print("\n" + "="*70)
print("MODEL COMPARISON: Does Adding Features Improve Performance?")
print("="*70)

print("\nSIMPLE Linear Regression (vehicle weight only):")
mse_slr = compute_mse(y, y_pred_slr)
r2_slr = compute_r_squared(y, y_pred_slr)
print(f"  Features: 1 (vehicle weight)")
print(f"  MSE:      {mse_slr:.4f}")
print(f"  R²:       {r2_slr:.4f}")
print(f"  → Explains {r2_slr*100:.1f}% of variance in MPG")

print("\nMULTIPLE Linear Regression (weight + horsepower):")
mse_mlr = compute_mse(y, y_pred_mlr)
r2_mlr = compute_r_squared(y, y_pred_mlr)
print(f"  Features: 3 (vehicle weight, horsepower)")
print(f"  MSE:      {mse_mlr:.4f}")
print(f"  R²:       {r2_mlr:.4f}")
print(f"  → Explains {r2_mlr*100:.1f}% of variance in MPG")

print("\nMULTIPLE Linear Regression (all features):")
mse_mlr_all = compute_mse(y, y_pred_mlr_all)
r2_mlr_all = compute_r_squared(y, y_pred_mlr_all)
print(f"  Features: {X_mlr_all.shape[1]}")
print(f"  MSE:      {mse_mlr_all:.4f}")
print(f"  R²:       {r2_mlr_all:.4f}")
print(f"  → Explains {r2_mlr_all*100:.1f}% of variance in MPG")

print("\n" + "-"*70)
print("IMPROVEMENT FROM ADDING 1 FEATURE:")
print("-"*70)
mse_reduction = mse_slr - mse_mlr
mse_reduction_pct = (mse_reduction / mse_slr) * 100
r2_increase = r2_mlr - r2_slr
r2_increase_pct = ((r2_mlr - r2_slr) / (1 - r2_slr)) * 100

print(f"  MSE reduction:     {mse_reduction:.4f} ({mse_reduction_pct:.1f}% decrease)"
print(f"  R² increase:       {r2_increase:.4f} (explains {r2_increase_pct:.1f}% more

if mse_mlr < mse_slr:
    print(f"\n  ✓ Adding horsepower IMPROVED the model!")
else:
    print(f"\n  ✗ Adding features did NOT improve the model (check implementation)")

print("="*70)
```

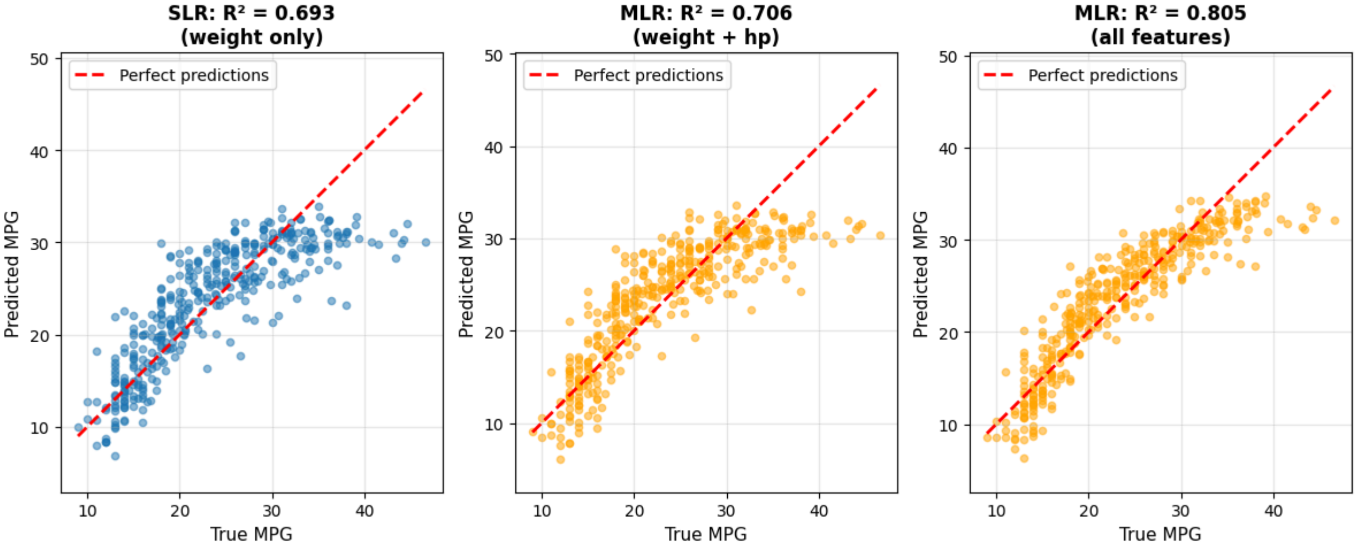
```
======================================================================
MODEL COMPARISON: Does Adding Features Improve Performance?
======================================================================

SIMPLE Linear Regression (vehicle weight only):
  Features: 1 (vehicle weight)
  MSE:      18.6766
  R²:       0.6926
  → Explains 69.3% of variance in MPG

MULTIPLE Linear Regression (weight + horsepower):
  Features: 3 (vehicle weight, horsepower)
  MSE:      17.8631
  R²:       0.7060
  → Explains 70.6% of variance in MPG

MULTIPLE Linear Regression (all features):
  Features: 6
  MSE:      11.8372
  R²:       0.8052
  → Explains 80.5% of variance in MPG

----------------------------------------------------------------------
IMPROVEMENT FROM ADDING 1 FEATURE:
----------------------------------------------------------------------
  MSE reduction:     0.8135 (4.4% decrease)
  R² increase:       0.0134 (explains 4.4% more of remaining variance)

  ✓ Adding horsepower IMPROVED the model!
======================================================================
```

In [75]:

```python
import matplotlib.pyplot as plt

# ============================================================
# Plot 1: Learning Curves - Convergence Comparison
# ============================================================

plt.figure(figsize=(14, 5))

plt.subplot(1, 3, 1)
plt.plot(slr_model.loss_history, alpha=0.7, linewidth=2)
plt.xlabel('Iteration', fontsize=11)
plt.ylabel('MSE Loss', fontsize=11)
plt.title('SLR Learning Curve\n(1 feature)', fontsize=12, fontweight='bold')
plt.grid(True, alpha=0.3)

plt.subplot(1, 3, 2)
plt.plot(mlr_model.loss_history, alpha=0.7, linewidth=2, color='orange')
plt.xlabel('Iteration', fontsize=11)
plt.ylabel('MSE Loss', fontsize=11)
plt.title('MLR Learning Curve\n(2 features)', fontsize=12, fontweight='bold')
plt.grid(True, alpha=0.3)

plt.subplot(1, 3, 3)
plt.plot(slr_model.loss_history, label='SLR (1 feature)', alpha=0.7)
plt.plot(mlr_model.loss_history, label='MLR (2 features)', alpha=0.7)
plt.xlabel('Iteration', fontsize=11)
plt.ylabel('MSE Loss', fontsize=11)
plt.title('Comparison:\nDoes MLR converge faster?', fontsize=12, fontweight='bold')
plt.legend()
plt.grid(True, alpha=0.3)

plt.tight_layout()
plt.savefig('learning_curves.png', dpi=150, bbox_inches='tight')
plt.show()
```

```python
# ============================================================
# Plot 2: Predictions vs. Actual - Accuracy Comparison
# ============================================================

plt.figure(figsize=(12, 5))

plt.subplot(1, 3, 1)
plt.scatter(y.flatten(), y_pred_slr.flatten(), alpha=0.5, s=20)
plt.plot([y.flatten().min(), y.flatten().max()], [y.flatten().min(), y.flatten().max(
         label='Perfect predictions')
plt.xlabel('True MPG', fontsize=11)
plt.ylabel('Predicted MPG', fontsize=11)
plt.title(f'SLR: R² = {r2_slr:.3f}\n(weight only)',
          fontsize=12, fontweight='bold')
plt.legend()
plt.grid(True, alpha=0.3)
plt.axis('equal')

plt.subplot(1, 3, 2)
plt.scatter(y.flatten(), y_pred_mlr.flatten(), alpha=0.5, s=20, color='orange')
plt.plot([y.flatten().min(), y.flatten().max()], [y.flatten().min(), y.flatten().max(
         label='Perfect predictions')
plt.xlabel('True MPG', fontsize=11)
plt.ylabel('Predicted MPG', fontsize=11)
plt.title(f'MLR: R² = {r2_mlr:.3f}\n(weight + hp)',
          fontsize=12, fontweight='bold')
plt.legend()
plt.grid(True, alpha=0.3)
plt.axis('equal')

plt.subplot(1, 3, 3)
plt.scatter(y.flatten(), y_pred_mlr_all.flatten(), alpha=0.5, s=20, color='orange')
plt.plot([y.flatten().min(), y.flatten().max()], [y.flatten().min(), y.flatten().max(
         label='Perfect predictions')
plt.xlabel('True MPG', fontsize=11)
plt.ylabel('Predicted MPG', fontsize=11)
plt.title(f'MLR: R² = {r2_mlr_all:.3f}\n(all features)',
          fontsize=12, fontweight='bold')
plt.legend()
plt.grid(True, alpha=0.3)
plt.axis('equal')

plt.tight_layout()
plt.savefig('predictions_vs_actual.png', dpi=150, bbox_inches='tight')
plt.show()
```





Observations to note in your report:

Convergence: Did both models converge (loss stopped decreasing)? Which converged to a lower loss?

Yes, MLR had less loss without overfitting

Accuracy: In the predictions vs. actual plot, which model has points closer to the red line?

MLR has more points closer to the red line so it preformed better.

MSE: How much did adding a single feature reduce the MSE? How about when all features were used?

One feature removed 4% of error. With all of them, we see MSE = 11.8372 which is a much bigger drop.

Comparison to R^2: How much more of the data's variance does the 2 feature model explain? How about when all features were used?

one adds= 0.0134 Many adds= 0.1126.