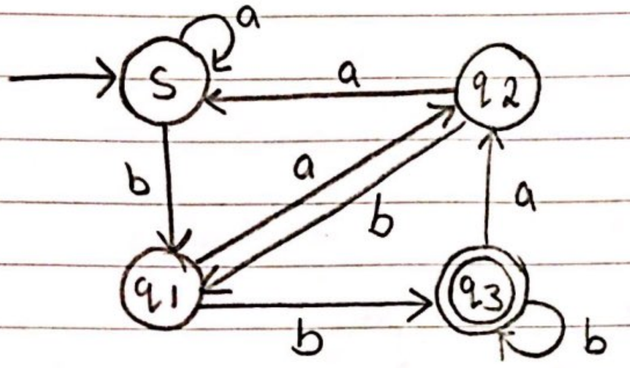
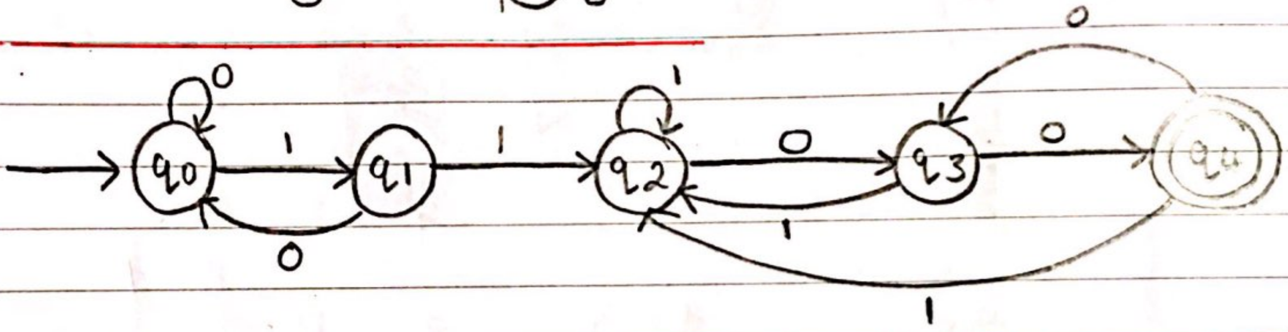
CSC2047 - Assessment : Individual Coursework 2

1. a) The DFA accepts all binary strings which contain the sequence 1101

b) 

c) 

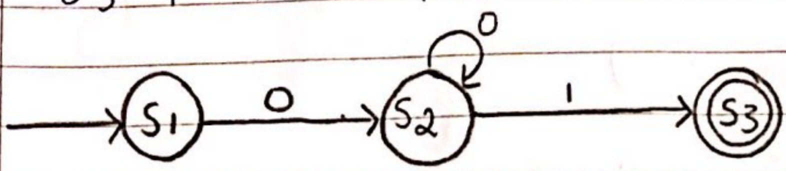
2. a) The NFA accepts all binary strings which start with any number of 0s and end with a 1.

b)

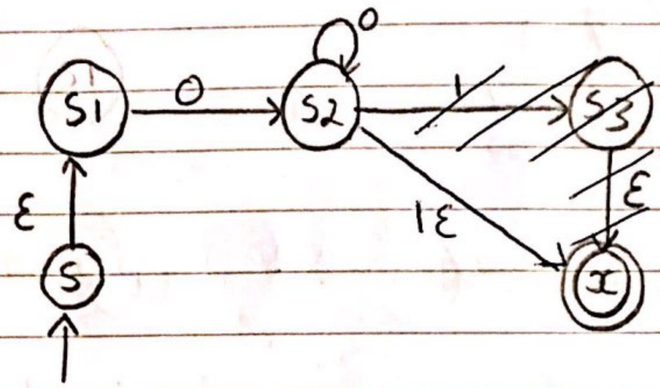
Q \ Σ	0	1	ε
q_0	$\{q_1, q_0\}$	$\emptyset$	$\emptyset$
q_1	$\{q_0, q_1\}$	$\{q_2\}$	$\{q_0\}$
q_2	$\emptyset$	$\emptyset$	$\emptyset$

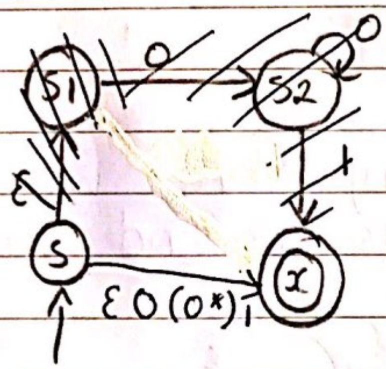
Q \ Σ	0	1	
$[q_0]$	$[q_1, q_0]$	$\emptyset$	
$[q_1, q_0]$	$[q_1, q_0]$	$[q_2]$	
$[q_2]$	$\emptyset$	$\emptyset$	

Q \ Σ	0	1
S_1	S_2	$\varnothing$
S_2	S_2	S_3
S_3	$\varnothing$	$\varnothing$

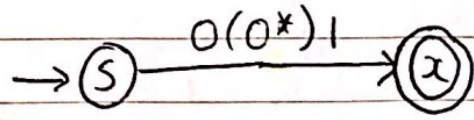


c)



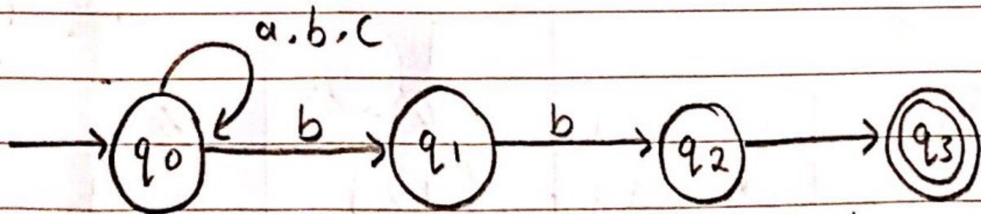


Regular Expression:



$= 0^+1$

d) NFA:



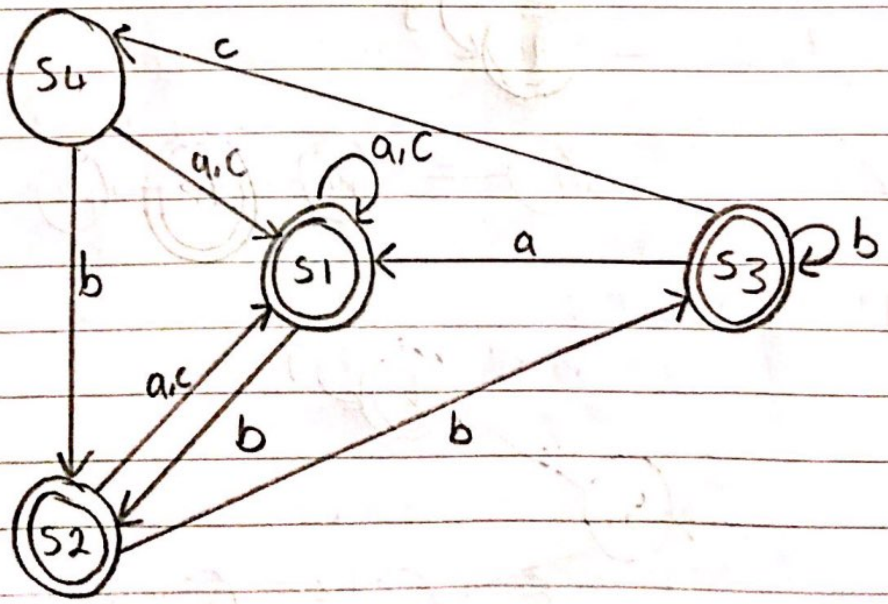
* This will accept strings ending in b b c

Q \ Σ	a	b	c
q_0	$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\emptyset$	$\{q_2\}$	$\emptyset$
q_2	$\emptyset$	$\emptyset$	$\{q_3\}$
q_3	$\emptyset$	$\emptyset$	$\emptyset$

Q \ Σ	a	b	c
$[q_0]$	$[q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0]$	$[q_0, q_1, q_2]$	$[q_0]$
$[q_0, q_1, q_2]$	$[q_0]$	$[q_0, q_1, q_2]$	$[q_0, q_3]$
$[q_0, q_3]$	$[q_0]$	$[q_0, q_1]$	$[q_0]$

*Negating accept states

Q \ Σ	a	b	c
S_1	S_1	S_2	S_1
S_2	S_1	S_3	S_1
S_3	S_1	S_3	S_4
S_4	S_1	S_2	S_1



3. a) i) $(xy)^* = \{xy, xyxy\}$

ii) $y^* \cup zx^+ = \{yy, zx\}$

iii) $\Sigma x \; \Sigma y \; \Sigma z$

$\{\Sigma \circ x\} \circ \{\Sigma \circ y\} \circ \{\Sigma \circ z\}$

$\downarrow$

$\{(x,y,z)\circ(x)\} \circ \{(x,y,z)\circ(y)\} \circ \{(x,y,z)\circ(z)\}$

$\downarrow$

$\{(xx),(yx),(zx)\} \circ \{(xy),(yy),(zy)\} \circ \{(xz),(yz),(zz)\}$

$= \{xxxyxz, \; yxyyyz\}$

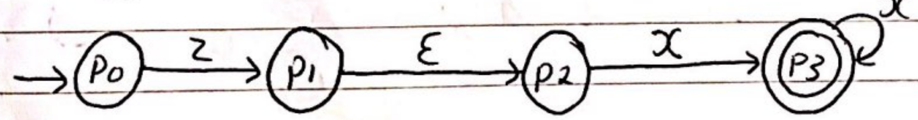
iv) $y^* z^* x^2 = \{yyzzx^2, \; yyyzzzx^2\}$

v) $(x \cup y) z^* = \{xzz, \; yzz\}$

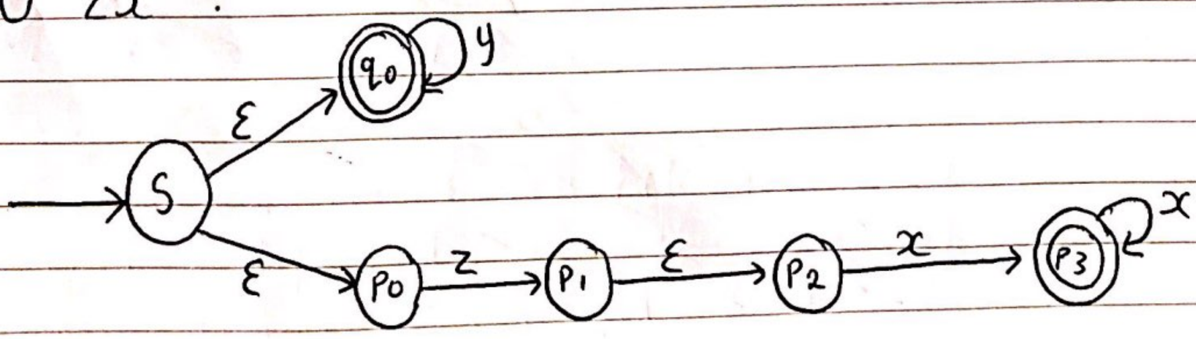
b) $y^* \cup zx^+$

y^* :

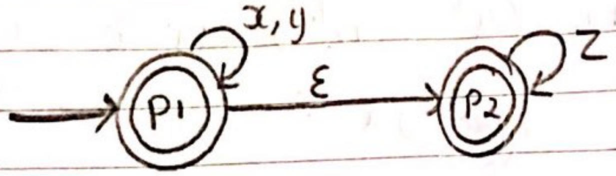
$z \circ x^+$:

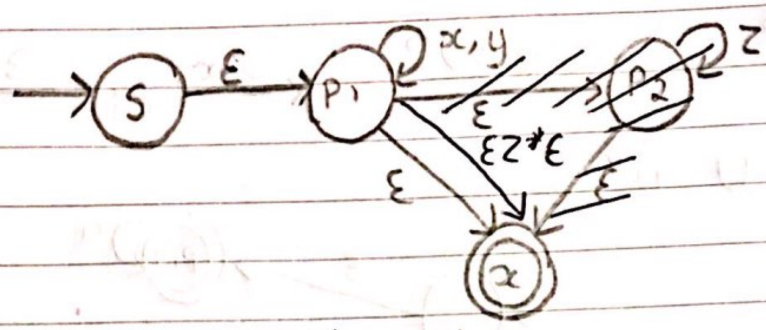


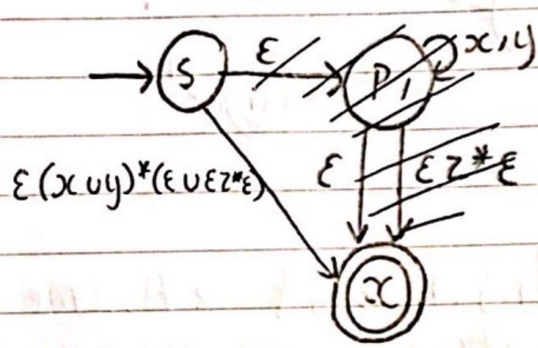
$y^* \cup zx^+$:



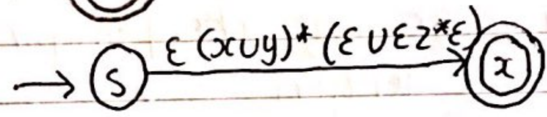
c) i) NFA:





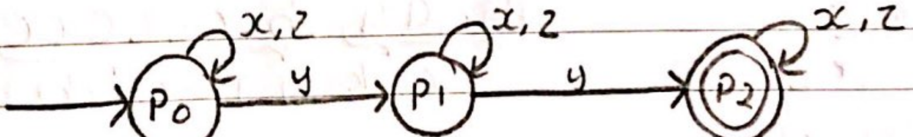


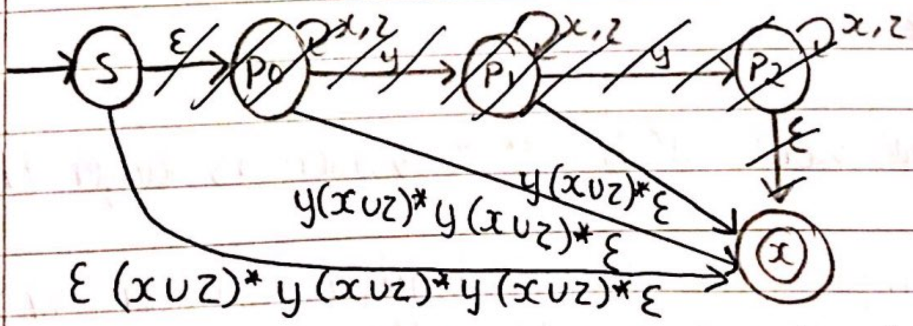
Regular expression:



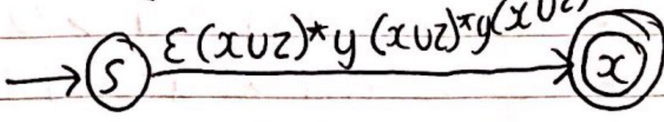
$= (x \cup y)^* z^*$

ii) NFA:





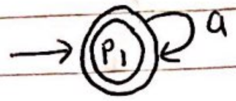
Regular expression:



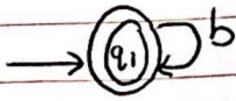
$= (x \cup z)^* y (x \cup z)^* y (x \cup z)^*$

4. a) $\{w | w \text{ in } (a^k \cup b^k) \text{ for } k \geq 0\}$ → regular

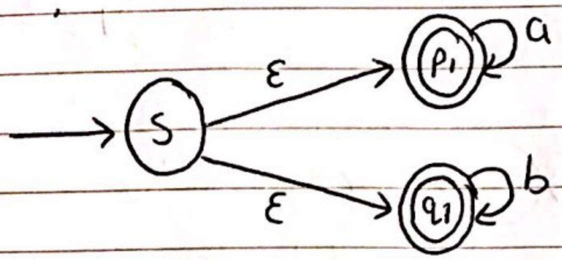
a^k :



b^k :



$a^k \cup b^k$:



b) $\{a^i b^i c^{2i+2j} | i,j,k \geq 0\}$ → Assume that the language is regular. By the pumping lemma, there exists a pumping length $k > 0$ such that any word in the language that is longer than k characters can be described as xyz.

Rules:
1. $xy^*z \in L$
2. $|y| > 0$
3. $|xy| \leq k$

eg. $\underbrace{aa}_{x=k}\underbrace{b}_{y=1}\underbrace{cccccc}_{z=2k+2} = a^k b^1 c^{2k+2}$

Consider the word $a^k b^1 c^{2k+2}$ which is larger than k:

In this example the 1st rule holds forall iterations of y
For $y = b$, $|y| = 1 > 0$ → the 2nd rule holds
For $y = b$ and $x = a^k$, $|xy| = k+1 > k$ ∴ the 3rd rule fails. Our assumption that the language is regular is wrong.