Assessment: Individual Coursework 1

Due date: Friday 9th November 2018, 11.55pm

Question Sheet

Total Marks: 100

Assignment Rules:

1. *Submission and late submission:* The solutions **must be submitted in pdf** only via QOL. You may wish to complete your coursework electronically using Word (utilising Microsoft Equation Editor where appropriate), LaTex, or other technology which facilitates mathematical typesetting. If you complete your coursework on paper you must create a digital copy of your work using a scanner. **Photographs of paper based coursework will not be accepted.** Late submissions will be deducted 5% marks per day of delay. No submission will be accepted more than 7 days later than deadline.
2. *No plagiarism allowed:* This is an individual assignment. No plagiarism is permitted: you should not copy your solutions from each other or any resource. If you need to refer to online sources/books (e.g. for some new definition), you must refer to them appropriately. If plagiarism is detected you may lose marks and/or face other action.
3. This is an open book and open resource assignment. You are allowed to access books and online resources. However, you must attribute sources (see point 4) and the solutions must be in your own words.
4. *Attribution:* If at all you need to cite any sources/books (*standard definitions do not require a citation*), have a separate **references** section at the end. All the references should be present using a single standardised reference style (e.g. IEEE, APA, Harvard etc.).

1.

a. Let A, B and C be sets. Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(4 marks)

b. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be functions such that

$$f(x) = 5x - 2 \text{ and } g(x) = x^2 - 4.$$

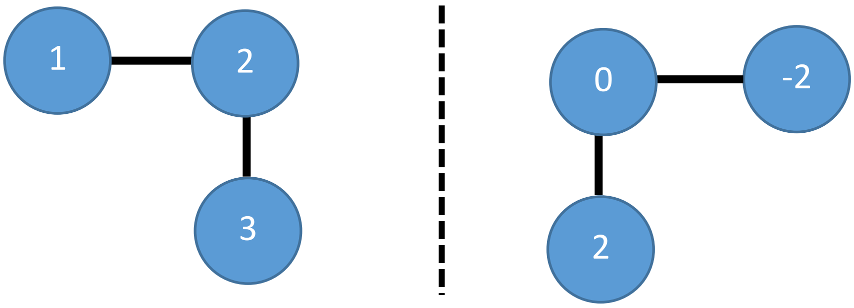
Give a definition for the composite function $f \circ g$.

(4 marks)

c. A graph $G = (V, E)$ is said to be isomorphic to a graph $H = (V', E')$ if there exists a bijective function, f, from the vertices of G to the vertices of H such that for two adjacent vertices in G, u and v, the corresponding vertices in H, $f(u)$ and $f(v)$, are adjacent.

Given the two graphs

$$G = (\{1,2,3\}, \{(1,2), (2,3)\}) \text{ and } H = (\{-2,0,2\}, \{(-2,0), (0,2)\},$$

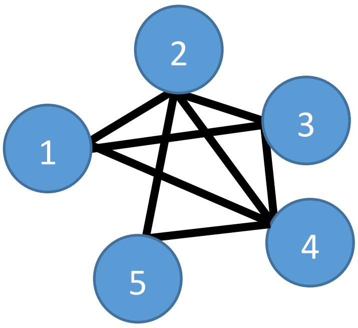


show that G is isomorphic to H.

(12 marks)

2.

a. Given the graph G, which can be illustrated as



Formally define the graph G' which is the largest possible subgraph of G where every vertex in G' has degree 3.

(5 marks)

b. Using appropriate propositional variables, construct a well-formed formula (WFF) for the expression:
"If Paul and Jay go to the shops then Alex will not go to the shops."
Make sure to clearly state how your propositional variables relate to the expression.

(7 marks)

c. Using the same propositional variables defined in (2b), construct a WFF for the expression:
"Either Alex or Paul go to the shops with Jay, which is the same as saying, Alex and Jay go to the shops and not Paul, or, Paul and Jay go to the shops and not Alex."

(8 marks)

3. Let A be an array of 15 integers and B be an array of 10 integers. The arrays A and B are indexed from 1 to 15 and 1 to 10 respectively. Construct predicates to assert that

a. The third element of A is larger than all elements of B.

(3 marks)

b. The elements of B form a consecutive sequence within A.

(3 marks)

c. If x is an element of A then $-x$ is an element of B.

(7 marks)

d. The value 7 occurs only once in A.

(7 marks)

4.

a. Let $x, y \in \mathbb{Z}$. Prove by construction that the sum of $x + y$ is even when

i. x and y are both even,

(4 marks)

ii. x and y are both odd.

(4 marks)

b. Prove by construction, that for any $n \in \mathbb{Z}$ that $(n^2 \bmod 2) = 0$ or 1.

(12 marks)

5.

a. Give a recursive definition for the following sequence which is defined over the natural numbers:

$$-2, 6, -18, 54, -162, 486, \ldots$$

(3 marks)

b. Use the pseudo code below to formulate an equivalent recursive definition

```
Function f(n)
   if n = 0:
       return 1;
   else:
        return f(n-1)+3n+2;
   end
end
```

(3 marks)

c. Using your recursive definition from (5b) compute $f(3)$.

(1 mark)

d. Find a closed-form solution to the function

$$f(n) = \begin{cases} 0 & if\ n = 0 \\ f(n-1) + 2n & otherwise \end{cases}$$

(5 marks)

e. Prove, by induction, that your closed-form solution in (5d) is correct.

(8 marks)