Optimal Control

Course Project #2

Optimal Control of an Aircraft

In this project, you have to develop an optimal control strategy for a simplified aircraft model.

The nonlinear dynamics for a simplified aircraft model is

$$
\begin{aligned}
\dot{x} &= V \cos\gamma \\
\dot{z} &= -V \sin\gamma \\
\dot{\theta} &= q \\
m\dot{V} &= -D(V,\alpha) - mg\sin\gamma + T\cos\alpha \\
mV\dot{\gamma} &= L(V,\alpha) - mg\cos\gamma + T\sin\alpha \\
J\dot{q} &= M
\end{aligned}
$$

where (x, z) represents the position of the aircraft center of mass on the longitudinal plane, α the angle of attack, γ the fligth path angle (between the velocity vector and the horizontal plane), $\theta = \gamma + \alpha$ the pitch angle. Moreover, $D(V,\alpha)$ e $L(V,\alpha)$ are respectively the lift and drag aerodynamical forces, defined as

$$
\begin{aligned}
D(V,\alpha) &= \frac{1}{2}\rho V^2 S (C_{d0} + C_{d\alpha}\alpha^2) \\
L(V,\alpha) &= \frac{1}{2}\rho V^2 S C_{l\alpha}\alpha
\end{aligned}
$$

with $C_{d0} = 0.1716, C_{d\alpha} = 2.395, C_{l\alpha} = 3.256$. The physical parameters are $m = 12$ kg, $g = 9.81\text{m/s}^2$, $S = 0.61\text{m}^2$, $\rho = 1.2\text{kg/m}^3$, $J = 0.24\text{kg}\cdot\text{m}^2$. Finally, T and M are the control inputs, i.e. the thrust force and the pitch moment, respectively.

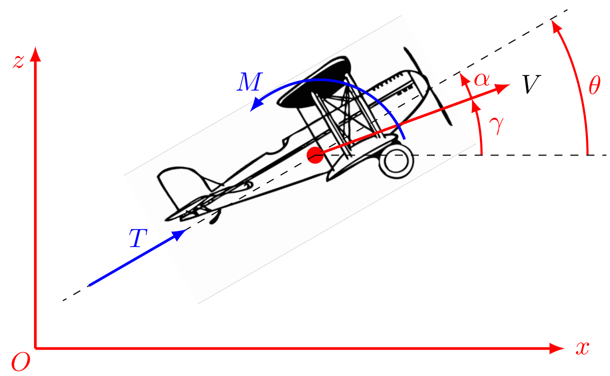

Figure 1: Simplified aircraft model.

Task 0 – Problem setup

Discretize the dynamics and write the `dynamics` function.

Task 1 – Trajectory exploration: gain altitude

Choose two equilibria from an altitude level to another and define a step between these two configurations. Compute the optimal transition to move from one equilibrium to another exploiting the Newton's algorithm for optimal control.

Task 2 – Trajectory optimization: acrobatic flight

Define a smooth reference trajectory (e.g., a loop trajectory) to perform a step ahead. Exploit the Newton's algorithm for optimal control to compute the optimal trajectory.

Task 3 – Trajectory tracking

Linearizing the system dynamics about the (optimal) trajectory $(\mathbf{x}^*, \mathbf{u}^*)$ computed in Task 2, exploit the LQR algorithm to define the optimal feedback controller to track this reference trajectory. In particular, you need to solve the LQ Problem

$$\begin{aligned}\min_{\substack{\Delta x_1,\dots,\Delta x_T\\ \Delta u_0,\dots,\Delta u_{T-1}}} \quad & \sum_{t=0}^{T-1} \Delta x_t^\top Q_t \Delta x_t + \Delta u_t^\top R_t \Delta u_t + \Delta x_T^\top Q_T \Delta x_T \\ \text{subj.to} \quad & \Delta x_{t+1} = A_t^* \Delta x_t + B_t^* \Delta u_t \qquad t = 0, \dots, T-1 \\ & x_0 = 0\end{aligned}$$

where A_t^*, B_t^* represent the linearization of the (nonlinear) system about the optimal trajectory. The cost matrices of the regulator are a degree-of-freedom you have.

Task 4 – Animation

Produce a simple animation of the system executing Task 3. You can use PYTHON, MATLAB or whatever software you like.

Task 5 – Bonus: physical limits

Try to introduce some physical constraints over states and inputs in the context of the smooth trajectory generation presented in Task 2. Use the Newton's method for optimal control and embed these constraints using a barrier function approach.

Hints

- As initial condition for the algorithm define a proper equilibrium trajectory. Try also some quasi-static trajectories for "more complex" initial trajectories. NOTE: to calculate the equilibria of the system you can use, if needed, *SQP*/nonlinear optimization algorithms.
- In the definition of the desired curve, try to calculate the desired trajectories using a simplified model, e.g., a simplified kinematic model.
- Consider the physical limits for states and inputs and study which desired curves exceed said limits.

Notes

1. Any other information and material necessary for the project development will be given during project "meetings".
2. The project report must be written in LaTeX and follow the main structure of the attached template.
3. Any email for project support must have the subject:

   "[OPTCON2022]-Group X: rest of the subject".
4. All the developed code must be handled in a `zip` folder.