A Survey on Temporal Logics

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Abstract

This paper surveys main and recent studies on temporal logics in a broad sense by presenting various logic systems, dealing with various time structures, and discussing important features, such as decidability (or undecidability) results, expressiveness and proof systems.

1 Introduction

Temporal logics are formal frameworks which describe statements whose truth values change over time. Despite the fact that classical logics do not include time element, temporal logics characterize state changes which depend on time. This makes temporal logics a richer notation than classical logics.

Temporal logics can be considered as extensions of classical propositional and first-order logic. In fact, propositional temporal logics are an extension of propositional logic with temporal operators. Similarly, first-order temporal logics are extension of first-order logic with temporal modalities. Temporal logics are also special type of modal logics, where statements are evaluated on 'worlds' which represent time instants.

Although various aspects of time and logic have been studied, an up-to date comprehensive analysis of logic of time does not exist in the literature. Some surveys (such as [106, 31, 105, 68]) can be found in the literature but these mainly concentrate on specific formal systems over specific structures of time; therefore, they do not contain a broad analysis. The aim of this paper is to outline main and recent developments in the field in a broad sense by presenting various formal systems dealing with various time structures, and discussing important features, such as (un)decidability results, expressiveness and axiomatization systems.

Temporal formalisms we will analyse include propositional/first-order linear temporal logics, branching temporal logics, partial-order temporal logics and interval temporal logics. We will summarize important results on decidability, axiomatizability, expressiveness, model checking, etc. for each logic analysed. We also provide a comparison of features of the temporal logics discussed.

Note that in some instances we think it is more convenient to refer to the original text for clarification purposes. In the following, we will use quotation marks to use the text from the original resources.

2 Preliminaries

We can classify temporal logics based on several criteria. The common dimensions are 'propositional versus first-order', 'point-based versus interval-based', 'linear versus branching', 'discrete versus continuous', etc [46, 141, 14]. Below we discuss the most important criteria to classify temporal logics.

Point versus interval structures: There are two structure types to model time in a temporal logic: points (instants) and intervals. A point structure \mathcal{T} can be represented as $\langle T, < \rangle$, where T is a nonempty time points, and < is a 'precedence' relation on T. Different temporal relationships can be described using different modal operators. Some logics include modal operators which can express quantification over time. However, a relationship between intervals is difficult to express using a point-based temporal logic [52].

Interval temporal logics are expressive, since these logics can express a relationship between two events, which are represented by intervals. Also, interval logics [128, 129, 103, 85, 99, 119, 74] have a simpler and neater syntax to define a relationship between intervals, which provides a higher level abstraction than a point-based logic when modeling a system. This makes interval logic formulas much simpler and more comprehensive than point-based logic formulas.

Some of the known interval operators are *meets*, *before*, *during* [4], which denote the ordering of intervals; *chop* modality [140], which denotes combining two intervals; and *duration*, which denotes a length of an interval [31].

Interval structures can be considered in two ways: (i) intervals are 'primitive' objects (ii) intervals are composed from points. [139, 101, 142] consider intervals as primitive objects of time. [139] defines a 'period structure' as the tuple $\langle \mathcal{I}, \subseteq, \prec \rangle$, where \mathcal{I} is a non-empty set of intervals, \subseteq is a sub-interval relation, and \prec is a precedence relation. One particular problem of this approach is that theoretical analyses are usually very difficult. Also, although it is very easy to define properties linearity, density, discreteness, unboundedness in a point-based logic, it is very difficult to define these properties in an interval logic where intervals are primitive objects.

[68, 74, 140] consider intervals as set of points, where the time flow is assumed as "a strict partial ordering of time points". Namely, an interval structure is defined as $\langle \mathcal{T}, \mathcal{I}(\mathcal{T}) \rangle$, where $\mathcal{T} = \langle T, < \rangle$ is a strict partial ordering and $\mathcal{I}(\mathcal{T})$ is a set of intervals. The properties mentioned above can be defined in an interval logic where intervals are composed of time instants.

We conclude this section with the historical development of interval-based temporal logics. The concept of time intervals was first studied by Walker [143]. Walker considered a non-empty set of intervals, which is partially orderd. However, his work does not cover aspects of temporal logic in a general sense. In [75] philosophical aspects of an interval ontology was analysed. In [79] an interval tense logic was introduced. [43, 80, 125, 28, 138, 63, 131] studied interval logics within the natural language domain. It was argued that interval-based semantics are more convenient for human language and reasoning, and interval-based approach is more suitable than point-based approach for temporal constructions of natural language. [4, 5, 7, 6] studied event relations and interval ordering. The authors introduced so-called Allen's thirteen interval relations and worked on axiomatisation and representation

of interval structures. Some further works on Allen's algebra were carried out by [85, 64]. Recently, [126] investigated the relation between Allen's logic and LTL. Interval based-logics have been also applied to other fields in computer science. [109, 114, 76] worked on process logic, where intervals are used as representation of information. Another important work was the development of interval temporal logic (ITL), and its application to design of hardware components [103, 73]. Since the development of ITL, various variations have been proposed so far. In particular, Duration Calculus [31] is an extension of interval temporal logic with "a calculus to specify and reason about properties of state durations".

Temporal Structure: There are important properties regarding the time flow and temporal domain structure. Some properties are summarized below:

Assume $\langle T, < \rangle$ represents a temporal structure, where T is a nonempty time points, and < is a 'precedence' relation on T. In a temporal logic the structure of time is *linear* if any two points can be compared. Mathematically, a strict partial ordering is called linear if any two distinct points satisfy the condition: $\forall x, y : x < y \lor x = y \lor x > y$. This definition suggests that in linear temporal logics each time point is followed by only one successor point.

Another class is the *branching-time structures*, where the underlying temporal structure is branching-like, and each point may have more than one successor points. The structure of time can be considered as a tree. A *tree* is a set of time points T ordered by a binary relation < which satisfies the following requirements [59]:

- $\langle T, < \rangle$ is irreflexive;
- $\langle T, < \rangle$ is transitive;
- $\forall t, u, v \in T \ u < t \ \text{and} \ v < t \rightarrow u < v, u = v \ \text{or} \ u > v \ \text{(i.e. the past of any point is linear)};$
- $\forall x, y \in T, \exists z \in T \text{ such that } z < x \text{ and } z < y \text{ (i.e. } \langle T, < \rangle \text{ is connected)}.$

One important characteristics of branching logics is that the syntax of these logics include path quantification which allows formulas to be evaluated over paths. However, linear temporal logics are restricted to only one path.

A temporal domain is discrete with respect to the precedence relation < if each non-final point is followed by a successor point. This can be formulated as follows: $\forall x, y \ (x < y \to \exists z \ (x < z \land \neg \exists w (x < w \land w < z)))$ [134]. Majority of temporal logics used for system specification are defined on discrete time, where points represent system states. A state sequence, as a result of a program execution, can be considered as isomorphic to discrete series of positive integers.

A temporal domain is *dense* if, between any two distinct points, there is another point. This can be formally denoted $\forall x, y (x < y \to \exists z (x < z < y))$ [134]. Above we mentioned that flow of discrete time can be represented as positive integers. Similarly, density can be represented as real numbers. It is noteworthy to mention that there is a distinction between *density* and *continuity*: "A model of dense time is isomorphic to a dense series of rational numbers, meaning that there is always a rational number between any two rational numbers; whereas a model of *continuous* time is isomorphic to a continuous series of real numbers" [141].

A temporal domain is bounded above (bounded below) if the temporal domain is bounded in the future (past) time. This can be formulated as follows: $\exists x \neg \exists y (x < y \ (\exists x \neg \exists y (y < x)))$ [134]. Similarly, a temporal domain is unbounded above (unbounded below) if each point has a successor (predecessor) point, which is formally denoted $\forall x \exists y (x < y \ (\forall x \exists y (y < x)))$ [134].

A temporal domain is *Dedekind complete* if all time point sets (non-empty) are bounded above, and they have a least upper bound.

Based on differences in temporal domain properties logics have different characteristics. For example, we can consider a temporal domain which is linear or branched; discrete or dense; finite/infinite in future and/or past, etc. All these choices result in different syntax, semantics, decidability and complexity.

3 Propositional Temporal Logics

An important success in temporal logic study was the introduction of the temporal operators into the classical logic [81]. In [113] Pnueli introduced a very influental Linear Temporal Logic (LTL). LTL can express properties of linear sequences of states. For example, properties such as 'p holds at some state in the sequence' or 'p holds at two consecutive states in the future' can be expressed in LTL. In [132] Sistla and Clarke proved that the satisfiability and model checking problems of LTL are PSPACE-complete. [132] shows that if the syntax is restricted only to \diamondsuit ('sometime') operator, or X ('next') operator, then the complexity of the satisfiability problem reduces to NP-complete. However, when both operators are included in the syntax PSPACE-completeness is preserved.

Recently, a quantitative extension of LTL was introduced in [88], where LTL is extended with counting quantification. [88] shows that satisfiability and model checking problems of this extension are both EXSPACE-complete.

In [62] the logic *Propositional Temporal Logic (PTL)* was introduced (over discrete time models with X ('next') and U ('until') operators. [62] shows that PTL is decidable, and it provides a sound and complete axiomatization. In [132] it was found that the satisfiability problem for PTL is PSPACE-complete. An automata theoretic technique for obtaining satisfiability can be found in [133].

[132] provides a complete axiomatic system for PTL. Among the proof systems existing in literature are Hilbert-style proof system [85], a Gentzen-style proof system [135] and a clausal resolution approach [54, 55]. These proof systems are all sound and complete. In [90] PTL was extended with the past operators, and a complete proof system for both future and past operators was presented (A detailed discussion can be found in [135, 89]). In [89] an EXPTIME tableau algorithm is presented for the satisfiability problem of PTL.

In the literature several examples of properties of programs expressible by means of temporal logics can be found [84, 95, 94]. Some important properties are expressed in PTL as follows [135]:

- $p \to \Box q$: q holds at all states after p holds.
- $\Box((\neg q) \lor (\neg p))$: p and q cannot hold at the same time.

- $p \to \Diamond q$: q holds at some time after p holds.
- $\Box \Diamond p \to \Diamond q$: If p repeatedly holds, q holds after some time.
- $\Box p \to \Diamond q$: If p always holds, q holds after some time.

Recently, more results have been presented on PTL. [123] showed that the satisfiability problem for PTL with the strict 'until' operator is PSPACE-complete. [93] extended the 'since-until' logic of real-line with the operators "sometime within n time units", and they showed that the new logic is PSPACE-complete. [124] showed that satisfiability problem for the logic with 'since-until' operators over real-numbers time is PSPACE-complete.

4 First-Order Temporal Logics

First-order temporal logics (FOTL) are extensions of propositional temporal logics. In addition to all propositional features these logics also allow arbitrary data structures and quantifiers. FOTLs have been extensively used in many areas including specification and verification of reactive systems, and analysis of hardware components. First-order logics provide an expressive formal framework for formalising the semantics of executable modal logics. They allow to obtain more robust techniques for reasoning about knowledge [51, 77]. FOTLs have also found applications in information systems. For example, temporal database query languages are mainly based on first-order like languages [36].

Although first-order temporal logics have proved to be useful in various areas, they suffer from high computational complexity because these logics are very expressive. Indeed, most of FOTLs are not even recursively enumerable [1, 10, 59]. Some axiomatisations of first-order temporal logics were studied in [120]. In some cases, fragments of first-order temporal logics with lower computational complexity are defined through restricted extensions to propositional temporal logics [1, 100, 35, 112].

One important logic is *monodic* fragment of first-order temporal logic defined in [77], which showed that it is an expressive logic with a feasible computational behaviour. In monodic formulas, one free variable is allowed at most. In [145] a finite axiomatic system is presented for the monodic fragment. In [11, 69] monodic guarded decidable fragments are introduced by restricting the quantification.

The first-order temporal logic is represented by QTL, which includes the following syntax (which does not comprise equality and function symbols): predicate symbols, variables, constants, boolean connectives, universal quantifier and temporal operators S ('since') and U ('until'). Let \mathbb{T} be the underlying time structures assumed for QTL constitutes strict linear orders. Then, $QTL(\mathbb{T})$ denotes the first-order temporal logic of \mathbb{T} , and $QTL_{fin}(\mathbb{T})$ denotes the logic of \mathbb{T} with finite domains.

4.1 Undecidable Fragments of QTL

In the literature, it has been known that both the monadic and two-variable fragments of first-order logic are decidable [19]. However, the computational complexities of their temporal counterparts are different. Let QTL^2 denote the two - variable fragment of QTL

(where every formula contains at most two variables), and QTL^{mo} denote the monadic fragment (not monodic) of QTL (where formulas contain only unary predicates). Assume \mathbb{T} be either $\{\langle \mathbb{N}, < \rangle\}$ or $\{\langle \mathbb{Z}, < \rangle\}$. Then, $QTL^2 \cap QTL^{mo} \cap QTL(\mathbb{T})$ and $QTL^2 \cap QTL^{mo} \cap QTL_{fin}(F)$ are not recursively enumerable [77].

4.2 Decidable Fragments of QTL

In the undecidable fragments given above, formulas can have the following quantification types: temporal modalities, path quantifiers and domain quantification. This causes a problem that these fragments of QTL are undecidable. It is known that the three-variable fragment of first-order logic is undecidable [19].

In order to preserve decidability, corresponding fragment of QTL, which is QTL_1 , contains all QTL-formulas φ , whose any subformula of the form $\varphi_1 U \varphi_2$ and $\varphi_1 S \varphi_2$ has at most one free variable. These formulas are *monodic* (not monadic) formulas. The monodic fragments of $QTL(\langle \mathbb{N}, < \rangle)$ and $QTL(\langle \mathbb{Z}, < \rangle)$ are recursively enumerable [78].

Let \mathbb{T}' be $\{\langle \mathbb{R}, < \rangle\}$ and \mathbb{T} be the following classes of time structures: " $\{\langle \mathbb{N}, < \rangle\}$, $\{\langle \mathbb{Z}, < \rangle\}$, $\{\langle \mathbb{Q}, < \rangle\}$, the class of all finite strict linear orders, and any first-order-definable class of strict linear orders" [78]. [77] proves that various fragments are decidable, such as $QTL(\mathbb{T}) \cap QTL_1$, $QTL(\mathbb{T}) \cap QTL_1^2$, $QTL(\mathbb{T}) \cap QTL_1^{mo}$, $QTL_{fin}(\mathbb{T}') \cap QTL_1^1$, $QTL_{fin}(\mathbb{T}') \cap QTL_1^2$ and $QTL_{fin}(\mathbb{T}') \cap QTL_1^{mo}$. They also provide some guarded fragment of first-order language (For a detailed discussion, see [77]).

In [60] it is shown that $QTL(\langle \mathbb{N}, < \rangle) \cap QTL_1$ is EXPSPACE-hard. It is also shown that "the satisfiability problem for QTL_1^{mo} -formulas in models based on $\langle \mathbb{N}, < \rangle$ is EXPSPACE-complete" [78].

Here we assumed that QTL and its fragments do not include equality and function symbols. It can be shown that undecidability is a major problem with the logic extended with function symbols [145]. For instance, "the set of one-variable formulas with one function symbol that are valid in models based on $\langle \mathbb{N}, < \rangle$ is not recursively enumerable" [78]. Moreover, "the set of monodic QTL formulas with equality that are valid in all temporal models based on $\langle \mathbb{N}, < \rangle$ is not recursively enumerable" [78]. [40] shows that the problem persists even for a simpler fragment. Namely, the authors prove that a fragment with "monodic monadic two-variable formulas" is not recursively enumerable. In [145] a finite Hilbert-style axiomatisation of monodic fragment of first-order temporal logic was presented. It was also proved that "the monodic fragment with equality is not recursively axiomatisable" [145].

Recent research results have showed that relatively expressive subsets of first-order temporal logic could be found. In [144, 77, 145] suggest that expressive power of monodic first-order temporal logic can be extended further. For example, temporal operators can be applied to formulas with more than one free variable [112]. The decidability results can be also be extended to temporalties description logics. Recently, tableau-based methods are presented for the satisfiability checking of temporal description logics [92]. Tableau-based methods can also be devised for the satisfiability checking of decidable monodic temporal logics. This can be done by extending the tableau methods for the propositional temporal logics to the first-order case [92]. An alternative approach is to use the resolution method. [41] introduces some resolution systems for monodic first-order temporal logics.

5 Branching Time Temporal Logics

A temporal logic system is called *branching time logic* if the underlying semantics of the structure of time is branching. The underlying structure of time in branching time logics is a tree-like structure. That is, every time instant can be followed by several immediate successor time instants. In branching time logics, there are two kinds of formulas: *state* formulas and *path* formulas. State formulas are interpreted over states and path formulas, containing all state formulas, are interpreted over paths.

Temporal logics with underlying branching time have found many applications in artificial intelligence study. In particular, they are very useful in planning systems, where agents formulate different plans and action strategies according to different future world states [98, 118].

Since very efficient model checking algorithms have been introduced for branching time logics, these logics have been extensively used to verify finite state systems. On the other hand, in linear time logics deductive proof systems are introduced for the verification of infinite state systems [106].

An initial work about branching time logics was done by [2]. Later, the unified branching time system (UB) was introduced in [15]. A simple branching time logic, CTL, was introduced in [37]. Thereafter, CTL* was introduced in [48]. CTL* is an extension over CTL by adding the properties of linear time temporal logic. CTL*[P], an extension over CTL*, was introduced in [86]. UB, CTL and CTL* include only future time temporal connectives. Whereas, CTL*[P] contains both past and future time temporal connectives.

5.1 Computational Tree Logic (CTL)

CTL is a point-based branching time logic, which is an extension of the logic UB by adding the operator U (until). Time is included implicitly within the temporal operators. CTL allows quantification over paths.

Some CTL formulas are given below [39]:

- $\exists \Diamond (p \land \neg q)$: There exists a state where p holds but q does not hold.
- $\forall \Box (p \to \forall \Diamond q)$: Whenever p holds, eventually q holds.
- $\forall \Box (\exists \Diamond p)$: At all paths p holds after some time.

CTL is a decidable logic [49]. It can be shown that CTL has the finite model property. That is, a satisfiable formula is satisfied in a finite model of size which is bounded by "some function of the length of the formula" [45]. [47] presents a tableau method for checking the satisfiability of CTL formulas. The complexity of this procedure is EXPTIME. Model checking problem of CTL is easier than the satisfiability problem. Indeed, model checking in CTL is linear in the size of the model and the formula [38]. This shows that model checking in CTL can be achieved very efficiently. In [110] a sound and complete axiomatic system is provided for CTL.

Recently, a quantitative extension of CTL was introduced in [87], where CTL is extended with counting quantification. [87] also provides an analysis of the expressiveness and the

complexity of the model-checking problem for a range of quantitative extensions. Depending on the extension, different complexity results are obtained.

5.2 Full Computational Tree Logic (CTL*)

The logic CTL* was introduced in [48]. CTL* is an extension over CTL by adding the properties of linear time temporal logic. That is, CTL* is a logic which unifies CTL and LTL. CTL* is a more expressive logic than CTL, which makes theoretical analyses more difficult. Although model checking for CTL is linear, CTL* model checking is PSPACE-complete [38]. Also, solving the satisfiability problem for CTL* is more difficult than solving the CTL satisfiability. [50] provides an algorithm for the satisfiability problem of CTL*, which has 2-EXPTIME complexity in the length of the formula. A sound and complete axiomatisation for CTL* has recently been defined by Reynolds in [121].

5.3 Full Computational Tree Logic with Past (CTL*[P])

In the logics CTL and CTL* we assumed that temporal operators are restricted to future time. [86] introduces a logic CTL*[P], which also includes past time operators. As in the linear case, addition of past operators to the language does not increase expressive power if we have a finite past; but this allows to express useful properties. CTL*[P] is a decidable logic, which can be easily observed from the decidability of CTL*. Until recently the axiomatizability of CTL*[P] has been a long-lasting open question. [122] gives a sound and complete axiomatisation system for CTL*[P].

5.4 Expressiveness of Branching Temporal Logics

One of the main reasons of using branching time logics is that the model-checking procedure is very efficient. The model checking task is simply to check whether a given a model satisfies a specification. CTL language allows to express useful system properties. Although model checking is expensive in linear time logics (for example, it is exponential for LTL), model checking complexity of CTL is very efficient, which is linear in the size of model and formula. However, model checking complexity of CTL* is higher than than of CTL, which is PSPACE-complete. The high complexity results from the recursive checking of all paths [61].

The branching logic systems can also be used to specify properties of concurrent programs; below are some example properties expressed in UB [110]:

- $\forall \Box p$: safety property: p is true at all states of each path.
- $\forall \Diamond p$: liveness property: p is true at some state of each path.
- $\exists \Diamond p$: possibility property: p is true at some state of some path.

Since CTL is an extension of UB, CTL subsumes the language of UB, and it can therefore express all properties which are specified in UB. CTL can express more properties such as relative ordering of events using the modality U [110]. Both UB and CTL cannot express

fairness constraints. CTL* has a more rich syntax than UB and CTL. This logic can be used in specification of more complex properties, which cannot be expressed either UB and CTL. Some examples are given below [110]:

• $\Box \Diamond p \rightarrow \Box \Diamond q$: fairness property

• $\Diamond \Box p \to \Box \Diamond q$: justice property

• $\exists ((pUq) \lor \Box p)$: weak until property

As seen above, different combinations of linear time operators result in more expressive power. This rich syntax enables to express more complex properties, such as fairness. The branching logics mentioned in this section can be made more expressive, while still keeping all their formulas as state formulas, by allowing classical operators between the temporal and path operators. If we add past operators, expressiveness does not increase; but the resulting logic allows more convenient notation to express some useful properties. Due to complexity and expressiveness considerations some other logics have been defined, such as CTL⁺ [49], ECTL [47], ECTL⁺ [47].

6 Partial-Order Temporal Logics

In concurrent systems computations are generally viewed as partially ordered sets. Since linear temporal logics are more suitable for totally ordered sets, it is difficult to apply them to concurrent and distributed systems [115]. Partial-order temporal logics are suitable to express partial orderings representing the behaviour of concurrent systems [111]. Partial order structures are similar to branching structures except that each time instant can be preceded by several previous time instants.

Initial attempts to define a logic based on partial orders were done in [111], where the logic POTL is introduced. POTL can express partially ordered computations without making any translation from totally orders sets. POTL can be considered as extension of the logic UB with past modalities. By adding 'backward' operators POTL allows quantification over backward paths, and it can express statements requiring states with several successors and predecessors. However, POTL framework is different than that of UB in the sense that a UB structure represents the "set of possible computations of a program" (where each computation is a totally ordered set); but a POTL structure represents a single computation (which is a partially ordered set) [111]. A POTL formula $q \to \forall \overleftarrow{\Diamond} p$ expresses that for all runs and backward fullpaths ending at states where q holds, there is a state where p holds [110].

[111] shows that POTL does not have the finite model property due to the addition of past operators; but in spite of this negative result the authors show that the logic has an exponential decision procedure, and a complete axiomatisation system.

[83] introduces the logic POTL[U,S], which extends POTL with 'until' and 'since' operators. Similar to the case of POTL, POTL[U,S] can be seen as an extension of CTL with past modalities. POTL[U,S] can express all properties POTL can express as well as the properties concerning the "relative order of events in the future and past" [110].

Since POTL[U,S] is an extension of POTL, it does not have the finite model property. However, [83] presents an exponential decision procedure. A sound and complete axiomatisation system for POTL[U,S] is also given in [83]. POTL[U,S] has a high model checking complexity, because formulas contain past modalities, and they are "interpreted over models corresponding to runs of concurrent systems" [110]. Indeed, [83] shows that the complexity is exponential in the model size and doubly exponential in the formulas size.

In literature, there are more recent results for logics with partial-order semantics. [17] presents a new temporal logic, where linear and partial order semantics are combined. Namely, a computation is modeled as a linear sequence of states, which are associated with "past partial-order history". The authors also give a sound and partially complete proof system for the logic. In [65] partial order reductions are studied for the logics CTL and CTL* based on the partial order techniques to reduce the state space. [3] introduces a new partial-order temporal logic based on different semantical model to increase the expressiveness. In [91] partial-order reduction techniques are applied to linear and branching time temporal logics for knowledge (without the next operator) to reduce the model size before applying model checking procedure.

7 Interval Temporal Logics

Interval temporal logics are temporal logics which allow reason about periods of time. Since representation of logical reasoning about periods are more expressive than reasoning about points, the interval-based scheme provides us with a richer representation formalism than the point-based approach.

In this section, we present a selection of well-known interval temporal logics. In literature, many similar logics can be found; but most of these logics are generalisations or specialisations of the ones we will discuss below.

7.1 Propositional Interval Temporal Logics

In this section we will present the well-known propositional interval logics, which involve unary or binary modal operators, and whose semantic structures are over partial orderings with linear interval property, i.e. "orderings in which every interval is linear" [68].

The syntax of propositional interval temporal logics is constructed from the following: the set of propositional variables, the truth values, the classical operators (boolean operators, negation, etc.), and a set of temporal operators defined for each logic.

7.1.1 The Logic HS

The logic HS [74] is a relatively expressive propositional interval temporal logic. All modal operators of HS are unary. The logic HS has enough expressive power to distinguish different temporal structures, such as of discrete, continuous, bound, linear or complete time structures. These are formally shown as follows [74]:

• $length0 \equiv [B] \perp$

- $length1 \equiv \langle B \rangle \top \wedge [B] length0$ (length1 holds at intervals with no proper subintervals.)
- $dense \equiv \neg length1$
- $discrete \equiv length0 \lor length1 \lor (\langle B \rangle length1 \land \langle E \rangle length1)$

where $\langle B \rangle \phi$ is true iff ϕ holds at some interval that begins with the current interval and ends before it ends; $\langle E \rangle \phi$ is true iff ϕ holds at some interval that begins after the current interval starts and ends when it ends; and $[B]\phi$ is defined as $\neg \langle B \rangle \neg \phi$.

HS is a quite expressive logic due to its large modal operator set. However, it is not axiomatisable and is highly undecidable [74]. The following theorems are taken from [74]:

- "The validity problem interpreted over any class of ordered structures with an infinitely ascending sequence is r.e.-hard (Thus, in particular, HS is undecidable for the class of all (non-strict) models, linear models, discrete linear models, dense linear models and unbounded linear models)."
- "The validity problem interpreted over any class of Dedekind complete ordered structures having an infinitely ascending sequence is Π₁-hard (For instance, the validity in any of the orderings of the natural numbers, integers, or reals is not recursively axiomatisable. Undecidability even occurs in the classes of structures with no infinitely ascending sequences)."
- "The validity problem interpreted over any class of Dedekind complete ordered structures having unboundedly ascending sequences is co-r.e.-hard."

Undecidability results given above are based on the observation that HS formulas encode the computation of a Turing machine. In [97] undecidability was proved by means of tiling problem.

In [96] some interesting results for the logic HS were presented. By using a geometrical representation for the modalities a sound and complete proof system for HS was introduced. [96] also proved that HS is a more expressive logic than any other temporal logic based on "linear orderings of time instants".

In [74] a translation machinery that converts an HS formula to its equivalent first-order formula on a corresponding first-order structure was provided. Such a translation is useful to reduce problems to well-known results in first-order logic.

7.1.2 The Logic CDT

The logic CDT was introduced by Venema in [140]. It is one of the most expressive propositional interval logic over linear orderings [68]. CDT includes the binary modal operators C, D and T. These operators subsume all unary modalities of propositional interval logics of Allen's interval relations [21]. CDT can distinguish different classes of temporal structures, such as of discrete, continuous, bound, linear or complete time structures. For example, an interval's being discrete can be specified in CDT as follows:

• $(length1 \ C \ \top) \land (\top \ C \ length1)$.

[140] gives an axiomatic system which is sound and complete for the logic CDT which is interpreted over non-strict linear models. This axiomatic system can be extended for the classes of discrete linear orderings, dense linear orderings, etc. [68]. Since CDT subsumes HS, the satisfiability problem for "CDT is not decidable over almost all classes of linear orderings, including discrete, dense, continuous, etc." [68].

The partial order semantics of CDT has been recently studied in [66], where the logic BCDT⁺ is introduced. BCDT⁺ uses the language of CDT with partial order semantics of linear intervals. To our best knowledge the decidability and axiomatizability of the strict versions of CDT and BCDT⁺ are still open.

7.1.3 The Logic PNL

Propositional Neighbourhood Logic (PNL) is the propositional fragment of First-Order Neighbourhood Logic introduced in [30]. It has been studied on both strict and non-strict linear structures in [67]. The language with non-strict semantics is called PNL^{π +} including the modalities \diamondsuit_r (met by) and \diamondsuit_l (meets), and the model constant π . The modal operators can have either strict or non-strict semantics.

Assume PNL⁺ denotes the non-strict PNL without the modal constant π , and PNL⁻ denotes the strict PNL without the modal constant π . The logic PNL^{π +} subsumes both PNL⁺ and PNL⁻ [21].

Given that formulas are interpreted over strict linear models, PNL⁻ has enough expressive power to distinguish the different classes of linear structures, such as discreteness, continuity, boundness, or completeness. For example, unboundness and density can be specified in PNL⁻ as follows [68]:

- $unbound \equiv \Box_r \phi \to \Diamond_r \phi$
- $dense \equiv (\lozenge_r \lozenge_r \phi \to \lozenge_r \lozenge_r \lozenge_r \phi) \land (\lozenge_r \square_r \phi \to \lozenge_r \lozenge_r \square_r \phi)$

In [67] several sound and complete axiomatic systems were provided for various classes of models. In addition to strict linear models [67] also provides sound and complete axiomatic systems for non-strict linear structures, complete unbounded linear structures, unbounded structures, dense structures, discrete structures, dense unbounded structures and discrete unbounded structures. As for decidability results, [24] shows that the satisfiability problem for PNL $^{\pi+}$, PNL $^{+}$ and PNL $^{-}$ over the integers is NEXPTIME-complete. [24] introduces a sound and complete tableau algorithm, and shows that it is optimal. In [22], the expressive power of PNL $^{\pi+}$, PNL $^{+}$ and PNL $^{-}$ is compared, and it is shown that PNL $^{\pi+}$ is strictly more expressive than PNL $^{+}$ and PNL $^{-}$. [22] proves that "the satisfiability problem for PNL $^{\pi+}$ over the class of all linear orders, as well as over some natural subclasses of it, such as the class of all well-orders and the class of all finite linear orders, can be decided in NEXPTIME by reducing it to the satisfiability problem for the two-variable fragment of first-order logic over the same classes of structures".

An important fragment of the PNL is the Right Propositional Neighbourhood Logic (RPNL) which is based on the right neighbourhood relation between intervals. The language with non-strict semantics is called RPNL^{π +}. The non-strict fragment without the modal constant π is denoted by RPNL⁺, and the strict fragment without the modal constant π is

denoted by RPNL⁻. As for decidability results, in [23] an EXSPACE tableau-based decision procedure is devised for RPNL⁻ interpreted over natural numbers. In [25] another NEXPTIME decision procedure is developed. This method works for all classes of RPNL, which are RPNL^{π +}, RPNL⁺, and RPNL⁻, interpreted over natural numbers. [25] also proves the optimality of the decision procedure.

7.1.4 Subinterval Logics

For many years, the high computational complexity of interval logics (such as HS and CDT) restricted these logics in practical applications and semantic investigation. Recently, the trend has shifted to finding expressive decidable fragments. The most important decidable fragments are PNL and its fragments, logics of neighbourhood [27], sub-interval and super-interval structures [26]. In [26] the logics of subinterval structures over dense linear orders is shown to be decidable. [26] also provides a tableau-based decision procedure, which is shown to be PSPACE-complete. [102] shows that "the satisfiability problem for interval logics of the reflexive sub-interval and super-interval relations interpreted over finite linear orders is PSPACE-complete".

Subinterval logics have also been investigated in natural language discourse. In [116] a sub-interval logic, which is used in capturing temporal prepositions of a natural language, is introduced. In [82] a quantitative extension of this logic is represented. Both logics are decidable, and their satisfiability problems are in NEXPTIME.

7.2 First-Order Interval Temporal Logics

First-order interval temporal logics were originally defined to formally specify and verify hardware components of real-time systems. ITL is the most commonly known first-order interval temporal logic. Numerous extensions of ITL, such as Duration Calculus [33], Neighbourhood Logic [33] etc., have been introduced. Below we will review well-known first-order interval temporal logics.

7.2.1 The Logic ITL

ITL was first introduced in [103] (which was "interpreted over discrete linear orderings with finite time intervals" [68]). The formulas of ITL are constructed from the following: an infinite set of global (independent of time and time intervals) variables, an infinite set of temporal variables, an infinite set of global function symbols, an infinite set of predicate symbols and an infinite set of temporal propositional letters.

Not surprisingly ITL is highly undecidable. A sound and complete axiomatic system is represented in [44]. [44, 104] consider some local variants of ITL, and provide sound and complete proof systems for ITL with locality constraint. [72] provides a complete proof system for ITL extended with projection. [71] studies probabilistic interval temporal logic.

7.2.2 The Logic NL

Although ITL is a very expressive logic, it has a limitation that it does not allow to reason about outside of the current interval. The logic NL, proposed in [33], solves this problem.

Its left neighbourhood modality \Diamond_l and right neighbourhood modality \Diamond_r can allow to look outside of the interval.

NL can express any of the Allen's interval relations; thus, it can represent important properties, such as discreteness, density, boundedness, etc; for example, the chop operator C can be expressed in terms of the modalities \diamondsuit_l and \diamondsuit_r as follows [68]:

•
$$\phi C \psi = \exists x, y \, (\ell = x + y) \land \Diamond_l \Diamond_r \, ((\ell = x) \land \phi \land \Diamond_r \, ((\ell = y) \land \psi))$$

NL is an undecidable logic like ITL. In [13] a sound and complete axiomatic system is given for the logic NL. In [12] up and down modalities, represented by \diamondsuit_u , \diamondsuit_d respectively, were introduced, and two dimensional version of NL was proposed.

7.2.3 The Logic DC

Duration Calculus (DC) [33] is a first-order interval temporal logic with the additional notion of *state*, which is characterised by a $duration^1$. DC is an extension of ITL that temporal variables other than ℓ have a state expression structure. The special interval variable ℓ denotes the interval length.

DC has been used in the specification and verification of various complex systems. As a specification example, we specify the real-time requirement of a gas burner system, which is "the proportion of leak time in an interval is not more than one-twentieth of the interval, if the interval is at least one minute long", which is expressed in DC as follows [33]:

• Req
$$\equiv \ell \geq 60 \Rightarrow 20 \int Leak \leq \ell$$

All axioms and inference rules of ITL can be adopted in DC. However, additional axioms are needed for temporal variables. In [31] an axiomatic system for Duration Calculus is given. The satisfiability problem for both first-order and propositional DC is shown to be undecidable [32].

Several fragments of DC have been investigated so far. In [32] a fragment of propositional DC, called RDC, was introduced. It was shown that RDC has a decidable satisfiability problem when interpreted over \mathbb{N} , \mathbb{Q} and \mathbb{R} . In [117] the satisfiability problems of several extensions of RDC were studied. In [56] an extension of RDC was presented on continuous time "with a restriction on the finite variability such that the number of discontinuous points of any state in any unit interval has a fixed upper bound". In [70] a decidable variant of DC was presented, where negation is removed from the syntax; but an iteration operator is introduced together with some form of inequalities. In [34] another fragment of propositional DC, which can capture Allen's relations [4], was introduced by imposing some syntactic restrictions. By proposing a sound, complete and terminating decision algorithm, it was shown that the satisfiability problem is decidable. In [107] a logic with quantification over states was introduced. It was shown that the satisfiability of formulas is decidable. This decision algorithm was implemented as a tool called DCVALID. In [30] Duration Calculus and first-order neighbourhood logic were combined, and a axiomatic systems for DC and

^{1&}quot;The duration of a state is the length of the time period during which the system remains in the state" [68].

NL were merged. It was proved that "the fragment of DC/NL obtained by restricting the formulas" of state expressions is decidable [68]. An extension with formulas with equality becomes undecidable.

Model checking problem for DC is a challenging task. In general, there has not been a general model checking technique for this logic. To have efficient model checking techniques, it is necessary to consider a fragment of the logic. In [56] some model checking tools were developed for a class of models which are restricted to some possible behaviours of real-time systems. In [146, 42, 136, 137] some techniques were developed to check if a timed automaton satisfies a formula of the type "linear duration invariants". In [127] some algorithms were developed to check the satisfiability over integer models. In [107] a DC validity checker, called DCVALID, to check the satisfiability of formulas which are interpreted over discrete-time. [57] suggested bounded validity checking [18] of "a discrete-time DC without timing constraints by polynomial-sized reduction to propositional SAT solving". In [58] a decidability result and a model-checking algorithm are presented "for a rich subset of DC through reductions to first-order logic over the real-closed field and to multi-priced timed automata (MPTA)".

7.2.4 The Logic IDL

Duration Calculus is a very expressive logic for specifying real-time requirements; but the automata theory for DC models is rather primitive and there are no available tools. By contrast, the state sequences with time has been widely used in real-time system behaviour [8]. The automata theory of timed state sequences have been applied to tools such as Hytec [9], Uppaal [16], Kronos [20] etc.

[108] introduced Interval Duration Logic (IDL), which is defined on timed state sequence models and incorporates formulas with cumulative amount of time. Due to its expressive syntax it can express complex real-time properties, e.g. scheduling and planning constraints. As an example, we give a specification example from a gas burner system. The property 'between two instances of Leak there is at least k seconds' is specified in IDL as follows:

•
$$\Box((\lceil Leak \rceil \cap \lceil \neg Leak \rceil \cap \lceil Leak \rceil^0) \Rightarrow \ell \geq k)$$

IDL is a very expressive logic; but it is undecidable. However, there are some methods which have been proposed for the satisfiability and model checking problems of IDL. [130] applies bounded validity checking technique [18] to IDL "by polynomially reducing this to checking unsatisfiability of lin-sat formulae". [130] also compares various methods for the satisfiability problem "including digitization technique [29], combined with an automata-theoretic analysis [107]; digitization technique [29] followed by pure propositional SAT solving [57]; and (c) lin-sat solving [53]".

[108] presents a decidable subset of IDL, which has a restriction that only *located time* constraints are allowed. The paper shows that the models of this subset can be considered as "timed words accepted by a finite state event-recording integrator automaton", which implies the satisfiability of the subset. It is also shown that the defined subset and event-recording automata have the same expressive power, which makes this logic an important decidable subset in the domain of DC.

8 Conclusion

In this survey paper we have outlined recent important developments on propositional/first-order linear temporal logics, branching temporal logics, partial-order temporal logics and interval temporal logics by presenting important features, such as (un)decidability results, expressiveness and axiomatization systems. For a comparison of features of the temporal logics we discussed see Table 1. Note that we use the following abbreviations: No^* : Undecidable in general, but decidable for some fragments or specific cases; No^{**} : No deduction system in general, but available for some fragments or specific cases; No^{***} : No model checking algorithm in general, but available for some fragments or specific cases; Yes^* : Decidable for some time domains; Yes^{***} : Available for some time domains.

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Table 1: A comparison of features of temporal logics.

Logic	Logic Order	Fund. Entity	Temp. Struc.	Metric for Time	Decidability	Deductive Sys.	Model Checking
LTL	Propositional	Point	Linear	No	Yes	Yes	Yes
PTL	Propositional	Point	Linear	No	Yes	Yes	Yes
QTL	First-order	Point	Linear	No	No*	No**	?
CTL	Propositional	Point	Branching	No	Yes	Yes	Yes
CTL*	Propositional	Point	Branching	No	Yes	Yes	Yes
CTL*[P]	Propositional	Point	Branching	No	Yes	Yes	Yes
POTL	Propositional	Point	Partial	No	Yes	Yes	?
HS	Propositional	Interval	Linear	No	No	No	No
CDT	Propositional	Interval	Linear	No	No	Yes	No
PNL	Propositional	Interval	Linear	No	Yes	Yes	No
ITL	First-order	Interval	Linear	No	No	Yes	No
NL	First-order	Interval	Linear	Yes	No*	Yes	No
DC	First-order	Interval	Linear	Yes	No*	Yes	No***
IDL	First-order	Interval	Linear	Yes	No*	No	No***

Acknowledgements.

This work was partially supported by EPSRC research project EP/F033567.

References

- [1] M. Abadi. The power of temporal proofs. *Theoretical Computer Science*, 65(1):35–83, 1989.
- [2] K. Abrahamson. Decidability and Expressiveness of Logics of Programs. PhD thesis, University of Washington, 1980.
- [3] Adrianna Alexander and Wolfgang Reisig. Compositional temporal logic based on partial order. In *Proceedings of the 11th International Symposium on Temporal Representation and Reasoning*, TIME '04, pages 125–132. IEEE Computer Society, 2004.
- [4] J. F. Allen. Maintaining knowledge about temporal intervals. *Communications of the ACM*, 26:832–843, 1983.
- [5] J. F. Allen. Towards a general theory of action and time. *Artificial Intelligence*, 23:123–154, 1984.
- [6] J. F. Allen and G. Ferguson. Actions and events in interval temporal logic. *Journal of Logic and Computation*, 4(5):531–57, 1994.
- [7] J. F. Allen and J. P. Hayes. Moments and points in an interval-based temporal logic. In *Computational Intelligence*, pages 225–238. Blackwell Publishers, 1989.
- [8] R. Alur and D.L. Dill. Automata-theoretic verification of real-time systems. Formal Methods for Real-Time Computing, pages 55–82, 1996.
- [9] R. Alur, T. A. Henzinger, and P. H. Ho. Automatic symbolic verification of embedded systems. *IEEE Transactions on Software Engineering*, 22(3):181–201, 1996.
- [10] H. Andreka, I. Nemeti, and I. Sain. Mathematical foundations of computer science. Lecture Notes in Computer Science, pages 208–218, 1979.
- [11] H. Andreka, I. Nemeti, and J. van Benthem. Modal languages and bounded fragments of predicate logic. *Journal of Philosophical Logic*, pages 217–274, 1998.
- [12] R. Barua and Z. Chaochen. Neighbourhood logics. Research Report 120, UNU/IIST, 1997.
- [13] R. Barua, S. Roy, and Z. Chaochen. Completeness of neighbourhood logic. *Journal of Logic and Computation*, 10(2):271–295, 2000.
- [14] P. Bellini, R. Mattolini, and P. Nesi. Temporal logics for real-time system specification. *ACM Computing Surveys*, 32(1), 2000.
- [15] M. Ben-Ari, Z. Manna, and A. Pnueli. The temporal logic of branching time. In Proceedings of the 8th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, pages 164–176. ACM, 1981.

- [16] J. Bengtsson, K. Larsen, F. Larsson, P. Pettersson, and W. Yi. Uppaal a tool suite for automatic verification of real-time systems. In *Proceedings of the DIMACS/SYCON Workshop on Hybrid systems III: Verification and Control*, pages 232–243. Springer-Verlag, 1996.
- [17] Girish Bhat and Doron Peled. Adding partial orders to linear temporal logic. In Antoni Mazurkiewicz and Jozef Winkowski, editors, CONCUR '97: Concurrency Theory, volume 1243 of Lecture Notes in Computer Science, pages 119–134. Springer Berlin / Heidelberg, 1997.
- [18] A. Biere, A. Cimatti, E. M. Clarke, and Y. Zhu. Symbolic model checking without bdds. In Proceedings of the 5th International Conference on Tools and Algorithms for Construction and Analysis of Systems, pages 193–207. Springer-Verlag, 1999.
- [19] E. Börger, E. Grädel, and Y. Gurevich. The Classical Decision Problem. Springer, 1997.
- [20] M. Bozga, C. Daws, O. Maler, A. Olivero, S. Tripakis, and S. Yovine. Kronos: A model-checking tool for real-time systems. In CAV '98: Proceedings of the 10th International Conference on Computer Aided Verification, pages 546–550. Springer-Verlag, 1998.
- [21] D. Bresolin. *Proof Methods for Interval Temporal Logics*. PhD thesis, University of Udine, 2007.
- [22] D. Bresolin, V. Goranko, A. Montanari, and G. Sciavicco. On decidability and expressiveness of propositional interval neighbourhood logics. In *Proceedings of the International Symposium on Logical Foundations of Computer Science*, pages 84–99. LNCS, 2007.
- [23] D. Bresolin and A. Montanari. A tableau-based decision procedure for right propositional neighbourhood logic. In *Proceedings of TABLEAUX 2005*, pages 63–77. LNAI, 2005.
- [24] D. Bresolin, A. Montanari, and P. Sala. An optimal tableau-based decision algorithm for propositional neighbourhood logic. In *Proceedings of STACS 2007: 24th Interna*tional Symposium on Theoretical Aspects of Computer Science, pages 549–560. LNCS, 2007.
- [25] D. Bresolin, A. Montanari, and G. Sciavicco. An optimal tableau-based decision procedure for right propositional neighbourhood logic. *Journal of Automated Reasoning*, 38:173–199, 2007.
- [26] Davide Bresolin, Valentin Goranko, Angelo Montanari, and Pietro Sala. Tableaux for logics of subinterval structures over dense orderings. J. Log. Comput., 20(1):133–166, 2010.
- [27] Davide Bresolin, Valentin Goranko, Angelo Montanari, and Guido Sciavicco. Propositional interval neighborhood logics: Expressiveness, decidability, and undecidable extensions. *Annals of Pure and Applied Logic*, 161(3):289 304, 2009.
- [28] J. P. Burgess. Axioms for tense logic 2: Time periods. *Notre Dame Journal of Formal Logic*, 23(2):375–383, 1982.

- [29] G. Chakravorty and P. Pandya. Digitizing interval duration logic. In *Computer Aided Verification*, pages 167–179. Springer-Verlag, 2003.
- [30] Z. Chaochen and M. Hansen. An adequate first order interval logic. In Compositionality: the Significant Difference, pages 584–608. LNCS, 1998.
- [31] Z. Chaochen and M. Hansen. Duration Calculus: A Formal Approach to Real-Time Systems. EATCS Series of Monographs in Theoretical Computer Science. Springer, 2004.
- [32] Z. Chaochen, M. Hansen, and P. Sestoft. Decidability and undecidability results for duration calculus. In *Proceedings of the 10th Symposium on Theoretical Aspects of Computer Science*, pages 58–68. LNCS, 1993.
- [33] Z. Chaochen, C. Hoare, and A. P. Ravn. A calculus of durations. *Information Processing Letters*, 40:269–276, 1991.
- [34] N. Chetcuti-Serandio and L. Del Cerro. A mixed decision method for duration calculus. *Journal of Logic and Computation*, 10(6):877–895, 2000.
- [35] J. Chomicki. Efficient checking of temporal integrity constraints using bounded history encoding. ACM Transactions on Database Systems, 20:149–186, 1995.
- [36] J. Chomicki and D. Toman. Temporal logic in information systems. pages 31–70, 1998.
- [37] E. M. Clarke and E. A. Emerson. Design and synthesis of synchronization skeletons using branching-time temporal logic. In *Logic of Programs*, *Workshop*, pages 52–71. Springer-Verlag, 1982.
- [38] E. M. Clarke, E. A. Emerson, and A. P. Sistla. Automatic verification of finite state concurrent systems using temporal logic. *ACM Transactions on Programming Languages and Systems*, 2(8):244–263, 1986.
- [39] E. M. Clarke, O. Grumberg, and D. A. Peled. Model Checking. MIT Press, 2000.
- [40] A. Degtyarev, M. Fisher, and A. Lisitsa. Equality and monodic first-order temporal logic. *Studia Logica*, 72(2):147–156, 2002.
- [41] C. Dixon, M. Fisher, B. Konev, and A. Lisitsa. Practical first-order temporal reasoning. In 15th International Symposium on Temporal Representation and Reasoning, TIME '08, pages 156–163. IEEE Computer Society Press, 2008.
- [42] L. X. Dong and D. V. Hung. Checking linear duration invariants by linear programming. In Concurrency and Parallelism, Programming, Networking, and Security, pages 321–332. Springer-Verlag, 1996.
- [43] D. Dowty. Word Meaning and Montague Grammar. Dordrecht: D. Reidel, 1979.
- [44] B. Dutertre. On first-order interval temporal logic. Technical Report CSD-TR-94-3, Department of Computer Science, Royal Holloway College, University of London, 1995.

- [45] E. Emerson and Jai Srinivasan. Branching time temporal logic. In Linear Time, Branching Time and Partial Order in Logics and Models for Concurrency, volume 354 of Lecture Notes in Computer Science, pages 123–172. Springer Berlin / Heidelberg, 1989.
- [46] E. A. Emerson. Temporal and modal logic. In *Handbook of Theoretical Computer Science*. North-Holland Pub. Co., 1995.
- [47] E. A. Emerson and E. Clarke. Using branching time temporal logic to synthesize synchronization skeletons. pages 241–266, 1982.
- [48] E. A. Emerson and J. Halpern. 'Sometimes' and 'Not Never' revisited: On branching versus linear time temporal logic. *Journal of the ACM*, 33(1):151–178, 1986.
- [49] E. A. Emerson and J. Y. Halpern. Decision procedures and expressiveness in the temporal logic of branching time. In *Proceedings of the Fourteenth Annual ACM Symposium on Theory of Computing*, pages 169–180. ACM, 1982.
- [50] E. A. Emerson and C. S. Jutla. The complexity of tree automata and logics of programs. SIAM Journal of Computation, 29(1):132–158, 2000.
- [51] R. Fagin, J. Halpern, Y. Moses, and M. Vardi. Reasoning About Knowledge. MIT Press, 1996.
- [52] J. L. Fiaderio and T. Maibum. Action refinement in a temporal logic of objects. In Temporal Logic. LNCS, 1994.
- [53] J. C. Filliatre, S. Owre, H. Ruess, and N. Shanka. Ics: Integrated canonizer and solver. In *Computer Aided Verification*, pages 246–249. Springer-Verlag, 2002.
- [54] M. Fisher. A resolution method for temporal logic. In Proceedings of Twelfth International Joint Conference on Artificial Intelligence (IJCAI). Morgan Kaufmann, 1991.
- [55] M. Fisher, C. Dixon, and M. Peim. Clausal temporal resolution. ACM on Transactions of Computational Logic, 2(1):12–56, 2001.
- [56] M. Fraenzle. Synthesizing controllers from duration calculus. In *Formal Techniques* in Real-Time and Fault-Tolerant Systems, pages 168–187. LNCS, 1996.
- [57] M. Fränzle. Take it np-easy: Bounded model construction for duration calculus. In Formal Techniques in Real-Time and Fault-Tolerant Systems, pages 245–264. Springer-Verlag, 2002.
- [58] Martin Fränzle and Michael R. Hansen. Deciding an interval logic with accumulated durations. In TACAS, pages 201–215, 2007.
- [59] D. Gabbay, I. Hodkinson, and M. Reynolds. Temporal Logic: Mathematical Foundations and Computational Aspects, volume 1. Clarendon Press, Oxford, 1994.
- [60] D. Gabbay, A. Kurucz, F. Wolter, and M. Zakharyaschev. Many-Dimensional Modal Logics: Theory and Applications. Elsevier, 2002.

- [61] D. Gabbay, M. Reynolds, and M. Finger. Temporal Logic: Mathematical Foundations and Computational Aspects, volume 2. Clarendon Press, Oxford, 2000.
- [62] D. M. Gabbay, A. Pnueli, S. Shelah, and J. Stavi. On the temporal analysis of fairness. Conference Record of the 7th Annual ACM Symposium on Principles of Programming Languages, pages 163–173, 1980.
- [63] A. Galton. The Logic of Aspect. Claredon Press, Oxford, 1984.
- [64] A. Galton. A critical examination of Allen's theory of action and time. *Artificial Intelligence*, 42:159–198, 1990.
- [65] Rob Gerth, Ruurd Kuiper, Doron Peled, and Wojciech Penczek. A partial order approach to branching time logic model checking. *Information and Computation*, 150(2):132 152, 1999.
- [66] V. Goranko, A. Montanari, and G. Sciavicco. A general tableau method for propositional interval temporal logics. In *Proceedings of the International Conference Tableaux 2003*, pages 102–116. LNAI, 2003.
- [67] V. Goranko, A. Montanari, and G. Sciavicco. Propositional interval neighbourhood temporal logics. *Journal of Universal Computer Science*, 9(9):1137–1167, 2003.
- [68] V. Goranko, A. Montanari, and G. Sciavicco. A road map of interval temporal and duration calculi. *Journal of Applied Non-Classical Logics*, 14, 2004.
- [69] E. Grädel. On the restraining power of guards. Journal of Symbolic Logic, 64:1719–1742, 1999.
- [70] D. Guelev and V. H. Dang. On the completeness and decidability duration calculus with iteration. In *Advances in Computer Science*, pages 139–150. LNCS, 1999.
- [71] D. P. Guelev. Probabilistic interval temporal logic. Technical Report 144, UNU/IIST, 1998.
- [72] D. P. Guelev. A complete proof system for first order interval temporal logic with projection. Technical Report 202, UNU/IIST, 2000.
- [73] J. Y. Halpern, Z. Manna, and B. Moszkowski. A high-level semantics based on interval logic. In *Proceedings of 10th ICALP*, pages 274–291, 1983.
- [74] J. Y. Halpern and Y. Shoham. A propositional modal logic of time intervals. *Journal* of the ACM, 38(4):935–962, 1991.
- [75] C. L. Hamblin. Instants and intervals. Stadium Generale, 27:127–134, 1971.
- [76] D. Harel, A. Pnueli, and Y. Stavi. Process logic: Expressiveness, decidability, completeness. *Journal of Computer and System Sciences*, 25:145–180, 1983.
- [77] I. Hodkinson, F. Wolter, and M. Zakharyaschev. Decidable fragments of first-order temporal logics. *Annals of Pure and Applied Logic*, pages 85–134, 2000.
- [78] Ian M. Hodkinson, Frank Wolter, and Michael Zakharyaschev. Monodic fragments of first-order temporal logics: 2000-2001 a.d. In *Proceedings of the Artificial Intelligence* on Logic for Programming, LPAR '01, pages 1–23. Springer-Verlag, 2001.

- [79] L. Humberstone. Interval semantics for tense logic: Some remarks. Journal of Philosophical Logic, 8:171–196, 1979.
- [80] H. Kamp. Events, instants and temporal reference. In Semantics from Different Points of View, pages 376–417. Springer, 1979.
- [81] J. A. Kamp. Tense Logic and the Theory of Linear Order. PhD thesis, University of California, Los Angeles, 1968.
- [82] S. Konur. A decidable temporal logic for events and states. In *Thirteenth International Symposium on Temporal Representation and Reasoning (TIME 2006)*, pages 36–41. IEEE Computer Society Press, 2006.
- [83] Y. Kornatzky and S. Pinter. An extension to partial order temporal logic (potl). Research Report 596, Department of Electrical Engineering, Technion-Israel Institute of Technology, 1986.
- [84] F. Kroger. Temporal Logic of Programs. Springer-Verlag, 1987.
- [85] P. Ladkin. Logical time pieces. AI Expert, 2(8):58–67, 1987.
- [86] F. Laroussinie and P. Schnoebelen. A hierarchy of temporal logics with past. *Theo-retical Computer Science*, 148(2):303–324, 1995.
- [87] François Laroussinie, Antoine Meyer, and Eudes Petonnet. Counting ctl. In FOS-SACS, pages 206–220, 2010.
- [88] François Laroussinie, Antoine Meyer, and Eudes Petonnet. Counting ltl. In *TIME* 2010, 2010.
- [89] O. Lichtenstein and A. Pnueli. Propositional temporal logics: Decidability and completeness. *Logic Journal of the IGPL*, 8(1):55–85, 2000.
- [90] O. Lichtenstein, A. Pnueli, and L. D. Zuck. The glory of the past. In *Proceedings of the Conference on Logic of Programs*, pages 196–218. Springer-Verlag, 1985.
- [91] Alessio Lomuscio, Wojciech Penczek, and Hongyang Qu. Partial order reductions for model checking temporal epistemic logics over interleaved multi-agent systems. In *Proceedings of the 9th International Conference on Autonomous Agents and Multiagent Systems: volume 1 Volume 1*, AAMAS '10, pages 659–666. International Foundation for Autonomous Agents and Multiagent Systems, 2010.
- [92] C. Lutz, H. Sturm, F. Wolter, and M. Zakharyaschev. A tableau decision algorithm for modalized \mathcal{ALC} with constant domains. *Studia Logica*, 72(2):199–232, 2002.
- [93] Carsten Lutz, Dirk Walther, and Frank Wolter. Quantitative temporal logics over the reals: Pspace and below. *Inf. Comput.*, 205(1):99–123, 2007.
- [94] Z. Manna and A. Pnueli. Verification of concurrent programs: Temporal proof principles. pages 200–252, 1981.
- [95] Z. Manna and A. Pnueli. Verification of concurrent programs: The temporal framework. pages 215–273, 1981.

- [96] D. Marx and Y. Venema. Multi-Dimensional Modal Logics. Kluwer Academic Press, 1997.
- [97] M. Marx and M. Reynolds. Undecidability of compass logic. *Journal of Logic and Computation*, 9(6):897–914, 1999.
- [98] D. McDermott. A temporal logic for reasoning about process and plans. *Cognitive Science*, 6:101–155, 1982.
- [99] P. M. Melliar-Smith. Extending interval logic to real time systems. In *Proceedings of the Conference on Temporal Logic Specification*, pages 224–242. Springer, 1987.
- [100] S. Merz. Decidability and incompleteness results for first-order temporal logics of linear time. *Journal of Applied Non-classical Logic*, pages 139–156, 1992.
- [101] A. Montanari, G. Sciavicco, and N. Vitacolonna. Decidability of interval temporal logics over split-frames via granularity. In *Proceedings of the 8th Europian Conference on Logics in AI*, pages 259–270. Springer, 2002.
- [102] Angelo Montanari, Ian Pratt-Hartmann, and Pietro Sala. Decidability of the logics of the reflexive sub-interval and super-interval relations over finite linear orders. In TIME 2010, 2010.
- [103] B. Moszkowski. *Reasoning about Digital Circuits*. PhD thesis, Computer Science Department, Stanford University, 1983.
- [104] B. Moszkowski. A complete axiomatization of interval temporal logic with infinite time. In *Proceedings of the 15th Annual IEEE Symposium on Logic in Computer Science*, pages 242–251. IEEE Computer Society Press, 2000.
- [105] P. Ohrstrom and P. F. Hasle. Temporal Logic: From Ancient Ideas to Artificial Intelligence. Kluwer Academic Publishers, Dordrecht, The Netherlands, 1983.
- [106] J. S. Ostroff. Formal methods for the specification and design of real-time safety critical systems. *Journal of Systems and Software*, 18(1):33–60, 1992.
- [107] P. K. Pandya. Specifying and deciding quantified discrete-time duration calculus formulas using DCVALID. In *Real-Time Tools*, 2001.
- [108] P. K. Pandya. Interval duration logic: Expressiveness and decidability. In *Proceedings of TPTS*, 2002.
- [109] R. Parikh. A decidability result for second order process logic. In *Proceedings of 19th FOCS*, pages 177–183. IEEE Computer Society Press, 1978.
- [110] W. Penczek. Branching time and partial-order in temporal logics. pages 179–228, 1995.
- [111] S. S. Pinter and P. Wolper. A temporal logic for reasoning about partially ordered computations (extended abstract). In *Proceedings of the Third Annual ACM Symposium on Principles of Distributed Computing*, pages 28–37. ACM, 1984.

- [112] R. Pliuskevicius. On the completeness and decidability of a restricted first order linear temporal logic. In *Proceedings of the 5th Kurt Gödel Colloquium on Computational Logic and Proof Theory*, pages 241–254. Springer-Verlag, 1997.
- [113] A. Pnueli. The temporal logic of programs. In *Proceedings of the 18th Annual IEEE Symposium on Foundations of Computer Science*, pages 46–57. IEEE Computer Society Press, 1977.
- [114] V. R. Pratt. Process logic. In Proceedings of 6th POPL, pages 93–100. ACM, 1979.
- [115] V. R. Pratt. On the composition of processes. In *Proceedings of the 9th ACM SIGPLAN-SIGACT symposium on Principles of programming languages*, POPL '82, pages 213–223. ACM, 1982.
- [116] Ian Pratt-Hartmann. Temporal prepositions and their logic. Artif. Intell., 166(1-2):1–36, 2005.
- [117] A. Rabinovich. Non-elementary lower bound for propositional duration calculus. *Information Processing Letters*, 66:7–11, 1998.
- [118] A. Rao and M. Georgeff. A model-theoretic approach to the verification of situated reasoning systems. In *Proceedings of IJCAI*, 1993.
- [119] R. Razouk and M. Gorlick. Real-time interval logic for reasoning about executions of real-time programs. SIGSOFT Software Engineering Notes 14, 8:10–19, 1989.
- [120] M. Reynolds. Axiomating first-order temporal logic: Until and since over linear time. pages 279–302, 1996.
- [121] M. Reynolds. An axiomatization of full computation tree logic. *Journal of Symbolic Logic*, 66:1011–1057, 2001.
- [122] M. Reynolds. An axiomatization of pctl. Information and Computation, 201(1):72–119, 2005.
- [123] Mark Reynolds. The complexity of the temporal logic with "until" over general linear time. J. Comput. Syst. Sci., 66(2):393–426, 2003.
- [124] Mark Reynolds. The complexity of temporal logic over the reals. Annals of Pure and Applied Logic, 161(8):1063–1096, 2010.
- [125] P. Roper. Intervals and tenses. Journal of Philosophical Logic, pages 451–469, 1980.
- [126] Grigore Ro?u and Saddek Bensalem. Allen linear (interval) temporal logic Đ translation to ltl and monitor synthesis. In *Computer Aided Verification*, volume 4144 of *Lecture Notes in Computer Science*, pages 263–277. Springer Berlin / Heidelberg, 2006.
- [127] M. Satpathy, D. V. Hung, and P. K. Pandya. Some decidability results for duration calculus under synchronous interpretation. In *Proceedings of the 5th International Symposium on Formal Techniques in Real-Time and Fault-Tolerant Systems*, pages 186–197. Springer-Verlag, 1998.

- [128] R. L. Schwartz and P. M. Melliar-Smith. From state machines to temporal logic: Specification methods for protocol standards. *IEEE Trans. Commun.* 30, pages 2486–2496, 1982.
- [129] R. L. Schwartz, P. M. Melliar-Smith, and F. H. Voght. An interval logic for higher-level temporal reasoning. In *Proceedings of the Second ACM Symposium on Principles of Distributed Computing*, pages 173–186. ACM Press, 1983.
- [130] B. Sharma, P. Paritosh, and C. Supratik. Bounded validity checking of interval duration logic. In *TACAS 2005: Tools and Algorithms for the Construction and Analysis of Systems*, pages 301–316. Springer-Verlag, 2005.
- [131] P. Simons. Parts, A Study in Ontology. Claredon Press, Oxford, 1987.
- [132] A. P. Sistla and E. M. Clarke. The complexity of propositional linear temporal logics. *Journal of the ACM*, 32:733–749, 1985.
- [133] A. P. Sistla, M. Y. Vardi, and P. Wolper. The complementation problem for büchi automata with applications to temporal logic (extended abstract). In *Proceedings* of the 12th Colloquium on Automata, Languages and Programming, pages 465–474. Springer-Verlag, 1985.
- [134] S. Spranger. Representation of temporal knowledge for web-based applications. Master's thesis, Ludwig Maximilians Universitaet Munchen, 2002.
- [135] A. Szalas. Temporal logic of programs: A standard approach. pages 1–50, 1995.
- [136] P. H. Thai and D. V. Hung. Checking a regular class of duration calculus models for linear duration invariants. In *Proceedings of the International Symposium on Software* Engineering for Parallel nd Distributed Systems, pages 61–71. IEEE Computer Society Press, 1998.
- [137] P. H. Thai and D. V. Hung. Verifying linear duration constraints of timed automata. In *Proceedings of ICTAC'04*, pages 295–309. Springer-Verlag, 2004.
- [138] J. F. van Benthem. *The Logic of Time*. Kluwer Academic Publishers, Dordrecht, 1983.
- [139] J. F. van Benthem. The Logic of Time: A Model-Theoretic Investigation into the Varieties of Temporal Ontology and Temporal Discourse. Kluwer, second edition, 1991.
- [140] Y. Venema. A modal logic for choppping intervals. *Journal of Logic and Computation*, 1:453–476, 1991.
- [141] Y. Venema. *Temporal Logic*. Blackwell Guide to Philosophical Logic, Blacwell Publishers, 1998.
- [142] N. Vitacolonna. *Intervals: Logics, Algorithms and Games*. PhD thesis, Department of Mathematics and Computer Science, University of Udine, 2005.
- [143] A. G. Walker. Durees et instants. La Revue Scientifique, (3266), 1947.

- [144] F. Wolter and M. Zakharyaschev. Modal description logics: Modalizing roles. Fundamenta Informaticae, pages 411–438, 1999.
- [145] F. Wolter and M. Zakharyaschev. Axiomatizing the monodic fragment of first-order temporal logic. *Annals of Pure and Applied Logic*, pages 133–145, 2002.
- [146] C. Zhou. Linear duration invariants. In Proceedings of the Third International Symposium Organized Jointly with the Working Group Provably Correct Systems on Formal Techniques in Real-Time and Fault-Tolerant Systems, pages 86–109. Springer-Verlag, 1994.