Can entropy-based image alignment metrics offer improved image aggregation of tissue density for mammographic risk assessment?

Final Report for CS39440 Major Project

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In signing below, I hereby agree to this dissertation being made available to other students and academic staff of the Aberystwyth Computer Science Department.

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Acknowledgements

I would like to thank my Supervisor Neil for his constant help and guidance throughout this project.

Ryan for being my constant sound-board throughout the process, always happy to lend an ear when I needed to work through an issue, or bounce programming ideas off of someone.

Harry, Fangyi

Charlie

Abstract

Include an abstract for your project. This should be no more than 300 words.

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Chapter 1

Introduction

1.1 Project Description

This project is concerned with the alignment of multiple mammographic images using an image-alignment technique called Congealing [1]. The aim will be to implement image-alignment software which allows the user to not only choose standard Entropy to align the images as in [1], but also 2 different light-weight Fuzzy Entropy metrics for alignment - Non-Probabilistic and Hybrid entropy. The User will be able to generate 3 mean images of the input set, 1 for each metric. By utilising different alignment metrics on the same images the result should be a range of average images, which further may be used to ascertain the most useful entropy algorithm for the alignment of mammographic images.

Each input set of images must belong to the same tissue density category, but from different women, to allow the resulting mean image to be an accurate depiction of the average breast structure within that category. Once a mean image is constructed of each category, this should aid radiographers in their qualitative categorisation of a new patient's scans.

Simple and accurate categorisation is important due to the increased risk factors associated with denser tissue breasts. Therefore if a radiographer can be confident in their categorisation of a patient's breast tissue, should the patient fall within the higher risk category they can receive more frequent, specialised scans to detect any abnormalities quicker should they arise.

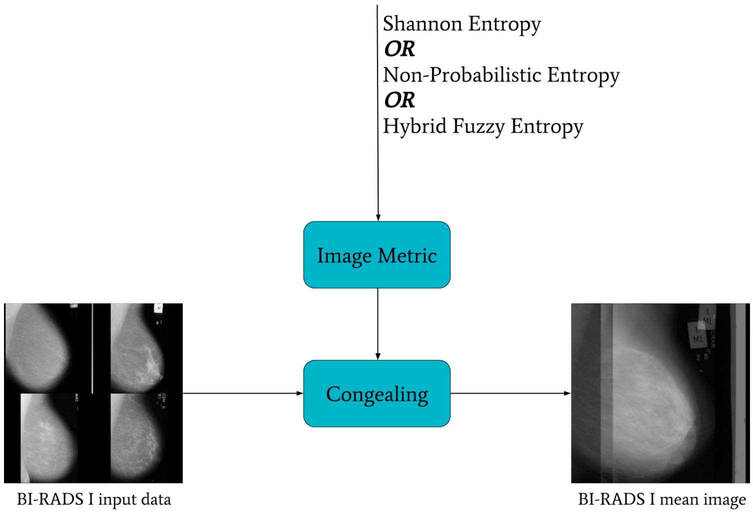


Figure 1.1: Graphical depiction of the project

1.2 Project Structure

This section will give a brief overview of the structure of the project.

1.2.1 Research

The main piece of research to be undertaken in this project will be evaluating which Fuzzy Entropy algorithms will be light-weight and simple enough to be run quickly on a radiographer's own laptop. Typically, research implementations of Fuzzy Entropy algorithms tend to be complex, and therefore computationally expensive, something not ideal when a patient has a short time-slot with a radiographer.

1.2.2 Software Implementation

In order to assess the usefulness of basic fuzzy entropy algorithms in the alignment of mammographic scans, a tool must be built to handle the input images and all the output data. This tool will be created using MATLAB and it's Fuzzy Logic and Image Processing toolboxes.

The main functions of the tool will be:

- Allow the user to input a large image containing all the scans they wish to align
- Allow the user to remove any medical markers as they see fit
- Allow the user to choose their alignment metric and number of iterations to run on the input images
- Output the final mean image, the adjusted input images (how they look after aligning) and the entropy of the final image set

1.2.3 Testing

The testing to be undertaken during this project will include scientific and software.

1.2.3.1 Scientific testing

This will be testing the output after the congealing process has been run using a fuzzy entropy alignment metric. One way to measure the result will be to evaluate the entropy value at the end of the alignment process - as the lower the entropy, the more aligned the images are. Another way in which to test the output of the experiments will be to visually inspect the final mean images produced to see how well aligned the input images are.

1.2.3.2 Software testing

Some software testing will be necessary to ensure the proper working of the tool developed for experimentation. Both Unit testing and acceptance testing off of the pre-defined user stories will be carried out.

Chapter 2

Background & Objectives

2.1 Background

In Europe, breast cancer is the leading cause of death through cancer for women, with 1 in 6 women dying from a cancer having it in the glandular breast tissue [12]. The UK is contained within the higher mortality band which runs across the EU, sitting alongside countries such as the Netherlands, North-West France and Western Germany (see Figure 2.1). However the reason behind why these countries have a higher breast cancer mortality rate than their neighbours to the north and south is unknown.

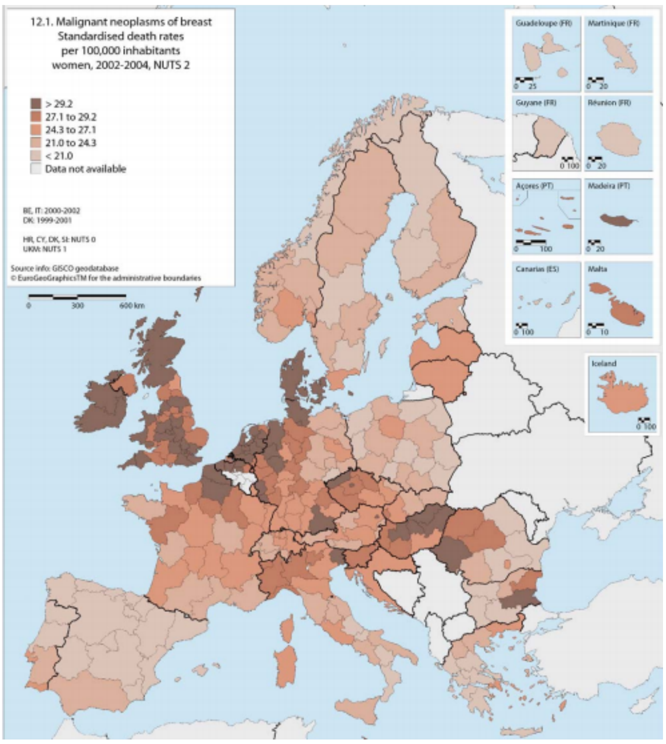


Figure 2.1: Image Source: EU Commission: Atlas on Mortality [12]

2.1.1 Tissue density classification

The internal breast structure consists of different kinds of tissue and glands [2]:

- Fatty and connective tissue: protects the lobules and ducts, gives shape to the breasts
- Lobules - milk-production glands
- Ducts - carry milk from Lobules to Nipple

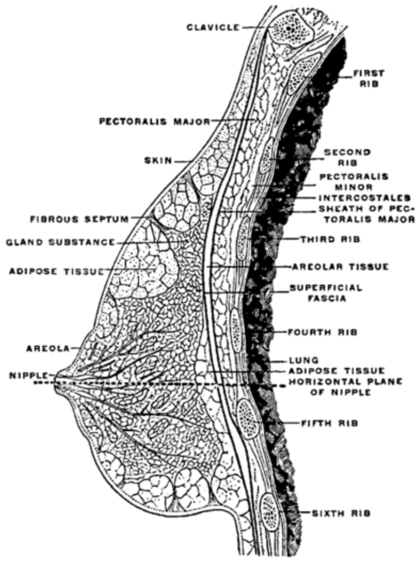


Fig. 1108.—Right breast in sagittal section, inner surface of outer segment. (Testut.)

Figure 2.2: Image Source: Gray's Anatomy [18]

Fatty and connective tissue density can vary widely between women. After extensive research into the links between a higher proportion of fibrous and glandular tissue versus fatty tissue and a higher risk of breast cancer, it is pretty widely accepted there is a strong link [10]

Reword this + add citations

. Therefore, simple classification of denser tissue is vital for both radiographers and patients alike.

2.1.1.1 Wolfe classification

Wolfe described the first qualitative means in which to classify breast tissue density in 1976 [35].

- **N1:** consisting mainly of fat (lowest risk)
- **P1:** fat plus linear densities occupying no more than 25% of the breast (low risk)
- **P2:** linear densities occupying >25% of breast (high risk)
- **DY:** dense (highest risk)

2.1.1.2 Boyd classification

Boyd and colleagues proposed a quantitative means to categorising breast tissue density, based on a percentage of 'dense' tissue assigned by a radiographer [10].

- **A:** 0%
- **B:** >0% - 10%
- **C:** >10% - 25%
- **D:** >25% - 50%
- **E:** >50% - 75%
- **F:** >75%

2.1.1.3 BI-RADS classification

A widely accepted quantitative tool for the classification and risk analysis of mammography and ultrasounds is BI-RADS (Breast Imaging-Reporting and Data System) system, defined by the American College of Radiology [31].

- **a:** almost entirely fatty
- **b:** scattered areas of fibroglandular density
- **c:** heterogeneously dense, which may obscure small masses
- **d:** extremely dense, which lowers the sensitivity of mammography

This is the classification of choice for this project due to it's wide-spread acceptance and usage in the industry.

2.1.1.4 Tabár classification

This technique is somewhat different from the previous 3 by utilising anatomic-mammographic correlations, as developed by Tabár [17].

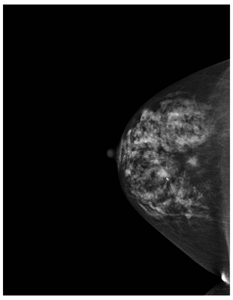
- **I:** balanced proportion of all components of breast tissue with a slight predominance of fibrous tissue
- **II:** predominance of fat tissue (fat breast)
- **III:** predominance of fat tissue with retroareolar residual fibrous tissue
- **IV:** predominantly nodular densities
- **V:** predominantly fibrous tissue (dense breast)

2.1.2 Mammograms

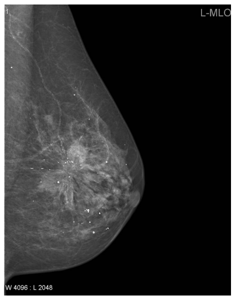
Quite simply, a Mammogram is an X-Ray of the breast tissue from a number of different angles. Below are a selection of the most common [28] [7]:

- Cranial-Caudal (CC) - taken from above (Figure 2.3a)
- Medio-Lateral Oblique (MLO) - from the side, at an angle (usually 45deg) (Figure 2.3b)
- Medio-Lateral (ML) - from the centre outwards (Figure 2.3d)
- Latero-Medial (LM) - from the side, into the centre (Figure 2.3c)

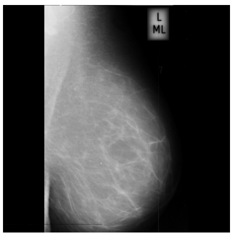
CC and MLO are generally the standard practice angles, with ML and LM adding more information for the radiographer to assess.



(a) Cranial-Caudal: Case courtesy of Dr Garth Kruger, Radiopaedia.org, rID: 18580



(b) Medio-Lateral Oblique: Case courtesy of A.Prof Frank Gaillard, Radiopaedia.org, rID: 12608



(c) Medio-Lateral: Case courtesy of Mini-MIAS dataset [32]

(d) Latero-Medial: Case courtesy of Dr Paresh K Desai , Radiopaedia.org, rID: 5873

Figure 2.3: Comparison of the 4 mammogram angles typically used

Organisations such as Breast Test Wales invite women between the ages of 50 and 70 to attend a scan every 3 years [8]. However women with higher-density breasts, which is ascertained during a mammogram, could be called back for more regular screening, to ensure to catch any abnormalities sooner.

2.1.2.1 Alternatives to Mammograms

Although the input data of choice for this project will be Mammographic scans, it is important to remember that for some women, and some circumstances, it may be more appropriate to use a different method of diagnosis.

Ultrasound

Women under 35 are often offered an ultrasound scan over a mammogram, due to their breasts being of a higher density naturally which makes obtaining a clear mammogram more difficult. Ultrasounds can also show the if the breast lump is a cyst, or if it is solid internally [33].

Biopsy

A Biopsy is usually a secondary step after diagnosis of a breast lump via mammogram or ultrasound. It can take a number of forms including:

- Needle biopsy
- Vacuum biopsy
- Needle aspiration
- Punch biopsy
- Wire guided biopsy

2.1.3 Existing Computer Systems

A lot of the current computer systems focus on mammography computer aided diagnosis due to the ease in which a Radiographer can misdiagnose cancer from a mammogram. Mammograms are difficult to read, or technological issues may occur and as a result, Radiographers can fail to detect between 10-15% of breast cancer cases [11].

However this is not the focus of this Project. This Project aims to focus upon healthy tissue, and the first steps towards computers classifying breast tissue into the correct density category. So what steps have been taken currently, in Industry and in research, towards this end goal?

Paper by Mohamed Abu ElSoud; Ahmed M. Anter - Automatic mammogram segmentation and computer aided diagnoses for breast tissue density according to BIRADS dictionary

2.2 Research Method

For this project, a literature review was undertaken to assess the work completed by researchers in the fields of Entropy, Fuzzy Entropy and image alignment methods to help better understand what has been investigated, and to gain a personal background understanding.

2.2.1 Entropy

In terms of Information Theory, the Merriam-Webster Dictionary defines Entropy to be [1]:

Entropy (noun): the degree of disorder or uncertainty in a system

Shannon entropy can be mathematically defined:

$$H(X) = -\sum_{i=0}^{N} p_i \log_2 p_i \tag{1}$$

Where p_i is the set of probabilities for all the variables in X.

Let us consider a fair coin toss. The probability of heads is exactly $\frac{1}{2}$, therefore, the entropy of landing on heads is:

$$\begin{aligned} H(heads) &= -\frac{1}{2}\log_2(\frac{1}{2}) - \frac{1}{2}\log_2(\frac{1}{2}) \\ &= 1.0 \end{aligned} \tag{2}$$

On the other side, if a system outputs solely the letter “M”, then the entropy of receiving the letter “M” is exactly 0. This is because when either the positive or the negative outcome is 100%, then both sides equal “0” when fed into the entropy equation.

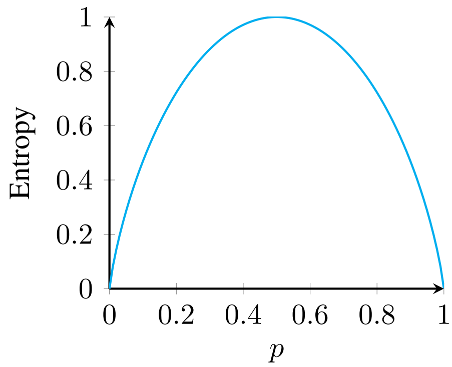


Figure 2.4: Entropy mapped against probability (p) of occurrence.

2.2.2 Uncertainty

A certain amount of uncertainty in life is to be expected and sometimes desired. A surprise party for many is the nice kind of uncertainty, however uncertainty associated with risk - i.e. "Will I lose my job in the recession?" - is uncertainty with a negative impact. Modeling uncertainty is especially important so as researchers we can understand it, and use it to our advantage in techniques such as Fuzzy Entropy.

2.2.2.1 Probabilistic Uncertainty

By definition:

> *Probability: the chance that something will happen [6]*

Probabilistic distribution is a widely accepted and used technique for representing expert judgements of uncertainty [23]. Early work carried out by DeGroot (1970) [15], built upon that of Savage (1954) [30], gave a simple layman's explanation:

> *For instance, if the person prefers decision A to B and B to C then they must also prefer A to C.*

2.2.2.2 Possibilistic Uncertainty

By definition:

> *Possibility: a chance that something might exist, happen, or be true : the state or fact of being possible [5]*

Possibilistic uncertainty (closely related to "fuzziness") indicates the lack of information we hold about the possible outcome values from a system - a sort of ambiguity. Possibilistic uncertainty models the possible outcomes from a system, as estimated by a decision maker because it is possibly impossible to determine beforehand [34]. For example,

2.2.2.3 Indiscernibility Uncertainty

By definition:

> *Indiscernibility: the quality or state of being indiscernible [3]*
> *Indiscernible: impossible to see, hear, or know clearly [4]*

2.2.3 Fuzzy Entropy

Fuzzy entropy stems from combining standard Entropy with the practices of Fuzzy Set Theory, discovered by Zadeh in 1965 [36]. This introduces the idea of "Membership" to a category, where an object can belong to more than one category to a certain degree.

One common example of this is listing someone as ‘Short’, ‘Average’ or ‘Tall’ in height. If a tall person is someone over 6 feet in height, would a person who measured 5foot 11inches not be classified as tall? Given crisp sets, then they would be classified as ‘Average’. In fuzzy set theory, they would be be a certain degree of tall, and a certain degree of average, with the highest membership likely to win out when categorising their height. Another example of this can be seen in Figure 2.5

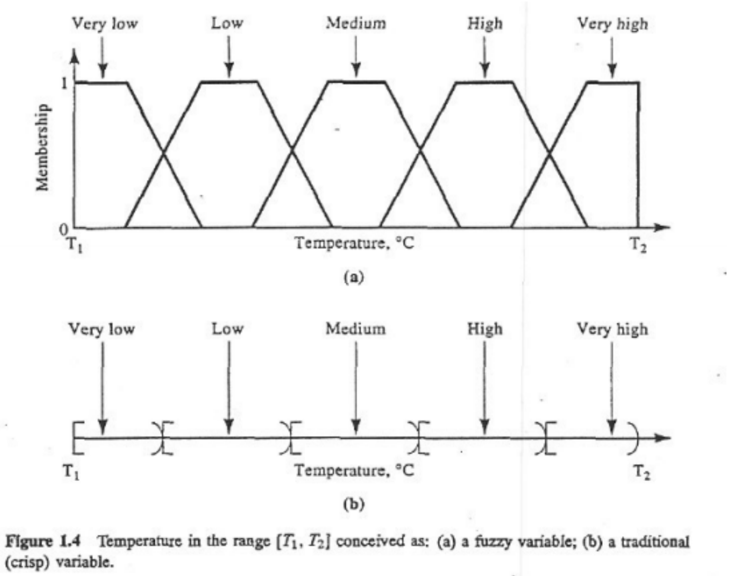


Figure 2.5: A comparison between Fuzzy Sets and Crisp sets. Image Source: Fuzzy Sets and Fuzzy Logic: Theory and Applications [16]

To combine Fuzzy Set Theory with Entropy, then the amount of fuzzy information gained from the fuzzy set(s) is known as Fuzzy Entropy.

2.2.3.1 Non-Probabilistic Entropy - 1972

De Luca and Termini are considered to be the first to have taken Shannon Entropy and extended it to include fuzziness [14]. They also defined properties which a fuzzy entropy must follow, in order to be classed as true.

Their non-probabilistic fuzzy entropy equation is as given:

$$H_A = -K \sum_{i=1}^{n} \{\mu_i \log(\mu_i) + (1 - \mu_i) \log(1 - \mu_i)\} \tag{3}$$

Where μ is the maximum membership across all the fuzzy sets.

The entropy given by equation (3) satisfies all 4 of De Luca and Termini’s defined properties:

P-1 $H_A = 0$ iff A is a crisp set ($\mu_i = 0$ o r$1 \forall x_i \in A$) (4a)

P-2 H_A is maximum iff $\mu_i = 0.5 \forall x_i \in A$ (4b)

P-3 $H \geq H^*$ where H^* is the entropy of A, a sharpened version of A (4c)

P-4 $H = \overline{H}$ where $\overline{H}$ is the entropy of the complement set $\overline{A}$ (4d)

Given the simplistic nature of this implementation, this was one of the chosen Fuzzy Entropy algorithms to be implemented in the project.

2.2.3.2 Fuzzy Shannon Entropy - 1989

Sander [29] presented a characterisation of a Fuzzy Entropy some time after De Luca and Termini's work was published. His implementation of Shannon Fuzzy Entropy is laid out in equation (5) below:

$$H(f) = -c \sum_{i=1}^{n} f(x_i) ln f(x_i), c > 0 \tag{5}$$

Where the power of a fuzzy set is defined as:

$$P(f) = \sum_{i=1}^{n} f(x_i) \tag{6}$$

Sander further went on to propose some properties, which must be imposed on a Fuzzy Entropy d to ensure that $d(f) = H(f)$:

1. Sharpness: $d(f) = 0 \Leftrightarrow f(X) \subset 0, 1, f \in [0, 1]^X$ (7a)

2. Valuation: $d(f \wedge g) + d(f \vee g) = d(f) + d(g), f, g \in [0, 1]^X$ (7b)

3. Generalised additivity: There exists two mappings s,t: $[0, \infty) \rightarrow [0, \infty)$ such that $d(fxg) = d(f)t(P(g)) + s(P(f))d(g)$ for all $f \in [0, 1]^X, g \in [0, 1]^Y$, where X and Y are finite sets. (7c)

The initial plan was to implement this algorithm in the project - however after further investigation which revealed that Fuzzy Shannon Entropy does not model Probabilistic uncertainty - it was decided that this algorithm was to be excluded.

2.2.3.3 Object-background segmentation using new definitions of entropy - 1989

Pal & Pal outlined their first Fuzzy Entropy algorithm in 1989 [24], which satisfies all 4 of De Luca and Termini's 4 conditions (outlined in Equations(4)). It as as follows:

$$H = -k \sum_{i=1}^{n} \{\mu_i exp(1 - \mu_i) + (1 - \mu_i) exp(\mu_i) \tag{8}$$

2.2.3.4 Higher Order Fuzzy Entropy & Hybrid Entropy - 1992

In Pal & Pal's paper "Higher order fuzzy entropy and hybrid entropy of a set" [25], they not only prove some of De Luca & Termini's work to be flawed, but also defined two new Fuzzy Entropy algorithms, and a new set of definitions.

Higher Order Fuzzy Entropy

As defined by Pal & Pal:

- $P =$ Fuzzy property set
- $\mu =$ the degree to which x_i possesses the property P
- $n =$ number of elements, with $r =$ a combination of elements from group n
- $S_i^r =$ denotes the ith element of such a combination
- $\mu(S_i^r) =$ the degree to which the combination S' as a whole possesses P
- There are $\left[\binom{n}{r}\right]$ such combinations

The entropy of order r of the fuzzy set A is defined as:

$$H' = \left(\frac{I}{\binom{n}{r}}\right)\sum_{i=1}^{\binom{n}{r}}\{\mu(S_i^r)exp(1-\mu(S_i^r))\} + \{1-\mu(S_i^r)\}log\{\mu(S_i^r)\} \tag{9}$$

If $r = 1$, then (9) reduces to Equations (8) and (3)

This project does not implement Higher Order Fuzzy Entropy due to the computational-overhead needed to run - especially on images with as much detail as a mammogram.

Hybrid Entropy

Another Fuzzy Entropy implementation outlined in Pal & Pal's paper was Hybrid Entropy. This algorithm is particularly useful as it combines Probabilistic and Possibilistic (fuzziness) uncertainty and if fuzziness is removed or not present, it returns to that of a classical set.

Let us define Hybrid Entropy.

- Let p_0 and p_1 be the probabilities of receiving 0 and 1 symbols over a noisy digital communication line respectively.
- Let μ denote the membership functions of the fuzzy set "Symbol close to 1"
- Both E_1 is a monotonically increasing function of μ - E_0 can be perceived as the likelihood (possibility) of receiving a "1" symbol
  - as μ increases from 0 to 1, then E_1 also increases
  - e.g. with an incoming "0" symbol, if μ increases, than the difficulty of correct interpretation also *increases* - a wrong interpretation of a "0" becomes likely

  - e.g. for an incoming "1" symbol, if μ increases, then the difficulty of correct interpretation *decreases* - improving likelihood of correct classification

- At the same time, E_0 can be perceived as the likelihood (possibility) of receiving the "0" symbol for the same reasoning

E_0 and E_1 can be defined as:

$$E_0 = \frac{1}{n} \sum_{i=1}^{n} (1 - \mu_i) exp(\mu_i) \tag{10a}$$

$$E_1 = \frac{1}{n} \sum_{i=1}^{n} \mu_i exp(1 - \mu_i) \tag{10b}$$

Therefore, the hybrid entropy of fuzzy set A can be defined as:

$$H_{hy} = -p_0 \log(1 - E_0) - p_1 \log(E_1) \tag{11}$$

This algorithm was chosen for implementation in this project due to it's hybrid nature (implementing both Probabilistic and Possibilistic uncertainty) and for it's simplification nature - in the absence of fuzziness, then E_0 and E_1 reduce to p_0 and p_1 respectively, therefore classical Shannon entropy. This is especially useful in image processing, and other such areas which deal with a lot of noise.

2.2.3.5 Fuzzy Entropy: a Brief Survey - 2001

Due to the older nature of some of the papers listed above, some were difficult to locate online. So when implementing the chosen algorithms (Non-Probalistic Entropy and Hybrid Entropy), Al-sharhan et al's paper "Fuzzy Entropy: a Brief Survey" [9] was a useful tool.

It's concise nature, and chronological listing ensured a strong understanding of the basic principles, before introducing the more complex algorithms (such as Higher Order Fuzzy Entropy). The paper also highlights advantages and flaws to each solution.

2.2.4 Joint Image Alignment

Image Alignment focuses on the alignment of several images, into one average image.

2.2.4.1 Learned-Miller's Congealing

Learned-Miller's Congealing [20] is often cited as being one of the first to truly align simple sets of data with minimal noise, no occlusions and illumination variation [37] [27] [26]. Many more robust image alignment techniques have been developed off of the basis of this work, however with more computational-expense.

This algorithm works by iteratively reducing the pixel-wise entropy over the input images, using a set of standard image transformations such as:

- x & y translations
- rotation
- x & y sheer
- x & y scale

The entropy is calculated by assessing each individual set of pixel-locations in the 'Pixel Stack' (see Figure 2.6), and by calculating the entropy of the empirical distribution of values in the Pixel Stack.

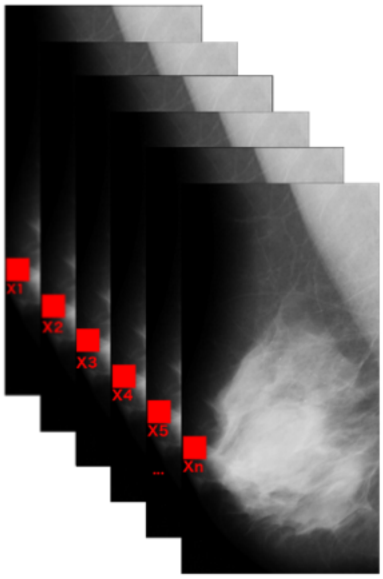


Figure 2.6: Each pixel from the same location throughout the set creates a 'Pixel Stack'

As this project will be working with mammograms, something with little variation nor inconsistency, Congealing is the perfect, light-weight image alignment algorithm to which to build upon, especially as the demonstration code available for research has an entropy implementation already developed.

2.2.4.2 Least squares congealing for unsupervised alignment of images

Further work was done upon the Congealing algorithm proposed by Learned-Miller by Cox et al. in 2008 [13]. They set out to address any performance issues and to remove the need for a pre-defined step size. It proposes to mitigate these issues by implementing an alternative method for aligning the images - utilising the Lucas & Kanade algorithm for aligning a single image to another using a gradient descent approach [21].

This alignment algorithm was disregarded for this project due to the preference to focus upon entropy-based alignment algorithms and the computational costs that the authors themselves regard to be a drawback of their algorithm.

2.2.5 Image Alignment using Fuzzy Entropy

Some work has already been undertaken to investigate image alignment using Fuzzy Entropy metrics, however typically they are computationally costly, and therefore slow to run. This project will be investigating whether there are simpler, more light-weight fuzzy entropy metrics which could be implemented, for more everyday use in image alignment. It will also be investigated if, and further how, the outputs of these alignments differ per each fuzzy entropy metric.

Some of this work which has implemented a more computationally-costly Congealing algorithm is that presented by Mac Parthaláin and Strange in their 2013 paper "Fuzzy-entropy based image congealing" [22]. Their implementation included dynamically-calculated fuzzy sets and a fuzzy similiarity relation matrix - allowing a comparison of all the objects to each other.

Chapter 3

Experiment Methods

3.1 Overview

3.2 Implementation tools

3.2.1 MATLAB

3.2.2 Version Control

3.3 Algorithms

3.3.1 Shannon Entropy

3.3.2 Non-Probabilistic Entropy

3.3.2.1 Fuzzy entropy description

De-Luca & Termini fuzzy entropy algorithm [14] is considered to be the first to build upon Shannon entropy. Their implementation takes into account a set of data, along with their various membership degrees.

$$H_A = -K \sum_{i=1}^{n} \{\mu_i \log(\mu_i) + (1 - \mu_i) \log(1 - \mu_i)\} \tag{1}$$

Al-sharhan et al's paper compiling several Fuzzy Entropy algorithms [9] contains a methodical,in-depth derivation of their algorithm, and has been instrumental in building my knowledge on the algorithm in question.

We will assume $-K$, the positive constant, is defined as $\frac{1}{n}$ as outlined in [14].

3.3.2.2 MATLAB implementation

After some research into current implementations of Fuzzy Entropy algorithms in MATLAB, it was concluded the best approach would be to implement De-Luca & Termini's algorithm from scratch. This entailed creating a membership class, which computes the grey-level membership of each pixel in the mean image (calculated from a set of input images).

This array of pixel memberships is fed into a 'De Luca' function where it is iteratively passed into latter part of equation 1 (after $\sum$). The output array is then summed and multiplied by $\frac{1}{n}$ as defined in Section 3.3.2.1. The final mean pixel entropy is calculated by taking the image entropy and dividing by the number of pixels in the image.

This is all relatively straight forward to implement in MATLAB, as it is designed to run mathematical equations.

3.3.2.3 Technical challenges

The main technical challenge for this implementation is ensuring maximum optimisation to keep running times to a minimum. Leveraging MATLAB's own functions for the membership saves a lot of time and lines of code, however it's been important to check what they call from within. One membership function was redrawing the trapeziums every time it was called, significantly slowing down the process - reducing the amount of times the initial function was called helped reduced the run-time by over 60seconds.

Another technical challenge faced whilst implementing the De Luca & Termini algorithm, isn't directly tied to the implementation of their specific equation, but more of my lack of experience in MATLAB, slowing down the programming rate. It has indeed been a steep learning curve, getting to grips with standard error messages, the debugger tool and knowing which 'Toolboxes' are needed to run specific MATLAB functions.

Finally, as can be ascertained from Figure 3.2, when writing the 4 seperate scans into 1 larger file, somewhere the images get rotated. This will be a reoccurring issue through the 3 Fuzzy entropy implementations, however as this is the first I will note it here. I think this is caused thanks to the swapping of the height and width values, however upon initial inspection of the file writing function, it is not clear as to which line is causing this issue. This issue has been marked low priority in the short term, due to all the scans being rotated in this fashion, and as such all have the same orientation. This means the congealing algorithm can work with no issues upon these images, the rotation is more merely an aesthetic issue.

3.3.2.4 Results

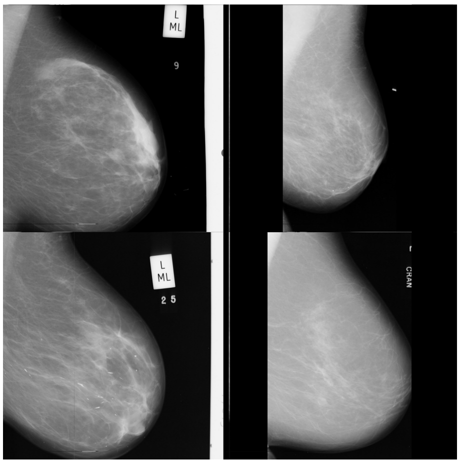
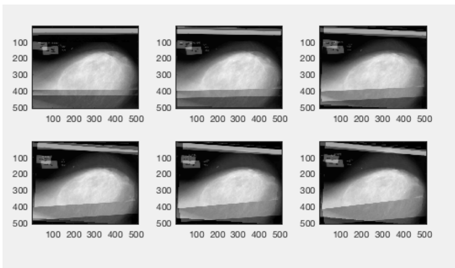
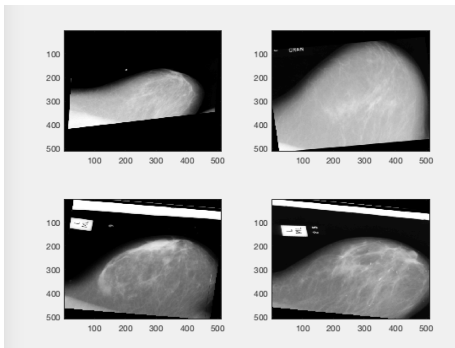


Figure 3.1: 4 input images of BI-RADS I classification



(a) Mean image over each iteration



(b) Adjusted original input images

Figure 3.2: Output of 5 congealing iterations

3.3.3 Entropy results

Iteration	Entropy
1	0.050519
2	0.043925
3	0.035679
4	0.029035
5	0.026194

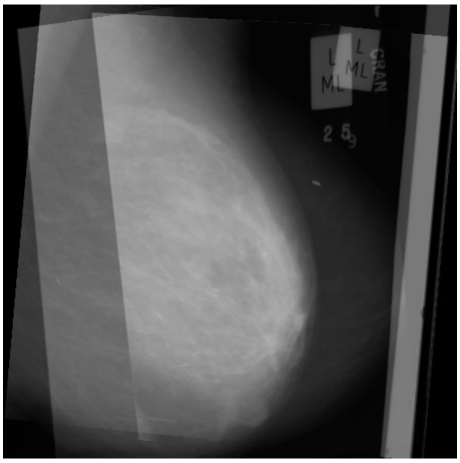
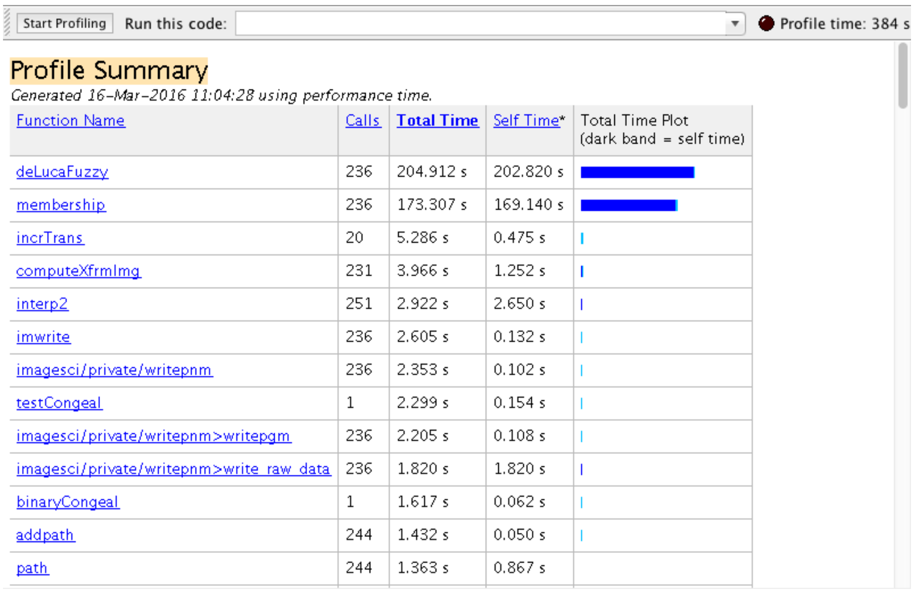


Figure 3.3: Final mean image after 5 iterations (bottom-right most in Figure 3.2)

3.3.4 Time to Run


Start Profiling Run this code: Profile time: 384 s

Profile Summary

Generated 16-Mar-2016 11:04:28 using performance time.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
deLucaFuzzy	236	204.912 s	202.820 s	
membership	236	173.307 s	169.140 s	
incrTrans	20	5.286 s	0.475 s	
computeXfrmImg	231	3.966 s	1.252 s	
interp2	251	2.922 s	2.650 s	
imwrite	236	2.605 s	0.132 s	
imagesci/private/writepnm	236	2.353 s	0.102 s	
testCongeal	1	2.299 s	0.154 s	
imagesci/private/writepnm>writepgm	236	2.205 s	0.108 s	
imagesci/private/writepnm>write_raw_data	236	1.820 s	1.820 s	
binaryCongeal	1	1.617 s	0.062 s	
addpath	244	1.432 s	0.050 s	
path	244	1.363 s	0.867 s	

Figure 3.4: Snapshot of run-time statistics

3.3.5 Hybrid Entropy

3.3.5.1 Fuzzy entropy description

Pal and Pal introduced Hybrid Entropy [25] in 1992 to help combat the issues faced by Non-Probablisitc entropy - mainly that it does not model probabilistic uncertainty. The name ‘Hybrid’ stems from the fact it models both Probabilistic and Possibilistic (fuzziness) uncertainty.

This could probably be covered in Lit review.

3.3.5.2 MATLAB implementation

Due to reasons covered in the Subsubsection 3.3.5.3, Hybrid Entropy membership was implemented using 2 trapeziums covering 2 fuzzy sets.

Two arrays are then fed into the Hybrid Entropy function - listing all the pixel membership values from the low trapezium, and the high trapezium. The final entropy is taken as a comparison between the low and high fuzzy sets.

3.3.5.3 Technical challenges

Whilst Hybrid Entropy utilises a membership function, much like Non-Probabilistic entropy, it was derived to work with binary entropy, not the ternary membership modeled for Non-Probabilistic. Because of the binary nature, the equation uses ‘inversion’ to depict if not this fuzzy set, then must belong to the other.

Experimentation was done as to whether the equation could be adapted in such a way to continue using three separate membership trapeziums - low, medium and high grey-level values.

Initial ideas - check email between me and neil

Logic would dictate that if the comparison of two fuzzy sets works, then to compare the low fuzzy set to the medium, the medium to the high and the high to the medium should work.

For example:

In theory, calculating E_0 and E_1 for each trapezium, calculating the hybrid entropy for each, and then combining them, should work:

$$E_0 = \frac{1}{No.\_of\_pixels\_in\_low\_trapezium} \sum_{i=1}^{n} (1 - Low\mu_i) exp(Low\mu_i) \tag{2}$$

$$E_1 = \frac{1}{No.\_of\_pixels\_in\_low\_trapezium} \sum_{i=1}^{n} Low\mu_i exp(1 - Low\mu_i) \tag{3}$$

Where $Low\mu$ is the membership of the pixels in the low fuzzy set.

$$H_{hy} = -p_0 \log_{10}(1 - E_0) - p_1 \log_{10}(E_1) \tag{4}$$

Where

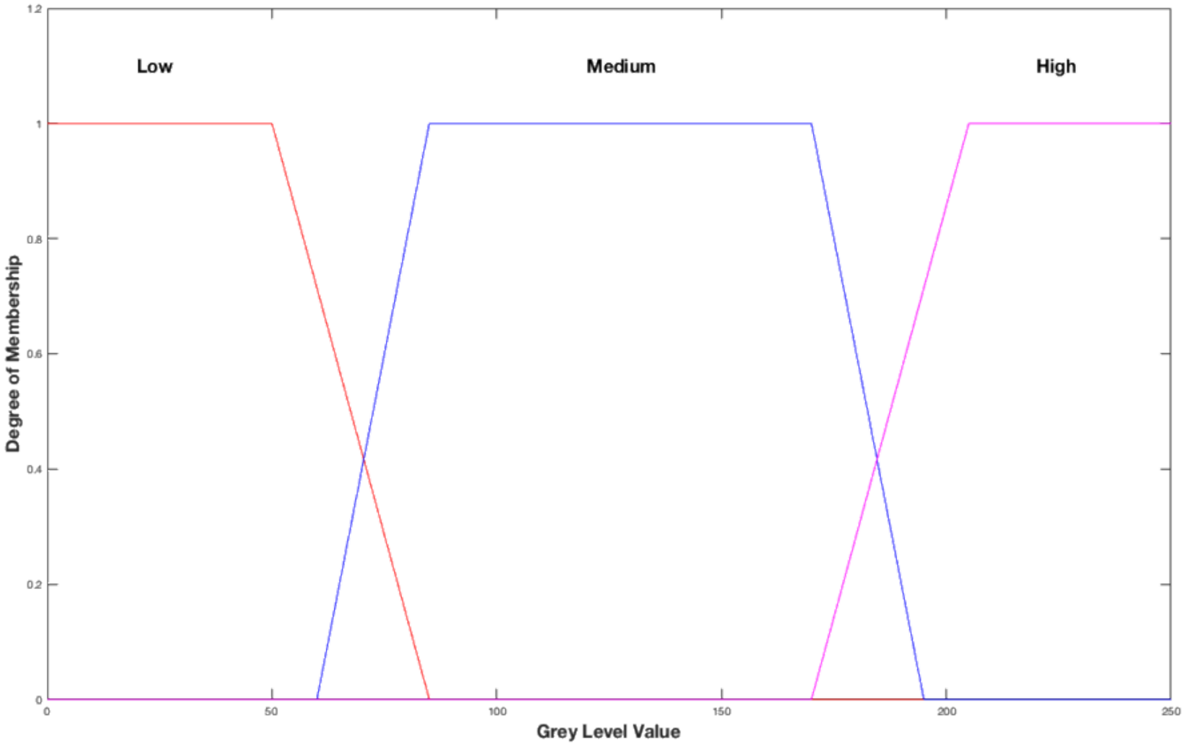


Figure 3.5: 3 fuzzy set trapeziums

$p_0 = \frac{No.\_of\_pixels\_in\_low\_trapezium}{No.\_of\_pixels\_in\_low\_trapezium+med\_trapezium}$

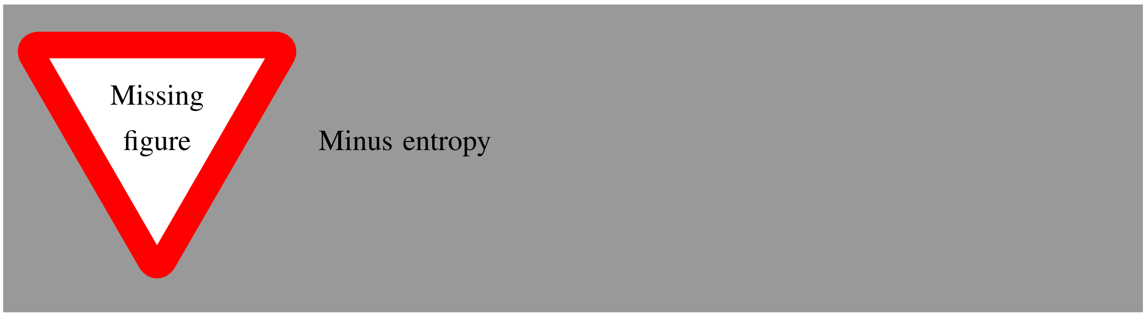
and

$p_1 = \frac{No.\_of\_pixels\_in\_med\_trapezium}{No.\_of\_pixels\_in\_low\_trapezium+med\_trapezium}$

This was done for all 3 trapeziums, then combined and divided by 3 (for the mean entropy). As the result for each trapezium should be between 0 and 1 (as each is an entropy value), then combining them should be no issue. However this was not the case.

First of all, the hybrid equation output was deemed to be 'NaN' - something which generally occurs when attempting to divide by 0. Anomalous outputs from the high trapezium was to be expected, as there are very few pixels which fall within the range nearer the white end of the grey-level scale. This was mitigated by setting the output equal to 0, in effect ignoring any output from the highest fuzzy set.

After this mitigation, the third and fourth iteration had suitable entropy values, however the fifth entropy value was a negative - something not possible in terms of entropy.



It was concluded that the implementation of three fuzzy sets within Hybrid Entropy would not be realistic within the remaining timeframe of the project, and the membership for Hybrid Entropy was redefined to the concept of 2 fuzzy sets, as set out by Pal and Pal.

3.3.5.4 Results

3.4 Software

3.4.1 Methodology

An adapted Scrum methodology has been undertaken for this project. This has been supported by the tool available at `taiga.io` - a beta web app.

- Burn down chart
- User stories
- Retrospectives
- Daily standup

3.4.2 Design

- CRC cards

3.4.3 Implementation

3.4.4 Testing

Chapter 4

Results and Conclusions

Chapter 5

Critical Evaluation

Appendices

Appendix A

Third-Party Code and Libraries

1.1 Congealing Code

The project focused on extending the existing Congealing Code implemented by Learned Miller et al in 2005. A Congealing demo is available on the Congealing website [19] which is open for experimentation. The original demo code was modified and extended to be able to read in mammograms and to work with 2 Fuzzy Entropy algorithms.

Appendix B

Ethics Submission

2.1 Ethics Application Number: 3958

AU Status
Undergraduate or PG Taught

Your aber.ac.uk email address
lac32@aber.ac.uk

Full Name
Laura Collins

Please enter the name of the person responsible for reviewing your assessment.
Reyer Zwiggelaar

Please enter the aber.ac.uk email address of the person responsible for reviewing your application
rrz@aber.ac.uk

Supervisor or Institute Director of Research Department
cs

Module code (Only enter if you have been asked to do so)
CS39440

Proposed Study Title
Entropy based metrics for joint image alignment

Proposed Start Date
25th January 2016

Proposed Completion Date
4th May 2015

Are you conducting a quantitative or qualitative research project?
Mixed Methods

Does your research require external ethical approval under the Health Research Authority?
No

Does your research involve animals?
No

Does your research involve human participants?
Yes

Are you completing this form for your own research?
Yes

Does your research involve human participants?
Yes

Institute
IMPACS

Please provide a brief summary of your project (150 word max)
I will be investigating the use of Congealing multiple MIAS dataset mammograms using several fuzzy entropy alignment metrics. If time permits I plan on speaking to a specialist (radiologist) to determine whether the output mean images of the congealing process are of any significant use to the research into breast cancer detection.

I can confirm that the study does not involve vulnerable participants including participants under the age of 18, those with learning/communication or associated difficulties or those that are otherwise unable to provide informed consent?
Yes

I can confirm that the participants will not be asked to take part in the study without their consent or knowledge at the time and participants will be fully informed of the purpose of the research (including what data will be gathered and how it shall be used during and after the study). Participants will also be given time to consider whether they wish to take part in the study and be given the right to withdraw at any given time.
Yes

I can confirm that there is no risk that the nature of the research topic might lead to disclosures from the participant concerning their own involvement in illegal activities or other activities that represent a risk to themselves or others (e.g. sexual activity, drug use or professional misconduct). Should a disclosure be made, you should be aware of your responsibilities and boundaries as a researcher and be aware of whom to contact should the need arise (i.e. your supervisor).
Yes

I can confirm that the study will not induce stress, anxiety, lead to humiliation or cause harm or any other negative consequences beyond the risks encountered in the participant's day-to-day lives.
Yes

Please include any further relevant information for this section here:

Where appropriate, do you have consent for the publication, reproduction or use of any unpublished material?
Yes

Will appropriate measures be put in place for the secure and confidential storage of data?
Yes

Does the research pose more than minimal and predictable risk to the researcher?
No

Will you be travelling, as a foreign national, in to any areas that the UK Foreign and Commonwealth Office advise against travel to?
No

Please include any further relevant information for this section here:

If you are to be working alone with vulnerable people or children, you may need a DBS (CRB) check. Tick to confirm that you will ensure you comply with this requirement should you identify that you require one.
Yes

Declaration: Please tick to confirm that you have completed this form to the best of your knowledge and that you will inform your department should the proposal significantly change.
Yes

Please include any further relevant information for this section here:

Appendix C

Code Examples

Annotated Bibliography

[1] [Online]. Available: http://www.merriam-webster.com/dictionary/entropy

Definition of Entropy with regards Information Theory (and thermodynamics)

[2] "The anatomy and physiology of the breast." [Online]. Available: http://www.cancer.ca/en/cancer-information/cancer-type/breast/anatomy-and-physiology/?region=on

Website outlining the make up of breast structure.

[3] "Definition of indiscernibility." [Online]. Available: http://www.merriam-webster.com/dictionary/indiscernibility

Helps define Indiscernibility uncertainty.

[4] "Definition of indiscernible." [Online]. Available: http://www.merriam-webster.com/dictionary/indiscernible

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[5] "Definition of possibility." [Online]. Available: http://www.merriam-webster.com/dictionary/possibility

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[6] "Definition of probability." [Online]. Available: http://www.merriam-webster.com/dictionary/probability

Helps define Probabilistic Entropy

[7] "Basic information about the typical mlo and cc views of mammography," Apr 2016. [Online]. Available: http://breast-cancer.ca/mammopics/

An informal website outlining some information about MLO and CC breast scans.

[8] I. C. about Cancer Screening, "Nhs breast screening: Helping you decide," Jun 2013. [Online]. Available: http://www.breasttestwales.wales.nhs.uk/sitesplus/documents/1025/1554%20Yogi%20-%20Breast%20Screening%20English%20v4.pdf

Leaflet about Breast Screening created by Informed Choice about Cancer Screening with Cancer Research UK

[9] S. Al-Sharhan, F. Karray, W. Gueaieb, and O. Basir, "Fuzzy entropy: a brief survey," in *Fuzzy Systems, 2001. The 10th IEEE International Conference on*, vol. 3. IEEE, 2001, pp. 1135–1139. [Online]. Available: http://dx.doi.org/10.1109/fuzz.2001.1008855

> Paper outlining the different implementations of Fuzzy Entropy, of which 3 will be selected and focused on during this Project.

[10] N. F. Boyd, J. W. Byng, R. A. Jong, E. K. Fishell, L. E. Little, A. B. Miller, G. A. Lockwood, D. L. Tritchler, and M. J. Yaffe, "Quantitative classification of mammographic densities and breast cancer risk: Results from the canadian national breast screening study," *Journal of the National Cancer Institute*, vol. 87, no. 9, p. 670675, May 1995, pMID: 7752271.

> This paper clearly outlines significant links between breast density and breast cancer risk.

[11] J. L. Champaign and G. J. Cederbom, "Advances in breast cancer detection with screening mammography," *The Ochsner Journal*, vol. 2, no. 1, p. 33, Jan 2000, pMID: 21765659.

> Paper outlining the advancements in screening mammography in the years up to 2000.

[12] E. Commission, "Health statistics - atlas on mortality in the european union - product - eurostat," 2009. [Online]. Available: http://ec.europa.eu/eurostat/en/web/products-statistical-books/-/KS-AC-04-000

> 2009 paper published by the European Commission on statistics into mortality rates and causes in the EU.

[13] M. Cox, S. Sridharan, S. Lucey, and J. Cohn, *Least squares congealing for unsupervised alignment of images*, Jun 2008, p. 18.

> A disregarded adaption of the Congealing algorithm - however was useful in highlighting performance issues in the original algorithm. Something which was near continuously faced when implementing heavier fuzzy entropy alignment metrics.

[14] A. De Luca and S. Termini, "A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory," *Information and Control*, vol. 20, no. 4, p. 301312, May 1972.

> De Luca & Termini's 1972 paper focuses on their definition of Non-probabilistic entropy, along with several example derivations.

[15] M. H. DeGroot, *Optimal Statistical Decisions*. Reprinted: McGraw-Hill, 2004.

> DeGroot gave a simple explanation about uncertainty in terms of probability.

[16] K. GEORGE J. and Y. BO, "Fuzzy sets and fuzzy logic, theory and applications," -, Mar 2008. [Online]. Available: http://digilib.uin-suka.ac.id/7049/

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> Tabár method for categorising scans using an anatomic-mammographic comparison technique.

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> Extremely influential book first written in 1858 by Henry Gray. It has been revised and republished, up until September 2015. Whilst this cited version is not 100% medically accurate by today's standards, the diagrams still hold true.

[19] E. Learned-Miller, "The congealing page." [Online]. Available: https://people.cs.umass.edu/~elm/congealing/

> This webpage was the source of the demo Congealing code modified and built upon in this project. It also contains several useful papers where Congealing has been used in research - such as Learned-Miller's own paper on Congealing MNIST handwriting data and MRI scans.

[20] E. G. Learned-Miller, "Data driven image models through continuous joint alignment," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 28, no. 2, pp. 236–250, Feb. 2006. [Online]. Available: http://dx.doi.org/10.1109/tpami.2006.34

> Learned-Miller's original Congealing method is the basis for this Project - however I am looking to further extend the alignment capabilities using fuzzy entropy metrics, rather than standard Shannon entropy as currently implemented. This paper was extremely useful for understanding of the basic concepts behind it, and will be a good reference guide throughout the project.

[21] B. D. Lucas and T. Kanade, *An iterative image registration technique with an application to stereo vision.* Morgan Kaufmann Publishers Inc., Aug 1981, p. 674679. [Online]. Available: http://dl.acm.org/citation.cfm?id=1623264.1623280

> Lucas and Kanade approach to aligning images using gradient-descent - this is what Cox et al based their Least Squares Congealing algorithm off of.

[22] N. Mac Parthalain and H. Strange, *Fuzzy-entropy based image congealing*, Jul 2013, p. 18.

> Paper outlining a Fuzzy Entropy congealing implementation - something more computationally costly than this project desires.

[23] A. OHagan, "Probabilistic uncertainty specification: Overview, elaboration techniques and their application to a mechanistic model of carbon flux," 2011. [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.319.9570

> A paper outlining a way in which to apply probabilistic uncertainty to an engineering problem. The opening sections of the paper give a detailed explanation of probabilistic uncertainty, enough to glean enough understanding to write a detailed analysis and apply it to my own case.

[24] N. R. Pal and S. K. Pal, "Object-background segmentation using new definitions of entropy," *IEE Proceedings E - Computers and Digital Techniques*, vol. 136, no. 4, p. 284295, Jul 1989.

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[25] ——, "Higher order fuzzy entropy and hybrid entropy of a set," *Information Sciences*, vol. 61, no. 3, p. 211231, Jun 1992.

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[26] Y. Peng, A. Ganesh, J. Wright, W. Xu, and Y. Ma, "Rasl: Robust alignment by sparse and low-rank decomposition for linearly correlated images," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 34, no. 11, p. 22332246, Nov 2012.

Alternative Joint Image alignment technique.

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Alternative Joint Image alignment technique.

[28] Radswiki, "Mammography views — radiology reference article — radiopaedia.org." [Online]. Available: http://radiopaedia.org/articles/mammography-views

Detailed online encyclopedia outlining the different types of mammogram scan which can be performed and their benefits/weaknesses.

[29] W. Sander, "On measures of fuzziness," *Fuzzy Sets and Systems - FSS*, vol. 29, no. 1, p. 4955, 1989.

Presents a characterisation of a fuzzy entropy - decided not to be included in the project due to the lack of probabilistic modeling.

[30] L. J. Savage, *The Foundations of Statistics*, 1954. [Online]. Available: https://books.google.co.uk/books/about/The_Foundations_of_Statistics.html?id=zSv6dBWneMEC

[31] E. Sickles, D. CJ, B. LW, *et al.*, *ACR BI-RADS® Mammography*. In: ACR BI-RADS® Atlas, Breast Imaging Reporting and Data System, 2013.

BI-RADS breast density classification commonly used today.

[32] J. Suckling, "The mammographic image analysis society digital mammogram database," *Exerpta Medica International Congress Series*, vol. 1069, 1994.

The Mini-MIAS dataset which was used during this project. These images have been reduced in size to allow a more sensible computation time when compared to the orignal MIAS scans.

[33] C. R. UK, "Breast cancer tests," Apr 2015. [Online]. Available: http://www.cancerresearchuk.org/about-cancer/type/breast-cancer/diagnosis/breast-cancer-tests

Webpage outlining the different types of breast scan which are available, including Mammogram via MRI, Ultrasound and needle biopsy.

[34] E. Untiedt, *A Parametrized Model for Optimization with Mixed Fuzzy and Possibilistic Uncertainty*. Springer Berlin Heidelberg, 2010. [Online]. Available: http://link.springer.com/chapter/10.1007/978-3-642-13935-2_9 978-3-642-13934-5.

Good description of possibilistic uncertainty versus fuzzy uncertainty.

[35] J. N. Wolfe, "Risk for breast cancer development determined by mammographic parenchymal pattern," *Cancer*, vol. 37, no. 5, p. 24862492, May 1976.

Paper outlining Wolfe's classification of breast tissue density.

[36] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, p. 338353, Jun 1965.

Zadeh's work into Fuzzy Set Theory paved the way for Fuzzy Entropy as implemented in this project.

[37] T. Zhou, Y. J. Lee, S. X. Yu, and A. A. Efros, *FlowWeb: Joint image set alignment by weaving consistent, pixel-wise correspondences*. IEEE, Jun 2015, p. 11911200. [Online]. Available: http://ieeexplore.ieee.org/articleDetails.jsp?arnumber=7298723

Alternative Joint Image alignment technique.